A Second-Best Argument for Low Optimal Tariffs *

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Abstract

We derive a new formula for the optimal uniform tariff in a heterogeneous-firm model with a traded and a nontraded sector, and with roundabout production in both. Two new terms appear in this formula: a M term reflecting the relative monopoly distortion between the traded and nontraded sectors, and a R term that reflects roundabout production in the traded sector, which amplifies the monopoly distortion. We find that roundabout production in the traded sector lowers the optimal tariff, especially when that amplified distortion exceeds the monopoly distortion in nontraded goods. We provide analytical results to obtain the optimal tariff as a fixed point.

Keywords: trade policy, monopolistic competition, gains from trade, input-output linkages

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1 Introduction

The use of tariffs to protect traded goods such as manufactures has a long history. In his famous *Report on Manufactures*, Alexander Hamilton argues for moderate tariffs combined with direct subsidies to promote manufacturing. Opposition to the proposed subsidies came from Thomas Jefferson and James Madison, who favored even higher tariffs, and Madison's administration produced the first protectionist tariff in the United States (Irwin, 2004). The administration of President Donald Trump enacted tariffs, often at 25%, to protect several manufacturing industries and against a broad range of products from China. Significantly, the Chinese products were initially selected to minimize the direct impact on consumer prices, leaving American businesses facing the brunt of tariffs on their imported inputs (Fajgelbaum, Goldberg, Kennedy and Khandelwal, 2020).

Does modern trade theory offer any new answer to this old question of whether to protect the traded sector? To answer this, we investigate the second-best role for tariffs in a model with two sectors – one traded and the other nontraded – and with heterogeneous firms, monopolistic competition and CES preferences (as in Melitz, 2003). We adopt a Pareto distribution for productivity (as in Chaney, 2008) and also roundabout production (as in Caliendo and Parro, 2015). The differentiated intermediate inputs in only one sector are traded, and also bundled into a finished good sold to home consumers and firms, while the final goods in both sectors are nontraded. It is known that in a single-sector model (without the nontraded sector and without roundabout production), the markup on home-produced varieties creates an argument for a similar import tariff, so as to equalize the distortion across domestic and imported goods (Gros, 1987, Demidova and Rodríguez-Clare, 2009, Felbermayr, Jung and Larch, 2013). This optimal tariff exploits the country's ability to improve its terms of trade, as occurs even for a small country under monopolistic competition.

But this terms-of-trade argument conflicts with the need to correct domestic monopoly distortions, in two respects. First, roundabout production means that firm purchases of the finished good should be subsidized to offset the markup on intermediate inputs. In other words, the equilibrium is not an optimal allocation, in contrast to the one-sector, closed-economy result with CES preferences – but *without* roundabout production – in Dhingra and Morrow (2019).[1] Indeed, roundabout pro-

[1]Celik, Karabay and McLaren (2018) recognize that the closed-economy equilibrium with CES preferences is inefficient in the presence of roundabout production.

duction means that the markup on differentiated inputs is amplified due to double-marginalization, so the *effective* markup is higher. Second, when we realistically introduce a nontraded sector (think of services), the effective markup there might be lower than in the traded sector (think of manufacturing) which has more roundabout production. It follows that the level of traded production is sub-optimal. But an import tariff worsens that distortion because, by Lerner symmetry, an import tariff on imported intermediate inputs is equivalent to an export tax, which we find leads to *exit* from the traded sector.

We obtain the optimal uniform tariff as a fixed-point of a formula that has two new terms: a M term that reflects the relative monopoly distortion between the traded and nontraded sectors; and a R term that reflects roundabout production in the traded sector, which amplifies the monopoly distortion there. With only the traded sector, the presence of roundabout production definitely lowers the optimal tariff. Adding a nontraded sector, but without roundabout production, also lowers the optimal tariff provided that the monopoly distortion there exceeds that in the nontraded sector (by a sufficient amount). When we combine two sectors with roundabout production, then we find that that even a small amount of roundabout is enough to lower the optimal tariff for a wide range of relative distortions between sectors. Thus, the second-best optimal tariff is lower than in the one-sector case (and even negative) under a wide range of parameter values, which we argue are consistent with the empirical evidence.

Costinot, Rodríguez-Clare and Werning (2020) have recently shown that tariffs can be lowered (and made negative) by having multiple sectors or with a non-Pareto distribution for productivity. Furthermore, when tariffs can differ across firms, then optimal policy for the importing country is to use an import subsidy on the least efficient foreign exports (but there is no role for export subsidies). Haaland and Venables (2016) demonstrated a potential second-best role for reduced trade taxes to offset a monopoly distortion. More recently, Lasharipour and Lugovsky (2019) analyze optimal tariffs with multiple sectors and input-output linkages. While they do not comment on the magnitude of these tariffs, they state that using tariffs in a second-best role to offset domestic monopoly distortions can conflict with the terms-of-trade motive. In view of this literature, we are not the first to point out that correcting a monopoly distortion can require a reduced optimal tariff. We are the first, however, to provide a formula for the optimal tariff that readily shows how

roundabout production in the traded sector (the R term), or a higher effective markup than in the nontraded sector (the M term), lead naturally to a low optimal tariff.

2 Two-Sector Model with Roundabout Production

We analyze a two-sector Melitz (2003)-Chaney (2008) model with roundabout production, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012, section IV) and Costinot and Rodríguez-Clare (2014). We summarize key equations here and the Appendix contains the full model. There are two countries, $k = i, j$, for $j \neq i$ and two sectors $s = 1, 2$, where sector 1 is traded and sector 2 is nontraded. In both sectors, firms produce differentiated inputs under monopolistic competition, which are costlessly bundled into a *finished good* in CES fashion, with elasticity $\sigma_s > 1$. The finished good is non-traded, and it is sold to domestic consumers as final goods and also to domestic firms as intermediate inputs, used to produce differentiated inputs (e.g. firms produce machinery parts using machines). In sector 1, the differentiated inputs are traded subject to iceberg costs, and both domestic and imported differentiated inputs are costlessly bundled into the finished good.

The finished output in each sector has quantity Q_{is}, price index P_{is}, and value $Y_{is} \equiv P_{is}Q_{is}$. Under the roundabout production structure, the marginal cost of producing a differentiated input for a firm with productivity $\varphi_s = 1$ in sector s is:

$$x_{is} \equiv w_i^{\gamma_{is}} P_{is}^{1-\gamma_{is}}, \tag{1}$$

where $0 < \gamma_{is} \leq 1$ is the labor share. We refer to (1) as the *input cost index*. Firms also face fixed labor costs of production and sunk labor costs to receive a productivity draw from a Pareto distribution, $G_s(\varphi_s) = 1 - \varphi_s^{-\theta_s}$, with $\varphi_s \geq 1$ and $\theta_s > \sigma_s - 1$.

Consumers have Cobb-Douglas preferences over final goods in sectors s:

$$U_i = C_{i1}^{\alpha_i} C_{i2}^{1-\alpha_i}, \tag{2}$$

where $\alpha_i > 0$ is the expenditure share on the traded sector 1. Consumer income I_i includes labor income (the only factor of production) $w_i L_i$, plus rebated tax revenue B_i, while free entry ensures

that expected firm profits equal zero.

Domestic consumer demand for finished goods equals $\alpha_i I_i = \alpha_i(w_i L_i + B_i)$ in sector 1 and $(1-\alpha_i)I_i = (1-\alpha_i)(w_i L_i + B_i)$ in sector 2. To determine domestic firm demand for finished goods, let λ_{ijs} denote the expenditure share of differentiated inputs that country j purchases from country i, where $\lambda_{ij2} \equiv 0$ in sector 2 so that $\lambda_{ii2} \equiv 1$. Then the total production cost of all differentiated inputs in sector 1 in country i is $\frac{\sigma_1 - 1}{\sigma_1} \sum_{k=i,j} \lambda_{ik1} Y_{k1}$; namely, the net-of-tariff value of sector s goods sold locally and abroad adjusted by markups. In sector 2 this is simply $\frac{\sigma_2 - 2}{\sigma_2} \lambda_{ii2} Y_{i2}$. Given the input-output coefficients γ_{is}, the total demand for the finished good in sector s of country i going to both consumers as final goods and to firms for intermediate use must equal total supply of that output:

$$Y_{is} = \alpha_{is}(w_i L_i + B_i) + \tilde{\gamma}_{is} \left(\lambda_{iis} Y_{is} + \lambda_{ijs} Y_{js} \right), \text{with } \tilde{\gamma}_{is} \equiv (1 - \gamma_{is}) \left(\frac{\sigma_s - 1}{\sigma_s} \right) < 1, \tag{3}$$

where once again we have $\lambda_{ij2} \equiv 0$ so that $\lambda_{ii2} \equiv 1$ in the nontraded sector. The parameter $\tilde{\gamma}_{is}$ eliminates markups from the value of intermediate inputs before computing the cost share, $(1-\gamma_{is})$, devoted to the finished good as an input.

We treat the home country as small and normalize the foreign wage at unity, and for simplicity we treat the foreign country as having a single traded sector where Y_{j1} is exogenous. The shares λ_{ij1} as well as λ_{ji1} depend on the cutoff productivities ϕ^*_{ij1} and ϕ^*_{ji1} that are determined in equilibrium (see Appendix A). With those shares, we can view (3) as solving for the value of production Y_{is} in sectors $s = 1, 2$ while a trade balance condition determines the home wage in the small country (Demidova and Rodríguez-Clare, 2013).

3 Optimal Consumer and Producer Taxes under Autarky

We first discuss the distortions arising in autarky from having monopolistic production of the differentiated inputs, and we now allow for $s = 1, ..., S$ sectors. The markup on the differentiated inputs is fully passed-through to the price of the bundled, finished good. That distortion then operates on two margins: consumer purchases of finished goods; and firm purchases of finished goods as inputs, where the higher price on the finished good is *further* passed-through to raise the price

of intermediate inputs, creating a double-marginalization of the markup on intermediate inputs. Rather than correcting the monopoly distortion at its source (i.e. in the price of differentiated inputs), it will be instructive to correct it by using tax/subsidies on purchases of the finished goods on these two margins. So we consider both consumer and producer tax/subsidies on purchases of the finished goods, where one plus the *ad valorem* rates are denoted by t^c_{is} and t^p_{is}, respectively. For simplicity, we assume a balanced budget with $B_i = 0$.

We consider two solutions for the planner's problem (see Appendix B): first, choosing both the consumer and producer tax/subsidies optimally; and second, using only the consumer tax/subsidy while setting $t^p_{is} \equiv 1$. When both tax/subsidy instruments are used, we obtain the solution:

$$\begin{aligned} t^p_{is} &= \left(\frac{\sigma_s - 1}{\sigma_s}\right) < 1, \\ t^c_{is} &= \mu \left(\frac{\sigma_s - 1}{\sigma_s}\right). \end{aligned} \tag{4}$$

where the a factor of proportionality $\mu > 0$ is specified so as to balance the budget. The optimal producer subsidies $t^p_{is} < 1$ exactly counteract the markups on differentiated inputs which would otherwise be fully passed-through to finished goods prices.[2] With these subsidies, firms pay prices for finished goods that reflect their marginal costs. In addition, optimal consumption tax/subsidies are needed to offset the markups implicit in finished goods' prices faced by consumers.

For example, if there is only a single sector *without* roundabout production, there is only one tax instrument so we must have $t^c_{i1} = 1$ to achieve a balanced budget, and it follows that $\mu = \frac{\sigma_1}{\sigma_1 - 1}$. Even without a balanced budget, we know from Dhingra and Morrow (2019) that the closed-economy market outcome in a one-sector Melitz-Chaney model is optimal, so no tax or subsidy is required. If we add roundabout production to that economy, however, then (4) shows that the markup on differentiated intermediates – that is fully passed-through to the price of the finished good purchased by firms – *must* be fully offset by a subsidy on those purchases, so that $t^p_{i1} < 1$. In that case, μ would be chosen so that $t^c_{i1} > 1$, so as to balance the budget. We see that even with a single sector, the presence of roundabout production leads to market inefficiency that must be corrected. More generally, with many sectors we have $t^p_{is} < 1$ in each so as to fully offset those

[2]The need for such subsidies in the context of a dynamic monopolistic competition model has been previously noted by Judd (1997, 2002).

monopoly distortions, and t^c_{is} are inversely proportional to the markups $\frac{\sigma_s}{\sigma_s - 1}$ but with a factor of proportionality so that the *relative* prices of finished goods equal their relative marginal costs.

In contrast to this first-best allocation, consider the second-best policy that involves consumption tax/subsidies only. In this case, the solution is:

$$t^c_{is} = \mu \frac{\gamma_{is}(\sigma_s - 1)}{\gamma_{is}(\sigma_s - 1) + 1}, \tag{5}$$

where μ is again chosen to balance the budget. To interpret (5), sectors with high roundabout production (low γ_{is}) or low elasticities must have the lowest tax/greatest subsidy to offset the *effective* monopoly distortion that is inversely measured by $\gamma_{is}(\sigma_s - 1)$. Even if $\sigma_s \equiv \sigma > 1$ so that markups are the same across sectors, then sectors with the strongest roundabout production (lowest γ_{is}) must be subsidized in consumption. This second-best policy promotes the final output of those sectors, which is too low because firms use too little of the finished good as an input (due to double marginalization of the markup on differentiated inputs). The intuition from the second-best case will be useful as we examine tariffs on trade, as we turn to next.

4 Trade with One Sector and no Roundabout Production

We now allow trade in the differentiated inputs. We assume that the home country is small, and that it applies a uniform tariff on all imports of the differentiated inputs, with that tariff revenue B_i redistributed to consumers, but we do not allow for any other policy instruments. The import tariff will perform the dual role of influencing the terms of trade and also offsetting monopoly distortions. As explained by Demidova and Rodríguez-Clare (2009, 2013), a small country in a monopolistic competition model faces a fixed foreign demand curve for its exported varieties, but not a *horizontal* export demand curve as under perfect competition. Rather, the home country retains monopoly power in trade and moves along its export demand curve by applying tariffs, but its actions have no impact on the location of the export demand curve. We normalize the foreign wage at unity, $w_j \equiv 1$, and denote the endogenous home wage by w_i.

In a one-sector small economy without roundabout production, Demidova and Rodríguez-Clare (2009) find that the first-best policy subsidizes domestic purchases of the differentiated good. An

equivalent policy is to tax imports: for example, the tariff could be chosen as equal to the markup on domestic goods, as $t_1 = \frac{\sigma_1}{(\sigma_1-1)}$ (see Gros, 1987). Demidova and Rodríguez-Clare (2009) find, however, that with heterogeneous firms this tariff would lead to too few imported varieties. For this reason, a one-sector model with heterogeneous firms has a smaller optimal tariff than t_1 (where we now omit the subscript 1 for simplicity):

$$t^{opt} \equiv \frac{\sigma_1}{\left[\sigma_1 - \frac{(\sigma_1-1)}{\theta_1}\right]} = \frac{\theta_1}{(\theta_1 - \rho_1)} > 1, \quad \text{with} \quad \rho_1 \equiv \frac{\sigma_1 - 1}{\sigma_1} < 1. \tag{6}$$

The same result as in (6) is found by Felbermayr, Jung and Larch (2013), who further show that the optimal tariff in a one-sector, large country is higher than (6).

5 Trade with Two Sectors and Roundabout Production

We now extend the one-sector model to two sectors, with sector 1 traded and sector 2 nontraded. As previously discussed, under autarky $\gamma_{is}(\sigma_s - 1)$ denotes the inverse of the effective monopoly distortion in sector s. We want to allow the distortion in the traded sector to exceed that in the nontraded sector, so that in terms of the inverse distortions we have $\gamma_{i1}(\sigma_1 - 1) < \gamma_{i2}(\sigma_2 - 1)$. We also want to allow for higher values of $\gamma_{i1}(\sigma_1 - 1)$, so the traded sector is less distorted, but we impose an upper-bound on that inverse distortion as compared to the nontraded sector:

$$\frac{\gamma_{i1}(\sigma_1 - 1)}{1 + \gamma_{i1}(\sigma_1 - 1)} \leq \kappa_i \frac{\gamma_{i2}(\sigma_2 - 1)}{1 + \gamma_{i2}(\sigma_2 - 1)}, \tag{7}$$

where the parameter $\kappa_i \geq 1$ will be specified in Theorem 1 below. Our aim is to choose κ_i high enough to include a wide range of effective distortions in (7).

5.1 Wages and Entry

There are two reasons to expect that the optimal tariff in this setting can be less than t^{opt}. First, even without the nontraded sector, roundabout production means that the tariff has a reduced impact on raising the wage in the importing country. We find (see Appendix C.1) that the marginal

impact of the tariff and entry on the wage is,

$$\hat{w}_i = \mathcal{E}_1 \hat{t}_i + \mathcal{E}_2 \hat{N}_{i1}, \tag{8}$$

where $\hat{N}_{i1}$ denotes entry into sector 1 and $\mathcal{E}_n,\ n = 1, ..., 5$ will denote various comparative statics elasticities that depend on the tariff and on sector 1 parameters, including γ_{i1}.[3] When holding entry fixed, the marginal impact of the tariff on wages is:

$$\mathcal{E}_1|_{\gamma_{i1}<1} \ < \ \mathcal{E}_1|_{\gamma_{i1}=1} \ = \ \left[\frac{1}{1 + \tilde{\lambda}_{ii1}^{-1} - \frac{\rho_1}{\theta_1}}\right] > 0, \tag{9}$$

where $\tilde{\lambda}_{ii1}$ denotes the share of *production* (value-added) of intermediate inputs that are purchased domestically in country i.[4] We see that the marginal impact of the tariff on the wage is *reduced* by the presence of roundabout production, i.e. $\gamma_{i1} < 1$. Roundabout production expands the demand for the finished good and the differentiated inputs used to produce it. Evidently, this additional demand makes the countries' excess demand curves for differentiated inputs more elastic, therefore muting the wage impact.

The wage elasticity $\mathcal{E}_1$ is not affected by the presence of a non-traded sector (i.e. it is independent of α_i), but adding that sector gives a second reason for the wage impact to be muted. Specifically, starting from free trade we find that an increase in t_i leads to the *exit* of firms from sector 1, so $\hat{N}_{i1} < 0$ in (9).[5] Because $\mathcal{E}_2 > 0$, that exit lowers the country i wage. This result follows from Lerner symmetry (Costinot and Werning, 2019), whereby an import tariff on imported intermediate inputs is equivalent to an export tariff.

This result is demonstrated by first solving for entry in sector 1 in both free trade and autarky, which gives (Appendix C.2):

$$N_{i1}^{FT|AUT} = \frac{\alpha_i L_i (\sigma_1 - 1)}{(1 - \tilde{\gamma}_{i1}) f_{i1}^E \theta_1 \, \sigma_1}. \tag{10}$$

[3]The comparative statics are performed in Appendix C.1, and all elasticity terms are shown in Appendix D.1.

[4]In the presence of tariffs this differs from the share of *expenditure* on domestically-produced intermediate inputs goods, λ_{ii1}, because expenditure on imports is evaluated at tariff-inclusive prices, but the export share of production is evaluated at free-trade prices.

[5]Our model allows for iceberg costs of trade between countries, so by *free trade* we mean the absence of tariffs.

To obtain entry in-between these two points, let us define a simple function of the tariff:

$$T(t_i) \equiv 1 - \tilde{\gamma}_{i1} + (t_i - 1)\,(1 - \tilde{\lambda}_{ii1}), \tag{11}$$

where $T(t_i) = 1 - \tilde{\gamma}_{i1}$ under both free trade ($t_i = 1$) and autarky ($t_i = +\infty$ with $\tilde{\lambda}_{ii1} = 1$).[6] In between these two points, we find that entry relative to that in 10 is:

$$\frac{N_{i1} - N_{i1}^{FT|AUT}}{N_{i1}^{FT|AUT}} = \frac{-(1-\alpha_i)[(t_i-1)(1-\tilde{\lambda}_{ii1})]^2}{(1-\alpha_i)T(t_i)^2 + \alpha_i\,(1-\tilde{\gamma}_{i1})\,T(t_i)} \leq 0. \tag{12}$$

Notice that if $\alpha_i = 1$, so there is no nontraded sector, then entry is unchanged due to the tariff.[7] But when $\alpha_i < 1$ so there is a nontraded sector, then the inequality in (12) is strict for $1 < t_i < +\infty$, and the tariff leads to exit from the traded sector relative to that in the free trade equilibrium. As the tariff is approaches its prohibitive level of infinity then the entry of firms returns to its free trade level, so there is a $\cup$-shaped relationship between entry and the tariff.

The exit of firms from the traded sector means there is a further effect on welfare, especially when the effective distortion in the traded sector differs from that in the nontraded sector. Let $D(t_i)$ denote the marginal welfare impact of firms moving from the nontraded to the traded sector, relative to the share of spending on that sector (α_i). We find that (Appendix C.1):

$$D(t_i) = \frac{\gamma_{i1}(\sigma_1 - 1)}{1 + \gamma_{i1}(\sigma_1 - 1)} \left[\frac{1 + \gamma_{i1}(\sigma_1 - 1)}{\gamma_{i1}(\sigma_1 - 1)} - \left(\frac{1 + \gamma_{i2}(\sigma_2 - 1)}{\gamma_{i2}(\sigma_2 - 1)} \right) \frac{(1 - \tilde{\gamma}_{i1})}{T(t_i)} - \mathcal{E}_3 \mathcal{E}_4 (1 - \lambda_{ii1}) \right], \tag{13}$$

where $\mathcal{E}_3, \mathcal{E}_4 > 0$. Because $T(t_i) \geq 1 - \tilde{\gamma}_{i1}$ when $t_i \geq 1$, the difference of the first two terms in square brackets in (13) is positive when $\gamma_{i1}(\sigma_1 - 1) < \gamma_{i2}(\sigma_2 - 1)$ holds, so the effective distortion in the traded sector exceeds that in the nontraded sector. There is a final negative term in (13) that could possibly offset this monopoly distortion, and that term arises because at low (high) tariffs, a *decrease* (*increase*) in the tariff is needed to induce entry, which has a revenue cost. This revenue cost means that $\gamma_{i1}(\sigma_1 - 1)$ must be *sufficiently below* $\gamma_{i2}(\sigma_2 - 1)$ to have $D(t_i) > 0$. For example,

[6] We find that $T(t_i)$ is an increasing monotonic transformation of tariff revenue B_i, so it has the same inverted-$\cup$ shape as B_i for $t_i \in [1, +\infty)$ (see Appendix C.2).

[7] The lack of an entry response in the one-sector model assumes that the tariff is applied to the value of imports including their markup and that tariff revenue is rebated. See our working paper (CFRT, 2020, Appendix A) and Felbermayr, Jung and Larch (2015).

when $\gamma_{is} = 1$ for $i = 1, 2$ then $D(t_i) > 0$ for $t_i \in [1, t^{opt}]$ provided that,

$$\sigma_1 < \sigma_2 \left[\frac{\sigma_1(\theta_1 - \rho_1)}{\sigma_1\theta_1 - \rho_1} \right]. \tag{14}$$

For the parameters $\sigma_1 = 4$ and $\theta_1 = 1.5(\sigma_1 - 1) = 4.5$, for example, we would need $\sigma_1 < 0.87\,\sigma_2$ to ensure that $D(t_i) > 0$.

5.2 Optimal Second-Best Tariff

We can now state a general formula for the optimal tariff t_i^* in our two-sector model with roundabout production, as compared to t^{opt} in the one-sector model without roundabout production. Specifically, t_i^* is obtained as a fixed point of the equation:

$$t_i^* = t^{opt}\, F(t_i^*), \text{ with } F(t_i) \equiv \left[\frac{1 - (1 - \gamma_{i1})R(t_i)}{1 + (1 - \alpha_i)M(t_i)} \right], \tag{15}$$

where $M(t_i)$ captures the impact of the higher monopoly distortion in the traded versus the nontraded sectors, and is defined by:

$$M(t_i) \equiv \mathcal{M} \times \left(\mathcal{E}_5 - \frac{(t_i - 1)}{t_i}\theta_1 \right) \frac{D(t_i)}{A(t_i)}, \tag{16}$$

where $\mathcal{E}_5 > 0$ and $\mathcal{M} > 0$ depend on sector 1 parameters and shares, and $A(t_i)$ is defined by:

$$A(t_i) \equiv \alpha_i - \tilde{\gamma}_{i1} + (1 - \alpha_i) \left[\left(1 - \tilde{\lambda}_{ii1} \right) t_i \left(1 - \mathcal{E}_4 \right) + \tilde{\lambda}_{ii1} \right], \tag{17}$$

with $A(t_i) > 0$ under conditions that we specify below; and $R(t_i)$ is defined by:

$$R(t_i) = \mathcal{R} \times \left[\left(\theta_1 - \rho_1 \left(1 - \lambda_{ii1} \right) \right) \left(T(t_i) + \tilde{\gamma}_{i1} \right) - \theta_1\rho_1 \right], \tag{18}$$

where $\mathcal{R} > 0$ depends on sector 1 parameters and shares.

To explain these terms more carefully, recall that the distortion term $D(t_i)$ measures the marginal welfare impact of firms moving from the nontraded to the traded sector, and notice that it enters $(1 - \alpha_i)M(t_i)$, which appears in the denominator of (15) and reflects the impact of the

relative monopoly distortion on the optimal tariff. When $\alpha_i = 1$ so there is only the traded sector, then this term vanishes, because there is no impact of the *relative* distortion between traded and nontraded goods. But there is still roundabout production in traded goods alone, and the impact of that roundabout production on the optimal tariff is captured by the term $R(t_i)$, appearing in the numerator of (15).

When $\alpha_i = 1$ and $\gamma_{i1} = 1$ in (15), then we are back in the one-sector, no-roundabout model and that formula immediately gives $t_i^* = t^{opt}$. Outside of that special case, there will be a lower optimal tariff, $t_i^* < t^{opt}$, whenever $(1-\alpha_i)M(t_i^*) \geq 0$ and $(1-\gamma_{i1})R(t_i^*) \geq 0$ with one of these inequalities holding strictly. For example, suppose that $\alpha_i = 1$ so there is only a traded sector, but $\gamma_{i1} < 1$ so there is some roundabout production. Then we can show that $R(t_i^*) > 0$ at the fixed point of (15), so that roundabout production contributes to lowering the optimal tariff: $t_i^* = F(t_i^*)t^{opt} < t^{opt}$ because the numerator of $F(t_i^*)$ equals $[1-(1-\gamma_{i1})R(t_i^*)] < 1$.

Next, suppose we add the nontraded sector that $\alpha_i < 1$, in which case the denominator of $F(t_i^*)$ equaling $[1+(1-\alpha_i)M(t_i^*)]$ comes into play. If the relative distortion in the traded sector is positive, $D(t_i^*) > 0$, then provided that the other terms in (16) are positive we will have $M(t_i^*) > 0$, so the denominator *further reduces* the optimal tariff. As we discussed in the previous section, however, it is quite possible to have $D(t_i^*) < 0$, either because the effective distortion is lower in the traded sector or because the revenue effect in (13) dominates. In that case, the two terms $M(t_i^*) < 0$ and $R(t_i^*) > 0$ have opposing effects on the optimal tariff, that we need to sort out.

Furthermore, with $M(t_i) < 0$ at some tariffs, we run into a technical difficulty because the denominator of $F(t_i)$ could be zero so this function has a discontinuity. A fixed point of (15) still exists under general conditions,[8] but we cannot use the function $F(t_i)$ to establish the properties of this fixed point. Instead, we work with a transformation that takes the *difference* between the numerator of $F(t_i)$ times t^{opt} and the denominator times t_i:

$$H(t_i) \equiv t^{opt}\left[1-(1-\gamma_{i1})R(t_i)\right] - t_i\left[1+(1-\alpha_i)\,M(t_i)\right]. \tag{19}$$

[8] Let $W(t_i)$ denote utility U_i as a function of the tariff. Provided that $W(t_i)$ is continuous and differentiable, then it will reach some maximum over the (compact) range of all possible tariffs and subsidies, and t_i^* at that maximum will satisfy the first-order condition (15).

The function $H(t_i)$ is a continuous function of the tariff provided that $A(t_i) > 0$ in the interval of tariffs we are interested in, so that $M(t_i)$ does not have any discontinuities. Our general approach in the next result (proved in Appendix D.2) is to find high and low tariffs at which the sign of $H(t_i)$ switches, and then we apply the intermediate value theorem to obtain a point where $H(t_i^*) = 0$, which by construction is a fixed-point of (15).

Theorem 1.

(a) ***Pure roundabout:*** *If $\alpha_i = 1$ and $\gamma_{i1} < 1$, then the optimal tariff is $t_i^* < t^{opt}$ with $R(t_i^*) > 0$.*

(b) ***No roundabout:*** *If $\gamma_{is} = 1$ for $i = 1, 2$ then: (i) the optimal tariff is $t_i^* < t^{opt}$ and $D(t_i^*) > 0$ when (14) holds; (ii) but if $\sigma_1 \geq \sigma_2$ then the optimal tariff is $t_i^* > t^{opt}$ and $D(t_i^*) < 0$.*

(c) ***Two sectors with roundabout:*** *Assume that $\alpha_i < 1$ and that the following two conditions hold:*

$$\gamma_{i1} \geq \frac{1}{1 + \frac{\sigma_1}{\rho_1}(\theta_1 - \rho_1)(1 - \rho_1)}, \tag{20}$$

$$\alpha_i \geq \min\left\{\tilde{\gamma}_{i1}, \frac{-\gamma_{i1}\theta_1 + \rho_1\left(1 + \frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}\right)}{\frac{\theta_1(1-\rho_1)}{\rho_1} + \rho_1\left(1 + \frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}\right)}\right\}. \tag{21}$$

Then $A(t_i) > 0$ for $t_i > t_i'$, where $t_i' < 1$ is an import subsidy. Furthermore, if there is enough roundabout production so that,

$$\gamma_{i1} \leq 1 - \frac{\rho_1}{\left[\theta_1(1 - \rho_1) + \rho_1^2\right](\theta_1 - \rho_1)} < 1, \tag{22}$$

and the bounds in (7) hold where we specify κ_i as,

$$\kappa_i = \left[\delta_1 + \frac{\tilde{\gamma}_{i1}\theta_1\left(\alpha_i(1 - \rho_1) + \gamma_{i1}\rho_1\right)\left(\theta_1(1 - \rho_1) + \left(t^{opt} - \tilde{\gamma}_{i1}\right)\rho_1\right)}{(1 - \alpha_i)\left(1 - \tilde{\gamma}_{i1}\right)^2}\right]\frac{(t^{opt} - \tilde{\gamma}_{i1})}{(1 - \tilde{\gamma}_{i1})}, \tag{23}$$

for $\delta_1 \equiv \frac{1 - \rho_1^2\gamma_{i1}(1-\gamma_{i1})\left(1 - \frac{\rho_1}{(t^{opt})^2}\right)^{-1}}{t^{opt} + \frac{1}{\sigma_1}}$, then the optimal tariff is $t_i^* < t^{opt}$ with $R(t_i^*) > 0$.

Part (a) has already been discussed, and shows that roundabout production in a one-sector model always lowers the optimal tariff. Part (b) deals with the opposite case where there is no roundabout production, in which case $A(t_i) > 0$ is guaranteed. Condition (14) used in part (b)(i)

ensures that the relative distortion in the traded sectors sufficiently exceeds that in the nontraded sector so that $D(t_i) > 0$ for $t_i \in [1, t^{opt}]$. In that case, the relative monopoly distortion is the only factor operating to reduce the optimal tariff and we find that that $D(t_i^*) > 0$ at the optimal tariff t_i^*, which is less that t^{opt} because the denominator of $F(t_i^*)$ exceeds unity. On the other hand, if the traded sector is less distorted that the nontraded sector, with $\sigma_1 \geq \sigma_2$, then we have the reverse outcome with $D(t_i^*) < 0$ and $t_i^* > t^{opt}$.

In part (c) we allow for two sectors and roundabout, and now we need to establish that $A(t_i) > 0$. Recall that we initially defined the term $D(t_i)$ as the marginal welfare impact of firms moving from the nontraded to the traded sector, relative to size of that sector (α_i). In (16), we instead measure the distortion $D(t_i)$ relative to the term $A(t_i)$: when $\alpha_i = 1$ then $A(t_i) > 0$ in (17), but as α_i falls then $A(t_i)$ could become negative. To ensure $A(t_i) > 0$ for $t_i > t_i'$, where $t_i' < 1$ is an import subsidy specified in the proof, we require conditions (20) and (21): the former is a lower-bound on γ_{i1};[9] and the latter is a lower-bound on α_i.[10] The shaded region of parameters violating these two conditions are shown at the bottom and the bottom-left side of Figure 2(a), which are drawn for $\sigma_1 = 4$ and $\theta_1 = 1.5(\sigma_1 - 1) = 4.5$.

The dots in Figure 1 are the values of α_i and γ_{i1} from the EORA dataset providing input-output matrices for 187 countries in 2020 and 15 sectors (Lenzen, Moran, Kanemoto and Geschke, 2013).[11] The traded sectors include manufacturing, agriculture and mining (including petroleum extraction) and those service industries that have greater than 5% of their output being traded.[12] Aggregating within these traded and nontraded sectors, we obtain the dots in Figure 1, and we see that conditions (20) and (21) do not exclude any countries.

Regardless of whether the traded sector is more or less distorted than the nontraded sector, we would like to know if a small amount of roundabout production is enough to overwhelm the relative distortion, so that we obtain $t_i^* < t^{opt}$ due to $R(t_i^*) > 0$ even though we don't know the

[9] Models with roundabout production can exhibit unusual properties when γ_{i1} is very low so that output is extremely high. See Celik, Karabay and McLaren (2018), for example, who restrict the extent of roundabout production. In this paper, allowing γ_{i1} to be very low in country i (only) would violate the assumption that it is a small country.

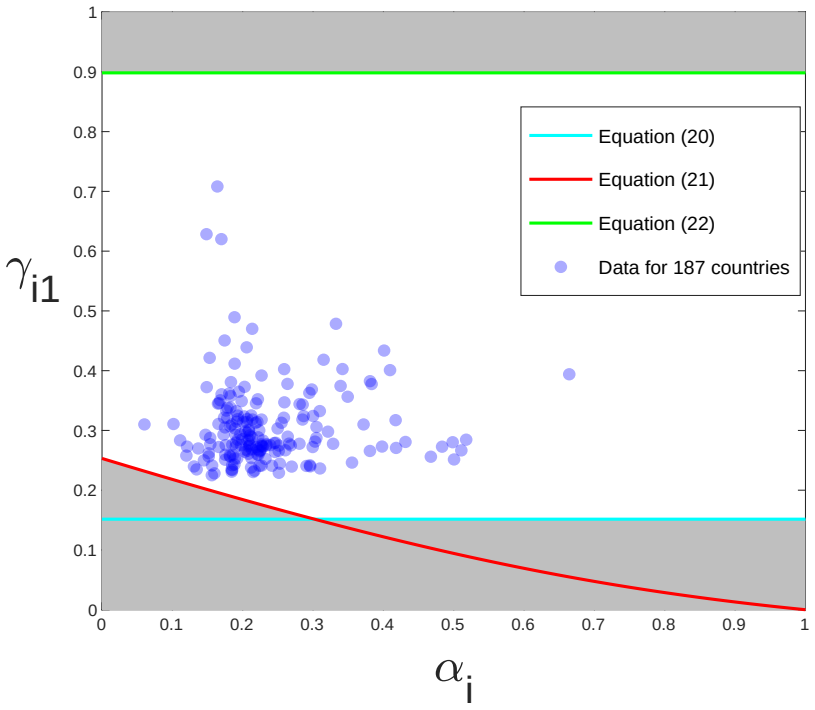
[10] Taking the minimum in (21) results in the kink shown for that equation in Figure 2(a).

[11] EORA has 190 countries including the Rest of the World, which we omit, along with Belarus and the Former Soviet Union because their input-output tables are nonsensical.

[12] Wholesale and retail trade have slightly less that 4% of their output being traded, so that industry and all other services with less than 5% of their output traded become the nontraded sector.

sign of $D(t_i^*)$. Part (c) answers that question in the affirmative. The needed amount of roundabout production is shown by the constraint (22), which is graphed in Figure 1 and is very weak: for our parameter values we only require that $\gamma_{i1} < 0.9$, which all countries easily satisfy.

Figure 1: Parameter Restrictions



The value of κ_i specified in (23) is greater than unity for all but a tiny sliver of $[\alpha_i, \gamma_{i1}]$ that are very close to $\gamma_{i1} = 1$. In most of the $[\alpha_i, \gamma_{i1}]$ space, the value of κ_i is substantially greater than unity, which allows for a wide range of values for relative distortion between the two sectors. Fro example, Gervais and Jensen (2019) find that services have elasticities of substitution about one-quarter *smaller* than for manufacturing. If we follow them, setting $\sigma_s = 3$ for services and, given $\theta_s/(\sigma_s - 1) = 1.5$, also setting $\theta_s = 3$. In that case we would have σ_s for traded goods exceeding σ_s for services, generating higher markups in the nontraded sector that includes most service industries. Nevertheless, the high value for κ_i means that *all countries* easily satisfy the constraint (7). We therefore find a low optimal tariff, $t_i^* < t^{opt}$, for a wide range of parameters that includes the empirically relevant values.

We conclude this section with two points. First, conditions (7), (20), (21) and (22) are sufficient to establish the low optimal tariff but not necessary. This point is suggested by Figure 1, where the shaded inadmissible regions are as shown for all $0 < \alpha_i < 1$, but when $\alpha = 1$ then the *only* inadmissible points are $\gamma_{i1} \in \{0, 1\}$. By continuity, the shaded inadmissible regions at the top and bottom of Figure 1 must surely shrink as $\alpha_i \to 1$, to become the points $\gamma_{i1} \in \{0, 1\}$, as would be obtained if the sufficient conditions to have $t_i^*(\alpha_i, \gamma_{i1}) < t^{opt}$ were also necessary.

Second, the optimal tariff can be negative. In our working paper (CFRT, 2020), we examine the conditions to ensure that the optimal tariff is negative, and we find that it occurs for two types of countries: a *Highly Linked Economy* that has high roundabout production (low γ_{i1}) and is very open (low λ_{ii1}); and a *Remote Economy*, with a small traded sector and with $\lambda_{ii1} \to 1$, so that the economy is nearly closed to trade due to high iceberg costs, as may occur for very distant countries. We have found examples of both types in our quantitative analysis.[13]

6 Conclusions

We began by asking whether modern trade theory has anything new to say about arguments for protecting the traded sector. We did not mention that there is a line of recent literature that to some extent argues in favor of such protection. Specifically, this is the *firm-delocation* literature that combines a monopolistically competitive traded sector with a competitive *traded* numeraire good (see e.g. Melitz and Ottaviano, 2008, section 4; Bagwell and Lee, 2018). The traded numeraire good pins down relative wages between countries, so the country applying tariffs is "small" in the sense that its wages do not respond to its tariff. In this literature, encouraging entry into traded goods requires positive import tariffs. Essentially, the ability to attract firms into the home country takes the place of a conventional terms-of-trade motive for tariffs, so that the optimal tariff is positive even though wages are fixed. Of course, with multiple countries pursuing this motive for protection, there is ample scope for trade agreements to reduce the deadweight losses due to the tariffs (Ossa,

[13] In our working paper (CFRT, 2020) we use the full 15 sector EORA model to numerically compute optimal tariffs for each country in 1990, using 182 countries that year. We start from global free trade and then alter the tariff for each country while holding other countries' *ad valorem* tariffs at zero. Ten economies in the sample – including China, Hong Kong, India, Israel, Vietnam, and five more remote countries – would have benefited from going beyond free trade to subsidizing their imports in 1990, since their optimal tariffs are negative.

2011; Bagwell and Staiger, 2015).

The major differences between this class of models and our own are: (i) our *nontraded* service sector does not fix relative wages between countries; and (ii) roundabout production. Lerner symmetry applies to the traded sector, so that import tariffs are equivalent to export taxes and inhibit entry into that sector. That logic does not apply when the numeraire good is traded, which gives firm-delocation models a very different flavor. Reduced entry into the traded sector sector leads to a welfare loss whenever that effective monopoly distortion (magnified by roundabout production) is greater than in services. Furthermore, roundabout production itself means that the market equilibrium is not efficient because of the markup on intermediate inputs (which is magnified by double-marginalization). We find that roundabout production on its own leads to a lower optimal tariff, and under quite weak conditions, that result holds for roundabout production combined with a range of effective distortions in the traded and nontraded sectors. We also expect that the political-economy motives for trade agreements will differ in this type of model (e.g. Dinopoulos, Heins and Unel, 2019), which deserves further attention.

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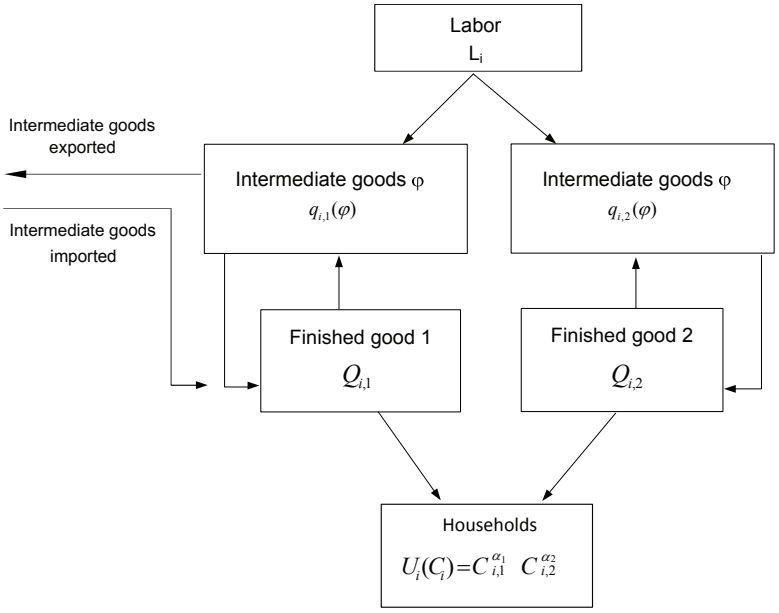
APPENDIX

A Two-Sector Small Open Economy Model

We focus on a small open economy model, with two sectors $s = 1, 2$ and roundabout production in both sectors. County i is the small open economy, and the foreign country $j \neq i$ is the rest of the world. We drop the sectoral subscript from the country i tariff, so that it becomes a tariff t_i applied to the traded sector 1, while sector 2 is nontraded. In the foreign country, for simplicity we assume a single traded sector denoted by $s = 1$, and no tariff.

A.1 Description of Economy

Figure 2: Schematic production structure



The structure of the country i economy is illustrated in Figure 3. Firms in sector 1 of country i can source differentiated inputs from countries $k = i, j$ for $i \neq j$, and the CES production functions

over the differentiated inputs purchased from each country k and in total are

$$Q_{i1} \equiv \left(\sum_{k=i,j} Q_{ki1}^{\frac{\sigma_1 - 1}{\sigma_1}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \text{ with } Q_{ki1} \equiv \left(N_{k1} \int_{\varphi_{ki1}^*}^{\infty} q_{ki1}(\varphi)^{\frac{\sigma_1 - 1}{\sigma_1}} g_1(\varphi) d\varphi \right)^{\frac{\sigma_1}{\sigma_1 - 1}}, \tag{24}$$

where N_{k1} are the mass of entrants in each of countries k who sell at the prices $p_{ki1}(\varphi)$, depending on their productivities φ with distribution $g_1(\varphi)$. The cutoff productivity $\varphi \geq \varphi_{ki1}^*$ needed to sell from country k to i will be derived below. The CES price indexes over the differentiated inputs purchased from each country k and in total are

$$P_{i1} \equiv \left(\sum_{k=i,j} P_{ki1}^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}} \text{ and } P_{ki1} \equiv \left(N_{k1} \int_{\varphi_{ki1}^*}^{\infty} p_{ki1}(\varphi)^{1-\sigma_1} g_1(\varphi) d\varphi \right)^{\frac{1}{1-\sigma_1}}. \tag{25}$$

The mass of varieties of inputs sold from country i to k for firms with productivity $\varphi \geq \varphi_{ik1}^*$ is

$$N_{ik1} \equiv N_{i1}[1 - G_1(\varphi_{ik1}^*)] = N_{i1} \varphi_{ik1}^{*-\theta_1}, \tag{26}$$

using the Pareto distribution $G_1(\varphi) = 1 - \varphi^{-\theta_1}$ with $\varphi \geq 1$. Notice that the entry of firms N_{k1} appearing in (24) and (25) can be converted into the mass of varieties by multiplying by $[1 - G_1(\varphi_{ki1}^*)]$. In that case, we divide the density functions $g_1(\varphi)$ by $[1 - G_1(\varphi_{ki1}^*)]$ to obtain conditional densities, as used below in (32).

The total value of production of the finished good in country i and sector s is $Y_{is} = P_{is} Q_{is}$. The demand for intermediates of variety φ sold from country i to $k = i, j$ is then given by

$$q_{iks}(\varphi) = \left(\frac{p_{iks}(\varphi)}{P_{ks}} \right)^{-\sigma_s} \frac{Y_{ks}}{P_{ks}}. \tag{27}$$

A firm in i supplying differentiated inputs has the marginal costs x_{is}/φ, with the costs of input bundle supply x_{is} given by (1). We assume that fixed and sunk costs of the firm require only labor and are denoted by f_{iks} and f_{is}^E, respectively. The profits from the firm in i supplying differentiated

inputs to market k are then

$$\pi_{iks}(\varphi) = \max_{p_{iks}(\varphi)\geq 0} \left\{ \frac{p_{iks}(\varphi)}{t_{iks}} q_{iks}(\varphi) - \frac{x_{is}}{\varphi} \tau_{iks}\, q_{iks}(\varphi) - w_i\, f_{iks} \right\}, \tag{28}$$

and likewise for $\pi_{jks}(\varphi)$, where $t_i \equiv t_{ji1}$ is one plus the *ad valorem* tariff charged for country i imports from j in sector 1, and all other tariff are unity: $t_{iis} \equiv t_{jjs} \equiv t_{ijs} \equiv 1$. The term τ_{iks} is iceberg trade costs, with $\tau_{iis} \equiv \tau_{jjs} \equiv 1$. The first-order conditions for profit maximization yield

$$\frac{p_{iks}(\varphi)}{t_{iks}} = \frac{\sigma_s}{\sigma_s - 1} \frac{x_{is}\, \tau_{iks}}{\varphi}, \tag{29}$$

$$q_{iks}(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1} \frac{x_{is}\, \tau_{iks} t_{iks}}{\varphi} \right)^{-\sigma_s} \frac{Y_{ks}}{P_{ks}^{1-\sigma_s}}, \text{ for } k = i, j. \tag{30}$$

For the the nontradable sector 2, we set $k = i$ and only use the equations for π_{iis}, p_{iis} and q_{iis}.

Substituting these expressions back into profits, we can readily solve for the cutoff productivity φ^*_{iks} at which profits are zero:

$$\pi_{iks}(\varphi^*_{iks}) = 0 \implies \varphi^*_{iks} = \left(\frac{\sigma_s}{\sigma_s - 1} \right) \left(\frac{\sigma_s w_i f_{iks} t_{iks}}{Y_{ks} P_{ks}^{\sigma_s - 1}} \right)^{\frac{1}{(\sigma_s - 1)}} x_{is}\, \tau_{iks} t_{iks}, \quad k = i, j, . \tag{31}$$

We follow Melitz (2003) in defining the average productivity as

$$\bar{\varphi}_{iks} \equiv \left(\int_{\varphi^*_{iks}}^{\infty} \varphi^{\sigma_s - 1} \frac{g_s(\varphi)}{\left[1 - G_s(\varphi^*_{iks})\right]} d\varphi \right)^{\frac{1}{\sigma_s - 1}} = K_s \varphi^*_{iks}, \text{with } K_s \equiv \left(\frac{\theta_s}{\theta_s - \sigma_s + 1} \right)^{\frac{1}{\sigma_s - 1}}, \tag{32}$$

where the constant K_s is obtained by computing the integral using the Pareto distribution.

We can substitute the above expressions into (24) to obtain the output of the finished good:

$$Q_{is} = K_s^{\sigma_s} \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{-\sigma_s} \left(\frac{Y_{is}}{P_{is}^{1-\sigma_s}} \right) \left[N_{is} \varphi_{iis}^{*-\theta_s} \left(\frac{x_{is}}{\varphi^*_{iis}} \right)^{1-\sigma_s} + N_{js} \varphi_{jis}^{*-\theta_s} \left(\frac{x_{js}\, \tau_{jis}\, t_{is}}{\varphi^*_{jis}} \right)^{1-\sigma_s} \right]^{\frac{\sigma_s}{\sigma_s - 1}}, \tag{33}$$

where in the nontraded sector 2 the second terms in brackets does not appear, and this should also be understood in the equations leading up to (35).

Using (26), (29), (32) we can also obtain an expression for P_{is}:

$$P_{is} = \frac{1}{K_s}\left(\frac{\sigma_s}{\sigma_s - 1}\right)\left[N_{is}\varphi_{iis}^{*(\sigma_s-1-\theta_s)} x_{is}^{1-\sigma_s} + \quad N_{js}\varphi_{jis}^{*(\sigma_s-1-\theta_s)}\left(x_{js}\,\tau_{jis}\,t_{is}\right)^{1-\sigma_s}\right]^{\frac{1}{1-\sigma_s}}. \tag{34}$$

We can multiply this by (33) to obtain a preliminary expression for the value of production of the finished goods in country i and sector s, $Y_{is} \equiv P_{is}Q_{is}$:

$$Y_{is} = K_s^{\sigma_s-1}\left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\left(\frac{Y_{is}}{P_{is}^{1-\sigma_s}}\right)\left[N_{is}\varphi_{iis}^{*-\theta_s}\left(\frac{x_{is}}{\varphi_{iis}^*}\right)^{1-\sigma_s} + \; N_{js}\varphi_{jis}^{*-\theta_s}\left(\frac{x_{js}\,\tau_{jis}\,t_{is}}{\varphi_{jis}^*}\right)^{1-\sigma_s}\right].$$

To simplify this expression, we can use (31) twice to obtain

$$\frac{Y_{is}}{P_{is}^{1-\sigma_s}} = \sigma_s w_i f_{iis}\left(\frac{\sigma_s}{\sigma_s-1}\frac{x_{is}}{\varphi_{iis}^*}\right)^{\sigma_s-1} = \sigma_s w_j f_{jis} t_{is}\left(\frac{\sigma_s}{\sigma_s-1}\frac{x_{js}\,\tau_{jis} t_{is}}{\varphi_{jis}^*}\right)^{\sigma_s-1}.$$

and substituting above we obtain

$$Y_{is} = K_s^{\sigma_s-1}\sigma_s\left(N_{is}\varphi_{iis}^{*-\theta_s}\, w_i\, f_{iis} + \; N_{js}\varphi_{jis}^{*-\theta_s}\, w_j\, f_{jis}\, t_{is}\right). \tag{35}$$

The value of finished output in each sector, Y_{is}, is sold to consumers and also back to domestic firms. That finished output is costlessly bundled from home and imported differentiated inputs. Let λ_{jis} denote the share of country's i total expenditure in sector s on intermediate goods from market j. In this share, integrating over sales of all varieties of s from j to i yields the numerator, and summing over all markets j gives the denominator. Using the conditions (29), (30), (32), and (34) we can obtain the following expression for the expenditure share on domestic inputs in both sectors $s = 1, 2$:

$$\lambda_{iis} = {\varphi_{iis}^*}^{-\theta_s} N_{is}\left(\frac{\sigma_s}{\sigma_s-1}\frac{x_{is}}{\bar{\varphi}_{iis}\,P_{is}}\right)^{1-\sigma_s}, \tag{36}$$

and on imported inputs in sector 1:

$$\lambda_{ji1} = {\varphi_{ji1}^*}^{-\theta_1} N_{j1}\left(\frac{\sigma_1}{\sigma_1-1}\frac{\tau_{ji1}\,x_{j1}\,t_{i1}}{\bar{\varphi}_{ji1}\,P_{i1}}\right)^{1-\sigma_1}. \tag{37}$$

Duty-free imports in sector 1 of country i are $E_{ji1} = (\lambda_{ji1}Y_{i1})/t_i$ while exports are $E_{ij1} =$

$\lambda_{ij1}Y_{j1}$, so that trade balance requires:

$$\frac{\lambda_{ji1}Y_{i1}}{t_i} = \lambda_{ij1}Y_{j1}. \tag{38}$$

Total variable costs of all differentiated inputs produced in sector s in country i is given by $\frac{\sigma_1-1}{\sigma_1}(\lambda_{ii1}Y_{i1} + \lambda_{ij1}Y_{j1})$; namely, the net-of-markup value of sector s goods that are sold locally and abroad. In sector 2 this is simply $\frac{\sigma_2-2}{\sigma_2}\lambda_{ii2}Y_{i2}$. We then obtain the total demand for the finished good in sector s of country i which goes to both consumers as final goods and to firms for intermediate use, and which must equal total supply of that output:

$$Y_{is} = \alpha_{is}I_i + \tilde{\gamma}_{is}\left(\lambda_{iis}Y_{is} + \lambda_{ijs}Y_{js}\right), \text{with } \tilde{\gamma}_{is} \equiv (1-\gamma_{iis})\left(\frac{\sigma_s - 1}{\sigma_s}\right), \tag{39}$$

where once again we have $\lambda_{ij2} \equiv 0$ so that $\lambda_{ii2} \equiv 1$ in the nontraded sector.

To close the model we need to solve for the mass of firms N_{is} entering in country i and sector s. We start with sector 1 by using (31) for $j \neq i$ and the average value $\bar{\varphi}_{ij1}$, and substituting into (37) to obtain an equation governing the cutoffs φ^*_{ij1},

$$\lambda_{ij1} = {\varphi^*_{ij1}}^{-\theta_1} N_{i1}\left(\frac{\sigma_1 w_i f_{ij1}}{Y_{j1}}\right)\left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1}\right). \tag{40}$$

Next, multiplying this equation by Y_{j1} and making use of home sales $E_{ii1} = (\lambda_{ii1}Y_{i1})$ and exports $E_{ij1} = \lambda_{ij1}Y_{j1}$, we obtain an expression for total domestic plus international sales of intermediate inputs in sector 1 by country i,

$$E_{ii1} + E_{ij1} = \sum_{k=i,j} {\varphi^*_{ik1}}^{-\theta_1} N_{i1}\left(\frac{\theta_1\sigma_1 w_i f_{ik1}}{\theta_1 + 1 - \sigma_1}\right).$$

Next, multiplying this equation by Y_{j1} and making use of home sales $E_{ii1} = (\lambda_{ii1}Y_{i1})$ and exports $E_{ij1} = \lambda_{ij1}Y_{j1}$, we obtain an expression for total domestic plus international sales of intermediate inputs in sector 1 by country i,

$$E_{ii1} + E_{ij1} = \sum_{k=i,j} {\varphi^*_{ik1}}^{-\theta_1} N_{i1}\left(\frac{\theta_1\sigma_1 w_i f_{ik1}}{\theta_1 + 1 - \sigma_1}\right).$$

This equation is simplified by making use of free entry in country i. Expected profits must equal the sunk costs of entry, so that for a country i firm:

$$\sum_{k=i,j} \int_{\varphi^*_{ik1}}^{\infty} \pi_{ik1}(\varphi) g_1(\varphi) d\varphi = w_i f^E_{i1}. \tag{41}$$

This expression can be simplified following the approach of Melitz and Redding (2014), who note that CES demand implies that $\pi_{ik1}(\varphi) + w_i f_{ik1} = [\pi_{ik1}(\varphi^*_{ik1}) + w_i f_{ik1}] (\varphi/\varphi^*_{ik1})^{\sigma_1 - 1}$. It follows from (28) that $\pi_{ik1}(\varphi) = \left[(\varphi/\varphi^*_{ik1})^{\sigma_1 - 1} - 1\right] w_i f_{ik1}$, and so the above entry condition becomes:

$$\sum_{k=i,j} J_1(\varphi^*_{ik1}) f_{ik1} = f^E_{i1} \quad \text{with} \quad J_1(\varphi^*) \equiv \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma_1 - 1} - 1 \right] g_1(\varphi) d\varphi. \tag{42}$$

Completing the integral above using the Pareto distribution, we arrive at:

$$\sum_{k=i,j} \varphi^{*-\theta_1}_{ik1} f_{ik1} = \left(\frac{\theta_1 - \sigma_1 + 1}{\sigma_1 - 1} \right) f^E_{i1}, \tag{43}$$

from which we can obtain an equation governing the mass of entrants N_{is}, namely:

$$N_{i1} = (E_{ii1} + E_{ij1}) \Big/ \left[w_i \, f^E_{i1} \left(\frac{\theta_1 \, \sigma_1}{\sigma_1 - 1} \right) \right]. \tag{44}$$

In sector 2 the mass of entrants is governed by the same equation but without E_{ij2} appearing. The productivity cutoff φ^*_{ii2} in (42) and (43) is likewise determined by summing over country $k = i$ only, and this version of (43) is shown below by setting $s = 2$ in (72).

We now derive expressions for labor market demand in sectors 1 and 2:

$$\begin{aligned} L_{i1} \;\; = \;\; & N_{i1} f^E_{i1} + N_{i1} f_{ii1} \int_{\varphi^*_{ii1}}^{\infty} g(\varphi) \, d\varphi + N_{i1} f_{ij1} \int_{\varphi^*_{ij1}}^{\infty} g(\varphi) \, d\varphi \\ & + \gamma_{i1} (\sigma_1 - 1) N_{i1} \sum\nolimits_{k=i,j} \left[\int_{\varphi^*_{ik1}}^{\infty} \frac{\pi_{ik1}}{w_i} (\varphi) \, g(\varphi) \, d\varphi + f_{ik1} \int_{\varphi^*_{ik1}}^{\infty} g(\varphi) \, d\varphi \right], \end{aligned}$$

and,

$$\begin{aligned} L_{i2} &= N_{i2} f_{i2}^E + N_{i2} f_{ii2} \int_{\varphi_{ii2}^*}^{\infty} g(\varphi)\, d\varphi \\ &\quad + \gamma_{i2} (\sigma_2 - 1) N_{i2} \left[\int_{\varphi_{ii2}^*}^{\infty} \frac{\pi_{ii2}}{w_i} (\varphi)\, g(\varphi)\, d\varphi + f_{ii2} \int_{\varphi_{ii2}^*}^{\infty} g(\varphi)\, d\varphi \right]. \end{aligned}$$

Then and using the free entry condition (41), we obtain

$$\frac{L_{i1}}{N_{i1}} = (1 + \gamma_{i1} (\sigma_1 - 1)) \left(\ f_{i1}^E + f_{ii1} {\varphi_{ii1}^*}^{-\theta_1} + f_{ij1} {\varphi_{ij1}^*}^{-\theta_1} \right)$$

$$\frac{L_{i2}}{N_{i2}} = (1 + \gamma_{i2} (\sigma_2 - 1)) \left(\ f_{i2}^E + f_{ii2} {\varphi_{ii2}^*}^{-\theta_2} \right)$$

Using (43), entry into sectors 1 and 2 becomes:

$$N_{i1} = \frac{(\sigma_1 - 1)}{(1 + \gamma_{i1} (\sigma_1 - 1))\, \theta_1} \frac{L_{i1}}{f_{i1}^E}, \tag{45}$$

$$N_{i2} = \frac{(\sigma_2 - 1)}{(1 + \gamma_{i2} (\sigma_2 - 1)) \theta_2} \frac{L_{i2}}{f_{i2}^E}. \tag{46}$$

Combining the expressions, we obtain

$$\frac{L_{i1}}{L_{i2}} = \frac{N_{i1}}{N_{i2}} \frac{\frac{\left((1+\gamma_{i1}(\sigma_1 - 1)) \theta_1 f_{i1}^E \right)}{(\sigma_1 - 1)}}{\frac{(1+\gamma_{i2}(\sigma_2 - 1)) \theta_2 f_{i2}^E}{(\sigma_2 - 1)}}. \tag{47}$$

Next we will characterize the labor allocation across sectors. Using free entry (44), trade balance (38) and recall that sector 2 is the nontraded sector, we obtain

$$\begin{aligned} N_{i1} &= \frac{\sigma_1 - 1}{\theta_1\, \sigma_1} \frac{\lambda_{ii1}\, Y_{i1} + \lambda_{ij1}\, Y_{j1}}{w_i\, f_{i1}^E} \\ &= \frac{\sigma_1 - 1}{\theta_1\, \sigma_1} \frac{\lambda_{ii1}\, Y_{i1} + \frac{\lambda_{ji1}}{t_i} Y_{i1}}{w_i\, f_{i1}^E} \\ &= \frac{\sigma_1 - 1}{\theta_1\, \sigma_1} \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i} \right) \frac{Y_{i1}}{w_i\, f_{i1}^E}. \end{aligned} \tag{48}$$

and,

$$N_{i2} = \frac{\sigma_2 - 1}{\theta_2 \sigma_2} \frac{Y_{i2}}{w_i f_{i2}^E} \tag{49}$$

Therefore, using (47), the ratio of labor in the two sectors is given by

$$\frac{L_{i1}}{L_{i2}} = \frac{\sigma_2 \left(1 + \gamma_{i1} \left(\sigma_1 - 1\right)\right)}{\sigma_1 (1 + \gamma_{i2}(\sigma_2 - 1))} \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right) \frac{Y_{i1}}{Y_{i2}}. \tag{50}$$

Recalling (46), we can also obtain

$$Y_{i2} = \frac{\sigma_2}{1 + \gamma_{i2}(\sigma_2 - 1)} w_i L_{i2} = \frac{1}{1 - \tilde{\gamma}_{i2}} w_i L_{i2}. \tag{51}$$

From market clearing condition (39), Y_{i1} and Y_{i2} can be written as

$$Y_{i1} = \alpha_i I_i + \tilde{\gamma}_{i1} \left(\lambda_{ii1} Y_{i1} + \lambda_{ij1} Y_{j1}\right), \tag{52}$$

$$Y_{i2} = (1 - \alpha_i) I_i + \tilde{\gamma}_{i2} Y_{i2}. \tag{53}$$

Using trade balance (38), we derive the expression for Y_{i1} depending on income:

$$Y_{i1} = \frac{\alpha_i}{1 - \tilde{\gamma}_{i1} \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right)} I_i. \tag{54}$$

This implies that

$$\frac{Y_{i1}}{Y_{i2}} = \frac{\frac{\alpha_i}{1 - \tilde{\gamma}_{i1}\left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right)} I_i}{\frac{1 - \alpha_i}{1 - \tilde{\gamma}_{i2}} I_i} = \frac{\alpha_i}{(1 - \alpha_i)} \frac{1 - \tilde{\gamma}_{i2}}{1 - \tilde{\gamma}_{i1} \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right)}.$$

Substituting this expression into (50) and recalling $\tilde{\gamma}_{is} \equiv (1 - \gamma_{is}) \left(\frac{\sigma_s - 1}{\sigma_s}\right)$, labor allocation across sectors can be written as

$$\frac{L_{i1}}{L_{i2}} = \frac{\alpha_i}{(1 - \alpha_i)} \frac{(1 - \tilde{\gamma}_{i1}) \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right)}{1 - \tilde{\gamma}_{i1} \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right)}. \tag{55}$$

We now introduce an expression that is going to be used several times in our derivations. We denote to the share of *production* (value-added) that is devoted to goods consumed domestically in

country i, by

$$\tilde{\lambda}_{ii1} = \frac{{\varphi^*_{ii1}}^{-\theta_1} f_{ii1}}{{\varphi^*_{ii1}}^{-\theta_1} f_{ii1} + {\varphi^*_{ij1}}^{-\theta_1} f_{ij1}}. \tag{56}$$

Note that this share differs from the domestic expenditure share (except when there are no tariffs). From (35), (40) and trade balance (38), the domestic expenditure shares are given by

$$\lambda_{ii1} = \frac{{\varphi^*_{ii1}}^{-\theta_1} f_{ii1}}{{\varphi^*_{ii1}}^{-\theta_1} f_{ii1} + {\varphi^*_{ij1}}^{-\theta_1} f_{ij1} t_i}. \tag{57}$$

For future reference it is worth noting from the above that

$$t_i = \frac{(1-\lambda_{ii1})}{\lambda_{ii1}} \frac{\tilde{\lambda}_{ii1}}{\left(1-\tilde{\lambda}_{ii1}\right)}, \tag{58}$$

and as a result

$$1 - t_i = \frac{\lambda_{ii1} - \tilde{\lambda}_{ii1}}{\lambda_{ii1}\left(1-\tilde{\lambda}_{ii1}\right)}. \tag{59}$$

From these two equations we can readily derive

$$\tilde{\lambda}_{ii1} = \frac{t_i \lambda_{ii1}}{1 + (t_i - 1)\lambda_{ii1}} \tag{60}$$

and,

$$1 - \tilde{\lambda}_{ii1} = \frac{1 - \lambda_{ii1}}{1 + (t_i - 1)\lambda_{ii1}} \tag{61}$$

These tariff formulas can be used to simplify the above expressions. Using (58) and (55), we have

$$\begin{aligned} \frac{L_{i1}}{L_{i2}} &= \frac{\alpha_i}{(1-\alpha_i)} \frac{(1-\tilde{\gamma}_{i1})\left(\frac{1+(t_i-1)\lambda_{ii1}}{t_i}\right)}{1-\tilde{\gamma}_{i1}\left(\frac{1+(t_i-1)\lambda_{ii1}}{t_i}\right)} \\ &= \frac{\alpha_i}{(1-\alpha_i)} \frac{(1-\tilde{\gamma}_{i1})\left(\frac{1}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1}}\right)}{1-\tilde{\gamma}_{i1}\left(\frac{1}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1}}\right)}. \end{aligned}$$

We can also express the labor allocation as a fraction of total labor supply

$$\frac{L_{i2}}{L_{i1}+L_{i2}} = \left(\frac{L_{i1}}{L_{i2}}+1\right)^{-1} = \frac{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1} + \frac{\alpha_i}{1-\alpha_i}\left(1-\tilde{\gamma}_{i1}\right)}, \tag{62}$$

$$\frac{L_{i1}}{L_{i1}+L_{i2}} = \frac{\frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right) + \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}. \tag{63}$$

Finally, we derive an expression for income I_i. First note that tariff revenue is given by

$$B_i = \left(t_i - 1\right)\frac{\lambda_{ji1}}{t_i}Y_{i1}, \tag{64}$$

and using (40) and (45), we obtain

$$\begin{aligned}
\frac{\lambda_{ji1}Y_{i1}}{t_i} &= {\varphi^*_{ji1}}^{-\theta_1} N_{j1}\left(\sigma_1 w_j f_{ji1}\right)\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right), \\
\lambda_{ij1}Y_{j,1} &= {\varphi^*_{ij1}}^{-\theta_1} N_{i1}\left(\sigma_1 w_i f_{ij1}\right)\left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right) \\
&= \frac{\sigma_1\left(\sigma_1-1\right)f_{ij1}}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)\left(\theta_1+1-\sigma_1\right)f^E_{i1}} w_i L_{i1} {\varphi^*_{ij1}}^{-\theta_1}.
\end{aligned}$$

Then income $I_i = w_i L_i + B_i$ can be using free entry (43) and $\tilde{\lambda}_{ii1}$ in (56) as

$$I_i = w_i L_i + \left(t_i-1\right)\frac{\sigma_1}{1+\gamma_{i1}\left(\sigma_1-1\right)}\frac{f_{ij1}{\varphi^*_{ij1}}^{-\theta_1}}{\frac{\left(\theta_1+1-\sigma_1\right)}{\left(\sigma_1-1\right)}f^E_{i1}} w_i L_{i1}. = w_i L_i + \left(t_i-1\right)\left(\frac{1}{1-\tilde{\gamma}_{i1}}\right)\left(1-\tilde{\lambda}_{ii1}\right) w_i L_{i1}.$$

Combining with (63), we have

$$\begin{aligned}
\frac{I_i}{w_i L_{i1}} &= \frac{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}{\frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right) + \tilde{\gamma}_{i1}} + \left(t_i-1\right)\left(\frac{1}{1-\tilde{\gamma}_{i1}}\right)\left(1-\tilde{\lambda}_{ii1}\right), \\
&= \frac{\left(1-\alpha_i\right)\left(t_i-1\right)\left(1-\tilde{\lambda}_{ii1}\right) + 1 - \tilde{\gamma}_{i1}}{\alpha_i\left(1-\tilde{\gamma}_{i1}\right)} + \left(t_i-1\right)\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{1-\tilde{\gamma}_{i1}} \\
&= \frac{1}{\alpha_i} + \frac{\left(t_i-1\right)\left(1-\tilde{\lambda}_{ii1}\right)}{\alpha_i\left(1-\tilde{\gamma}_{i1}\right)}.
\end{aligned}$$

Using $T(t_i) \equiv 1 - \tilde{\gamma}_{i1} + (t_i - 1)\left(1 - \tilde{\lambda}_{ii1}\right)$ from (11), we then obtain

$$I_i = \frac{w_i L_{i1}}{\alpha_i} \frac{T(t_i)}{(1 - \tilde{\gamma}_{i1})}. \tag{65}$$

Next we derive the expression for intermediate demand in sector 1,

$$\text{Intermediate demand} = N_{i1}\left(1 - \gamma_{i1}\right)\left(\int_{\varphi^*_{ii1}}^{\infty} \frac{x_{i1} q_{ii1}(\varphi)}{\varphi} g(\varphi)\, d\varphi + \int_{\varphi^*_{ij1}}^{\infty} \frac{x_{i1}\tau_{ij1} q_{ij1}(\varphi)}{\varphi} g(\varphi)\, d\varphi\right),$$

and using the expression for profits,

$$\frac{x_{i1}\tau_{ij1} q_{ij1}(\varphi)}{\varphi} = (\sigma_1 - 1)\,\pi_{ij1}(\varphi) + (\sigma_1 - 1)\, w_i f_{ij1},$$

we obtain,

$$\text{Intermediate demand} = N_{i1}\left(1 - \gamma_{i1}\right)(\sigma_1 - 1) \sum\nolimits_{k=i,j} \int_{\varphi^*_{ik1}}^{\infty} \left(\pi_{ik1}(\varphi) + w_i f_{ik1}\right) g(\varphi)\, d\varphi.$$

Then using the free entry condition (41), we obtain

$$\text{Intermediate demand} = N_{i1}\left(1 - \gamma_{i1}\right)(\sigma_1 - 1)\, w_i \left(f^E_{i1} + f_{ii1}{\varphi^*_{ii1}}^{-\theta_1} + f_{ij1}{\varphi^*_{ij1}}^{-\theta_1}\right).$$

Finally using (43) and the labor market clearing condition, the intermediate demand is given by

$$\text{Intermediate demand} = w_i L_{i1} \frac{\left(1 - \gamma_{i1}\right)(\sigma_1 - 1)}{\left(1 + \gamma_{i1}(\sigma_1 - 1)\right)}.$$

Then the total demand for finished goods in sector 1 is

$$Y_{i1} = \alpha_i I_i + w_i L_{i1} \frac{\left(1 - \gamma_{i1}\right)(\sigma_1 - 1)}{\left(1 + \gamma_{i1}(\sigma_1 - 1)\right)}.$$

Now use (35), (38) and (40), we obtain trade balance for sector 1:

$$\varphi_{ij1}^{*}{}^{-\theta_1} N_{i1} w_i f_{ij1} = \varphi_{ji1}^{*}{}^{-\theta_1} N_{j1} w_j f_{ji1}. \tag{66}$$

After combining these expressions, I_i is given in terms of sector 1 variables by

$$I_i = \frac{w_i L_{i1}}{\alpha_i} \frac{(\sigma_1 - 1)}{(1 + \gamma_{i1}(\sigma_1 - 1))} \left(K_1^{\sigma_1 - 1} \frac{\sigma_1}{\theta_1 f_{i1}^E} \left(\varphi_{ii1}^{*-\theta_1} f_{ii1} + \varphi_{ij1}^{*}{}^{-\theta_1} f_{ij1} t_i \right) - (1 - \gamma_{i1}) \right). \tag{67}$$

Recalling (51) and (53), income is written in terms of sector 2 as

$$I_i = \frac{w_i L_{i2}}{(1 - \alpha_i)}. \tag{68}$$

A.2 Equilibrium Conditions

We use the definition of small open economy following Demidova and Rodríguez-Clare (2009) where we impose a fixed import demand curve from foreign . In particular from the point of view of country j, country i is small such that wages, prices, entry, and expenditure at county j are not affected by changes in policy of i. Formally, the equilibrium conditions of the SOE are the following

Definition 1. *An equilibrium of small open economy two sectors roundabout model is characterized for a set of prices* $(w_i, x_{i1}, x_{i2}, P_{i1}, P_{i2})$ *productivity thresholds* $\left(\varphi_{ii1}^{*}, \varphi_{ii2}^{*}, \varphi_{ji1}^{*}, \varphi_{ij1}^{*}\right)$, *expenditures* (Y_{i1}, Y_{i2}), *mass of firms* (N_{i1}, N_{i2}), *and expenditure shares* $(\lambda_{ii1}, \lambda_{ji1})$ *such that solve the following equilibrium conditions:*

(zero cut-off)

$$\varphi_{ii1}^{*} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right) \left(\frac{\sigma_1 w_i f_{ii1}}{Y_{i1}}\right)^{\frac{1}{\sigma_1 - 1}} \frac{x_{i1}}{P_{i1}},$$

$$\varphi_{ji1}^{*} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right) \left(\frac{\sigma_1 w_j f_{ji1}}{Y_{i1}}\right)^{\frac{1}{\sigma_1 - 1}} \frac{x_{j1} (t_i)^{\frac{\sigma_1}{\sigma_1 - 1}}}{P_{i1}},$$

$$\varphi_{ii2}^{*} = \left(\frac{\sigma_2}{\sigma_2 - 1}\right) \left(\frac{\sigma_2 w_i f_{ii2}}{Y_{i2}}\right)^{\frac{1}{\sigma_2 - 1}} \frac{x_{i2}}{P_{i2}},$$

$$\varphi_{ij1}^{*} = \left(\frac{\sigma_1}{\sigma_1 - 1}\right) (\sigma_1 w_i f_{ij1})^{\frac{1}{\sigma_1 - 1}} x_{i1} \frac{(Y_{j1})^{\sigma_1 - 1}}{P_{j1}},$$

(input costs)

$$x_{i1} = (w_{i1})^{\gamma_{i1}} (P_{i1})^{1-\gamma_{i1}}$$

$$x_{i2} = (w_{i2})^{\gamma_{i2}} (P_{i2})^{1-\gamma_{i2}}$$

(expenditures)

$$Y_{i1} = \alpha \frac{w_i L_i}{1 - \tilde{\gamma}_1 \left(\frac{1+(t_i-1)\lambda_{ii1}}{t_i} \right) - \alpha (t_i - 1) \frac{1-\lambda_{ii1}}{t_i}},$$

$$Y_{i2} = \tilde{\gamma}_i Y_{i2} + (1-\alpha) \left(\frac{t_i - 1}{t_i} (1 - \lambda_{ii1}) Y_{i1} + w_i L_i \right),$$

with $\tilde{\gamma}_1 = (1 - \gamma_{i1}) \frac{\sigma_1 - 1}{\sigma_1}$

(price index)

$$P_{i1} = \left({\varphi^*_{ii1}}^{-\theta_1} N_{i1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{i1}}{\tilde{\varphi}_{ii1}} \right)^{1-\sigma_1} + {\varphi^*_{ji1}}^{-\theta_1} N_{j_1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{j1} t_i}{\tilde{\varphi}_{ji1}} \right)^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}},$$

$$P_{i2} = \left({\varphi^*_{ii2}}^{-\theta_2} N_{i2} \left(\frac{\sigma_2}{\sigma_2 - 1} \frac{x_{i2}}{\tilde{\varphi}_{ii2}} \right)^{1-\sigma_2} \right)^{\frac{1}{1-\sigma_2}},$$

(entry)

$$N_{i1} = \frac{\frac{1+\lambda_{ii1}(t_i-1)}{t_i}}{w_{i1} f^E_{i1} \left(\frac{\theta_1 \sigma_1}{\sigma_1 - 1} \right)} Y_{i1},$$

$$N_{i2} = \frac{Y_{i2}}{w_{i2} f^E_{i2} \left(\frac{\theta_2 \sigma_2}{\sigma_2 - 1} \right)},$$

(expenditure shares)

$$\lambda_{ii1} = {\varphi^*_{ii1}}^{-\theta_1} N_{i1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{i1}}{P_{i1} \tilde{\varphi}_{ii1}} \right)^{1-\sigma_1},$$

note that $\tilde{\varphi}_{iis} = K_s \varphi^*_{iis}$, *with* $K_s = \left(\frac{\theta_s}{\theta_s + 1 - \sigma_s} \right)^{\frac{1}{\sigma_s - 1}}$,

(trade balance)

$$w_i = \frac{{\varphi^*_{ji1}}^{-\theta} N_{j1} w_{j1} f_{ji1}}{{\varphi^*_{ij1}}^{-\theta} N_{i1} f_{ij}},$$

taken as given $\{x_{j1}, P_{j1}, Y_{j1}, N_{j1}, w_{j,1}\}$.

B Autarky Multisector Model

In the autarky model we use α_{is} to denote the consumption share across for multiple sectors $s = 1, ..., S$, where we now introduce producer and consumer tax/subsidies on purchases of the finished good. The producer tax/subsidy means that the input cost index is modified from (1) as:

$$x_{is} = w_i^{\gamma_{is}} (t_{is}^p P_{is})^{1-\gamma_{is}}, \tag{69}$$

where P_{is} denotes the price of the finished good *net* of taxes. In the market clearing condition (3), there is no trade so that $\lambda_{iis} = 1$ and $\lambda_{ijs} = 0$, and the consumer and firm purchases must be divided by t_{is}^c and t_{is}^p, respectively, to obtain the net-of-tax purchases. Further multiplying these purchases by the *ad valorem* tax rates $t_{is}^c - 1$ and $t_{is}^p - 1$, respectively, we obtain the condition for a balanced budget:

$$0 = (t_{is}^c - 1)\frac{\alpha_{is} w_i L_i}{t_{is}^c} + (t_{is}^p - 1)\frac{\tilde{\gamma}_s Y_{is}}{t_{is}^p}. \tag{70}$$

The term $\alpha_{is} w_i L_i / t_{is}^c$ on the right of (70) is the value of consumer purchases of the finished good. Dividing this by the duty-free price index of the finished good, P_{is}, we obtain consumption in each sector, and so the objective function for the planner is:

$$\max_{t_{is}^c, t_{is}^p > 0} \prod_{s=1}^S C_{is}^{\alpha_{is}} = \prod_{s=1}^S \left(\frac{\alpha_{is} w_i L_i}{t_{is}^c P_{is}} \right)^{\alpha_{is}}, \tag{71}$$

subject to the constraint (70), where $\alpha_{is} > 0$ are the consumption shares with $\sum_{s=1}^S \alpha_{is} = 1$.

To determine the optimal policies, we need an expression for the price index in each sector under autarky, which is obtained from (34) but ignoring the terms involving the foreign country j, and also simplifying this expression using (26) and (32). The cutoff and average productivities are not affected by the consumer or producer taxes, because they are determined by the free-entry condition like in (42), but simplified to the autarky equilibrium in country i:

$$J_s(\varphi_{iis}^*) f_{iis} = f_{is}^E. \tag{72}$$

Entry into each sector, N_{is}, is endogenous and is determined by (44), where the expenditure

on the differentiated inputs in autarky, E_{iis}, equals the net-of-tax value of the final good that are bundled from them, Y_{is}, and we ignore the term E_{ijs} under autarky. In the market clearing condition (3), with no trade then $\lambda_{iis} = 1$ and $\lambda_{ijs} = 0$, and the consumer and firm purchases must be divided by t^c_{is} and t^p_{is}, respectively, to obtain the net-of-tax purchases:

$$Y_{is} = \frac{\alpha_{is}}{t^c_{is}} w_i L_i + \frac{\tilde{\gamma}_{is}}{t^p_{is}} Y_{is}, \tag{73}$$

recalling that we have set tax revenue at zero, so that $w_i L_i$ is consumer income. Then we solve for $Y_{is} = (\alpha_{is} w_i L_i)/\{t^c_{is}[1 - (\tilde{\gamma}_{is}/t^p_{is})\}$, so that entry from (44) is:

$$N_{is} = (\alpha_{is} L_i) \Big/ \left[t^c_{is} \left(1 - \frac{\tilde{\gamma}_{is}}{t^p_{is}} \right) f^E_{is} \left(\frac{\theta_s \, \sigma_s}{\sigma_s - 1} \right) \right]. \tag{74}$$

Substituting (74) into (26) and (71) and ignoring constants, we obtain the objective function:

$$\max_{t^c_{is}, t^p_{is} > 0} \prod_{s=1}^{S} \left\{ t^c_{is} \left[t^c_{is} \left(1 - \frac{\tilde{\gamma}_{is}}{t^p_{is}} \right) \right]^{\frac{1}{\gamma_{is}(\sigma_s - 1)}} (t^p_{is})^{\frac{(1-\gamma_{is})}{\gamma_{is}}} \right\}^{-\alpha_{is}}. \tag{75}$$

As explained in the main text, we solve the problem (75) subject to (70) twice: first, by choosing the optimal consumer and producer tax/subsidies; and second, by choosing the consumer tax t^c_{is} without any producer tax, $t^p_{is} \equiv 1$. The solutions are recorded in (4) and (5).

C Changes in the Tariff

In Appendices and , we return to the small open economy model described in Appendix A. In Appendix C.1 we perform the comparative statics with respect to a change in the tariff to obtain the first-order condition for the optimal tariff, and in Appendix C.2 we examine how entry into sector i changes with the tariff. In Appendix D.1 we summarize the elasticities that arise from the comparative statics, and in Appendix D.2 we develop the proof of Theorem 1.

C.1 First-order Condition for the Optimal Tariff

We now derive an expression for the price index that is going to be used in order to express welfare as a function of productivity thresholds. From (34) the sector 1 price index is

$$P_{i1} = \frac{1}{K_1}\left(\frac{\sigma_1}{\sigma_1 - 1}\right)\left[N_{i1}\varphi_{ii1}^{*(\sigma_1-1-\theta_1)}x_{i1}^{1-\sigma_1} + N_{j1}\varphi_{ji1}^{*(\sigma_1-1-\theta_1)}\left(x_{j1}\,\tau_{ji1}\,t_i\right)^{1-\sigma_1}\right]^{\frac{1}{1-\sigma_1}}.$$

We combine the entry thresholds φ_{ji1}^*, and φ_{ii1}^*, to obtain

$$\left(\frac{\sigma_1}{\sigma_1-1}\frac{x_{j1}\tau_{ji1}\,t_{i1}}{\varphi_{ji1}^*}\right)^{1-\sigma_1} = \frac{\sigma_1\,w_j\,f_{ji1}\,t_i}{\sigma_1\,w_i\,f_{ii1}}\left(\frac{\sigma_1}{\sigma_1-1}\frac{x_{i1}}{\varphi_{ii1}^*}\right)^{1-\sigma_1}. \tag{76}$$

Using this expression together with trade balance (66) and (1) we obtain

$$P_{i1} = \left(\frac{\sigma_1}{\sigma_1-1}\frac{1}{K_1}\left(\frac{N_{i1}}{f_{ii1}}\right)^{\frac{1}{(1-\sigma_1)}}\right)^{\frac{1}{\gamma_{i1}}}(\varphi_{ii1}^*)^{-\frac{1}{\gamma_{i1}}}\,w_i\left(\,f_{ii1}\varphi_{ii1}^{*-\theta_1} + \;\varphi_{ij1}^{*\;-\theta_1}\;f_{ij1}t_i\right)^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}. \tag{77}$$

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Similarly, we obtain

$$P_{i2} = \left(\frac{\sigma_2}{\sigma_2-1}\frac{1}{K_2}\left(\frac{N_{i2}}{f_{ii2}}\right)^{\frac{1}{(1-\sigma_2)}}\right)^{\frac{1}{\gamma_{i2}}}(\;\varphi_{ii2}^*)^{-\frac{1}{\gamma_{i2}}}\,w_i\left(\,f_{ii2}\varphi_{ii2}^{*-\theta_2}\right)^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}.$$

Using expressions (67) and (68) for income and the above expressions for the price indexes, we substitute these into indirect utility or welfare, which from (2) is given by

$$U_i = \left(\frac{\alpha_i I_i}{P_{i1}}\right)^{\alpha_i}\left(\frac{(1-\alpha_i)I_i}{P_{i2}}\right)^{1-\alpha_i}.$$

Define the term,

$$\Theta \equiv \left(\frac{\frac{(\sigma_1-1)}{(1+\gamma_{i1}(\sigma_1-1))}}{\left(\frac{\sigma_1}{\sigma_1-1}\frac{1}{K_1}\left(\frac{1}{f_{ii1}}\right)^{\frac{1}{(1-\sigma_1)}}\right)^{\frac{1-\sigma_1}{\gamma_{i1}(1-\sigma_1)}}}\right)^{\alpha_i}\left(\frac{1}{\left(\frac{\sigma_2}{\sigma_2-1}\frac{1}{K_2}\left(\frac{1}{f_{ii2}}\right)^{\frac{1}{(1-\sigma_2)}}\right)^{\frac{1}{\gamma_{i2}}}(\varphi_{ii2}^*)^{-\frac{1}{\gamma_{i2}}}\left(\,f_{ii2}\;\varphi_{ii2}^{*-\theta_2}\right)^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}}\right)^{1-\alpha_i}$$

which is a constant because φ^*_{ii2} is constant from (42). We then obtain the welfare expression,

$$\begin{aligned} U_i &= \Theta \left(\frac{\frac{k_1^{\sigma_1-1}\sigma_1}{\theta_1 f^E_{i1}} \left(\varphi^{*-\theta_1}_{ii1} \; f_{ii1} + \; \varphi^{*}_{ij1}{}^{-\theta_1} \; f_{ij1}\, t_i \right) - (1-\gamma_{i1})}{\left(f_{ii1} \varphi^{*-\theta_1}_{ii1} + \varphi^{*}_{ij1}{}^{-\theta_1} \; f_{ij1} t_i \right)^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}} (\varphi^*_{ii1})^{\frac{1}{\gamma_{i1}}} \right)^{\alpha_i} \left(\frac{L_{i1}}{(N_{i1})^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}} \right)^{\alpha_i} \\ &\times \left(\frac{L_{i2}}{(N_{i2})^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}} \right)^{1-\alpha_i}. \end{aligned}$$

There is new term in this expression, given by

$$\begin{aligned} &\left(\frac{L_{i1}}{(N_{i1})^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}} \right)^{\alpha_i} \left(\frac{L_{i2}}{(N_{i2})^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}} \right)^{1-\alpha_i} = \\ &\left(\frac{(1+\gamma_{i1}(\sigma_1-1))\,\theta_1 f^E_{i1}}{(\sigma_1-1)} (N_{i1})^{\frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)}} \right)^{\alpha_i} \left(\frac{(1+\gamma_2(\sigma_2-1))\,\theta_2 f^E_{i1}}{(\sigma_2-1)} (N_{i2})^{\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}} \right)^{1-\alpha_i} \end{aligned}$$

using (45), (46).

Totally differentiating welfare, $\hat{U}_i$ can be written as

$$\begin{aligned} \hat{U}_i &= \alpha_i \left(1 + \frac{1-\gamma_{i1}}{\frac{\alpha_i I_i}{w_i L_{i1}} \frac{(1+\gamma_{i1}(\sigma_1-1))}{(\sigma_1-1)}} + \frac{1}{\gamma_{i1}(\sigma_1-1)} \right) \left((1-\lambda_{ii1}) \left(-\theta_1 \hat{\varphi}^*_{ij1} + \hat{t}_i \right) - \theta_1 \lambda_{ii1} \hat{\varphi}^*_{ii1} \right) \\ &\quad + \frac{\alpha_i}{\gamma_{i1}} \hat{\varphi}^*_{ii1} + \alpha_i \frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)} \hat{N}_{i1} + (1-\alpha_i) \frac{1+\gamma_2(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)} \hat{N}_{i2} \\ &= \alpha_i \left(1 + \frac{(1-\gamma_{i1})(\sigma_1-1)}{(1+\gamma_{i1}(\sigma_1-1)) + \sigma_1(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)} + \frac{1}{\gamma_{i1}(\sigma_1-1)} \right) \\ &\quad \left((1-\lambda_{ii1}) \left(-\theta_1 \hat{\varphi}^*_{ij1} + \hat{t}_i \right) - \theta_1 \lambda_{ii1} \hat{\varphi}^*_{ii1} \right) + \frac{1}{\gamma_{i1}} \hat{\varphi}^*_{ii1} \\ &\quad + \alpha_i \frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)} \hat{N}_{i1} + (1-\alpha_i) \frac{1+\gamma_2(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)} \hat{N}_{i2}, \end{aligned} \tag{78}$$

where the equality is obtained by using the following expression

$$\begin{aligned} 1 + \frac{1-\gamma_{i1}}{\frac{\alpha_i I_i}{w_i L_{i1}} \frac{(1+\gamma_{i1}(\sigma_1-1))}{(\sigma_1-1)}} &+ \frac{1}{\gamma_{i1}(\sigma_1-1)} = 1 + \frac{1-\gamma_{i1}}{\left(1 + \frac{\sigma_1(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}{(1+\gamma_{i1}(\sigma_1-1))} \right) \frac{(1+\gamma_{i1}(\sigma_1-1))}{(\sigma_1-1)}} + \frac{1}{\gamma_{i1}(\sigma_1-1)} \\ &= 1 + \frac{(1-\gamma_{i1})(\sigma_1-1)}{(1+\gamma_{i1}(\sigma_1-1)) + \sigma_1(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)} + \frac{1}{\gamma_{i1}(\sigma_1-1)}, \end{aligned}$$

Now the strategy is to obtain expressions for $\hat{\varphi}^*_{ii1}$, and $\hat{\varphi}^*_{ij1}$. First, totally differentiate the free entry condition (43) and use (56) to obtain

$$\hat{\varphi}^*_{ii1} = -\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\hat{\varphi}^*_{ij1}. \tag{79}$$

Then we totally differentiate the price index (77) to obtain

$$\begin{aligned}\hat{P}_{i1} \quad &= \frac{1}{\gamma_{i1}\left(1-\sigma_1\right)}\hat{N}_{i1} + \hat{w}_i + \frac{1}{\gamma_{i1}}\left(\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\left(\frac{\theta_1}{\left(1-\sigma_1\right)}\lambda_{ii1} + 1\right) - \frac{\theta_1\left(1-\lambda_{ii1}\right)}{\left(1-\sigma_1\right)}\right)\hat{\varphi}^*_{ij1} \\ &+ \frac{1}{\gamma_{i1}\left(1-\sigma_1\right)}\left(1-\lambda_{ii1}\right)\hat{t}_i. \end{aligned} \tag{80}$$

Next, totally differentiate the expression for φ^*_{ij1} in (31) and recall that country i is a small open economy so that the country j price index, value of output and input-cost index are fixed. It follows that $\hat{\varphi}^*_{ij1}$ is given by

$$\hat{\varphi}^*_{ij1} - \left(\frac{1}{\sigma_1 - 1} + \gamma_{i1}\right)\hat{w}_i = \left(1-\gamma_{i1}\right)\hat{P}_{i1}. \tag{81}$$

Now combine (80) and (81) to obtain

$$\begin{aligned}&\frac{\hat{\varphi}^*_{ij1}}{1-\gamma_{i1}} - \frac{1}{1-\gamma_{i1}}\left(\frac{1}{\sigma_1-1} + \gamma_{i1}\right)\hat{w}_i = \\ &\frac{1}{\gamma_{i1}\left(1-\sigma_1\right)}\hat{N}_{i1} + \hat{w}_i + \frac{1}{\gamma_{i1}}\left(\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\theta_1\lambda_{ii1}}{1-\sigma_1} + 1\right) - \frac{\theta_1\left(1-\lambda_{ii1}\right)}{1-\sigma_1}\right)\hat{\varphi}^*_{ij1} \\ &+ \frac{1-\lambda_{ii1}}{\gamma_{i1}\left(1-\sigma_1\right)}\hat{t}_i. \end{aligned} \tag{82}$$

From the trade balance (66), we obtain

$$\hat{\varphi}^*_{ji1} = \hat{\varphi}^*_{ij1} - \frac{1}{\theta_1}\hat{w}_i - \frac{1}{\theta_1}\hat{N}_{i1}. \tag{83}$$

From the equality for $\hat{\varphi}^*_{ji1}$ and $\hat{\varphi}^*_{ii1}$ (76), we can see that

$$\hat{\varphi}^*_{ji1} = \hat{\varphi}^*_{ii1} - \left(\frac{1}{\sigma_1-1} + \gamma_{i1}\right)\hat{w}_i - \left(1-\gamma_{i1}\right)\hat{P}_{i1} + \frac{\sigma_1}{\sigma_1-1}\hat{t}_i.$$

Combining (79) and (81), $\hat{\varphi}^*_{ji1}$ is given by

$$\hat{\varphi}^*_{ji1} = -\frac{1}{\tilde{\lambda}_{ii1}}\left(\frac{1}{\sigma_1 - 1} + \gamma_{i1}\right)\hat{w}_i - \frac{1}{\tilde{\lambda}_{ii1}}\left(1 - \gamma_{i1}\right)\hat{P}_{i1} + \frac{\sigma_1}{\sigma_1 - 1}\hat{t}_i,$$

and after using (83), we obtain

$$\hat{\varphi}^*_{ij1} - \frac{1}{\theta_1}\hat{w}_i - \frac{1}{\theta_1}\hat{N}_{i1} = -\frac{1}{\tilde{\lambda}_{ii1}}\left(\frac{1}{\sigma_1 - 1} + \gamma_{i1}\right)\hat{w}_i - \frac{1}{\tilde{\lambda}_{ii1}}\left(1 - \gamma_{i1}\right)\hat{P}_{i1} + \frac{\sigma_1}{\sigma_1 - 1}\hat{t}_i.$$

Then from (81) we have

$$\hat{\varphi}^*_{ij1} = \frac{\tilde{\lambda}_{ii1}}{1 + \tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(\hat{w}_i + \hat{N}_{i1}\right) + \frac{\tilde{\lambda}_{ii1}}{1 + \tilde{\lambda}_{ii1}}\frac{\sigma_1}{\sigma_1 - 1}\hat{t}_i. \tag{84}$$

Using (82) and multiplying both sides by $(1 - \gamma_{i1})$

$$\begin{aligned} -\left(\frac{\sigma_1}{\sigma_1 - 1}\right)\hat{w}_i &= \left(-1 + \frac{1 - \gamma_{i1}}{\gamma_{i1}}\left(\frac{\theta_1}{(1 - \sigma_1)}\left(\frac{\lambda_{ii1} - \tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) + \frac{\left(1 - \tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right)\right)\hat{\varphi}^*_{ij1} \\ &+ \frac{1 - \gamma_{i1}}{\gamma_{i1}\left(1 - \sigma_1\right)}\hat{N}_{i1} + \frac{\left(1 - \gamma_{i1}\right)}{\gamma_{i1}\left(1 - \sigma_1\right)}\left(1 - \lambda_{ii1}\right)\hat{t}_i. \end{aligned}$$

Combine this expression with (84) and use the change of notation that $\rho_1 \equiv \frac{\sigma_1 - 1}{\sigma_1}$, we finally obtain

$$\hat{w}_i = \mathcal{E}_1\hat{t}_i + \mathcal{E}_2\hat{N}_{i1},$$

where,

$$\mathcal{E}_1 = \frac{1 - \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(1 - \frac{1}{\sigma_1}\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}} + \left(\frac{1}{\sigma_1} - \frac{\theta_1}{\sigma_1 - 1}\right)\left(1 - t_i\right)\lambda_{ii1}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} - \frac{1}{\sigma_1}\left(1 - t_i\right)\lambda_{ii1}\right)}, \tag{85}$$

$$\mathcal{E}_2 = \frac{\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{\rho_1}{\theta_1} - \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} - \frac{1}{\sigma_1}\left(1 - t_i\right)\lambda_{ii1}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} - \frac{1}{\sigma_1}\left(1 - t_i\right)\lambda_{ii1}\right)}. \tag{86}$$

Notice that at free trade

$$\mathcal{E}_1 = \frac{1 - \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(1 - \frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\frac{\rho_1}{\theta_1}}$$

which is less than the value with $\gamma_{i1} = 1$ provided that

$$\left(1 - \frac{1}{\sigma_1}\left(1 + \tilde{\lambda}_{ii1}\right)\right) > 0 \text{ which holds if } \sigma_1 > 2.$$

Now substituting these expressions into (84), we obtain

$$\begin{aligned}
\hat{\varphi}^*_{ij1} &= \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(1+\mathcal{E}_2\right)\hat{N}_{i1} + \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{\mathcal{E}_1}{\theta_1}+\frac{1}{\rho_1}\right)\hat{t}_i \\
&= \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}} + \left(t_i-1\right)\lambda_{ii1}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}\left(t_i-1\right)\lambda_{ii1}\right)}\right)\hat{t}_i \\
&+\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(1+\mathcal{E}_2\right)\hat{N}_{i1}. \qquad (87)
\end{aligned}$$

Note that

$$\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(1+\mathcal{E}_2\right) = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} - \frac{1}{\sigma_1}\left(1-t_i\right)\lambda_{ii1}\right)}.$$

Then from the welfare equation (78), using (79) we obtain

$$\begin{aligned}
\hat{U}_i &= \alpha_i\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\hat{t}_i + \left(\mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right)\hat{\varphi}^*_{ij1}\right] \\
&+\alpha_i\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_{i1}\left(\sigma_1-1\right)}\hat{N}_{i1} + \left(1-\alpha_i\right)\frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\hat{N}_{i2}, \qquad (88)
\end{aligned}$$

where,

$$\mathcal{E}_3 \equiv \left(1 + \frac{\left(1-\gamma_{i1}\right)\left(\sigma_1-1\right)}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right) + \sigma_1\left(t_i-1\right)\left(1-\tilde{\lambda}_{ii1}\right)} + \frac{1}{\gamma_{i1}\left(\sigma_1-1\right)}\right). \qquad (89)$$

Inverting (87), $\hat{t}_i$ is given by

$$\begin{aligned}
\hat{t}_i = &\frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}\left(t_i-1\right)\lambda_{ii1}\right)}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}} + \left(t_i-1\right)\lambda_{ii1}\right)\right)}\hat{\varphi}^*_{ij1} \\
&-\frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}} + \left(t_i-1\right)\lambda_{ii1}\right)\right)}\hat{N}_{i1}.
\end{aligned}$$

Write this expression for $\hat{t}_i$ as

$$\hat{t}_i = -\mathcal{E}_4 \hat{N}_{i1} + \mathcal{E}_5 \hat{\varphi}^*_{ij1}, \tag{90}$$

where

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}} + (t_i-1)\,\lambda_{ii1}\right)\right)},$$

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_i-1)\,\lambda_{ii1}\right)}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}} + (t_i-1)\,\lambda_{ii1}\right)\right)}.$$

Using (59), $\mathcal{E}_4$ and $\mathcal{E}_5$ can be written as

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})}, \tag{91}$$

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_i-1)\,\lambda_{ii1}\right)}{\frac{1}{\rho_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})}. \tag{92}$$

Now we work on with the welfare expression in (88). First note that using (90), we obtain

$$\begin{aligned}\hat{U}_i &= \alpha_i\left[\mathcal{E}_3(1-\lambda_{ii1})\,\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right]\hat{\varphi}^*_{ij1} \\ &\quad +\alpha_i\left[\frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_1(\sigma_1-1)} - \mathcal{E}_3(1-\lambda_{ii1})\,\mathcal{E}_4\right]\hat{N}_{i1} + (1-\alpha_i)\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\hat{N}_{i2}.\end{aligned} \tag{93}$$

Now we seek to express $\hat{N}_{i1}$, $\hat{N}_{i2}$ as a function of $\hat{\varphi}^*_{ij1}$. From the labor market clearing condition $L_{i1} + L_{i2} = L_i$ and using (45) and (46), we obtain

$$0 = \frac{L_{i2}}{L_i}\hat{N}_{i2} + \frac{L_{i1}}{L_i}\hat{N}_{i1},$$

which implies that

$$\hat{N}_{i2} = -\frac{L_{i1}}{L_{i2}}\hat{N}_{i1}. \tag{94}$$

In addition, recalling (49), (51) and (62), we obtain

$$N_{i2}\, w_i f_{i2}^E \left(\frac{\theta_2\sigma_2}{\sigma_2-1}\right) = \frac{1}{1-\tilde{\gamma}_{i2}} w_i L_{i2} = \frac{1}{1-\tilde{\gamma}_{i2}} w_i L_i \left(\frac{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}\right)$$

Here we define

$$l_{i2} \equiv \frac{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)} = \frac{L_{i2}}{L_i},$$

and then $\hat{N}_{i2}$ is given by

$$\hat{N}_{i2} = \hat{l}_{i2},$$

where

$$\hat{l}_{i2} = (1-l_{i2}) \frac{(1-t_i)\,\tilde{\lambda}_{ii1}\hat{\tilde{\lambda}}_{ii1} + \left(1-\tilde{\lambda}_{ii1}\right) t_i \hat{t}_i}{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}},$$

Combining this expression with (94), $\hat{N}_{i1}$ can be written as

$$\begin{aligned}
\hat{N}_{i1} &= -\frac{L_{i2}}{L_{i1}}(1-l_{i2}) \left(\frac{(1-t_i)\,\tilde{\lambda}_{ii1}\hat{\tilde{\lambda}}_{ii1} + \left(1-\tilde{\lambda}_{ii1}\right) t_i \hat{t}_i}{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}\right) \\
&= -\left(\frac{(1-t_i)\,\tilde{\lambda}_{ii1}\hat{\tilde{\lambda}}_{ii1} + \left(1-\tilde{\lambda}_{ii1}\right) t_i \hat{t}_i}{\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}.\right)
\end{aligned}$$

From (56) and (76), we can use

$$\hat{\tilde{\lambda}}_{ii1} = \theta_1 \frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}} \hat{\varphi}^*_{ij1}, \tag{95}$$

and combining with (90), we obtain

$$\begin{aligned}
&-\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right) \hat{N}_{i1} \\
&= (1-t_i)\,\tilde{\lambda}_{ii1}\theta_1 \frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\, \hat{\varphi}^*_{ij1} + \left(1-\tilde{\lambda}_{ii1}\right) t_i (\mathcal{E}_5 \hat{\varphi}^*_{ij1} - \mathcal{E}_4 \hat{N}_{i1}).
\end{aligned}$$

Then we finally get

$$\hat{N}_{i1} = \frac{\left(1-\tilde{\lambda}_{ii1}\right)\left(\left(1-t_i\right)\theta_1 + t_i\mathcal{E}_5\right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right)}\hat{\varphi}^*_{ij1}. \tag{96}$$

Next we obtain $\hat{N}_{i2}$. From (62), (63) and (94), $\hat{N}_{i2}$ can be written as

$$\hat{N}_{i2} = -\frac{\alpha_i}{1-\alpha_i}\frac{\left(1-\tilde{\gamma}_1\right)}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}\hat{N}_{i1}. \tag{97}$$

We substitute (96) and (97) into the second term of welfare in (93),

$$\begin{aligned}
&\alpha_i\left(\tfrac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_1(\sigma_1-1)} - \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right)\hat{N}_{i1} + \left(1-\alpha_i\right)\tfrac{\sigma_2}{\sigma_2-1}\hat{N}_{i2} \\
&= \left[\tfrac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)} - \tfrac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\tfrac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)(t_i-1)+(1+\gamma_{i1}(\sigma_1-1))} - \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right]\alpha_i\hat{N}_{i1}.
\end{aligned}$$

Then $\hat{U}_i$ can be written as

$$\begin{aligned}
\hat{U}_i &= \alpha_i\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\tfrac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \tfrac{1}{\gamma_{i1}}\tfrac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right]\hat{\varphi}^*_{ij1} \\
&+ \left[\tfrac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)} - \tfrac{1-\tilde{\gamma}_{i2}}{\gamma_{i2}\rho_2}\tfrac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)(t_i-1)+(1+\gamma_{i1}(\sigma_1-1))} - \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right]\alpha_i\hat{N}_{i1}.
\end{aligned}$$

At the optimum, $\hat{U}_i/\left(\alpha_i\hat{\varphi}^*_{ij1}\right) = 0$, which implies that

$$\begin{aligned}
&\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right] \\
= &-\left[\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_{i1}\left(\sigma_1-1\right)} - \frac{1-\tilde{\gamma}_{i2}}{\gamma_{i2}\rho_2}\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i-1\right)+\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)} - \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right] \\
&\times\frac{\left(1-\tilde{\lambda}_{ii1}\right)\left(\left(1-t_i\right)\theta_1 + t_i\mathcal{E}_5\right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i-\tilde{\gamma}_{i1}\right)\right)}.
\end{aligned}$$

This expression can also be written as

$$\begin{aligned}
&\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5+\mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right)-\frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right] \\
= \quad & -\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_{i1}\left(\sigma_1-1\right)}\frac{\left(1-\tilde{\lambda}_{ii1}\right)\left(\left(t_i-1\right)\theta_1-t_i\mathcal{E}_5\right)}{\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(1-\mathcal{E}_4\right)+\tilde{\lambda}_{ii1}+\frac{1}{1-\alpha_i}\left(\alpha_i-\tilde{\gamma}_{i1}\right)}D(t_i), \qquad (98)
\end{aligned}$$

where

$$D(t_i)\equiv\left[1-\frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_2\left(\sigma_2-1\right)}\frac{\gamma_{i1}\left(\sigma_1-1\right)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i-1\right)+\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}-\frac{\gamma_{i1}\left(\sigma_1-1\right)}{1+\gamma_{i1}\left(\sigma_1-1\right)}\left(1-\lambda_{ii1}\right)\mathcal{E}_3\mathcal{E}_4\right]. \qquad (99)$$

We define

$$\tilde{M}(t_i)=\frac{1-\tilde{\gamma}_{i1}}{\rho_1}\frac{\left(\mathcal{E}_5-\frac{\left(t_i-1\right)}{t_i}\theta_1\right)}{\left(1-\alpha_i\right)\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(1-\mathcal{E}_4\right)+\tilde{\lambda}_{ii1}\right)+\alpha_i-\tilde{\gamma}_{i1}}\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}}D(t_i), \qquad (100)$$

and then the first-order condition (98) becomes

$$\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5+\mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right)-\frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right]=\frac{\left(1-\lambda_{ii1}\right)\left(1-\alpha_i\right)}{\gamma_{i1}}\tilde{M}.$$

Using $\frac{1-t_i}{t_i}\left(1-\lambda_{ii1}\right)=\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}$ from (58), we get

$$\frac{\left(1-\lambda_{ii1}\right)}{\gamma_{i1}}\left[\gamma_{i1}\mathcal{E}_3\mathcal{E}_5+\gamma_{i1}\mathcal{E}_3\theta_1\left(\frac{1-t_i}{t_i}\right)-\frac{1}{\lambda_{ii1}t_i}\right]=\frac{\left(1-\lambda_{ii1}\right)\left(1-\alpha_i\right)}{\gamma_{i1}}\tilde{M}(t_i),$$

$$\mathcal{E}_5+\theta_1\left(\frac{1-t_i}{t_i}\right)-\frac{1}{\lambda_{ii1}t_i\gamma_{i1}\mathcal{E}_3}=\left(1-\alpha_i\right)\frac{\tilde{M}(t_i)}{\gamma_{i1}\mathcal{E}_3}.$$

Using (92), we obtain

$$\begin{aligned}
&\left(\frac{1-t_i}{t_i}\right)\frac{\theta_1}{\rho_1}+\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}-\frac{\rho_1}{\theta_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\frac{\rho_1}{\theta_1}-\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}{\lambda_{ii1}t_i\gamma_{i1}\mathcal{E}_3} \\
= \quad & \left(1-\alpha_i\right)\frac{\tilde{M}(t_i)}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right).
\end{aligned}$$

We multiply both sides by t_i and use (58) to get

$$\begin{aligned}
&(1-t_i)\frac{\theta_1}{\rho_1}+\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}t_i-\frac{\rho_1}{\theta_1}t_i+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})}{\lambda_{ii1}\gamma_{i1}\mathcal{E}_3}\\
= \;& (1-\alpha_i)\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})\right).
\end{aligned}$$

Next we add and subtract $\frac{1}{\lambda_{ii1}}$ and use $\frac{t_i}{\tilde{\lambda}_{ii1}}=t_i-1+\frac{1}{\lambda_{ii1}}$, to obtain

$$\begin{aligned}
&(1-t_i)\left(\frac{\theta_1-\rho_1}{\rho_1}\right)+t_i-\frac{\rho_1}{\theta_1}t_i+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-\left(\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})}{\gamma_{i1}\mathcal{E}_3}-1\right)\frac{1}{\lambda_{ii1}}\\
= \;& (1-\alpha_i)\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})\right).
\end{aligned}$$

Note that

$$\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})}{\gamma_{i1}\mathcal{E}_3}-1=(1-\gamma_{i1})\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})+\frac{\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\gamma_{i1}\mathcal{E}_3}.$$

Then the first-order condition becomes

$$\begin{aligned}
&\left(\frac{\theta_1}{\theta_1-\rho_1}-t_i\right)\frac{(\theta_1-\rho_1)^2}{\theta_1\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}\\
&-(1-\gamma_{i1})\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})+\frac{\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\gamma_{i1}\mathcal{E}_3}\frac{1}{\lambda_{ii1}}\\
= \;& (1-\alpha_i)\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})\right).
\end{aligned}$$

Now we find the common denominator for the second terms on the left hand side of the the first-

order condition and use (89)

$$\begin{aligned}
&\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-(1-\gamma_{i1})\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})+\frac{\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\gamma_{i1}\mathcal{E}_3}\frac{1}{\lambda_{ii1}}\\
= &\frac{(1-\gamma_{i1})}{\gamma_{i1}\mathcal{E}_3}\rho_1\left(\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\mathcal{E}_3}{\theta_1}-\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})+\frac{\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\rho_1}\frac{1}{\lambda_{ii1}}\right)\\
= &\frac{(1-\gamma_{i1})}{\gamma_{i1}\mathcal{E}_3}\frac{1}{\theta_1}\frac{1}{\lambda_{ii1}}\frac{\theta_1-\theta_1\rho_1-(1-\lambda_{ii1})\rho_1+(\theta_1-\rho_1(1-\lambda_{ii1}))(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}.
\end{aligned}$$

Then the first-order condition becomes

$$\left(\frac{\theta_1}{\theta_1-\rho_1}-t_i\right)\frac{(\theta_1-\rho_1)^2}{\theta_1\rho_1}-\frac{(1-\gamma_{i1})}{\gamma_{i1}\mathcal{E}_3}\frac{\tilde{R}(t_i)}{\theta_1}=(1-\alpha_i)\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}(1-\lambda_{ii1})\right),$$

where

$$\tilde{R}(t_i)=\frac{1}{\lambda_{ii1}}\left(\frac{\theta_1-\theta_1\rho_1-\rho_1(1-\lambda_{ii1})+(\theta_1-\rho_1(1-\lambda_{ii1}))(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}{1-(1-\gamma_{i1})\rho_1+(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}\right). \tag{101}$$

The first-order condition can then be written using $=\frac{\theta_1}{\theta_1-\rho_1}$ as

$$t^{opt}[1-(1-\gamma_{i1})R(t_i)]=t_i[1+(1-\alpha_i)M(t_i)], \tag{102}$$

where we define $M(t_i)$ as

$$\begin{aligned}
M(t_i) &\equiv \frac{\theta_1}{(\theta_1-\rho_1)^2}\frac{\tilde{M}(t_i)}{\gamma_{i1}\mathcal{E}_3}\left(1+\frac{(1-\gamma_{i1})}{\gamma_{i1}}\frac{\rho_1}{\theta_1\sigma_1}(1-\lambda_{ii1})\right)\\
&= \frac{\theta_1(1-\tilde{\gamma}_{i1})}{(\theta_1-\rho_1)^2}\frac{\left(\mathcal{E}_5-\frac{(t_i-1)}{t_i}\theta_1\right)}{\rho_1\gamma_{i1}\mathcal{E}_3}\left(1+\frac{\rho_1(1-\gamma_{i1})}{\theta_1\sigma_1\gamma_{i1}}(1-\lambda_{ii1})\right)\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}}\frac{D(t_i)}{A(t_i)}\\
&= \mathcal{M}\times\left(\mathcal{E}_5-\frac{(t_i-1)}{t_i}\theta_1\right)\frac{D(t_i)}{A(t_i)},
\end{aligned} \tag{103}$$

and $A(t_i)$ is given by (17), which is rewritten using $T(t_i)$ from (11) as

$$A(t_i) = \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\left[\left(T(t_i) + \tilde{\gamma}_{i1} - \tilde{\lambda}_{ii1}\right)(1-\mathcal{E}_4) + \tilde{\lambda}_{ii1}\right], \tag{104}$$

while $\mathcal{M}$ is defined by

$$\mathcal{M} \equiv \frac{\theta_1(1-\tilde{\gamma}_{i1})}{(\theta_1-\rho_1)^2\,\rho_1\gamma_{i1}\mathcal{E}_3}\left(1 + \frac{\rho_1(1-\gamma_{i1})}{\theta_1\sigma_1\gamma_{i1}}(1-\lambda_{ii1})\right)\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} > 0, \tag{105}$$

and we define $R(t_i)$ as

$$R(t_i) \equiv \frac{\rho_1}{\theta_1(\theta_1-\rho_1)}\frac{\tilde{R}(t_i)}{\gamma_{i1}\mathcal{E}_3} = \frac{\rho_1}{\theta_1(\theta_1-\rho_1)}\frac{1}{\lambda_{ii1}}\frac{(\theta_1-\rho_1(1-\lambda_{ii1}))(T+\tilde{\gamma}_{i1}) - \theta_1\rho_1}{(1-\tilde{\gamma}_{i1})\frac{T}{\rho_1} + \gamma_{i1}\tilde{\gamma}_{i1}},$$

where the equality follows using $T(t_i)$ from (11) in (101). We rewrite this as in (18)

$$R(t_i) = \mathcal{R} \times \left[(\theta_1 - \rho_1(1-\lambda_{ii1}))(T(t_i)+\tilde{\gamma}_{i1}) - \theta_1\rho_1\right],\ \mathcal{R} \equiv \frac{\frac{\rho_1}{\theta_1(\theta_1-\rho_1)\lambda_{ii1}}}{(1-\tilde{\gamma}_{i1})\frac{T(t_i)}{\rho_1} + \gamma_{i1}\tilde{\gamma}_{i1}} > 0. \tag{106}$$

Finally, we can write the first-order condition (102) as in (15), or alternatively as

$$H(t_i^*) \equiv t^{opt}\left[1 - (1-\gamma_{i1})R(t_i^*)\right] - t_i\left[1 + (1-\alpha_i)M(t_i^*)\right] = 0. \tag{107}$$

Throughout this Appendix we will use (107) because we do not need to be concerned about dividing by $[1+(1-\alpha_i)M(t_i)]$ in (15) when it might be zero. Finding a fixed-point t_i^* of (15) is equivalent to finding a value t_i^* at which (107) holds.

C.2 Entry

Our goal is to show that starting from free trade, an increase in the tariff starting from free trade reduces entry into the traded sector, as asserted in the main text. Combining (48) and (54) we get

$$N_{i1} = \frac{\alpha_i(\sigma_1-1)}{f_{i1}^E\theta_1\,\sigma_1}\frac{\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}}{1-\tilde{\gamma}_{i1}\left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i}\right)}\frac{I_i}{w_i}. \tag{108}$$

We can eliminate the shares in this expression by using (58) and (59) to obtain

$$t_i = \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} \frac{(1-\lambda_{ii1})}{(1-\tilde{\lambda}_{ii1})} = [(t_i-1)(1-\tilde{\lambda}_{ii1})+1] \frac{(1-\lambda_{ii1})}{(1-\tilde{\lambda}_{ii1})}.$$

Using this expression and also (11), with $\lambda_{ji1} = 1 - \lambda_{ii1}$ we can solve for

$$\lambda_{ii1} + \frac{\lambda_{ji1}}{t_i} = 1 - \frac{(t_i-1)[(T(t_i)+\tilde{\gamma}_{i1})-1]}{(t_i-1)(T(t_i)+\tilde{\gamma}_{i1})} = \frac{1}{(T(t_i)+\tilde{\gamma}_{i1})}. \tag{109}$$

Substituting this into (108), we obtain

$$N_{i1} = \frac{\alpha_i(\sigma_1-1)}{T(t_i) f_{i1}^E \theta_1 \, \sigma_1} \left(\frac{I_i}{w_i}\right) = \frac{\alpha_i(\sigma_1-1)}{T(t_i) \, f_{i1}^E \theta_1 \, \sigma_1} \left(L_i + \frac{B_i}{w_i}\right). \tag{110}$$

Notice that entry depends on the share of real spending $\alpha_i I_i / w_i$ in sector 1, adjusted by the term $T(t_i)$ reflecting the tariff $T(t_i) \equiv 1 - \tilde{\gamma}_{i1} + (t_i - 1)\left(1 - \tilde{\lambda}_{ii1}\right)$. In free trade we have $t_i = 1$, and with a prohibitive tariff of infinity then $\tilde{\lambda}_{ii1} = 1$, so in both cases we see that $T = 1 - \tilde{\gamma}_{i1} > 0$. In between these two points, $T > 1 - \tilde{\gamma}$ and so $T(t_i)$ is an inverted-$\cup$-shaped function of the tariff. If tariff revenue B_i in (110) did not change, then entry would become a $\cup$-shaped function of the tariff, i.e. falling from free trade and later rising so that entry in autarky equals that in free-trade.

Of course, tariff revenue does change with the tariff, and it can be expressed as

$$B_i = (t_i - 1)\frac{\lambda_{ji1} Y_{i1}}{t_i} = \left(\frac{T(t_i) + \tilde{\gamma}_{i1} - 1}{T(t_i) + \tilde{\gamma}_{i1}}\right) Y_{i1}, \tag{111}$$

where we make use of (109) to obtain $\frac{(t_i-1)\lambda_{ji1}}{t_i} = \frac{T(t_i)+\tilde{\gamma}_{i1}-1}{T(t_i)+\tilde{\gamma}_{i1}}$. To solve for Y_{i1}, we substitute B_i into market clearing (52) to obtain

$$Y_{i1} \;\; = \;\; \alpha_i \left[w_i L_i + (t_i - 1)\frac{\lambda_{ji1} Y_{i1}}{t_i}\right] + \tilde{\gamma}_{i1}\left(\lambda_{ii1} Y_{i1} + \frac{\lambda_{ji1}}{t_i} Y_{i1}\right). \tag{112}$$

Substituting (109) and $\frac{(t_i-1)\lambda_{ji1}}{t_i} = \frac{T(t_i)+\tilde{\gamma}_{i1}-1}{T(t_i)+\tilde{\gamma}_{i1}}$ into (112), we get

$$Y_{i1} = \frac{\alpha_i w_i L_i}{1 - \alpha_i + \frac{(\alpha_i - \tilde{\gamma}_{i1})}{(T(t_i)+\tilde{\gamma}_{i1})}}. \tag{113}$$

Substituting this and (111) back into (110), we obtain

$$N_{i1} = \frac{\alpha_i L_i(\sigma_1 - 1)}{T(t_i) f_{i1}^E \theta_1 \, \sigma_1} \left(\frac{(1-\alpha_i)(T(t_i) + \tilde{\gamma}_{i1} - 1) + T(t_i)}{(1-\alpha_i)(T(t_i) + \tilde{\gamma}_{i1} - 1) + 1 - \tilde{\gamma}_{i1}} \right). \tag{114}$$

Evaluating entry under autarky and free trade, we obtain (10) in the main text. Taking the difference between (10) and (114) we find

$$\frac{N_{i1}^{FT|AUT} - N_{i1}}{N_{i1}^{FT|AUT}} = \left(\frac{(T(t_i) - (1 - \tilde{\gamma}_{i1}))}{T(t_i)} \right) \frac{(1-\alpha_i)(T(t_i) + \tilde{\gamma}_{i1} - 1)}{(1-\alpha_i)(T(t_i) + \tilde{\gamma}_{i1} - 1) + (1 - \tilde{\gamma}_{i1})}.$$

Using $T(t_i) - (1 - \tilde{\gamma}_{i1}) = (t_i - 1)(1 - \tilde{\lambda}_{ii1})$ we simplify this expression as

$$\frac{N_{i1}^{FT|AUT} - N_{i1}}{N_{i1}^{FT|AUT}} = \frac{(1-\alpha_i)[(t_i - 1)(1 - \tilde{\lambda}_{ii1})]^2}{(1-\alpha_i) T(t_i)^2 + \alpha_i \, (1 - \tilde{\gamma}_{i1}) \, T(t_i)},$$

which is shown as (12) in the main text. We can obtain a final expression for $T(t_i)$ by substituting (113) back into (111) to obtain

$$B_i = \alpha_i w_i L_i \left(\frac{T(t_i) + \tilde{\gamma}_{i1} - 1}{(1-\alpha_i)(T(t_i) + \tilde{\gamma}_{i1} - 1) + 1 - \tilde{\gamma}_{i1}} \right), \tag{115}$$

from which we solve for

$$T(t_i) = \frac{\alpha_i \, (w_i L_i + B_i) \, (1 - \tilde{\gamma}_{i1})}{[\alpha_i w_i L_i - (1-\alpha_i) B_i]}. \tag{116}$$

We show in Appendix D.2 that $T(t_i) > 0$ for all t_i exceeding a minimum value $t_i^{\min}$, and (116) shows that $T(t_i)$ is a monotonically increasing function of B_i in this range.

D Properties of the Optimal Tariff

D.1 Elasticities and Transformation of Terms

We begin by recording here the elasticities that arise from the comparative statics of the model, and which appear in the first-order condition, and we transform these terms using $T(t_i)$ from (11). For the elasticity of wages, $\mathcal{E}_1$ and $\mathcal{E}_2$ shown in (9), the elasticity terms are in (85) and (86). The elasticity terms $\mathcal{E}_3$ and $\mathcal{E}_4$ appear within $M(t_i)$ and were defined in (89) and (91), which are repeated

here as

$$\mathcal{E}_3 = \left(\frac{T(t_i) + \tilde{\gamma}_{i1}}{T(t_i)} + \frac{1}{\gamma_{i1}(\sigma_1 - 1)} \right) > 0.$$

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1} \frac{1+\gamma_{i1}(\sigma_1 - 1)}{\sigma_1 \gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\gamma_{i1}} \frac{1}{\sigma_1} (1 - \lambda_{ii1})} > 0.$$

Another elasticity term $\mathcal{E}_5$ also appear within $M(t_i)$, and was defined in (92) and is repeated here as

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}} \frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} \left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1} (t_i - 1) \lambda_{ii1} \right)}{\frac{1}{\rho_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\gamma_{i1}} \frac{1}{\sigma_1} (1 - \lambda_{ii1})} > 0.$$

We transform the terms appearing above using the term $T(t_i)$ from (11) to obtain

$$\mathcal{E}_3 = \left(\frac{T(t_i) + \tilde{\gamma}_{i1}}{T(t_i)} + \frac{1}{\gamma_{i1}(\sigma_1 - 1)} \right) > 0, \tag{117}$$

and

$$\mathcal{E}_3 T(t_i) = \left(1 + \frac{1}{\gamma_{i1} (\sigma_1 - 1)} \right) T + \tilde{\gamma}_{i1} = (1 - \tilde{\gamma}_{i1}) \frac{T(t_i)}{\gamma_{i1} \rho_1} + \tilde{\gamma}_{i1}. \tag{118}$$

We also use (11) and (13) to obtain the alternative expression

$$D(t_i) T(t_i) = \left[T(t_i) - \frac{1 + \gamma_{i2} (\sigma_2 - 1)}{\gamma_{i2} (\sigma_2 - 1)} \gamma_{i1} \rho_1 - \left(T(t_i) + \gamma_{i1} \rho_1 \frac{\tilde{\gamma}_{i1}}{1 - \tilde{\gamma}_{i1}} \right) (1 - \lambda_{ii1}) \mathcal{E}_4 \right]. \tag{119}$$

D.2 Proof of Theorem 1

In order to prove Theorem 1, we need to consider values of t_i below unity, meaning an import subsidy, so the revenue cost of the subsidy needs to be deducted from labor income $w_i L_i$ to obtain net income I_i. With enough roundabout production, it seems possible that at a very low tariff – meaning a very high import subsidy – the revenue-cost of the subsidy could exhaust the labor income of the economy, so that net income $I_i = w_i L_i - B_i$ is zero. In that case, the there is no consumption by consumers in country i (though the labor endowment L_i is still provided), which is run like an overseas factory solely for the benefit of foreign consumers. We need to check whether this extreme case can occur, and if it does, we denote that minimal tariff (maximum import subsidy) by $t_i^{\min} \geq 0$. We give a more formal definition with:

Definition 2. $t_i^{\min} \equiv \arg\max_{t_i \geq 0} \{T(t_i) | T(t_i) = 0\}$, *with* $\lambda_{ii1}^{\min}$ *denoting the value of* λ_{ii1} *at* $t_i^{\min}$.

To explain this definition, notice that using B_i from the main text in (111) and Y_{i1} from (113), we can solve for income I_i as

$$I_i = w_i L_i + \frac{T(t_i) + \tilde{\gamma}_{i1} - 1}{T(t_i) + \tilde{\gamma}_{i1}} Y_{i1} = w_i L_i \left[\frac{T(t_i)}{T(t_i) + \alpha_i (T(t_i) + \tilde{\gamma}_{i1} - 1)} \right]. \tag{120}$$

We see that $I_i = 0 \iff T(t_i) = 0$. We do not rule out in Definition 2 the possibility that there might be multiple tariffs at which $T(t_i) = 0$, in which case $t_i^{\min}$ is chosen as the maximum of these.[14] To solve for $t_i^{\min}$, we use the market clearing condition (52) together with trade balance (38) to get

$$Y_{i1} = \alpha_i I_i + \tilde{\gamma}_{i1} \left(\lambda_{ii1} Y_{i1} + \frac{\lambda_{ji1}}{t_i} Y_{i1} \right).$$

If we set $I_i = 0$ and use $\lambda_{ii1} + \lambda_{ij1} = 1$, then we solve for

$$1 = \tilde{\gamma}_{i1} \lambda_{ii1}^{\min} + \tilde{\gamma}_{i1} \frac{1 - \lambda_{ii1}^{\min}}{t_i^{\min}} \implies t_i^{\min} = \frac{\tilde{\gamma}_{i1}(1 - \lambda_{ii1}^{\min})}{(1 - \tilde{\gamma}_{i1} \lambda_{ii1}^{\min})}. \tag{121}$$

We see that $t_i^{\min} = 0$ for $\gamma_{i1} = 1$ because then $\tilde{\gamma}_{i1} = (1 - \gamma_{i1})\rho_1 = 0$. More generally for $0 \leq \gamma_{i1} \leq 1$ we have $0 \leq \tilde{\gamma}_{i1} \leq \rho_1$, and it follows from (121) that $0 \leq t_i^{\min} \leq \tilde{\gamma}_{i1}$. Because we solved for $t_i^{\min}$ from the market clearing condition when $I_i = 0$, it follows from (120) that $T(t_i^{min}) = 0$. Negative income is not possible, so at higher tariffs we have $I_i > 0$ and then $T(t_i) > 0$ from (120).

Remark 2. *We henceforth restrict our attention to tariffs in the range* $t_i \geq t_i^{\min}$, *where* $T^{\min} \equiv T(t_i^{\min}) = 0$ *and* $T(t_i) > 0 \iff t_i > t_i^{\min}$.

From (18) and (106) we have $R(t_i) = \mathcal{R} \times [(\theta_1 - \rho_1 (1 - \lambda_{ii1})) (T(t_i) + \tilde{\gamma}_{i1}) - \theta_1 \rho_1]$, where $\mathcal{R} > 0$. It appears that the term multiplying $\mathcal{R}$ can be zero, particularly as $T(t_i)$ is low, so that $R(t_i) = 0$. For the proof of parts (a) and (c) in Theorem 1, we will make extensive use of this point, which is defined more formally as follows:

Definition 3. *Define* $t_i^{R0} \equiv \arg\max_{t_i \geq t_i^{min}} \{R(t_i) | R(t_i) = 0\}$ *and denote* $T^{R0} \equiv T(t_i^{R0})$, *where it is understood that* T^{R0} *uses the shares* $\tilde{\lambda}_{ii1}^{R0}$ *and* λ_{ii1}^{R0} *which are evaluated at* t_i^{R0}.

[14] If $T(t_i)$ is increasing in t_i then $t_i^{\min}$ will be unique, and conditions to ensure that are provided in Lemma 8.

This definition allows for the possibility that there could be multiple tariffs at which $R(t_i) = 0$, in which case t_i^{R0} is chosen as the maximum of these points.[15]

Lemma 3. *The tariff t_i^{R0} is given by*

$$t_i^{R0} = 1 + \frac{\rho_1}{\left(1 - \tilde{\lambda}_{ii1}^{R0}\right)} \frac{\rho_1 \left(1 - \lambda_{ii1}^{R0}\right)}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} - \frac{(1 - \rho_1)}{\left(1 - \tilde{\lambda}_{ii1}^{R0}\right)}, \tag{122}$$

with $t_i^{\min} < t_i^{R0} < 1$ and $R(t_i) > 0$ for $t_i > t_i^{R0}$.

Proof: Because $\mathcal{R} > 0$ in (106), then $R(t_i) = 0$ implies $[(\theta_1 - \rho_1 (1 - \lambda_{ii1}))(T(t_i) + \tilde{\gamma}_{i1}) - \theta_1 \rho_1] = 0$. This condition is rewritten as

$$T^{R0} = \frac{\theta_1 \rho_1}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} - \tilde{\gamma}_{i1} = \left(\frac{\theta_1}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} - (1 - \gamma_{i1}) \right) \rho_1 > 0, \tag{123}$$

because the first ratio on the right is greater than 1 and so it exceeds $(1 - \gamma_{i1})$. It follows from Remark 2 that $t_i^{R0} > t_i^{\min}$, and from Definition 3 that $R(t_i) > 0$ for $t_i > t_i^{R0}$.

Using (11) we can solve for t_i^{R0} to obtain obtain (122), which can also be written as

$$t_i^{R0} = 1 + \frac{1}{\left(1 - \tilde{\lambda}_{ii1}^{R0}\right)} \left(\frac{-(1 - \rho_1)\theta_1 + \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} \right) < 1$$

where the final inequality follows from $\theta_1 > (\sigma_1 - 1) \Rightarrow \theta_1 (1 - \rho_1) > \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)$. QED

We now prove Theorem 1 by a series of Lemmas and Remarks.

PROOF OF PART (a)

We assume that $\alpha = 1$, and then $H(t_i)$ from (19) becomes $H(t_i) = t^{opt} - t_i - t^{opt}(1 - \gamma_{i1})R(t_i)$. With $R(t_i^{R0}) = 0$ for $t_i^{R0} < 1$, we obtain $H(t_i^{R0}) = t^{opt} - t_i^{R0} > 0$. Checking the sign of $R(t^{opt})$, because $T(t^{opt}) > 1 - \tilde{\gamma}_{i1}$ it readily follows from (106) that $R(t^{opt}) > 0$. In that case we obtain $H(t_i^{opt}) = -t^{opt}(1 - \gamma_{i1})R(t^{opt}) < 0$ for $\gamma_{i1} < 1$. Using the continuity of $R(t_i)$ and therefore of $H(t_i)$, it follows from the intermediate value theorem that there exists a tariff t_i^* with $t_i^{R0} < t_i^* < t^{opt}$ at which $H(t_i^*) = 0$. By construction, this tariff is a fixed point of (15). QED

[15] We will show that there is a unique tariff at which $R(t_i) = 0$ under the conditions of Lemma 12.

PROOF OF PARTS (b) AND (c)

From (16) and (103), we have $M(t_i) = \mathcal{M} \times \left(\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1\right) \frac{D(t_i)}{A(t_i)}$, where $\mathcal{M} > 0$ from (105). It appears to be possible that $M(t_i) = 0$ for two reasons: either $D(t_i) = 0$ at some tariff; or $\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1$ at some tariff. For the proof of parts (b) and (c) in Theorem 1, we will make extensive use of the first of these points, where $D(t_i) = 0$, which is defined more formally as follows:

Definition 4. *Define*

$$t_i^{D0} \equiv \begin{cases} \arg\min_{t_i \geq t_i^{\min}} \{D(t_i) = 0\} & \textit{if this value exists,} \\ +\infty & \textit{otherwise,} \end{cases}$$

and denote $T^{D0} \equiv T(t_i^{D0})$ *and likewise for the shares* $\tilde{\lambda}_{ii1}^{D0}$ *and* λ_{ii1}^{D0} *evaluated at* t_i^{D0}.

Once again, we allow for multiple solutions for the tariff where $D(t_i) = 0$, and in this case we choose t_i^{D0} as the minimum of them. Next, we establish a result for the term $\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1$ that also appears within $M(t_i) = \mathcal{M} \times \left(\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1\right) \frac{D(t_i)}{A(t_i)}$, and could possibly make this expression equal to zero.

Lemma 4. $\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1 > 0$ *for all* $t_i \in [t_i^{\min}, t_i^{opt}]$. *In addition, if* $\gamma_{i1} = 1$ *then* $\mathcal{E}_5 - \frac{(t_i''-1)}{t_i''}\theta_1 = 0$ *at a tariff* $t_i'' > t_i^{opt}$.

Proof: From (92) we see that

$$\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_i-1)\lambda_{ii1}\right)}{\frac{1}{\rho_1} + \frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}} - \frac{(t_i-1)}{t_i}\theta_1. \tag{124}$$

Notice that the final term on the right is increasing in t_i, so it takes its highest value over $t_i \in$

$[t_i^{\min}, t_i^{opt}]$ at $t_i = t^{opt}$, in which that term equals ρ_1. It follows that

$$\begin{aligned}
\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1 &\geq \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_i-1)\lambda_{ii1}\right)}{\frac{1}{\rho_1} + \frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}} - \rho_1 \\
&= \frac{\rho_1}{\tilde{\lambda}_{ii1}}\left(\frac{1 - \frac{\rho_1}{\theta_1}\tilde{\lambda}_{ii1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}\sigma_1}\left(\left(1-\tilde{\lambda}_{ii1}\right)(t_i-1)\lambda_{ii1} - \frac{\rho_1}{\theta_1}(1-\lambda_{ii1})\tilde{\lambda}_{ii1}\right)}{1+\rho_1\frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}}\right) \\
&= \frac{\rho_1}{\tilde{\lambda}_{ii1}}\left(\frac{1 - \frac{\rho_1}{\theta_1}\tilde{\lambda}_{ii1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}\sigma_1}\left(1-\tilde{\lambda}_{ii1}\right)\lambda_{ii1}\left(\frac{t_i}{t^{opt}}-1\right)}{1+\rho_1\frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}}\right)
\end{aligned}$$

where the final line follows using (58). The second two terms in the numerator are

$$\frac{1-\gamma_{i1}}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\left(\frac{\rho_1}{\theta_1} - \frac{\lambda_{ii1}}{\sigma_1}\left(1-\frac{t_i}{t^{opt}}\right)\right) \geq \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1\sigma_1}\left(1-\tilde{\lambda}_{ii1}\right)(t_i(\theta_1-\rho_1) - (\theta_1-(\sigma_1-1)))$$

which is positive for $t_i \geq 1$ and proves that $\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1 > 0$ for all $t_i \in [t_i^{\min}, t_i^{opt}]$.

To evaluate (124) at higher levels of the tariff, note that with $\gamma_{i1} = 1$ we have that

$$\lim_{t_i\to\infty}\left(\mathcal{E}_5|_{\gamma_{i1}=1} - \frac{(t_i-1)}{t_i}\theta_1\right) = \rho_1\left(2-\frac{\rho_1}{\theta_1}\right) - \theta_1 < 0$$

because $\left(2-\frac{\rho_1}{\theta_1}-\frac{\theta_1}{\rho_1}\right) < 0$ for $\frac{\theta_1}{\rho_1} > 1$. It follows that for $\gamma_{i1} = 1$ then there exists a tariff $t_i'' > t_i^{opt}$ at which $\mathcal{E}_5 - \frac{(t_i''-1)}{t_i''}\theta_1 = 0$. QED

PROOF OF PART (b)(i)

If $\gamma_{is} = 1$ for $s = 1, 2$ then from (91) and (92) we have we have $\mathcal{E}_3 = \frac{\sigma_1}{(\sigma_1-1)}$ and $\mathcal{E}_4 = \frac{\rho_1}{\theta_1\sigma_1}$. Substituting these into (13) we obtain

$$\begin{aligned}
D(t_i) &= \frac{(\sigma_1-1)}{\sigma_1}\left[\frac{\sigma_1}{(\sigma_1-1)} - \left(\frac{\sigma_2}{(\sigma_2-1)}\right)\frac{1}{T(t_i)} - \frac{1}{\sigma_1\theta_1}(1-\lambda_{ii1})\right] \\
&> \frac{(\sigma_1-1)}{\sigma_1}\left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)} - \frac{1}{\sigma_1\theta_1}\right],
\end{aligned}$$

$\cup$where the inequality follows from $T(t_i) \geq 1$ for $t_i \in [1, t^{opt}]$ and $\lambda_{ii1} < 1$. It follows that $D(t_i) > 0$ when condition (14) holds.

Notice that when $\gamma_{i1}=1$ and $\mathcal{E}_4=\frac{\rho_1}{\theta_1\sigma_1}$ then $A(t_i)$ in (17) becomes

$$A(t_i)\equiv\alpha_i+(1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(1-\frac{\rho_1}{\theta_1\sigma_1}\right)+\tilde{\lambda}_{ii1}\right]>\alpha_i>0, \tag{125}$$

which is bounded away from zero so that $M(t_i)$ is continuous. Then because $\mathcal{E}_5-\frac{(t_i-1)}{t_i}\theta_1>0$ for $t_i\in[1,t^{opt}]$ from Lemma 4, it follows that $M(t_i)>0$ in that interval, and in particular $M(t_i^{opt})>0$. From (19) with $\gamma_{i1}=1$ it follows that $H(t^{opt})=-t^{opt}\left(1-\alpha_i\right)M(t^{opt})<0$.

Now we make use of t_i^{D0} which in this simplified setting where $\gamma_{i1}=1$ is solved from

$$\frac{\sigma_1}{(\sigma_1-1)}-\left(\frac{\sigma_2}{(\sigma_2-1)}\right)\frac{1}{T(t_i^{D0})}-\frac{1}{\sigma_1\theta_1}(1-\lambda_{ii1})=0. \tag{126}$$

It follows that

$$T(t_i^{D0})=\frac{\frac{\sigma_2}{\sigma_2-1}}{\frac{\sigma_1}{(\sigma_1-1)}-\frac{(1-\lambda_{ii1})}{\sigma_1\theta_1}}<\frac{\frac{\sigma_2}{\sigma_2-1}}{\frac{\sigma_1}{(\sigma_1-1)}-\frac{1}{\sigma_1\theta_1}}<1$$

since condition (14) implies $\frac{\sigma_1}{\sigma_1-1}-\frac{1}{\sigma_1\theta_1}>\frac{\sigma_2}{(\sigma_2-1)}$. Because $T(t_i^{D0})=1+\left(t_i^{D0}-1\right)\left(1-\tilde{\lambda}_{ii1}\right)$ when $\gamma_{i1}=1$, it follows immediately that $t_i^{D0}<1$.

The proof of part (b)(i) is now completed much like the proof of part (a). We have already found $H(t^{opt})=-t^{opt}\left(1-\alpha_i\right)M(t^{opt})<0$. Then at $t_i^{D0}<1$ we have $M(t_i^{D0})=0$ and so $H(t_i^{D0})=t^{opt}-t_i^{D0}\left[1+\left(1-\alpha_i\right)M(t_i^{D0})\right]=t^{opt}-t_i^{D0}>0$. Using the continuity of $M(t_i)$ and therefore of $H(t_i)$, it follows from the intermediate value theorem that there exists a tariff t_i^* with $t_i^{D0}<t_i^*<t^{opt}$ at which $H(t_i^*)=0$. By construction, this tariff is a fixed point of (15). QED

PROOF OF PART (b)(ii)

Under $\sigma_1\geq\sigma_2$, we have $D(t_i^{opt})<0$. Using Lemma 4 it follows that $M(t_i^{opt})<0$, and therefore from $H(t_i)$ in (19) with $\gamma_{i1}=1$ we have $H(t^{opt})=-t^{opt}\left(1-\alpha_i\right)M(t^{opt})>0$.

Now we check a higher tariff $t_i''>t^{opt}$ from Lemma 4 at which $\mathcal{E}_5-\frac{(t_i-1)}{t_i}\theta_1=0$ and therefore $M(t_i'')=0$. From $H(t_i)$ in (19) with $\gamma_{i1}=1$ we have $H(t_i'')=t^{opt}-t_i''[1+\left(1-\alpha_i\right)M(t_i'')]=t^{opt}-t_i''<0$. Using from the continuity of $M(t_i)$ and therefore of $H(t_i)$, it follows from the intermediate value theorem that there exists a tariff exists a tariff t_i^* with $t_i^{opt}<t_i^*<t_i^{opt}<t_i''$ at which $H(t_i^*)=0$. By construction, this tariff is a fixed point of (15). QED

PROOF OF PART (c)

We first establish conditions to ensure that $A(t_i) > 0$, starting with the region $t_i \geq t^{opt}$.

Lemma 5. *$(1-\mathcal{E}_4)t^{opt} \geq \rho_1$ when condition (20) holds, where $\mathcal{E}_4$ can be evaluated at any tariff. It follows that $A(t_i) > \alpha_i(1-\rho_1) + \gamma_{i1}\rho_1 > 0$ for all $t_i \geq t^{opt}$.*

Proof: We want to ensure that $(1-\mathcal{E}_4) \geq \frac{\rho_1}{t^{opt}} = \frac{\rho_1(\theta_1-\rho_1)}{\theta_1}$. Use (91) to obtain

$$1 - \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})} \geq \frac{\rho_1(\theta_1-\rho_1)}{\theta_1},$$

Then we take $\lambda_{ii1} = 1$ to get a sufficient condition

$$1 - \frac{\rho_1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}} \geq \frac{\rho_1(\theta_1-\rho_1)}{\theta_1} \iff \frac{\sigma_1}{\rho_1}(\theta_1-\rho_1)(1-\rho_1) \geq \frac{1-\gamma_{i1}}{\gamma_{i1}}. \tag{127}$$

which is equivalent to 20 in the main text. Now the magnitude of $A(t_i)$ is established from

$$\begin{aligned}
A(t_i) &= \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right)t_i(1-\mathcal{E}_4) + \tilde{\lambda}_{ii1}\right] \\
&\geq \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right)t^{opt}(1-\mathcal{E}_4) + \tilde{\lambda}_{ii1}\right] \\
&\geq \alpha_i - (1-\gamma_{i1})\rho_1 + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right)\rho_1 + \tilde{\lambda}_{ii1}\right] \\
&> \alpha_i(1-\rho_1) + \gamma_{i1}\rho_1,
\end{aligned}$$

where the first inequality follows from second inequality follows from $t_i \geq t^{opt}$, the second from $(1-\mathcal{E}_4)t^{opt} \geq \rho_1$, and the final inequality from $[(1-\tilde{\lambda}_{ii1})\rho_1 + \tilde{\lambda}_{ii1}] > \rho_1$. QED

Next, we define the tariff t_i^{A0} at which $A(t_i)$ becomes zero, if it exists:

Definition 5. *a) Define*

$$t_i^{A0} \equiv \begin{cases} \arg\max_{t_i \geq t_i^{\min}} \{A(t_i) = 0\} & \textit{if this value exists,} \\ t_i^{\min} & \textit{otherwise,} \end{cases}$$

and denote $T^{A0} \equiv T(t_i^{A0})$ and likewise for the shares $\tilde{\lambda}_{ii1}^{A0}$ and λ_{ii1}^{A0}, which are evaluated at t_i^{A0}.

In this definition we are looking for tariffs at which $A(t_i) = 0$, but there will be no such tariffs if $A(t_i) > 0$ for all $t_i \geq t_i^{\min}$. In that case, $t_i^{A0} = t_i^{\min} < 1$. On the other hand, if there are multiple tariffs at which $A(t_i) = 0$, then t_i^{A0} is the maximum of these. From Lemma 5 which relies on condition (20) we know that $t_i^{A0} < t^{opt}$. In Lemma 12 below, we will further show that condition (21) ensures that $t_i^{A0} < 1$.

Remark 6. *The tariff t_i^{A0} is the import subsidy referred to as t_i' in the statement of Theorem 1(c).*

In Definition 4 we defined the tariff t_i^{D0}, which we solved for in (126) under the simplifying assumption that $\gamma_{is} = 1$ for $s = 1, 2$. Now we drop that assumption, so we need to solve for t_i^{D0} once again. We make use of the alternative expression in (119):

$$D(t_i)T(t_i) = \left[T(t_i) - \frac{1 + \gamma_{i2}(\sigma_2 - 1)}{\gamma_{i2}(\sigma_2 - 1)}\gamma_{i1}\rho_1 - \left(T(t_i) + \gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1 - \tilde{\gamma}_{i1}}\right)(1 - \lambda_{ii1})\,\mathcal{E}_4\right].$$

It follows that

$$T^{D0} - \left(T^{D0} + \gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1 - \tilde{\gamma}_{i1}}\right)\left(1 - \lambda_{ii1}^{D0}\right)\mathcal{E}_4^{D0} - \frac{1 + \gamma_{i2}(\sigma_2 - 1)}{\gamma_{i2}(\sigma_2 - 1)}\gamma_{i1}\rho_1 = 0. \tag{128}$$

Using this equation and $T(t_i)$ in (11), we can solve for t_i^{D0} as follows:

Lemma 7. *The solution for t_i^{D0} is given by*

$$t_i^{D0} = 1 + \frac{\rho_1}{\left(1 - \tilde{\lambda}_{ii1}^{D0}\right)}\frac{\rho_1\left(1 - \lambda_{ii1}^{D0}\right)}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{D0}\right)} - \frac{(1 - \rho_1)}{\left(1 - \tilde{\lambda}_{ii1}^{D0}\right)} + \frac{\tilde{\gamma}_{i1}(1 - \Gamma)}{\left(1 - \tilde{\lambda}_{ii1}^{D0}\right)}, \tag{129}$$

where $\Gamma < 1$.

Proof: We can rewrite (128) as

$$T^{D0} = \frac{\theta_1\rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{D0}\right)} - \tilde{\gamma}_{i1}\Gamma, \tag{130}$$

where

$$\Gamma \equiv \left(\frac{1}{\tilde{\gamma}_{i1}} - \left(\begin{array}{c}\frac{\tilde{\gamma}_{i1}}{1 - \tilde{\gamma}_{i1}}\left(1 - \lambda_{ii1}^{D0}\right)\mathcal{E}_4^{D0} \\ + \frac{1 + \gamma_{i2}(\sigma_2 - 1)}{\gamma_{i2}(\sigma_2 - 1)}\end{array}\right)\left(\frac{\gamma_{i1}}{\tilde{\gamma}_{i1}} + \frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1 - \lambda_{ii1}^{D0}\right)\right)\right)\frac{\theta_1\rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{D0}\right)}. \tag{131}$$

Using (11), we therefore obtain (129). To establish that $\Gamma < 1$, we evaluate $\mathcal{E}_4$ at t_i^{D0}, to obtain

$$\mathcal{E}_4^{D0} = \frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}} \frac{\frac{1}{\theta_1}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}^{D0}\right)},$$

and after substituting into (131) we obtain

$$\Gamma = \left(\frac{1}{\tilde{\gamma}_{i1}} - \left(\left(1-\lambda_{ii1}^{D0}\right)\frac{\tilde{\gamma}_{i1}}{1-\gamma_{i1}}\frac{1}{\theta_1} + \frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\left(\frac{\gamma_{i1}}{\tilde{\gamma}_{i1}} + \frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}^{D0}\right)\right)\right)\right)\frac{\theta_1\rho_1}{\theta_1 - \rho_1\left(1-\lambda_{ii1}^{D0}\right)}.$$

It follows that $\Gamma < 1$ if

$$\frac{1}{\tilde{\gamma}_{i1}} - \frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\left(\frac{1}{\tilde{\gamma}_{i1}}\gamma_{i1} + \frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}^{D0}\right)\right) < \frac{1}{\rho_1} + \left(1-\lambda_{ii1}^{D0}\right)\frac{\left(\rho_1-1\right)}{\theta_1}.$$

We can simplify this equation as

$$1 - \frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\gamma_{i1} < \tilde{\gamma}_{i1}\left(\frac{1}{\rho_1} + \left(1-\lambda_{ii1}^{D0}\right)\frac{\left(\rho_1-1\right)}{\theta_1} + \frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}^{D0}\right)\right).$$

or,

$$-\frac{1}{\gamma_{i2}(\sigma_2-1)} < \frac{\tilde{\gamma}_{i1}}{\gamma_{i1}}\left(1-\lambda_{ii1}^{D0}\right)\left(\rho_1 - 1 + \frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\frac{1}{\sigma_1}\right)\frac{1}{\theta_1}.$$

Note that the left-hand side of this equation is negative, and the right-hand is positive because

$$\rho_1 - 1 + \frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\frac{1}{\sigma_1} = \frac{1}{\gamma_{i2}\left(\sigma_2-1\right)}\frac{1}{\sigma_1} > 0,$$

where the equality is obtained because $\rho_1 - 1 = -\frac{1}{\sigma_1}$. Therefore the inequality above holds and it follows that $\Gamma < 1$. QED

Lemma 8. *For $t_i \in (t_i^{\min}, 1)$, $T(t_i)$ is monotonically increasing in t_i provided that $A(t_i) > 0$.*

Proof: From (59) combining with (11), T is given by

$$T = \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} - \tilde{\gamma}_{i1}, \tag{132}$$

which we differentiate to obtain,

$$dT = \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} \left(\hat{\tilde{\lambda}}_{ii1} - \hat{\lambda}_{ii1} \right).$$

Totally differentiate (58), we can show that

$$\hat{\lambda}_{ii1} = \frac{(1-\lambda_{ii1})}{\left(1-\tilde{\lambda}_{ii1}\right)} \left(\hat{\tilde{\lambda}}_{ii1} - \left(1-\tilde{\lambda}_{ii1}\right) \hat{t}_i \right) \tag{133}$$

Then combining with (95), we obtain

$$\hat{\lambda}_{ii1} - \hat{\tilde{\lambda}}_{ii1} = \frac{\theta_1}{\tilde{\lambda}_{ii1}} \left(\tilde{\lambda}_{ii1} - \lambda_{ii1} \right) \hat{\varphi}^*_{ij1} - (1-\lambda_{ii1}) \hat{t}_i$$

It follows that,

$$dT = -\frac{\theta_1}{\lambda_{ii1}} \left(\tilde{\lambda}_{ii1} - \lambda_{ii1} \right) \hat{\varphi}^*_{ij1} + \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} (1-\lambda_{ii1}) \hat{t}_i \tag{134}$$

Notice that the coefficient of $\hat{t}_i$ in the final term is positive. We now show that $\hat{\varphi}_{ij1}/\hat{t}_i$ is positive, so then because $\tilde{\lambda}_{ii1} < \lambda_{ii1}$ for $t_i < 1$, we have established monotonicity of T.

Using (90) and (96), we have

$$\hat{t}_i = \left(\mathcal{E}_5 - t_i \mathcal{E}_4 \frac{\left(1-\tilde{\lambda}_{ii1}\right) \left(\mathcal{E}_5 - \frac{(t_i-1)}{t_i} \theta_1 \right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(\mathcal{E}_4 - 1 \right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i} \left(\alpha_i - \tilde{\gamma}_{i1} \right) \right)} \right) \hat{\varphi}^*_{ij1},$$

and so

$$\hat{\varphi}^*_{ij1} = \frac{\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(\mathcal{E}_4 - 1 \right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i} \left(\alpha_i - \tilde{\gamma}_{i1} \right) \right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(\mathcal{E}_4 - 1 \right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i} \left(\alpha_i - \tilde{\gamma}_{i1} \right) \right) \mathcal{E}_5 - \mathcal{E}_4 \left(1-\tilde{\lambda}_{ii1}\right) \left(t_i \mathcal{E}_5 - (t_i - 1) \theta_1 \right)} \hat{t}_i.$$

Multiply the numerator and denominator by $-(1-\alpha_i)$ and use (17) to obtain

$$\hat{\varphi}^*_{ij1} = \frac{A}{A\mathcal{E}_5 + (1-\alpha_i) \mathcal{E}_4 \left(1-\tilde{\lambda}_{ii1}\right) \left(t_i \mathcal{E}_5 - (t_i - 1) \theta_1 \right)} \hat{t}_i.$$

Because $\mathcal{E}_4 > 0$ and $\mathcal{E}_5 > 0$, then for $t_i < 1$ we have $\hat{\varphi}_{ij1}/\hat{t}_i > 0$. QED

Lemma 9. *$A(t_i) > 0$ for $t_i \in [t_i^{R0}, t^{opt}]$ provided that (20) and (21) hold.*

Proof:

There are two cases to consider. The first case is where $t_i^{A0} = t_i^{min}$ so that $A(t_i) > 0$ for all $t_i > t_i^{min}$. In that case, the lemma holds trivially.

The second case is where $t_i^{A0} > t_i^{min}$. Then according to (17), $A(t_i^{A0}) = 0$ at the tariff

$$(1-\alpha_i)\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i^{A0}(1-\mathcal{E}_4)+\tilde{\lambda}_{ii1}\right)+\alpha_i-\tilde{\gamma}_{i1}=0,$$

so that,

$$\left(1-\tilde{\lambda}_{ii1}\right)t_i^{A0}=-\frac{\alpha_i\left(1-\tilde{\lambda}_{ii1}\right)+\tilde{\lambda}_{ii1}-\tilde{\gamma}_{i1}}{(1-\mathcal{E}_4)(1-\alpha_i)}. \tag{135}$$

Using the definition of $T(t_i)$ in (11), we can rewrite (17) as

$$(1-\alpha_i)\left(T^{A0}-\left(1-\tilde{\lambda}_{ii1}\right)t_i^{A0}\mathcal{E}_4\right)+\alpha_i(1-\tilde{\gamma}_{i1})=0, \quad \text{so that,}$$

$$T^{A0}=\left(1-\tilde{\lambda}_{ii1}\right)t_i^{A0}\mathcal{E}_4-\frac{\alpha_i}{1-\alpha_i}(1-\tilde{\gamma}_{i1}).$$

Combining with (135), T^{A0} can be written as

$$\begin{aligned} T^{A0} &= -\frac{1}{1-\alpha_i}\left(\frac{\mathcal{E}_4(1-\alpha_i)\left(\tilde{\lambda}_{ii1}-\tilde{\gamma}_{i1}\right)+\alpha_i(1-\tilde{\gamma}_{i1})}{(1-\mathcal{E}_4)}\right) \\ &= -\left(\tilde{\lambda}_{ii1}-\tilde{\gamma}_{i1}\right)\frac{\mathcal{E}_4}{(1-\mathcal{E}_4)}-\frac{\alpha_i(1-\tilde{\gamma}_{i1})}{(1-\alpha_i)(1-\mathcal{E}_4)}. \end{aligned} \tag{136}$$

We know that the tariff t_i^{R0} at which $R(t_i^{R0}) = 0$ occurs at:

$$T^{R0}=\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}-\tilde{\gamma}_{i1}.$$

Our goal is to show that $T^{A0} \leq T^{R0}$, which will ensure T is invertible in the range $[t_i^{A0}, 1]$ using Lemma 8 with $A(t_i) > 0$ in that range. The condition $T^{A0} \leq T^{R0}$ holds if,

$$-\left(\tilde{\lambda}_{ii1}-\tilde{\gamma}_{i1}\right)\frac{\mathcal{E}_4}{(1-\mathcal{E}_4)}-\frac{\alpha_i(1-\tilde{\gamma}_{i1})}{(1-\alpha_i)(1-\mathcal{E}_4)}\leq\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}-\tilde{\gamma}_{i1}, \quad \text{or,}$$

$$-\tilde{\lambda}_{ii1}\frac{\mathcal{E}_4}{(1-\mathcal{E}_4)}-\frac{\alpha_i\left(1-\tilde{\gamma}_{i1}\right)}{(1-\alpha_i)\left(1-\mathcal{E}_4\right)}\leq\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}-\tilde{\gamma}_{i1}\frac{1}{1-\mathcal{E}_4}.$$

Drop the share on the left and we will get the sufficient condition:

$$\frac{1}{(1-\mathcal{E}_4)}\frac{(\tilde{\gamma}_{i1}-\alpha_i)}{(1-\alpha_i)}\leq\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}. \tag{137}$$

If $\tilde{\gamma}_{i1}<\alpha_i$, this restriction is satisfied. However, for $\tilde{\gamma}_{i1}>\alpha_i$, then we need $\mathcal{E}_4$ sufficient small so that the above sufficient condition holds.

From the Lemma 5 we know that $(1-\mathcal{E}_4)t^{opt}<1$ and it follows that $\mathcal{E}_4<1$. Then the sufficient condition for (137) is

$$(1-\mathcal{E}_4)\geq\frac{(\tilde{\gamma}_{i1}-\alpha_i)}{(1-\alpha_i)}\left[\frac{1}{\rho_1}-\frac{\left(1-\lambda_{ii1}^{R0}\right)}{\theta_1}\right].$$

If $\alpha_i\geq\tilde{\gamma}_{i1}$ then this condition is automatically satisfied, since then the right-hand side is less than or equal to zero, while the left-hand side is positive. For $\alpha_i<\tilde{\gamma}_{i1}$, we can take $\lambda_{ii1}^{R0}=1$ to get the sufficient condition:

$$1-\mathcal{E}_4\geq\frac{(\tilde{\gamma}_{i1}-\alpha_i)}{(1-\alpha_i)}\left[\frac{1}{\rho_1}-\frac{1}{\theta_1}+\frac{1}{\theta_1}\right]\geq\frac{(\tilde{\gamma}_{i1}-\alpha_i)}{(1-\alpha_i)}\left[\frac{1}{\rho_1}-\frac{1}{\theta_1}+\frac{\lambda_{ii1}^{R0}}{\theta_1}\right].$$

We can substitute for $\mathcal{E}_4$ and the sufficient condition becomes

$$1-\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}\geq\frac{(\tilde{\gamma}_{i1}-\alpha_i)}{(1-\alpha_i)\rho_1}.$$

A sufficient condition for this inequality is obtained by taking $\lambda_{ii1}=1$ on the left,

$$1-\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}}\geq\frac{(\tilde{\gamma}_{i1}-\alpha_i)}{(1-\alpha_i)\rho_1}\implies\alpha_i\geq\frac{-\gamma_{i1}+\rho_1\left(\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}\right)}{\frac{1}{\rho_1}-1+\rho_1\left(\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}\right)}.$$

We therefore obtain (21) as the sufficient condition for $t_i^{A0}<t_i^{R0}$, which ensures the $A(t_i)>0$ for $t_i\in[t_i^{R0},t^{opt}]$. QED

From Lemma 7, the function T is invertible so that if $T_i^{D0}>T_i^{R0}$ then $t_i^{D0}>t_i^{R0}$. From (130) and (123), it would appear that $T_i^{D0}>T_i^{R0}$ because $\Gamma<1$. The difficulty with this argument, however, is that T_i^{D0} and T_i^{R0} also depend on their respective shares, $\tilde{\lambda}_{ii}^{D0}$ and $\tilde{\lambda}_{ii}^{R0}$, which vary

with the tariffs and with the general equilibrium change in wages and all other variables. In order to conclude that $T_i^{D0} > T_i^{R0}$ so that $t_i^{D0} > t_i^{R0}$, we need to take into account the changes in these shares. We accomplish that through the following definition and result.

Lemma 10. *Define* $\tilde{T}(t_i)$ *as*

$$\tilde{T}(t_i) \equiv T(t_i) - \frac{\theta_1 \rho_1}{\theta_1 - \rho_1 (1 - \lambda_{ii1})}. \tag{138}$$

Provided that,

$$\frac{\hat{\varphi}^*_{ij1}}{\hat{t}_i} > \frac{\tilde{\lambda}_{ii1}}{\theta_1} \Delta \tag{139}$$

within a non-empty interval $(t_i^0, 1)$*, where*

$$\Delta \equiv \frac{\left(\frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} - \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} \right)}{\left(\frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} - \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} + \frac{\tilde{\lambda}_{ii1}(1 - \tilde{\lambda}_{ii1})}{\lambda_{ii1}(1 - \lambda_{ii1})} \right)}, \tag{140}$$

then $\tilde{T}$ *is monotonically increasing in* t_i*. Therefore* $\tilde{T}$ *is invertible in this interval.*

Proof: We differentiate (138) to obtain

$$d\tilde{T} = dT + \frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} \hat{\lambda}_{ii1}.$$

From (133) and (95) we know that

$$\hat{\lambda}_{ii1} = (1 - \lambda_{ii1}) \left[\frac{\theta_1}{\tilde{\lambda}_{ii1}} \hat{\varphi}^*_{ij1} - \hat{t}_i \right].$$

Substituting this expression and using (134) in the above, we obtain

$$\begin{aligned} d\tilde{T} &= -\frac{\theta_1}{\lambda_{ii1}} \left(\tilde{\lambda}_{ii1} - \lambda_{ii1} \right) \hat{\varphi}^*_{ij1} + \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} (1 - \lambda_{ii1}) \hat{t}_i \\ &+ \frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} \frac{\theta_1 (1 - \lambda_{ii1})}{\tilde{\lambda}_{ii1}} \hat{\varphi}^*_{ij1} - \frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} (1 - \lambda_{ii1}) \hat{t}_i, \end{aligned}$$

and then

$$\begin{aligned} d\tilde{T} &= \left(\frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} - \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} + \frac{\tilde{\lambda}_{ii1} \left(1 - \tilde{\lambda}_{ii1}\right)}{\lambda_{ii1} (1 - \lambda_{ii1})} \right) \frac{(1 - \lambda_{ii1})}{\tilde{\lambda}_{ii1}} \theta_1 \ \hat{\varphi}^*_{ij1} \\ &+ \left(\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} - \frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}{(\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2} \right) (1 - \lambda_{ii1}) \hat{t}_i. \end{aligned}$$

The first expression in parentheses is positive because $\lambda_{ii1} > \tilde{\lambda}_{ii1}$ for $t_i < 1$. So, dividing by that expression, we see that $d\tilde{T}/\hat{t}_i > 0$ provided that (139) holds. QED

Lemma 11. $\Delta < 0$ *at free trade, where* $t_i = 1$ *and* $\tilde{\lambda}_{ii1} = \lambda_{ii1}$.

Proof:We evaluate Δ at free trade to obtain

$$\Delta|_{t_i=1} = \frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1} - (\theta_1 - \rho_1 (1 - \lambda_{ii1}))^2}{\theta_1 \rho_1 \rho_1 \lambda_{ii1}}$$

which is positive if $0 < (\theta_1 - \rho_1)^2 + 2\theta_1 \rho_1 \lambda_{ii1} - \rho_1 \rho_1 2\lambda_{ii1} + \rho_1 \rho_1 \lambda_{ii1} \lambda_{ii1} - \theta_1 \rho_1 \rho_1 \lambda_{ii1}$. Multiplying both sides of this inequality by $\frac{1}{\lambda_{ii1} \rho_1}$, we get

$$0 < \frac{(\theta_1 - \rho_1)^2}{\lambda_{ii1} \rho_1} + (2 - \rho_1) \theta_1 - \rho_1 (2 - \lambda_{ii1}).$$

A sufficient condition is obtained by taking $\lambda_{ii1} = 1$,

$$0 < \theta_1^2 + \rho_1^2 - 2\theta_1 \rho_1 + (2 - \rho_1)\theta_1 \rho_1 - \rho_1^2 \iff 0 < \theta_1(\theta_1 - \rho_1^2).$$

This inequality is satisfied because $\theta_1 > \rho_1 = \frac{\sigma_1 - 1}{\sigma_1}$, so $\Delta < 0$ at free trade. QED

From the proof of Lemma 8, we know that $\frac{\hat{\varphi}^*_{ij1}}{\hat{t}_i} > 0$. It follows that condition (139) will hold whenever $\Delta < 0$, so that Lemma 10 holds. Furthermore, Lemma 11 shows that $\Delta < 0$ holds at free trade. Then considering lower values for t_i in the range $t_i \in [t_i^{R0}, 1]$, we want to know whether $\Delta < 0$ so that (139) continues to hold. That will be shown in the next result.

Definition 6. *a) Define*

$$t_i^{\Delta 0} \equiv \begin{cases} \arg\max_{t_i \geq t_i^{\min}} \{\Delta = 0\} & \textit{if this value exists,} \\ t_i^{\min} & \textit{otherwise,} \end{cases}$$

and denote $T^{\Delta 0} \equiv T(t_i^{\Delta 0})$ *and likewise for the shares* $\tilde{\lambda}_{ii1}^{\Delta 0}$ *and* $\lambda_{ii1}^{\Delta 0}$ *evaluated at* $t_i^{\Delta 0}$.

b) Define $t_i^0 \equiv \max\left\{t_i^{A0}, t_i^{\Delta 0}\right\}$

Lemma 12. *Under (20) and (21), then* $t_i^{D0} > t_i^{R0} > t_i^0$, *and* t_i^{R0} *is unique.*

Proof: Using $\tilde{T}$, we can solve for t_i^{D0}, t_i^{R0}, and $t_i^{\Delta 0}$, as follows

$$\tilde{T}\left(t_i^{D0}\right) = T^{D0} - \frac{\theta_1 \rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{D0}\right)} = -\tilde{\gamma}_1 \Gamma, \tag{141}$$

$$\tilde{T}\left(t_i^{R0}\right) = T^{R0} - \frac{\theta_1 \rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{R0}\right)} = -\tilde{\gamma}_1. \tag{142}$$

From (140) we have that $\Delta(t_i^{\Delta 0}) = 0$ if and only if

$$\frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}^{\Delta 0}}{\left(\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{\Delta 0}\right)\right)^2} = \frac{\tilde{\lambda}_{ii1}^{\Delta 0}}{\lambda_{ii1}^{\Delta 0}}.$$

Using the expression for T in (132) and $\Delta(t_i^{\Delta 0}) = 0$, we get

$$\begin{aligned} \tilde{T}\left(t_i^{\Delta 0}\right) &= \frac{\theta_1 \rho_1 \rho_1 \lambda_{ii1}^{\Delta 0}}{\left(\theta_1 \rho_1\left(1 - \lambda_{ii1}^{\Delta 0}\right)\right)^2} - \tilde{\gamma}_{i1} - \frac{\theta_1 \rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{\Delta 0}\right)} \\ &= \frac{\theta_1 \rho_1 (\rho_1 - \theta_1)}{\left(\theta_1 \rho_1\left(1 - \lambda_{ii1}^{\Delta 0}\right)\right)^2} - \tilde{\gamma}_{i1} < -\tilde{\gamma}_{i1}, \end{aligned}$$

so we see that $\tilde{T}\left(t_i^{\Delta 0}\right) < \tilde{T}(t_i^{R0})$. In addition note that

$$\tilde{T}\left(t_i^{\min}\right) = T^{\min} - \frac{\theta_1 \rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{\min}\right)} = -\frac{\theta_1 \rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{\min}\right)},$$

where the final equality follows from Remark 2. Then in order to have $\tilde{T}\left(t_i^{\min}\right) < \tilde{T}(t_i^{R0})$, we need

$$\frac{\theta_1 \rho_1}{\theta_1 - \rho_1\left(1 - \lambda_{ii1}^{\min}\right)} > \tilde{\gamma}_{i1}.$$

A sufficient condition to ensure this inequality is obtained by taking $\lambda_{ii1}^{\min}=1$, in which case we obtain $\frac{\theta_1\rho_1}{\theta_1}=\rho_1>\tilde{\gamma}_{i1}=(1-\gamma_{i1})\rho_1$, which always holds. It follows that,

$$\tilde{T}\left(t_i^{D0}\right)>\tilde{T}\left(t_i^{R0}\right)>\tilde{T}\left(t_i^{0}\right)=\max\left\{\tilde{T}\left(t_i^{A0}\right),\tilde{T}\left(t_i^{\Delta 0}\right)\right\}. \tag{143}$$

Because $\tilde{T}(t_i)$ is monotonically increasing and invertible in the interval $(t_i^0,1)$, while $t_i^{R0}<1$ was shown in (122), it follows that $t_i^{R0}>t_i^0$.

Then considering t_i^{D0}, there are two possibilities. If $t_i^{D0}\geq 1$, then we immediately obtain the ranking $t_i^{D0}>t_i^{R0}>t_i^0$ because $t_i^{R0}<1$. On the other hand, if $t_i^{D0}<1$, then we can use (143) together with the invertibility of $\tilde{T}(t_i)$ in the interval $(t_i^0,1)$ to again obtain $t_i^{D0}>t_i^{R0}>t_i^0$. QED

Lemma 13. *$D(t_i^{R0})<0$. It follows by also using conditions (20) and (21) together with Lemma 4 that $M(t_i^{R0})<0$.*

Proof: We evaluate (13) at t_i^{R0}.

Evaluating $D(t_i)$ at t_i^{R0} we obtain

$$D(t_i^{R0})\equiv\left[1-\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)}\frac{\gamma_{i1}(\sigma_1-1)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0}-1\right)+(1+\gamma_{i1}(\sigma_1-1))}-\frac{\gamma_{i1}(\sigma_1-1)}{1+\gamma_{i1}(\sigma_1-1)}(1-\lambda_{ii1})\mathcal{E}_3\mathcal{E}_4\right]$$

where we also evaluate the elasticities $\mathcal{E}_3$, and $\mathcal{E}_4$ at t_i^{R0}. $D(t_i^{R0})<0$ if the following expression is negative

$$1-\frac{\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)}\gamma_{i1}(\sigma_1-1)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0}-1\right)+(1+\gamma_{i1}(\sigma_1-1))}-\frac{\gamma_{i1}(\sigma_1-1)(1-\lambda_{ii1})}{1+\gamma_{i1}(\sigma_1-1)}\frac{\left(\frac{T(t_i)+\tilde{\gamma}_{i1}}{T(t_i)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})}<0$$

use

$$T^{R0}=\frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}-\tilde{\gamma}_{i1}$$

$$1-\frac{\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)}\gamma_{i1}(\sigma_1-1)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0}-1\right)+(1+\gamma_{i1}(\sigma_1-1))}-\frac{\gamma_{i1}(\sigma_1-1)(1-\lambda_{ii1})}{1+\gamma_{i1}(\sigma_1-1)}\frac{\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1+\gamma_{i1}($$
$$\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})$$

given that $\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)} > 1$ then sufficient if

$$\frac{1}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0}-1\right)+\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}+\frac{\left(1-\lambda_{ii1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)} \geq \frac{1}{\gamma_{i1}\left(\sigma_1-1\right)}$$

use $t_i^{R0} = 1+\frac{\rho_1}{\left(1-\tilde{\lambda}_{ii1}^{R0}\right)}\frac{\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}-\frac{(1-\rho_1)}{\left(1-\tilde{\lambda}_{ii1}^{R0}\right)}$

$$\frac{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{\left(1-\lambda_{ii1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\rho_1\frac{\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\rho_1\frac{1}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\gamma_{i1}\left(\sigma_1-1\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\frac{1}{\sigma_1}\left(1-\lambda\right.\right.}$$

multiply devide by $\gamma_{i1}\left(\sigma_1-1\right)$ second term

$$\frac{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{\left(1-\lambda_{ii1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\rho_1\frac{\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\rho_1\rho_1\left(1-\lambda_{ii1}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right)} \geq$$

with simplications we obtain

$$\frac{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{\left(\frac{\theta_1\rho_1(1-\lambda_{ii1})(\sigma_1-1)}{\gamma_{i1}\theta_1+(1-\gamma_{i1})\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+-\sigma_1\theta_1+\rho_1\left(1-\lambda_{ii1}\right)\right)}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)} \geq 0$$

$$\frac{1}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)} \geq \frac{\frac{(\sigma_1-1)\theta_1\gamma_{i1}}{\gamma_{i1}\theta_1+(1-\gamma_{i1})\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+1}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)}$$

combining terms

$$\frac{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)} \geq \frac{\left(\sigma_1-1\right)\theta_1\gamma_{i1}}{\gamma_{i1}\theta_1+\left(1-\gamma_{i1}\right)\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+1$$

$$1+\frac{\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1-1\right)\theta_1}{\sigma_1\rho_1\rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)} \geq \frac{\left(\sigma_1-1\right)\theta_1\gamma_{i1}}{\gamma_{i1}\theta_1+\left(1-\gamma_{i1}\right)\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+1$$

so we obtain that

$$\gamma_{i1}\theta_1 + (1-\gamma_{i1})\,\rho_1\left(1-\lambda_{ii1}^{R0}\right) \geq \rho_1\left(1-\lambda_{ii1}^{R0}\right) + \gamma_{i1}\left(\theta_1 - \rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)$$

$$\theta_1 - \rho_1\left(1-\lambda_{ii1}^{R0}\right) \geq \theta_1 - \rho_1\left(1-\lambda_{ii1}^{R0}\right)$$

QED.

To prove Theorem 1(c), we rely on following, final Lemmas.

Lemma 14. *Provided that condition (22) holds, then*

$$D^{opt} > \delta_1 - \frac{1+\gamma_{i2}\,(\sigma_2-1)}{\gamma_{i2}\,(\sigma_2-1)}\frac{\gamma_{i1}\rho_1}{t^{opt}-\tilde{\gamma}_{i1}}, \tag{144}$$

where

$$\delta_1 \equiv \frac{1-\rho_1^2\gamma_{i1}(1-\gamma_{i1})\left(1-\frac{\rho_1}{(t^{opt})^2}\right)^{-1}}{t^{opt}+\frac{1}{\sigma_1}}. \tag{145}$$

Proof: We define $T^{opt} \equiv T(t^{opt})$ and $D^{opt} \equiv D(t^{opt})$. It follows from substituting the share $\tilde{\lambda}_{ii1}$ from (60) into (11) that

$$T^{opt} = 1-\tilde{\gamma}_{i1} + \frac{\left(t^{opt}-1\right)(1-\lambda_{ii1})}{1+(t^{opt}-1)\lambda_{ii1}}, \tag{146}$$

and also from (119)

$$D^{opt}T^{opt} = T^{opt}\left[1-(1-\lambda_{ii1})\,\mathcal{E}_4\right] - \frac{1+\gamma_{i2}\,(\sigma_2-1)}{\gamma_{i2}\,(\sigma_2-1)}\gamma_{i1}\rho_1 - \gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\,(1-\lambda_{ii1})\,\mathcal{E}_4. \tag{147}$$

It should be understood that the shares appearing in these equations are also evaluated at t^{opt}. Our strategy, however, is to treat these shares as parameters and differentiate $D^{opt}T^{opt}$ with respect to the share λ_{ii1} so as to obtain a lower-bound on $D^{opt}T^{opt}$. During this process, we are allowing $\tilde{\lambda}_{ii1}$, which was substituting to also adjust parametrically according to (60).

The value $D^{opt}T^{opt}$ changes with the share λ_{ii1} according to

$$\frac{\partial D^{opt}T^{opt}}{\partial\lambda_{ii1}} = \frac{\partial T^{opt}}{\partial\lambda_{ii1}}\left[1-(1-\lambda_{ii1})\,\mathcal{E}_4\right] - \left(T^{opt}+\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)\frac{\partial\,(1-\lambda_{ii1})\,\mathcal{E}_4}{\partial\lambda_{ii1}}. \tag{148}$$

From (146) we have,

$$\begin{aligned}\frac{\partial T^{opt}}{\partial \lambda_{ii1}} &= -\frac{\left(t^{opt}-1\right)}{1+(t^{opt}-1)\lambda_{ii1}} - \frac{\left(t^{opt}-1\right)^2(1-\lambda_{ii1})}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2} \\ &= -\frac{\left(t^{opt}-1\right)t^{opt}}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2}.\end{aligned}$$

Also from (91)

$$(1-\lambda_{ii1})\,\mathcal{E}_4 = \frac{\frac{(1-\lambda_{ii1})}{\theta_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}(1-\lambda_{ii1})}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})} = \frac{\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}},$$

so that,

$$\begin{aligned}\frac{\partial(1-\lambda_{ii1})\,\mathcal{E}_4}{\partial\lambda_{ii1}} &= -\frac{\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]^2}\frac{1}{\rho_1(1-\lambda_{ii1})^2} \\ &= -\mathcal{E}_4\frac{\frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]}.\end{aligned}$$

Substituting these expressions into (148), we obtain

$$\begin{aligned}\frac{\partial D^{opt}T^{opt}}{\partial\lambda_{ii1}} &= -\frac{\left(t^{opt}-1\right)t^{opt}}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2}\left[1-(1-\lambda_{ii1})\,\mathcal{E}_4\right] \\ &+ \left(T^{opt}+\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)\mathcal{E}_4\frac{\frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]} \\ &= -\frac{\left(t^{opt}-1\right)t^{opt}}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2}\left[\frac{\frac{1}{\rho_1(1-\lambda_{ii1})} - \frac{1}{\theta_1}}{\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}}\right] \\ &+ \left(T^{opt}+\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)\mathcal{E}_4\frac{\frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]} \\ &> \left\{-\frac{\left(t^{opt}-1\right)t^{opt}}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2} + \left(T^{opt}+\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)\mathcal{E}_4\right\}\frac{\frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]}.\end{aligned}$$

Using (146) it follows that $\frac{\partial D^{opt}T^{opt}}{\partial\lambda_{ii1}} > 0$ if

$$\mathcal{E}_4\frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}} > \frac{\left(t^{opt}-1\right)}{1+(t^{opt}-1)\lambda_{ii1}}\left(\frac{t^{opt}}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]} - \mathcal{E}_4(1-\lambda_{ii1})\right). \tag{149}$$

Substituting $t^{opt} = \frac{\theta_1}{\theta_1 - \rho_1}$ so that $t^{opt} - 1 = \frac{\rho_1}{\theta_1 - \rho_1}$, we simplify to obtain

$$\left(\mathcal{E}_4 - \frac{\rho_1}{\theta_1 - \rho_1(1-\lambda_{ii1})}\right)\left(\frac{\rho_1(1-\lambda_{ii1})}{\theta_1 - \rho_1(1-\lambda_{ii1})} + 1\right) > \mathcal{E}_4\left(1 - \frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)$$

Then we substitute (91) on the left and we use the bound from Lemma 5 on the right, which implies that $\mathcal{E}_4 \leq \frac{\theta_1 - \rho_1(\theta_1-\rho_1)}{\theta_1}$, to obtain the suficient condition

$$\left(\frac{\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})} - \frac{\rho_1}{\theta_1 - \rho_1(1-\lambda_{ii1})}\right)\left(\frac{\rho_1(1-\lambda_{ii1})}{\theta_1 - \rho_1(1-\lambda_{ii1})} + 1\right)$$

$$> \frac{\theta_1 - \rho_1(\theta_1-\rho_1)}{\theta_1}\left(1 - \frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right).$$

We set $\lambda_{ii1} = 0$ on the left to obtain a sufficient condition:

$$\frac{\frac{1}{\theta_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}} - \frac{\rho_1}{\theta_1 - \rho_1} > \frac{\theta_1 - \rho_1(\theta_1-\rho_1)}{\theta_1}\left(1 - \frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)$$

After extensive simplification, this inequality is written as

$$(1-\gamma_{i1})^2\rho_1(\theta_1 - \rho_1(\theta_1-\rho_1))(1+\gamma_{i1}\rho_1)\frac{\theta_1-\rho_1}{\theta_1} + (1-\gamma_{i1})(\theta_1 - 2\rho_1)(1-(1-\gamma_{i1})\rho_1)$$

$$> \sigma_1\gamma_{i1}\rho_1(1-(1-\gamma_{i1})\rho_1) - \sigma_1\gamma_{i1}(1-\gamma_{i1})(\theta_1 - \rho_1(\theta_1-\rho_1))(1+\gamma_{i1}\rho_1)(\theta_1-\rho_1)$$

This inequality fails to hold at $\gamma_{i1} = 1$, so lower values are needed. The first set of terms on the left will involve a cubic in γ_{i1}, so to avoid that a sufficient condition is obtained by ignoring those (positive) terms, resulting in

$$(1-\gamma_{i1})(\theta_1 - 2\rho_1)(1-(1-\gamma_{i1})\rho_1) - \sigma_1\gamma_{i1}\rho_1(1-(1-\gamma_{i1})\rho_1)$$

$$\geq -\sigma_1\gamma_{i1}(1-\gamma_{i1})(\theta_1 - \rho_1(\theta_1-\rho_1))(1+\gamma_{i1}\rho_1)(\theta_1-\rho_1).$$

A further simplification is obtained by observing that $(1+\gamma_{i1}\rho_1)$ on the right is made smaller by

replacing it with $(1-(1-\gamma_{i1})\rho_1)$, and dividing out that common term to obtain

$$(1-\gamma_{i1})(\theta_1-2\rho_1) \geq \sigma_1\gamma_{i1}\left[\rho_1-(1-\gamma_{i1})(\theta_1-\rho_1(\theta_1-\rho_1))(\theta_1-\rho_1)\right].$$

A sufficient condition for this inequality to hold is provided by (22).

It follows that we can take $\lambda_{ii1}=0$ to obtain a lower-bound for D^{opt}, and also $\tilde{\lambda}_{ii1}=0$ from (60), and so we set both these shares at zero in (146) and (147) to obtain $T^{opt}|_{\lambda_{ii1}=0}=t^{opt}-\tilde{\gamma}_{i1}$ and using this in (147),

$$D^{opt}|_{\lambda_{ii1}=0}T^{opt}|_{\lambda_{ii1}=0}=(t^{opt}-\tilde{\gamma}_{i1})(1-\mathcal{E}_4|_{\lambda_{ii1}=0})+\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\gamma_{i1}\rho_1-\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\mathcal{E}_4|_{\lambda_{ii1}=0}.$$

We use these expressions to obtain a bound on D^{opt} of

$$\begin{aligned} D^{opt} &= \frac{D^{opt}T^{opt}}{T^{opt}} \geq \frac{D^{opt}|_{\lambda_{ii1}=0}T^{opt}|_{\lambda_{ii1}=0}}{T^{opt}} \\ &= \frac{(t^{opt}-\tilde{\gamma}_{i1})(1-\mathcal{E}_4|_{\lambda_{ii1}=0})+\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\gamma_{i1}\rho_1-\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\mathcal{E}_4|_{\lambda_{ii1}=0}}{1-\tilde{\gamma}_{i1}+(t^{opt}-1)\left(1-\tilde{\lambda}_{ii1}\right)}. \end{aligned}$$

We further set $\tilde{\lambda}_{ii1}$ in the denominator at zero to obtain another lower bound on D^{opt} of

$$\begin{aligned} D^{opt} &> \frac{(t^{opt}-\tilde{\gamma}_{i1})(1-\mathcal{E}_4|_{\lambda_{ii1}=0})-\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\gamma_{i1}\rho_1-\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\mathcal{E}_4|_{\lambda_{ii1}=0}}{t^{opt}-\tilde{\gamma}_{i1}} \\ &= 1-\mathcal{E}_4|_{\lambda_{ii1}=0}\left(1+\frac{\gamma_{i1}\rho_1}{(t^{opt}-\tilde{\gamma}_{i1})}\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)-\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\frac{\gamma_{i1}\rho_1}{t^{opt}-\tilde{\gamma}_{i1}}. \end{aligned} \tag{150}$$

Evaluating the second term in this expression, we apply 20 which is equivalent to 127 to obtain

$$\mathcal{E}_4|_{\lambda_{ii1}=0} \leq \frac{\rho_1+(\theta_1-\rho_1)(1-\rho_1)}{\theta_1+(\theta_1-\rho_1)(1-\rho_1)} = \frac{t^{opt}-\rho_1}{t^{opt}+\frac{1}{\sigma_1}}.$$

In addition, the denominator of the term $1+\frac{\gamma_{i1}\rho_1}{(t^{opt}-\tilde{\gamma}_{i1})}\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}$ is bounded below by using 20 again to obtain $\tilde{\gamma}_{i1}=(1-\gamma_{i1})\rho_1 \leq \rho_1/t^{opt}$ and so $\left(t^{opt}-\tilde{\gamma}_{i1}\right)(1-\tilde{\gamma}_{i1}) \geq \left(t^{opt}-\frac{\rho_1}{t^{opt}}\right)\left(1-\frac{\rho_1}{t^{opt}}\right)$. It follows that

$$1-\mathcal{E}_4|_{\lambda_{ii1}=0}\left(1+\frac{\gamma_{i1}\rho_1}{(t^{opt}-\tilde{\gamma}_{i1})}\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right) \geq \frac{1-\rho_1^2\gamma_{i1}(1-\gamma_{i1})\left(1-\frac{\rho_1}{(t^{opt})^2}\right)^{-1}}{t^{opt}+\frac{1}{\sigma_1}} \equiv \delta_1.$$

Substituting these results into (150) we have shown (144), with δ_1 defined as above and in (145). QED

Lemma 15. *When conditions (20) and (22) hold, then $H^{opt} < 0$ for all parameters satisfying (7) when κ_i is chosen as stated in part (c) of Theorem 1.*

Proof: Using (19), the needed condition is that

$$-(1-\alpha_i)\,\mathcal{M}^{opt}\left(\mathcal{E}_5^{opt} - \frac{\left(t^{opt}-1\right)}{t^{opt}}\theta_1\right)\frac{D^{opt}}{A^{opt}} < (1-\gamma_{i1})R^{opt},$$

where $\mathcal{M}^{opt}$, $\mathcal{E}_5^{opt}$, D^{opt}, A^{opt} and R^{opt} are all evaluated at t^{opt}. Using Lemma 14, we can rewrite this expression as

$$\frac{\gamma_{i1}(\sigma_1-1)}{1+\gamma_{i1}(\sigma_1-1)} < \left[\delta_1 + \frac{(1-\gamma_{i1})R^{opt}A^{opt}}{(1-\alpha_i)\,\mathcal{M}^{opt}\left(\mathcal{E}_5^{opt} - \frac{(t^{opt}-1)}{t^{opt}}\theta_1\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})\gamma_{i2}\,(\sigma_2-1)}{(1-\tilde{\gamma}_{i1})[1+\gamma_{i2}\,(\sigma_2-1)]}.$$

Therefore a sufficient condition is given by $\frac{\gamma_{i1}(\sigma_1-1)}{1+\gamma_{i1}(\sigma_1-1)} \leq \kappa_i \frac{\gamma_{i2}(\sigma_2-1)}{1+\gamma_{i2}(\sigma_2-1)}$, with

$$\kappa_i > \left[\delta_1 + \frac{(1-\gamma_{i1})R^{opt}A^{opt}}{(1-\alpha_i)\,\mathcal{M}^{opt}\left(\mathcal{E}_5^{opt} - \frac{(t^{opt}-1)}{t^{opt}}\theta_1\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}.$$

Using the method in the proof of Lemma 4, we readily obtain

$$\mathcal{E}_5^{opt} - \frac{\left(t^{opt}-1\right)}{t^{opt}}\theta_1 = \frac{\rho_1}{\tilde{\lambda}_{ii1}}\left(\frac{1-\frac{\rho_1}{\theta_1}\tilde{\lambda}_{ii1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{\rho_1}{\theta_1}\left(1-\tilde{\lambda}_{ii1}\right)}{1+\rho_1\frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}}\right).$$

We substitute this along with the lower-bound for $A(t^{opt})$ from Lemma 5, $A(t^{opt}) > \alpha_i(1-\rho_1)+\gamma_{i1}\rho_1$, together with the expressions for $\mathcal{M}^{opt}$, D^{opt} and R^{opt}, to obtain

$$\kappa_i > \left[\delta_1 + \frac{(1-\gamma_{i1})\theta_1\left(\theta_1-\rho_1\right)^2\rho_1\left(\alpha_i(1-\rho_1)+\gamma_{i1}\rho_1\right)\left(\begin{array}{c}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)\frac{1}{\theta_1-\rho_1}\\ +\frac{-\theta_1\rho_1+(\theta_1-\rho_1(1-\lambda_{ii1}))(1-\gamma_{i1})\rho_1}{\left(\theta_1-\rho_1+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)-(\theta_1-\rho_1)(1-\gamma_{i1})\rho_1\right)}\end{array}\right)}{(1-\alpha_i)\left(1-\tilde{\gamma}_{i1}\right)\left(\theta_1-\rho_1+\frac{\rho_1}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}.$$

Now we use that $\frac{\theta_1-\rho_1+\rho_1\lambda_{ii1}}{\theta_1-\rho_1} > 1$ and that $(\theta_1-\rho_1(1-\lambda_{ii1}))\frac{1}{\theta_1-\rho_1} < \frac{\theta_1}{\theta_1-\rho_1}$ to obtain the sufficient condition

$$\kappa_i > \left[\frac{(1-\gamma_{i1})\theta_1(\theta_1-\rho_1)^2\rho_1(\alpha_i(1-\rho_1)+\gamma_{i1}\rho_1)\left(\frac{\theta_1}{\theta_1-\rho_1}+\frac{-\theta_1\rho_1+(\theta_1-\rho_1(1-\lambda_{ii1}))(1-\gamma_{i1})\rho_1}{(\theta_1-\rho_1+\rho_1(1-\tilde{\lambda}_{ii1})-(\theta_1-\rho_1)(1-\gamma_{i1})\rho_1)}\right)}{(1-\alpha_i)(1-\tilde{\gamma}_{i1})\left(\theta_1-\rho_1+\frac{\rho_1}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}.$$

Notice that

$$\frac{\theta_1}{\theta_1-\rho_1}+\frac{-\theta_1\rho_1+(\theta_1-\rho_1(1-\lambda_{ii1}))(1-\gamma_{i1})\rho_1}{\left(\theta_1-\rho_1+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)-(\theta_1-\rho_1)(1-\gamma_{i1})\rho_1\right)}=\frac{\theta_1(1-\rho_1)-\rho_1\tilde{\gamma}_{i1}(1-\lambda_{ii1})+\frac{\theta_1}{(\theta_1-\rho_1)}\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1})+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}$$

and we can replace λ_{ii1} by $\tilde{\lambda}_{ii1}$ to obtain the sufficient condition

$$\begin{aligned}\frac{\theta_1(1-\rho_1)-\rho_1\tilde{\gamma}_{i1}(1-\lambda_{ii1})+\frac{\theta_1}{(\theta_1-\rho_1)}\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1})+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)} &> \frac{\theta_1(1-\rho_1)+\left(\frac{\theta_1}{\theta_1-\rho_1}-\tilde{\gamma}_{i1}\right)\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1})+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}\\ &> \frac{\theta_1(1-\rho_1)+\left(\frac{\theta_1}{\theta_1-\rho_1}-\tilde{\gamma}_{i1}\right)\rho_1}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1})}.\end{aligned}$$

Another sufficient condition is obtained by using $\tilde{\lambda}_{ii1}=0$ in numerator and $\tilde{\lambda}_{ii1}=1$ in the denominator above, and again using $\tilde{\lambda}_{ii1}=1$ in the denominator of κ_i to obtain

$$\kappa_i \geq \left[\delta_1+\frac{\tilde{\gamma}_{i1}\theta_1(\alpha_i(1-\rho_1)+\gamma_{i1}\rho_1)\left(\theta_1(1-\rho_1)+\left(t^{opt}-\tilde{\gamma}_{i1}\right)\rho_1\right)}{(1-\alpha_i)(1-\tilde{\gamma}_{i1})^2}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}.$$

In the statement of Theorem 1(c), we use the equality above to specify κ_i. QED

To complete the proof of part (c) we need to establish the tariff $t_i^* \in (t_i^{R0}, t_i^{opt})$ with $H(t_i^*)=0$. Because $R(t_i^{R0})=0$, it follows from (19) that $H(t_i^{R0})=\left(t^{opt}-t_i^{R0}\right)-t_i^{R0}(1-\alpha_i)M(t_i^{R0})>0$ from Lemma 13 because $M(t_i^{R0})<0$ since $D\left(t_i^{R0}\right)<0$. From Lemma 15 we have $H\left(t_i^{opt}\right)<0$. It follows from the continuity of $H(t_i)$ that there exists a tariff $t_i^* < t^{opt}$ at which $H(t_i^*)=0$. QED