Plants in Space

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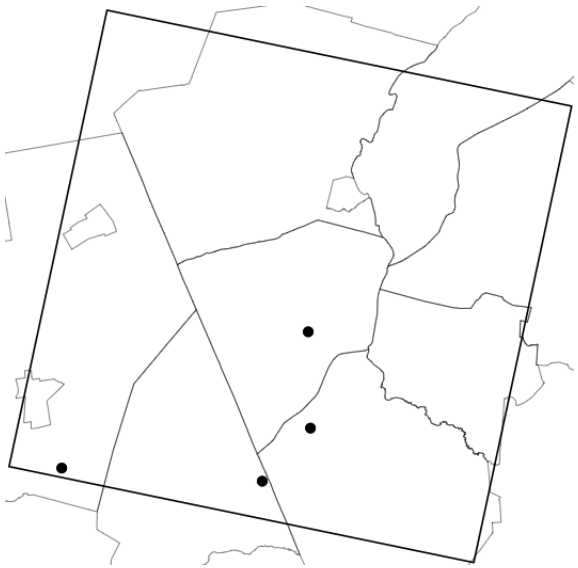
Introduction

- A fundamental part of a firm's production problem is to determine the number, size, and location of its plants
- More plants, closer to consumers, imply lower **transport costs** but larger **fixed and managerial span-of-control costs**
  - Plants **cannibalize** each other's markets, particularly if they are close
- There are **no general insights** on the solution to this problem **when locations are heterogeneous**
  - Large **combinatorial problem**, so most of the analysis is purely numerical
  - Solution is known when space is homogeneous
- Important to study this problem to understand firm characteristics as well as consumer access to a firm's output
- In equilibrium, plant location decisions are an essential determinant of local concentration and the distribution of economic activity in space

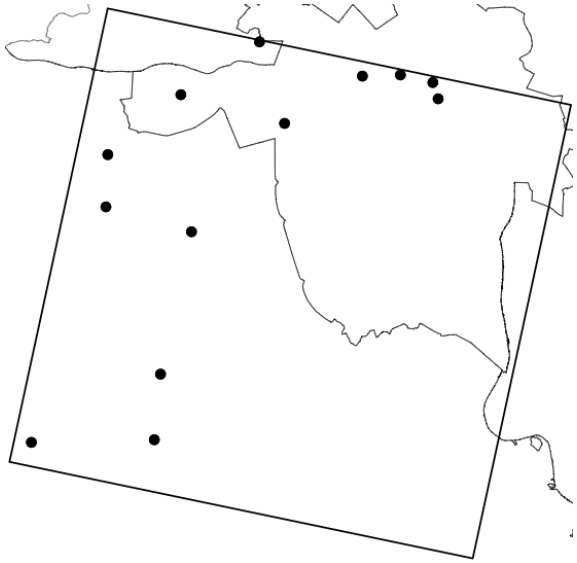
Example: Starbucks' Stores Density and Size

- Density and arrangement of plants differs tremendously across locations
  - **Starbucks** operated around 14000 stores across the U.S. in 2014
  - Consider their location in 12 miles squares with different population density

Princeton



Richmond



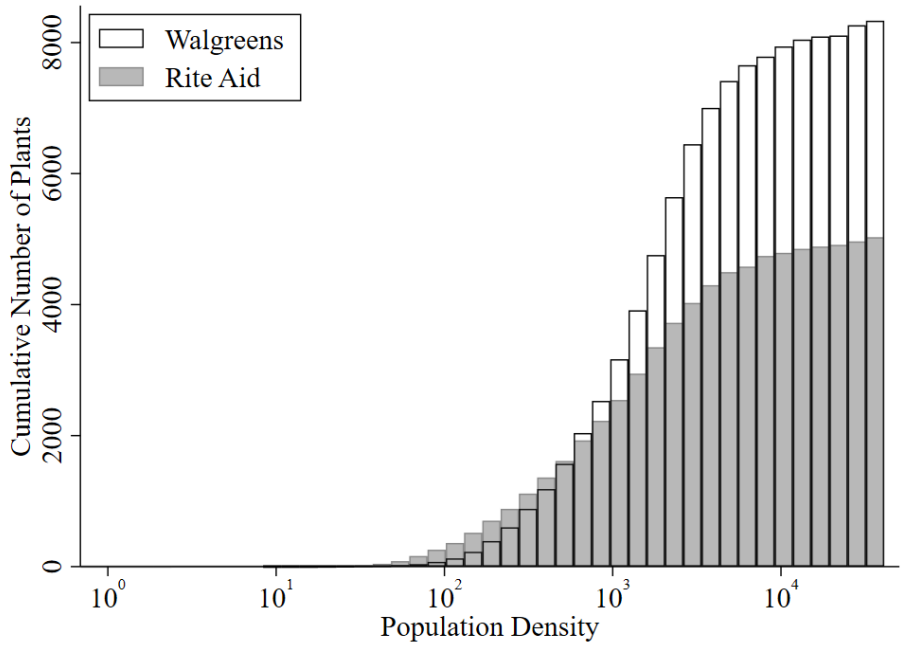
New York



- Size is also not constant: New York stores are 23% larger than Richmond's

Example: Sorting of Pharmacies

- Perhaps firm location is simply monotone in population (expenditure) density
  - With more productive firms reaching more marginal markets
- Data suggests otherwise: **Sorting**
  - Consider the location of **Walgreens** and **Rite Aid** pharmacies
    - ★ Walgreens is a larger, and presumably more productive, firm
  - Productive (large) firm has **more** plants in denser locations, but **fewer** plants in lower density ones



Density | Map | Auto Parts

What We Do

- Propose a model of a firm's location decisions
  - Heterogeneous firms locate multiple plants in heterogeneous locations
  - Firms face transport, span of control, and fixed costs
  - Key tradeoff: **minimize transport cost** vs. **cannibalization**
- Problem of the firm is a large combinatorial discrete-choice problem
- Our contribution is to **propose a tractable limit case**
  - All key forces and trade-offs remain relevant
  - Solution method inspired by central place theory, discrete geometry
  - Amounts to a many-to-many matching problem that can be partially characterized analytically
- Limit case can be embedded in an equilibrium framework
  - Study effect of changes in technology and transport costs on sorting and local characteristics
- We verify many of the implications using the NETS data set

Related Literature

- Firm with multiple plants
  - Over time: Luttmer (2011); Cao, et al. (2019); Aghion, et al. (2019)
  - Across space: Rossi-Hansberg, et al. (2018), Hsieh and Rossi-Hansberg (2020)
- Multinationals and export platforms
  - Ramondo (2014), Ramondo and Rodriguez Clare (2013), Tintelnot (2017), Arkolakis, et al. (2018)
- Solutions to plant location problem
  - Homogeneous space: Christaller (1933), Fejes Toth (1953), Bollobas (1972)
  - Numerical: Jia (2008), Holmes (2011), Arkolakis and Eckert (2018), Hu and Shi (2019)
- Assignment in space
  - Firm location: Gaubert (2018), Ziv (2019)
  - Worker location: Behrens, et al. (2014), Eeckhout et al. (2014), Davis and Dingel (2019), Bilal and Rossi-Hansberg (2019)

The Environment

- Customers distributed across locations $s \in \mathcal{S} = [0,1]^2 \subset \mathbb{R}^2$
- Each location s characterized by
  - Exogenous local productivity, B_s
  - Residual demand, $D_s(p) = D_s p^{-\varepsilon}$, with $\varepsilon > 1$
    - ⋆ Later, D_s a function of local price index and exogenous amenities
  - Wage rate, W_s
  - Commercial rent, R_s
- Firms take local equilibrium as given

Firms

- Each firm $j \in J$ produces a unique variety
- Chooses set of locations $O_j \in S$ where to produce
  - Let $N_j = |O_j|$, denote the number of locations where j produces
- Firm productivity in location $o \in O_j$ is $B_o Z(q_j, N_j)$
  - where q_j is an exogenous component of firm productivity
  - and $Z_N(q_j, N_j) < 0$ and $Z(q_j, 0) < \infty$ (**Span-of-control costs**)
- Each plant takes ξ units of commercial real estate, with rental cost R_s per unit of space
- Iceberg cost, $T(\delta)$, to deliver good to customer at distance δ

The Firm's Problem

- Minimal cost of delivering one unit to s is $\Lambda_{js}(O_j) \equiv \min_{o \in O_j} \frac{W_o T(\delta_{so})}{B_o Z(q_j, N_j)}$
- Optimal price is then $\max_{p_{js}} D_s(p_{js})(p_{js} - \Lambda_{js})$
- Total profit of firm j is then given by

$$\pi_j = \max_{O_j} \left\{ \int_s \max_{p_{js}} D_s p_{js}^{-\varepsilon} \left(p_{js} - \Lambda_{js}(O_j)\right) ds - \sum_{o \in O_j} R_o \xi \right\}$$

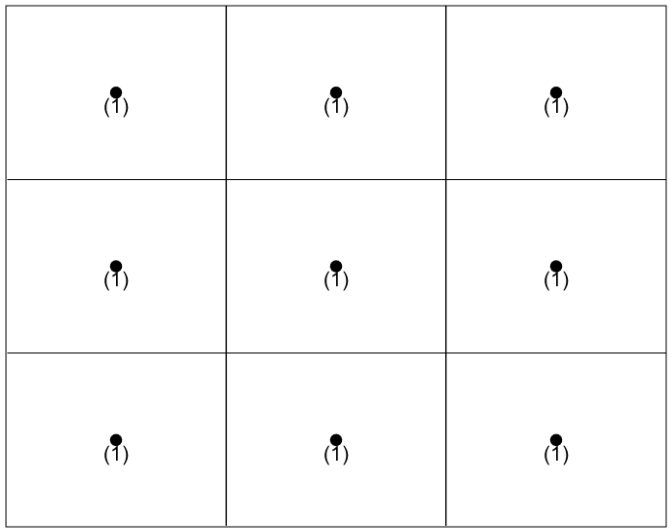
$$= \max_{O_j} \left\{ Z(q_j, N_j)^{\varepsilon - 1} \frac{(\varepsilon - 1)^{\varepsilon - 1}}{\varepsilon^{\varepsilon}} \int_s D_s \max_{o \in O_j} \left(\frac{B_o / W_o}{T(\delta_{so})} \right)^{\varepsilon - 1} ds - \sum_{o \in O_j} R_o \xi \right\}$$

Catchment Areas

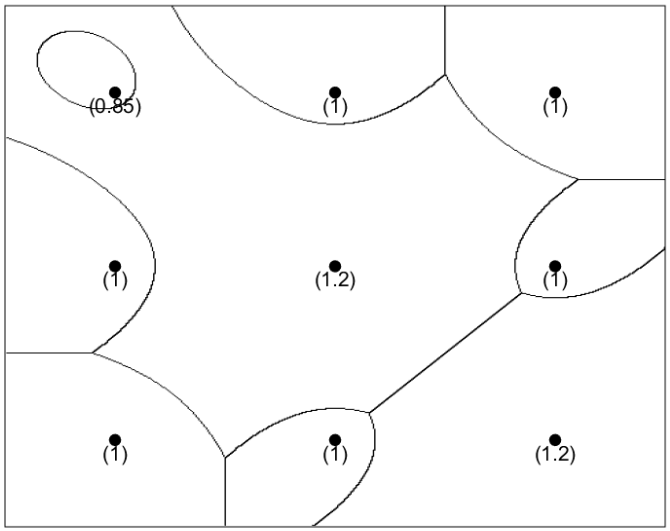
- **Given plant locations**, catchment areas only depend on $T(\delta_{so})$ and B_o/W_o

$$CA(o) = \left\{ s \in \mathcal{S} \text{ for which } o = \arg\max_{\tilde{o} \in O_j} \left\{ \frac{B_{\tilde{o}}/W_{\tilde{o}}}{T\left(\delta_{s\tilde{o}}\right)} \right\} \right\}$$

- Example: $T(\delta_{so}) = 1 + \delta_{so}$



$B_o/W_o = 1 \; \forall \; o$



B_o/W_o vary with o

A Limit Case

- In general, placement of plants in space is a hard problem
  - Catchment areas depend on local characteristics of plant locations
  - Plant locations depend on the whole distribution of demand across space
- Our approach is to study a limit case in which firms choose to have many plants, with small catchment areas
  - Consider an environment indexed by Δ, in which

$$\xi^{\Delta} = \Delta^2$$

$$T^{\Delta}(\delta) = t\left(\frac{\delta}{\Delta}\right)$$

$$Z^{\Delta}(q, N) = z(q, \Delta^2 N)$$

  - Study limit case as $\Delta \to 0$
- **Tradeoffs** between the fixed and span-of-control costs of setting up plants and the cost of reaching consumers **remain relevant**
  - Plants continue to cannibalize each other's customers
  - But forces will apply at local level

The Core Result

Proposition

Suppose that R_s, D_s, and B_s/W_s are continuous functions of s. Then, in the limit as $\Delta \to 0$,

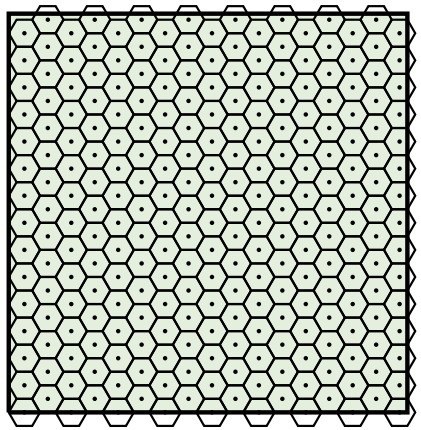
$$\pi_j = \sup_{n:\mathcal{S}\to\mathbb{R}^+} \int_s \left[x_s z \left(q_j, \int n_{\tilde{s}} d\tilde{s} \right)^{\varepsilon-1} n_s g\left(1/n_s\right) - R_s n_s \right] ds$$

where $x_s \equiv \frac{(\varepsilon-1)^{\varepsilon-1}}{\varepsilon^\varepsilon} D_s \left(B_s/W_s\right)^{\varepsilon-1}$ and where $g(u)$ is the integral of $t\left(\cdot\right)^{1-\varepsilon}$ over the distances of points to the center of a regular hexagon with area u.

- x_s combines local demand and effective labor costs
- $\kappa(n) \equiv ng\left(\frac{1}{n}\right)$ represents the **local efficiency of distribution**
- In the limit:
  - Maximum profits are attained by placing plants so that catchment areas are, locally, uniform infinitesimal hexagons
  - Firm's problem is one of calculus of variations which is much simpler

Elements of the Proof

- When economic characteristics are **uniform across space** solution is known
  - Fejes Toth (1953) shows that if number of plants grows large, catchment areas are uniform regular hexagons



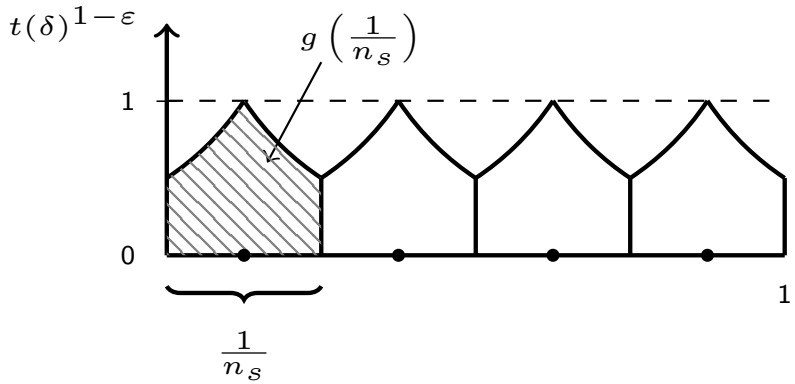
- We show that logic can be generalized to **heterogeneous space**
  - Construct upper and lower bounds for π_j in original problem for all Δ
  - Use hexagons for the bounds, as $\Delta \downarrow 0$
  - Upper and lower bound approach the same limit value. Thus, so does π_j

The Local Efficiency of Distribution

- We can write the firm's problem in the limit case as

$$\pi_j = \sup_{N_j, n_j : \mathcal{S} \to \mathbb{R}^+} \int_s \left[x_s z(q_j, N_j)^{\varepsilon - 1} x_s \kappa(n_{js}) - R_s n_s \right] ds, \quad \text{s.t. } N_j = \int_s n_{js} ds$$

- The local efficiency of distribution, $\kappa(n) \equiv n g\left(\frac{1}{n}\right)$, is
  - $\kappa(0) = 0$
  - Strictly increasing and strictly concave
  - $\lim_{n \to \infty} \kappa(n) = 1$ (**Saturation**)
  - $1 - \kappa(n) \underset{n \to \infty}{\sim} n^{-1/2}$ (**Asymptotic power law**)
- If, additionally, $\lim_{\delta \to \infty} T(\delta) \delta^{-4/(\varepsilon - 1)} = \infty$, then
  - $\kappa''(0) = 0$
  - $\kappa'(0) < \infty$ (**No INADA condition**)



FOCs and Span-of-Control Costs

- The problem in the limit case is

$$\pi_j = \sup_{N_j, n_j : \mathcal{S} \to \mathbb{R}^+} \int_s \left[x_s z(q_j, N_j)^{\varepsilon - 1} x_s \kappa(n_{js}) - R_s n_s \right] ds, \quad \text{s.t. } N_j = \int_s n_{js} ds$$

- Differentiating with respect to the number of plants in s, n_{js}, we obtain

$$x_s z(q_j, N_j)^{\varepsilon - 1} \kappa'(n_{js}) \leq R_s + \lambda_j, \qquad \text{with } \text{“}=\text{”} \text{ if } n_{js} > 0$$

- Differentiating with respect to the total number of plants, N_j, we obtain

$$\lambda_j = -\frac{dz(q_j, N_j)^{\varepsilon - 1}}{dN_j} \int_s x_s \kappa(n_{js}) ds$$

  - where the Lagrange multiplier of the constraint, λ_j, can be interpreted as the **marginal span-of-control cost** of firm j

Sorting

- The FOC implies that more productive firms have larger span-of control costs
  - That is, if $z_1 < z_2$ then $\lambda_1 < \lambda_2$, in fact $\frac{\lambda_1}{z_1^{\varepsilon-1}} < \frac{\lambda_2}{z_2^{\varepsilon-1}}$
- This implies that firms sort in space according to rents, namely,

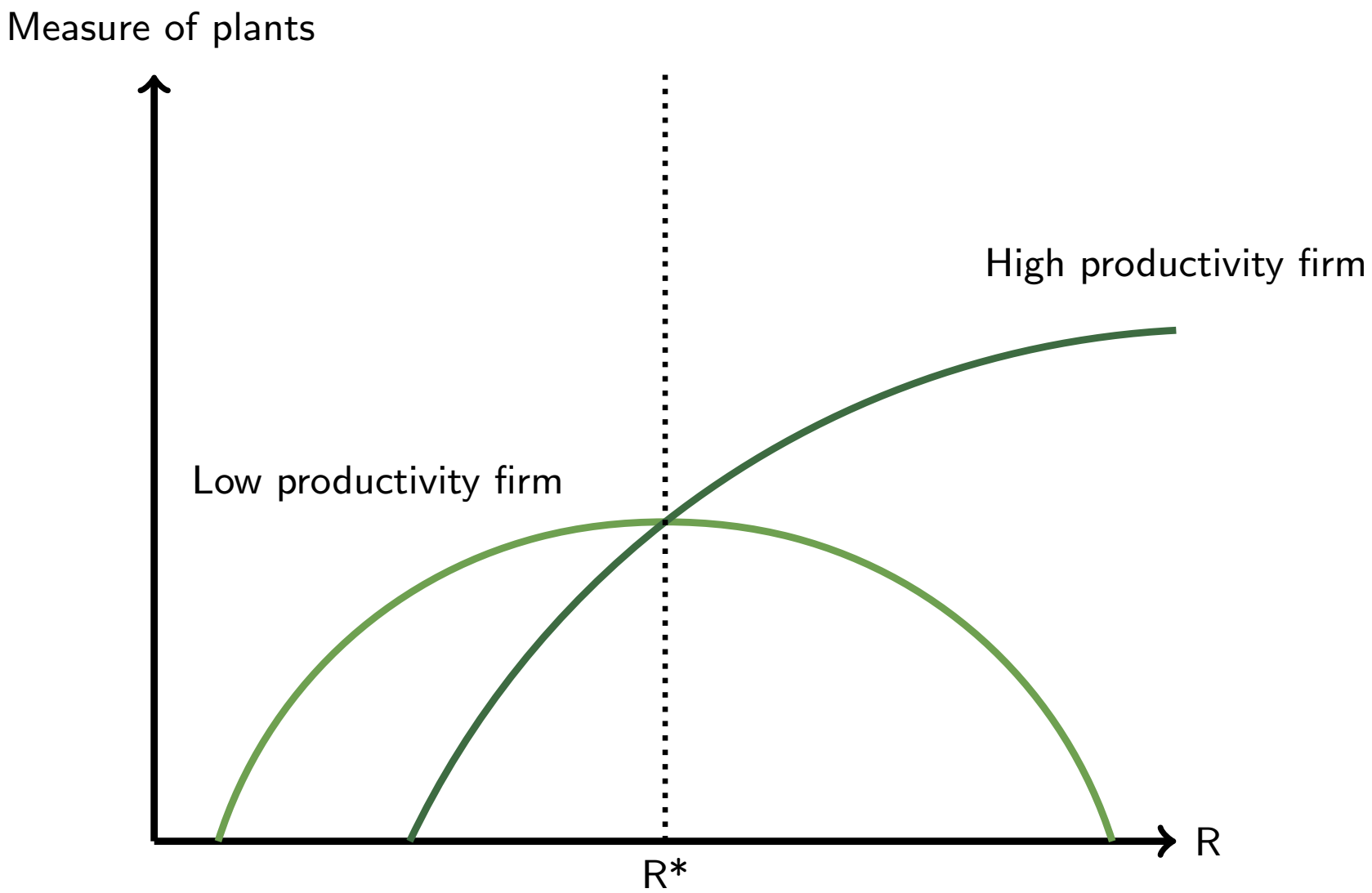
Proposition

If $z_1 < z_2$, there is a unique cutoff $R^*(z_0, z_1)$ for which $\frac{R^*(z_1,z_2)+\lambda_2}{R^*(z_1,z_2)+\lambda_1} = \frac{z_2^{\varepsilon-1}}{z_1^{\varepsilon-1}}$.

- *If $R_s > R^*(z_1, z_2)$ then $n_{2s} \geq n_{1s}$, with strict inequality if $n_{2s} > 0$*
- *If $R_s < R^*(z_1, z_2)$ then $n_{1s} \geq n_{2s}$, with strict inequality if $n_{1s} > 0$.*
- *If $R_s = R^*(z_1, z_2)$ then $n_{1s} = n_{2s}$.*

- Firms also sort based on local profitability, x_s

Sorting


Measure of plants
High productivity firm
Low productivity firm
R
R*

Sorting and Span-of-Control

- In virtually all existing models, more productive firms enter more marginal markets
- Here, less productive firms have more plants in worse locations. **Why?**
  - Productive firms have more profits per plant, but also larger effective fixed costs

$$x_s z_j^{\varepsilon-1} \kappa'(n_{js}) = \underbrace{R_s + \lambda_j}_{\text{effective fixed cost}}$$

  - High productivity firms are less sensitive to rents since

$$\lambda_j \uparrow \Rightarrow \frac{d \ln(R_s + \lambda_j)}{d \ln R_s} \downarrow$$

    so they sort into high-rent locations

Equilibrium

We specify the rest of the economy as follows:

- Locations characterized by exogenous **amenities**, A_s, and **productivity**, B_s
- Mass $\mathcal{L}$ of workers
  - Freely mobile across s, live and work at same location, supply labor inelastically
  - Preferences from consumption and housing are given by $u(c, h, a) = Ac^{1-\eta}h^{\eta}$
    - ⋆ with c Dixit-Stiglitz with elasticity ε
  - Budget constraint is $P_s c + R_s^H h \leq W_s + \Upsilon$
    - ⋆ where Υ are the mutual fund proceeds from land and firms
  - Hence, $D_s = \mathcal{L}_s c_s P_s^{\varepsilon}$
- Unit measure of land in each location
  - Competitive developers rent land to firms and workers
  - $H_s + \mathcal{N}_s \leq 1$, where $H_s = h_s \mathcal{L}_s$ and $\mathcal{N}_s = \int_j n_{js} dj$

Aggregation

- In equilibrium we can define **local productivity** as

$$\mathcal{Z}_s \equiv \left(\int_j z_j^{\varepsilon-1} \kappa(n_{js}) dj \right)^{\frac{1}{\varepsilon-1}}$$

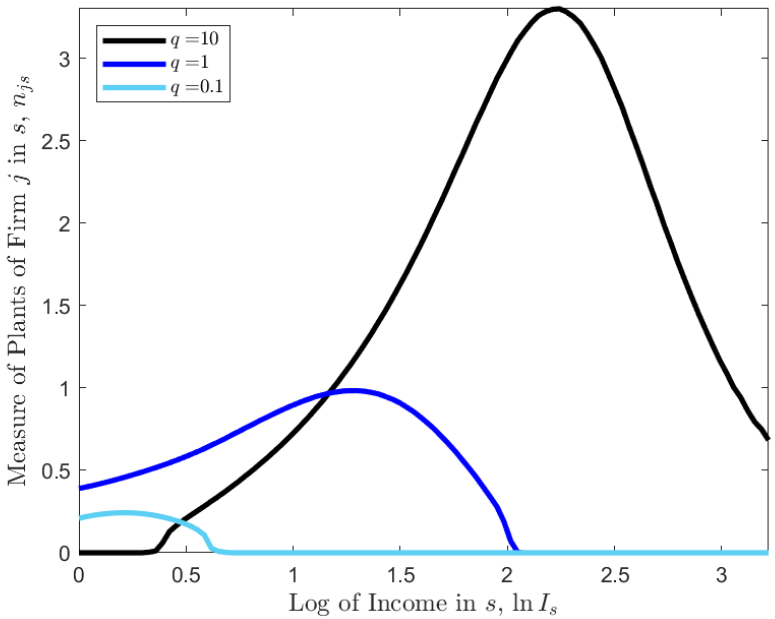
  - Then, the consumption bundle is given by $c_s = B_s \mathcal{Z}_s$
  - the price index by $P_s = \frac{\varepsilon}{\varepsilon-1} \frac{W_s}{B_s \mathcal{Z}_s}$
  - **local profitability** by $x_s = \frac{1}{\varepsilon-1} \frac{W_s \mathcal{L}_s}{\mathcal{Z}_s^{\varepsilon-1}}$
  - and the share of labor in location s is then

$$\frac{\mathcal{L}_s}{\mathcal{L}} = \frac{\left[A_s B_s^{1-\eta} \mathcal{Z}_s^{1-\eta} H_s \right]^{1/\eta}}{\int_s \left[A_{\tilde{s}} B_{\tilde{s}}^{1-\eta} \mathcal{Z}_{\tilde{s}}^{1-\eta} H_s \right]^{1/\eta} d\tilde{s}}$$

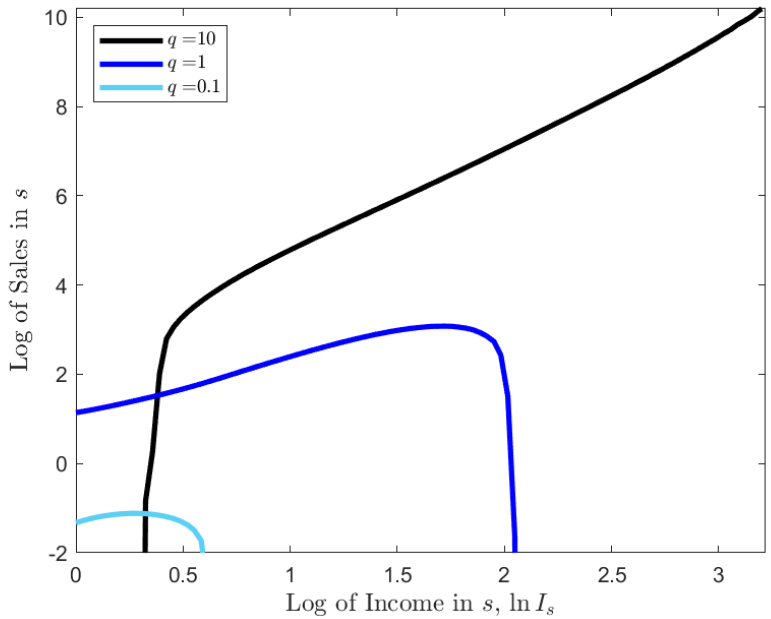
Numerical Illustration: Industry Equilibrium

- Continuum of industries and symmetric Cobb-Douglas preferences
- Total income, I_s, is truncated Pareto and $R(I_s) = e^{\log(I_s)^2}$
- Productivity, q_j, is also truncated Pareto with $z(q, N) = qe^{-N/\sigma}$
- Transportation costs are given by $t(\delta; \phi) = e^{\gamma/\sqrt{\phi}}$

Plants

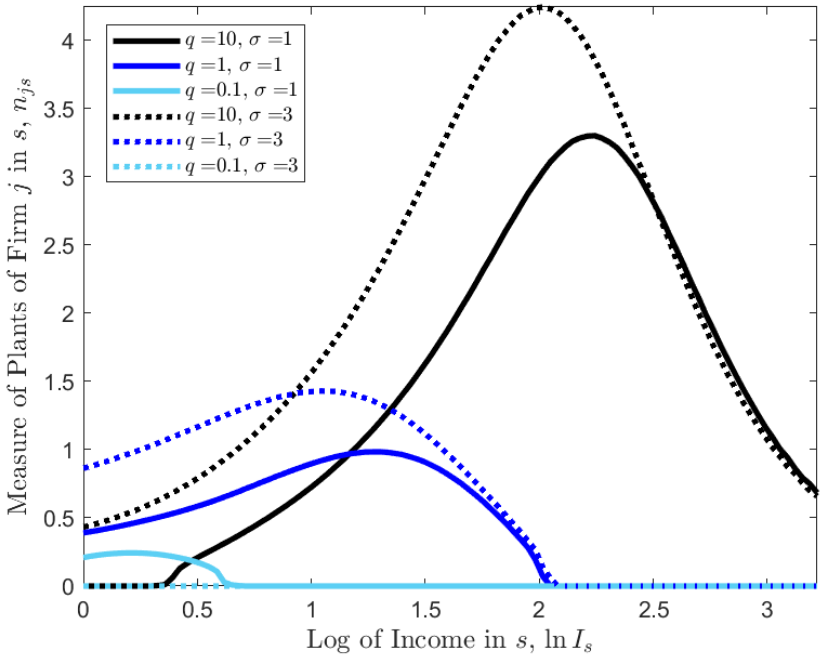


Sales

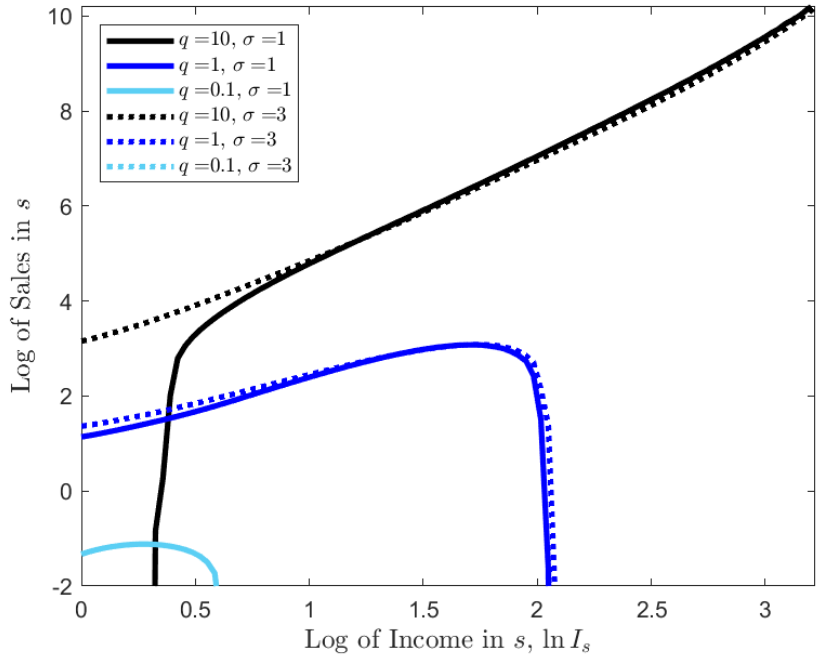


Improvements in Span-of-Control: $z(q, N) = qe^{-N/\sigma}$

Plants



Sales

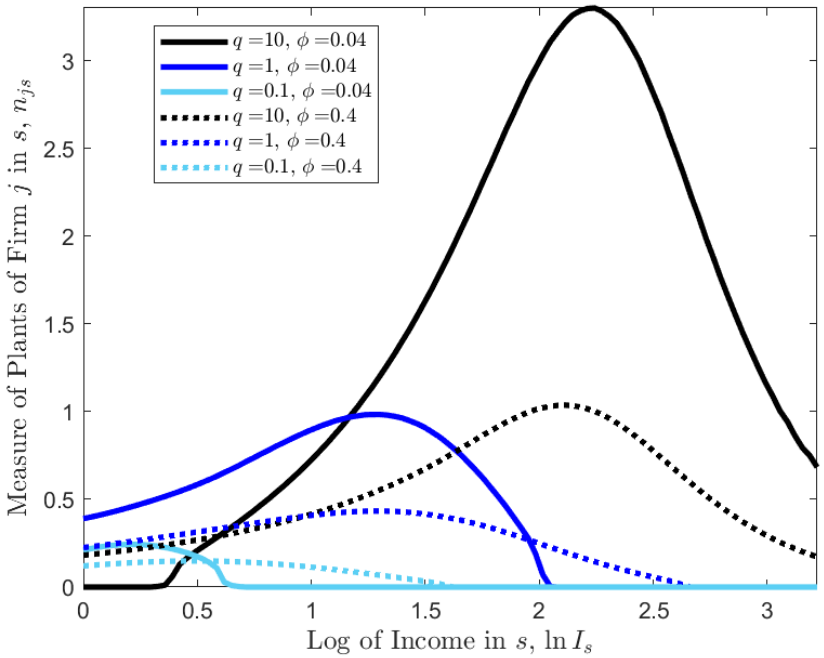


- **Top firms** expand to low income locations
  - Also, contract presence in top locations due to competition
- **Worse firms** exit

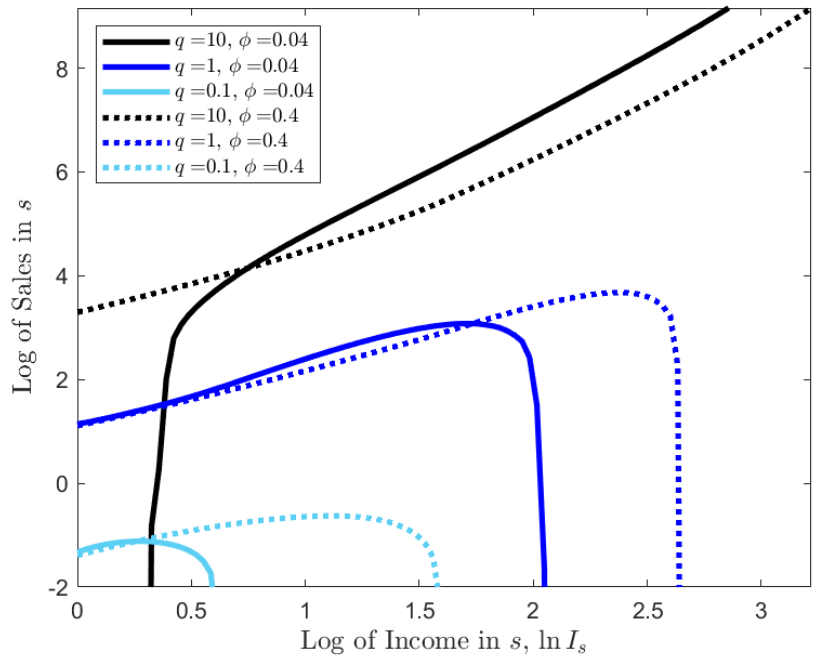
Effect on λ_j and x_s

Improvements in Transportation: $t(\delta;\phi) = e^{\gamma/\sqrt{\phi}}$

Plants



Sales



- **Increase in catchment areas** reduces effective fixed cost of new plants
  - **Top firms** suffer more from increased cannibalization and competition
  - **Worse firms** benefit disproportionately from lower fixed costs

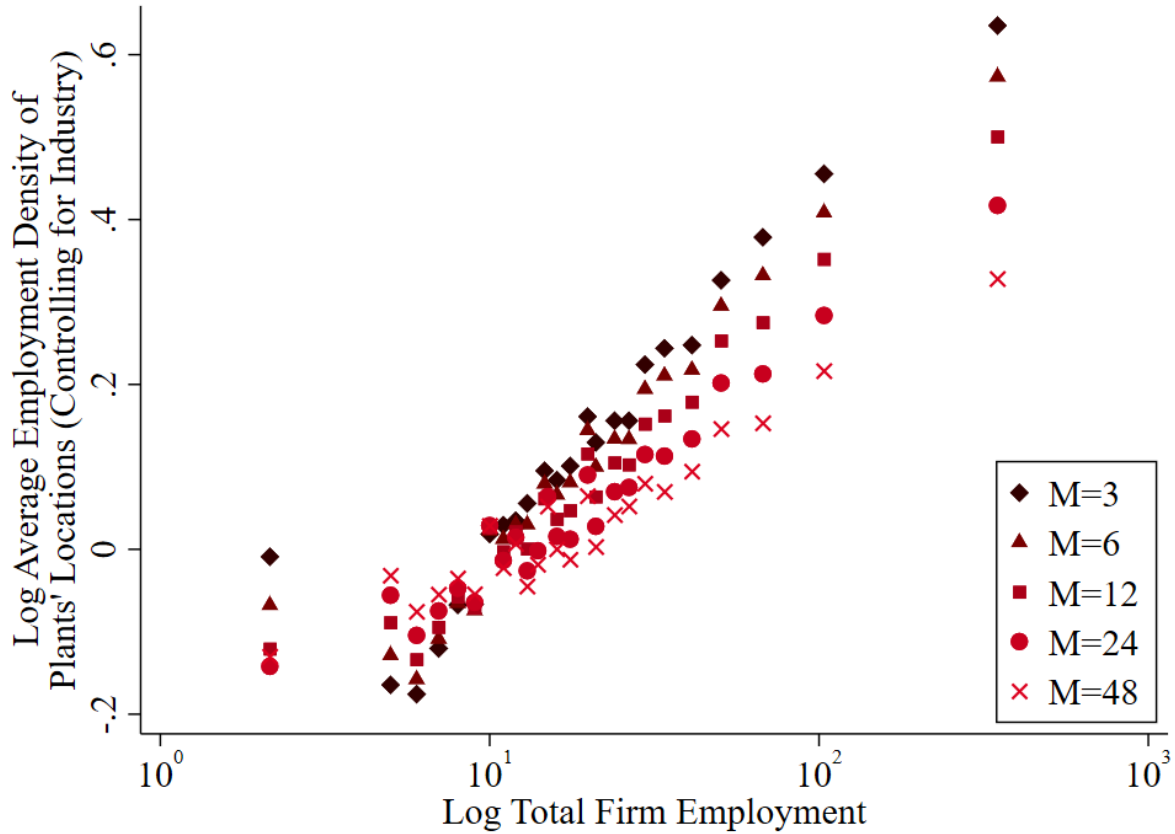
Empirical Evidence

- We use the National Establishment Time Series (NETS) dataset in 2014
  - Private sector source of business microdata (Dunn & Bradstreet)
  - We have establishment's precise location, employment, and link to parent company
    - ★ Drop plants with less than 5 employees
  - Definition of a location is a square of resolution M miles $\times$ M miles
- In the data we do not observe
  - Firm productivity: Monotone in total employment if $z(q, N) = qe^{-N/\sigma}$
  - Location 'quality' index, $A_s B_s^{1-\eta}$: Monotone in population density
- In Zillow data rents are increasing in population density

Rents and Density

Sorting in the Data: Average Density and Firm Size

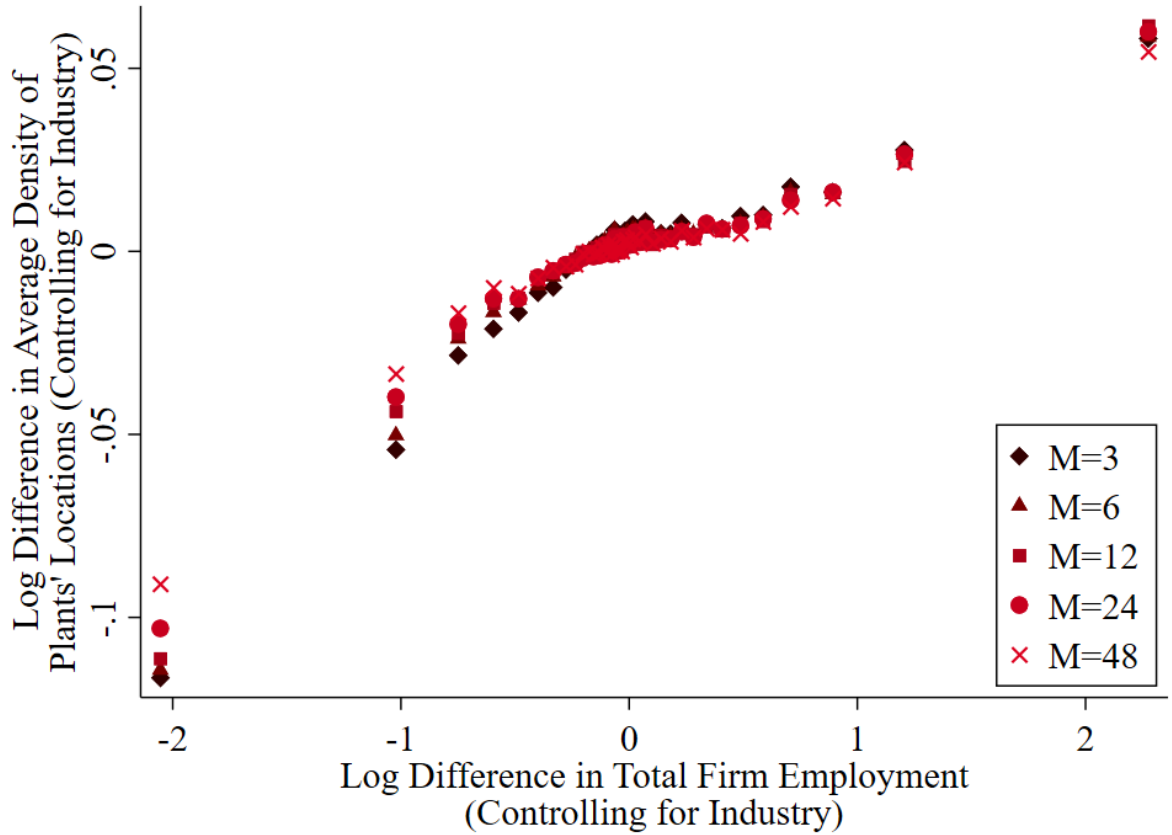
- Compute average weighted population density of plant locations of firms
  - In the baseline we use share of plants as weights



Alternative weights | Netting out firm's contribution | Sectors | Non-imputed data

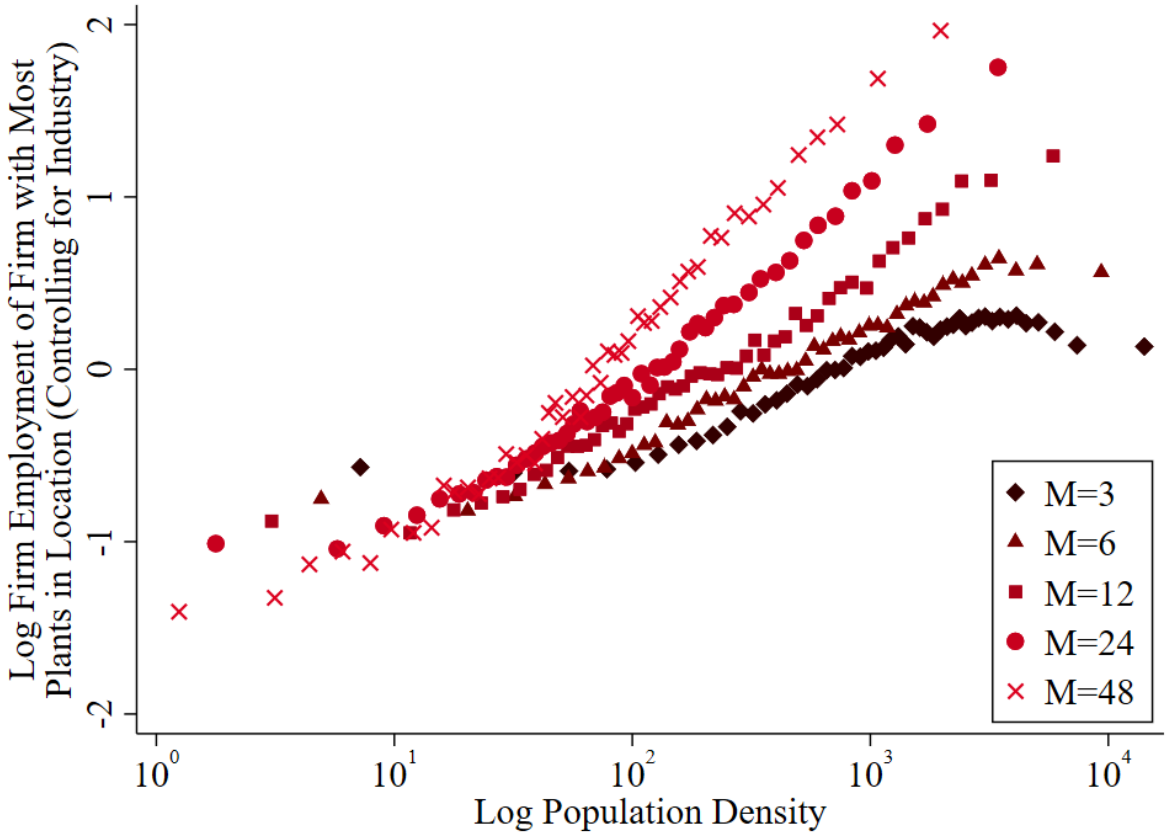
Sorting in the Data: Changes over Time

- Calculate change in firm's plant location density between 2000 and 2014
  - In both years, use 2000 local density levels
- Calculate change in total firm size between 2000 and 2014
- Subtract industry fixed effects from both variables



Sorting in the Data: National Size of the Top Firm in Town

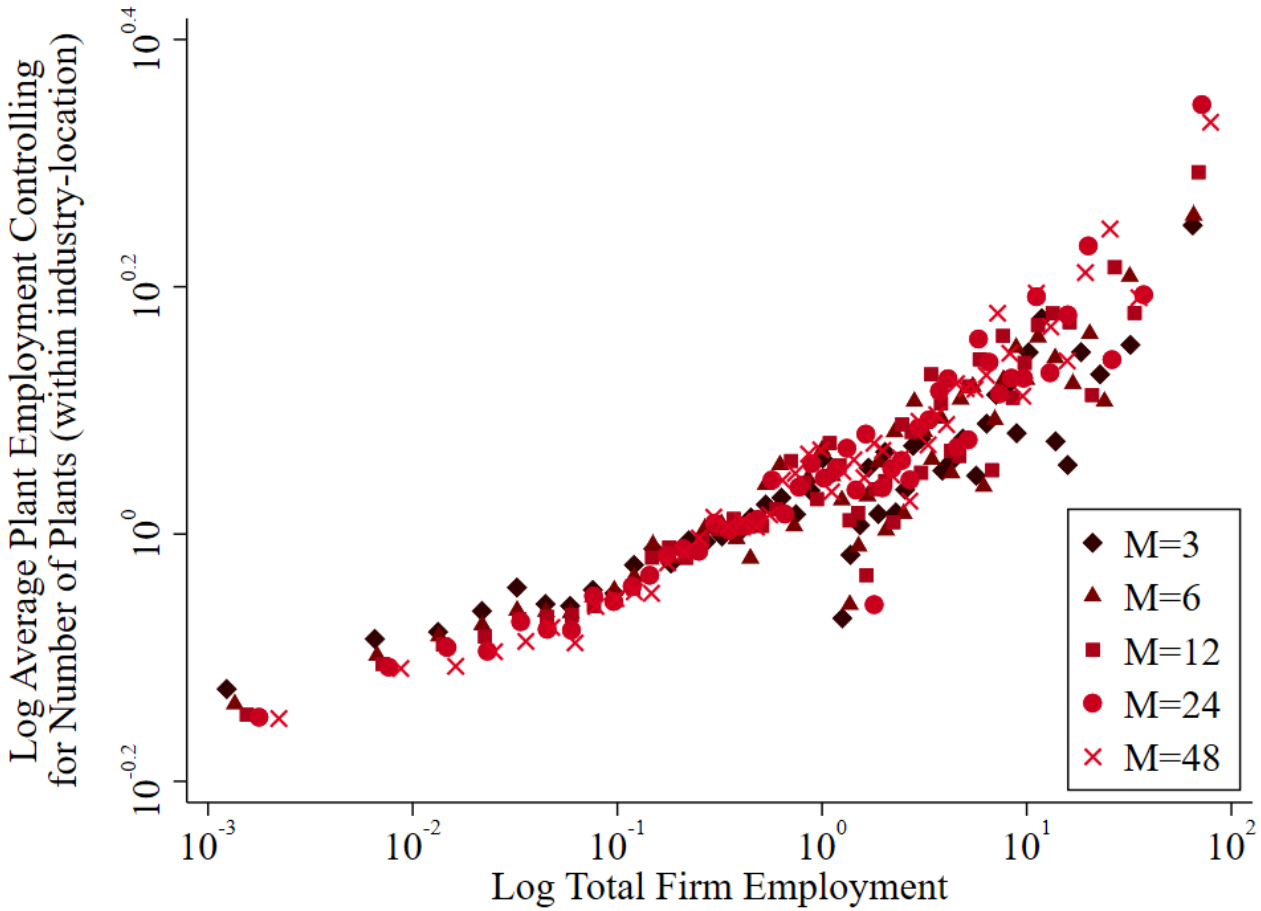
- For each location and industry, find firm with most plants



Alternative tie-breaking rules | Netting out location's contribution | Sectors | Non-imputed data

The Role of Span-of-Control in the Data

- If two firms with different z_j have the same n_{js}
  - Low z_j, low λ_j, firm's n_{js} is limited by low productivity
  - High z_j, high λ_j, firm's n_{js} is limited by high span-of-control cost
- Since firm size rises with z_j, **large firms have larger plants, given** n_{js}



Saturation in the data

- The local size of a firm's plants is given by $l_{js} = (\varepsilon - 1) z_j^{\varepsilon - 1} \frac{x_s}{W_s} \kappa(n_{js}) / n_{js}$
  - Remember, $\kappa(n_{js})$ is increasing, convex, and converges to 1
- **The more saturation, the more cannibalization, and so the more plant size declines with extra plants**
  - Important to control for firm and location fixed effects

	(1) $\Delta \ln l_{js}$	(2) $\Delta \ln l_{js}$	(3) $\Delta \ln l_{js}$	(4) $\Delta \ln l_{js}$	(5) $\Delta \ln l_{js}$
$\ln n_{js,2000}$	-0.0792***	-0.0467***	-0.0402***	-0.0634***	-0.0661***
	(0.0260)	(0.0169)	(0.0136)	(0.0115)	(0.0101)
$\Delta \ln n_{js}$	0.0729***	0.0460**	0.0768***	0.0610***	0.0639***
	(0.0262)	(0.0190)	(0.0157)	(0.0135)	(0.0126)
$\ln n_{js,2000} \times \Delta \ln n_{js}$	**-0.0954*****	**-0.00369**	**-0.0194**	**-0.0192****	**-0.0225*****
	(0.0308)	(0.0203)	(0.0132)	(0.00970)	(0.00787)
Observations	20,230	30,583	41,246	49,888	56,170
R-squared	0.628	0.621	0.604	0.589	0.571
SIC8-location FE	✓	✓	✓	✓	✓
SIC8-firm FE	✓	✓	✓	✓	✓
M	3	6	12	24	48

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Local Characteristics and Plant Growth in the Data

- Suppose local profitability in s, x_s, increases, which makes rents, R_s, rise
- Remember the FOC, $x_s z(q_j, N_j)^{\varepsilon-1}\kappa'(n_{js}) \leq R_s + \lambda_j$
- Hence, conditional on n_{js}, and firm and local fixed effects
  - Nationally large firms expand the number of plants more when x_s rises
  - Rents are a smaller part of large firms' fixed costs, $R_s + \lambda_j$

	(1) Growth in n_{js}	(2) Growth in n_{js}	(3) Growth in n_{js}	(4) Growth in n_{js}	(5) Growth in n_{js}
$\ln n_{js,2000}$	0.00438	-0.00302	-0.00213	0.00150	0.00511*
	(0.00646)	(0.00457)	(0.00357)	(0.00303)	(0.00268)
$\ln L_{j,2000} \times \Delta \ln \mathcal{L}_s$	**0.0100***	**0.0126****	**0.0171*****	**0.0237*****	**0.0427*****
	(0.00540)	(0.00522)	(0.00549)	(0.00622)	(0.00717)
Observations	272,506	360,721	418,772	443,227	442,352
R-squared	0.793	0.788	0.782	0.780	0.775
SIC8-firm FE	✓	✓	✓	✓	✓
SIC8-location FE	✓	✓	✓	✓	✓
M	3	6	12	24	48

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Transport efficiency and Plants in the Data

- Effect of **transport efficiency**, ϕ, on n_{js} is ambiguous
  - Higher transport efficiency enlarges CA but increases L_{js}
- **Saturation** $(\kappa'' < 0)$ implies that **cross-effect of L_{js} and ϕ is negative**

VARIABLES	(1) $\ln n_{js}$	(2) $\ln n_{js}$	(3) $\ln n_{js}$	(4) $\ln n_{js}$	(5) $\ln n_{js}$	(6) $\ln n_{js}$	(7) $\ln n_{js}$	(8) $\ln n_{js}$
$\ln L_{js}$	0.276***	0.244***	0.276***	0.351***	0.129***	0.271***	0.271***	0.272***
	(0.00145)	(0.00131)	(0.00145)	(0.00219)	(0.00555)	(0.00152)	(0.00183)	(0.00150)
X Gini		-0.151***						
		(0.00171)						
X Ellison-Glaeser			-0.0413***					
			(0.00607)					
X Consumer Gravity				-0.0970***				
				(0.00150)				
X Freight Cost					0.0200***			
					(0.00584)			
X Trade Cost						0.0250***		
						(0.00118)		
X Speed Score							-0.00355***	
							(0.00131)	
X Travel Time								0.00881***
								(0.00117)
Observations	366,979	366,979	366,979	209,700	8,166	345,771	207,155	323,782
R-squared	0.747	0.769	0.748	0.798	0.692	0.750	0.769	0.753
SIC8-location FE	✓	✓	✓	✓	✓	✓	✓	✓
SIC8-firm FE	✓	✓	✓	✓	✓	✓	✓	✓
M	24	24	24	24	24	24	24	24

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

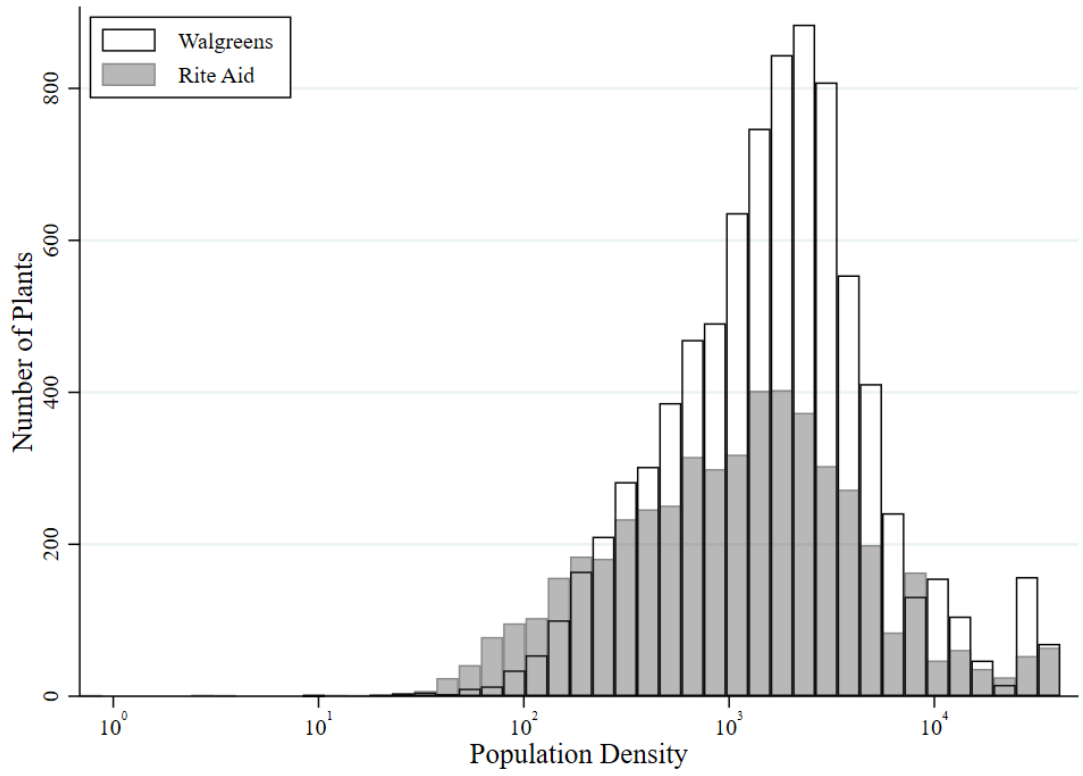
Robustness for M

Conclusions

- We propose methodology to analyze the location, number, and size of a firm's plants across heterogeneous locations
  - Original problem intractable but limit problem much simpler to analyze
  - Important that limit problem preserves all the relevant tradeoffs but locally
  - Easy to incorporate in a quantitative spatial economic framework
- Problem yields multiple insights: Sorting, as well as the role of saturation, span-of-control, and transport technology
  - We corroborate these implications using U.S. NETS data
- We study numerically the effect of changes in span-of-control and transport technology in a 'small' industry
  - Interesting to study economy-wide or 'large' industry changes
  - In particular the effects of secular changes of technology on economic activity and local competition

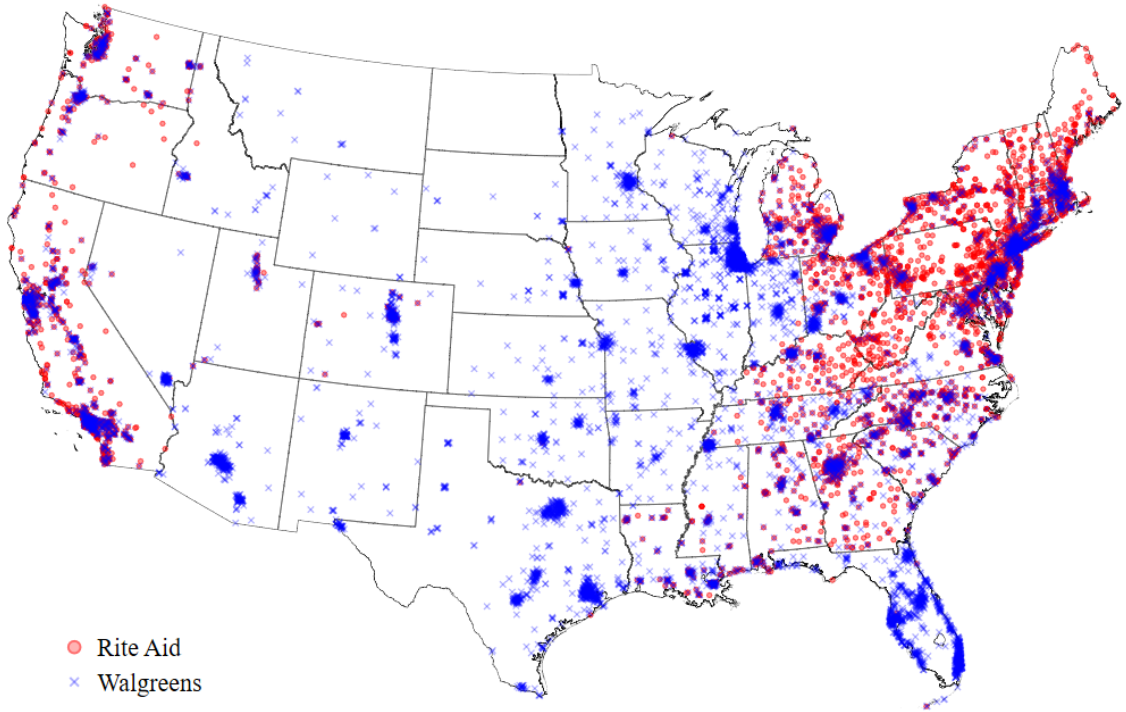
Thank You

Sorting Example: Drug Stores



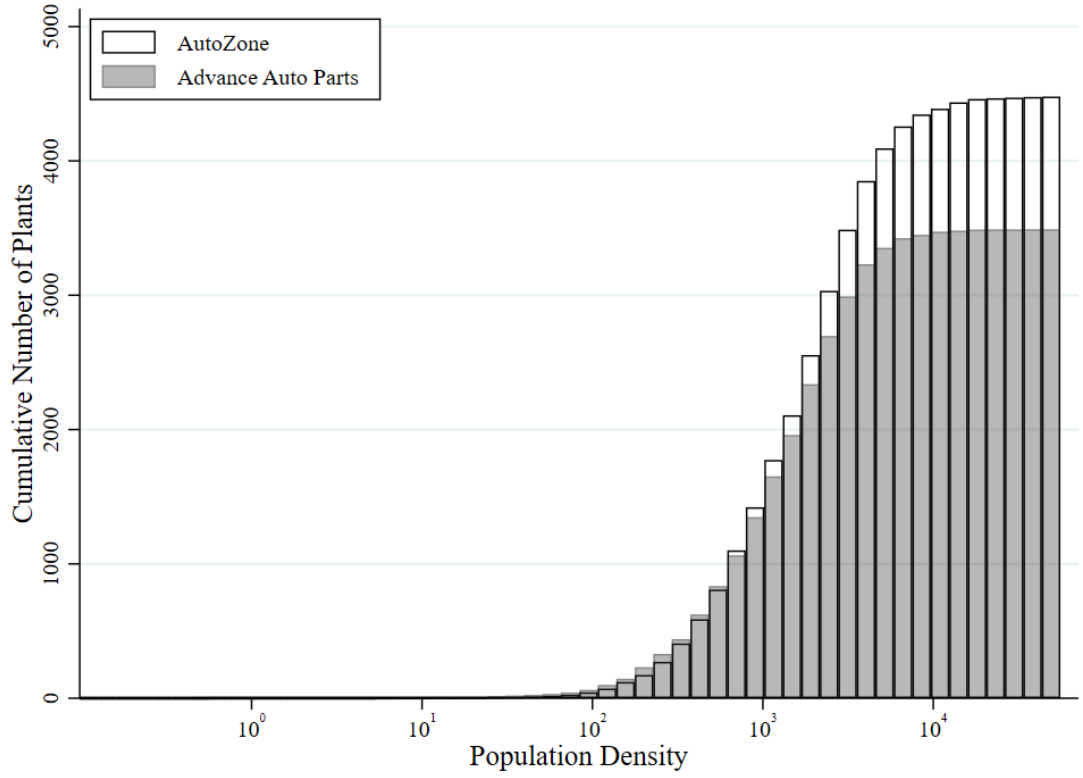
Back to Intro

Sorting Example: Drug Stores



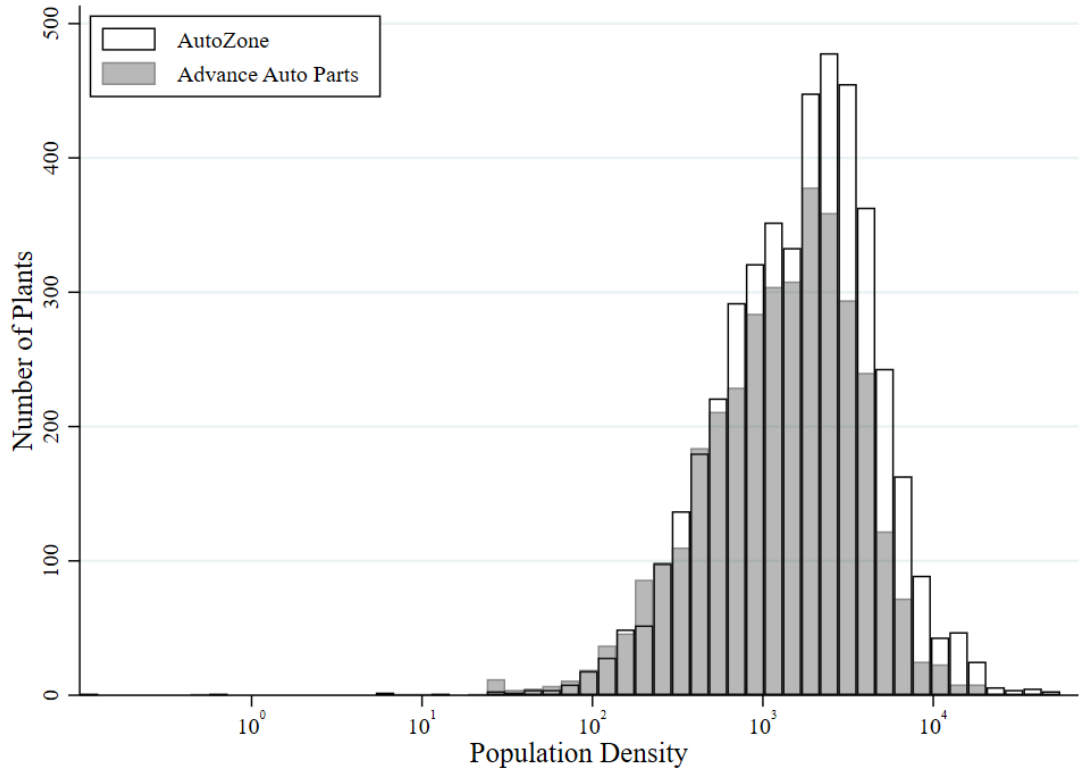
Back to Intro

Sorting



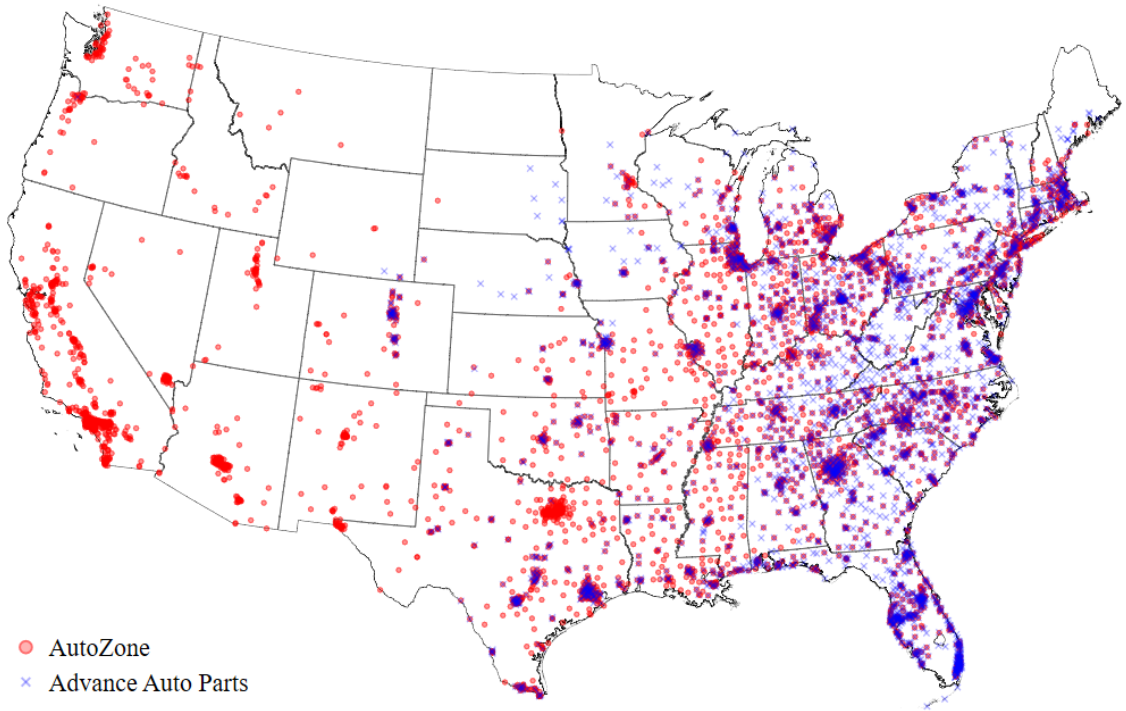
Density Map Back to Intro

Sorting Example: Auto Parts



Back to Auto Parts
Back to Intro

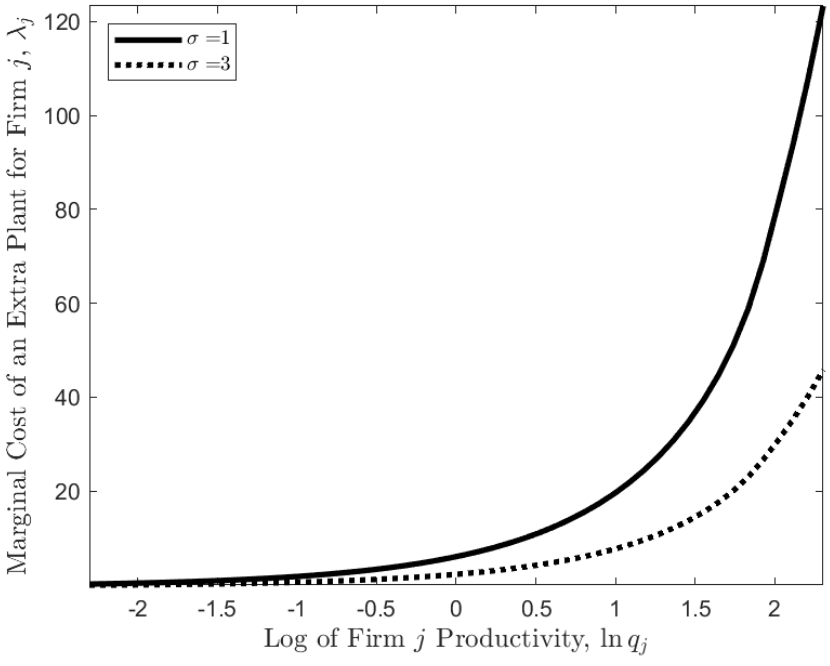
Sorting Example: Auto Parts



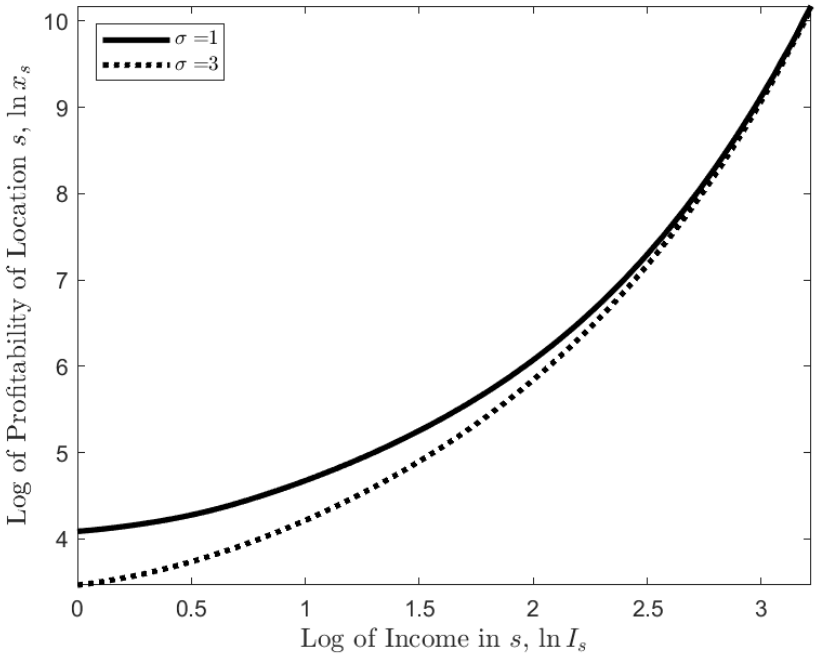
Back to Auto Parts Back to Intro

Improvements in Span of Control: $z(q, N) = qe^{-N/\sigma}$

λ_j

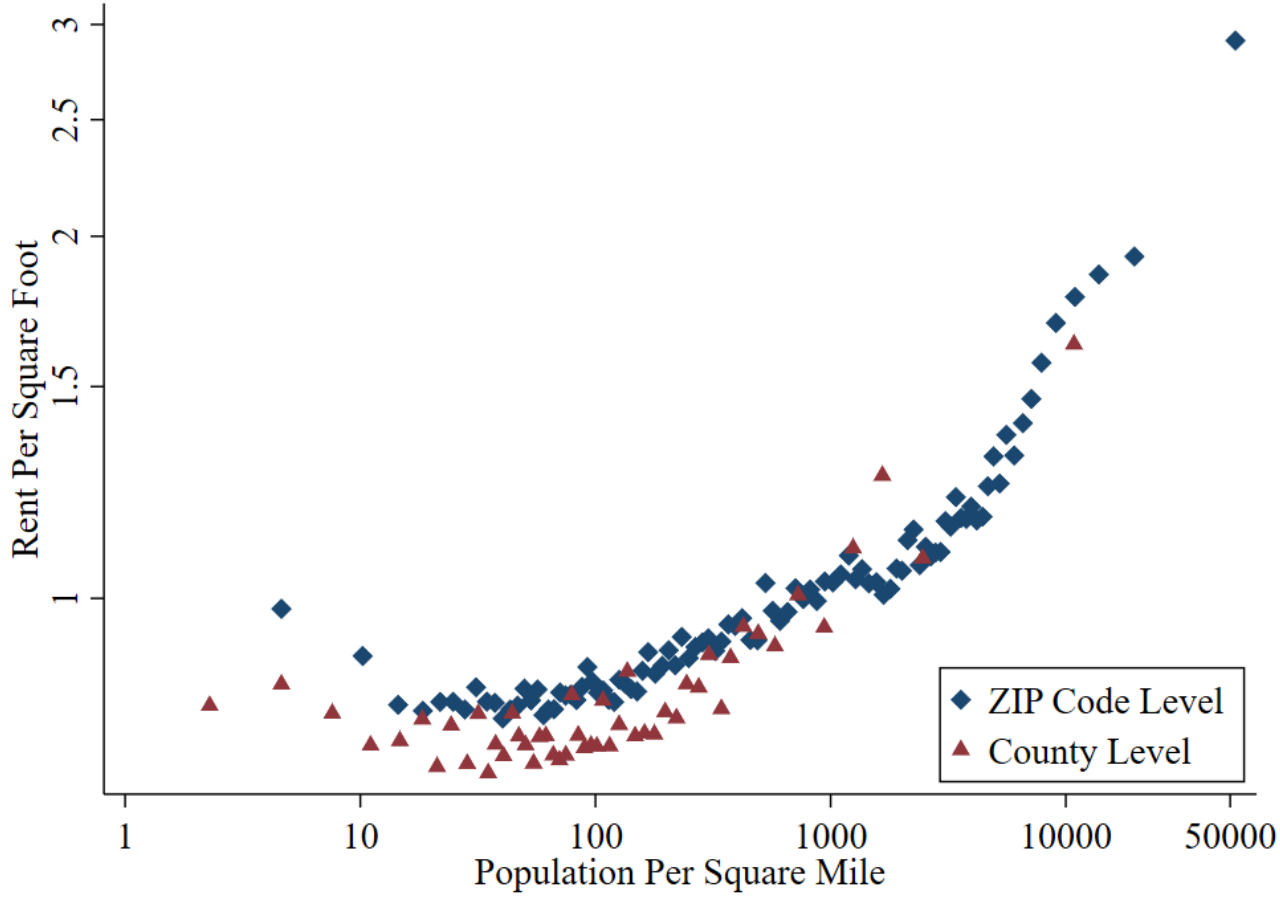


x_s



back

Denser Locations Have Higher Rents



Data source: Zillow

back

Sorting in the cross-section

Weighted by Employment



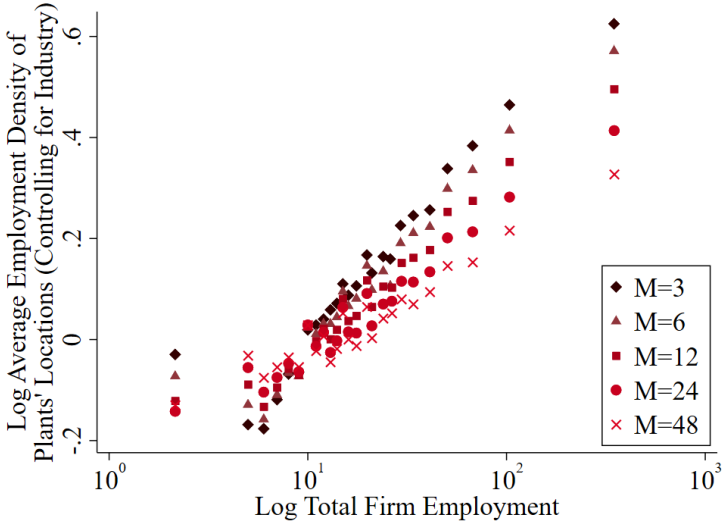
Locations equally weighted



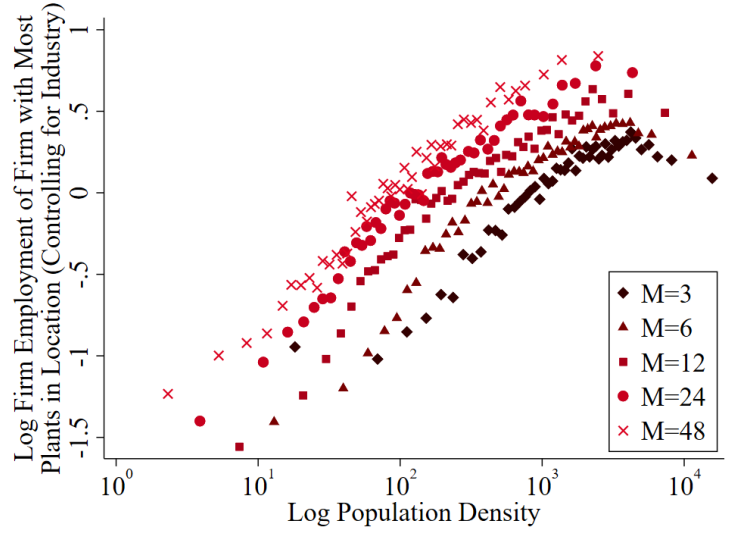
back

(Net) Sorting and National Size

Sorting



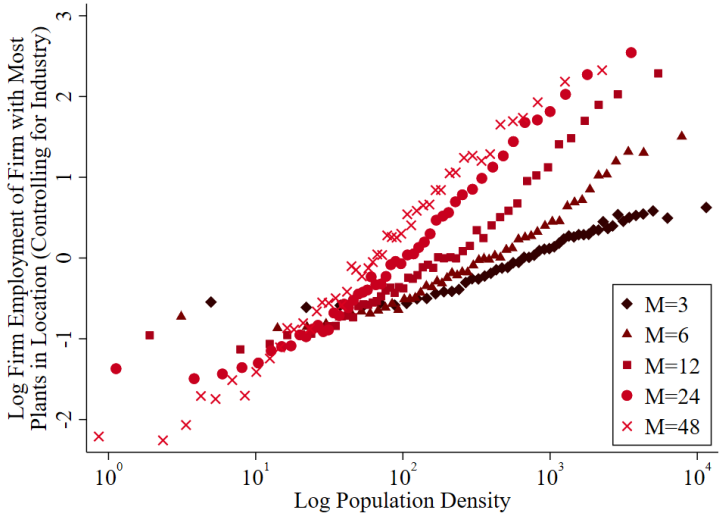
National size of Largest firm in town



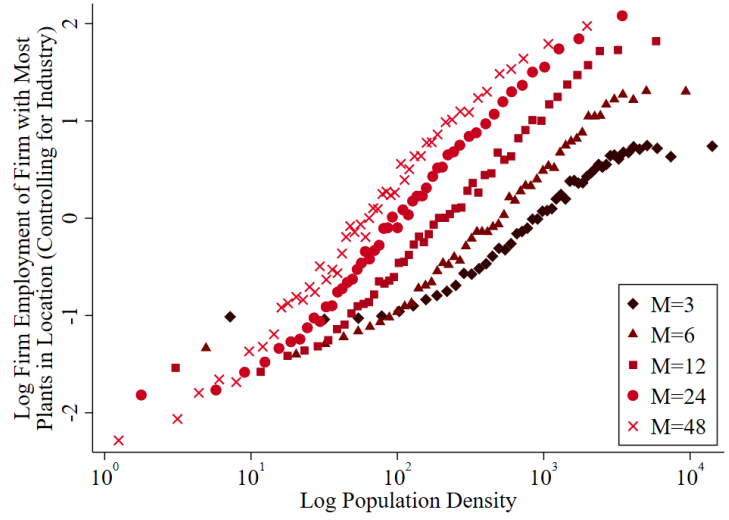
back to sorting back to National Size

National Size, alternative tie-breakers

Discarding Locations with Ties



Using Largest Firm



back to National Size

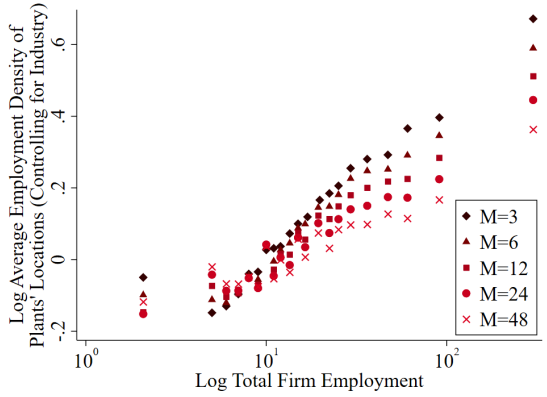
Sorting, by sector

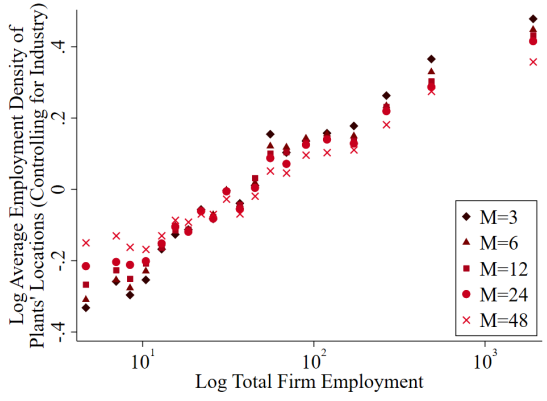
Manufacturing



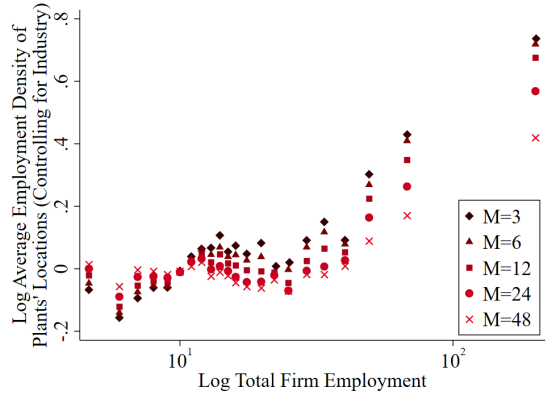


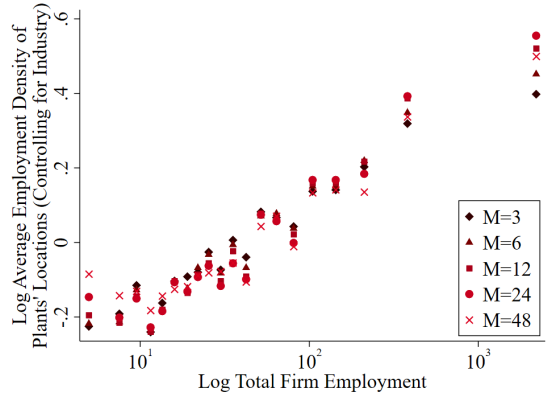
Services





Retail Trade



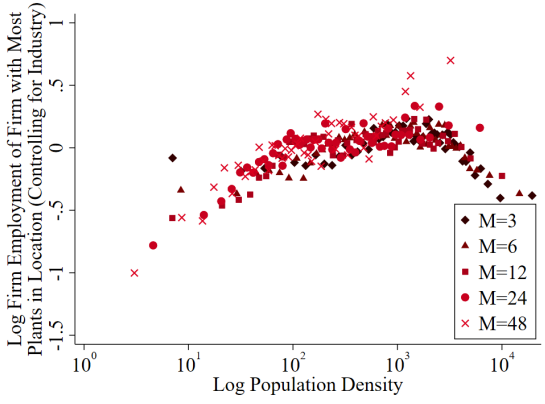


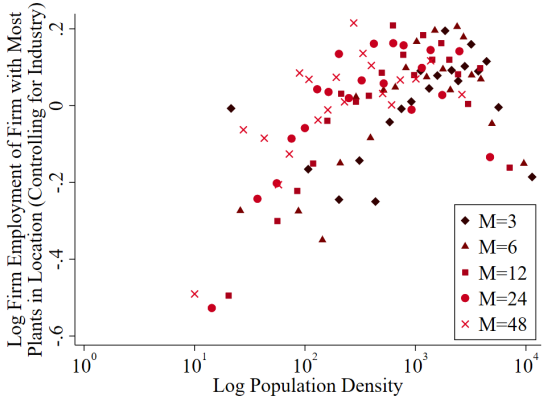
- Second row excludes single-plant firms

back to Sorting

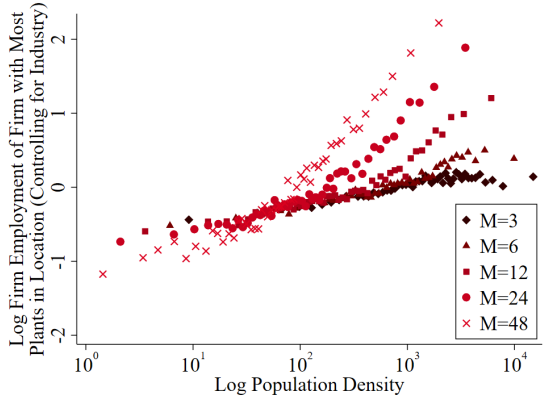
National size, by sector

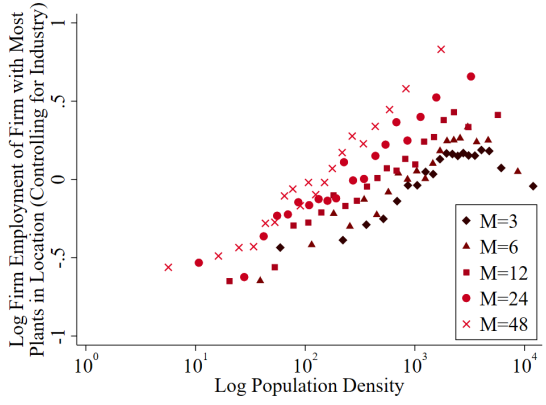
Manufacturing



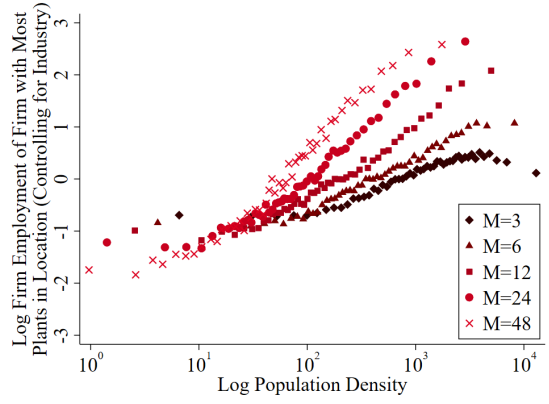


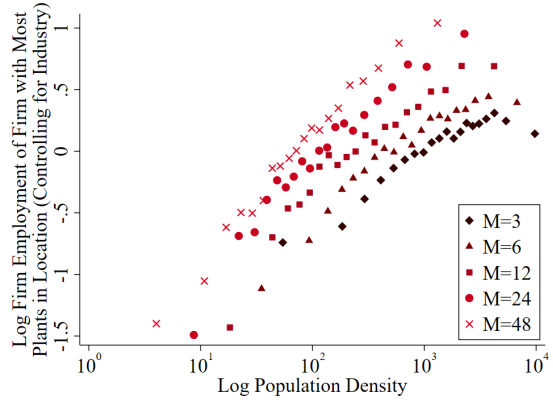
Services





Retail Trade

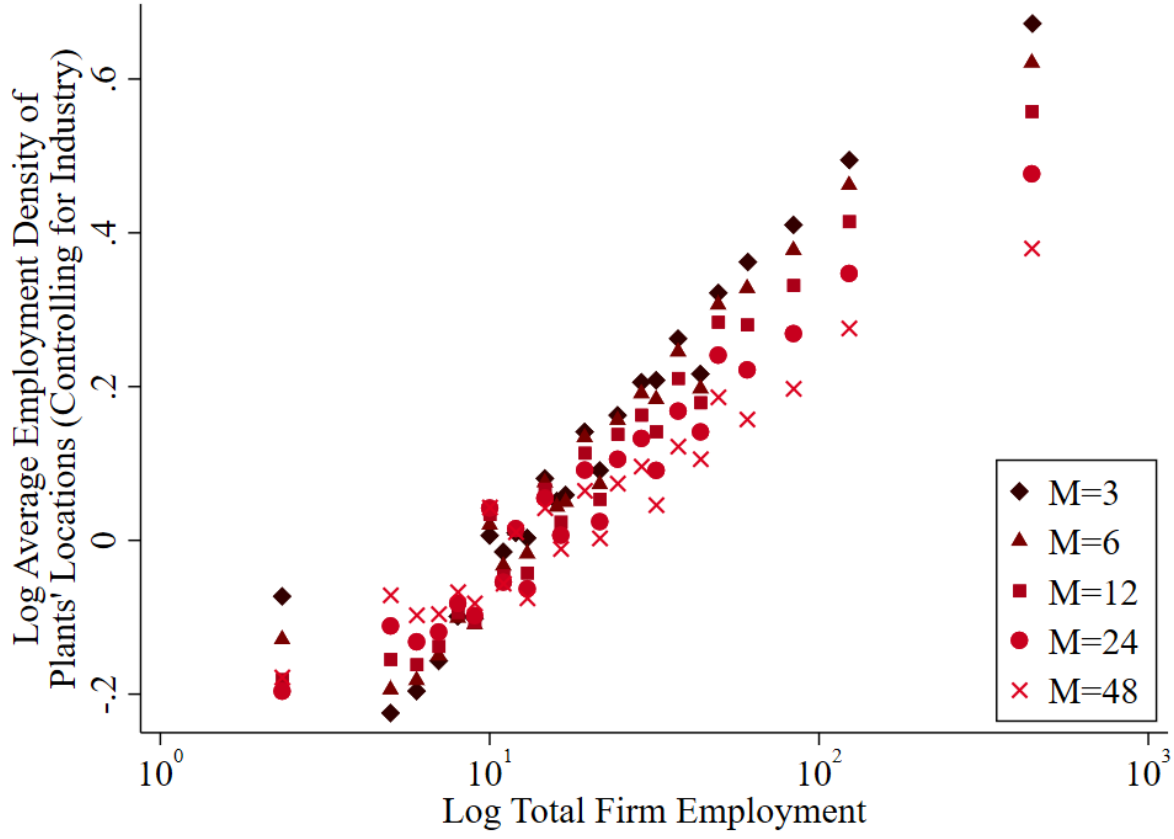




- Second row excludes single-plant firms

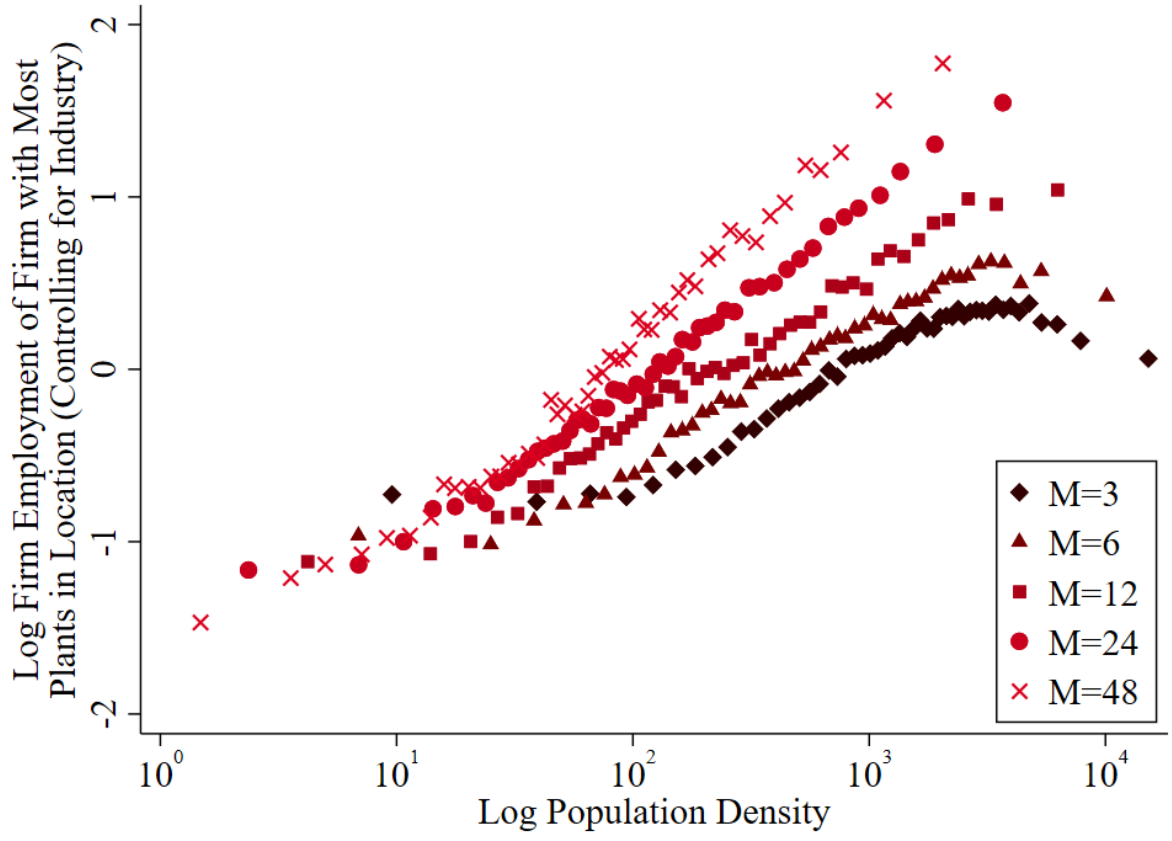
back to National Size

Sorting, non-imputed data



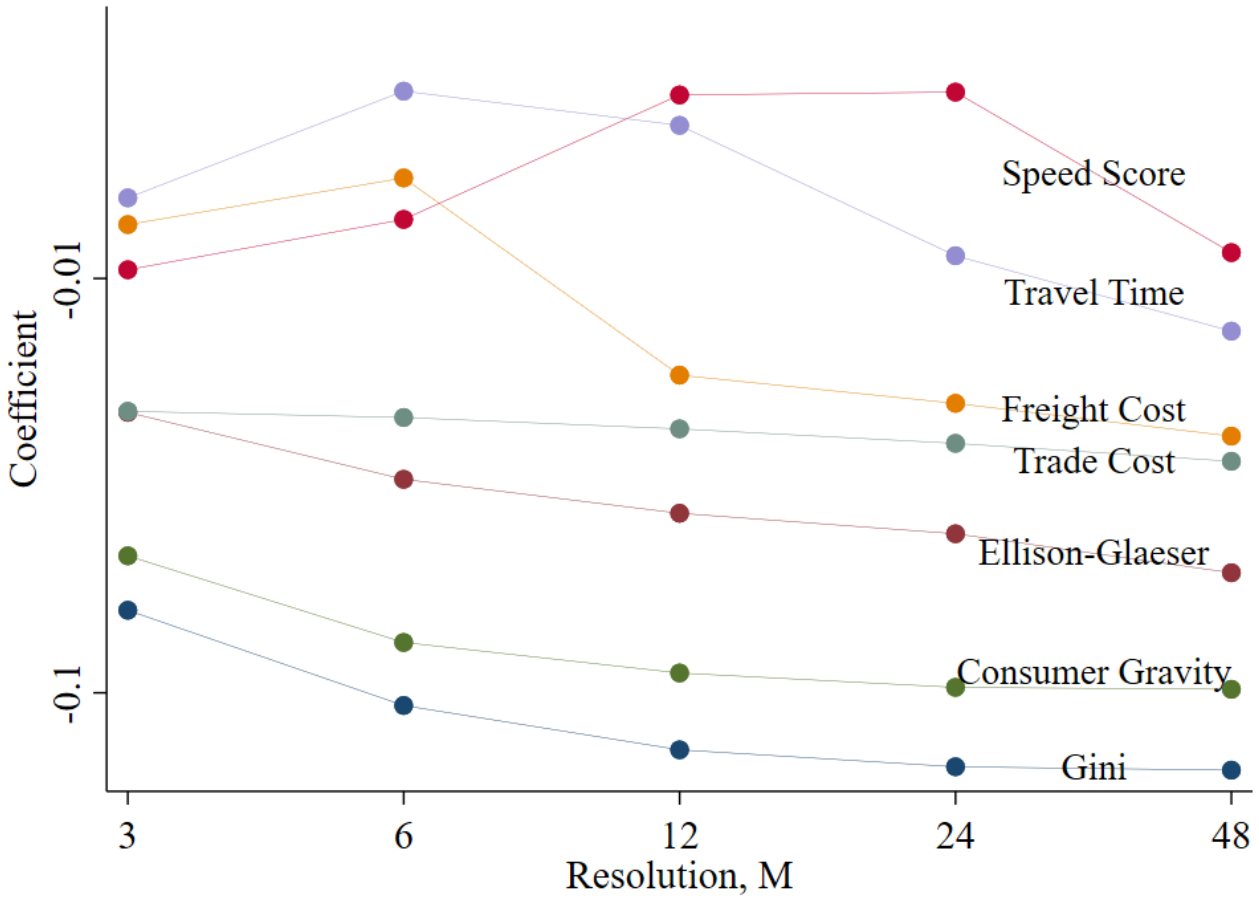
back to sorting

National Size of the Top Firm in Town, non-imputed data



back to National Size

Transportation efficiency, plants, and catchment areas



back