RESTRUCTURING VS. BANKRUPTCY*

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PRELIMINARY

Abstract

We develop a model of a firm in financial distress. Distress can be mitigated by filing for bankruptcy, which is costly, or preempted by restructuring, which is impeded by a collective action problem. We find that bankruptcy and restructuring are complements, not substitutes: Reducing bankruptcy costs facilitates restructuring, rather than crowding it out. And so does making bankruptcy more debtor-friendly, under a condition that seems likely to hold now in the United States. The model gives new perspectives on current relief policies (e.g., subsidized loans to firms in bankruptcy) and on long-standing legal debates (e.g., the efficiency of the absolute priority rule).

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1 Introduction

When a firm enters financial distress, it has several options to avoid liquidation. One is bankruptcy reorganization; another is an out-of-court restructuring agreement with creditors. Both options reduce leverage by exchanging existing debt for new securities (debt or equity). The main difference between them is that restructuring agreements avoid the deadweight costs of an immediate bankruptcy. However, they do not preclude a future bankruptcy case. Restructurings constitute about forty percent of corporate defaults, roughly the same share as bankruptcy filings (Moody's (2020)). About seventeen percent of restructurings are followed by a bankruptcy during the next three years (Moody's (2017)).

Although the restructuring and bankruptcy options are well understood, much of the literature conflates them or treats them as substitutes.[1] In this paper, we study the firm's choice between restructuring and bankruptcy and show how key parameters of the bankruptcy environment—its deadweight costs and the extent to which it is "creditor friendly"—affect the probability of restructuring.

Two insights guide our analysis. When a firm's debt is dispersed, restructurings are inhibited by a collective action problem among its creditors, each of which has an incentive to hold out. This problem can be overcome through a type of restructuring—a "distressed exchange"—that offers creditors new debt with lower face value but higher priority than the original debt, as shown in Bernardo and Talley (1996) and Gertner and Scharfstein (1991). Higher priority is valuable, however, only if (i) the firm is still somewhat likely to file for bankruptcy following the restructuring—if the firm never goes bankrupt, then even low-priority debt is paid in full–and (ii) high priority in bankruptcy ensures a high percentage recovery on the new debt. Thus, the likelihood that creditors accept a restructuring offer depends on (i) the probability of bankruptcy and (ii) the parameters of the bankruptcy environment (deadweight costs and creditor friendliness). That is our first insight.

Our second insight arises from the observation that shareholders decide whether to file a bankruptcy case. They control (i) the probability of bankruptcy, and their choice depends on (ii) the parameters of the bankruptcy environment. The greater their expected recovery in bankruptcy, the more likely they are to file for bankruptcy. This means that a key parameter (creditor friendliness) affects both the shareholders' decision to file for bankruptcy and the creditors' decision to accept a restructuring offer, though the effect on the latter decision is not obvious.

Building on these insights, we develop a model of a firm in financial distress. We obtain several novel results. First, restructuring and bankruptcy are complements. Policies

[1] See, e.g., Asquith, Gertner, and Scharfstein (1994), Becker and Josephson (2016), Favara, Schroth, and Valta (2012), Franks and Torous (1994), Gertner and Scharfstein (1991), and Gilson, John, and Lang (1990). In some papers, such as Fan and Sundaresan (2000) and Hart and Moore (1994, 1998), bankruptcy serves as the outside option for renegotiation. These papers, however, typically do not model the bankruptcy choice; it is instead synonymous with liquidation.

that reduce the deadweight costs of bankruptcy, for example, will facilitate out-of-court restructurings. Second, under conditions that are likely to hold today, policies that make bankruptcy more debtor friendly (and less creditor friendly) will facilitate out-of-court restructurings.

Our model allows us to assess recent policies aimed at alleviating corporate financial distress during the COVID-19 pandemic. We find that lending programs, such as the Primary Market Corporate Credit Facility or the Main Street Lending Program, can impede restructuring, potentially doing more harm than good. Grants are better, as are loans that can be forgiven, such as those advocated by Blanchard, Philippon, and Pisani-Ferry (2020) and associated with the Paycheck Protection Program.[2] Better still are policies that either directly facilitate restructuring agreements, as proposed by Blanchard, Philippon, and Pisani-Ferry (2020) and Greenwood and Thesmar (2020), or make subsidized loans to firms in bankruptcy through, for example, the Debtor-in-Possession Financing Facility (DIPFF) proposed by DeMarzo, Krishnamurthy, and Rauh (2020).

Our model also contributes to recent debates on the design of corporate bankruptcy laws. A number of scholars and policymakers have advocated limiting the priority of senior (secured) creditors and/or the control they exercise over the bankruptcy process. Examples include Bebchuk and Fried (1996), Casey (2011), Jacoby and Janger (2014), and Ayotte and Ellias (2020). We show that any policy reducing the value of priority to secured creditors can undermine out-of-court restructurings. Our analysis thus implies that the welfare effects of the absolute priority rule (APR) are more complex than prior literature acknowledges:[3] In fact, deviations from the APR undermine out-of-court restructurings when they favor unsecured creditors at the expense of secured creditors. Finally, our model raises questions about proposals to elevate the priority of involuntary creditors (especially tort victims), who are treated as unsecured creditors under current law. Giving them priority over secured creditors can undermine out-of-court workouts (which is harmful to the victims), but giving them priority over unsecured creditors facilitates workouts.

In the remainder of this introduction, we preview our model and results and then discuss related literature. Section 2 presents the model and Section 3 derives the first two main results. In Section 4, we analyze alternative policies for alleviating financial distress. Section 5 explores extensions: We consider the effect of (i) secured creditor power, including APR deviations (favoring unsecured creditors at the expense of secured creditors), (ii) court congestion, (iii) endogenous asset values and debt overhang, and (iv) creditor concentration. In Section 6, we conclude with a discussion of the model's broader implications. All proofs

[2]E.g., United Airlines will receive a total of $5 billion through the Payroll Protection Program. Of the $5 billion the airline expects to receive, approximately $3.5 billion will be a direct grant and approximately $1.5 billion will be a low interest rate loan.

[3]One useful example is Bebchuk (2002), which summarizes much of the literature and explores ex ante effects of the APR, but does not consider the APR's effects on restructuring.

and omitted derivations appear in the Appendix.

1.1 Results Preview

We present a two-period model in which a single firm has risky assets v and unsecured debt D_0 held by dispersed creditors. At date 0, before v is realized, the firm can propose a restructuring of its debt. At date 1, v is realized, and the firm has a choice: Repay the debt or default. If it defaults, creditors will liquidate the firm unless it files for bankruptcy. Both liquidation and bankruptcy are costly in the sense that they generate deadweight costs. We assume that the costs of bankruptcy, $(1-\lambda)v$, are lower than the costs of liquidation.

In bankruptcy, creditors bargain with equity holders to capture a fraction θ of the value available for distribution (λv). This fraction θ measures the "creditor friendliness" of bankruptcy and reflects both creditors' bargaining power in bankruptcy and their recovery in liquidation (which is their outside option as well as their legally mandated minimum recovery in bankruptcy).

Restructuring to a lower debt level mitigates distress costs because it reduces the likelihood of default. Thus, it has the potential to make everyone better off, including the creditors who have their debt written down. But it can be impeded by a collective action problem: Each creditor decides whether to accept a restructuring offer, taking other creditors' decisions as given. If others are accepting the offer, a creditor should reject it ("hold out") because acceptance by the others will lead to a successful restructuring that avoids default and renders the firm able to pay the non-restructured debt. Thus, each creditor has incentive to free ride on others' write-downs, leading the offer to be rejected in equilibrium.

In our model, as in prior literature including Bernardo and Talley (1996) and Gertner and Scharfstein (1991), the firm can restructure only if it offers *seniority* to creditors: Creditors will accept a write-down in the face value of the debt (which decreases what they are paid when the firm does not default) only in exchange for an increase in their priority (which increases what they are paid when the firm does default). Seniority ensures they are first in line for repayments in bankruptcy, when only a subset of creditors are paid.

In the literature, seniority in bankruptcy is essential to solving the hold-out problem, but it is generally assumed that bankruptcy occurs automatically whenever cash flows are low. In our model, as in practice, bankruptcy is instead a strategic decision of the firm. Hence, the value of seniority at the time of restructuring depends on the firm's future decision to file for bankruptcy or not. Of course, if the firm's asset value v is sufficiently high at date 1, the firm repays its outstanding debt (which could be the outcome of prior restructuring). But if v is below a threshold ($\hat{v}$), it prefers to default and file for bankruptcy protection. The threshold $\hat{v}$ depends on the parameters of the bankruptcy environment: Bankruptcy is more attractive to the firm when bankruptcy costs are low (λ is high) and when the Code is debtor-friendly (θ is low). Hence, the bankruptcy filing threshold, $\hat{v}$, is increasing in λ

and decreasing in θ.

This leads to our first main result: A decline in bankruptcy costs (an increase in λ) facilitates restructuring. To see why, recall that restructuring is feasible only insofar as creditors are willing to accept write-downs in exchange for seniority. An increase in λ makes seniority more valuable in two ways: (i) It increases recovery values for senior debt in bankruptcy (a direct effect), and (ii) it makes filing for bankruptcy more attractive to the firm (an indirect effect). Since seniority is valuable only insofar as bankruptcy is probable, the indirect effect, like the direct one, makes seniority more valuable.

Our second main result is a characterization of the level of creditor friendliness (θ) that facilitates restructuring. Like λ, the optimal θ should maximize the value of seniority. Unlike λ, θ must balance two effects. One is the direct effect we just saw: Increasing θ increases recovery values for senior debt. But now there is a countervailing indirect effect: It makes filing for bankruptcy less attractive to the firm. As the likelihood of bankruptcy declines, the value of seniority in bankruptcy declines as well.

We derive a "sufficient statistics" condition to test whether the creditor friendliness of bankruptcy is inefficiently high in the sense that a small decrease in θ would make restructuring easier. A back-of-the-envelope calculation, drawing on estimates from the literature, suggests this condition is likely satisfied now in the United States: Bankruptcy law is likely too creditor friendly. Any further increase in creditor friendliness is likely to have a minor effect on creditor recovery values, but a decrease could have a significant effect on the filing probability. The net effect is that restructurings, which avoid the deadweight costs of bankruptcy, would be more common if U.S. law were less creditor friendly.

We use our model to evaluate policy interventions that could mitigate financial distress. This leads to our third main result: The most effective policies are those that allocate the marginal subsidy dollar to facilitate restructuring by, for example, rewarding creditors for accepting restructurings directly. In the past, this has been done directly via the tax code.[4] But we show it can be done just as easily by subsidizing firms in bankruptcy—by subsidizing debtor in possession (DIP) loans, for example. A policy that increases payoffs in bankruptcy makes seniority more valuable, facilitating restructuring before bankruptcy. Cash injections/grants are less effective, in part because they decrease the likelihood of bankruptcy, which makes restructuring harder. Subsidized loans are even worse, because they increase leverage without facilitating restructuring.[5]

We explore several extensions of our model, which generate further results. (i) We allow secured creditors to exercise control over the bankruptcy process. We show that such

[4] In 2012, for example, IRS Regulation TD9599 reduced the taxes that creditors owe upon restructuring. Campello, Ladika, and Matta (2018) show that this policy led bankruptcy risk to fall by nearly 20 percent and restructurings to double.

[5] A caveat to our policy analysis, which takes the firm's initial debt D_0 as given, is that anticipated policy interventions could affect how much the firm borrows in the first place. An ex post analysis seems especially appropriate for unanticipated crises like the COVID-19 pandemic.

control can facilitate or deter restructuring, depending on how control is exercised. If secured creditors manipulate the bankruptcy process to divert value from unsecured creditors without reducing (direct or indirect) payoffs to equity (as in Ayotte and Ellias (2020)), a marginal increase in creditor control can facilitate the likelihood of restructuring. But if secured creditors induce excessive liquidations that reduce payoffs to all investors, including equity (as in Ayotte and Morrison (2009) and Antill (2020)), a marginal increase in secured creditor control reduces the likelihood of restructuring. This extension also allows us to explore the effect of deviations from the APR between senior and junior debt as well as between debt and equity. We find that debt-debt deviations are never optimal, whereas debt-equity deviations can be. This gives a new perspective on long-standing policy debates about the APR (see, e.g., Bebchuk and Fried (1996)) and potentially rationalizes observed practice.[6] (ii) We capture court congestion by allowing the costs of bankruptcy to increase with the probability that firms file for bankruptcy. We show that this can generate financial instability in the form of multiple equilibria and argue that bankruptcy policy thus matters for financial stability. (iii) We allow for ex ante costs of financial distress, arising from debt overhang or risk-shifting, as well as ex post costs arising from judicial errors, bargaining frictions, or court congestion. We find that, although these costs unambiguously increase the benefits of restructuring, their effect on the likelihood of restructuring is complex: Restructuring is more likely under some conditions and less likely under others. We therefore add nuance to Brunnermeier and Krishnamurthy's (2020) argument that an efficient bankruptcy system helps resolve debt-overhang problems. (iv) Finally, we allow creditors to be concentrated as well as dispersed. We find that restructurings will include debt-for-equity swaps when creditors are sufficiently concentrated, but only debt-for-debt swaps (swapping junior unsecured debt for senior secured debt) when they are dispersed. This offers a testable explanation for the composition of observed exchange offers, which sometimes include debt-for-equity swaps (e.g., Asquith, Gertner, and Scharfstein (1994)).

1.2 Literature Review

Our paper bridges two strands of the bankruptcy literature. One focuses on the hold-out problem as an impediment to restructuring.[7] Roe (1987) was among the first to focus on this problem in the context of bondholders, whose inability to coordinate (exacerbated by

[6] Deviations from the priority of secured debt over unsecured debt are rare, occurring in only 12 percent of the Chapter 11 bankruptcies in Bris, Welch, and Zhu (2006), whereas those of unsecured debt over equity seem to be somewhat more common (see Eberhart, Moore, and Roenfeldt (1990), Franks and Torous (1989), and Weiss (1990)).

[7] Our paper complements papers studying other restructuring frictions, such as asymmetric information (Bulow and Shoven (1978), Giammarino (1989), and White (1980, 1983)). Our work departs from papers in which such frictions are absent and, as a result, Coasean bargaining among investors leads to efficiency (e.g., Baird (1986), Haugen and Senbet (1978), Jensen (1986), and Roe (1983)).

federal law) can prevent efficient restructuring and render bankruptcy necessary.[8] Gertner and Scharfstein (1991) study the problem more formally, showing that a debtor can induce claimants to agree to a restructuring via an "exchange offer" that offers seniority to consenting creditors (and thereby demotes non-consenting creditors).[9] Bernardo and Talley (1996) show that the ability to make such exchange offers can distort management investment incentives.[10] In these papers, however, bankruptcy is not a choice; it is an automatic consequence of the firm's inability to pay its debts.

A separate strand of the literature focuses on the bankruptcy decision and explores the effects of bankruptcy rules, such as the APR, on this decision. Baird (1991) and Picker (1992), for example, assess whether these rules induce firms to enter Chapter 11 when doing so maximizes recoveries to dispersed unsecured creditors. Picker (1992) concludes that, because the filing decision is held by shareholders, optimal rules might permit violations of the APR in order to induce filings that maximize ex post recoveries. These papers, however, do not consider how rules affecting the bankruptcy filing decision also affect the likelihood of a successful restructuring ex ante.[11]

Our paper is also related to a vast literature studying the effects of creditor priority on bankruptcy outcomes and ex ante investment decisions (examples include Adler (1995) and Bebchuk (2002)).

2 Model

We set up a model of a firm that could enter financial distress and face either costly liquidation or costly bankruptcy. Out-of-court debt restructuring can mitigate the costs of distress. However, such restructuring is inhibited by a collective action problem because the firm cannot negotiate with creditors collectively, but must negotiate with each bilaterally.

In the model, there are two dates, date 0 and date 1. The firm starts with initial debt D_0 to dispersed creditors and risky assets v in place. At date 0, before v is realized, the firm can try to restructure its debt to $D < D_0$, deleveraging to reduce the likelihood of future distress. At date 1, v is realized, and the firm has a choice. It can either repay its debt or default. In the event of default, it risks being liquidated by its creditors, but can file for bankruptcy as protection.

[8] In corporate finance, this idea is also central to Grossman and Hart's (1980) model in which free-riding shareholders refuse efficient takeovers.

[9] Roe and Tung (2016) also study exchange offers and show that a successful exchange can nonetheless be followed by a bankruptcy filing.

[10] Haugen and Senbet (1988) discuss ways to solve the coordination problem contractually (though some of the solutions could run afoul of the Trust Indenture Act). For example, the indenture could permit the firm to repurchase the bonds at any time at a specified price (e.g., the price quoted in the most recent trade).

[11] Another strand of the literature is exemplified by Mooradian (1994), Povel (1999), and White (1994), who view bankruptcy as a screening device that can induce liquidation of inefficient firms and the reorganization or restructuring of efficient firms.

2.1 The Firm and its Capital Structure

There is a single firm. It has assets with random positive value $v \sim F$ and initial debt D_0 owed to identical, dispersed, risk-neutral creditors. The firm is controlled by risk-neutral equity holders, who seek to maximize their final payoff, equal to the value of the assets that remain after repaying creditors and incurring distress costs (defined below).

2.2 Restructuring

Because distress is the result of high leverage, the firm can potentially avoid it by deleveraging. To do so, it can restructure its debt to D, an amount less than its initial debt D_0. We allow it to restructure at any time, either at date 0 (before v is realized) or at date 1 (after it is realized). To do so, the firm makes a take-it-or-leave-it offer to exchange its creditors' debts for new instruments (which we shall call "claims" for expositional convenience). We focus on the most common claims in real-world restructurings: Equity and senior debt (Gilson, John, and Lang (1990)). However, we argue in Appendix B.1 that our analysis is robust and applies to more general claims.

The main friction in the model is that there is a collective action problem among creditors. Each decides whether to accept the firm's offer, taking others' decisions as given. Distress costs are another friction, which we define next.

2.3 Financial Distress: Liquidation and Bankruptcy

We capture financial distress by the costs of (out-of-court) liquidation or bankruptcy that arise when the firm does not repay its debt D. Here, D denotes the firm's debt at the end of date 1; it can be the outcome of a restructuring, if one has taken place, or the initial debt D_0, if one has not.

If the firm pays D in full, creditors get D and equity holders get the residual $v - D$. But if the realization of v is low relative to D, the firm could choose to default. In the event of default, there are two possibilities: liquidation or bankruptcy.

1. **Liquidation.** If the firm defaults and does not file for bankruptcy, creditors can seize the firm's assets. We assume that their liquidation (or redeployment) value is less than the value to incumbent equity holders,[12] leading to deadweight costs $(1 - \mu)v$. All of the remaining value μv goes to creditors; equity holders get nothing.[13] Moreover, we assume that seizure takes place in an uncoordinated "creditor race." This means that a restructuring or going-concern sale cannot be used to avoid these costs in liquidation.

[12] For the microfoundations of this wedge in value, see, e.g., Aghion and Bolton (1992), Hart (1995), and Shleifer and Vishny (1992).

[13] This is just a normalization that does not affect the results; see footnote 15.

2. **Bankruptcy.** To avoid liquidation, the firm can file for bankruptcy.[14] We assume that bankruptcy is costly, leading to deadweight costs $(1-\lambda)v$, which may derive from professional fees, inefficient judicial decisions, separations from suppliers/trade creditors/customers, and other factors (e.g., Titman (1984)). The remaining value is determined by bargaining in bankruptcy. As in practice, bankruptcy allows creditors to act collectively, avoiding the creditor race; liquidation is just their outside option. We capture this using the generalized Nash bargaining protocol: Creditors get their liquidation value $\mu\lambda v$ plus a fraction $\hat{\theta}$ of the surplus created by avoiding liquidation, where $\hat{\theta}$ is their bargaining power. (Below, we show that a single parameter, denoted by θ, captures the effects of both μ and $\hat{\theta}$ and thereby measures the "creditor friendliness" of the bankruptcy environment.)

To summarize, the deadweight costs of distress are $(1-\mu)v$ if the firm is liquidated out of court and $(1-\lambda)v$ if it files for bankruptcy. (Although we focus on ex post/direct costs of distress in our baseline model, we extend it to include ex ante/indirect costs in Section 5.3.)

Observe that we focus on asset values, not cash flows. The reason is that, for the type of firms the model captures, which have dispersed debt holdings, solvency problems (low asset values) are likely a necessary condition for financial distress. Liquidity problems (low cash flows) are insufficient because such firms are likely to be able to raise capital to meet liquidity problems for at least three reasons: (i) They are likely to be owned by deep-pocketed equity holders who will inject capital to preserve going-concern value if asset values are high (as in, e.g., Leland (1994)); (ii) they are likely to have access to capital markets, and creditors will lend against collateral if asset values are high (see, e.g., Chaney, Sraer, and Thesmar (2012)); and (iii) they are likely to be able to sell/liquidate capital, and buyers will pay high prices if asset values are high (see, e.g., Asquith, Gertner, and Scharfstein (1994)).

2.4 Timeline

In summary, the timing is as follows:

1. Debt can be restructured or not (which we refer to as "ex ante restructuring").
2. The asset value v is realized.
3. Debt can, again, be restructured or not (which we refer to as "ex post restructuring").
4. The firm repays its debt or defaults; if it defaults, it can file for bankruptcy (and bargain with creditors) or not (and risk liquidation by creditors).

[14] We assume the firm has exclusive authority to commence a bankruptcy case. In footnote 16, we discuss this assumption, which precludes involuntary bankruptcy filings by creditors.

3 Results

Here, we derive our results, working backward from the payoffs in bankruptcy/liquidation, to the bankruptcy filing decision, to ex post restructuring, to ex ante restructuring. Our main insights follow from comparative statics on the condition for an individual creditor to accept a restructuring.

3.1 Bargaining and Payoffs in Bankruptcy

When a firm reorganizes in bankruptcy, creditors bargain collectively and are guaranteed (via the "best interests test") a payoff no lower than what they would receive in a liquidation ($\mu\lambda v$). The extent to which their payoff exceeds $\mu\lambda v$ depends on the value available for distribution in a reorganization (λv) and their bargaining power ($\hat{\theta}$). Thus,

$$\begin{aligned} \text{creditors' payoff} &= \text{liquidation value} + \hat{\theta} \times \text{surplus from reorganization} && (1)\\ &= \mu\lambda v + \hat{\theta}\big(\lambda v - \mu\lambda v\big) && (2)\\ &= \big(\mu + (1-\mu)\hat{\theta}\big)\lambda v. && (3) \end{aligned}$$

Equity holders receive the residual, that is, λv minus the creditors' payoff above.

We will see that what matters in our analysis is just the fraction of bankruptcy value, λv, that goes to creditors. We denote this by

$$\theta := \mu + (1-\mu)\hat{\theta}. \tag{4}$$

We refer this to as a measure of the "creditor friendliness" of the bankruptcy system. The complementary fraction, $1-\theta$, which goes to equity holders, is a measure of the system's "debtor friendliness." θ captures creditors' overall strength in bankruptcy, reflecting both the value of their outside option (μ) and their direct bargaining power in bankruptcy court ($\hat{\theta}$).

Since equity holders get $(1-\theta)\lambda v > 0$ in bankruptcy and zero in creditor liquidation, they always prefer to file than to default and be liquidated out of court.[15] (Liquidation still matters, because it is creditors' outside option in bankruptcy reorganization.) Thus, if the

[15] If we relax the assumption that equity gets nothing in out-of-court liquidation, and assume instead that it gets a fraction $1-\delta$ of the liquidation value μv, calculations analogous to those above give $\theta = \delta\mu + (1-\mu)\hat{\theta}$. The analysis below is unaffected as long as $(1-\delta)\mu < (1-\theta)\lambda$, which ensures that equity holders prefer a bankruptcy filing to out-of-court liquidation.

firm has assets worth v and debt D, total payoffs to equity holders and creditors are:

$$\text{equity payoff} = \begin{cases} v - D & \text{if repayment,} \\ (1-\theta)\lambda v & \text{if bankruptcy,} \end{cases} \tag{5}$$

and

$$\text{debt payoff} = \begin{cases} D & \text{if repayment,} \\ \theta\lambda v & \text{if bankruptcy.} \end{cases} \tag{6}$$

3.2 Default and the Bankruptcy Filing Decision

To solve backwards, we consider the firm's choice between repayment and filing for bankruptcy, given assets v and debt D at date 1. Comparing the equity holders' payoffs in equation (5), the firm prefers to file when the payoff from filing, $(1-\theta)\lambda v$, is higher than the payoff from repaying, $v - D$, or

$$v \leq \hat{v}(D) := \frac{D}{1-(1-\theta)\lambda}. \tag{7}$$

Notice that, if the deadweight costs of bankruptcy destroy all value ($\lambda = 0$) or the bankruptcy system is perfectly creditor friendly ($\theta = 1$), firms will file for bankruptcy only when the value of the firm's assets v is less than its debt D (i.e., when the firm is "insolvent"). But if bankruptcy preserves at least some value ($\lambda > 0$) and yields some payoff to equity ($\theta < 1$), a firm may file even when it is solvent ($v > D$). The more debtor friendly the law is, the more likely the firm is to file when it is solvent.[16]

3.3 Ex Post Restructuring

Restructuring is a way to avoid bankruptcy and its associated deadweight costs, but it is generally infeasible when there is no uncertainty about firm value.

To see why restructuring is a way to avoid bankruptcy, suppose the firm could convert

[16]These characteristics of the firm's filing decision imply that a firm will rarely, if ever, be forced into bankruptcy by its creditors. Under U.S. law, creditors can file an "involuntary" bankruptcy case against a firm, but they must be prepared to prove that the firm is "generally not paying such debtor's debts as such debts become due" 11 U.S.C. §303(h)(1). Courts have not given a precise or consistent definition of "generally not paying," but it appears to describe a situation where the firm has defaulted on multiple debts that account for a substantial fraction of total debt (Levin and Sommer (2020)). This is a situation close to insolvency, that is, $v \leq D$. Equation (7), however, shows that the firm will choose to file when $v \leq \hat{v}(D)$. As discussed above, $\hat{v}(D)$ will exceed D whenever θ and λ are greater than 0. This suggests that creditor power to start a case is relevant only in the (unusual) situation where the bankruptcy law offers no payout to equity or has no deadweight costs. In practice, involuntary filings account for less than 0.1 percent of all bankruptcy filings (see, e.g., *In re Murray*, 543 B.R. 484, 497 (Bankr. S.D.N.Y. 2018); Hynes and Walt (2020)).

its debt into equity and that this equity will have claim on a fraction $1-\alpha$ of the assets. In the bankruptcy region ($v < \hat{v}$), creditors are better off if the proportion $1-\alpha$ of the assets is higher than the proportion $\theta\lambda$ they get in bankruptcy:

$$(1-\alpha)v \geq \theta\lambda v. \tag{8}$$

The equity holders are also better off if their residual claim to the fraction α of the assets is higher than the proportion $(1-\theta)\lambda$ they get in bankruptcy:

$$\alpha v \geq (1-\theta)\lambda v. \tag{9}$$

Together, these inequalities imply that a restructuring that converts debt to equity is a strict Pareto improvement whenever $v < \hat{v}$:

Result 1. *Suppose $v < \hat{v}$. For any α such that*

$$\lambda - \theta\lambda < \alpha < 1 - \theta\lambda, \tag{10}$$

restructuring debt by converting it to equity worth a fraction $1-\alpha$ of the assets makes the firm and creditors strictly better off.

This result is a corollary of the Coase Theorem: Ex post inefficiencies can be avoided by assigning property rights appropriately. Unfortunately, however, the collective action problem can make it hard to agree on how to assign property rights. In the setting we analyze, creditors cannot coordinate. Thus, they accept the equity share $1-\alpha$ when it makes them better offer individually, which may not coincide with when it makes them better off collectively.

To see why such a Pareto-improving restructuring is nonetheless infeasible, recall that each creditor accepts only if it is better off, assuming other creditors accept. That is, an individual creditor must prefer getting a fraction $1-\alpha$ of the assets to getting repaid the face value D:

$$(1-\alpha)v \geq D. \tag{11}$$

Recall that, because each creditor is infinitesimally small, no single creditor's action has any effect on others' payoffs. Equityholders, too, must be better off from a restructuring than a bankruptcy. Assuming equity holders act collectively, this means that their residual claim on the firm after a restructuring (α) is more valuable than their payoff in bankruptcy, $(1-\theta)\lambda$:

$$\alpha v \geq (1-\theta)\lambda v. \tag{12}$$

These inequalities are incompatible whenever restructuring can avoid bankruptcy (i.e., whenever $v < \hat{v}$). The first implies that αv must be less than $v - D$. The second im-

plies that αv must be greater than $(1-\theta)\lambda v$. But we know from condition (7) that the firm files for bankruptcy only if $(1-\theta)\lambda v$ is greater than $v - D$. Thus, a restructuring is not feasible because, when bankruptcy is credible, no payoff to creditors (αv) can satisfy both inequalities.

Result 2. *There is no ex post restructuring that (uncoordinated) creditors are willing to accept and that the firm is willing to offer.*

Although the argument so far has applied only to restructurings of debt to equity, it is easy to see that no restructuring of old debt to new debt can help either, due to the same hold-out problem we just saw: Conditional on other creditors' taking write-downs to prevent default, an individual creditor anticipates having its original debt repaid in full, and hence is not willing to take a write-down itself.

3.4 Ex Ante Restructuring

Although restructuring is not feasible after the firm's asset value v has been realized (Result 2), a firm could attempt a restructuring preemptively while v remains uncertain. Here we show that preemptive restructurings are feasible, but not necessarily efficient, provided they have two key features: (1) They offers creditors the option to convert old debt to new debt, and (2) the new debt will have priority over the old debt. We start by showing that debt-to-equity restructurings are efficient but infeasible and then turn to debt-to-debt restructurings.

3.4.1 Debt-to-Equity Restructurings

A preemptive restructuring that converts debt to *equity* is Pareto-improving, but infeasible. Assume again that the firm could convert its debt into equity with a claim on $1-\alpha$ of the assets. Creditors are better off if the value of this equity is greater than their expected payoff in bankruptcy:

$$(1-\alpha)\mathbb{E}[v] \geq \mathbb{E}\big[\mathbf{1}_{\{v\geq\hat{v}\}}D_0 + \mathbf{1}_{\{v<\hat{v}\}}\theta\lambda v\big]. \tag{13}$$

Similarly, equity holders are better off if their residual claim (α of the assets) is worth more than what they expect in bankruptcy:

$$\alpha\mathbb{E}[v] \geq \mathbb{E}\big[\mathbf{1}_{\{v\geq\hat{v}\}}(v-D_0) + \mathbf{1}_{\{v<\hat{v}\}}(1-\theta)\lambda v\big] \equiv \mathbb{E}\big[\max\{v-D_0,(1-\theta)\lambda v\}\big]. \tag{14}$$

These inequalities can be re-written and combined as:

$$\mathbb{E}\big[v - \mathbf{1}_{\{v\geq\hat{v}\}}D_0 - \mathbf{1}_{\{v<\hat{v}\}}\lambda\theta v\big] \geq \alpha\mathbb{E}[v] \geq \mathbb{E}\big[v - \mathbf{1}_{\{v\geq\hat{v}\}}D_0 - \mathbf{1}_{\{v<\hat{v}\}}\lambda\theta v\big] - (1-\lambda)\mathbb{E}\big[\mathbf{1}_{\{v<\hat{v}\}}v\big]. \tag{15}$$

Since the left-most term is always strictly greater than the right-most term, an appropriate debt-to-equity restructuring can implement a strict Pareto improvement (and avoid all costs of financial distress):

Result 3. *For any α satisfying condition (15) strictly, restructuring debt to equity worth a fraction of $1-\alpha$ of the assets makes the firm and creditors both strictly better off.*

This result again recalls the Coase Theorem: Ex post inefficiencies are avoided by assigning property rights appropriately.[17] Again, however, the collective action problem can make it hard to agree on how to assign property rights. Even though restructuring debt to equity can fully eliminate inefficiencies (i.e., the costs of financial distress), creditors might not accept due to the same hold-out problem.

To see why, recall that an individual creditor accepts only if it is better off, given that other creditors accept. That is, it must prefer getting a fraction $1-\alpha$ of the assets to holding its original debt with face value D_0. If all other creditors agree to the restructuring, the firm is effectively all equity (assuming the individual creditor is infinitesimally small). A creditor therefore accepts if:

$$(1-\alpha)\mathbb{E}[v] \geq D_0. \tag{16}$$

Similarly, equity holders are better off in a restructuring if their residual claim on the fraction α of the assets is worth more than their bankruptcy payoff, as in inequality (14). These inequalities can be re-written and combined as:

$$\mathbb{E}[v] - D_0 \geq \alpha\mathbb{E}[v] \geq \mathbb{E}[v] - D_0 + \mathbb{E}\big[\mathbf{1}_{\{v<\hat{v}\}}\big\{(1-\theta)\lambda v - (v-D_0)\big\}\big]. \tag{17}$$

The last expectation is positive, because the term in braces is positive for $v < \hat{v}$ by the definition of $\hat{v}$ (equation (7)). Hence, the right-most term is greater than the left-most term; no restructuring of debt to equity is feasible:

Result 4. *There is no ex ante restructuring of debt to equity that (uncoordinated) creditors are willing to accept and the firm is willing to offer.*

3.4.2 Debt-to-Debt Restructurings

So far, we have assumed that the firm's creditors have equal priority in bankruptcy, sharing pro rata in any distribution of assets. In practice, a firm can issue debt with different levels of *seniority*. The value of seniority is that, in the event of liquidation or bankruptcy, senior creditors must be paid before junior creditors receive anything. As we will show below, ex ante restructuring becomes feasible—though not necessarily efficient—when the firm is able

[17] Unlike Result 1, however, the result here is not an immediate corollary of the Coase Theorem because the Theorem does not imply that the same securities eliminate the inefficiency for each realization of v. Equity securities do that here, though, because debt is the only source of inefficiency (i.e., the costs of financial distress).

to issue senior debt. By offering to exchange senior debt (with a lower face value) for existing junior debt, the firm can induce individual creditors to assent to the restructuring because hold-outs will be diluted.[18] This kind of restructuring is feasible ex ante (when v is still uncertain) but not ex post (when v is realized). Ex post, individual creditors do *not* gain from giving up (i) low-priority, high-face-value claims for (ii) high-priority, low-face-value claims if the restructuring eliminates the risk of a bankruptcy filing. Assuming all other creditors agree to the restructuring, the individual creditor should retain its original (low-priority) claim, which will be paid in full after the restructuring. Priority, in other words, is only as valuable as a bankruptcy is likely. This is why a debt-for-debt restructuring is feasible ex ante, when v is uncertain: Even after a restructuring, a bankruptcy filing is still possible in the future because restructuring prevents distress for some but not all realizations of v. Priority is therefore valuable: It puts senior creditors ahead of junior creditors if a bankruptcy occurs.

We begin by showing the feasibility of a restructuring that swaps old debt (D_0) for more senior debt (D). An individual creditor will accept this restructuring if the value of senior debt with face value D is greater than the value of junior debt with face value D_0, given that other creditors are accepting the terms of the restructuring:

$$\Big(1 - F\big(\hat{v}(D)\big)\Big)D + F\big(\hat{v}(D)\big)\mathbb{E}\left[\theta\lambda v \,\middle|\, v < \hat{v}(D)\right] \geq \Big(1 - F\big(\hat{v}(D)\big)\Big)D_0. \tag{18}$$

The right-hand side of the inequality measures the expected payoff to junior debt, which is paid D_0 if there is no future default and zero otherwise. The reason the payoff is zero in default is that in bankruptcy not all debt is paid in full ($D > \hat{v}$ by equation (7)) and, since all other debt is senior, the payoff to junior debt is zero.[19]

Equity holders will also accept a restructuring—swapping junior debt for senior debt—if their residual claim on the assets is more valuable than their expected payoff in the absence of a restructuring. Note that the seniority of debt has no effect on the payoff to equity: Whether debt is senior or junior, it still has priority over equity. This means that equity holders will accept the restructuring whenever it reduces the face value of the debt, or $D \leq D_0$.

Rearranging these inequalities, we obtain the following result, which describes the feasibility of a restructuring that reduces the face value of debt by $D_0 - D$:

[18] Optimal "dilutable debt" also appears in Diamond (1993), Donaldson, Gromb, and Piacentino (2018, 2020), Donaldson and Piacentino (2020), and Hart and Moore (1995).

[19] We are assuming that senior debt is always paid ahead of junior debt. That is, there are no deviations from the APR that favor junior creditors at the expense of senior creditors. Although this assumption appears to be a good approximation of reality (Bris, Welch, and Zhu (2006)), we relax it in Section 5.1. Moreover, we show that deviations favoring junior creditors at the expense of senior debt are suboptimal from a welfare point of view.

Result 5. *For any D such that*

$$D_0 - D \leq \frac{F\big(\hat{v}(D)\big)}{1 - F\big(\hat{v}(D)\big)} \mathbb{E}\left[\theta\lambda v \,\middle|\, v \leq \hat{v}(D)\right], \tag{19}$$

restructuring the initial debt D_0 to senior debt with face value $D < D_0$ is accepted by creditors and makes the firm strictly better off.

Inequality (19) makes clear that the feasibility of a restructuring increases with both (i) the likelihood of a bankruptcy filing $F(\hat{v})$ and (ii) creditors' recovery value in default $\mathbb{E}\left[\theta\lambda v \,\middle|\, v \leq \hat{v}(D)\right]$. This is intuitive because both increase the value of priority. Indeed, if a firm never goes bankrupt, priority has no value—even the last creditor will be repaid in full. And, similarly, if creditors' recovery value is low, priority has no value—even the first creditor might not be repaid in full. These two pieces underlie all of our main results.

Building on this result, we explore how the parameters of the bankruptcy environment—deadweight costs (λ) and creditor friendliness (θ)—affect the ability to restructure. Despite the intuitive pieces underlying them, the comparative statics can be counter-intuitive because θ and λ affect restructuring not only directly, but also indirectly via $\hat{v}$ (cf. equation (7)).

Before turning to comparative statics, we pause to note that, although restructuring increases efficiency because it decreases financial distress costs, it is possible (at least theoretically) that a restructuring will not yield a Pareto improvement. It could instead constitute a "coercive exchange" in which creditors accept a restructuring that makes them worse off because they want to avoid being diluted by new senior debt. This so-called "hold in" problem appears to be more of a theoretical possibility than a practical reality. Restructurings generally do not harm creditors (Chatterjee, Dhillon, and Ramãrez (1995)). Additionally, restructurings do implement Pareto improvements in our model whenever distress is sufficiently costly because, in such a case, creditors benefit more from avoiding distress than they suffer from write-downs.[20]

[20]It may be useful to illustrate how a marginal decrease in debt can still make creditors better off, because they prefer to get paid a smaller amount with higher probability than a larger amount with lower probability. Creditors with debt D_0 are better off decreasing debt if

$$\frac{\partial}{\partial D}\bigg|_{D=D_0} \left(\Big(1 - F\big(\hat{v}(D)\big)\Big) D + F\big(\hat{v}(D)\big) \mathbb{E}\left[\lambda\theta v \,\middle|\, v < \hat{v}(D)\right] \right) < 0 \tag{20}$$

or, computing,

$$\frac{1 - F\big(\hat{v}(D_0)\big)}{f\big(\hat{v}(D_0)\big)\hat{v}(D_0)} < \frac{1-\lambda}{1-(1-\theta)\lambda}. \tag{21}$$

Let us make two observations. (i) The condition can be satisfied only if λ is sufficiently small: If $\lambda = 1$, there are no bankruptcy costs to avoid by reducing debt, so creditors are always better off with more debt. (ii) It can be satisfied more easily when $f\big(\hat{v}(D_0)\big)$ is large—that is, when a small reduction in debt from D_0 has a significant impact on the probability of default. At any rate, even if they do not implement Pareto improvements,

3.5 Write-downs and Secured Credit Spreads

Before turning to comparative statics, we pause to interpret the bound on the feasible write-down $D_0 - D$ (inequality (19)) in terms of market prices. To do so, we can re-write the condition for creditors to accept a restructuring (inequality (18)) in terms of continuously compounded yields-to-maturity, y^s and y^u, conditional on the write-down:

$$De^{-y^s} \geq D_0 e^{-y^u}. \tag{22}$$

Here, y^s is the yield on secured/senior debt (which creditors receive in a restructuring); y^u is the yield on unsecured/junior bonds (which they exchange). This is equivalent to:

$$\log\left(\frac{D_0}{D}\right) \leq y^u - y^s. \tag{23}$$

The left-hand side is approximately the proportion of debt that can be written down $(D_0 - D)/D_0$. The right-hand side is the spread between secured and unsecured credit. We thus have the following approximation:[21]

$$\text{max \% write-down} \;\leq\; \text{secured credit spread}. \tag{25}$$

This inequality captures the basic intuition of the hold-out problem in our model. Creditors are willing to accept write-downs only to the extent that seniority is valuable (as measured by the secured credit spread). This inequality also suggests a way to estimate feasible write-downs. Secured credit spreads are, in principle, observable.[22]

Finally, inequality (25) offers an unusual perspective on the timing of restructuring. Observed spreads are substantial only when firms are in trouble (Benmelech, Kumar, and Rajan (2020)). Thus, debt restructuring could be rare in good times not only because it is not valuable (because expected distress costs are low), but also because it is infeasible (because secured credit spreads are low).

3.6 How the Costs of Bankruptcy Affect Restructuring

A restructuring is feasible if the debt write-down, $D_0 - D$, satisfies the constraint in inequality (19). The maximum feasible write-down renders this an equality. Here we explore

restructurings do always increase total surplus in our model.

[21] Using $\log(1-x) \approx -x$, we can re-write the left-hand side of inequality (23):

$$\log\left(\frac{D_0}{D}\right) = -\log\left(1 - \frac{D_0 - D}{D_0}\right) \approx \frac{D_0 - D}{D_0}. \tag{24}$$

[22] Although, to be precise, the spread must be conditional on successful restructuring. Additionally, to measure the spread in practice, the firm must have some other debt that is not restructured.

how the maximum feasible write-down varies with the deadweight costs of bankruptcy (λ). Is the new face value D that makes creditors indifferent between accepting and rejecting a restructuring—which makes inequality (19) bind—increasing or decreasing in λ?

Define an individual creditor's gain from restructuring relative to bankruptcy, given others accept, as Δ. Using inequality (19), we can write Δ as follows:

$$\Delta := \Big(1 - F\big(\hat{v}(D)\big)\Big)\big(D - D_0\big) + F\big(\hat{v}(D)\big)\mathbb{E}\left[\theta\lambda v \,\middle|\, v < \hat{v}(D)\right]. \tag{26}$$

This is the creditors' incentive compatibility constraint (IC). The maximum write-down, or lowest face value D^*, corresponds to $\Delta = 0$. Differentiating D^* with respect to λ, we obtain the next result:

Result 6. Bankruptcy Costs*: Reducing bankruptcy costs (increasing λ) facilitates restructuring in the sense that the maximum write-down $D_0 - D^*$ is increasing in λ.*

This is a central result of our paper: Restructuring and bankruptcy are complements, not substitutes. This is true for two reasons:

1. The more efficient bankruptcy is, the more likely the firm is to file, and priority in bankruptcy is more valuable when it is more likely.
2. The more efficient bankruptcy is, the more creditors get in bankruptcy, and priority in bankruptcy is more valuable when recovery values are higher.[23]

In other words, as bankruptcy costs fall, priority in bankruptcy becomes more valuable, which increases the likelihood that creditors will accept write-downs in exchange for priority. Hence, contrary to common intuition, policies that reduce bankruptcy costs actually facilitate out-of-court restructuring. This adds support to Brunnermeier and Krishnamurthy's (2020, p. 6) conclusion that "reducing the cost of bankruptcy is unambiguously beneficial to society."

3.7 How the Creditor Friendliness of Bankruptcy Affects Restructuring

The feasibility of a restructuring also depends on the creditor friendliness of the bankruptcy system. As in the previous subsection, we focus on the maximum feasible write-down, D^*, corresponding to $\Delta = 0$ in equation (26). Differentiating D^* with respect to θ, we obtain our next result:

[23] To see why, recall, from the IC in inequality (18), that creditors accept a restructuring only if the payoff they get in bankruptcy from accepting senior debt is high relative to the payoff they get in bankruptcy from holding out, which, conditional on others accepting, is zero. Thus, as senior creditors' payoff in bankruptcy increases, so does the write-down creditors are willing to accept.

Result 7. CREDITOR FRIENDLINESS*: An increase in creditor friendliness (θ) facilitates restructuring, in the sense that the maximum feasible write-down $D_0 - D^*$ increases, if and only if $\partial\Delta/\partial\theta$ is positive (see equation (53) in the Appendix). Moreover, if*

$$\frac{1 - F\big(D^*_{\theta=1}\big)}{D^*_{\theta=1} f\big(D^*_{\theta=1}\big)} < \lambda, \tag{27}$$

where $D^*_{\theta=1}$ denotes the solution to $\Delta = 0$ with $\theta = 1$, then there is an interior level of creditor friendliness $\theta^* \in (0,1)$ that maximizes the feasible write-down.

The ambiguity in this result stems from the fact that, although restructuring is facilitated when priority in bankruptcy becomes more valuable, creditor friendliness has two effects on the value of priority:

1. By increasing what creditors receive in the event of bankruptcy, creditor friendliness makes priority *more* valuable.
2. By reducing the payoff to equity holders, creditor friendliness reduces their incentive to file for bankruptcy, which makes priority less valuable.

Condition (27) is important because it tells us that, when creditor friendliness is very high (θ is near 1), further increases in θ reduce the likelihood of a successful restructuring. This implies that the optimal level of creditor friendliness is less than 1 ($\theta^* < 1$). In other words, the optimal bankruptcy system does not maximize creditor recoveries. Systems that are too creditor friendly are inefficient because they discourage cost-reducing restructurings. The U.S. may be one such system, as we illustrate in the next section. Before that, however, we illustrate the result above with an example.

3.7.1 Example: Optimal Creditor Friendliness If v Is Uniform

Here, we illustrate the results above for v uniform, $F(v) \equiv v/\bar{v}$ on $[0, \bar{v}]$. In this case, creditors' binding IC ($\Delta = 0$ in equation (26)) becomes:

$$\frac{\lambda\theta}{2\bar{v}}\big(\hat{v}(D^*)\big)^2 = (D_0 - D^*)\left(1 - \frac{\hat{v}(D^*)}{\bar{v}}\right). \tag{28}$$

Substituting for $\hat{v}$ from equation (7) and solving gives:[24]

$$D^* = \frac{\big(1 - (1-\theta)\lambda\big)\bar{v} + D_0 - \sqrt{\Big(D_0 - \big(1 - (1-\theta)\lambda\big)\bar{v}\Big)^2 + 2\lambda\theta\bar{v}D_0}}{2 - \frac{\lambda\theta}{1-(1-\theta)\lambda}}. \tag{29}$$

This expression illustrates that the write-down $D_0 - D^*$ is increasing in λ, as per Result 6, and is hump-shaped in θ, as per Result 7; see Figure 1.

[24]The other root of the quadratic is larger than D_0 and is omitted.

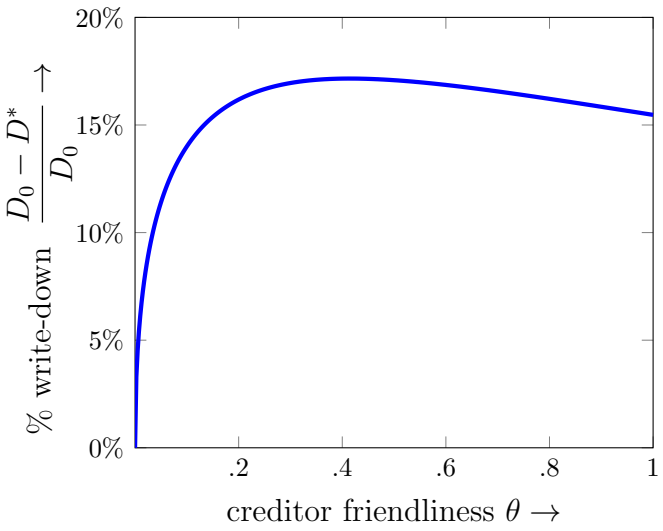


Figure 1: The percentage write-down for uniform v on $[0, \bar{v}]$, with $\bar{v} = 100$, $D_0 = 50$, and $\lambda = 1/2$.

3.8 Is the U.S. Bankruptcy Code Too Creditor Friendly?

We can use our model to assess whether existing laws are excessively creditor friendly in the sense that a marginal reduction in creditor friendliness would increase the feasibility of restructurings. To do this, we assume that D^* is a continuous function of θ with a unique local minimum.[25] Under this assumption, we can say that a bankruptcy system is too creditor friendly if an increase in θ leads to an increase in D^*, the minimum debt level attainable in restructuring:

$$\frac{\partial D^*}{\partial \theta} > 0. \tag{30}$$

In Appendix B.2, we calculate that a sufficient condition for this inequality to hold is:

$$1 < \lambda \theta \hat{v} \frac{\partial F(\hat{v})}{\partial D}. \tag{31}$$

Focusing on the terms on the right-hand side, this condition says that the Code is too creditor friendly if (i) creditors' total recovery value for the marginal bankruptcy firm is already high (i.e., $\lambda\theta\hat{v}$ is high), or (ii) the likelihood that the firm files is highly sensitive to its indebtedness ($\partial F/\partial D$ is high). When $\lambda\theta\hat{v}$ is already high, the marginal value of a further increase in recovery value is low. When $\partial F/\partial D$ is high, the marginal value of a decrease in debt is high. That is, even a small reduction in the face value of debt, via a restructuring, would yield a large reduction in the probability of bankruptcy, thereby avoiding deadweight costs.

A back-of-the-envelope calculation allows us to get a preliminary sense of whether con-

[25] That is, an increase in creditor friendliness either always increases, always decreases, or first increases and then decreases the maximum feasible write-down (based on numerical examples, we think that this assumption is satisfied for commonly-used distributions).

dition (31) is satisfied in the U.S. today. The empirical literature provides approximations for each term on the right hand side:

- λ: This term captures the direct costs of bankruptcy. In studies of corporate reorganizations (mostly involving large corporations), the literature consistently estimates $\lambda > 90$ percent. (See Hotchkiss et al. (2008), Table 1, for a summary of twelve studies.)
- θ: A number of papers investigate the value retained by equity holders in bankruptcy. They suggest $\theta > 85$ percent is a conservative lower bound—in most cases, creditors are paid in full before equity is paid anything. (See Hotchkiss et al. (2008), Section 5.1, for a summary of estimates.)
- $\hat{v}\partial F(\hat{v})/\partial D$: To approximate this term, suppose a firm is near bankruptcy, in the sense that the current value of assets, which we denote by v_0, is close to the threshold $\hat{v}$. In this case, we can write:

$$\hat{v}\frac{\partial F(\hat{v})}{\partial D} \approx \frac{\partial F(\hat{v})}{\partial (D/v_0)}. \tag{32}$$

  This term thus measures the sensitivity of the default probability to the level of leverage, D/v_0 (holding the current value of assets constant). An estimate of this sensitivity can be derived from Campbell, Hilscher, and Szilagyi (2008). In a logistic regression of the bankruptcy-filing probability against leverage (and other controls, including market capitalization), the authors estimate a coefficient on leverage equal to 5.38. Using the "divide-by-four" rule (Gelman and Hill (2007)), this coefficient translates to a marginal effect equal to 1.35. We view this as a lower-bound estimate of condition (32) because Campbell, Hilscher, and Szilagyi (2008) study a sample of healthy and distressed firms, whereas our model pertains to distressed firms. The sensitivity of bankruptcy to leverage is likely highest for distressed firms. Indeed, healthy firms take on leverage for a variety of reasons, such as financing profitable investments or controlling managerial free cash-flow problems, that could even be negatively related to the probability of bankruptcy.

Taking these numbers at face value, we can calculate the right-hand side of condition (31):

$$\lambda\theta\hat{v}\frac{\partial F(\hat{v})}{\partial D} > 90\% \times 85\% \times 135\% \approx 103\%. \tag{33}$$

This is greater than one, implying that the condition (31) is satisfied in the U.S. Note that this condition is sufficient, but far from necessary, and that the numbers we plug in are conservative. This leads us to believe that current law is likely too creditor friendly. Giambona, Lopez-de Silanes, and Matta (2019) provide evidence supporting this conclusion. They find that an exogenous increase in creditor protection led to an increase in bankruptcy

filings. This could be surprising because nearly all bankruptcies are initiated by debtors—why should they file bankruptcy more often when they expect less in bankruptcy?—but it is consistent with our calculations, which show that creditor-friendly bankruptcy rules can impede restructuring, resulting in more bankruptcies.

4 Relief Policy

Here, we turn to the policy implications of our model. We consider the vantage point of a (utilitarian) social planner choosing how to allocate a marginal dollar ("subsidy"). We think this is a useful exercise because, during the ongoing COVID-19 pandemic, policymakers have implemented subsidies to aid firms outside of bankruptcy (e.g., the CARES Act) and various commentators have proposed programs to aid firms in bankruptcy (e.g., DeMarzo, Krishnamurthy, and Rauh (2020)). We begin by showing that the social planner must take into account not only the effect of the subsidy on the likelihood of financial distress, but also its effect on restructuring. Then, we consider specific policies, including subsidies granted unconditionally, conditional on restructuring, and conditional on bankruptcy. Finally, we compare these policies and show that the most effective policies are those that facilitate restructuring either by subsidizing it directly or by subsidizing bankruptcy.

4.1 Planner's Problem for a Marginal Dollar

In our model, social welfare is firm value. Since firm value is maximized when the default probability $F(\hat{v})$ is minimized, the planner's objective is simply to minimize $\hat{v}$. We denote the set of subsidies the planner can deploy by the vector $\mathbf{s}$ and their associated costs by $\mathbf{q}$. For example, s_i could capture a direct subsidy to the firm's assets, such as cash grants or forgivable loans (both of which have been deployed during the pandemic). In this case, $q_i = 1$, because the planner pays a dollar to increase the firm's assets by a dollar. Similarly, s_j could capture a subsidy to the firm's assets conditional on filing for bankruptcy. In this case, $q_j = F(\hat{v})$, because the planner pays a dollar to increase the firm's assets by a dollar only if the firm files for bankruptcy, which happens with probability $F(\hat{v})$. Thus, if the planner's budget is ε, its budget constraint is $\mathbf{q} \cdot \mathbf{s} = \varepsilon$.

Although we allow the planner to make its subsidies contingent on bankruptcy and restructuring, we do not allow it to force creditors to discharge debt outside of bankruptcy. Put differently, the planner must respect creditors' IC, shown in inequality (18) (equivalently, $\Delta(\mathbf{s}) = 0$).

Thus, the planner's problem is:

$$\begin{cases} \min & \hat{v}(\mathbf{s}) \\ \text{s.t.} & \Delta(\mathbf{s}) = 0 \\ \& & \mathbf{q} \cdot \mathbf{s} = \varepsilon \end{cases} \tag{34}$$

over feasible subsidies $\mathbf{s}$.

Because we are interested in how the planner can best spend a marginal dollar, we focus on ε near zero. Writing the optimal policy as a function of the budget, $\mathbf{s} = \mathbf{s}(\varepsilon)$, we can approximate the social planner's objective as:

$$\hat{v}\big(\mathbf{s}(\varepsilon)\big) = \hat{v}\big(\mathbf{s}(0)\big) + \varepsilon \frac{\mathrm{d}}{\mathrm{d}\varepsilon}\hat{v}\big(\mathbf{s}(0)\big). \tag{35}$$

This captures the idea that the planner can allocate its budget little by little: It should select subsidies to maximize the marginal impact on the objective—to maximize $\mathrm{d}\hat{v}/\mathrm{d}\varepsilon$—subject to the creditors' IC ($\Delta(\mathbf{s}) = 0$).

Below, we analyze and compare policies. As a shorthand, we let $\mathbf{s}_i$ be the policy that gives subsidy s_i to the ith policy and nothing to the others. For such one-dimensional policies, the planner's objective is to minimize

$$\left.\frac{\mathrm{d}\hat{v}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left.\frac{\partial \hat{v}}{\partial s_i}\frac{\mathrm{d}s_i}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0}, \tag{36}$$

over feasible policies $\mathbf{s}_i$, subject to the constraints. This expression lends itself to interpretation. Begin with the second term on the right-hand side ($\frac{\mathrm{d}s_i}{\mathrm{d}\varepsilon}$). If we differentiate the budget constraint, we see that this term is the reciprocal of the cost of the ith policy,

$$\frac{\mathrm{d}s_i}{\mathrm{d}\varepsilon} = \frac{1}{q_i}. \tag{37}$$

The planner's objective is therefore:

$$\left.\frac{\partial \hat{v}}{\partial s_i}\right|_{s_i=0} \times \frac{1}{q_i} \equiv \text{“policy efficacy”} \times \text{“bang for the buck”}. \tag{38}$$

subject to creditors' IC.

This IC plays a central role in our policy analysis below. The planner must account for the effect of each policy on creditors' private incentives to accept a restructuring. To see how, observe that both $\hat{v}$ and the IC depend on the equilibrium level of debt post-restructuring, which in turn depends on the subsidy scheme: $D = D(\mathbf{s})$. Taking this into

account and applying the chain rule twice, we can re-write the objective as:

$$\left.\frac{\mathrm{d}\hat{v}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left(\frac{\partial \hat{v}}{\partial s_i} + \frac{\partial \hat{v}}{\partial D}\frac{\partial D}{\partial s_i}\right)\frac{1}{q_i}\Bigg|_{s_i=0} \tag{39}$$

$$= \left(\frac{\partial \hat{v}}{\partial s_i} - \frac{\partial \hat{v}}{\partial D}\frac{\partial \Delta/\partial s_i}{\partial \Delta/\partial D}\right)\frac{1}{q_i}\Bigg|_{s_i=0}. \tag{40}$$

The first term captures the direct effect of a subsidy. But it is far from the whole story. The second term captures its indirect effect via restructuring: The product of how the debt level affects the default threshold ($\frac{\partial \hat{v}}{\partial D}$) and how the subsidy affects the debt level via the IC ($\frac{\partial \Delta/\partial s_i}{\partial \Delta/\partial D}$), all times the "bang for the buck" ($1/q_i$).

4.2 Feasible Policies

We use expression (40) to evaluate two groups of policies: ex post policies that target firms in bankruptcy and ex ante policies that target firms prior to bankruptcy. Ex post policies have the advantage of lower fiscal costs (due to the lower probability of actually paying the subsidy), but the potential disadvantage of distorting firms' decisions, as we characterize below.

Ex post policies include:

1. **Asset subsidies**. These include, for example, cash subsidies to firms in bankruptcy.
2. **Equity subsidies.** These include policies that permit shareholders to retain ownership interests during a bankruptcy reorganization. For example, small-business bankruptcy laws were recently amended to permit reorganization plans that allow owners to retain their interests, as discussed in Morrison and Saavedra (2020).
3. **Loan subsidies.** These policies increase both assets and debt by extending credit at below-market rates. One example is DeMarzo, Krishnamurthy, and Rauh's (2020) proposed debtor-in-possession financing facility (DIPFF), which would offer subsidized financing to firms in bankruptcy. The distributional impact of these subsidies depends on the priority of the new loans: if the loans are junior to senior debt, they are a subsidy to senior debt; if they are senior to existing debt, they function as a subsidy to equity. Below we focus on ex post loan subsidies that benefit creditors.

Ex ante policies are similar, but broader in scope:

4. **Asset subsidies**. These include include cash grants (such as those paid to the airlines under the CARES Act) and forgivable loans (such as those paid under the Paycheck Protection Program).
5. **Loan subsidies.** These include the various facilities launched by the Federal Reserve during the current crisis (e.g., Primary Market Corporate Credit Facility, Main Street Lending Program).

6. **Debt subsidies.** These policies reduce debt by repurchasing it at the market price (before restructuring) and then forgiving it. They bear some resemblance to quantitative easing programs in which central banks purchase corporate debt, with the twist that the central bank then does not enforce repayment on the debt.

7. **Restructuring subsidies.** The government could simply reward creditors who participate in a restructuring. One way to do this is to alter the tax consequences of restructurings, as discussed in Campello, Ladika, and Matta (2018). Another is for the government to announce that, if creditors agree to write-downs, the government will agree to even larger write-downs of its own claims, as discussed in Blanchard, Philippon, and Pisani-Ferry (2020).

In Appendix B.3, we formalize these policies. For each Policy i, we describe its direct cost q_i (the probability the subsidy is paid), its direct effect via the filing decision (the change in the bankruptcy threshold $\hat{v}$), and its indirect effect via restructuring (the way D is affected through the IC). We find that we can rank the policies by simply comparing $\mathrm{d}\hat{v}_{\mathbf{s}_i}/\mathrm{d}\varepsilon$ for each policy. This leads to our next result:

Result 8. Policy Comparison*: Ex ante debt subsidies (Policy 6) and restructuring subsidies (Policy 7) are equivalent to ex post loan subsidies (Policy 3) and are strictly preferred to all other policies. Further, ex ante asset subsidies (Policy 4) are preferred to both ex ante subsidized lending (Policy 5) and ex post bankruptcy subsidies for equity (Policy 2). Finally, ex post bankruptcy subsidies for assets (Policy 1) are preferred to ex post bankruptcy subsidies for equity (Policy 2). (The rest of the ranking is ambiguous.)*

In other words, the best policies—ex ante restructuring subsidies, ex ante debt purchases, and ex post loan subsidies—are those that facilitate restructuring alone. Indeed, none of these policies has any direct effect on filing: Each affects welfare indirectly by increasing creditors' payoff from accepting a restructuring by ε, thereby slackening the IC:

- Because restructuring subsidies (Policy 7) go directly to creditors who accept restructuring, they increase creditors' payoff from accepting by ε. This mechanically slackens the IC by ε.
- Ex post loan subsidies (Policy 3) increase the value of priority. This is because, if creditors accept a restructuring, they get the entire asset value in bankruptcy. The subsidies are similar to a reduction in the costs of bankruptcy borne by creditors. It turns out that this slackens the IC by ε.[26]

[26]These subsidies increase creditors' payoff from accepting senior debt by $F \cdot s_3$, where F is the probability of bankruptcy. Given $s_3 = \varepsilon/F$ ($q_3 = F$ in the planner's budget constraint), this slackens the IC by ε.

- Debt subsidies (Policy 6) decrease each creditor's initial debt.[27] Because of this, they decrease creditors' payoff from holding out. It turns out this slackens the IC by ε.[28]

5 Extensions

5.1 Secured Creditor Power and Priority Rules

We have assumed thus far that senior debt is paid strictly before junior debt in bankruptcy. In other words, there are no deviations from the "absolute priority rule" (APR) that favor unsecured creditors at the expense of secured creditors.[29] Although this is a good first approximation, in practice the division of surplus between secured and unsecured creditors depends on post-filing decisions, such as the decision to liquidate or reorganize. Liquidation is likely to favor secured creditors who seek quick payouts, whereas reorganization is likely to favor unsecured creditors who want to gamble on the going concern. (Other decisions affecting the division of surplus include whether to incur post-petition (DIP) financing, liquidate assets or the entire firm, or litigate priority disputes.) And, even though all bankruptcy decisions are overseen by a bankruptcy judge, many scholars have shown that secured creditors exert substantial influence over the bankruptcy process.

Here, we extend the model to capture different levels of secured creditor power, which we denote by ρ. Specifically, as in the baseline model, we assume that there are two classes of debt—senior/secured and junior/unsecured. Unlike the baseline model, however, we also assume that senior creditors are more likely to be paid first as their power (ρ) increases. Specifically, we assume that senior debt is paid first with probability ρ, but shares pro-rata with junior debt with probability $1-\rho$ (i.e., they are treated as if they are equal in priority). We still assume that equity gets a fraction $1-\theta$ of the value in bankruptcy. What is changing here is how the fraction θ is divided among creditors.

To explore how ρ affects restructuring, we start with the creditors' IC to accept a write-

[27] Here, we interpret the subsidy as decreasing each creditor's debt by, for example, buying a small amount from every debt holder. We could also interpret it as decreasing the total amount of debt by, for example, buying all the debt of a small number of debt holders. The two interpretations are mathematically equivalent, but give different perspectives on the creditors' IC. The latter slackens the IC because it sweetens the deal from accepting, since the bankruptcy payoff is divided among fewer creditors. In contrast, the former slackens the IC because it sours the deal for hold-outs, as described above.

[28] The policy decreases creditors' payoff from holding out by $(1-F)s_6$, where $1-F$ is the probability of repayment. Given $s_6 = \varepsilon/(1-F)$ ($q_6 = 1-F$ in the planner's budget constraint), this slackens the IC by ε.

[29] At the same time, we have assumed that equity recovers something in bankruptcy ($(1-\theta)\lambda v$) even if debt is not paid in full. We have, in other words, assumed that there are deviations from the APR that favor equity at the expense of debt. Our analysis above suggests that high θ could impede restructuring (Result 7). This means that violations of debt-equity priority can be optimal. Here we show the opposite is true for violations of secured-unsecured priority: They make restructuring harder.

down from D_0 to D:

$$\begin{aligned}\Big(1-F\big(\hat{v}(D)\big)\Big)D+F\big(\hat{v}(D)\big)\mathbb{E}\left[\theta\lambda v\,\middle|\,v<\hat{v}(D)\right]\\ \geq\Big(1-F\big(\hat{v}(D)\big)\Big)D_0+(1-\rho)F\big(\hat{v}(D)\big)\mathbb{E}\left[\theta\lambda v\,\middle|\,v<\hat{v}(D)\right]\frac{D_0}{D}.\end{aligned}\tag{41}$$

The difference between the above and condition (18) is that, with probability $1-\rho$, a hold-out creditor's junior debt receives a positive recovery value in bankruptcy (an accepting creditor's payoff is unchanged because it takes as given that others accept). Rearranging, we see that a write-down $D-D_0$ is feasible if:

$$D_0-D\leq\frac{\theta\lambda F\big(\hat{v}(D)\big)}{1-F\big(\hat{v}(D)\big)}\mathbb{E}\left[v\,\middle|\,v\leq\hat{v}(D)\right]\left(1-(1-\rho)\frac{D_0}{D}\right).\tag{42}$$

This is identical to the original feasibility condition (19) except for the final expression in brackets on the right-hand side. Indeed, when the APR is enforced strictly ($\rho=1$), inequality (42) reduces to the original feasibility condition. Because the right-hand side is increasing in ρ, we have the next result:

Result 9. *Strict enforcement of the priority of senior over junior debt, i.e. $\rho=1$, facilitates restructuring, in the sense that it maximizes the feasible write-down in inequality (42).*

This suggests a counterpoint to arguments that the APR between secured and unsecured debt should be relaxed (e.g., Bebchuk and Fried (1996)). Advocates of this view often emphasize that the APR gives secured debt power to dilute unsecured debt. Our model reflects this power, but suggests that this power is not necessarily inefficient because it facilitates restructuring and thereby helps circumvent financial distress. Thus, our model helps rationalize observed practice: Equity–debt violations are more common than secured–unsecured violations (Bris, Welch, and Zhu (2006)).

We now ask how secured creditor power affects the write-down-maximizing level of creditor friendliness: If secured creditors get more relative to unsecured creditors (ρ is higher), should creditors as a whole get more or less relative to equity (i.e., should θ be higher or lower) to maximize the write-down D_0-D? We find that they should get more:

Result 10. *Suppose that the write-down is maximized at a unique interior level of creditor friendliness θ^* that is not an inflection point (as in, e.g., the uniform case in Figure 1). Increasing the secured creditor power ρ increases the optimal level of creditor friendliness. That is,* $\mathrm{d}\theta^*/\mathrm{d}\rho>0$.

To see the intuition for this result, recall that θ^* is chosen to maximize the value of priority, balancing the increase in creditor recovery value against the decrease in the filing probability. Because high secured creditor power ρ increases recovery value without affecting the filing probability, θ^* increases to balance the two effects.

5.1.1 Inefficiencies of Secured Creditor Control

Our analysis so far has assumed that secured creditor control amounts to a transfer from unsecured creditors. It could instead reduce total surplus. For example, secured creditors could force quick sales, potentially at fire sale prices, at the expense of other claimants as Ayotte and Ellias (2020), Antill (2020), and Ayotte and Morrison (2009) document. In light of this evidence, we now relax the assumption that liquidation costs do not depend on the division of surplus. We assume instead that secured creditor power can lead to inefficient liquidation.

To capture the inefficiencies resulting from secured creditor power, we assume that unsecured debt gets only a fraction ζ of what is left after secured debt and equity are paid. Thus, $1-\zeta$ captures the inefficiency of secured creditor power. With this modification, the creditors' IC in equation (41) becomes

$$\begin{aligned}\Big(1-F\big(\hat{v}(D)\big)\Big)D+F\big(\hat{v}(D)\big)\mathbb{E}\left[\theta\lambda v\,\middle|\,v<\hat{v}(D)\right]\\ \geq\Big(1-F\big(\hat{v}(D)\big)\Big)D_0+\zeta(1-\rho)F\big(\hat{v}(D)\big)\mathbb{E}\left[\theta\lambda v\,\middle|\,v<\hat{v}(D)\right]\frac{D_0}{D}.\end{aligned}\tag{43}$$

Observe that ζ above plays exactly the same role as $1-\rho$ in equation (42). Therefore, Results 9 and 10 imply that an increase in ζ makes restructuring harder and reduces the optimal level of creditor friendliness θ^*. In words, the inefficiencies of secured creditor power $1-\zeta$ are actually good for restructuring and suggest that a more creditor-friendly Code is optimal. The intuition mirrors that for the results above: As the bankruptcy payoff to unsecured debt decreases, their payoff from holding out decreases, inducing them to accept write-downs.

But what happens when secured creditor power imposes inefficiencies on equity holders? We address this question in Appendix B.4 and show that, more conventionally, the Code should be less creditor friendly when creditors impose greater deadweight costs.

5.1.2 Tort claimants

Priority rules appear in another place in policy debates: Should tort claimants—"accidental" or "involuntary" creditors—be treated on par with or ahead of other creditors in bankruptcy? Our model allows us to evaluate the effects of alternative priority rules on the likelihood of restructuring before bankruptcy.

To address this question in reduced form, suppose the firm has outstanding tort claims equal to T. If T is not paid in full prior to a bankruptcy filing, different priority rules correspond to different types of taxes in bankruptcy: If tort claims are treated on-par with secured claims, they are equivalent to tax τ_s on senior debt. If they are treated on-par with unsecured debt, they represent a tax τ_j on junior debt. If they are junior to unsecured

debt, there is no tax on creditors (and they will often go unpaid).

With this set-up, we can return to creditors' incentive to accept a restructuring. Their IC becomes:

$$\begin{aligned}&\big(1-F(\hat{v}(D+T))\big)D+F\big(\hat{v}(D+T)\big)\mathbb{E}\left[(1-\tau_s)\theta\lambda v\,\middle|\,v<\hat{v}(D+T)\right]\geq\\&\big(1-F(\hat{v}(D+T))\big)D_0+(1-\rho)F(\hat{v}(D+T))\mathbb{E}\left[(1-\tau_j)\theta\lambda v\,\middle|\,v<\hat{v}(D+T)\right]\frac{D_0}{D}.\end{aligned}\tag{44}$$

This condition is easiest to satisfy if τ_s is small and τ_j is large. This suggests that, to facilitate restructuring, tort claimants should be paid behind secured debt, but ahead of unsecured debt. That ordering makes priority valuable by increasing (i) the value of secured debt and (ii) decreasing the value of unsecured debt. Put differently, by giving tort claimants priority above junior debt but below senior debt, the law increases the difference in payoffs between junior and senior debt. This makes priority more valuable and facilitates restructuring.

5.2 Court Congestion

We have assumed thus far that the costs of bankruptcy do not depend on whether restructuring occurs. This is reasonable for an individual firm because a single restructuring is unlikely to affect the efficiency of courts. However, taken in aggregate, restructurings can affect the costs of bankruptcy—if there are more out-of-court restructurings, fewer firms will file for bankruptcy, and courts are likely to be less congested and more efficient. In this section, we allow the costs of bankruptcy to be increasing in the number of firms that file and we show that this creates a feedback loop that amplifies the effects of bankruptcy costs on the hold-out problem.

We assume a unit of identical firms and that the costs of bankruptcy increase with the number of firms that file. By the law of large numbers, the number of firms that file for bankruptcy is equal to the probability that any individual firm files, or $F(\hat{v})$. We assume that a bankruptcy court can receive a maximum number of filings (κ) before experiencing "congestion costs" that increase the deadweight costs of bankruptcy (see, e.g., Iverson (2018)). A simple way to express this idea is to write bankruptcy costs as follows:

$$\text{bankruptcy costs } = 1-\lambda_H-\mathbb{1}_{\{F(\hat{v})>\kappa\}}(\lambda_L-\lambda_H),\tag{45}$$

which says that bankruptcy costs are equal to $1-\lambda_H$ if the number of bankruptcies $F(\hat{v})$ is below the threshold "court capacity" κ and increase to $1-\lambda_L$ if they are above it. When $\kappa=1$, this corresponds to the baseline model with $\lambda=\lambda_H$; when $\kappa=0$. it corresponds to the baseline model with $\lambda=\lambda_L$.

We can analyze the effects of congestion on the feasibility of restructuring by appealing

to Result 6: High bankruptcy costs impede restructuring, making it hard to reduce debt, and hence making bankruptcy itself more likely. This can create an amplification spiral, with high bankruptcy filings having a feedback effect on restructurings, inducing even more filings. As bankruptcy filings exceed the court's threshold ($F(\hat{v}) > \kappa$), bankruptcy costs increase by assumption. As bankruptcy costs increase, priority in bankruptcy becomes less valuable. This reduces the feasibility of restructuring (by Result 6). As restructurings become less feasible, bankruptcy filings increase.

This spiral has the potential to generate financial instability in the form of multiple equilibria:

Result 11. *Suppose*

$$\hat{v}\big(D^*_{\lambda=\lambda_H}\big) < F^{-1}(\kappa) < \hat{v}\big(D^*_{\lambda=\lambda_L}\big), \tag{46}$$

where D^* is the face value that makes creditors' IC (inequality (18)) bind in the baseline model for the indicated value of λ. There are two equilibria:

- *There is a "good" equilibrium, in which the probability of filing is low, courts are not congested, and the costs of bankruptcy are low; and*
- *there is a "bad" equilibrium, in which the probability of filing is high, courts are congested, and the costs of bankruptcy are high.*

Condition (46) suffices for the equilibria to be self-fulfilling. If creditors believe that bankruptcy costs are low ($\lambda = \lambda_H$), they accept a large restructuring to a low debt level $D^*_{\lambda=\lambda_H}$ (Result 6). As a result, firms file rarely, courts are not congested ($F(\hat{v}) \leq \kappa$), and, by the first inequality, bankruptcy costs are indeed low (equation (45)). Conversely, if creditors believe bankruptcy costs are high, they accept only a smaller restructuring. As a result, firms file often, courts are congested, and, by the second inequality, bankruptcy costs are indeed high.

This result suggests that bankruptcy policy cannot be separated from financial stability regulation: Congestion itself can create panic-like coordination failures. Bankruptcy policy is not just about mitigating the costs of filings at the margin, but about preventing mass filings altogether. Indeed, increasing court capacity κ so that the second inequality in condition (46) is violated can eliminate the "bad" equilibrium. This adds support to the argument that avoiding court congestion should be a policy priority in response to COVID-19 (see Iverson, Ellias, and Roe (2020)).

5.3 Endogenous Asset Values and Debt Overhang

We have also assumed that, although v is uncertain ex ante, the ex post distribution is exogenous. We have therefore focused on the ex post costs of financial distress, often called the "direct" costs of liquidation and bankruptcy, which are meaningful in practice (see

Section 3.8). However, "indirect" costs can arise when asset values are endogenous. Debt overhang, for example, can lead equity holders to forgo positive net-present-value (NPV) investments (Myers (1977)). Here, we incorporate endogenous asset values and show how debt overhang can amplify or attenuate our results.

We assume that the firm can make an investment before v is realized. Specifically, as in Holmstrom and Tirole (1997), we assume that the firm can exert effort η to improve the distribution of v. Greater effort improves outcomes in the sense that increasing η to $\eta' > \eta$ improves the distribution of asset values from F^{η} to $F^{\eta'} \succ F^{\eta}$, where "$\succ$" indicates first-order stochastic dominance. We also assume that the firm will exert less effort when it has high levels of debt because the costs of effort are borne solely by equity holders while the benefits are shared with creditors. Rather than model effort directly, however, we simply assume the firm exerts more effort when it has less debt.

Here, we want to ask whether the fact that restructuring solves a debt overhang problem (increases η) makes it easier or harder to write down debt. In order to formulate a comparative-statics result, as well as to relate to the parameters of the bankruptcy environment, we define λ^{η} as follows: It is the smallest λ for which it is feasible to write down debt sufficiently to reduce $\hat{v}$ to a given level (where we use $\hat{v}$ instead of the debt itself only because it makes the math significantly easier). [30] Our question therefore becomes: How does λ^{η} depend on η? The answer is given by the next result:

Result 12. *If the probability of default $F(\hat{v})$ does not depend on η, then λ^{η} is decreasing in η. On the other hand, if the tail conditional expectation $\mathbb{E}[v \,|\, v \leq \hat{v}(D)]$ does not depend on η, then λ^{η} is increasing in η.*

This result reveals that the effect of debt overhang on restructuring depends on how the overhang affects the distribution of values ex post. High effort can have two effects. On the one hand, it can reduce the probability of default and bankruptcy, which makes priority less valuable, thereby discouraging restructuring ("Effect (1)"). On the other hand, high effort can increase creditor recoveries in bankruptcy, which makes priority more valuable, encouraging restructurings ("Effect (2)"). More precisely:

- If effort does not affect the probability of default $F(\hat{v})$, then increasing η makes restructuring easier. In this situation, Effect (1) is absent. As effort increases, Effect (2) kicks in: Higher effort increases the probability of high asset values, increasing creditor recoveries in bankruptcy and making priority more valuable. In this case, the more there is to gain from restructuring, the more likely it is to occur—creditors restructure to access these gains.
- If, on the other hand, effort does not affect the value of the firm's assets in bankruptcy $\mathbb{E}[v|v \leq \hat{v}]$, then increasing η makes restructuring harder. In this situation, Effect (2)

[30] Result 6, which implies that high λ facilitates restructuring, implies such a λ^{η} is well-defined.

is absent. As effort increases, Effect (1) kicks in: Higher effort reduces the probability of bankruptcy, reducing the value of seniority and making priority less valuable. In this case, the more there is to gain from restructuring, the less likely it is to occur—creditors hold out to free ride on these gains.

5.4 Concentrated Debt Holdings and Debt-Equity Exchanges

So far, we have focused on debt held by dispersed and infinitesimally small creditors. In this setting, we find that the feasible restructurings involve swapping junior debt for senior debt. Although this is a reasonable approximation of reality, we do observe some exchange offers that swap debt for equity (see, e.g., Asquith, Gertner, and Scharfstein (1994)). In this section, we relax the assumption that debt is dispersed and show that debt-for-equity exchanges occur if and only if debt holdings are sufficiently concentrated.

To show this, we introduce a measure of creditor concentration, ξ. Specifically, ξ measures the probability that the firm's debt is held by a single large creditor (or a group of creditors acting in concert). The debt is held by a unit of dispersed creditors with probability $1-\xi$. The firm does not know the distribution of creditors when it makes its exchange offer, but creditors know the distribution when they accept or reject the offer (we focus exclusively on ex ante restructuring).[31]

To explore how ξ affects restructuring, we need to separately analyze the creditors' IC when debt is dispersed and when it is concentrated. With dispersed debt, the hold-out condition is the same as in our baseline model (equation (18)). As we saw there, a debt-for-equity exchange is infeasible; the firm just offers a minimum amount of senior debt. With concentrated debt, by contrast, a feasible restructuring may include swapping debt for equity. Assume the firm offers the creditor a combination of new debt D and new equity $1-\alpha$. The creditor will accept if it exceeds its outside option:

$$\begin{aligned} F\big(\hat{v}(D)\big)\mathbb{E}\left[\big(\theta+(1-\theta)(1-\alpha)\big)\lambda v \,\middle|\, v\le\hat{v}(D)\right] \\ \qquad + \big(1-F(\hat{v}(D))\big)\mathbb{E}\left[D+(1-\alpha)(v-D)\,\middle|\, v\ge\hat{v}(D)\right] \\ \ge F\big(\hat{v}(D_0)\big)\mathbb{E}\left[\theta\lambda v \,\middle|\, v\le\hat{v}(D_0)\right] + \big(1-F(\hat{v}(D_0))\big)D_0. \end{aligned} \tag{47}$$

The left-hand side resembles expressions we have seen before: It is the combined payoff of debt with face value D and a fraction $1-\alpha$ of the equity. If the creditor accepts the restructuring, the firm may subsequently file for bankruptcy (first term) or avoid that outcome (second term). Either way, the creditor receives a payoff on account of both its

[31] We are assuming that a firm's debt can become more or less concentrated as it approaches default and bankruptcy. The measure ξ reflects the firm's expectations regarding future concentration. This framework can accommodate situations where there is no uncertainty about creditor concentration (e.g., ξ is equal to zero or one).

new debt claim and its new equity interest. The right-hand side of inequality (47) differs from what we have seen before: It is the payoff to debt if no write-down takes place. Unlike dispersed creditors, the concentrated creditor internalizes the fact that, if it does not accept, the restructuring will fail.

The firm knows these IC conditions, but does not know whether its creditors are dispersed or concentrated when it makes a restructuring offer. Because it can offer a mix of (i) senior debt and (ii) equity, it has three options:

1. It can offer (i) the smallest amount of senior debt such that dispersed creditors accept but (ii) no equity. In this case, the large creditor may or may not accept. Either way, this will be an attractive option when ξ is low (creditors are likely dispersed).
2. It can offer (i) no debt but (ii) the smallest amount of equity such that the large creditor accepts. In this case, the dispersed creditors will not accept, but this will nonetheless be an attractive option when ξ is high (creditors are likely concentrated).
3. It can offer (i) the smallest amount of senior debt that makes dispersed creditors accept and (ii) enough equity to make the large creditor accept too. This could entail leaving some rent to the dispersed creditors, but could be optimal to ensure the offer is accepted.

The firm's choice among these options depends on a trade-off. If it makes an offer that is accepted all of the time (case 3), it minimizes its exposure to the deadweight costs of bankruptcy but the offer may be overly generous, allowing creditors to capture rents. If the firm makes a less generous offer (cases 1 and 2), it can reduce the rents paid to creditors, but it will expose itself to the deadweight costs of bankruptcy if the offer is rejected.

Comparing the firms' payoffs from these options gives the following result:

Result 13. *Suppose D_0 is sufficiently large. There are thresholds, $\tilde{D}$, $\underline{\xi}$, and $\bar{\xi}$ (given explicitly in equations (92), (114) and (115) in the proof) such that:*

- *For $D^* \geq \tilde{D}$ and $\underline{\xi} < \bar{\xi}$, there are three regions:*
  - *If $\xi \leq \underline{\xi}$, the firm offers senior debt only (and only dispersed creditors accept).*
  - *If $\underline{\xi} < \xi \leq \bar{\xi}$, the firm offers a mix of senior debt and equity (and all creditors accept).*
  - *If $\xi > \bar{\xi}$, it offers equity only (and only the concentrated creditor accepts).*
- *Otherwise, there are two regions: Below a threshold, the firm offers senior debt (and all creditors accept) and, above the threshold, it offers equity only (and only the concentrated creditor accepts).*

Overall, this result says that our baseline analysis is robust to some creditor concentration, but that higher levels of concentration induce the firm to use equity as well. Thus, our

framework can explain the use of equity in exchange offers observed in practice. Our result is consistent with James's (1995) finding that banks take equity in restructuring because they, unlike dispersed bondholders, internalize the effect of write-downs.

6 Discussion and Conclusion

Our results build upon two simple observations about debt restructuring when a firm has dispersed creditors. The first is that restructuring is difficult, if not impossible, unless the firm can dilute existing debt by offering high-priority (senior) debt in an exchange offer. This is well-understood, as illustrated by Bernardo and Talley (1996) and Gertner and Scharfstein (1991). The second observation is that priority is only as valuable as bankruptcy is likely: Creditors do not care about priority if the firm will never enter bankruptcy, but they do care when bankruptcy is likely, and the value they place on priority increases with the probability of a bankruptcy filing. Although intuitive, this observation yields a counterintuitive implication: Bankruptcy and restructuring are complements. Policies that increase the attractiveness of bankruptcy to equity holders will also increase the likelihood of restructuring. This counterintuitive implication drives most of the results in our paper, which show how key parameters of the bankruptcy environment, such as its deadweight costs (λ) and creditor friendliness (θ), affect the probability of a restructuring.

Our model has concrete payoffs for policymakers. First, it provides a simple way to evaluate whether current bankruptcy law is excessively creditor friendly (or debtor friendly). Our calculations suggest that U.S. law is too creditor friendly: A reduction in θ would likely increase the frequency of restructurings.

Second, as policymakers consider policies to aid struggling businesses during the COVID-19 pandemic, our model shows that they should prioritize policies that facilitate restructuring. One powerful way to do this, we show, is to announce a commitment to increase court capacity if bankruptcy filings increase substantially. Currently, many observers expect a surge in filings during the months ahead, congesting the bankruptcy courts. That expectation will increase the expected costs of bankruptcy, reduce restructurings, and—much like a self-fulfilling prophecy—generate the very congestion that observers expect. But if Congress simply announced a commitment to increase the capacity of the bankruptcy courts if filings increase above a threshold, that announcement could reduce expected bankruptcy costs, facilitate restructurings, and avoid the congestion.

Our analysis points to other policies that could facilitate restructuring during the pandemic. These include debt purchases, policies (e.g., tax subsidies) that reward creditors for restructuring debts, and policies that increase the payoff to senior lenders in bankruptcy (e.g., a government backstop to DIP loans extended by senior lenders).

It also has broad implications for the design of bankruptcy policy outside of a crisis. For

example, senior creditor control in bankruptcy can have beneficial effects on the likelihood of restructuring. These benefits must be weighed against the costs frequently discussed in recent literature. Equally important, our model shows that the APR facilitates restructuring to a limited extent: Restructuring is facilitated by an APR that protects senior creditors from dilution but permits deviations in favor of equity holders at the expense of junior creditors. From a restructuring perspective, some APR deviations are bad; others are good.

The model can shed light on other recent controversies in bankruptcy policy. Scholars and practitioners have expressed concern about loans ("DIP loans") extended to firms in bankruptcy. The vast majority of DIP loans are extended by pre-existing senior lenders, the rates of return on these loans are thought to be highly (perhaps excessively) profitable (as argued by Eckbo and Wang (2019)), and the terms of the DIP loans allow senior lenders to exercise control over speed and outcomes of the bankruptcy process (as discussed by Ayotte and Morrison (2009), among others). Our model suggests a different perspective on DIP loans. These loans increase the payoff to senior lenders in bankruptcy, and protect senior lenders from dilution (because they allow the lenders to exercise control over the process). Seen this way, the criticized features of DIP loans can actually facilitate restructuring, thereby avoiding the deadweight costs of bankruptcy.

Finally, our findings could even shed light on the evolution of the U.S. bankruptcy law and practice: During the late 19th century, the U.S. lacked a stable bankruptcy law. In that void, lawyers developed techniques for reorganizing companies using other (non-bankruptcy) devices, including what is called an "equity receivership." The equity receivership was a proto-Chapter 11 procedure, but often criticized because many companies (especially railroads) used it as a device to (i) maximize the returns to secured creditors, (ii) give a payoff to equity, and (iii) squeeze out unsecured creditors (Miller and Berkovich (2006)). In other words, private parties developed a technique that seems to resemble what our model recommends.

Appendix

A Proofs

A.1 Proof of Result 1

The proof is in the text. □

A.2 Proof of Result 2

The proof is in the text. □

A.3 Proof of Result 3

The proof is in the text. □

A.4 Proof of Result 4

The proof is in the text. □

A.5 Proof of Result 5

The proof is in the text. □

A.6 Proof of Result 6

First, we re-write Δ in equation (26) as:

$$\Delta := \Big(1 - F\big(\hat{v}(D)\big)\Big)\big(D - D_0\big) + \lambda\theta \int_0^{\hat{v}(D)} v \mathrm{d}F(v). \tag{48}$$

The maximum write-down, or lowest face value D^*, corresponds to $\Delta = 0$. To see how D^* depends on λ, we use the chain rule to write:

$$\frac{\partial D^*}{\partial \lambda} = -\frac{\partial \Delta / \partial \lambda}{\partial \Delta / \partial D}. \tag{49}$$

Computing, we see that the denominator is positive:

$$\frac{\partial \Delta}{\partial D} = f(\hat{v}) \left(\frac{\partial \hat{v}}{\partial D} D_0 - \hat{v} \right) + \big(1 - F(\hat{v})\big) + \lambda\theta\hat{v} f(\hat{v}) \frac{\partial \hat{v}}{\partial D} > 0, \tag{50}$$

given that all terms are positive.[32] The numerator is positive too:

$$\frac{\partial \Delta}{\partial \lambda} = \theta \int_0^{\hat{v}(D^*)} v \mathrm{d}F(v) + \Big(\lambda\theta\hat{v} + D_0 - D^*\Big) f(\hat{v}) \frac{\partial \hat{v}}{\partial \lambda} > 0. \tag{51}$$

This proves the result in the text. □

A.7 Proof of Result 7

To prove the first part of the result—how D^* depends on θ—we use the chain rule to write:

$$\frac{\partial D^*}{\partial \theta} = -\frac{\partial \Delta / \partial \theta}{\partial \Delta / \partial D}. \tag{52}$$

The denominator is as in equation (50) above. Recall that it is positive. The numerator is:

$$\frac{\partial \Delta}{\partial \theta} = \lambda \int_0^{\hat{v}} v \mathrm{d}F(v) + \big(\lambda\theta\hat{v} + D_0 - D^*\big) f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta}. \tag{53}$$

Given that the last term is negative, that is,

$$\frac{\partial \hat{v}}{\partial \theta} = -\frac{\lambda D}{\big(1 - (1-\theta)\lambda\big)^2} < 0, \tag{54}$$

this expression can change sign depending on parameters. This proves the first part of the result.

To prove the second part of the result on the existence of an interior θ^*, we show that it follows from continuity: We show that θ^* is always decreasing in θ at $\theta = 0$ and, under the condition in the result, is increasing in θ at $\theta = 1$, so D^* must be minimized for an interior value $\theta^* \in (0, 1)$.

From equations (50) and (52), we know that $\partial D^*/\partial\theta$ has the opposite sign of $\partial\Delta/\partial\theta$, which is given in equation (53). We compute:

- At $\theta = 0$, we have $\Delta = (1 - F(\hat{v}))(D - D_0)$; hence, $D^* = D_0$. Now, substituting into equation (53),

$$\left.\frac{\partial \Delta}{\partial \theta}\right|_{\theta=0} = \lambda \int_0^{\hat{v}} v \mathrm{d}F(v) + \big(D_0 - D^*\big) f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta} \tag{55}$$

$$= \lambda \int_0^{\hat{v}} v \mathrm{d}F(v) > 0. \tag{56}$$

[32] To see why, note that

$$\frac{\partial \hat{v}}{\partial D}(D) D_0 = \hat{v}(D_0)$$

and $D_0 > D^*$.

- At $\theta = 1$, we have $\hat{v} = D$ and $\Delta = (1 - F(D))(D - D_0) + \lambda \int_0^D v dF(v)$; hence, $\lambda \int_0^{D^*} v dF(v) = (1 - F(D^*))(D_0 - D^*)$. Now, substituting into equation (53),

$$\left.\frac{\partial \Delta}{\partial \theta}\right|_{\theta=1} = \lambda \int_0^{D^*} v \mathrm{d}F(v) - (D_0 - D^*)\lambda D^* f(D^*) \tag{57}$$

$$= (D_0 - D^*)\Big[1 - F(D^*) - \lambda D^* f(D^*)\Big] \tag{58}$$

Given $D_0 > D^*$, a sufficient condition for this to be negative (and hence for the existence of an interior minimum) is for the term in square brackets to be negative (at $\theta = 1$), which is the condition in the statement of the result. □

A.8 Proof of Result 8

Here, we simply compare the marginal effect on welfare $\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_i}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0}$ for each policy $i \in \{1, ..., 7\}$ as calculated in Appendix B.3 (see equations (143), (146), (157), (165), (174), (179), and (184)).

From these equations we already know that $\mathbf{s}_7, \mathbf{s}_3$, and $\mathbf{s}_1$ are equivalent.

To compare them to the other policies, it is convenient to multiply by $\left.\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}}\right|_{\varepsilon=0}$, which, as it turns out, is the same for all $\mathbf{s}_i$ (see equations (139), (150), (156), (163), (178), and (183)). So we want to:

$$\text{minimize } \left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_i}}{\mathrm{d}\varepsilon}\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}}\right|_{\varepsilon=0} \tag{59}$$

over $\mathbf{s}_i$. To do so, we compare directly as follows: We start with policies 2, 3, and 6. For all these policies, the objective above is -1. We compute the objective directly for policies $\mathbf{s}_4$, $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_5$ and show that in each case the objective is greater than -1, implying $\mathbf{s}_7$, $\mathbf{s}_3$, and $\mathbf{s}_6$ are preferred. The calculations also reveal that $\mathbf{s}_1$ is preferred to $\mathbf{s}_2$.

- **Policies 3, 6, and 7:**

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_i}}{\mathrm{d}\varepsilon}\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}}\right|_{\varepsilon=0} = -1 \tag{60}$$

for $i \in \{3, 6, 7\}$.

- **Policy 1:**

$$\frac{\mathrm{d}\hat{v}_{\mathbf{s}_1}}{\mathrm{d}\varepsilon}\frac{\partial\Delta}{\partial\hat{v}_{\mathbf{0}}}\bigg|_{\varepsilon=0} = \frac{(1-\theta)\Big((1-F(\hat{v}_{\mathbf{0}}))(1-\lambda(1-\theta)) + (D_0-D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}})\Big)}{(1-\lambda(1-\theta))F(\hat{v}_{\mathbf{0}})} \tag{61}$$

$$- \left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right)\frac{1-\theta}{1-\lambda(1-\theta)}\frac{f(\hat{v}_{\mathbf{0}})}{F(\hat{v}_{\mathbf{0}})} - \theta \tag{62}$$

$$= -1 + \frac{1-\theta}{F(\hat{v}_{\mathbf{0}})} > -1. \tag{63}$$

- **Policy 2:**

$$\frac{\mathrm{d}\hat{v}_{\mathbf{s}_2}}{\mathrm{d}\varepsilon}\frac{\partial\Delta}{\partial\hat{v}_{\mathbf{0}}}\bigg|_{\varepsilon=0} = \frac{(1-F(\hat{v}_{\mathbf{0}}))(1-\lambda(1-\theta)) + (D_0-D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}})}{\big(1-\lambda(1-\theta)\big)F(\hat{v}_{\mathbf{0}})} \tag{64}$$

$$- \left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right)\frac{1}{1-\lambda(1-\theta)}\frac{f(\hat{v}_{\mathbf{0}})}{F(\hat{v}_{\mathbf{0}})} \tag{65}$$

$$= -1 + \frac{1}{F(\hat{v}_{\mathbf{0}})} > -1. \tag{66}$$

To see that it is also inferior to $\mathbf{s}_4$, simply compare the expression in equation (66) here directly with that in equation (70). The result follows from $1/F(\hat{v}_{\mathbf{0}}) > 1 > \lambda(1-\theta) + (1-\lambda)F(\hat{v}_{\mathbf{0}})$.

- **Policy 4:**

$$\frac{\mathrm{d}\hat{v}_{\mathbf{s}_4}}{\mathrm{d}\varepsilon}\frac{\partial\Delta}{\partial\hat{v}_{\mathbf{0}}}\bigg|_{\varepsilon=0} = -(1-F(\hat{v}_{\mathbf{0}}))(1-\lambda(1-\theta)) \tag{67}$$

$$-(D_0-D)f(\hat{v}_{\mathbf{0}}) - \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}}) \tag{68}$$

$$+\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right)f(\hat{v}_{\mathbf{0}}) - \lambda\theta F(\hat{v}_{\mathbf{0}}) \tag{69}$$

$$= -1 + \lambda(1-\theta) + (1-\lambda)F(\hat{v}_{\mathbf{0}}) > -1 \tag{70}$$

- **Policy 5:**

$$\begin{aligned}\frac{\mathrm{d}\hat{v}_{\mathbf{s}_5}}{\mathrm{d}\varepsilon}\frac{\partial\Delta}{\partial\hat{v}_{\mathbf{0}}}\bigg|_{\varepsilon=0} &= -1 + \frac{1}{F(\hat{v}_{\mathbf{0}})} - \lambda\theta \\ &\quad + \left(1 + \frac{1}{1-\lambda(1-\theta)}\right)\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right)\frac{f(\hat{v}_{\mathbf{0}})}{F(\hat{v}_{\mathbf{0}})} > -1\end{aligned} \tag{71}$$

since $1/F(\hat{v}_{\mathbf{0}}) > 1 > \lambda\theta$ and the last term is positive.

To see that it is also inferior to $\mathbf{s}_4$, simply compare the expression in equation (71) here directly with that in equation (70), keeping in mind that it is hardest to satisfy when $f(\hat{v}_{\mathbf{0}}) = 0$. The result follows from $1/F(\hat{v}_{\mathbf{0}}) > 1 > \lambda + (1-\lambda)F(\hat{v}_{\mathbf{0}})$. □

A.9 Proof of Result 9

Note that the result is not quite as immediate as it might seem from equation (42), because we have to take into account that the equilibrium debt level D^* and, hence, the default threshold $\hat{v} = \hat{v}(D^*)$ depend on ρ. That said, the immediate intuition does hold: Increasing ρ increases the write-down. To prove it, we use implicit differentiation.

First, define the difference in a creditor's payoffs from accepting versus rejecting an offer, given that other creditors accept:

$$\Delta = (1 - F(\hat{v}))(D - D_0) + F(\hat{v})\mathbb{E}[\lambda\theta v | v < \hat{v}] \left(1 - (1-\rho)\frac{D_0}{D}\right) \tag{72}$$

$$= (1 - F(\hat{v}))(D - D_0) + \left(1 - (1-\rho)\frac{D_0}{D}\right) \int_0^{\hat{v}} \lambda\theta v \mathrm{d}F(v). \tag{73}$$

Now compute the derivative $\mathrm{d}D^*/\mathrm{d}\rho$ in two steps:

$$\begin{aligned}\frac{\partial \Delta}{\partial D} = (1 - F(\hat{v})) + (1-\rho)D_0 D^{-2} \int_0^{\hat{v}} \lambda\theta v \mathrm{d}F(v) + \\ + \underbrace{\left(D_0 - D + \left(1 - (1-\rho)\frac{D_0}{D}\right)\lambda\theta\hat{v}\right)}_{=(1-\lambda+\rho\lambda\theta)\hat{v} - (1-\lambda)\hat{v} > 0} f(\hat{v})\frac{\partial \hat{v}}{\partial D} > 0\end{aligned} \tag{74}$$

and

$$\frac{\partial \Delta}{\partial \rho} = \frac{D_0}{D} \int_0^{\hat{v}} \lambda\theta v \mathrm{d}F(v) > 0. \tag{75}$$

So,

$$\frac{\mathrm{d}D^*}{\mathrm{d}\rho} = -\frac{\partial\Delta/\partial\rho}{\partial\Delta/\partial D} < 0 \tag{76}$$

and $\rho = 1$ maximizes the write-down.

□

A.10 Proof of Result 10

The IC in inequality (43) can be re-written as

$$\Delta = (1 - F(\hat{v}))(D - D_0) + \left(1 - (1-\rho)\frac{D_0}{D}\right)\lambda\theta \int_0^{\hat{v}} v \mathrm{d}F(v) \geq 0. \tag{77}$$

The binding IC $\Delta = 0$ defines the written-down debt level D^*; minimizing D^* over θ defines the optimal level of creditor friendliness. Thus, by the chain rule, the effect of ρ on θ^* is

given by:

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\rho} = -\frac{\frac{\partial}{\partial\rho}\left(\frac{\partial\Delta}{\partial\theta}\right)}{\frac{\partial}{\partial\theta}\left(\frac{\partial\Delta}{\partial\theta}\right)} = -\frac{\frac{\partial^2\Delta}{\partial\rho\partial\theta}}{\frac{\partial^2\Delta}{\partial\theta^2}}. \tag{78}$$

Note that the denominator is negative at θ^* given that we have assumed that θ^* is an interior local minimum and not an inflection point.

We compute the denominator directly, step by step:

- First,

$$\begin{aligned}\frac{\partial\Delta}{\partial\theta} =& \left[f(\hat{v})(D_0 - D) + \left(1 - (1-\rho)\frac{D_0}{D}\right)\lambda\theta\hat{v}f(\hat{v})\right]\frac{\partial\hat{v}}{\partial\theta} \\ &+ \left(1 - (1-\rho)\frac{D_0}{D}\right)\lambda\int_0^{\hat{v}} v\mathrm{d}F(v) \qquad (79)\\ =& -\lambda\hat{v}f(\hat{v})(\hat{v}_0 - \hat{v}) + \left(1 - (1-\rho)\frac{D_0}{D}\right)\lambda\left(\int_0^{\hat{v}} v\mathrm{d}F(v) - \frac{\lambda\theta\hat{v}f(\hat{v})}{1-\lambda(1-\theta)}\hat{v}\right), \qquad (80)\end{aligned}$$

  having used that $D = (1-\lambda(1-\theta))\hat{v}$, $D_0 = (1-\lambda(1-\theta))\hat{v}_0$, and

$$\frac{\partial\hat{v}}{\partial\theta} = -\frac{\lambda\hat{v}}{1-\lambda(1-\theta)}. \tag{81}$$

- Second,

$$\frac{\partial^2\Delta}{\partial\rho\partial\theta} = \frac{D_0}{D}\lambda\left(\int_0^{\hat{v}} v\mathrm{d}F(v) - \frac{\lambda\theta\hat{v}f(\hat{v})}{1-\lambda(1-\theta)}\hat{v}\right). \tag{82}$$

  To determine its sign, it turns out that we can use two facts:

  - Using $D < D_0$ in $\Delta = 0$ above, we get that

$$1 - (1-\rho)\frac{D_0}{D} > 0. \tag{83}$$

  - Given θ^* is optimal, $\partial\Delta/\partial\theta|_{\theta=\theta^*} = 0$. This, together with the last inequality, implies that

$$\int_0^{\hat{v}} v\mathrm{d}F(v) > \frac{\lambda\theta\hat{v}f(\hat{v})}{1-\lambda(1-\theta)}\hat{v}. \tag{84}$$

    This implies that $\partial^2\Delta/\partial\rho\partial\theta$ in equation (82) is positive and, thus, given the above, that

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\rho} > 0. \tag{85}$$

□

A.11 Proof of Result 11

The proof is in the text (following the statement of the result). □

A.12 Proof of Result 12

Let's re-write the creditors' biding IC in equation (18) as a function of η as:

$$\Delta = \int_{\hat{v}(D)}^{\infty} \Big((1-\lambda^{\eta}(\hat{v})(1-\theta))\hat{v} - D_0\Big)\mathrm{d}F^{\eta}(v) + \int_{0}^{\hat{v}(D)} \theta\lambda^{\eta}(\hat{v})v\mathrm{d}F^{\eta}(v) = 0, \tag{86}$$

having substituted for $D = (1-\lambda^{\eta}(\hat{v})(1-\theta))\hat{v}$ from equation (7).

Solving, we can write the minimum $\lambda^{\eta}(\hat{v})$ needed to restructure to the debt level D as:

$$\lambda^{\eta}(\hat{v}) = \frac{\int_{\hat{v}(D)}^{\infty}(D_0-\hat{v})\mathrm{d}F^{\eta}(v)}{\int_{0}^{\hat{v}(D)}\theta v\mathrm{d}F^{\eta}(v) - \int_{\hat{v}}^{\infty}(1-\theta)\hat{v}\mathrm{d}F^{\eta}(v)} \tag{87}$$

$$= \frac{(D_0-\hat{v})(1-F^{\eta}(\hat{v}))}{\theta F^{\eta}(\hat{v})\mathbb{E}^{\eta}[v|v<\hat{v}] - (1-\theta)(1-F^{\eta}(\hat{v}))\hat{v}}, \tag{88}$$

where $\mathbb{E}^{\eta}$ denotes the expectation given the distribution F^{η}.

Tail expectation $\mathbb{E}^{\eta}[v|v<\hat{v}]$ does not depend on η. Given that, by assumption $F^{\eta'} \succ F^{\eta}$, we have that $F^{\eta'}(\hat{v}) < F^{\eta}(\hat{v})$. This implies that the numerator in equation (88) is increasing in η and the denominator is decreasing in η. Thus, $\lambda^{\eta'}(\hat{v}) > \lambda^{\eta}(\hat{v})$.

Default probability $F^{\eta}(\hat{v})$ does not depend on η. That is, $F^{\eta'}(\hat{v}) = F^{\eta}(\hat{v})$. Given that, by assumption $F^{\eta'} \succ F^{\eta}$, we have that $F^{\eta'}(\hat{v})\mathbb{E}^{\eta'}[v|v<\hat{v}] > F^{\eta}(\hat{v})\mathbb{E}^{\eta}[v|v<\hat{v}]$.[33] This implies that the denominator of equation (88) is increasing in η. And, since $F^{\eta'}(\hat{v}) = F^{\eta}(\hat{v})$, the numerator is the same under η and η'. Thus, $\lambda^{\eta'}(\hat{v}) < \lambda^{\eta}(\hat{v})$. □

[33] Given $F^{\eta}(\hat{v})$ does not depend on η, this is akin to the fact that $F^{\eta'} \succ F^{\eta}$ implies $\mathbb{E}^{\eta'}[v] > \mathbb{E}^{\eta}[v]$ and can be proved via a change of variables, $v := (F^{\eta})^{-1}\big(F^{\eta'}(\tilde{v})\big)$:

$$\begin{aligned} F^{\eta}(\hat{v})\mathbb{E}^{\eta}[v|v<\hat{v}] \equiv \int_0^{\hat{v}} v\mathrm{d}F^{\eta}(v) &= \int_0^{\hat{v}} (F^{\eta})^{-1}\big(F^{\eta'}(\tilde{v})\big)\mathrm{d}F^{\eta}\Big((F^{\eta})^{-1}\big(F^{\eta'}(\tilde{v})\big)\Big) \\ &= \int_0^{\hat{v}} (F^{\eta})^{-1}\big(F^{\eta'}(\tilde{v})\big)\mathrm{d}F^{\eta'}(\tilde{v}) \\ &\leq \int_0^{\hat{v}} \tilde{v}\mathrm{d}F^{\eta'}(\tilde{v}) \equiv F^{\eta'}(\hat{v})\mathbb{E}^{\eta'}[v|v<\hat{v}] \end{aligned}$$

where the last inequality follows from the definition of stochastic dominance. That is, $F^{\eta}(\tilde{v}) \geq F^{\eta'}(\tilde{v})$ or, equivalently, $\tilde{v} \geq (F^{\eta})^{-1}\big(F^{\eta'}(\tilde{v})\big)$. Note, critically, that the assumption that $F^{\eta}(\hat{v})$ does not depend on η allowed us to change variables without changing the bounds of integration (the result would not obtain without that assumption).

A.13 Proof of Result 13

Here, we solve for the restructuring offer that equity holders make if they face a large, concentrated creditor (denoted by L in the proof) with probability ξ and small, dispersed creditors (denoted by S in the proof) with probability $1-\xi$.

Ultimately, there are three possibilities:

1. The firm makes an offer that L accepts and S reject.
2. The firm makes an offer the L rejects and S accept.
3. The firm makes an offer that both L and S accept.

The proof involves simply (i) computing current equity's optimal restructuring given each of these possibilities and (ii) comparing its payoffs (denoted by u in the proof) from each possibility given these optimal restructurings. But this involves some steps:

Step 1: Write L's and S's ICs to accept a restructuring offer.

Step 2: Write current equity's payoffs u in benchmarks in which $\xi = 1$ and $\xi = 0$. That is, creditors are concentrated for sure or dispersed for sure (these expressions are useful in the subsequent comparisons).

Step 3: Calculate current equity's payoffs u for each possibility 1–3 above.

Step 4: Compare these payoffs to determine the optimal restructuring offer.

Step 1: IC constraints. Here, we define the conditions for each type of creditor to accept the firm's offer of a restructuring to debt D and equity $1-\alpha$.

- *Large creditor's IC.* Define the difference in its payoffs from accepting and rejecting the offer (inequality (47) with the expectations expanded as integrals) as Δ_L:

$$\begin{aligned}\Delta_L := &\int_0^{\hat{v}(D)} \big(\theta + (1-\theta)(1-\alpha)\big)\lambda v f(v)\mathrm{d}v + \int_{\hat{v}(D)}^{\infty} \big(D + (1-\alpha)(v-D)\big) f(v)\mathrm{d}v \\ &- \int_0^{\hat{v}(D_0)} \theta\lambda v f(v)\mathrm{d}v - \int_{\hat{v}(D_0)}^{\infty} D_0 f(v)\mathrm{d}v.\end{aligned} \tag{89}$$

  Its IC is thus $\Delta_L \geq 0$.

- *Small creditors' IC.* Define their difference in payoffs from accepting and rejecting the offer as Δ_S:

$$\begin{aligned}\Delta_S := &\int_0^{\hat{v}(D)} (\theta + (1-\theta)(1-\alpha))\lambda v f(v)\mathrm{d}v \\ &+ \int_{\hat{v}(D)}^{\infty} (D + (1-\alpha)(v-D)) f(v)\mathrm{d}v - \int_{\hat{v}(D)}^{\infty} D_0 f(v)\mathrm{d}v.\end{aligned} \tag{90}$$

Their IC is thus $\Delta_S \geq 0$.

Note the difference between these conditions: Whereas L takes into account the effect of restructuring on the default probability, S do not (this creates the hold-out problem in the baseline model).

When equity holders make a restructuring offer, they can choose D and $1-\alpha$ that will be accepted if there is a large creditor, if there are small creditors, or in both cases. Thus, they have to take into account which IC is tighter.

Comparing Δ_L and Δ_S reveals that L's IC is tighter than S's if

$$\Delta_S - \Delta_L = \int_0^{\hat{v}(D_0)} \lambda\theta v f(v)\mathrm{d}v - \int_{\hat{v}(D)}^{\hat{v}(D_0)} D_0 f(v)\mathrm{d}v \geq 0. \tag{91}$$

Because this inequality does not depend on α, it is satisfied whenever D is above a threshold, $\tilde{D}$, which solves

$$\Delta_L - \Delta_S \Big|_{D=\tilde{D}} = 0. \tag{92}$$

Step 2: Current equity's payoff u for $\xi = 1$ and $\xi = 0$. To find the firm's payoff, we define its payoff in the following benchmark cases:

- u_L is current equity holders' payoff if there is a large concentrated creditor (i.e., if $\xi = 1$):

$$u_L := \int_0^{\hat{v}(D)} \lambda v f(v)\mathrm{d}v + \int_{\hat{v}(D)}^{\infty} v f(v)\mathrm{d}v - \left(\int_0^{\hat{v}(D_0)} \lambda\theta v f(v)\mathrm{d}v + \int_{\hat{v}(D_0)}^{\infty} D_0 f(v)\mathrm{d}v \right). \tag{93}$$

- u_S is current equity's payoff if there are small dispersed creditors (i.e., if $\xi = 0$ as in the baseline model):

$$u_S := \int_0^{\hat{v}(D)} \lambda v f(v)\mathrm{d}v + \int_{\hat{v}(D)}^{\infty} v f(v)\mathrm{d}v - \int_{\hat{v}(D)}^{\infty} D_0 f(v)\mathrm{d}v. \tag{94}$$

- $u_\varnothing$ is current equity's payoff if there is no restructuring (i.e., if $D = D_0$):

$$u_\varnothing := \int_0^{\hat{v}(D_0)} \lambda(1-\theta) v f(v)\mathrm{d}v + \int_{\hat{v}(D_0)}^{\infty} (v - D_0) f(v)\mathrm{d}v. \tag{95}$$

These expressions are useful, because equity holders' payoff for $\xi \in (0,1)$ will be a weighted average of them.

Step 3: Current equity payoff calculation. There are three possible cases, depending on which IC binds, which we consider in turn.

Case 1: $\Delta_L \geq 0 > \Delta_S$. In this case, only L accepts the restructuring but S do not. So

the restructuring succeed with probability ξ and equity holders' problem is to

$$\text{maximize } u = \xi u_L + (1-\xi)u_\varnothing \tag{96}$$

over α and D. Given $u_\varnothing$ does not depend on α or D, this is the same as maximizing u_L. Differentiating with respect to D, we find:

$$-(1-\lambda)\hat{v}f(\hat{v}) < 0, \tag{97}$$

for all D. So the solution is $D = 0$. We can find $1-\alpha$ from the binding IC ($\Delta_L = 0$):

$$(1-\alpha)\mathbb{E}[v] = \int_0^{\hat{v}(D_0)} \lambda\theta v f(v)\mathrm{d}v + \int_{\hat{v}(D_0)}^{\infty} D_0 f(v)\mathrm{d}v. \tag{98}$$

Defining $u_L^{\max}$ as the maximum of u_L in this case, we have:

$$u_L^{\max} = \int_0^{\infty} v f(v)\mathrm{d}v - \left(\int_0^{\hat{v}(D_0)} \lambda\theta v f(v)\mathrm{d}v + \int_{\hat{v}(D_0)}^{\infty} D_0 f(v)\mathrm{d}v \right). \tag{99}$$

So the expected payoff is:

$$u = \xi u_L^{\max} + (1-\xi)u_\varnothing. \tag{100}$$

Case 2: $\Delta_S \geq 0 > \Delta_L$. In this case, only the small creditors' IC is satisfied. So the restructuring succeed with probability $1-\xi$ and and equity holders' problem is to

$$\text{maximize } u = \xi u_\varnothing + (1-\xi)u_S \tag{101}$$

over α and D. Given $u_\varnothing$ does not depend on α or D, this is the same as maximizing u_S. This is the baseline case of dispersed creditors studied in Section B.1: Where $1-\alpha = 0$ and $D = D^*$.

Defining $u_S^{\max}$ as the maximum of u_S in this case, we have:

$$u_S^{\max} = \int_0^{\hat{v}(D)} \lambda v f(v)\mathrm{d}v + \int_{\hat{v}(D)}^{\infty} v f(v)\mathrm{d}v - \int_{\hat{v}(D)}^{\infty} D_0 f(v)\mathrm{d}v. \tag{102}$$

So the expected payoff is

$$u = \xi u_\varnothing + (1-\xi)u_S^{\max}. \tag{103}$$

Case 3: $\Delta_S \geq 0$ *and* $\Delta_L \geq 0$. In this case, both ICs are satisfied and equity holders' problem is to

$$\text{maximize } u = \xi u_L + (1-\xi)u_S \tag{104}$$

over α and D.

Here, there are three sub-cases:

- Case 3(i): $\Delta_L > \Delta_S = 0$ ($\implies D < \tilde{D}$). In this case, the firm chooses α and D so that S's IC binds and L accepts the restructuring on S's terms. Hence, the equity holders problem is to

$$\text{maximize } u = \xi u_S + (1-\xi)u_S = u_S \tag{105}$$

  over α and D. From case 2, we know that to maximize u_S, equity holders set $1-\alpha = 0$ and $D = D^*$. So the expected payoff is:

$$u = u_S^{\max}. \tag{106}$$

- Case 3(ii): $\Delta_S > \Delta_L = 0$ ($\implies D > \tilde{D}$). This cannot arise in equilibrium: Given S's IC is slack ($\Delta_S > 0$), equity holders can increase their surplus by decreasing D (and increasing $1-\alpha$ so as not to violate L's IC).
- Case 3(iii): $\Delta_S = \Delta_L = 0$ ($\implies D = \tilde{D}$). Given both ICs are binding, $D = \tilde{D}$ by definition. Here, the expected payoff is:

$$u = u_S(\tilde{D}) = u_L(\tilde{D}). \tag{107}$$

  In this case, the offer includes both debt $\tilde{D}$ and an equity stake $1-\alpha$, which we can solve for from the binding ICs:

$$1-\alpha = \frac{\int_0^{\hat{v}_{D_0}} \lambda\theta v f(v)\mathrm{d}v + \int_{\hat{v}(D_0)}^{\infty} D_0 f(v)\mathrm{d}v - \int_0^{\hat{v}(\tilde{D})} \theta\lambda v f(v)\mathrm{d}v - \int_{\hat{v}(\tilde{D})}^{\infty} \tilde{D} f(v)\mathrm{d}v}{\int_0^{\hat{v}(\tilde{D})} (1-\theta)\lambda v f(v)\mathrm{d}v + \int_{\hat{v}(\tilde{D})}^{\infty} (v-\tilde{D}) f(v)\mathrm{d}v}. \tag{108}$$

Combining these sub-cases, we have that equity's expected payoff is:

$$u = \begin{cases} u_S^{\max} & \text{if } D^* < \tilde{D}, \\ u_S(\tilde{D}) & \text{if } D^* \geq \tilde{D}. \end{cases} \tag{109}$$

Step 4: Payoff comparison. Combining the cases above, we have that equity's expected payoff is:

$$u = \begin{cases} \max\left\{\xi u_L^{\max} + (1-\xi)u_\varnothing\,,\, \xi u_\varnothing + (1-\xi)u_S^{\max}\,,\, u_S^{\max}\right\} & \text{if } D^* < \tilde{D}, \\ \max\left\{\xi u_L^{\max} + (1-\xi)u_\varnothing\,,\, \xi u_\varnothing + (1-\xi)u_S^{\max}\,,\, u_S(\tilde{D})\right\} & \text{if } D^* \geq \tilde{D}. \end{cases} \tag{110}$$

To simplify the thresholds above, first observe that, from the definitions of $u_L^{\max}, u_S^{\max}$,

and $u_S(\tilde{D})$ and the assumption that D_0 is sufficiently large (and hence $u_\varnothing$ sufficiently small), we have:

$$u_L^{\max} \geq u_L(\tilde{D}) = u_S(\tilde{D}) > u_\varnothing \tag{111}$$

and

$$u_S^{\max} \geq u_L(\tilde{D}) = u_S(\tilde{D}) > u_\varnothing. \tag{112}$$

With this, we can re-write u as:

$$u = \begin{cases} \max\left\{\xi u_L^{\max} + (1-\xi)u_\varnothing\,,\, u_S^{\max}\right\} & \text{if } D^* < \tilde{D} \\ \max\left\{\xi u_L^{\max} + (1-\xi)u_\varnothing\,,\, \xi u_\varnothing + (1-\xi)u_S^{\max}\,,\, u_S(\tilde{D})\right\} & \text{if } D^* \geq \tilde{D}. \end{cases} \tag{113}$$

To give the formulation in the statement of the result, we divide the expression above into cases for $D^* \lessgtr \tilde{D}$ and $\underline{\xi} \lessgtr \bar{\xi}$, where:

$$\underline{\xi} := \frac{u_S^{\max} - u_S(\tilde{D})}{u_S^{\max} - u_\varnothing}, \tag{114}$$

$$\bar{\xi} := \frac{u_S(\tilde{D}) - u_\varnothing}{u_L^{\max} - u_\varnothing}. \tag{115}$$

- If $D^* \geq \tilde{D}$ and $\underline{\xi} < \bar{\xi}$, then:

$$u = \begin{cases} \xi u_\varnothing + (1-\xi)u_S^{\max} & \text{if } \xi \leq \underline{\xi}, \\ u_S(\tilde{D}) & \text{if } \xi \in \left(\underline{\xi}, \bar{\xi}\right], \\ \xi u_L^{\max} + (1-\xi)u_\varnothing & \text{if } \xi > \bar{\xi} \end{cases} \tag{116}$$

  The analysis above implies this corresponds to debt for $\xi \leq \underline{\xi}$, a mix of debt and equity for $\xi \in (\underline{\xi}, \bar{\xi}]$, and equity for $\xi > \bar{\xi}$.

- If $D^* \geq \tilde{D}$ and $\underline{\xi} \geq \bar{\xi}$, then:

$$u = \begin{cases} \xi u_\varnothing + (1-\xi)u_S^{\max} & \text{if } \xi \leq \dfrac{u_S^{\max} - u_\varnothing}{u_L^{\max} + u_S^{\max} - 2u_\varnothing}, \\ \xi u_L^{\max} + (1-\xi)u_\varnothing & \text{if } \xi > \dfrac{u_S^{\max} - u_\varnothing}{u_L^{\max} + u_S^{\max} - 2u_\varnothing}. \end{cases} \tag{117}$$

  The analysis above implies this corresponds to debt for ξ below the threshold $(u_S^{\max} - u_\varnothing)/(u_L^{\max} + u_S^{\max} - 2u_\varnothing)$ and equity above it.

- If $D^* < \tilde{D}$, then:

$$u = \begin{cases} u_S^{\max} & \text{if } \xi \leq \frac{u_S^{\max} - u_\varnothing}{u_L^{\max} - u_\varnothing} \\ \xi u_L^{\max} + (1-\xi) u_\varnothing & \text{if } \xi > \frac{u_S^{\max} - u_\varnothing}{u_L^{\max} - u_\varnothing}. \end{cases} \tag{118}$$

The analysis above implies this corresponds to debt for ξ below the threshold $(u_S^{\max} - u_\varnothing)/(u_L^{\max} - u_\varnothing)$ and equity above it. □

B Omitted Derivations

B.1 Mixed Offers

In our baseline model, we study exchange offers including a single security, equity or debt. In Section 5.4, we show that offers of a mix and debt and equity could arise with a concentrated creditor. Here, we show they never arise in our baseline model with dispersed creditors.

Suppose that the firm offers creditors a mix of a proportion of equity $1-\alpha$ and senior debt with face value D in exchange for their junior debt D_0.

Noting that the bankruptcy decision condition in equation (7) is unchanged by new equity (i.e., that the firm will file whenever $v \leq \hat{v}(D)$), we can write a creditor's payoffs as follows:

- If it accepts, it gets:

$$\begin{aligned} \text{payoff}\big|_{\text{acc.}} = \big(1 - F(\hat{v})\big) D + \int_0^{\hat{v}} (\theta + (1-\alpha)(1-\theta)) \lambda v \mathrm{d}F(v) \\ + \int_{\hat{v}}^{\infty} (1-\alpha)(v - D) \mathrm{d}F(v). \end{aligned} \tag{119}$$

- If it rejects, it gets:

$$\text{payoff}\big|_{\text{rej.}} = \big(1 - F(\hat{v})\big) D_0. \tag{120}$$

So the firm chooses α and D to

$$\text{maximize} \int_0^{\hat{v}} \alpha \lambda (1-\theta) v f(v) \mathrm{d}v + \int_{\hat{v}}^{\infty} \alpha (v - D) f(v) \mathrm{d}v \tag{121}$$

subject to creditors' IC that

$$\text{payoff}\big|_{\text{acc.}} \geq \text{payoff}\big|_{\text{rej.}} \tag{122}$$

Supposing that the constraint binds and substituting it in the objective, we can re-write

the problem as:

$$\text{maximize } \int_0^{\hat{v}} \lambda v \mathrm{d}F(v) + \int_{\hat{v}}^{\bar{v}} v \mathrm{d}F(v) - (1 - F(\hat{v}(D)))D_0. \tag{123}$$

Now observe that this does not depend on α and that D appears only in $\hat{v}$. Hence, if there is an interior optimum, we can maximize the objective with respect to $\hat{v}$ directly, to get:

$$-\lambda f(\hat{v})\hat{v} + f(\hat{v})\hat{v} + f(\hat{v})D_0 = 0. \tag{124}$$

Substituting in for $\hat{v}$ from equation (7) and solving, we find that:

$$D = \frac{1 - \lambda(1-\theta)}{1-\lambda} D_0 > D_0. \tag{125}$$

Still supposing an interior optimum, substitute D_0 into the binding constraint in (122) to get that:

$$\begin{aligned}(1-\alpha)\left(\int_0^{\hat{v}} + (1-\theta)\lambda v \mathrm{d}F(v) + \int_{\hat{v}}^{\infty} (v - D)\mathrm{d}F(v) \right) = \\ = -\big(1 - F(\hat{v})\big)\theta\lambda\hat{v}(D) - \int_0^{\hat{v}} \theta\lambda v \mathrm{d}F(v).\end{aligned} \tag{126}$$

But this implies that new equity is negative. So we conclude that there is not an interior optimum, but, rather, that $1 - \alpha = 0$. Substituting into the constraint (122), we get:

$$(1 - F(\hat{v}))D + \int_0^{\hat{v}} \theta\lambda v f(v)\mathrm{d}v \geq (1 - F(\hat{v}))D_0, \tag{127}$$

which is the usual constraint.

B.2 Omitted derivation from Section 3.8

Given equations (52) and (53), inequality (30) is equivalent to:

$$\frac{\partial \Delta}{\partial \theta} = \lambda \int_0^{\hat{v}} v \mathrm{d}F(v) + \big(\lambda\theta\hat{v} + D_0 - D^*\big) f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta} < 0. \tag{128}$$

To massage this expression, we re-write $\partial\hat{v}/\partial\theta$ using equation (7) as follows:

$$\frac{\partial\hat{v}}{\partial\theta} = -\frac{\lambda D}{\left(1-(1-\theta)\lambda\right)^2} \tag{129}$$

$$= -\lambda\,\frac{D}{1-(1-\theta)\lambda}\,\frac{1}{1-(1-\theta)\lambda} \tag{130}$$

$$= -\lambda\hat{v}\frac{\partial\hat{v}}{\partial D}. \tag{131}$$

With this, inequality (128) becomes:

$$\frac{1}{\hat{v}}\int_0^{\hat{v}} v\mathrm{d}F(v) < \lambda\theta\hat{v}\frac{\partial F(\hat{v})}{\partial D} + (D_0 - D)\frac{\partial F(\hat{v})}{\partial D}. \tag{132}$$

Observe that the left-hand side is less than one[34] and that the second term on the right-hand side is positive, so we can write a sufficient condition:

$$1 < \lambda\theta\hat{v}\frac{\partial F(\hat{v})}{\partial D}. \tag{133}$$

This is the condition in the text.

B.3 Omitted Derivations from Section 4

We compute the marginal effect of a dollar for each policy $\mathbf{s}_i$, as in equation (40). To streamline some of the calculations, we change variables, and write things in terms of the default threshold in the baseline model $\hat{v}_{\mathbf{0}}$ instead of the debt level D. Since $\hat{v}_{\mathbf{0}} \propto D$ (equation (7)), equation (40) can be re-written as:

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_i}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left(\frac{\partial\hat{v}_{\mathbf{s}_i}}{\partial s_i} - \frac{\partial\hat{v}_{\mathbf{s}_i}}{\partial\hat{v}_{\mathbf{0}}}\frac{\partial\Delta/\partial s_i}{\partial\Delta/\partial\hat{v}_{\mathbf{0}}}\right)\frac{1}{q_i}. \tag{134}$$

We go through the omitted steps for each policy in turn:

Policy 1: Bankruptcy Subsidies for Assets.

- *Cost:* s_1 is paid conditional on bankruptcy and bankruptcy occurs with probability $F(\hat{v}_{\mathbf{s}_1})$, so $q_1 = F(\hat{v}_{\mathbf{s}_1})$.
- *Filing:* s_1 increases the firm's asset value in bankruptcy, so, given equation (5), the firm files if

$$(1-\theta)\left(\lambda v + s_1\right) \geq v - D, \tag{135}$$

[34]The average of v given $v \leq \hat{v}$ is less than $\hat{v}$.

giving filing threshold

$$\hat{v}_{\mathbf{s}_1} := \frac{D + (1-\theta)s_1}{1-\lambda(1-\theta)} \equiv \hat{v}_{\mathbf{0}} + \frac{(1-\theta)s_1}{1-\lambda(1-\theta)}. \tag{136}$$

From equation (136)

$$\frac{\partial \hat{v}_{\mathbf{s}_1}}{\partial \hat{v}_{\mathbf{0}}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{\mathbf{s}_1}}{\partial s_1} = \frac{1-\theta}{1-\lambda(1-\theta)}. \tag{137}$$

- *Restructuring:* The binding IC is:

$$\Delta = \big(1 - F(\hat{v}_{\mathbf{s}_1})\big)(D - D_0) + \int_0^{\hat{v}_{\mathbf{s}_1}} \theta\left(\lambda v + s_1\right) \mathrm{d}F(v) = 0, \tag{138}$$

where the subsidy affects both the probability that the creditors are repaid in full (via $\hat{v}_{\mathbf{s}_1}$) and what they get if they are not (because they share s_1).

We differentiate Δ in equation (138) w.r.t. s_1 and $\hat{v}_{\mathbf{0}}$:

$$\frac{\partial \Delta}{\partial s_1} = -f(\hat{v}_{\mathbf{s}_1})(D - D_0)\frac{\partial \hat{v}_{\mathbf{s}_1}}{\partial s_1} + \int_0^{\hat{v}_{\mathbf{s}_1}} \theta \mathrm{d}F(v) + \theta(\lambda \hat{v}_{\mathbf{s}_1} + s_1) f(\hat{v}_{\mathbf{s}_1})\frac{\partial \hat{v}_{\mathbf{s}_1}}{\partial s_1} \tag{139}$$

$$\to \left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right)\frac{1-\theta}{1-\lambda(1-\theta)} f(\hat{v}_{\mathbf{0}}) + \theta F(\hat{v}_{\mathbf{0}}), \tag{140}$$

as $\varepsilon \to 0$, given $\hat{v}_{\mathbf{s}_1} = \hat{v}_{\mathbf{0}}$ when $\varepsilon = 0$.

And

$$\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{s}_1})\big)\frac{\partial D}{\partial \hat{v}} - (D - D_0) f(\hat{v}_{\mathbf{s}_1}) + \lambda\theta(\hat{v}_{\mathbf{s}_1} + s_1) f(\hat{v}_{\mathbf{s}_1})\frac{\partial \hat{v}_{\mathbf{s}_1}}{\partial \hat{v}} \tag{141}$$

$$\to (1 - F(\hat{v}_{\mathbf{0}}))(1 - \lambda(1-\theta)) + (D_0 - D) f(\hat{v}_{\mathbf{0}}) + \lambda\theta \hat{v}_{\mathbf{0}} f(\hat{v}_{\mathbf{0}}), \tag{142}$$

as $\varepsilon \to 0$.

Substituting the above and $q_1 = F(\hat{v}_{\mathbf{0}})$ into equation (134), we find the marginal benefit of investing in policy $\mathbf{s}_1$:

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_1}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left(\frac{1-\theta}{1-\lambda(1-\theta)} - \frac{\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right)\frac{1-\theta}{1-\lambda(1-\theta)} f(\hat{v}_{\mathbf{0}}) + \theta F(\hat{v}_{\mathbf{0}})}{(1 - F(\hat{v}_{\mathbf{0}}))(1 - \lambda(1-\theta)) + (D_0 - D) f(\hat{v}_{\mathbf{0}}) + \lambda\theta \hat{v}_{\mathbf{0}} f(\hat{v}_{\mathbf{0}})}\right)\frac{1}{F(\hat{v}_{\mathbf{0}})}. \tag{143}$$

Policy 2: Bankruptcy Subsidies for Equity.

- *Cost:* s_2 is paid conditional on bankruptcy, and bankruptcy occurs with probability $F(\hat{v}_{\mathbf{s}_2})$, so $q_2 = F(\hat{v}_{\mathbf{s}_2})$.

- *Filing:* s_2 increases equity holders' payoff in bankruptcy, so, given equation (5), the firm files if

$$(1-\theta)\lambda v + s_2 \geq v - D, \tag{144}$$

giving the filing threshold

$$v_{\mathbf{s}_2} := \hat{v}_{\mathbf{0}} + \frac{s_2}{1-\lambda(1-\theta)}. \tag{145}$$

From equation (145), we have:

$$\frac{\partial \hat{v}_{\mathbf{s}_2}}{\partial \hat{v}_{\mathbf{0}}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{\mathbf{s}_2}}{\partial s_2} = \frac{1}{1-\lambda(1-\theta)}. \tag{146}$$

- *Restructuring:* The binding IC is

$$\Delta = \big(1 - F(\hat{v}_{\mathbf{s}_2})\big)(D - D_0) + \int_0^{\hat{v}_{\mathbf{s}_2}} \lambda\theta v \mathrm{d}F(v), \tag{147}$$

where the subsidy affects the probability that the creditors are repaid in full (via $\hat{v}_{\mathbf{s}_2}$), but does not affect what they receive if they are not paid in full (because the subsidy goes entirely to equity).

We differentiate Δ in equation (147) with respect to s_2 and $\hat{v}_{\mathbf{0}}$:

$$\frac{\partial \Delta}{\partial s_2} = -f(\hat{v}_{s_4})(D - D_0)\frac{\partial \hat{v}_{\mathbf{s}_2}}{\partial s_2} + \lambda\theta \hat{v}_{s_2} f(\hat{v}_{s_2})\frac{\partial \hat{v}_{\mathbf{s}_2}}{\partial s_2} \tag{148}$$

$$\to \left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)D}\right)\frac{1}{1-\lambda(1-\theta)} f(v_{\mathbf{0}}), \tag{149}$$

as $\varepsilon \to 0$, given $\hat{v}_{\mathbf{s}_2} \to \hat{v}_{\mathbf{0}}$ as $\varepsilon \to 0$.

And

$$\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{s}_2})\big)\frac{\partial D}{\partial \hat{v}_{\mathbf{0}}} - f(\hat{v}_{\mathbf{s}_2})(D - D_0) + \lambda\theta \hat{v}_{s_2} f(\hat{v}_{\mathbf{s}_2})\frac{\partial \hat{v}_{\mathbf{s}_2}}{\partial \hat{v}_{\mathbf{0}}} \tag{150}$$

$$\to \big(1 - F(\hat{v}_{\mathbf{0}})\big)\big(1 - \lambda(1-\theta)\big) + (D_0 - D) f(\hat{v}_{\mathbf{0}}) + \lambda\theta \hat{v}_{\mathbf{0}} f(\hat{v}_{\mathbf{0}}), \tag{151}$$

as $\varepsilon \to 0$.

Substituting the above and $q_2 = F(\hat{v}_{\mathbf{0}})$ into equation (134) above, we can find the marginal benefit of investing in policy $\mathbf{s}_2$:

$$\frac{\mathrm{d}\hat{v}_{\mathbf{s}_2}}{\mathrm{d}\varepsilon}\Big|_{\varepsilon=0} = \left(\frac{1}{1-\lambda(1-\theta)} - \frac{\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)D}\right)\frac{1}{1-\lambda(1-\theta)} f(\hat{v}_{\mathbf{0}})}{(1 - F(\hat{v}_{\mathbf{0}}))(1-\lambda(1-\theta)) + (D_0 - D) f(\hat{v}_{\mathbf{0}}) + \lambda\theta \hat{v}_{\mathbf{0}} f(\hat{v}_{\mathbf{0}})}\right)\frac{1}{F(\hat{v}_{\mathbf{0}})}. \tag{152}$$

Policy 3: Bankruptcy Subsidies for Debt.

- *Cost:* s_3 is paid conditional on bankruptcy and bankruptcy occurs with probability $F(\hat{v}_{\mathbf{s}_3})$, so $q_3 = F(\hat{v}_{\mathbf{s}_3})$.
- *Filing:* s_3 does not affect equity holders' payoffs (only creditors'). Hence, $\hat{v}_{\mathbf{s}_3} = \hat{v}_{\mathbf{0}}$. Thus, we have:
$$\frac{\partial \hat{v}_{s_3}}{\partial \hat{v}_{\mathbf{0}}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{s_3}}{\partial s_3} = 0. \tag{153}$$
- *Restructuring:* The binding IC is:
$$\Delta = \big(1 - F(\hat{v}_{\mathbf{s}_3})\big)(D - D_0) + \int_0^{\hat{v}_{\mathbf{s}_3}} (\lambda\theta v + s_3)\, \mathrm{d}F(v) = 0, \tag{154}$$
where the subsidy affects both the probability that the creditors are repaid (via $\hat{v}_{\mathbf{s}_3}$) and what they get if they are not repaid in full (because they get s_3).
We differentiate Δ in equation (154) with respect to s_3 and $\hat{v}_{\mathbf{0}}$:
$$\frac{\partial \Delta}{\partial s_3} = F(\hat{v}_{\mathbf{0}}). \tag{155}$$
And,
$$\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{0}})\big)\big(1 - \lambda(1-\theta)\big) + (D_0 - D) f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}} f(\hat{v}_{\mathbf{0}}). \tag{156}$$

Substituting the above and $q_3 = F(\hat{v}_{\mathbf{0}})$ into equation (134) above, we can find the marginal benefit of investing in policy $\mathbf{s}_3$:
$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_3}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = -\frac{1}{(1 - F(\hat{v}))(1 - \lambda(1-\theta)) + (D_0 - D) f(\hat{v}) + \lambda\theta\hat{v} f(\hat{v})}. \tag{157}$$

Policy 4: Asset Subsidies.

- *Cost:* s_4 is paid unconditionally, so $q_4 = 1$.
- *Filing:* s_4 increases the firm's asset value in the firm's payoff (equation (5)), giving the filing threshold
$$\hat{v}_{\mathbf{s}_4} := \frac{D}{1 - (1-\theta)\lambda} - s_4 \equiv \hat{v}_{\mathbf{0}} - s_4, \tag{158}$$
where $\hat{v}_{\mathbf{0}}$ denotes the filing threshold in the baseline model with no policy intervention, as given in equation (7).
From equation (158), we have:
$$\frac{\partial \hat{v}_{\mathbf{s}_4}}{\partial \hat{v}_{\mathbf{0}}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{\mathbf{s}_4}}{\partial s_4} = -1. \tag{159}$$

- *Restructuring:* The binding IC is:

$$\Delta = \big(1 - F(\hat{v}_{\mathbf{s}_4})\big)(D - D_0) + \int_0^{\hat{v}_{\mathbf{s}_4}} \lambda\theta(v + s_4)\mathrm{d}F(v) = 0, \tag{160}$$

where the subsidy affects both the probability that the creditors are repaid in full (via $\hat{v}_{\mathbf{s}_4}$) and what they get if they are not repaid in full (because they share s_4).[35]

We differentiate the Δ in equation (160) with respect to s_4 and $\hat{v}_{\mathbf{0}}$:

$$\frac{\partial\Delta}{\partial s_4} = -f(v_{\mathbf{s}_4})(D - D_0)\frac{\partial\hat{v}_{\mathbf{s}_4}}{\partial s_4} + \int_0^{\hat{v}_{\mathbf{s}_4}} \lambda\theta\mathrm{d}F(v) + \lambda\theta(\hat{v}_{\mathbf{s}_4} + s_4)f(v_{\mathbf{s}_4})\frac{\partial\hat{v}_{\mathbf{s}_4}}{\partial s_4} \tag{161}$$

$$\to -\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right) f(\hat{v}_{\mathbf{0}}) + \lambda\theta F(\hat{v}_{\mathbf{0}}), \tag{162}$$

as $\varepsilon \to 0$, given $\hat{v}_{\mathbf{s}_4} \to \hat{v}_{\mathbf{0}}$ as $\varepsilon \to 0$.

And,

$$\frac{\partial\Delta}{\partial\hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{s}_4})\big)\frac{\partial D}{\partial\hat{v}_{\mathbf{0}}} - f(\hat{v}_{\mathbf{s}_4})(D - D_0) + \lambda\theta(\hat{v}_{\mathbf{s}_4} + s_4)f(\hat{v}_{\mathbf{s}_4})\frac{\partial\hat{v}_{\mathbf{s}_4}}{\partial\hat{v}_{\mathbf{0}}} \tag{163}$$

$$\to \big(1 - F(\hat{v}_{\mathbf{0}})\big)\big(1 - \lambda(1-\theta)\big) + (D_0 - D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}}), \tag{164}$$

as $\varepsilon \to 0$.

Substituting the above and $q_4 = 1$ into equation (134) above, we can find the marginal benefit of investing in policy $\mathbf{s}_4$:

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_4}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = -1 + \frac{\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right) f(\hat{v}_{\mathbf{0}}) - \lambda\theta F(\hat{v}_{\mathbf{0}})}{(1 - F(\hat{v}_{\mathbf{0}}))(1 - \lambda(1-\theta)) + (D_0 - D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}})}. \tag{165}$$

Policy 5: Subsidized Lending.

- *Cost:* s_5 is paid unconditionally and repaid conditional on solvency, which occurs with probability $1 - F(\hat{v}_{\mathbf{s}_5})$ (it is not repaid in bankruptcy, since it is diluted in restructuring). Thus,

$$q_5 = 1 - \big(1 - F(\hat{v}_{\mathbf{s}_5})\big) = F(\hat{v}_{\mathbf{s}_5}). \tag{166}$$

- *Filing:* s_5 increases both the firm's assets and liabilities in the firm's payoff (equation (5)), giving the filing threshold

$$\hat{v}_{\mathbf{s}_5} := \frac{D}{1 - (1-\theta)\lambda} + \frac{s_5}{1 - (1-\theta)\lambda} \equiv \hat{v}_{\mathbf{0}} + \frac{s_5}{1 - (1-\theta)\lambda}. \tag{167}$$

[35] Note that the lower bound on the integral is 0 (instead of $-s_4$) because $f = 0$ on $[-s_4, 0)$.

From equation (167), we have:

$$\frac{\partial \hat{v}_{\mathbf{s}_5}}{\partial \hat{v}_{\mathbf{0}}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{\mathbf{s}_5}}{\partial s_5} = \frac{1}{1-\lambda(1-\theta)}. \tag{168}$$

- *Restructuring:* The binding IC is:

$$\Delta = \big(1 - F(\hat{v}_{\mathbf{s}_5})\big)(D - D_0) + \int_0^{\hat{v}_{\mathbf{s}_5}} \lambda\theta(v + s_5)\mathrm{d}F(v) = 0, \tag{169}$$

where the subsidy affects both the probability that the creditors are repaid in full (via $\hat{v}_{\mathbf{s}_5}$) and what they get if they are not repaid in full (because they share s_5).

We differentiate Δ in equation (169) with respect to s_5 and $\hat{v}_{\mathbf{0}}$:

$$\frac{\partial \Delta}{\partial s_5} = -f(v_{\mathbf{s}_5})(D - D_0)\frac{\partial \hat{v}_{\mathbf{s}_5}}{\partial s_5} + \int_0^{\hat{v}_{\mathbf{s}_5}} \lambda\theta \mathrm{d}F(v) + \lambda\theta(v_{\mathbf{s}_5} + s_5)f(v_{\mathbf{s}_5})\frac{\partial \hat{v}_{\mathbf{s}_5}}{\partial s_5} \tag{170}$$

$$\to -\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right) f(\hat{v}_{\mathbf{0}}) + \lambda\theta F(\hat{v}_{\mathbf{0}}), \tag{171}$$

as $\varepsilon \to 0$, given $\hat{v}_{\mathbf{s}_5} \to \hat{v}_{\mathbf{0}}$ as $\varepsilon \to 0$.

And,

$$\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{s}_5})\big)\frac{\partial D}{\partial \hat{v}_{\mathbf{0}}} - f(\hat{v}_{\mathbf{s}_5})(D - D_0) + \lambda\theta(\hat{v}_{\mathbf{s}_5} + s_5)f(\hat{v}_{\mathbf{s}_5})\frac{\partial \hat{v}_{\mathbf{s}_5}}{\partial \hat{v}_{\mathbf{0}}} \tag{172}$$

$$\to \big(1 - F(\hat{v}_{\mathbf{0}})\big)\big(1 - \lambda(1-\theta)\big) + (D_0 - D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}}), \tag{173}$$

as $\varepsilon \to 0$.

Substituting the above and $q_5 = F(\hat{v}_{\mathbf{0}})$ into equation (134) above, we can find the marginal benefit of investing in policy $\mathbf{s}_5$:

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_5}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left(\frac{1}{1-\lambda(1-\theta)} + \frac{\left(D_0 - \frac{1-\lambda}{1-\lambda(1-\theta)}D\right) f(\hat{v}_{\mathbf{0}}) - \lambda\theta F(\hat{v}_{\mathbf{0}})}{(1 - F(\hat{v}_{\mathbf{0}}))(1-\lambda(1-\theta)) + (D_0 - D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}})}\right)\frac{1}{F(\hat{v}_{\mathbf{0}})}. \tag{174}$$

Policy 6: Debt Purchases (and Forgiveness).

- *Cost:* The cost is the market price of debt, which, given debt holders take into account the debt reduction and future restructuring, is just $q_6 = 1 - F(\hat{v}_{\mathbf{s}_6})$.
- *Filing:* s_6 does not affect equity holder's payoffs directly. Hence, $\hat{v}_{\mathbf{s}_6} = \hat{v}_{\mathbf{0}}$. Thus, we have:

$$\frac{\partial \hat{v}_{s_6}}{\partial \hat{v}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{s_6}}{\partial s_6} = 0. \tag{175}$$

- *Restructuring:* The binding IC is:

$$\Delta = \big(1 - F(\hat{v}_{\mathbf{s}_6})\big)\big(D - (D_0 - s_6)\big) + \int_0^{\hat{v}_{\mathbf{s}_6}} \lambda\theta v \mathrm{d}F(v), \tag{176}$$

where the subsidy affects the probability that the creditors are repaid in full (via $\hat{v}_{\mathbf{s}_6}$) and what they get if they hold out (because s_6 reduces the face value).

We differentiate Δ in equation (154) with respect to s_3 and $\hat{v}_{\mathbf{0}}$:

$$\frac{\partial \Delta}{\partial s_6} = 1 - F(\hat{v}_{\mathbf{0}}). \tag{177}$$

And,

$$\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{0}})\big)\big(1 - \lambda(1-\theta)\big) + (D_0 - D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}}). \tag{178}$$

Substituting the above and $q_6 = 1 - F(\hat{v}_{\mathbf{0}})$ into equation (134) above, we can find the marginal benefit of investing in policy $\mathbf{s}_6$. We get the same for policy $\mathbf{s}_3$ (equation (157)):

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_6}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_3}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0}. \tag{179}$$

Policy 7: Restructuring Subsidies.

- *Cost:* s_7 is paid conditional on a restructuring, but restructuring occurs with probability one in equilibrium, so $q_7 = 1$.
- *Filing:* s_7 does not affect equity holders' payoffs directly. Hence, $\hat{v}_{\mathbf{s}_7} = \hat{v}_{\mathbf{0}}$. Thus, we have:
$$\frac{\partial \hat{v}_{s_7}}{\partial \hat{v}_{\mathbf{0}}} = 1 \quad \text{and} \quad \frac{\partial \hat{v}_{s_7}}{\partial s_7} = 0 \tag{180}$$
- *Restructuring:* The binding IC is:
$$\Delta = s_7 + \big(1 - F(\hat{v}_{\mathbf{0}})\big)(D - D_0) + \int_0^{\hat{v}_{\mathbf{0}}} \lambda\theta v \mathrm{d}F(v) = 0, \tag{181}$$
where the subsidy does not affect the probability that the creditors are repaid (because $\hat{v}_{\mathbf{s}_7} = \hat{v}_{\mathbf{0}}$) but only affects what they get if they accept the restructuring (because they get s_7).

  We differentiate Δ in equation (181) with respect to s_7 and $\hat{v}_{\mathbf{0}}$:
$$\frac{\partial \Delta}{\partial s_7} \equiv 1. \tag{182}$$
  And
$$\frac{\partial \Delta}{\partial \hat{v}_{\mathbf{0}}} = \big(1 - F(\hat{v}_{\mathbf{0}})\big)\big(1 - \lambda(1-\theta)\big) + (D_0 - D)f(\hat{v}_{\mathbf{0}}) + \lambda\theta\hat{v}_{\mathbf{0}}f(\hat{v}_{\mathbf{0}}). \tag{183}$$

Substituting the above and $q_7 = 1$ into equation (134) above, we find the marginal benefit of investing in policy $\mathbf{s}_7$ is the same as for policies $\mathbf{s}_3$ and $\mathbf{s}_6$:

$$\left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_7}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_3}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \left.\frac{\mathrm{d}\hat{v}_{\mathbf{s}_6}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0}. \tag{184}$$

B.4 What if Powerful Secured Creditors Destroy Value for Equity Holders?

Here, we assume that for creditors to gain \$1 in bankruptcy, equity holders must forgo more than \$1. Specifically, for every $(1-\gamma/2)\theta\lambda v$ that creditors get, equity gives up $(1-\theta)\lambda v$. Thus, γ measures the inefficiencies they induce ex post. If $\gamma = 0$, the model is the same as the baseline. Increasing γ decreases the total surplus.

To explore how the inefficiencies of creditor power could affect restructuring, we explore how the optimal level of creditor friendliness depends on the inefficiencies induced by secured creditor power γ (cf. Section 3.7 and Section 3.8). This gives the next result:

Result 14. *Suppose that the write-down is maximized at a unique interior level of creditor friendliness θ^* that is not an inflection point (as in, e.g., the uniform case in Figure 1). Increasing the inefficiency of secured creditor power γ decreases the optimal level of creditor friendliness. That is,* $\mathrm{d}\theta^*/\mathrm{d}\gamma < 0$.

Intuitively, if giving creditors power destroys value in bankruptcy, then giving them more power is likely to make them even less willing to accept a restructuring. Hence, the larger is γ, the larger is the region of θ for which making the code more creditor friendly makes restructuring harder.

Proof. We begin from the creditors' IC:

$$\Delta = (1-F(\hat{v}))(D-D_0) + \lambda\theta\left(1-\frac{\gamma\theta}{2}\right)\int_0^{\hat{v}} v\mathrm{d}F(v), \tag{185}$$

which is just the creditors' IC in equation (26) modified to include the inefficiencies captured by γ . The binding IC, $\Delta = 0$, defines the written-down debt level D^* and minimizing D^* over θ defines the optimal level of creditor friendliness θ^*. Thus, by the chain rule, the effect of γ on θ^* is given by:

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\gamma} = -\frac{\frac{\partial}{\partial\gamma}\left(\frac{\partial\Delta}{\partial\theta}\right)}{\frac{\partial}{\partial\theta}\left(\frac{\partial\Delta}{\partial\theta}\right)} = -\frac{\partial^2\Delta/(\partial\gamma\partial\theta)}{\partial^2\Delta/\partial\theta^2}. \tag{186}$$

Note that the denominator is negative given that we have assumed that θ^* is an interior local minimum and not an inflection point.

We compute the numerator $\frac{\partial^2\Delta}{\partial\gamma\partial\theta}$ directly, step by step:

- First,

$$\frac{\partial \Delta}{\partial \theta} = \lambda (1-\gamma\theta) \int_0^{\hat{v}} v \mathrm{d}F(v) + \left[D_0 - D + \lambda\theta \left(1 - \frac{\gamma\theta}{2}\right) \hat{v} \right] f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta} \tag{187}$$

$$= \lambda (1-\gamma\theta) \int_0^{\hat{v}} v \mathrm{d}F(v) - \lambda \left[\hat{v}_0 - \hat{v} + \frac{\lambda\theta (2-\gamma\theta)}{2(1-\lambda(1-\theta))} \hat{v} \right] \hat{v} f(\hat{v}), \tag{188}$$

having used that $D = (1-\lambda(1-\theta))\hat{v}$, $D_0 = (1-\lambda(1-\theta))\hat{v}_0$, and

$$\frac{\partial \hat{v}}{\partial \theta} = \frac{-\lambda \hat{v}}{1-\lambda(1-\theta)}. \tag{189}$$

- Second,

$$\frac{\partial^2 \Delta}{\partial \gamma \partial \theta} = -\lambda\theta \left(\int_0^{\hat{v}} v \mathrm{d}F(v) - \frac{\lambda\theta}{2(1-\lambda(1-\theta))} \hat{v}^2 f(\hat{v}) \right). \tag{190}$$

To determine the sign, it turns out that we can we can use the fact that θ^* is optimal, so $\partial\Delta/\partial\theta|_{\theta=\theta^*} = 0$. That is, the derivative in equation (188) is zero. Manipulating, we get:

$$\lambda (1-\gamma\theta) \left(\int_0^{\hat{v}} v \mathrm{d}F(v) - \frac{\lambda\theta}{2(1-\lambda(1-\theta))} \hat{v}^2 f(\hat{v}) \right) - \lambda \left[\hat{v}_0 - \hat{v} + \frac{\lambda\theta}{2(1-\lambda(1-\theta))} \hat{v} \right] \hat{v} f(\hat{v}) = 0 \tag{191}$$

and, thus, that:

$$-\int_0^{\hat{v}} v \mathrm{d}F(v) + \frac{\lambda\theta}{2(1-\lambda(1-\theta))} \hat{v}^2 f(\hat{v}) = -(1-\gamma\theta)^{-1} \left[\hat{v}_0 - \hat{v} + \frac{\lambda\theta}{2(1-\lambda(1-\theta))} \hat{v} \right] \hat{v} f(\hat{v}), \tag{192}$$

which is the numerator in equation (190). It is negative given that $\hat{v}_0 > \hat{v}$.

Substituting into equation (186), we see that $\mathrm{d}\theta^*/\mathrm{d}\gamma$ is negative.

□

References

Adler, B. E. (1995). A re-examination of near-bankruptcy investment incentives. *The University of Chicago Law Review 62*(2), 575–606.

Aghion, P. and P. Bolton (1992). An incomplete contracts approach to financial contracting. *The Review of Economic Studies 59*(3), 473–494.

Antill, S. (2020). Do the right firms survive bankruptcy? Working paper, Harvard Business School.

Asquith, P., R. Gertner, and D. Scharfstein (1994). Anatomy of financial distress: An examination of junk-bond issuers. *The Quarterly Journal of Economics 109*(3), 625–658.

Ayotte, K. and J. Ellias (2020). Bankruptcy process for sale. Research paper no. 382, UC–Hastings.

Ayotte, K. M. and E. R. Morrison (2009). Creditor control and conflict in chapter 11. *Journal of Legal Analysis 1*(2), 511–551.

Baird, D. (1986). The uneasy case for corporate reorganizations. *The Journal of Legal Studies 15*(1), 127–147.

Baird, D. G. (1991). The initiation problem in bankruptcy. *International Review of Law and Economics 11*(2), 223 – 232.

Bebchuk, L. A. (2002). Ex ante costs of violating absolute priority in bankruptcy. *The Journal of Finance 57*(1), 445–460.

Bebchuk, L. A. and J. M. Fried (1996). The uneasy case for the priority of secured claims in bankruptcy. *The Yale Law Journal 105*(4), 857–934.

Becker, B. and J. Josephson (2016). Insolvency resolution and the missing high-yield bond markets. *The Review of Financial Studies 29*(10), 2814–2849.

Benmelech, E., N. Kumar, and R. Rajan (2020). Secured credit spreads. Working paper, Northwestern University.

Bernardo, A. E. and E. L. Talley (1996). Investment policy and exit-exchange offers within financially distressed firms. *The Journal of Finance 51*(3), 871–888.

Blanchard, O., T. Philippon, and J. Pisani-Ferry (2020). A new policy toolkit is needed as countries exit COVID-19 lockdowns. *Peterson Institute for International Economics Policy Brief 20-8*, 1–19.

Bris, A., I. Welch, and N. Zhu (2006). The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization. *The Journal of Finance 61*(3), 1253–1303.

Brunnermeier, M. and A. Krishnamurthy (2020). Corporate debt overhang and credit policy. BPEA Conference Drafts, Brookings Papers on Economic Activity.

Bulow, J. I. and J. B. Shoven (1978). The bankruptcy decision. *The Bell Journal of Economics 9*(2), 437–456.

Campbell, J. Y., J. Hilscher, and J. Szilagyi (2008). In search of distress risk. *The Journal of Finance 63*(6), 2899–2939.

Campello, M., T. Ladika, and R. Matta (2018). Renegotiation Frictions and Financial Distress Resolution: Evidence from CDS Spreads. *Review of Finance 23*(3), 513–556.

Casey, A. J. (2011). The creditors' bargain and option-preservation priority in chapter 11. *University of Chicago Law Review 78*, 759–807.

Chaney, T., D. Sraer, and D. Thesmar (2012). The collateral channel: How real estate shocks affect corporate investment. *The American Economic Review 102*(6), 2381–2409.

Chatterjee, S., U. S. Dhillon, and G. G. Ramãrez (1995). Coercive tender and exchange offers in distressed high-yield debt restructurings an empirical analysis. *Journal of Financial Economics 38*(3), 333–360.

DeMarzo, P., A. Krishnamurthy, and J. Rauh (2020). Debtor-in-possession financing facility (DIPFF) proposal. Working paper, Stanford GSB.

Diamond, D. W. (1993). Seniority and maturity of debt contracts. *Journal of Financial Economics 33*(3), 341–368.

Donaldson, J. R., D. Gromb, and G. Piacentino (2018). The paradox of pledgeability. *Journal of Financial Economics,* forthcoming.

Donaldson, J. R., D. Gromb, and G. Piacentino (2020). Conflicting priorities: A theory of covenants and collateral. Working paper, Washington University in St. Louis.

Donaldson, J. R. and G. Piacentino (2020). Netting. Working paper, Washington University in St. Louis.

Eberhart, A., W. Moore, and R. Roenfeldt (1990). Security pricing and deviations from the absolute priority rule in bankruptcy proceedings. *The Journal of Finance 45*(5), 1457–1469.

Eckbo, B. E. and W. Wang (2019). Rent extraction by super-priority lenders. Working paper, Tuck School of Business.

Fan, H. and S. Sundaresan (2000). Debt valuation, renegotiation, and optimal dividend policy. *The Review of Financial Studies 13*(4), 1057–1099.

Favara, G., E. Schroth, and P. Valta (2012). Strategic default and equity risk across countries. *The Journal of Finance 67*(6), 2051–2095.

Franks, J. and W. Torous (1989). An empirical investigation of U.S. firms in reorganization. *The Journal of Finance 44*(3), 747–769.

Franks, J. R. and W. N. Torous (1994). A comparison of financial recontracting in distressed exchanges and Chapter 11 reorganizations. *Journal of Financial Economics 35*(3), 349–370.

Gelman, A. and J. Hill (2007). *Data analysis using regression and multilevel/hierarchical models*. New York, NY: Cambridge University Press.

Gertner, R. and D. Scharfstein (1991). A theory of workouts and the effects of reorganization law. *The Journal of Finance 46*(4), 1189–1222.

Giambona, E., F. Lopez-de Silanes, and R. Matta (2019). Stiffing the creditor: The effect of asset verifiability on bankruptcy. Working paper, Syracuse University.

Giammarino, R. (1989). The resolution of financial distress. *The Review of Financial Studies 2*(1), 25–47.

Gilson, S. C., K. John, and L. H. Lang (1990). Troubled debt restructurings: An empirical study of private reorganization of firms in default. *Journal of Financial Economics 27*(2), 315 – 353.

Greenwood, R. and D. Thesmar (2020). Sharing the economic pain of the coronavirus. Working paper, Harvard Business School.

Grossman, S. J. and O. D. Hart (1980). Takeover bids, the free-rider problem, and the theory of the corporation. *The Bell Journal of Economics 11*(1), 42–64.

Hart, O. (1995). *Firms, Contracts, and Financial Structure*. Oxford University Press.

Hart, O. and J. Moore (1994, November). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics 109*(4), 841–79.

Hart, O. and J. Moore (1995). Debt and seniority: An analysis of the role of hard claims in constraining management. *American Economic Review 85*(3), 567–85.

Hart, O. and J. Moore (1998, February). Default and renegotiation: A dynamic model of debt. *The Quarterly Journal of Economics 113*(1), 1–41.

Haugen, R. A. and L. W. Senbet (1978). The insignificance of bankruptcy costs to the theory of optimal capital structure. *The Journal of Finance 33*(2), 383–393.

Haugen, R. A. and L. W. Senbet (1988). Bankruptcy and agency costs: Their significance to the theory of optimal capital structure. *Journal of Financial and Quantitative Analysis 23*(1), 27–38.

Holmstrom, B. and J. Tirole (1997). Financial Intermediation, Loanable Funds, and the Real Sector. *The Quarterly Journal of Economics 112*(3), 663–91.

Hotchkiss, E. S., K. John, R. M. Mooradian, and K. S. Thorburn (2008). Chapter 14 – Bankruptcy and the resolution of financial distress. In B. E. Eckbo (Ed.), *Handbook of Empirical Corporate Finance*, pp. 235 – 287. San Diego: Elsevier.

Hynes, R. and S. Walt (2020). Revitalizing involuntary bankruptcy. *Iowa Law Review 105*, 1127–1185.

Iverson, B. (2018). Get in line: Chapter 11 restructuring in crowded bankruptcy courts. *Management Science 64*(11), 4967–5460.

Iverson, B., J. Ellias, and M. Roe (2020). Estimating the need for additional bankruptcy judges in light of the COVID-19 pandemic. *Harvard Business Law Review,* forthcoming.

Jacoby, M. B. and E. J. Janger (2014). Ice cube bonds: Allocating the price of process in Chapter 11 bankruptcy. *Yale Law Journal 123*, 862–1117.

James, C. (1995). When do banks take equity in debt restructurings? *The Review of Financial Studies 8*(4), 1209–1234.

Jensen, M. C. (1986). Agency costs of free cash flow, corporate finance, and takeovers. *The American Economic Review 76*(2), 323–329.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance 49*(4), 1213–1252.

Levin, R. and H. J. Sommer (Eds.) (2020). *Collier on Bankruptcy* (16th ed.). New Providence, NJ: LexisNexis.

Miller, H. and R. Berkovich (2006). The implications of the Third Circuit's *Armstrong* decision on creative corporate restructuring: Will strict construction of the Absolute Priority Rule make Chapter 11 consensus less likely? *American University Law Review 55*(5), 1345–1426.

Moody's (2017). A closer look at distressed exchanges.

Moody's (2020). Recoveries in a pandemic-driven default cycle.

Mooradian, R. M. (1994). The effect of bankruptcy protection on investment: Chapter 11 as a screening device. *The Journal of Finance 49*(4), 1403–1430.

Morrison, E. R. and A. C. Saavedra (2020). Bankruptcy's role in the COVID-19 crisis. In K. Pistor (Ed.), *Law in the Time of COVID-19*, pp. 127–140. New York, NY: Columbia Law School.

Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics 5*(2), 147–175.

Picker, R. C. (1992). Voluntary petitions and the creditors' bargain. *Cincinnati Law Review 61*, 519–541.

Povel, P. (1999). Optimal "soft" or "tough" bankruptcy procedures. *Journal of Law, Economics, & Organization 15*(3), 659–684.

Roe, M. J. (1983). Bankruptcy and debt: A new model for corporate reorganization. *Columbia Law Review 83*(3), 527–602.

Roe, M. J. (1987). The voting prohibition in bond workouts. *Yale Law Journal 97*, 232–279.

Roe, M. J. and F. Tung (2016). *Bankruptcy and Corporate Reorganization, Legal and Financial Materials*. St. Paul, MN: Foundation Press.

Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–66.

Titman, S. (1984). The effect of capital structure on a firm's liquidation decision. *Journal of Financial Economics 13*(1), 137–151.

Weiss, L. A. (1990). Bankruptcy resolution: Direct costs and violation of priority of claims. *Journal of Financial Economics 27*(2), 285–314.

White, M. J. (1980). Public policy toward bankruptcy: Me-first and other priority rules. *Bell Journal of Economics 11*(2), 550–564.

White, M. J. (1983). Bankruptcy costs and the new bankruptcy code. *Journal of Finance*, 477–488.

White, M. J. (1994). Corporate bankruptcy as a filtering device: Chapter 11 reorganizations and out-of-court debt restructurings. *Journal of Law, Economics, & Organization 10*(2), 268–295.