# The Risks of Safe Assets

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US government bonds exhibit characteristics often attributed to safe assets. A long literature documents significant convenience yields in scarce US Treasuries, suggesting that rising Treasury supply and government debt comes with a declining liquidity premium and a fall in firms' relative cost of debt financing. We empirically document and theoretically identify a dual role for government debt and novel fiscal cost. Through a *liquidity channel* an increase in government debt improves liquidity and lowers liquidity premia by facilitating debt rollover, thereby reducing credit spreads. Through an *uncertainty channel*, however, rising government debt creates policy uncertainty, raising default risk premia. We interpret and quantitatively evaluate these two channels through the lens of a general equilibrium asset pricing model with risk-sensitive agents subject to liquidity shocks, in which firms issue defaultable bonds and the government issues tax-financed bonds that endogenously enjoy liquidity benefits. The calibrated model generates quantitatively realistic liquidity spreads and default risk premia, and suggests that rising government debt reduces liquidity premia, but crowds out corporate debt financing and investment, and creates endogenous tax volatility, reflected in higher credit spreads, risk premia, and consumption volatility. Therefore, increasing safe asset supply can be risky, and come at a significant fiscal cost.

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# 1 Introduction

US government bonds exhibit many characteristics often attributed to safe assets: They are very liquid and lenders readily accept them as collateral. Indeed, as pointed out in the recent literature, for example, by Longstaff (2003), Bansal, Coleman, and Lundblad (2012), Krishnamurthy and Vissing-Jorgensen (2012), or Nagel (2016), Treasuries exhibit many money-like features suggesting that investors attach a 'liquidity premium' or 'convenience yield' to holding these assets. Arguably, therefore, by issuing Treasuries and raising debt, the US government can provide liquidity services to investors and facilitate transactions in the economy. Such an increase in the supply of safe assets comes with a declining liquidity premium and thus a fall in firms' relative cost of debt financing, providing further stimulus to the economy. These ideas have risen to further prominence around the recent financial crisis, which also manifested itself as a dramatic liquidity crisis in financial markets. Indeed, in its aftermath, economists argued that safety attributes of Treasuries lower interest rates below expected growth rates, so that the fiscal costs of debt servicing become negligible. For example, Blanchard (2019) forcefully argues 'that the current U.S. situation in which safe interest rates are expected to remain below growth rates for a long time, is more the historical norm than the exception', so that, 'put bluntly, public debt may have no fiscal cost'.

In this paper, we empirically document and theoretically evaluate a dual role of government debt for credit markets. Indeed, through a *liquidity channel* an increase in government debt improves liquidity and lowers liquidity premia by facilitating transaction services and debt rollover, thereby reducing credit spreads. Through an *uncertainty channel*, however, rising government debt creates future tax commitments and policy uncertainty regarding the government's intertemporal budget balance, that raises credit spreads, default risk premia, and expected corporate bond excess returns and eventually leads to rising relative costs of firms' debt financing. Under such a dual view, we identify a novel fiscal cost through a fiscal risk channel associated with rising US government debt. Ultimately therefore, increasing safe asset supply can be risky.

Our analysis starts from the empirical observation that the government debt to GDP ratio has a dual role in predicting the costs of debt financing. While, indeed, it exhibits significantly negative predictive power for money market instruments, confirming its negative impact on liquidity premia, we provide novel empirical evidence that it significantly positively predicts future corporate bond credit spreads and excess returns. More formally, we also present econometric evidence from estimating a VAR that suggests that liquidity premia and credit risk premia respond to innovations

to the debt to GDP ratio with opposite sign. Similarly, in terms of quantities, we find that corporate debt issuance declines significantly with unexpected surges in government debt. Taken together, this evidence suggests that a rising debt to GDP ratio is correlated with future risks in the economy reflected in rising risk premia[1].

To interpret and quantify the dual role of government debt for credit markets, and to understand the sources and effects of fiscal risks, we introduce a novel general equilibrium asset pricing model that can rationalize our evidence. In the model, risk-sensitive agents with Epstein-Zin preferences invest in government bonds, corporate bonds as well as stocks to smooth their consumption. The government finances debt by levying taxes on wages and corporate income, while corporations issue defaultable bonds to finance investment according to their advantageous tax treatment, in line with the US tax code. Households are subject to sporadic liquidity shocks that create funding needs. While households can always liquidate their asset holdings to cover their funding needs, they can also trade their asset positions subject to transaction costs in the market place. In our model, different asset classes endogenously provide differential liquidity benefits to investors across time and states reflected in endogenously time-varying liquidity premia, liquidity risk premia and trading volumes.

Increasing the supply of government bonds facilitates covering liquidity needs in the market place and thus endogenously leads to a decline in liquidity premia on safe assets in the model, in line with the empirical evidence. However, issuing debt also raises the government's future funding needs through the government budget constraint. In particular, critically, higher debt does not only lead to higher average future tax obligations going forward, but it also renders them more *volatile*. Intuitively, the present value of future tax commitments becomes more sensitive to shocks as government debt grows[2]. A rising supply of safe assets, therefore, does not only facilitate transactions in the economy, but it also gives rise to policy uncertainty and therefore constitutes a source of fiscal risk. In the presence of elevated tax uncertainty, firms in our model exploit the tax advantage of debt financing more cautiously, which raises their overall costs of financing. This effect depresses corporate investment, so that there is not just 'financial crowding out' of private debt through government activity, but in our production economy, also 'real crowding out'. Ultimately, we show that in our model the dual role of government debt in terms of enhanced

[1]See Liu (2018) and Croce, Nguyen, Raymond, and Schmid (2019) for further evidence that measures of government debt predict positive excess returns in various asset classes.

[2]A similar mechanism is at work in, for example, Croce, Nguyen, and Schmid (2012) and Croce, Nguyen, Raymond, and Schmid (2019).

liquidity services and elevated policy uncertainty is reflected in rising consumption volatility.

Quantitatively, our dual mechanism of liquidity provision versus policy uncertainty provides a realistic account of the empirical evidence. It rationalizes liquidity premia declining with safe asset supply, and credit spreads rising with the latter, in line with the data. The model also endogenously generates time-varying risk premia in that the conditional volatility of the stochastic discount factor reflects the tax volatility that endogenously moves with the supply of government debt. The latter feature of the model also gives rise to a realistic description of credit spreads, in that a sizeable component of spreads is a credit risk premium, accounted for not just by expected losses in default, but by the observation that losses in default tend to occur in downturns, which bondholders require compensation for. Finally, our model with endogenous leverage also delivers a sizeable equity premium, in a setting with realistically modest macroeconomic risks.

Our model, in which safety attributes of government debt and expected growth are jointly determined and linked through the government budget constraint, also provides some perspective on the fiscal costs of rising public debt. While, on average, our model indeed is quantitatively consistent with the recent low interest rate environment, and elevated expected growth rates, our equilibrium policies imply that rising government debt can lead to episodes in which Treasury yields dwarf expected growth. Indeed, as government debt rises, Treasury yields increase as well, as liquidity premiums and safety attributes decline. At the same time, the government budget constraint dictates that with a rising debt burden tax pressure and tax risk increase, thereby depressing expected growth. Therefore, a growing government burden can push Treasury yields and thus debt servicing costs above expected growth rates. To the extent that public debt may become unsustainable[3] in such an environment, our model suggests that increasing 'safe' asset supply can be quite risky.

## 1.1 Related Literature

Our work is related to and links several strands of literature. We build on the empirical observation, well-known e.g. from Longstaff (2003), Bansal, Coleman, and Lundblad (2012), Krishnamurthy

[3]The term 'unsustainable' is only vaguely defined, but here is a revealing comment from Tom Sargent's Nobel Prize Press Conference:'Here's a phrase that you hear. You hear that 'US fiscal policy is unsustainable'. You hear it from both parties. That can't possibly be true, because government budget constraints are going to make it sustainable. What they mean is that certain promises people have made (taxes, entitlements, medicare, medicaid) those are incredible, they're not going to fit together. So US fiscal policy is sustainable, [but] it's very uncertain. It's uncertain because it's not clear which of these incredible promises is going to be broken first'. We would like to thank Espen Henriksen for pointing it out to us.

and Vissing-Jorgensen (2011), Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), Du, Im, and Schreger (2018), Jiang, Krishnamurthy, and Lustig (2018) or Jiang, Krishnamurthy, and Lustig (2019), that U.S. Treasuries are arguably among the worlds safest and most liquid financial assets and investors attach a 'liquidity premium' or 'convenience yield' to holding these assets. While this literature recognizes the specialness of U.S. Treasuries, in recent work, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) document that commonly estimated convenience yields alone cannot explain the valuation of government debt, so that these have to be either larger than previously estimated, or U.S. Treasury markets have failed to enforce the no-bubble condition. We connect this stylized fact to the recent, and growing, evidence in Liu (2018) and Croce, Nguyen, Raymond, and Schmid (2019) that a rising supply of Treasuries, while lowering liquidity premia, significantly predicts rising excess returns in a variety of asset classes. While we present novel evidence in the context of credit, the papers cited provide further evidence across asset classes.

In our model, we connect the empirical evidence of rising risk premia with growing Treasury supply to the pricing of volatility risks. In this sense, our work builds on and exploits the ideas regarding pricing of uncertainty risks with recursive preferences in Bansal and Yaron (2004), and Drechsler and Yaron (2011). While these mechanisms operate in endowment economies, volatility risks arise endogenously in our work in a fully fledged general equilibrium production economy. Tax and fiscal risks thus emerge as an endogenous foundation for priced uncertainty risks.

Our emphasis on fiscal risks also relates to recent work that points out that US government itself may have become risky. Indeed, as documented and examined in Chernov, Schmid, and Schneider (2019) and Augustin, Chernov, Schmid, and Song (2019), CDS premiums, that is protection against the event of a default on US government debt traded in derivatives markets, have surged after the financial crisis. While we do not explicitly allow for sovereign default in our general equilibrium setting for reasons of tractability, doing so would likely exacerbate our results.

Our model embeds defaultable corporate debt along the lines of Gomes, Jermann, and Schmid (2016), Miao and Wang (2010), and Corhay (2015) into a general equilibrium asset pricing model with a rich fiscal sector, similar to Croce, Kung, Nguyen, and Schmid (2012) or Gomes, Michaelides, and Polkovnichenko (2013). We follow Croce, Kung, Nguyen, and Schmid (2012) in assuming that taxes have a negative long-run effect on productivity growth, so that effectively fiscal policy provides a source of 'long-run productivity risk' as specified in Croce (2014) and micro-founded in a setting with endogenous growth in Croce, Nguyen, and Schmid (2012), Kung and Schmid (2015),

or Corhay, Kung, and Schmid (2017).

Our work also contributes to the literature on equilibrium models of corporate bond pricing, motivated by the observation, often referred to as the 'credit spread puzzle' that credit spreads tend to be high relative to the average losses bondholders have to expect in default. Our model gives a general equilibrium perspective on the recent literature that attributes a large component of credit spreads to a default risk premium compensating bondholders for incurring losses in high marginal utility episodes, as spearheaded by Chen, Collin Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010b), Bhamra, Kuehn, and Strebulaev (2010a), and Chen (2010). While we abstract from significant cross-sectional heterogeneity as in Gomes and Schmid (2019), we emphasize the liquidity attributes of bonds similar to Chen, Cui, He, and Milbradt (2018) and He and Milbradt (2014). In that respect, our model of liquidity attributes builds on and generalizes the work of Amihud and Mendelson (1986) and, more recently, He and Xiong (2012). In particular, our work contributes to the literature on liquidity premia by integrating a model of differential liquidity attributes across assets into a quantitative general equilibrium asset pricing model.

More broadly, our work contributes to the literature on production-based asset pricing in general equilibrium models, along the lines of Jermann (1998), Kaltenbrunner and Lochstoer (2010), Kuehn (2007), or Kuehn, Petrosky-Nadeau, and Zhang (2014). Relative to that work, our model also implies that part of the resolution to the low risk-free rate puzzle embedded in the equity premium, may stem from the liquidity services or the convenience yields that safe assets provide, similar to Bansal and Coleman (1996).

# 2 Empirical Motivation

We start by collecting and documenting some stylized facts regarding the links between safe asset supply and liquidity and default premiums, respectively. Similar, and richer, results have been reported previously in the literature. Therefore, the objective of this section is to set the stage and provide some context on the empirical patterns our model is meant to capture and explore.

Figure 1 provides some first suggestive graphical evidence regarding the links between safe asset supply and liquidity and default premiums. We focus on the GZ spread in Gilchrist and Zakrajŝek (2012) as the relevant corporate bond spread, and the spreads between general collateral repo rate

(Repo) and treasury bill rate as a measure of the liquidity premium. The figure illustrates the dynamic relationship of liquidity and default premiums in our sample by plotting the demeaned corporate bond spread in Gilchrist and Zakrajŝek (2012) and the spreads between general collateral repo rate (Repo) and treasury bill rate. While naturally default premiums jump up during recessions (indicated by the shaded bars), when government debt tends to rise, liquidity premiums tend to fall. This pattern is especially pronounced in more recent recessions, such as the great recession following the financial crisis which constituted a severe liquidity crisis. In the recessions at the beginning of our sample, the pattern is somewhat weaker, but nevertheless, there tends to be downward pressure on liquidity premiums in the earlier stages of the downturns.

In Table 1 we report some preliminary regression evidence using our main default and liquidity measures[4]. Panels A and B in the table document that government debt, as measured by the log debt-to-GDP ratio, is significantly positively related to default premia , while significantly negatively so to liquidity premia. This holds both in levels as well as in first differences. Moreover, the results get stronger when controlling for another well-known determinant of both liquidity and default premia, namely volatility, as measured by realized stock return volatility. In terms of predictive regressions, panel C documents that government debt predicts significantly higher expected corporate bond excess returns going forward. This result complements earlier results, e.g. in Liu (2018) and Croce, Nguyen, Raymond, and Schmid (2019), that government debt predicts higher excepted stock returns in the time series, and further corroborates the strong connection between the debt-to-gdp ratio and risk premia.

Together, these results provide suggestive evidence that increasing the supply of government bonds does indeed provide liquidity services to investors and facilitate transactions in the economy by lowering liquidity premiums, as suggested previously in the literature, but at the same time, raises default risk and default premia in the corporate sector.

We next elaborate on this link by providing some more formal econometric evidence on the dynamic relationship between government debt and yield spreads. We do so by analyzing the impulse response functions in a vector autoregression framework. Going beyond the mere correlations documented in Table 1, the VAR framework allows to trace out the responses of liquidity premia and credit spreads to empirically identified innovations to the Treasury supply, thereby giving a sense of causality in these links subject to the identification strategy. In particular, we estimate a

[4]See Liu (2018) for rich alternative specifications, and robustness.

seven-variable VAR of the following form

$$Z_t = \Phi Z_{t-1} + u_t$$

$$Z_t = [ffr_t,\ \Delta ip_t,\ r_t^{ex},\ by_t,\ GZ_t,\ GZp_t,\ repobill_t]$$

The VAR includes fed funds rate ($ffr_t$), industrial production growth ($\Delta ip_t$), corporate bond excess return ($r_t^{ex}$), debt-to-GDP ratio ($by_t$), corporate bond spread ($GZ_t$) and premium ($GZp_t$) in Gilchrist and Zakrajšek (2012), and the spreads between general collateral repo rate and treasury bill rate ($repobill_t$). The corporate bond premium, ($GZp_t$), is constructed in Gilchrist and Zakrajšek (2012) to provide an empirical measure of the component of credit spreads that reflects compensation for systematic default risk, as defaults tend to cluster in downturns. We use this measure as our primary empirical proxy for default risk premia, capturing systematic rather than idiosyncratic default risk.

Following the literature, we use an identification strategy that recursively orders the variables as above. Accordingly, we identify innovations to the fourth variable as a non-discretionary increase of government debt, that is, a debt shock. This shock increases the debt-to-GDP ratio but is orthogonal to feds fund rate, IP and corporate bond excess return contemporaneously. Therefore, by construction, the shock is not driven by conditions in the macroeconomy, monetary policy, treasury market, and corporate bond market. The estimated impulse responses of yield spreads therefore provide evidence on the effects of debt shocks on credit market conditions.

Figure 2 shows the impulse response of the spreads in the corporate bond market where credit risks are important. In that market, the debt shock significantly increases not only credit spreads, but also default risk premia (as measured by ($GZp_t$)) and expected excess returns on corporate bonds. The latter distinction is important as credit spreads also reflect compensation for idiosyncratic default risk, while the risk premium measures isolate compensation for systematic risk, so that this observation suggests a debt shock indeed creates macroeconomic risk compensated in corporate bond markets. In contrast, we find a negative response of the Repo/Bill spreads in the money market where credit risks are of second order, in line with the notion of falling liquidity premia familiar from the literature. These results confirm that government debt has differential effects on different markets.

We expect movements in credit spreads and premia to be reflected in firms' financing choices,

and especially debt issuance. To provide evidence on such effects, we augment the VAR with two other variables, namely the net increases of corporate bond and commercial paper of nonfinancial corporate business, normalized by GDP. As shown in the leftmost panel in Figure 3, the debt shock significantly reduces the issuance of corporate bonds and commercial paper. This suggests substantial changes in firms' issuance and financing activities subsequent to movements in government debt.

In our VAR framework, we can similarly estimate the response of corporate debt issuance activity to unexpected moevements in other variables. We focus on innovations prevalent in the literature, such as credit shocks (e.g. Jermann and Quadrini (2012)) and liquidity shocks. Following the recursive identification strategy adopted above, we identify the innovations to the corporate bond spread as credit shocks and the innovations to the spreads between general collateral repo rate and treasury bill rate as liquidity shocks. As shown in the middle and right panels in Figure 3, the corresponding responses of corporate issuance activity are negative to both shocks. The responses to a liquidity shocks are somewhat muted, but statistically significantly negative over medium horizons in case of credit shocks.

These observations prompt us to develop a formal model to examine, and to quantify, the role of government debt supply, for liquidity and default premiums, and the macroeconomy.

# 3 Model

We develop a general equilibrium asset pricing model with endogenous liquidity and default premiums. There is a consumer sector with risk-sensitive households, a production sector in which firms finance investment with equity and defaultable bonds, and a government which finances expenditures by levying corporate taxes and issuing bonds. Households face stochastic liquidity needs which they can cover by selling off financial assets, subject to transaction costs. These liquidity needs lead to endogenous, state and asset dependent liquidity premia that households attribute to various financial assets.

We start by describing the household, production, and government sectors, and then detail the pricing of financial assets.

## 3.1 Households

The economy is populated by a continuum of households of measure one. Households have Epstein-Zin recursive preferences preferences defined over a composite of aggregate consumption, $C_t$, and labor, $L_t$, defined as $\tilde{C}_t = C_t^{\vartheta}(1-L_t)^{1-\vartheta}$, so that

$$U_t = \left[(1-\delta)\tilde{C}_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}},$$

where $\delta$ is the time discount factor, $\gamma$ is the relative risk aversion, $\psi$ denotes the intertemporal elasticity of substitution (IES), and $\theta \equiv \frac{1-\gamma}{1-/\psi}$. We assume that $\psi > \frac{1}{\gamma}$, so that the agent has a preference for early resolution of uncertainty following the long-run risks literature.

Households maximize utility by supplying labor and by participating in financial markets. Specifically, the household can take positions in the stock market, $S_t$, in corporate bonds, $B_t$, and in government bond markets, $B_t^g$. Let the values of stocks, corporate bonds, and government bonds be denoted by $V_{e,t} = P_t^e S_t, V_{c,t} = Q_t B_{t+1}, V_{g,t} = Q_t^g B_{t+1}^g$ respectively. Here, $P_t$ denotes the price per share of equity, $Q_t$ is the price of a corporate bond, and $Q_t^g$ is the price of a government bond. These prices will be determined endogenously below. Participating in financial markets exposes households to liquidity needs in the magnitude $\xi_{t^+}$ with probability $\lambda_t$, covering which involves trading in financial assets that is associated with costs $\lambda_t \nu_t(V_{g,t}, V_{c,t}, \xi_{t^+})$ that we endogenize below. Moreover, wage bills are subject to income taxes $\tau_{l,t}$. Accordingly, households' budget constraint becomes

$$\begin{aligned} &C_t + V_{g,t} + V_{c,t} + V_{e,t} + \lambda_t \nu_t(V_{g,t}, V_{c,t}, \xi_{t^+}) \\ =&V_{g,t-1}R_{g,t} + V_{c,t-1}R_{c,t} + V_{e,t-1}R_{e,t} + w_t L_t(1-\tau_{l,t}), \end{aligned}$$

so that the stochastic discount factor is given, in a standard manner, by

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1-1/\psi}\left(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}])}\right)^{1-1/\theta}.$$

### 3.1.1 Endogenous Liquidity

A critical feature of our model is that all financial assets endogenously exhibit different liquidity attributes, and thus, liquidity premiums. We now describe how we embed a model of endogenous

liquidity in our equilibrium asset pricing model. Our market structure is similar to Amihud and Mendelson (1986) and He and Xiong (2012). The key innovation is that our agents can choose between several different assets to sell when they are hit by a liquidity shock, while agents are forced to sell one asset in the early literature. This feature generates interdependence of liquidity across different markets. To that end, we assume that every period $t$ contains an intra-period $t^+$ in which agents in each household serve distinct roles as workers, firm managers, asset managers, and intermediaries, respectively.

**Timeline** We start by detailing the timeline.

- Time $t$.

Household take their asset allocation decisions. Holdings of government bonds and corporate bonds are $V_g$, and $V_c$, respectively.

- Time $t^+$. The intra-period.

Within each household, each asset manager is hit by a liquidity shock with probability $\lambda_t$ with size $\xi_{t^+}$. We assume that liquidity shocks follow a log-normal distribution, so that $\xi \sim \log N(\mu_\xi,\ \sigma_\xi^2)$. While, assuming so, we are implicitly effectively attributing all corporate bond trading to liquidity trades, we can, realistically perhaps, interpret the liquidity shocks in our model as capturing funding shocks, and portfolio rebalancing needs, shocks to individual beliefs, or idiosyncratic preference shocks, more broadly. Liquidity shocks bring about liquidity needs, which asset managers can choose to cover either by selling off an amount of $\xi_{t^+}$ of total assets to the competitive intermediaries, or by liquidating subject to liquidation costs $\varphi_l$. Liquidated assets are returned in form of cash to workers who deposit the proceeds with intermediaries. The intermediaries buy these assets using deposits. The intermediaries, thus, essentially provide a technology of liquidity transformation.

To liquidate the assets, asset managers have to sell them off at a price lower than the equilibrium price at time $t$ and incur liquidation costs $\varphi$. We assume that government bond and corporate bond come with different transaction costs in that $\varphi_g < \varphi_c$. Intermediaries are competitive and use households' stochastic discount factor to value assets, so that $Q_{t^+} = Q_t$. They bid at $Q_{t^+}(1-\varphi)$, so that they make the profits $Q_{t^+}\varphi$, which they return back to the household.

- Time $t+1$.

Workers, firm managers, asset managers, and intermediaries all convene back at the household and make consumption decisions. We have perfect consumption risk sharing in the households.

**Household's Liquidation Problem** When hit by a liquidity shock, households need to decide how to optimally cover their liquidity needs by either selling off government bond holdings of size $u_g$ and corporate bond holdings of size $u_c$, or to liquidate some of their positions. We assume that households choose $u_g$ and $u_c$ to minimize liquidation costs. This problem is static. More formally, therefore, their liquidation choices satisfy

$$\min_{u_g,u_c} \varphi_l \max[\xi - (u_g + u_c), 0] + \varphi_g u_g + \varphi_c u_c$$

In our setup, households' liquidation problem has a straightforward solution. In particular, because the liquidation cost exceeds the transaction cost in that $\varphi_g < \varphi_c < \varphi_l$, the solution follows a pecking order:

$$\begin{cases} u_g = \xi & \xi < V_g \\ u_g = V_g,\ u_c = \xi - V_g & V_g < \xi < V_g + V_c \\ u_g = V_g,\ u_c = V_c & \xi > V_g + V_c \end{cases}$$

In other words households find it optimal to first sell off government bonds, then cover the remaining liquidity needs by selling corporate bonds, and only liquidate assets in case liquidity needs exceed joint government and corporate bond holdings.

**Liquidity Premium** In our model, liquidity premiums will arise endogenously from the marginal savings of liquidation costs given some government and corporate bonds holdings. We can determine expected liquidation costs by forming expectations over $\xi$. In other words, formally, we have

$$\begin{aligned} \nu(V_g, V_c, \xi) = &\int_0^{V_g} \varphi_g \xi d\Phi_\xi + \int_{V_g}^{V_g+V_c} \left[\varphi_g V_g + \varphi_c(\xi - V_g)\right] d\Phi_\xi \\ &+ \int_{V_g+V_c}^{\infty} \left[\varphi_g V_g + \varphi_c V_c + \varphi_l(\xi - V_g - V_c)\right] d\Phi_\xi \end{aligned}$$

Given our assumption that $\xi$ has a continuous cumulative distribution function $\Phi_\xi$, it follows that $\nu(V_g, V_c, \xi)$ is differentiable, so that we can determine the marginal benefits of a government bond

as

$$\frac{\partial}{\partial V_g}\nu(V_g, V_c, \xi) = -(\varphi_c - \varphi_g)\int_{V_g}^{V_g+V_c} d\Phi_\xi - (\varphi_l - \varphi_g)\int_{V_g+V_c}^{\infty} d\Phi_\xi$$

Accordingly, the benefits of having an additional unit of government bonds stem from saving liquidation costs if households either sell corporate bonds (first term) or sell everything (second term). Similarly, the marginal benefits of corporate bond holdings are

$$\frac{\partial}{\partial V_c}\nu(V_g, V_c, \xi) = -(\varphi_l - \varphi_c)\int_{V_g+V_c}^{\infty} d\Phi_\xi$$

so that the benefits of holding an additional corporate bond stem from saving liquidation costs if households have to sell everything. Overall, the liquidation costs that emerge endogenously in our model share many properties with common reduced-form specifications of transaction costs, in that, formally, we have that $\nu > 0, \nu' < 0, \lim \nu' \to 0$, and $\nu'' > 0$. Moreover, a number of important economic properties of our liquidation costs are straightforward to establish. To begin, increasing the supply of government bonds decreases the liquidity benefits, in that it renders government bonds less useful assets to buffer liquidity shocks. Indeed, as

$$\frac{\partial^2}{\partial V_g^2}\nu(V_g, V_c, \xi) = (\varphi_l - \varphi_c)\phi_\xi(V_g + V_c) + (\varphi_c - \varphi_g)\phi_\xi(V_g),$$

given the pecking order of transaction and liquidation costs, we have $\frac{\partial^2}{\partial V_g^2}\nu(V_g, V_c, \xi) > 0$. Thus, a higher government bond supply reduces its benefit in buffering liquidity shocks in our setting.

Similarly, our specification implies that

$$\frac{\partial^2}{\partial V_c \partial V_g}\nu(V_g, V_c, \xi) = (\varphi_l - \varphi_c)\phi_\xi(V_g + V_c),$$

so that a higher government bond supply reduces the relevance of corporate bond holdings in buffering liquidity shocks, and higher corporate bond holdings render government bonds less attractive securities to buffer liquidity shocks.

**Trading Volume** Our model also has implications for the endogenous trading volumes of government and corporate bonds. In particular, the expected trading volume, conditional on the

arrival of a liquidity shock, of government bonds is straightforward to determine as

$$E_t^{\lambda}(u_{g,t^+}) = \int_0^{V_g} \xi d\Phi_\xi + V_g \int_{V_g}^{\infty} d\Phi_\xi.$$

That is, as long as the liquidity shock $\xi$ is sufficiently small, so that it can be covered by government bonds alone, realized volume in Treasuries is precisely the size of the shock, while for larger liquidity shocks, all government bonds will be soild off to begin with. Similarly, we find that the expected trading volume of corporate bonds satisfies

$$E_t^{\lambda}(u_{c,t^+}) = \int_{V_g}^{V_g+V_c} (\xi - V_g) d\Phi_\xi + V_c \int_{V_g+V_c}^{\infty} d\Phi_\xi.$$

## 3.2 Firms

There is a continuum of ex ante identical firms. Firms invest, hire labor, and produce according to a constant returns to scale technology. Given advantageous tax treatment in line with the US tax code, firms issue debt as well as equity to finance expenditures. Ex post, firms are subject to an iid cash flow shock, which may be potentially large, and can lead firms to declare bankruptcy. The trade-offs between tax advantages, liquidity benefits and default costs determine firms' capital structure decisions.

### 3.2.1 Production

Firms use capital, $K_t$ and labor, $L_t$, to produce according to the constant returns to scale production technology

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha},$$

where $A_t$ is a stochastic productivity process, whose evolution is given as

$$\Delta a_{t+1} = \mu + x_t - \phi_\tau(\tau_t - \tau_{ss}) + \sigma_a \eta_{a,t+1}.$$

Here $x_t$ is persistent long-run productivity, with $x_{t+1} = \rho_x x_t + \sigma_x \eta_{x,t+1}$ and $\tau_t$ is the prevailing tax rate, which will be pinned down endogenously below through the government's budget constraint. This specification captures the long-run effects of elevated taxation on economic growth in a parsimonious and tractable way. While the notion that rising tax rates exert a negative effect

on productivity growth is consistent with the empirical evidence, such as that documented in Jaimovich and Rebelo (2017), we specify that link directly here. However, as shown for example in Croce, Nguyen, and Schmid (2012), it is easily endogenized in the context of a model with endogenous growth.

After solving the static labor choice problem, we can define firms' profit function in a straightforward manner as follows

$$\Pi_t = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha}.$$

To introduce firm heterogeneity in a meaningful and tractable manner, we assume that firms are subject to additive, idiosyncratic shocks on their cash flows in the overall amount of $-(1-\tau_t)z_{i,t}q_{k,t}K_t$. The shock $z_{i,t}$ is scaled by capital price and capital. The scaling is important for aggregation. We assume that these shocks are i.i.d. across firms and time, and follow a normal distribution, so $z_{i,t} \sim N(0, \sigma_{z,t}^2)$. Moreover, we specify idiosyncratic volatility as countercyclical in that $\sigma_{z,t} = \sigma_{z,0}\exp(-\phi_{\sigma,a}(\Delta a_t - \mu))$.

We think of these as direct shocks to firms' operating income and not necessarily output. They summarize the overall firm specific component of their business risk. Although they average to zero in the cross section, they can potentially be very large for any individual firm.

### 3.2.2 Investment and Financing

We assume that capital adjustment is costly in that investment is subject to convex adjustment costs. Firm level capital accumulation is thus given by

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t$$

where $\Phi$ denotes the adjustment cost function.

Given the advantageous tax treatment of debt in the tax code, firms fund investment by issuing both equity and defaultable debt. For tractability, we assume that debt comes in form of one-period securities and refer to the stock of outstanding defaultable debt at the beginning of the period as $B_t$. In addition to the principal, the firm is also required to pay a coupon $C$ per unit of outstanding debt. Let $Q_t$ denote the price of a new bond issue that comes due at time $t+1$. We will determine the bond pricing function endogenously below.

With this notation at hand, we can, taking into account investment expenses and net debt outlays, write firms' equity distributions as

$$D_{i,t} = (1-\tau_t)(\Pi(K_t) - z_{i,t}q_{k,t}K_t) - I_{i,t} + Q_t B_{t+1} - (1+(1-\tau_t)C)B_t.$$

The last term reflects the fact that interest payments are tax deductible, in line with the tax code.

### 3.2.3 Firms' Problem

Firms' objective is to maximize equity value, that is, $V_{i,t}(K_t, B_t, z_{i,t}; S_t)$. The individual state variables are capital $K_t$, corporate bonds $B_t$, and the idiosyncratic shock $z_{i,t}$. We denote the aggregate state variables as $S_t$, which contains the long-run productivity $x_t$ and fiscal policies specified below. If a firm does not default, it invests, issues new debt, and pays dividends. We can therefore write the equity value function as

$$V_{i,t}(K_t, B_t, z_{i,t}; S_t) = \max_{I_{i,t}, K_{t+1}, B_{t+1}} D_{i,t} + E_t[M_{t+1} \int^{z^*_{t+1}} V_{i,t+1}(K_{t+1}, B_{t+1}, z_{i,t+1}; S_{t+1}) dF]$$

The truncation of the integral reflects the possibility of default: a sufficiently severe cash flow shock implies an equity value of zero. In this case, equity holders are unwilling to inject further capital in the firm, and are better off defaulting. In our setup, default occurs whenever cash flow shocks exceed an endogenous and state-dependent cutoff level of $z^*_t$, which is implicitly defined by the condition $V_{i,t}(K_t, B_t, z^*_t; S_t) = 0$.

We note that given our assumption of iid cash flow shocks, outside default, all firms make identical investment and financing decisions.

**Optimality Conditions** Given our assumptions, firms optimal policies satisfy the relevant Euler equations that we examine now. Denoting the capital price by $q_{k,t}$, corporate policies satisfy the Euler conditions

$$q_{k,t}\Phi'_t(\frac{I_t}{K_t}) = 1,$$

and

$$q_{k,t} = \frac{\partial Q_t}{\partial K_{t+1}} B_{t+1} + E_t \left[ M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial K_{t+1}} dF \right],$$

so that at the optimum, the cost of investment is offset by the increase of the bond price $\frac{\partial Q_t}{\partial K_{t+1}} B_{t+1}$ and the increase in future equity value $E_t \left[ M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial K_{t+1}} dF \right]$). Indeed, we have that $\frac{\partial Q_t}{\partial K_{t+1}} B_{t+1} > 0$, as higher corporate bond prices reflect higher collateral.

Similarly, we have

$$\frac{\partial Q_t}{\partial B_{t+1}} B_{t+1} + Q_t + E_t \left[ M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial B_{t+1}} dF \right] = 0,$$

so that the fall in bond prices $\frac{\partial Q_t}{\partial B_{t+1}} B_{t+1}$ and future equity values $E_t \left[ M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial B_{t+1}} dF \right]$) is offset by the increasing debt financing in the magnitude of $Q_t$. Here, additional debt financing depresses future equity values through higher default probablilites, stemming from a falling default cutoff $z^*_{t+1}$. Formally, indeed, we find that $\frac{\partial z^*_{t+1}}{\partial B_{t+1}} < 0$.

Defining the capital return to be $R_{k,t} = \frac{1}{q_{k,t-1}} \left[ (1-\tau_t)\Pi_{K,t} + q_{k,t}((1-\delta) - \Phi'_t \frac{I_t}{K_t} + \Phi_t) \right]$, we can write the envelope conditions compactly as

$$\frac{\partial V_{i,t+1}}{\partial K_{t+1}} = q_{k,t-1} R_{k,t} - (1-\tau_t) z_{i,t} q_{k,t}, \qquad \text{and} \qquad \frac{\partial V_{i,t+1}}{\partial B_{t+1}} = -(1 + (1-\tau_t)C).$$

The former expression illustrates the impact of cash flow shocks on equity values, while the latter captures the tax shielding effects of corporate debt, reflected in equity values.

The default boundary $z^*_t$ satisfies $V_{i,t}(K_t B_t, z^*_t; S_t) = 0$, so that

$$V_{i,t}(K_t B_t, z^*_t; S_t) = q_{k,t-1} R_{k,t} K_t - q_{k,t} K_{t+1} - (1-\tau_t) z_{i,t} K_t + Q_t B_{t+1} - (1 + (1-\tau_t)C) B_t + V^{ex}_t = 0,$$

and we can solve for

$$z^*_t = \frac{q_{k,t-1} R_{k,t} K_t - q_{k,t} K_{t+1} + Q_t B_{t+1} - (1 + (1-\tau_t)C) B_t + V^{ex}_t}{(1-\tau_t) q_{k,t} K_t}.$$

### 3.3 Government

We assume that the government faces an exogenous and stochastic expenditure stream that evolves as follows

$$\frac{G_t}{Y_t} = \mu_g + \rho_g \frac{G_{t-1}}{Y_t} + \sigma_g \varepsilon_{b,t}.$$

Moreover, the government also faces an exogenous and stochastic stream of transfers that we specify as follows

$$\frac{TR_t}{A_t} = \mu_{tr} + \sigma_b \varepsilon_{b,t}.$$

Both spending and transfers are required outlays that the government needs to finance by issuing debt or raising taxes, at an endogenous, and possibly, time-varying tax rate $\tau_t$. Spending and transfers exhibit some relevant economic differences.[5] Spending affects the resource constraint so that it raises aggregate demand and has to be met by a higher supply of goods in equilibrium. However, transfers *within* representative households do not affect aggregate demand directly, so that these shocks purely affect government outlays.

The government issues one-period zero-coupon bonds with price $Q_t^g$. We assume that the government conducts fiscal policy by sticking to a debt rule. In particular, we assume that the market value $Q_t^g B_{t+1}^g$ (detrended by $Y_t$) follows the law of motion

$$\frac{Q_t^g B_{t+1}^g}{Y_t} = \mu_b + \rho_b \frac{B_t^g}{Y_t} + \kappa_\tau (\sigma_g + \sigma_b) \varepsilon_{b,t}. \tag{1}$$

Here, $\kappa_\tau$ captures the tax smoothing policy in that a part of the spending and transfer shock $\varepsilon_{b,t}$ is financed by issuing debt. The rest will be financed by taxes, implied by the government budget constraint. In particular, the government is subject to a standard budget constraint of the form

$$Q_t^g B_{t+1}^g = B_t^g + G_t + TR_t - T_t.$$

where $T_t$ denotes the government's overall tax income. Given our specification of the spending, transfer and debt issuance policies, these tax receipts are endogenously determined by the govern-

[5] We assume that spending and transfers are driven by the same shock $\varepsilon_{b,t}$. Otherwise, we need to introduce another state variable to an already large model.

ment budget constraint. Formally, we have

$$\frac{T_t}{Y_t} = \mu_g + \mu_{tr} - \mu_b + (1-\rho_b)\frac{Y_{t-1}}{Y_t}\frac{B_t^g}{Y_{t-1}} + \rho_g \frac{G_{t-1}}{Y_t} + (1-\kappa_\tau)\kappa_\tau(\sigma_g + \sigma_b)\varepsilon_{b,t}.$$

The tax base is the sum of capital and labor income subtracting the corporate tax-deductible interest payments, so that $T_t = \tau_t(\Pi(K_t) + w_t L_t - CB_t)$. Given the tax receipts and the tax base, we can compute the corresponding equilibrium tax rate.

Although the tax rate is endogenous and depends on the state of the economy and other policy choices, it follows some intuitive dynamics. First, tax rates increase with the spending and transfer shocks, though the increases are not one-for-one. Second, tax rates increase with the existing government debt as a form of fiscal consolidation. Indeed, from the term, $(1-\rho_b)\frac{Y_{t-1}}{Y_t}\frac{B_t^g}{Y_{t-1}}$, higher debt implies high taxes in that

$$(1-\rho_b)\frac{Y_{t-1}}{Y_t} > 0.$$

Third, the volatility of tax rates also increases with the existing government debt, in that $(1-\rho_b)^2(\frac{B_t^g}{Y_{t-1}})^2 Var_t(\frac{Y_{t-1}}{Y_t}) > 0$. Given the same tax base, a large stock of debt amplifies the shocks so that tax rates have to be more responsive. We will illustrate these dynamics by means of our numerical solution below.

## 3.4 Equilibrium and Asset Prices

To complete the model, we require the goods markets to clear. We assume that the liquidation costs effectively are the profits of the intermediaries. On the other hand, losses in default are absorbed as profits of the law firms. These profits are also part of output. Given these assumptions, the aggregate resource constraint takes the standard form

$$Y_t = C_t + I_t + G_t.$$

This specification embeds the arguably extreme assumption that government spending is effectively entirely waste.

A critical feature of our model is the interplay between securities' liquidity benefits and their default risk. We now turn to a detailed examination of the endogenous linkages that emerge in

our setup.

**Government bonds** We can use households' optimality conditions to determine their valuations of a government bond, and find that its price $Q_t^g$ satisfies

$$Q_t^g(1+\lambda_t\nu_{g,t}) = E_t\left[M_{t+1}\right],$$

where $\nu_{g,t} \equiv \frac{\partial}{\partial V_g}\nu(V_g, V_c, \xi)$ denotes the marginal value of government bonds' liquidity services. The expression shows that households do not only value government bonds because of their future payments, but also because they are valuable in covering households' liquidity needs in case they are hit by a liquidity shock of size $\lambda_t$.

**Corporate bonds** Corporate bond values $Q_t$ depend on default probabilities and costs of default, as well as on the liquidity benefits they provide to households. Regarding default costs $\zeta_t$, we assume that they are countercyclical in line with the evidence in Chen (2010). Therefore, we specify $\zeta_t$ to follow the process

$$\zeta_t = \zeta_0 \exp(-\phi_{\zeta,a}(\Delta a_t - \mu)).$$

Accordingly, corporate bond prices satisfy

$$Q_t B_{t+1}(1+\lambda_t\nu_{c,t}) = E_t\left[M_{t+1}\left(\int^{z^*_{t+1}}(1+C)B_{t+1}dF + (1-\zeta_t)\int_{z^*_{t+1}}(V_{i,t+1}+(1+C)B_{t+1})dF\right)\right],$$

where $\nu_{c,t} \equiv \frac{\partial}{\partial V_c}\nu(V_g, V_c, \xi)$ denotes the marginal liquidity services that corporate bonds offer to households. The first term on the right hand side denotes debt service outside default, while the second term shows that bondholders recover firm value net of default costs $\zeta_t$ after a sufficiently adverse cash flow shock. This expression highlights that corporate bonds also provide liquidity benefits to households, but possibly in different magnitudes, and different states.

More compactly, we can thus write the corporate bond pricing equation as

$$Q_t(1+\lambda_t\nu_{c,t}) = E_t\left[M_{t+1}\left((1+C)F(z^*_{t+1}) + Q_t R_{rec,t+1}\right)\right],$$

where $R_{rec,t+1}$ denotes the recovery value.

$$R_{rec,t+1} = (1-\zeta_t)\frac{\int_{z^*_{t+1}}(V_{i,t+1}+(1+C)B_{t+1})dF}{Q_t B_{t+1}}$$

**Corporate yield spread** To determine credit spreads, we first note that coporate bond yields can be computed as

$$\frac{1+C}{Q_t} = \frac{1+\lambda_t\nu_{c,t} - E_t[M_{t+1}R_{rec,t+1}]}{E_t[M_{t+1}F(z^*_{t+1})]}$$

so that comparing with the yield on a government bond with the same coupon, that is $\frac{1+\lambda_t\nu_{g,t}}{E_t[M_{t+1}]}$ gives

$$y^c_t - y^g_t = E_t[M_{t+1}]E_t[\frac{1-F(z^*_{t+1})}{E_t[M_{t+1}]} - R_{rec,t+1}] + Cov_t[M_{t+1}, \frac{1-F(z^*_{t+1})}{E_t[M_{t+1}]} - R_{rec,t+1}] + \lambda_t\nu_{c,t} - \lambda_t\nu_{g,t}.$$

The first term captures expected losses in default, while the second term is a default risk premium in that it captures to what extent losses arise in high marginal utility states. We note that the first term also reflects idiosyncratic default risk, while the second only captures systematic exposure. Finally, the last term captures the differential liquidity services that government, and corporate bonds, respectively, provide to households. This liquidity spread increases with the probability of liquidity shocks $\lambda_t$ and the liquidity advantage of government bonds over corporate bonds, measured by the differential of the marginal values of endogenous liquidity services $\nu_{c,t} - \nu_{g,t}$.

# 4 Quantitative Analysis

Most of our quantitative analysis is based on model simulations. We use a global approximation technique to solve for the model policy functions. We describe our numerical approach in the next section, along with our parameter choices.

## 4.1 Computation and Calibration

The possibility of default induces strong nonlinearities in payoffs, discount factor, and policies. Therefore, we use a global, nonlinear solution method. More specifically, we solve the model globally using a collocation approach. Since we face multiple state variables and the curse of

dimensionality, we use Smolyak polynomials on sparse grids as the basis functions to approximate the policy functions.

Briefly, we approximate $N_c$ control variables as functions of $N_s$ state variables using $N_p$ Smolyak polynomials and $N_p \times N_c$ coefficients $\beta$. There are $N_s = 6$ state variables $K_t,\ x_t,\ B_t,\ B_t^g,\ \varepsilon_{b,t},\ G_t/Y_t$, $N_c = 5$ control variables $L_t,\ U_t,\ B_{t+1},\ Q_t,\ Q_t^G$, and $N_p = 85$ Smolyak polynomials. We choose the approximation level of the Smolyak method to be 2 so that the highest order polynomial of each state variable is 5. We compute the system of $N_c$ equilibrium conditions over a grid of $N_p \times N_c$. We solve the system of equations and obtain $\beta$. This process involves projecting state variable one period forward, computing the implied approximation errors, and minimizing these with respect to $\beta$. We then simulate the model and compute the approximation arrors in the state space, and repeat the process until convergence. A more detailed description of the algorithm is provided in the appendix.

The model is calibrated at quarterly frequency. We report our parameter choices in table 2. Regarding preference and technology parameters, such as risk aversion $\gamma$, intertemporal elasticity of substitution $\psi$, time discount $\beta$, leisure parameter $\vartheta$, capital share $\alpha$, and depreciation $\delta$, we pick standard values in line with the literature. Our parameter choices for preferences imply that households have a preference for early resolution of uncertainty, so that they are concerned about shocks to long-run growth prospects. These choices are in line with the long-run risk literature pioneered in Bansal and Yaron (2004), or Drechsler and Yaron (2011), involving a large intertemporal elasticity of substitution.

In line with the production-based asset pricing literature, as in Jermann (1998), we specify the adjustment cost function to have the form $\Phi(\frac{I}{K}) = \left[\frac{a_1}{1-1/\xi_k}(\frac{I}{K})^{1-1/\xi_k} + a_2\right]$, where the coefficients $a_1$ and $a_2$ are chosen such that $\Phi(\frac{I}{K}) = 0$ and $\Phi'(\frac{I}{K}) = 0$ at the steady state. The coupon rate on corporate bonds is set at 1.5%.

The quarterly growth rate of productivity $\mu$ is 0.45% to match average trend growth in the postwar sample. The volatility of the productivity shock $\sigma_a$ is set to match the volatility of consumption growth. The long run productivity has a persistence $\rho_a$ of 0.965 and its shock volatility $\sigma_x$ is 5% of the short run shock volatility, in line with the estimates in Croce (2014). We pick the idiosyncratic shock volatility $\sigma_{z,0}$ to match the average default rate. The default loss $\zeta$ is set at 0.3 of the total asset value, which is at the lower bound of estimates reported in the literature, so that we are erring on the conservative side. Both idiosyncratic volatility and default losses exhibit

mild countercyclicality governed by the parameters $\phi_{\sigma,a}$ and $\phi_{\zeta,a}$, consistent with the evidence in Chen (2010) for example.

The long run productivity growth effects of taxation $\phi_\tau$ are set at 0.05, consistent with the estimates in Croce, Kung, Nguyen, and Schmid (2012). This parameter choice captures the notion that raising taxes has detrimental effects on productivity growth in the long run, consistent with the evidence in Jaimovich and Rebelo (2017). The processes for the government debt and the spending are chosen to match their data counterparts. The parameter $\kappa_\tau$ determines the degree of tax smoothing and is set to match the tax rate persistence and volatility.

We calibrate the liquidation cost $\varphi_g$ and $\varphi_c$ to match bid ask spreads of government and corporate bonds. The liquidity shock probability $\lambda$ is calibrated to the the absolute deviation of the money market mutual fund flow relative to the fund size. This data moment, estimated to be 0.12, captures that around 12 pecent of the money market mutual fund flows in and out on a quarterly basis. The distribution of the liquidity shock determines the turnover of government and corporate bonds. We discipline $\mu_\xi$ and $\sigma_\xi$ by matching the relative turnover of treasury and corporate bonds, and the liquidity premium on treasury bills, measured by the Repo/Bill spread.

## 4.2 Quantitative Results

We start by assessing the overall quantitative relevance of our model for liquidity and credit spreads by inspecting a wide range of credit and asset market statistics, along with macroeconomic moments. We then illustrate the basic intuition and discuss the economic mechanisms more succinctly by evaluating the relevant equilibrium policies.

### 4.2.1 Moments

Table 3 reports basic moments from model simulations regarding some of the main building blocks of our model. To illustrate the quantitative relevance of our liquidity model, we report statistics obtained from a model specification in which we abstract from liquidity considerations alongside, labeled 'Default Only'. Panel A shows that our calibrated model is consistent with relevant aspects of the dynamics of fiscal variables. While overall government debt dynamics are targeted through our specification of the fiscal rule, the levels and dynamics of taxes are endogenously determined through the government's budget constraint. In particular, levels, volatilities, and persistence of taxes are matched quite well in our model. A quantitatively relevant account of tax dynamics is

critical in our context, as taxes emerge as an endogenous source of long-run productivity risk in our model, priced in equity and, importantly, credit markets.

Panel B reports statistics regarding default risk in corporate credit markets. At around forty percent, recovery rates in default on corporate bonds are roughly in line with their empirical counterparts in the data. At the same time, default rates in the model are low, and match up very close with the data. In spite of this low default risk, leverage ratios are well matched in the model, at around forty percent. This joint observation is often labeled as the 'low leverage puzzle' in the empirical literature,referring to the question why in the presence of significant tax advantages of debt, and low default probabilities, firms use leverage rather moderately.

Our explanation in this model is related to the one explored in the recent literature on the 'credit spread puzzle' literature (see, for example, Chen, Collin Dufresne, and Goldstein (2009), Chen (2010), or Bhamra, Kuehn, and Strebulaev (2010b)), which observes that credit spreads are relatively high in spite of low expected losses in default, although with a twist. In our risk-sensitive model, in which households have recursive preferences and are subject to long-run productivity risks, defaults tend to cluster in downturns, so that bondholders incur losses precisely when their marginal valuations are highest. Through that mechanism, credit spreads in the model are realistically matched at around a hundred basis points, because a substantial fraction of spreads, namely about twenty percent, is made up by a default premium that investors require as compensation for countercyclical losses.

In contrast to the credit spread puzzle literature, the default premium in our model here emerges in a fully fledged general equilibrium production economy. As panel B shows, moreover, the component of credit spreads that compensates investors for average losses is about fifty percent. Our model also gives rise to a novel twist in determining credit spreads, in that corporate bonds provide less valuable liquidity services to investors than do government bonds. This differential liquidity benefit is priced into corporate bonds and contributes a quantitatively significant amount to spreads. Indeed, in our calibrated setting it makes up for about one fourth of the overall credit spread.

In panel C, we turn to a more detailed investigation of the quantitative implications of the model for liquidity premia, and the dual role of safe asset supply for liquidity and risk premia more specifically. We first document a liquidity premium on government bonds of about 0.3, in line with the empirical estimates obtained in the recent literature. This number suggests,

therefore, that yields on traded government bonds are significantly below the equilibrium risk free rate due to the liquidity services they provide. The table also shows that corporate bonds also enjoy some liquidity benefits in spite of their inherent default risk. That liquidity premium, however, is, at 0.06, substantially smaller than the one in government bonds. Intuitively, in the context of the model, they trade with higher transaction costs in the market for liquidity services. While our model falls short of matching the turnover ratios in both government and corporate bond markets, it captures their relative magnitudes. In particular, it is quantitatively consistent with the intuitive observation that turnover in Treasury markets is substantially higher than in corporate bond markets.

Next, we evaluate the potential of our model to capture some of the stylized facts about the differential effects of safe asset supply on liquidity and default spreads, respectively. As demonstraded in section 2, a rising government debt supply lowers liquidity premia, but does raise default premia on corporate bonds at the same time. Our model is consistent with that observation. In particular, as reported in panel C, the regression coefficients on default spreads and liquidity spreads on the debt to GDP ratio in simulated data have positive and negative signs, respectively. Moreover, the magnitudes are roughly in range of the empirical counterparts. In the next section, we discuss the economic mechanism underlying this quantitative result in more detail.

In panel D, we provide some quantitative evidence on the magnitude of 'crowding out' of corporate debt through the issuance of government debt. Indeed, in the data the regression coefficient of the aggregate market value of corporate debt on the government debt-to-GDP ratio is negative, suggesting that government debt crowds out some of public debt market activity. A similar pattern obtains in the model. Quantitatively, the model overstates the magnitude somewhat, but it is broadly in line with the data.

Beyond credit market statistics, our model is quantitatively broadly consistent with a wide range of stylized facts about aggregate fluctuations and stock returns, as table 4 shows. In particular, in spite of a realistically moderate amount of aggregate consumption risk, our model produces a significant equity premium of about five percent annually, and annual return volatility of close to ten percent, thus giving rise to a realistic Sharpe ratio in the range of 0.5. While this is to a large extent due to the exogenous and tax-based endogenous movements in long-run productivity risk, as explored in the literature previously, it obtains in spite of realistically low corporate leverage.

Figure 7 provides further support for the quantitative implications of our model by examining

its dynamic properties more closely. Specifically, it shows that the equilibrium policies give rise to empirically plausible impulse response functions, in line with those obtained in the data and documented earlier in figure 2. We show the responses of expected corporate bond excess returns, credit spreads, the default risk premium, as well as the Treasury liquidity premium, to a one standard deviation government debt shock $\varepsilon_b$. Qualitatively, the model reproduces the evidence quite well in that expected excess returns, credit spreads, and the default risk premium rise, while liquidity premia decline with an increasing supply of safe assets.

### 4.2.2 Equilibrium Policies

We now illustrate and examine the basic model mechanisms by means of the equilibrium policy functions in the following figures. Figure 5 illustrates the policy functions of various key macroeconomic variables with respect to government debt, holding the other state variables fixed. Not surprisingly, overall tax pressure endogenously rises when government debt is increasing, as the top left panel shows. Critically, moreover, not only does our model predict rising tax levels, but also higher tax volatility. This is because given the same tax base, a large stock of debt amplifies the shocks so that tax rates have to be more responsive. Intuitively, the present value of future tax commitments becomes more sensitive to shocks as government debt grows. While we have presented this result analytically from the government budget constraint, the bottom left panel demonstrates that the implied tax volatility effects are quantitatively significant.

The remaining panels of figure 5 show that rising tax risk spills over into the remaining macroeconomic variables. The top middle panel documents that the rising tax pressure with a growing debt burden is detrimental to growth. This result stems both from a level effect associated with higher average taxation,as induced by our productivity specification, but also from a volatility channel owing to elevated tax risk. The lower middle panel shows that such rising tax risk comes with higher consumption risk. Endogenous fiscal dynamics therefore create endogenous stochastic volatility in our model as they are tied together through the government budget constraint. Such higher consumption risk is reflected in increasing volatility of the stochastic discount factor, which is further amplified through the endogenous effects of tax dynamics on productivity growth. Indeed, as a source of endogenous long-run productivity risks and long-run productivity volatility risk, tax volatility amplifies these low frequency risks that are priced in the presence of Epstein-Zin preferences. Overall, therefore, the market price of risk is therefore endogenously increasing with

government debt in our model.

In our trade-off based leverage model, movements in taxes and tax volatility feed back into firms' capital structure and financing choices. Higher tax risk is manifested in falling corporate debt levels, as the top right panel shows. A reluctance to tap external financing is reflected in real outcomes at the corporate level as well, in our model with financial frictions. Indeed, the bottom right panel shows that higher tax volatility is also reflected in falling aggregate investment rates. While higher aggregate risk is naturally reflected in low investment given a precautionary motive, in our model, this effect is amplified given firms' cost of debt financing, as we show next.

Figure 6 illustrates various credit market variables with respect to movements in government debt. To begin with, critically, the overall corporate credit spread non-monotonic, and in fact U-shaped, with respect to government debt. This pattern documents succinctly the dual effects that government debt has on firms' cost of capital. Decomposing credit spreads into its components illustrates the underlying drivers. In line with the stylized empirical evidence presented earlier, default probabilities rise with government debt, as the bottom left panel shows, owing to the endogenous tax dynamics illustrated in figure 5. Rising tax volatility creates not only higher aggregate risk leading to higher default probabilities, but also to higher long-run productivity risk. This is because in our model taxes are an endogenous source of long-run productivity risk. As illustrated above, this additional source of priced risk leads to higher risk premia in our setup. The required compensation for the additional default risk therefore goes up, and leads to a higher default risk premium, as shown in the top middle panel. Ultimately, this is a reflection of our fiscal uncertainty channel.

In contrast, liquidity premia fall, as the bottom middle panel documents. This is an illustration of the classic liquidity channel of government debt provision that is also at work in our model. It is intuitive that given a higher supply of government bonds, liquidity shocks are more easily absorbed leading to falling liquidity premia. This diminishes the liquidity benefits of additional Treasury supply, and makes corporate debt relatively more attractive with respect to government debt, and leads to a fall in corporations' cost of capital. Quantitatively, in our calibration, the liquidity effect dominates in parts of the state space in which government is relatively moderate, while the fiscal uncertainty channel becomes increasingly important when government debt rises. In particular, it leads to rapidly rising costs of firms' external financing when the net debt burden starts exceeding fifty percent.

Clearly, a rising debt burden also increases the government debt servicing costs. The top right panel shows that Treasury yields rise significantly with government debt. That reflects falling liquidity premia. While, all else equal, a rising debt burden also leads to lower growth and rising volatility, effects that tend to lower interest rates, quantitatively, the liquidity channel dominates here. Rising debt thus makes it harder for the government to roll over debt, and such episodes tend to coincide with slowdowns in growth and rises in volatility, as figure 5 shows.

A higher supply of safe assets has further implications for government and corporate bond markets. First, as shown in the bottom middle panel, government bond turnover falls, as liquidity shocks are more easily absorbed. Second, as documented previously, corporate leverage falls. In this sense, rising government debt indeed crowds out corporate debt. Importantly, in our setup with financial frictions, this decline in corporate leverage has real effects. Rising costs of debt financing increase firms' overall cost of capital, leading to a decline in corporate investment, as illustrated above. In this sense, our model also predicts a sort of 'real crowding out'.

**'r versus g'** Under the label 'r vs g', a growing literature in policy circles argues that the fiscal costs of servicing the current large stock of government debt may be low in an environment with low interest rates. Indeed, as Blanchard (2019) argues,'the current U.S. situation in which safe interest rates are expected to remain below growth rates for a long time, is more the historical norm than the exception'.

Our model, in which safety attributes of government debt and expected growth ('g') are jointly determined and linked through the government budget constraint, provides some perspective on this link. While, on average, our model indeed is quantitatively consistent with the recent low interest rate environment, and elevated expected growth rates[6], our equilibrium policies imply that rising government debt can lead episodes in which Treasury yields dwarf expected growth. Indeed, as government rises, Treasury yields rise as well, as liquidity premiums and safety attributes decline. At the same time, the government budget constraint dictates that with a rising debt burden tax pressure and tax risk growth, thereby depressing expected growth. Therefore, a growing government burden can push Treasury yields and thus debt servicing costs above expected growth rates.

Figure 7 illustrates such tensions. It displays realized Treasury yields and expected growth rates in relation to the government debt-to-gdp ratio from the simulated ergodic distribution

[6]In our calibration, the transversality condition is always satisfied, so that the debt-to-gdp ratio remains bounded.

of the model. Indeed, Treasury yields rise with government debt, while expected growth rates decline, in line with our previous discussion. Quantitatively, in the model, when net debt goes beyond roughly seventy percent, yields exceeding expected growth become increasingly common. Such periods of elevated debt servicing costs therefore create episodes of fiscal stress, and notably, coincide with high taxes and volatility. In this sense, the safety attributes of Treasuries rapidly decline.

## 4.3 Liquidity Shocks and Crises

The financial crisis of 2008 prominently manifested itself also as a dramatic liquidity crisis as investors and financial institutions tried to liquidate assets to cover funding needs. As a consequence, liquidity spreads in financial markets were subject to sudden spikes. To give an account of that episode through the lens of our model, and to evaluate possible remedies, we now examine an extension of the benchmark specification, in which stochastic liquidity needs can give rise to liquidity crises. We use that specification to extract historical liquidity shocks that can rationalize the financial crisis, and more generally, the recent history of U.S. liquidity premia in the time series.

We introduce aggregate liquidity shocks into the model, by modeling the probability of being exposed to a liquidity demand shock, $\lambda_t$ as a persistent stochastic process. We entertain different specifications in section 4.3.2 below. More specifically, we assume it follows an AR(1) process with constant volatility. Our calibration attempts to disentangle movements in liquidity spreads induced by changes in government debt supply, from pure liquidity demand shocks. To that end, we first fit an autoregressive process to the government debt to gdp ratio and extract the corresponding innovations. We the regress liquidity spreads on debt to filter out the liquidity state, to which we fit an autoregressive process and use it extract the shocks. The implied process displays an autoregressive coefficient of 0.98.

A first set of results is displayed in figure 8. The figure displays the evolution of the liquidity state (top right panel) and with the historical debt to gdp ratio (top left panel) when we extract the historical innovations in the blue lines along with the theoretically implied ones when these shocks are fed into the model in the red lines. By construction, the model fits the debt to gdp series almost perfectly. The implied liquidity state is volatile and persistent, suggesting significant risk surrounding investors' liquidity demand. This risk allows the model to provide a fairly realistic

account of the historical evolution of U.S. liquidity spreads, as the red line in the lower left panel shows. In particular, the model captures spikes in liquidity spreads around the recent financial crisis well. The red line in the lower right panel shows the model implied evolution of credit spreads. While, very clearly, the model does not capture the volatility in corporate bond markets as the model implied spreads are much smoother than those in the data, it captures the directions of the movements relatively well, as spikes in the data coincide with spread increases in the model.

Overall, the model provides a reasonably realistic account of the recent evolution of the liquidity and default risk in U.S. bond markets. A natural question that we can address through the lens of the model is to what extent the debt policy of the U.S. Treasury affects these dynamics. We address this question next.

### 4.3.1 Counterfactuals: Liquidity-sensitive Debt policies

The negative link between liquidity premia and the supply of safe assets that has been documented in the literature before and confirmed in the empirical part of this paper suggests that the debt policy of the government should affect movements in liquidity spreads. In particular, while it appears to be standard practice of the Treasury to only adjust to only let debt issuance respond to spending shocks, as considered in the baseline specification of the model, it may appear beneficial to allow it to respond to liquidity states as well in an attempt to reduce spikes in spreads. Indeed, the empirical pattern that the model reproduces, would suggest that if the government were to issue debt aggressively when liquidity demand is high, it could reduce smooth out movements in liquidity spreads. Specifically, we modify the government's debt policy rule in expression (1) by allowing it not only to respond to expenditure shocks as captured by the parameter $\kappa_\tau$, but also to innovations to the liquidity states captured by a parameter $\kappa_{liq}$.

The yellow lines in figure 8 show a first set of results. They represent the implied historical evolution of variables, when the government's debt issuance, counterfactually, responded to movements in liquidity demand by setting $\kappa_{liq} = 0.04$. Clearly, such a policy smoothes out fluctuations in liquidity spreads, and reduces their overall level. Historically, however, as the top left panel and the lower right panel show, this would have come along with a more volatile debt to gdp series, as well as significantly more volatile credit spreads.

Figure 9 illustrates and quantifies the real implications of the underlying trade-off. It provides sensitivity of average credit spreads and their default components, as well as average liquidity

spreads and consumption risks with respect to the responsiveness of the debt policy of the government with respect to liquidity states, as captured by varying the parameter $\kappa_{liq}$. It illustrates the real fiscal cost of a more accommodating debt policy trough a binding government budget constraint in that this raises tax commitments and their volatility, thereby raising credit spreads and firms' funding costs, and ultimately, consumption risk.

### 4.3.2 Further Liquidity Determinants

We further explore the determinants of liquidity premiums that supports our analysis of the effects of time-varying liquidity. To provide sensitivity, we perform comparative statics on four liquidity parameters: the probability of arrival $\lambda_t$ , the mean $\mu_\xi$ and standard deviation $\sigma_\xi$ of liquidity shocks, and transaction costs $\varphi_g, \varphi_c, \varphi_l$.

The results are presented in Table 5. In the "High" columns, we set the parameter to be twice the value in the benchmark specification, while in the "Low" columns, we set the parameter to be half of the value in the benchmark. We emphasize a number of observations. First, the probability $\lambda_t$ acts as a scaling factor. An increase in the probability increases the liquidity premiums $\nu_g$ and $\nu_c$, liquidity spreads and turnover proportionally. Second, an increase in the mean $\mu_\xi$ increases liquidity spreads and turnover. This is intuitive, as given larger average shocks, relatively higher funding needs have to be covered by either selling corporate bonds, or even outright liquidation. Moreover, the effect is no longer linear because of that shift of the liquidity shock distribution. The turnover ratio decreases since the fatter tail leads to a higher transaction volume and frequency in the corporate bond market. It also has a nontrivial effect on the default spread as rollover risk rises alongside. Third, an increase in the standard deviation $\sigma_\xi$ increases the liquidity premia $\nu_g$ and $\nu_c$, as there is a higher likelihood of funding needs of a size that require transacting corporate bonds, or even liquidation. On the other hand, quantitatively, the effects on the spread and the turnover are modest, given our calibration. In this sense, second moment variation in liquidity needs is second order relative to first moment variation. Finally, transactions costs also act as a scaling factor. While an increase in transaction costs increases the spreads, it does not materially affect turnover.

To sum up, the four parameters play distinct roles. This facilitates our identification as well as the calibrations.

# 5 Conclusion

We empirically and theoretically examine the impact of safe asset supply through government bonds on credit markets, and firms' cost of debt financing. Our results emphasize a dual role of government debt in credit market activity. Through a *liquidity* channel an increase in government debt improves liquidity and lowers liquidity premia by facilitating debt rollover, thereby reducing credit spreads. Through an *uncertainty* channel, however, rising government debt creates policy uncertainty,credit spreads, default risk premia, and expected corporate bond excess returns and eventually leads to rising relative costs of firms' debt financing. We first present empirical evidence regarding the dual role of government debt in credit markets, and interpret it through the lens of a novel general equilibrium asset pricing model with endogenous credit markets and a rich role for the government in setting fiscal policy. We use the model to identify and quantify a novel fiscal risk channel associated with rising US government debt.

Such a dual view also provides some perspective on the fiscal costs of rising public debt. While, on average, our model indeed is quantitatively consistent with the recent low interest rate environment, and elevated expected growth rates, our equilibrium policies imply that rising government debt can lead to episodes in which Treasury yields dwarf expected growth. This is because, in our model, safety attributes of government debt and expected growth are jointly determined and linked through the government budget constraint. Indeed, as government debt rises, Treasury yields increase as well, as liquidity premiums and safety attributes decline. At the same time, the government budget constraint dictates that with a rising debt burden tax pressure and tax risk increase, thereby depressing expected growth. Therefore, a growing government burden can push Treasury yields and thus debt servicing costs above expected growth rates. To the extent that public debt may become unsustainable in such an environment, our model suggests that increasing 'safe' asset supply can be quite risky.

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Table 1: Regression Analysis

| | *by* | (*t*-stat) | $R^2$ | *by* | (*t*-stat) | *vol* | (*t*-stat) | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| *A. Level* | | | | | | | | |
| GZ Spread | 0.84 | (2.40) | 0.10 | 0.93 | (4.36) | 0.68 | (3.42) | 0.46 |
| Repo/Tbill | -0.77 | (-4.30) | 0.29 | -0.76 | (-4.42) | 0.10 | (1.46) | 0.32 |
| *B. First Diff* | | | | | | | | |
| GZ Spread | 3.41 | (2.12) | 0.10 | 3.44 | (2.32) | 0.08 | (2.57) | 0.17 |
| Repo/Tbill | -0.81 | (-1.72) | 0.00 | -0.80 | (-1.70) | 0.02 | (1.17) | 0.01 |
| *C. Predictive* | | | | | | | | |
| Excess Return | 0.01 | (2.39) | 0.02 | 0.01 | (2.40) | 0.00 | (0.03) | 0.02 |

The table reports estimates from OLS regressions of yield spreads and corporate bond excess returns on log debt-to-GDP ratio and stock market realized vol.
In Panel A, $spread_t = \beta_0 + \beta_1 by_t + \beta_2 vol_t + u_t$
In Panel B, $\Delta spread_t = \beta_0 + \beta_1 \Delta by_t + \beta_2 \Delta vol_t + u_t$
In Panel C, $r_{corp,t+1} - r_{f,t} = \beta_0 + \beta_1 by_t + \beta_2 vol_t + u_{t+1}$
GZ spread is the corporate bond spread in Gilchrist and Zakrajŝek (2012). Repo/Bill is the spreads between general collateral repo rate (Repo) and treasury bill rate. $r_{corp,t+1} - r_{f,t}$ is the excess corporate bond return. *by* is the log debt-to-GDP ratio. *vol* is the stock return realized volatility. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1973:1 to 2014:12.

Table 2: Parameter Values

| Parameter | Description | Value |
|---|---|---|
| A. Preferences | | |
| $\beta$ | Subjective discount factor | 0.997 |
| $\psi$ | Elasticity of intertemporal substitution | 2.00 |
| $\gamma$ | Risk aversion | 10 |
| $\theta$ | Labor elasticity | 0.23 |
| B. Production | | |
| $\alpha$ | Capital share | 0.345 |
| $\delta$ | Depreciation rate of capital stock | 0.15 |
| $\xi_k$ | Adjustment cost parameter | 0.8 |
| $\mu$ | Mean productivity growth | 0.0045 |
| $\sigma_a$ | Conditional volatility | 0.0113 |
| $\rho_x$ | Long-run persistence | 0.965 |
| $\sigma_x$ | Long-run conditional volatility | 0.00056 |
| $\sigma_{z,0}$ | Idiosyncratic volatility | 0.28 |
| $\phi_{\sigma,a}$ | Idiosyncratic Cylicality | 20 |
| C. Financing | | |
| $C$ | Corporate coupon rate | 0.015 |
| $\zeta_0$ | Default loss mean | 0.3 |
| $\phi_{\zeta,a}$ | Default loss cyclicality | 20 |
| $\phi_l$ | Liquidation cost | 0.05 |
| $\phi_g$ | Treasury transaction cost | 0.001 |
| $\phi_c$ | Corporate transaction cost | 0.01 |
| $\lambda$ | Liquidity shock mean arrival rate | 0.4 |
| $\mu_\xi$ | Liquidity shock size mean | 0.25 |
| $\sigma_\xi$ | Liquidity shock size std | 0.45 |
| D. Fiscal Policy | | |
| $\phi_\tau$ | Long-run tax effects | 0.05 |
| $\mu_g$ | Spending constant | 0.008 |
| $\rho_b$ | Spending persistence | 0.96 |
| $\sigma_g$ | Spending volatility | 0.0028 |
| $\mu_{tr}$ | Transfer constant | 0.186 |
| $\sigma_g$ | Transfer volatility | 0.14 |
| $\mu_b$ | Debt constant | 0.064 |
| $\rho_b$ | Debt persistence | 0.96 |
| $\kappa_\tau$ | Tax smoothing | 0.93 |

This table summarizes the parameter values used in the benchmark calibration of the model. The table is divided into four categories: Preferences, Production, Financing, and Fiscal Policy.

Table 3: Moments I

| Variable | Model | Default Only | Data | Notes |
|---|---|---|---|---|
| A. Fiscal | | | | |
| $Q^g B^g/Y$ | 0.37 | 0.37 | 0.37 | government debt-to-GDP ratio |
| $sd(Q^g B^g/Y)$ | 0.11 | 0.11 | 0.15 | |
| $AR1(Q^g B^g/Y)$ | 0.96 | 0.96 | 0.98 | |
| $\tau$ | 0.30 | 0.30 | 0.33 | corporate tax rate |
| $sd(\tau)$ | 3.45 | 3.45 | 8.90 | |
| $AR1(\tau)$ | 0.95 | 0.95 | 0.95 | |
| B. Credit | | | | |
| $r_{rec}$ | 0.39 | 0.39 | 0.40 | recovery rate of corporate bond |
| $F(z)$ | 1.05 | 1.03 | 1.00 | default rate |
| $Q_t B_{t+1}/(Q_t B_{t+1} + V_t^{ex})$ | 0.35 | 0.35 | 0.40 | leverage |
| yield spread | 1.13 | 0.82 | 1.00 | |
| spread: default | 0.83 | 0.82 | | |
| spread: default loss | 0.61 | 0.60 | | |
| spread: default premium | 0.21 | 0.21 | | |
| spread: liquidity | 0.30 | 0 | | $\nu_c - \nu_g$ |
| C. Liquidity | | | | |
| $\nu_g$ | -0.36 | 0 | -0.30 | risk-free rate/treasury bill spread |
| $\nu_c$ | -0.06 | 0 | | liquidity premium of corporate bond |
| $u_{g,t+\epsilon}/Q_t^g B_{t+1}^g$ | 0.46 | 0 | 19.75 | government bond turnover |
| $u_{c,t+\epsilon}/Q_t B_{t+1}$ | 0.15 | 0 | 0.85 | treasury bond turnover |
| turnover ratio | 23.24 | | 23 | |
| $QB/Y$ | 0.30 | 0.30 | 0.30 | corporate debt-to-GDP ratio |
| $\beta_{csd}$ | 1.09 | 1.31 | 0.84 | ols of spread default on $Q^g B^g/Y$ |
| $\beta_{csl}$ | -1.12 | 0 | -0.77 | ols of spread liquidity on $Q^g B^g/Y$ |
| D. Crowding out | | | | |
| $\beta_{Vc}$ | -0.25 | -0.21 | -0.08 | ols of $QB$ on $Q^g B^g/Y$ |

This table summarizes the main statistics obtained from model simulations. We report moments of fiscal variables in panel A, credit variables in panel B, and liquidity moments in panel C.

Table 4: Moments II

| Variable | Model |
|---|---|
| $\Delta c$ | 2.27 |
| $r_d$ | 5.06 |
| $r_f$ | 0.15 |
| $r_d - r_f$ | 4.90 |
| $sd(\Delta c)$ | 2.75 |
| $sd(r_d)$ | 8.82 |
| $sd(r_f)$ | 0.35 |

This table summarizes the main statistics obtained from model simulations. We report moments of macroeconomic and return variables.

Table 5: Moments III

| Variable | Bench | High $\lambda$ | Low $\lambda$ | High $\mu_\xi$ | Low $\mu_\xi$ | High $\sigma_\xi$ | Low $\sigma_\xi$ | High $\varphi's$ | Low $\varphi's$ |
|---|---|---|---|---|---|---|---|---|---|
| A. Credit | | | | | | | | | |
| $r_{rec}$ | 0.39 | 0.40 | 0.39 | 0.42 | 0.39 | 0.40 | 0.39 | 0.40 | 0.39 |
| $F(z)$ | 1.05 | 1.07 | 1.04 | 1.56 | 1.02 | 1.13 | 1.02 | 1.07 | 1.04 |
| leverage | 0.35 | 0.35 | 0.35 | 0.39 | 0.34 | 0.36 | 0.34 | 0.35 | 0.35 |
| yield spread | 1.13 | 1.46 | 0.97 | 1.58 | 0.98 | 1.17 | 1.12 | 1.46 | 0.97 |
| spread: default | 0.83 | 0.84 | 0.82 | 1.15 | 0.81 | 0.88 | 0.81 | 0.84 | 0.82 |
| default loss | 0.61 | 0.62 | 0.61 | 0.87 | 0.60 | 0.65 | 0.60 | 0.62 | 0.61 |
| default premium | 0.21 | 0.22 | 0.21 | 0.28 | 0.21 | 0.23 | 0.21 | 0.22 | 0.21 |
| spread: liquidity | 0.30 | 0.62 | 0.15 | 0.43 | 0.17 | 0.29 | 0.31 | 0.62 | 0.15 |
| B. Liquidity | | | | | | | | | |
| $\nu_g$ | -0.36 | -0.72 | -0.18 | -1.63 | -0.17 | -0.52 | -0.32 | -0.72 | -0.18 |
| $\nu_c$ | -0.06 | -0.10 | -0.03 | -1.20 | 0.00 | -0.23 | -0.01 | -0.10 | -0.03 |
| $u_{g,t+\epsilon}/Q_t^g B_{t+1}^g$ | 0.46 | 0.92 | 0.23 | 0.48 | 0.41 | 0.44 | 0.46 | 0.46 | 0.46 |
| $u_{c,t+\epsilon}/Q_t B_{t+1}$ | 0.15 | 0.30 | 0.07 | 0.43 | 0.05 | 0.17 | 0.14 | 0.15 | 0.14 |
| turnover ratio | 23.24 | 21.33 | 24.27 | 1.14 | 8536 | 3.41 | $6.8\times10^7$ | 21.33 | 24.27 |
| $QB/Y$ | 0.30 | 0.31 | 0.30 | 0.36 | 0.30 | 0.31 | 0.30 | 0.31 | 0.30 |
| $\beta_{csd}$ | 1.09 | 0.91 | 1.19 | 0.52 | 1.32 | 0.96 | 1.31 | 0.91 | 1.19 |
| $\beta_{csl}$ | -1.12 | -2.18 | -0.56 | -0.04 | -1.30 | -0.90 | -1.23 | -2.18 | -0.56 |

This table summarizes the main statistics obtained from model simulations. We report moments of credit variables in panel A, and liquidity moments in panel B. In the "Bench" column, we use our benchmark calibration. In the "High" columns, we set the parameter to be twice the value in the benchmark. In the other "Low" columns, we set the parameter to be half of the value in the benchmark. In the "$\varphi's$" columns, we change all the three transaction cost parameters $\varphi_g$, $\varphi_c$ and $\varphi_l$.

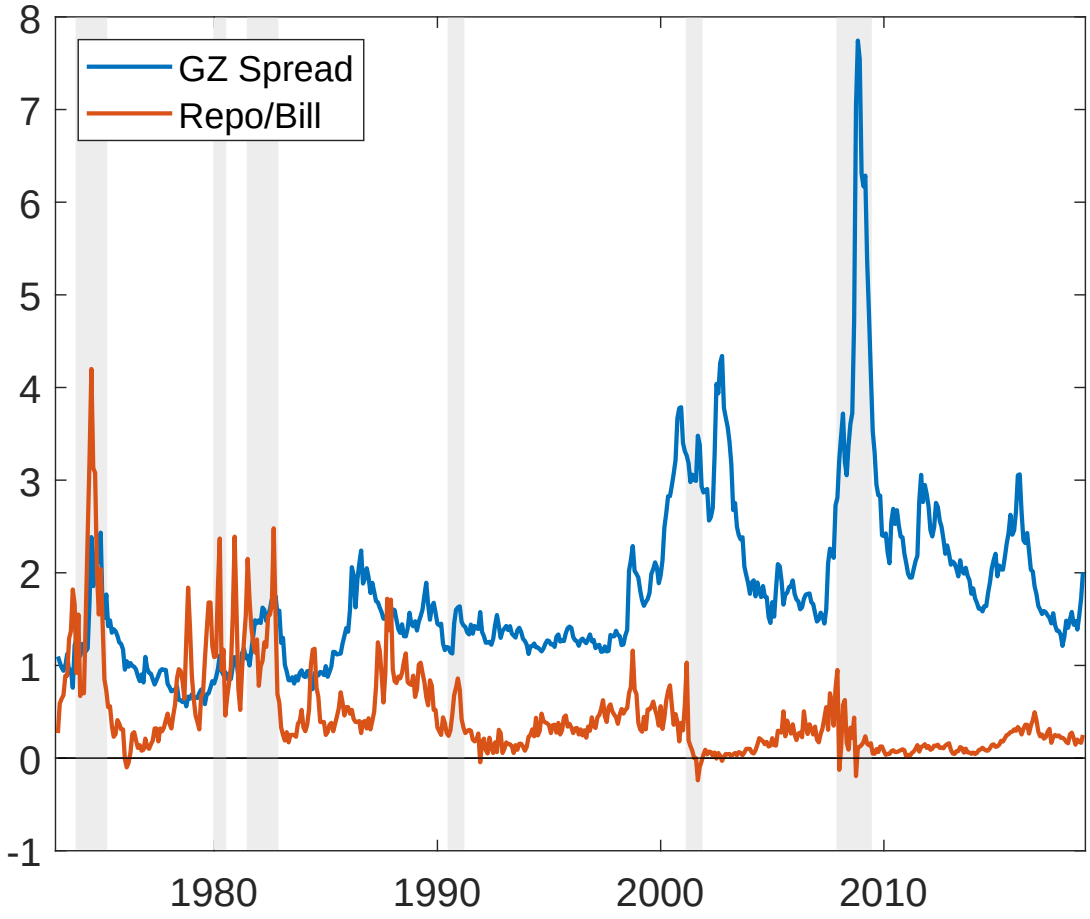



Figure 1: **Debt and Yield.** The figure plots the corporate bond spread in Gilchrist and Zakrajŝek (2012) and the spreads between general collateral repo rate (Repo) and treasury bill rate. The sample period is from 1973M1 to 2014M12.

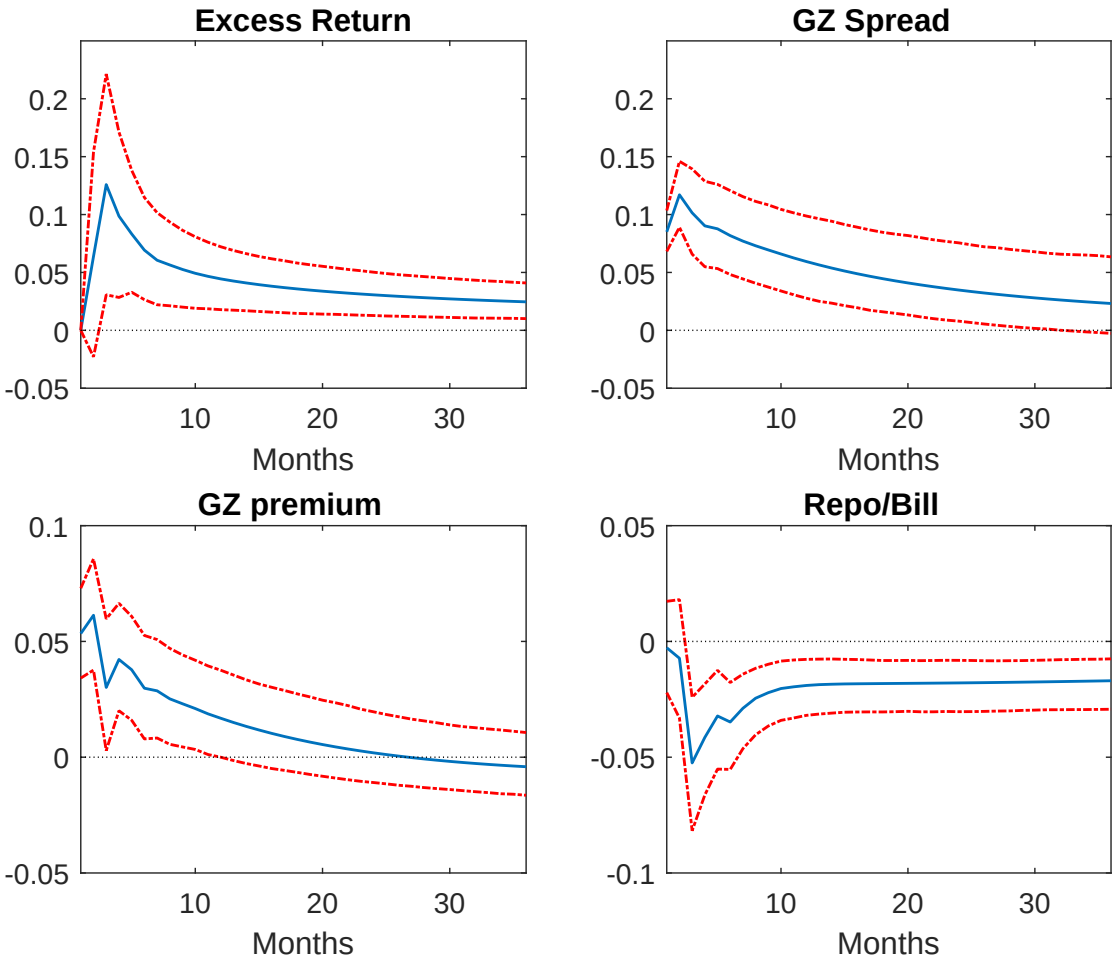



Figure 2: **Impulse Response Functions.** The figure plots the impulse response functions to a shock to debt-to-GDP ratio, based on our estimated VAR. The VAR includes fed funds rate ($ffr_t$), industrial production growth ($\Delta ip_t$), corporate bond excess return ($r_t^{ex}$), debt-to-GDP ratio ($by_t$), corporate bond spread ($GZ_t$) and premium ($GZp_t$) in Gilchrist and Zakrajšek (2012), and the spreads between general collateral repo rate and treasury bill rate. We use an recursive identification strategy and identify orthorganalized innovations to the debt-to-GDP (the fourth variable) as a non-discretionary increase of government debt, that is, a debt shock . The sample period is from 1973M1 to 2014M12.

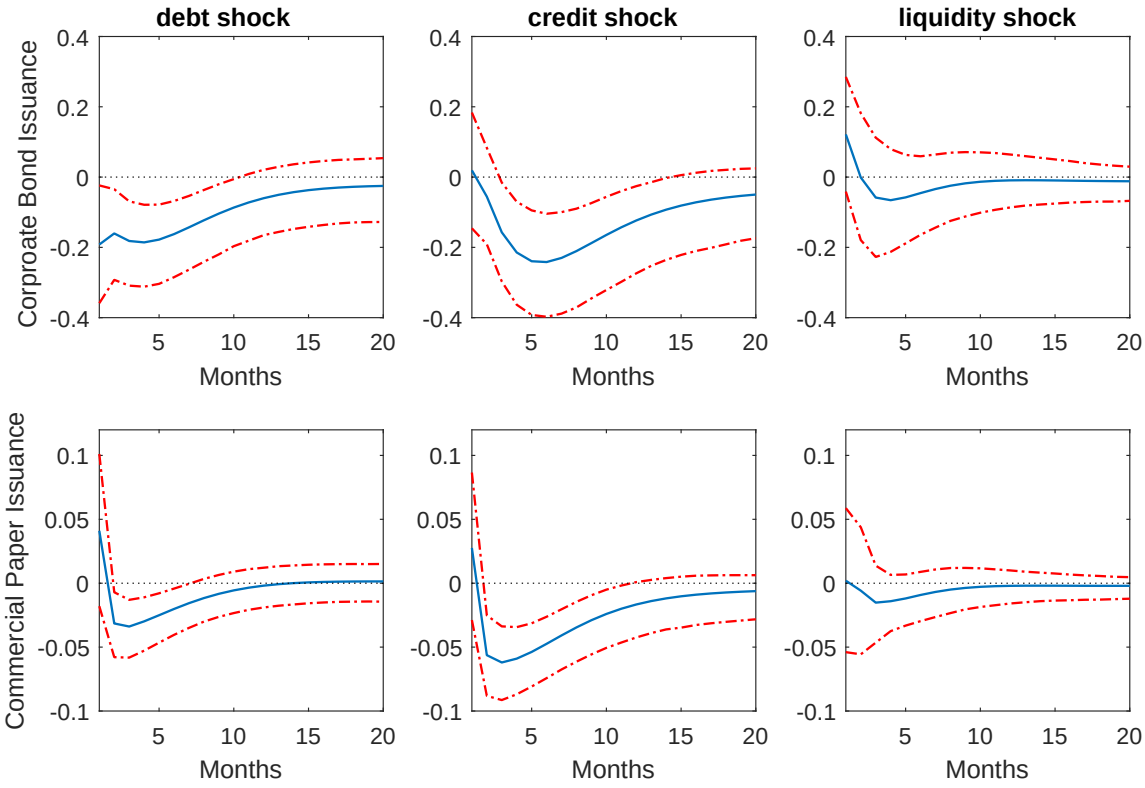



Figure 3: **Impulse Response Functions.** The figure plots the impulse response functions to a shock to debt-to-GDP ratio, corporate bond spread, and liquidity spread, based on our estimated VAR. The VAR includes fed funds rate ($ffr_t$), real GDP growth ($\Delta y_t$), corporate bond excess return ($r_t^{ex}$), debt-to-GDP ratio ($by_t$), corporate bond spread ($GZ_t$) and premium ($GZp_t$) in Gilchrist and Zakrajšek (2012), the spreads between general collateral repo rate and treasury bill rate, and the net increases of corporate bond and commercial paper of nonfinancial corporate business, normalized by GDP. We use an recursive identification strategy and identify orthorganalized innovations to the debt-to-GDP (the fourth variable) as a non-discretionary increase of government debt, that is, a debt shock. The innovations to the corporate bond spread and the repo-bill spread are identified as a credit shock and a liquidity shock . The sample period is from 1973Q1 to 2014Q12.

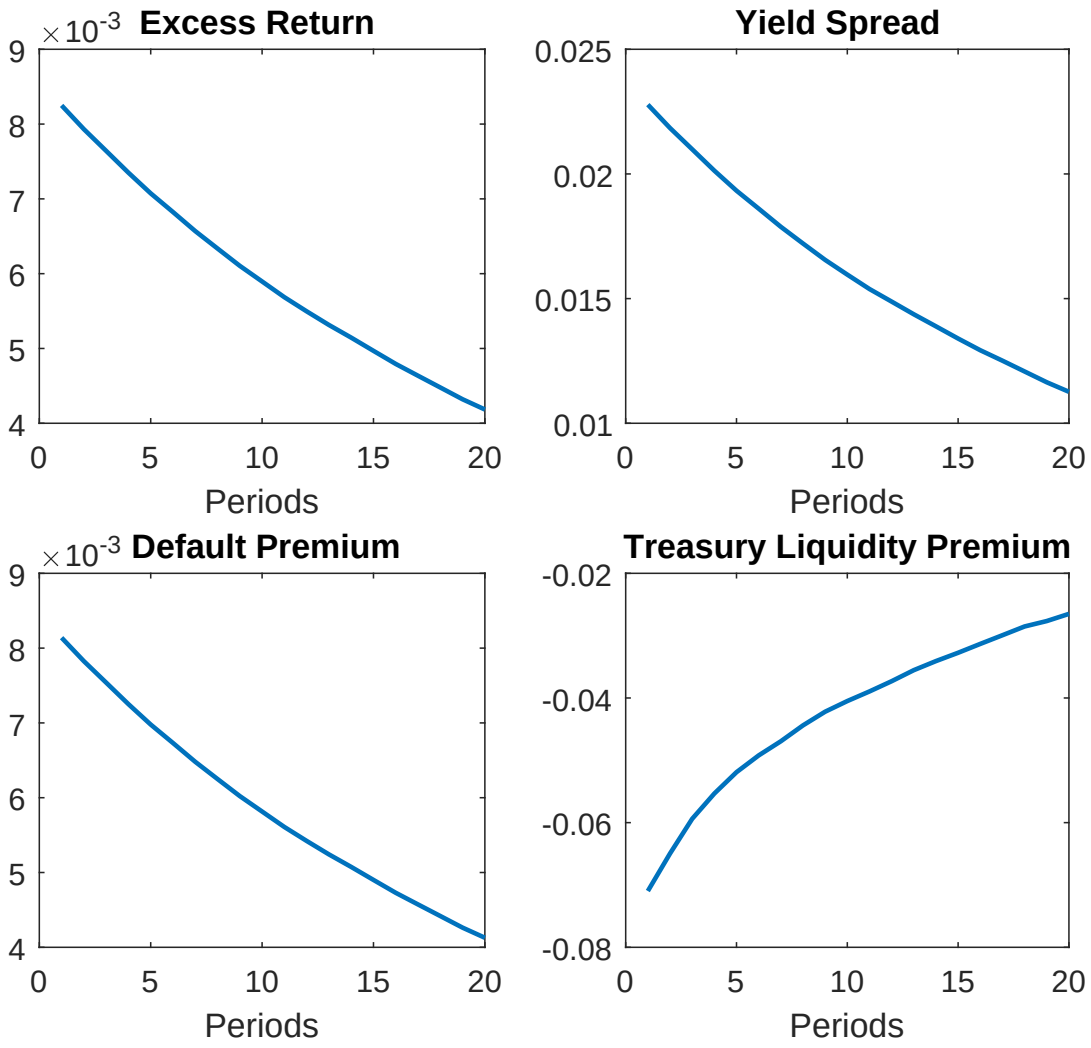



Figure 4: **Impulse Response Functions.** The figure plots the impluse response functions of expected corporate bond excess return, corporate yield spread, credit default premium, and liquidity premium on treasury bond to one s.d. shock of government debt, in the calibrated model.

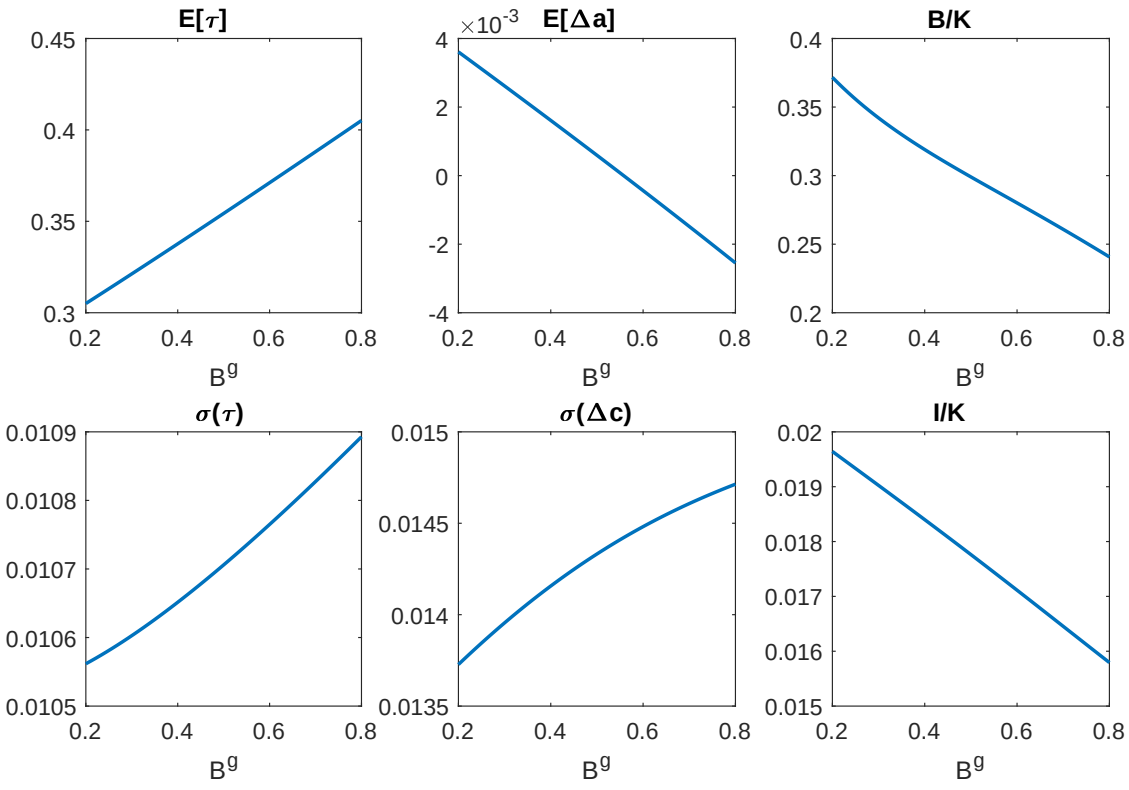



Figure 5: **Policy Functions I.** The figure plots the policy functions of expected tax rate ($\mathrm{E}[\tau]$), conditional tax rate volatility ($\sigma(\tau)$), expected TFP growth ($\mathrm{E}[\Delta a]$), conditional consumption growth volatility ($\sigma(c)$), corporate debt over capital ($B/K$), and investment over capital ($I/K$) on government debt ($B_g$), holding other state variables at the mean.

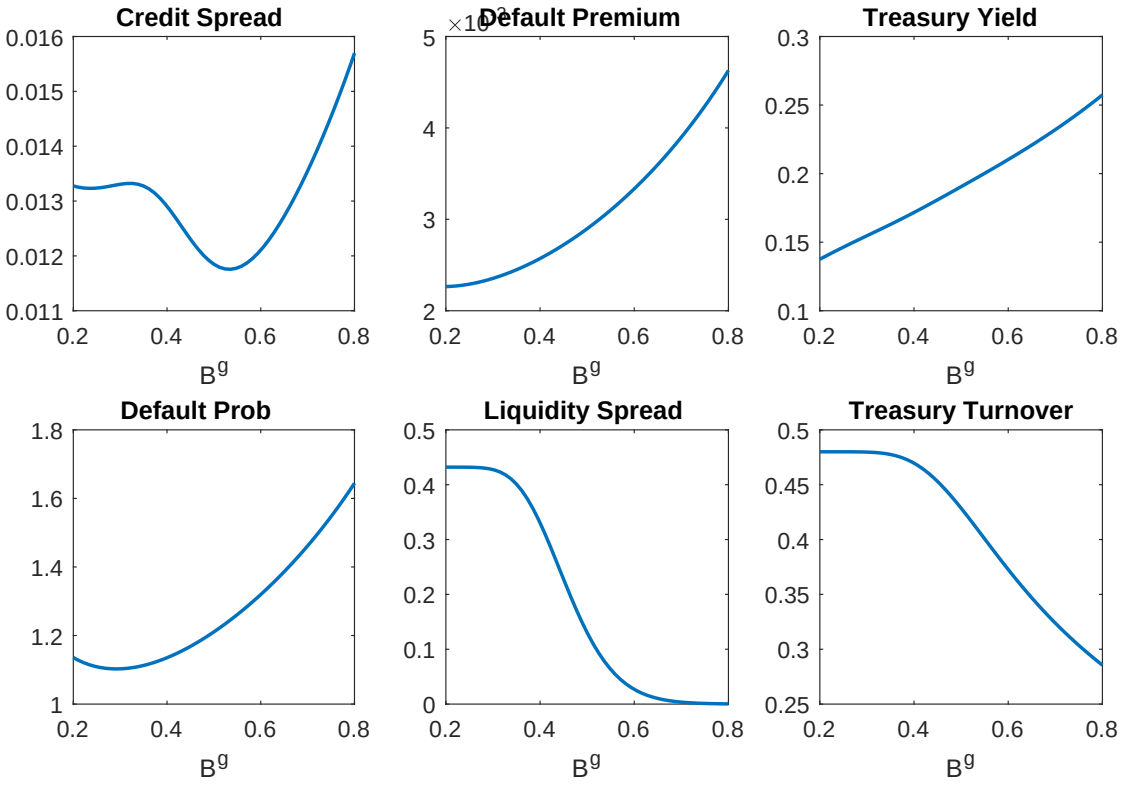



Figure 6: **Policy Functions II.** The figure plots the policy functions of credit spread, default probability, default premium, liquidity spread, Treasury yield and Treasury turnover on government debt ($B_g$), holding other state variables at the mean.

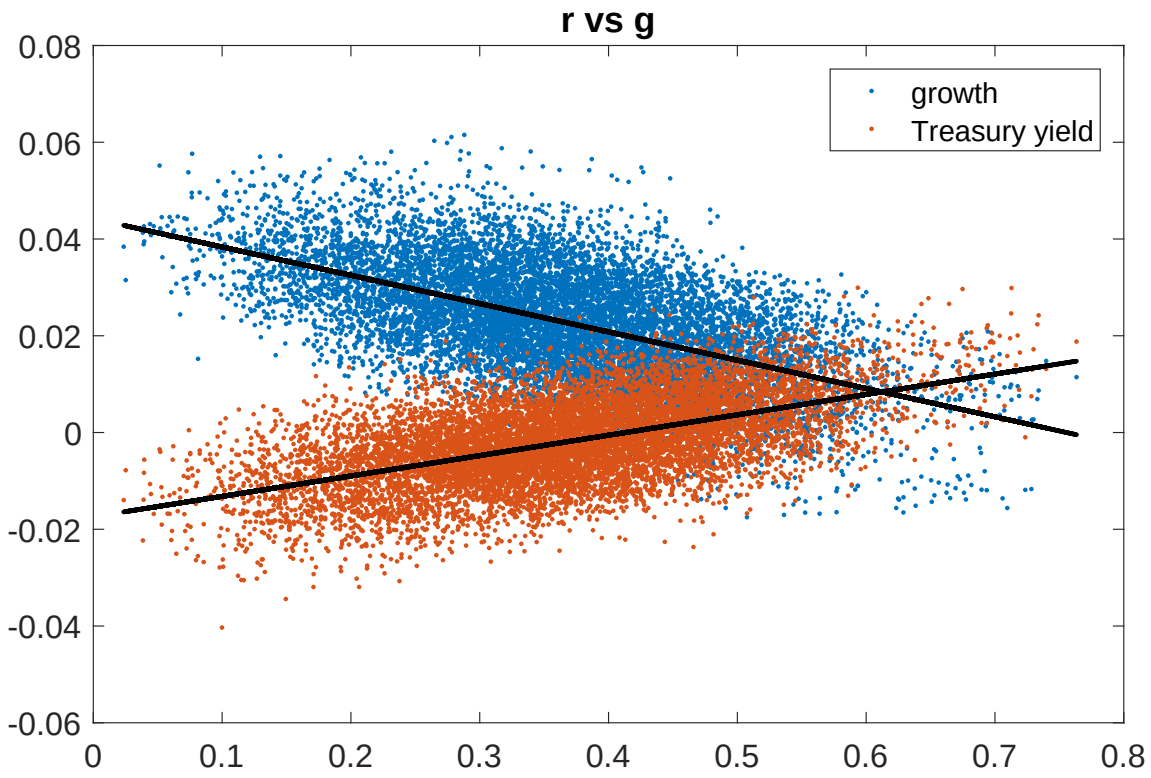



Figure 7: **Distribution of Yields and Expected Growth.** The figure plots realized Treasury yields and expected growth rates as a function of government debt from the ergodic distribution of model simulations.

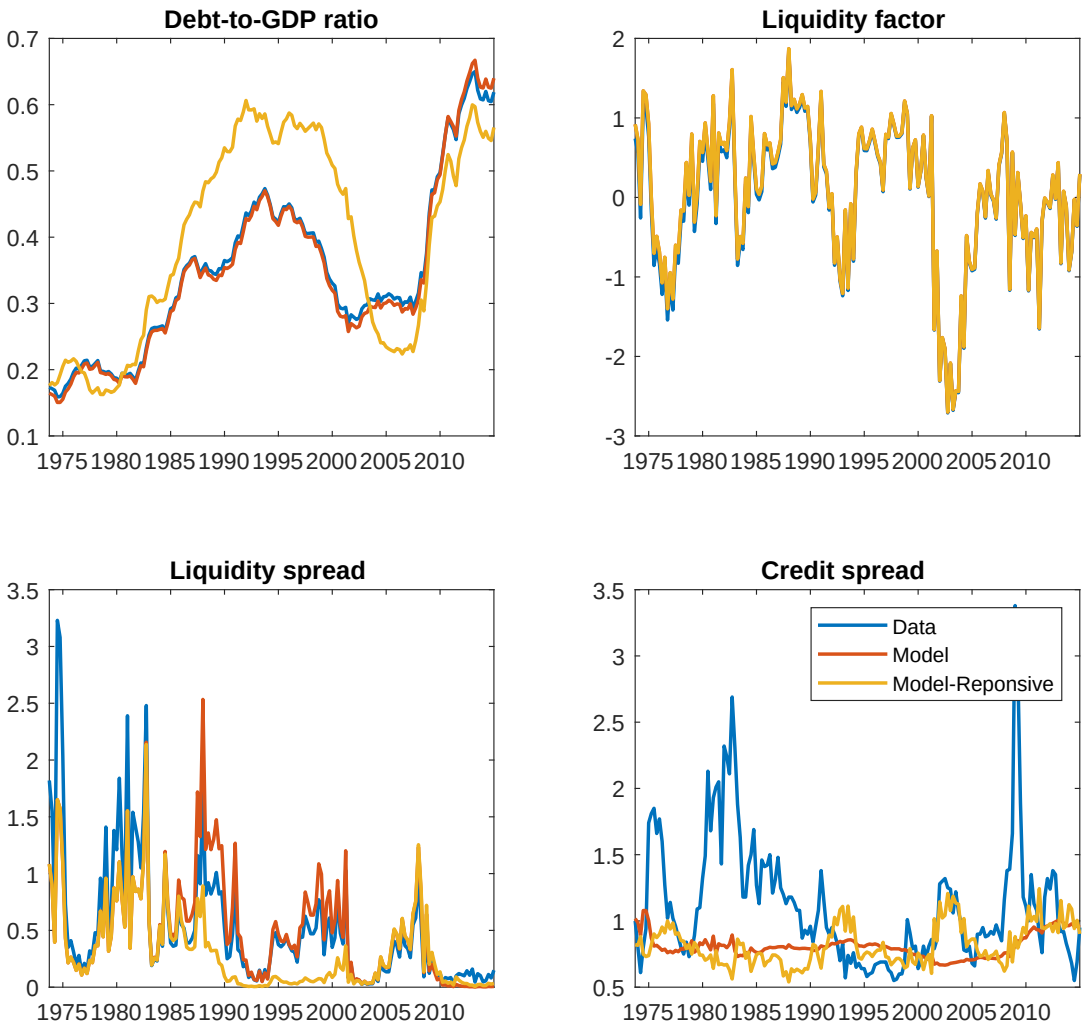



Figure 8: **Time Series of Debt, Liquidity and Spreads.** The figure plots the time series of debt-to-GDP ratio, the liquidity factor, the liquidity spread, and the credit spread in the data, in the baseline model, and in the model that debt issuance responds to liquidity demand ($\kappa_{liq} = 0.04$).

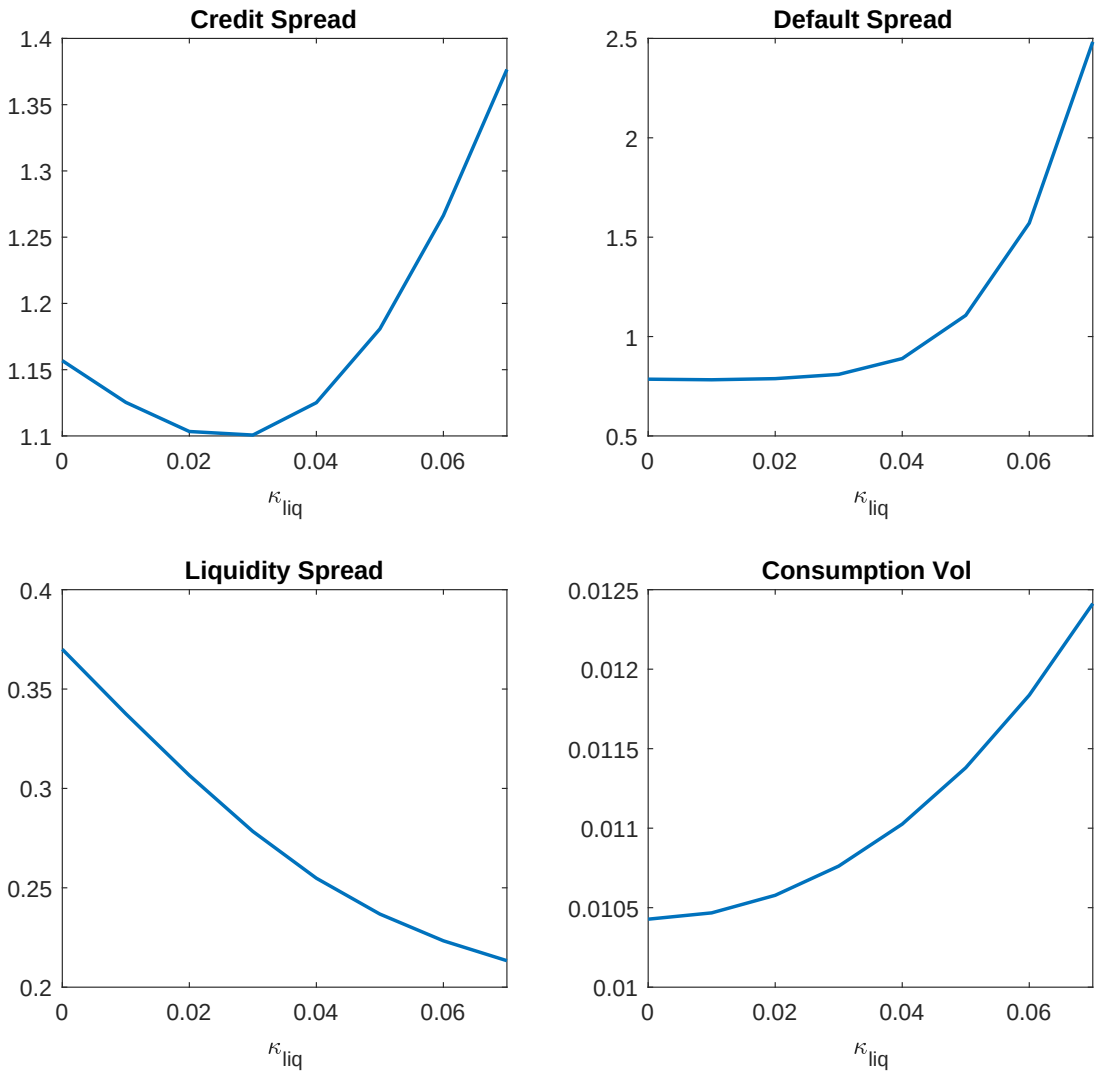



Figure 9: **Responsiveness of The Debt Policy to Liquidity Demand.** The figure plots mean of the credit spread, default spread, liquidity spread and the volatility of consumption growth in models that the debt policy have degree of Responsiveness $\kappa_{liq}$ to liquidity demand.

# Appendix A. Computational Algorithm

This section presents a brief overview of our computational algorithm. The possibility of default induces strong nonlinearities in both payoffs and the discount factor. Therefore, we use a global, nonlinear solution method. Endogenous variables are approximated using Smolyak polynomials and solved for using projection methods.

## A.1 Projection Method

We approximate $N_c$ control variables as functions of $N_s$ state variables using $N_p$ Smolyak polynomials and $N_p \times N_c$ coefficients $b$. $N_p$ increases with $N_s$.

We compute the system of $N_c$ equilibrium conditions over a grid of $N_p \times N_c$. We solve the system of equations and obtain $b$.

In our case,there are 6 state variables $X = [K_t,\ x_t,\ B_t,\ B_t^g,\ \varepsilon_{b,t},\ G_t/Y_t]$ and 5 control variables $L_t,\ U_t,\ B_{t+1},\ Q_t,\ Q_t^G$. $N_s = 6$. $N_c = 5$. $N_p = 85$.

## A.2 Algorithm

**Step 1. Compute the policy function** Given coefficients $b$ and grid $X$.

Use rescale function $\Phi : R^2 \to [-1, 1]^{N_s}$ to rescale the state variables. For example,

$$\Phi(K_t) = -1 + 2\frac{K_t - K_{min}}{K_{max} - K_{min}}$$

Use the Smolyak basis functions $\Psi_n(X)$ to compute policy function $\hat{f}(X; b^{(i)}) = \sum_{n=1}^{N_p} b_n \Psi_n(\Phi(X))$

$$[L_t,\ U_t,\ IB_t,\ Q_t,\ Q_t^G]' = \hat{f}(X; b^{(i)}) = \sum_{n=1}^{N_p} b_n \Psi_n(\Phi(X_n))$$

**Step 2. Compute the state variables in the next period** We use the equilibrium conditions to compute the state variables.

$Y_t,\ F_t = K_t^\alpha (A_t L_t)^{1-\alpha}$

$C_t,\ F_{L,t} = \frac{(1-\nu)C_t}{\nu(1-L_t)}$

$I_t,\ Y_t = C_t + I_t$

$K_{t+1},\ K_{t+1} = (1-\delta)K_t + \Phi_k(\frac{I_t}{K_t})K_t$

The other state variables follows their law of motions.

Table 6: Number of Polynomials

| Dimension | 1th Smolyak | 2th Smolyak | 3th Chebyshev | 5th Chebyshev |
|---|---|---|---|---|
| $N$ | $1+2d$ | $1+4d+2d(d-1)$ | $3^d$ | $5^d$ |
| 1 | 3 | 5 | 3 | 5 |
| 2 | 5 | 13 | 9 | 25 |
| 3 | 7 | 25 | 27 | 125 |
| 4 | 9 | 41 | 81 | 625 |
| 5 | 11 | 61 | 243 | 3125 |
| 6 | 13 | 85 | 729 | 15625 |
| 7 | 15 | 113 | 2187 | 78125 |

**Step 3. Compute approximation errors** Given the Gaussian quadrature, compute conditional expectation using $J$ integration nodes and weights, $\epsilon_{t+1,j}$ and $\omega_{t,j}$. At each node $X_{t+1,j}$, compute $U_{t+1,j}$

$$E_t[U_{t+1}^{1-\gamma}] = \sum_{j=1}^{J} \omega_{t,j} \left\{U_{t+1,j}^{1-\gamma}\right\}$$

compute $L_{t+1,j}$, $Y_{t+1,j}$, $C_{t+1,j}$, $I_{t+1,j}$,$M_{t+1,j}$, $q_{k,t+1,j}$,$Q_{t+1,j}$, $K_{t+2,j}$, $B_{t+2,j}$, $V_{t+1,j}^{ex}$, $\int^{z_{t+1}^*} V_{i,t+1,j} dF$, $\frac{\partial Q_{t+1}}{\partial K_{t+1}}$,$R_{k,t+1,j}$, $z_{t+1,j}^*, ...$

Use the variables at $t+1$ and node $j$ to compute all the expectations.

**Step 4. Solve system of equations of approximation errors with respect to b**

**Step 5. Simulate the model and compute approximation errors in the simulated state space**

## A.3 Smolyak polynomials

Smolyak polynomials are a carefully-selected subset of Chebyshev polynomials. It has an approximation level $\mu$. The maximum order of one dimension is $2^\mu + 1$. For example, the 2th Smoyak polynomials have the highest order of 5, the same the 5th Chebyshv polynomials. However, the number of polynomials are significantly smaller than tensor product.

# Appendix B. Data Sources

Our government debt data are from the Federal Reserve Bank of Dallas. We use the FRED database to collect the following data: GDP, Industrial Production, 3-month treasury bill rates and banker's acceptance rate. Returns on corporate bonds are obtained from the investment grade bond return index from Barclay. General collateralized Repo rates are obtained from Bloomberg. We augment the repo rate with banker's acceptance rate before 1991. The GZ spread and credit risk premium are from Simon Gilchrist's website. We collect the total outstanding and annual trading volume of government and corporate debt from the Securities Industry and Financial Markets Association.