# Hitting the Elusive Inflation Target*


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**Abstract**

Since the 2001 recession, average core inflation has been below the Federal Reserve's 2% target. This deflationary bias is a predictable consequence of the current symmetric monetary policy strategy that fails to recognize the risk of encountering the zero-lower-bound. An asymmetric rule according to which the central bank responds less aggressively to above-target inflation corrects the bias, improves welfare, and reduces the risk of deflationary spirals – a pathological situation in which inflation keeps falling indefinitely. This approach does not entail any history dependence or commitment to overshoot the inflation target and can be implemented with an asymmetric target range.



JEL Codes: E31, E52.

Keywords: Deflationary bias, asymmetric rules, opportunistic reflation, welfare, natural rate, zero lower bound, disanchoring of inflation expectations, inflation targeting, liquidity traps, macroeconomic uncertainty.



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# 1 Introduction

Since the 2001 recession, core inflation has been on average below the Federal Reserve's implicit 2% target. This phenomenon has become even more severe in the aftermath of the 2008 recession. In other words, the "conquest of US inflation" that started with the Volcker disinflation seems to have gone too far. Inflation, instead of stabilizing around the desired 2% inflation target, has kept falling down. This deflationary bias is a predictable consequence of a low nominal interest rate environment in which the central bank follows a symmetric strategy to stabilize inflation. We argue that a low inflation target should be combined with an asymmetric monetary policy strategy calling for more aggressive actions when inflation is below target than when inflation is above target.

Figure 1 provides evidence for the stylized fact that we are interested in. The year-to-year PCE core inflation is reported with its ten-year moving average. In the early 1990s inflation was still well above 2%. By the end of the same decade, the Federal Reserve had completed the long process that had started with the Volcker disinflation. Around this time the Federal Reserve started discussing the possibility of moving to an explicit inflation targeting regime. While an explicit 2% target was only announced on 25 January 2012 by Federal Reserve Chairman Ben Bernanke, the existence of an implicit 2% target predates this historical shift. However, as the graph illustrates, inflation has not stabilized around the desired target, instead it has kept on falling. As of today, the ten-year moving average is around 1.6%. Importantly, a similar picture emerges even when removing the 2001 and 2008 recessions. Furthermore, survey-based measures of long-term inflation expectations also declined in recent years. The University of Michigan's survey-based expectations on inflation five to ten years out have fallen by 80 basis points since 2007. The survey of professional forecasters' ten-year-ahead expectations on CPI inflation has followed a similarly declining pattern since 2012.

The deflationary bias poses serious challenges to the central bank. For instance, it may entail a considerable reputation loss if the private sector loses confidence in the Federal Reserve's ability to bring inflation back to target in an expansion. This outcome may be very costly as it could impair the central bank's capability to credibly commit to future actions, which is particularly critical to stimulate the economy when the current interest rate is at its zero lower bound (ZLB) constraint (Krugman 1998;

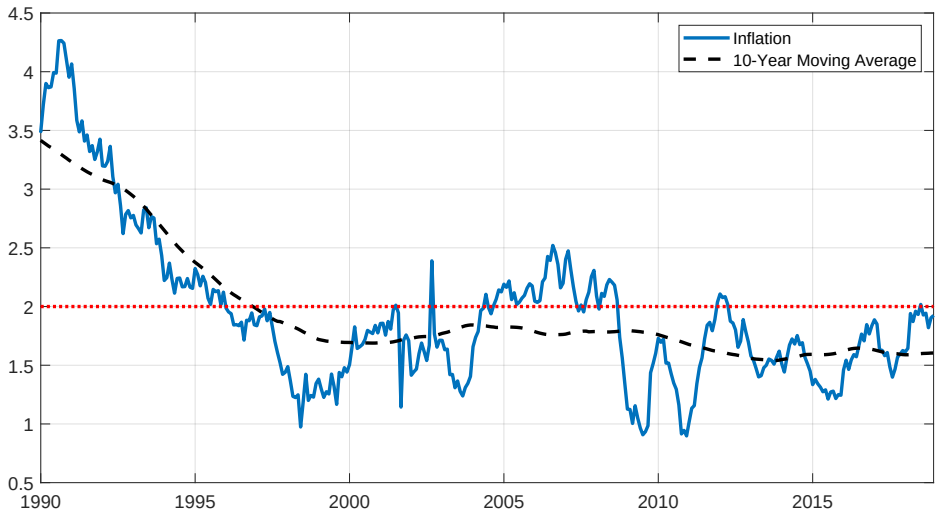



Figure 1: Year-to-year PCE core inflation and its ten-year moving average. Unit: Annualized percentage rates.

Eggertsson and Woodford 2003; and Bassetto 2019). Furthermore, a prolonged period of low inflation might cast doubts about whether or not the Federal Reserve is in fact committed to a symmetric 2% inflation target, as opposed to a two-percent ceiling on the inflation rate. Such an interpretation of the Federal Reserve's commitment can be shown to exacerbate the bias.

In addition to these challenges, we show that the deflationary bias is the harbinger of deflationary spirals. Deflationary spirals represent a pathological situation in which inflation keeps falling unboundedly. The deflationary bias arises when the probability of hitting the zero lower bound is nonzero. To counteract this deflationary pressure, the central bank keeps the interest rate low even when the economy is healthy and away from the zero lower bound. This deflationary pressure can become so large that the ZLB becomes binding also in good states. Lacking the offsetting effects of monetary policy, the real interest rate starts increasing and, in doing so, depresses aggregate demand, exacerbating the deflationary pressure. This vicious circle of low inflation, rising real interest rates, and even lower inflation sets the stage for deflationary spirals and implies that no stable rational expectations equilibrium exists. Note that this scenario does not require any recessionary shock to materialize. All it takes is a sufficiently large risk of encountering the ZLB constraint in the future, which could be driven by an increase in macroeconomic uncertainty or a fall in the natural interest rate. Given the persistent and increasing deflationary bias observed

in the last twenty years, the US economy might currently be in the proximity of this scenario, implying that remedying the deflationary bias is an issue of first order importance.

The interaction of the following two factors explains the deflationary bias: (*i*) the remarkably low long-run nominal interest rates and (*ii*) the perfect symmetry of the current monetary policy framework, which treats positive and negative deviations of inflation from the central bank's target on equal footing. We formalize our argument using a prototypical non-linear New Keynesian model, which we solve with global methods to show that in the absence of either one of these two factors the bias would not emerge.

When the long-run real interest rate is calibrated to the low values that seem plausible today (Laubach and Williams 2003), the model predicts that average inflation will remain below target even during expansions. Forward-looking price setters anticipate that in case of a large negative shock the central bank will be unable to fully stabilize inflation due to the ZLB constraint on nominal rates. These beliefs bring about deflationary pressures and depress inflation dynamics even when the economy is away from the ZLB. All changes in the macroeconomic environment that make ZLB episodes more likely or more persistent also cause the deflationary bias to become more severe. Thus, a decline in the long-term real interest rate raises the probability of hitting the ZLB in the future and consequently makes the deflationary bias larger. Similarly, heightened macroeconomic uncertainty causes or prolongs the ZLB and, hence, contributes to exacerbating the deflationary bias.

We argue that the symmetric approach to inflation stabilization, which is currently followed by the Federal Reserve, loses efficacy in a low interest rates environment because it contributes to the formation of the deflationary bias. An example of the Federal Reserve's symmetric strategy is in the *Statement on Longer-Run Goals and Monetary Policy Strategy*, which reads: "The Committee would be concerned if inflation were running persistently above or below this objective. Communicating this symmetric inflation goal clearly to the public helps keep longer-term inflation expectations firmly anchored [...]". We show that in the current low interest rate environment, it is advantageous for the Federal Reserve to be more concerned about inflation running below target than about inflation going above target.

The central bank can remove the deflationary bias and can raise social welfare by committing to adjust the policy rate less aggressively when inflation is above target

than when inflation is below target. By removing the deflationary bias, this asymmetric strategy raises the long-term inflation expectations and hence makes deflationary spirals less likely. The proposed strategy raises the probability of inflation on the upside and, in doing so, offsets the downside risk due to the ZLB. Thus, an apparent paradox emerges: In order to interpret its inflation target as symmetric, the central bank should follow an asymmetric strategy. This paradox is only apparent, because the asymmetric strategy corrects for the constraint represented by the ZLB.

Of course, in practice, it may not be easy for the central bank to convince agents that it has adopted an asymmetric strategy. When inflation is below target, announcing to be less aggressive in countering future upswings in inflation is time inconsistent.[1] In this context, the central bank can conduct an *opportunistic reflation* to demonstrate its commitment to the asymmetric strategy. To conduct an opportunistic reflation, the central bank announces the adoption of the asymmetric strategy in the aftermath of a shock that pushes inflation above target. Even though this action leads to a higher inflation rate in the short run, which entails a welfare loss, this rise in inflation offers the central bank the opportunity to show to the public that the central bank is now committed to follow the asymmetric strategy, which raises welfare in the long-run by removing the deflationary bias.[2] We show that in our calibrated model an opportunistic reflation improves welfare, unless the size of the shock is implausibly large.

In the minutes of the meeting of September 17-18 2019, the Federal Open Market Committee (FOMC) discussed whether its current long-run framework can be improved by adopting asymmetric strategies that require to "respond more aggressively to below-target inflation than to above-target inflation," in line with what advocated in this paper. Furthermore, according to the minutes, several participants suggested a target range as an effective way to communicate this asymmetric strategy. We use the model to show that the introduction of such a range can indeed close the deflationary bias and hence reduce the risk of deflationary spirals provided that the range itself is asymmetric around the desired inflation objective. For instance, if the central bank

[1] If the announcement is believed by the public, the deflationary bias disappears and once this happens, the central bank has an incentive to renege on its announcement and to respond aggressively to future upswings in inflation.

[2] Under the asymmetric rule, the weaker systematic response to positive deviations of inflation from target raises agents' long-run uncertainty about inflation and hence, everything else being equal, lowers welfare in the long-run. However, in our model these losses are dominated by the gains from removing the deflationary bias.

is committed not to respond to inflation when inflation is within the target range, specifying a range between 1.5 percent and 2.85 percent will remove the deflationary bias. We show that while the degree of asymmetry in the range required to remove the bias depends on the strength of the central bank's in-range response to inflation, the required degree of asymmetry is generally fairly modest.

Kiley and Roberts (2017) and Bernanke et al. (2019) study a set of rules to mitigate the severity of recurrent ZLB episodes. Mertens and Williams (2019) evaluate a large varieties of monetary policy rules (including dynamic rules such as price-level-targeting rules, average-inflation-rate rules, and shadow-rate rules) and conclude that dynamic rules, which make up for forgone accommodation after the ZLB episode, can eliminate the deflationary biases and deliver better macroeconomic outcomes than static rules (such as the Taylor rule). Unlike dynamic rules, the asymmetric strategy we propose does not rely on history dependence to solve the deflationary bias. Therefore, the central bank is not committed to engineer deflation following a period of above-target inflation. Similarly, the asymmetric strategy does not contemplate inflation overshooting; that is, a contingency in which the central bank maneuvers the policy rate so as to create positive deviations of inflation from its target to make up for past periods in which inflation ran below the central bank's target. Unlike the standard approach in this literature that studies linearized models with a kink in the monetary policy reaction function, we solve the fully non-linear specification of the model with global methods. This approach allows us to capture the effects of the asymmetric rules considered in the paper.

Adam and Billi (2007) and Nakov (2008) were among the first to formally show that the deflationary bias and the corresponding output bias arise in New Keynesian models in which the nominal interest rate is occasionally constrained by the zero lower bound. With respect to the existing literature, we emphasize that the symmetry of standard monetary policy rules (e.g., the Taylor rule) plays an important role for these biases to arise and show that adopting an asymmetric strategy can remove these biases.

The paper is organized as follows. In Section 2, we present a prototypical New Keynesian model to study the deflationary bias, the solution method, and the calibration of the model to U.S. data. In Section 3, we introduce a simplified version of the model to illustrate the conditions that give rise to the deflationary bias and when the deflationary bias turns into deflationary spirals. In Section 4, we show that given

the low long-run real interest rate, inflation fails to converge to the Federal Reserve's 2% inflation target in the long run. We also assess that the sensitivity of the bias to the level of macroeconomic uncertainty and to the natural rate of interest. In Section 5, we introduce the asymmetric strategy and show that it can remove the deflationary bias. We also show that this strategy improves households' welfare compared to following a symmetric Taylor rule. We also show how the asymmetric strategy can be implemented in the aftermath of an inflationary shock (opportunistic reflation) to avoid time inconsistency. In Section 6, we use the model to evaluate the effects of introducing a target range, which was recently discussed by the FOMC as a way to implement asymmetric strategies of the kind proposed in this paper. In Section 7, we conclude.

# 2 The Model

In this section, we introduce a prototypical New Keynesian model in the tradition of Clarida, Galí, and Gertler (2000), Woodford (2003), and Galí (2008) augmented with a zero lower bound constraint for the nominal interest rate set by the monetary authority. The model is solved with global methods in its non-linear specification.

## 2.1 Model description

The economy consists of households, final goods producers, a continuum of monopolistic intermediate goods firms, a monetary authority, and a fiscal authority. Households buy and consume the final goods from producers, trade one-period government bonds, and supply labor to firms. The final goods producers buy intermediate goods and aggregate them into a homogenous final good using a CES aggregation technology. The intermediate goods firms set the price of their differentiated good subject to price adjustment costs a la Rotemberg. They demand labor to produce the amount of differentiated goods to be sold to households in a monopolistic competitive market. Labor is the only factor of production. The fiscal authority balances its budget in every period. The monetary authority sets the interest rate for the government bonds.

**The Representative Household** In every period, the representative household chooses consumption $C_t$, labor $H_t$, and government bonds $B_t$ so as to maximize the expected discounted stream of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^d \left[ (1-\sigma)^{-1} C_t^{1-\sigma} - \chi (1+\eta) H_t^{1+\eta} \right] \tag{1}$$

subject to the flow budget constraint

$$P_t C_t + B_t = P_t W_t H_t + R_{t-1} B_{t-1} + T_t + P_t Div_t \tag{2}$$

where $P_t$ is the price level, $W_t$ is the real wage, $R_t$ is the gross interest rate, $T_t$ are lump-sum taxes and $Div_t$ are real profits from the intermediate good firms. $B_t$ denotes the one-period government bonds in zero net supply. The preference shock $\zeta_t^d$ follows an AR(1) process in logs $\ln(\zeta_t^d) = \rho_\zeta \ln(\zeta_{t-1}^d) + \sigma^{\zeta^d} \epsilon_t^{\zeta^d}$.

**Final Goods Producers** Final goods producers transform intermediate goods into the homogeneous good, which is obtained by aggregating intermediate goods using the following technology:

$$Y_t = \left( \int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where $Y_t(j)$ is the consumption of the good of the variety produced by firm $j$.

The price index for the aggregate homogeneous good is:

$$P_t = \left[ \int_0^1 P_t(j)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}, \tag{4}$$

and the demand for the differentiated good $j \in (0,1)$ is

$$Y_t(j) = (P_t(j)/P_t)^{-\epsilon} Y_t. \tag{5}$$

**Intermediate Goods Firms** The firm $j$ produces output with labor as the only input

$$Y_t(j) = A \; H_t(j)^{\alpha} \tag{6}$$

| Parameters | | Value | Parameters | | Value |
|---|---|---|---|---|---|
| $\beta$ | Steady state discount rate | 0.9975 | $\varphi$ | Rotemberg pricing | 79.41 |
| $\alpha$ | Production Function | 1 | $\theta_{\Pi}$ | MP inflation response | 2 |
| $\sigma$ | Relative risk aversion | 1 | $\theta_{Y}$ | MP output response | 0.25 |
| $\eta$ | Inverse Frisch elasticity | 1 | $4\log(\Pi)$ | Annualized Inflation target | 2% |
| $\epsilon$ | Price elasticity of demand | 7.67 | $\rho_{\zeta^d}$ | Persistence preference shock | 0.60 |
| $\chi$ | Disutility labor | 0.87 | $100\sigma_{\zeta^d}$ | Std. dev. preference shock | 1.175 |

Table 1: Benchmark calibration: Parameter Values

where $A$ denotes total factor productivity, which follows an exogenous process. The firm $j$ sets the price $P_t(j)$ of its differentiated goods $j$ so as to maximize its profits:

$$Div_t(j) = P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t} - \alpha\, mc_t\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}Y_t - \frac{\varphi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)Y_t, \quad (7)$$

subject to the downward sloping demand curve for intermediate goods. The parameter $\varphi > 0$ measures the cost of price adjustment in units of the final good.

**Policy makers** The monetary authority sets the interest rate $R_t$ responding to inflation and output from their corresponding targets. The monetary authority faces a zero lower bound constraint. The policy rule reads as follows

$$R_t = \max\left[1, R\left(\Pi_t/\Pi\right)^{\theta_{\Pi}}\left(Y_t/Y\right)^{\theta_Y}\right]. \quad (8)$$

where $\Pi$ and $Y$ denote the inflation target which pins down the inflation rate in the deterministic steady state and the natural output level, which is the level output that would arise if prices were flexible. The fiscal authority sets taxes to balance the budget in every period $T_t = B_t - R_{t-1}B_{t-1}$.

**Resource Constraint** The resource constraint is

$$C_t = Y_t\left[1 - .5\varphi\left(\Pi_t/\Pi - 1\right)^2\right]. \quad (9)$$

## 2.2 Model Solution and Calibration of Parameters

We solve the model with time iterations and linear interpolation as in Richter et al. (2014). Expectations are evaluated with Gauss-Hermite Quadrature. A detailed

description about how we solve the model is provided in Appendix A. See Fernández-Villaverde et al. (2016) for a review of alternative solution methods based on perturbation methods.

We set the discount factor $\beta$ to 0.9975 that corresponds to an annualized real interest rate of one percent, which is in line with the FOMC's Summary of Economic Projections (SEP) of September 2018. The standard deviation of preference shocks $\sigma^{\zeta^d}$ is chosen to be in line with the standard deviation of the U.S. real GDP growth rate over a period ranging from the first quarter of 1983 through the fourth quarter of 2007. This period has been characterized by record low macroeconomic volatility and therefore the calibrated value of the standard deviation of preference shocks should be regarded as low by historical standards. For instance, the standard deviation of the U.S. real GDP growth rate was twice as big in the 1970s. We will show how trend inflation and the long-term real interest rate vary under different assumptions about the Post-Great Recession macroeconomic volatility.

The Rotemberg parameter $\varphi$ is the equivalent to a Calvo parameter of 0.75 in case of a first-order approximation. The calibrated value for the demand elasticity $\epsilon$ implies a steady-state markup of 15 percent. The parameter controlling the disutility of labor $\chi$ is set to normalize the steady-state level of employment to unity.

The persistence of preference shocks $\rho_\zeta$ is set to 0.60. Higher values for this parameter prevents us from solving the model. The same problem occurs if the variance of the preference shock is too high. Both parameters lift the unconditional volatility of preference shocks and hence the number of future periods agents expect monetary policy to be passive because of the ZLB constraint. We set the inflation target to 2%.[3] The remaining parameters are standard and are listed in Table 1.

# 3 Deflationary Bias and Deflationary Spirals

To gain intuition about the causes of the deflationary bias and its relation with the deflationary spirals, we consider a simplified version of the model presented in the previous section. We assume that the central bank does not respond to the output gap ($\theta_Y = 0$) and that the preference shock can only take two values low (bad state)

[3]There is some disagreement about what the Federal Reserve's effective inflation objective was before 2012 (Shapiro and Wilson 2019). However, there is a strong consensus that the objective has been 2 percent since 2010.

and high (good state); i.e., $\zeta_t^d \in \left\{\zeta_L^d, \zeta_H^d\right\}$ with $\zeta_H^d > \zeta_L^d$. When the realizations of the preference shock are binary, equilibrium outcomes can be conditioned on the high or low value of the preference shock and hence can be characterized by solving a set of nonlinear equations as explained in greater detail in Appendix B. This simplified version of the model will prove useful for understanding the causes of the deflationary bias and those of the deflationary spirals and why these two outcomes are intertwined. Once we have established these points, we will go back to the benchmark model and the calibration introduced in the previous section.

Given the structure of the simplified model, we can partition the model equilibrium conditions into two blocks of equations, one for the good state and one for the bad state. In what follows, we focus on the equilibrium in the good state because - as we will see - this is the state where the deflation bias arises. The red dashed line in Figure 2 represents the interest rate $R^H$ as function of inflation $\Pi^H$ as implied by the Taylor rule in the good state, subject to the ZLB constraint. The blue line in the same figure conflates the restrictions imposed on the inflation rate and the nominal interest rate in the good state by all the other equations. Importantly, this curve also takes into account the equilibrium conditions for the bad state because agents in the model are forward looking. The intersections between the red dashed line and the blue solid line give us the (stable) Rational Expectations equilibria in the good state. Appendix B describes how these two lines are worked out.

The blue line is upward sloping because a fall in the equilibrium inflation rate in the good state, $\Pi^H$, lowers inflation expectations and hence the nominal interest rate in the good state, $R^H$.[4] The blue line also presents a kink and gets steeper for low values of inflation in the good state. When inflation in the good state declines, the partial equilibrium effect is such that expected inflation declines under both states, depressing inflation in the bad state. When the ZLB is not binding, the central bank responds by lowering the interest rate in the bad state. However, for sufficiently low levels of inflation *in the good state*, the central bank encounters the zero lower bound *in the bad state*. The existence of this threshold creates the kink in the blue line. When inflation is below this threshold, the ZLB constraint is binding in the bad state and any further decline in inflation in the good state implies an increase in the

[4]Next period's inflation expectations are the weighted average of the equilibrium inflation expectations in the two states. In symbols, $E_t\Pi_{t+1} = p_{HH}\ \Pi^H + (1 - p_{HH})\ \Pi^L$, where $p_{HH}$ is the probability that the economy will stay in the good state in the next period and $\Pi^i$, $i \in \{H, L\}$, denotes the equilibrium inflation in the state $\zeta_{t+1}^d = \zeta_i$.

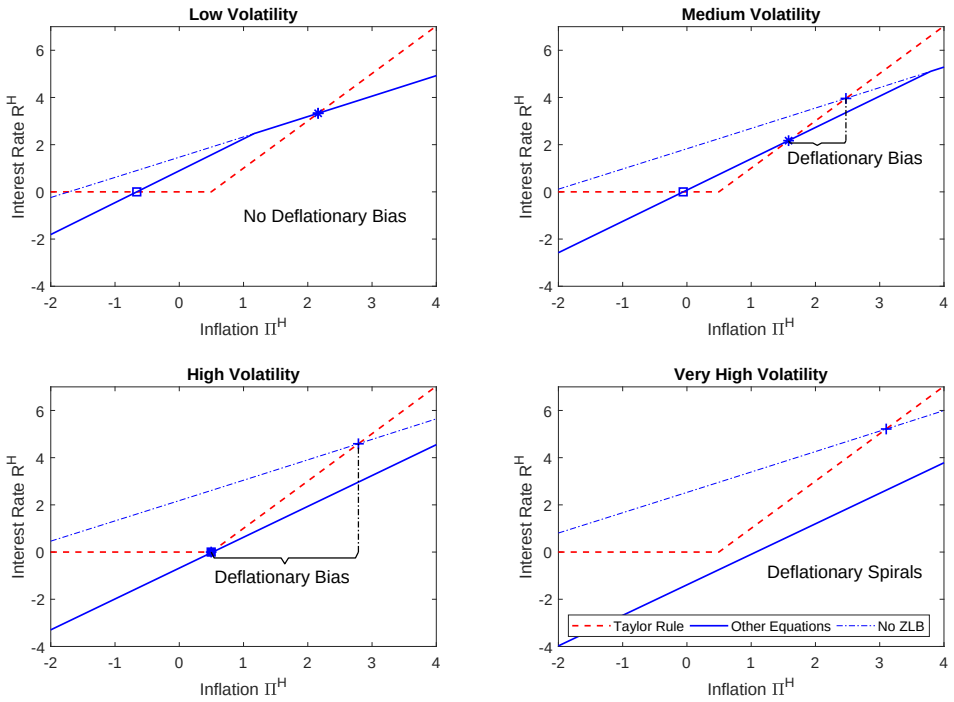



Figure 2: Equilibrium interest rate and inflation when the preference shock is high (good state). The intersections of the blue line and the red line mark the rational expectations equilibria.

real interest rate in the bad state, which exacerbates the recession and the drop in inflation in the bad state. In the good state, agents anticipate that the recession and deflation in the bad state will be more severe and these beliefs determine a steeper decline in inflation expectations and the nominal interest rate in the good state. For comparison, the blue dashed-dotted line captures the counterfactual case in which we do not impose the ZLB constraint on the nominal interest rate in the bad state and hence the slope of the blue dashed-dotted line does not change.

The four plots of Figure 2 show the equilibrium in the good state for various levels of volatility of the discrete shock.[5] In particular, we consider scenarios of low volatility ($\zeta_L^d = 0.9875$, $\zeta_H^d = 1.0025$), medium volatility ($\zeta_L^d = 0.9625$, $\zeta_H^d = 1.0075$), high volatility ($\zeta_L^d = 0.9375$, $\zeta_H^d = 1.0125$) and very high volatility ($\zeta_L^d = 0.9125$, $\zeta_H^d = 1.0175$) with a transition probability of staying in the good state $p = 0.9$ and staying in the bad state $q = 0.5$ fixed for all the levels that are considered. Across

[5] The mean of the binary random variable $\zeta_t^d$ is unchanged when we raise its variance throughout this exercise. So when we raise the volatility of the preference shock, we effectively make the negative and positive realizations of the shock bigger.

the four panels, we can see that as the volatility of the demand shock increases, the kink in the blue line occurs for larger values of $\Pi^H$, implying that the ZLB becomes a more relevant concern, even if the economy is currently in the good state.

In the upper left graph of Figure 2, we consider a low-volatility scenario. The volatility is relatively low and hence the severity of the negative preference shock is contained. In this case, there are two equilibria in the good state of the economy. One equilibrium implies that the nominal interest rate is not constrained (the star mark in the plot) and the other one is constrained by the ZLB (the square mark in the plot) in the good state.[6] In what follows, we disregard the equilibrium implying that the ZLB is binding in the good state and focus on the other equilibrium, corresponding to the star mark in the plot. In the upper-left plot, the economy is away from the ZLB. Furthermore, in this case the negative preference shock is too small to make the ZLB constraint binding in the bad state. This can be seen by observing that the equilibrium of interest, which is denoted by the star mark in the graph, lies on the flatter part of the blue line.

We now slightly increase the volatility of the preference shock, which implies that the negative preference shock is now larger than what it was in the previous case. Now the target equilibrium lies on the steeper part of the blue line, implying that the economy will go to the ZLB if a negative preference shock will hit tomorrow. These expectations have important effects on today's equilibrium outcomes. Now inflation is lower than what it would have been if the blue line were less steep as in the case in which we do not impose the ZLB constraint (the dashed-dotted blue line in the graph). We call the lower inflation rate in the good state due to the binding ZLB constraint in the bad state the *deflationary bias*. The magnitude of the deflationary bias is shown in the graph.

A further increase in the volatility of the binary preference shock causes the nominal rate and inflation to fall further, as illustrated in the lower left graph of Figure 2. Now the deflationary consequences of hitting the ZLB in the bad state are even more severe. As a result, the inflation rate in the good state falls further down and the deflationary bias widens. To respond to this large deflationary bias, the central bank has to drive the nominal interest rate to the ZLB even in the good state. This can

[6] This result is reminiscent of the two steady-state equilibria characterized in a perfect-foresight environment in the influential paper by Benhabib et al. (2001). However, the equilibria in upper left plot are derived in a stochastic environment where agents take into account the probability that the economy may be hit by preference shocks in future periods.

be seen in the graph where the solid blue line intersects the kink of the red dashed line, implying that the two good-state equilibria now coincide in the graph and the ZLB is binding in both. Furthermore, note that the deflationary bias is now larger than that in the previous case.

What happens if the volatility increases even further and the realization of the preference shock in the bad state becomes even worse? The central bank would like to lower the nominal interest rate further in the good state in order to mitigate the deflationary pressures owing to the severe deflation expected in the bad state. However, the binding ZLB constraint in the good state prevents the central bank from doing so. As a result, the fall in inflation expectations combined with the forced inaction of the central bank lead to an increase in the real interest rate in the good state, which depresses inflation expectations even further. We call this vicious circle of lower and lower inflation *deflationary spirals*. In the lower right graph, the blue solid line and the dashed red line do not intersect, implying that no stable Rational Expectations equilibrium exists.

This exercise illustrates two important points. First, the deflationary bias emerges when agents expect with some probability that the interest rate will become constrained by the ZLB in the future. Second, the deflationary bias and the deflationary spirals are intertwined: deflationary spirals occur when the deflationary bias is so large that the central bank cannot prevent inflation expectations from spiraling down.

# 4 ZLB Risk and Macroeconomic Biases

The previous section illustrated the origins of the deflationary bias and the link between the deflationary bias and deflationary spirals. We can now return to our benchmark calibration, which are shown in Table 1, with a continuum of possible realizations for the preference shock.

Hitting the inflation target is more challenging for the central bank when the probability of encountering the ZLB is non-negligible. Even in tranquil times and away from the ZLB, the mere risk that monetary policy might become constrained in the future hinders the convergence of inflation to the central bank's inflation target. This is because forward-looking price setters anticipate that in case of a large negative shock the central bank will be unable to fully stabilize inflation due to the ZLB constraint. These beliefs cause inflation expectations to become disanchored from

the central bank's target and to depress inflation dynamics.

The existence of this deflationary bias constitutes an important anomaly that should concern policymakers. Failure to acknowledge this anomaly may lead the central bank to conduct expansionary monetary policy that ends up overheating the economy and creating more macroeconomic biases. These macroeconomic biases are broadly consistent with the recent performance of the U.S. economy.[7] Moreover, the size of these biases increases exponentially as the volatility of the macroeconomic environment rises and the natural rate of interest declines. In the subsequent sections, we will show that the symmetric approach followed by the central bank to inflation stabilization is responsible for these biases.

**Probability of encountering the zero lower bound** The left plot of Figure 3 shows the percentage of periods spent at the ZLB when the model is simulated for a long period of time (300,000 periods). In technical jargon, this is the ergodic probability of being constrained by the ZLB. As shown in the figure, this probability is affected by how volatile the shocks are (x-axis). The different lines are associated with different assumptions about the long-run annualized real rate of interest $r^* = \beta^{-1}$. Our benchmark calibration for this parameter is one percent, which is in line with the FOMC's projections (SEP) of September 2018. The red stars on the lines denote the calibrated standard deviation of the preference shock.

A lower long-term real interest rate raises the expected frequency of the ZLB as it shrinks the central bank's room of maneuver to counter the deflationary effects of recessionary shocks. We are closer to the bound on average so the central bank is expected to hit the lower bound more often. Note that the expected frequency of the ZLB as a function of macroeconomic volatility grows at an increasing speed as the long-term real interest rate $r^*$ falls. Symmetrically, a given drop in the long term real interest rate $r^*$ implies larger increases in the probability of encountering the ZLB if the volatility of the shock is higher. Thus, the more volatile shocks are and the lower $r^*$ is, the higher the expected frequency of the ZLB, with the two effects reinforcing each other.

The graph on the right shows how likely it is for monetary policy to become constrained by the ZLB in the next period conditional on being currently at the

[7] See Hills et al. (2016) for an empirical investigation of the magnitude of the deflationary bias and the associated output bias.

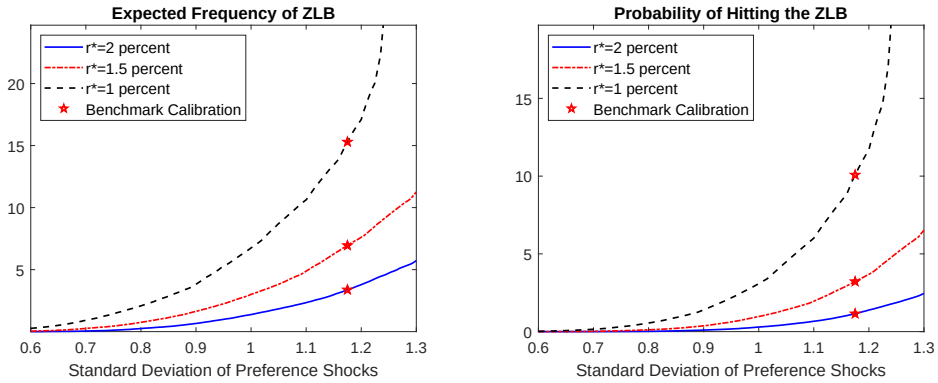



Figure 3: The risk of the zero lower bound. Left graph: Expected frequency of the zero lower bound as the variance of preference shocks varies and for different values of the long-run real rate. The frequency is in percentage points and it is computed as the ratio between the number of periods spent at the zero lower bound and the total sample size (300,000). Right graph: Probability of hitting the zero lower bound next period conditional on being at the stochastic steady state in the current period for different values of the variance of preference shocks and of the steady-state real rate. The probability is expressed in percentage points.

(stochastic) steady state. As for the expected frequency of the ZLB, we study how this probability varies as we change the standard deviation of the preference shocks and the steady-state real rate of interest $r^*$. The larger the volatility of the shock, the more likely it is that the ZLB will be binding in the next period. It should be noted that the probability rises exponentially with the volatility of the shock. Lowering the long-term real rate of interest leads to similar results.

The worrying finding highlighted by both graphs is that in a low real-interest rate environment (low $r^*$, black dashed lines) the two functions are very steep. This means that even a small increase in the volatility of the shocks can lead to substantial increases in the probability of encountering the zero lower bound. Recall that our benchmark calibration for the volatility of the preference shock is arguably very low for the U.S., given that it was chosen to match the level of volatility during the Great Moderation. The results above imply that even a small increase in macroeconomic volatility may lead agents to believe that the ZLB constraint has become a pervasive problem for monetary policy. These beliefs cause serious macroeconomic biases and distortions and can potentially lead to deflationary spirals.

**Deterministic and stochastic steady state** To show that inflation fails to converge to the central bank's target in the absence of inflationary shocks, it is useful to

define the stochastic steady-state equilibrium of the model.[8] We define the deflationary bias as the difference between the rate of inflation at the stochastic steady-state equilibrium and the central bank's inflation target, which coincides with the rate of inflation at the deterministic steady state. The deflationary bias arises when inflation at the stochastic steady state is lower than the central bank's target. A large deflationary bias implies serious hurdles for the central bank to hit its inflation objective.

Both the deterministic and stochastic steady states define an economy that has not been hit by shocks for a sufficiently long number of periods, so that their variables have stabilized around their steady-state values and do not vary anymore (unless a shock suddenly hits). However, in the deterministic steady state, agents fail to appreciate the macroeconomic risk due to future realizations of the shocks. Instead, in the stochastic steady state, agents appreciate the macroeconomic risks due to future realizations of the shocks and adjust their behavior accordingly. While in a linear model these two concepts of steady-state equilibria lead to the same macroeconomic outcome, in non-linear models whether agents act in response to future macroeconomic risks matters.

Unlike the stochastic steady state equilibrium, the deterministic steady-state equilibrium of our model can be characterized analytically.[9] The real interest rate in the deterministic steady state, $r^*$, coincides with $\beta^{-1}$ and captures the long-run level of the real interest rate in the absence of risk. Importantly, $r^* = -\ln(\beta)$ also coincides with the deterministic steady state of the natural interest rate. The deterministic steady state of inflation is pinned down by the inflation target of the central bank, $\Pi$, and can be effectively dealt with as a parameter. Since the price adjustment cost function takes into account the deterministic steady-state inflation rate, the chosen value of the inflation target does not affect any macroeconomic outcomes either at the deterministic steady state or away from the deterministic steady state. Thus, the deterministic steady state for output $Y$ is purely determined by the level of TFP.

Unlike the stochastic steady state, the deterministic steady state is not affected by

[8] Some scholars use the terms "risky steady state" to refer to what we call stochastic steady state. See, for instance, Coeurdacier et al. (2011).

[9] As shown by Benhabib et al. (2001), there exist two deterministic steady-state equilibria once the zero lower bound on nominal interest rates is taken into account. The first steady state is characterized by positive inflation and a positive policy rate. The second steady state is characterized by a liquidity trap, that is, a situation in which the nominal interest rate is near zero and inflation is possibly negative. In line with most of the literature studying new-Keynesian models, we focus on the positive-inflation deterministic steady state.

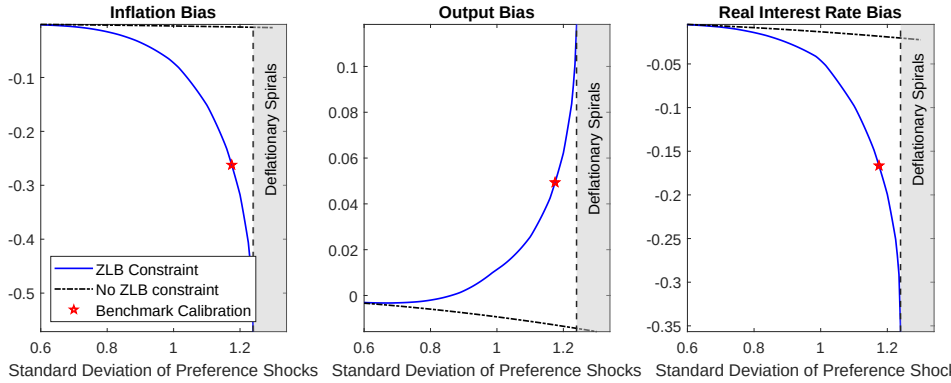



Figure 4: Macroeconomic distortions due the zero lower bound as the volatility of the preference shocks varies. Left graph: The inflationary bias due to model's non-linearities. The red star denotes the calibrated value of the standard deviation of this shock. The difference between the blue solid line and the black dot-dashed line captures the deflationary effects of a risk of a recession that pushes the nominal interest rate to its lower bound. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. The gray area marks the region of the values for the standard deviation of the preference which trigger deflationary spirals. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

macroeconomic uncertainty, which influences the optimal behavior of rational agents in non-linear models. Such volatility drives a wedge between the outcomes of these two steady-state equilibria and hence fuels the deflationary bias. In this section, we will show that among the many sources of non-linearity in the model (e.g., the non-linearities that give rise to precautionary savings), the zero lower bound constraint is the main culprit behind the formation of the deflationary bias and all the associated macroeconomic biases.

**The Deflationary Bias** The left graph of Figure 4 shows the difference between the inflation rate at the stochastic steady state and inflation at the deterministic steady state with (blue solid line) and without the zero lower bound constraint (black dash-dotted line). Comparing the blue solid line with the black dash-dotted line allows us to isolate the effects of the ZLB constraint on the inflation bias. From the figure, it is easy to conclude that when removing the ZLB constraint, the gap between the deterministic and stochastic steady state is quite low. Instead, the risk of hitting the zero lower bound can lead to large discrepancies between the desired and realized levels of inflation.

The red star denotes the deflationary bias that arises at the benchmark value of the standard deviation of the preference shock (Table 1). Inflation undershoots the central bank's inflation target by 27 basis points because of the risk of hitting the ZLB in the

future. As the macroeconomic volatility increases, the bias widens up exponentially. A one-percentage-point increase in the standard deviation of shocks causes a 15-basis-points reduction in the model's long-run inflation rate. Furthermore, it would take just a two-percentage-point increase in the standard deviation of preference shocks to make deflationary spirals possible. Given that our benchmark calibration reflects a record-low macroeconomic volatility, this result is a reason for concern.

It should also be noted that the steepness of the function of the deflationary bias has to be chiefly imputed to the presence of the ZLB constraint. Indeed, the slope of the black dash-dotted line, which captures the counterfactual case where the ZLB constraint is not enforced and nominal rates are allowed to become negative, is tiny and close to constant for different values of the standard deviation of the shocks.

What if the central bank realizes that inflation is in general below the desired target and decides to lower its inflation target to make it coincide with average inflation? The deflationary bias induced by the ZLB constraint would become even larger because lowering the target would make the probability of encountering the zero lower bound even larger. We discuss below what the central bank can do to bring inflation in line with the desired target.

**The Output Bias** The center graph of Figure 4 shows the effects of the risk of hitting the ZLB on the long-run level of output. As before, the long-term output bias due to the zero lower bound is given by the vertical difference between the blue solid line and solid dashed-dot line, which gives us the bias when the ZLB constraint is not imposed. The output bias is positive because the central bank has a two percent inflation target but inflation fluctuates around its stochastic steady state that is lower than the central bank's target (see the left graph of Figure 4). As a result the central bank keeps the interest rate lower than its deterministic steady-state level to close the negative inflation gap. Since the central bank applies the Taylor principle ($\theta_{\Pi} > 1$), this expansionary monetary policy leads to a negative bias in the real interest rate, as shown in the right graph of Figure 4. This monetary stimulus drives a positive wedge between the level of output at the stochastic steady state and that at the deterministic steady state.

It should be noted that if we relax the ZLB constraint, the other non-linearities in the model would imply a level of output *lower* than the deterministic steady-state value. The difference between the two would be increasing in the volatility

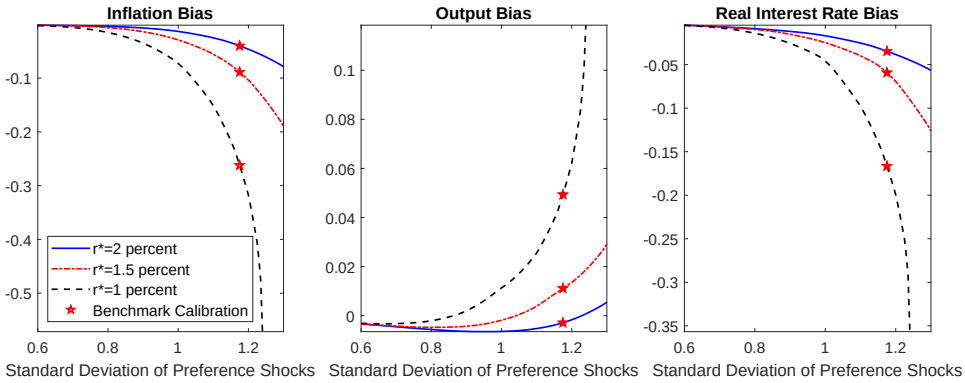



Figure 5: Macroeconomic distortions due the zero lower bound as the standard deviation of preference shocks varies (x-axis) and for alternative values of the steady-state real rate of interest. Left graph: The inflationary bias due to the zero lower bound constraint. The red star denotes the calibrated value of the standard deviation of this shock. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

of the shock. In the absence of the ZLB constraint, the deflationary bias becomes tiny (see the graph on the left) and therefore the central bank will respond to this by lowering the interest rate only by a little. Moreover, in this case precautionary motives, which prompt households to save more to shelter themselves against future risks, become the driver of the negative long-term output bias. The positive bias due to the lower bound constraint dominates these effects for our benchmark calibration of the standard deviation of preference shocks, which is marked by the red star in the plot.

**Implications of a Low Natural Interest Rate.** The results we have discussed so far rely on the assumption that the long-run natural rate of interest is fixed and equal to one percent. Now we show that the combination of a low interest rates environment and the presence of the zero lower bound gives rise to the deflationary bias. Increasing values of the real rate of interest would mitigate or even completely eliminate the bias on inflation because it would be less likely that monetary policy will become constrained by the ZLB, as shown in the right plot of Figure 3.

Figure 5 precisely illustrates these results by showing the effects of changing both the standard deviation of shocks and the long-term real rate of interest $r^*$. The important takeaway from this graph is that as the long-term real interest rate $r^*$ increases sufficiently, the long-term inflation and output biases disappear. The intuition is straightforward: when the long-term real interest rate is higher the central bank

has more room to counteract the deflationary effects of a contractionary shock and hence is less likely to become constrained by the zero lower bound (see Figure 3).

It is worth noting that a slightly lower real interest rate $r^*$ than that of our benchmark calibration can lead to deflationary spirals (the gray area). In such an unfavorable state of the world, the central bank loses control over inflation expectations because the binding ZLB constraint becomes so pervasive that the central bank cannot prevent inflation expectations from being swallowed by the deflationary spirals, as shown in Section 3.

Moreover, a higher real rate of interest $r^*$ would make the function of the deflationary bias less steep and therefore would increase the threshold of the volatility of shocks that triggers the deflationary spirals. It is also interesting to notice that an increase in the long-term real rate of interest of one percentage point more than halves the deflationary bias in our benchmark calibration, denoted by the red star in the graph.

The size of the bias due to non-linearities in the model other than the ZLB does not vary with the long-term real interest rate (not shown), suggesting that the long-term macroeconomic biases linked to a low-interest-rate environment is entirely due to one specific source of non-linearity in the New Keynesian model: the zero lower bound.

To sum up, the deflationary bias brought about by the presence of the ZLB can generate first-order distortions for a central bank that tries to stabilize inflation around the target. Furthermore, we noticed that the combination of a low long-term real interest rate, $r^*$, and moderate macroeconomic risk can trigger the long-run bias in inflation and output or, even worse, deflationary spirals.

**The Unconditional Bias** The previous section has shown that even when the economy is at the stochastic steady-state equilibrium and thus away from the zero lower bound, a deflationary bias arises because of the risk of encountering the zero lower bound in the future. This, in turn, triggers a bias in the real interest rate, as the central bank tries to lift inflation closer to the target and drives a wedge between actual output and optimal output. While the deflationary bias can be defined and measured within the context of a structural model, it cannot be directly measured in the data. A concept of deflationary bias that can be observed more directly in the data is the *unconditional deflationary bias*, which we define as the difference

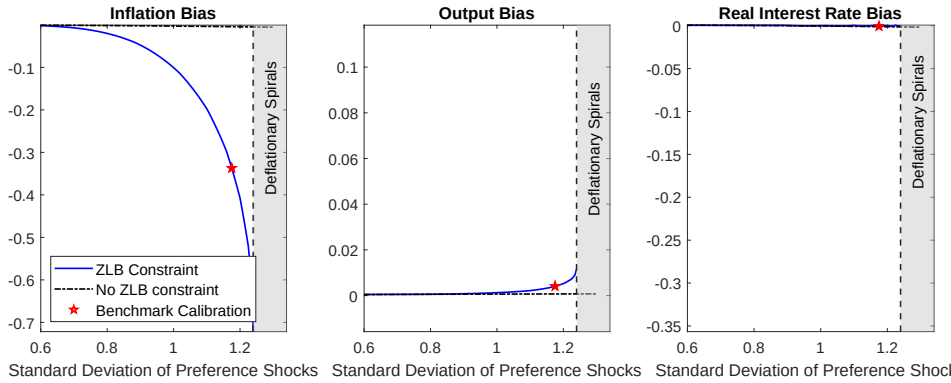



Figure 6: Average macroeconomic biases as the volatility of the preference shock varies. The bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 1,000,000 periods. We drop the first 100,000 observations to minimize the effects of initial conditions. The biases are reported on the same scale used in Figure 3.

between the model's unconditional mean of inflation and inflation at the deterministic steady-state equilibrium (i.e., the central bank's inflation target $\Pi$). This alternative concept of gap does not only reflect the risk of hitting the ZLB but it also reflects the inflation outcomes observed when ZLB episodes actually materialize. As such, the unconditional deflationary gap is more closely related to the bias shown in Figure 1 than the concept used in the previous section.

To compute the unconditional inflation bias, we simulate the model for several periods and then compute the mean of the variables of interest. Figure 6 reports the average bias as the volatility of the preference shock varies. The bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 1,000,000 periods. We drop the first 100,000 observations to minimize the effects of initial conditions. The biases are reported on the same scale used in Figure 4.

The unconditional deflationary bias is even larger than the deflationary bias shown in Figure 4. When computing the unconditional bias, the zero lower bound is not a mere possibility, but an event that occasionally occurs and, in fact, depresses the dynamics of inflation. Thus, average inflation is even further away from the desired inflation target because the economy experiences the deflationary pressures associated with the ZLB period.

This behavior of inflation seems consistent with what is reported in Figure 1. In the late 1990s, the conquest of US inflation was completed. The central bank was successful in convincing agents about the 2% inflation target. In terms of the

model, this event can be captured as convergence toward the stochastic steady state associated with a 2% inflation target. Such a low target, combined with the low natural rate of interest leads to a deflationary bias − even if the zero lower bound is not binding − and causes inflation to drift down and away from the desired 2% target. In fact, during those years the Federal Reserve was genuinely concerned about the risk of deflation (Krugman 2003). With the 2008 recession, the ZLB risk materialized. The model predicts in this case a further reduction in inflation, as in the data. Finally, as the economy recovers, the model predicts that inflation does not return to a 2% target, but it stabilizes around a lower value corresponding to the stochastic steady state.

When it comes to the behavior of output and the real interest rate, the bias is largely gone. When looking at the average bias for the real interest rate, there is a countereffect that pushes the bias to be positive. This countereffect is brought about by the presence of the ZLB itself that truncates the left tail of the distribution of the nominal interest rate. Thus, the negative bias that arises away from the zero lower bound is compensated by the fact that at the zero lower bound the central bank cannot further lower the interest rate, making the effective real interest rate too high. Importantly, the two phenomena are just the two sides of the same coin: The negative bias away from the zero lower bound is generated by the deflationary pressure that arises exactly because at the zero lower bound the central bank is not able to lower the interest rate to mitigate the fall in inflation.

# 5 The Asymmetric Rule

We have shown that the deflationary bias induced by the ZLB increases when the natural interest rate $r^*$ declines or macroeconomic volatility rises. We now turn our attention to what the central bank can do to address the deflationary bias.

## 5.1 The Policy Proposal

In the academic literature and in policy circles, there has been an ample discussion about the possibility of increasing the inflation target as a way to avoid the perils of the zero lower bound. An increase in the target would reduce the possibility of hitting the zero lower bound and the associated bias, as shown by Coibion et al.

(2012) and Nakata and Schmidt (2016). However, Nakamura et al. (2018) show that standard models are unreliable when it comes to assess the welfare implications for the optimal inflation target. Moreover, policymakers have been quite reluctant to reconsider the target of inflation because they fear losses of reputation and argue that higher inflation is historically associated with more volatile inflation. Another line of research has proposed price or nominal GDP targeting or average-inflation targeting (Mertens and Williams 2019). However, such policies are perceived as risky because they may require the central bank to engineer a deflation over certain periods of time.

In this paper, we propose a different approach that does not require the central bank to explicitly aim at hitting a time-varying inflation target. Instead, the central bank reacts less aggressively to positive deviations of inflation from the target than to negative deviations. We will show that embracing this asymmetric strategy can effectively remove the deflationary bias.

The policy strategy that we propose implies a smaller response to inflation when inflation is above target. Specifically, we consider the following modified policy rule:

$$R_t = \max\left[1, \left[\mathbf{1}_{\Pi_t<\Pi}\left(\frac{\Pi_t}{\Pi}\right)^{\underline{\theta_\Pi}} + (1-\mathbf{1}_{\Pi_t<\Pi})\left(\frac{\Pi_t}{\Pi}\right)^{\overline{\theta_\Pi}}\right]\left(\frac{Y_t}{Y}\right)^{\theta_Y} R\right] \tag{10}$$

where $\underline{\theta_\Pi}$ denotes the response of inflation when inflation is below target, $\overline{\theta_\Pi}$ stands for the response to inflation when inflation is above target, and $\mathbf{1}_{\Pi_t<\Pi}$ is an indicator function that is equal to one when inflation is below target ($\Pi_t < \Pi$). In what follows, we set $\underline{\theta_\Pi} = 2$ as in the benchmark calibration of Section 2.2 and study how the average and stochastic steady state biases vary in response to changes in $\overline{\theta_\Pi}$.

The asymmetric rule in equation (10) can be interpreted as a strategy according to which the central bank is slower in raising rates when inflation goes above target. This strategy reduces the risk of encountering the zero lower bound and its undesirable effects. It is therefore particularly effective in a low interest rates environment, like the current one, in which the biases on key macroeconomic variables can be sizable.

Figure 7 shows how the macroeconomic distortions due to the zero lower bound vary as a function of the central bank's response to above-target inflation. We examine the behavior of the bias away from the zero lower bound (stochastic steady state, blue solid line) and its unconditional mean (red dashed-dotted line). The red stars denote

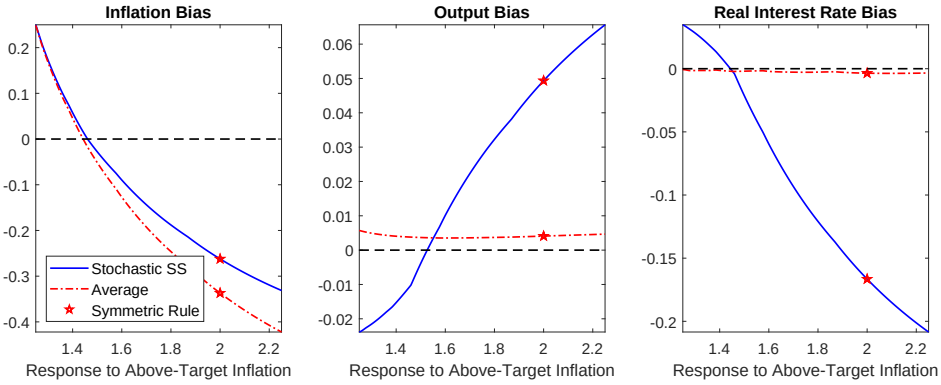



Figure 7: Macroeconomic biases due to the ZLB constraint as the central bank varies its response to positive deviations of inflation from target. The inflation bias (left plot), the output bias (center plot), and the real interest rate bias (the right plot) are computed by taking the difference between these variables at the stochastic steady state and their value at the deterministic steady state (blue solid line). These biases are also computed as the difference between the unconditional mean of these three variables and their value at the deterministic steady state (red dashed-dotted line). The response when inflation is below target is always equal to 2 as in the benchmark calibration. The red star marks the symmetric case in which the central bank responds with equal strength to inflation or deflation. Units: The inflation and the real interest rate biases are expressed in annualized percentage points and the output gap in percentage points.

the distortion under a symmetric rule with a response to inflation equal to two, as in the benchmark calibration.

We observe that being less aggressive when inflation is above target helps to mitigate all three biases. Specifically, for a response $\overline{\theta_\Pi}$ around 1.5, the ZLB-driven macroeconomic distortions become negligible. In a nutshell, to remove the macroeconomic distortions due to the ZLB constraint, policymakers need to be willing to tolerate inflation above the target for longer periods of time. By raising the long-run inflation expectations, the asymmetric strategy also makes the deflationary spirals less likely to happen. Graphically, this makes the gray areas in Figure 4 smaller. This is an important point to which we will return in Section 5.3.

It is worth emphasizing that this policy effectively reduces the probability of hitting the ZLB. This is reflected in the reduction of the distance between the inflation target (deterministic steady state) and the stochastic steady state of inflation. As explained above, in this case, the economy is currently away from the zero lower bound. The reduction in the bias is therefore a result of a lower risk of hitting the zero lower bound in the future.

**The Asymmetric Strategy Is Not a Makeup Strategy** The asymmetric strategy proposed in this paper removes the deflationary bias because it raises the probability of inflation on the upside and, in doing so, offsets the downside risk due to

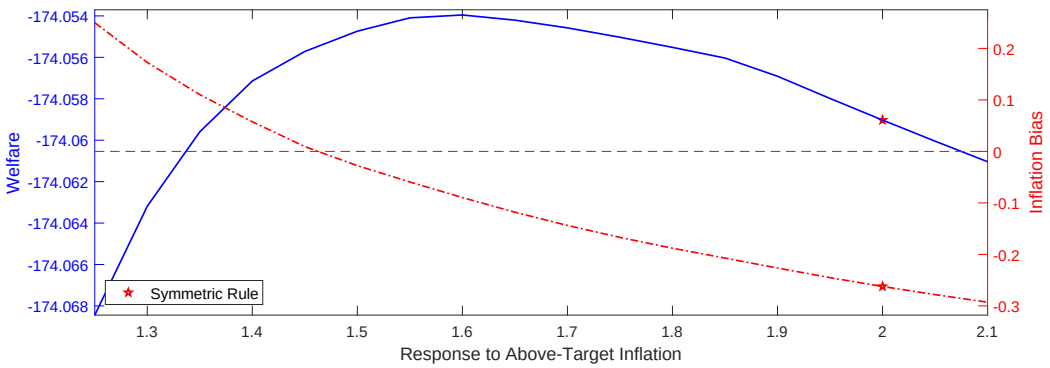



Figure 8: Welfare and inflation bias as the response to positive deviations of inflation from target varies in magnitude. Welfare is evalutated at the stochastic steady state and reported on the left axis. The inflation bias is defined as the difference between the annualized percentage rate of inflation at the stochastic steady state and the annualized percentage rate of inflation at the deterministic steady state and is reported on the right axis.

the ZLB. Hence, our strategy differs from the so-called makeup strategies (e.g., price-level targeting, and average inflation targeting) that correct the deflationary bias by committing the central bank to overheat the economy after the ZLB episodes. Consequently, makeup strategies rely on history dependence which − it is often argued − makes these strategies hard to communicate to the public.

While both approaches require the central bank to make some sort of commitment, the nature of the commitment is very different. The asymmetric strategy commits the central bank to respond asymmetrically to *current* deviations of inflation from the central bank's target with no account for the past dynamics of inflation. The asymmetric strategy never requires the central bank to engineer an overshooting in inflation. In Appendix C, this important property of the asymmetric strategy is illustrated using a simulation exercise. The challenges in communicating the asymmetric strategy are discussed in the next section.

## 5.2 Welfare Analysis

We evaluate the appeal of the asymmetric strategy by measuring its impact on households' welfare $W_0$, which reads as follows:

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^d \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{11}$$

Figure 8 shows welfare $W_t$ (left axis) and the inflation bias (right axis). As the

central bank deviates from the symmetric strategy (the red star) by lowering the response to above-target inflation, welfare increases. When this response is around 1.6, the welfare peaks and then it declines as the response to positive inflation deviations from target is further decreased. It should be noticed that to close the deflationary bias, the central bank has to respond more weakly to inflation than optimal. The asymmetric strategy that completely removes the deflationary bias, is suboptimal in that it allows too large and persistent positive deviations of inflation from the central bank's target. To see this, note that the optimal asymmetric rule solves the following trade-off. On the one hand, by tolerating some persistent positive deviations of inflation from its target the central bank manages to mitigate the deflationary bias. On the other hand, the central bank allows larger positive deviations of inflation from its target.

**Opportunistic Reflation** Announcing that the central bank will respond less aggressively to inflation when inflation will be above target is time inconsistent if this announcement is made when inflation is below target. Therefore, the central bank needs an opportunity to show the public its commitment to the new asymmetric rule. The arrival of a shock that pushes inflation above target is such an opportunity. We call this scenario opportunistic reflation. We now investigate the implications for welfare and the macroeconomic outcomes of a central bank pursuing an opportunistic reflation.

Let us assume that the economy is initially at the stochastic steady state associated with the symmetric rule when it gets hit by a positive preference shock that boosts consumption and aggregate demand. The central bank receives now the opportunity to show to the private sector that it is willing to commit to the optimal asymmetric rule by responding less aggressively to the inflation consequences of this shock. It is assumed that by observing the muted response to inflation, the private sector immediately believes that the central bank will follow the asymmetric rule forever.

In Figure 9, we show the impulse response function of welfare and the macroeconomic gaps (inflation and output) to a two standard deviation positive preference shock under the symmetric rule and under the optimal asymmetric rule. The output gap is measured in deviations from the flexible price economy whereas the inflation gap is expressed in deviations from the central bank's two-percent target. The optimal asymmetric rule raises the output and inflation gaps in the short run relative

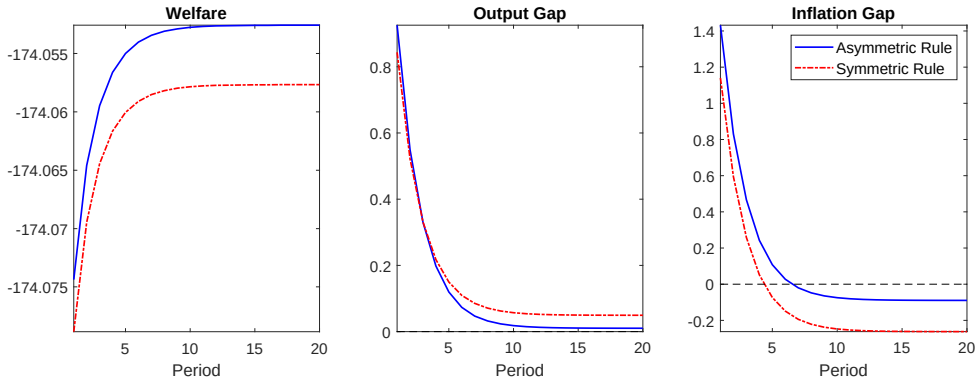



Figure 9: The dynamics of welfare, the output gap, and the inflation gap after a two-standard-deviation positive preference shock hits the economy in period 1. Two cases are reported: the case in which the central bank adopts the optimal asymmetric rule and conducts an opportunistic reflation of the economy (solid blue line) and the case in which the central bank does not take this opportunity and sticks to the symmetric rule (red dashed-dotted line). In both cases, the economy is initialized at its stochastic steady state. Units: Inflation gap is measured in percentage points of annualized rates while the output bias is expressed in percentage points.

to the symmetric rule whereas it mitigates the macroeconomic gaps in the longer run. Welfare is reported in the left graph of Figure 9, which shows that the optimal asymmetric rule raises welfare both in the short run and in the longer run.

Why is welfare higher in every period when the central bank adopts the asymmetric rule even though this rule causes output and inflation gaps to widen more at the beginning? Welfare does not depend only on the current inflation and output gaps but it is also affected by the expected discounted stream of welfare gains that will be accrued over time. The short-term responses of social welfare to a two-standard-deviation positive preference shock implies that the long-term welfare gains associated with the mitigation of the macroeconomic biases outweigh the short-term welfare losses.[10]

The opportunistic reflation involves a trade-off between short-term and long-term macroeconomic stabilization. Hence, a myopic central bank may refrain from seizing this opportunity as welfare costs are mostly front-loaded.[11] To further investigate this issue, we tweak the welfare function (11) to study the behaviors of a myopic central banker who only cares about the welfare gains accrued up to a finite time horizon $k$. The welfare of the myopic central banker is denoted by $\widetilde{W}_0^k$, which is defined as

[10] Under the asymmetric rule, the weaker systematic response to inflation raises agents' long-run uncertainty about inflation and hence, everything else being equal, lowers welfare in the long-run. However, in our model these losses are dominated by the gains from removing the deflationary bias.

[11] In what follows, a myopic central bank can also be interpreted as a conservative central bank that cares too much about the short-term inflation consequences of its actions.

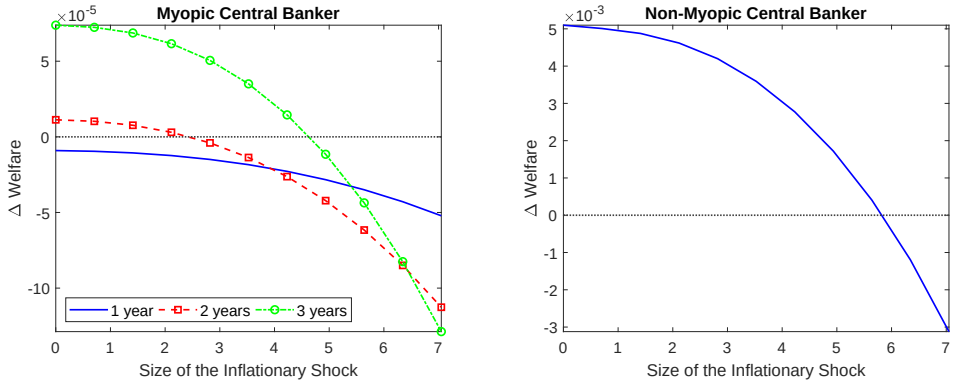



Figure 10: Welfare gains/losses from carrying out an opportunistic reflation as the size of the inflationary shock varies under different assumptions about how forward looking the central banker is. The left plot shows the myopic central banker's case and the different lines refer to different degrees of myopia; that is, the horizon k the central banker cares about when computing welfare gains/losses. The right plot shows the case of the benevolent central banker who maximizes the households' utility and thereby cares about the welfare gains at all horizons. Welfare gains/losses are computed as the difference between the welfare associated with adopting the optimal asymmetric rule and the welfare associated with sticking to the benchmark symmetric rule in the period when the inflationary shock hits the economy.

follows:

$$\widetilde{W}_0^k = E_0 \sum_{t=0}^{k} \beta^t \zeta_t^d \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{12}$$

The left plot of Figure 10 shows the myopic central bank's welfare gains from carrying out an opportunistic reflation following a positive preference shock as the size of the shock varies. The gains are computed by taking the difference of the welfare under the asymmetric rule and welfare under the benchmark symmetric rule at the time the inflationary shock hits the economy. The level of asymmetry is the one we find to be optimal for the non-myopic central banker. The different lines are associated with four degrees of the central banker's myopia, which is captured by the relevant horizons $k = 4$, 8, and 12 quarters. The shorter the horizon $k$, the more myopic the central banker. The gains are shown as a function of the size of the shock. The myopic central banker's gains decline as the size of the preference shocks increases and, hence, the short-run response of inflation to the shock is more pronounced. The speed of this decline increases as the myopia of the central banker becomes less severe.

If the relevant horizon is less or equal than four quarters ($k \leq 4$), gains are negative for all positive shock sizes. Such high levels of myopia dissuade the central bank from seizing the opportunity of reflating the economy as the policymaker is more allured by the short-run welfare gains, which stem from mitigating the immediate inflationary consequences of the shock. If the myopic central bank has a horizon of two years, it will opportunistically reflate the economy if the standard deviation of preference

shocks is lower than two. Lower degrees of myopia (higher $k$) lead the central bank to carry out the opportunistic reflation even when the magnitude of the shock is large and the likely short-run inflationary consequences of the shock are considerable.

The right plot of Figure 10 shows the welfare gains from opportunistic reflation for the case of the non-myopic/benevolent central banker ($k \longrightarrow \infty$). In this case, the optimal asymmetric rule dominates the symmetric rule if the size of the shock is less than 6 times the calibrated standard deviations of the shocks (i.e., $100\sigma_{\zeta^d} = 1.175$). We consider this value as fairly high, which suggests that opportunistic reflation increases the economy's welfare by removing the deflationary bias, as long as the central bank internalizes the long term benefits of the policy.

Finally, if no opportunity to reinflate the economy occurs, the central bank can implement the asymmetric strategy by cutting the rate more aggressively when inflation is below target. This action shows to the public that the central bank has credibly adopted an asymmetric strategy. Appendix D shows that this alternative asymmetric strategy also removes the deflationary bias by lowering the probability of hitting the ZLB.

## 5.3 Asymmetric Rules and Deflationary Spirals

As already discussed in Section 4, adopting an asymmetric strategy does not only remove the deflationary bias but it also lowers the risk for the economy of experiencing deflationary spirals. Since in our model parameters are fixed, welfare is not directly affected by this risk. Nevertheless, falling into a deflationary spiral may be very costly for the economy. The gray areas in Figure 11 denote the values of the standard deviation of preference shocks (upper panels) and the values of the long-term real interest rate (lower panels) that trigger the deflationary spirals for any given above-target response to inflation (left panels) and for any given below-target response to inflation (right panels). The bigger the asymmetry in the parameters of the rule, the bigger the macroeconomic uncertainty (the smaller the real rate of interest) has to be to trigger deflationary spirals. This is because asymmetric rules make the risk of encountering the ZLB lower.

Mertens and Williams (2019) study a rule according to which the Federal Reserve enforces an upper bound on the federal funds rate to resolve the deflationary bias. This rule, while correcting the bias, would imply an increase in the probability of

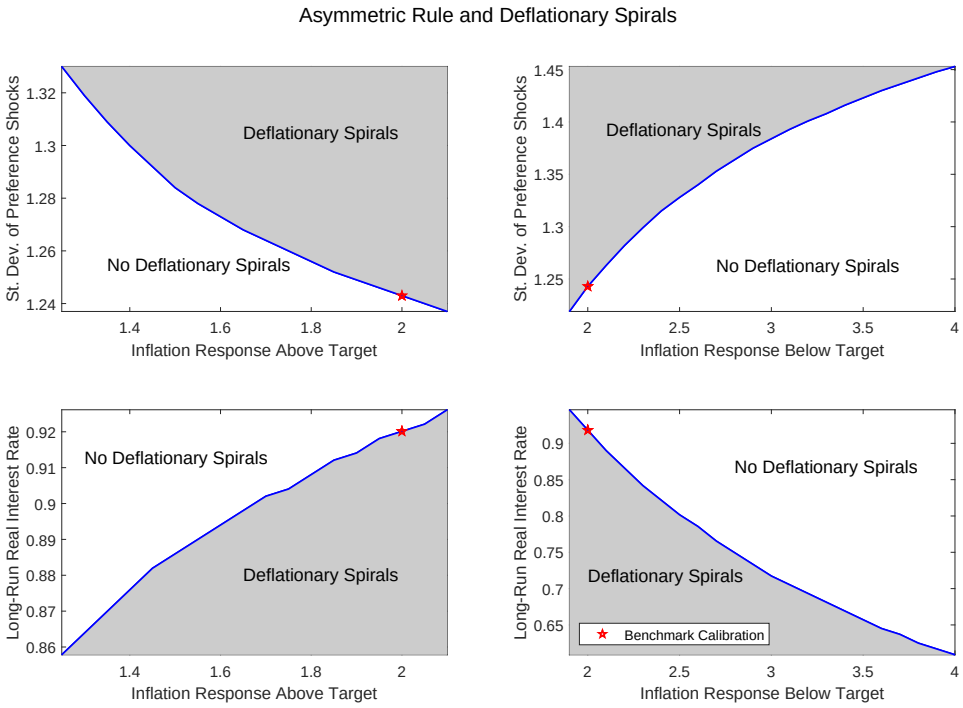



Figure 11: Asymmetric Rule and Deflationary Spirals. Upper left plot: the values of the standard deviation of preference shocks above which deflationary spirals arise as the above-target response to inflation varies and the below-target response is set to be equal to 2.0. Upper right plot: the value of the standard deviation of preference shocks above which deflationary spirals arise as the below-target response to inflation varies and the above-target response is set to be equal to 2.0. Lower left plot: the values of the real long-term interest rate below which deflationary spirals arise as the above-target response to inflation varies and the below-target response is set to be equal to 2.0. Lower right plot: the values of the real long-term interest rate below which deflationary spirals arise as the below-target response to inflation varies and the above-target response is set to be equal to 2.0. The red stars mark the the thresholds for the standard deviation of the preference shock and for the real interest rate under the benchmark calibration (symmetric rule).

inflationary spirals because effectively monetary policy becomes passive when inflation goes above a certain level. Therefore, such a rule reduces the risk of deflationary spirals at the cost of increasing the risk of triggering inflationary spirals. Instead, our asymmetric rule always implies active responses to inflation deviations from the target and hence does not expose the economy to the risk of indeterminately large increases in inflation.

# 6 Target Ranges

In a recent meeting, the FOMC focused on two classes of alternative proposals to revisit the long-run monetary policy framework. The first class involves dynamic

strategies that make up for periods of below-target inflation. The second class is in line with what advocated in this paper and it includes "those [strategies] that respond more aggressively to below-target inflation than to above-target inflation," (minutes of the FOMC meeting, September 17–18, 2019). According to the minutes, several FOMC members also proposed a specific way to implement the asymmetric strategy: "In this context, several participants suggested that the adoption of a target range for inflation could be helpful in achieving the Committee's objective of 2 percent inflation, on average, as it could help communicate to the public that periods in which the Committee judged inflation to be moderately away from its 2 percent objective were appropriate." In what follows, we show that the asymmetric strategy proposed in this paper can in fact be implemented using target ranges as long as the target range is in itself asymmetric around the inflation objective.

To illustrate this point, we consider the following policy rule:

$$R_t = \max\left[1, \left[\mathbf{1}_{\Pi_t \notin [\Pi_L, \Pi_H]} \left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi^O} + (1 - \mathbf{1}_{\Pi_t \notin [\Pi_L, \Pi_H]}) \left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi^I}\right] \left(\frac{Y_t}{Y}\right)^{\theta_Y} R\right] \quad (13)$$

This policy rule prescribes a different response to deviations of inflation from the objective $\Pi$ depending on how far inflation is from the desired level. Specifically, when inflation is inside the target range $[\Pi_L, \Pi_H]$, the central bank adjusts the interest rate less aggressively than what it does when inflation is outside the target range: $\theta_\Pi^I < \theta_\Pi^O$.[12] Such a rule is arguably easy to communicate. For example, if the in-range response $\theta_\Pi^I$ is set to zero, the central bank could simply announce that levels of inflation inside the target range are not reason of concern. However, an asymmetric target range is required to correct the deflationary bias.

In the left panel of Figure 12, we fix the in-range response to inflation to zero ($\theta_\Pi^I = 0$), while keeping the out-of-range response unchanged with respect to the benchmark case ($\theta_\Pi^O = 2$). We then report the target ranges that remove the deflationary bias (the solid blue line). Specifically, for each value of the lower bound of the target range, $\Pi_L$, we report on the y-axis the upper bound, $\Pi_H$, that corrects the deflationary bias. Thus, the U-shaped line reported in the panel represents all the pairs $[\Pi_L, \Pi_H]$ such that the deflationary bias is fully corrected.

[12] The target range rule could also be expressed in deviations from the boundaries of the target range. We prefer this formulation because it nests both a standard Taylor rule and the asymmetric rule presented above.

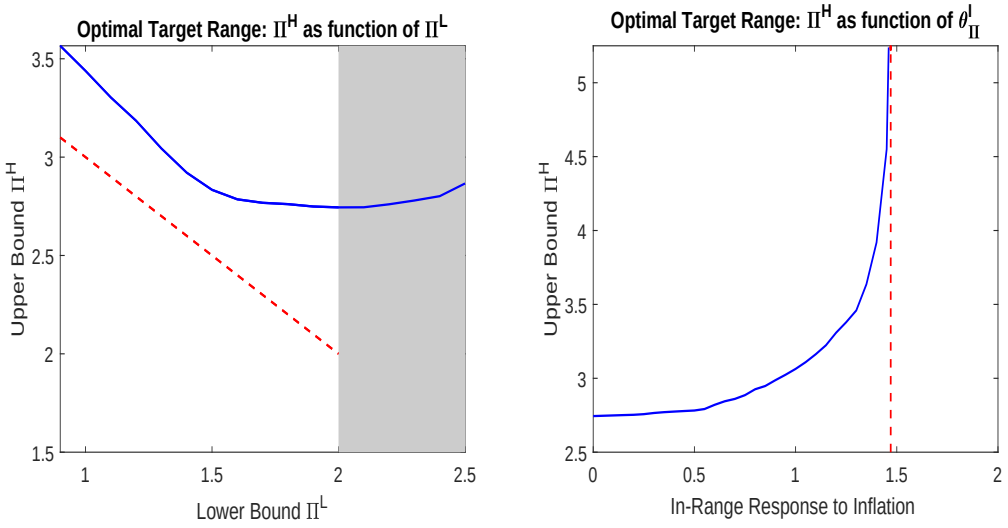



Figure 12: The target range required to close the deflationary bias. The left plot: the blue line shows the lower and upper bounds of the range that closes the deflationary bias when the central bank's in-range response to inflation is zero. The dashed red line marks the bounds implied by the symmetric target range. The right plot: the blue line shows the upper bound of the range as the central bank's in-range response to inflation varies on the horizontal axis. The lower bound of the range is fixed to 2 percent. The vertical red-dashed line is an asymptote that arises when the in-range response to inflation equals the above-target response to inflation in the asymmetric rule that removes the deflationary bias.

We start with a lower-bound $\Pi_L$ equal to 1%. In this case the upper bound needs to be only slightly larger than 3.5%, implying a modest level of asymmetry around the 2% objective. As the lower bound keeps increasing, the upper bound starts declining, but the asymmetry always remains. For instance, a target range $[1.5\%, 2.85\%]$ would also allow the central bank to remove the deflationary bias. To see this, note that the solid blue curve is always above the red-dashed line that implies a symmetric target range.

When the lower bound reaches the 2% objective, the upper bound is around 2.7%. Thus, a target region $[2\%, 2.7\%]$ is necessary to achieve the 2% objective under the assumption of an in-range response to inflation equal to zero. To understand why, it is worth emphasizing that a target region with a lower bound equal to the 2% objective is conceptually very similar to the asymmetric rule presented in Section 5. When inflation is below the objective, the response of the policy rate is strong. When inflation is above the target the response is weaker, but in a piecewise fashion. In fact, the rule presented in Section 5 can also be thought as a degenerate target range rule in which the upper bound of the target range goes to infinity. The advantage of the target range is arguably that it preserves the message that excessively high levels of inflation will not be tolerated.

The gray area of the graph denotes values of the lower bound $\Pi_L$ that are *larger* than the objective 2%. While these target ranges also succeed in eliminating the deflationary bias, we believe that they are less interesting because they are not so easy to communicate: The target range now excludes the inflation objective ($\Pi_L > \Pi$). Nevertheless, we review this case for completeness. Once the lower bound become larger than the inflation objective, the upper bound of the target range starts increasing again. This is consistent with the results presented so far. Recall that in order to correct the deflationary bias, a rule needs to feature more tolerance to high inflation than to low inflation. When the target range is above the desired objective, higher and higher levels of inflation become progressively acceptable.

The right panel of Figure 12 shows that the amount of asymmetry required to correct the deflationary bias depends on the strength with which the central bank responds to inflation inside the target range. In this exercise, the lower bound of the target range is fixed to 2%. On the x-axis, we report different values of the in-range response to inflation $\theta_\Pi^I$. For each of them, the y-axis reports the upper-bound $\Pi_H$ required to remove the deflationary bias. When the in-range response is equal to zero, the upper bound is around 2.7%, implying only a mild level of asymmetry around the 2% objective: $[2\%, 2.7\%]$. However, as the in-range response $\theta_\Pi^I$ increases, the required level of asymmetry of the target range increases. For example, with an in-range response $\theta_\Pi^I$ equal to 1, the required target range becomes: $[2\%, 3.06\%]$. This pattern accelerates as the inside-range response is raised until the blue line approaches a vertical asymptote. The level of asymmetry goes to infinity as the in-range response $\theta_\Pi^I$ approaches 1.47 and the target range rule collapses to the asymmetric rule of Section 5 that removes the deflationary bias.

Summarizing, a target range can be an effective way to implement an asymmetric policy strategy. However, the target range needs to be asymmetric around the desired objective for inflation. The extent of the asymmetry depends on the response to inflation inside the target range. In the benchmark case of a zero response inside the range, we show that the range needed to remove the deflationary bias is only modestly asymmetric. A target range also allows the central bank to preserve the message that excessively high inflation will trigger a strong policy response.

# 7 Conclusions

In an environment in which monetary policy faces the risk of encountering the zero lower bound, inflation tends to remain persistently below target, even if monetary policy is not constrained. This is because agents anticipate the possibility of low inflation in the future. We showed that an asymmetric policy strategy eliminates the macroeconomic biases due to the ZLB. A strategy according to which the central bank reacts less aggressively to positive deviations of inflation from the 2% target than to negative deviations can effectively remove the macroeconomic biases, improve social welfare, and reduce the risk for the economy to fall into highly costly deflationary spirals.

We argue that convincing agents that the central bank will abandon the old symmetric strategy to embrace the asymmetric one is non-trivial when inflation is below target. A myopic central banker might put too much wight on current inflation volatility and stick to the symmetric rule once inflation increases. This lowers the short-run volatility of inflation, but causes the deflationary bias. A way to address this time inconsistency is to conduct an opportunistic reflation. We show that an opportunistic reflation is welfare improving in a standard New Keynesian model. Nevertheless, the welfare gains are back-loaded and hence the policymaker needs to be sufficiently forward looking to be willing to conduct an opportunistic reflation. Finally, we showed that the asymmetric strategy can be implemented with a target range, as long as the target range is in itself asymmetric. A target range is arguably easy to communicate and allows the central bank to preserve the message that excessively high inflation will not be tolerated.

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# A Non-linear Solution Method

Solving the representative household's problem yields the Euler equation

$$1 = \beta R_t \mathbb{E}_t \left[ \frac{\zeta_{t+1}^d}{\zeta_t^d} \left( \frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{1}{\Pi_{t+1}} \right], \tag{14}$$

where $\Pi_t = P_t / P_{t-1}$ is gross inflation, and the labor supply

$$W_t = \chi N_t^{\eta} c_t^{\sigma}, \tag{15}$$

The firm $j$ produces output with labor as the only input

$$Y_t(j) = A \; H_t(j)^{\alpha} \tag{16}$$

where $A_t$ denotes the total factor productivity, which follows an exogenous process. The firm $j$ sets the price $P_t(j)$ of its differentiated goods $j$ so as to maximize its profits:

$$Div_t(j) = P_t(j) \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - \alpha \; mc_t \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t - \frac{\varphi}{2} \left( \frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right) Y_t, \tag{17}$$

subject to the downward sloping demand curve for intermediate goods. The parameter $\varphi > 0$ measures the cost of price adjustment in units of the final good.

The first order condition is

$$\begin{aligned}(\epsilon - 1) \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} = \epsilon \; \alpha \; MC_t \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon - 1} \frac{Y_t}{P_t} - \varphi \left( \frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right) \frac{Y_t}{\Pi P_{t-1}(j)} + \\ \varphi E_t \Lambda_{t,t+1} \left( \frac{P_{t+1}(j)}{\Pi P_t(j)} - 1 \right) \frac{P_{t+1}(j)}{\Pi P_t(j)} \frac{Y_{t+1}}{P_t(j)}\end{aligned} \tag{18}$$

where the stochastic discount factor $\Lambda_{t,t+1}$ is

$$\Lambda_{t,t+1} = \beta E_t \left[ \left( \frac{\zeta_{t+1}^d}{\zeta_t^d} \right) \left( \frac{C_t}{C_{t+1}} \right)^{\sigma} \right] \tag{19}$$

In equilibrium all firms choose the same price. Thus, the New Keynesian Phillips

curve is

$$\mathbb{E}_t\left[\varphi\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\frac{\Pi_t}{\Pi}\right]=(1-\epsilon)+\epsilon\ \alpha\ MC_t+\varphi\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\frac{\Pi_{t+1}}{\Pi}\frac{Y_{t+1}}{Y_t}\right] \tag{20}$$

The monetary authority sets the interest rate $R_t$ responding to inflation and output from their corresponding targets. The monetary authority faces a zero lower bound constraint. The policy rule reads as follows

$$R_t=\max\left[1,R\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}\right]. \tag{21}$$

where $\Pi$ and $Y$ denote the inflation target which pins down the inflation rate in the deterministic steady state and the natural output level, which is the level output that would arise if prices were flexible.

The resource constraint is

$$C_t=Y_t\left[1-\frac{\varphi}{2}\left(\frac{\Pi_t}{\Pi}-1\right)^2\right] \tag{22}$$

The model is solved with global methods. The agents take the presence of the zero lower bound into account and form their expectations accordingly. Therefore, the possibility of hitting the zero lower bound in the future affects potentially the equilibrium outcome in times of unconstrained monetary policy. We use time iteration (Coleman 1990 and Judd 1998) with piecewise linear interpolation of policy functions as in Richter et al. (2014).[13] Expectations are calculated using numerical integration based on Gauss-Hermite quadrature.

The state variable is $\zeta_t^d$, while the policy variables are $\Pi_t$ and labor $H_t$:

$$\Pi_t=g^1(\zeta_t^d) \tag{23}$$

$$H_t=g^2(\zeta_t^d) \tag{24}$$

where $g=(g^1,g^2)$ and $g^i:R^1\to R^1$. To solve the model, we approximate the

[13]This approach can handle the non-linearities associated with zero lower bound. Richter et al. (2014) demonstrate that linear interpolation outperforms Chebyshev interpolation, which is a popular alternative, for models with zero lower bound. The kink in the policy functions is more accurately located which gives a more precise solution.

unknown policy functions with piecewise linear functions $\tilde{g}^i$ that can be written as:

$$\Pi_t = \tilde{g}^1(\zeta_t^d) \tag{25}$$

$$H_t = \tilde{g}^2(\zeta_t^d) \tag{26}$$

The time iteration algorithm to solve for the policy functions is summarized below:

1. Define a discretized state grid $[\underline{\zeta_t^d}, \overline{\zeta_t^d}]$ and integration nodes $\epsilon^{\zeta^d} = [\underline{\epsilon_t^{\zeta^d}}, \overline{\epsilon_t^{\zeta^d}}]$.

2. Guess the piece-wise linear policy functions $\tilde{g}(\zeta_t^d)$.

3. Solve for all time $t$ variables for a given state vector $\zeta_t^d$. The policy variables are:

$$\Pi_t = \tilde{g}^1(\zeta_t^d) \tag{27}$$

$$H_t = \tilde{g}^2(\zeta_t^d) \tag{28}$$

so that the remaining variables are given as:

$$Y_t = AH_t^{1-\alpha} \tag{29}$$

$$C_t = Y_t(1 - \varphi\left(\frac{\Pi_t}{\Pi} - 1\right)^2/2) - g \tag{30}$$

$$R_t = \max\left[1, R\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}\right] \tag{31}$$

$$W_t = \chi N_t^\eta c_t^\sigma \tag{32}$$

$$MC_t = \frac{W_t}{(1-\alpha)AH_t(j)^{-\alpha}} \tag{33}$$

Calculate the state variable for period $t+1$ at each integration node $i$:

$$\zeta_{t+1}^{i,d} = \exp\left(\rho_\zeta \log(\zeta_t^d) + \epsilon_{t+1}^{i,\zeta^d}\right)$$

For each integration node $\zeta_{t+1}^{i,d}$, calculate the policy variables and solve for out-

put and consumption:

$$\Pi^i_{t+1} = \tilde{g}^1(\zeta^{i,d}_t) \tag{34}$$
$$H^i_{t+1} = \tilde{g}^2(\zeta^{i,d}_t) \tag{35}$$
$$Y^i_{t+1} = A {H^i_{t+1}}^{1-\alpha} \tag{36}$$
$$C^i_{t+1} = Y^i_{t+1}(1 - \varphi\Big(\frac{\Pi^i_{t+1}}{\Pi} - 1\Big)^2/2) - g \tag{37}$$

Calculate the errors for the Euler Equation and the New Keynesian Phillips curve

$$err_1 = 1 - \beta R_t E_t\Big[\frac{\zeta^d_{t+1}}{\zeta^d_t}\Big(\frac{C_t}{C_{t+1}}\Big)^\sigma \frac{1}{\Pi_{t+1}}\Big],$$
$$err_2 = \varphi\Big(\frac{\Pi_{t+1}}{\Pi} - 1\Big)\frac{\Pi_t}{\Pi} - (1-\epsilon) - \epsilon MC_t(1-\alpha) - E_t\varphi\Lambda_{t,t+1}\Big(\frac{\Pi_{t+1}}{\Pi} - 1\Big)\Big(\frac{\Pi_{t+1}}{\Pi}\Big)\frac{Y_{t+1}}{Y_t}.$$

where the expectations are numerically integrated across the integration nodes. The nodes and weights are based on Gaussian-Hermite quadrature.

4. Use a numerical root finder to minimize the errors for the equations.

5. Update the policy functions until the errors at each point of the discretized state are sufficiently small.

# B A Model with Binary Realizations of the Shock

In this binary case, we treat the Taylor rule in the good state and all the other remaining equilibrium equations separately. Using different candidates of inflation for the good state ($\Pi^H$), we calculate two nominal interest rates for the good state $R^{H1}(\Pi^H)$ and $R^{H2}(\Pi^H)$. The first one stems from the Taylor rule, while the other one results from the other remaining equations.

The candidate for the nominal interest rate $R^{H1}(\Pi^H)$ resulting from of the Taylor rule in the good state reads as follows:

$$R^{H1} = \max\left[1, R\Big(\frac{\Pi^H}{\Pi}\Big)^{\theta_\Pi}\right]$$

This equation corresponds to the red line in Figure 2.

The other equilibrium equations in the good state give another solution for the nominal interest conditionally on $\Pi^H$. The remaining equations in the good state are given as:

$$1 = \beta R^{H2}\Big[(1-p)\frac{\zeta_L^d}{\zeta_H^d}\Big(\frac{C^H}{C^L}\Big)^{\sigma}\frac{1}{\Pi^L} + p\frac{1}{\Pi^H}\Big], \tag{38}$$

$$Y^H = A(H^H)^{1-\alpha}, \tag{39}$$

$$(1-\alpha)MC^H A = \chi H_t^{H\eta} c^{H\sigma}, \tag{40}$$

$$C^H = Y^H\Big(1 - \varphi\Big(\frac{\Pi^H}{\Pi} - 1\Big)^2/2\Big) \tag{41}$$

$$\varphi\Big(\frac{\Pi^H}{\Pi} - 1\Big)\frac{\Pi^H}{\Pi} = (1-\epsilon) + \epsilon MC^H(1-\alpha) \tag{42}$$
$$+ \varphi\beta\left[(1-p)\frac{\zeta_L^d}{\zeta_H^d}\Big(\frac{C^H}{C^L}\Big)^{\sigma}\Big(\frac{\Pi^L}{\Pi} - 1\Big)\Big(\frac{\Pi^L}{\Pi}\Big)\frac{Y^L}{Y^H} + p\Big(\frac{\Pi^H}{\Pi} - 1\Big)\Big(\frac{\Pi^H}{\Pi}\Big)\right]$$

Since the good-state equilibrium outcomes depend on the bad state, we have to solve for the equilibrium in the bad state. An equilibrium in the bad state satisfies the following equations:

$$R^L = \max\left[1, R\Big(\frac{\Pi^L}{\Pi}\Big)^{\theta_\Pi}\right] \tag{43}$$

$$1 = \beta R^L\Big[(1-q)\frac{\zeta_H^d}{\zeta_L^d}\Big(\frac{C^L}{C^H}\Big)^{\sigma}\frac{1}{\Pi^H} + q\frac{1}{\Pi^L}\Big], \tag{44}$$

$$Y^L = A(H^L)^{1-\alpha}, \tag{45}$$

$$(1-\alpha)MC^L A = \chi H_t^{L\eta} c^{L\sigma}, \tag{46}$$

$$C^L = Y^L\Big(1 - \varphi\Big(\frac{\Pi^L}{\Pi} - 1\Big)^2/2\Big) \tag{47}$$

$$\varphi\Big(\frac{\Pi^L}{\Pi} - 1\Big)\frac{\Pi^L}{\Pi} = (1-\epsilon) + \epsilon MC^L(1-\alpha) \tag{48}$$
$$+ \varphi\beta\left[(1-q)\frac{\zeta_H^d}{\zeta_L^d}\Big(\frac{C^L}{C^H}\Big)^{\sigma}\Big(\frac{\Pi^H}{\Pi} - 1\Big)\Big(\frac{\Pi^H}{\Pi}\Big)\frac{Y^H}{Y^L} + q\Big(\frac{\Pi^L}{\Pi} - 1\Big)\Big(\frac{\Pi^L}{\Pi}\Big)\right]$$

Equations (38) to (43) give us a solution for the nominal interest rate $R^{H2}(\Pi^H)$. The nonlinear root solver is applied at this step as this system cannot be solved

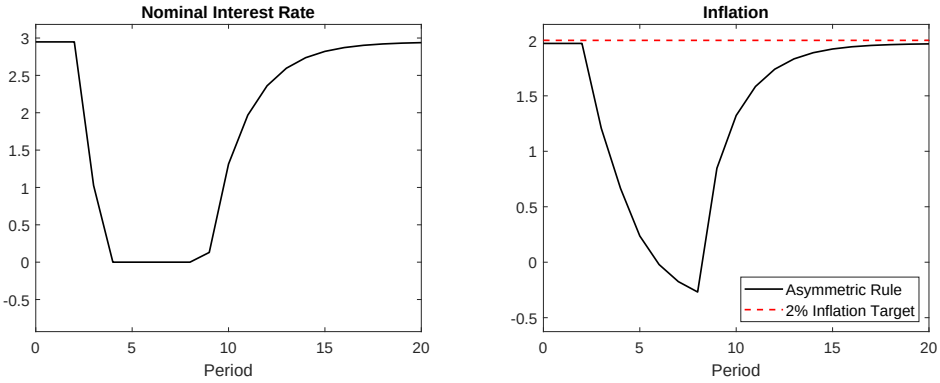



Figure 13: Simulations of inflation and nominal interest rate during an artificial recession. The economy is at its stochastic steady state in period 0, 1, and 2. From period 3 through period 8, the economy is hit by a one-standard-deviation negative preference shock in every period. Starting from period 9 no more shocks occur and the economy evolves back to its stochastic steady-state equilibrium. Units: percentage points of annualized rates.

analytically.[14] The mapping of $\Pi^H$ to $R^{H2}$ corresponds to the blue solid line in Figure 2. To calculate a hypothetical economy without a zero lower bound in the bad state, we we assume that the ZLB constraint is not binding in that state. This gives us the dash-dotted blue line in Figure 2.

An equilibrium for the economy exists for a given inflation in the good state $\Pi^H$ if $R^{H1}(\Pi^H) = R^{H2}(\Pi^H)$. This corresponds to an intersection of the red and the blue line in Figure 2. Looping over $\Pi^H$ allows to check the existence of equilibria and find all possible solutions of the economy with binary realizations of the preference shock.

# C The Asymmetric Strategy is Not a Makeup Strategy

In this appendix we will show that the asymmetric strategy does not require the central bank to engineer an overshooting in inflation after a ZLB episode as makeup strategies (e.g., price-level targeting, average inflation targeting. etc.) do. To this end, we simulate the economy under a sequence of negative shocks large enough to bring the economy to the zero lower bound for a certain number of periods. We assume that the central bank is following the asymmetric rule that removes the deflationary bias.

[14] To handle the kink in the Taylor rule in the low state, we use a guess and verify approach in practice. First, we solve the whole system assuming that the Taylor rule is not binding in the bad state. We keep the results if the result does not violate the zero lower bound in the bad state. Then, we guess that zero lower bound is binding in the bad state and keep the results if this is indeed the bad-equilibirum outcome.

Figure 13 shows the path for the endogenous variables in the three cases. We assume that the economy is initially at its stochastic steady states and the size of the each shock is one standard deviation. In period 3, a sequence of negative demand shocks hits the economy. Starting from period 9 no more shocks occur and the economy slowly goes back to the stochastic steady state.

In the left plot of Figure 13, the ZLB is binding when the negative preference shocks hit the economy. After the ZLB period, no more shocks hit the economy and the central bank lifts the nominal interest rate off the ZLB constraint. In the right plot of Figure 13, the dynamics of inflation in the simulation is reported. Inflation falls as the economy is hit by the negative preference shocks. As the effects of these shocks fade away, the inflation rate converges to the desired two-percent inflation target. Note that inflation converges to the desired target from below because the central bank does not try to overshoot its inflation target as it would have done if it had adopted a makeup strategy.

# D Strategic Interest Rate Cuts

We showed that if the central bank seizes the opportunity of reflating the economy by adopting an asymmetric rule after an inflationary shock arises, social welfare generally increases. If no opportunity to reflating the economy arises, the central bank can still remove the deflationary bias and improves welfare by cutting more aggressively the interest rate if inflation is below target while clarifying that the response to inflation above target is unchanged.

This alternative asymmetric rule also eliminates the macroeconomic biases. The upper panels of Figure 14 report the behavior of the macroeconomic biases defined with respect to the stochastic steady state (blue solid lines) and the observable averages (red dashed lines) as the response to below-target inflation, $\theta_{\Pi}$, varies. The response to positive deviations of inflation from the target is the same as in the symmetric rule ($\overline{\theta_{\Pi}} = 2$). The red star denotes the distortions under a symmetric rule ($\underline{\theta_{\Pi}} = \overline{\theta_{\Pi}} = 2$) as in the baseline calibration. The response to inflation below target that zeroes the biases is approximately three.

The effects of adopting this asymmetric rule on the probability of hitting the ZLB and the frequency of ZLB episodes is ambiguous ex ante. On the one hand, lowering more vigorously the nominal interest rate to fight against deflationary pressures

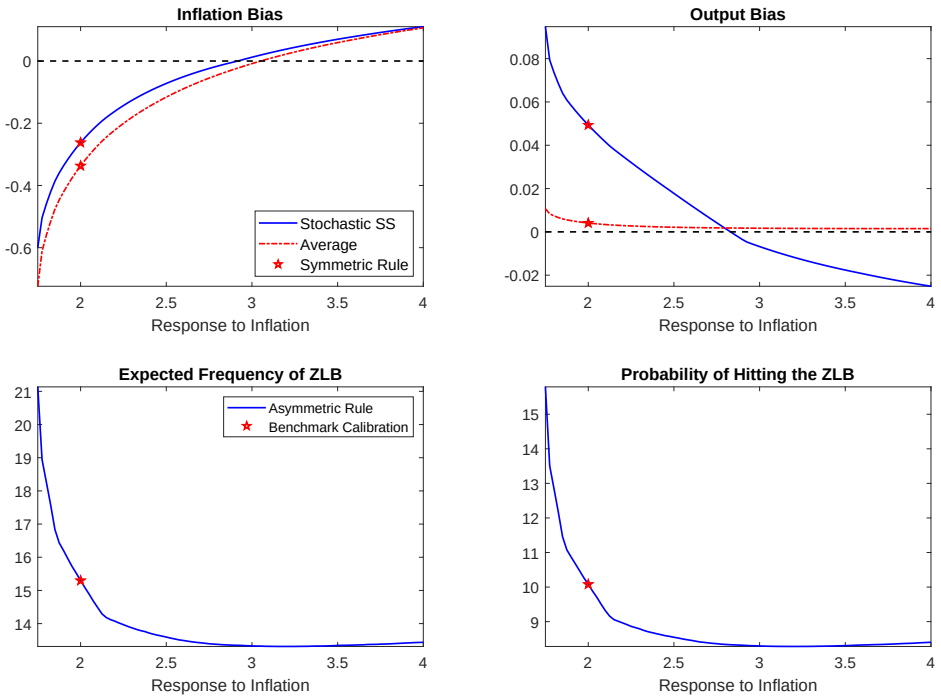



Figure 14: Macroeconomic biases due to risk of hitting ZLB under the asymmetric rule. The biases are computed relatively to the stochastic steady state (blue solid line) or the average inflation (red dashed-dotted line) and are shown in the upper panels. The output gap is expressed in percentage points and inflation gap is expressed in percentage points of annualized rates. The lower panels show the risk of hitting the ZLB in the next period (left) and the expected frequency of the ZLB (right) as the response to inflation below target varies. The frequency is in percentage points and it is computed as the ratio between the number of periods spent at the zero lower bound and the total sample size (300,000). The probability of hitting the zero lower bound in the next period is conditional on being at the stochastic steady state in the current period and is expressed in percentage points.

could increase the probability of hitting the zero lower bound. On the other hand, committing to respond more aggressively to negative deviations of inflation from target eliminates the deflationary bias and thereby raises the long-term nominal interest rate. Higher nominal rates cause the likelihood of hitting the ZLB to fall. As shown in the lower panels of Figure 14, the asymmetric rule that allows the central bank to remove the macroeconomic bias ($\theta_\Pi = 3$) lowers the probability of hitting the ZLB and the expected frequency of ZLB episodes.