Optimal Unilateral Carbon Policy

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We consider climate policy by a country (called Home) in a world with international trade in energy and in manufactured goods produced with energy and labor. In this setting we derive an optimal unilateral policy to confront the global externality from manufacturing's combustion of carbon-based energy.

Our solution strategy combines techniques from Markusen (1975) and from Costinot, Donaldson, Vogel, and Werning (2015; henceforth CDVW). We interpret the policy as a particular set of taxes and subsidies. The key features are: (i) Home institutes a carbon tax equal to its damages from emissions, raising the cost of energy for its manufacturers relative to the price received by its energy extractors; (ii) this Pigouvian tax is the sum of an extraction tax and a production tax, with a border adjustment on imported manufactured goods equal to the production tax; (iii) the mix of extraction tax and production tax is optimized to reduce carbon leakage and to improve Home's terms of trade; (iv) the border adjustment on the energy content of imported manufactured goods leaves Home's consumption decisions undistorted; (v) energy taxes are not removed at the border for exports, rather Home subsidizes exports of goods in which its comparative advantage is weak while taxing those where its comparative advantage is strong; (vi) Home expands exports of manufactures on the extensive margin, potentially even exporting goods that it also imports.

Features (i)-(iii) of this optimal policy are reminiscent of Markusen, in a model with trade only in energy. Features (iv)-(v) are reminiscent of CDVW, in a model with no externalities. A novel property, captured in (vi), is how Home can exploit international trade in manufactured goods to expand the

reach of its climate policy. Through its tax policy Home indirectly controls how energy is used in producing both its imports and its exports.

1 Basic Model

Two countries, Home and Foreign, are endowed with labor, L and L^*, as well as energy deposits, E and E^*. The $*$ distinguishes Foreign from Home. There are three sectors: services s, goods manufacturing g, and energy e. Services are produced with labor only, goods are manufactured by combining labor and energy, and energy is extracted by combining labor and energy deposits.

After laying out the basic elements of the model here, in the following section we consider a social planner in Home orchestrating an optimal carbon policy. Foreign is modeled as a decentralized economy with no carbon policy, taking as given the choices of Home's social planner.

1.1 Preferences

We denote consumption of services by C_s and consumption across all manufactured goods $j \in [0,1]$ by $c = \{c_j\}$. Preferences in Home are:

$$U(C_s, c) = C_s + \int_0^1 u(c_j)dj - \varphi Q_e^W,$$

where $Q_e^W = Q_e + Q_e^*$ is global emissions (proportional to global energy extraction), φ is marginal damages from carbon emissions (assumed constant for simplicity), and

$$u(c_j) = \alpha^{1/\sigma} \frac{c_j^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where $\sigma > 0$ is the elasticity of demand for individual goods. Defining the consumption index of manufactured goods as:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)},$$

it follows that:

$$U(C_s, c) = C_s + \alpha^{1/\sigma} \frac{C_g^{1-1/\sigma} - 1}{1 - 1/\sigma} - \varphi Q_e^W.$$

Preferences in Foreign are the same as those in Home, except with α^* in place of α and σ^* in place of σ. Since Foreign takes global emissions Q_e^W as given, its marginal damage parameter φ^* has no effect on the optimal carbon policy. To simplify the analysis, we will assume that any solution we consider satisfies $C_s > 0$ and $C_s^* > 0$, a condition that can be easily checked.

We will restrict $\sigma^* \leq 1$ since assuming inelastic Foreign demand simplifies the solution for Home's optimal exports of individual manufactured goods. Furthermore, inelastic demand for manufactured goods, taken as a whole, is plausible. In the case of $\sigma = 1$ (or $\sigma^* = 1$), the usual Cobb-Douglas limits arise. To avoid carrying around two cases, the equations that follow hold for $\sigma \neq 1$ and $\sigma^* < 1$ (but are easily adapted to the Cobb-Douglas limit).

1.2 Technology

Production and trade are Ricardian. Services are produced in both countries with unit labor productivity. Manufactured goods are produced with a Cobb-Douglas combination of labor and energy. Good j is produced with efficiency A_j in Home and A_j^* in Foreign.

The production function for good j in Home is:

$$q_j = A_j \left(\frac{l_j}{\gamma}\right)^{\gamma} \left(\frac{e_j}{1-\gamma}\right)^{1-\gamma},$$

where l_j is labor input e_j is energy input, and γ is the labor share parameter. In all that follows, the energy intensity of manufacturing production

$$z_j = e_j / l_j$$

plays a prominent role. To highlight that role, we can rewrite the production function, for a given energy intensity z, as:

$$q_j = \nu A_j z^{1-\gamma} l_j, \tag{1}$$

where:

$$\nu = \gamma^{-\gamma} (1-\gamma)^{-(1-\gamma)}. \tag{2}$$

In Foreign, given energy intensity z, we have:

$$q_j^* = \nu A_j^* z^{1-\gamma} l_j^*.$$

While these expressions are evaluated at a particular z, there is no restriction on how energy intensity may vary across different goods or countries.

Energy is produced in Home with technology:

$$Q_e = \left(\frac{L_e}{\beta}\right)^{\beta} (E)^{1-\beta},$$

where E denotes Home's energy deposits. In Foreign:

$$Q_e^* = \left(\frac{L_e^*}{\beta^*}\right)^{\beta^*} (E^*)^{1-\beta^*}.$$

Energy deposits E and E^* are exogenous country endowments.

1.3 International Trade

We assume that services and energy are costlessly traded between Home and Foreign. Manufactured goods are traded subject to iceberg costs.

Energy is traded at price p_e. We take services to be the numéraire, with price 1. We will generally assume that $Q_s^* > 0$ so that given the unit labor requirement for services the wage in Foreign is $w^* = 1$. (This outcome is guaranteed with a large enough labor endowment in Foreign.)

Trade in manufactured goods follows Dornbusch, Fisher, and Samuelson (1977). Goods are arranged in decreasing order of Home's comparative advantage:

$$\frac{A_j}{A_j^*} = F(j), \tag{3}$$

where we assume that $F(j)$ is a strictly decreasing continuous function.[1]

Due to the iceberg trade cost $\tau \geq 1$ on Home's exports, Home supplies good j to Foreign at efficiency A_j/τ. Home's imports are subject to the iceberg trade cost $\tau^* \geq 1$ so that Foreign supplies good j to Home at efficiency A_j^*/τ^*.

1.4 Input Requirements

Having described technology and trade for manufactured goods, we now adopt a convenient notation for input requirements. At any energy inten-

[1] In order to have well defined integrals in what follows, we also assume that A_j and A_j^* can be treated as continuous functions of j.

sity z, we can invert the production function (1) to express the unit labor requirement to produce good j in Home:

$$a_j^l(z) = \frac{a_j}{\nu} z^{-(1-\gamma)}, \tag{4}$$

where $a_j = 1/A_j$. The corresponding unit energy requirement is simply:

$$a_j^e(z) = z a_j^l(z) = \frac{a_j}{\nu} z^{\gamma}. \tag{5}$$

Input requirements in Foreign, $a_j^{l*}(z)$ and $a_j^{e*}(z)$, are the same, but with $a_j^* = 1/A_j^*$ in place of a_j.

In what follows, we will work with these input requirements rather than the corresponding efficiencies. Total input requirements satisfy:

$$\frac{a_j^*}{a_j} = F(j),$$

where F is from (3). Home's input requirements are multiplied by τ for its exports and Foreign's are multiplied by τ^* for Home's imports.

1.5 Carbon Accounting

We take a unit of energy to be a unit of carbon. Carbon is extracted in both countries. Home may export some of what it extracts or else may import some from Foreign. Carbon is used by manufacturers and emitted in the production process. Manufactured goods, implicitly embodying carbon emissions, are also traded. Some goods are exported and others imported before they are consumed by households. We can therefore trace carbon from its extraction through its use in manufacturing and finally to its implicit consumption by households.

We define M_e as the use of energy by manufacturers in Home and M_e^* by manufacturers in Foreign. Home's net exports of energy, which can be positive or negative, are given by:

$$X_e = Q_e - M_e.$$

Global energy market clearing implies:

$$X_e = M_e^* - Q_e^*.$$

These expressions account for the first level of trade in carbon.

The second level of trade in carbon (embodied in manufactured goods) is more intricate, requiring some additional notation. For each manufactured good j we distinguish between Home's exports, $0 \leq x_j \leq q_j$, and Home's imports, $0 \leq m_j \leq q_j^*$ (forcing each to be weakly positive). These quantities connect to consumption via:

$$x_j = \max\{q_j - c_j, 0\}$$

and

$$m_j = \max\{c_j - q_j, 0\}.$$

Home's production for its domestic market is:

$$y_j = q_j - x_j,$$

and Foreign's production for its domestic market is:

$$y_j^* = q_j^* - m_j.$$

We need to separately consider four lines of production of good j: y_j, x_j, m_j, and y_j^*. The corresponding energy intensities are z_j^y, z_j^x, z_j^m, and z_j^*. (We use $*$ rather than y^* as the superscript, slightly abusing notation.) Thus producers in Home and Foreign may vary their energy intensity by good and separately for the domestic market and export market.

Home manufacturers use energy to serve the domestic market:

$$C_e^{HH} = \int_0^1 a_j^e(z_j^y) y_j dj$$

and to serve the export market (subject to the iceberg cost τ):

$$C_e^{FH} = \tau \int_0^1 a_j^e(z_j^x) x_j dj,$$

where, in each case, we have made use of equation (5). Total use of energy by Home's manufacturers is therefore:

$$M_e = C_e^{HH} + C_e^{FH}.$$

Foreign manufacturers use energy to serve their domestic market:

$$C_e^{FF} = \int_0^1 a_j^{e*}(z_j^*) y_j^* dj$$

and to meet Home's demand for imports (subject to the iceberg cost τ^*):

$$C_e^{HF} = \tau^* \int_0^1 a_j^{e*}(z_j^m) m_j dj.$$

Total use of energy by Foreign's manufacturers is therefore:

$$M_e^* = C_e^{FF} + C_e^{HF}.$$

The ultimate consumer of energy (or carbon) is by the households purchasing manufactured goods. To capture this concept, we defined:

$$C_e = C_e^{HH} + C_e^{HF}$$

and

$$C_e^* = C_e^{FF} + C_e^{FH}.$$

These last two terms are crucial for a full accounting of carbon.

2 Free-Market Outcome

To be completed

3 Social Planner Problem

We now consider Home's optimal carbon policy. We consider a social planner that allocates all resources in Home so as to maximize Home's welfare. Foreign outcomes are determined as a competitive equilibrium, given the choices made by planner. After solving this problem, we'll deduce the set of taxes on extraction, production, consumption, and trade that implement Home's optimal unilateral carbon policy.

The first step is to express the world price of each manufactured good j in terms of Home's trade in good j. To do so, note that the social planner will dictate the quantities y_j, x_j, and m_j. It will also dictate the corresponding energy intensities of production: z_j^y, z_j^x, and z_j^m. Even though Home's imports of good j are produced in Foreign, the planner can set the standards for how they are produced. Of course the price at which Foreign sells these goods to Home will reflect those standards.

Foreign consumers choose y_j^* to maximize utility and Foreign producers choose the energy intensity z_j^* for producing these goods so as to minimize cost. These decisions are based on the energy price p_e, which the social planner also chooses. We draw out that dependence on p_e in what follows.

3.1 Prices for International Trade

As mentioned above, Home and Foreign trade services at a price of 1. They trade energy at a price of p_e. The set of prices at which they trade manufactured goods is more subtle.

Foreign is either producing manufactured good j or else it is consuming only what Home exports to it. When Foreign is producing, its cost-minimizing energy intensity is:

$$z_j^* = z^*(p_e) = \frac{1-\gamma}{\gamma p_e}, \tag{6}$$

since the wage there is $w^* = 1$. It serves domestic consumers at its unit cost of production:[2]

$$p_j^*(p_e) = a_j^* p_e^{1-\gamma}. \tag{7}$$

We represent the cost minimizing value of z^* and the resulting price as a functions of p_e since the social planner can control them indirectly, by manipulating the price of energy.

When Home is exporting to Foreign, the price is:

$$p_j^x(p_e) = \min\left\{p_j^*(p_e), u^{*\prime}(x_j)\right\}.$$

The first term in the minimization is simply Foreign's cost of supplying itself, which puts an upper bound on what Home can charge for exports to Foreign. The second term is Foreign's marginal utility:

$$u^{*\prime}(c_j^*) = \left(c_j^*/\alpha^*\right)^{-1/\sigma^*}$$

[2] We can derive this expression using our expressions for input requirements by adding together the cost of the required labor and energy:

$$\begin{aligned} a_j^{l*}(z^*(p_e)) + p_e a_j^{e*}(z^*(p_e)) &= \frac{a_j^*}{\nu}\left(\nu\gamma p_e^{1-\gamma} + \nu(1-\gamma)p_e^{1-\gamma}\right) \\ &= a_j^* p_e^{1-\gamma}. \end{aligned}$$

evaluated at $c_j^* = x_j$. Since demand is inelastic, Home will never export so much of manufactured good j that Foreign's marginal utility is pushed below p_j^*. Hence, we end up with a simple limit price:

$$p_j^x(p_e) = p_j^*(p_e).$$

When Home imports from Foreign, the price is determined by Foreign's cost of production. This cost depends on both labor cost and energy cost in a combination determined by the energy intensity z_j^m that the social planner dictates. It also depends on the iceberg cost τ^*. The price of Home's imports of manufactured good j is thus:

$$p_j^m(p_e) = \tau^* a_j^{l*}(z_j^m) + p_e \tau^* a_j^{e*}(z_j^m).$$

We now have expressions for all the prices necessary for evaluating trade balance.

3.2 Trade Balance Constraint

The value of Home's net exports of energy, which can be positive or negative, is $V_e = p_e X_e$. The value of its exports of services, again positive or negative, is simply:

$$V_s = p_s \left(Q_s - C_s\right) = Q_s - C_s = X_s,$$

since the price of services is 1.

The value of Home's manufacturing exports to Foreign is:

$$V_g^{FH}(p_e) = \int_0^1 p_j^x(p_e) x_j dj,$$

while the value of Home's imports from Foreign is:

$$V_g^{HF}(p_e) = \int_0^1 p_j^m(p_e) m_j dj.$$

Hence, the value of Home's net exports of manufactures (positive or negative) is:

$$V_g(p_e) = V_g^{FH}(p_e) - V_g^{HF}(p_e).$$

Trade in the three sectors is connected via trade balance:

$$V_s + V_e + V_g(p_e) = 0,$$

which we can rearrange as:

$$X_s = \int_0^1 p_j^m(p_e) m_j dj - \int_0^1 p_j^x(p_e) x_j dj - p_e X_e. \tag{8}$$

3.3 Energy Constraints

Setting up the social planner's problem will require that we derive expressions for terms that the social planner can't control directly. Employing (7), Foreign consumers demand for good j is:

$$c_j^*(p_e) = \alpha^* \left(a_j^* p_e^{1-\gamma}\right)^{-\sigma^*}, \tag{9}$$

of which

$$y_j^*(p_e) = \max\left\{c_j^*(p_e) - x_j, 0\right\}.$$

is produced in Foreign, with energy intensity $z^*(p_e)$. We can therefore express C_e^{FF} as a function of p_e:

$$C_e^{FF}(p_e) = \int_0^1 a_j^{e*}(z^*(p_e)) y_j^*(p_e) dj.$$

Given the social planner's choices of (x_j), (m_j), and $\left(z_j^m\right)$, we can express energy use by Foreign manufacturers as:

$$M_e^*(p_e) = C_e^{FF}(p_e) + C_e^{HF},$$

which is decreasing in p_e.

Foreign extraction of energy is increasing in the energy price:

$$Q_e^*(p_e) = p_e^{\beta^*/(1-\beta^*)} E^*. \tag{10}$$

Market clearing for energy implies Home's energy exports are a decreasing function of p_e:

$$X_e(p_e) = M_e^*(p_e) - Q_e^*(p_e).$$

Instead of inverting $X_e(p_e)$, as in Markusen (1975), we will simply impose a constraint linking Foreign's production of energy to Home's net exports of energy:[3]

$$Q_e^*(p_e) = -X_e + M_e^*(p_e). \tag{11}$$

[3]Markusen's (1975) strategy was to write the inverse as:

$$p_e = H(X_e),$$

Likewise, Home's total energy production must satisfy:

$$Q_e = X_e + M_e. \tag{12}$$

We will impose each of these constraints separately in the social planner's problem below. Of course, adding them together gives global market clearing for energy:

$$Q_e^*(p_e) + Q_e = M_e^*(p_e) + M_e. \tag{13}$$

3.4 Labor Constraint

Finally, we turn to Home's labor constraint. The labor required to extract Q_e is:

$$L_e = \beta E^{-(1-\beta)/\beta} Q_e^{1/\beta}.$$

The labor required to manufacture goods for the domestic and export market is:

$$L_g = \int_0^1 \left(a_j^l(z_j^y) y_j + \tau a_j^l(z_j^x) x_j \right) dj.$$

Thus, Home's labor constraint is:

$$Q_s + \beta E^{-(1-\beta)/\beta} Q_e^{1/\beta} + \int_0^1 \left(a_j^l(z_j^y) y_j + \tau a_j^l(z_j^x) x_j \right) dj \leq L. \tag{14}$$

3.5 Social Planner's Lagrangian

The social planner chooses Q_s, X_s, Q_e, X_e, p_e, $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^x\}$, and $\{z_j^m\}$ to maximize welfare:

$$Q_s - X_s + \frac{\alpha^{1/\sigma}}{1 - 1/\sigma} \int_0^1 \left((y_j + m_j)^{1-1/\sigma} - 1 \right) dj - \varphi \left(Q_e + Q_e^*(p_e) \right),$$

subject to: Home's labor constraint (14), trade balance (8), Home's energy constraint (12), and Foreign's energy constraint (11).

The first two constraints can be simply substituted into the objective function. The other two could also be substituted in, but it's easier to instead

making the price of energy dependent on the social planner's choice of X_e. That strategy appeared to be more complicated in the present setting, so we have the social planner choose the price of energy directly.

apply a lagrange multiplier λ_e to the third and λ_e^* to the fourth. The resulting Lagrangian is (dropping constants such as L):

$$\begin{aligned}\mathcal{L} &= \frac{\alpha^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j+m_j)^{1-1/\sigma}\,dj - \varphi\left(Q_e + Q_e^*(p_e)\right) \\ &\quad -\beta E^{-(1-\beta)/\beta} Q_e^{1/\beta} - \int_0^1 \left(a_j^l(z_j^y)y_j + \tau a_j^l(z_j^x)x_j\right)dj \\ &\quad -\int_0^1 \tau^*\left(a_j^{l*}(z_j^m) + p_e a_j^{e*}(z_j^m)\right)m_j dj + \int_0^1 p_j^x(p_e)x_j dj + p_e X_e \\ &\quad -\lambda_e\left(\int_0^1 \left(a_j^e(z_j^y)y_j + \tau a_j^e(z_j^x)x_j\right)dj - Q_e + X_e\right) \\ &\quad -\lambda_e^*\left(\int_0^1 \left(a_j^{e*}(z^*(p_e))y_j^*(p_e) + \tau^* a_j^{e*}(z_j^m)m_j\right)dj - Q_e^*(p_e) - X_e\right).\end{aligned}$$

The terms in the Lagrangian are, line by line: (i) Home's welfare less services consumption, (ii) what remains of Home's labor constraint after substituting out services production, (iii) what remains of Home's trade-balance constraint after substituting out services exports, (iv) Home's energy constraint, and (v) Foreign's energy constraint.

We want to maximize this Lagrangian by the optimal choice of Q_e, X_e, p_e, λ_e, λ_e^*, $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^x\}$, and $\{z_j^m\}$.

4 Solution

We now turn to the myriad first order conditions of the Lagrangian. We start with what Costinot, Donaldson, Vogel, and Werning (2015) call the *inner problem*, involving conditions for an individual manufactured good. Later we turn to the outer problem, involving conditions for aggregates.

4.1 The Inner Problem

Solving the inner problem consists of first order conditions with respect to the variables that are specific to some manufactured good j, which are: y_j, x_j, m_j, z_j^y, z_j^x, and z_j^m. These first order conditions, and their implications given Q_e, X_e, p_e, λ_e, and λ_e^*, can be considered one good at time. We therefore

define a Lagrangian for manufactured good j:

$$\begin{aligned}\mathcal{L}_j &= \frac{\alpha^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} \\ &\quad -\frac{a_j}{\nu}\left(y_j\left(z_j^y\right)^{-(1-\gamma)}+\tau x_j\left(z_j^x\right)^{-(1-\gamma)}\right) \\ &\quad -\frac{a_j^*\tau^*}{\nu}\left(\left(z_j^m\right)^{-(1-\gamma)}+p_e\left(z_j^m\right)^{\gamma}\right)m_j+p_j^x(p_e)x_j \\ &\quad -\lambda_e\frac{a_j}{\nu}\left(y_j\left(z_j^y\right)^{\gamma}+\tau x_j\left(z_j^x\right)^{\gamma}\right) \\ &\quad -\lambda_e^*\frac{a_j^*}{\nu}\left(\max\left\{c_j^*(p_e)-x_j,0\right\}\left(z^*(p_e)\right)^{\gamma}+\tau^* m_j\left(z_j^m\right)^{\gamma}\right),\end{aligned}$$

where we have substituted in the expressions for unit input requirements (4) and (5) in Home (as well as their analogs in Foreign). We consider the variables relevant to Home consumers first, then turn to those relevant to Foreign consumers.

4.1.1 Manufactured Goods for Home Consumers

The first order condition for z_j^y implies:

$$z_j^y = z^y = \frac{1-\gamma}{\gamma\lambda_e}.$$

The social planner has all Home manufacturers that serve the domestic market produce with the same energy intensity.

The FOC for y_j is:

$$\left(\left(y_j+m_j\right)/\alpha\right)^{-1/\sigma} \leq \frac{a_j}{\nu}\left(z_j^y\right)^{-(1-\gamma)}+\lambda_e\frac{a_j}{\nu}\left(z_j^y\right)^{\gamma} = \frac{a_j}{\nu}\left(z_j^y\right)^{-(1-\gamma)}\left(1+\lambda_e z_j^y\right).$$

Substituting in the solution for z_j^y and applying (2) this FOC reduces to:

$$\left(\left(y_j+m_j\right)/\alpha\right)^{-1/\sigma} \leq a_j\lambda_e^{1-\gamma},$$

with equality if $y_j > 0$. If Home produces good j for the domestic market the social planner equates the marginal utility of consumption of good j to the shadow cost of production. If the FOC for y_j holds with strict inequality then $y_j = 0$ and Home imports the good.

The first order condition for z_j^m implies:

$$z_j^m = z^m = \frac{1-\gamma}{\gamma\left(p_e + \lambda_e^*\right)}.$$

All Foreign manufacturers serving consumers in Home also produce with the same energy intensity.

The FOC for m_j is:

$$\left(\left(y_j + m_j\right)/\alpha\right)^{-1/\sigma} \leq \frac{a_j^* \tau^*}{\nu}\left(z_j^m\right)^{-(1-\gamma)}\left(1 + \left(p_e + \lambda_e^*\right) z_j^m\right).$$

Substituting in the solution for z_j^m this FOC reduces to:

$$\left(\left(y_j + m_j\right)/\alpha\right)^{-1/\sigma} \leq a_j^* \tau^* \left(p_e + \lambda_e^*\right)^{1-\gamma},$$

with equality if $m_j > 0$.

To distill these results about consumption in Home, define a cutoff good $\bar{j}_m$ satisfying:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\left(\frac{\lambda_e}{p_e + \lambda_e^*}\right)^{1-\gamma}. \tag{15}$$

1. For any good $j < \bar{j}_m$, Home's comparative advantage leads it to produce for itself:
$$y_j = \alpha\left(a_j \lambda_e^{1-\gamma}\right)^{-\sigma},$$
while importing nothing, $m_j = 0$.

2. For any good $j > \bar{j}_m$, Foreign's comparative disadvantage leads Home to import:
$$m_j = \alpha\left(a_j^* \tau^* \left(p_e + \lambda_e^*\right)^{1-\gamma}\right)^{-\sigma},$$
while producing nothing for itself, $y_j = 0$.

Note that the iceberg cost of moving goods from Foreign to Home shrinks the range of goods that Home imports. For any good that Home does import, this iceberg cost also reduces the quantity imported. Because the relative productivity curve (3) is strictly monotonic, we can ignore the pattern of trade for the measure-zero cutoff good $j = \bar{j}_m$ itself.

4.1.2 Manufactured Goods for Foreign Consumers

We now turn to Home's exports to Foreign. The first order condition for z_j^x implies:

$$z_j^x = z^x = \frac{1-\gamma}{\gamma\lambda_e}.$$

All Home manufacturers serving the export market produce with the same energy intensity, which is also the same as their energy intensity z^y for serving the domestic market.

The derivative with respect to x_j is:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\frac{a_j}{\nu}\tau\left(z_j^x\right)^{-(1-\gamma)} - \lambda_e\frac{a_j}{\nu}\tau\left(z_j^x\right)^{\gamma} + p_j^x(p_e) + \lambda_e^*\frac{a_j^*}{\nu}\left(z^*(p_e)\right)^{\gamma}.$$

The terms on the right-hand side are: (i) the marginal value of Home's labor used to produce more of x_j, (ii) the value of the energy used, (iii) the value of the revenue obtained, and (iv) the value to Home of Foreign reducing its energy use (by importing instead of producing). Substituting in the solution for z_j^x, $z^*(p_e)$, and $p_j^x(p_e)$ and then combining the first two terms, this derivative simplifies to:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -a_j\tau\lambda_e^{1-\gamma} + a_j^* p_e^{1-\gamma} + a_j^*\lambda_e^*\left(1-\gamma\right)p_e^{-\gamma}. \tag{16}$$

The last term plays a novel role in the solution. It represents the value that the social planner places on the energy Foreign would use to produce an additional unit of good j if it didn't import the good from Home.[4] We'll call it the *Foreign emissions cost*, and will treat it as a positive value (as it will be for $\lambda_e^* > 0$).

The solution for x_j depends on the sign of (16). If

$$a_j\tau\lambda_e^{1-\gamma} < a_j^* p_e^{1-\gamma} + a_j^*\lambda_e^*\left(1-\gamma\right)p_e^{-\gamma} \tag{17}$$

[4]Foreign's energy intensity of production satisfies:

$$(z^*)^{\gamma} = \nu\left(1-\gamma\right)p_e^{-\gamma}.$$

Therefore, its use of energy per unit of good j produced is:

$$a_j^{e*}\left(z^*\right) = a_j^*\left(1-\gamma\right)p_e^{-\gamma}.$$

The social planner places a shadow value of λ_e^* on each unit of energy used by Foreign.

then $x_j > 0$. In that case x_j is determined by the marginal utility of Foreign if they were to produce the good for themselves:

$$(x_j/\alpha^*)^{-1/\sigma^*} = a_j^* p_e^{1-\gamma}.$$

If inequality (17) is reversed then $x_j = 0$ and y_j^* solves:

$$\left(y_j^*/\alpha^*\right)^{-1/\sigma^*} = a_j^* p_e^{1-\gamma}.$$

In this case Foreign produces all of the good j that it consumes.

To distill these results about consumption in Foreign, define a cutoff good $\bar{j}_x$ satisfying:

$$F(\bar{j}_x) = \frac{\tau \left(\frac{\lambda_e}{p_e}\right)^{1-\gamma}}{1 + (1-\gamma)\frac{\lambda_e^*}{p_e}}. \tag{18}$$

1. For any good $j < \bar{j}_x$, Home's comparative advantage leads it to export:

   $$x_j = \alpha^* \left(a_j^* p_e^{1-\gamma}\right)^{-\sigma^*},$$

   while Foreign produces nothing for itself, $y_j^* = 0$.

2. For any good $j > \bar{j}_x$, Foreign's comparative advantage leads it to produce for itself:

   $$y_j^* = \alpha^* \left(a_j^* p_e^{1-\gamma}\right)^{-\sigma^*},$$

   while demanding no exports from Home, $x_j = 0$.

Note that the iceberg cost τ has no direct effect on Home's exports of good j, in contrast to the direct effect of τ^* on Home's imports. That's because Home determines the quantity of good j to export based on Foreign's marginal utility evaluated at the cost to Foreign of producing good j itself. A larger iceberg cost does, however, narrow the range of goods that Home exports. Again, we can ignore the pattern of trade for the measure-zero cutoff good $j = \bar{j}_x$ itself.

4.1.3 Novel Properties of the Solution

Before turning to the outer problem, it is worth taking a moment to dwell on some unusual properties of the solution to the inner problem. The first concerns how the optimal policy alters the cost share of energy in the manufacturing sector even though the underlying production technologies are Cobb Douglas. The second concerns the possibility that Home will simultaneously import and export the same manufactured goods. Both properties provide an insight into the logic of an optimal unilateral carbon policy.

Energy Shares Consider the share of energy in production costs for each of the four production processes, which vary with the location of production and the destination of goods. For producers in Foreign, the cost of labor is 1 and the cost of energy is p_e. For producers in Home, the cost of labor is 1 and the shadow price of energy is λ_e.

If producers were choosing labor and energy to minimize cost, the energy share would be $1-\gamma$ since that's the Cobb-Douglas exponent on energy input. Producers in Foreign serving the Foreign market are simply minimizing cost. Thus:

$$\frac{p_e a_j^{e*}(z^*)}{p_e a_j^{e*}(z^*) + a_j^{l*}(z^*)} = \frac{p_e z^*}{p_e z^* + 1} = \frac{(1-\gamma)/\gamma}{(1-\gamma)/\gamma + 1} = 1 - \gamma.$$

Likewise, producers in Home, whether producing for the domestic or export market, have energy cost share:

$$\frac{\lambda_e a_j^e(z^y)}{\lambda_e a_j^e(z^y) + a_j^l(z^y)} = \frac{\lambda_e a_j^e(z^x)}{\lambda_e a_j^e(z^x) + a_j^l(z^x)} = \frac{\lambda_e z^x}{\lambda_e z^x + 1} = 1 - \gamma.$$

Foreign producers serving Home's import market, however, will choose an energy share of:

$$\frac{p_e a_j^{e*}(z^m)}{p_e a_j^{e*}(z^m) + a_j^{l*}(z^m)} = \frac{p_e z^m}{p_e z^m + 1} = \frac{\frac{p_e}{\lambda_e}(1-\gamma)/\gamma}{\frac{p_e}{\lambda_e}(1-\gamma)/\gamma + 1} = \frac{1-\gamma}{1-\gamma+\gamma\frac{\lambda_e}{p_e}} < 1 - \gamma.$$

The lower energy share result reflects Home dictating a low energy intensity of production for its imports from Foreign. That energy intensity is below the intensity z^* that Foreign would choose based on the price of energy there.

Crosshauling Comparing the results for consumption in Home and in Foreign, a novel result emerges base on the inequality:

$$(p_e + \lambda_e^*)^{1-\gamma} < p_e^{1-\gamma} + (1-\gamma)\, p_e^{-\gamma} \lambda_e^*,$$

which holds for $\lambda_e^* > 0$ (which we assume here). The right-hand side is the first-order expansion of the left-hand side, around $\lambda_e^* = 0$. The inequality then follows from the concavity (in λ_e^*) of the left-hand side. Rearranging the inequality yields:

$$\frac{1}{p_e^{1-\gamma}\left(1 + (1-\gamma)\,\lambda_e^*/p_e\right)} < \frac{1}{(p_e + \lambda_e^*)^{1-\gamma}}.$$

Multiplying each side by $\lambda_e^{1-\gamma}$, we obtain:

$$\frac{\left(\frac{\lambda_e}{p_e}\right)^{1-\gamma}}{1 + (1-\gamma)\,\frac{\lambda_e^*}{p_e}} < \left(\frac{\lambda_e}{p_e + \lambda_e^*}\right)^{1-\gamma}. \tag{19}$$

If iceberg cost are close enough to 1, then (19) will continue to hold even if the left-hand side is mulitiplied by the product of the iceberg costs, $\tau\tau^*$. In this case, we can substitute from (18) and (15) to get:

$$F(\bar{j}_x) < F(\bar{j}_m).$$

Since F is monotonically decreasing, it follows that $\bar{j}_m < \bar{j}_x$. Under these conditions of low trade costs, there is a range of goods that Home imports but also exports. The optimal policy can imply crosshauling.

The economic rationale is that Home controls the energy intensity of manufacturing not only for all production in Home, but also for production in Foreign that Home imports. In contrast, goods produced in Foreign, for consumption there, escape Home's climate policy. Increased trade gives Home more control over the use of energy, helping it to lower global emissions. In particular, if trade costs are low enough, Home is willing to supply a range of goods to Foreign at a price below the shadow price of those goods to Home consumers. Consider good j at which $a_j = a_j^* = a$. With $\lambda_e^* = 0$, Home would be indifferent between exporting this good or having Foreign produce it for itself. But, with $\lambda_e^* > 0$ global energy use is reduced if the good is produced in Home and exported to Foreign. The energy requirement

is $a\left(1-\gamma\right)\left(p_e+\lambda_e^*\right)^{-\gamma}$ which is less than if Foreign produced for itself, with energy requirement $a\left(1-\gamma\right)\left(p_e\right)^{-\gamma}$ per unit produced.

Trade costs mute this effect. With high enough trade costs, so that

$$\tau\tau^*\frac{\left(\frac{\lambda_e}{p_e}\right)^{1-\gamma}}{1+\left(1-\gamma\right)\frac{\lambda_e^*}{p_e}}>\left(\frac{\lambda_e}{p_e+\lambda_e^*}\right)^{1-\gamma},$$

the inherent inefficiency of crosshauling overcomes its advantages in reducing global emissions.

4.2 The Outer Problem

We now turn to the first order conditions with respect to Q_e, X_e, p_e, λ_e, and λ_e^*. For this purpose, we can rewrite the Lagrangian in terms of aggregate magnitudes:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{\sigma\alpha^{1/\sigma}}{\sigma-1}C_g^{(\sigma-1)/\sigma}-\varphi\left(Q_e+Q_e^*(p_e)\right)\\ & -\beta E^{-(1-\beta)/\beta}Q_e^{1/\beta}-L_g\\ & +V_g(p_e)+p_eX_e\\ & -\lambda_e\left(M_e-Q_e+X_e\right)\\ & -\lambda_e^*\left(M_e^*(p_e)-Q_e^*(p_e)-X_e\right).\end{aligned}$$

We now turn to the first order conditions one by one.

4.2.1 Home Energy Exports

The first order condition with respect to X_e is:

$$\frac{\partial\mathcal{L}}{\partial X_e}=p_e-\lambda_e+\lambda_e^*=0,$$

which implies:

$$\lambda_e=p_e+\lambda_e^*. \tag{20}$$

Thus, the optimal energy export policy requires that the shadow value of energy in Home, λ_e, is the same as the shadow value of energy faced by producers in Foreign that serve customers in Home, $p_e+\lambda_e^*$.

Combined with the results from the inner problem, an implication is that optimal energy intensity is the same for all manufacturers producing for consumers in Home (whether the manufacturers are in Home or Foreign) and for all manufacturers in Home (whether producing for consumers in Home or Foreign):

$$z^y = z^x = z^m = \frac{1-\gamma}{\gamma\lambda_e},$$

where from the inner problem we established that these energy intensities do not vary by good. The exception to equalization is z^*, the common energy intensity for any goods manufactured in Foreign for consumption there.

Another implication is that the extensive margin in Home, of what goods are produced locally and what goods are imported, are the same as in the Free-Market equilibrium. In particular, the condition for the import cutoff (15) simplifies to:

$$F(\bar{j}_m) = 1/\tau^*,$$

which is invariant to the optimal climate policy.

Yet another implication is that under the optimal policy Home actually expands the range of goods it exports (given our presumption of $\lambda_e^* > 0$). Substituting (20) into the right-hand side of the crosshauling inequality (19) and multiplying both sides by the iceberg cost τ:

$$\frac{\tau\left(\frac{\lambda_e}{p_e}\right)^{1-\gamma}}{1+(1-\gamma)\frac{\lambda_e^*}{p_e}} < \tau. \tag{21}$$

Combining (21) and (18) we have:

$$F(\bar{j}_x) < \tau.$$

The export threshold $\bar{j}_x$ under the optimal policy therefore exceeds its Free-Market level.

4.2.2 Home Energy Extraction

The first order condition with respect to Q_e is:

$$\frac{\partial\mathcal{L}}{\partial Q_e} = -\varphi - E^{-(1-\beta)/\beta}Q_e^{(1-\beta)/\beta} + \lambda_e = 0,$$

which implies:

$$\left(\frac{Q_e}{E}\right)^{(1-\beta)/\beta} = \lambda_e - \varphi. \tag{22}$$

To interpret (22), note that Home's labor requirement for energy extraction is:

$$L_e = \beta E^{-(1-\beta)/\beta} Q_e^{1/\beta}.$$

The marginal labor requirement is thus:

$$\frac{\partial L_e}{\partial Q_e} = E^{-(1-\beta)/\beta} Q_e^{(1/\beta)-1} = \left(\frac{Q_e}{E}\right)^{(1-\beta)/\beta},$$

which is the left-hand side of (22). We can interpret this equation as Home's optimal extraction equation. Since we already have Foreign's extraction as a function of p_e, we've thus determined global energy supply given λ_e and p_e.

4.2.3 Energy Price

Two new expressions are useful in deriving the first order condition with respect to the energy price. The first is the *value* of Foreign's net exports of manufactured goods:

$$V_g^*(p_e) = \tau^* \int_0^1 \left(a_j^{l*}(z_j^m) + p_e a_j^{e*}(z_j^m)\right) m_j dj - \int_0^1 p_j^x(p_e) x_j dj.$$

The second is Foreign's net exports of energy, which is increasing in the price of energy:

$$X_e^*(p_e) = Q_e^*(p_e) - \int_0^1 \left(a_j^{e*}(z^*(p_e)) y_j^*(p_e) + \tau^* a_j^{e*}(z_j^m) m_j\right) dj.$$

Either expression may be positive or negative.

The first order condition with respect to p_e is thus:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\varphi \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial V_g^*}{\partial p_e} - X_e^* + \lambda_e^* \frac{\partial X_e^*}{\partial p_e} = 0,$$

or:

$$\lambda_e^* \frac{\partial X_e^*}{\partial p_e} = \varphi \frac{\partial Q_e^*}{\partial p_e} + X_e^* + \frac{\partial V_g^*}{\partial p_e}.$$

We can rewrite this condition as:

$$\lambda_e^* = \varphi \frac{\partial Q_e^*/\partial p_e}{\partial X_e^*/\partial p_e} + \frac{X_e^* + \partial V_g^*/\partial p_e}{\partial X_e^*/\partial p_e}. \tag{23}$$

The first term on the right-hand side is the optimal policy with respect to the global externality, which depends on the damage parameter φ. The second term is Home's optimal manipulation of the terms of trade for energy, which arises even if $\varphi = 0$.

To get more insight into this optimality condition, we consider the relevant derivatives of these integrals, with respect to the energy price. In particular, for Foreign's net exports of energy:

$$\partial X_e^*(p_e)/\partial p_e = \partial Q_e^*(p_e)/\partial p_e - \partial M_e^*(p_e)/\partial p_e > 0.$$

The first term on the left-hand side is simply:

$$\frac{\partial Q_e^*(p_e)}{\partial p_e} = \frac{\beta^*}{1-\beta^*}\frac{Q_e^*}{p_e}, \tag{24}$$

which is strictly positive for $\beta^* > 0$ (and zero if $\beta^* = 0$). The second term is:

$$\frac{\partial M_e^*(p_e)}{\partial p_e} = -\left(\gamma + \sigma^*(1-\gamma)\right)\frac{C_e^{FF}}{p_e}, \tag{25}$$

which is strictly positive since $\sigma^* > 0$.

We treat C_e^{HF} as a constant when differentiating $M_e^*(p_e)$, since it is determined by Home's choice of $\{z_j^m\}$ and $\{m_j\}$ from the inner problem. For the same reason, in the derivative $\partial V_g^*(p_e)/\partial p_e$, we take Foreign's labor use as constant. Thus, we have:

$$\frac{\partial V_g^*(p_e)}{\partial p_e} = C_e^{HF} - (1-\gamma)\frac{V_g^{*FH}}{p_e}. \tag{26}$$

4.2.4 Energy Constraints

The first order condition with respect to λ_e yields Home's energy constraint:

$$Q_e = M_e + X_e.$$

The first order condition with respect to λ_e^* yields Foreign's energy constraint:

$$X_e^*(p_e) = X_e. \tag{27}$$

The price of energy p_e must clear the global market for energy.

4.3 Assessment

We are now in a position to compute the optimal policy. The inner problem gives V_g, M_e, and M_e^* as functions of p_e, λ_e, and λ_e^*. Equations (10) and (22), give Q_e^* and Q_e as functions of p_e and λ_e, respectively. Together with (20), (23), and (12) from the outer problem we can solve jointly for p_e, λ_e, λ_e^*. Along the way we've also solved for Q_e, X_e, Q_s, and X_s. Before computing the solution, we reinterpret the optimality conditions in terms of a set of taxes.

5 Competitive Equilibrium with Taxes

We consider in turn the various taxes and subsidies that bring about the optimal policy.

5.1 A Production Tax with Border Adjustments

Starting with (20), we can interpret λ_e^* as a specific tax on the use of energy by manufacturing producers. This production tax is at rate:

$$t_p = \lambda_e^* = \lambda_e - p_e, \tag{28}$$

raising the cost of energy to producers in Home from p_e to λ_e. A border tax on the energy content of imported manufactured goods, also at rate:

$$t_b = t_p,$$

effectively raises the cost of energy to Foreign producers when serving Home consumers from p_e to λ_e as well. As a result the set of manufactured goods that Home imports under the optimal policy is the same as it would be with no taxes. The derivation of y_j and m_j in the inner problem confirms this result.

In this parameterization of taxes we are treating p_e as the global energy price, which applies whether Home imports or exports energy. In either case the production tax t_p is applied to users of energy in Home, raising the price they pay to $p_e + t_p$. The border adjustment of $t_b = t_p$ is applied to energy embodied in manufactured imports, not to energy imports themselves.

5.2 Export Taxes and Subsidies

Home's exporters are also subject to the production tax, which raises the cost of their energy input from p_e to $\lambda_e = p_e + t_p$. This production tax is not removed at the border for exporters because it is optimal for Home to give exporters an incentive to reduce energy use. Yet an exporter may receive a subsidy per unit exported. We turn now to that situation. Other exporters will be taxed.

The marginal Home producer for the export market has relative productivity (the lowest among Home's exporters) of:

$$\frac{a_j^*}{a_j} = \frac{\tau \left(\frac{\lambda_e}{p_e}\right)^{1-\gamma}}{1 + (1-\gamma)\frac{t_p}{p_e}},$$

where we have substituted t_p in place of λ_e^* from the derivation in the inner problem. Its unit cost is therefore:

$$a_j \tau \lambda_e^{1-\gamma} = \left(1 + (1-\gamma)\frac{t_p}{p_e}\right) a_j^* p_e^{1-\gamma}.$$

Under the optimal policy, this producer must be able to sell to Foreign at price:

$$p_j^* = a_j^* p_e^{1-\gamma}.$$

Thus, it requires an export subsidy that reduces its unit costs by the factor $1 + (1-\gamma)\, t_p/p_e$. Since $1-\gamma$ is the share of energy costs in production costs, this subsidy is just enough to offset the overall cost disadvantage of the marginal Home exporter due to the production tax. But, as noted above, the export subsidy is not a rebate of the production tax on energy, but rather a per-unit subsidy. It does not change the producer's optimal energy intensity.

The level of this subsidy declines as we move to goods where Home's comparative advantage is stronger. It hits zero on the good j for which

$$\frac{a_j^*}{a_j} = \tau \left(\frac{\lambda_e}{p_e}\right)^{1-\gamma},$$

as the Home manufacturer of this good needs no subsidy to compete with Foreign. The subsidy then switches over to an export tax for goods in which Home's comparative advantage is even stronger.

5.3 An Extraction Tax

Consider now the interpretation of (22). Recall that the left-hand side is the marginal labor requirement for extracting an additional unit of energy. In a market setting, the extractor would equate this magnitude to its net revenue from selling an extra unit of energy. Since extractors would receive a price p_e in a competitive equilibrium with no taxes, Home can implement (22) with an extraction tax t_e that equates $p_e - t_e$ to the right-hand side, $\lambda_e - \varphi$. When combined with (28) we have:

$$\begin{aligned} t_e &= \varphi - \lambda_e + p_e \\ &= \varphi - \lambda_e^* \\ &= \varphi - t_p. \end{aligned}$$

More intuitively:

$$t_e + t_p = \varphi.$$

The combined extraction and production tax is at the Pigouvian rate φ, equal to marginal damages to Home from global emissions.[5] While we have not yet determined the split between t_e and t_p, we have determined the overall wedge between the price paid for energy by Home manufacturers and the price received for energy by Home extractors.[6]

5.4 Optimal Level of the Production Tax

While we've determined that the production tax is at rate $t_p = \lambda_e^*$ and that $t_p + t_e = \varphi$, we have yet to determine the level of the production tax itself.

[5] Markusen (1975) obtains the same result, disguised by two differences in his parameterization. First, Markusen treats taxes as ad valorem rather than specific as we do here. Second, what Markusen refers to as the tax on production, converted to a specific rate t_τ, is what we think of as the sum of the extraction and production tax, $t_p + t_e$. In either case, $t_\tau = t_p + t_e$ is the wedge between the price paid by the energy user in Home and the price received by the energy extractor in Home. In either case this wedge is equal to marginal damages, φ.

[6] So far, we've treated the first order condition (22) as holding with equality. It's possible that even at $Q_e = 0$ the marginal value of increasing Q_e is negative. In that case the solution would be $Q_e = 0$ with $t_e = p_e$. Home would choose to shut down its extraction sector. We have yet to find a set of parameters in which this outcome is the optimal policy.

That's where we can employ equation (23), substituting in $t_p = \lambda_e^*$ to get:

$$t_p = \varphi \frac{\partial Q_e^*/\partial p_e}{\partial X_e^*/\partial p_e} - \frac{X_e + \partial V_g/\partial p_e}{\partial X_e^*/\partial p_e}. \tag{29}$$

Taken together with the border adjustment $t_b = t_p$ and the extraction tax $t_e = \varphi - t_p$, equation (29) is strikingly similar to Markusen (1975).[7] The basic reasoning is most familiar in the case of no global externality.

5.4.1 Interpretation with No Global Externality

With no global externality we have $\varphi = t_e + t_p = 0$ and

$$t_p = -\frac{X_e + \partial V_g/\partial p_e}{\partial X_e^*/\partial p_e}.$$

The denominator is always positive.

Suppose the numerator is negative as typically occurs if Home is a net importer of energy and manufactured goods. The optimal policy has Home impose a positive production tax $t_p > 0$, a border adjustment of $t_b = t_p > 0$, and a negative extraction tax of $t_e = -t_p < 0$. The subsidy to extraction, together with the tax on energy used by Home's manufacturers, lowers the price of energy to extractors and cost of manufacturing in Foreign, improving Home's terms of trade. These parts of the optimal policy are equivalent to Home imposing a tariff on energy imports that raises the energy price to Home's extractors and manufacturers relative to Foreign's. The border adjustment makes sure that the energy embodied in Home's imports of manufactured goods does not avoid this tariff.

If the numerator is positive, for example if Home is a net exporter of both energy and manufactured goods, Home will impose a negative production tax $t_p < 0$, a border adjustment of $t_b = t_p < 0$, and a positive extraction tax of $t_e = -t_p > 0$. The positive extraction tax, together with the subsidy to energy use by Home's manufacturers, raises the price of energy in Foreign relative to Home, improving Home's terms of trade. The border adjustment

[7] Without trade in manufactures, Markusen's formula doesn't have the term $\partial V_g/\partial p_e$. Furthermore his tax rates are ad valorem rather than specific. Finally, he names the taxes differently. He refers to wedge between the price paid by producers and users of energy in Home (which in our notation is $t_p + t_e$) as the "production" tax. He refers to the wedge between the price paid by users of energy in Home and Foreign (which in our notation is t_p) as the "trade" tax.

subsidizes the energy content of Foreign's exports of manufactured goods, eliminating any distortions to Home's import margin.[8]

5.4.2 Interpretation with No Terms-of-Trade Manipulation

Now consider marginal damages from the global externality of $\varphi > 0$. We have already established that Home will impose a Pigouvian tax of $t_e + t_p = \varphi$. The question is how it is split between a production tax and an extraction tax, where the production tax is always imposed with an equal border adjustment $t_b = t_p$.

We'll focus on the first term of (29), ignoring the second for now. Foreign energy extraction is clearly non-decreasing in the energy price $\partial Q_e^*/\partial p_e \geq 0$, and since $X_e^* = Q_e^* - M_e^*$ and $\partial M_e^*/\partial p_e < 0$, we have:

$$0 \leq \frac{\partial Q_e^*/\partial p_e}{\partial X_e^*/\partial p_e} < 1.$$

Thus, the externality on its own will lead Home to impose positive production and extraction taxes satisfying:

$$0 \leq \frac{t_p}{t_e + t_p} = \frac{\partial Q_e^*/\partial p_e}{\partial X_e^*/\partial p_e} < 1.$$

If extraction in Foreign is very responsive to the price of energy, Home's optimal Pigouvian tax will be weighted towards a production tax, with only a small extraction tax. The reason is that the production tax lowers the energy price, reducing Foreign's extraction of energy. This policy requires that Home impose a substantial border tax on the energy content of imported manufactured goods.

On the other hand, if Foreign extraction is not very responsive to the energy price, Home's Pigouvian tax will be weigted towards an extraction tax. The extraction tax raises the energy price, reducing energy use in Foreign. In the extreme of a fixed extraction level in Foreign, i.e. with $\beta^* = 0$, Home

[8] These results are standard in the optimal tariff literature. To see that most clearly we can shut down trade in manufactured goods by imposing infinite iceberg costs, τ, $\tau^* \to \infty$. As a result $\partial V_g/\partial p_e = 0$, while there is still trade in services and energy. In this case we get the classic result that the ad valorem trade tax t_p/p_e (together with $t_b/p_e = t_p/p_e$ and $t_e/p_e = -t_p/p_e$) is equal to the inverse net energy supply elasticity of Foreign, $(X_e^*/p_e) / (\partial X_e^*/\partial p_e)$. (Note that we can impose $X_e^* = -X_e$ because the energy market clears.)

would impose an extraction tax of $t_e = \varphi$ with no production tax or border adjustment.

5.5 Assessment

Let's summarize what we've achieved by interpreting the optimal policy in terms of a set of taxes. To be concrete, suppose Home is an energy importer, both directly and embodied in manufactured goods. Its optimal policy will be a positive production tax $t_p > 0$ on manufacturers use of energy and a positive extraction tax $t_e > 0$ on extraction of energy in Home, with the sum equal to marginal damages, φ. The production tax is combined with full border adjustments on the energy content of Home's imports of manufactured goods, $t_b = t_p$. Home's energy taxes are not rebated to manufacturing exporters. Rather, Home imposes a set of subsidies and taxes per unit exported, without regard to the energy content. Exports of manufactures are subsidized for goods in which Home's comparative advantage is weak and taxed for goods in which Home's comparative advantage is strong. The subsidy can potentially be large enough that Home exports and imports the same good. That's what we can say qualitatively. We now turn to the quantitative solution.

6 Computation

The basic computational strategy is to: (i) guess tax rates, (ii) solve for the price p_e that equates world supply and demand for energy given tax rates, and (iii) given the equilibrium magnitudes, use the optimal tax formulas to update the tax rates. We iterate between steps (ii) and (iii) until the tax rates converge.

6.1 Integration

Steps (ii) and (iii) require that we solve some of the integrals that appear in the Lagrangian. We begin with some preliminary steps in that direction.

6.1.1 Energy Intensity

We can make some progress by using the results on optimal energy intensity from the inner problem. Employing (2), as well as our optimal tax results,

we have:

$$\left(z^y\right)^{-(1-\gamma)} = \left(z_j^x\right)^{-(1-\gamma)} = \left(z^m\right)^{-(1-\gamma)} = \nu\gamma\left(p_e + t_p\right)^{1-\gamma},$$

$$\left(z^y\right)^{\gamma} = \left(z^x\right)^{\gamma} = \left(z^m\right)^{\gamma} = \nu\left(1-\gamma\right)\left(p_e + t_p\right)^{-\gamma},$$

and:

$$\left(z^*(p_e)\right)^{\gamma} = \nu\left(1-\gamma\right)p_e^{-\gamma}.$$

Substituting in these results simplifies the following integrals that are key to computing global energy demand and the optimal tax rate:

1. Energy demand by Home:

$$\begin{aligned} M_e &= \int_0^1 \frac{a_j}{\nu}\left(y_j\left(z_j^y\right)^{\gamma} + \tau x_j\left(z_j^x\right)^{\gamma}\right)dj \\ &= \left(1-\gamma\right)\left(p_e + t_p\right)^{-\gamma}\int_0^1 a_j\left(y_j + \tau x_j\right)dj, \end{aligned} \tag{30}$$

2. Energy demand by Foreign:

$$\begin{aligned} M_e^* &= \int_0^1 \frac{a_j^*}{\nu}\left(\max\left\{c_j^*(p_e) - x_j, 0\right\}\left(z^*(p_e)\right)^{\gamma} + \tau^* m_j\left(z_j^m\right)^{\gamma}\right)dj \\ &= \left(1-\gamma\right)\left(p_e^{-\gamma}\int_0^1 a_j^* y_j^* dj + \left(p_e + t_p\right)^{-\gamma}\tau^*\int_0^1 a_j^* m_j dj\right), \end{aligned} \tag{31}$$

3. The value of Home's net exports of manufactured goods:

$$\begin{aligned} V_g &= p_e^{1-\gamma}\int_0^1 a_j^* x_j dj - \int_0^1 \frac{a_j^*\tau^*}{\nu}\left(\left(z_j^m\right)^{-(1-\gamma)} + p_e\left(z_j^m\right)^{\gamma}\right)m_j dj \\ &= p_e^{1-\gamma}\int_0^1 a_j^* x_j dj - \left(p_e + \gamma t_p\right)\left(p_e + t_p\right)^{-\gamma}\tau^*\int_0^1 a_j^* m_j dj. \end{aligned} \tag{32}$$

6.1.2 Efficiency Thresholds

To simplify further, we employ convenient functional forms for bundle requirements:

$$a_j = \frac{1}{A_j} = \left(\frac{j}{A}\right)^{1/\theta},$$

$$a_j^* = \frac{1}{A_j^*} = \left(\frac{1-j}{A^*}\right)^{1/\theta},$$

where A and A^* determine absolute advantage in either country, and θ determines (inversely) the scope of comparative advantage. Taking the ratio, we get Home's relative productivity:

$$F(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta},$$

where $F(j)$ is continuous and strictly decreasing as specified in (3).

These functional forms allow us to solve for the two key thresholds separating goods. The import threshold is:

$$\bar{j}_m = \frac{A}{A + (\tau^*)^{-\theta} A^*}. \tag{33}$$

The export threshold is:

$$\bar{j}_x = \frac{\tau^{-\theta}\left(1 + (1-\gamma)\, t_p/p_e\right)^{\theta} A}{\tau^{-\theta}\left(1 + (1-\gamma)\, t_p/p_e\right)^{\theta} A + \left(1 + t_p/p_e\right)^{(1-\gamma)\theta} A^*}. \tag{34}$$

6.1.3 Integration Results

We can now solve, at least partially, each of the four integrals above.

For (30) we have:

$$M_e = (1-\gamma)\left(p_e + t_p\right)^{-\gamma}\left[\int_0^1 a_j y_j dj + \tau \int_0^1 a_j x_j dj\right].$$

The first integral is:

$$\begin{aligned}
\int_0^1 a_j y_j dj &= \int_0^{\bar{j}_m} a_j \alpha \left(a_j \left(p_e + t_p\right)^{1-\gamma}\right)^{-\sigma} dj \\
&= \alpha \left(p_e + t_p\right)^{-\sigma(1-\gamma)} \int_0^{\bar{j}_m} \left(\frac{j}{A}\right)^{(1-\sigma)/\theta} dj \\
&= \alpha \left(p_e + t_p\right)^{-\sigma(1-\gamma)} A^{-(1-\sigma)/\theta} \frac{\theta}{\theta + 1 - \sigma} \left(\bar{j}_m\right)^{(\theta+1-\sigma)/\theta}.
\end{aligned}$$

The second integral is:

$$\begin{aligned}\int_0^1 a_j x_j dj &= \int_0^{\bar{j}_x} a_j \alpha^* \left(a_j^* p_e^{1-\gamma}\right)^{-\sigma^*} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \int_0^{\bar{j}_x} \left(\frac{j}{A}\right)^{1/\theta} \left(\frac{1-j}{A^*}\right)^{-\sigma^*/\theta} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \frac{(A^*)^{\sigma^*/\theta}}{A^{1/\theta}} \int_0^{\bar{j}_x} j^{1/\theta} (1-j)^{-\sigma^*/\theta} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \frac{(A^*)^{\sigma^*/\theta}}{A^{1/\theta}} B\left(\bar{j}_x, \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right),\end{aligned}$$

where and $B(x,a,b)$ is the incomplete beta function.[9] Combining these results:

$$\begin{aligned}M_e &= \alpha \frac{(1-\gamma)\,\theta A^{-(1-\sigma)/\theta}}{\theta+1-\sigma} (p_e+t_p)^{-(\gamma+(1-\gamma)\sigma)} \left(\bar{j}_m\right)^{(\theta+1-\sigma)/\theta} \\ &\quad + \tau\alpha^* \frac{(1-\gamma)\,(A^*)^{\sigma^*/\theta}}{A^{1/\theta}} (p_e+t_p)^{-\gamma} p_e^{-\sigma^*(1-\gamma)} B\left(\bar{j}_x, \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right),\end{aligned}$$

where $\bar{j}_m$ is given by (33) and $\bar{j}_x$ by (34).

For (31) we have:

$$M_e^* = (1-\gamma)\, p_e^{-\gamma} \int_0^1 a_j^* y_j^* dj + (1-\gamma)\,(p_e+t_p)^{-\gamma}\,\tau^* \int_0^1 a_j^* m_j dj.$$

The first integral is:

$$\begin{aligned}\int_0^1 a_j^* y_j^* dj &= \int_{\bar{j}_x}^1 a_j^* \alpha^* \left(a_j^* p_e^{1-\gamma}\right)^{-\sigma^*} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \int_{\bar{j}_x}^1 \left(\frac{1-j}{A^*}\right)^{(1-\sigma^*)/\theta} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \frac{\theta\,(A^*)^{-(1-\sigma^*)/\theta}}{\theta+1-\sigma^*} (1-\bar{j}_x)^{(\theta+1-\sigma^*)/\theta}.\end{aligned}$$

[9] The incomplete beta function is:

$$B(x,a,b) = \int_0^x i^{a-1}(1-i)^{b-1} di,$$

for $0 \le x \le 1$, $a > 0$, and $b > 0$.Setting $x = 1$ gives the beta function itself, $B(a,b)$.

The second integral becomes:

$$\begin{aligned}\int_0^1 a_j^* m_j dj &= \int_{\bar{j}_m}^1 a_j^* \alpha \left(a_j^* \tau^* \left(p_e + t_p\right)^{1-\gamma}\right)^{-\sigma} dj \\ &= \alpha \left(\tau^*\right)^{-\sigma} \left(p_e + t_p\right)^{-\sigma(1-\gamma)} \int_{\bar{j}_m}^1 \left(\frac{1-j}{A^*}\right)^{(1-\sigma)/\theta} dj \\ &= \alpha \left(\tau^*\right)^{-\sigma} \left(p_e + t_p\right)^{-\sigma(1-\gamma)} \frac{\theta \left(A^*\right)^{-(1-\sigma)/\theta}}{\theta + 1 - \sigma} \left(1 - \bar{j}_m\right)^{(\theta+1-\sigma)/\theta}.\end{aligned}$$

Combining these results:

$$\begin{aligned}M_e^* &= \alpha^* \frac{(1-\gamma)\,\theta \left(A^*\right)^{-(1-\sigma^*)/\theta}}{\theta + 1 - \sigma^*} p_e^{-(\gamma+(1-\gamma)\sigma^*)} \left(1 - \bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta} \\ &\quad + \left(\tau^*\right)^{1-\sigma} \alpha \frac{(1-\gamma)\,\theta \left(A^*\right)^{-(1-\sigma)/\theta}}{\theta + 1 - \sigma} \left(p_e + t_p\right)^{-(\gamma+(1-\gamma)\sigma)} \left(1 - \bar{j}_m\right)^{(\theta+1-\sigma)/\theta},\end{aligned}$$

where

$$1 - \bar{j}_x = \frac{\left(1 + t_p/p_e\right)^{(1-\gamma)\theta} A^*}{\tau^{-\theta} \left(1 + (1-\gamma)\, t_p/p_e\right)^{\theta} A + \left(1 + t_p/p_e\right)^{(1-\gamma)\theta} A^*}$$

and

$$1 - \bar{j}_m = \frac{\left(\tau^*\right)^{-\theta} A^*}{A + \left(\tau^*\right)^{-\theta} A^*}.$$

Turning to the value of Home's net exports of manufactured goods (32), we have:

$$V_g = p_e^{1-\gamma} \int_0^1 a_j^* x_j dj - \left(p_e + \gamma t_p\right) \left(p_e + t_p\right)^{-\gamma} \tau^* \int_0^1 a_j^* m_j dj,$$

where the second integral was computed above and the first is:

$$\begin{aligned}\int_0^1 a_j^* x_j dj &= \int_0^{\bar{j}_x} a_j^* \alpha^* \left(a_j^* p_e^{1-\gamma}\right)^{-\sigma^*} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \int_0^{\bar{j}_x} \left(\frac{1-j}{A^*}\right)^{(1-\sigma^*)/\theta} dj \\ &= \alpha^* p_e^{-\sigma^*(1-\gamma)} \frac{\theta \left(A^*\right)^{-(1-\sigma^*)/\theta}}{\theta + 1 - \sigma^*} \left(1 - \left(1 - \bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta}\right).\end{aligned}$$

Combining these results:

$$\begin{aligned} V_g &= \alpha^* \frac{\theta \left(A^*\right)^{-(1-\sigma^*)/\theta}}{\theta + 1 - \sigma^*} p_e^{(1-\sigma^*)(1-\gamma)} \left(1 - \left(1 - \bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta}\right) \\ &\quad - \left(\tau^*\right)^{1-\sigma} \alpha \frac{\theta \left(A^*\right)^{-(1-\sigma)/\theta}}{\theta + 1 - \sigma} \left(p_e + \gamma t_p\right) \left(p_e + t_p\right)^{-(\gamma+(1-\gamma)\sigma)} \left(1 - \bar{j}_m\right)^{(\theta+1-\sigma)/\theta}. \end{aligned}$$

Having gone as far as we can go with integration, we turn to assembling the pieces.

6.2 Supply and Demand

For step (ii) of the computation, we need to evaluate (27) based on World energy supply and demand. On the supply side, extractors in Foreign face a price p_e while energy extractors in Home get:

$$\lambda_e - \varphi = (p_e + t_p) - (t_p + t_e) = p_e - t_e.$$

World energy supply is thus:

$$Q_e^W = Q_e + Q_e^*(p_e) = \left(p_e - t_e\right)^{\beta/(1-\beta)} E + p_e^{\beta^*/(1-\beta^*)} E^*. \tag{35}$$

From the expressions above, World energy demand is:

$$\begin{aligned} C_e^W &= C_e^{HH} + C_e^{FH} + C_e^{FF} + C_e^{HF} = M_e + M_e^* \\ &= \alpha \frac{(1-\gamma)\,\theta A^{-(1-\sigma)/\theta}}{\theta + 1 - \sigma} \left(p_e + t_p\right)^{-(\gamma+(1-\gamma)\sigma)} \left(\bar{j}_m\right)^{(\theta+1-\sigma)/\theta} \\ &\quad + \tau\alpha^* \frac{(1-\gamma)\left(A^*\right)^{\sigma^*/\theta}}{A^{1/\theta}} \left(p_e + t_p\right)^{-\gamma} p_e^{-\sigma^*(1-\gamma)} B\left(\bar{j}_x, \frac{1+\theta}{\theta}, \frac{\theta - \sigma^*}{\theta}\right) \\ &\quad + \alpha^* \frac{(1-\gamma)\,\theta \left(A^*\right)^{-(1-\sigma^*)/\theta}}{\theta + 1 - \sigma^*} p_e^{-(\gamma+(1-\gamma)\sigma^*)} \left(1 - \bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta} \\ &\quad + \left(\tau^*\right)^{1-\sigma} \alpha \frac{(1-\gamma)\,\theta \left(A^*\right)^{-(1-\sigma)/\theta}}{\theta + 1 - \sigma} \left(p_e + t_p\right)^{-(\gamma+(1-\gamma)\sigma)} \left(1 - \bar{j}_m\right)^{(\theta+1-\sigma)/\theta} \end{aligned} \tag{36}$$

Given t_p we can solve for the value of p_e that brings C_e^W into equality with Q_e^W.

6.3 Optimal Taxes

We now turn to the problem of computing the optimal production and extraction tax. For the production tax, we can start with (29). Substituting in the derivative expressions (24), (25), and (26) we obtain:

$$t_p = \varphi \frac{\frac{\beta^*}{1-\beta^*} Q_e^* - p_e X_e - (1-\gamma) V_g^{FH} + p_e C_e^{HF}}{\frac{\beta}{1-\beta} Q_e^* + (\gamma + (1-\gamma)\,\sigma)\, C_e^{FF}}.$$

Each of the terms on the right-hand side can be evaluated within the computational algorithm. The optimal extraction tax is then simply:

$$t_e = \varphi - t_p.$$

In an iterative procedure, we would then return to equating global supply and demand given these tax rates.

The computational scheme outlined above should deliver values for p_e, t_p, and t_e that are fully consistent with each other. The more intricate taxes on exports are already embodied in the equation for Home's exports, but can be computed explicitly along the way. We will have computed the optimal climate policy.

6.4 Home's Welfare

With a little work, we can also obtain an expression for Welfare. Doing so will allow us to compare welfare under the optimal policy relative to various alternatives, including a baseline with no taxes. Welfare in Home is:

$$\begin{aligned} U &= C_s + \alpha^{1/\sigma} \frac{C_g^{1-1/\sigma} - 1}{1 - 1/\sigma} - \varphi Q_e^W \\ &= \frac{\sigma \alpha^{1/\sigma}}{\sigma - 1} C_g^{(\sigma-1)/\sigma} - \varphi Q_e^W + L - \beta E^{-(1-\beta)/\beta} Q_e^{1/\beta} - L_g + V_g + p_e X_e. \end{aligned}$$

The two terms we have not yet computed are:

$$C_g = \left(\int_0^1 (y_j + m_j)^{(\sigma-1)/\sigma}\, dj \right)^{\sigma/(\sigma-1)}$$

and:

$$L_g = \int_0^1 \left(a_j^l(z_j^y) y_j + \tau a_j^l(z_j^x) x_j \right) dj.$$

Note that the total labor force L is just a constant.

First, we compute:

$$\alpha^{1/\sigma} C_g^{(\sigma-1)/\sigma} = \alpha \left(p_e + t_p\right)^{(1-\sigma)(1-\gamma)} \left(\int_0^{\bar{j}_m} \left(\frac{j}{A}\right)^{(1-\sigma)/\theta} dj + \int_{\bar{j}_m}^1 \left(\frac{1-j}{A^*}\right)^{(1-\sigma)/\theta} dj \right),$$

where

$$\begin{aligned} \int_0^{\bar{j}_m} \left(\frac{j}{A}\right)^{(1-\sigma)/\theta} dj &= \frac{A}{1-(\sigma-1)/\theta} \left(\frac{j}{A}\right)^{1-(\sigma-1)/\theta} \Bigg|_0^{\bar{j}_m} \\ &= \frac{\theta A}{\theta+1-\sigma} \left(A + (\tau^*)^{-\theta} A^*\right)^{-(1-\sigma)/\theta-1}, \end{aligned}$$

and, applying the change of variable $j' = 1 - j$:

$$\begin{aligned} \int_{\bar{j}_m}^1 \left(\frac{1-j}{A^*}\right)^{(1-\sigma)/\theta} di &= \int_0^{1-\bar{j}_m} \left(\frac{j'}{A^*}\right)^{(1-\sigma)/\theta} dj' \\ &= \frac{A^*}{1-(\sigma-1)/\theta} \left(\frac{j'}{A^*}\right)^{1+(1-\sigma)/\theta} \Bigg|_0^{1-\bar{j}_m} \\ &= \frac{\theta (\tau^*)^{-\theta} A^*}{\theta+1-\sigma} \left(A + (\tau^*)^{-\theta} A^*\right)^{-(1-\sigma)/\theta-1}. \end{aligned}$$

Assembling the pieces:

$$\alpha^{1/\sigma} C_g^{(\sigma-1)/\sigma} = \alpha \left(p_e + t_p\right)^{(1-\sigma)(1-\gamma)} \frac{\theta}{\theta+1-\sigma} \left(A + (\tau^*)^{-\theta} A^*\right)^{-(1-\sigma)/\theta}.$$

Second, we have:

$$L_g = \gamma \left(p_e + t_p\right)^{1-\gamma} \int_0^1 a_j \left(y_j + \tau x_j\right) dj,$$

where, substituting in two results from above:

$$\begin{aligned} L_g &= \gamma\alpha \left(p_e + t_p\right)^{-(1-\sigma)(1-\gamma)} \frac{\theta A^{-(1-\sigma)/\theta}}{\theta+1-\sigma} \left(\bar{j}_m\right)^{(\theta+1-\sigma)/\theta} \\ &\quad + \tau\alpha^*\gamma \left(p_e + t_p\right)^{1-\gamma} p_e^{-\sigma^*(1-\gamma)} \frac{(A^*)^{\sigma^*/\theta}}{A^{1/\theta}} B\left(\bar{j}_x, \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right). \end{aligned}$$

We now have all the expressions needed to compute welfare.

6.5 Zero-Tax Baseline for Calibration

We want to calibrate the model to a zero-tax baseline. In a baseline with $t_e = t_p = 0$ and $\beta = \beta^*$ global energy supply (35) reduces to:

$$Q_e^W = (p_e)^{\beta/(1-\beta)} (E + E^*) .$$

On the demand side, the import threshold (33) is unchanged:

$$\bar{j}_m = \frac{A}{A + (\tau^*)^{-\theta} A^*}$$

while the export threshold (34) simplifies to

$$\bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*}.$$

Without export taxes and subsidies, Home exports any good $j \in [0, \bar{j}_x]$ at a price equal to its cost of production,

$$p_j = a_j \tau p_e^{1-\gamma},$$

so that:

$$x_j = \alpha^* \left(a_j \tau p_e^{1-\gamma}\right)^{-\sigma^*},$$

which depends on Home's iceberg cost. We get a simpler expression for:

$$C_e^{FH} = (1-\gamma) p_e^{-\gamma} \tau \int_0^1 a_j x_j dj,$$

where

$$\begin{aligned}
\int_0^1 a_j x_j dj &= \int_0^{\bar{j}_x} a_j \alpha^* \left(a_j \tau p_e^{1-\gamma}\right)^{-\sigma^*} dj \\
&= \alpha^* \tau^{-\sigma^*} p_e^{-\sigma^*(1-\gamma)} \int_0^{\bar{j}_x} \left(\frac{j}{A}\right)^{(1-\sigma^*)/\theta} dj \\
&= \alpha^* \tau^{-\sigma^*} p_e^{-\sigma^*(1-\gamma)} \frac{\theta (A)^{-(1-\sigma^*)/\theta}}{\theta + 1 - \sigma^*} \left(\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta} .
\end{aligned}$$

Hence, for the baseline:

$$C_e^{FH} = \tau^{1-\sigma^*} \alpha^* \frac{(1-\gamma)\theta (A)^{-(1-\sigma^*)/\theta}}{\theta + 1 - \sigma^*} p_e^{-(\gamma+(1-\gamma)\sigma^*)} \left(\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta} .$$

Global demand for energy becomes:

$$\begin{aligned} C_e^W &= \alpha \frac{(1-\gamma)\,\theta A^{-(1-\sigma)/\theta}}{\theta+1-\sigma} p_e^{-(\gamma+(1-\gamma)\sigma)} \left(\bar{j}_m\right)^{(\theta+1-\sigma)/\theta} \\ &\quad + \tau^{1-\sigma^*}\alpha^* \frac{(1-\gamma)\,\theta\,(A)^{-(1-\sigma^*)/\theta}}{\theta+1-\sigma^*} p_e^{-(\gamma+(1-\gamma)\sigma^*)} \left(\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta} \\ &\quad + \alpha^* \frac{(1-\gamma)\,\theta\,(A^*)^{-(1-\sigma^*)/\theta}}{\theta+1-\sigma^*} p_e^{-(\gamma+(1-\gamma)\sigma^*)} \left(1-\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta} \\ &\quad + \left(\tau^*\right)^{1-\sigma} \alpha \frac{(1-\gamma)\,\theta\,(A^*)^{-(1-\sigma)/\theta}}{\theta+1-\sigma} p_e^{-(\gamma+(1-\gamma)\sigma)} \left(1-\bar{j}_m\right)^{(\theta+1-\sigma)/\theta}. \end{aligned}$$

The other outcome that needs to be re-derived in the absence of export taxes and subsidies is the value of Home's exports to Foreign. With Home pricing at its marginal cost, we get:

$$\begin{aligned} V_g^{FH} &= \int_0^{\bar{j}_x} \alpha^* \left(a_j \tau p_e^{1-\gamma}\right)^{1-\sigma^*} dj \\ &= \tau^{1-\sigma^*}\alpha^* p_e^{(1-\sigma^*)(1-\gamma)} \int_0^{\bar{j}_x} \left(\frac{j}{A}\right)^{(1-\sigma^*)/\theta} dj \\ &= \tau^{1-\sigma^*}\alpha^* \frac{\theta A^{-(1-\sigma^*)/\theta}}{\theta+1-\sigma^*} p_e^{(1-\sigma^*)(1-\gamma)} \left(\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta}. \end{aligned}$$

7 Quantitative Illustration

Two be written.

8 Conclusion

While the model in this paper is very stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy. The main new finding is the extent to which international trade can broaden the reach of unilateral carbon policy.

To see whether such effects are of first-order importance, it is critical to push the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. For the first extension, the multi-country model of Eaton and Kortum (2002) retains the

Ricardian structure of trade in manufactures used here while the model of Larch and Wanner (2019) is a natural multi-country generalization of the energy sector. On the second extension, the dynamic analysis in Golosov, Hassler, Krusell, and Tsyvinski (2014) appears amenable to nesting multiple countries.

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