Sustainable Investing in Equilibrium

Ľuboš Pástor

Robert F. Stambaugh

Lucian A. Taylor*

February 4, 2020

Abstract

We present a model of investing based on environmental, social, and governance (ESG) criteria. In equilibrium, green assets have negative CAPM alphas, whereas brown assets have positive alphas. Green assets' negative alphas stem from investors' preference for green holdings and from green stocks' ability to hedge climate risk. Green assets can nevertheless outperform brown ones during good performance of the ESG factor, which captures shifts in customers' tastes for green products and investors' tastes for green holdings. The latter tastes produce positive social impact by making firms greener and shifting real investment from brown to green firms. The ESG investment industry is at its largest, and the alphas of ESG-motivated investors are at their lowest, when there is large dispersion in investors' ESG preferences.

JEL classifications: G11, G12

Keywords: sustainable investing, socially responsible investing, ESG, social impact

---

*Pástor is at the University of Chicago Booth School of Business. Stambaugh and Taylor are at the Wharton School of the University of Pennsylvania. Pástor and Stambaugh are also at the NBER. Pástor is additionally at the National Bank of Slovakia and the CEPR. The views in this paper are the responsibility of the authors, not the institutions they are affiliated with. We are grateful for comments from Rui Albuquerque, Malcolm Baker, Alex Edmans, Gene Fama, Sam Hartzmark, Yrjo Koskinen, Jeff Wurgler, Josef Zechner, and seminar audiences at WU Vienna and the National Bank of Slovakia.

1. Introduction

Sustainable investing is an investment approach that considers not only financial but also environmental, social and governance (ESG) objectives. This approach initially gained popularity by imposing negative screens under the umbrella of socially responsible investing (SRI), but its scope has expanded significantly in recent years. Assets managed with an eye on sustainability have grown to tens of trillions of dollars, and they seem poised to grow further.[1] Given the rapid growth of ESG-driven investing, it seems important to understand its effects on asset prices and corporate investment.

We analyze both financial and real effects of sustainable investing through the lens of a general equilibrium model. The model features many heterogeneous firms and many heterogeneous agents, yet it is highly tractable, yielding simple and intuitive expressions for the quantities of interest. The model illuminates the key channels through which agents' preferences for sustainability can move asset prices, tilt portfolio holdings, determine the size of the ESG investment industry, and cause real impact on society.

In the model, firms differ in the sustainability of their activities. "Green" firms generate positive externalities on society, "brown" firms generate negative externalities, and there are different shades of green and brown. Agents differ in their preferences for sustainability, or "ESG preferences." These preferences have multiple dimensions. In the baseline model, agents derive utility from holdings of green firms and disutility from holdings of brown firms. In one model extension, they also care about firms' social impact. In another extension, they care about climate risk. Naturally, agents also care about financial wealth.

Agents' tastes for green holdings affect asset prices. The greener the asset, the lower is its CAPM alpha in equilibrium. Green assets have negative alphas, whereas brown assets have positive alphas. Consequently, agents with stronger ESG preferences, whose portfolios tilt more toward green assets and away from brown assets, earn lower expected returns. Yet such agents are not unhappy because they derive utility from their holdings.

The model implies three-fund separation, whereby each agent holds the market portfolio, the risk-free asset, and an "ESG portfolio," which is largely long green assets and short brown assets. Agents with stronger-than-average ESG preferences go long the ESG portfolio, agents with weaker preferences go short, and agents with average preferences hold the market

[1] According to the 2018 Global Sustainable Investment Review, sustainable investing assets exceeded $30 trillion globally at the start of 2018, a 34% increase in two years. As of November 2019, more than 2,600 organizations have become signatories to the United Nations Principles of Responsible Investment (PRI), with more than 500 new signatories in 2018/2019, according to the 2019 Annual Report of the PRI.

portfolio. If there is no dispersion in ESG preferences, all agents simply hold the market. Even if all agents derive a large amount of utility from green holdings, they end up holding the market if their ESG preferences are equally strong, because asset prices adjust to reflect those preferences. In this equal-preference case, the ESG investment industry does not exist: despite the strong demand for green holdings, a market index fund is all that is needed to satisfy investors. For the ESG industry to exist, some dispersion in ESG preferences is necessary. The larger the dispersion, the larger the industry.

We illustrate the economic significance of the above effects by calibrating a case in which there are two types of investors, those sharing equal concerns about ESG and those not concerned at all. The key free parameter is Δ, the maximum certain return an ESG-concerned investor is willing to forego in exchange for investing in her desired portfolio instead of the market. The negative alpha such investors earn is greatest, and the ESG industry is largest, when dispersion in ESG preferences is greatest (here meaning ESG investors constitute half of total wealth). That worst-case alpha is substantially smaller than Δ, however, because equilibrium prices adjust to ESG demands, thereby pushing the market portfolio toward the portfolio desired by ESG investors. For example, when ESG investors have a Δ of 4%, their worst-case alpha is only -2%. This difference between Δ and alpha provides ESG investors with an "investor surplus": In order to hold their desired portfolio, they sacrifice less return than they are willing to. The same price-adjustment mechanism lessens the impact of ESG investing on the ESG industry's size, which we measure as the aggregate ESG dollar tilt away from the market portfolio. For example, if the ESG industry reaches 33% of the stock market's value when Δ is 1%, then doubling the strength of ESG concerns (raising Δ to 2%) increases that maximum industry size by less than half, to 46% of the market's value.

Our model implies that sustainable investing leads to positive social impact. We define social impact as the product of a firm's ESG characteristic and the firm's operating capital. We show that agents' tastes for green holdings increase firms' social impact, through two channels. First, firms choose to become greener, because greener firms have higher market values. Second, real investment shifts from brown to green firms, due to shifts in firms' cost of capital (up for brown firms, down for green firms). We obtain positive aggregate social impact even if agents have no direct preference for it, shareholders do not engage with management, and managers simply maximize market value.

We introduce an "ESG factor" that captures unexpected changes in ESG concerns. These concerns can change in two ways: customers may shift their demands for goods of green providers, or investors may change their appreciation for green holdings. The ESG factor affects the relative performance of green and brown assets, both ex post and ex ante. Ex

post, the factor's positive realizations boost green assets while hurting brown ones. If ESG concerns strengthen unexpectedly, green assets can outperform brown ones despite having lower expected returns. Ex ante, green and brown assets have opposite ESG factor exposures, which push these assets' market betas in opposite directions.

Finally, we extend the model by allowing climate to enter investors' utility. Expected returns then depend not only on market betas and investors' tastes, but also on climate betas, which measure firms' exposures to climate shocks. Evidence suggests that brown assets have higher climate betas than green assets (e.g., Choi, Gao, and Jiang, 2019, and Engle et al., 2019), which pushes up brown assets' expected returns in our model. The idea is that investors dislike unexpected deteriorations in the climate. If the climate worsens unexpectedly, brown assets lose value relative to green assets (e.g., due to new government regulation that penalizes brown firms). Because brown firms lose value in states of the world investors dislike, they are riskier, so they must offer higher expected returns. Greener stocks thus have lower CAPM alphas not only because of investors' tastes for green holdings, but also because of their ability to better hedge climate risk.

The climate factor is likely to be related to the ESG factor because climate shocks are likely to affect both customers' tastes for green products and investors' tastes for green holdings. In the special case where climate shocks are the only reason behind shifts in agents' tastes, the two factors coincide. A two-factor asset pricing model then prices all stocks, with the factors being the market portfolio and the ESG factor. In general, though, a multi-factor approach may be necessary to capture the risk associated with ESG investing because agents' tastes may shift also for reasons unrelated to climate shocks.

Our theoretical treatment of climate risk is related to recent empirical work on the implications of such risk for asset prices. Hong, Li, and Xu (2019) analyze the response of food stock prices to climate risks. Bolton and Kacperczyk (2019) argue that investors demand compensation for exposure to carbon risk in the form of higher returns on carbon-intensive firms. Ilhan, Sautner, and Vilkov (2019) show that firms with higher carbon emissions exhibit more tail risk and more variance risk. Engle et al. (2019) develop a procedure to dynamically hedge climate risk by constructing mimicking portfolios that hedge innovations in climate news series obtained by textual analysis of news sources. Bansal, Ochoa, and Kiku (2016) model climate change as a long-run risk factor. Krueger, Sautner, and Starks (2019) find that institutional investors consider climate risks to be important investment risks.

Besides climate risk, studies have identified multiple other aspects of ESG-related risk. Hoepner et al. (2018) find that ESG engagement reduces firms' downside risk, as well as

their exposures to a downside-risk factor. Luo and Balvers (2017) find a premium for boycott risk. We complement these studies with a theoretical contribution. We construct an ESG risk factor that is driven by unexpected shifts in ESG concerns of firms' customers and market investors. We show that green and brown assets have opposite exposures to this factor, and we link these exposures to market betas.

Prior studies report that green assets underperform brown assets, in various contexts. Hong and Kacperczyk (2009) find that "sin" stocks (i.e., stocks of public firms producing alcohol, tobacco, and gaming, which we would classify as brown) outperform non-sin stocks. They argue that social norms lead investors to demand compensation for holding sin stocks. Barber, Morse, and Yasuda (2018) find that venture capital funds that aim not only for financial return but also for social impact earn lower returns than other funds. They argue that investors derive nonpecuniary utility from investing in dual-objective funds. Baker et al. (2018) and Zerbib (2019) find that green bonds tend to be priced at a premium, offering lower yields than traditional bonds. Both studies argue that the premium is driven by investors' environmental concerns. Similarly, Chava (2014) and El Ghoul et al. (2011) find that greener firms have a lower implied cost of capital. All of these results are consistent with the effects of tastes for green holdings on the cost of capital in our model.

Some studies find the opposite result, based on different definitions of green and brown. Firms perform better if they are better-governed, judging by employee satisfaction (Edmans, 2011) or strong shareholder rights (Gompers, Ishii, and Metrick, 2003), or if they have higher ESG ratings in the 1992–2004 period (Kempf and Osthoff, 2007). These results are consistent with our model if ESG concerns strengthened unexpectedly over the sample period.

Our model is related to prior theoretical studies of sustainable investing. Heinkel, Kraus, and Zechner (2001) build an equilibrium model in which exclusionary ethical investing affects firm investment. They consider two types of investors, one of which refuses to hold shares in polluting firms. The resulting reduction in risk sharing increases the cost of capital of polluting firms, depressing their investment. Albuquerque, Koskinen, and Zhang (2019) construct a model in which a firm's socially responsible investments increase customer loyalty, giving the firm more pricing power. This power makes the firm less risky and thus more valuable. Unlike these models, ours features neither a lack of risk sharing nor pricing power; instead, the main force is investors' tastes for holding green assets.

Tastes for holding assets can affect prices, as emphasized by Fama and French (2007). Baker et al. (2018) build a model featuring two types of investors with mean-variance preferences, where one type also has tastes for green assets. Their model predicts that green

assets have lower expected returns and more concentrated ownership, and they find support for these predictions in the universe of green bonds. Pedersen, Fitzgibbons, and Pomorski (2019) consider the same two types of mean-variance investors, but also add a third type that is unaware of firms' ESG scores. This lack of awareness is costly if firms' ESG scores predict their profits. The authors show that stocks with higher ESG scores can have either higher or lower expected returns, depending on the wealth of the third type of investors. They obtain four-fund separation and derive the ESG-efficient frontier characterizing the tradeoff between the ESG score and the Sharpe ratio.

While these studies share some modeling features with ours, we also offer additional novel insights that do not appear in those studies. We have a continuum of investors with multiple dimensions of ESG preferences. We show that the size of the ESG investment industry, as well as investors' alphas, crucially depend on the dispersion in investors' ESG tastes. We relate market betas to assets' exposures to an ESG risk factor. We show that this factor, along with the market, prices assets in a two-factor model. We also show that positive realizations of this factor, which result from shifts in customers' and investors' tastes, can result in green assets outperforming brown. We analyze the pricing of climate risk. Finally, we show that ESG investing has positive social impact.

Positive social impact also emerges from the model of Oehmke and Opp (2020), but through a different channel. Key ingredients to generating impact in their model are financing constraints and coordination among agents. Our model does not include those ingredients, but it produces social impact nevertheless, through tastes for green holdings. To emphasize these tastes, we do not model shareholder engagement with management, which is another channel through which ESG investing can potentially increase market value (e.g., Dimson, Karakas, and Li, 2015). In our model, value-maximizing managers make their firms greener voluntarily, without pressure from shareholders, because greener firms command higher market values.[2]

Our assumption that some investors derive nonpecuniary benefits from green holdings has a fair amount of empirical support in the mutual fund literature. Mutual fund flows respond to ESG-salient information, such as Morningstar sustainability ratings (Hartzmark and Sussman, 2019) and environmental disasters (Bialkowski and Starks, 2016). Flows to SRI mutual funds are less volatile than flows to non-SRI funds (Bollen, 2007) and less responsive to negative past performance (Renneboog, ter Horst, and Zhang, 2011). Investors in SRI

[2]Theoretical work on sustainable investing also includes Friedman and Heinle (2016), Gollier and Pouget (2014), and Luo and Balvers (2017). Empirical work includes Geczy, Stambaugh, and Levin (2005), Hong and Kostovetsky (2012), and Cheng, Hong, and Shue (2016), among others. For surveys of the early literature, see Bauer, Koedijk, and Otten (2005) and Renneboog, ter Horst, and Zhang (2008).

funds also indicate willingness to forgo financial performance to accommodate their social preferences (Riedl and Smeets, 2017).

This paper is organized as follows. Section 2 presents our baseline model. Section 3 explores the model's quantitative implications. Section 4 extends the baseline model by allowing ESG concerns to shift over time, giving rise to the ESG factor. Section 5 extends the baseline model by letting agents care about the climate, showing that climate risk commands a risk premium. Section 6 discusses social impact. Section 7 concludes.

2. Model

The model considers a single period, from time 0 to time 1, in which there are N firms, $n = 1, \ldots, N$. Let $\tilde{r}_n$ denote the return on firm n's shares in excess of the riskless rate, r_f, and let $\tilde{r}$ be the $N \times 1$ vector whose nth element is $\tilde{r}_n$. We assume $\tilde{r}$ is normally distributed:

$$\tilde{r} = \mu + \tilde{\epsilon} \ , \tag{1}$$

where μ contains equilibrium expected excess returns and $\tilde{\epsilon} \sim N(0, \Sigma)$. In addition to financial payoffs, firms produce social impact. Each firm n has an observable "ESG characteristic" g_n, which can be positive (for "green" firms) or negative (for "brown" firms). Firms with $g_n > 0$ have positive social impact, meaning they generate positive externalities (e.g., cleaning up the environment). Firms with $g_n < 0$ have negative social impact, meaning they generate negative externalities (e.g., polluting the environment). In Section 6, we model firms' social impact in greater detail.

There is a continuum of agents who trade firms' shares and the riskless asset. The riskless asset is in zero net supply, whereas each firm's stock is in positive net supply. Let X_i denote an $N \times 1$ vector whose nth element is the fraction of agent i's wealth invested in stock n. Agent i's wealth at time 1 is $\tilde{W}_{1i} = W_{0i}\,(1 + r_f + X_i'\tilde{r})$, where W_{0i} is the agent's initial wealth. Besides liking wealth, agents also derive utility from holding green stocks and disutility from holding brown stocks.[3] Each agent i has exponential (CARA) utility

$$V(\tilde{W}_{1i}, X_i) = -e^{-A_i \tilde{W}_{1i} - b_i' X_i} \ , \tag{2}$$

where A_i is the agent's absolute risk aversion and b_i is an $N \times 1$ vector of nonpecuniary benefits that the agent derives from her stock holdings. Holding the riskless asset brings no

[3]We frame the discussion in terms of green and brown stocks, but our main ideas apply more broadly to any set of green and brown assets, such as bonds and private equity investments.

such benefit. The benefit vector has agent-specific and firm-specific components:

$$b_i = d_i g \ , \tag{3}$$

where g is an $N \times 1$ vector whose nth element is g_n and $d_i \geq 0$ is a scalar measuring agent i's "ESG taste." Agents with higher values of d_i care more about the ESG characteristics of their holdings. Agents also care about firms' aggregate social impact, but that component of preferences does not affect agents' portfolio choice, as we show in Section 6.

2.1. Expected Returns

Due to their infinitesimal size, agents take asset prices (and thus also the return distribution) as given when choosing their optimal portfolios, X_i, at time 0. To derive the first-order condition for X_i, we compute the expectation of agent i's utility in equation (2) and differentiate it with respect to X_i. As we show in the Appendix, agent i's portfolio weights are

$$X_i = \frac{1}{a_i}\Sigma^{-1}\left(\mu + \frac{1}{a_i}b_i\right) \ , \tag{4}$$

where $a_i \equiv A_i W_{0i}$ is agent i's relative risk aversion. For tractability, we assume that $a_i = a$ for all agents. We define w_i to be the ratio of agent i's initial wealth to total initial wealth: $w_i \equiv W_{0i}/W_0$, where $W_0 = \int_i W_{0i} di$. The market-clearing condition requires that x, the $N \times 1$ vector of weights in the market portfolio, satisfies

$$\begin{aligned} x &= \int_i w_i X_i \, di \\ &= \frac{1}{a}\Sigma^{-1}\mu + \frac{\bar{d}}{a^2}\Sigma^{-1} g \ , \end{aligned} \tag{5}$$

where $\bar{d} \equiv \int_i w_i d_i di \geq 0$ is the wealth-weighted mean of ESG tastes d_i across agents. Note that $\bar{d} > 0$ unless the mass of agents who care about ESG is zero. Solving for μ gives

$$\mu = a\Sigma x - \frac{\bar{d}}{a} g \ . \tag{6}$$

Premultiplying by x' gives the market equity premium, $\mu_M = x'\mu$:

$$\mu_M = a\sigma_M^2 - \frac{\bar{d}}{a} x'g \ , \tag{7}$$

where $\sigma_M^2 = x'\Sigma x$ is the variance of the market return. In general, the equity premium depends on average ESG preferences, $\bar{d}$, through $x'g$, which is the overall "greenness" of the market portfolio. If the market is net green ($x'g > 0$) then stronger ESG preferences (i.e., larger $\bar{d}$) reduce the equity premium. If the market is net brown ($x'g < 0$), stronger ESG

preferences increase the premium. For simplicity, we make the natural assumption that the market portfolio is ESG-neutral,

$$x'g = 0 \ , \tag{8}$$

so that the equity premium is independent of agents' ESG preferences. In this case, equation (7) implies $a = \mu_M/\sigma_M^2$. Combining this with equation (6) and noting that the vector of market betas is $\beta = (1/\sigma_M^2)\Sigma x$, we obtain our first proposition.

Proposition 1. *Expected excess returns in equilibrium are given by*

$$\mu = \mu_M \beta - \frac{\bar{d}}{a} g \ . \tag{9}$$

We see that expected excess returns deviate from their CAPM values, $\mu_M\beta$, due to ESG preferences for holding green stocks.

Corollary 1. *If $\bar{d} > 0$, the expected return on stock n is decreasing in g_n.*

As long as the mass of agents who care about sustainability is nonzero, $\bar{d}$ is positive, and expected returns are decreasing in stocks' ESG characteristics. Because the alpha of stock n is defined as $\alpha_n \equiv \mu_n - \mu_M\beta_n$, equation (9) yields the following corollary.

Corollary 2. *The alpha of stock n is given by*

$$\alpha_n = -\frac{\bar{d}}{a} g_n \ . \tag{10}$$

If $\bar{d} > 0$, green stocks have negative alphas, and brown stocks have positive alphas. Moreover, greener stocks have lower alphas.

As long as some agents care about sustainability, equation (10) implies that the CAPM alphas of stocks with $g_n > 0$ are negative, the alphas of stocks with $g_n < 0$ are positive, and α_n is decreasing with g_n. Furthermore, the negative relation between α_n and g_n is stronger when risk aversion, a, is lower and when the average ESG taste, $\bar{d}$, is higher.

Proposition 2. *The mean and variance of the excess return on agent i's portfolio are*

$$\mathrm{E}(\tilde{r}_i) = \mu_M - \delta_i \left(\frac{\bar{d}}{a^3} g'\Sigma^{-1} g \right) \tag{11}$$

$$\mathrm{Var}(\tilde{r}_i) = \sigma_M^2 + \delta_i^2 \left(\frac{1}{a^4} g'\Sigma^{-1} g \right) \ , \tag{12}$$

where $\delta_i \equiv d_i - \bar{d}$.

Both equations are derived in the Appendix. Agents with $\delta_i > 0$ earn below-market expected returns because their portfolios tilt toward stocks with negative alphas. In contrast, agents with $\delta_i < 0$ earn above-market returns because they tilt toward positive-alpha stocks. All agents with $\delta_i \neq 0$ hold portfolios more volatile than the market.

Corollary 3. *If $\bar{d} > 0$ and $g \neq 0$, agents with larger δ_i earn lower expected returns.*

Under the conditions of this corollary, the term in parentheses in equation (11) is strictly positive. Therefore, agents with stronger ESG tastes (i.e., larger δ_i) earn lower expected returns. The conditions are not satisfied if no agents care about ESG ($\bar{d} = 0$) or if all firms are ESG-neutral ($g = 0$); in that case, $\mathrm{E}(\tilde{r}_i)$ is independent of δ_i because all agents hold the market. The effect of δ_i on $\mathrm{E}(\tilde{r}_i)$ is stronger when the average ESG taste is stronger (i.e., when $\bar{d}$ is larger), when risk aversion a is smaller, and when $g'\Sigma^{-1}g$ is larger.

The low expected returns earned by ESG-sensitive agents do not imply these agents are unhappy. As we show in the Appendix, agent i's expected utility in equilibrium is given by

$$\mathrm{E}\left\{V(\tilde{W}_{1i})\right\} \;=\; \bar{V}e^{-\frac{\delta_i^2}{2a^2}\,g'\Sigma^{-1}g}\;, \tag{13}$$

where $\bar{V}$ is the expected utility if the agent has $\delta_i = 0$. We see that expected utility is increasing in δ_i^2 (note from equation (2) that $\bar{V} < 0$), so it is larger for agents with larger absolute values of δ_i. The more an agent's ESG taste d_i deviates from the average in either direction, the more ESG preferences contribute to the agent's utility. High-δ_i investors derive utility from their holdings of green stocks, while low-δ_i investors derive utility from the positive alphas of brown stocks.

2.2. Portfolio Tilts

Substituting for μ from equation (9) into equation (4), we obtain an agent's portfolio weights:

Proposition 3. *Agent i's equilibrium portfolio weights are given by*

$$X_i = x + \frac{\delta_i}{a^2}\left(\Sigma^{-1}g\right)\;. \tag{14}$$

Proposition 3 implies three-fund separation as each agent's portfolio can be implemented with three assets: the riskless asset, the market portfolio x, and an "ESG portfolio" whose weights are proportional to $\Sigma^{-1}g$. Agents with $\delta_i > 0$ go long the ESG portfolio; agents with $\delta_i < 0$ short the portfolio. Agent i's portfolio departs from the market portfolio due to the second term in equation (14), which we refer to as agent i's "ESG tilt."

The ESG tilt is zero for agents whose ESG taste is average, in that $d_i = \bar{d}$. Agents with average ESG preferences hold the market portfolio. Interestingly, agents with $d_i = 0$ hold a portfolio that departs from the market in the direction away from ESG. It is suboptimal for an investor to say "I don't care about ESG, so I'm just going to hold the market." Investors who do not care about ESG must tilt away from ESG, otherwise they are not optimizing. The market portfolio is optimal for investors who care about ESG to an average extent, but not for those who do not care about ESG at all.

Corollary 4. *If there is no dispersion in ESG preferences across agents then all agents hold the market portfolio.*

No dispersion in ESG preferences implies $d_i = \bar{d}$, and so zero ESG tilt, for all i. Interestingly, even if all agents derive a large amount of utility from green holdings, they end up holding the market if their preferences are equally strong. The reason is that stock prices adjust to reflect those preferences, making the market everybody's optimal choice. Some dispersion in ESG preferences is necessary for an ESG investment industry to exist.

If the covariance matrix Σ is diagonal, meaning all risk is idiosyncratic, then the ESG portfolio weights are positive for green stocks (whose $g_n > 0$), negative for brown stocks (whose $g_n < 0$), and lower for stocks with more volatile returns. A similar result obtains when Σ has a simple one-factor structure, allowing systematic risk:

$$\Sigma = \sigma^2 \iota\iota' + \eta^2 I_N \ , \tag{15}$$

where ι is an $N \times 1$ vector of ones and I_N is an identity matrix, because in that case[4]

$$\Sigma^{-1} g = \frac{1}{\eta^2} \left(g - \frac{\bar{g}}{\frac{\eta^2}{N\sigma^2} + 1} \iota \right) \ , \tag{16}$$

where $\bar{g} = \iota' g / N$ is the mean g_n across firms. As N gets large, the ESG portfolio goes long stocks that are greener than average ($g_n > \bar{g}$) and short stocks that are browner than average ($g_n < \bar{g}$). The ESG portfolio's positions are smaller when idiosyncratic risk η^2 is higher, because tilting toward the ESG portfolio exposes investors to more idiosyncratic risk.

In general, the ESG tilt depends also on the covariances among stocks. If a stock is positively correlated with a greener stock, the former stock may be shorted by agents who want to hold the greener stock and hedge their risk exposure to it. In principle, even a green stock could be shorted if it is sufficiently correlated with a stock that is even greener.

[4]With Σ given in equation (15), we have $\Sigma^{-1} = \frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2 + N}\iota\iota'\right)$.

The ESG tilt is smaller when risk aversion, a, is higher, because the tilt exposes agents to additional risk (see equation (12)). Holding ESG preferences (δ_i) constant, those preferences are reflected less strongly in agents' portfolios if agents' risk aversion is higher.

3. Quantitative Implications

To explore the model's quantitative implications, we consider a special case with two types of agents: ESG investors, for whom $d_i = d > 0$, and non-ESG investors, for whom $d_i = 0$. ESG investors thus consume nonpecuniary benefits dg, whereas non-ESG investors consume no benefits (see equation (3)). Let λ denote the fraction of total wealth belonging to ESG investors, so that $1 - \lambda$ is the corresponding fraction for non-ESG investors.

3.1 Expected Returns and Portfolio Tilts

In this setting, $\bar{d} = \lambda d$, so from equation (9) the vector of expected excess returns becomes

$$\mu = \mu_M \beta - \lambda \frac{d}{a} g \ . \tag{17}$$

As λ increases, expected returns on green stocks decrease, whereas expected returns on brown stocks increase. In this comparative static sense, growing interest in ESG increasingly pushes stock prices in the direction of their ESG characteristics.

The portfolio weights for each type of investor follow directly from equation (14), with $\delta_i = (1 - \lambda)d$ for an ESG investor and $\delta_i = -\lambda d$ for a non-ESG investor:

$$X_{esg} = x + (1 - \lambda)\frac{d}{a^2}\Sigma^{-1} g \tag{18}$$

$$X_{non} = x - \lambda \frac{d}{a^2}\Sigma^{-1} g \ . \tag{19}$$

Both ESG tilts depend on λ in an interesting way. As $\lambda \to 0$, all investing is non-ESG, and all capital is invested in the market portfolio x because $X_{non} \to x$. As $\lambda \to 1$, all investing is ESG, and again, all capital is invested in the market because $X_{esg} \to x$. In other words, whether $\lambda \to 0$ or $\lambda \to 1$, all portfolios converge to the market portfolio. When $\lambda \to 0$, everybody holds the market because there are no ESG investors. When $\lambda \to 1$, everybody holds the market because ESG preferences are fully embedded in market prices.

From equation (11), the difference in expected excess returns earned by the two types of investors is

$$\mathrm{E}(\tilde{r}_{esg}) - \mathrm{E}(\tilde{r}_{non}) = -\frac{\lambda d^2}{a^3} g' \Sigma^{-1} g \ . \tag{20}$$

An ESG investor thus earns a lower expected return than a non-ESG investor. The performance gap is larger when there is a greater presence of ESG investors (i.e., when λ is larger). In this comparative static sense, growth in ESG investing deepens the performance gap. The gap is also larger when the two types of investors are further apart in their ESG preferences (i.e., when d is larger), when risk aversion a is smaller, and when $g'\Sigma^{-1}g$ is larger.

3.2. Parameter Specifications

We further simplify our setting by assuming that Σ has the simple one-factor structure given in equation (15) and setting $\beta = \iota$, $x = (1/N)\iota$, $\iota' g = 0$ (i.e., $x'g = 0$), and $g'g = 1$. With these assumptions, as the number of assets (N) grows large, the mean and variance of market returns, the certainty-equivalent return of ESG investors, and other aggregate quantities of interest do not depend on N, as will be evident below.

This simple setting has five free parameters: λ, a, σ, η, and d. We present results over the entire $(0, 1)$ range of λ. We specify a and σ so that the return on the market portfolio has a mean of $\mu_M = 0.08$ and a standard deviation of $\sigma_M = 0.20$, corresponding roughly to annual empirical estimates. To translate these values of μ_M and σ_M into implied values for a and σ, first note that the above assumptions imply the variance of the market, $x'\Sigma x$, is

$$\sigma_M^2 \;=\; \frac{1}{N^2}\,\iota'\left(\sigma^2\iota\iota' + \eta^2 I_N\right)\iota \;=\; \sigma^2 + \frac{\eta^2}{N}\;, \tag{21}$$

so we set $\sigma^2 = \sigma_M^2$, taking the limit as N grows large. Next, recall from equation (7) that $a = \mu_M/\sigma_M^2$. We set $\eta^2 = (0.7/0.3)\sigma^2$, so that the common market factor explains 30% of the variance of each individual stock's return.

The remaining free parameter, d, reflects the strength of ESG preferences. We calibrate this parameter by choosing Δ, the maximum rate of return that an ESG investor is willing to sacrifice, for certain, in order to invest in her desired portfolio rather than in the market portfolio. The sacrifice is greatest when there are no other ESG investors, i.e., when $\lambda \approx 0$, because that is when the ESG investor's portfolio most differs from the market portfolio.

Specifically, we define $\Delta \equiv r^*_{esg} - r^*_M$, where r^*_{esg} is the ESG investor's certainty equivalent excess return when investing in the optimal ESG portfolio, and r^*_M is the same investor's corresponding certainty equivalent if forced to hold the market portfolio instead. Both certainty equivalents are computed for $\lambda = 0$. For a given Δ, the corresponding value of d is

$$d = \sqrt{2\Delta a^3\eta^2}\;. \tag{22}$$

We derive this equation, along with the expressions for r^*_{esg} and r^*_M, in the Appendix. In the following analysis, we consider four values of Δ: 1, 2, 3, and 4% per year.

3.3. Correlation Between the ESG Return and the Market Return

The correlation between the return on an ESG investor's portfolio and the return on the market portfolio is derived in the Appendix:

$$\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right) = \frac{\sigma}{\sqrt{\sigma^2 + \frac{2\Delta}{a}(1-\lambda)^2}} \ . \tag{23}$$

Figure 1 plots the value of $\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right)$ as λ goes from 0 to 1. The correlation takes its lowest value at $\lambda = 0$. For $\Delta = 0.01$, that value is nearly 0.9, whereas for $\Delta = 0.04$, it is just over 0.7. As Δ increases, indicating that ESG investors feel increasingly strongly about ESG, those investors' portfolios become increasingly different from the market portfolio in terms of $\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right)$, and this effect is strongest when $\lambda = 0$. However, as λ approaches 1, so does $\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right)$. When ESG investors hold an increasingly large fraction of wealth, market prices adjust to their preferences, and all portfolios converge to the market portfolio.

3.4. ESG versus Non-ESG Expected Portfolio Returns

The difference in expected excess returns on the portfolios of the two investor types is

$$\mathrm{E}\{\tilde{r}_{esg}\} - \mathrm{E}\{\tilde{r}_{non}\} = -2\lambda\Delta \ , \tag{24}$$

as shown in the Appendix. Figure 2 plots this difference as λ goes from 0 to 1. The difference is zero at $\lambda = 0$, but it declines linearly as λ increases. At $\lambda = 1$, ESG preferences are fully reflected in prices, and the difference is at its largest in magnitude. In that scenario, the difference is -2% when $\Delta = 0.01$, but it is -8% when $\Delta = 0.04$. ESG investors thus earn significantly lower returns than non-ESG investors when the former account for a large fraction of wealth (i.e., λ is large) and their ESG preferences are strong (i.e., Δ is large).

The certainty equivalent returns of the two types, r^*_{esg} for ESG investors and r^*_{non} for non-ESG investors, are both increasing in Δ, but r^*_{esg} decreases with λ whereas r^*_{non} increases with λ, as we show in the Appendix. As λ increases, stock prices are affected more by ESG investors' tastes, so these investors must pay more for the green stocks they desire. The resulting drop in r^*_{esg} need not imply, however, that an ESG investor is made less happy by an increased presence of ESG investors. With the latter, there is also greater

social impact of ESG investing, as we discuss in Section 6. The additional utility that the ESG investor derives from the greater social impact, as in equation (54), can exceed the drop in utility corresponding to the lower r^*_{esg}. Non-ESG investors, on the other hand, do prefer to be lonely in their ESG preferences. A non-ESG investor is happiest when all other investors are ESG ($\lambda = 1$), because that scenario maximizes deviations of prices from pecuniary fundamentals, which the non-ESG investor exploits to her advantage. This investor's preference for loneliness in ESG preferences is even stronger if she derives utility from social impact because that impact is maximized when $\lambda = 1$.

3.5. Alphas and the Investor Surplus

The alphas of the ESG and non-ESG investors' portfolios are derived in the Appendix:

$$\alpha_{esg} = -2\lambda(1-\lambda)\Delta \tag{25}$$

$$\alpha_{non} = 2\lambda^2\Delta\ . \tag{26}$$

Panel A of Figure 3 plots α_{esg} as λ goes from 0 to 1. ESG investors earn zero alpha at both extremes of λ. Their portfolio differs most from the market portfolio when $\lambda = 0$, but all stocks have zero alphas in that scenario, because there is no impact of ESG investors on prices. At the other extreme, when $\lambda = 1$, many stocks have non-zero alphas, due to the price impacts of ESG investors, but ESG investors hold the market, so again they earn zero alpha. Otherwise, ESG investors earn negative alpha, which is greatest in magnitude when $\lambda = 0.5$. At that peak, $\alpha_{esg} = -0.5\%$ when $\Delta = 0.01$, but $\alpha_{esg} = -2\%$ when $\Delta = 0.04$.

Interestingly, these worst-case alphas are substantially smaller than the corresponding Δ's. For example, when ESG investors are willing to give up 2% certain return to hold their portfolio rather than the market (i.e., $\Delta = 0.02$), their worst-case alpha is only -1%. The reason is that equilibrium stock prices adjust to ESG demands. These demands push the market portfolio toward the portfolio desired by ESG investors, thereby bringing those investors' negative alphas closer to zero. Through this adjustment of market prices, ESG investors earn an "investor surplus" in that they do not have to give up as much return as they are willing to in order to hold their desired portfolio.

The magnitude of this investor surplus is easy to read off Panel B of Figure 3, which plots α_{esg} as a function of Δ. For any given value of λ, investor surplus is the difference between the corresponding solid line and the dashed line, which has a slope of -1. The surplus increases with Δ because the stronger the ESG investors feel about greenness, the

more they move market prices. The relation between the surplus and λ is richer. Formally, investor surplus $\mathcal{I} \equiv \alpha_{esg} + \Delta$ follows quickly from equation (25):

$$\mathcal{I} \;=\; \Delta[1 - 2\lambda(1-\lambda)] \;. \tag{27}$$

Because $0 \leq \lambda \leq 1$, the value in brackets is always between 0.5 and 1, so $\mathcal{I}$ is always between $\Delta/2$ and Δ. It reaches its smallest value of $\Delta/2$ when $\lambda = 0.5$ and its largest value of Δ when $\lambda = 0$ or 1. For example, when $\Delta = 0.02$, $\mathcal{I}$ ranges from 1% to 2% depending on λ.

Figure 4 plots α_{non} as a function of λ and Δ. Like ESG investors, non-ESG investors earn zero alpha when $\lambda = 0$ or $\Delta = 0$. However, α_{non} increases in both λ or Δ. This alpha can be as large as 8% when $\lambda = 1$ and $\Delta = 0.04$. A non-ESG investor earns the highest alpha when all other investors are ESG (i.e., $\lambda = 1$) and when those investors' ESG preferences are strong (i.e., Δ is large) because the price impact of ESG is then particularly large. By going long brown stocks, whose alphas are positive and large, and short green stocks, whose alphas are negative and large, the non-ESG investor earns a large positive alpha.

Given the simplifying assumption that all assets have unit betas, the differences between the alphas plotted in Figures 3 and 4 are equal to the differences in expected returns plotted in Figure 2. Specifically, from equations (24) through (26), $\alpha_{esg} - \alpha_{non} = \mathrm{E}\{\tilde{r}_{esg}\} - \mathrm{E}\{\tilde{r}_{non}\}$.

3.6. Size of the ESG Investment Industry

We define the size of the ESG investment industry by the aggregate amount of ESG-driven investment that deviates from the market portfolio, divided by the stock market's total value. In general, this aggregate ESG tilt is given by

$$T \;=\; \int_{i:d_i>0} w_i T_i \, di \;, \tag{28}$$

where

$$T_i \;=\; \frac{1}{2}\iota' |X_i - x| \;. \tag{29}$$

The aggregate ESG tilt, T, is a wealth-weighted average of agent-specific tilts, T_i, across all agents who care at least to some extent about ESG (i.e., $d_i > 0$). Each T_i is one half of the sum of the absolute values of the N elements of agent i's ESG tilt, $|X_i - x|$. We compute absolute values of portfolio tilts because ESG-motivated investors both over- and under-weight stocks relative to the market. We divide by two because we do not want to double-count: for each dollar that an agent moves into a green stock, she must move a dollar out of another stock. The value of T_i is formally equivalent to agent i's active

share (Cremers and Petajisto, 2009), with the market portfolio as the benchmark, but its interpretation is different: instead of measuring the activeness of the agent's portfolio, T_i measures the portfolio's ESG-induced tilt away from the market.

With two types of agents, the expression for T simplifies to

$$T = \frac{1}{2}\lambda\iota'|X_{esg} - x| = \lambda(1-\lambda)\sqrt{\frac{\Delta}{2a}}\,\iota'|g| \;, \tag{30}$$

as we show in the Appendix. The aggregate tilt depends on the absolute values of the elements of g. To evaluate $\iota'|g|$ in this quantitative exercise, we further assume that the elements of g are normally distributed across stocks, in addition to the previous assumptions that these elements have a mean of zero and a variance of $1/N$ (recall $x'g = (1/N)\iota'g = 0$ and $g'g = 1$). Then $\iota'|g| = N\mathrm{E}(|g_n|) = \sqrt{2N/\pi}$. Therefore,

$$T = \lambda(1-\lambda)\sqrt{\frac{\Delta N}{a\pi}} \;. \tag{31}$$

We set the number of assets here to $N = 100$. That number is considerably smaller than the actual number of stocks in the U.S. market, but recall that we assume equal market weights across stocks. We reduce N as a concession to the fact that the actual distribution of firm size in the U.S. market is quite disperse. Another reason to choose a small N is that we do not impose any investment constraints. As investors go long and short, the sum of the absolute values of their short positions increases with N, without bounds. In reality, however, investors often face short-sale or margin constraints that would prevent this from happening. Choosing a smaller N helps offset the effect of a growing number of short positions on T. Given the arbitrariness inherent in the choice of N, we are more interested in the dependence of T on λ and Δ than in the magnitude of T per se. The overall level of T depends on N, but the patterns with respect to λ and Δ do not.

Figure 5 plots T for different values of λ and Δ. In Panel A, λ goes from 0 to 1. At both $\lambda = 0$ and $\lambda = 1$, we have $T = 0$ because all investors hold the market portfolio. The maximum value of T in equation (31) always occurs at $\lambda = 0.5$, the maximum of $\lambda(1-\lambda)$. In Panel B, Δ goes from 0 to 0.04. Larger values of Δ produce larger values of T. This relation between Δ and T is concave (see also equation (31)). For example, the ESG industry peaks at 46% of the stock market's value when $\Delta = 0.02$, but doubling the strength of ESG preferences (raising Δ to 0.04) increases that maximum industry size by less than half, to 65% of the market's value. We see that the price impact of ESG preferences weakens their impact on the size of the ESG investment industry.

4. The ESG Factor

In this section, we extend our model from Section 2 to show how firms' ESG characteristics can emerge as sensitivities to a risk factor—the ESG factor. The strength of ESG concerns can change over time, both for investors in firms' shares and for the customers who buy the firms' goods and services. If ESG concerns strengthen, customers may shift their demands for goods and services to greener providers (the "customer" channel), and investors may derive more utility from holding the stocks of greener firms (the "investor" channel). Both channels contribute to the ESG factor's risk in our framework.

To model the investor channel, we assume that the average ESG taste $\bar{d}$ shifts unpredictably from time 0 to time 1. To model the customer channel, we need to model firm profits. Let $\tilde{u}_n$ denote the financial payoff (profit in our one-period setting) that firm n produces at time 1, for each dollar invested in the firm's stock at time 0. We assume a two-factor structure for the $N \times 1$ vector of these payoffs:

$$\tilde{u} - \mathrm{E}_0\{\tilde{u}\} = \tilde{z}_h h + \tilde{z}_g g + \tilde{\zeta} \ , \tag{32}$$

where $\mathrm{E}_0\{\ \}$ denotes expectation as of time 0, the random variables $\tilde{z}_h$ and $\tilde{z}_g$ have zero means, and $\tilde{\zeta}$ is a mean-zero vector that is uncorrelated with $\tilde{z}_h$, $\tilde{z}_g$, and $\bar{d}$ and has a diagonal covariance matrix, Λ. The shock $\tilde{z}_h$ can be viewed as a macro output factor, with the elements of h being firms' sensitivities to that pervasive shock. The shock $\tilde{z}_g$ represents the effect on firms' payoffs of unanticipated shifts in customers' demands. These shifts can result not only from changes in consumers' tastes but also from revisions of government policy. For example, pro-environmental regulations may subsidize green products, leading to more customer demand, or handicap brown products, leading to less demand. A positive $\tilde{z}_g$ shock increases the payoffs of green firms but hurts those of brown firms.

To assess how shifts in ESG preferences affect asset prices, we need to price stocks not only at time 0, as we have done so far, but also at time 1, after the preference shift in $\bar{d}$ occurs. To make this possible in our simple framework, we split time 1 into two times, 1^- and 1^+, that are close to each other. We calculate prices p_1 as of time 1^-, by which time ESG preferences have shifted and all risk associated with $\tilde{u}$ has been realized. Stockholders receive $\tilde{u}$ at time 1^+. During the instant between times 1^- and 1^+, these payoffs are riskless. For economy of notation, we assume the risk-free rate $r_f = 0$.

There are two generations of agents, Gen-0 and Gen-1. Gen-0 agents live from time 0 to time 1^-; Gen-1 agents live from time 1^- to 1^+. Gen-1 agents have identical tastes of $d_i = \bar{d}_1$, a condition that gives them finite utility, given the absence of both risk and

position constraints during their lifespan. Neither a nor g change across generations. At time 1^-, Gen-0 agents sell stocks to Gen-1 agents at prices p_1, which depend on Gen-1 ESG preferences $\bar{d}_1$ and the financial payoff $\tilde{u}$. This simple setting maintains single-period payoff uncertainty while also allowing risk stemming from shifts in ESG preferences to enter via both channels described earlier.

4.1. Two Channels

Given that the payoff $\tilde{u}_n$ is known at the time when the price $p_{1,n}$ is computed, $p_{1,n}$ is equal to $\tilde{u}_n$ discounted at the expected return implied by equation (9) with β_n set to zero:

$$p_{1,n} \;=\; \frac{\tilde{u}_n}{1-\frac{g_n}{a}\bar{d}_1} \;\approx\; \tilde{u}_n + \frac{g_n}{a}\bar{d}_1 \;. \tag{33}$$

The approximation above holds well for typical discount rates, which are not too far from zero.[5] Representing it as an equality for all assets gives

$$p_1 = \tilde{u} + \frac{1}{a}\bar{d}_1 g \;, \tag{34}$$

which is the vector of payoffs to Gen-0 agents. Its expected value at time 0 equals

$$\mathrm{E}_0\{p_1\} = \mathrm{E}_0\{\tilde{u}\} + \frac{1}{a}\mathrm{E}_0\{\bar{d}_1\} g \;. \tag{35}$$

Note that $p_1 - \mathrm{E}_0\{p_1\}$ equals the vector of unexpected returns for Gen-0 agents, because $\tilde{u}_n$ is the firm's payoff per dollar invested in its stock at time 0. From equations (32) through (35), these unexpected returns, $\tilde{\epsilon} = \tilde{r} - \mathrm{E}_0\{\tilde{r}\}$, are given by

$$\tilde{\epsilon} \;=\; \tilde{z}_h h + \tilde{f}_g g + \tilde{\zeta} \tag{36}$$

$$\;=\; B\tilde{f} + \tilde{\zeta} \;, \tag{37}$$

where $B = [h \; g]$, $\tilde{f} = [\tilde{z}_h \;\; \tilde{f}_g]'$, with $\mathrm{E}_0\{\tilde{f}\} = 0$, and $\tilde{f}_g$ denotes the "ESG factor" given by

$$\tilde{f}_g = \tilde{z}_g + \frac{1}{a}\left[\bar{d}_1 - \mathrm{E}_0\{\bar{d}_1\}\right]. \tag{38}$$

The two components of $\tilde{f}_g$ correspond to the two ESG risk channels discussed earlier: $\tilde{z}_g$ represents the customer channel while the other term represents the investor channel. While the customer channel follows closely from the structure assumed in equation (32), the investor channel emerges from the equilibrium dependence of stock prices on $\bar{d}$.

The elements of $\tilde{f}_g g$ in equation (36) drive a wedge between expected and realized returns for ESG-motivated agents in Gen-0. We thus have the following proposition.

[5] For arbitrary rates $\rho_1 \approx 0$ and $\rho_2 \approx 0$, we have $\frac{1+\rho_1}{1-\rho_2} = \frac{(1+\rho_1)(1+\rho_2)}{1-\rho_2^2} \approx (1+\rho_1)(1+\rho_2) \approx 1+\rho_1+\rho_2$, neglecting ρ_2^2 and $\rho_1\rho_2$. Setting $\tilde{u}_n = 1+\rho_1$ and $\frac{g_n}{a}\bar{d}_1 = \rho_2$ gives the approximation in equation (33).

Proposition 4. *Green (brown) stocks perform better (worse) than expected if ESG concerns strengthen unexpectedly via either the customer channel or the investor channel.*

As noted earlier, green stocks generally have lower expected returns than brown stocks. If $\tilde{f}_g$ is positive, however, such an outcome boosts the realized performance of green stocks while hurting that of brown stocks. If one computes average returns over a sample period when ESG concerns consistently strengthened more than investors expected, so that the average of $\tilde{f}_g$ over that period is strongly positive, then green stocks could outperform brown stocks, contrary to what is expected.

4.2. The ESG Factor's Effects on Market Betas

Besides affecting stock returns ex post, the ESG factor also affects returns ex ante by influencing market betas. As we show in the Appendix, the vector of market betas is

$$\beta \;=\; h\beta_h + g\beta_g + \frac{1}{\sigma_M^2}\Lambda x \;, \tag{39}$$

where $\beta_h \equiv \text{Cov}\{\tilde{\epsilon}_M, \tilde{z}_h\}/\sigma_M^2$, $\beta_g \equiv \text{Cov}\{\tilde{\epsilon}_M, \tilde{f}_g\}/\sigma_M^2$, and $\tilde{\epsilon}_M \equiv x'\tilde{\epsilon}$ is the unexpected market return. In words, a stock's market beta depends on the stock's loading on the macro factor (h_n) times that factor's loading on the market (β_h), plus the stock's loading on the ESG factor (g_n) times that factor's loading on the market (β_g), plus a term reflecting idiosyncratic risk. Substituting from equation (36) into $\tilde{\epsilon}_M \equiv x'\tilde{\epsilon}$, we immediately obtain

$$\beta_g = (x'h)\text{Cov}\{\tilde{z}_h, \tilde{f}_g\}/\sigma_M^2 + (x'g)\text{Var}(\tilde{f}_g)/\sigma_M^2 \;. \tag{40}$$

The overall stock market surely loads positively on the macro factor, $\tilde{z}_h$, meaning $x'h > 0$. Also, recall from equation (8) that $x'g = 0$, so the second term in equation (40) drops out.

Proposition 5. *If* $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} > 0$ *then ESG factor risk raises the market betas of green stocks but lowers the betas of brown stocks. These effects are reversed if* $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} < 0$.

If $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} > 0$ then $\beta_g > 0$ in equation (40), which then increases β's of green stocks, and reduces β's of brown stocks, through equation (39). If $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\} < 0$ then $\beta_g < 0$, and the effect on the β's is the opposite. The sign of $\text{Cov}\{\tilde{z}_h, \tilde{f}_g\}$, the covariance between the macro factor and the ESG factor, is unclear. On the one hand, a positive covariance is supported by the evidence of Bansal, Wu, and Yaron (2018) that green stocks outperform brown stocks in good times but underperform in bad times. Those authors argue that green stocks are similar to luxury goods in that they are in higher demand when the economy

does well and thus financial concerns matter less. On the other hand, the covariance could be negative if $\tilde{z}_h$ and $\tilde{f}_g$ have opposite exposures to climate risk. Adverse climate shocks are likely to be accompanied by positive realizations of $\tilde{f}_g$, as we discuss in Section 5. If such shocks are large enough to reduce aggregate output then they are also accompanied by negative realizations of $\tilde{z}_h$. This common dependence on climate shocks then makes $\tilde{z}_h$ and $\tilde{f}_g$ negatively correlated. Albuquerque, Koskinen, and Zhang (2019) find empirically that greener firms, as measured by high corporate social responsibility scores, have lower market betas. Their evidence is consistent with a negative correlation between $\tilde{z}_h$ and $\tilde{f}_g$.

Finally, if we relax the assumption from equation (8) that $x'g = 0$, the role of ESG factor risk depends on the overall greenness of the market portfolio. If the market is net green, so that $x'g > 0$, then the second term in equation (40) is positive, further increasing the covariance between the market return and the ESG factor. As the economy becomes greener, $x'g$ rises, pushing up β_g in equation (40). The greenifying of the economy thus makes green stocks increasingly exposed to the market, and brown stocks decreasingly so.

4.3. Two-Factor Pricing

Under the above setting, the ESG factor, with its mean shifted to a non-zero value, also produces near-zero alphas in a two-factor model in which the market return is the other factor. This result obtains when the market portfolio is neither green nor brown ($x'g = 0$) and is also well diversified, in that $x_n \approx 0$ for all n (a large-N scenario). Those conditions imply that excess returns are closely approximated by the regression relation

$$\tilde{r} = \theta \tilde{r}_M + g(\tilde{f}_g + \mu_g) + \tilde{\nu} \ , \tag{41}$$

with $\mathrm{E}\{\tilde{\nu} | \tilde{r}_M, \tilde{f}_g\} = 0$, $\theta = (1/x'h)h$, and

$$\mu_g = \mu_M \beta_g - \bar{d}/a \ , \tag{42}$$

as we show in the Appendix. We thus obtain the following proposition.

Proposition 6. *Each stock has zero alpha with respect to a two-factor model with the market factor and the ESG factor, with stock n's loading on the ESG factor equal to g_n.*

The premium associated with the ESG factor, μ_g in equation (42), has two components. The first component, $\mu_M \beta_g$, is a part of the market risk premium. The second component, $-\bar{d}/a$, is not a risk premium; it is characteristic-based. Recall from equation (10) that $-g\bar{d}/a$ is a vector of stocks' CAPM alphas. The variation in $\tilde{f}_g$ represents shifts in tastes;

it represents common risk, but this risk is not priced, except for the part that comoves with the market. Only market risk, captured by β, is priced in Section 4. The ESG factor affects expected returns only through its effect on β, which we discuss earlier in Section 4.2.

The reason why ESG factor risk is not priced in this section, above and beyond its co-movement with the market, is that its variation is unrelated to agents' utility function in equation (2). However, ESG factor risk is priced if the utility function is modified so that agents care about the realizations of $\tilde{f}_g$. In the following section, we modify the utility function by adding a concern about the global climate. We show that climate risk is priced, and we draw parallels between climate shocks and ESG factor realizations. If agents care about climate shocks and those shocks are correlated with $\tilde{f}_g$—a scenario we consider plausible—then we should observe the ESG factor's risk being priced in any empirical setting that does not fully control for climate shocks.

5. Climate Risk

Sustainable investing is motivated in part by concerns about global climate change (part of "E" in ESG). Many experts expect climate change to impair quality of life, essentially lowering utility of the typical individual beyond what is captured by climate's effect on wealth. Unanticipated climate changes present investors with an additional source of risk. This section extends our model from Section 2 to include such risk.

Let $\tilde{C}$ denote climate at time 1, which is unknown at time 0. We modify the utility function for individual i in equation (2) to include $\tilde{C}$ as follows:

$$V(\tilde{W}_{1i}, X_i, \tilde{C}) = -e^{-A_i \tilde{W}_{1i} - b_i' X_i - c_i \tilde{C}} \ , \tag{43}$$

where $c_i \geq 0$, so that agents dislike low realizations of $\tilde{C}$. Define $\bar{c} \equiv \int_i w_i c_i di$, the wealth-weighted mean of climate sensitivity across agents. We assume $\tilde{C}$ is normally distributed, and without loss of generality we set $\mathrm{E}\{\tilde{C}\} = 0$ and $\mathrm{Var}\{\tilde{C}\} = 1$. Besides replacing equation (2) by equation (43), we maintain all other assumptions from Section 2.

5.1. Expected Returns

Climate risk has a transparent effect on equilibrium stock returns.

Proposition 7. *Expected excess returns in equilibrium are given by*

$$\mu = \mu_M \beta - \frac{\bar{d}}{a} g + \bar{c} \left(1 - \rho_{MC}^2\right) \psi \ , \tag{44}$$

where ψ is the $N \times 1$ vector of "climate betas," that is slope coefficients on $\tilde{C}$ in a multivariate regression of $\tilde{\epsilon}$ on both $\tilde{\epsilon}_M$ and $\tilde{C}$, and ρ_{MC} is the correlation between $\tilde{\epsilon}_M$ and $\tilde{C}$.

Expected returns depend on climate betas, ψ, which represent firms' exposures to non-market climate risk. To understand the regression defining ψ, recall that $\tilde{\epsilon}$ is an $N \times 1$ vector of unexpected stock returns from equation (1) and $\tilde{\epsilon}_M$ is the unexpected market return. A firm's climate beta is its loading on $\tilde{C}$ after controlling for the market return.

Compared to equation (9), expected excess returns contain an additional component given by the last term on the right-hand side of equation (44). Stock n's climate beta, ψ_n, enters expected return positively. Thus, a stock with a negative ψ_n, which provides investors with a climate-risk hedge, has a lower expected return than it would in the absence of climate risk. Vice versa, a stock with a positive ψ_n, which performs particularly poorly when the climate worsens unexpectedly, has a higher expected return.

Climate betas, ψ_n, are likely to be related to ESG characteristics, g_n, as we argue next.

5.2. Green Stocks as Climate Hedges

Green stocks seem more likely than brown stocks to hedge climate risk. This hedging asymmetry can be motivated through both channels described in Section 4. First, consider the customer channel. Unexpected worsening of the climate can heighten consumers' climate concerns, prompting greater demands for goods and services of greener providers. These demands can arise not only from consumers' choices but also from government regulation. Negative climate shocks can lead governments to adopt regulations that favor green providers or penalize brown ones. Half of the institutional investors surveyed by Krueger, Sautner, and Starks (2019) state that climate risks related to regulation have already started to materialize. Second, consider the investor channel. Unexpected worsening of the climate can strengthen investors' preference for green holdings (i.e., increase $\bar{d}$). For example, Choi, Gao, and Jiang (2019) show that retail investors sell carbon-intensive firms in extremely warm months, consistent with $\bar{d}$ rising in such months. Climate shocks are thus likely to correlate negatively with both components of the ESG factor, $\tilde{f}_g$, in equation (38). Green stocks, which have positive exposures to $\tilde{f}_g$, are likely to have negative exposures to $\tilde{C}$. In other words, the correlation between g_n and ψ_n across firms is likely to be negative.

Evidence also suggests that greener stocks are better climate hedges. Empirical studies find that returns on green (brown) stocks have positive (negative) correlations with adverse climate shocks. For example, Choi, Gao, and Jiang (2019) show that green firms, as measured

by low carbon emissions, outperform brown firms during months with abnormally warm weather, which the authors argue alert investors to climate change. Similarly, Engle et al. (2019) report that green firms, as measured by high E-Scores from Sustainalytics, outperform brown firms in periods with negative climate news. Both studies thus show that a high-minus-low g_n stock portfolio is a good hedge against climate risk, indicating that g_n is negatively correlated with ψ_n across firms.

In the special case where this negative correlation is perfect, so that

$$\psi_n = -\xi g_n \ , \tag{45}$$

where $\xi > 0$ is a constant, equation (44) simplifies to

$$\mu = \mu_M \beta - \left[\frac{\bar{d}}{a} + \bar{c} \left(1 - \rho_{MC}^2 \right) \xi \right] g \ . \tag{46}$$

Stock n's CAPM alpha is then given by

$$\alpha_n = - \left[\frac{\bar{d}}{a} + \bar{c} \left(1 - \rho_{MC}^2 \right) \xi \right] g_n \ . \tag{47}$$

Both terms inside the brackets are positive, so the negative relation between α_n and g_n is stronger than in Corollary 2. Greener stocks now have lower CAPM alphas not only because of investors' preferences for green holdings, but also because of greener stocks' ability to better hedge climate risk. Climate risk thus represents another reason to expect green stocks to underperform brown ones over the long run. For the same reason, green stocks have a lower cost of capital than brown stocks relative to the CAPM.

5.3. Climate-Hedging Portfolio

Finally, climate risk has interesting implications for investors' asset holdings.

Proposition 8. *Agent i's equilibrium portfolio weights are given by*

$$X_i = x + \frac{\delta_i}{a^2} \left(\Sigma^{-1} g \right) - \frac{\gamma_i}{a} \left(\Sigma^{-1} \sigma_{\epsilon C} \right) \ , \tag{48}$$

where $\gamma_i \equiv c_i - \bar{c}$ and $\sigma_{\epsilon C}$ is an $N \times 1$ vector of covariances between $\tilde{\epsilon}_n$ and $\tilde{C}$.

Proposition 8 implies four-fund separation. The first three funds are the same as in Proposition 3; the fourth one is a climate-hedging portfolio whose weights are proportional to $\Sigma^{-1}\sigma_{\epsilon C}$. Agents with $\gamma_i > 0$, whose climate sensitivity is above average, short the hedging portfolio, whereas agents with $\gamma_i < 0$ go long.

The climate-hedging portfolio, $\Sigma^{-1}\sigma_{\epsilon C}$, is a natural mimicking portfolio for $\tilde{C}$. To see this, note that the N elements of $\Sigma^{-1}\sigma_{\epsilon C}$ are the slope coefficients from the multiple regression of $\tilde{C}$ on $\tilde{\epsilon}$. Therefore, the return on the hedging portfolio has the highest correlation with $\tilde{C}$ among all portfolios of the N stocks. Investors in our model hold this maximum-correlation portfolio, to various degrees determined by their γ_i, to hedge climate risk.

The climate-hedging portfolio is likely to favor green stocks over brown. The reason is that green stocks are generally better climate hedges than brown stocks, as discussed earlier in Section 5.2. Yet the climate-hedging portfolio is not necessarily simply long green stocks and short brown ones. Even if $\sigma_{\epsilon C}$ were perfectly correlated with g across firms, $\sigma_{\epsilon C}$ in equation (48) is pre-multiplied by Σ^{-1}, which in general makes the climate-hedging portfolio weights imperfectly aligned with stocks' ESG characteristics.

5.4. Climate Risk versus ESG Risk

In this section, we relate the climate variable, $\tilde{C}$, from Section 5, to the ESG factor, $\tilde{f}_g$, from Section 4. At first sight, the two variables are not comparable because Sections 4 and 5 are different extensions of the baseline model in Section 2. To make $\tilde{C}$ and $\tilde{f}_g$ comparable, we embed them in the same modeling framework. Specifically, we rederive the results in Section 4 after replacing the utility function in equation (2) by that in equation (43) and maintaining all other assumptions of Section 4. We continue to assume that firm payoffs are given by the two-factor structure in equation (32), so that the effect of $\tilde{C}$ on firm payoffs is spanned by $\tilde{z}_h$ and $\tilde{z}_g$. In this modified setting, the derivation in Section 4.1 remains unchanged, except that the last term in equation (44), $\bar{c}\,(1-\rho_{MC}^2)\,\psi$, appears on the right-hand sides of equations (33) through (35). For simplicity, we assume that this term, which represents the premium for climate risk, is constant over time. Therefore, this term subtracts out in equation (36) and the ESG factor, $\tilde{f}_g$, remains the same as in equation (38). Therefore, $\tilde{C}$ from equation (43) and $\tilde{f}_g$ from equation (38) are directly comparable, after all.

The two factors, $\tilde{C}$ and $\tilde{f}_g$, are likely to be negatively correlated. Recall that $\tilde{f}_g$ from equation (38) has two components reflecting the customer and investor channels. Both components are likely to be negatively correlated with $\tilde{C}$, as discussed in Section 5.2. When the climate worsens unexpectedly, consumers may shift their demands toward green products and away from brown products, either of their own volition or prompted by new government regulations subsidizing green products or taxing, possibly even prohibiting, brown products. In addition, worse climate can elevate investors' tastes for green holdings, for example, as a result of stronger public pressure on institutional investors to divest from brown assets.

If climate shocks are the only reason behind shifts in customers' and investors' tastes, $\tilde{C}$ and $\tilde{f}_g$ are perfectly negatively correlated. In this special case, the two factors coincide. Without loss of generality, we scale them so that $\tilde{f}_g = -\tilde{C}$. Stocks' loadings on the single factor are then given by the ESG characteristics: $\psi = -g$. Moreover, stocks are priced by only two factors: the market portfolio and the ESG factor. As we show in the Appendix, the two-factor asset pricing formula in equation (41) holds also in this setting, except that the ESG factor's premium in that equation is redefined to include an extra term:

$$\mu_g = \mu_M \beta_g - \bar{d}/a - \bar{c}\left(1 - \rho_{MC}^2\right) \ . \tag{49}$$

The last term in μ_g, which is absent from equation (42), reflects compensation for climate risk. This compensation is negative because greener firms are better hedges against this risk. The ESG factor's premium thus has two risk-based components, $\mu_M \beta_g$ for market risk and $-\bar{c}\,(1 - \rho_{MC}^2)$ for climate risk, and one taste-based component, $-\bar{d}/a$.

This special case is appealing because it can serve as motivation for empiricists to use a single factor to capture the risk associated with ESG investing. Moreover, it shows that firms' loadings on the single factor are simply proportional to firms' ESG characteristics, g_n. However, in general, a multi-factor approach may be required because the tastes of customers and investors may shift also for reasons unrelated to climate news, such as environmental activism and political pressure. Whether a single factor is sufficient to capture all ESG-related risk is an interesting question for future empirical research.

6. Social Impact

Does sustainable investing produce real social impact? We explore this question by adding firms' choices of investment and ESG characteristics to the baseline model from Section 2.

We define the social impact of firm n as

$$S_n \equiv g_n K_n \ , \tag{50}$$

where K_n is the firm's operating capital. Social impact captures the firm's total amount of externalities, which depends on both the nature of the firm's operations (g_n) and their scale (K_n). The firm is initially endowed with operating capital $K_{0,n} > 0$ and an ESG characteristic $g_{0,n}$. The firm's manager maximizes firm value at time 0 by choosing how much additional capital ΔK_n to buy, and by choosing Δg_n, the change in the ESG characteristic. For example, a coal power producer can increase its g_n by installing scrubbers, and it can

increase its K_n by building more power plants. The firm's capital investment produces time-0 cash flows of $-\Delta K_n - \frac{\kappa_n}{2}(\Delta K_n)^2$, where $\kappa_n > 0$ controls capital-adjustment costs. The firm uses capital to produce expected gross cash flows at time 1 equal to $A_n K_n$, where A_n denotes the firm's gross profitability.[6] Adjusting g_n is also costly: it reduces the firm's time-1 cash flow by a fraction $\frac{\omega_n}{2}(\Delta g_n)^2$, where $\omega_n > 0$ controls ESG-adjustment costs.

These extra assumptions change none of the previous sections' predictions. Since investors are atomistic, they still take asset prices and firms' ESG characteristics as given, even though firms now choose those characteristics. Firms' choices of K_n and g_n affect their market values, which are consistent with the expected returns derived in Section 2.

We prove the following proposition in the Appendix.

Proposition 9. *Firm n's value-maximizing choices of ESG adjustment and investment are*

$$\Delta g_n\left(\overline{d}\right) \approx \frac{\overline{d}}{a\omega_n} \tag{51}$$

$$\Delta K_n\left(\overline{d}\right) = \frac{1}{\kappa_n}\left[\frac{A_n\left(1-\frac{\omega_n}{2}(\Delta g_n(\overline{d}))^2\right)}{1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}g_n(\overline{d})}-1\right], \tag{52}$$

where $g_n(\overline{d}) = g_{0,n} + \Delta g_n(\overline{d})$*, and the approximation uses* $\log(1+x) \approx x$ *for small* x*.*

Both choices are intuitive. When $\overline{d} > 0$, investors value ESG characteristics, so all firms choose to become greener (i.e, $\Delta g_n > 0$) to increase their market value. This effect is especially strong when risk aversion a is low because ESG characteristics then have large effects on market values. Firms also adjust g_n by more when doing so is less costly.

ESG preferences tilt investment away from brown firms and toward green firms. The term in brackets in equation (52) is the NPV of an additional dollar of investment, ignoring capital adjustment costs. The higher is this NPV, the more the firm invests. The denominator in equation (52) shows that ESG preferences reduce the cost of capital for firms with $g_n > 0$ (i.e., green firms), which increases their projects' NPV and hence investment. And vice versa, ESG preferences increase brown firms' cost of capital, reducing their projects' NPV and investment. In addition, ESG preferences affect expected cash flows in the numerator of equation (52). Stronger ESG preferences induce all firms, green and brown, to adjust their g_n by more, which reduces their expected cash flows, and hence also their investment.

For a given firm n, the social impact induced by ESG-motivated investment is $S_n(\overline{d}) -$

[6] We assume A_n is high enough so that the firm chooses positive investment for any value $\overline{d} \geq 0$. In other words, we assume the firm's marginal project has positive net present value (NPV).

$S_n(0)$, the difference between the firm's actual social impact and its hypothetical impact if agents did not care about ESG.

Proposition 10. *The social impact induced by ESG-motivated investment is nonnegative for all firms. It is strictly positive for all firms as long as* $\bar{d} > 0$.

A simple decomposition helps explain this result:

$$S_n\left(\overline{d}\right) - S_n\left(0\right) = K_{n,0}\Delta g_n\left(\overline{d}\right) + \Delta K_n\left(\overline{d}\right)\Delta g_n\left(\overline{d}\right) + g_{n,0}\left(\Delta K_n\left(\overline{d}\right) - \Delta K_n\left(0\right)\right). \quad (53)$$

ESG-motivated investment increases social impact via three channels: by making existing capital greener (first term), by making new capital greener (second term), and by tilting investment away from brown firms and toward green firms (third term). The first term is strictly positive as long as $\overline{d} > 0$, which produces $\Delta g_n(\overline{d}) > 0$ for all firms. The second term is strictly positive for the same reason, under the assumption in footnote 6 that A_n is sufficiently high to make $\Delta K_n > 0$. To understand the third term, recall that ESG preferences change firms' cost of capital so that $\Delta K_n(\overline{d}) - \Delta K_n(0)$ tends to be positive for green firms (i.e., $g_n > 0$) and negative for brown firms (i.e., $g_n < 0$), which makes the third term positive for both types of firms. One complication is that a firm can start out brown and turn green, but we prove in the Appendix that $S_n\left(\overline{d}\right) - S_n\left(0\right) > 0$ even for those firms.

Figure 6 plots the social impact induced by ESG-motivated investors across firms with different initial ESG characteristics. The three shaded regions indicate the three sources of social impact from equation (53). All firms have positive social impact, mainly because all firms' existing capital becomes greener (bottom blue region). The effect of new capital becoming greener (middle red region) is modest given these parameter values. The third source of social impact, tilting investment from brown to green firms, equals zero for an ESG-neutral firm, but it is large for very green or very brown firms, which experience the largest shift in investment. Due to this non-monotonic third channel, the overall social impact induced by ESG-motivated investors is largest for firms with extreme ESG characteristics, but it is strictly positive even for ESG-neutral firms.

The aggregate social impact induced by ESG investors, denoted $S(\overline{d}) - S(0)$, is the sum of $S_n(\overline{d}) - S_n(0)$ across firms n. This sum can be computed off the curve in Figure 6. Since this curve is convex in $g_{0,n}$, $S(\overline{d}) - S(0)$ is greater when there is more dispersion in ESG characteristics across firms. A larger dispersion in $g_{0,n}$ deepens the cost-of-capital differentials between green and brown firms, leading to larger investment differentials. With green firms investing more and brown firms investing less, aggregate social impact increases.

Figure 7 illustrates how aggregate social impact varies with the strength of ESG preferences. We assume firms differ only in their initial ESG characteristics $g_{n,0}$, which we assume are uniformly distributed with mean zero. The figure shows that $S(\overline{d}) - S(0)$ increases as average ESG preferences strengthen, which is intuitive. We also see that all three social-impact channels from equation (53) become stronger as ESG preferences strengthen. These results hold whether ESG preferences strengthen because more investors are ESG (Panel A), or because ESG investors have stronger ESG tastes (Panel B).

In our model, ESG preferences tilt real investment from brown to green firms because those preferences generate alphas, which affect the cost of capital, which in turn affects investment. There is considerable empirical support for this mechanism. Baker and Wurgler (2012) survey papers that find a negative relation between corporate investment and alpha. Most of these papers interpret alpha as mispricing, whereas our paper's ESG-induced alphas do not reflect mispricing. We expect ESG-induced alphas to have an especially strong effect on investment. Whereas mispricing is transient, firms' ESG traits are highly persistent, which makes ESG-induced alphas highly persistent. Van Binsbergen and Opp (2019) show that when alphas are more persistent, they have stronger effects on investment.

We have made the standard assumption that managers maximize the firm's market value. This assumption makes sense if, for example, managers wish to maximize the value of their stock-based compensation. Our assumption may not be appropriate, however, in a world where investors care directly about firms' ESG characteristics. A manager may then maximize shareholder welfare, which depends not just on market value but also on the firm's ESG characteristics (e.g., Hart and Zingales, 2017). Such behavior could result from shareholders engaging actively with the firm, so that managers run the firm as shareholders desire (e.g., Dyck et al., 2019), or from shareholders appointing managers whose preferences match their own. Our model arguably provides a lower bound on social impact. Extending the model so that managers additionally care about their firms' ESG characteristics should produce Δg_n values (and hence social impact) even larger than we currently predict. Put differently, we show that ESG-motivated investors can generate social impact even without direct engagement by shareholders, and even if managers do not care directly about firms' ESG characteristics. Even a "selfish" manager who cares only about market value behaves in a way that increases social impact.

Finally, we enrich the model by allowing agents to care about aggregate social impact, S. Specifically, we assume each agent i's utility is increasing in S:

$$U(\tilde{W}_{1i}, X_i, S) = V(\tilde{W}_{1i}, X_i) + h_i(S) \ , \tag{54}$$

where $h_i'(S) > 0$ and V is the original utility function from equation (2). (The additive specification is not needed; our results are identical if S enters utility multiplicatively.) Agents now care about sustainability in two ways: they derive utility from their holdings as well as from firms' aggregate social impact.

Proposition 11. *If agents derive utility also from aggregate social impact (equation (54)), all of our results in Propositions 1 through 10 and Corollaries 1 through 4 continue to hold.*

The inclusion of S in the utility function does not affect any of our prior results. The reason is that agents are infinitesimally small. Small agents take stock prices, and hence S, as given when choosing their portfolios. Therefore, agents' preference for S does not affect their portfolio choice. When an agent tilts toward green stocks, she generates a positive externality on other agents via the $h_i(S)$ term in their utility. Being small, though, she does not internalize this effect. As the preference for S does not affect portfolio choice, it does not affect equilibrium asset prices, real investment, or S. In the model of Oehmke and Opp (2020), agents' preference for social impact does lead to impact because agents are assumed to coordinate. In our model, agents cannot coordinate. Social impact is caused by the inclusion of X_i, not S, in the utility function in equation (54).

7. Conclusion

We analyze both financial and real effects of sustainable investing in a highly tractable general equilibrium model. The model produces a number of empirical implications regarding asset prices, portfolio holdings, the size of the ESG investment industry, climate risk, and the social impact of sustainable investing. We summarize those implications below.

First, ESG preferences move asset prices. Stocks of greener firms have lower ex ante CAPM alphas, especially when risk aversion is low and the average ESG preference is strong. Green stocks have negative alphas, whereas brown stocks have positive alphas. Green stocks' negative alphas stem from two sources: investors' tastes for green holdings and such stocks' ability to hedge climate risk. Green and brown stocks have opposite exposures to an ESG risk factor, which captures unexpected changes in ESG concerns of customers and investors. If either kind of ESG concerns strengthen unexpectedly over a given period of time, green stocks can outperform brown stocks over that period, despite having lower alphas. Under plausible assumptions, stocks are priced by a two-factor asset pricing model, where the factors are the market portfolio and the ESG factor. In general, though, multiple factors may be required to fully capture asset exposures to ESG and climate risks.

Second, portfolio holdings exhibit three-fund separation. Investors with stronger-than-average ESG tastes hold portfolios that have a green tilt away from the market portfolio, whereas investors with weaker-than-average ESG tastes have a brown tilt. These tilts are larger when risk aversion is lower. Investors with stronger ESG tastes earn lower expected returns, especially when risk aversion is low and the average ESG taste is high. In the model extension that adds climate risk, we obtain four-fund separation, with the fourth fund representing a climate-hedging portfolio with a green tilt.

Third, the size of the ESG investment industry—the aggregate dollar amount of ESG-driven investment that deviates from the market portfolio, scaled by total market value— is increasing in the heterogeneity of investors' ESG preferences. If there is no dispersion, there is no ESG industry because everyone holds the market.

Finally, sustainable investing generates positive social impact, in two ways. First, it leads firms to become greener. Second, it induces more real investment by green firms and less investment by brown firms.

While the model's predictions for alphas have been examined empirically by prior studies, most of its other predictions remain untested, presenting opportunities for future empirical work. One challenge is that our model aims to describe the world of the present and the future, but not necessarily the world of the past. Although the "sin" aspects of investing have been recognized for decades, the emphasis on ESG criteria is a recent phenomenon. How the model fits in various time periods is a question for empirical work.

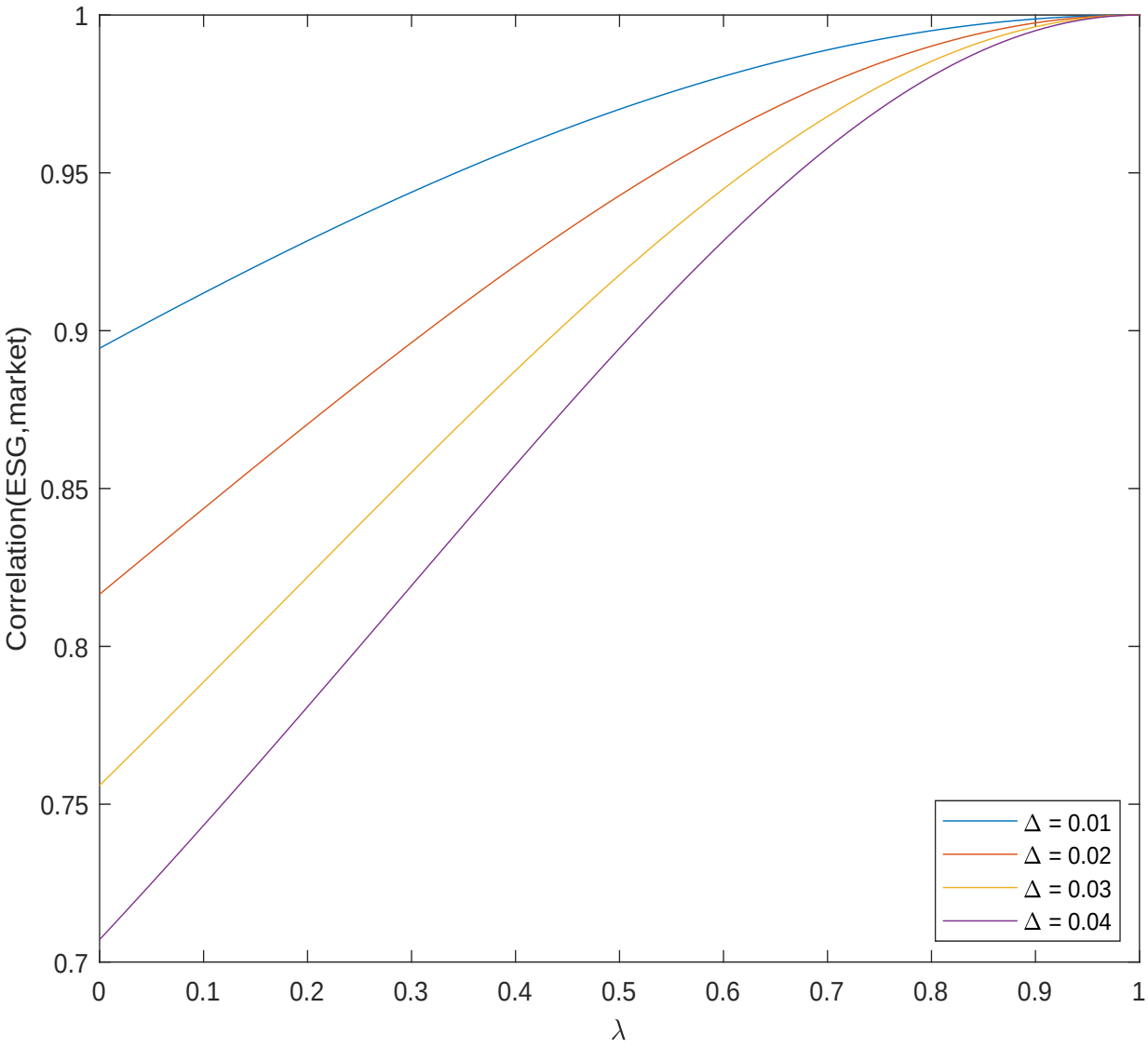


Figure 1. Correlation of ESG investor's portfolio return with the market return. The figure plots the correlation between the returns on the ESG investor's portfolio and the market portfolio. Results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio.

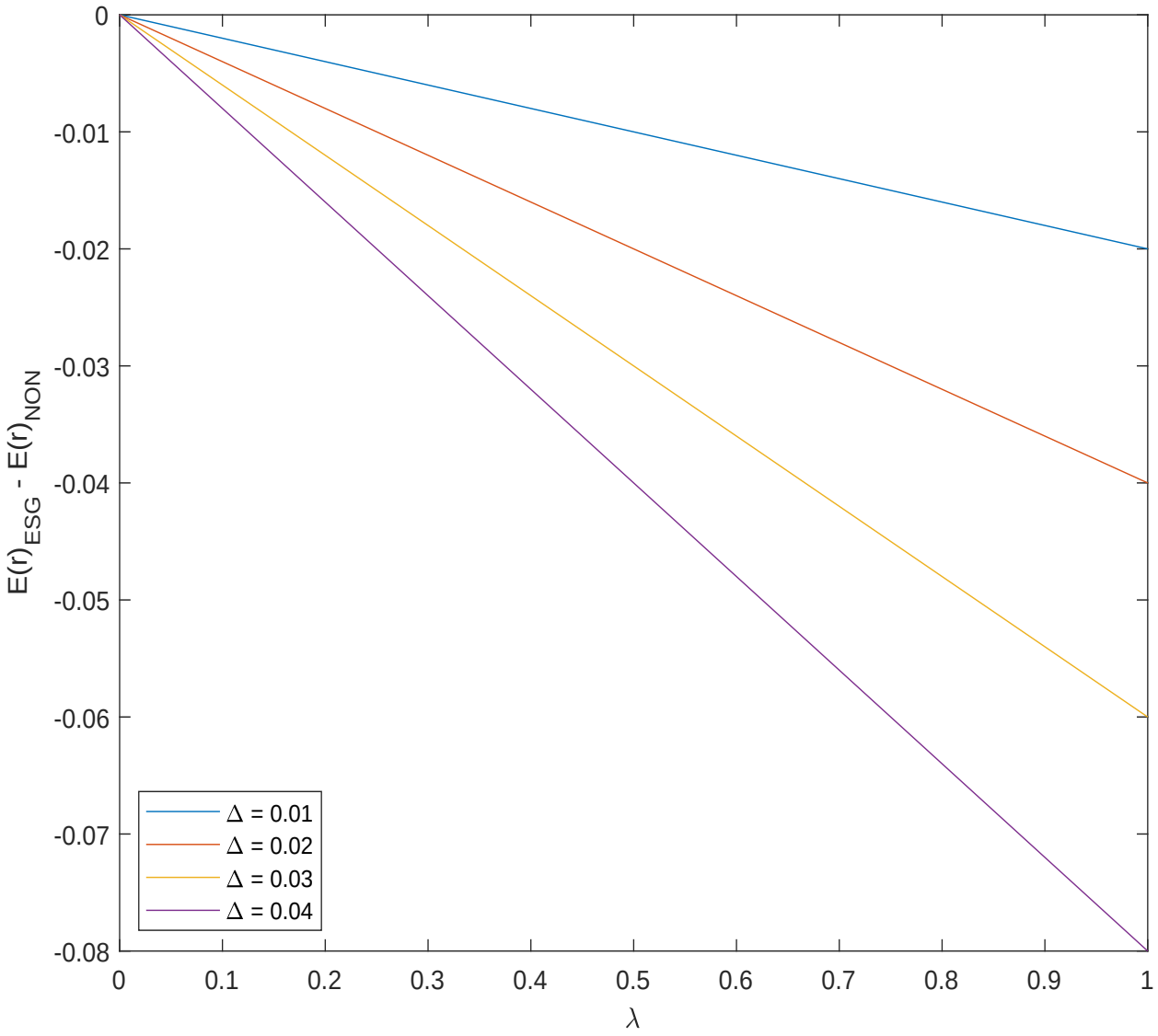
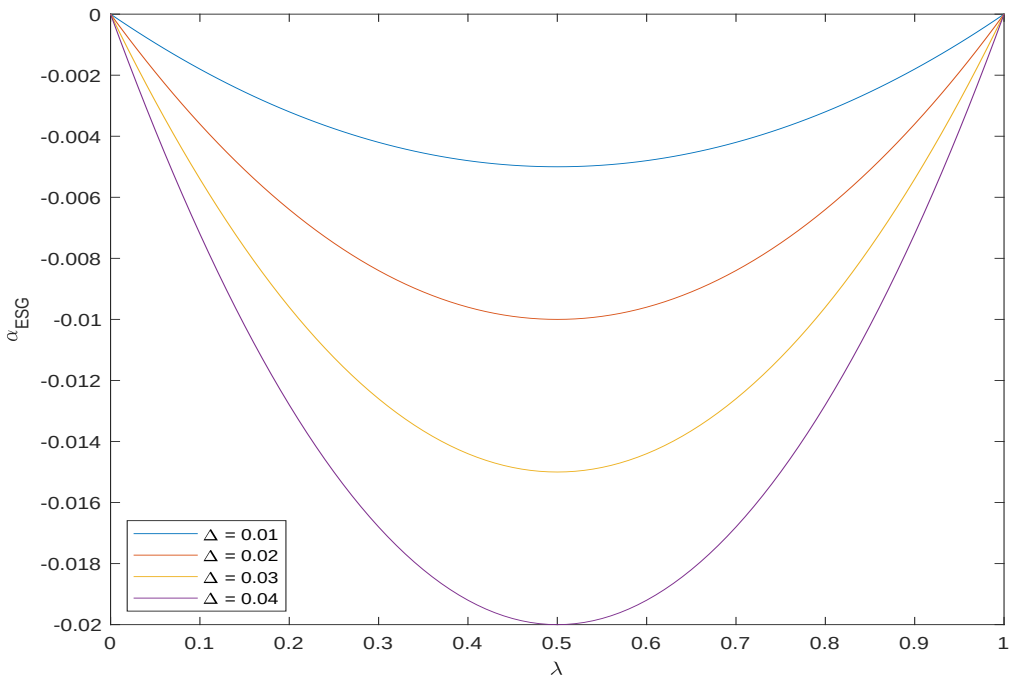


Figure 2. ESG versus non-ESG expected portfolio return. The figure plots the expected excess return on the portfolio of ESG investors minus the corresponding value for non-ESG investors. Results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio.

Panel A. The Role of λ



Panel B. The Role of Δ

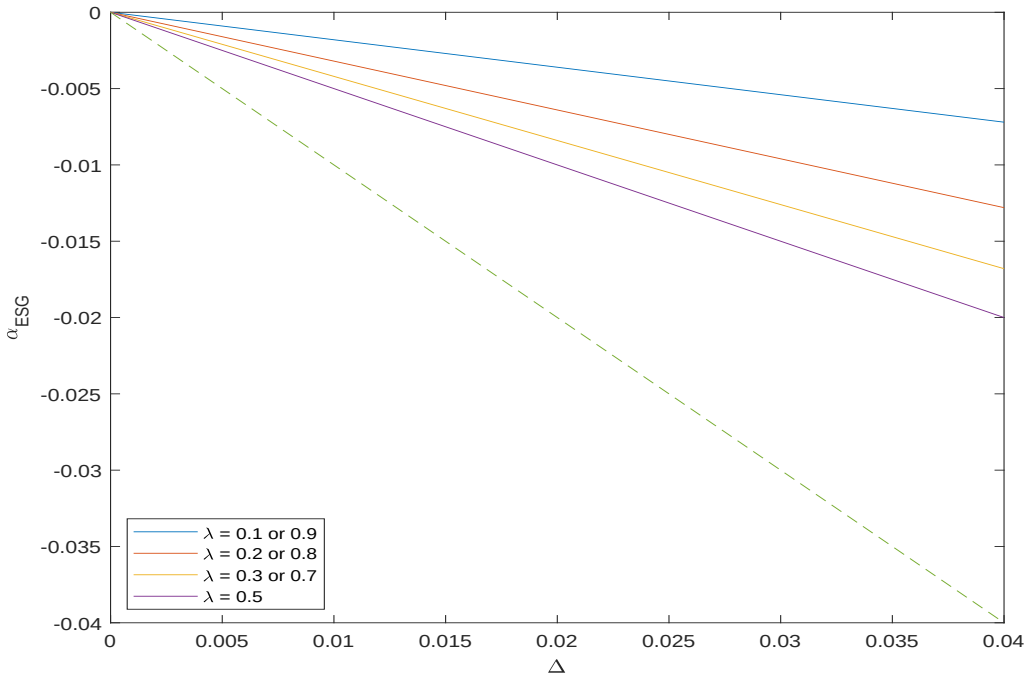
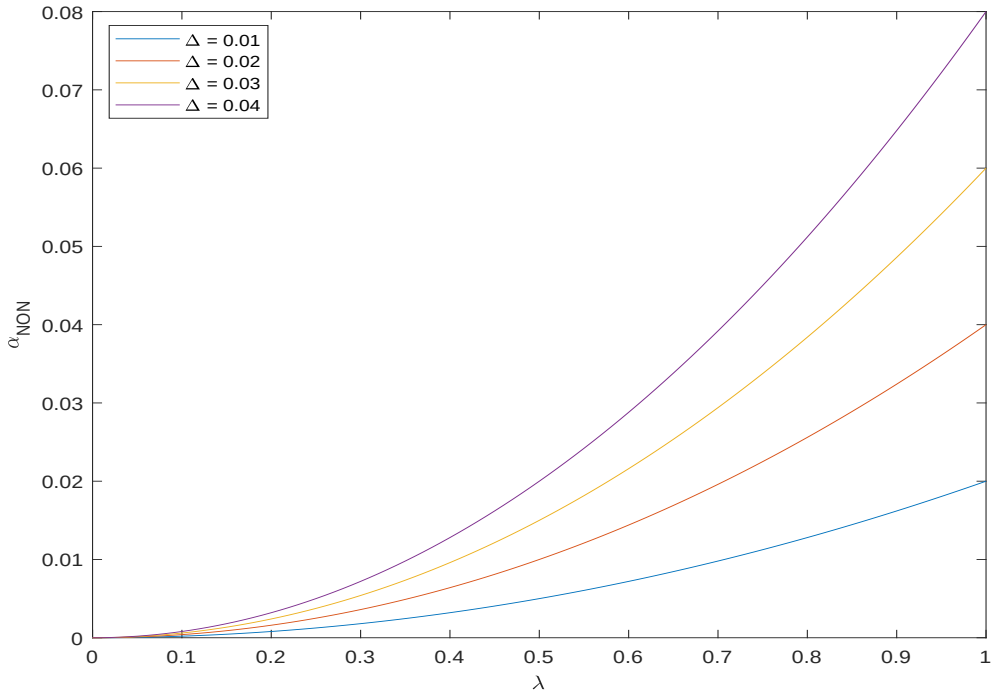


Figure 3. Alphas of ESG Investors. This figure plots the alpha for the portfolio held by ESG investors as a function of λ, the fraction of wealth belonging to ESG investors, and Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio. Panel A plots the ESG alpha as a function of λ for four different values of Δ; Panel B flips the roles of λ and Δ. The dashed line in Panel B has a slope of -1. The differences between the solid lines and the dashed line represent investor surplus.

Panel A. The Role of λ



Panel B. The Role of Δ

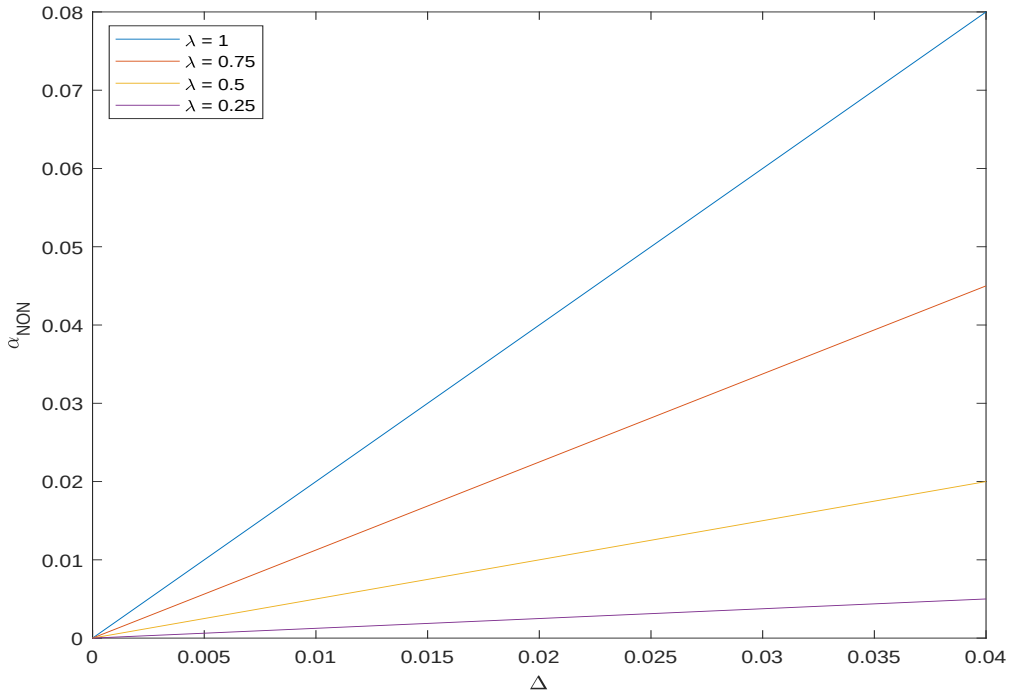
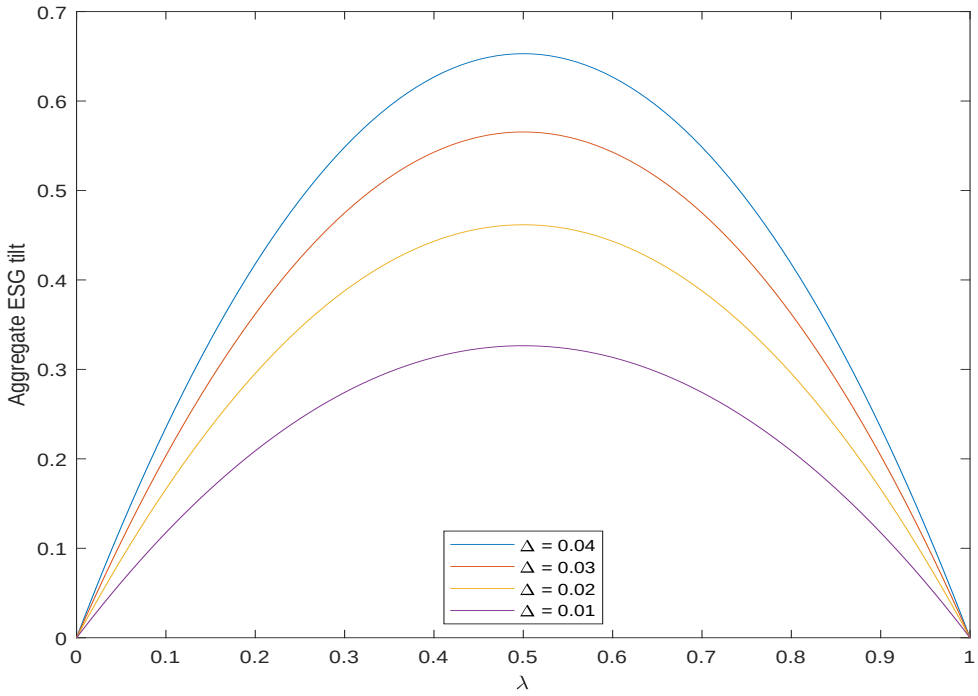


Figure 4. Alphas of Non-ESG Investors. This figure plots the alpha for the portfolio held by non-ESG investors as a function of λ, the fraction of wealth belonging to ESG investors, and Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio. Panel A plots the ESG alpha as a function of λ for four different values of Δ; Panel B flips the roles of λ and Δ.

Panel A. The Role of λ



Panel B. The Role of Δ

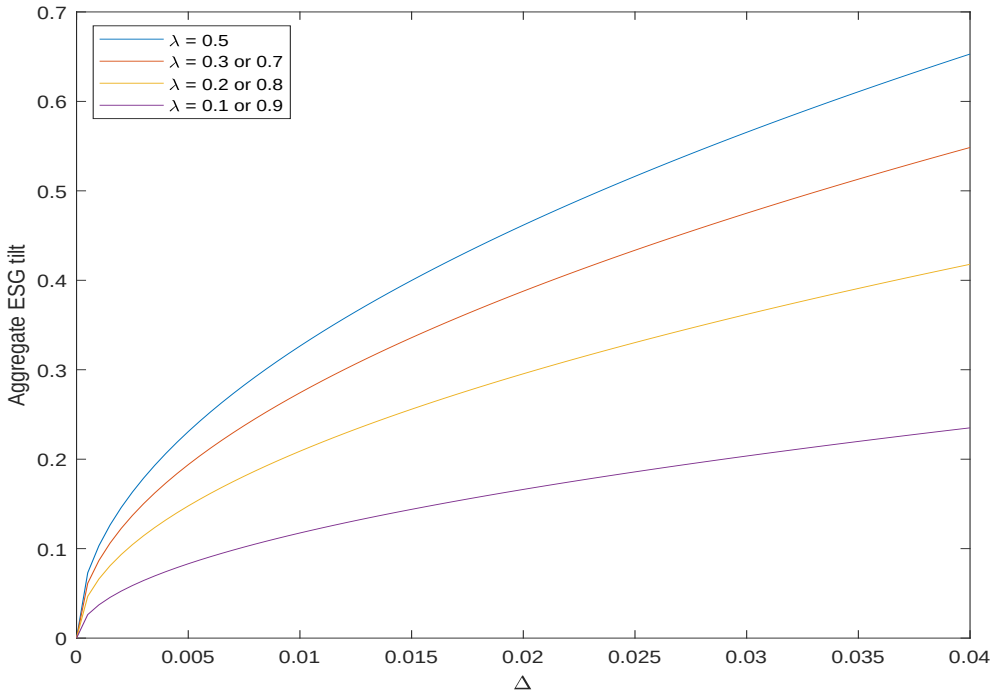


Figure 5. Size of the ESG Industry. The figure plots the aggregate dollar size of ESG investors' deviations from the market portfolio (the ESG "tilt"), expressed as a fraction of the market's total capitalization. In Panel A, results are plotted against λ, the fraction of wealth belonging to ESG investors, and for different values of Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio. In Panel B, results are plotted against Δ and for different values of λ.

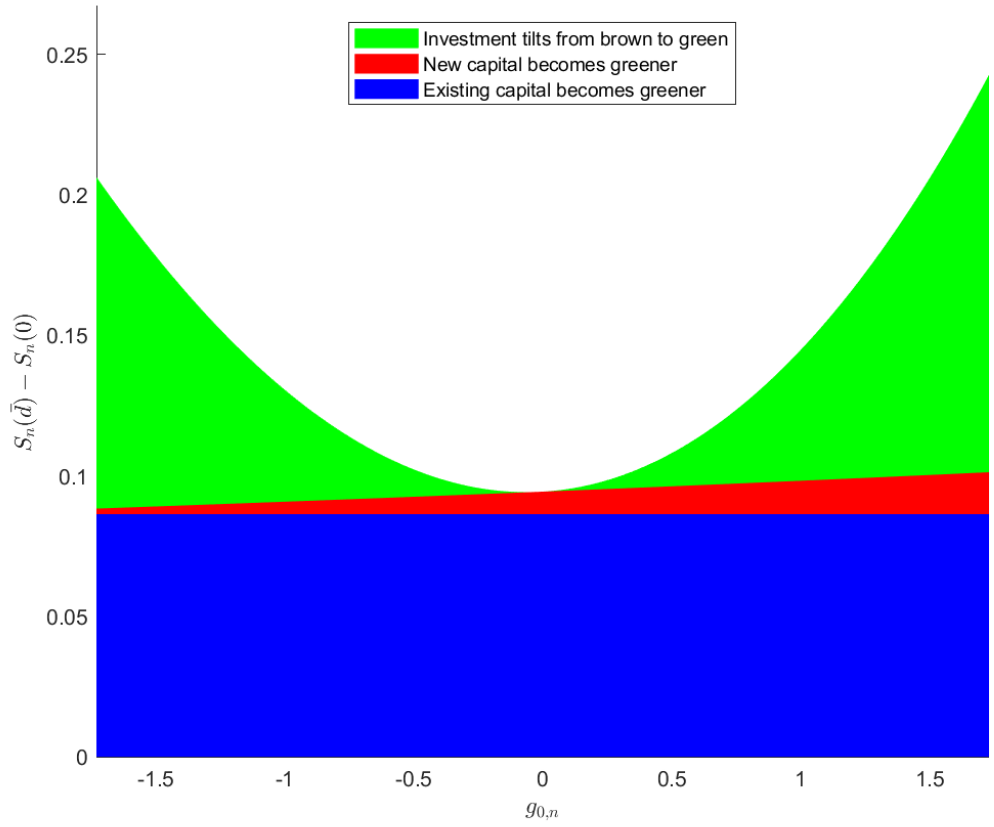


Figure 6. Firm-Level Social Impact. This figure plots $S_n(\bar{d}) - S_n(0)$, the social impact induced by ESG-motivated investors, for different firms n. The horizontal axis indicates the firm's initial ESG characteristic, $g_{n,0}$. The three regions indicate the components of $S_n(\bar{d}) - S_n(0)$ from equation (53). This figure uses the same parameters as the previous figures, with $\lambda = 0.5$ and $\Delta = 0.02$, as well as $r_f = 0.02$, $K_{0,n} = 1$, $A_n = 1.2$, $\omega_n = 0.5$, and $\kappa_n = 1$. These parameter values produce $\Delta g_n(\bar{d}) = 0.0864$, $\Delta K_n(0) = 0.0909$, and $\Delta K_n(\bar{d})$ ranging from 0.0228 to 0.1726.

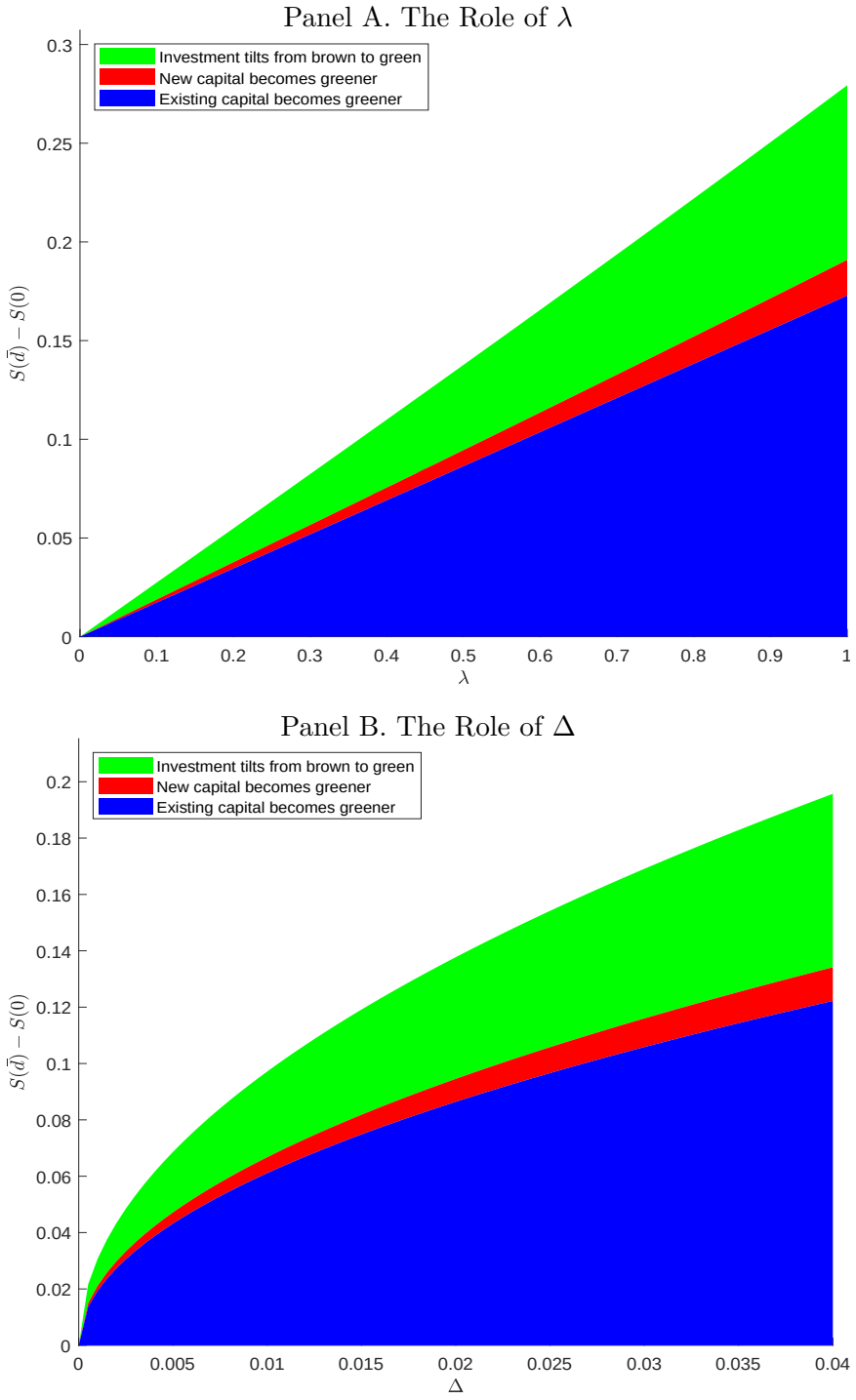


Figure 7. Aggregate Social Impact. The figure plots $S(\bar{d}) - S(0)$, the aggregate social impact induced by ESG-motivated investors. We assume the firms' initial ESG characteristics $g_{0,n}$ are uniformly distributed in $[-\sqrt{3}, \sqrt{3}]$. (These endpoints maintain $g_0'\iota = 0$ and $g_0'g_0 = 1$.) The three shaded regions indicate the components of $S_n(\bar{d}) - S_n(0)$ from equation (53), averaged across firms. In Panel A, results are plotted against λ, the fraction of wealth belonging to ESG investors, assuming $\Delta = 0.02$. In Panel B, results are plotted against Δ, the maximum certain return an ESG investor would sacrifice to invest in the ESG portfolio instead of the market portfolio, assuming $\lambda = 0.5$. All remaining parameter values are the same as in Figure 6.

REFERENCES

Albuquerque, Rui, Yrjo Koskinen, and Chendi Zhang, 2019, Corporate social responsibility and firm risk: Theory and empirical evidence, *Management Science* 65, 4451–4469.

Baker, Malcolm, Daniel Bergstresser, George Serafeim, and Jeffrey Wurgler, 2018, Financing the response to climate change: The pricing and ownership of U.S. green bonds, Working paper.

Baker, Malcolm, and Jeffrey Wurgler, 2012, Behavioral corporate finance: An updated survey. In George Constantinides, Milton Harris, and René Stulz, eds., *Handbook of the Economics of Finance*, vol. 2. North-Holland: Elsevier.

Bansal, Ravi, Marcelo Ochoa, and Dana Kiku, 2016, Climate change and growth risks, Working paper.

Bansal, Ravi, Di A. Wu, and Amir Yaron, 2018, Is socially responsible investing a luxury good? Working paper.

Barber, Brad M., Adair Morse, and Ayako Yasuda, 2018, Impact investing, Working paper.

Bauer, Rob, Kees Koedijk, and Roger Otten, 2005, International evidence on ethical mutual fund performance and investment style, *Journal of Banking and Finance* 29, 1751–1767.

Bialkowski, Jedrzej, and Laura T. Starks, 2016, SRI funds: Investor demand, exogenous shocks and ESG profiles, Working paper.

van Binsbergen, Jules H., and Christian C. Opp, 2019, Real anomalies, *Journal of Finance* 74, 1659–1706.

Bollen, Nicolas PB, 2007, Mutual fund attributes and investor behavior, *Journal of Financial and Quantitative Analysis* 42, 683–708.

Bolton, Patrick, and Marcin T. Kacperczyk, 2019, Do investors care about carbon risk?, Working paper.

Chava, Sudheer, 2014, Environmental externalities and cost of capital, *Management Science* 60, 2223–2247.

Cheng, Ing-Haw, Harrison G. Hong, and Kelly Shue, 2016, Do managers do good with other peoples' money?, Working paper.

Cremers, Martijn, and Antti Petajisto, 2009, How active is your fund manager? A new measure that predicts performance, *Review of Financial Studies* 22, 3329–3365.

Dimson, Elroy, Oguzhan Karakas, and Xi Li, 2015, Active ownership, *Review of Financial Studies* 28, 3225–3268.

Dyck, Alexander, Karl V. Lins, Lukas Roth, and Hannes F. Wagner, 2019, Do institutional investors drive corporate social responsibility? International evidence, *Journal of Financial Economics* 131, 693–714.

Edmans, Alex, 2011, Does the stock market fully value intangibles? Employee satisfaction and equity prices, *Journal of Financial Economics* 101, 621–640.

El Ghoul, Sadok, Omrane Guedhami, Chuck CY Kwok, and Dev R. Mishra, 2011, Does corporate social responsibility affect the cost of capital?, *Journal of Banking and Finance* 35, 2388–2406.

Engle, Robert F., Stefano Giglio, Bryan T. Kelly, Heebum Lee, and Johannes Stroebel, 2019, Hedging climate change news, *Review of Financial Studies*, forthcoming.

Fama, Eugene F., and Kenneth R. French, 2007, Disagreement, tastes, and asset prices, *Journal of Financial Economics* 83, 667–689.

Friedman, Henry L., and Mirko S. Heinle, 2016, Taste, information, and asset prices: Implications for the valuation of CSR, *Review of Accounting Studies* 21, 740–767.

Geczy, Christopher, Robert F. Stambaugh, and David Levin, 2005, Investing in socially responsible mutual funds, Working paper.

Gollier, Christian, and Sebastien Pouget, 2014, The "Washing Machine": Investment Strategies and Corporate Behavior with Socially Responsible Investors, Working paper.

Gompers, Paul, Joy Ishii, and Andrew Metrick, 2003, Corporate governance and equity prices, *Quarterly Journal of Economics* 118, 107–156.

Hart, Oliver, and Luigi Zingales, 2017, Companies should maximize shareholder welfare not market value, *Journal of Law, Finance, and Accounting*, 247-274.

Hartzmark, Samuel M., and Abigail B. Sussman, 2019, Do investors value sustainability? A natural experiment examining ranking and fund flows, *Journal of Finance*, forthcoming.

Heinkel, Robert, Alan Kraus, and Josef Zechner, 2001, The effect of green investment on corporate behavior, *Journal of Financial and Quantitative Analysis* 36, 431–449.

Hoepner, Andreas GF, Ioannis Oikonomou, Zacharias Sautner, Laura T. Starks, and Xiaoyan Zhou, 2018, ESG shareholder engagement and downside risk, Working paper.

Hong, Harrison, and Marcin Kacperczyk, 2009, The price of sin: The effects of social norms on markets, *Journal of Financial Economics* 93, 15–36.

Hong, Harrison, and Leonard Kostovetsky, 2012, Red and blue investing: Values and finance, *Journal of Financial Economics* 103, 1–19.

Hong, Harrison, Frank Weikai Li, and Jiangmin Xu, 2019, Climate risks and market efficiency, *Journal of Econometrics* 208, 265-281.

Ilhan, Emirhan, Zacharias Sautner, and Grigory Vilkov, 2019, Carbon tail risk, Working paper.

Kempf, Alexander, and Peer Osthoff, 2007, The effect of socially responsible investing on portfolio performance, *European Financial Management* 13, 908–922.

Krueger, Philipp, Zacharias Sautner, and Laura T. Starks, 2019, The importance of climate risks for institutional investors, *Review of Financial Studies*, forthcoming.

Luo, Arthur R., and Ronald J. Balvers, 2017, Social screens and systematic investor boycott risk, *Journal of Financial and Quantitative Analysis* 52, 365-399.

Oehmke, Martin, and Marcus M. Opp, 2020, A theory of socially responsible investment, Working paper.

Pedersen, Lasse Heje, Shaun Fitzgibbons, and Lukasz Pomorski, 2019, Responsible investing: The ESG-efficient frontier, Working paper.

Renneboog, Luc, Jenke Ter Horst, and Chendi Zhang, 2008, Socially responsible investments: Institutional aspects, performance, and investor behavior, *Journal of Banking and Finance* 32, 1723–1742.

Renneboog, Luc, Jenke Ter Horst, and Chendi Zhang, 2011, Is ethical money financially smart? Nonfinancial attributes and money flows of socially responsible investment funds, *Journal of Financial Intermediation* 20, 562–588.

Riedl, Arno, and Paul Smeets, 2017, Why do investors hold socially responsible mutual funds, *Journal of Finance* 72, 2505–2550.

Zerbib, Olivier David, 2019, The effect of pro-environmental preferences on bond prices: Evidence from green bonds, *Journal of Banking and Finance* 98, 39–60.

Appendix. Proofs and Derivations

Derivation of Equation (4):

To compute agent i's expected utility, we rely on equation (2), the relation $\tilde{W}_{1i} = W_{0i}(1+r_f + X_i'\tilde{r})$, and the fact that $\tilde{r}$ is normally distributed, $\tilde{r} \sim N(\mu, \Sigma)$:

$$\begin{aligned}
\mathrm{E}\left\{V(\tilde{W}_{1i}, X_i)\right\} &= \mathrm{E}\left\{-e^{-A_i\tilde{W}_{1i} - b_i'X_i}\right\} \\
&= \mathrm{E}\left\{-e^{-A_i[W_{0i}(1+r_f+X_i'\tilde{r})] - b_i'X_i}\right\} \\
&= -e^{-a_i(1+r_f)}\mathrm{E}\left\{e^{-a_iX_i'[\tilde{r}+\frac{1}{a_i}b_i]}\right\} \\
&= -e^{-a_i(1+r_f)}e^{-a_iX_i'[E(\tilde{r})+\frac{1}{a_i}b_i]+\frac{1}{2}a_i^2X_i'Var(\tilde{r})X_i} \\
&= -e^{-a_i(1+r_f)}e^{-a_iX_i'[\mu+\frac{1}{a_i}b_i]+\frac{1}{2}a_i^2X_i'\Sigma X_i} \qquad \text{(A1)}
\end{aligned}$$

where $a_i \equiv A_iW_{0i}$ is agent i's relative risk aversion. Agents take μ and Σ as given. Differentiating with respect to X_i, we obtain the first-order condition

$$-a_i[\mu + \frac{1}{a_i}b_i] + \frac{1}{2}a_i^2(2\Sigma X_i) = 0\ , \qquad \text{(A2)}$$

from which we obtain agent i's portfolio weights

$$X_i = \frac{1}{a_i}\Sigma^{-1}\left(\mu + \frac{1}{a_i}b_i\right). \qquad \text{(A3)}$$

Derivation of Equation (5):

The nth element of agent i's portfolio weight vector, X_i, is given by

$$X_{i,n} = \frac{W_{0i,n}}{W_{0i}} \qquad \text{(A4)}$$

where $W_{0i,n}$ is the dollar amount invested by agent i in stock n. Let $W_{0,n} \equiv \int_i W_{0i,n}di$ denote the total amount invested in stock n by all agents. Then the nth element of the market-weight vector, x, is given by

$$x_n = \frac{W_{0,n}}{W_0} = \frac{1}{W_0}\int_i W_{0i,n}di = \frac{1}{W_0}\int_i W_{0i}X_{i,n}di = \int_i \frac{W_{0i}}{W_0}X_{i,n}di = \int_i w_iX_{i,n}di \qquad \text{(A5)}$$

Note that $\sum_{n=1}^N x_n = 1$ because $\sum_{n=1}^N W_{0,n} = W_0$, which follows from the riskless asset being in zero net supply. Plugging in for X_i from equation (A3) and imposing $a_i = a$, we have

$$\begin{aligned}
x &= \int_i w_iX_idi \\
&= \int_i w_i\left[\frac{1}{a}\Sigma^{-1}\left(\mu + \frac{1}{a}b_i\right)\right]]di \\
&= \frac{1}{a}\Sigma^{-1}\mu\left(\int_i w_idi\right) + \frac{1}{a^2}\Sigma^{-1}g\left(\int_i w_id_idi\right) \\
&= \frac{1}{a}\Sigma^{-1}\mu + \frac{\bar{d}}{a^2}\Sigma^{-1}g\ . \qquad \text{(A6)}
\end{aligned}$$

Derivation of Equation (11):

Agent i's expected excess return is given by $\mathrm{E}(\tilde{r}_i) = X_i'\mu$. We take μ from equation (9) and express X_i in terms of x by subtracting equation (5) from equation (4). Recalling the assumption $x'g = 0$ from equation (8), we obtain agent i's expected excess return as

$$\begin{aligned}
\mathrm{E}(\tilde{r}_i) &= X_i'\mu \\
&= \left[x' + \frac{\delta_i}{a^2} g'\Sigma^{-1}\right]\left[\mu_M\beta - \frac{\bar{d}}{a} g\right] \\
&= \left[x' + \frac{\delta_i}{a^2} g'\Sigma^{-1}\right]\left[\frac{\mu_M}{\sigma_M^2}\Sigma x - \frac{\bar{d}}{a} g\right] \\
&= \mu_M - \frac{\bar{d}}{a} x'g + \frac{\delta_i\mu_M}{a^2\sigma_M^2} g'x - \frac{\delta_i\bar{d}}{a^3} g'\Sigma^{-1}g \\
&= \mu_M - \frac{\delta_i\bar{d}}{a^3} g'\Sigma^{-1}g. \qquad \text{(A7)}
\end{aligned}$$

Derivation of Equation (12):

Recall that agent i's excess portfolio return is $\tilde{r}_i = X_i'\tilde{r}$, where $\tilde{r} \sim N(\mu, \Sigma)$. Therefore,

$$\begin{aligned}
\mathrm{Var}(\tilde{r}_i) &= X_i'\,\Sigma\, X_i \\
&= \left[x' + \frac{\delta_i}{a^2} g'\Sigma^{-1}\right]\Sigma\left[x + \frac{\delta_i}{a^2}\Sigma^{-1}g\right] \\
&= x'\Sigma x + \frac{\delta_i}{a^2} g'\Sigma^{-1}\Sigma x + x'\Sigma\frac{\delta_i}{a^2}\Sigma^{-1}g + \frac{\delta_i^2}{a^4} g'\Sigma^{-1}\Sigma\Sigma^{-1}g \\
&= x'\Sigma x + \frac{\delta_i}{a^2} g'x + \frac{\delta_i}{a^2} x'g + \frac{\delta_i^2}{a^4} g'\Sigma^{-1}g\ .
\end{aligned}$$

Recognizing that $x'\Sigma x = \sigma_M^2$ and $x'g = 0$, we have

$$\mathrm{Var}(\tilde{r}_i) = \sigma_M^2 + \frac{\delta_i^2}{a^4} g'\Sigma^{-1}g\ ,$$

which is equation (12). We see that $\mathrm{Var}(\tilde{r}_i) > \sigma_M^2$ as long as $\delta_i \neq 0$.

Derivation of Equation (13):

The second exponent in agent i's expected utility in equation (A1) contains the terms $-aX_i'\mu$, $-X_i'b_i$, and $(a^2/2)X_i'\Sigma X_i$. The first of these is simply minus a times the expression in equation (A7). The second is given by

$$\begin{aligned}
-X_i'b_i &= -\left[x' + \frac{\delta_i}{a^2} g'\Sigma^{-1}\right][d_i g] \\
&= -\frac{d_i\delta_i}{a^2} g'\Sigma^{-1}g, \qquad \text{(A8)}
\end{aligned}$$

and the third is given by

$$\begin{aligned}\frac{a^2}{2}X_i'\Sigma X_i &= \frac{a^2}{2}\left[x' + \frac{\delta_i}{a^2}g'\Sigma^{-1}\right]\Sigma\left[x + \frac{\delta_i}{a^2}\Sigma^{-1}g\right] \\ &= \frac{a^2}{2}\sigma_M^2 + \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g, \end{aligned} \tag{A9}$$

recalling $x'g = 0$ in both cases. Adding the three terms then gives

$$\begin{aligned}-aX_i'\mu - X_i'b_i + (a^2/2)X_i'\Sigma X_i &= -a\mu_M + \frac{\delta_i\bar{d}}{a^2}g'\Sigma^{-1}g - \frac{d_i\delta_i}{a^2}g'\Sigma^{-1}g + \frac{a^2}{2}\sigma_M^2 + \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g \\ &= -a\mu_M + \frac{a^2}{2}\sigma_M^2 + \frac{1}{a^2}\left(\delta_i\bar{d} - d_i\delta_i + \frac{1}{2}\delta_i^2\right)g'\Sigma^{-1}g \\ &= -a\left(\mu_M - \frac{a}{2}\sigma_M^2\right) - \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g. \end{aligned} \tag{A10}$$

Substituting this exponent into equation (A1) gives

$$\begin{aligned}\mathrm{E}\left\{V(\tilde{W}_{1i}, X_i)\right\} &= -e^{-a(1+r_f)}e^{-a\left(\mu_M - \frac{a}{2}\sigma_M^2\right) - \frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g} \\ &= \left[-e^{-a(1+r_f)}e^{-a\left(\mu_M - \frac{a}{2}\sigma_M^2\right)}\right]e^{-\frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g} \\ &= \bar{V}e^{-\frac{\delta_i^2}{2a^2}g'\Sigma^{-1}g}, \end{aligned} \tag{A11}$$

noting that the bracketed term is, $\bar{V}$, the agent's expected utility if $\delta_i = 0$.

Derivation of Equation (22):

To implement the approach for calibrating d, we first note that under the assumptions given, the vector of the ESG investor's portfolio weights in equation (18) becomes

$$\begin{aligned}X_{esg} &= \frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2}\left[\sigma^2\iota\iota' + \eta^2 I_N\right]^{-1}g \\ &= \frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2}\left[\frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2 + N}\iota\iota'\right)\right]g \\ &= \frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2\eta^2}g, \end{aligned} \tag{A12}$$

and the variance of the ESG investor's portfolio return, for large N, is

$$\begin{aligned}X_{esg}'\Sigma X_{esg} &= \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[\sigma^2\iota\iota' + \eta^2 I_N\right]\left[\frac{1}{N}\iota + (1-\lambda)\frac{d}{a^2\eta^2}g\right] \\ &= \sigma^2 + (1-\lambda)^2\frac{d^2}{a^4\eta^2}. \end{aligned} \tag{A13}$$

With expected utility as given by equation (A1), an ESG investor's certainty equivalent excess return from holding the ESG portfolio is then, for large N,

$$r_{esg}^* = X_{esg}'(\mu + \frac{d}{a}g) - \frac{a}{2}X_{esg}'\Sigma X_{esg}$$

$$
\begin{aligned}
&= X'_{esg}(\mu_M\beta - \frac{\lambda d}{a}g + \frac{d}{a}g) - \frac{a}{2}X'_{esg}\Sigma X_{esg} \\
&= \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[a\sigma_M^2\iota - \lambda\frac{d}{a}g + \frac{d}{a}g\right] - \frac{a}{2}\left[\sigma_M^2 + (1-\lambda)^2\frac{d^2}{a^4\eta^2}\right] \\
&= \frac{1}{2}\left[a\sigma_M^2 + \frac{(1-\lambda)^2d^2}{a^3\eta^2}\right]. \qquad \text{(A14)}
\end{aligned}
$$

If the ESG investor is instead constrained to hold the market portfolio, the resulting certainty equivalent excess return is given by

$$
\begin{aligned}
r_M^* &= x'\mu - \frac{a}{2}x'\Sigma x \\
&= \mu_M - \frac{a}{2}\sigma_M^2 \\
&= \frac{a\sigma_M^2}{2}. \qquad \text{(A15)}
\end{aligned}
$$

The ESG investor's certainty-equivalent gain from investing as desired, versus investing in the market, is therefore

$$
r_{esg}^* - r_M^* = \frac{(1-\lambda)^2d^2}{2a^3\eta^2}. \qquad \text{(A16)}
$$

when N is large. As noted, this difference in certainty equivalents is largest when $\lambda = 0$. Solving for d with $\Delta \equiv r_{esg}^* - r_M^*$ then gives equation (22).

Derivation of the Certainty Equivalent Excess Return of a Non-ESG Investor:

The non-ESG investor's portfolio weights in equation (19) are

$$
\begin{aligned}
X_{non} &= \frac{1}{N}\iota - \lambda\frac{d}{a^2}\left[\sigma^2\iota\iota' + \eta^2 I_N\right]^{-1}g \\
&= \frac{1}{N}\iota - \lambda\frac{d}{a^2}\left[\frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2 + N}\iota\iota'\right)\right]g \\
&= \frac{1}{N}\iota - \lambda\frac{d}{a^2\eta^2}g, \qquad \text{(A17)}
\end{aligned}
$$

and the variance of the non-ESG investor's portfolio return, for large N, is

$$
\begin{aligned}
X'_{non}\Sigma X_{non} &= \left[\frac{1}{N}\iota' - \lambda\frac{d}{a^2\eta^2}g'\right]\left[\sigma^2\iota\iota' + \eta^2 I_N\right]\left[\frac{1}{N}\iota - \lambda\frac{d}{a^2\eta^2}g\right] \\
&= \sigma^2 + \lambda^2\frac{d^2}{a^4\eta^2}. \qquad \text{(A18)}
\end{aligned}
$$

A non-ESG investor's certainty equivalent excess return from holding the non-ESG portfolio is then, for large N,

$$
\begin{aligned}
r_{non}^* &= X'_{non}\mu - \frac{a}{2}X'_{non}\Sigma X_{non} \\
&= X'_{non}(\mu_M\beta - \frac{\lambda d}{a}g) - \frac{a}{2}X'_{non}\Sigma X_{non}
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\frac{1}{N}\iota' - \lambda\frac{d}{a^2\eta^2}g'\right]\left[a\sigma_M^2\iota - \lambda\frac{d}{a}g\right] - \frac{a}{2}\left[\sigma_M^2 + \lambda^2\frac{d^2}{a^4\eta^2}\right] \\
&= \frac{1}{2}\left[a\sigma_M^2 + \frac{\lambda^2 d^2}{a^3\eta^2}\right]. \qquad \text{(A19)}
\end{aligned}
$$

Derivation of Equation (23):

The correlation between the ESG investor's return and the market return is equal to

$$
\begin{aligned}
\rho\left(\tilde{r}_{esg}, \tilde{r}_M\right) &= \frac{\mathrm{Cov}\left(X'_{esg}\tilde{\epsilon}, x'\tilde{\epsilon}\right)}{\sqrt{\mathrm{Var}\left(X'_{esg}\tilde{\epsilon}\right)}\sqrt{\mathrm{Var}\left(x'\tilde{\epsilon}\right)}} \\
&= \frac{X'_{esg}\Sigma x}{\sqrt{X'_{esg}\Sigma X_{esg}}\sqrt{x'\Sigma x}}. \qquad \text{(A20)}
\end{aligned}
$$

From equations (15) and (A12), recalling that $x = (1/N)\iota$ and $\iota' g = 0$, we obtain

$$
X'_{esg}\Sigma x = \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[\sigma^2\iota\iota' + \eta^2 I_N\right]\left[\frac{1}{N}\iota\right] = \sigma^2, \qquad \text{(A21)}
$$

for large N. Substituting from equations (A13) and (A21) into equation (A20), recalling $x'\Sigma x = \sigma^2$ and equation (22), gives equation (23).

Derivation of Equation (24):

Applying equation (20) gives

$$
\begin{aligned}
\mathrm{E}\{\tilde{r}_{esg}\} - \mathrm{E}\{\tilde{r}_{non}\} &= -\frac{\lambda d^2}{a^3}g'\Sigma^{-1}g \\
&= -\frac{\lambda d^2}{a^3}g'\left[\sigma^2\iota\iota' + \eta^2 I_N\right]^{-1}g \\
&= -\frac{\lambda d^2}{a^3}g'\left[\frac{1}{\eta^2}\left(I_N - \frac{1}{\eta^2/\sigma^2}\iota\iota'\right)\right]g \\
&= -\frac{\lambda d^2}{a^3\eta^2}. \qquad \text{(A22)}
\end{aligned}
$$

Plugging in for d^2 from equation (22), we obtain equation (24).

Derivations of Equations (25) and (26):

Let α denote the $N \times 1$ vector of alphas given by equation (10). Taking X_{esg} from equation (A12), the alpha of the ESG investor is given by

$$
\begin{aligned}
\alpha_{esg} &= X'_{esg}\alpha \\
&= \left[\frac{1}{N}\iota' + (1-\lambda)\frac{d}{a^2\eta^2}g'\right]\left[-\frac{\lambda d}{a}g\right] \\
&= -\lambda(1-\lambda)\frac{d^2}{a^3\eta^2}. \qquad \text{(A23)}
\end{aligned}
$$

By using X_{non} from equation (A17), we obtain the alpha of the non-ESG investor:

$$\begin{aligned}\alpha_{non} &= X'_{non}\alpha \\ &= \left[\frac{1}{N}\iota' - \lambda\frac{d}{a^2\eta^2}g'\right]\left[-\frac{\lambda d}{a}g\right] \\ &= \lambda^2\frac{d^2}{a^3\eta^2}. \end{aligned} \tag{A24}$$

Plugging in for d^2 from equation (22), we obtain equations (25) and (26).

Derivation of Equation (30): Using equation (A12) and $x = (1/N)\iota$,

$$\begin{aligned}T &= \frac{1}{2}\lambda\iota'|X_{esg} - x| \\ &= \frac{1}{2}\lambda\iota'|(1-\lambda)\frac{d}{a^2\eta^2}g| \\ &= \frac{1}{2}\lambda(1-\lambda)\frac{d}{a^2\eta^2}\iota'|g|. \end{aligned} \tag{A25}$$

Plugging in for d from equation (22), we obtain equation (30).

Derivation of Equations (39) and (40):

From equation (37), the return covariance matrix is $\Sigma = B\text{Cov}\{\tilde{f}, \tilde{f}'\}B' + \Lambda$. The $N \times 1$ vector of market betas can therefore be written as

$$\begin{aligned}\beta &= \frac{1}{\sigma_M^2}\Sigma x = \frac{1}{\sigma_M^2}\left\{B\text{Cov}\{\tilde{f}, \tilde{f}'\}B' + \Lambda\right\}x = \frac{1}{\sigma_M^2}\left\{B\text{Cov}\{\tilde{f}, \tilde{f}'B'\} + \Lambda\right\}x \\ &= \frac{1}{\sigma_M^2}\left\{B\text{Cov}\{\tilde{f}, \tilde{\epsilon}'\} + \Lambda\right\}x = \frac{1}{\sigma_M^2}\left\{B\text{Cov}\{\tilde{f}, \tilde{\epsilon}'x\} + \Lambda x\right\} = \frac{1}{\sigma_M^2}\left\{B\text{Cov}\{\tilde{f}, \tilde{\epsilon}_M\} + \Lambda x\right\} \\ &= \frac{1}{\sigma_M^2}\left\{B\text{Cov}\{\begin{pmatrix}\tilde{z}_h \\ \tilde{f}_g\end{pmatrix}, \tilde{\epsilon}_M\} + \Lambda x\right\} = \frac{1}{\sigma_M^2}\left\{[h\ g]\begin{pmatrix}\text{Cov}\{\tilde{z}_h, \tilde{\epsilon}_M\} \\ \text{Cov}\{\tilde{f}_g, \tilde{\epsilon}_M\}\end{pmatrix} + \Lambda x\right\} \\ &= h\frac{\text{Cov}\{\tilde{z}_h, \tilde{\epsilon}_M\}}{\sigma_M^2} + g\frac{\text{Cov}\{\tilde{f}_g, \tilde{\epsilon}_M\}}{\sigma_M^2} + \frac{1}{\sigma_M^2}\Lambda x = h\beta_h + g\beta_g + \frac{1}{\sigma_M^2}\Lambda x\ , \end{aligned} \tag{A26}$$

which is equation (39). Substituting from equation (36) into $\tilde{\epsilon}_M \equiv x'\tilde{\epsilon}$ yields

$$\tilde{\epsilon}_M = (x'h)\tilde{z}_h + (x'g)\tilde{f}_g + x'\tilde{\zeta}\ , \tag{A27}$$

which immediately implies equation (40).

Derivation of Equations (41) and (42):

Combining equations (9) and (36) gives

$$\begin{aligned}\tilde{r} &= \mu + \tilde{\epsilon} \\ &= \beta\mu_M - g\frac{\bar{d}}{a} + h\tilde{z}_h + g\tilde{f}_g + \tilde{\zeta}. \end{aligned} \tag{A28}$$

With $x'\beta = 1$ and $x'g = 0$, premultiplying the above by x' gives the excess market return as

$$\tilde{r}_M = \mu_M + (x'h)\tilde{z}_h + x'\tilde{\zeta}, \tag{A29}$$

implying

$$\tilde{z}_h = \left(\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right)/x'h. \tag{A30}$$

Substituting into equation (A28) and then using equation (39) gives

$$\begin{aligned}
\tilde{r} &= \beta\mu_M - g\frac{\bar{d}}{a} + h\left(\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right)/x'h + g\tilde{f}_g + \tilde{\zeta} \\
&= \left(h\beta_h + g\beta_g + \frac{1}{\sigma_M^2}\Lambda x\right)\mu_M - g\frac{\bar{d}}{a} + h\left[\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right]/x'h + g\tilde{f}_g + \tilde{\zeta} \\
&= h\left(\beta_h\mu_M + \left[\tilde{r}_M - \mu_M - x'\tilde{\zeta}\right]/x'h\right) + g\left(\tilde{f}_g + \beta_g\mu_M - \frac{\bar{d}}{a}\right) + \frac{\mu_M}{\sigma_M^2}\Lambda x + \tilde{\zeta} \\
&= \theta\tilde{r}_M + g\left(\tilde{f}_g + \beta_g\mu_M - \frac{\bar{d}}{a}\right) + \tilde{\nu},
\end{aligned} \tag{A31}$$

where $\theta = (1/x'h)h$, and

$$\tilde{\nu} = h\mu_M\left(\beta_h - \frac{1}{x'h}\right) + \frac{\mu_M}{\sigma_M^2}\Lambda x - h\left(\frac{x'\tilde{\zeta}}{x'h}\right) + \tilde{\zeta}. \tag{A32}$$

Equation (A31) provides the desired relation, but it remains to show that $\mathrm{E}\{\tilde{\nu}|\tilde{r}_M, \tilde{f}_g\} \approx 0$. Recall that $\beta_h \equiv \mathrm{Cov}\{\tilde{\epsilon}_M, \tilde{z}_h\}/\sigma_M^2$. Substituting for $\tilde{\epsilon}_M$ from equation (A27) and recalling $x'g = 0$, we obtain

$$\beta_h = \frac{(x'h)var(\tilde{z}_h)}{(x'h)^2 var(\tilde{z}_h) + var(x'\tilde{\zeta})}. \tag{A33}$$

If the market is well diversified with N large, such that $x_n \approx 0$, then $var(x'\tilde{\zeta}) = x'\Lambda x \approx 0$, and thus $\beta_h \approx 1/x'h$, thereby making the first term in equation (A32) approximately zero. The second term in that equation is also approximately zero if $x_n \approx 0$, as the nth element of that vector is $\left(\mu_M \mathrm{Var}(\tilde{\zeta}_n)/\sigma_M^2\right)x_n$. The third and fourth terms in equation (A32) have zero means, so we have $\mathrm{E}\{\tilde{\nu}\} \approx 0$. Because $\mathrm{Cov}\{\tilde{\zeta}, \tilde{f}_g\} = 0$, it remains to show that $\mathrm{Cov}\{\tilde{\zeta}, \tilde{r}_M\} \approx 0$. That result follows from equation (A29), which implies that the nth element of $\mathrm{Cov}\{\tilde{\zeta}, \tilde{r}_M\}$ equals $\mathrm{Var}(\tilde{\zeta}_n)x_n$, approximately zero if $x_n \approx 0$.

Derivation of Equation (44):

Modifying the earlier derivation of equation (4), we obtain

$$\begin{aligned}
\mathrm{E}\left\{V(\tilde{W}_{1i}, X_i, \tilde{C})\right\} &= -e^{-a_i(1+r_f)}\mathrm{E}\left\{e^{-a_i X_i'[\tilde{r} + \frac{1}{a_i}b_i] - c_i\tilde{C}}\right\} \\
&= -e^{-a_i(1+r_f)}e^{-a_i X_i'[E(\tilde{r}) + \frac{1}{a_i}b_i] + \frac{1}{2}a_i^2 X_i' Var(\tilde{\epsilon})X_i + a_i c_i X_i' Cov(\tilde{\epsilon}, \tilde{C}) + \frac{1}{2}c_i^2 Var(\tilde{C})} \\
&= -e^{-a_i(1+r_f)}e^{-a_i X_i'[\mu + \frac{1}{a_i}b_i] + \frac{1}{2}a_i^2 X_i'\Sigma X_i + a_i c_i X_i'\sigma_{\epsilon C} + \frac{1}{2}c_i^2\sigma_C^2},
\end{aligned} \tag{A34}$$

where $\sigma_{\epsilon C} \equiv \text{Cov}(\tilde{\epsilon}, \tilde{C})$. Differentiating with respect to X_i gives the first-order condition

$$-a_i[\mu + \frac{1}{a_i}b_i] + a_i^2 \Sigma X_i + a_i c_i \sigma_{\epsilon C} = 0 \tag{A35}$$

from which we obtain agent i's portfolio weights

$$X_i = \frac{1}{a_i}\Sigma^{-1}\left(\mu + \frac{1}{a_i}b_i - c_i\sigma_{\epsilon C}\right). \tag{A36}$$

Again imposing the market-clearing condition and $a_i = a$ gives

$$\begin{aligned} x &= \int_i w_i X_i \, di \\ &= \frac{1}{a}\Sigma^{-1}\mu + \frac{\bar{d}}{a^2}\Sigma^{-1}g - \frac{\bar{c}}{a}\Sigma^{-1}\sigma_{\epsilon C} \ , \end{aligned} \tag{A37}$$

which implies

$$\mu = a\Sigma x - \frac{\bar{d}}{a}g + \bar{c}\sigma_{\epsilon C} \ . \tag{A38}$$

Premultiplying by x', again imposing the assumption $x'g = 0$, gives

$$\mu_M = a\sigma_M^2 + \bar{c}\sigma_{MC} \ , \tag{A39}$$

where $\sigma_{MC} \equiv \text{Cov}(\tilde{\epsilon}_M, \tilde{C}) = x'\sigma_{\epsilon C}$. Solving equation (A39) for a and substituting into the first term on the right-hand side of equation (A38) gives

$$\begin{aligned} \mu &= \frac{\mu_M - \bar{c}\sigma_{MC}}{\sigma_M^2}\Sigma x - \frac{\bar{d}}{a}g + \bar{c}\sigma_{\epsilon C} \\ &= (\mu_M - \bar{c}\sigma_{MC})\beta - \frac{\bar{d}}{a}g + \bar{c}\sigma_{\epsilon C} \\ &= \mu_M\beta - \frac{\bar{d}}{a}g + \bar{c}\left(\sigma_{\epsilon C} - \frac{\sigma_{MC}}{\sigma_M^2}\sigma_{\epsilon M}\right), \end{aligned} \tag{A40}$$

noting $\beta = (1/\sigma_M^2)\sigma_{\epsilon M} = (1/\sigma_M^2)\Sigma x$. To see that the third term on the right-hand side of equation (A40) is the same as that in equation (44), first observe that in the multivariate regression of $\tilde{\epsilon}$ on $\tilde{\epsilon}_M$ and $\tilde{C}$, the $N \times 2$ matrix of slope coefficients is given by

$$[\sigma_{\epsilon M} \ \sigma_{\epsilon C}] \begin{bmatrix} \sigma_M^2 & \sigma_{MC} \\ \sigma_{MC} & \sigma_C^2 \end{bmatrix}^{-1} = \frac{1}{\sigma_M^2\sigma_C^2 - \sigma_{MC}^2}\left[\sigma_C^2\sigma_{\epsilon M} - \sigma_{MC}\sigma_{\epsilon C} \quad \sigma_M^2\sigma_{\epsilon C} - \sigma_{MC}\sigma_{\epsilon M}\right],$$

so the second column is given by

$$\psi = \frac{\sigma_M^2\sigma_{\epsilon C} - \sigma_{MC}\sigma_{\epsilon M}}{\sigma_M^2\sigma_C^2 - \sigma_{MC}^2} \ . \tag{A41}$$

Using equation (A41), we can rewrite the third term on the right-hand side of equation (A40) as

$$\begin{aligned} \bar{c}\left(\sigma_{\epsilon C} - \frac{\sigma_{MC}}{\sigma_M^2}\sigma_{\epsilon M}\right) &= \bar{c}\frac{\sigma_M^2\sigma_C^2 - \sigma_{MC}^2}{\sigma_M^2}\psi \\ &= \bar{c}\left(1 - \rho_{MC}^2\right)\psi \ , \end{aligned} \tag{A42}$$

recalling that $\sigma_C = 1$.

Derivation of Equation (48):

Substituting for μ from equation (A40) into equation (A36) and setting $a_i = a$, we obtain

$$\begin{aligned} X_i &= \frac{1}{a}\Sigma^{-1}\left(\mu + \frac{1}{a}b_i - c_i\sigma_{\epsilon C}\right) \\ &= \frac{1}{a}\Sigma^{-1}\left(\mu_M\beta - \frac{\bar{d}}{a}g + \bar{c}\left(\sigma_{\epsilon C} - \frac{\sigma_{MC}}{\sigma_M^2}\sigma_{\epsilon M}\right) + \frac{1}{a}b_i - c_i\sigma_{\epsilon C}\right) \\ &= \frac{\mu_M}{a}\Sigma^{-1}\beta - \frac{1}{a}\Sigma^{-1}\bar{c}\frac{\sigma_{MC}}{\sigma_M^2}\sigma_{\epsilon M} + \frac{1}{a}\Sigma^{-1}\left(\frac{d_i}{a}g - \frac{\bar{d}}{a}g\right) - \frac{1}{a}\Sigma^{-1}\left(c_i - \bar{c}\right)\sigma_{\epsilon C} \\ &= \frac{\mu_M}{a}\Sigma^{-1}\beta - \frac{1}{a}\Sigma^{-1}\left(\bar{c}\sigma_{MC}\right)\frac{\sigma_{\epsilon M}}{\sigma_M^2} + \frac{1}{a}\Sigma^{-1}\frac{\delta_i}{a}g - \frac{c_i - \bar{c}}{a}\Sigma^{-1}\sigma_{\epsilon C} \end{aligned} \tag{A43}$$

Noting from equation (A39) that $\bar{c}\sigma_{MC} = \mu_M - a\sigma_M^2$, and that $\beta = \frac{\sigma_{\epsilon M}}{\sigma_M^2} = \frac{\Sigma x}{\sigma_M^2}$, we have

$$\begin{aligned} X_i &= \frac{\mu_M}{a}\Sigma^{-1}\beta - \frac{1}{a}\Sigma^{-1}\left(\mu_M - a\sigma_M^2\right)\beta + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{c_i - \bar{c}}{a}\Sigma^{-1}\sigma_{\epsilon C} \\ &= \sigma_M^2\Sigma^{-1}\beta + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{c_i - \bar{c}}{a}\Sigma^{-1}\sigma_{\epsilon C} \\ &= x + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{c_i - \bar{c}}{a}\Sigma^{-1}\sigma_{\epsilon C}\ , \end{aligned} \tag{A44}$$

which is equation (48).

Derivation of Equation (49):

This derivation follows exactly the same steps as our prior derivation of equations (41) and (42), except that instead of using μ from equation (9) we use μ from equation (44). As a result, we obtain a counterpart of equation (A31) with an extra term:

$$\tilde{r} = \theta\tilde{r}_M + g\left(\tilde{f}_g + \beta_g\mu_M - \frac{\bar{d}}{a}\right) + \bar{c}\left(1 - \rho_{MC}^2\right)\left[\psi - \theta(x'\psi)\right] + \tilde{\nu}\ , \tag{A45}$$

where θ and $\tilde{\nu}$ are the same as before. In the special case to which equation (49) pertains, $\psi = -g$. To see this, note from equation (A45) that g is the slope on $\tilde{f}_g$ from the multiple regression of stock returns on market returns and $\tilde{f}_g$, whereas ψ is defined in the context of equation (44) as the slope on $\tilde{C}$ in the multiple regression of stock returns on market returns and $\tilde{C}$. In this special case, $\tilde{f}_g = -\tilde{C}$, which implies $\psi = -g$. It then follows that $x'\psi = x'g = 0$, and equation (A45) simplifies to

$$\tilde{r} = \theta\tilde{r}_M + g\left(\tilde{f}_g + \beta_g\mu_M - \frac{\bar{d}}{a} - \bar{c}\left(1 - \rho_{MC}^2\right)\right) + \tilde{\nu}\ , \tag{A46}$$

which immediately delivers equation (49).

Derivation of Equations (51) and (52):

The firm's value at time 0 is

$$\upsilon_n = -\Delta K_n - \frac{\kappa_n}{2}(\Delta K_n)^2 + \frac{A_n\left(K_{0,n}+\Delta K_n\right)\left(1-\frac{\omega_n}{2}(\Delta g_n)^2\right)}{1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}(g_{n,0}+\Delta g_n)} \ . \tag{A47}$$

The manager maximizes υ_n by choosing Δg_n and ΔK_n. The choice of Δg_n depends only on the third term of equation (A47), and we can maximize its log. Using the approximation that $\log(1+x)\approx x$ and ignoring terms without Δg_n, the choice of Δg_n simplifies to

$$\max_{\Delta g_n} -\frac{\omega_n}{2}(\Delta g_n)^2 + \frac{\overline{d}}{a}\Delta g_n \ . \tag{A48}$$

The first-order condition delivers equation (51). Without taking logs, the first-order condition for ΔK_n is

$$-1-\kappa_n\Delta K_n + \frac{A_n\left(1-\frac{\omega_n}{2}(\Delta g_n)^2\right)}{1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}g_n} = 0 \ , \tag{A49}$$

which delivers equation (52).

Proof of Proposition 10:

The amount of social impact induced by ESG preferences is

$$\begin{aligned}
S_n\left(\overline{d}\right) - S_n\left(0\right) &= K_n\left(\overline{d}\right)g_n\left(\overline{d}\right) - K_n\left(0\right)g_n\left(0\right) && \text{(A50)}\\
&= \left(K_{n,0}+\Delta K_n\left(\overline{d}\right)\right)\left(g_{n,0}+\Delta g_n\left(\overline{d}\right)\right) - && \text{(A51)}\\
&\quad \left(K_{n,0}+\Delta K_n\left(0\right)\right)\left(g_{n,0}+\Delta g_n\left(0\right)\right)\\
&= K_{n,0}\Delta g_n\left(\overline{d}\right) + \Delta K_n\left(\overline{d}\right)\Delta g_n\left(\overline{d}\right) + && \text{(A52)}\\
&\quad g_{n,0}\left(\Delta K_n\left(\overline{d}\right)-\Delta K_n\left(0\right)\right),
\end{aligned}$$

since $\Delta g_n(0)=0$. The first term is strictly positive if $\overline{d}>0$, since $\Delta g_n\left(\overline{d}\right)=\frac{\overline{d}}{a\omega_n}>0$. The second term is strictly positive for the same reason, and also because of the assumption that A_n is high enough so that $\Delta K_n\left(\overline{d}\right)>0$. We focus on the third term next. The ESG-induced change in investment is

$$\begin{aligned}
\Delta K_n\left(\overline{d}\right) - \Delta K_n\left(0\right) &= \frac{A_n}{\kappa_n}\left[\frac{1-\frac{1}{2}\frac{\overline{d}^2}{a^2\omega_n}}{1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}\left(g_{0,n}+\frac{\overline{d}}{a\omega_n}\right)} - \frac{1}{1+r_f+\mu_M\beta_n}\right] && \text{(A53)}\\
&= \frac{A_n}{\kappa_n}\left[\frac{\left(1-\frac{1}{2}\frac{\overline{d}^2}{a^2\omega_n}\right)\left(1+r_f+\mu_M\beta_n\right)-\left(1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}\left(g_{0,n}+\frac{\overline{d}}{a\omega_n}\right)\right)}{\left(1+r_f+\mu_M\beta_n\right)\left(1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}\left(g_{0,n}+\frac{\overline{d}}{a\omega_n}\right)\right)}\right]\\
&= \frac{A_n}{\kappa_n}\left[\frac{-\frac{1}{2}\frac{\overline{d}^2}{a^2\omega_n}\left(1+r_f+\mu_M\beta_n\right)+\frac{\overline{d}}{a}\left(g_{0,n}+\frac{\overline{d}}{a\omega_n}\right)}{\left(1+r_f+\mu_M\beta_n\right)\left(1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}\left(g_{0,n}+\frac{\overline{d}}{a\omega_n}\right)\right)}\right] && \text{(A54)}
\end{aligned}$$

$$= \frac{A_n}{\kappa_n}\left[\frac{\frac{\overline{d}}{a}g_{0,n}+\frac{1}{2}\frac{\overline{d}^2}{a^2\omega_n}\left(1-r_f-\mu_M\beta_n\right)}{\left(1+r_f+\mu_M\beta_n\right)\left(1+r_f+\mu_M\beta_n-\frac{\overline{d}}{a}\left(g_{0,n}+\frac{\overline{d}}{a\omega_n}\right)\right)}\right] \tag{A55}$$

The denominator is positive for reasonable parameter values. The numerator equals zero, implying the third term in equation (A52) is zero, if $\overline{d}=0$. If $\overline{d}>0$, the numerator is strictly positive if and only if

$$\frac{\overline{d}}{a}g_{0,n}+\frac{1}{2}\frac{\overline{d}^2}{a^2\omega_n}\left(1-r_f-\mu_M\beta_n\right)>0 \tag{A56}$$

$$g_{0,n} > -\frac{1}{2}\frac{\overline{d}}{a\omega_n}\left(1-r_f-\mu_M\beta_n\right). \tag{A57}$$

Therefore, $\Delta K_n\left(\overline{d}\right)-\Delta K_n\left(0\right)$ has the same sign as $g_{0,n}$, implying $g_{0,n}\left(\Delta K_n\left(\overline{d}\right)-\Delta K_n\left(0\right)\right)>0$, if $g_{0,n}>0$ or $g_{0,n}<-\frac{1}{2}\frac{\overline{d}}{a\omega_n}\left(1-r_f-\mu_M\beta_n\right)<0$. This result establishes that all three terms in equation (A52) are strictly positive, implying $S_n\left(\overline{d}\right)-S_n\left(0\right)>0$, for firms with $g_{n,0}>0$ or $g_{0,n}<-\frac{1}{2}\frac{\overline{d}}{a\omega_n}\left(1-r_f-\mu_M\beta_n\right)<0$, if $\overline{d}>0$.

Next, we show that $S_n\left(\overline{d}\right)-S_n\left(0\right)>0$ for the remaining firms, meaning those with $g_{n,0}\in\left[-\frac{1}{2}\frac{\overline{d}}{a\omega_n}\left(1-r_f-\mu_M\beta_n\right),\ 0\right]$. Despite these firms starting with $g_{n,0}\leq 0$, their chosen $g_n\left(\overline{d}\right)\geq 0$, because

$$\begin{aligned} g_n\left(\overline{d}\right) &= g_{n,0}+\Delta g_n\left(\overline{d}\right)\approx g_{n,0}+\frac{\overline{d}}{a\omega_n}\geq -\frac{1}{2}\frac{\overline{d}}{a\omega_n}\left(1-r_f-\mu_M\beta_n\right)+\frac{\overline{d}}{a\omega_n} \\ &= \frac{1}{2}\frac{\overline{d}}{a\omega_n}\left(1+r_f+\mu_M\beta_n\right)\geq 0. \end{aligned} \tag{A58}$$

In other words, these firms switch from brown (or ESG-neutral) to green (or ESG-neutral). The first term in equation (A52) is still strictly positive for these firms if $\overline{d}>0$. We can rewrite the last two terms of equation (A52) as

$$\Delta K_n\left(\overline{d}\right)\Delta g_n\left(\overline{d}\right)+g_{n,0}\left(\Delta K_n\left(\overline{d}\right)-\Delta K_n\left(0\right)\right)=\Delta K_n\left(\overline{d}\right)g_n\left(\overline{d}\right)-g_{n,0}\Delta K_n\left(0\right). \tag{A59}$$

The first term on the right-hand side is greater or equal to zero for these firms, because we showed that their $g_n\left(\overline{d}\right)\geq 0$, and we assumed A_n is large enough so that $\Delta K_n\left(\overline{d}\right)>0$. The second term is less than or equal to zero, because these firms have $g_{n,0}\leq 0$, and we assume A_n is large enough so that $\Delta K_n\left(0\right)>0$. Since the second term is preceded by a minus sign, the quantity in equation (A59) is greater than or equal to zero. Since the first term in equation (A52) is strictly positive even for these firms, it follows that $S_n\left(\overline{d}\right)-S_n\left(0\right)>0$ even for this set of firms.