Integrated Monetary and Financial Policies for Small Open Economies*

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Abstract

In the Mundell-Fleming framework, fully flexible exchange rates are optimal because they insulate economies from shocks through expenditure switching. However, that model abstracts from imperfections that are supported by empirical evidence and cited by many emerging markets among the reasons for resorting to policies other than plain vanilla monetary policy and exchange rate flexibility. To fill this gap, we develop a micro-founded model that characterizes optimal monetary policy, capital controls and foreign exchange intervention. The framework incorporates a menu of real and financial shocks, and allows countries to differ across a number of dimensions such as the specific form of nominal rigidities, prominence of currency mismatches, tightness of borrowing constraints as well as the depth of foreign exchange markets. The framework then maps these shocks and country characteristics to optimal policies. The analysis yields a number of broad principles. For example, if an additional instrument becomes available, it should not necessarily be deployed because it may not be the right tool to address the imperfection at hand. The use of a new policy instrument can lead to more or less use of the existing ones as instruments interact with each other in nontrivial ways.

*Preliminary and incomplete. Please do not circulate. Thanks to Javier Bianchi, Marcos Chamon, Giovanni Dell'Ariccia, Emmanuel Farhi, Olivier Jeanne, Matteo Maggiori, Enrique Mendoza, Pau Rabanal and Ludwig Straub for insightful discussions. Thanks to Zidong An for excellent research assistance. The views expressed in this paper are those of the authors and do not necessarily represent those of the IMF, its Executive Board, or management.

1 Introduction

The Mundell-Fleming model remains the workhorse framework for the analysis of small open economies in the policy space. In that model, flexible exchange rates are optimal, and can deliver full employment together with low inflation after domestic and external shocks. This is partly because of a powerful expenditure-switching channel that operates on both the import side and the export side. However, the world is more complex than modeled in the Mundell-Fleming framework: the inflexibility of prices can take different forms, and international and domestic financial imperfections are prevalent. These complexities can make monetary policymaking more involved than setting a policy rate and allowing the exchange rate to flexibly adjust. This paper provides an analytical framework to guide policymakers with the joint configuration of monetary policy, capital controls and foreign exchange intervention.

This work is motivated by two observations. First, the empirical evidence is inconsistent with some of the assumptions underlying the Mundell-Fleming framework. Most emerging markets have dollar invoicing shares above 80 percent.[1] This empirical finding implies that we need to consider a dominant currency pricing paradigm where export prices are sticky in a dominant currency, which is most often the dollar and in some cases the euro. Similarly, on the financial side, a large literature establishes an array of imperfections in capital markets. For example, foreign currency borrowing is prevalent and generates a link between the exchange rate and the macroeconomy through currency mismatches and borrowing constraints. Indeed, currency mismatches have been cited as a reason for using alternative tools to limit exchange rate movements in Brazil, Indonesia and Peru. Another imperfection on the financial side relates to the foreign exchange markets where international financial intermediaries generally have a limited appetite for taking on emerging markets' currency exposure and uncovered interest parity condition breaks down. Those countries which intervened heavily in FX markets during the depreciation episodes tend to be countries where balance sheet concerns prevailed, and where financial markets were not deep enough to provide hedging opportunities.

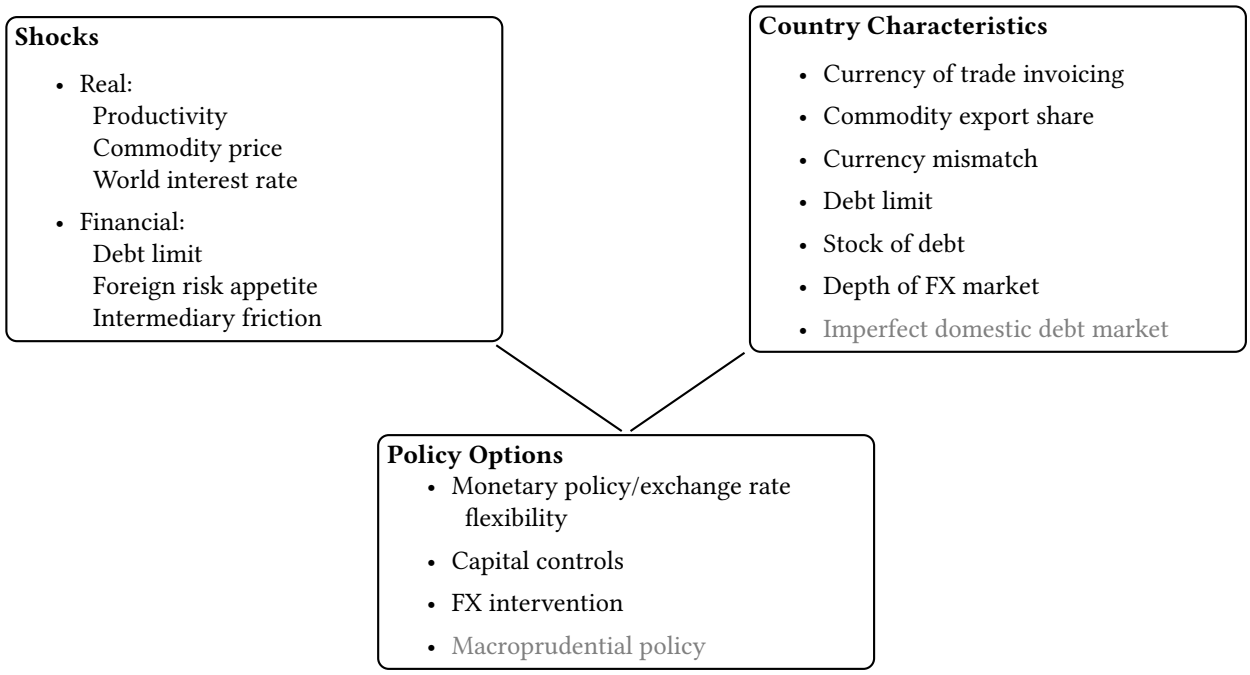
[1] Exception to this are European Union countries that rely heavily on the euro.

The second observation is that many countries adopt more eclectic approaches than plain-vanilla monetary policy and floating exchange rates to cope with shocks. When faced with capital outflow pressures, most emerging markets use multiple tools to achieve macroeconomic stabilization with substantial variation across countries. For example, during the emerging market stress episodes induced by the dollar appreciation in 2018, policy rates were raised in some countries to stem outflows, but in other countries, they were lowered. While some countries allowed the exchange rate to depreciate, other central banks intervened in the FX market to lean against the depreciation. Given their frequent and heterogeneous use, one of the objectives of this paper is to better understand these alternative tools: what they do, how they interact, the trade-offs involved, and characterize policy counterfactuals. In other words, we aim to establish under what conditions the standard prescription of flexible exchange rates still holds and when it may instead be optimal to limit exchange rate flexibility and rely on other tools. Clearly, doing so requires going beyond the Mundell-Fleming framework.

The novelty of our analysis is that we develop an integrated model that considers jointly the role of monetary policy, capital controls and foreign exchange intervention in small open economies. We characterize the use of these policy instruments as a function of shocks, and both real and nominal frictions. The diagram in figure 1 demonstrates this visually. The top left box in the diagram captures a range of real and financial shocks. In the top right box, we list the country characteristics that our model aims to capture. Ultimately, our goal is to complete this picture fully, to study combinations of shocks and country characteristics and map them to the optimal policy mix, listed in the box at the bottom of the diagram.

To achieve these objectives, we develop a model of a small open economy with three sectors: the tradable goods sector, the commodity sector and the non-tradable goods sector. The price of the non-commodity traded goods is sticky. The price may be sticky in the producer's currency (PCP) or in the dominant currency (DCP). The country is a price taker in commodity markets. On the financial side, households go to the domestic bond market to satisfy their borrowing needs. Financial

Figure 1: Modeling Agenda



Note: Items in gray are to be incorporated.

intermediaries borrow in foreign currency in world markets and meet the households' supply of local currency bonds. These intermediaries are partly foreign and partly owned by domestic agents; this domestic ownership is the source of the economy's currency mismatch. The first inefficiency in the financial market is a borrowing constraint that limits the households' debt to a fraction of their pledgable income. This constraint is not binding in normal times, but it might become binding after a sufficiently bad shock. Importantly, households do not internalize the effects of their individual actions on the exchange rate and on the tightness of the constraint ex-post, which may lead to overborrowing. The second inefficiency in financial markets is that the intermediaries have a limited capacity for bearing the country's currency exposure, which leads to deviations from the uncovered interest parity condition. We refer to this inefficiency as *shallow* FX markets.

Our model captures the key trade-offs associated with monetary policy/exchange rate flexibility, capital controls and FX intervention. Monetary policy, working through changes in interest rates, has its traditional New-Keynesian impact. It affects the interest rate faced by domestic households

when they make borrowing and saving decisions. Flexible exchange rates have some expenditure-switching benefits but can generate costs if they exacerbate financial imperfections. When the exchange rate depreciates, imports become more expensive relative to home-produced goods. Households therefore switch away from imports towards home goods. Under PCP, expenditure switching is also operational through exports: an exchange rate depreciation boosts demand by making exports more competitive. Under DCP, exchange rate adjustment becomes a weaker tool because while it continues to affect import consumption, it no longer affects the competitiveness of exports on world markets, as the dollar price remains unchanged.

Capital controls are modeled as state contingent taxes on capital inflows. Prudential capital controls work to prevent overborrowing by curbing debt and consumption ex-ante, which is desirable when a depreciation can make the borrowing constraint bind by worsening the country's balance sheet through currency mismatches. When FX markets are shallow, another incentive to curb debt through capital controls arises because doing so reduces the losses incurred due to the inefficiency in the intermediation of debt. This use of capital controls can happen in an ex-ante or ex-post fashion. On the downside, capital controls can distort capital flows relative to the efficient benchmark, which may generate welfare costs since those flows can be beneficial for the recipient countries.

Sterilized FX intervention changes the external premium and hence affects the exchange rate by changing how much of the households' debt needs to be absorbed by financial intermediaries. By doing so, it can allow monetary policy to be aimed at stabilizing households' borrowing and saving decisions. However, reserve accumulation involves buying low-return foreign currency bonds and selling high-return domestic currency bonds, therefore, a carry cost.

For a class of shock-country characteristic combinations, we find the Mundell-Fleming prescription of fully flexible exchange rates to be optimal. This class of shock-country characteristic combinations are those that do not trigger any of the financial market imperfections. On the trade side, while pricing in the dominant currency reduces the benefits of exchange rate flexibility and generally features under- or over-exporting, flexible exchange rates remain optimal. This is because alternative

policies such as capital controls do not help improve efficiency since they do not eliminate the source of the problem which lies at stickiness of export prices in the dominant currency. Interestingly, the DCP economy is characterized by more volatile exchange rates than the PCP economy. This is because under DCP, achieving the same benefits from exchange rate flexibility as PCP requires *larger* exchange rate movements.

When future shocks can lead to a binding borrowing constraint, the pecuniary externality creates a role for prudential capital controls with the intensity of their use depending on the form of nominal rigidity. For example, adverse shocks to the country's debt limit can make the constraint bind and lead to overborrowing ex-ante. Such shocks alter the trade-offs for monetary policy and make the use of prudential capital controls optimal. For monetary policy, depreciating the exchange rate after a bad shock becomes costlier as a depreciation could further tighten the constraint. Thus, monetary policy needs to weigh the macro benefits of depreciation against the financial costs. Capital controls, which can attenuate the overborrowing problem before the shock, are optimally larger under DCP than PCP because of the larger exchange rate deprecation required under DCP. Foreseeing a larger exchange rate depreciation after an adverse shock under DCP, the social planner finds it optimal to impose higher capital controls before the shock.

Inefficiency in financial intermediation, i.e. shallow FX markets, generates an incentive to stabilize the premium on debt. In these cases, monetary autonomy may be enhanced by the use of ex-post capital controls and FX intervention. Positive external debt, either in the steady state or in response to shocks, leads to a reduction in the resources available to the economy due to the premia paid to the foreign-owned intermediaries as they finance external debt. This loss of resources creates an incentive to impose capital controls (ex-ante and/or ex-post) and ex-post FX intervention. By the same token, shocks that are optimally dealt with via exchange rate adjustments without any associated changes in external debt, such as permanent productivity and commodity price shocks, do not warrant ex-post capital controls or FX intervention. More generally, we find that the use of FX intervention depends on the comparison between the severity of the intermediation inefficiency

ex-ante and ex-post. If this inefficiency is severe enough ex-ante, perhaps due to high initial levels of debt, we find that the optimal policy is not to accumulate reserves ex-ante, because the carry cost of reserves is not justified when the marginal welfare cost of ex-ante FX accumulation is larger than the expected marginal welfare benefit of ex-post FX sales.

We synthesize the above results into broad principles to help guide policy design. First, policy instruments are not created equal; they operate through different margins. If an additional tool becomes available, it does not mean one should use it because it just may not be the right tool. A good example of this is the case of under-/over-exporting under DCP where capital controls are not useful. Second, instruments generally affect multiple imperfections, so the use of an existing policy may be reduced or increased after a new tool becomes available, that is, tools may be substitutes or complements. This is well exemplified by the ambiguous effects of prudential capital controls on the use of monetary policy under deep FX markets.

Our paper contributes to three strands of literature. First, we build on the insights developed by Gopinath (2015), Casas et al. (2016) and Gopinath et al. (2019) on the dominant currency paradigm. Following Casas et al. (2016), we compare and contrast the monetary policy implications of producer and dominant currency pricing for a small open economy. Unlike Casas et al. (2016), we consider a smaller scale model that can be solved nonlinearly and develop a rich financial market structure that allows us to analyze policies other than interest rate policy.

Second, our paper contributes to the literature on aggregate demand and pecuniary externalities, and the joint study of monetary and macroprudential policies. Similar to Farhi and Werning (2016), we build a small scale model with nominal rigidities and monetary policy to form the backbone of our model environment. Also similar to Farhi and Werning (2016), we benefit from the findings of the vast literature on pecuniary externalities and the use of macroprudential policies, exemplified by Mendoza (2010) and Bianchi (2011). We build on these papers by considering alternative forms of nominal rigidities as well as further financial frictions.

Third, our paper borrows elements from the literature on exchange rate determination and foreign

exchange intervention in the presence of inefficient financial intermediation. The intermediation inefficiency we consider follows that developed by Gabaix and Maggiori (2015). Similar to Fanelli-Straub (2019) and Cavallino (2019), this inefficiency provides a rationale for FX intervention and at the same time determines its effectiveness. Unlike, these papers, we nest the intermediation inefficiency into a larger setting that features other frictions and policies.

In ongoing analysis, we have been working to include domestic macroprudential measures into the policy mix. This requires enriching the domestic financial market by introducing further heterogeneity of domestic agents and frictional lending among them. Examples of such models in the literature include Korinek and Sandri (2016), and Caballero and Krishnamurthy (2001) who consider frictions that pertain to cross-border flows separately from flows among domestic agents.

The rest of this paper proceeds as follows. First, section 2 lays out the model environment. Sections 3 and 4 describe our results for deep and shallow foreign exchange markets, respectively, under different shocks and pricing paradigms. Section 5 conducts additional policy experiments such as FX bailouts and banning of open FX positions. Finally, section 6 concludes.

2 A Three-Period Small Open Economy

We construct a three-period model of a small open economy composed of households, firms, a governent and financial intermediaries, a fraction of which is owned by domestic households. Firms use labor to produce the tradable-sector good. The economy receives endowments of nontradable goods and exportable commodities. Tradable good prices are sticky, and following Gopinath (2015), export prices of home-produced tradable goods may follow producer currency pricing (PCP, which means that export prices are denominated in domestic currency) or dominant currency pricing (DCP, which means that export prices are denominated in dollars). Under both PCP and DCP, import and commodity prices are denominated in dollars. An occasionally-binding borrowing constraint limits households' debt to a fraction of their income, following Mendoza (2010), Bianchi (2011) and Farhi and Werning (2016). There are two noncontingent assets—a domestic currency bond and a dollar bond—and asset market segmentation: households can only trade the domestic currency bond, while international financial intermediaries can trade in both bonds. This financial market structure follows Gabaix and Maggiori (2015). The constrained social planner maximizes households' welfare while taking as given the decisions of private agents.

The stylized timeline for our model is shown in figure 2 and the financial structure of our model is shown in figure 3. A variety of shocks strike in period 1 of the model (taking high, H, or low, L, values), after which all uncertainty is resolved. The planner can implement policies either in period 0 in anticipation of possible shocks (i.e., prudential or ex-ante policy) or in period 1 after the shock has been realized (i.e., ex-post policy):

- Monetary policy. The planner sets the policy rate, which is equal to the interest rate on domestic currency bonds, between periods 0 and 1 and between periods 1 and 2.

- Capital inflow controls. The planner can set taxes/subsidies on inflows which generate a spread between the interest rate for households and the interest rate earned by financial intermediaries. Such controls can be used in a prudential fashion, between periods 0 and 1, or in an

Figure 2: Timeline of Events

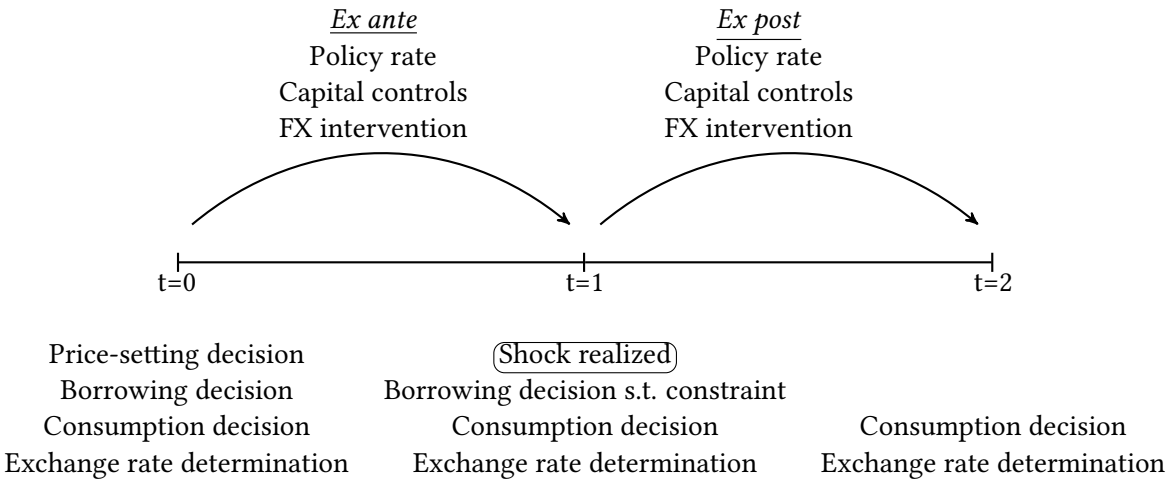
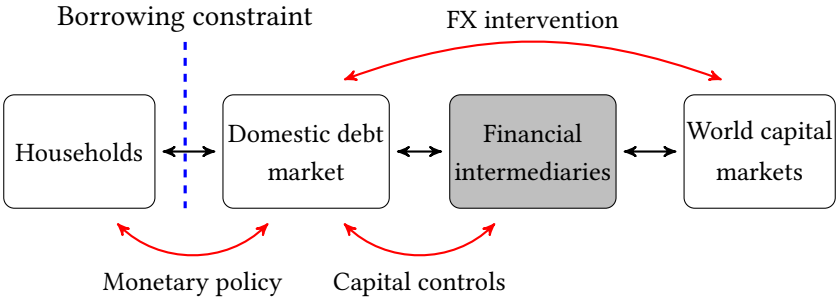


Figure 3: Structure of the Financial Market



ex-post fashion, between periods 1 and 2. Prudential capital controls are similar to those studied by Bianchi (2011), Korinek (2011), and Farhi and Werning (2016).

- FX intervention. The planner can intermediate between the domestic currency bond and the dollar bond, circumventing the financial intermediaries and their inefficiency, between periods 0 and 1 and between periods 1 and 2. Such intervention is similar to that in Gabaix and Maggiori (2015), Cavallino (2019), and Fanelli and Straub (2019).

The three-period structure is attuned to our objective of characterizing optimal combinations of policy instruments as a function of shocks and characteristics relevant for the economy under consideration. It is rich enough to distinguish between prudential and ex-post policies. At the same time, it keeps the model tractable to study the interactions of policies and frictions, and between the policies themselves.

Next, we lay out the environment for the private sector agents and derive their optimization decisions before turning to the constrained social planner problem.

Households

Households maximize a welfare function which follows the Cole-Obstfeld (1991) formulation over consumption, and the Gali-Monacelli (2005) special case of linear disutility of labor:

$$\mathbb{E}_0 \left[\sum\nolimits_{t=0}^{2} \beta^t U\left(C_{Ht}, C_{Ft}, C_{NTt}, N_t\right) \right]$$

where $U\left(C_{Ht}, C_{Ft}, C_{NTt}, N_t\right) = \alpha_H \log C_{Ht} + \alpha_F \log C_{Ft} + (1 - \alpha_H - \alpha_F) \log C_{NTt} - N_t$

Their maximization is subject to a budget constraint:

$$\begin{aligned} & W_t N_t + \Pi_{Tt} + \lambda \Pi_{FIt} + T_{FXIt} - T_{Gt} + P_{NTt} Y_{NTt} + E_t P^*_{Zt} Z_t + D_{t+1} \\ & \geq P_{Ht} C_{Ht} + E_t P^*_{Ft} C_{Ft} + P_{NTt} C_{NTt} + \left(1 + i_{t-1}\right) D_t, \end{aligned} \tag{1}$$

and a borrowing constraint:

$$D_2 \leq \kappa P_{H1}. \tag{2}$$

Starting with the right hand side of the budget constraint, P_{Ht} and C_{Ht} are the domestic price and consumption of the tradable good, E_t is the exchange rate in units of domestic currency per dollar, $E_t P^*_{Ft}$ (i.e., the exchange rate multiplied by the dollar price of imports) is the domestic currency price of imports, C_{Ft} is consumption of imports, C_{NTt} is consumption of nontradable goods, D_{t+1} is the domestic-currency debt at the end of period t, and i_t is the domestic policy rate which is set in period t and applies on domestic currency debt between periods t and $t+1$. N_t is labor supply, W_t is the wage, Π_{Tt} is the profit from firms, λ is the fraction of international financial intermediaries owned by domestic households while Π_{FIt} is the profit of each of them, T_{FXIt} is the profit of the planner from FX operations, T_{Gt} is the lump-sum tax levied by the planner, P_{NTt}, C_{NTt} and Y_{NTt} are the price, consumption, and endowment of nontradable goods, $E_t P^*_{Zt}$ (i.e., the exchange rate multiplied by the dollar price of commodities) is the domestic price of commodity exports, and Z_t is the endowment of commodities.

The borrowing constraint can bind between periods 1 and 2 and takes a simple form: κ is a parameter governing the pledgability of domestic tradable goods between periods 1 and 2, and it multiplies the domestic-currency price P_{H1}, which means that in dollar terms, the constraint becomes tighter when the exchange rate depreciates. This formulation brings our model closer to practical concerns of policymakers around the world, and it also echoes the constraint in Farhi and Werning (2016).[2]

The households' first order conditions (FOCs) lead to the following intratemporal conditions:

$$\frac{\alpha_H}{C_{Ht}P_{Ht}} = \frac{\alpha_F}{E_t P^*_{Ft} C_{Ft}} = \frac{\alpha_{NT}}{C_{NTt}P_{NTt}} = \frac{1}{W_t} \tag{3}$$

$$\Rightarrow C_{Ht} = \frac{\alpha_H}{\alpha_F} p_{Ht} C_{Ft} \text{ where } p_{Ht} = \frac{E_t P^*_{Ft}}{P_{Ht}},$$

where p_{Ht} is the price of foreign goods relative to home goods. The FOCs also yield the Euler con-

[2] In subsection 5.3 of Farhi and Werning (2016), the nontradable good has a sticky price, and households can borrow up to a specific fraction of the value of nontradable output. Instead, in our model, we assume that households can borrow up to a specific fraction of the sticky price of the home-produced tradable good. Both constraints become tighter when the exchange rate depreciates.

ditions:

$$\frac{\alpha_F}{P^*_{F0}C_{F0}} = \beta\left(1+i_0\right)\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right] \text{ and } \frac{\alpha_F}{P^*_{F1}C_{F1}} \geq \beta\left(1+i_1\right)\frac{E_1}{E_2}\frac{\alpha_F}{P^*_{F2}C_{F2}}, \tag{4}$$

where the condition between periods 1 and 2 holds with inequality if and only if the borrowing constraint binds.

Firms

Firms are monopolistically competitive and set prices at the beginning of period $t = 0$, after which prices are fully rigid (so we can remove the time subscripts on tradable good prices).[3] Following the New-Keynesian tradition, we assume that they produce a variety $j \in [0,1]$ of tradable goods, $Y_{Tt}\left(j\right)$, using labor, $N_t\left(j\right)$. The varieties may be consumed domestically, $Y_{Ht}\left(j\right)$, or exported, $Y_{Xt}\left(j\right)$:

$$Y_{Tt}\left(j\right) = Y_{Ht}\left(j\right) + Y_{Xt}\left(j\right) = A_t N_t\left(j\right). \tag{5}$$

Firms face downward-sloping demands for their output from domestic consumption and from export demand. Domestic consumption involves combining the tradable varieties into an aggregate tradable good:

$$Y_{Ht} = \left(\int_0^1 Y_{Ht}\left(j\right)^{(\varepsilon-1)/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}.$$

The corresponding domestic demand curve is:

$$Y_{Ht}\left(j\right) = \left(\frac{P_H\left(j\right)}{P_H}\right)^{-\varepsilon} Y_{Ht},$$

where $P_H\left(j\right)$ is the sticky domestic-currency price of each variety and P_H is the price index for the aggregate tradable good. We assume that the export demand curve follows the same form over each traded variety, and a unit-elastic expression for the aggregate traded good:

$$Y_{Xt}\left(j\right) = \left(\frac{P_X\left(j\right)}{P_X}\right)^{-\varepsilon} Y_{Xt} \text{ and } Y_{Xt} = \omega p_{Xt},$$

[3] This extreme price-setting assumption keeps the model tractable, and allows us later on to interpret the optimal exchange rate policy as being related to the planner's desire to mitigate static price pressures, i.e., to ensure that the domestic price level is at an appropriate level relative to the price level of foreign goods. However, this assumption prevents us from considering the welfare costs of price dispersion and credibility-damaging inflation dynamics.

where $P_X(j)$ is the price fixed by firms for each exported variety, P_X is the corresponding price index for the exported tradable good, and p_{Xt} is the relative price of foreign goods to exports. The denomination of $P_X(j)$ and P_X, and the formula for p_{Xt} depend on the pricing paradigm.

Under PCP, firms set identical domestic-currency prices for all their output, regardless of whether the good is consumed domestically or exported, i.e., $P_X(j) = P_H(j)$. In other words, the law of one price holds, as in Gali and Monacelli (2005). Under DCP, firms set a domestic-currency price, $P_H(j)$, for the domestically-consumed portion of the tradable good, and a separate dollar price, $P_X(j)$, for the exported portion of the good. As a result, the relative price of foreign goods to exports, i.e. the terms of trade, follows separate formulae under PCP and DCP:

$$p_{Xt}^{PCP} = \frac{E_t P_{Ft}^*}{P_H} \text{ and } p_{Xt}^{DCP} = \frac{P_{Ft}^*}{P_X}.$$

Profit maximization for firms *under PCP* is given by:

$$\max \Pi_{Tt}(j) = \Pi_{Ht}(j) + \Pi_{Xt}(j)$$

$$= \max \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j) \left(Y_{Ht}(j) + Y_{Xt}(j) \right) - (1+\phi) W_t N_t(j) \right] \right]$$
$$= \max \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j) - (1+\phi) \frac{W_t}{A_t} \right] (Y_{Ht} + Y_{Xt}) \left(\frac{P_H(j)}{P_H} \right)^{-\varepsilon} \right],$$

where ϕ is a constant labor tax applied on all home production of tradable goods. We assume that firms have perfect access to dollar debt markets, so we set discount rates for the firm following the uncovered interest parity (UIP) conditions: $\Lambda_0 = 1$, $\Lambda_1 = \frac{1}{(1+i_0^*)} \frac{E_0}{E_1}$, and $\Lambda_2 = \frac{1}{(1+i_0^*)(1+i_1^*)} \frac{E_0}{E_2}$. The FOC of the above expression produces a formula for $P_H(j)$—and, since all varieties are identical, for P_H—as a function of home demand, export demand, and the labor tax:

$$P_H = P_H(j) = (1+\phi) \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_0 \left[\sum_{t=0}^{2} \Lambda_t \frac{W_t}{A_t} (Y_{Ht} + Y_{Xt}) \right]}{\mathbb{E}_0 \left[\sum_{t=0}^{2} \Lambda_t (Y_{Ht} + Y_{Xt}) \right]} \tag{6}$$

The optimal price trades off the profit-maximizing positions the firm wants to target on the two separate home and export demand schedules. By changing the labor tax, ϕ, the planner can control the domestic price level and the export price level, both given by P_H.

Profit maximization for firms *under DCP* follows:

$$\max \Pi_{Tt}(j) = \Pi_{Ht}(j) + \Pi_{Xt}(j)$$

where

$$\Pi_{Ht}(j) = \mathbb{E}_0\left[\sum_{t=0}^{1}\Lambda_t\left[P_H(j) - (1+\phi)\frac{W_t}{A_t}\right]Y_{Ht}\left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon}\right]$$

$$\Pi_{Xt}(j) = \mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\left[E_tP_X(j) - (1+\phi)\frac{W_t}{A_t}\right]Y_{Xt}\left(\frac{P_X(j)}{P_X}\right)^{-\varepsilon}\right].$$

The fact that the labor tax is commonly applied on all home production of tradable goods, and not differentiated across goods according to their final destination, imposes a connection between the domestic price, P_H, and the export price, P_X, in equilibrium. Taking FOCs of the above expressions and rearranging:

$$P_H = P_H(j) = (1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Ht}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tY_{Ht}\right]},\ P_X = P_X(j) = (1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Xt}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tE_tY_{Xt}\right]}$$

$$\Rightarrow P_X = P_H\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Xt}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tE_tY_{Xt}\right]}\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tY_{Ht}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Ht}\right]}$$

$$= P_X\left(P_H, C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}\right). \tag{7}$$

The planner needs to take into account that the expression for the export price, P_X, is not an independent choice variable, but rather a function of the domestic price, P_H, and also the levels of tradable consumption and exchange rates in all periods and states. The formula for P_X is provided in Appendix A.1.

Financial intermediaries

Financial intermediaries take positions of q_{t+1} in domestic currency bonds and $-\frac{q_{t+1}}{E_t}$ in dollar bonds in period t in order to maximize their dollar profits subject to a balance sheet friction echoing the one considered by Gabaix and Maggiori (2015):[4]

$$\max_{q_{t+1}} \frac{1}{(1+i_t^*)} \frac{q_{t+1}}{E_t} \mathbb{E}_t \left[(1-\varphi_t)(1+i_t) \frac{E_t}{E_{t+1}} - (1+i_t^*) \right]$$

$$\text{subject to } \frac{1}{(1+i_t^*)} \frac{q_{t+1}}{E_t} \mathbb{E}_t \left[(1-\varphi_t)(1+i_t) \frac{E_t}{E_{t+1}} - (1+i_t^*) \right] \geq \frac{1}{(1+i_t^*)} \Gamma_t \left(\frac{q_{t+1}}{E_t} \right)^2,$$

where $\Gamma_t \geq 0$ captures the severity of the balance sheet friction, and φ_t is the capital inflow tax announced in period t and applies to the repayments on those inflows made to the financial intermediaries in period $t+1$. A fraction λ of the intermediaries are owned by domestic households and the remaining fraction $(1-\lambda)$ are owned by foreigners. Capital controls distort the decisions of all intermediaries, but since the planner rebates all tax revenues to households, only the foreign-owned fraction of the intermediaries end up paying taxes in net terms.

The constraint for financial intermediaries always binds. Integrating, we derive the intermediaries' demand for domestic currency bonds:

$$\frac{Q_{t+1}}{E_t} = \frac{1}{\Gamma_t} \mathbb{E}_t \left[(1-\varphi_t)(1+i_t) \frac{E_t}{E_{t+1}} - (1+i_t^*) \right].$$

The intermediaries' realized profit in domestic currency in period $t+1$ is:

$$\Pi_{FIt+1} = Q_{t+1} \left[(1-\varphi_t)(1+i_t) - (1+i_t^*) \frac{E_{t+1}}{E_t} \right].$$

We assume that there is a separate group of non-optimizing foreigners who have exogenous and stochastic demands for domestic-currency debt. Their exogenous debt holdings are L_{t+1} in domestic currency bonds, amounting to $S_t = \frac{L_{t+1}}{E_t}$ in dollar value. They are not subject to the balance sheet friction described above, and their decisions to purchase domestic-currency debt do not depend on the expected returns.

[4] We assume that intermediaries maximize the dollar value of profits, not the domestic-currency value of profits.

In our model, FX intervention involves the planner taking a position of O_{t+1} in local currency bonds and $FXI_t = -\frac{O_{t+1}}{E_t}$ in dollar bonds. The realized profit for the planner from this transaction is:

$$T_{FXIt+1} = O_{t+1}\left[(1+i_t) - (1+i_t^*)\,\frac{E_{t+1}}{E_t}\right]. \tag{8}$$

The use of one-period bonds in FX accumulation and sales keeps the model tractable by ensuring that the period-0 asset positions are liquidated at the beginning of period 1, before shocks strike. However, in practice, an important concern regarding FX intervention is that the stock of FX reserves cannot typically fall below zero. FX reserves cannot be borrowed after shocks strike, so the volume of possible intervention cannot exceed what has been accumulated in previous periods. Recognizing this problem, policymakers accumulate FX reserves of varying maturities ex-ante, to have on hand when shocks strike. To capture this concern in our framework while preserving tractability, we keep the assumption of one-period bonds but additionally impose that the cumulative FX accumulation must be non-negative:

$$R_1 = FXI_0 \geq 0 \text{ and } R_2 = FXI_0 + FXI_1 \geq 0. \tag{9}$$

Market clearing in the domestic-currency debt market requires:

$$Q_{t+1} = D_{t+1} - O_{t+1} - L_{t+1},$$

which produces the "Gamma equations" that relate expected excess premia to capital inflows:

$$\Gamma_0\left(\frac{D_1}{E_0} + FXI_0 - S_0\right) = \mathbb{E}_0\left[\eta_1 - (1+i_0^*)\right] \tag{10}$$

$$\Gamma_1\left(\frac{D_2}{E_1} + FXI_1 - S_1\right) = \eta_2 - (1+i_1^*)\,, \tag{11}$$

where we define the gross return on domestic assets in dollar terms:

$$\eta_{t+1} = (1-\varphi_t)\,(1+i_t)\,\frac{E_t}{E_{t+1}} > 0.$$

Since the gross external return is a combination of the ex-ante policy rate, the ex-ante capital controls, and the ex-ante and ex-post exchange rates, it must inherit the contingency properties of its constitutent components. Using H and L superscripts for the values of variables after the period-1 realizations of high and low shocks respectively, we derive the following "contingency constraint:"

$$\frac{\eta_1^H}{\eta_1^L} = \frac{E_1^L}{E_1^H} \Rightarrow E_1^H \eta_1^H = E_1^L \eta_1^L. \tag{12}$$

Finally, the planner's proceeds from labor taxes and capital inflow taxes are distributed to households via a lump-sum transfer:

$$T_{Gt+1} + \phi W_t N_t + \varphi_t \left(1 + i_t\right) Q_{t+1} = 0. \tag{13}$$

The endowment of commodities is entirely exported, while the market for nontradable goods must clear domestically:

$$C_{NTt} = Y_{NTt}. \tag{14}$$

Competitive equilibrium

Definition A competitive equilibrium for this economy is a set of quantities $\{C_{Ht}, C_{Ft}, C_{NTt}, N_t, Y_{Ht}, Y_{Xt}, Y_{NTt}\}_{t=0}^2$ and prices $\left\{P_H, P_X, \{W_t, E_t\}_{t=0}^2\right\}$ that satisfy the households' constraints and FOCs (1)-(4), the firms' production and price-setting decisions (5) and either (6) or (7), the domestic-currency bond-market clearing conditions (10)-(11), the FX intervention constraints (9), the contingency constraint for gross external returns (12), the lump-sum-transfer constraints (8) and (13), and the nontradable goods market clearing condition (14), taking as given the planner's choice of the policy instruments $\{i_t, \varphi_t, FXI_t\}_{t=0}^1$.

Substituting the competitive equilibrium equations into the households' budget constraints, we obtain the economy-wide resource constraint for tradable goods:

$$D_{t+1} \geq -E_t P_{Ft}^* \left[\omega C_t^* - C_{Ft}\right] - \left(1 - \lambda\right) O_t \left[\left(1 + \widehat{i}_{t-1}\right) - \left(1 + i_{t-1}^*\right) \frac{E_t}{E_{t-1}}\right]$$

$$- E_t P^*_{Zt} Z_t + \lambda \left(1 + i^*_{t-1}\right) \frac{E_t}{E_{t-1}} D_t + (1-\lambda) \left(1 + \widehat{i}_{t-1}\right) D_t, \tag{15}$$

where $\left(1 + \widehat{i}_t\right) = (1 - \varphi_t)(1 + i_t)$. Combining the capital controls and the policy rate as an "effective interest rate" is useful for analytical simplicity. Once the effective rate is pinned down, the planner can decompose it into the two separate policy instruments using the households' Euler condition and the information on whether the borrowing constraint is binding or not.

The resource constraint highlights the importance of the parameter λ, which captures the fraction of the financial intermediaries which is owned by domestic households. Since all households are identical, their net domestic-currency debt position must be entirely absorbed by the intermediaries. Households own a fraction λ of the intermediaries, and those intermediaries borrow in dollars to purchase the domestic-currency debt that the households issue, so the fraction λ of the domestic-currency debt position nets out in the expression for the households' income, and the other fraction $(1 - \lambda)$ represents the external debt position. Therefore, if $\lambda > 0$, then households' income moves as if the households have issued some dollar bonds themselves: there is a currency mismatch, and a depreciation in the exchange rate increases the domestic-currency value of the households' external debt repayments, which may tighten their borrowing constraints. This connection of the economy-wide borrowing constraint to the exchange rate becomes more evident when the constraint is written in dollar terms:

$$\frac{D_2}{E_1} \leq \kappa \frac{P_H}{E_1}. \tag{16}$$

Constrained Efficient Allocations

We can write the indirect utility function in period t as follows:

$$V\left(C_{Ft}, p_{Ht}, p_{Xt}, Y_{NTt}\right) = U\left(\frac{\alpha_H}{\alpha_F} p_{Ht} C_{Ft}, C_{Ft}, Y_{NTt}, \frac{\frac{\alpha_H}{\alpha_F} p_{Ht}}{A_t} C_{Ft} + \frac{\omega p_{Xt}}{A_t} C^*_t\right)$$

where $V_{Ft} = \frac{\alpha_F}{C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F}\left(1 - \frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}\right)\right]$, $V_{p_H t} = \frac{\alpha_H}{p_{Ht}}\left(1 - \frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}\right)$, and $V_{p_X t} = -\frac{\omega}{A_t} C^*_t$.

We define three sets of wedges that summarize the distance of the allocations from the efficient

frontier. The first wedge is for home consumption, as in Farhi and Werning (2016), and arises from the stickiness of the tradable-good price when sold for domestic consumption:

$$\tau_{Ht} = 1 + \frac{1}{A_t}\frac{U_{Nt}}{U_{Ht}} = 1 - \frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}.$$

This "aggregate demand (AD) wedge" captures the impact of aggregate demand externalities, and is positive if P_H is too high relative to domestic aggregate demand. Households do not internalize the impact of their consumption decisions on the time path of aggregate demand. The level of aggregate demand affects the appropriateness of the pre-set domestic price, P_H, and the level of the exchange rate E_1 which enters the borrowing constraint.

The second wedge relates to export production and varies depending on the price-setting paradigm:

$$\tau_{Xt}^{PCP} = \left(1 - \frac{\omega p_{Xt}^{PCP}}{p_{Xt}^{PCP}\frac{d}{dp_{Xt}^{PCP}}\left(\omega p_{Xt}^{PCP}\right)}\right) + p_{Xt}^{PCP}\frac{1}{A_t}\frac{U_{Nt}}{U_{Ft}} = -p_{Xt}^{PCP}\frac{1}{A_t}\frac{C_{Ft}}{\alpha_F}$$

$$\tau_{Xt}^{DCP} = \left(1 - \frac{\omega p_{Xt}^{DCP}}{p_{Xt}^{DCP}\frac{d}{dp_{Xt}^{DCP}}\left(\omega p_{Xt}^{DCP}\right)}\right) + p_{Xt}^{DCP}\frac{1}{A_t}\frac{U_{Nt}}{U_{Ft}} = -p_{Xt}^{DCP}\frac{1}{A_t}\frac{C_{Ft}}{\alpha_F}.$$

This "TOT wedge" highlights that there is a TOT externality because while firms do take into account that the demand curve for their own export variety is downward-sloping, they do not internalize that the demand curve for the aggregate export good is also downward-sloping. Under the unit elastic demand assumption for export demand, the first term in the above expressions is zero. These wedges are always negative because firms set the export price, P_X, lower than the level that maximizes the economy-wide TOT. In other words, the planner wishes to push the economy to an allocation with a higher export price, P_X, and a lower export volume, Y_{Xt}, while earning the same dollar value of export revenues.

The third wedge captures the state-by-state deviation of the gross external return from the level that would prevail if all households could borrow using dollar bonds:

$$\tau_{\Gamma t} = \eta_t - \left(1 + i_{t-1}^*\right).$$

This "UIP wedge" enters the economy-wide resource and borrowing constraints. If the wedge is positive in a particular state and some of the intermediaries are foreign-owned, i.e., $\lambda \in [0, 1)$, then there is a net loss of resources from the domestic economy to those foreign-owned intermediaries in that state.

In this economy, the first-best allocation is not feasible: all the wedges $\{\tau_{Ht}, \tau_{Xt}, \tau_{\Gamma t}\}$ cannot be equal to zero in every state. In a deterministic closed economy, the planner can set the labor tax, ϕ, to manipulate the domestic price level, P_H, such that the distortion owing to monopolistic competition is perfectly eliminated and the AD wedge is zero, i.e., $\tau_{Ht} = 0$. In a deterministic open economy under PCP, the planner cannot perfectly eliminate this distortion. Instead, it has to set the labor tax to balance the AD and TOT wedges, and, as in Gali and Monacelli (2005), some of both distortions remain in equilibrium, i.e., $\tau_{Ht} \neq 0$ and $\tau_{Xt} \neq 0$. The same principle applies under DCP because of the fact that the export price, P_X, is still tied to the domestic price, P_H. The addition of shocks to the economy, as well as the introduction of the UIP wedge, $\tau_{\Gamma t}$, reinforces the result that all distortions cannot be entirely eliminated.

This insight leads us to focus on deriving the constrained efficient allocations.

Definition A constrained efficient allocation is a set of quantities $\{C_{Ht}, C_{Ft}, C_{NTt}, N_t, Y_{Ht}, \ Y_{Xt}, Y_{NTt}\}_{t=0}^{2}$, prices $\left\{P_H, P_X, \{W_t, E_t\}_{t=0}^{2}\right\}$, and policy instruments $\{i_t, \varphi_t, FXI_t\}_{t=0}^{1}$ which solve under full commitment:

$$\max_{\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t\}} \left\{ \begin{array}{cc} \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t V \left(C_{Ft}, \frac{E_t P^*_{Ft}}{P_H}, \frac{E_t P^*_{Ft}}{P_H}, Y_{NTt} \right) \right] & \text{if PCP} \\ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t V \left(C_{Ft}, \frac{E_t P^*_{Ft}}{P_H}, \frac{P^*_{Ft}}{P_X}, Y_{NTt} \right) \right] & \text{if DCP,} \\ \text{with } P_X = P_X \left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H \right) & \end{array} \right\}$$

subject to the restriction that the allocation constitutes a competitive equilibrium. The full set of equations, including the dollar forms of all the constraints, is listed in Appendix A.2.

Our model allows us to assess the optimal combinations of policy instruments as a function of shocks and structural characteristics, and in particular whether it is optimal for the planner to

deviate from the "traditional Mundell-Fleming prescription" of just using the domestic policy rate and allowing exchange rate flexibility. It also allows us to derive optimal policies when different sets of instruments are in the planner's policy toolkit, which means that we can assess the complementarity and/or substitutability of the instruments. Every time an instrument is removed from the planner's policy toolkit, additional constraints need to be added onto the planner problem:

- If FX intervention is not permitted, then we need to set $FXI_0 = FXI_1 = 0$ and remove the FOCs with respect to FX intervention, FXI_t.
- If capital controls are not permitted, then the household Euler conditions (4) need to be added as constraints with the capital control terms set to zero.
- If the domestic policy rate cannot be used, then we set it equal to the foreign interest rate, i.e., $i_t = i_t^*$, in the formula for the gross external premium, η_{t+1}.

Our solution approach is as follows. We assume that the constrained planner problem is convex in the region of interest, and correspondingly, we derive the FOCs for the problem in Appendix A.2. In the next two sections, we summarize the salient properties of these FOCs, indexing our results by the pricing paradigm:

$$\mathbb{I}^{PCP} = \left\{ \begin{array}{ll} 1 & \text{if PCP} \\ 0 & \text{if DCP} \end{array} \right\} \text{ and } \mathbb{I}^{DCP} = \left\{ \begin{array}{ll} 0 & \text{if PCP} \\ 1 & \text{if DCP} \end{array} \right\}.$$

Then in each section, we aim to provide a mix of analytical and numerical results which qualitatively characterize the optimal combinations of policy instruments as a function of shocks and structural characteristics.

3 Deep Foreign Exchange Markets

In this section, we focus on the optimal policy problem for a country with deep FX markets. Such a case covers most advanced economies and some emerging markets such as Mexico and South Africa. These countries' currencies are traded by a substantial number of financial intermediaries, except possibly during episodes of severe global financial stress.

As we explain below, we study the deep FX markets case by setting $\Gamma_t = 0$ (no balance sheet frictions) and $\lambda = 1$ (households own all intermediaries), and by removing FX intervention from the planner's toolkit.

3.1 Policy Instruments and Wedges

The deep FX markets case formally corresponds to setting $\Gamma_t = 0$ in the constraints and FOCs summarized in Appendix A.2. This Gamma value means that financial intermediaries face no balance sheet constraints, so their capacity to hold domestic-currency debt is unlimited, and the country's external debt position does not matter for the country's gross external return, η_t. The Gamma equations reduce to the UIP conditions:

$$\mathbb{E}_0 \left[\tau_{\Gamma 1}\right] = 0 \text{ and } \tau_{\Gamma 2} = 0. \tag{17}$$

UIP wedges, $\tau_{\Gamma t}$, paid by the domestic economy to intermediaries generate welfare losses if a fraction of the intermediaries are foreign-owned, i.e., $\lambda \in [0, 1)$. However, since the UIP wedges average out to zero, the average external premium is zero. Therefore, to simplify the algebra, we ignore foreign ownership of the intermediaries in this section and set $\lambda = 1$. Note that if domestic households own all financial intermediaries, the economy's liabilities are effectively entirely in dollars.

Equation (17) establishes that FX intervention does not affect the exchange rate, so we defer further consideration of FX intervention to section 4.

The households' Euler conditions can be rewritten:

$$\frac{\alpha_F}{P^*_{F0}C_{F0}} = \beta\frac{(1+i^*_0)}{(1-\varphi_0)}\frac{1}{\mathbb{E}_0\left[\frac{E_0}{E_1}\right]}\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right] \text{ and } \frac{\alpha_F}{P^*_{F1}C_{F1}} \geq \beta\frac{(1+i^*_1)}{(1-\varphi_1)}\frac{\alpha_F}{P^*_{F2}C_{F2}}. \tag{18}$$

These Euler conditions demonstrate that capital controls are effective instruments under deep FX markets, as they raise the domestic policy rate above the foreign interest rate, and thereby reduce domestic borrowing. Exchange rates E_0 and E_1 enter the Euler condition between periods 0 and 1 because households have access to domestic-currency bonds only, and not dollar bonds, and the extent of possible risk-sharing depends on the contingency of the available bonds.[5] They do not enter the Euler condition between periods 1 and 2 because there is no uncertainty between those periods.

Next, we turn to the conditions characterizing the constrained efficient allocation, to understand which externalities arise under deep FX markets and how policies should be used to alleviate them. The planner's Euler conditions for $t \in \{0,1\}$ are:

$$\begin{aligned}
&\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] - \mathbb{I}^{DCP}\cdot\left(\frac{1}{\beta\pi_1}\right)^t\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{P^*_{Ft}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft}}\\
&= \beta\left(1+i^*_t\right)\mathbb{E}_t\left\{\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right]\right\} + \left(\Psi_t\frac{1}{\beta\left(1+i^*_{t-1}\right)}\right)^t\\
&- \mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_t\left[\left(\frac{1}{\beta\pi_1}\right)^t\frac{(1+i^*_t)}{P^*_{Ft+1}}\frac{1}{P_X}\frac{\pi_{t-1}}{\pi_t}\frac{\partial P_X}{\partial C_{Ft+1}}\right],
\end{aligned} \tag{19}$$

where Ψ_t is the multiplier on the borrowing constraint. The first term on the left hand side represents the marginal utility of consumption in period t, taking into account the AD wedge. The second term on the left hand side captures an effect which only arises under DCP: the impact of the period-t consumption decision on welfare via the period-0 export-price-setting decision. The first term on the right hand side represents the marginal utility of consumption in period $t+1$, taking into account the AD wedge. The second term on the right hand side captures the distortion in the Euler conditions

[5] If the shock is such that period-1 imports, C_{F1}, is perfectly stabilized across high and low realizations of the shock, or if the period-1 exchange rate, E_1, is perfectly stabilized across realizations, then the state-contingency of the bond is no longer important, and the exchange rates do not appear in the Euler conditions. The Euler conditions become identical to the Euler conditions of households who are able to participate directly in the dollar bond market subject to capital controls.

if the borrowing constraint binds. The third term on the right hand side captures the impact of the period-$t+1$ consumption decision on welfare via the period-0 export-price-setting decision.

Comparing the household and planner Euler conditions, we can see that two of our wedges, τ_{Ht} and τ_{Xt} may provide a rationale for intervention in the deep FX markets case.

We have mentioned in section 2 that it is not feasible for these two wedges to be equal to zero in every state. Nevertheless, for illustrative purposes, we observe that if we set all wedges to zero, then both the households' and the planner's Euler conditions would reduce to:

$$\frac{\alpha_F}{P^*_{Ft}C_{Ft}} = \beta\left(1+i^*_t\right)\mathbb{E}_t\left[\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right] \text{ for } t\in\{0,1\},$$

which is identical to the Euler condition of households who are able to participate without restriction in the dollar bond market. In this case, the planner would set a domestic policy rate consistent with zero capital controls, and would allow full flexibility of the exchange rate.

When there are non-zero wedges, there may be a case for the planner to move the domestic policy rate in a different manner, and also to add capital controls into the toolkit in order to stabilize the wedges over time.

The FOCs for exchange rates in each state are:

$$\underbrace{\frac{(\beta\pi_t)^t}{\pi_{t-1}}\alpha_H\tau_{Ht}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{array}{c}-\mathbb{I}^{PCP}\cdot\left\{\frac{(\beta\pi_t)^t}{\pi_{t-1}}\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{PCP}_{Xt}\right\}\\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\left(-\frac{E_t}{P_X}\frac{\partial P_X}{\partial E_t}\right)\right\}\end{array}}_{\text{Optimize TOT on export goods}}$$
$$+\underbrace{\frac{(\beta\pi_t)^t}{\pi_{t-1}}\Psi_t\frac{1}{I_{t-1}}\kappa\frac{P_H}{E_t}}_{\text{Relax binding borrowing constraint}} \pm \underbrace{\Lambda_t E_t\left[\left(1+i^*_{t-1}\right)+\tau_{\Gamma t}\right]}_{\text{Prevent excessive contingency of exchange rate}}. \tag{20}$$

The planner sets the exchange rate to balance price pressures and binding borrowing constraints within each state. The first term, which includes the AD wedge τ_{Ht}, represents the benefit of moving the exchange rate to generate import substitution and stabilize domestic demand for the home-produced tradable good. The second term, which includes the TOT wedges τ_{Xt}, represents the benefit

of moving the exchange rate to optimize the TOT on export goods by altering the export volume—either just within a specific state (under PCP), or on average across all states via the impact on the period-0 export-price-setting decision (under DCP). Price pressures are balanced if, for example, the first term is positive because prices are too high for consumption purposes, but the second term (including the minus sign in front) is also positive because prices are too low for export purposes. The third term represents the effect of exchange rate movements on the tightness of the borrowing constraint. There is also a technical term ensuring that the exchange rate volatility across states is consistent with a non-contingent domestic policy rate. In line with condition (17), the domestic policy rate moves inversely to the expected exchange rate depreciation.

The expression for capital controls is:

$$\varphi_t = \begin{cases} 1 - \dfrac{\frac{1}{\mathbb{E}_t\left\{\frac{E_t}{E_{t+1}}\right\}}\mathbb{E}_t\left\{\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right\}\left[\begin{array}{c}\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] \\ -\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\left(\frac{1}{\beta\pi_1}\right)^t\frac{1}{P^*_{Ft}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft}}\end{array}\right]}{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[\begin{array}{c}\mathbb{E}_t\left\{\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right]\right\} \\ -\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_t\left[\left(\frac{1}{\beta\pi_1}\right)^t\frac{\pi_{t-1}}{\beta\pi_t}\frac{1}{P^*_{Ft+1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft+1}}\right]\end{array}\right]} & \text{if } \Psi_t = 0 \\ 0 & \text{if } \Psi_t > 0, \end{cases} \tag{21}$$

where capital controls are ineffective, and therefore set to zero, when the borrowing constraint binds.

Capital controls are non-zero if and only if the numerator and denominator of the fraction in equation (21) are unbalanced. The expressions for the numerator and denominator are obtained by substituting for the AD wedge, τ_{Ht}, using equation (20). Therefore, there are two possible rationales for capital controls in our model.

The first potential rationale for capital controls arises if there is a "pecuniary AD externality" from an occasionally-binding borrowing constraint, i.e., $\Psi_t > 0$, which captures the concerns of many emerging-market policymakers. Households do not internalize that their borrowing in period

t may generate lower aggregate demand and a more depreciated exchange rate in period $t+1$, making the borrowing constraint binding. When the borrowing constraint binds, equation (20) indicates that the AD wedge, τ_{Ht}, is kept higher than the TOT wedges, τ_{Xt}, would justify, i.e., the exchange rate is more appreciated and the domestic policy rate is kept higher. Therefore, monetary policy and exchange rate flexibility are no longer able to fully address the AD externality, and the pecuniary component of the externality remains. As a result, non-zero capital controls become optimal. This finding captures Farhi and Werning's (2016) insights regarding the case for capital controls with occasionally-binding borrowing constraints. Our model additionally allows for the strength of the externality to be related to the pricing structure (PCP versus DCP).

The second potential rationale for capital controls arises from the TOT externality. Capital controls are non-zero if the weighted TOT wedges, τ_{Xt}, are not balanced over time (as in Costinot, Lorenzoni, and Werning, 2014). We refer to this rationale as "intertemporal TOT manipulation," as it relates to the time path rather than the level of the TOT wedges. Our finding nests the results of Farhi and Werning (2015) and extends them to the DCP case. This rationale naturally arises in any open-economy framework with price-setting, but policymakers do not emphasize this channel, so we focus on insights that do not hinge on this motive.

Equations (19)-(21) show that an integrated model is necessary to characterize the optimal use of multiple instruments: the use of each policy instrument affects several wedges and, as a result, the optimal use of other policy instruments. Specifically, we can see that the level of the domestic policy rate affects exchange rates and thereby the optimal use of capital controls, and vice versa.

Next, we illustrate how the constrained efficient policy response varies with different shocks, and for each shock, we highlight both the impact of the pricing structure (PCP or DCP) and other structural characteristics such as commodity dependence and the level of external debt.

3.2 Real Shocks

In this subsection, we consider domestically- and externally-generated shocks which affect the real income of domestic households.

3.2.1 Productivity Shocks

A permanent productivity shock is a shock to the value of $A_1 = A_2$ in the production function of the home tradable good. Since we focus on insights which do not hinge on intertemporal TOT manipulation, we set a parameterization of the shock to neutralize this motive across the PCP and DCP cases:[6]

$$\mathbb{E}_0 \left[\frac{A_0}{A_1} \right] = 1.$$

Under PCP, it is optimal to let the exchange rate move to absorb the shock without moving the level of any policy instruments. Figure 5 shows that the optimal domestic policy rate remains unchanged, while both ex-ante and ex-post capital controls are zero. After the high realization of the productivity shock, there is an increase in domestic aggregate supply. To close the gap between aggregate demand and aggregate supply, the exchange rate depreciates in order to make exports more competitive. Given the unit elastic demand for exports, there is no increase in the dollar value of export revenues, so domestic households are not more wealthy in dollar terms, and imports remain unchanged whether there is a high or low shock. Households consume more of the home tradable good after positive productivity shocks, as they become cheaper relative to imports. The exchange rate depreciates by the same percentage as the increase in productivity, so $\frac{E_t}{A_t}$ is constant across states of nature. Employment is perfectly stabilized across high and low shocks. The AD and TOT wedges are fully stabilized across shocks and over time, which means that there is no case for capital controls. This result does not depend on the level of initial external debt, B_0.

Under DCP, the key difference is that employment and the TOT wedges are no longer stabilized across high and low shocks by the exchange rate (figure 6). Nevertheless, capital controls do not

[6] The PCP results do not depend on this parameterization, while the DCP results do.

become optimal. Just because there is an unstabilized wedge and an extra available instrument does not mean that the instrument should be used. The instrument should only be used if it can help stabilize the wedge. The imperfect macro stabilization under DCP arises from the stickiness of the dollar price of exports across states. Capital controls alter the level of borrowing and the time path of consumption between periods, but they do not address the source of the stabilization problem.

Proposition 1. *Countries facing permanent productivity shocks should rely solely on exchange rate flexibility, under both PCP and DCP.*

Remark 1. *After permanent productivity shocks, TOT wedges are stabilized under PCP and not under DCP, but capital controls are optimally zero in both cases.*

3.2.2 Commodity Price Shocks

A permanent commodity price shock is a shock to the value of P^*_{Zt}, the dollar price of commodities, and is relevant for countries with large commodity sectors. Since commodities are undifferentiated while the traded goods are differentiated across countries, the effect of the shock should be different to the effect of the productivity shock. We consider two types of countries: one with no financial imperfections, i.e., $\kappa \to \infty$, and one with financial imperfections, i.e., $\kappa \in (0, 1)$.

Under PCP and $\kappa \to \infty$, i.e., for countries with large commodity sectors but without financial imperfections, capital controls are only optimal insofar as the intertemporal TOT externality is relevant. Figure 7 shows that after shocks, it is optimal to allow exchange rate flexibility without moving the policy rate, and there are small ex-ante capital controls arising solely from the intertemporal TOT manipulation motive. After the high realization of the shock, there is an increase in aggregate demand, and domestic households become wealthier in dollar terms. Therefore, the planner allows an exchange rate appreciation, which generates an increase in imports. The lower export demand from the exchange rate appreciation exactly offsets the wealth-driven increase in domestic consumption of the home traded good, so that employment in the traded sector does not change. Indeed, traded-sector employment is stabilized across high and low shocks, insulated from the commodity sector.

When κ is low enough and the initial external debt B_0 is high enough, however, i.e., for countries with large commodity sectors, financial imperfections, and high external FX debt, capital controls become optimal to address pecuniary AD externalities. Figure 8 shows that after the low realization of the shock, the borrowing constraint becomes binding, because the associated exchange rate depreciation causes a contraction in the dollar value of the debt limit. The forced deleveraging of households causes a contraction in aggregate demand, and the policy rate is lowered. However, since the multiplier on the borrowing constraint is positive, the planner recognizes that depreciation tightens the constraint (16). Therefore, equation (20) implies that the exchange rate is more appreciated, and the domestic policy rate is kept higher, than the price pressures from the AD and TOT wedges would justify after the low realization of the shock. As a result, the domestic consumption of the home good, and employment in the traded sector, both decline. The planner internalizes the AD externality and sets larger ex-ante capital controls than in the economy without financial imperfections.

Under DCP, the main results from PCP on the optimal use of instruments continue to hold (figures 9 and 10). The main difference is that the exchange rate is more volatile under DCP than PCP. After the high realization of the shock, the planner wants to implement the same increase in imports under DCP as under PCP. However, when the exchange rate appreciates under DCP, there is no decrease in export demand, so domestic households must absorb less home goods to keep employment stable. Inducing households to consume the same level of imports as under PCP, but fewer home goods than under PCP, means that the exchange rate appreciation must be larger under DCP. In the end, despite the larger appreciation, households consume less home goods, and employment becomes destabilized.

Owing to the larger exchange rate volatility under DCP than PCP, countries with financial imperfections face larger contractions in the debt limit after low realizations of the shock, and the planner sets larger ex-ante capital controls under DCP than PCP.

Proposition 2. *Countries with large commodity sectors should not impose capital controls to address pecuniary AD externalities if there are no financial imperfections, but they should impose positive ex-*

ante capital controls if there are such imperfections and initial external FX debt is high.

Remark 2. *After commodity price shocks, exchange rates are more volatile under DCP than PCP.*

Remark 3. *For countries with large commodity sectors, financial imperfections, and high initial external FX debt, ex-ante capital controls to address AD externalities are larger under DCP than PCP.*

3.3 Financial Shocks

In this subsection, we consider shocks which affect borrowing/lending transactions between domestic households and the rest of the world. We wish to capture a range of shocks which unpack the global financial cycle—shocks to world interest rates, foreign appetite for domestic-currency assets, and the FX debt limit—because different instruments may be necessary to address different dimensions of the cycle. Each of the shocks could also strike on a country-specific basis.

Since we set the intermediaries' balance sheet frictions to zero, i.e., $\Gamma_t = 0$, throughout this section, we do not consider shocks to intermediary frictions.

3.3.1 World Interest Rate Shocks

A world interest rate shock is a shock to the value of $(1 + i_1^*)$. Such a shock could be triggered by a change in U.S. monetary policy, expected global growth, or global risk appetite.

Under PCP, it is optimal to use the domestic policy rate combined with exchange rate flexibility, and capital controls are only optimal insofar as intertemporal TOT manipulation is relevant. Consistent with the households' Euler equation between periods 1 and 2, figure 11 shows that if the world interest rate experiences a high realization, then it is optimal for imports to decline in period 1 and increase in period 2. There is a decline in aggregate demand in period 1, which the planner cushions by allowing an exchange rate depreciation (the planner does increase the domestic policy rate, but less than the increase in the world interest rate), which stimulates export demand and import substitution. Employment is stabilized across the high and low shocks.

The high realization of the shock does not tend to make financial constraints binding in our

simulations. The reason is that although the exchange rate depreciation does cause the debt limit to contract, the desired debt level falls more rapidly.

Under DCP, the main difference is that the exchange rate is more volatile under DCP than PCP (figure 12), for the same reason as discussed above for the commodity price shock: export demand is no longer boosted by exchange rate movements, so import substitution must play a larger role in order to attempt to stabilize employment. This exchange rate volatility is achieved by not moving the policy rate in response to the shock. Even with the higher exchange rate volatility, employment does become destabilized. The ex-post capital controls which were optimal under PCP owing to the intertemporal TOT manipulation motive disappear under DCP, while the same motive generates small ex-ante capital controls.

Proposition 3. *Countries facing world interest rate shocks should not use capital controls to address AD externalities under PCP or DCP.*

Remark 4. *After world interest rate shocks, exchange rates are more volatile under DCP than PCP.*

3.3.2 Foreign Appetite Shocks

A foreign appetite shock is a shock to the value of S_1. For the case of deep FX markets, allocations are not affected by this shock. We defer further consideration of foreign appetite shocks to section 4.

Proposition 4. *Countries with deep FX markets do not need to respond to shocks to the foreign appetite for domestic-currency debt.*

3.3.3 Debt Limit Shocks

A debt limit shock is a shock to the value of κ, the pledgability parameter in the households' borrowing constraints. A decline in κ reflects either a downgrade in the perceived creditworthiness of a particular borrower country, or a reversal in the willingness of international banks to extend credit to all borrower countries.

Under PCP, when the initial external debt B_0 is high enough, and a low value of κ is possible ex-post, it is optimal to impose ex-ante capital controls to address pecuniary AD externalities. First, look at the solid lines in figure 13, which show the allocations when the planner uses both monetary policy (MP) and capital controls (CC). The figure shows that after the low realization of the shock, the borrowing constraint becomes binding. As described above in the example of the commodity price shock with financial imperfections, the binding constraint implies that the exchange rate is more appreciated, and the domestic policy rate is kept higher, than the price pressures from the AD and TOT wedges would justify. The planner internalizes this AD externality and sets positive ex-ante capital controls. There is no TOT-manipulation-based rationale for capital controls.

These results echo some of the concerns of emerging-market policymakers about the global financial cycle: if there is high external FX debt, they keep policy rates higher than standard AD concerns would suggest after global capital outflow shocks, and they choose to implement ex-ante policies to mitigate the overborrowing problem.

For the debt limit shock, and unlike the commodity price shock, the pecuniary AD externality is the sole rationale for capital controls. Therefore, variations in the optimal level of capital controls and the domestic policy rate arise solely owing to AD considerations, which facilitates the examination of the complementarity/substitutability of the domestic policy rate and capital controls.

Compare the dashed lines in figure 13, which show the allocation when only monetary policy is permitted, to the solid lines, which allow for both monetary policy and capital controls. When the domestic policy rate is the only instrument, the policy rate is kept elevated after the low realization of the shock in order to prevent excessive depreciation and the associated tightening of borrowing constraints. In principle, the imposition of ex-ante capital controls has an ambiguous impact on the ex-post policy rate. On the one hand, i.e., the "financial channel," capital controls reduce ex-ante borrowing, which makes the constraint less binding ex-post, and reduces the need to increase the policy rate so as to relax the constraint. On the other hand, i.e., the "macro channel," capital controls shift aggregate demand from period 1 to period 2, which induces higher policy rates in all states in

period 2. Whether imposing capital controls causes a higher or lower ex-post policy rate is therefore theoretically ambiguous. In our simulation, ex-ante capital controls are associated with a higher ex-post policy rate after the low realization of the shock.

Under DCP, the main results from PCP on the optimal use of instruments continue to hold (figure 14). The key difference is that the exchange rate is more depreciated after the debt limit shock, again owing to the pressure on import substitution to substitute for export adjustment. As a result, the ex-ante capital controls are larger.

Proposition 5. *Countries with high initial external FX debt who are subject to debt limit shocks should impose positive ex-ante capital controls.*

Remark 5. *For countries with high initial external FX debt and vulnerability to debt limit shocks, ex-ante capital controls are larger under DCP than PCP.*

Remark 6. *The imposition of ex-ante capital controls shifts both sides of the macro-financial trade-off for monetary policy, with ambiguous results on whether the policy rate and capital controls are substitutes or complements.*

3.4 Summary

For countries with deep FX markets, we have identified several shocks and structural characteristics which justify a deviation from the traditional Mundell-Fleming prescription of relying solely on the domestic policy rate and exchange rate flexibility. Focusing on the pecuniary-AD-externality rationale for capital controls, countries should impose positive ex-ante capital controls to reduce ex-ante overborrowing if they have high external FX debt near their debt limits, combined with an exposure to commodity price and/or debt limit shocks. If external FX debt limits are irrelevant, there is no case for the use of capital controls to address AD externalities for countries facing productivity shocks, commodity price shocks, world interest rate shocks, and foreign appetite shocks.

Mapping the global financial cycle to our model

Consider a retrenchment of the global financial cycle which encompasses three component shocks: a rise in world interest rates, a decline in foreign appetite for domestic-currency debt, and a contraction in debt limits for borrower countries. We can derive lessons for the policy rate and capital controls.

The optimal policy rate response is heterogeneous across different types of countries and retrenchment episodes. The planner wishes to accomodate the world interest rate shock by raising the policy rate less than the increase in the world interest rate under PCP, and having minimal change in the policy rate under DCP, so that the exchange rate is allowed to depreciate and cushion aggregate demand. The planner wishes to ignore the foreign appetite shock and keep the policy rate unchanged. The planner wishes to lower the policy rate, but keep it high enough to defend the exchange rate, after the debt limit shock. Given that each retrenchment episode has a different blend of component shocks, and that countries have heterogeneous vulnerabilities to the different component shocks, there should be heterogeneous directions of policy rate responses in the cross-section of countries in response to global financial cycle shocks.

Capital controls to address pecuniary AD externalities are used in only one direction, but not evenly across countries and episodes. The only modality of usage of such capital controls is a positive ex-ante capital inflow tax. The tax is only welfare-improving to the extent that the debt-limit-contraction component of the global financial cycle is relevant, and is only used by countries with high FX external debt.

4 Shallow Foreign Exchange Markets

We study next the constrained efficient allocations for a country with shallow FX markets. Such a case is relevant for most emerging markets, including countries such as Brazil, Peru, and Malaysia, as their currencies are traded by a limited subset of financial intermediaries. Even in normal times in the absence of shocks, these countries may only be able to finance their external debt by offering an external premium. Additionally, these countries are especially vulnerable to risk-on/risk-off episodes of the global financial cycle, as the willingness of foreigners to participate in the domestic-currency debt market is prone to boom-bust dynamics.

4.1 Policy Instruments and Wedges

The shallow FX markets case is characterized by setting $\Gamma_t > 0$ in the constraints and FOCs summarized in Appendix A.2. The relevant households' Euler conditions and Gamma equations are equations (4) and (10)-(11).

For countries with shallow FX markets, the Gamma equations indicate that the UIP conditions in equation (17) can be violated, with the level of gross external returns depending on the quantity of domestic-currency debt that financial intermediaries must be induced to hold on their balance sheets. These gross external returns must be provided through a combination of the domestic policy rate (which sets the returns available to households), capital controls, and the expected exchange rate movements (the latter two of which create a gap between households' and intermediaries' returns). For shallowness to matter in welfare terms, we need to impose that domestic households do not own all the intermediaries, i.e., $\lambda \in [0, 1)$.

Moving from the deep to the shallow FX markets case, the instrument of FX intervention becomes effective through the portfolio balance channel. Under shallow FX markets, the planner can use FX intervention to absorb some of the debt inflows and outflows and alter the equilibrium exposure of financial intermediaries to domestic-currency debt. In this manner, FX intervention changes the necessary level of the gross external returns on domestic-currency debt. Therefore, FX intervention

can now have an impact on exchange rates and allocations via the portfolio balance channel. Bear in mind that as in Gabaix-Maggiori (2015), Cavallino (2019), and Fanelli and Straub (2019), our modeling choice of the portfolio balance approach means that both the positive impact of, and normative rationale for, FX intervention will be focused on debt and premia, not on the level of the exchange rate per se.

Next we turn to the FOCs for the constrained efficient allocation to understand which additional externalities emerge as we move from deep to shallow FX markets. The planner's Euler conditions for $t \in \{0, 1\}$ are now:

$$\begin{aligned}
&\frac{\alpha_F}{P^*_{Ft} C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht}\right] - \mathbb{I}^{DCP} \cdot \left(\frac{1}{\beta \pi_1}\right)^t \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt}\right] \frac{1}{P^*_{Ft}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft}} \\
&= \beta \mathbb{E}_t \left\{ \frac{\alpha_F \left[(1 + i^*_t) + (1 - \lambda) \tau_{\Gamma t+1}\right]}{P^*_{Ft+1} C_{Ft+1}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht+1}\right] \right\} \\
&- \mathbb{I}^{DCP} \cdot \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt}\right] \mathbb{E}_t \left[\left(\frac{1}{\beta \pi_1}\right)^t \frac{(1 + i^*_t) + (1 - \lambda) \tau_{\Gamma t+1}}{P^*_{Ft+1}} \frac{1}{P_X} \frac{\pi_{t-1}}{\pi_t} \frac{\partial P_X}{\partial C_{Ft+1}} \right] \\
&+ \left(\Psi_t \frac{1}{\beta \left[\left(1 + i^*_{t-1}\right) + (1 - \lambda) \tau_{\Gamma t}\right]} \right)^t + \left(\frac{1}{\beta}\right)^t \Gamma_t \Omega_t. \qquad (22)
\end{aligned}$$

All three wedges, $\{\tau_{Ht}, \tau_{Xt}, \tau_{\Gamma t}\}$, now enter the planner's Euler conditions and generate divergences from the households' Euler conditions. Exchange rates continue to follow equation (20).

Relative to the deep FX markets Euler condition (19), there are two main additions: the term $\left(\frac{1}{\beta}\right)^t \Gamma_t \Omega_t$, where Ω_t is the multiplier on the Gamma equation; and the dependence of the discount factor in period $t + 1$ on the UIP wedges in the same period, $\tau_{\Gamma t+1}$. These UIP wedges also appear in the exchange rate equation (20).

Focusing first on the $\left(\frac{1}{\beta}\right)^t \Gamma_t \Omega_t$ term, we observe that the multiplier on the Gamma equation is positive when the external debt level in period t is forcing the UIP wedge, $\tau_{\Gamma t+1}$, to be higher than the planner would otherwise like it to be. We refer to this term as the "financial TOT" externality. When deciding on their level of borrowing, each household takes returns as given, without internalizing the impact of its borrowing decision on the returns facing all households. It does not internalize

that since the economy as a whole is the sole supplier of domestic-currency bonds to the financial intermediaries, the level of debt determines the UIP wedge in equilibrium. High UIP wedges lower welfare because they constitute excessive premia paid by domestic households to the foreign-owned fraction of the financial intermediaries.

The expression for capital controls is updated relative to the deep FX markets case as follows:

$$\varphi_t = \begin{cases} 1 - \dfrac{\frac{1}{\mathbb{E}_t\left\{\frac{E_t}{E_{t+1}}\right\}}\mathbb{E}_t\left\{\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right\}\left[\begin{array}{c}\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] \\ -\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\left(\frac{1}{\beta\pi_1}\right)^t\frac{1}{P^*_{Ft}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft}}\end{array}\right]}{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[\begin{array}{c}\mathbb{E}_t\left\{\frac{(1+i^*_t)+(1-\lambda)\tau_{\Gamma t+1}}{(1+i^*_t)}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right]\right\} \\ -\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right] \\ \times\mathbb{E}_t\left[\frac{(1+i^*_t)+(1-\lambda)\tau_{\Gamma t+1}}{(1+i^*_t)}\left(\frac{1}{\beta\pi_1}\right)^t\frac{\pi_{t-1}}{\beta\pi_t}\frac{1}{P^*_{Ft+1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft+1}}\right] \\ +\left(\frac{1}{\beta}\right)^t\Gamma_t\Omega_t\end{array}\right]} & \text{if } \Psi_t = 0 \\ 0 & \text{if } \Psi_t > 0, \end{cases} \tag{23}$$

If Ω_t is positive, the level of capital controls tends to be larger, as the planner discourages households' borrowing in order to reduce the UIP wedge.

Turning next to the UIP wedges in the planner's Euler condition (22) and in the exchange rate equation (20), we observe that if households do not own all the financial intermediaries, i.e., $\lambda \in [0, 1)$, then their effective external liabilities are partially in domestic currency. The planner can improve welfare by redistributing resources across states using the exchange rate: specifically, the planner should depreciate away the dollar value of repayments on external liabilities in states when domestic households' dollar resources are reduced by shocks, and increase the dollar value of repayments when households' dollar resources are enhanced by shocks. Using all policy instruments, the planner therefore selects different values of the UIP wedges in the exchange rate equation (20), which alters the optimal use of capital controls from equation (23).

The following trade-off determines the optimal UIP wedge, $\tau_{\Gamma t+1}$:

$$\underbrace{\Omega_t}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\frac{\Phi_{t+1}+\Psi_{t+1}}{\prod_{s=0}^{t} I_s}\left(B_{t+1}+FXI_t\right)}_{\text{Higher repayments tomorrow}} \pm \underbrace{\frac{1}{\pi_{t+1}}\Lambda_{t+1}E_{t+1}}_{\text{Prevent excessive contingency of premium}}$$

$$+ \underbrace{(1-\lambda)\,\Omega_{t+1}\Gamma_{t+1}\left(B_{t+1}+FXI_t\right)}_{\text{Higher premium tomorrow owing to rollover needs}}, \tag{24}$$

where $B_{t+1} \equiv \frac{D_{t+1}}{E_t}$ is the dollar value of debt, and Φ_t, Ψ_t, and Λ_t are the multipliers on the resource constraint, borrowing constraint, and contingency-check equation in period t.

The planner understands that under shallow FX markets, increasing the UIP wedge in order to allow a higher level of debt in period t worsens consumption in period $t+1$ owing to higher external debt repayments, and also requires a higher UIP wedge in period $t+1$ if the debt is rolled over. In addition, the possibility of FX intervention alters the expression for the gross external debt position: when the planner borrows in the domestic-currency debt market in order to accumulate dollar assets abroad, a fraction $(1-\lambda)$ of the debt ends up on the balance sheet of the foreign-owned intermediaries, so it constitutes external debt, and it enters the equation above.

The following trade-off determines the optimal level of FX intervention:

$$\underbrace{\Gamma\Omega_t}_{\text{Lower premium today}} + \underbrace{(1-\lambda)\,\mathbb{E}_t\left[\frac{\Phi_{t+1}+\Psi_{t+1}}{\prod_{s=0}^{t} I_s}\tau_{\Gamma t+1}\right]}_{\text{Change in carry cost}} + \underbrace{(1-\lambda)\,\mathbb{E}_t\left[\Omega_{t+1}\tau_{\Gamma t+1}\right]}_{\text{Change in premium tomorrow owing to change in carry cost}}$$

$$= \Theta_t + \mathbb{E}_t\Theta_{t+1}, \tag{25}$$

where $\{\Theta_t, \Theta_{t+1}\}$ represent the multipliers on the constraints associated with FX intervention, equation (9).

By absorbing some of the capital inflow or outflow, FX intervention can reduce the external debt that foreign-owned intermediaries have to absorb, and it can thereby reduce the UIP wedge. There is a marginal benefit from relaxing households' borrowing constraints if and only if the constraints are binding, i.e., $\Psi_t > 0$. This marginal benefit should be combined with any carry costs incurred

by the FX intervention, taking into account that higher carry costs incurred between periods t and $t+1$ may result in higher external debt, which needs to be rolled over in period $t+1$. If the planner can undertake FX intervention in an unconstrained manner, i.e., $\Theta_t = \Theta_{t+1} = 0$, then it should intervene until the net marginal benefit is pushed down to zero. However, if some or all of the FX intervention constraints in equation (9) are binding, and $\Theta_t, \Theta_{t+1} > 0$, then not all the marginal benefits of intervention are exhausted.

Since the case with shallow FX markets features a more complex problem than the deep FX markets case, with more policy instruments and equilibrium conditions, let us take stock of the system of equations and highlight some of their insights regarding the potential complementarity and substitutability of instruments.

Firstly, the equations reveal several interactions between FX intervention and capital controls. FX intervention has two effects on the optimal capital control expression: (i) intervention affects the UIP wedge, $\tau_{\Gamma t}$, which alters the time path of consumption and the AD wedge, τ_{Ht}; and (ii) intervention alters the multiplier on the Gamma equation, Ω_t. Capital controls have two effects on the optimal level of FX intervention: (i) capital controls reduce gross external returns, η_t, which affect the carry cost of intervention; and (ii) they reduce households' debt, which alters the incentive to absorb the debt via FX intervention. The complementarity/substitutability of FX intervention and capital controls depends on the relative strength of all these channels, which should vary according to shocks and structural characteristics.

Secondly, the equations show that the domestic policy rate also interacts with FX intervention and capital controls. Complementarity/substitutability on this dimension again depends on shocks and structural characteristics. If FX intervention and capital controls can insulate allocations from a particular shock, then the policy rate may need to move less when those instruments are available. If the planner wishes to alter allocations as a result of the shock, but FX intervention and capital controls alone can only stabilize the UIP wedge while exacerbating the volatility of the AD wedges, then the policy rate may need to move more when all instruments are used together.

With these insights in mind, we proceed to document and discuss our simulation results on the constrained-efficient policy responses as a function of different shocks and structural characteristics. We again highlight deviations from the Mundell-Fleming prescription of solely using the domestic policy rate and exchange rate flexibility, and the potential complementarity and substitutability of instruments. In the deep FX markets case, we found that it is optimal to use capital controls to address pecuniary AD externalities when shocks trigger occasionally-binding borrowing constraints owing to financial imperfections. In the shallow FX markets case, financial imperfections in the FX market are always binding, so we should expect to find a greater role for capital controls and FX intervention.

The simulations for the shallow FX markets cases are expansive and currently in progress, so we do not yet have results for all combinations of instruments and imperfections. Nevertheless, we present a wide range of results with insights that are new relative to the deep FX markets case.

4.2 Real Shocks

Under shallow FX markets, the optimal responses to real shocks now depend to a greater degree on the level of external debt, B_0, than they did under deep FX markets. Therefore, we present separate simulations for $B_0 = 0$ and $B_0 > 0$ below.

4.2.1 Productivity Shocks

We consider permanent shocks to the value of productivity, $A_1 = A_2$. The parameterization of the shock which neutralizes the intertemporal TOT manipulation now changes to the following condition:[7]

$$\mathbb{E}_0 \left[\frac{A_0}{A_1} \frac{\lambda \left(1 + i_0^*\right) + \left(1 - \lambda\right) \eta_1}{\left(1 + i_0^*\right)} \right] = 1.$$

Under PCP, the optimal policy depends on the level of initial debt, B_0. Figure 15 shows solid and dashed lines corresponding respectively to $B_0 = 0$ and $B_0 > 0$. When there is no initial external

[7] As in the case with deep FX markets, the PCP results do not depend on this parameterization, while the DCP results do. The new condition under shallow FX markets is more complicated than under deep FX markets, because it relates the exogenous shock A_1 to the endogenous variable η_1. However, the condition is easily implemented as follows: (i) we fix the values of η_1 at the values which arise from the specification $\lambda = 1$, (ii) then we allow λ to vary within $[0, 1)$, and find that the equilibrium values of η_1 remain unchanged, so the condition continues to be satisfied.

debt, i.e., $B_0 = 0$, it is optimal to let the exchange rate move to absorb the shock without moving the ex-post level of any policy instruments, like in the case of deep FX markets. Since imports are fully stabilized over time using exchange rate flexibility alone, the planner does not need to induce households to issue external debt for consumption-smoothing purposes after the shock. Without new external debt entering the FX market, the depth of the FX market—i.e., the ability of intermediaries to absorb new debt without requiring a premium—is not relevant for ex-post policy responses.

From the ex-ante perspective, capital controls and FX intervention are close to zero. The logic from the deep FX markets case continues to apply, pushing both to zero. However, there is one new channel owing to the presence of less-than-full currency mismatch, i.e., $\lambda < 1$: the "repayment contingency" motive. Some of any ex-ante external debt is effectively in domestic-currency, so the dollar value of ex-post repayments is altered by ex-post exchange rate movements. Since welfare is not identical across ex-post shocks (as there is higher consumption of the home good after high productivity shocks), the planner wishes to generate an ex-ante asset position so that the ex-post exchange rate movements transfer resources from high to low states. The desired asset position is necessarily small owing to the desire to smooth imports between periods 0 and 1, and in the figure it is almost undetectable. It shows up in the simulations as being implemented by an elevated policy rate combined with small capital controls and FX intervention. In practice, the contingency of ex-post debt repayments is realistic, but the desire to distort the ex-ante debt level is not, so we do not focus on ex-ante policy recommendations which hinge on this motive.

With positive initial external debt, i.e., $B_0 > 0$, we derive three additional insights. Firstly, there is a case for steady-state capital controls. Intermediaries require a premium to hold the debt even in the absence of shocks, and this premium drives households to deleverage ex-ante and ex-post. Each household does not internalize that their debt position affects the interest rate for all other households. The planner does internalize this financial TOT externality, and sets capital controls both ex-ante and ex-post to optimize the pace of develeraging. In the PCP case, it appears that households deleverage too fast, so the steady-state capital inflow taxes are negative. Secondly, it is not optimal

to use FX intervention. The declining debt path means that UIP wedges are higher ex-ante than ex-post. Therefore, the cost of ex-ante FX accumulation (which increases the UIP wedge) is larger than the benefit of ex-post FX sales (which reduce the UIP wedge), so FX reserves are never accumulated to begin with.[8] Thirdly, the contingency of the dollar value of debt repayments is amplified when external debt is positive, although it still remains small. The dollar value of households' wealth diverges slightly across high and low shocks, which generates a slight divergence in imports as well.

Under DCP, the main results from PCP continue to hold (figure 16). When there is no initial external debt, i.e., $B_0 = 0$, the optimal policy responses under deep and shallow FX markets coincide ex-post and vary slightly ex-ante. When there is positive initial external debt, i.e., $B_0 > 0$, the same considerations regarding steady-state capital controls and FX intervention apply as they did under PCP. The main difference is that under DCP, it appears that households deleverage too slowly, so the optimal steady-state capital inflow taxes are positive.

Proposition 6. *If initial external debt is low, countries facing permanent productivity shocks should rely solely on exchange rate flexibility, under both PCP and DCP. If initial external debt is high, then exchange rate flexibility should be supplemented with non-zero steady-state capital controls. Regardless of the level of external debt, it is not optimal to accumulate or spend FX reserves.*

Remark 7. *Productivity shocks which generate exchange rate fluctuations without generating changes in external debt do not rationalize ex-post capital controls or FX intervention.*

4.2.2 Commodity Price Shocks

We consider permanent shocks to the dollar price of commodities, P^*_{Zt}. So far, we only have results for the case with no debt limits, i.e., $\kappa \to \infty$.

Under PCP, the optimal policy depends on the level of initial debt, B_0. Figure 17 shows that

[8] If the planner's ability to sell dollar bonds and lend in domestic currency is unlimited, it would issue dollar bonds and absorb all of the domestic-currency debt B_0 in every period, removing the need for the intermediaries and reducing the premium to zero. With a zero premium, there is no need to deleverage, and household debt would remain at B_0 in every period. This allocation is unrealistic, as the planner can always circumvent the need for intermediaries. Imposing the FX intervention constraint (9) makes the allocation no longer feasible.

when there is no external debt, i.e., $B_0 = 0$, it is optimal to let the exchange rate move to absorb the shock without moving the ex-post policy rate, like in the case of deep FX markets. Exchange rate flexibility stabilizes imports over time after the shock, so ex-post external debt issuance is not necessary. Therefore, the depth of the FX market is not relevant, and neither capital controls nor FX intervention are used ex-post (in other words, the argument above for the permanent productivity shock applies here as well ex-post).

From the ex-ante perspective, the repayment contingency motive (as described above for the permanent productivity shock) continues to apply, but rationalizes only small and unrealistic levels for ex-ante capital controls and FX intervention. In addition, as in the deep FX markets case, ex-ante capital controls may also be motivated by the intertemporal TOT externality.

Moving from zero to positive external debt alters the allocations in exactly the same way as was applicable for the permanent productivity shock.

Under DCP, the main results from PCP continue to hold (figure 18). When there is no initial external debt, i.e., $B_0 = 0$, the optimal policy responses under deep and shallow FX markets coincide ex-post and vary slightly ex-ante. Moving from zero to positive initial external debt alters the allocations in the same manner as was applicable for the permanent productivity shock. The main difference between the PCP and DCP cases is that exchange rates are more volatile under DCP.

Proposition 7. *Suppose that external debt is not subject to borrowing constraints. If initial external debt is low, countries with large commodity sectors should not impose capital controls to address pecuniary AD externalities. If initial external debt is high, then exchange rate flexibility should be supplemented with non-zero steady-state capital controls. Regardless of the level of initial external debt, it is not optimal to accumulate or spend FX reserves.*

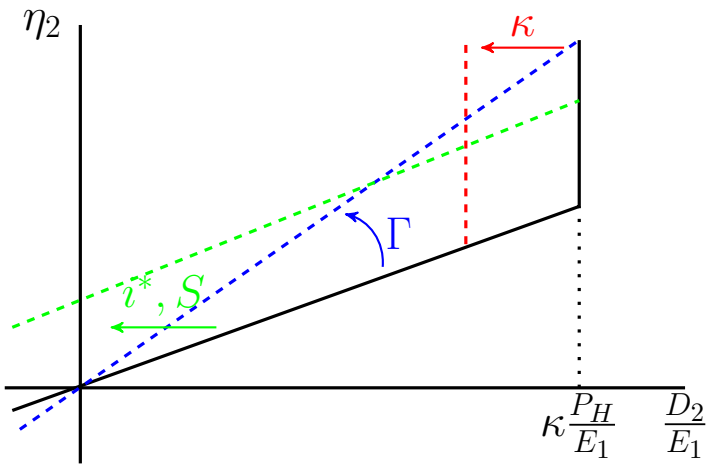
Remark 8. *After commodity price shocks, exchange rates are more volatile under DCP than PCP.*

Remark 9. *Commodity price shocks which generate exchange rate fluctuations without generating changes in external debt do not rationalize ex-post capital controls or FX intervention.*

4.3 Financial Shocks

Under shallow FX markets, financial shocks interact with both the occasionally-binding debt limits and the always-binding intermediation frictions. Therefore, some shocks which can be ignored by countries with deep FX markets cannot be ignored by countries with shallow FX markets. Figure 4 illustrates how our array of financial shocks affects the net demand curve for bonds in the domestic-currency debt market.

Figure 4: Foreign Appetite Shock under PCP with MP and CC



The dependence of the optimal policy responses on the level of initial external debt, B_0, has already been discussed above for real shocks, and the insights do not change for financial shocks. Therefore, for simplicity, we fix $B_0 = 0$ below except for the intermediary friction and debt limit shocks, where $B_0 > 0$ is necessary for the shocks to have any impact.

4.3.1 World Interest Rate Shocks

We consider a shock to the value of the world interest rate, $(1 + i_1^*)$. We do not have simulation results yet for the case with the FX intervention constraint (9), so we solve for the allocations without this constraint and then explain our conjecture regarding the likely implications of imposing the constraint.

Under PCP, the planner optimally accommodates the shock using all policy instruments in a complementary fashion. To begin, consider the allocation with only monetary policy (MP) and capital controls (CC), represented by the dashed lines in figure 19. When the world interest rate declines, then from the Euler condition, it is optimal for imports to increase in period 1 and decline in period 2. However, an increase in imports requires external borrowing, which increases the UIP wedges that foreign-owned intermediaries earn at the expense of the domestic economy. Recognizing this problem, the planner does not stimulate a large increase in debt: it lowers the domestic policy rate slightly. Capital controls are small because both the new financial TOT externality and the old intertemporal TOT manipulation motive are small at such low levels of debt.

Next, consider the solid lines in figure 19, which plot the allocation when FX intervention is also available, but ignoring the FX intervention constraint (9). The planner can now reduce the premium accruing to intermediaries by conducting ex-post FX sales and purchasing domestic-currency bonds. Since the premium is now lower, the planner can reduce the ex-post policy rate more, and stimulate more ex-post debt. At higher levels of debt, capital controls become larger only insofar as intertemporal TOT manipulation is relevant.

Therefore, the planner does not lean against the wind, but instead uses a combination of looser monetary policy and expansionary FX intervention in order to take advantage of the lower world interest rate. The aim is to boost aggregate demand while minimizing the UIP wedge.

We conjecture that if the FX intervention constraint (9) were to be imposed, the constrained-efficient allocation would lie somewhere in-between the dashed and solid lines. Our rationale lies in the time path of the UIP wedges. In the absence of FX intervention, external debt and the UIP wedge are higher in some ex-post states than in the ex-ante period. Therefore, under certain parameterizations, the marginal welfare cost of ex-ante FX accumulation (which increases the UIP wedge) is smaller than the expected marginal welfare benefit of ex-post FX sales (which reduce the UIP wedge), and FX reserves would indeed be accumulated ex-ante. Owing to the ex-ante carry cost of accumulation, ex-post FX intervention would take a value between zero and the level of intervention indicated

by the solid lines in figure 19.

Under DCP, FX intervention is again used to accommodate the low realization of the shock, increasing debt and aggregate demand while minimizing the UIP wedge (figure 20). However, there are two main differences. Firstly, the exchange rate is more volatile under DCP than PCP, like in the deep FX markets case. Secondly, the introduction of FX intervention reduces rather than increases the ex-post movement in the policy rate. The reason is that FX intervention makes ex-post policy rate changes move closer to their deep-FX-market equivalents, and remember that under deep FX markets, the policy rate is kept constant in response to the shock in order to facilitate a larger exchange rate volatility.

Proposition 8. *Countries facing world interest rate shocks should use ex-post FX intervention to accommodate the shocks under PCP and DCP.*

Remark 10. *After world interest rate shocks, exchange rates are more volatile under DCP than PCP.*

4.3.2 Foreign Appetite Shocks

A foreign appetite shock is a shock to the value of S_1. These shocks are changes to the foreign demand for domestic-currency bonds motivated by factors other than returns. Potential triggers includes booms/busts in foreigners' "animal spirits," changes in perceived creditworthiness, and/or the abrupt entry of large institutional investors into the thin market for a borrower country's debt.

As with the previous shock, we solve for the allocations without the FX intervention constraint (9), and then explain our conjecture regarding the likely implications of imposing the constraint.

Under PCP, the domestic policy rate appears to be a poor substitute for capital controls and FX intervention. Figures 21 and 22 trace out the allocations as we add instruments one by one to the planner's toolkit.

The dashed lines in figure 21 plot the allocation with only monetary policy. When foreigners' appetite improves, there is a decline in the UIP wedge, because the entry of the new category of foreigners reduces the debt held, and the premium demanded, by the profit-maximizing interme-

diaries. Domestic households increase their borrowing to finance a large increase in imports and aggregate demand. Their borrowing is excessive, because each household does not internalize that their increase in debt is increasing the interest rate for all other households. As the shock is insensitive to the premium, monetary policy is not an effective instrument to insulate the economy from the shock: raising the policy rate to deter overborrowing simply results in larger UIP wedges for the intermediaries. The exchange rate appreciation leads households to consume less of the home tradable good, so the planner reduces the policy rate to limit the appreciation, stimulating export demand and stabilizing employment.

The solid lines in figure 21 plot the allocation with monetary policy and capital controls. When foreigners' appetite improves, positive ex-post capital controls are more effective than monetary policy. Capital controls cannot absorb the full impact of all the new foreigners, but they can address the financial TOT externality. Ex-post overborrowing is significantly reduced. Since the shock is being tackled closer to its source, there are less symptoms to remedy, and the ex-post policy rate needs to move less to stabilize employment.

The dashed lines in figure 22 plot the allocation with monetary policy and FX intervention. They show that ex-post FX accumulation is also more effective than monetary policy. FX intervention cannot address the financial TOT externality at any given level of debt, but it can limit the impact of foreigners' entry. By selling domestic-currency bonds and purchasing dollar bonds, the planner absorbs some of the high appetite shock instead of letting it be absorbed in an inefficient manner by domestic households. Again, the ex-post policy rate needs to move less to stabilize employment. Even though we have not imposed the FX intervention constraint (9), FX intervention does not absorb all of the appetite shock: the planner internalizes that it is earning carry profits on its intervention, and those profits would diminish if all of the shock is absorbed.

The solid lines in figure 22 plot the allocation with monetary policy, capital controls, and FX intervention. Capital controls can now address the financial TOT externality while FX intervention absorbs some of the appetite shock, and when both are used together, allocations are perfectly stabi-

lized across states and over time. Capital controls and FX intervention appear to be mild substitutes with each other under PCP, as both are used a little less together than when they were used alone. They are strong substitutes for the domestic policy rate: if both capital controls and FX intervention are available, then they insulate the economy from the volatility of the foreign appetite shocks, and the domestic policy rate need not move at all.

The use of capital controls and FX intervention increases monetary autonomy by tackling the appetite shock close to its source, and freeing up monetary policy to be used for other purposes.

We conjecture that if the FX intervention constraint (9) were to be imposed, the constrained-efficient allocation would lie somewhere in-between the solid lines in figures 21 and 22. There would be a rationale for FX accumulation under some parameterizations because the UIP wedges are higher in some ex-post states than in the ex-ante period. However, owing to the ex-ante carry cost of accumulation, ex-post FX intervention would take a value between zero and the level of intervention indicated by the solid lines in figure 22.

Under DCP, the the main results from PCP on the optimal use of instruments and their complementarity/substitutability continue to hold (figures 23 and 24). There are two main differences. Firstly, exchange rates are more volatile under DCP and PCP. Secondly, the policy rate increases rather than decreases after the high appetite shock.

Proposition 9. *Countries facing foreign appetite shocks should use ex-post capital controls and ex-post FX intervention to insulate the domestic economy and increase ex-post monetary autonomy under PCP and DCP.*

Remark 11. *After foreign appetite shocks, capital controls and FX intervention are substitutes for the domestic policy rate.*

4.3.3 Intermediary Friction Shocks

An intermediary friction shock is a shock to Γ_1, the parameter capturing the balance sheet frictions of intermediaries. In contrast to the foreign appetite shocks, these intermediary friction shocks are

changes to the sensitivity of the demand for domestic-currency bonds to the returns on those bonds.

Under PCP, it is optimal to use the domestic policy rate, capital controls, and under some circumstances FX intervention. Figure 25 shows solid and dashed lines corresponding respectively to low and high values of initial external debt, specifically $B_0 = 0.2$ and $B_0 = 0.3$. Financing a positive debt position requires provision of UIP wedges to intermediaries in all periods. A high shock to the intermediary friction amounts to a higher UIP wedge, and in response, households deleverage their debt position. Aggregate demand declines and the exchange rate depreciates, stimulating exports and employment. The planner raises the domestic policy rate to provide a higher UIP wedge to intermediaries so that they continue to provide some financing to domestic households. The planner tightens capital controls a little in order to mitigate the financial TOT externality, whose salience is higher when the shock is higher.

Since the UIP wedges are higher ex-post than ex-ante, there may be a case for ex-ante FX accumulation followed by ex-post FX sales. The solid lines show that when initial external debt is low, i.e., $B_0 = 0.2$, the marginal welfare cost of ex-ante FX accumulation (which increases the UIP wedge) is smaller than the expected marginal welfare benefit of ex-post FX sales (which reduce the UIP wedge), so FX reserves are indeed accumulated ex-ante and spent ex-post. However, the dashed lines show that when initial external debt is high, i.e., $B_0 = 0.3$, the marginal welfare cost of ex-ante FX accumulation is too high for any accumulation to occur, and there are no FX reserves to be spent after shocks.

We have not yet simulated this shock for the DCP case.

Proposition 10. *Suppose that the pricing paradigm is PCP and that initial external debt is positive, i.e., $B_0 > 0$. Countries facing intermediary friction shocks should use the ex-post policy rate and ex-post capital controls, alongside steady-state capital controls. If ex-ante UIP wedges are low enough, then FX reserves should be accumulated ex-ante to be used ex-post.*

4.3.4 Debt Limit Shocks

We consider a shock to the value of the pledgability parameter in the households' borrowing constraints, κ.

Under PCP, when the initial external debt B_0 is high enough, and a low value of κ is possible ex-post, the main insights from the deep FX markets case continue to hold. Figure 26 shows that it remains optimal to impose ex-ante capital controls to address pecuniary AD externalities, and to keep the domestic policy rate higher than the price pressures from the AD and TOT wedges would justify after the low realization of the shock. The only difference is that since the external debt is positive and FX markets are shallow, we need to take into account that there is a case for steady-state capital controls, following the same logic as for the permanent productivity shock. From an ex-ante perspective, the level of the ex-ante capital controls needed to address pecuniary AD externalities should be added onto the level of steady-state capital controls for shallow-FX-market reasons.

This similarity between the cases for deep and shallow FX markets may be puzzling, as it would seem intuitive that using FX intervention to defend the exchange rate ex-post should help relax borrowing constraints while letting the domestic policy rate address aggregate demand. However, there is a flaw in this intuition. When the debt limit binds, the Euler condition between periods 1 and 2 no longer holds for households, so the domestic policy rate has no impact on aggregate demand through the domestic-currency bond market. In the case of deep FX markets, the policy rate is assigned solely to managing the level of the exchange rate after low realizations of the shock, as it has no other use. In the case of shallow FX markets, both the policy rate and FX intervention can manage the level of the exchange rate when the debt limit binds. The optimal instrument to manage the exchange rate is actually the policy rate, because the policy rate is not needed for any other purpose, while FX intervention has a carry cost. Therefore, it is not optimal to use ex-post FX intervention, and there is little difference between the cases of deep and shallow FX markets.

Under DCP, the main results from PCP on the optimal use of instruments continue to hold (figure 27). The key difference is that the exchange rate is more depreciated after the debt limit shock, and

as a result, the ex-ante capital controls are larger.

Proposition 11. *Countries with high initial external FX debt who are subject to debt limit shocks should impose positive ex-ante capital controls.*

Remark 12. *After debt limit shocks, FX intervention does not improve monetary autonomy.*

Remark 13. *For countries with high initial external FX debt and vulnerability to debt limit shocks, ex-ante capital controls are larger under DCP than PCP.*

4.4 Summary

Countries with shallow FX markets have more reasons to deviate from the traditional Mundell-Fleming prescription, and to use capital controls and FX intervention, than countries with deep FX markets. Our model offers three main insights on optimally integrated policies which apply across all the shocks.

Firstly, countries with shallow FX markets and positive external debt should impose capital controls in steady-state, i.e., both ex-ante and ex-post. These capital controls address the financial TOT externality which arises from the need to continuously provide positive UIP wedges to the foreign-owned intermediaries who finance external debt.

Secondly, there is a case for ex-post capital controls and ex-post FX intervention if and only if shocks generate changes in external debt and UIP wedges. Shocks which generate exchange rate fluctuations without generating changes in external debt, such as permanent productivity and commodity price shocks, do not rationalize ex-post capital controls or FX intervention. This result underlines that the rationale for deviating from the traditional Mundell-Fleming prescription is focused on debt and premia, not on the level of the exchange rate per se. Countries worried about whether to respond to capital inflows and outflows should monitor UIP wedges.

Thirdly, ex-ante FX accumulation and ex-post FX sales are conducted only if UIP wedges are higher under some shocks ex-post than they are ex-ante. The carry costs of reserves can only be justified if there is some possibility that the marginal welfare cost of ex-ante FX accumulation (which

increases the UIP wedge) is smaller than the expected marginal welfare benefit of ex-post FX sales (which reduce the UIP wedge). If UIP wedges are high ex-ante, owing to high external debt financing needs ex-ante, our model does not recommend the ex-ante accumulation of FX reserves, even if the country is vulnerable to the interaction of shocks and intermediary frictions ex-post.

Mapping the global financial cycle to our model

Consider a retrenchment of the global financial cycle which encompasses four component shocks: a rise in world interest rates, a decline in foreign appetite for domestic-currency debt, an increase in intermediary frictions, and a contraction in debt limits for borrower countries. We can derive lessons for the policy rate, capital controls, and FX intervention.

The optimal responses are again heterogeneous across different types of countries and retrenchment episodes. Whether the planner wishes to accommodate, ignore, or fight each component shock has parallels to the case of deep FX markets, but the policy implementation is different under shallow FX markets.

The planner wishes to accommodate the world interest rate shock, and under shallow FX markets, it optimally uses ex-post FX intervention to reinforce the shock rather than to lean against it.

The planner wishes to ignore the foreign appetite and intermediary friction shocks and set monetary policy based on domestic considerations only. Unfortunately, the shallowness of FX markets compromises monetary autonomy, and the policy rate is forced to respond to external factors. Since the foreign appetite shock is not based on returns, monetary policy is ineffective at addressing the shock at its source, and the policy rate is assigned to address the employment destabilization generated by the shock; while for the intermediary friction shock, capital flows respond to returns, and monetary policy is more effective. For both shocks, the use of ex-post capital controls and ex-post FX intervention restores monetary autonomy, so the economy is insulated from the global-financial-cycle shocks while the policy rate can be set based on domestic considerations only.

The planner's response to the debt limit shock is similar under deep and shallow FX markets, because FX intervention is unable to increase monetary autonomy in the state of nature when the

economy is most stressed. When the debt limit binds, there is no benefit from using FX intervention to target the exchange rate so that monetary policy can be set based on domestic considerations only: the policy rate is not effective in influencing households' borrowing/lending decisions anyway, because the domestic monetary transmission mechanism has broken down.

5 Additional Policy Experiments

In this section, we extend our model in order to shed light on the positive and normative impact of some additional policy instruments which have been used in practice.

5.1 FX Bailouts

First, after shocks which make the borrowing constraint bind, we allow the planner to provide ex-post FX bailouts.

We have previously assumed that in period 1, the planner's position of O_2 in domestic-currency bonds is financed via borrowing $-FXI_1 = \frac{O_2}{E_1}$ in dollar bonds. Now, we assume that the planner can provide an exogenous fraction $\mu \in (0, 1]$ of those dollar bonds directly to domestic households, so that even if intermediaries are not willing to lend, the households can use the dollars to consume imports. The households' borrowing constraint changes to the following:

$$D_2 \leq \kappa P_{H1} + \mu O_2,$$

and the economy-wide borrowing constraint changes accordingly:

$$\frac{D_2}{E_1} \leq \kappa \frac{P_H}{E_1} - \mu FXI_1. \tag{26}$$

Replacing condition (16) with the above constraint means that in the constraints and FOCs summarized in Appendix A.2, all the $\kappa \frac{P_H}{E_1}$ terms should be replaced by $\left[\kappa \frac{P_H}{E_1} - \mu FXI_1\right]$. The trade-off which determines the optimal level of FX intervention under shallow FX markets changes to the following:

$$\underbrace{\Gamma \Omega_t}_{\text{Lower premium today}} + \underbrace{\Psi_t \frac{\mu}{I_0}}_{\text{Relax binding constraints via FX bailout}}$$

$$+ \underbrace{(1-\lambda)\,\mathbb{E}_t\left[\frac{\Phi_{t+1}+\Psi_{t+1}}{\prod_{s=0}^{t} I_s}\tau_{\Gamma t+1}\right]}_{\text{Change in carry cost}} + \underbrace{(1-\lambda)\,\mathbb{E}_t\left[\Omega_{t+1}\tau_{\Gamma t+1}\right]}_{\text{Change in premium tomorrow owing to change in carry cost}}$$

$$= \Theta_t + \mathbb{E}_t \Theta_{t+1}. \tag{27}$$

Relative to the previous trade-off condition (25), the new condition takes into account the benefit of FX intervention in directly relaxing households' borrowing constraints, provided that the borrowing constraint is binding, i.e., $\Psi_t > 0$, and that FX bailouts are possible, i.e., $\mu > 0$.

Figure 28 shows that the possibility of ex-post FX bailouts reduces the optimal level of ex-ante capital controls. The dashed lines on the figure plot the allocations when FX bailouts are not possible, and the solid lines plot the allocations when FX bailouts are possible. FX bailouts improve welfare after the low debt limit shock by directly relaxing households' borrowing constraints. The optimality of FX intervention no longer depends only on the time path of UIP wedges, but also on the multiplier on the borrowing constraint. If the multiplier is positive, i.e., $\Psi_t > 0$, and the carry cost of ex-ante FX accumulation is not too high, it becomes optimal to conduct ex-ante FX accumulation, and then provide ex-post FX bailouts after low shocks.[9]

Since ex-post FX bailouts reduce the multiplier on the borrowing constraint, the optimal level of ex-ante capital controls to address pecuniary AD externalities declines. In our PCP simulation, when steady-state capital controls are negative, the AD-externality-motivated ex-ante capital controls and the steady-state component of capital controls sum to a number which is negative overall.

The figure is plotted for the PCP case. We expect that under DCP, the possibility of ex-post FX bailouts again reduces the optimal level of ex-ante capital controls. However, the steady-state capital controls are positive under DCP, so we expect the overall ex-ante capital controls to be positive.

Proposition 12. *The possibility of ex-post FX bailouts reduces the optimal level of ex-ante capital controls.*

[9] In the figure, it looks like bailouts are provided after both high and low debt limit shocks. In fact, after high shocks, the borrowing constraint is not binding, so FX intervention only affects allocations via the portfolio balance channel and not via bailouts. The reason for the FX intervention after high shocks is simply that once ex-ante FX accumulation has occurred, motivated by the possibility of ex-post FX bailouts after low shocks, those FX reserves are also automatically available to reduce UIP wedges after high shocks. We have seen in section 2 that the reduction of UIP wedges after high shocks is not sufficiently welfare-improving to on its own justify ex-ante FX accumulation.

5.2 Ban on Open FX Exposures

Next, we consider the impact of a ban on open FX positions for those intermediaries which are domestically owned. The economy acquires currency mismatch through its ownership of intermediaries who have dollar liabilities. A ban on FX positions for those intermediaries may then be seen as a way to reduce the economy's vulnerability to debt limit shocks.

In our model, the mechanical impact of such a ban is to remove the fraction of intermediaries which are domestically owned, λ, from participation in the FX market. As a result, domestic-currency debt can only be absorbed by those intermediaries who are foreign-owned, and the Gamma equations (10) and (11) need to be replaced with the following equations:

$$\frac{\Gamma_0}{(1-\lambda)}\left(\frac{D_1}{E_0}+FXI_0-S_0\right)=\mathbb{E}_0\left[\eta_1-(1+i_0^*)\right] \tag{28}$$

$$\frac{\Gamma_1}{(1-\lambda)}\left(\frac{D_2}{E_1}+FXI_1-S_1\right)=\eta_2-(1+i_1^*)\,. \tag{29}$$

The economy-wide resource constraint (15) should be replaced as well:

$$\begin{aligned} D_{t+1} \geq & -E_t P_{Ft}^* \left[\omega C_t^* - C_{Ft}\right] - O_t\left[\left(1+\widehat{i}_{t-1}\right)-\left(1+i_{t-1}^*\right)\frac{E_t}{E_{t-1}}\right] \\ & - E_t P_{Zt}^* Z_t + \left(1+\widehat{i}_{t-1}\right) D_t. \end{aligned} \tag{30}$$

Relative to equation (15), there are two main differences. Firstly, there is no fraction λ of debt with currency mismatch, so an exchange rate depreciation does not increase the domestic-currency value of debt repayments. Secondly, the effective carry cost of FX intervention is larger, because all the carry profits accrue to foreign-owned intermediaries and none to domestically-owned intermediaries.

The amended system of constraints and FOCs is summarized in Appendix 5.2.

A ban on FX positions does indeed reduce the economy's vulnerability to debt limit shocks and remove the need for ex-ante capital controls to address AD externalities. Figure 29 shows the impact of banning FX positions on the allocations after debt limit shocks, with the dashed lines representing

allocations without the ban and the solid lines representing allocations with the ban. With the ban in effect, the borrowing constraint is no longer binding after the debt limit shock. This result matches the stated motivation of policymakers who impose such bans, and is captured here because our model includes currency mismatch and occasionally-binding borrowing constraints.

However, taking an integrated approach, and considering together the occasionally-binding borrowing constraints and the always-binding intermediary frictions, we can derive new insights.

Firstly, the debt financing problem worsens, and steady-state capital controls change. Open FX positions only arise in equilibrium for countries who need intermediaries to finance domestic-currency debt. If markets are shallow, this financing generates positive UIP wedges. The removal of domestically-owned intermediaries from the FX market makes the market shallower, which worsens the debt financing problem. In equilibrium, either UIP wedges must increase or debt must decrease. The latter occurs in our simulation. The level of steady-state capital controls changes when the financing problem changes. Under our simulation's assumption of DCP, there is a large increase in steady-state capital controls.

Secondly, the ban on open FX positions increases the vulnerability of the economy to any shocks which generate external debt, such as foreign appetite shocks, and increases the marginal value of ex-post FX intervention. Figure 30 shows the impact of banning FX positions on the allocations after foreign appetite shocks, with the dashed lines representing allocations without the ban and the solid lines representing allocations with the ban. Since we already know that a combination of ex-post capital controls and FX intervention fully stabilizes allocations after foreign appetite shocks in the absence of the constraint (9), let us consider instead the allocations with monetary policy (MP) and capital controls (CC) only, and then infer the marginal value of FX intervention. Since FX markets are shallower owing to the ban, allocations become more volatile in response to foreign appetite shocks. Both the ex-post policy rate and ex-post capital controls also become more volatile.

The marginal value of ex-post FX sales can be assessed using the values of $\left\{\Theta_1^H, \Theta_1^L\right\}$, the multipliers on the period-1 component of the constraint (9). Moving from the allocations with the ban to

the allocations with the ban, the values of the multipliers increase from $\left\{\Theta_1^H = -0.03, \Theta_1^L = 0.02\right\}$ to $\left\{\Theta_1^H = -0.04, \Theta_1^L = 0.12\right\}$, signifying that the value of ex-post FX intervention increases. We expect that if FX intervention were allowed in this economy, more FX reserves would be accumulated ex-ante and spent ex-post.

The figures are plotted for the DCP case. We expect that our insights on the vulnerability of the economy to different kinds of shocks and on the debt financing problem would continue to hold under PCP. However, the level of steady-state capital controls may be different under PCP, and could even be negative.

Proposition 13. *Banning open FX exposures reduces the economy's vulnerability to debt limit shocks and removes the need for ex-ante capital controls to address AD externalities. However, it has side-effects: (i) it makes debt harder to finance in steady state, and it alters steady-state capital controls; (ii) it increases the economy's vulnerability to any shocks which generate external debt, such as foreign appetite shocks, and may make the economy more dependent on FX accumulation and intervention.*

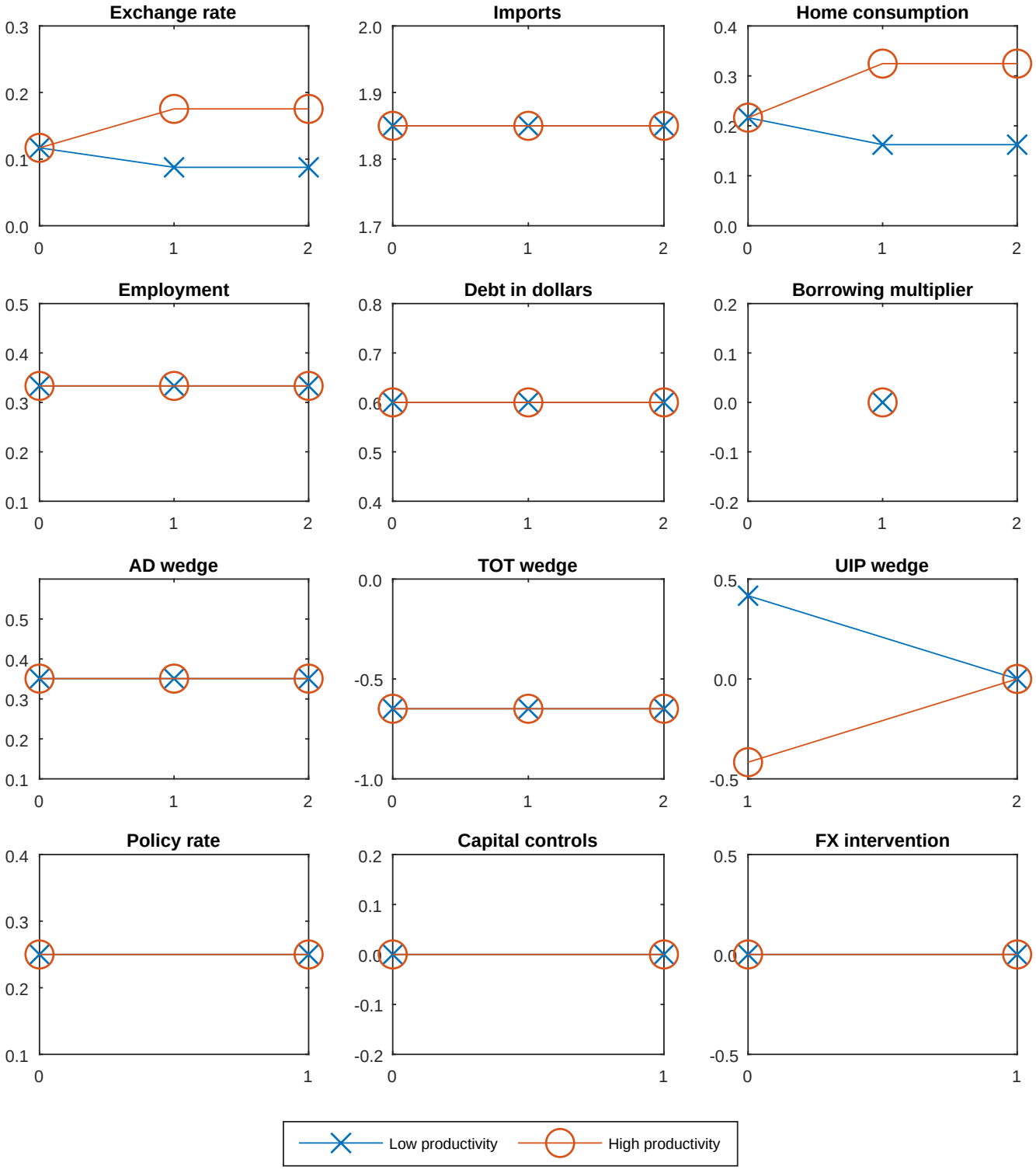
6 Conclusion

In this paper we built a model of a small open economy that features real and nominal frictions. We used this framework to characterize the optimal use of monetary policy, capital controls and FX intervention for different shocks and for countries that differ in terms of currency of trade invoicing, debt levels, severity of currency mismatches and borrowing constraints, and the depth of their FX markets. In broad terms, we found that not just the number but the workings of individual policy instruments matter. In addition, instruments interact with each other in complex, sometimes unexpected, ways, making it essential that they are designed jointly.

Our work is not yet complete. We still need to better establish the robustness of our results to our modeling choices. Furthermore, we need to complete the diagram shown in Figure 1 to introduce frictions in domestic debt markets to study the role of macroprudential tools and their interaction with the policies we have already considered.

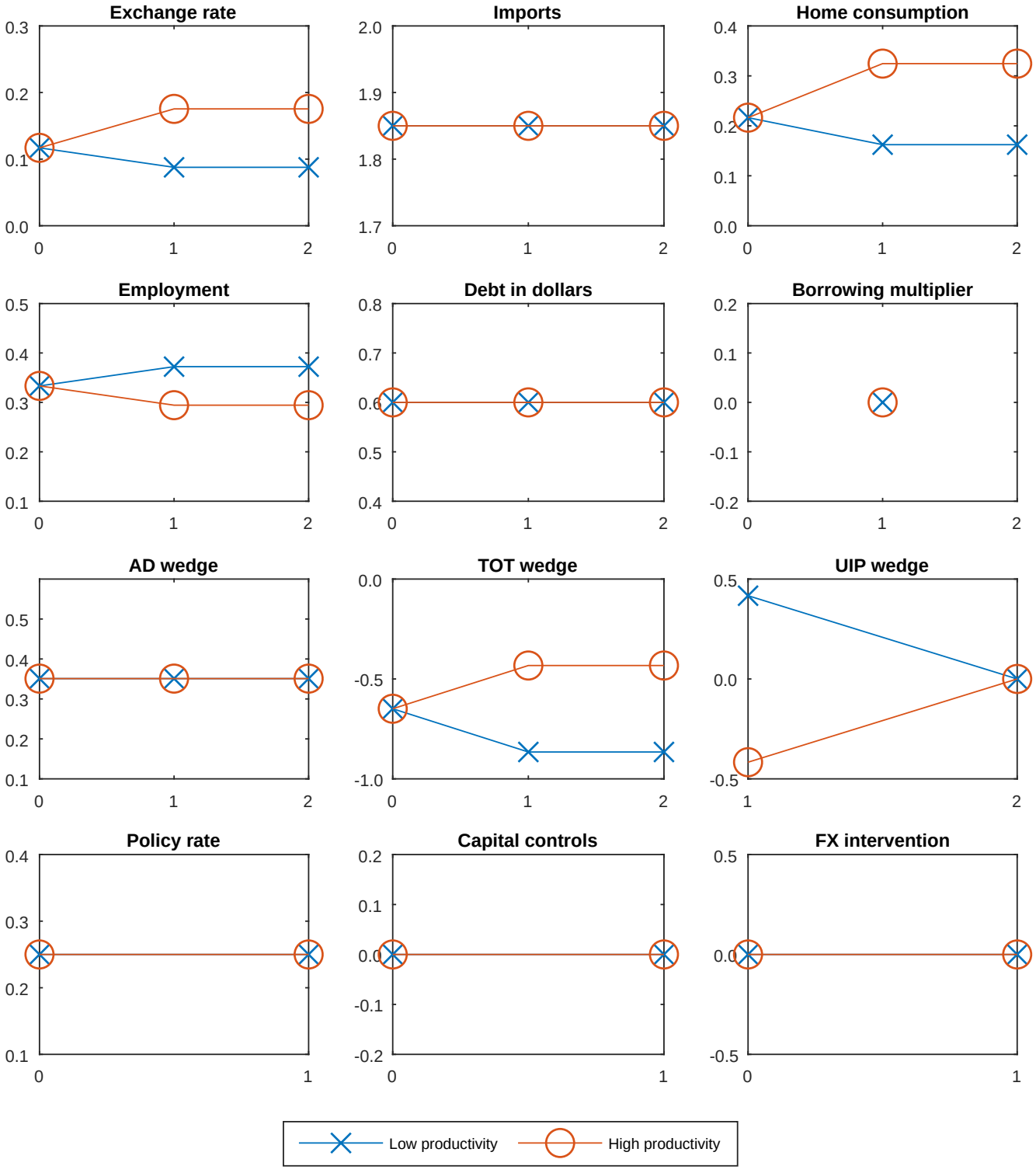
In applying the results of our model in practice, we would emphasize that while each policy tool could have its own merit in certain circumstances as outlined in this paper, in practice central banks will have to consider how to carefully incorporate multiple objectives and tools into their policy strategy, operational framework, and communication strategy. Central banks affect the economy both through immediate policy actions and by affecting the public's expectations about future policy actions. The stabilization of expectations requires sustained clear communication of how policy tools will be used in different states of nature in the future, and how to verify that the central bank is sticking to its previous promises. With multiple instruments, this communication problem becomes more complex, and transitional arrangements (such as building credibility with some instruments before adding others into the toolkit) may be necessary. Central banks also need to be careful to monitor any side-effects from the use of unorthodox policy instruments on risk-taking by domestic and foreign financial institutions, and on long-term market development. FX intervention, for instance, can exacerbate currency mismatches.

Figure 5: Productivity Shock under PCP with Deep FX Markets



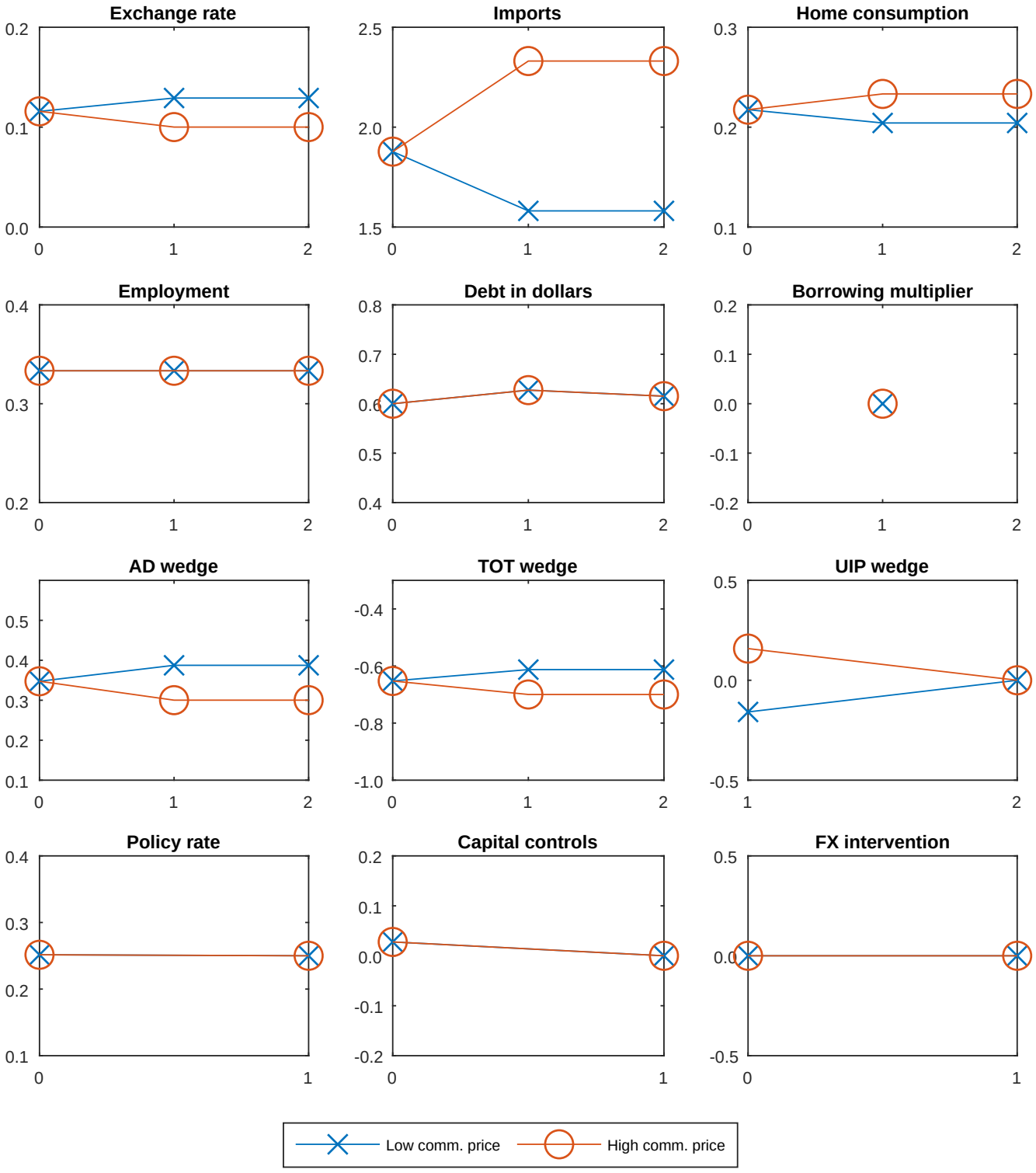
Notes: This figure plots the responses of key variables to a permanent productivity shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.75, 1.5]$. Such calibration of the shock neutralizes the intertemporal TOT externality.

Figure 6: Productivity Shock under DCP with Deep FX Markets



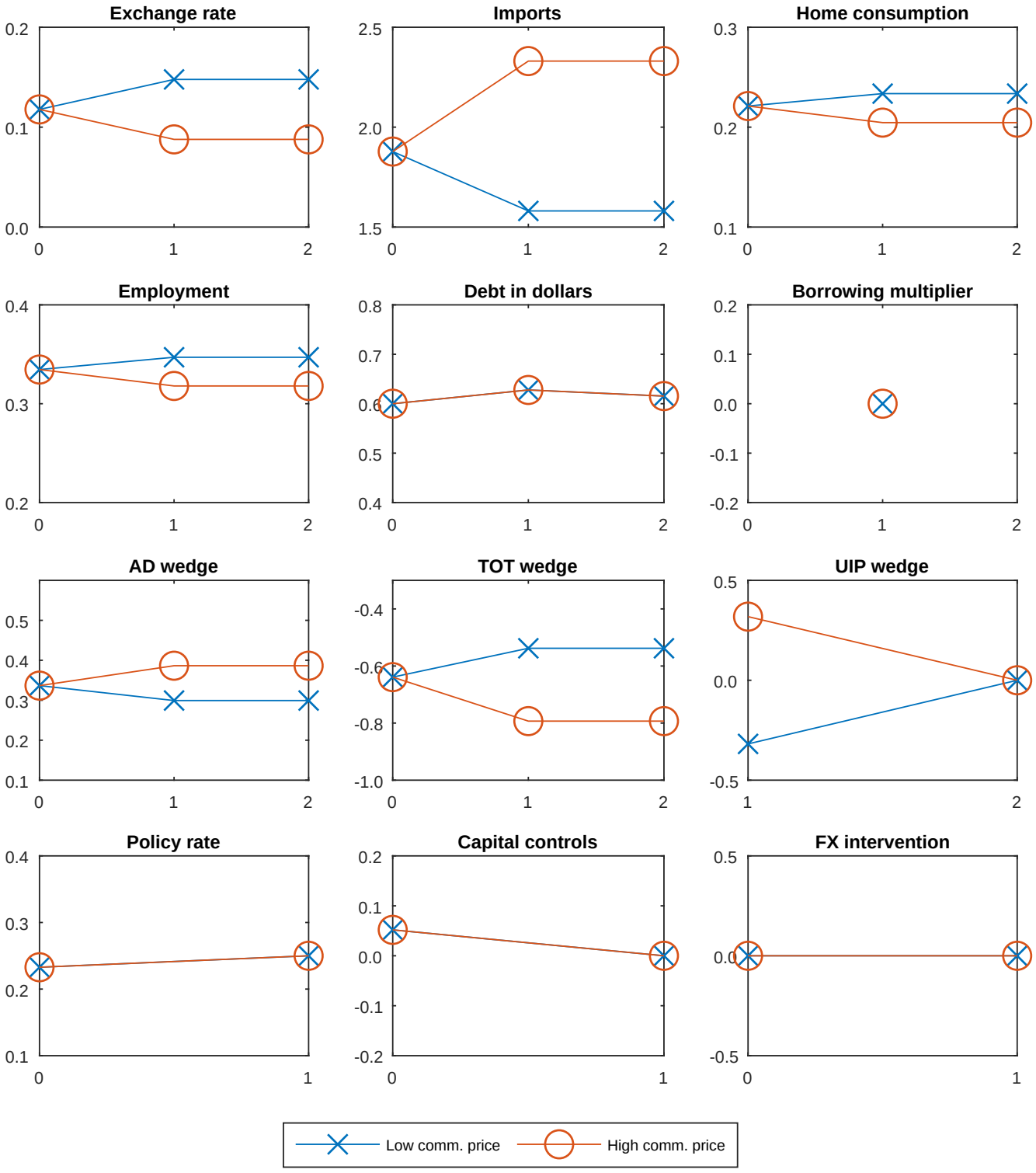
Notes: This figure plots the responses of key variables to a permanent productivity shock under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.75, 1.5]$. Such calibration of the shock neutralizes the intertemporal TOT externality.

Figure 7: Commodity Price Shock under PCP with Deep FX Markets



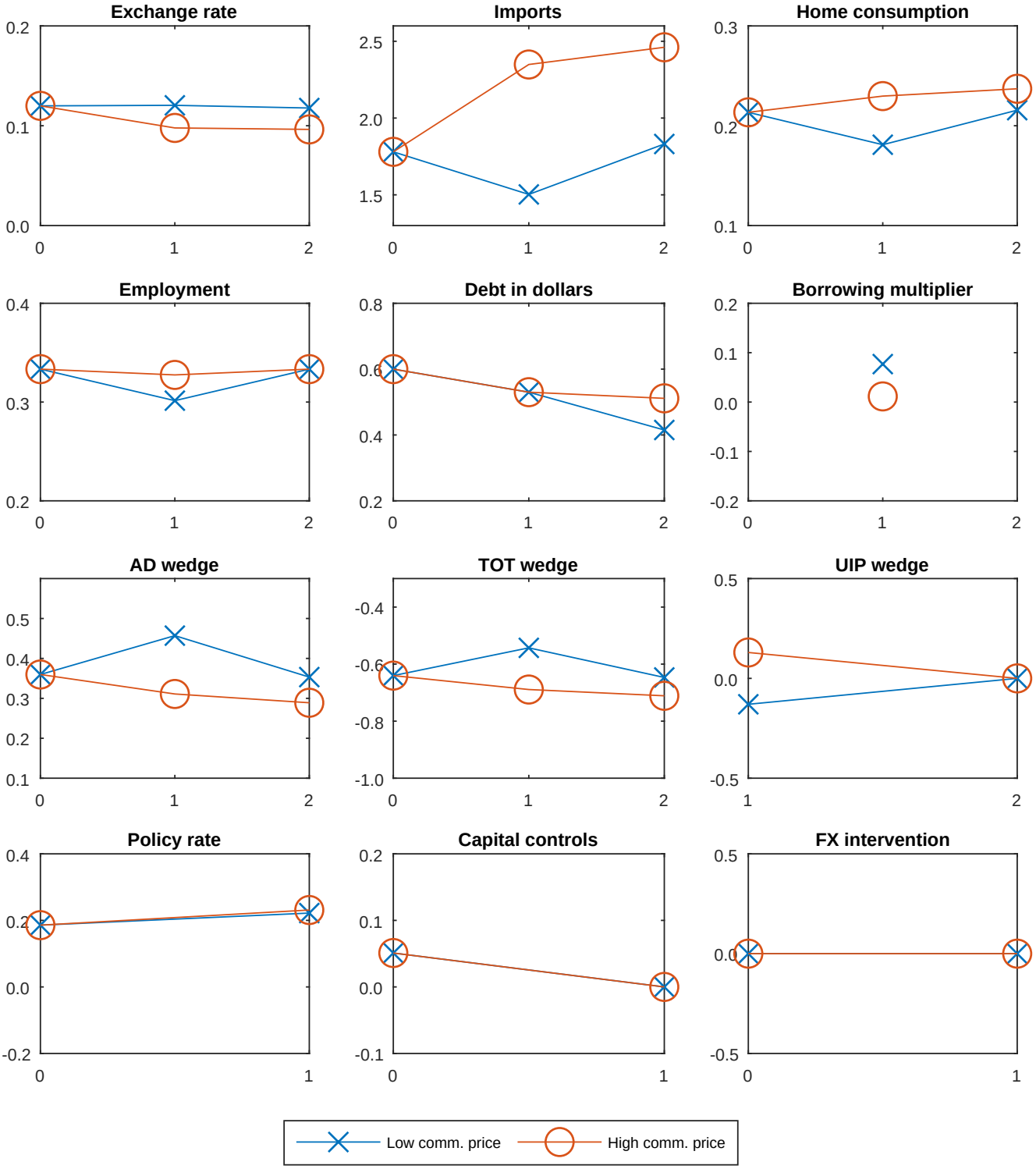
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 8: Commodity Price Shock under DCP with Deep FX Markets



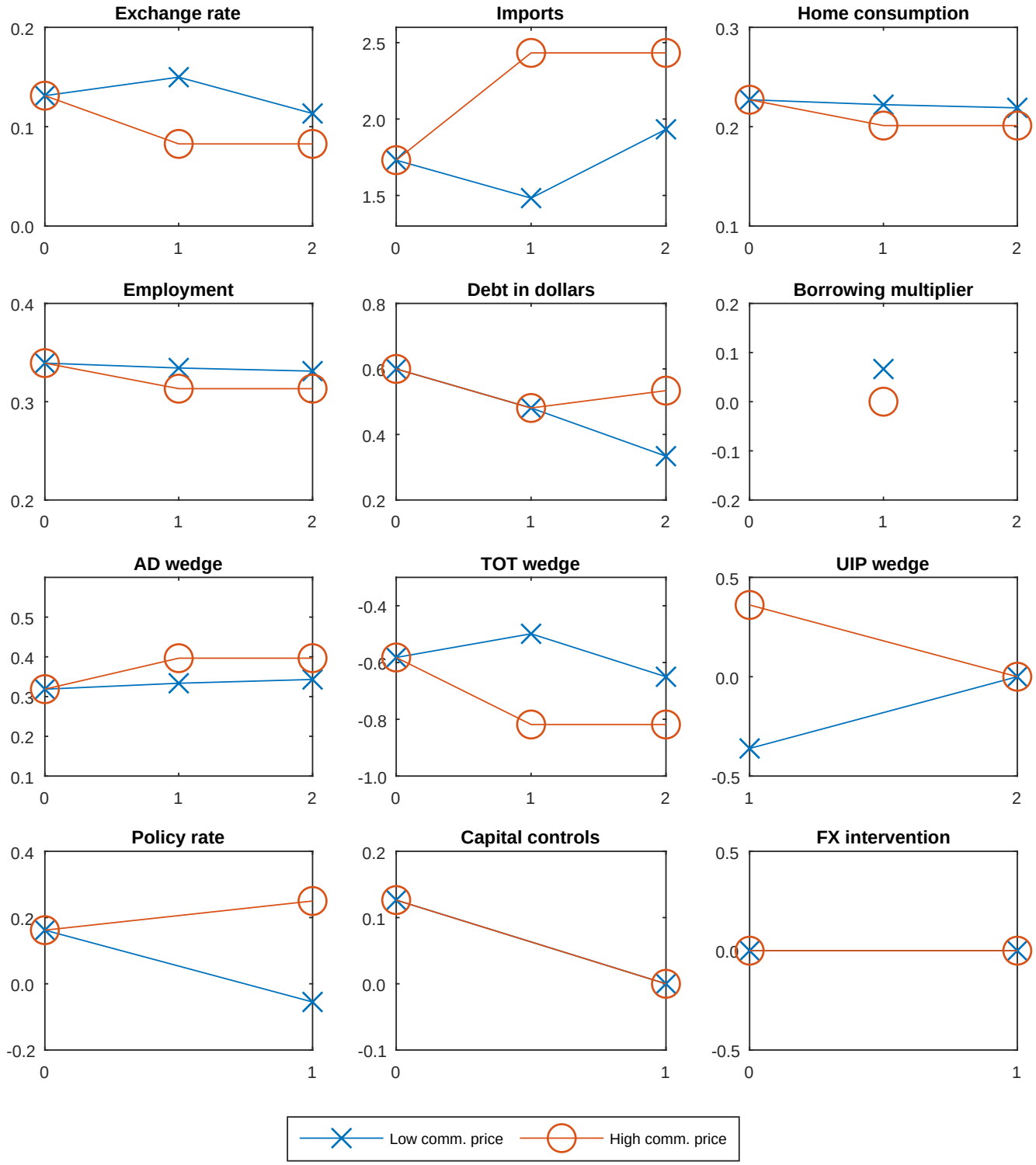
Notes: This figure plots the responses of key variables to a permanent commodity price shock under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 9: Commodity Price Shock under PCP with Debt Limits and Deep FX Markets



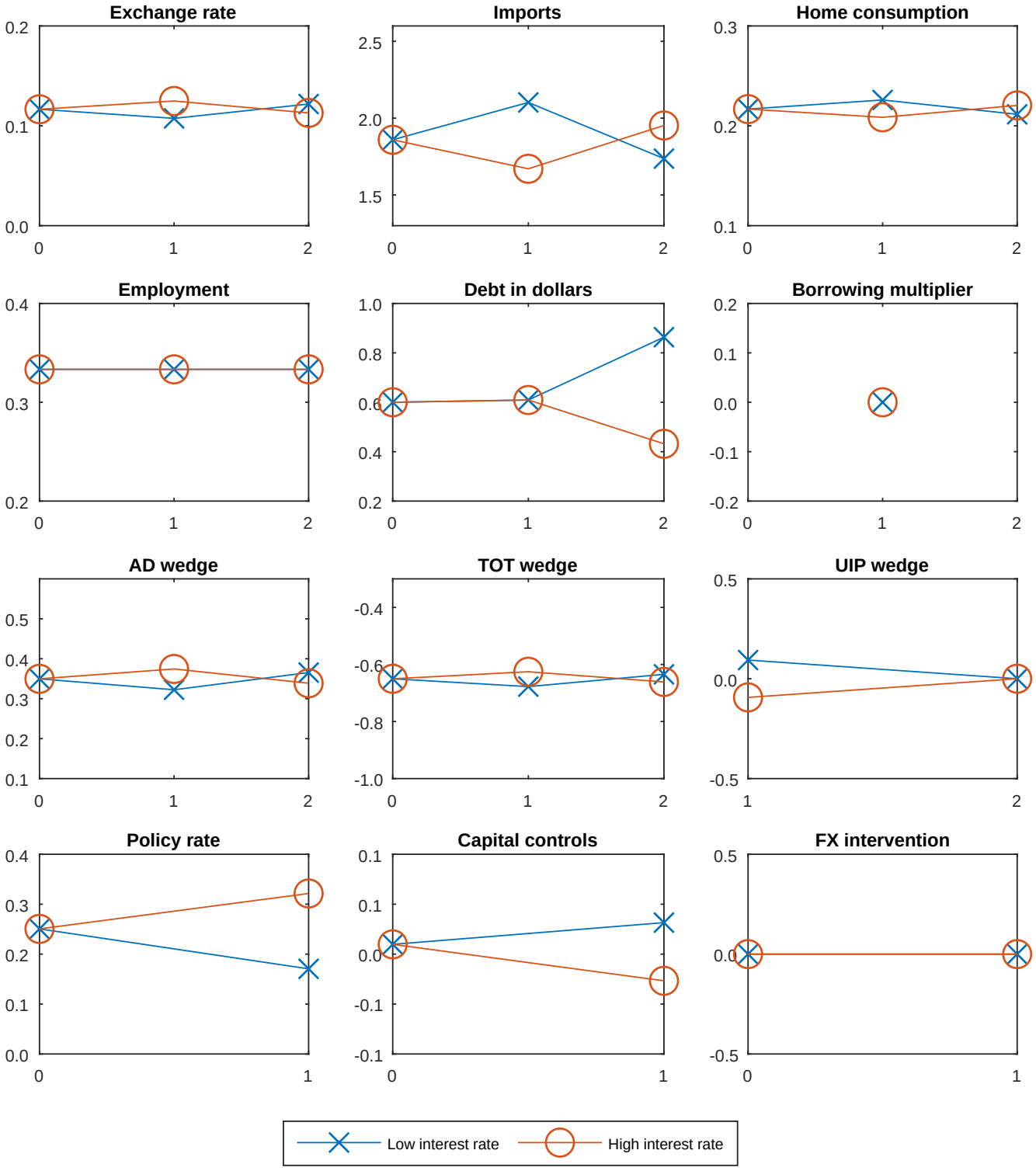
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with deep FX markets in presence of borrowing constraints. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. The borrowing constraint is calibrated using $\kappa = 0.05$ and binds after a low commodity price realization.

Figure 10: Commodity Price Shock under DCP with Debt Limits and Deep FX Markets



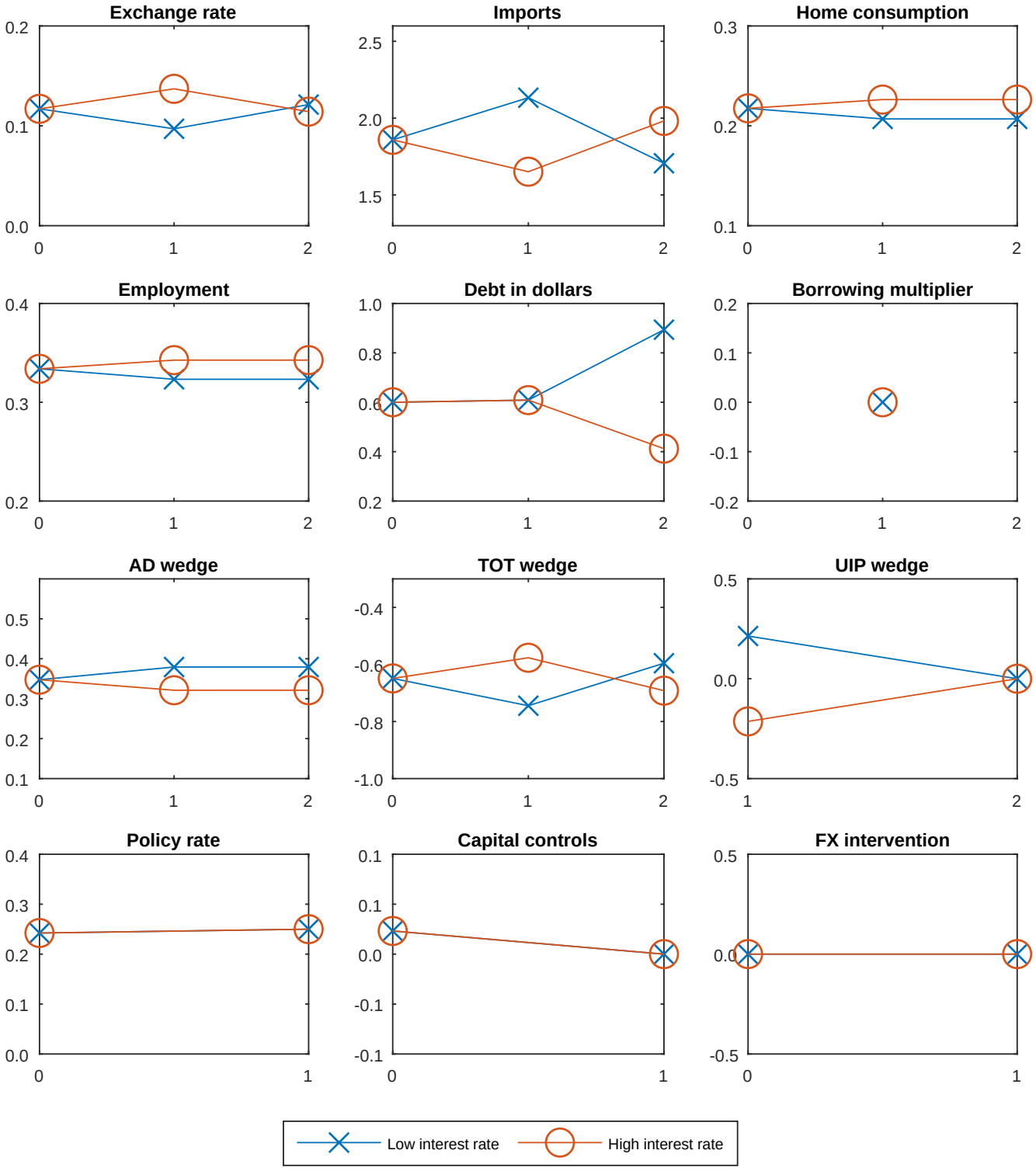
Notes: This figure plots the responses of key variables to a permanent commodity price shock under DCP with deep FX markets in presence of borrowing constraints. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. The borrowing constraint is calibrated using $\kappa = 0.05$ and binds after a low commodity price realization.

Figure 11: Interest Rate Shock under PCP with Deep FX Markets



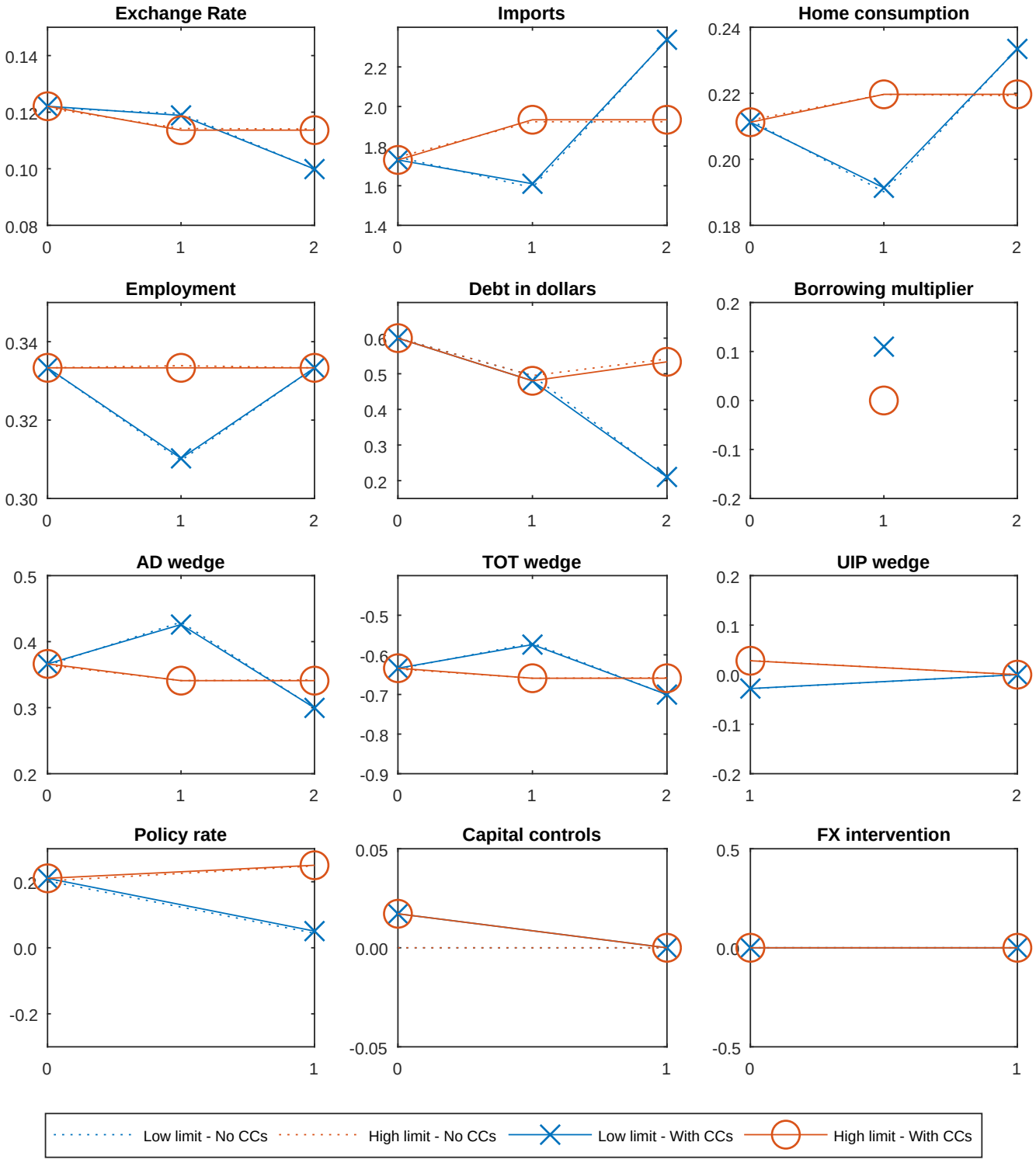
Notes: This figure plots the responses of key variables to a world interest rate shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 12: Interest Rate Shock under DCP with Deep FX Markets



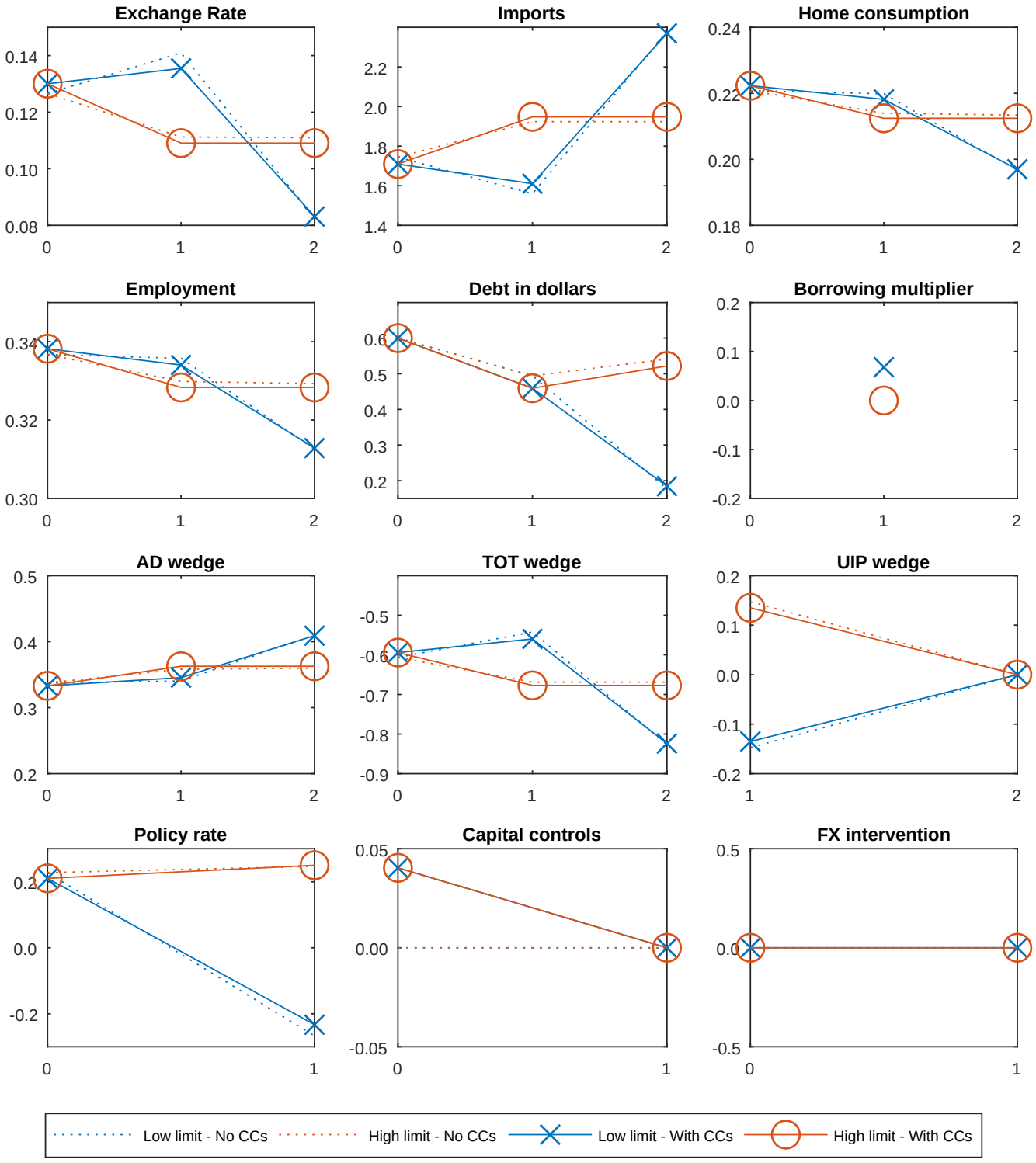
Notes: This figure plots the responses of key variables to a world interest rate shock under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 13: Debt Limit Shock under PCP with Deep FX Markets



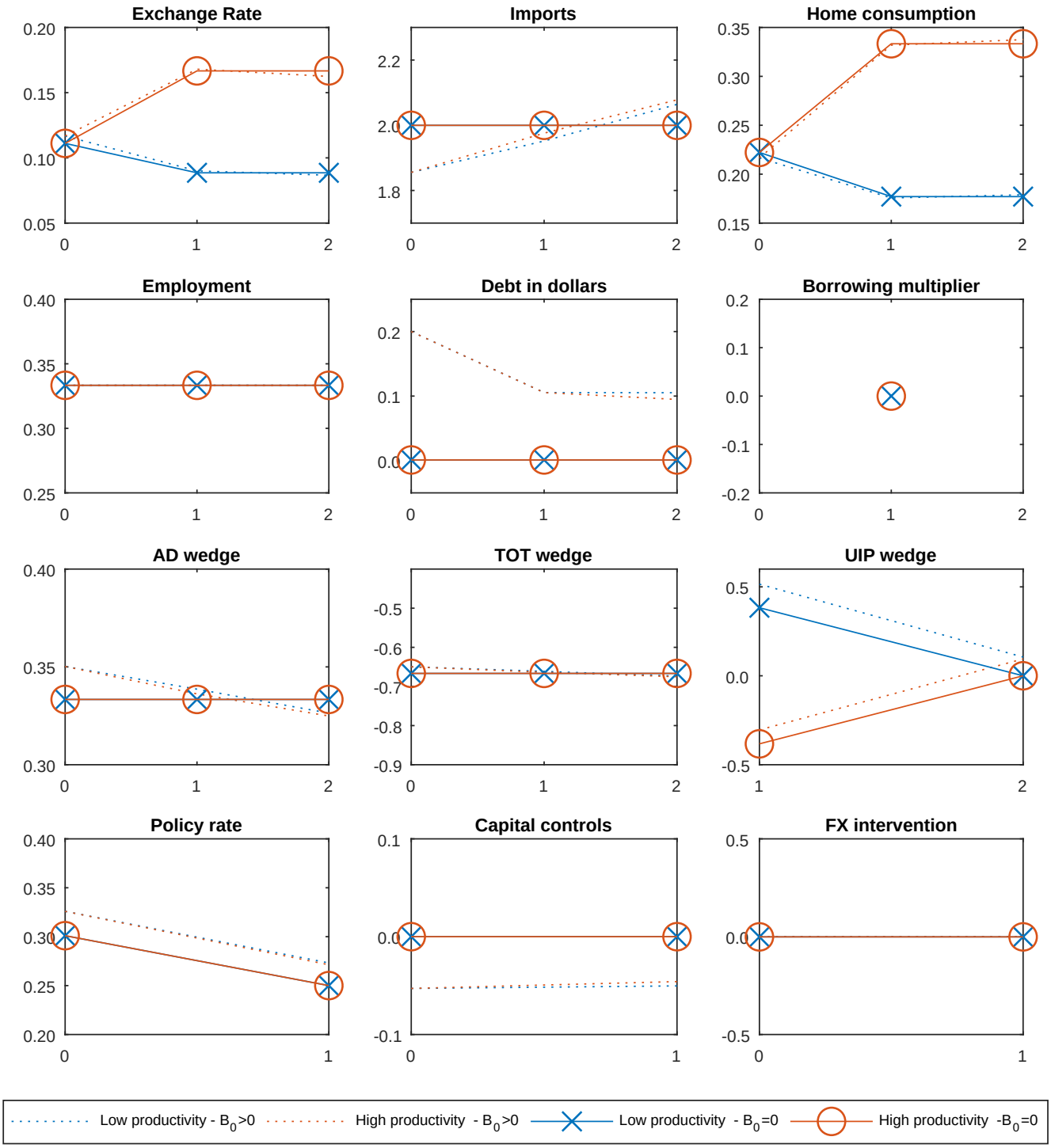
Notes: This figure plots the responses of key variables to a debt limit shock under under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 14: Debt Limit Shock under DCP with Deep FX Markets



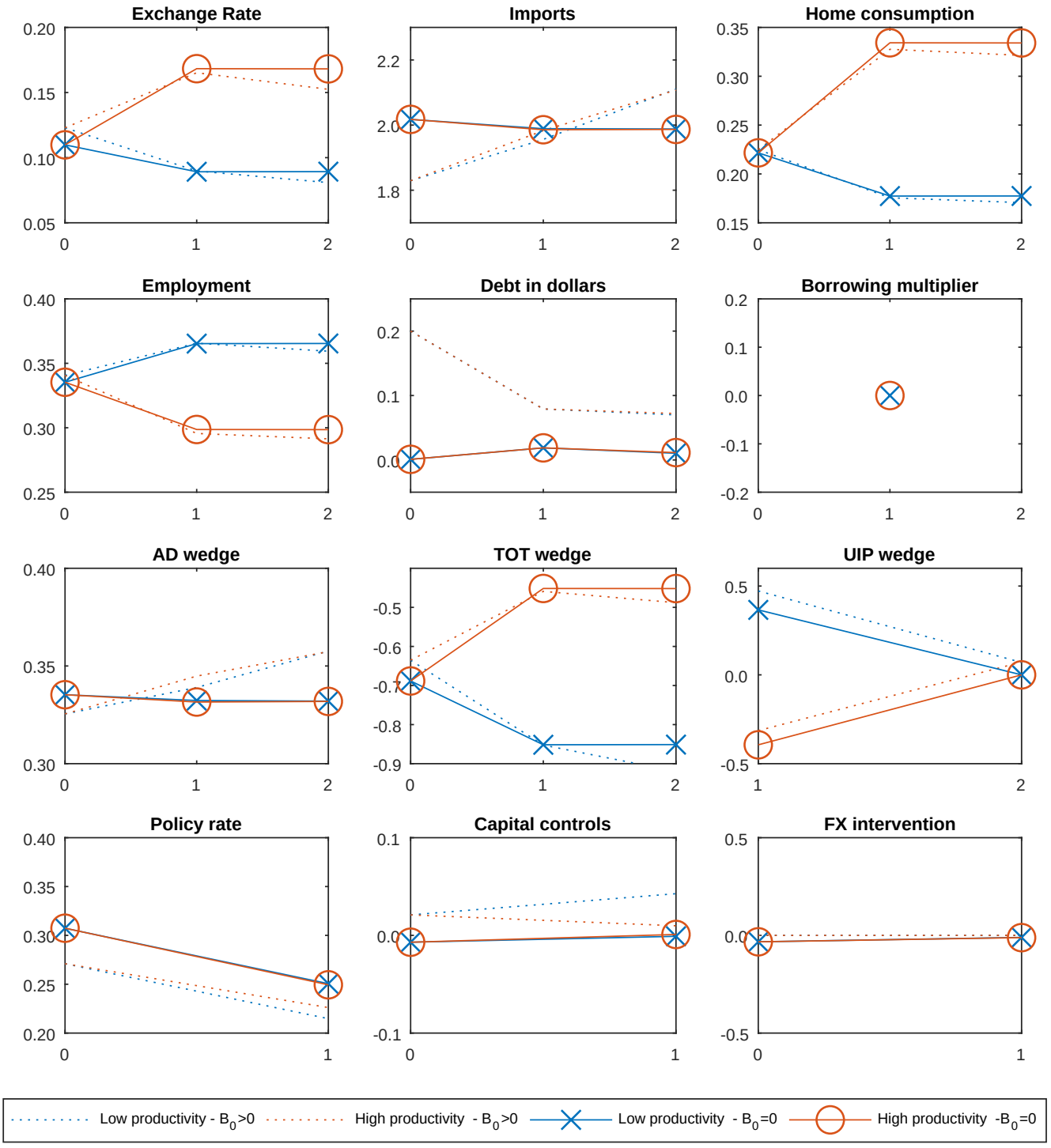
Notes: This figure plots the responses of key variables to a debt limit shock under under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 15: Productivity Shock under PCP with Shallow FX Markets



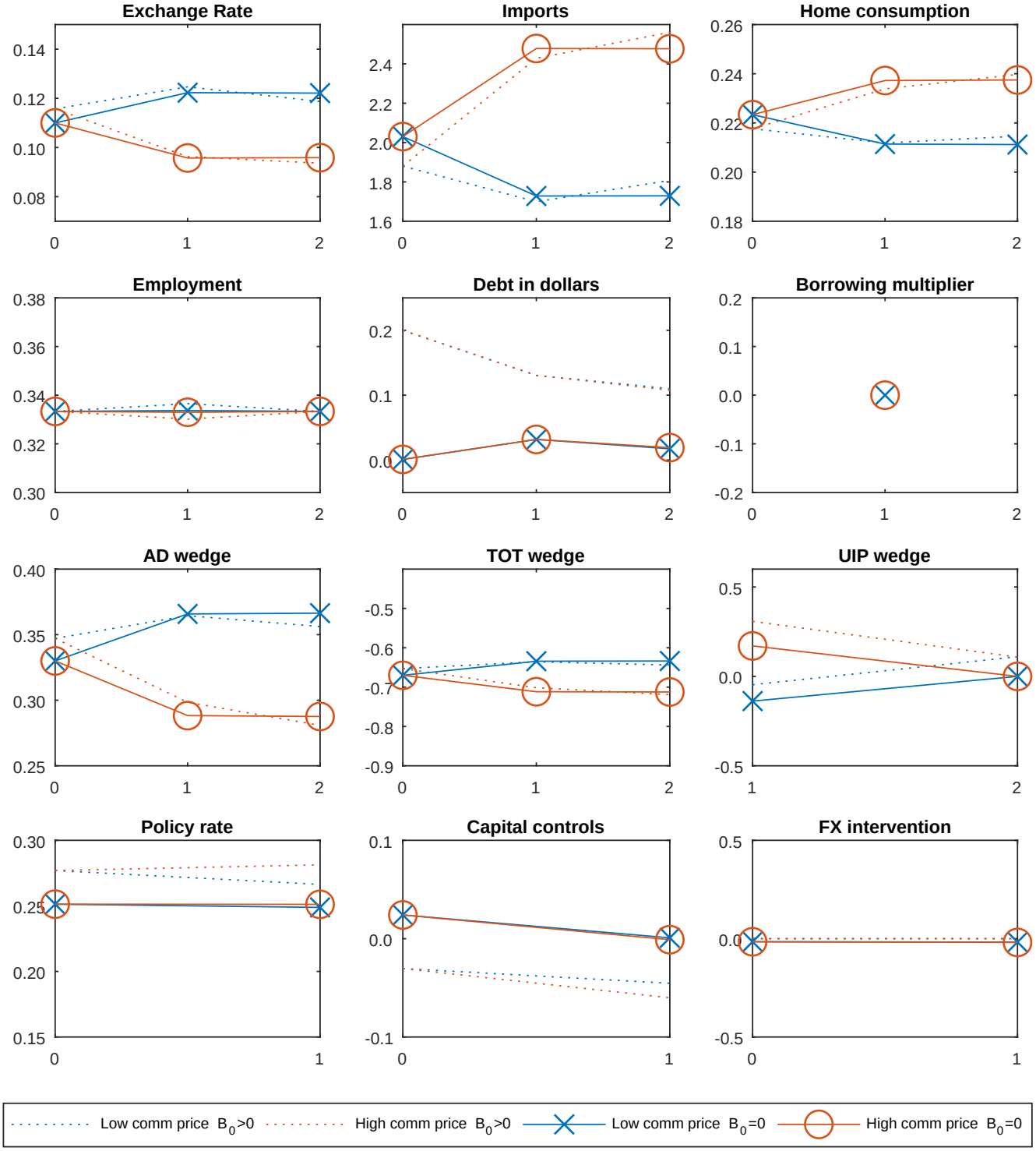
Notes: This figure plots the responses of key variables to a permanent productivity shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.8, 1.5]$. Such calibration of the shock neutralizes the intertemporal TOT externality. When positive, $B_0 = 0.2$.

Figure 16: Productivity Shock under DCP with Shallow FX Markets



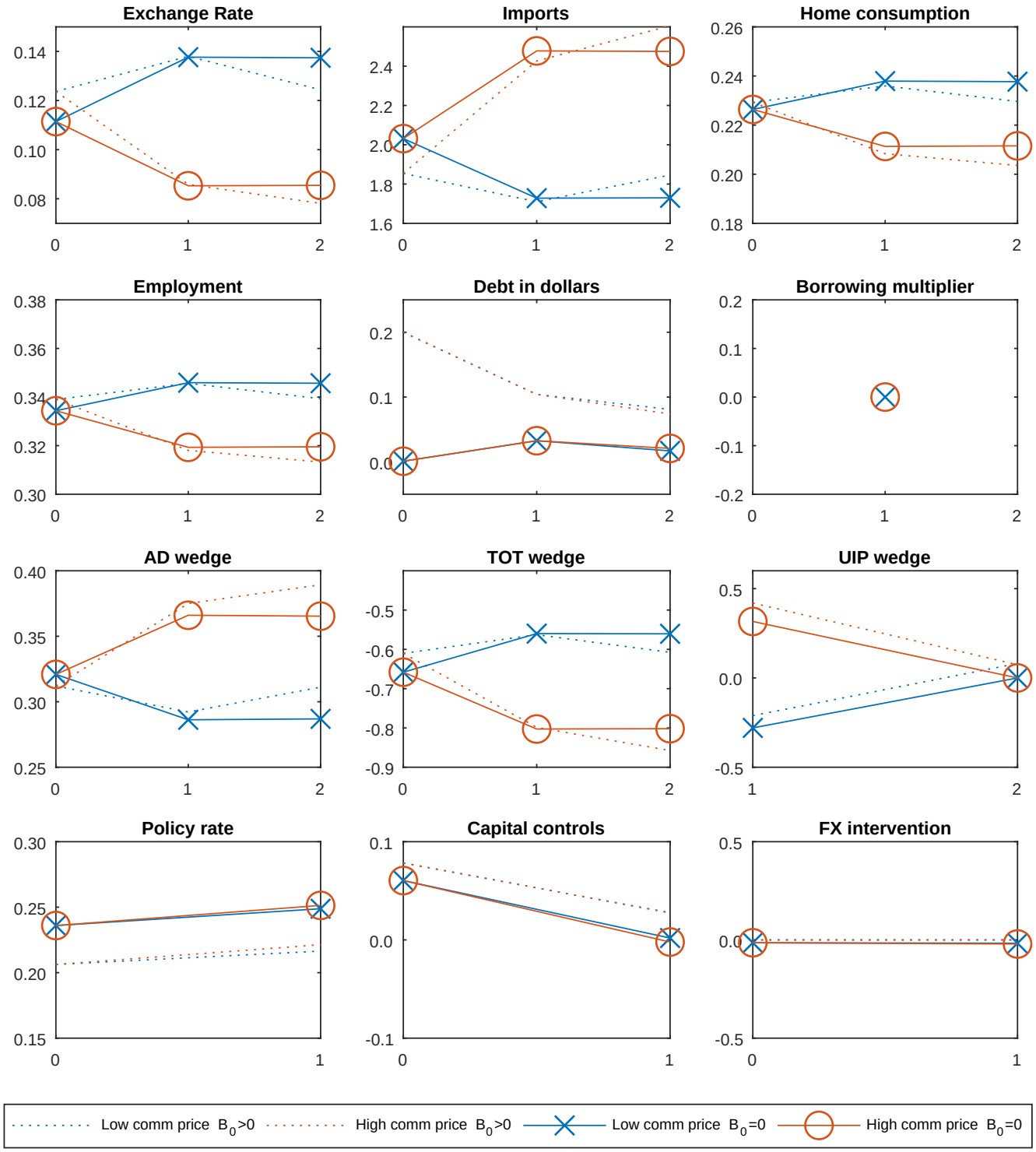
Notes: This figure plots the responses of key variables to a permanent productivity shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.8, 1.5]$. Such calibration of the shock neutralizes the intertemporal TOT externality. When positive, $B_0 = 0.2$.

Figure 17: Commodity Price Shock under PCP with Shallow FX Markets



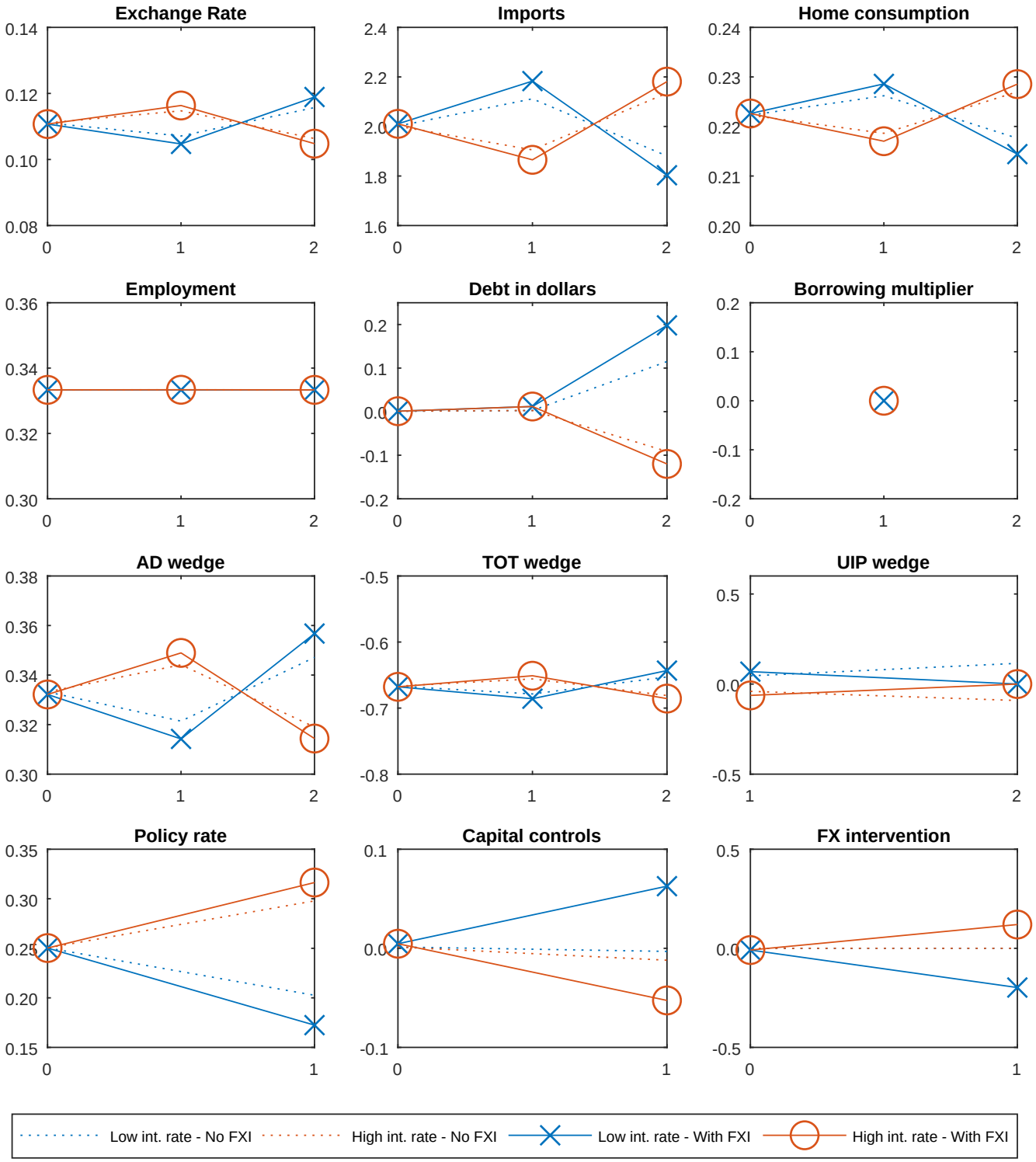
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. When positive, $B_0 = 0.2$.

Figure 18: Commodity Price Shock under DCP with Shallow FX Markets



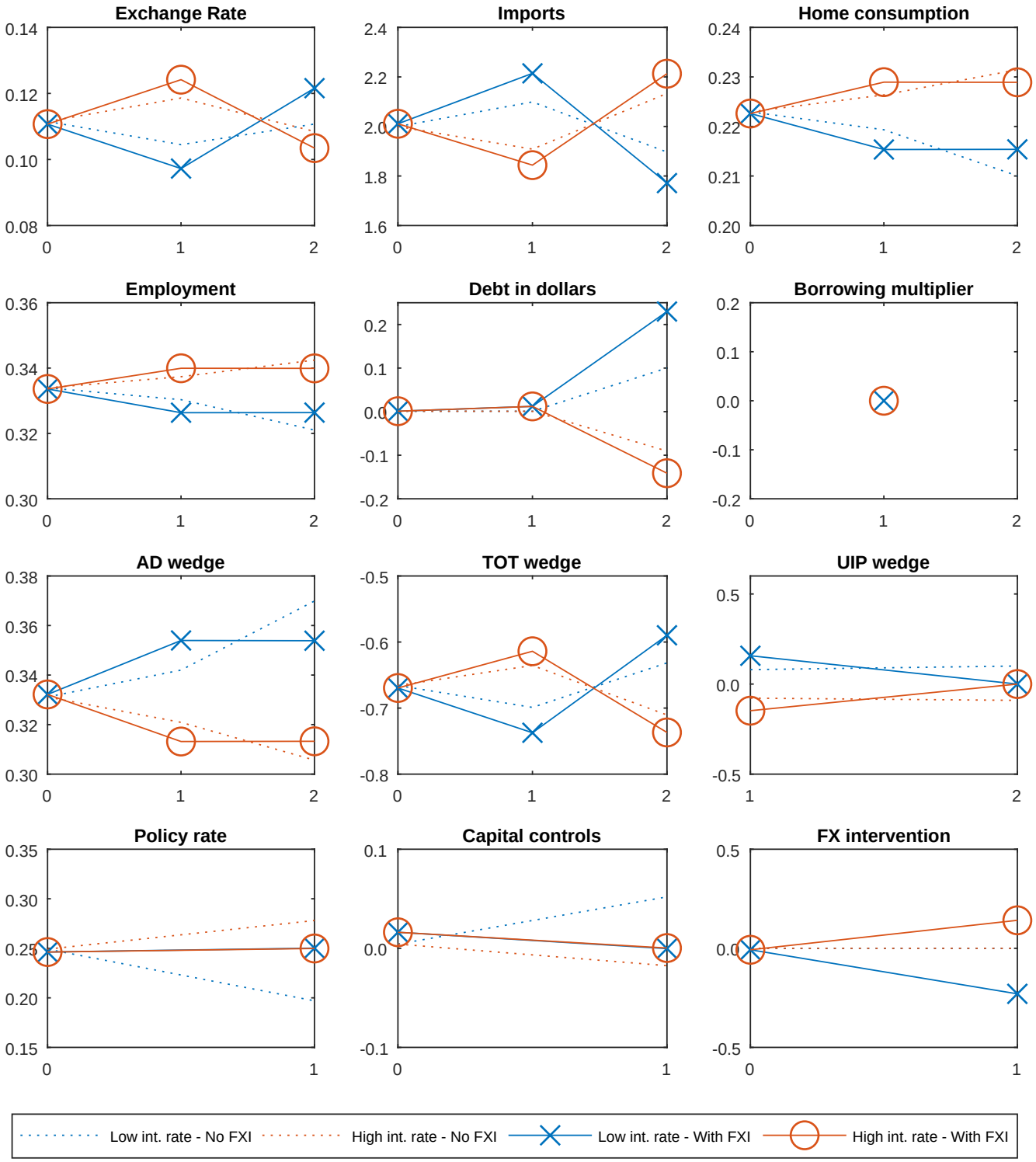
Notes: This figure plots the responses of key variables to a permanent commodity price shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. When positive, $B_0 = 0.2$.

Figure 19: Interest Rate Shock under PCP with Shallow FX Markets



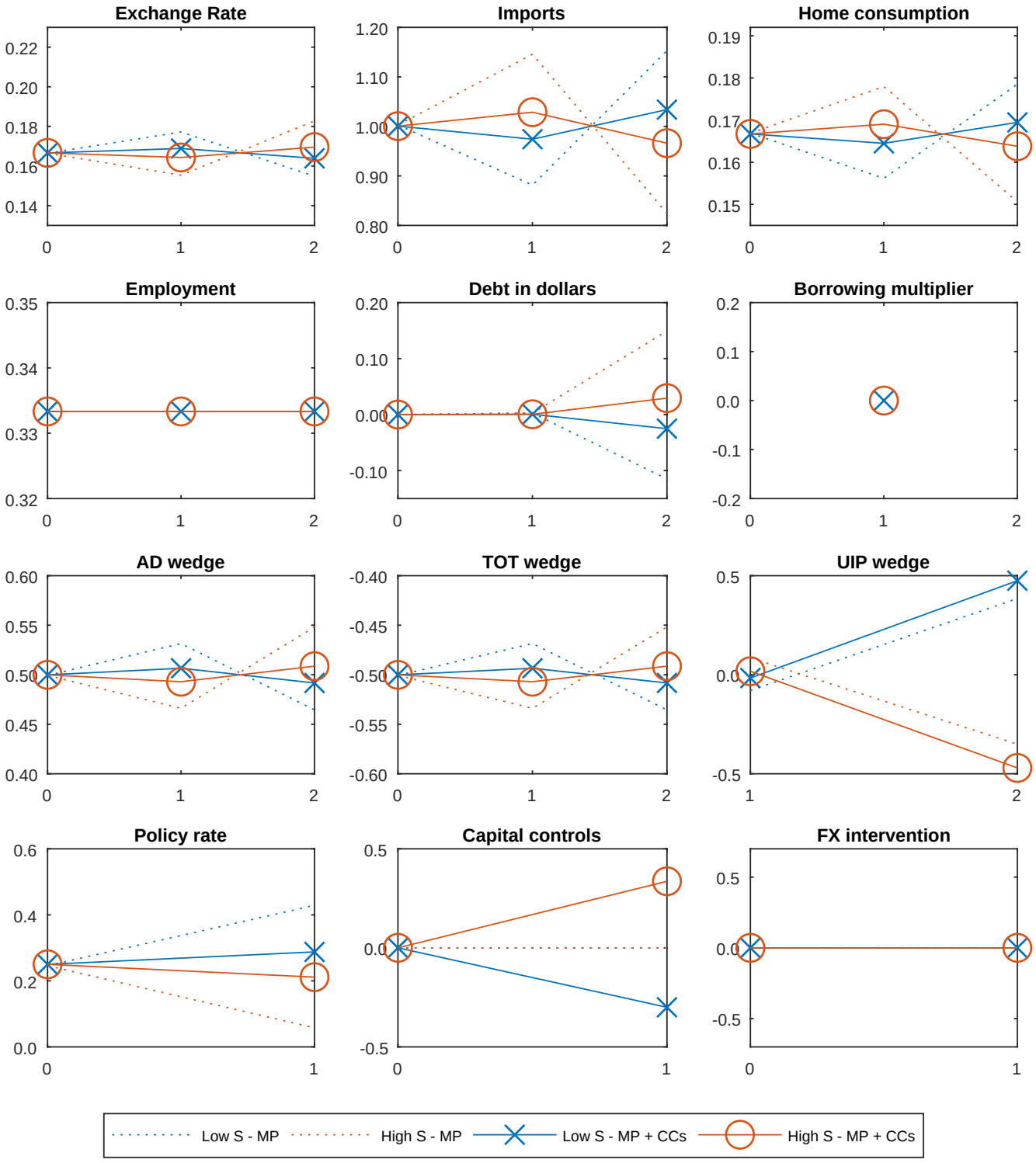
Notes: This figure plots the responses of key variables to a shock to the world interest rate under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 20: Interest Rate Shock under DCP with Shallow FX Markets



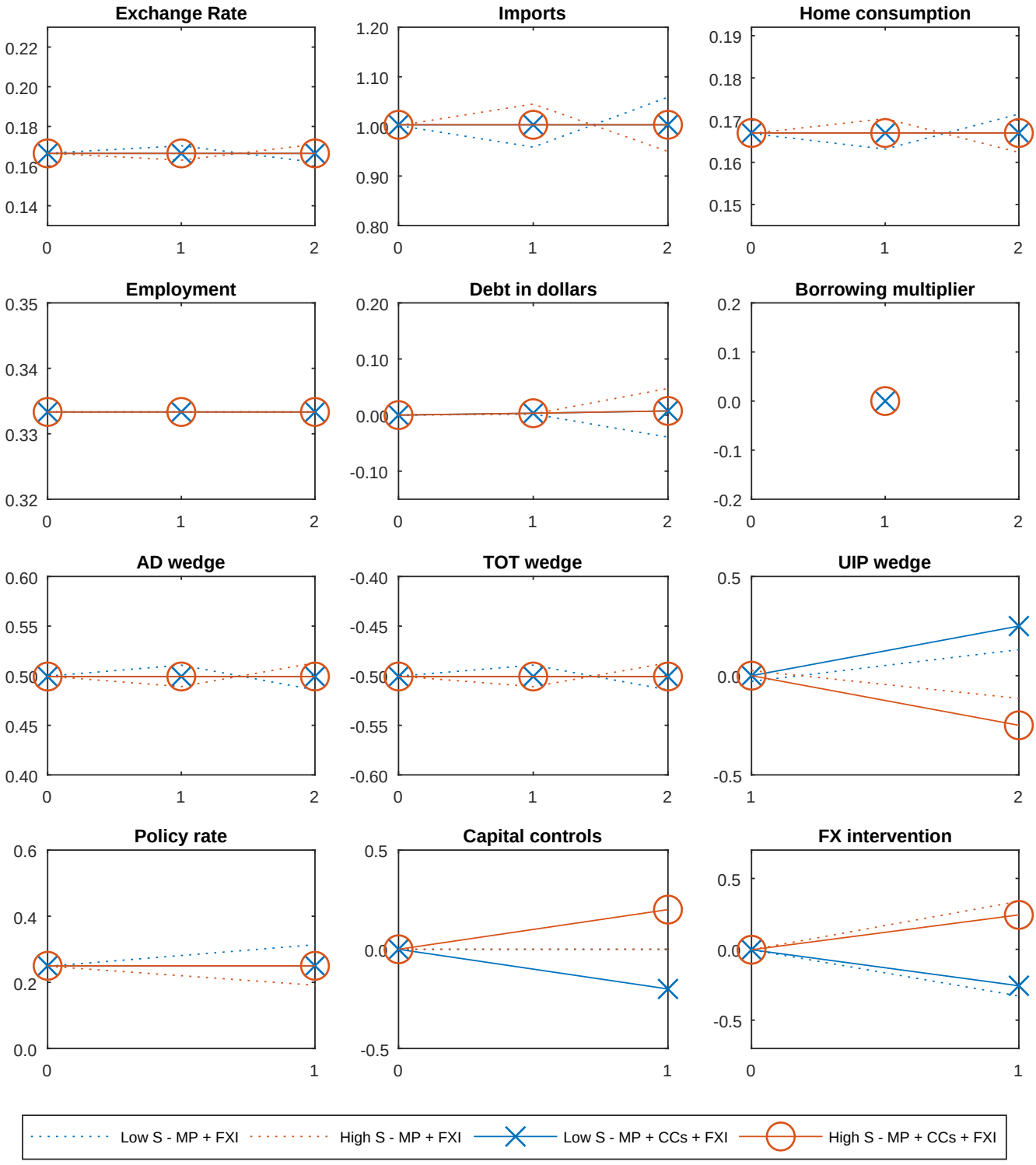
Notes: This figure plots the responses of key variables to a shock to the world interest rate under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 21: Foreign Appetite Shock under PCP with MP and CC



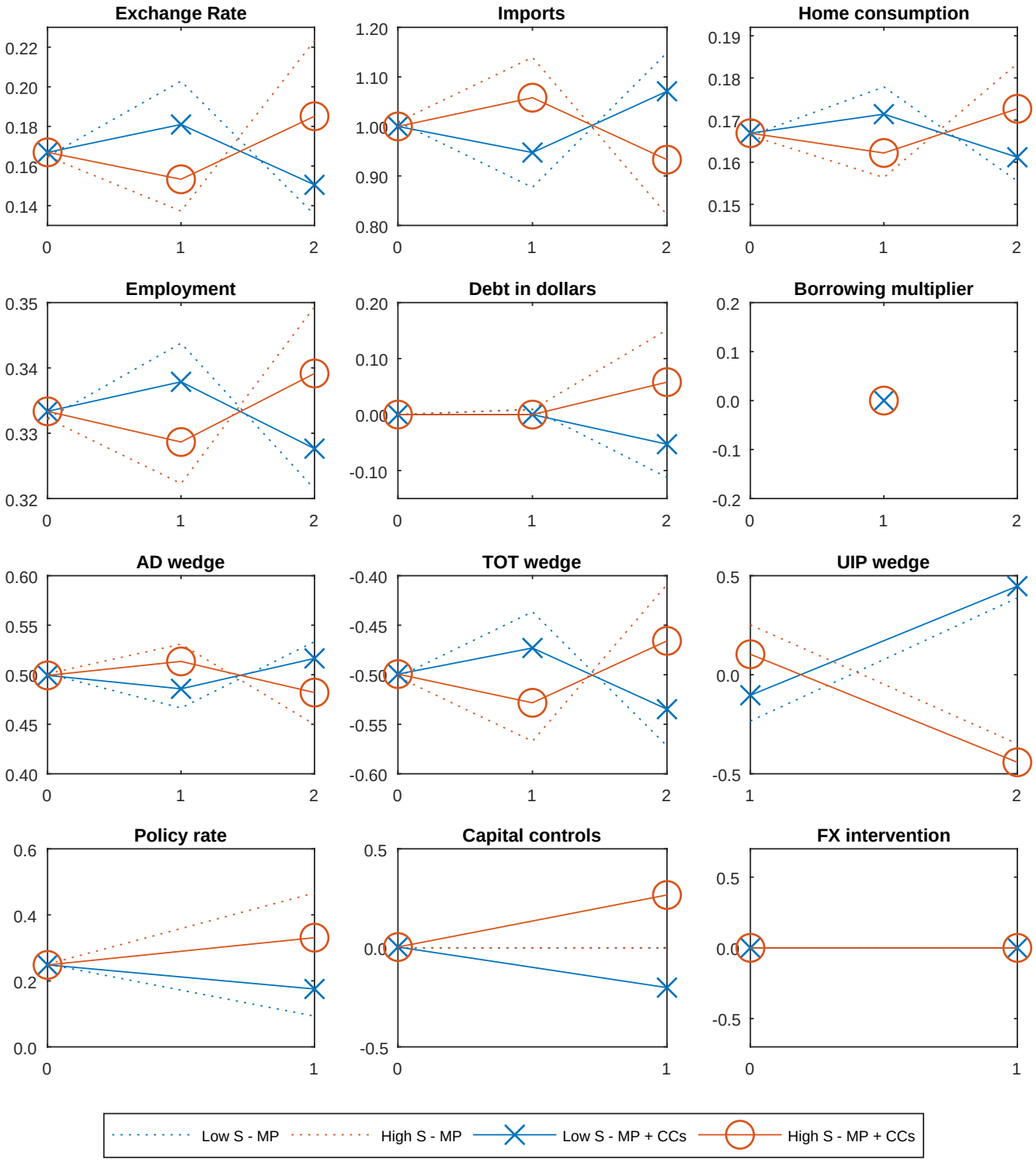
Notes: This figure plots the responses of key variables to a foreign appetite shock under PCP with shallow FX markets, monetary policy (MP) and capital controls (CC). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 22: Foreign Appetite Shock under PCP with MP, CC, and FXI



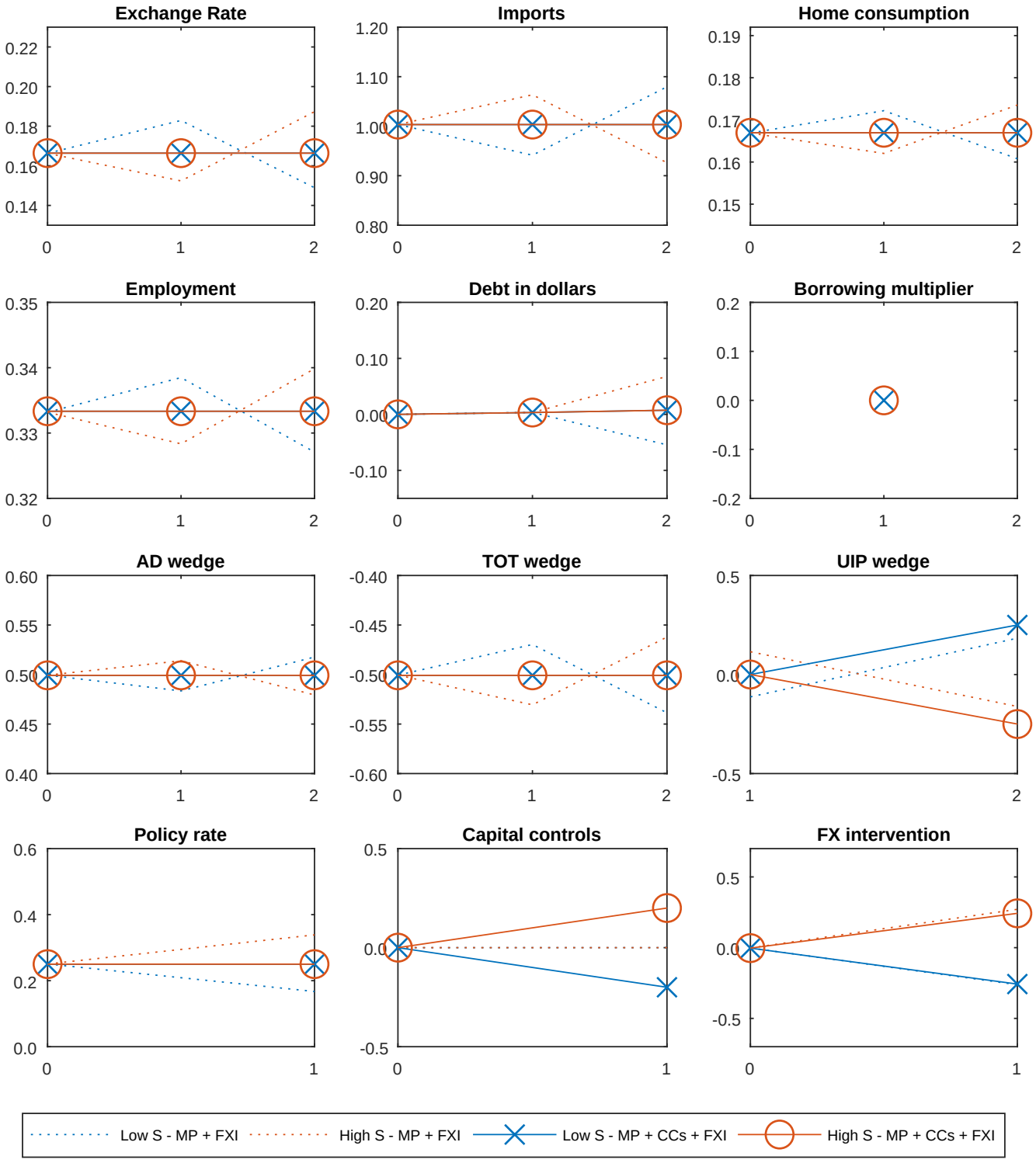
Notes: This figure plots the responses of key variables to a foreign appetite shock under PCP with shallow FX markets, monetary policy (MP), capital controls (CC), and FX intervention (FXI) without imposing the FX intervention constraint (9). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 23: Foreign Appetite Shock under DCP with MP and CC



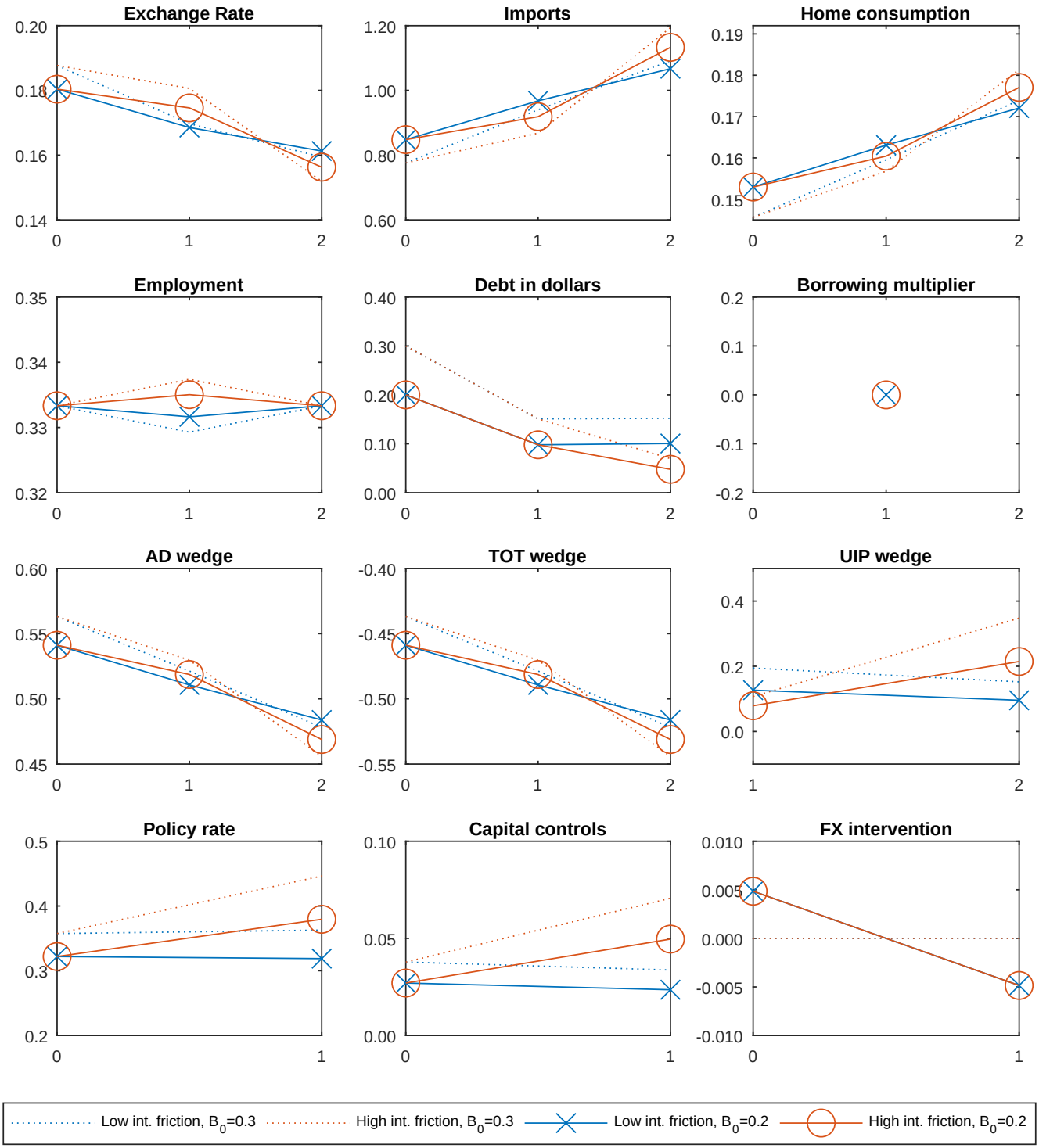
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets, monetary policy (MP) and capital controls (CC). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 24: Foreign Appetite Shock under DCP with MP, CC, and FXI



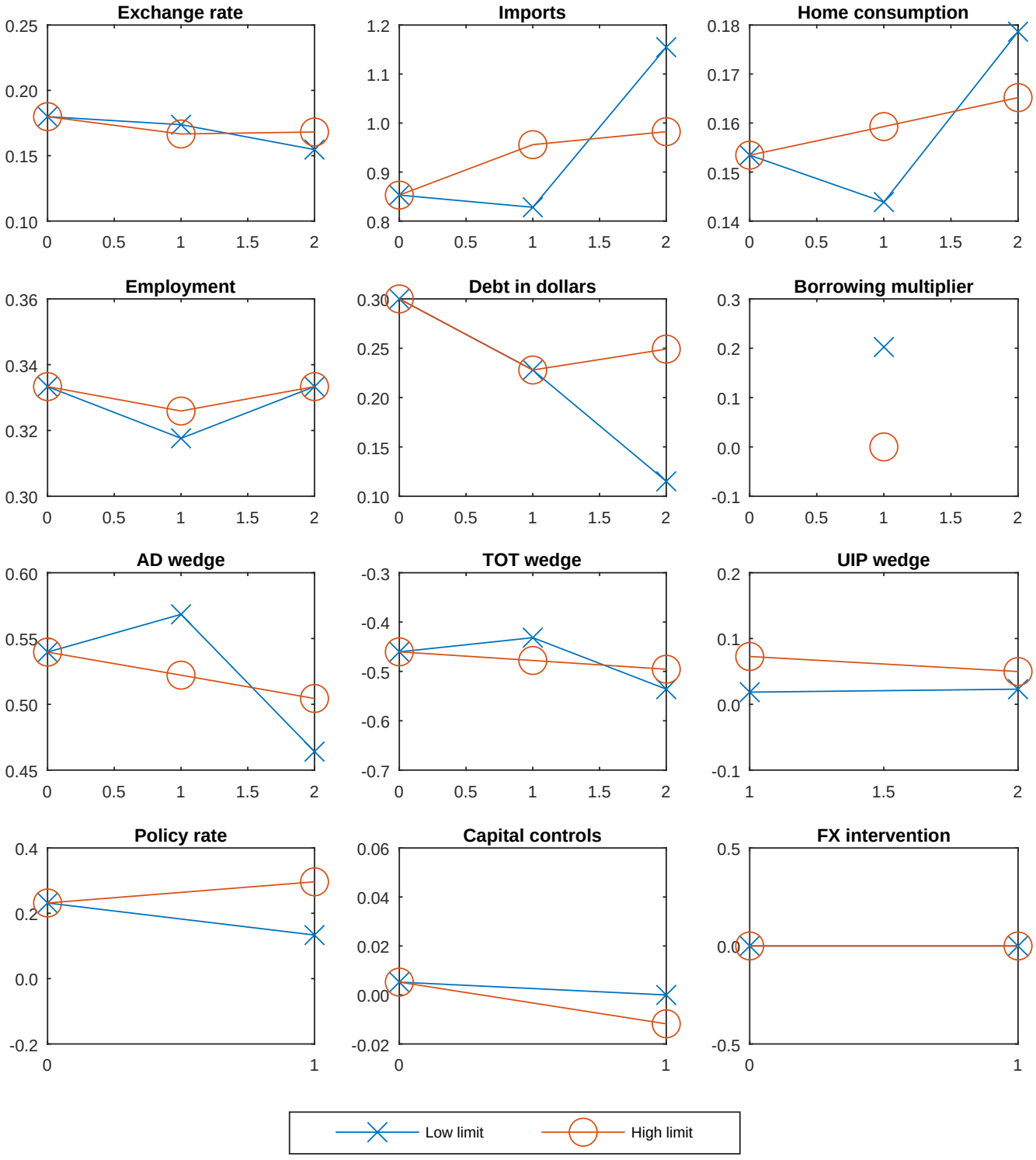
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets, monetary policy (MP), capital controls (CC), and FX intervention (FXI) without imposing the constraint (9). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 25: Intermediation Friction Shock under PCP



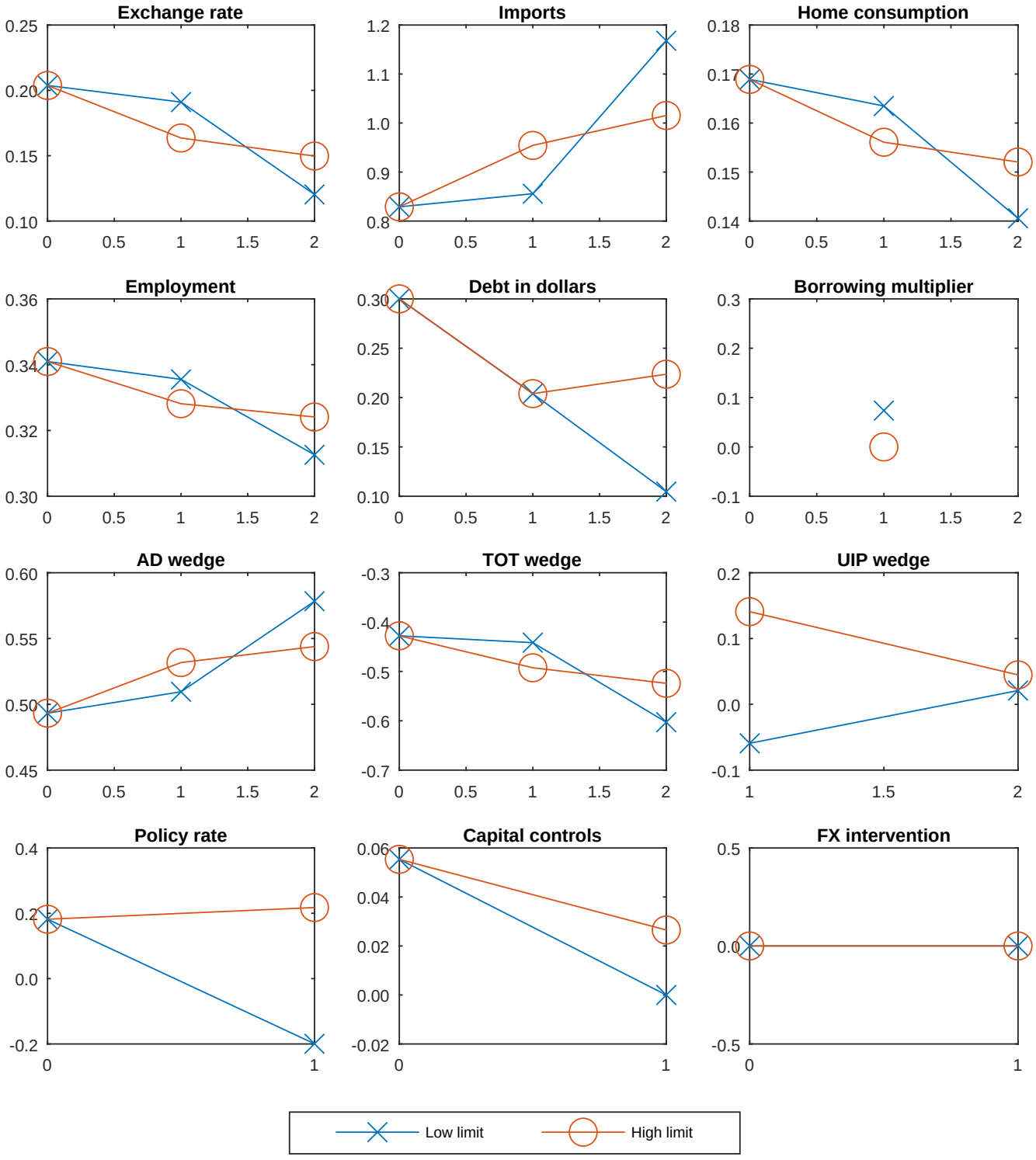
Notes: This figure plots the responses of key variables to a shock to intermediation friction under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $\Gamma_1 \in [1, 5]$.

Figure 26: Debt Limit Shock under PCP with Shallow FX Markets



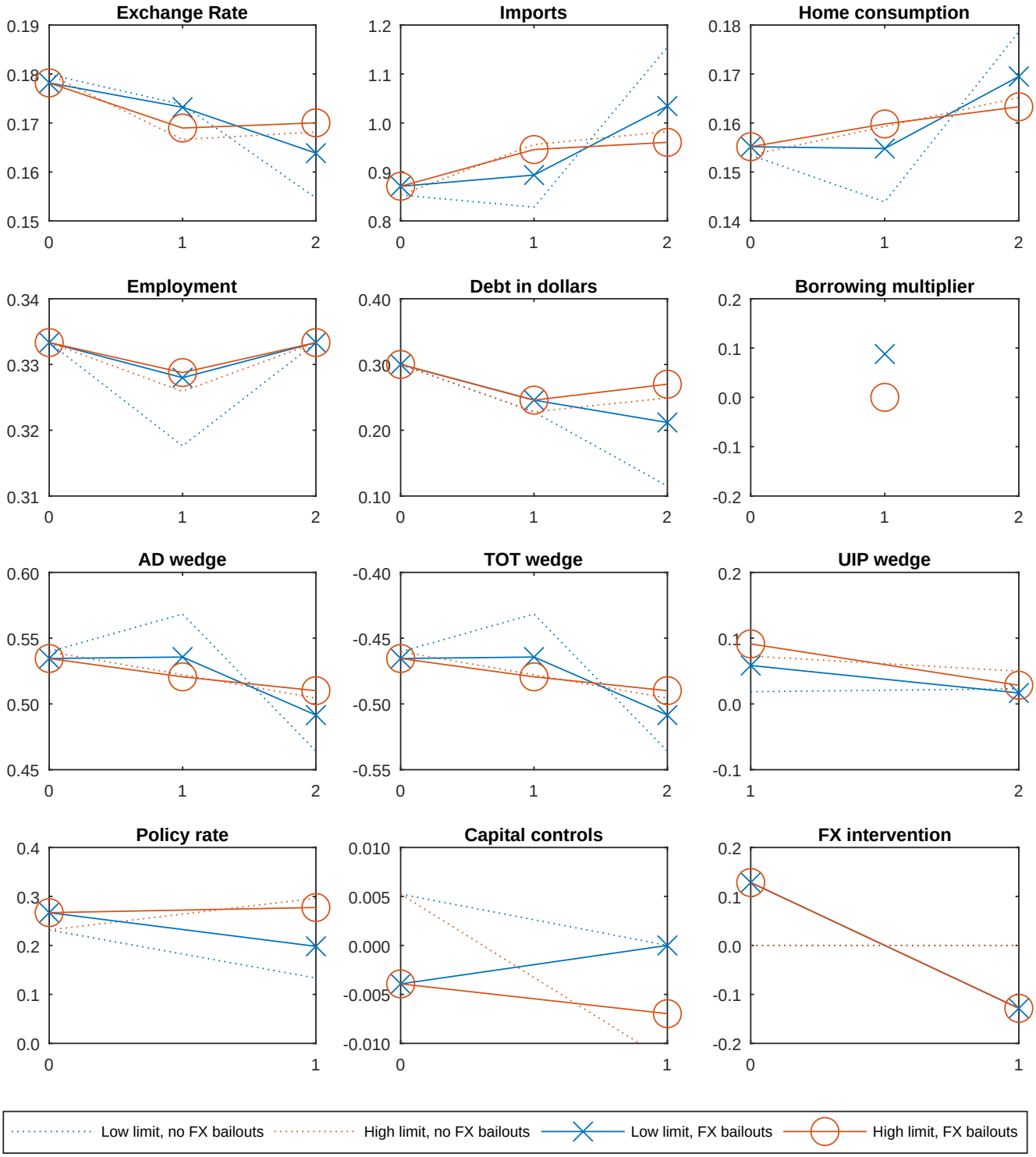
Notes: This figure plots the responses of key variables to a debt limit shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.02, 0.5]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 27: Debt Limit Shock under DCP with Shallow FX Markets



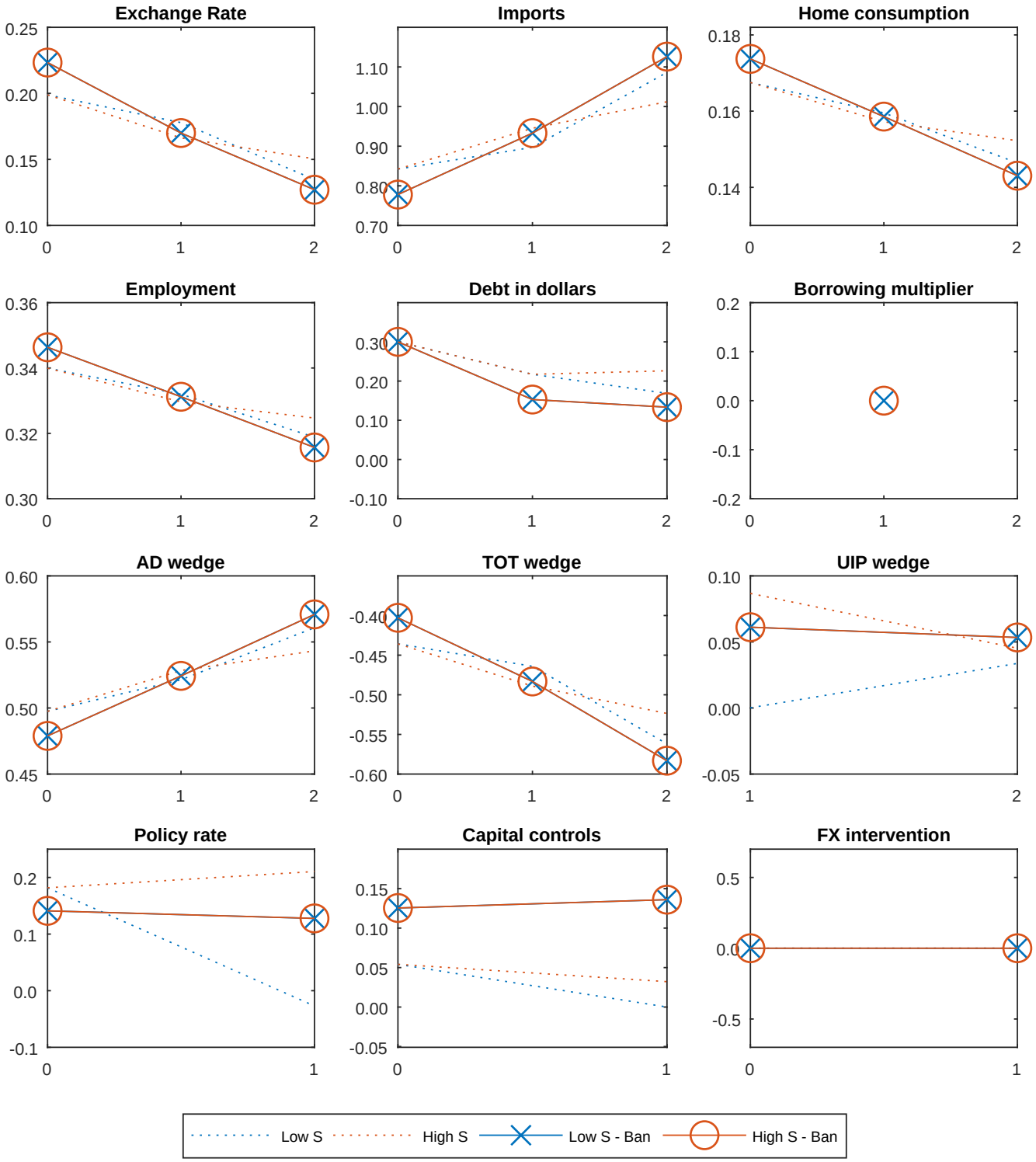
Notes: This figure plots the responses of key variables to a debt limit shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.02, 0.5]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 28: Debt Limit Shock under PCP with Shallow FX Markets and FX Bailouts



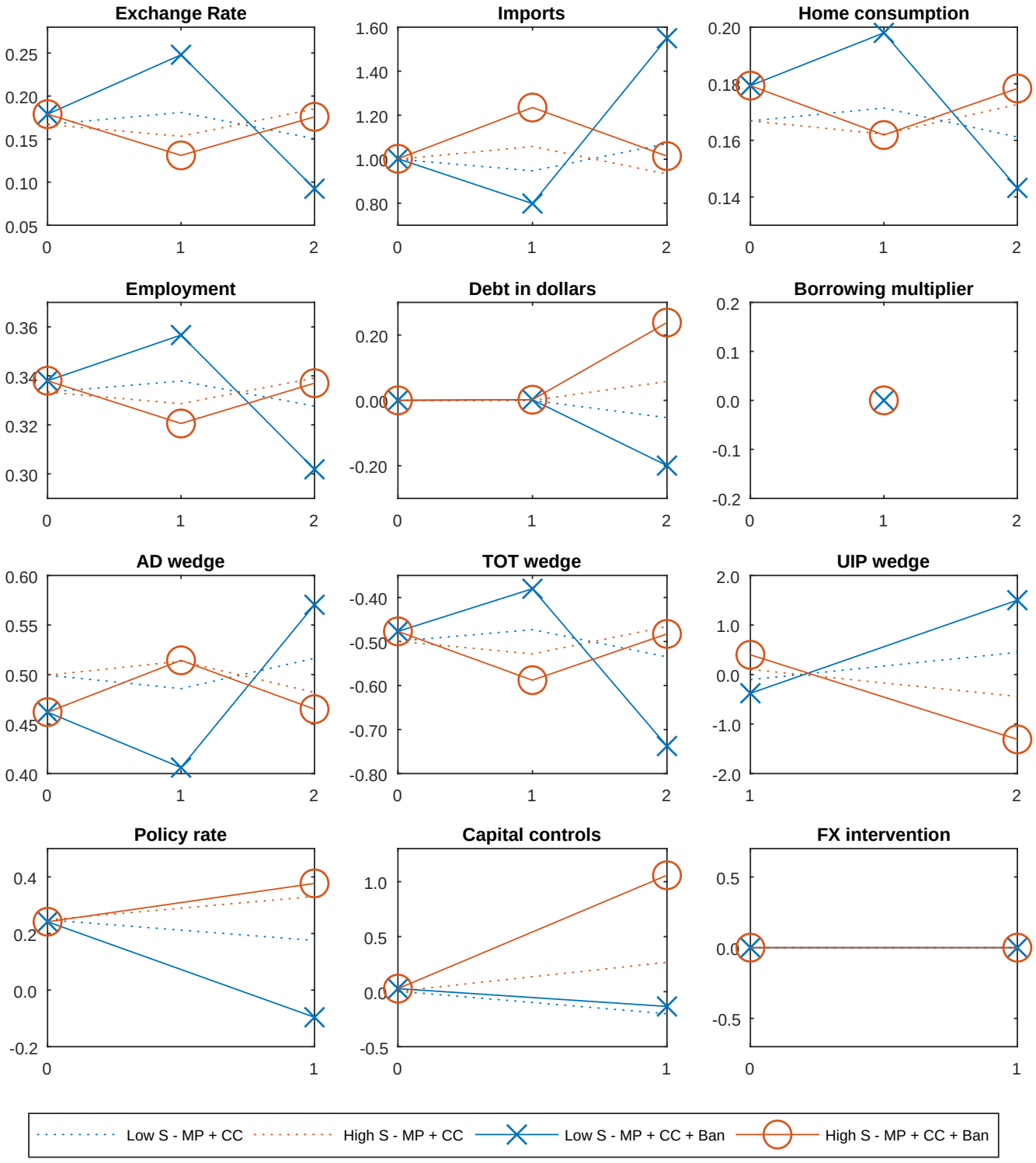
Notes: This figure plots the responses of key variables to a debt limit shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.02, 0.5]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization. For the case with FX bailouts, $\mu = 0.75$.

Figure 29: Debt Limit Shock under DCP and Banning FX Exposures



Notes: This figure plots the responses of key variables to a debt limit shock under DCP with shallow FX markets, banning of open FX positions. The shock hits at date-1 and is calibrated as $\kappa \in [0.03, 0.5]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 30: Foreign Appetite Shock under DCP with MP, CC, and Banning FX Exposures



Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets, monetary policy (MP), capital controls (CC) and banning of open FX positions. The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Table 1: Parameter Values

Parameter	Description	Value				
		Deep FX All shocks	Shallow FX A_1, P^*_{Z1}, i^*_1	Shallow FX S_1	Shallow FX Γ_1	Shallow FX κ
α_H	Expenditure share of tradable goods	1/3	1/3	1/3	1/3	1/3
α_F	Expenditure share of imports	1/3	1/3	1/3	1/3	1/3
α_{NT}	Expenditure share of nontradable goods	1/3	1/3	1/3	1/3	1/3
β	Discount factor	0.8	0.8	0.8	0.8	0.8
ω	Elasticity of export demand	1	1	1	1	1
P^*_F	Dollar price of imports	1	1	1	1	1
C^*	World demand level	1	1	1	1	1
Y_{NT}	Endowment of nontradable goods	1	1	1	1	1
Z	Endowment of commodities	0	0	0	0	0
P^*_{Z0}	Initial dollar price of commodity exports	1	1	1	1	1
i^*_0	Initial world interest rate	$1/\beta$-1	$1/\beta$-1	$1/\beta$-1	$1/\beta$-1	$1/\beta$-1
A_0	Initial level of productivity	1	1	1	1	1
B_0	Initial debt level	0.6	[0, 0.2]	0	0.3	0.3
λ	Domestic share of intermediaries	1	0.6	0.8	0.2	0.5
Γ	Balance sheet friction	0	1	1	1	0.2
Shocks	Description	Value				
π	Probability of good/bad shock	0.5	0.5	0.5	0.5	0.5
A	Productivity	[0.75, 1.50]	[0.8, 1.50]	NA	NA	NA
P^*_{Z1}	Commodity price	[0.75, 1.50]	[0.75, 1.50]	NA	NA	NA
i^*_1	World interest rate	[0, 0.5]	[0, 0.5]	NA	NA	NA
κ	Debt limit	[0.025, 10]	NA	NA	NA	[0.02, 5]
S_1	Foreign risk appetite	NA	NA	[-0.5, 0.5]	NA	NA
Γ_1	Intermediation friction	NA	NA	NA	[1, 5]	NA

Notes: The additional policy experiments discussed in Section 5 use the same calibration for the corresponding shocks reported above.

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[10] Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Diez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Moller, "Dominant Currency Paradigm," *American Economic Review* (forthcoming).

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A APPENDIX

A.1 Price Setting Condition under DCP

The functional form for $P_X = P_X\left(P_H, C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}\right)$ is as follows:

$$P_X = P_H \frac{X_1}{X_2}\frac{X_3}{X_4}$$

$$\text{where } X_1 = \frac{1}{A_0}\left(P^*_{F0}\right)^2 C^*_0 C_{F0} + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[\frac{1}{A_1}\left(P^*_{F1}\right)^2 C^*_1 C_{F1}\right] + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[\frac{1}{(1+i^*_1)}\frac{1}{A_2}\left(P^*_{F2}\right)^2 C^*_2 C_{F2}\right]$$

$$X_2 = P^*_{F0}C^*_0 + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[P^*_{F1}C^*_1\right] + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[\frac{1}{(1+i^*_1)}P^*_{F2}C^*_2\right]$$

$$X_3 = P^*_{F0}C_{F0} + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[P^*_{F1}C_{F1}\right] + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[\frac{1}{(1+i^*_1)}P^*_{F2}C_{F2}\right]$$

$$X_4 = \frac{1}{A_0}E_0\left(P^*_{F0}C_{F0}\right)^2 + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[\frac{1}{A_1}E_1\left(P^*_{F1}C_{F1}\right)^2\right] + \frac{1}{(1+i^*_0)}\mathbb{E}_0\left[\frac{1}{(1+i^*_1)}\frac{1}{A_2}E_2\left(P^*_{F2}C_{F2}\right)^2\right]$$

This price-setting constraint on the planner captures the fact that when setting the export price at the beginning of period 0, firms take into account the planner's anticipated actions in all future periods. The solution of the constrained efficient allocation will require the following derivatives:

$$\frac{\partial P_X}{\partial P_H} = \frac{P_X}{P_H}$$

$$\begin{aligned}\frac{\partial P_X}{\partial C_{F0}} &= P_H \frac{\frac{1}{A_0}\left(P^*_{F0}\right)^2 C^*_0}{X_2}\frac{X_3}{X_4} + P_H\frac{X_1}{X_2}\frac{X_4 P^*_{F0} - X_3\frac{2}{A_0}E_0\left(P^*_{F0}\right)^2 C_{F0}}{\left(X_4\right)^2}\\ &= P_X\left[\frac{\frac{1}{A_0}\left(P^*_{F0}\right)^2 C^*_0}{X_1} + \frac{X_4 P^*_{F0} - X_3\frac{2}{A_0}E_0\left(P^*_{F0}\right)^2 C_{F0}}{X_3 X_4}\right], \text{ a single equation}\end{aligned}$$

$$\begin{aligned}\frac{\partial P_X}{\partial C_{F1}} &= \pi_1 P_H \frac{\frac{1}{(1+i^*_0)}\frac{1}{A_1}\left(P^*_{F1}\right)^2 C^*_1}{X_2}\frac{X_3}{X_4}\\ &\quad + \pi_1 P_H \frac{X_1}{X_2}\frac{X_4\frac{1}{(1+i^*_0)}P^*_{F1} - X_3\frac{1}{(1+i^*_0)}\frac{2}{A_1}E_1\left(P^*_{F1}\right)^2 C_{F1}}{\left(X_4\right)^2}\\ &= \frac{\pi_1}{(1+i^*_0)}P_X\left[\frac{\frac{1}{A_1}\left(P^*_{F1}\right)^2 C^*_1}{X_1} + \frac{X_4 P^*_{F1} - X_3\frac{2}{A_1}E_1\left(P^*_{F1}\right)^2 C_{F1}}{X_3 X_4}\right],\\ &\quad \text{one equation per period-1 state } s_1\end{aligned}$$

$$\begin{aligned}\frac{\partial P_X}{\partial C_{F2}} &= \pi_1 P_H \frac{\frac{1}{(1+i^*_0)(1+i^*_1)}\frac{1}{A_2}\left(P^*_{F2}\right)^2 C^*_2}{X_2}\frac{X_3}{X_4}\\ &\quad + \pi_1 P_H \frac{X_1}{X_2}\frac{X_4\frac{1}{(1+i^*_0)(1+i^*_1)}P^*_{F2} - X_3\frac{1}{(1+i^*_0)(1+i^*_1)}\frac{2}{A_2}E_2\left(P^*_{F2}\right)^2 C_{F2}}{\left(X_4\right)^2}\end{aligned}$$

$$= \frac{\pi_1}{(1+i_0^*)(1+i_1^*)} P_X \left[\frac{\frac{1}{A_2}\left(P_{F2}^*\right)^2 C_2^*}{X_1} + \frac{X_4 P_{F2}^* - X_3 \frac{2}{A_2} E_2 \left(P_{F2}^*\right)^2 C_{F2}}{X_3 X_4} \right],$$

one equation per period-1 state s_1

$$\frac{\partial P_X}{\partial E_0} = -P_H \frac{X_1}{X_2} \frac{X_3}{\left(X_4\right)^2} \frac{1}{A_0} \left(P_{F0}^* C_{F0}\right)^2 = -P_X \frac{\frac{1}{A_0}\left(P_{F0}^* C_{F0}\right)^2}{X_4}, \text{ a single equation}$$

$$\begin{aligned} \frac{\partial P_X}{\partial E_1} &= -\pi_1 P_H \frac{X_1}{X_2} \frac{X_3}{\left(X_4\right)^2} \frac{1}{(1+i_0^*)} \frac{1}{A_1} \left(P_{F1}^* C_{F1}\right)^2 \\ &= -\frac{\pi_1}{(1+i_0^*)} P_X \frac{\frac{1}{A_1}\left(P_{F1}^* C_{F1}\right)^2}{X_4}, \end{aligned}$$

one equation per period-1 state s_1

$$\begin{aligned} \frac{\partial P_X}{\partial E_2} &= -\pi_1 P_H \frac{X_1}{X_2} \frac{X_3}{\left(X_4\right)^2} \frac{1}{(1+i_0^*)(1+i_1^*)} \frac{1}{A_2} \left(P_{F2}^* C_{F2}\right)^2 \\ &= -\frac{\pi_1}{(1+i_0^*)(1+i_1^*)} P_X \frac{\frac{1}{A_2}\left(P_{F2}^* C_{F2}\right)^2}{X_4}, \end{aligned}$$

one equation per period-1 state s_1.

A.2 FOCs for Constrained Efficient Allocations

Constrained efficient allocation

The constrained efficient allocation under full commitment is:

$$\max_{\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t\}} \left\{ \begin{array}{cc} \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t V \left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{E_t P_{Ft}^*}{P_H}, Y_{NTt} \right) \right] & \text{if PCP} \\ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t V \left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{P_{Ft}^*}{P_X}, Y_{NTt} \right) \right] & \text{if DCP,} \\ \text{with } P_X = P_X \left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H \right) & \end{array} \right\}$$

subject to the following constraints:

$$\begin{aligned} \left(1+i_{-1}^*\right) B_0 \leq\ & P_{F0}^* \left[\omega C_0^* - C_{F0}\right] + P_{Z0}^* Z_0 \\ & + \frac{P_{F1}^* \left[\omega C_1^* - C_{F1}\right] - (1-\lambda) FXI_0 \left[\eta_1 - (1+i_0^*)\right] + P_{Z1}^* Z_1}{\lambda (1+i_0^*) + (1-\lambda)\eta_1} \\ & + \frac{P_{F2}^* \left[\omega C_2^* - C_{F2}\right] - (1-\lambda) FXI_1 \left[\eta_2 - (1+i_1^*)\right] + P_{Z2}^* Z_2 + B_3}{\left[\lambda (1+i_0^*) + (1-\lambda)\eta_1\right]\left[\lambda (1+i_1^*) + (1-\lambda)\eta_2\right]}, \end{aligned}$$

one equation per period-1 state s_1 $[\Phi]$

$$\begin{aligned} \left(1+i_{-1}^*\right) B_0 \leq\ & P_{F0}^* \left[\omega C_0^* - C_{F0}\right] + P_{Z0}^* Z_0 + F_1 - \left(1+i_{-1}^*\right) F_0 \\ & + \frac{P_{F1}^* \left[\omega C_1^* - C_{F1}\right] - (1-\lambda) FXI_0 \left[\eta_1 - (1+i_0^*)\right] + P_{Z1}^* Z_1}{\lambda (1+i_0^*) + (1-\lambda)\eta_1} \end{aligned}$$

$$+\frac{\kappa\frac{P_H}{E_1}}{\lambda\left(1+i_0^*\right)+\left(1-\lambda\right)\eta_1},$$
one equation per period-1 state s_1 $[\Psi]$

$$\Gamma_0\left(\begin{array}{c}\left(1+i_{-1}^*\right)B_0+FXI_0-S_0\\-P_{F0}^*\left[\omega C_0^*-C_{F0}\right]-P_{Z0}^*Z_0-F_1+\left(1+i_{-1}^*\right)F_0\end{array}\right)$$
$$=\mathbb{E}_0\left[\eta_1-\left(1+i_0^*\right)\right],\text{ a single equation }[\Omega_0]$$

$$\Gamma_1\left(\begin{array}{c}\left[\begin{array}{c}\left(1+i_{-1}^*\right)B_0-P_{F0}^*\left[\omega C_0^*-C_{F0}\right]\\-P_{Z0}^*Z_0-F_1+\left(1+i_{-1}^*\right)F_0\end{array}\right]\left[\lambda\left(1+i_0^*\right)+\left(1-\lambda\right)\eta_1\right]+FXI_1-S_1\\-P_{F1}^*\left[\omega C_1^*-C_{F1}\right]+\left(1-\lambda\right)FXI_0\left[\eta_1-\left(1+i_0^*\right)\right]-P_{Z1}^*Z_1-F_2+\left(1+i_0^*\right)F_1\end{array}\right)$$
$$=\eta_2-\left(1+i_1^*\right),\text{ one equation per period-1 state }s_1\ [\Omega_1]$$

$$E_1^H\eta_1^H=E_1^L\eta_1^L,\text{ a single equation }[\Lambda]$$
$$FXI_0\geq 0,\text{ a single equation }[\Theta_0]$$
$$FXI_0+FXI_1\geq 0,\text{ one equation per period-1 state }s_1\ [\Theta_1],$$

where we define all the constraints in dollar terms. We fix the dollar value of initial debt repayments at $\left(1+i_{-1}^*\right)B_0$ in order to avoid the artefact depreciating away domestic-currency debt repayments at time 0, and we fix the dollar value of final debt at $B_3=B_0$ in order to normalize the debt path. We indicate the multipliers in capital Greek letters in square brackets after each constraint above.

The above planner problem assumes that all instruments (i.e., the policy rate, capital controls, and FX intervention) are available. The optimal allocations from the problem can be used to produce the implied optimal domestic policy rates and capital controls:

$$\left(1+\widehat{i}_t\right)=\left(1-\varphi_t\right)\left(1+i_t\right)=\eta_{t+1}\frac{E_{t+1}}{E_t}$$

$$\left(1+i_0\right)=\frac{\frac{\alpha_F}{P_{F0}^*C_{F0}}}{\beta\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\right]}\text{ and }\varphi_0=1-\frac{\left(1+\widehat{i}_0\right)}{\left(1+i_0\right)}$$

$$\left(1+i_1\right)=\frac{\frac{\alpha_F}{P_{F1}^*C_{F1}}}{\beta\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}}=\frac{E_2P_{F2}^*C_{F2}}{\beta E_1P_{F1}^*C_{F1}}\text{ and }\varphi_1=1-\frac{\left(1+\widehat{i}_1\right)}{\left(1+i_1\right)}\quad\text{if households' borrowing constraint is slack}$$
$$\left(1+i_1\right)=\left(1+\widehat{i}_1\right)\text{ and }\varphi_1=0\quad\text{if households' borrowing constraint is binding}$$

If FX intervention is not permitted, then we need to set:

$$FXI_0=FXI_1=0,$$

and remove the FOCs with respect to FXI_t. If capital controls are not permitted, then the household Euler conditions need to be added as constraints:

$$\frac{\alpha_F}{P_{F0}^*C_{F0}}=\beta E_1^H\eta_1^H\mathbb{E}_0\left[\frac{1}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\right],\text{ a single equation }[\Upsilon_0]$$

$$\frac{\alpha_F}{P_{F1}^*C_{F1}}\geq\beta\eta_2\frac{\alpha_F}{P_{F2}^*C_{F2}},\text{ one equation per period-1 state }s_1\ [\Upsilon_1]$$

If capital controls are not permitted and the domestic policy rate cannot be used, then the additional constraints are:

$$E_1^H \eta_1^H = \frac{1}{\beta} E_0, \text{ a single equation } [\Xi_0]$$

$$\eta_2 E_2 = \frac{1}{\beta} E_1, \text{ one equation per period-1 state } s_1 \ [\Xi_1]$$

FOCs
The FOCs for the constrained efficient allocation are:

$$\begin{aligned} C_{F0} : & \frac{\alpha_F}{P_{F0}^* C_{F0}} \left[1 + \frac{\alpha_H}{\alpha_F} \left(1 - \frac{1}{A_0} \frac{C_{H0}}{\alpha_H} \right) \right] + \mathbb{I}^{DCP} \cdot \left\{ \frac{1}{P_{F0}^*} \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2} \right] \frac{\partial P_X}{\partial C_{F0}} \right\} \\ & = \mathbb{E}_0 \left(\Phi + \Psi \right) + \Gamma_0 \Omega_0 + \mathbb{E}_0 \left(\Gamma_1 I_0 \Omega_1 \right) + \Upsilon_0 \frac{\alpha_F}{\left(P_{F0}^* C_{F0} \right)^2}, \text{ a single equation} \end{aligned}$$

$$\begin{aligned} C_{F1} : & \beta I_0 \frac{\alpha_F}{P_{F1}^* C_{F1}} \left[1 + \frac{\alpha_H}{\alpha_F} \left(1 - \frac{1}{A_1} \frac{C_{H1}}{\alpha_H} \right) \right] + \mathbb{I}^{DCP} \cdot \left\{ \frac{I_0}{\pi_1 P_{F1}^*} \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2} \right] \frac{\partial P_X}{\partial C_{F1}} \right\} \\ & = \Phi + \Psi \left[1 - \kappa_T \frac{\alpha_H}{\alpha_F} \right] + \Gamma_1 I_0 \Omega_1 + I_0 \frac{\alpha_F}{\left(P_{F1}^* C_{F1} \right)^2} \left[\Upsilon_1 - \Upsilon_0 \beta \eta_1 \right], \\ & \text{one equation per period-1 state } s_1 \end{aligned}$$

$$\begin{aligned} C_{F2} : & \beta^2 I_0 I_1 \frac{\alpha_F}{P_{F2}^* C_{F2}} \left[1 + \frac{\alpha_H}{\alpha_F} \left(1 - \frac{1}{A_2} \frac{C_{H2}}{\alpha_H} \right) \right] + \mathbb{I}^{DCP} \cdot \left\{ \frac{I_0 I_1}{\pi_1 P_{F2}^*} \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2} \right] \frac{\partial P_X}{\partial C_{F2}} \right\} \\ & = \Phi - \Upsilon_1 \beta I_0 I_1 \eta_2 \frac{\alpha_F}{\left(P_{F2}^* C_{F2} \right)^2}, \text{ one equation per period-1 state } s_1 \end{aligned}$$

$$\begin{aligned} E_0 : & \alpha_H \left(1 - \frac{1}{A_0} \frac{C_{H0}}{\alpha_H} \right) - \Xi_0 \frac{E_0}{\beta} \\ & = \mathbb{I}^{PCP} \cdot \left\{ \frac{E_0 P_{F0}^*}{P_H} \frac{\omega}{A_0} C_0^* \right\} + \mathbb{I}^{DCP} \cdot \left\{ E_0 \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2} \right] \left(- \frac{\partial P_X}{\partial E_0} \right) \right\}, \\ & \text{a single equation} \end{aligned}$$

$$\begin{aligned} E_1^H : & \beta \alpha_H \left(1 - \frac{1}{A_1} \frac{C_{H1}}{\alpha_H} \right) \pi_1 - \Psi \frac{1}{I_0} \kappa_{NT} \frac{P_H}{E_1} \pi_1 + \Lambda E_1 \eta_1 - \Upsilon_0 \beta \eta_1^L \frac{\alpha_F}{P_{F1}^* C_{F1}^L} \pi_1^L + \Xi_0 E_1 \eta_1 - \Xi_1 \frac{E_1}{\beta} \pi_1 \\ & = \mathbb{I}^{PCP} \cdot \left\{ \beta \frac{E_1 P_{F1}^*}{P_H} \frac{\omega}{A_1} C_1^* \pi_1 \right\} + \mathbb{I}^{DCP} \cdot \left\{ E_1 \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2} \right] \left(- \frac{\partial P_X}{\partial E_1} \right) \right\}, \\ & \text{a single equation for the } H\text{-state} \end{aligned}$$

$$E_1^L : \beta\alpha_H\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1 - \Psi\frac{1}{I_0}\kappa_{NT}\frac{P_H}{E_1}\pi_1 - \Lambda E_1\eta_1 + \Upsilon_0\beta\eta_1\frac{\alpha_F}{P_{F1}^* C_{F1}}\pi_1 - \Xi_1\frac{E_1}{\beta}\pi_1$$
$$= \mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1 P_{F1}^*}{P_H}\frac{\omega}{A_1}C_1^*\pi_1\right\} + \mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\},$$
a single equation for the L-state

$$E_2 : \beta^2\alpha_H\left(1-\frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\pi_1 + \Xi_1 E_2\eta_2\pi_1$$
$$= \mathbb{I}^{PCP}\cdot\left\{\beta^2\frac{E_2 P_{F2}^*}{P_H}\frac{\omega}{A_2}C_2^*\pi_1\right\} + \mathbb{I}^{DCP}\cdot\left\{E_2\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_2}\right)\right\},$$
one equation per period-1 state s_1

$$\eta_1^H : (\Phi+\Psi)(1-\lambda)\frac{B_1+FXI_0}{I_0} + \Psi(1-\lambda)\frac{\kappa\frac{P_H}{E_1}-B_2}{(I_0)^2}$$
$$= \Omega_0 - \Omega_1\Gamma_1(1-\lambda)(B_1+FXI_0) + \frac{1}{\pi_1}\Lambda E_1 - \frac{1}{\pi_1}\Upsilon_0\beta E_1\mathbb{E}_0\left\{\frac{1}{E_1}\frac{\alpha_F}{P_{F1}^* C_{F1}}\right\} + \frac{1}{\pi_1}\Xi_0 E_1^H,$$
a single equation for the H-state

$$\eta_1^L : (\Phi+\Psi)(1-\lambda)\frac{B_1+FXI_0}{I_0} + \Psi(1-\lambda)\frac{\kappa\frac{P_H}{E_1}-B_2}{(I_0)^2}$$
$$= \Omega_0 - \Omega_1\Gamma_1(1-\lambda)(B_1+FXI_0) - \frac{1}{\pi_1}\Lambda E_1, \text{ a single equation for the } L\text{-state}$$

$$\eta_2 : \Phi(1-\lambda)\frac{B_2+FXI_1}{I_0 I_1} = \Omega_1 - \Upsilon_1\beta\frac{\alpha_F}{P_{F2}^* C_{F2}} + \Xi_1 E_2, \text{ one equation per period-1 state } s_1$$

$$FXI_0 : 0 = -(1-\lambda)\mathbb{E}_0\left\{(\Phi+\Psi)\frac{[\eta_1-(1+i_0^*)]}{I_0}\right\}$$
$$-\Gamma_0\Omega_0 - (1-\lambda)\mathbb{E}_0\left\{\Gamma_1\Omega_1[\eta_1-(1+i_0^*)]\right\} + \Theta_0 + \mathbb{E}_0\Theta_1, \text{ a single equation}$$

$$FXI_1 : -\Phi\frac{(1-\lambda)[\eta_2-(1+i_1^*)]}{I_0 I_1} - \Gamma_1\Omega_1 + \Theta_1 = 0,$$
one equation per period-1 state s_1,

where we define:

$$I_t = \lambda(1+i_t^*) + (1-\lambda)\eta_{t+1}$$
$$B_1 \equiv \frac{D_1}{E_0} = \left(1+i_{-1}^*\right)B_0 - \left(P_{F0}^*[\omega C_0^* - C_{F0}] + P_{Z0}^* Z_0\right)$$
$$B_2 \equiv \frac{D_2}{E_1} = B_1 I_0 - \left(P_{F1}^*[\omega C_1^* - C_{F1}] - (1-\lambda)FXI_0[\eta_1-(1+i_0^*)] + P_{Z1}^* Z_1\right)$$

$$B_3 \equiv \frac{D_3}{E_2} = B_2 I_1 - \left(P^*_{F2}\left[\omega C^*_2 - C_{F2}\right] - (1-\lambda)\, FXI_1\left[\eta_2 - (1+i^*_1)\right] + P^*_{Z2} Z_2\right),$$

and where $\Theta_t \geq 0$ and $\sum_{s=0}^{t} FXI_t \geq 0$ with complementary slackness. The FOCs with respect to P_H are redundant, so we normalize $P_H = 1$.

A.3 Additional Experiment: Ban on FX Positions

If domestically-owned intermediaries are prohibited from taking open FX positions, then the constrained planner problem changes:

$$\max_{\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t\}} \left\{ \begin{array}{c} \mathbb{E}_0\left[\sum_{t=0}^{2} \beta^t V\left(C_{Ft}, \frac{E_t P^*_{Ft}}{P_H}, \frac{E_t P^*_{Ft}}{P_H}, Y_{NTt}\right)\right] \quad \text{if PCP} \\ \mathbb{E}_0\left[\sum_{t=0}^{2} \beta^t V\left(C_{Ft}, \frac{E_t P^*_{Ft}}{P_H}, \frac{P^*_{Ft}}{P_X}, Y_{NTt}\right)\right] \quad \text{if DCP} \\ \text{with } P_X = P_X\left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H\right) \end{array} \right\}$$

$$\begin{aligned} \left(1+i^*_{-1}\right) B_0 \leq\ & P^*_{F0}\left[\omega C^*_0 - C_{F0}\right] + P^*_{Z0} Z_0 + F_1 - \left(1+i^*_{-1}\right) F_0 \\ & + \frac{P^*_{F1}\left[\omega C^*_1 - C_{F1}\right] - FXI_0\left[\eta_1 - (1+i^*_0)\right] + P^*_{Z1} Z_1 + F_2 - (1+i^*_0) F_1}{\eta_1} \\ & + \frac{P^*_{F2}\left[\omega C^*_2 - C_{F2}\right] - FXI_1\left[\eta_2 - (1+i^*_1)\right] + P^*_{Z2} Z_2 + F_3 - (1+i^*_1) F_2 + B_3}{\eta_1 \eta_2}, \\ & \text{one equation per period-1 state } s_1 \ \ [\Phi] \end{aligned}$$

$$\begin{aligned} \left(1+i^*_{-1}\right) B_0 \leq\ & P^*_{F0}\left[\omega C^*_0 - C_{F0}\right] + P^*_{Z0} Z_0 + F_1 - \left(1+i^*_{-1}\right) F_0 \\ & + \frac{P^*_{F1}\left[\omega C^*_1 - C_{F1}\right] - FXI_0\left[\eta_1 - (1+i^*_0)\right] + P^*_{Z1} Z_1 + F_2 - (1+i^*_0) F_1}{\eta_1} \\ & + \frac{\kappa \frac{P_H}{E_1}}{\eta_1}, \text{ one equation per period-1 state } s_1 \ \ [\Psi] \end{aligned}$$

$$\begin{aligned} & \frac{\Gamma_0}{(1-\lambda)} \begin{pmatrix} \left(1+i^*_{-1}\right) B_0 + FXI_0 - S_0 \\ -P^*_{F0}\left[\omega C^*_0 - C_{F0}\right] - P^*_{Z0} Z_0 - F_1 + \left(1+i^*_{-1}\right) F_0 \end{pmatrix} \\ & = \mathbb{E}_0\left[\eta_1 - (1+i^*_0)\right], \text{ a single equation } \ [\Omega_0] \end{aligned}$$

$$\begin{aligned} & \frac{\Gamma_1}{(1-\lambda)} \begin{pmatrix} \begin{bmatrix} \left(1+i^*_{-1}\right) B_0 - P^*_{F0}\left[\omega C^*_0 - C_{F0}\right] \\ -P^*_{Z0} Z_0 - F_1 + \left(1+i^*_{-1}\right) F_0 \end{bmatrix} \eta_1 + FXI_1 - S_1 \\ -P^*_{F1}\left[\omega C^*_1 - C_{F1}\right] + FXI_0\left[\eta_1 - (1+i^*_0)\right] - P^*_{Z1} Z_1 - F_2 + (1+i^*_0) F_1 \end{pmatrix} \\ & = \eta_2 - (1+i^*_1), \text{ one equation per period-1 state } s_1 \ \ [\Omega_1] \end{aligned}$$

$$E^H_1 \eta^H_1 = E^L_1 \eta^L_1, \text{ a single equation } \ [\Lambda]$$

$$FXI_0 \geq 0, \text{ a single equation } \ [\Theta_0]$$

$$FXI_0 + FXI_1 \geq 0, \text{ one equation per period-1 state } s_1 \ \ [\Theta_1]$$

$$\frac{\alpha_F}{P^*_{F0}C_{F0}} = \beta E^H_1 \eta^H_1 \mathbb{E}_0 \left[\frac{1}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right], \text{ a single equation } [\Upsilon_0]$$

$$\frac{\alpha_F}{P^*_{F1}C_{F1}} \geq \beta\eta_2 \frac{\alpha_F}{P^*_{F2}C_{F2}}, \text{ one equation per period-1 state } s_1 \ [\Upsilon_1]$$

$$E^H_1 \eta^H_1 = \frac{1}{\beta}E_0, \text{ a single equation } [\Xi_0]$$

$$\eta_2 E_2 = \frac{1}{\beta}E_1, \text{ one equation per period-1 state } s_1 \ [\Xi_1]$$

where we define all the constraints in dollar terms, we fix the dollar value of initial debt repayments at $\left(1+i^*_{-1}\right)B_0$, and we set $B_3 = B_0$.

The FOCs also change:

$$C_{F0}: \frac{\alpha_F}{P^*_{F0}C_{F0}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_0}\frac{C_{H0}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP}\cdot\left\{\frac{1}{P^*_{F0}}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C^*_t\frac{P^*_{Ft}}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F0}}\right\}$$
$$= \mathbb{E}_0\left(\Phi+\Psi\right) + \frac{\Gamma_0}{(1-\lambda)}\Omega_0 + \mathbb{E}_0\left(\frac{\Gamma_1}{(1-\lambda)}I_0\Omega_1\right) + \Upsilon_0\frac{\alpha_F}{\left(P^*_{F0}C_{F0}\right)^2}, \text{ a single equation}$$

$$C_{F1}: \beta I_0\frac{\alpha_F}{P^*_{F1}C_{F1}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP}\cdot\left\{\frac{I_0}{\pi_1 P^*_{F1}}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C^*_t\frac{P^*_{Ft}}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F1}}\right\}$$
$$= \Phi + \Psi\left[1-\kappa_T\frac{\alpha_H}{\alpha_F}\right] + \frac{\Gamma_1}{(1-\lambda)}I_0\Omega_1 + I_0\frac{\alpha_F}{\left(P^*_{F1}C_{F1}\right)^2}\left[\Upsilon_1 - \Upsilon_0\beta\eta_1\right],$$
one equation per period-1 state s_1

$$C_{F2}: \beta^2 I_0 I_1\frac{\alpha_F}{P^*_{F2}C_{F2}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP}\cdot\left\{\frac{I_0 I_1}{\pi_1 P^*_{F2}}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C^*_t\frac{P^*_{Ft}}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F2}}\right\}$$
$$= \Phi - \Upsilon_1\beta I_0 I_1\eta_2\frac{\alpha_F}{\left(P^*_{F2}C_{F2}\right)^2}, \text{ one equation per period-1 state } s_1$$

$$E_0: \alpha_H\left(1-\frac{1}{A_0}\frac{C_{H0}}{\alpha_H}\right) - \Xi_0\frac{E_0}{\beta}$$
$$= \mathbb{I}^{PCP}\cdot\left\{\frac{E_0 P^*_{F0}}{P_H}\frac{\omega}{A_0}C^*_0\right\} + \mathbb{I}^{DCP}\cdot\left\{E_0\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C^*_t\frac{P^*_{Ft}}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_0}\right)\right\},$$
a single equation

$$E^H_1: \beta\alpha_H\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1 - \Psi\frac{1}{I_0}\kappa_{NT}\frac{P_H}{E_1}\pi_1 + \Lambda E_1\eta_1 - \Upsilon_0\beta\eta^L_1\frac{\alpha_F}{P^*_{F1}C^L_{F1}}\pi^L_1 + \Xi_0 E_1\eta_1 - \Xi_1\frac{E_1}{\beta}\pi_1$$
$$= \mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1 P^*_{F1}}{P_H}\frac{\omega}{A_1}C^*_1\pi_1\right\} + \mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C^*_t\frac{P^*_{Ft}}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\},$$
a single equation for the H-state

$$E_1^L : \beta\alpha_H \left(1 - \frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1 - \Psi\frac{1}{I_0}\kappa_{NT}\frac{P_H}{E_1}\pi_1 - \Lambda E_1\eta_1 + \Upsilon_0\beta\eta_1\frac{\alpha_F}{P_{F1}^* C_{F1}}\pi_1 - \Xi_1\frac{E_1}{\beta}\pi_1$$
$$= \mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1 P_{F1}^*}{P_H}\frac{\omega}{A_1}C_1^*\pi_1\right\} + \mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\},$$
a single equation for the L-state

$$E_2 : \beta^2\alpha_H\left(1 - \frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\pi_1 + \Xi_1 E_2\eta_2\pi_1$$
$$= \mathbb{I}^{PCP}\cdot\left\{\beta^2\frac{E_2 P_{F2}^*}{P_H}\frac{\omega}{A_2}C_2^*\pi_1\right\} + \mathbb{I}^{DCP}\cdot\left\{E_2\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_2}\right)\right\},$$
one equation per period-1 state s_1

$$\eta_1^H : (\Phi+\Psi)\frac{B_1 + FXI_0}{I_0} + \Psi\frac{\kappa\frac{P_H}{E_1} - B_2}{(I_0)^2}$$
$$= \Omega_0 - \Omega_1\frac{\Gamma_1}{(1-\lambda)}(B_1 + FXI_0) + \frac{1}{\pi_1}\Lambda E_1 - \frac{1}{\pi_1}\Upsilon_0\beta E_1\mathbb{E}_0\left\{\frac{1}{E_1}\frac{\alpha_F}{P_{F1}^* C_{F1}}\right\} + \frac{1}{\pi_1}\Xi_0 E_1^H,$$
a single equation for the H-state

$$\eta_1^L : (\Phi+\Psi)\frac{B_1 + FXI_0}{I_0} + \Psi\frac{\kappa\frac{P_H}{E_1} - B_2}{(I_0)^2}$$
$$= \Omega_0 - \Omega_1\frac{\Gamma_1}{(1-\lambda)}(B_1 + FXI_0) - \frac{1}{\pi_1}\Lambda E_1, \text{ a single equation for the } L\text{-state}$$

$$\eta_2 : \Phi\frac{B_2 + FXI_1}{I_0 I_1} = \Omega_1 - \Upsilon_1\beta\frac{\alpha_F}{P_{F2}^* C_{F2}} + \Xi_1 E_2, \text{ one equation per period-1 state } s_1$$

$$FXI_0 : 0 = -\mathbb{E}_0\left\{(\Phi+\Psi)\frac{[\eta_1 - (1+i_0^*)]}{I_0}\right\}$$
$$-\frac{\Gamma_0}{(1-\lambda)}\Omega_0 - \mathbb{E}_0\left\{\frac{\Gamma_1}{(1-\lambda)}\Omega_1[\eta_1 - (1+i_0^*)]\right\} + \Theta_0 + \mathbb{E}_0\Theta_1, \text{ a single equation}$$

$$FXI_1 : -\Phi\frac{[\eta_2 - (1+i_1^*)]}{I_0 I_1} - \frac{\Gamma_1}{(1-\lambda)}\Omega_1 + \Theta_1 = 0,$$
one equation per period-1 state s_1,

where we define:

$$I_t = \eta_{t+1}$$
$$B_1 \equiv \frac{D_1}{E_0} = \left(1 + i_{-1}^*\right)B_0 - \left(P_{F0}^*[\omega C_0^* - C_{F0}] + P_{Z0}^* Z_0\right)$$
$$B_2 \equiv \frac{D_2}{E_1} = B_1 I_0 - \left(P_{F1}^*[\omega C_1^* - C_{F1}] - FXI_0[\eta_1 - (1+i_0^*)] + P_{Z1}^* Z_1\right)$$

$$B_3 \equiv \frac{D_3}{E_2} = B_2 I_1 - \left(P^*_{F2}\left[\omega C^*_2 - C_{F2}\right] - FXI_1\left[\eta_2 - (1+i^*_1)\right] + P^*_{Z2} Z_2\right),$$

and where $\Theta_t \geq 0$ and $\sum_{s=0}^{t} FXI_t \geq 0$ with complementary slackness. The FOCs with respect to P_H are redundant, so we normalize $P_H = 1$.

A.4 Numerical Solution

We start from the relevant set of planner constraints and FOCs from the preceding subsections. First, we select the set of policy instruments available to the planner:

- If all policy instruments (i.e., the policy rate, capital controls, and FX intervention) are available to the planner, then use all the FOCs above but set $\Upsilon_0 = \Upsilon_1 = \Xi_0 = \Xi_1 = 0$.
- If FX intervention is not permitted, then set $FXI_0 = FXI_1 = 0$, and remove the FOCs with respect to FXI_t.
- If capital controls are not permitted, then use all the FOCs above but set $\Xi_0 = \Xi_1 = 0$.
- If capital controls and the domestic policy rate are not permitted, then use all the FOCs above.

Next, we characterize the solution numerically by running the following iterative process to convergence.

1. Fix guess on whether the borrowing constraint (16) is slack or binding in every period-1 state. For states where the constraint is slack, fix $\Psi = 0$ and remove the borrowing constraint. For states where the constraint is binding, set the borrowing constraint to be satisfied with equality and allow $\Psi \neq 0$. Run the following iterative process to convergence.

    - Fix guess on whether the FXI constraint (9) is slack or binding in every period-0 and period-1 state. For states where the constraint is slack, fix $\Theta_t = 0$ and allow $\sum_{s=0}^{t} FXI_t \neq 0$. For states where the constraint is binding, set $\sum_{s=0}^{t} FXI_t = 0$ and allow $\Theta_t \neq 0$.
    - Numerically solve the system of equations.
    - Verify that $\sum_{s=0}^{t} FXI_t \geq 0$ for states where the constraint was guessed to be slack; otherwise, change the guess. Verify that $\Theta_t \geq 0$ for states where the constraint was guessed to be binding; otherwise, change the guess.

2. Verify that the borrowing constraint is slack for states where the constraint was guessed to be slack; otherwise, change the guess. Verify that $\Psi \geq 0$ for states where the constraint was guessed to be binding; otherwise, change the guess.