Debt versus Equity in Liquidity Provision*

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Abstract

We propose a unified framework to study liquidity provision by debt-issuing versus equity-issuing financial intermediaries. We show that both types of intermediaries provide liquidity by insuring against idiosyncratic liquidity risks as in Diamond and Dybvig (1983) but with distinct frictions. The fixed value of debt induces *panic runs* whereas the flexible payoff of equity renders investor redemptions more sensitive to news on fundamentals, i.e., a *flow-to-fundamentals* relationship. Both frictions constrain liquidity provision by generating inefficient premature liquidation of long-term investments. Informed by the theory, we develop the Liquidity Provision Index (LPI) as a unified measure of liquidity provision. We find that, at the end of 2017, the per dollar liquidity provision by bond mutual fund shares amounts to a quarter of that by uninsured bank deposits. The majority of the gap arises from the difference in contract forms instead of regulatory features such as deposit insurance. Further applying the LPI to money market funds (MMFs), we identify a 20% drop in liquidity provision due to the change from debt to equity funding around the 2016 MMF Reform.

1 Introduction

A predominant function of financial intermediaries is liquidity provision. In a world where investors face idiosyncratic liquidity risks, banks provide liquidity by issuing debt claims redeemable at short notice, that is, liquid demand deposits, from investing in a portfolio of illiquid loans. Liquidity provision is achieved by pooling and thus insuring against idiosyncratic liquidity shocks across investors, and so that collectively, more illiquid assets can be funded using liquid short-term claims (Diamond and Dybvig, 1983).

Since then, liquidity provision has primarily been attributed to debt-issuing financial intermediaries such as banks. While banks are undoubtedly an important part of the economy, equity-issuing intermediaries have played an increasingly important role. Open-end mutual funds, for example, saw their total assets increase from 5% of GDP in the 1980s to 100% in 2016 (see Goldstein, Jiang and Ng, 2017, Chernenko and Sunderam, 2017), while commercial banks' assets remained at around 65% of GDP during the same time period.

The above trends beg the question of whether liquidity provision is really only confined to debt. After all, equity-issuing intermediaries such as mutual funds also invest in substantial amounts of illiquid assets like corporate bonds, loans, commodities, and even real estates, and their shares are also redeemable at short notice. Moreover, if equity-issuing intermediaries also provide liquidity, is it any different from that of debt-issuing intermediaries such as banks?[1]

This paper provides a unified framework to study liquidity provision by debt- versus equity-issuing financial intermediaries., We theoretically show that intermediaries issuing redeemable equity at short notice like open-end mutual funds indeed create liquidity through smoothing idiosyncratic liquidity shocks in the same way as banks do in Diamond and Dybvig (1983).[2] Intuitively, as long as a sufficiently large number of investors hold their funds in the intermediary, their idiosyncratic liquidity risks can be pooled within the intermediary, which holds cash to meet withdrawals and redemptions at short notice. Thus, those who require funds early can receive more than what they would have achieved by liquidating the underlying portfolio on their own. As long as liquidity risk is pooled, the issuance of debt versus equity per se is not an economic prerequisite for liquidity provision.

[1] To be exact, we consider open-end equity contracts redeemable at short notice so although banks also issue equity, bank equity does not directly fall into our consideration of liquidity provision.

[2] Our baseline model compares demandable equity with demandable debt absent other regulatory components and frictions such as deposit insurance. We further disentangle these components in the empirical section.

While the source of liquidity provision is the same for both equity- and debt-issuing intermediaries, the contractual form affects how liquidity is distributed among investors, and in turn, the magnitude of and frictions underlying liquidity provision. On the upside, equity's flexibly adjusting value eliminates belief-fueled runs on the intermediary, or so called *panic runs*, which have long been viewed as a major drawback of primitive debt contracts. On the downside, equity's flexibly adjusting value also exposes investors to news of economic fluctuations, leading to more volatile *flows to fundamentals*. Both panic runs and the flow-to-fundamentals relationship induce inefficient asset liquidations and thereby constrain liquidity provision. Hence, their relative magnitude pins down the liquidity provision capacity of debt- and equity-issuing intermediaries.[3]

To quantify the amount of liquidity provided by different types of intermediaries, we develop the first unified empirical measure of liquidity provision: the Liquidity Provision Index (LPI). Using commercial banks and bond mutual funds as a laboratory, we estimate that the average dollar invested in open-end bond mutual fund shares creates about 20% of the liquidity as the average dollar invested in uninsured bank deposits. Our measurement does not include insured deposits and we further verify that the bulk of the gap arises from the difference in contract forms instead of indirect effects from explicit and implicit guarantees. In the time series, the gap shrinks from 2011 onwards in part due to unconventional monetary policy and post-crisis liquidity regulation.

A key novelty of our paper is to jointly analyze liquidity provision by bank debt and fund equity under a common theoretical framework. Similar to Diamond and Dybvig (1983), investors are subject to idiosyncratic liquidity shocks in an incomplete market. They invest through an intermediary, which chooses a portfolio of liquid cash and illiquid long-term investment projects at the beginning ($t = 0$), considering investors' liquidity shocks in the short run ($t = 1$) and the uncertain but potentially higher project return in the long run ($t = 2$). Notably, as in Goldstein and Pauzner (2005), there is aggregate uncertainty over the long-run return of the long-term project, that is, economic fundamentals. Investors receive a noisy private signal about the future

[3]Of course, there are many institutional differences between banks and mutual funds that could induce fundamentals-driven, non-panic-based runs in banks (e.g., Allen and Gale, 1998) and panic-based runs in mutual funds (Goldstein, Jiang and Ng, 2017, Zeng, 2017). Our baseline results highlight that the use of debt per se exposes banks to panic runs whereas the use of redeemable equity with perfectly adjusting values per se induces a flow-to-fundamentals relationship in funds. Please refer to the literature review for a more detailed discussion.

economic fundamentals at $t = 1$, which they use to decide whether to withdraw their funds invested in the intermediary.[4]

In this setting, liquidity provision can be thought of as the difference between the intermediary's expected contract payment to investors withdrawing at short notice ($t = 1$) and the direct liquidation value of the same intermediary's underlying asset portfolio. Bank debt provides liquidity because idiosyncratic liquidity risks are pooled across investors, allowing the bank to invest more in the illiquid long-term project and offer a higher debt payment to investors hit by liquidity shocks than what they would have enjoyed by directly liquidating the bank portfolio. In the same context, an intermediary issuing equity contracts redeemable at flexible net asset values (NAVs) also provides liquidity because as long as investors' resources are pooled at the intermediary level, the intermediary can invest in more long term projects and meet early redemptions at an NAV higher than the direct liquidation value of the underlying assets. The use of debt funding is therefore not a necessary condition for liquidity provision by intermediaries.

Nevertheless, the choice of debt versus equity funding determines how contract payments are distributed among investors, which affects the efficiency of liquidity provision. On one hand, in the absence of deposit insurance, debt-issuing intermediaries are susceptible to panic runs because the fixed face value of debt induces a first-mover advantage to withdraw.[5] Applying the global games technique a la Goldstein and Pauzner (2005), we show that panic runs occur only when fundamentals fall below a threshold. In contrast, even under relatively poor fundamentals, panic runs would not happen with equity contracts because NAVs drop with early redemptions, dampen the incentive of further withdrawals and thus remove any first-mover advantage. The comparative advantage of equity over debt in sufficiently bad states of the world can be seen in Figures 2 and 3.

On the other hand, equity with flexible NAVs has the disadvantage of rendering liquidity provision more sensitive to fluctuations in fundamentals. When equity investors receive private news about project returns, i.e., fundamentals, they decide whether to redeem based on the signaled

[4]To be exact, only those without a liquidity shock consider to withdraw or not. Investors hit with a liquidity shock always consume at $t = 1$.

[5]Specifically, banks meets withdrawal requests by prematurely selling the long-term projects and paying out the face value of debt on a first-come-first-serve basis. Knowing that funds will eventually run out, investors withdraw early in the hopes of getting paid first once they believe that others will. A belief-based panic run results.

return of staying in the fund versus the payoff from redeeming and storing cash on their own.[6] Redemption decisions are continuously revised according to the news on fundamentals, resulting in a *flow-to-fundamentals relationship*. It leads to premature liquidations, which ultimately limit liquidity provision. In contrast, bank debt promises a fixed payment and is less sensitive to fundamentals driven withdrawals than equity as long as fundamentals exceed the run threshold as displayed in Figures 2 and 3.

Having derived the comparative advantage and disadvantage of debt and equity, we ask how they play out to affect the quantity of liquidity produced by financial intermediaries in the real world. One advantage of our theoretical notion of liquidity provision - the difference between an intermediary's expected contract payment to investors withdrawing at short notice and the direct liquidation value of the intermediary's asset portfolio - is the direct mapping to publicly available data. This allows us to construct the Liquidity Provision Index (LPI) as the first empirical measure of liquidity production by both debt- and equity-issuing financial intermediaries.

The LPI estimation follows a three-step approach. We first determine the model-implied contract payment of uninsured bank deposits and fund shares for any given proportion of early redemptions. For example, when outflows are very small, funds can first use cash to meet redemptions, which do not incur any haircuts, so the contract payment is 100% of the initial NAV. As outflows increase, funds resort to selling increasingly illiquid assets with higher haircuts, continuously decreasing the NAV paid out. For banks, incurred haircuts also increase with withdrawals but uninsured depositors continue to receive the same face value until funds run out and the bank defaults. Given these differences, the expected contract payment depends on the distribution of outflows. Therefore, in the second step, we use the observed distribution of fund and bank level outflows as the equilibrium outcome to calculate the expected contract payment of a fund's shares and a bank's deposits. Finally, to determine the amount of liquidity provision, we deduct the direct liquidation value of the underlying portfolio.

Applying the LPI in a laboratory of commercial banks and bond mutual funds, we find that both uninsured deposits and fund shares provide a positive and significant amount of liquidity. At the end of 2017, for a dollar invested in uninsured bank deposits, investors expect to obtain \$0.190 more upon early withdrawal than under direct liquidation of the underlying portfolio; whereas for the average dollar invested in redeemable bond shares, they expect \$0.043 more.

[6]The private storage technology can be thought of as the physical cost of storing cash by investors or as the return obtainable from investing in riskless assets in capital markets e.g. Treasuries.

This shows that at the end of our sample in 2017, the liquidity provided by bond mutual fund shares amounts to about a quarter of liquidity provided by bank deposits. Howevever, the gap in liquidity production capacity between banks and funds has increasingly narrowed over our sample period from 2011 to 2017. The fall in LPI is more pronounced for banks absorbing more central bank reserves after Quantitative Easing and those more exposed to the Liquidity Coverage Ratio because the high proportion of liquid assets on bank balance sheets raises the direct liquidation value of the portfolio and limits the contribution to liquidity provision by banks.

The LPI is carefully constructed to only map to the uninsured portion of bank deposits. Uninsured deposits are consistent with the demandable debt in our model and of high empirical relevance. Egan, Hortacsu and Matvos (2017) estimate that uninsured comprise half of all consumer deposits in large US commercial banks and show that they are frequently impaired in bank defaults and subject to runs.[7] Nevertheless, explicit and implicit guarantees may still indirectly affect the estimated LPI of uninsured deposits through equilibrium flows. To further isolate the effect of contract forms on liquidity provision, we perform two additional tests.

We first use cross-sectional variation in bank liability structure and LPI to project that a zero deposit insurance and zero non-deposit liabilities bank provides two thirds of the estimated average bank LPI, which is still higher than the average mutual fund LPI. This result implies that the bulk of the difference in bank versus fund liquidity provision arises from the use of demandable debt versus equity.

We further apply the LPI to a sample of money market funds around the 2016 MMF Reform to identify the effect of debt versus equity funding on liquidity provision. The reform required institutional prime MMFs to switch from fixed to floating NAVs, which is effectively a transition from debt-funding to equity-funding.[8] At the same time, retail prime MMFs not affected by the reform provide a natural control group for other changes in the economy during the reform period. Our difference-in-differences estimates suggest an economically and statistically significant 20%

[7]One notable example of modern commercial bank run is that on Washington Mutual (WaMu). WaMu's first wave of depositor run took place on July 12, 2008 mostly in Southern California after the federal government seized IndyMac. The second wave of run was more wildly spread and started on September 11, 2008, when Moody's downgraded WaMu. Another notable example is the depositor run on Northern Rock on September 14 and 17, 2007 amid its attempts to tap Bank of England as the Lender of the Last Resort. Although Bank of England eventually injected liquidity to compensate the depositors, it did not stop the run in the first place.

[8]Institutional prime MMFs were indeed subject to panic runs before the reform, notable examples including the Reserve Primary Fund, the first MMF that "broke the buck," and the Putnam Fund, the first MMF that suddenly closed, both due to severe runs in 2008.

drop in liquidity provision due to the reform, corroborating that both demandable debt and equity provide liquidity but with debt performing relatively better.

In the cross-section, the within-fund and within-bank variations are also consistent with the theory. Funds with flows less sensitive to changes in returns, i.e., with a less pronounced flows-to-fundamentals relationship, generate a higher LPI. For banks, a higher incidence of deposit insurance, which reduces the incentive of panic runs, relates to a higher bank LPI.

Although we test our theoretical framework in the context of commercial banks, bond mutual funds, and MMFs, the economic insights we uncover and the LPI we construct can be extended to demandable debt- and equity-issuing financial intermediaries in general.[9,10,11] Further, while this paper focuses on justifying and quantifying liquidity provision by individual intermediaries and intermediary-types, an important but separate question is to understand the coexistence and competition between them. This is addressed in Ma, Xiao and Zeng (2019), where we jointly consider intermediaries' liquidity provision capacity and the costs of liquidity provision, e.g., the returns obtained from and fees incurred by investing in deposits or mutual fund shares.

Through analyzing the role of demandable equity versus debt in providing liquidity, our paper bridges the seminal banking literature on liquidity provision and a large literature on mutual fund flows and their financial stability implications. The banking literature on liquidity provision has mostly centered around deposit-issuing banks as in Diamond and Dybvig (1983), Diamond and Rajan (2001), Kashyap, Rajan and Stein (2002), and Goldstein and Pauzner (2005), for example.[12] More recently, Hanson, Shleifer, Stein and Vishny (2015) consider debt claims issued

[9]Debt-issuing intermediaries include commercial banks, prime money market funds (MMF) pre MMF reform, asset-backed commercial paper (ABCP) conduits, and leveraged hedge funds; whereas equity-issuing intermediaries comprise of open-end mutual funds, prime money market funds post MMF reform, unleveraged hedge funds, pension funds, and insurance companies.

[10]Note that our framework does not consider closed-end funds or hidden trades as in Jacklin (1987). We do not focus on closed-end funds because, as argued by Stein (2005), the competition among mutual funds renders most funds open-ending, which is consistent with empirical evidence that closed-end funds only account for less than 2% of the mutual fund sector. On hidden trades, Farhi, Golosov and Tsyvinski (2009) show that the resulting inefficiency from hidden trades can be simply corrected by implementing a liquidity floor.

[11]Our framework does not directly apply to exchange traded funds (ETFs). This is because the mechanism for ETF shares to track the value of the underlying assets relies on designated arbitragers, which differs from the liquidation technology by open-end mutual funds and is contingent on market conditions (Pan and Zeng, 2019).

[12]Other classic papers on the role of banks, to name a few, include Diamond (1984) on bank monitoring, Gorton and Pennacchi (1990) on bank liabilities as a medium of exchange, and Rajan (1992) on relationship banking. Recently, He and Krishnamurthy (2013) analyze macroeconomic implications of banking and Drechsler, Savov and Schnabl (2017) analyze how market power renders banks special in deposit-taking. They do not focus on liquidity provision as Diamond and Dybvig (1983).

by both banks and shadow banks that differ in their access to deposit insurance,[13] whereas Dang, Gorton, Holmström, and Ordoñez (2017) compare banks to financial markets focusing on banks' ability to keep information on fundamentals secret. Our paper zooms in on the contractual form of intermediary liabilities, which is a fundamental difference between banks and non banks. We jointly analyze liquidity provision by equity and debt in a common theoretical framework based on Diamond and Dybvig (1983) to show that equity also insures against liquidity risk but with different frictions.

We find that the comparative disadvantage of equity is the presence of more volatile flows to fundamentals. A related concept is that of information insensitivity proposed by Gorton and Pennacchi (1990), who show that debt enjoys higher market liquidity when traded because its value is less subject to underlying cash flow fluctuations. We show that a less sensitive flows-to-fundamentals relationship is also a key advantage of debt over equity for liquidity provision in the sense of Diamond and Dybvig (1983). By doing so, we relate the two long-standing, but somewhat separate literatures on bank runs and debt information insensitivity.

Demonstrating the presence and implications of flows to fundamentals for equity-funded intermediaries also contributes to a large literature on mutual fund flows (see Christoffersen, Musto and Wermers, 2014, for a review). Starting from Berk and Green (2004), numerous empirical papers have examined the interplay between fund liquidity holdings and the flow-to-performance relationship (e.g., Chordia, 1996, Edelen, 1999, Wermers, 2000, Jiang, Li and Wang, 2017, Pástor, Stambaugh and Taylor, 2019). The novelty of our paper is to relate flows to information on economic fundamentals to the efficiency of liquidity provision.

The comparative advantage of equity with flexible NAVs in eliminating panic runs speaks to the literature on mutual funds' financial stability implications. Chen, Goldstein and Jiang (2010) and Goldstein, Jiang and Ng (2017) find that panic runs may occur when adjustments in fund NAVs are imperfect. Using a dynamic model, Zeng (2017) illustrates that fund managers' cash management and an inherent non-commitment problem may also lead to panic runs. Chernenko and Sunderam (2017) empirically examine the relationship between fund cash management and maturity mismatch, focusing on financial stability concerns. We take a step back and formally analyze equity in its most primitive form with fully flexible NAVs. In fact, within the framework we establish, sticky and non-forward looking NAVs can be seen as deviations away from a bench-

[13] Other papers on shadow bank liquidity provision includes Gorton and Metrick (2010), Stein (2012), Sunderam (2015), Nagel (2016) and Xiao (2019).

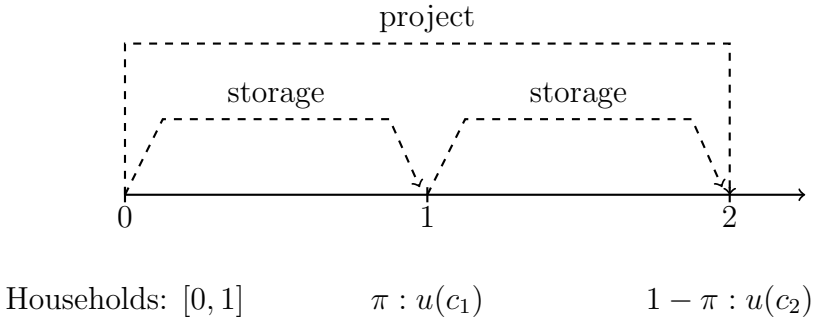
mark equity contract towards a debt contract with fixed face value. This would bring along frictions of debt contracts in liquidity provision, i.e. the potential of runs, but would further improve the liquidity provision capacity of mutual funds according to our empirical estimates.

Finally, our paper contributes to the measurement of liquidity provision by financial intermediaries. The prior literature has focused on the banking sector (Berger and Bouwman, 2009, Brunnermeier, Gorton and Krishnamurthy, 2012, Bai, Krishnamurthy and Weymuller, 2018), as the theoretical foundation of liquidity provision by other types of intermediaries has not yet being laid out. We derive the LPI as a unified measure of liquidity provision which applies to both debt-issuing and equity-issuing intermediaries. The LPI provides a useful tool for researchers and policy-makers to assess the functions and risks of non-bank financial institutions.

2 Theoretical Framework

The economy has three dates, $t = 0, 1, 2$, with no time discount. There is a $[0, 1]$ continuum of ex-ante identical households, each of which has one unit of consumption good at $t = 0$ and no endowment afterwards. Each household is uncertain about her preferences over consumption at $t = 1$ and $t = 2$. At the beginning of $t = 1$ a household learns her preferences privately: with probability π she is an early-type and gets utility $u(c_1)$ from date-1 consumption only, while with probability $1 - \pi$ she is a late-type and gets utility $u(c_2)$ from date-2 consumption only. Let the primitive flow utility function, $u(c)$, be increasing, concave, and satisfy the Inada conditions.[14]

Figure 1: Timeline



[14]Notably, Inada conditions require $u(0) = 0$.

The market is incomplete in the sense that no Arrow-Debreu securities are available. There is one consumption good, called "cash", which serves as the numeraire. Cash can be directly consumed at any given date or transferred to the next date via one of two technologies: 1) a long-term, illiquid investment project denoted as "project", and 2) a short-term liquid asset denoted as "storage".[15,16]

The project is risky, illiquid, and available for investment at $t = 0$ only. One unit of cash invested in the project at $t = 0$ yields R units of cash at $t = 2$, where R is a random variable that follows a distribution of $G(\cdot)$ with support $[0, +\infty)$. Denote R as the fundamentals of the economy; since R is uncertain, the economy entails aggregate risks.[17] Only the distribution of R, $G(\cdot)$, is common knowledge to households at $t = 0$. At $t = 1$, the project has not yet come to fruition and retains a value of one. If it is prematurely liquidated at this point, a liquidation discount will be incurred.[18] Denote the cash value obtainable when l of the project is liquidated as:

$$C(l) = l - \frac{\phi}{2} l^2 \,, \tag{2.1}$$

where $0 < \phi \leq 1$ captures the extent of project illiquidity. This parametric form (2.1) can be micro-founded by a downward-sloping demand for the illiquid project: the more of the project is prematurely liquidated, the lower the marginal liquidation value.[19]

Storing cash is riskless. An intermediary always yields one unit of cash on the next date when using the storage technology. Households are less efficient at storing cash and only obtain

[15]Note that we do not attach any exogenous utility or convenience value to holding cash per se. Instead, liquidity creation arises endogenously through liquidity insurance in an incomplete market setting as in Diamond and Dybvig (1983).

[16]Although the original Diamond and Dybvig (1983) model does not have storage, the portfolio choice between storage and an illiquid project has been introduced to the Diamond and Dybvig (1983) framework since Cooper and Ross (1998) and Ennis and Keister (2006).

[17]The original Diamond and Dybvig (1983) model does not have aggregate risks, but they have been introduced in the follow-up literature, notably, Allen and Gale (1998) and Goldstein and Pauzner (2005).

[18]The original Diamond and Dybvig (1983) model does not have this liquidation cost. But it has been introduced by Cooper and Ross (1998) to consider inefficient liquidation and its interplay with bank liquidity holdings, and has been so far considered as a standard element of the Diamond and Dybvig (1983) paradigm.

[19]Formally, let the marginal liquidation value of the project be $1 - \phi z$, where z is the total unit of projects being liquidated. As a result, if a total of l illiquid project were prematurely liquidated at $t = 1$, the amount of cash raised would be

$$\int_0^l (1 - \phi z) dz \,.$$

Calculating this integral yields (2.1).

γ units when investing one unit from $t = 1$ to $t = 2$. Specifically, we assume

$$\gamma = 1 - \kappa n \,, \tag{2.2}$$

where $0 \leq \kappa < 1$ captures the decreasing returns to scale when a late household operates the storage and n is the population of late households who use this storage.[20] This can be thought of as households finding it more costly to physically store cash or being less efficient at investing cash in capital market securities. It is important to note that intermediaries' storage efficiency does not determine the presence of liquidity provision. In other words, all predictions still carry through if the relative inefficiency parameter κ is equal to 0. The purpose of introducing κ is to capture the extent of the flow-to-fundamentals relationship, which will eventually influence the magnitude of liquidity provision.

At the beginning of $t = 1$, every household i receives a private signal of R:

$$s_i = \theta(R) + \varepsilon_i \,.$$

where $\theta(R) \in [0, 1)$ is strictly increasing in R, and ε_i is i.i.d. and arbitrarily small.[21]

To uncover the similarities and differences between debt and equity in liquidity provision, we compare two scenarios with different intermediary arrangements, keeping the same underlying economy. In both scenarios, a representative financial intermediary is present who makes portfolio choices $(x_{k,0}, y_{k,0})$ at $t = 0$ on households' behalf, where $x_{k,0}$ is the amount of cash and $y_{k,0}$ the amount of projects. The subscript $k \in \{b, f\}$ denotes the intermediary type: a bank or a fund. Since the intermediary is representative, it maximizes households' utility and breaks even in equilibrium. Another interpretation is that the intermediary is mutually owned by all the households without any agency frictions.

[20] An alternative specification is to explicitly assume γ to be a decreasing function of the units of cash stored by late households. This specification would not affect the economic insight because it is mathematically equivalent to assume γ to be decreasing in n in a non-linear way. To see the intuition, notice that any household using this storage always stores a positive amount of cash, and hence specification (2.2) already implies that γ is decreasing in cash stored by late households.

[21] This information structure is similar to that in Allen and Gale (1998) and more formally that in Goldstein and Pauzner (2005).

In the first scenario, a representative bank offers a standard demandable debt contract $(c_{b,1}, c_{b,2})$ to households, where the cash payment at $t = 1$, $c_{b,1}$, is subject to a sequential service constraint as in Diamond and Dybvig (1983).[22]

In the second scenario, a representative open-end mutual fund offers an equity contract $(c_{f,1}(\lambda(R)), c_{f,2}(\lambda(R)))$ in which the cash payments are the end-of-date net asset values (NAVs). These payments are contingent on the number of households redeeming at $t = 1$, λ, which is in turn determined by economic fundamental R in equilibrium. Thus, this payment structure represents a redeemable equity contract.

It is important to note that we derive optimal debt and equity contracts based on the type of debt and equity most closely aligned with those in the real world, i.e., demandable deposits and open end mutual fund shares with flexible NAV. In this sense, we solve for constrained optimal contracts just as in Diamond and Dybvig (1983). We do not focus on an unconstrained optimal contract or mechanism design problem over a general contract space.

2.1 Liquidation Value and Liquidity Provision

Given our focus on liquidity provision by intermediaries, we first define a benchmark case of how much short-term consumption a household can enjoy by liquidating a given portfolio at short notice without an intermediary. Formally, we denote this as the liquidation value. Note that the liquidation value is general in the sense that it can be defined on any portfolio and thus is a function of (x_0, y_0); it does not necessarily rely on any equilibrium concept.

Definition 1. *Given any t-0 portfolio* (x_0, y_0)*, its liquidation value at* $t = 1$ *is given by*

$$\begin{aligned} c_1(x_0, y_0) &\doteq x_0 + y_0 - \frac{\phi}{2} y_0^2 \\ &= 1 - \frac{\phi}{2} y_0^2 . \end{aligned} \tag{2.3}$$

As (2.3) indicates, the liquidation value is lower than the fair value of the portfolio at $t = 1$ due to the liquidation discount as specified in (2.1). The less liquid a portfolio, the lower the liquidation value.

[22] Our model insights will not change if we instead assume a pro rata rule following, for example, Allen and Gale (1998). We do not consider other versions of the sequential service constraint such as that in Green and Lin (2003).

According to Diamond and Dybvig (1983), creating deposits that are more liquid than the underlying portfolio held by banks can be viewed as a liquidity insurance arrangement in which depositors share the risk of liquidating an illiquid portfolio at short notice and at a loss. Definition 1 allows us to naturally extend this insight to non-bank, equity-issuing financial intermediaries to define a unified notion of liquidity provision as the difference between the expected intermediary contract payment and the liquidation value of the underlying portfolio. Formally, the equilibrium amount of liquidity provision for any intermediary is defined as follows.

Definition 2. *For a dollar invested in claims issued by an intermediary $k \in \{b, f\}$, the amount of liquidity provision is defined as*

$$E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0}), \tag{2.4}$$

where the contract payment $c^*_{k,1}(R)$ and intermediary portfolio holdings $(x^*_{k,0}, y^*_{k,0})$ are equilibrium outcomes given the intermediary arrangement (bank or fund) and the underlying economy, the expectation $E[\cdot]$ is taken over fundamental R, and $c_1(\cdot, \cdot)$ is the liquidation value function as defined in Definition 1.

Intuitively, definition 2 indicates that the amount of liquidity provision to a household subject to realized liquidity shocks is affected by both sides of the intermediary's balance sheet. Specifically, liquidity provision is the difference between the expected contract payment (liability side of the intermediary) and the liquidation value of the underlying portfolio (asset side of the intermediary). It is positive if the claims issued by the intermediary are more liquid, i.e., have a higher value upon liquidation at short notice, than the underlying assets it holds. Looking ahead, we will show that the two sides of the intermediary balance sheet are jointly determined in equilibrium, and that the equilibrium amount of liquidity provision is characterized by the endogenous frictions stemming from the different contractual forms.

One nice feature about Definition 2 is that, since the liquidation value $c_1(x^*_{k,0}, y^*_{k,0})$ is independent to fundamental R, (2.4) can be re-expressed as

$$E[c^*_{k,1}(R) - c_1(x^*_{k,0}, y^*_{k,0})],$$

which (before taking the expectation) allows our theory to capture ex-post liquidity provision for any given realization of R. It thus allows us to compare the relative frictions of bank debt versus fund equity in liquidity provision for any given state of the underlying economy.

Our definition of liquidation value and liquidity provision has two advantages. Theoretically, it is in line with the original insight of liquidity provision by Diamond and Dybvig (1983) and allows for a unified comparison across debt- and equity-issuing financial intermediaries.[23] Empirically, all essential inputs for calculating the liquidation value and the magnitude of liquidity provision are observable, which allows for a direct application of the model to measure liquidity provision by real-world intermediaries such as banks, open-end mutual funds, and money market funds (MMFs).

2.2 Bank Debt

At $t = 0$, the representative bank offers a demandable debt contract $(c_{b,1}, c_{b,2})$, which is subject to a sequential service constraint at $t = 1$, to households. Notably, $c_{b,1}$ represents the face value of debt, which is agreed on at $t = 0$. Because the bank breaks even, $c_{b,2}$ is automatically determined once $c_{b,1}$ is chosen so that the face value of debt, $c_{b,1}$, is sufficient to capture the contract offered to depositors.[24] At $t = 0$, the bank also chooses the optimal portfolio $(x_{b,0}, y_{b,0})$ to maximize ex-ante expected household utility.

Although the debt face value $c_{b,1}$ is fixed and independent of the number of withdrawals as long as the bank is solvent, the sequential service constraint implies that the actual payments of $(c_{b,1}, c_{b,2})$ implicitly depend on λ_b given any initial bank portfolio choice $(x_{b,0}, y_{b,0})$ as shown in Table 1:

where

$$q(\lambda_b) = \frac{1 - \frac{\phi}{2} y_{b,0}^2}{\lambda_b c_{b,1}} \tag{2.5}$$

[23] In Appendix A, we further show that the notion of liquidation value is tightly linked to the equilibrium outcome in autarky, which serves as a direct welfare benchmark but is not empirically observable.

[24] Following Diamond and Dybvig (1983), debt means that $c_{b,1}$ does not depend on λ_b (unless a run occurs), regardless of whether $c_{b,2}$ depends on λ_b or R. This focus on t-1 consumption is consistent with our definition of fund equity, which hinges on date-1 NAV $c_{f,1}(\lambda_f)$ directly depending on λ_f and and in turn on R in equilibrium because λ_f depends on R.

Table 1: Ex-post bank debt payments

t-1 withdrawal	$\lambda_b \leq \dfrac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$	$\lambda_b > \dfrac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$
t-1 payment	$c_{b,1}$	$c_{b,1}$ with probability $q(\lambda_b)$
t-2 payment	$\dfrac{x_{b,1}+y_{b,1}R}{1-\lambda_b}$	0

is the probability of withdrawing households to be served when the bank fails and the sequential service constraint is binding, and $(x_{b,1}, y_{b,1})$ is the bank's remaining portfolio after meeting date-1 withdrawals:

$$x_{b,1} = \max\{x_{b,0} - \lambda_b c_{b,1}, 0\}\,,$$

and

$$y_{b,1} = \begin{cases} y_{b,0} & \text{if} \quad x_{b,0} \geqslant \lambda_b c_{b,1}\,, \\ y_{b,0} - l_b & \text{if} \quad x_{b,0} < \lambda_b c_{b,1}\,, \end{cases} \tag{2.6}$$

where l_b, the unit of projects that the bank has to liquidate at $t=1$, is determined by

$$\lambda_b c_{b,1} = x_{b,0} + l_b - \frac{\phi}{2} l_b^2\,, \tag{2.7}$$

if the bank is solvent at $t=1$, and clearly $l_b = y_{b,0}$ if the bank fails.

According to Definition 2, bank debt provides liquidity when $E[c_{b,1}^*] > c_1(x_{b,0}^*, y_{b,0}^*)$. Looking forward, we provide sufficient conditions to ensure that this is true in equilibrium at the end of this subsection. Nevertheless, the actual debt payment schedule shown in Table 1 suggests that the debt-issuing bank is indeed vulnerable to potential panic runs due to strategic uncertainty among late households.

The intuition of the strategic uncertainty and the underlying first-mover advantage is the following. suppose the bank has enough cash in the sense that $x_{b,0}$ is large enough to meet withdrawal by the π population of early households.[25] If a late household expects all other households to wait until $t=2$, it is her best response to wait because that would allow her to get a strictly positive t-2 consumption level. Nevertheless, if she expects all other households to

[25]The coordination failure among late households is essentially the same when $x_{b,0}$ is small but the verbal discussion is more involved. And in a t-0 equilibrium, the bank always holds a sufficiently high $x_{b,0}$ to meet early household withdrawal.

withdraw at $t = 1$, nothing would remain at $t = 2$ so that her best response would be to run as well. Thus, the design of $(c_{b,1}, c_{b,2})$ must take the probability of panic runs at $t = 1$ into account.

We work backwards by analyzing late households' run decision, the bank's optimal choice of debt face value $c_{b,1}$ and the portfolio choice problem. In doing so, we apply the global games technique following Goldstein and Pauzner (2005) to pin down the probability of panic runs. The advantage of using global games is to link bank run probability to fundamentals while preserving the panic-based nature of runs. This is important in our framework because it enables a parallel comparison between bank debt and fund equity in liquidity provision given the same fundamentals.

To formulate the analysis, we introduce a mild distributional assumption regarding households' private signal s_i. Specifically, we assume θ to be uniformly distributed, and that ε_i, the i.i.d. noise term, is uniformly distributed over $[-\varepsilon, \varepsilon]$, where ε is arbitrarily small. As shown in Goldstein and Pauzner (2005), under these mild distributional assumptions, there exists a unique t-1 equilibrium in which a panic run occurs when R is below a threshold R^*, and any equilibrium must be such a threshold equilibrium.[26] Thus, we follow the same logic to show that such a threshold equilibrium exists in our framework, and then analyze the property of the threshold R^*.

Proposition 1. *Given the date-0 fund position $(x_{b,0}, y_{b,0})$, the promised debt value $c_{b,1}$, and the signal s_i received by households at $t = 1$, there exists a unique run threshold $R^* > \underline{R}$, where $\underline{R} > 0$ is given in the appendix.*

Proposition 1 shows that bank debt subjects the bank to panic runs, which is coordinated by fundamentals. An important observation from Proposition 1 is that the run threshold R^* can be either larger or smaller than 1. We depict the idea of Proposition 1 in Figure 2, where the red step curve illustrates the relationship between late households' withdrawal decision and economic fundamentals.

After illustrating that bank liquidity provision entails panic runs, we analyze how bank contract design and portfolio choice affect the magnitude of runs given economic fundamentals:

[26] More formally, following Goldstein and Pauzner (2005), we need to assume an "upper dominance region," which means that late households never run when $R \to +\infty$, to ensure the existence of a threshold run equilibrium. This is a technical assumption to ensure that the global games technique works. Since this is a well understood technical point in the literature and is not crucial to our economic mechanism, we omit the details and refer interested readers to Goldstein and Pauzner (2005) for the economic motivation of that upper dominance region.

Proposition 2. *Given the date-0 fund position $(x_{b,0}, y_{b,0})$, the promised debt value $c_{b,1}$, and the signal s_i received by households at $t = 1$, the unique run threshold $R^*(c_{b,1}, x_{b,0})$ is increasing in $c_{b,1}$ and decreasing in $x_{b,0}$.*

Proposition 2 shows that a higher promised debt value $c_{b,1}$ renders panic runs more likely. On one hand, a higher $c_{b,1}$ directly indicates more liquidity provision at $t = 1$. On the other hand, the induced higher run threshold suggests more inefficient premature liquidation, hurting liquidity provision. It follows that panic runs are a disadvantage of using bank debt in providing liquidity – a key conceptual point of this paper. Proposition 2 also implies that stored cash $x_{b,0}$ helps mitigate panic runs. This observation stems from the role of stored cash in helping liquidity provision at $t = 1$. A higher amount of stored cash helps reduce the first-mover advantage and thus decreases the run threshold.

Proposition 2 is economically important because it illustrates that panic runs indeed comprise the main friction for bank debt to provide liquidity. To see this, consider the bank's optimization problem at $t = 0$. The representative bank solves the optimal debt face value $c_{b,1}$ and portfolio allocation $(x_{b,0}, y_{b,0})$ to maximize the expected utility of households:

$$\max_{c_{b,1}, x_{b,0}, y_{b,0}} \int_0^{R^*} u(c_{b,1}) q(1) dG(R) + \int_{R^*}^{+\infty} \left(\pi u(c_{b,1}) + (1-\pi) u(c_{b,2})\right) dG(R) \tag{2.8}$$

where the function q is given by (2.5) and R^* is determined according to Proposition 1. As (2.8) indicates, a higher run possibility (i.e., a higher run threshold R^*) unambiguously leads to a lower ex-ante expected household utility. Thus, according to Proposition 2, at the margin, the bank has to either provide a lower deposit value $c^*_{b,1}$ or increase its cash position $x^*_{b,0}$ to reduce panic runs, both leading to lower equilibrium liquidity provision according to Definition 2.

Finally, we ensure that the bank in our model indeed provides liquidity ex-ante according to Definition 2. It suffices to give a set of sufficient conditions that ensures $E[c^*_{b,1}(R)] > c_1(x^*_{b,0}, y^*_{b,0})$ without fully solving (2.8).[27] When the distribution of $G(R)$ reflects sufficiently good fundamentals (e.g., $G(R)$ follows an exponential distribution with a sufficiently small rate), we have $c^*_{b,1} \to \frac{1}{\pi} > 1$ whereas $c_1(x^*_{b,0}, y^*_{b,0}) < 1$. Thus, by standard continuity argument, there exists a

[27] This approach is standard in the literature, for example, Theorem 3 in Goldstein and Pauzner (2005).

distribution $G(R)$ that ensures the bank to provide liquidity in our model. For any distribution $G'(R)$ that first-order stochastic dominates $G(R)$, the bank provides liquidity ex-ante.[28]

2.3 Fund Equity

The representative open-end mutual fund offers an equity contract $(c_{f,1}(\lambda_f(R)), c_{f,2}(\lambda_f(R)))$, which is demandable at the end of each date and whose payments are the end-of-day NAVs.[29] To capture the essence of equity, we consider fully adjustable NAV in the sense that the contract payment is explicitly written on the number of shareholders who actually redeem at $t = 1$, denoted by λ_f. In practice, open-end mutual funds achieve this by "striking the NAV" – a standard industry practice during the trading day to form the estimated amount of redemption requests, perform the necessary asset transactions, and pre-calculate the end-of-day NAVs. We essentially take this process to be frictionless in the model. In equilibrium, withdrawals λ_f will in turn be determined by fundamentals R, which is why contract payments are indirectly affected by R and take the form of $(c_{f,1}(\lambda_f(R)), c_{f,2}(\lambda_f(R)))$.[30]

We work backwards on the fund's optimal contract design and portfolio choice, taking household decisions into account. First, at $t = 1$, the fund pays out the end-of-day NAV_1 to redeeming households. The representative fund, which maximizes expected shareholder utility, will first deploy stored cash to meet redemptions at $t = 1$ because this avoids premature liquidation of the project. If the cash stored no longer suffices to pay all redeeming households, the fund will resort to liquidating the project prematurely, raise cash, adjust the end-of-day NAV downwards, and pay all redeeming households the resulting NAV.[31] Hence, at $t = 1$, the fund NAV is determined

[28] Theoretically, we can also easily show that, under the same set of sufficient conditions, the bank provides a higher expected t-1 consumption than the autarky. See Appendix A for a formal analysis.

[29] Relatedly, Green and Lin (2003) considers an alternative, hypothetical equity contract implied by a direct mechanism design problem in the Diamond and Dybvig (1983) setting. As acknowledged by Green and Lin (2003), however, that hypothetical equity contract is not observed in reality.

[30] Notice that this is an equilibrium outcome and not because contract payments are directly written on households' private signal s_i.

[31] In reality, the Investment Company Act of 1940 prohibits open-end mutual funds from borrowing or lending (i.e., using or extending credit lines), even within the same fund family. Within-fund-family overnight borrowing and lending is legal only upon application to and approval from the US SEC under emergency situations. The same legal restriction applies to using redemption gates to suspend daily redemption to shareholders. Mutual funds may also opt to redeem in-kind rather than redeem in-cash; however, to protect their reputation to facilitate future share distribution, mutual funds are extremely reluctant to use redemption in-kind even in bad times. Consistent with this argument, there has been no evidence of open-end mutual funds using redemption in-kind.

by

$$\begin{aligned} c_{f,1}(\lambda_f) &= NAV_1(\lambda_f) \\ &= x_{f,0} + y_{f,0} - \frac{\phi}{2} l_f^2(\lambda_f) \\ &= 1 - \frac{\phi}{2} l_f^2(\lambda_f) \\ &= 1 - \frac{\phi}{2}(y_{f,0} - y_{f,1}(\lambda_f))^2 \,, \end{aligned} \tag{2.9}$$

where $l_f = y_{f,0} - y_{f,1}$ is the unit of prematurely liquidated projects at $t = 1$. $y_{f,1}$ is the fund's remaining position in the illiquid project after meeting date-1 redemptions, which is a function of λ_f in equilibrium. l_f also depends on λ_f and is determined by

$$\lambda_f NAV_1(\lambda_f) = \min\left\{\lambda_f, x_{f,0} + l_f - \frac{\phi}{2} l_f^2\right\} , \tag{2.10}$$

where the LHS is the total amount of cash distributed to redeeming investors and the RHS is the amount of available cash, both evaluated at the end of $t = 1$. And clearly, we have

$$y_{f,1} = \begin{cases} y_{f,0} & \text{if} \quad x_{f,0} \geqslant \lambda_f \,, \\ y_{f,0} - l_f & \text{if} \quad x_{f,0} < \lambda_f \,, \end{cases} \tag{2.11}$$

Two important remarks are in order. First, although households' private signal s_i at $t = 1$ is not contractable and cannot be written into the NAV, fundamentals R influence equilibrium payments through affecting the number of households who actually redeem at $t = 1$. Thus, the equilibrium equity contract payment is written in the form of $(c_{f,1}(\lambda_f(R)), c_{f,2}(\lambda_f(R)))$. Second, the value of projects that are not liquidated at $t = 1$ remains at 1 because they have not yet come to fruition. This valuation is reflected in the NAVs as a result of its flexibly adjusting contract value.

Without fully solving for the equilibrium, we first show that fund equity indeed provides liquidity in the sense of Diamond and Dybvig (1983) in any equilibrium:

Proposition 3. *Given any equilibrium fund portfolio $(x^*_{f,0}, y^*_{f,0})$, the fund provides liquidity ex-ante as defined in Definition 2, that is,*

$$E[c^*_{f,1}(\lambda_f(R))] > c_1(x^*_{f,0}, y^*_{f,0}) \,,$$

where the t-1 NAV function $c^*_{f,1}(\cdot)$ *is given by* (2.9).

The proof of Proposition 3 is straightforward and thus we give it here to help build intuition. Directly comparing (2.9) and the liquidation value as defined in Definition 1 shows that $c_{f,1}(\lambda_f(R)) \geq c_1(x_{f,0}, y_{f,0})$, and this inequality takes a strict form if $y_{f,1} > 0$. Intuitively, unless the fund is fully liquidated, the end-of-day NAV promised to households who redeem at $t = 1$ is strictly higher than the liquidation value of the underlying fund portfolio. Notice that this statement is true regardless of how many households actually redeem, and thus is also true regardless of the fundamental. Taking expectations with respect to the fundamental thus yields the result.

The intuition behind Proposition 3 is that, by pooling resources among early and late households at the intermediary level, issuing demandable equity, and paying out early households at the NAV, the fund is able to provide liquidity insurance among all households exactly in the spirit of Diamond and Dybvig (1983).[32] Notably, the fund equity under its payment structure (2.9) does not have any debt-like feature, suggesting that a debt contract is not a necessary condition for liquidity provision. Again, the key is to create a pool of resources to share idiosyncratic liquidity risks, be it a bank or a fund.

Nevertheless, the characteristics of liquidity provision by bank debt and fund equity differ because the former pays a fixed face value unless default whereas the latter promises a flexible and information sensitive payment. The following lemma offers a first insight into the difference:

Lemma 1. *Given any fund portfolio* $(x_{f,0}, y_{f,0})$*, the consumption promised to early households is decreasing in the number of redeeming households, that is,* $\partial c_{f,1}(\lambda_f)/\partial \lambda_f \leq 0$*, and this inequality takes a strict form if* $l_f > 0$*.*

Lemma 1 illustrates that, by issuing equity, funds are able to flexibly adjust their actual, ex-post liquidity provision at $t = 1$ once liquidation losses realize. In contrast, banks would have to honor the fixed debt value $c_{b,1}$ to redeeming households unless the bank fails. As we will show later, the flexible contract value of equity allows funds to eliminate panic runs but comes at the cost of inefficient flows to fundamentals, which represent the comparative advantage and disadvantage of liquidity provision by equity-issuing intermediaries respectively.

[32] Similarly to the analysis of bank above, the ability of fund equity liquidity provision is general and holds qualitatively if we instead compare the optimally determined $E[c^*_{f,1}(\lambda_f)]$ to the autarky outcome $c^*_{a,1}$ in equilibrium under certain sufficient conditions. See Appendix A for a formal analysis.

Having analyzed the NAV at $t = 1$, the NAV at $t = 2$ is determined by

$$c_{f,2}(\lambda_f) = NAV_2(\lambda_f) = \frac{1}{1-\lambda_f}\left(x_{f,1} + y_{f,1}R\right) , \tag{2.12}$$

where

$$x_{f,1} = \max\{x_{f,0} - \lambda_f c_{f,1}(\lambda_f), 0\} ,$$

and $y_{f,1}$ is given by (2.11).

Taken together, the NAV rules (2.9), (2.10), and (2.12) and Lemma 1 lead to an important result:

Proposition 4. *$NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R > 1$.*

Proposition 4 has two important implications. The first one is that fund equity with fully flexible NAV is not subject to panic runs at all. This is because any late households' redemption decision is uniquely determined by the comparison between NAV_2 and NAV_1, which depends on economic fundamentals R but not any strategic motives by households. In contrast, in the original Diamond and Dybvig (1983) framework, bank liquidity provision goes hand in hand with panic-based bank runs.

The second implication of Proposition 4 is that the comparison between NAV_2 and NAV_1 is directly linked to fundamentals R, which households infer from the signal they receive. This direct link between fund NAVs and fundamentals immediately leads to the friction underlying fund liquidity provision, as we illustrate below. In contrast, for a bank, the deposit value is only indirectly linked to fundamentals through the potential for bank runs.

The above two implications play a key role in understanding late households' optimal redemption decision at $t = 1$ because they compare NAV_2 and NAV_1 to decide whether to redeem or not. Suppose that after observing s_i, w_f late households choose to redeem at $t = 1$, the total portion of redeeming households will be

$$\lambda_f = \pi + w_f$$

since early households always redeem. Given (2.9) and (2.12), the amount of late householders w_f who choose to redeem at $t = 1$ will be

$$\begin{cases} w_f = 0 & \text{if} \quad u(c_{f,1}(\lambda_f)) < E[u(c_{f,2}(\lambda_f))|s_i]\,, \\ w_f \in (0, 1-\pi) & \text{if} \quad u(c_{f,1}(\lambda_f)(1-\kappa w_f)) = E[u(c_{f,2}(\lambda_f))|s_i]\,, \\ w_f = 1-\pi & \text{if} \quad u(c_{f,1}(\lambda_f)(1-\kappa w_f))) > E[u(c_{f,2}(\lambda_f))|s_i]\,. \end{cases} \tag{2.13}$$

Solving (2.13) with Proposition 4 in mind yields the late households' optimal redemption decision at $t = 1$:

Proposition 5. *Given date-0 fund position $(x_{f,0}, y_{f,0})$ and signal s_i received by households at $t = 1$, late households withdrawing from the fund amount to*

$$w_f^* = \begin{cases} 0 & \text{if} \quad R \geq 1 \\ \dfrac{1}{2}\left(1 - \pi - \sqrt{\dfrac{2}{4(1-x_{f,0})(R-1) + \kappa(1-\pi)^2}}\right) & \text{if} \quad R < 1 \text{ and } x_{f,0} \geq \pi \\ \dfrac{1-R}{\kappa} & \text{if} \quad R < 1 \text{ and } x_{f,0} < \pi \end{cases} \tag{2.14}$$

subject to $w_f^* \leq 1 - \pi$.

We denote the negative relationship between redemptions w_f and fundamentals R as the flows-to-fundamentals relationship.[33] Notice that in contrast to withdrawals induced by bank runs, there is no strategic element driving the flows to fundamentals. Figure 2 illustrates the result of Proposition 5 for funds and Proposition 1 for banks. The blue and red lines indicate the relationship between equilibrium outflows and economic fundamentals for debt-issuing banks and equity-issuing funds respectively. The abrupt increase in liquidations at R^* for the red line indicates the presence of bank runs whereas the negative slope of the blue line corresponds to the flows-to-fundamentals relationship for funds.

Given the flows-to-fundamentals relationship, the fund liquidates the project in order to meet redemption requests. The following relationship between fund flows and fundamentals results:

[33] Further, w_f^* is also increasing in $1 - \pi$ and ϕ while decreasing in $x_{f,0}$ and κ.

Proposition 6. *Given the date-0 fund position* $(x_{f,0}, y_{f,0})$ *and the signal* s_i *received by households at* $t = 1$*, the fund liquidates*

$$l_f^* = \begin{cases} 0 & \text{if} \quad x_{f,0} \geq \pi + w_f^* \\ \dfrac{1 - \sqrt{1 + 2\phi\left(\pi^2 + x_{f,0} - \pi(1 + x_{f,0})\right)}}{(1-\pi)\phi} & \text{if} \quad R \geq 1 \text{ and } x_{f,0} < \pi + w_f^* \\ \dfrac{1 - \sqrt{1 + 2\phi\left((\pi + w_f^*)^2 + x_{f,0} - (\pi + w_f)(1 + x_{f,0})\right)}}{(1 - \pi - w_f^*)\phi} & \text{if} \quad R < 1 \text{ and } x_{f,0} < \pi + w_f^* \end{cases} \tag{2.15}$$

units of project at $t = 1$ *subject to* $l_f^* \leq y_{f,0}$*, where* w_f^* *is given by Proposition 5.*

Proposition 6 is graphically illustrated in Figure 3 along with the corresponding result for debt-issuing banks. It shows both bank debt and fund equity are subject to inefficient liquidations albeit with different characteristics arising from panic runs at banks and the flows-to-fundamentals relationship at funds.[34]

Taken together, Propositions 5 and 6 show that liquidity provision by equity-issuing funds is exposed to inefficient premature liquidations arising from the sensitivity of equity valuation to fluctuations in economic fundamentals. This flow-to-fundamentals friction hurts NAV_1 and thereby limits equilibrium liquidity provision according to Definition 2.

Proposition 6 also suggests that more stored cash between $t = 0$ and 1 can alleviate the sensitivity of liquidations to fundamentals through three channels. First, when $R \geq 1$ but $x_{f,0} < \pi$, the fund can directly use stored cash to meet early household redemptions, incurring a lower liquidation l_f. Second, a higher $x_{f,0}$ translates to lower $y_{f,0}$, which leads to a lower premature project liquidation l_f. Finally, because the marginal project liquidation value is decreasing in the unit of projects liquidated, a higher $x_{f,0}$ also implies a weaker flow-to-fundamentals relationship. The fund will take these channels into account when maximizing ex-ante household utility to choose a storage-project mix at $t = 0$ that optimally trades off liquidity provision to early households and return generation to late households.

[34] Also note that l_f^* is increasing in ϕ, decreasing in $x_{f,0}$, and concave in R due to the flexible adjustment of NAV_1.

Taking the above analysis into account, the representative fund solves the optimal portfolio allocation $(x_{f,0}, y_{f,0})$ at $t = 0$ to maximize the expected utility of all households:

$$\max_{x_{f,0}, y_{f,0}} E\left[\lambda_f u(c_{f,1}(\lambda)) + (1-\lambda_f) u(c_{f,2}(\lambda))\right] \tag{2.16}$$

subject to (2.9), (2.11), (2.12), and (2.15). As we have already shown in Proposition 3, fund equity indeed provides liquidity ex-ante in any equilibrium.

3 Liquidity Provision Index

Our model shows that both deposit-funded banks and equity-funded funds are able to pool resources to provide liquidity but that they are subject to different endogenous frictions. To quantify how much liquidity is produced by the two types of financial intermediaries, we map our model to the data and develop the Liquidity Provision Index (LPI). In line with Definition 2, the LPI aims to measure by how much more a household cashing out at short notice expects to obtain through holding and liquidating deposit claims of a bank (shares of a fund) relative to directly holding and liquidating the bank's (fund's) underlying assets themselves. In other words, it is the empirical proxy for $E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0})$ as in Definition 2.[35]

While most components for the LPI can be directly obtained from balance sheet data and market haircuts, one challenge remains in calculating $E[c^*_{k,1}(R)]$ because economic fundamentals R are not directly measurable. Fortunately, our theory, in particular, Propositions 1 (for bank debt) and 5 (for fund equity), suggests that the fundamental R affects liquidity provision through affecting late households' withdrawal (redemption) decisions, captured by λ in the model. Thus, as long as we observe the distribution of actual household withdrawals (redemptions), that is, outflows from the respective intermediaries in the data, we are able to calculate $E[c^*_{k,1}(R)]$ through calculating $E[c^*_{k,1}(\lambda(R))]$ without relying on the distribution of R per se. We therefore change the state variable from the fundamentals, R, to the the fundamentals driven outflows, λ, which have unambiguous empirical counterparts, to facilitate the construction of the LPI.

[35] Notice that the LPI remains agnostic about the long run consumption, or $E[c^*_{k,2}]$ in the model. As discussed in the introduction, the long run return, or, in other words, the cost of liquidity provision, is an important but separate question we address in Ma, Xiao and Zeng (2019).

Below we first provide a simple example to illustrate the construction of the LPI. Then we provide a formal step-by-step explanation of the construction and discuss the connection to the theory in detail.

3.1 An Example

Consider a hypothetical mutual fund that holds 10% cash and 90% of illiquid assets, where the illiquid asset can only be converted to a fraction, say 70%, of their fair value upon early redemption. When all investors demand to redeem their shares early, the per unit contract payment would only be $1 \times 0.1 + 0.7 \times 0.9 = 0.73$ because all illiquid assets would be prematurely liquidated and incur a discount. Notice that an investor directly holding 0.1 of cash and 0.9 of the illiquid assets receives the same 0.73 if she has to liquidate early because she would also have to sell her entire portfolio as well. In this sense, holding a share at the fund does not improve the consumption value when all other fund investors also withdraw early because all risk is systematic and liquidity insurance is ineffective.

In any other case when not all investors redeem early, however, idiosyncratic liquidity risk is pooled at the fund level and holding fund shares reduces the average discount suffered and improves the amount of cash obtainable upon short notice. For example, if total outflows amount to less than 10% of the funds assets, they can be met by just using the fund's cash and the the investor can obtain the full contract payment according to fund NAV, which is 1, when she liquidates early. If redemptions exceed 10%, more illiquid assets would have to be liquidated and the contract payment would decrease until it reaches 0.73 at a 100% outflows. This trend is depicted graphically by the blue line in Figure 4. The dotted line indicates the liquidation value of the underlying portfolio 0.73.

Similarly, a hypothetical bank with 10% cash and 90% of illiquid assets also provides a higher liquidation value than 0.73 as long as not all depositors prematurely withdraw from the bank because of the pooling of idiosyncratic liquidity shocks. However, as the red line in Figure 4 shows, the liquidation value does not continuously decrease with the percentage of outflows because the face value of deposit contracts is fixed until the default threshold is crossed whereas the equity value of mutual fund shares flexibly adjusts with the actual amount of liquidation.

For a given level of outflows, the difference between the red (blue) line and the dotted line shows the increase in liquidation value when the bank's (fund's) asset are held through the bank (fund) relative to directly held by the investor herself. It captures the contribution of the bank (fund) to liquidity available and hence its liquidity provision for a given amount of outflows.

Finally, to calculate expected liquidity provision, we need to know the distribution of redemptions/withdrawls. Note that Figure 4 does not imply that all outflow volumes will actually occur. In fact, our model specifically predicts how redemptions in equilibrium depends on the use of debt versus equity contracts and the subsequent portfolio choice on the asset side. We therefore let the data speak by taking the empirical distribution of fund and bank deposit flows as the equilibrium outcome of early redemptions. This allows us to calculate the expected liquidity provision for each bank and fund as its Liquidity Provision Index.

3.2 Construction and Data

More formally, we can generalize the LPI construction into three steps.

Step 1: The first step is to calculate the contract payment by outflows as in Figure 4. This requires knowing how much of each asset is liquidated to meet a given proportion of redemptions/withdrawals and at what cost.

Let Ω_i be the amount of outflows at bank (fund) i as a percentage of total assets. Ω_i is the empirical counterpart to the number of investors that withdraw in the model, λ. We also extend the two-asset world of the model to allow for a complete portfolio distribution. Let vector A_{it} be bank (fund) i's asset portfolio distribution at time t with A_{ijt} denoting the portfolio weight on asset $j \in J$. The asset index, j, is ranked in increasing order with the haircut of the asset h_{jt}, i.e., $h_{jt} \leq h_{j't}$ for any $j \leq j'$. In other words, more liquid assets have smaller indexes. The ranking of liquidation haircuts and the distribution of assets determine which assets are to be liquidated for a given outflow as banks (funds) meet redemption requests by selling more liquid assets first.[36] To illustrate, recall the above example where a bank (fund) holding 10% of cash can meet outflows of up to 10% without losses. As outflows increase, banks (funds) use increasingly more illiquid asset categories to meet their redemption requests until even the most illiquid assets are liquidated when outflows (hypothetically) reach 100%.

[36] Jiang, Li and Wang (2017), among others, identify this order of liquidation in the data.

While the contract payment of fund shares follows that of fund assets due to flexible NAVs, bank deposits do not lose value until there are not enough funds left to pay the face value. Notice that, to focus on the difference in contract forms, we only consider those deposit contracts without deposit insurance throughout our calculations, for which the contract payment will fall below the face value if proceeds from asset liquidations are insufficient to pay the face value of debt. But the LPI construction is indeed general enough to handle insured deposits, as well as to capture and isolate the effect of deposit insurance on uninsured deposits (see Appendix B for details). Denoting $1 - H_t(\Omega_i, A_{it})$ as the contract payment for investors withdrawing (redeeming) a dollar of deposits (fund shares) when outflows are Ω_i under bank (fund) asset distribution A_{it}, we have for bank i:

$$H_t(\Omega_i, A_{it}) = \begin{cases} 0 & \text{if the bank survives} \\ \sum_{j=1}^{J} A_{ijt} h_{jt} & \text{if the bank fails}\,, \end{cases} \tag{3.1}$$

and for fund i:

$$H_t(\Omega_i, A_{it}) = \sum_{j=1}^{J_\Omega - 1} A_{ijt} h_{jt} + \widehat{A_{iJ_\Omega t}} h_{J_\Omega t}\,, \tag{3.2}$$

where $\widehat{A_{iJ_\Omega t}}$ is the position of the least liquid asset being liquidated to meet Ω_i while assets more liquid than J_Ω are fully liquidated.

Regarding data, we use bank call reports and mutual fund holdings from the CRSP database to obtain the distribution of assets A_{it} for banks and funds respectively.[37] For haircuts, we use collateral haircuts in repo markets and discounts on loan sales in secondary markets. Following Bai, Krishnamurthy and Weymuller (2018), we collect securities haircuts from the New York Fed's repo data series, commercial loan haircuts from the Loan Syndications and Trading Association and real estate loan haircuts from the Federal Home Loan Banks.[38] Also similar to Bai, Krishnamurthy and Weymuller (2018), we smooth the series to reduce the impact of outliers.[39] Figure 5 plots the predicted haircut series for different asset categories over our sample period. As expected, safe assets such as Treasuries have smaller and less volatile haircuts whereas relatively illiquid assets such as loans have higher and more volatile haircuts.

Step 2: Since contract payments vary with outflows, the expected contract payment depends on the distribution of outflows. We take the empirical distribution of bank deposit flows (mutual

[37] Please refer to the Online Appendix for details.

[38] We combine different data sources similar to Bai, Krishnamurthy and Weymuller (2018) as there is no one market in which all types of assets under consideration are traded and observed.

[39] Please refer to the Online Appendix for details.

fund flows) to proxy for early withdrawals (redemptions) realized in equilibrium.[40] With the estimated distribution $F(\cdot)$, we can derive the bank and fund level expected contract payment as

$$\int (1 - H_t(\Omega_i, A_{it}))dF(\Omega_i)\,. \tag{3.3}$$

Viewed through the lens of the model, this expected contract payment of bank deposits (fund shares) proxies for the expected consumption available to bank depositors (fund share holders) withdrawing prematurely at the end of the first period, $E[c^*_{k,1}(\lambda)]$, taking into the possibility of panic runs for a bank (the flow-to-fundamentals relationship for a fund). Importantly, the difference between $t = 0$ and 1 in Diamond and Dybvig (1983)-type models captures an ultra-short-term period in reality. Thus, when we take the model to data to measure liquidity provision in reality, the construction of the contract payment as in (3.3) also reflects an ultra-short time period in which the net return on assets are negligible[41,42] and focuses on the shape of the function $c^*_{k,1}(\lambda)$, which directly depends on flows.

For banks, we calculate flows as percentage changes in total bank liabilities over total bank assets, using bank call reports from 2010 to 2017. Since deposit flows account for the vast bulk of short-term changes in bank liabilities, our results are robust to alternative approaches such as directly measuring deposit flows or non-equity flows. We augment the resulting flow distribution with data on bank failures, in which case we assign an outflow of 100%.[43] For each mutual fund, flows are calculated using its monthly (portfolio-level) total net asset changes using the CRSP database. For positive flows, we assume that no assets are prematurely liquidated and that the effective contract payment is one.

Step 3: Finally, we calculate and subtract the liquidation value an investor expects to obtain by directly holding the bank's (fund's) assets on her own as the benchmark. In this case, she has

[40] Our theory predicts how early redemptions/withdrawals vary with the use of debt versus equity and its subsequent choice in asset portfolio. Our empirical approach directly measures the realized flows, which can be seen as the equilibrium outcome of this decision-making process.

[41] This is consistent with our model that between $t = 0$ and 1, the investment project has not yet come to fruition and thus retains a value of one, and thus the fund's NAV is also one at the beginning of $t = 1$. For a bank, although the level of the optimal deposit value at $t = 1$ can be theoretically different from one in a static setting following Diamond and Dybvig (1983), the effective ultra-short-term net asset return is still negligible.

[42] While the focus of the LPI is short-term liquidity provision, or $E[c^*_{k,1}]$, the long run consumption, $E[c^*_{k,2}]$, is also an important element of our model. It relates to the cost of liquidity provision and maps to observed deposit rates (fund returns). The cost effectiveness of liquidity provision is analyzed in a separate paper, Ma, Xiao and Zeng (2019).

[43] This is consistent with our model that when a panic run occurs, all late households withdraw at $t = 1$.

to sell the entire portfolio to meet her own withdrawal needs, which is equivalent to the effective contract payment of the bank (fund) with outflows equal to one, i.e., $1 - H_t(1, A_{it})$.

Taken together, the LPI of bank (fund) i at time t is the expected liquidity it provides relative to direct holding:

$$LPI_{it} = \int (1 - H_t(\Omega_i, A_{it}))dF_i(\Omega_i) - (1 - H_t(1, A_{it})), \tag{3.4}$$

where $F_i(\Omega_i)$ is the estimated distribution of bank deposit (fund) outflow Ω_i and $H_t(\Omega_i, A_{it})$ is given by (3.1) and (3.2).

To summarize, we have the following close mapping between the theory and empirics as shown in Table 2:

Table 2: Mapping between equilibrium outcomes and empirical counterparts

	Outflows	Asset holdings	Contract Payment	Liquidation Value
Theory	λ	y_0^*	$E[c_{k,1}^*(\lambda)]$	$c_1(x_{k,0}^*, y_{k,0}^*)$
Empirics	Ω	A	$\int(1 - H_t(\Omega_i, A_{it}))dF(\Omega_i)$	$1 - H_t(1, A_{it})$

4 Liquidity Provision by Bank Debt and Fund Equity

We apply the LPI to compare and contrast liquidity provision by two important intermediaries using debt and equity financing respectively: commercial banks and bond mutual funds. We focus on uninsured deposits and tease out any indirect effects from regulatory features such as deposit insurances and non-deposit liabilities. We also apply the LPI to money market funds (MMFs) as an identification, focusing on around the 2016 MMF Reform when institutional MMFs were required to switch from debt to equity funding. It also serves as an independent application to demonstrate the generality of the LPI – it applies to any intermediaries issuing demandable claims.

4.1 Construction and Aggregate Results

We follow the procedure described in Section 3.2 to construct the LPI for each bank-quarter and each fund-quarter in our sample. We report the aggregate results to illustrate the magnitude.

We first construct the contract payment as a function of outflow for uninsured bank deposits and mutual funds shares in our sample. We report the aggregate results in Figure 7. This aggregate result can be thought of as a representative bank holding the average portfolio of the banking sector and a representative fund holding the same proportion of assets as the bond mutual fund sector.[44] Similar to the Example in Section 3.1, the contract payment for funds continuously declines with early redemptions because fund shares flexibly adjust to reflect the incurred haircuts when increasingly illiquid assets are liquidated. Banks also sell more illiquid assets as deposit withdrawals increase but the uninsured debt contract guarantees a constant contract payment until the bank defaults, i.e, deposit withdrawals cannot be met at their promised value.

When outflows reach 100%, an intermediary has to fully liquidate the underlying portfolio, and the contract payment of uninsured bank deposits and mutual fund shares reach 78% and 95% respectively, as shown in Figure 7. These two numbers, by construction, are also the liquidation values of the underlying portfolios. These two numbers differ because the assets that banks and funds hold in equilibrium are different with banks' portfolio being more illiquid. One rationale provided by our theory is that the use of debt versus equity contracts induce different incentives for holding liquid assets. As our theoretical framework implies, deposit funded banks hold liquid assets to reduce the incidence of panic runs whereas equity funded funds hold liquid assets to reduce the flow to fundamentals sensitivity. The aggregate result is consistent with the latter concern being more pronounced in the time period we examine.

We then estimate the flow distribution for each bank and fund in our sample. Figure 6 plots the distribution of bank and fund flows from 2011 to 2017 for a representative bank and a representative fund. As we argued before, these empirical flow distributions indeed take into account the impact of fundamentals on flows in equilibrium, that is, panic runs for a bank and flow-to-fundamentals for a fund. For example, the two panels show that the distribution of fund flows Figure 6 is more dispersed than that of bank flows, consistent with our theory prediction

[44] The size of the sector does not affect the result because both outflows and contract payments are expressed as percentages.

that fund equity is subject to a more volatile flow-to-fundamentals relationship (whereas panic runs are relatively less seen in the sample period we focus).

Finally, we use the estimated flow distribution for each intermediary to calculate the LPI for each intermediary-quarter, using the constructed contract payment function and the calculated liquidation value in the first step. Up to this point, we are able to assemble a panel of LPIs at the intermediary-quarter level.

4.2 Liquidity Provision in the Cross-section

Figure 8 plots the distribution of bank and fund level LPIs in the cross-section. It shows that an average dollar invested in funds and banks contributes 0.04 and 0.22 more to early consumption respectively. In other words, according to the LPI, liquidity provision in mutual funds is economically significant and comprises about a fifth of that by commercial banks per unit investment.

Although our contribution of bank LPI only consider the uninsured portion of deposits, one may ask whether regulatory features such as deposit insurances and non-deposit liabilities may indirectly affect the estimated LPI through bank portfolio choice. As explained in Appendix B in detail, deposit insurance, along with other similar regulatory policies, does not affect the nature and validity of the LPI construction, but it indeed affects the magnitude of LPI through affecting investors outflows and intermediary portfolio choices in equilibrium.

Empirically, to perfectly isolate the indirect effects of deposit insurance and other regulatory policies requires the ideal experiment of the same bank operating without deposit insurance. Although this ideal experiment does not exist, we perform two tests to show that regulation has some influence but does only partially explain the difference in liquidity provision by debt versus equity funded intermediaries; the bulk of the difference arises from the difference in contract forms.

We first directly relate bank-level LPIs to the ratio of insured deposits in the data and project the LPI that would have applied with zero-insured deposits in Table 3. The constant term in column (1), which is statistically significant, indicates that without deposit insurance, the LPI for uninsured deposits in an average bank in our sample would be about 0.16, which is a 27% drop from the average LPI of 0.22 for banks. In other words, deposit insurance has indirectly

contributed 0.08 to the LPI of uninsured bank deposits, but the remaining 0.16 is still significantly higher than 0.04, the average LPI of mutual fund shares. Similarly, column (2) indicate that a linear-projected bank without insured deposits or non-deposit liabilities would have had an LPI of 0.15 for its uninsured deposits.

There may be remaining concerns of regulatory differences other than deposit insurance, some of which cannot be directly observed such as implicit guarantees by the government. To this end, we apply the LPI to a laboratory of money market funds (MMFs) before and after the Money Market Fund Reform as an identification of the pure effect arising from debt versus equity. Implemented in October 2016, the Securities and Exchange Commission's MMF Reform requires institutional prime MMFs to switch from reporting a \$1 fixed share price to floating net asset values, which effectively corresponds to a switch from debt to equity contracts in our framework. Importantly, among other dimensions of the reform, only the floating NAV rule did not apply to retail prime funds who continued reporting a \$1 fixed share price. This setting naturally lends itself to a difference-in-differences analysis with institutional and retail prime funds as treatment and control groups to study the difference in liquidity provision by debt and equity funded intermediaries.

Figure 11 plots the LPI of institutional and retail prime MMFs from three years before to three years after the reform. To account for changes in the sensitivity of fund flows, the flow distribution for the LPI calculation are constructed separately for the pre and post reform periods respectively. The lower panel repeats the analysis with the subset of funds that appeared in both pre and post reform periods. For both samples, institutional and retail prime fund LPI largely followed the same pattern before October 2016, agreeing with the parallel trends assumption. Since the onset of the reform, institutional MMFs experienced a significantly larger drop in LPI than retail funds. This trend is corroborated by a formal difference-in-differences test in Table 4. The coefficient on the interaction term reflects the difference in LPI for funds using debt versus equity contracts. Using the full sample of funds, the baseline result in column (1) shows that the LPI is 42/(203+9)=20% lower when the institutional MMFs switches from fixed to floating NAVs. Columns (3) and (4) repeat the analysis excluding flows around the implementation from August to October 2016 columns (5) and (6) shifts the treatment date a year back to account for potential anticipation effects. Even-numbered columns focus on the subsample of funds appearing in both pre and post period respectively.

The aforementioned results all point to the same message: the use of equity over debt in funding MMFs decreases liquidity provision, consistent with our bank-fund comparison that bank debt tends to provide more liquidity than fund equity, even in the absence of deposit insurance and other institutional and regulatory features. Independently, the application of LPI to MMFs also demonstrates the generality of LPI – it can be applied to any debt- and equity-issuing financial intermediaries.

In addition to the LPI gap between banks and funds, Figure 8 also displays dispersion in liquidity provision within the fund and banking sector respectively. For banks, as seen in Table 3, deposit insurance cannot fully explain but has a positive effect on the LPI, which is consistent with lower bank run probability allowing for more liquidity provision in the theory. The same trend is also depicted in the upper panel of Figure 10. Other factors, such as leverage ratio and the proportion of non-deposit types of funding may also affect the probability of runs while banks of different asset sizes may be subject to different regulatory constraints. The lower panel of Figure 10 shows that the relationship between LPI and deposit insurance ratio is robust to these variables.

Regarding mutual fund fund LPIs, our model shows that liquidity provision by equity is constrained by flows to fundamentals. In line with the theory, we find that the more sensitive fund flows are to changes in fund returns, the lower its liquidity provision as shown in Figure 9. While fund returns are used as an empirical measure for economic fundamentals, other features of the fund, such as size, age and expense ratio, may simultaneously influence outflows. The lower panel of Figure 9 shows that the negative relationship between liquidity provision and fund flow sensitivity is preserved upon including fund-level characteristics as controls. The corresponding regression results are in Table 5.

4.3 Liquidity Provision in the Time-series

We proceed to examine how liquidity provision has evolved over time. Plotting the quarterly weighted average of bank and fund level LPIs, Figure 12 shows that the difference has been narrowing over time with bank LPI sharply decreasing from 0.285 in 2011 to 0.190 in 2017 and fund LPI increasing from 0.039 in 2011 to 0.043 in 2017. In other words, within six years, liquidity provided by an average dollar invested in funds has approximately increased from a seventh to a quarter of an dollar invested in commercial banks.

While a full characterization of the trend's determinants is beyond the scope of this paper, we highlight that changes in the regulatory landscape have had significant impacts on the capacity of liquidity provision by commercial banks.

First is the increase in central bank reserves following Quantitative Easing. Excess reserves held with the Federal Reserve are liquid assets redeemable at short notice. The overall effect of more reserves on liquidity provision can go in both ways. It could have a positive effect because, as Proposition 2 suggests, more liquid bank balance sheets are less prone to runs. This is illustrated by the rightward shift of the default threshold for outflows from Ω to Ω' in Figure 13. On the other hand, a portfolio with more reserves also has a higher liquidation value under direct holding by investors, which is reflected by the increase in liquidation value of the bank portfolio from C to C' in Figure 13. Correspondingly, the potential increase and decrease in liquidity provision are indicated by the areas shaded in green and orange respectively.

Empirically, we find evidence consistent with the latter effect being dominant, that is, QE decreases the capacity of liquidity provision by banks. As shown in Figure 14, the expansion in excess reserves from 1 trillion in 2011Q3 to more then 2.5 trillion in 2014Q3 is mirrored by a corresponding sharp fall in bank LPI during the same period. Sorting banks into quartiles of reserve uptake as a proportion of balance sheet size, we observe that the LPI drops consistently more for banks in the upper quartile, which lends further support for the aggregate effect. Therefore, QE lowers bank LPI through raising the liquidation value under direct holding, which limits how much banks can contribute to liquidity provision in non-default states. In the limit, this argument also suggests that narrow banks, which only hold liquid assets such as reserves backed by deposits, create much less liquidity than traditional commercial banks.

In this context, the Liquidity Coverage Ratio (LCR) has similar effect on bank liquidity provision as Quantitative Easing. In the U.S., the LCR stipulates that banks with $250 billion or more in total assets or $10 billion or more in on-balance sheet foreign exposures are required to hold sufficient High Quality Liquid Assets (HQLA), which include cash, central bank reserves and some agency MBS, to cover expected net cash outflows for a 30-day stress period. Banks with $50 billion or more in consolidated assets are also subject to a less stringent LCR requirement. As a result, large banks hold a higher fraction of liquid assets on their balance sheets, which raises the default threshold in terms of outflows but also increases the liquidation value of the benchmark asset portfolio as in the case of QE shown in Figure 13.

We find evidence for an overall negative impact of the LCR on bank liquidity provision within our sample period. Figure 15 shows that banks most impacted by the LCR, i.e., above $250 billion in total assets, also experience the most pronounced decline in LPI relative to those without and with a less stringent LCR requirement. This is again consistent with the interpretation that the LCR moves commercial banks more towards a narrow-banking business model, for which the gap between the liquidation value of bank assets and the contract payment of bank liabilities (i.e., deposits) is reduced and liquidity provision is diminished.

5 Conclusion

This paper demonstrates that open-end equity issued by non-bank intermediaries is able to provide liquidity in the sense of Diamond and Dybvig (1983) just like demandable debt issued by the traditional banking sector. Liquidity creation stems from the pooling of idiosyncratic liquidity shocks at the intermediary level, which occurs independently of the claims issued by the intermediary as long as they are redeemable at short notice. The characteristics of liquidity provision, however, are different. Equity is not prone to panic runs as in the case of debt because flexible NAVs commit investors to internalize the externalities of their withdrawal decisions. However, the continuous adjustment of equity's contract value also renders investor outflows more sensitive to fluctuations in the economy.

Based on the theory, we develop the Liquidity Provision Index (LPI) as a parsimonious measure of liquidity creation across different types of intermediaries. It captures the extra proceeds an investor expects to obtain by withdrawing a debt or equity claim from an intermediary relative to directly holding and selling the underlying portfolio of assets herself. Applied to deposit-issuing commercial banks and equity-issuing bond mutual funds, we find that the average LPI of funds is positive and about a quarter of the LPI of banks at the end of 2017. The LPI gap between banks and funds has been continuously narrowing over time, coinciding with an increase in liquid assets on bank balance sheets following Quantitative Easing and the Liquidity Coverage Ratio. These results highlight a new side effect of unconventional monetary policy and post crisis liquidity regulation.

An important question that remains is how liquidity provision costs. This translates into the fees investors pay as well as the returns they expect to obtain when investing in a bank

or fund. The amount and cost of liquidity provision determine the competitive environment between different types of intermediaries in the real world because investors value both liquidity and returns while intermediaries optimally choose liquidity production and prices to cater to investors' demand. These questions are explored in Ma, Xiao and Zeng (2019) to shed light on the implications of financial intermediation moving beyond the traditional banking system.

Figure 2: Investor Withdrawals and Economic Fundamentals

This graph depicts withdrawals (ω) by late households against variation in economic fundamentals (R). The blue line represents the equilibrium withdrawal decisions of fund investors, where the negative slope indicates a flows-to-fundamentals relationship for funds. The red line corresponds to withdrawal decisions of bank investors, where the abrupt rise in withdrawals at R^* indicates the presence of panic runs, with R^* being the run threshold.

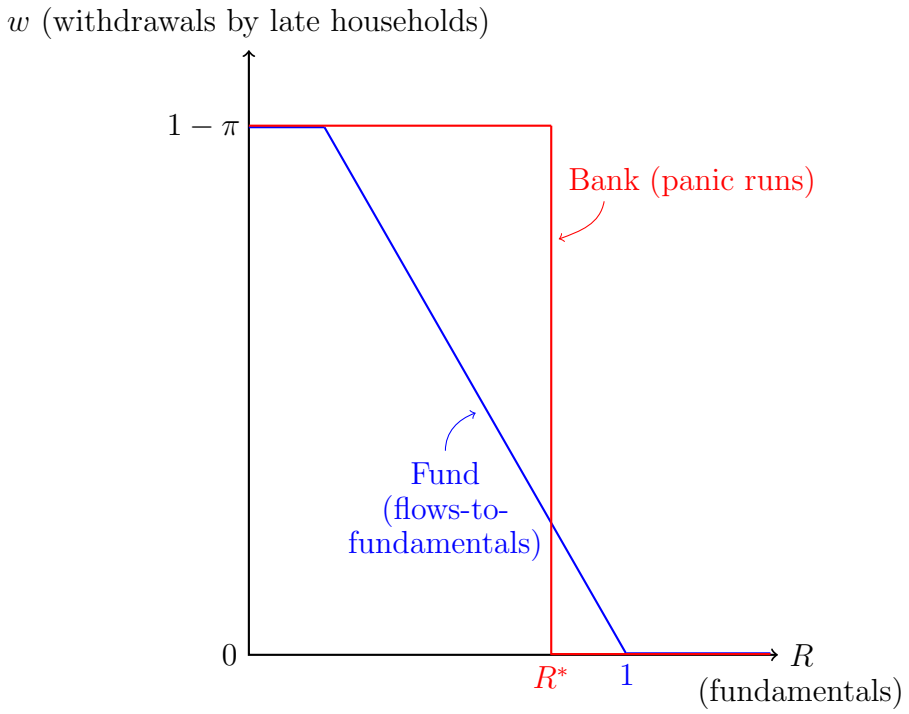


Figure 3: Premature Liquidations and Economic Fundamentals

This graph depicts the level of premature liquidations (l) against economic fundamentals (R). The blue line represents the premature liquidations at the fund, where the negative slope arises from a flows-to-fundamentals relationship. The red line corresponds to premature liquidations at the bank, where the abrupt rise in liquidations at R^* indicates the presence of panic runs, with R^* being the run threshold. Note that the fund curve is currently depicted under $x_{f,0} \geq \pi$. The flat portion for funds (when $R > 1$) is strictly higher than 0 if $x_{f,0} < \pi$.

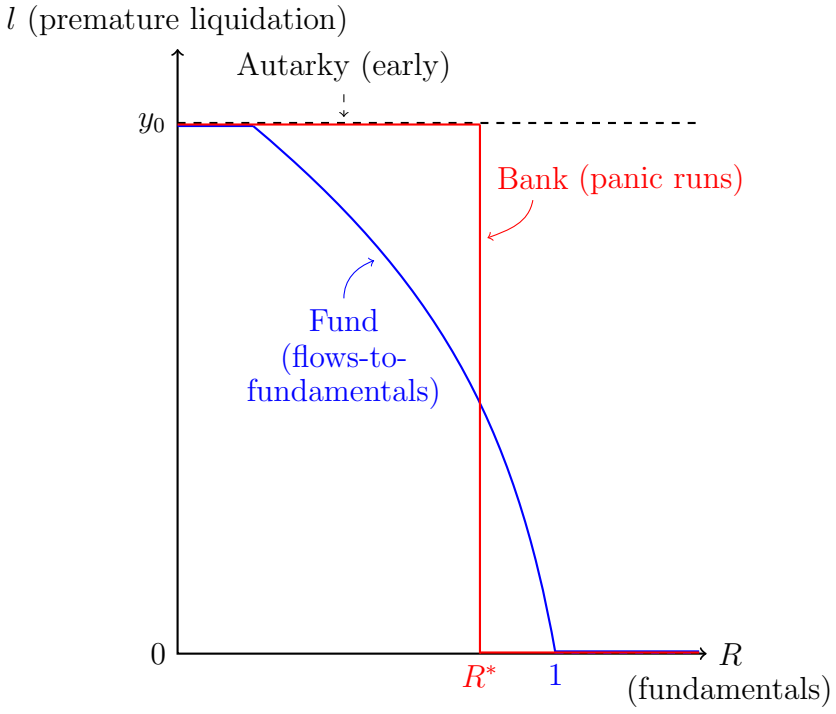


Figure 4: Contract Payment, Liquidation Value and Liquidity Provision (Example)

This graph plots the contract payment of bank deposits (fund shares) when a given percentage of the bank assets (fund assets) has been withdrawn. The hypothetical bank and fund considered both hold 10% of cash and 90% of illiquid assets, where the latter incur a haircut of 30% upon early redemption. The dotted line is the liquidation value of the portfolio of assets when directly held and sold by an investor.

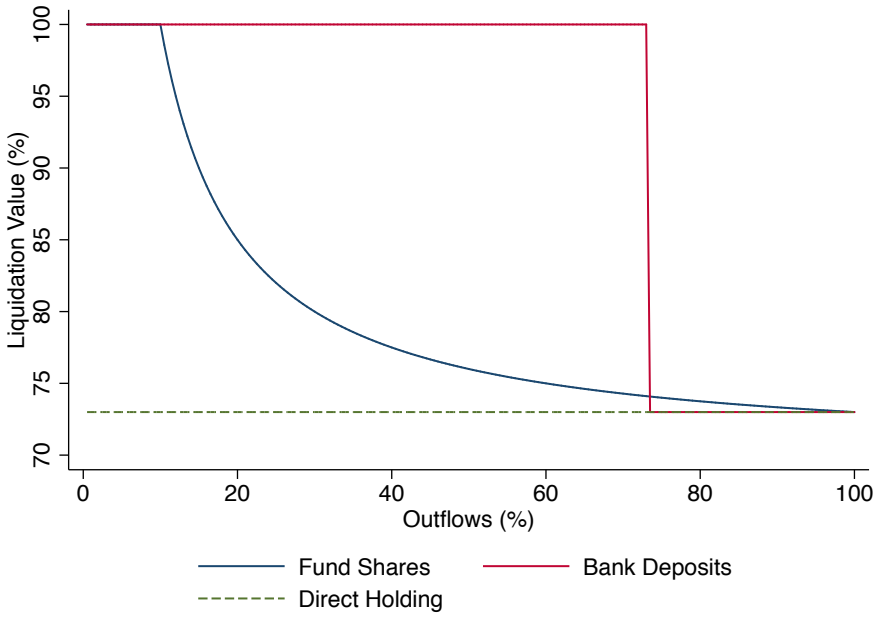
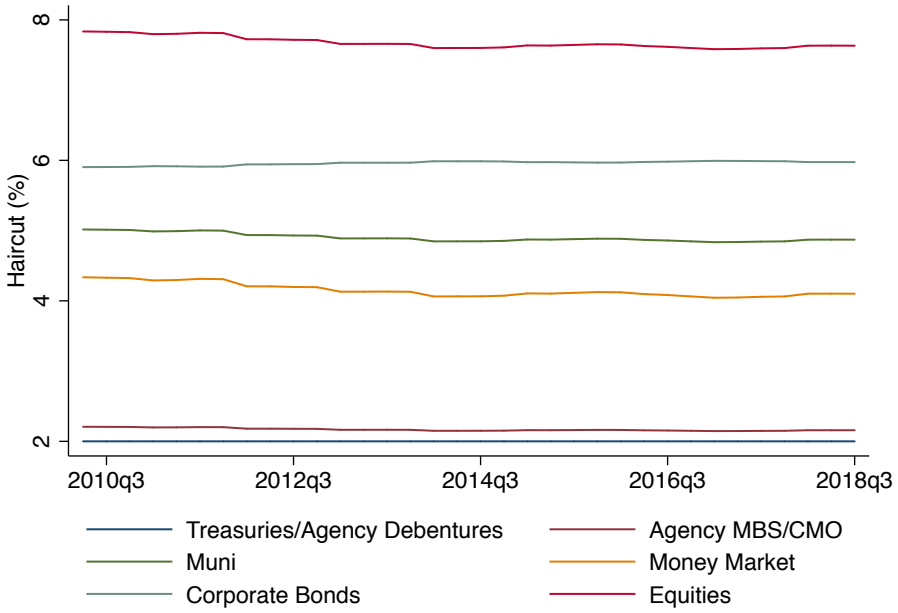


Figure 5: Haircuts

This graph plot the average market haircuts for different asset categories over time. Securites haircut data (upper panel) is obtained from the Federeal Reserve Bank of New York's pulished repo series. Haircuts for commercial loans, real estate loans and personal loans (lower panel) are from the Loan Syndications and Trading Association (LSTA), the Federal Home Loan Banks website and the Federal Reserve respectively. To remove outliers in the original data, we calculate the first principal component of the underlying series and plot the predicted value from the loadings regression $h_k = a_k + b_k PC_t + \epsilon_{kt}$ for each asset category k.



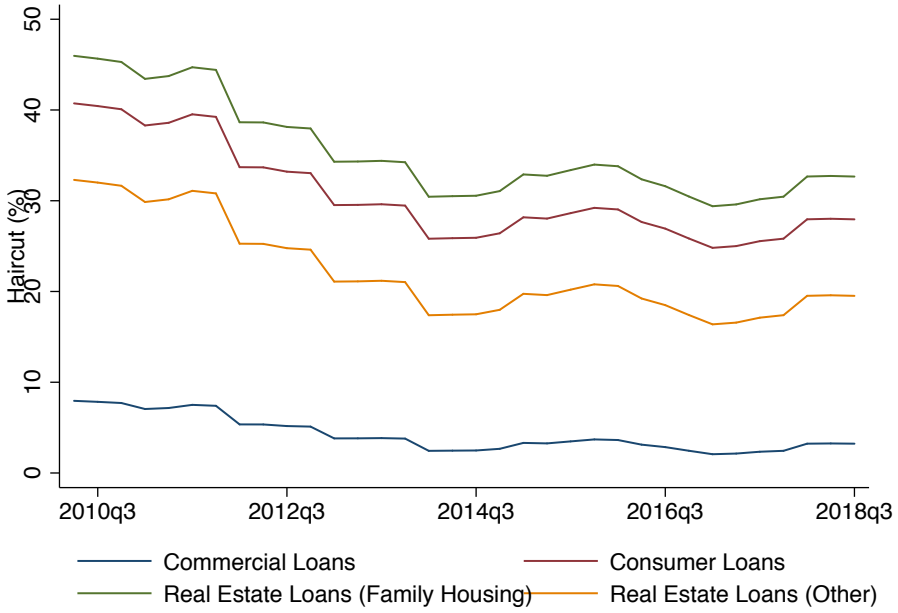
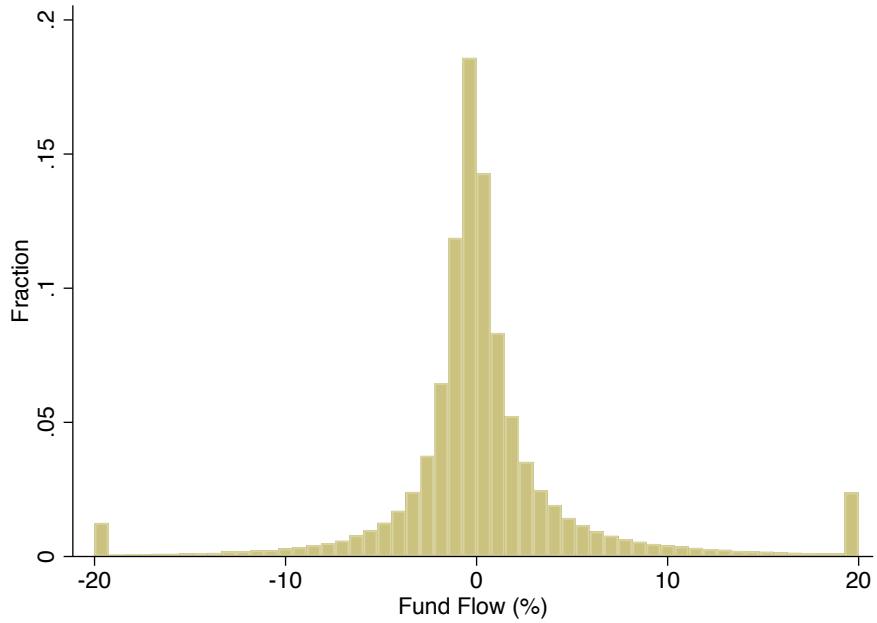


Figure 6: Distribution of Bank and Fund Flows

This graph plots frequency distribution of monthly flows of funds (upper panel) and bank liabilities (lower panel). The sample covers all commercial banks and mutual funds in our sample from 2011 to 2017. For presentation purposes, the distribution is truncated at -20% and 20% with observations in the tail assigned to the last bin.



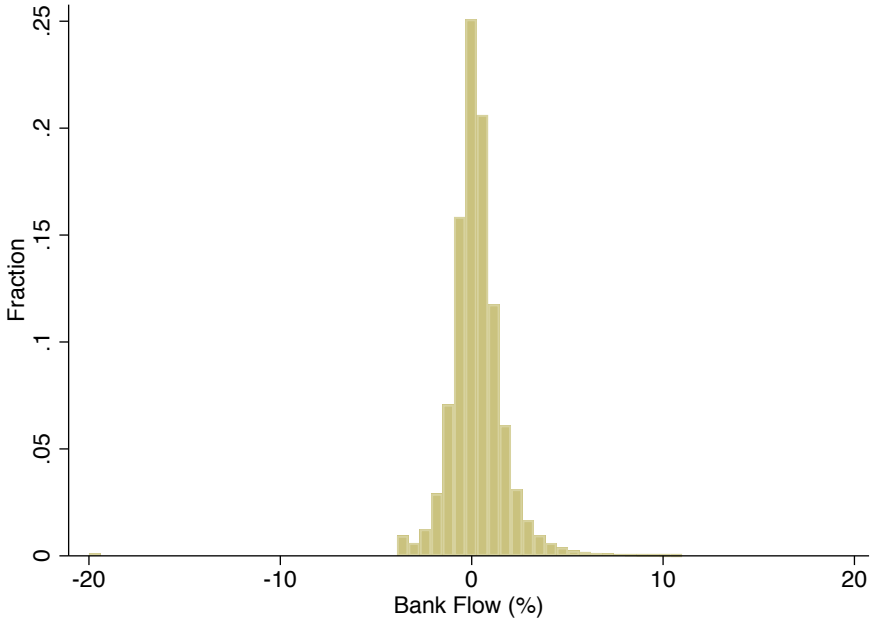


Figure 7: Contract Payment, Liquidation Value and Liquidity Provision

This graph plots the contract payment of bank deposits (fund shares) when a given percentage of bank assets (fund assets) have been withdrawn. The bank (fund) asset porfolio reflects the volume weighted average of all commercial banks' (bond mutual funds') portfolios during our sample period from 2011 to 2017. The dotted lines represent the liquidation value of the portfolio of bank (fund) assets when directly held and sold by an investor.

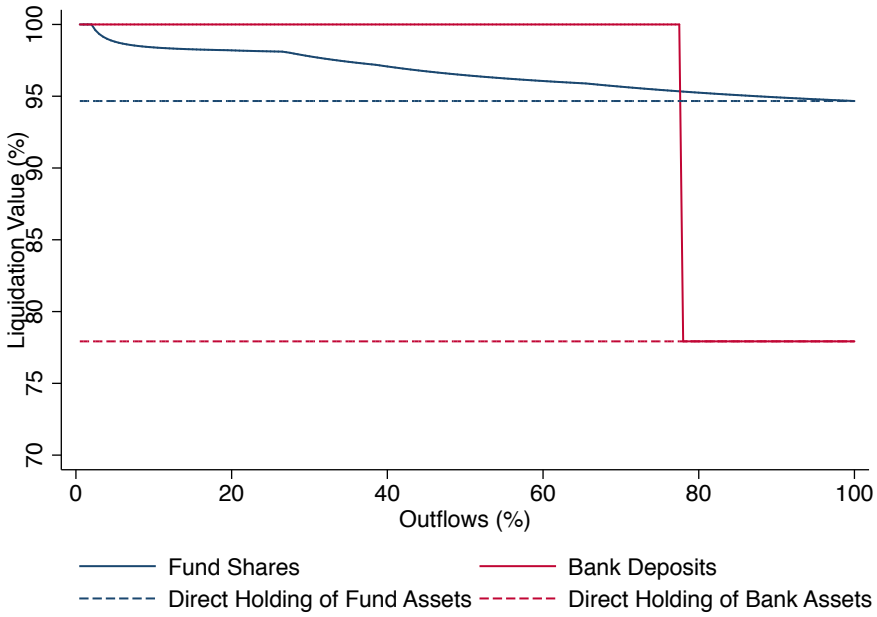


Figure 8: Cross-section of Bank and Fund Liquidity Provision

This graph plots the distribution of average commercial bank and bond mutual fund LPIs in the cross-section. The LPI for each bank and fund is calculated as the average LPI over the sample period from 2011 to 2017.

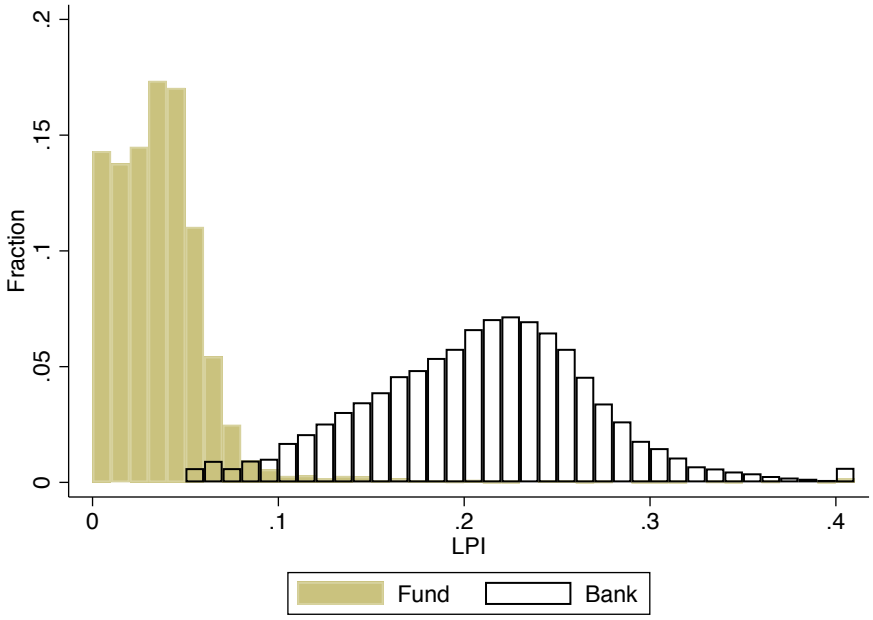
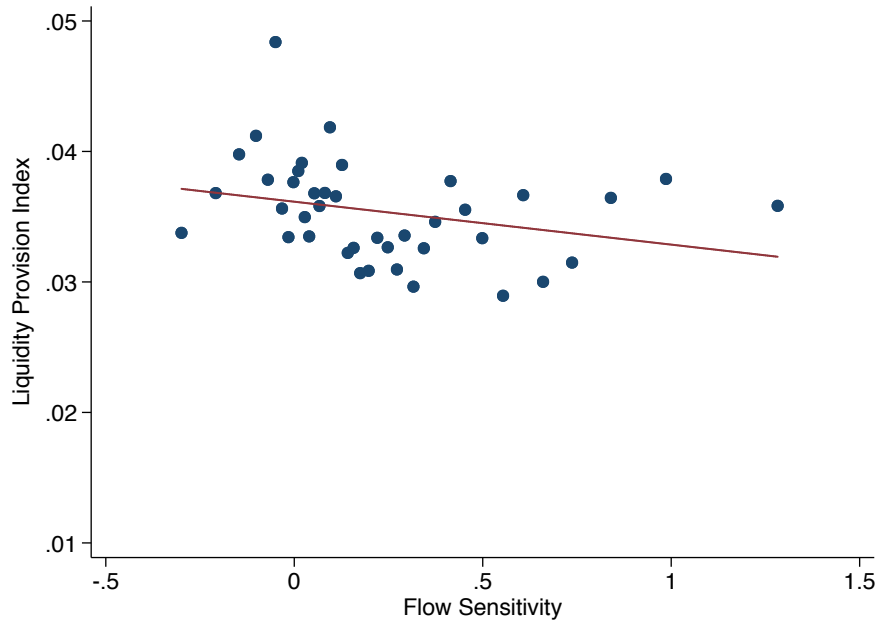


Figure 9: Sensitivity of Fund Liquidity Provision

This graph plots average LPI against fund flow sensitivity at the fund level for bond mutual funds. Fund level LPIs are averaged over the sample period from 2011 to 2017. Fund flow sensitivity is obtained by regressing fund flows against lagged fund returns and lagged fund flows from 2011 to 2017. The upper panel is a univariate binned plot while the lower panel also controls for log(fund age), log(assets) and expense ratio by residualizing the variables on the controls before binning and plotting.



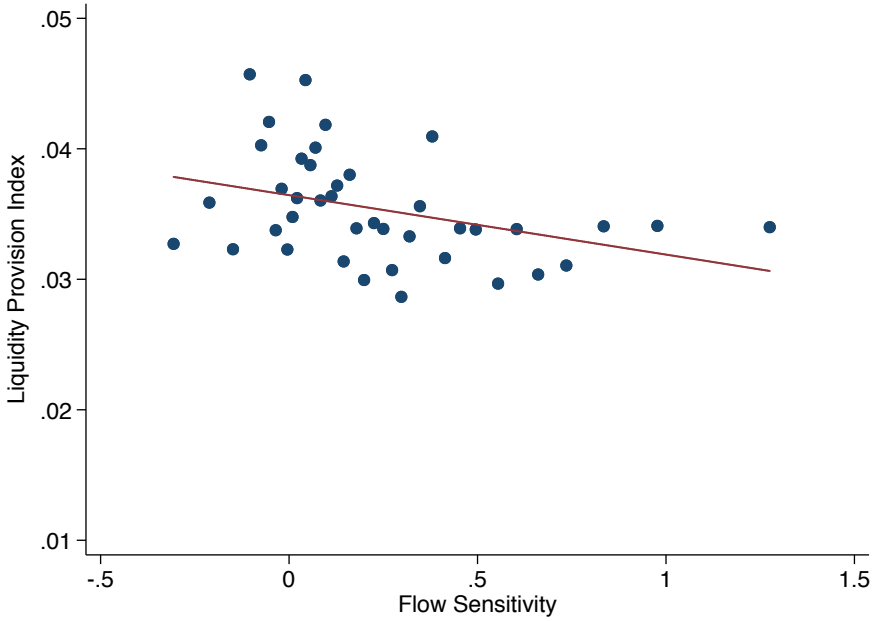
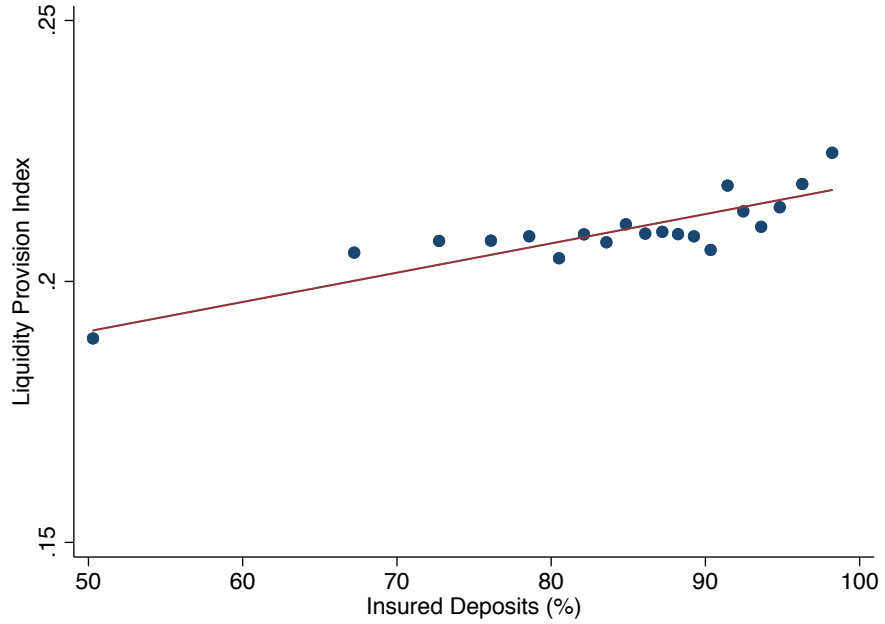


Figure 10: Sensitivity of Bank Liquidity Provision

This graph plots average LPI against the ratio of insured deposits over total deposits at the bank level for commercial banks. Bank level LPIs and insured deposit ratios are averaged over the sample period from 2011 to 2017. The upper panel is a univariate binned plot while the lower panel also controls for log(assets), equity ratio and the ratio of non-deposit liabilities over total assets by residualizing the variables on the controls before binning and plotting.



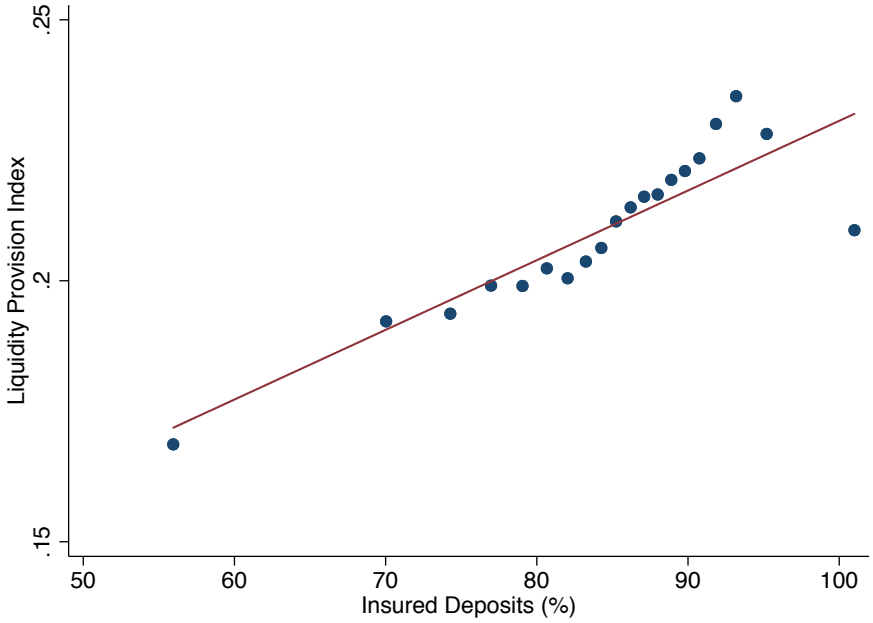
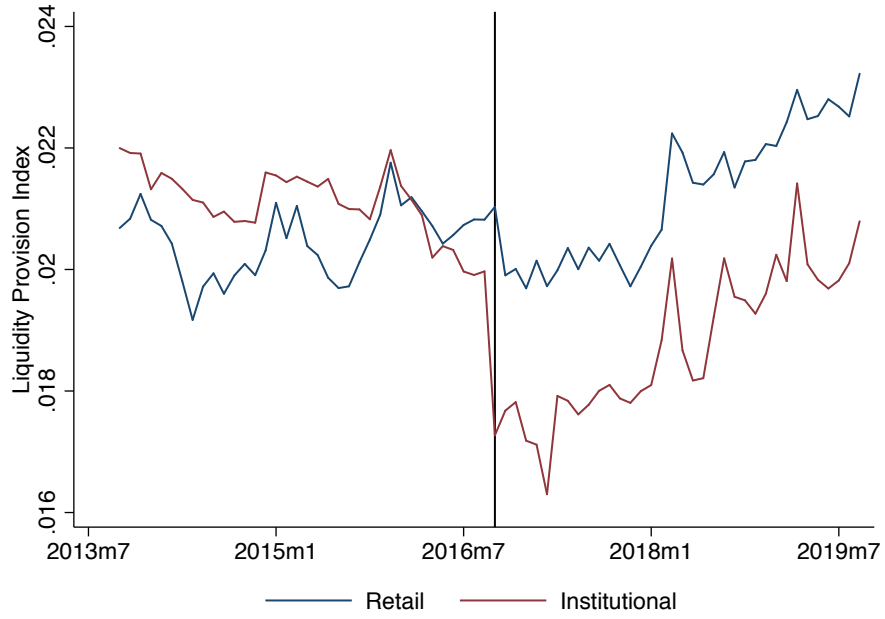


Figure 11: MMF Liquidity Provision and Floating NAVs

This graph plots average LPIs of institutional and retail prime MMFs from September 2013 to September 2019. The vertical line marks the implementation of the MMF Reform, which requires institutional prime funds to report floating NAVs. The upper panel covers the full sample whereas the lower panel restricts fund share-classes that appear at least once in both the pre and post reform period.



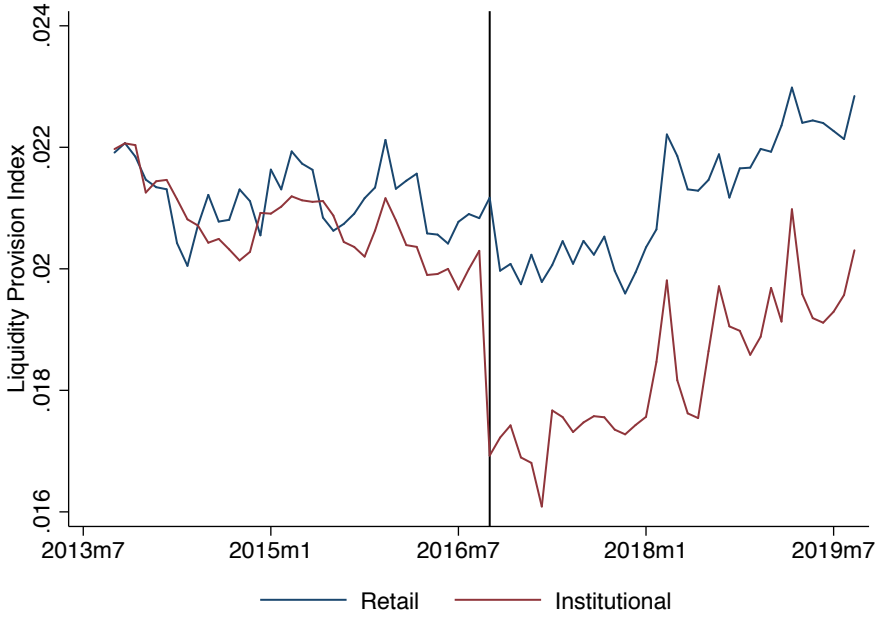


Figure 12: Liquidity Provision Index from 2011 to 2017

This graph plots the average LPI for commercial banks and bond mutual funds from 2011 to 2017. We first calculate the LPI for each bank and fund in each quarter and then plot the asset-size weighted LPI from 2011 to 2017.

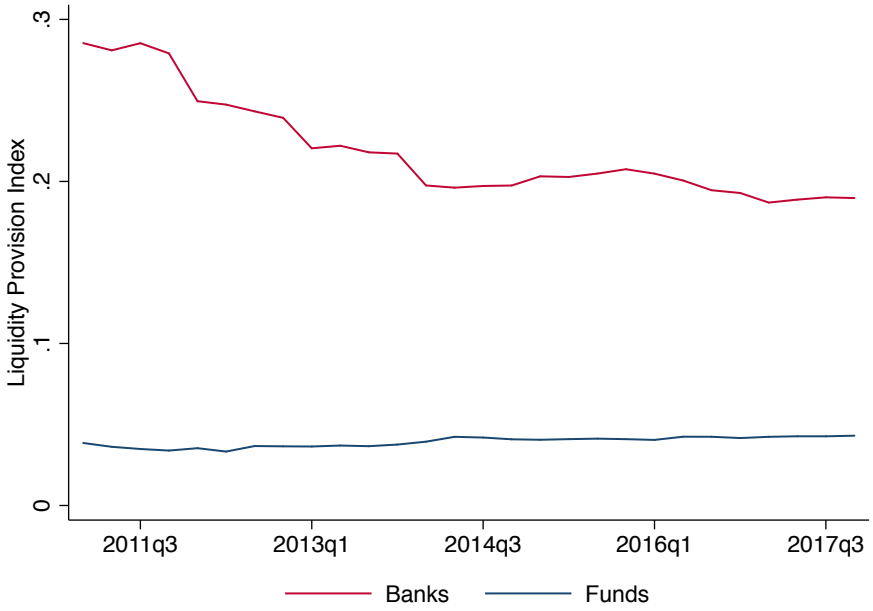


Figure 13: Liquidity Provision Pre- and Post-QE and LCR

This graph plots illustrates the contract payment of bank deposits when a given percentage of bank assets have been withdrawn before and after the implementation of QE (LCR). The liquidation value follows the orange line pre QE (LCR) and shifts to the green line post QE (LCR). This shifts the default threshold for outflows from Ω to $\Omega^{'}$ and the liquidation value of the asset portfolio under direct holding from C to $C^{'}$.

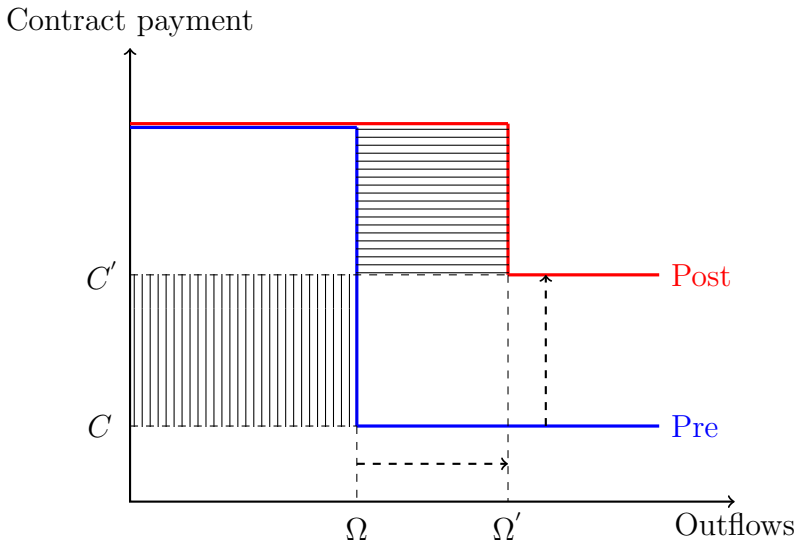


Figure 14: Bank Liquidity Provision and Excess Reserves

This graph plots the LPI for commercial banks by reserve uptake quartile (left axis) and the aggregate volume of excess reserves (right axis) from 2011 to 2017. Reserve uptake is measured by the percentage change in reserves as a fraction of total assets from 2011Q1, the beginning of the sample, to 2014Q3, when reserve levels peak. The median LPI in each quartile is selected, normalized by its initial value in 2011Q1 and plotted.

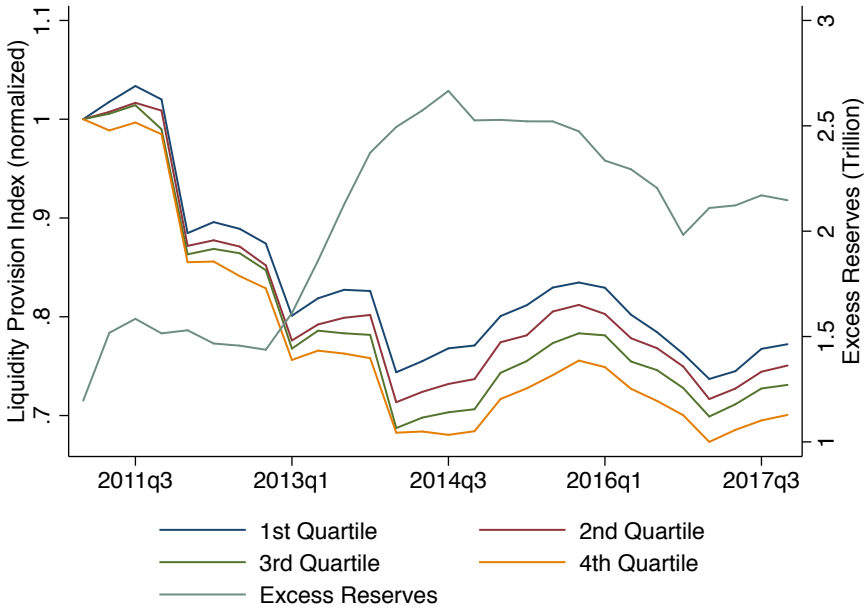


Figure 15: Bank Liquidity Provision and the Liquidity Coverage Ratio

This graph plots the LPI for commercial banks by asset size groups for the Liquidity Coverage Ratio. Banks are sorted by asset size into those above 250 billion, between 50 and 250 billion and below 50 billion. The median LPI in each asset group is selected, normalized by its initial value in 2011Q1 and plotted.

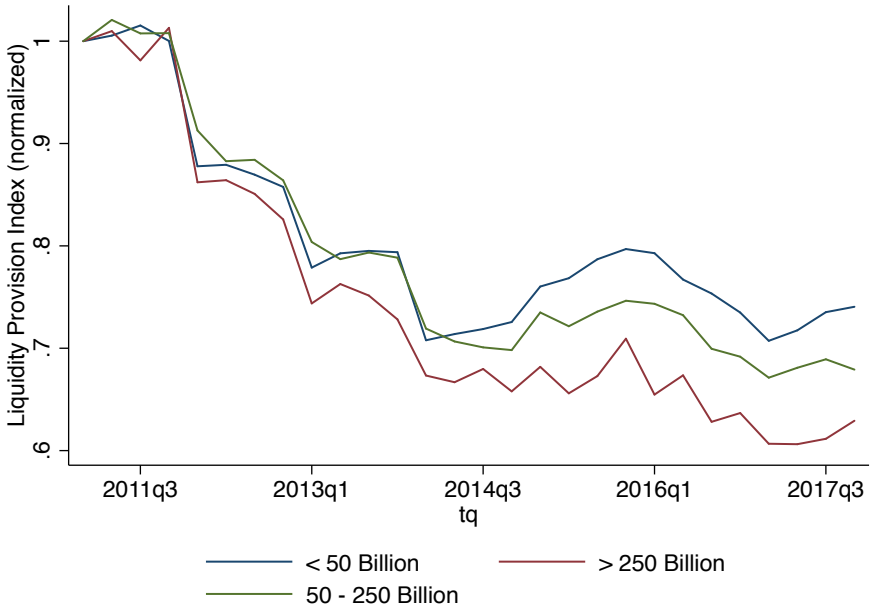


Table 3: Bank LPI and Insured Deposits Ratios

This table shows the relationship between bank LPI and the ratio of insured deposits. Control variables include non deposit ratio over all liabilities, and bank size measured by log(assets). Bank LPI and all other variables are averaged over the sample period from 2011 to 2017. Note that the constant term in the second column represents the expected LPI of banks without insured deposits or non-deposit liabilities.

	(1)	(2)	(3)
	LPI	LPI	LPI
Insured Deposits Ratio	0.055***	0.064***	0.144***
	[0.006]	[0.006]	[0.006]
Non-deposits Ratio		0.052***	0.034***
		[0.008]	[0.008]
Log(assets)			0.017***
			[0.001]
Constant	0.164***	0.147***	-0.127***
	[0.005]	[0.006]	[0.010]
Observations	7535	7535	7535

Table 4: The Effect of Floating NAV on MMF Liquidity Provision

This table shows the effect of the 2016 MMF Reform on liquidity provision by institutional prime funds versus retail prime funds. *Institutional Fund* is a dummy variable for the treatment group as the floating NAV requirement of the MMF reform only applies to institutional but not retail share-classes . *Post Reform* is an indicator variable for the treatment period. For columns (1) to (2), the treatment period begins with the official implementation date of the reform in October 2016. Columns (3) and (4) repeat the analysis in columns (1) and (2) but exclude flows around the implementation period from August to October 2016. Columns (5) and (6) account for potential anticipation effects by setting the treatment period to begin one year earlier in October 2015. Columns (2), (4) and (6) restrict the sample to the set of fund share classes that appear in both the pre and post period, that is, those funds which survived the reform. The dependent variable is share-class level LPI averaged for both the pre and post period. The coefficient on the interaction variable, *Post Reform* $*$ *Institutional Fund*, corresponds to the difference-in-differences result. Standard errors are clustered at the share-class level.

	(1)	(2)	(3)	(4)	(5)	(6)
	LPI	LPI	LPI	LPI	LPI	LPI
	All Oct 16	Survived Oct 16	All Oct 16 Excl. Aug-Oct	Survived Oct 16 Excl. Aug-Oct	All Oct 15	Survived Oct 15
Post Reform	0.0006	-0.0009	0.0004	-0.0012	-0.0003	-0.0018***
	[0.0009]	[0.0009]	[0.0009]	[0.0010]	[0.0007]	[0.0007]
Institutional Fund	0.0009*	-0.0005	0.0014**	0.0000	0.0023***	0.0016***
	[0.0006]	[0.0007]	[0.0006]	[0.0007]	[0.0006]	[0.0006]
Post Reform * Institutional Fund	-0.0042***	-0.0030**	-0.0040***	-0.0030**	-0.0040***	-0.0032***
	[0.0012]	[0.0013]	[0.0012]	[0.0013]	[0.0009]	[0.0009]
Constant	0.0203***	0.0213***	0.0208***	0.0221***	0.0210***	0.0219***
	[0.0005]	[0.0006]	[0.0005]	[0.0006]	[0.0005]	[0.0005]
Observations	659	452	646	429	745	624

Table 5: Fund LPI and Fund Flow Sensitivity

This table shows the relationship between fund LPI and the sensitivity of fund flows to lagged fund returns. Other control variables include the expense ratio, log(total net assets) and log(fund age). Fund LPI and all other variables are averaged over the sample period from 2011 to 2017.

	(1) LPI	(2) LPI	(3) LPI	(4) LPI
Flow Sensitivity	-0.329**	-0.328**	-0.330**	-0.456***
	[0.158]	[0.158]	[0.158]	[0.154]
Expense Ratio		0.014	0.014	0.013
		[0.020]	[0.020]	[0.019]
Log(Total Net Assets)			0.000	0.001***
			[0.000]	[0.000]
Log(Fund Age)				-0.009***
				[0.001]
Constant	0.036***	0.036***	0.035***	0.049***
	[0.001]	[0.001]	[0.002]	[0.002]
Observations	2626	2626	2626	2626

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Appendix

A Autarky, Liquidation Value, and Liquidity Provision

In this appendix, we first formally characterize an autarky outcome and show that it is tightly linked to the definition of liquidation value as in Definition 1. Then we show that our definition of liquidity provision, that is, Definition 2, is tightly related but preferable to an alternative definition based on the autarky.

Consider households live in autarky without access to financial intermediaries. They choose their portfolio $(x_{a,0}, y_{a,0})$ at $t = 0$, where $x_{a,0}$ is the amount of cash stored in the storage and $y_{a,0}$ the amount of projects invested before knowing their types.

At the beginning of $t = 1$, the household learns her type and receives the signal s_i. An early household always liquidates all her projects regardless of s_i, and consume the cash:

$$\begin{aligned} c_{a,1} &= x_{a,0} + y_{a,0} - \frac{\phi}{2} y_{a,0}^2 \\ &= 1 - \frac{\phi}{2} y_{a,0}^2 \,, \end{aligned} \tag{A.1}$$

where $(x_{a,0}, y_{a,0})$ represents her portfolio choice at $t = 0$. An important observation from (A.1) is that what an early household can get in the autarky is indeed the liquidation value of her portfolio, as defined in Definition 1.

Late households, however, will only liquidate the project and store the proceeds as cash if she expects a sufficiently poor performance after observing s_i. Formally, conditional on s_i, the late household's optimal portfolio choice problem at $t = 1$ is given by

$$\max_{l_a \geqslant 0} E\left[u(c_{a,2}) | s_i\right] \,,$$

where

$$c_{a,2} = x_{a,1}(1 - \kappa) + y_{a,1} R \,, \tag{A.2}$$

where in turn

$$\begin{cases} x_{a,1} = x_{a,0} + l_a - \dfrac{\phi}{2} l_a^2 \,, \\ y_{a,1} = y_{a,0} - l_a \end{cases}$$

are the late household's position at the end of $t = 1$.[45] It is easy to show that, given the date-0 position $(x_{a,0}, y_{a,0})$ and the signal R received at $t = 1$, an early household always liquidates all the illiquid asset holding $y_{a,0}$ regardless of R, while a late household optimally liquidates

$$l_a = \frac{1}{\phi}\left(1 - \frac{R}{1-\kappa}\right) \tag{A.3}$$

units of project at $t = 1$, subject to $l_a \geq 0$ and $l_a \leq y_{a,0}$.

Taking the decisions at $t = 1$ into account, households optimally choose their portfolios at $t = 0$ before knowing their type:

$$\max_{x_{a,0}} E[\pi u(c_{a,1}) + (1-\pi)u(c_{a,2})]\,,$$

subject to conditions (A.1), (A.2), (A.3). Without fully solving for the equilibrium, it is straightforward to see that the optimal $c^*_{a,1}$ is strictly lower than 1 when the household invests in a positive amount of projects at $t = 0$, that is, $y^*_{a,0} > 0$, and hence $x^*_{a,0} < 1$.

The analysis above about the autarky allows us to link our definition of provision, as in Definition 2, to an alternative definition in which one compares the t-1 consumption level promised by an intermediary and the autarky outcome. Formally, by the Principle of Optimization, we have:

Proposition 7. *If an intermediary $k \in \{b, f\}$ provides liquidity in the sense that $E[c^*_{k,1}] - c^*_{a,1} > 0$, it must provide liquidity in the sense that $E[c^*_{k,1}] - c_1(x^*_{k,0}, y^*_{k,0}) > 0$.*

Proposition 7 suggests that liquidity provision based on Definition 2 is a necessary condition of liquidity provision under the alternative definition based on autarky. Thus, given the advantages of Definition 2 that 1) it is conceptually tightly linked to the insight of Diamond and Dybvig (1983) and 2) is all relies on empirically observable outcomes and thus can be more directly mapped to empirics, we use it as our preferred definition of liquidity provision.

B Deposit Insurance

In this appendix, we provide a detailed example showing that our framework, both theoretically and empirically, is robust to the consideration of deposit insurance and other similar institutional and regulatory features. Our empirical LPI construction is indeed general enough to handle deposit insurance. It also illustrates that deposit insurance contributes to bank liquidity provision but only partially.

[45] One unit of cash invested in the storage yields $1 - \kappa$ units of cash since one unit of late households is using the storage.

Consider a hypothetical bank that holds 10% cash and 90% of illiquid loans, where the loans, if liquidated at short notice, can be only recovered at 60% of their fair value (i.e., the haircut is 40%). Different from our baseline model, suppose this bank has 50% insured deposits and 50% uninsured deposits. Also suppose that the empirical distribution of outflows is uniform on [0,1]. (To be precise, we only take the 50-50 mix of insured and uninsured deposits as exogenously given in this example; the distribution of outflows and the bank portfolio should be viewed as endogenously determined by household and bank optimization.) Under this example, the liquidation value of the bank portfolio is $1 \times 0.1 + 0.6 \times 0.9 = 0.64$. Thus, for an insured depositor, a dollar invested in the insured deposit generates an LPI of $1 - 0.64 = 0.36$. While for an uninsured depositor, a dollar invested in the uninsured deposit generates an LPI of $(1 - 0.64) \times 0.64 = 0.2304$.

Now suppose for this same bank, all deposits become uninsured. According to the theoretical model of Allen, Carletti, Goldstein and Leonello (2018),[46] few insured deposits lead to a higher run threshold (i.e., a higher run probability), and consequently, a more liquid bank portfolio. Consistent with this argument, suppose the distribution of outflows becomes a triangular distribution with a densify function of $f(\lambda) = 2\lambda$, and the bank portfolio becomes 20% cash and 80% loans. Under this case, the liquidation value of the bank portfolio becomes $1 \times 0.2 + 0.6 \times 0.8 = 0.68$, which is higher than before. For an uninsured depositor, a dollar invested in the uninsured deposit generates an LPI of $\int_0^{0.68}(1-0.68)2\lambda d\lambda = 0.1479$. Compared to the case above, the LPI decreases by $0.2304 - 0.1479 = 0.0825$, which suggests that for this actual bank with a 50-50 mix of insured and uninsured deposits, deposit insurance contributes 0.0825 towards its total LPI of 0.2304.

The example above has two important implications. First, it clearly illustrates that, what our LPI captures is indeed the amount of liquidity provision by uninsured bank deposits. In other words, by design, the LPI is independent to the consideration of deposit insurances. Second, because the LPI construction uses information about the two equilibrium outcomes of 1) the empirical distribution of flows and 2) the bank portfolio, and deposit insurance affects bank liquidity through affecting these two equilibrium outcomes, our LPI as an empirical measure of bank liquidity provision is indeed robust to the consideration of deposit insurances.

[46] Similar to our baseline model, Allen, Carletti, Goldstein and Leonello (2018) build a global-games based model with a different focus on the interplay between general government guarantees (including deposit insurance) and bank runs.

C Proofs

Proof of Proposition 1. Denote the run threshold as $R' = R(\theta')$, that is, if household i observes a private signal $s_i < \theta'$ she runs; otherwise she stays. Then the population of households who runs, λ_b, can be written as

$$\lambda_b(\theta, \theta') = \begin{cases} 1 & \text{if } \theta \leq \theta' - \varepsilon \\ \pi + (1-\pi)\left(\frac{\theta' - \theta + \varepsilon}{2\varepsilon}\right) & \text{if } \theta' - \varepsilon < \theta \leq \theta' + \varepsilon \\ \pi & \text{if } \theta > \theta' + \varepsilon \end{cases}.$$

Let $v(R(\theta), \lambda_b)$ be the difference of utilities between staying and running, then

$$v(R(\theta), \lambda_b) = \begin{cases} u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R(\theta)}{1-\lambda_b}\right) - u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right) & \text{if } \pi \leq \lambda_b < \frac{x_{b,0}}{c_{b,1}} \\ u\left(\frac{\left(y_{b,0} - l_b\left(\lambda_b, c_{b,1}\right)\right) R(\theta)}{1-\lambda_b}\right) - u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right) & \text{if } \frac{x_{b,0}}{c_{b,1}} \leq \lambda_b < \frac{1 - \frac{\phi}{2} y_{b,0}^2}{c_{b,1}} \\ -q(\lambda_b)\, u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right) & \text{if } \frac{1 - \frac{\phi}{2} y_{b,0}^2}{c_{b,1}} < \lambda_b \leq 1 \end{cases},$$

where $l_b(\lambda_b, c_{b,1})$ satisfies $\lambda_b c_{b,1} = x_{b,0} + l_b - \frac{\phi}{2} l_b^2$. If household i observes signal s_i, given that other households use the threshold strategy, she will run if $\int_{s_i - \varepsilon}^{s_i + \varepsilon} v(R(\theta), \lambda_b(\theta, \theta'))\, d\theta > 0$; or stay otherwise.

To prove that there exists a unique run threshold R^*, we need to prove that there is a unique θ^* such that if $\theta' = \theta^*$, the household who observes signal $s_i = \theta' = \theta^*$ is indifferent between run and stay. That is,

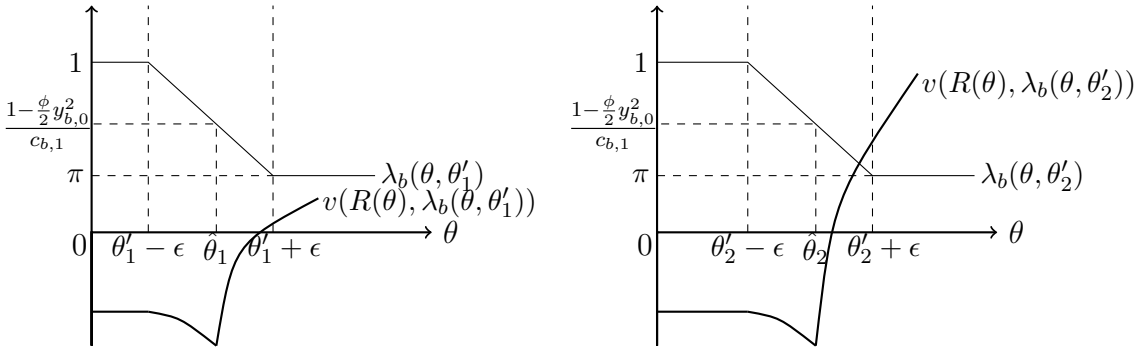
$$V(\theta^*) \equiv \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} v(R(\theta), \lambda_b(\theta, \theta^*))\, d\theta = 0.$$

The graph of $v(R(\theta), \lambda_b(\theta, \theta'))$ is depicted in Figure A1, where $\hat{\theta}$ satisfies $\lambda_b\left(\hat{\theta}, \theta'\right) = \frac{1 - \frac{\phi}{2} y_{b,0}^2}{c_{b,1}}$. It is easy to check that v is constant on $(0, \theta' - \varepsilon)$, decreasing on $\left(\theta' - \varepsilon, \hat{\theta}\right)$ and increasing on $(\hat{\theta}, 1)$. Figure A1 also illustrates how v changes when θ' increases. As θ' increases, the integral of v on $(0, \theta' - \varepsilon)$ remains the same since v does not directly depends on θ in this interval and the length of the interval is constant given ε; on $(\hat{\theta}, \theta' + \varepsilon)$, for any given λ_b, if θ' goes up, v increases because all θ in this interval goes up. Thus the integral on $(\hat{\theta}, \theta' + \varepsilon)$ increases. In summary, $V(\theta')$ is increasing in θ'.

Since $G(\cdot)$ is supported on $[0, +\infty)$, $R \to +\infty$ when $\theta \to 1$. That is, $\lim_{\theta' \to 1} V(\theta') = +\infty$. Furthermore, $V(0) < 0$. Then by the intermediate value theorem, there exists θ^* such that $V(\theta^*) = 0$. The uniqueness of θ^* follows by the monotonicity of $V(\cdot)$ and so does R^*.

Intuitively, the threshold R^* must larger than the lower dominance region $\underline{R}$ since the households face more risks in the sense that bankrupt may happen. $\underline{R}$ is pinned down by letting households be in-

Figure A1: The graph of $v(R(\theta), \lambda_b(\theta, \theta'))$



different between run and stay if there is no withdrawal, that is, $u\left(\frac{x_{b,1}+y_{b,1}\underline{R}}{1-\lambda_b}\right) = u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)$, or, $\underline{R} = \frac{c_{b,1}(1-\kappa)(1-\pi)+\pi c_{b,1}-x_{b,0}}{1-x_{b,0}}$. ∎

Proof of Proposition 2. In the proof of Proposition 1, we know that $V(\cdot)$ is increasing and θ^* is pinned down by $V(\theta^*) = 0$. Thus, to prove that $\frac{d\theta*}{dc_{b,1}} > 0$, it is sufficient to prove that $V(\theta')$ is decreasing in $c_{b,1}$. Note that when $\theta \in (\theta' - \varepsilon, \theta' + \varepsilon)$, $\pi + (1-\pi)\left(\frac{\theta'-\theta+\varepsilon}{2\varepsilon}\right) = \lambda_b$, so we have $d\theta = -\frac{2\varepsilon}{1-\pi}d\lambda_b$. Then $V(\theta')$ can be rewritten as

$$
\begin{aligned}
V(\theta') &= \int_{\pi}^{1} v\left(R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda-\pi}{1-\pi}\right)\right), \lambda_b\right)\frac{2\varepsilon}{1-\pi}d\lambda_b \\
&= \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \frac{2\varepsilon}{1-\pi}\left(u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0}R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda-\pi}{1-\pi}\right)\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
&+ \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \frac{2\varepsilon}{1-\pi}\left(u\left(\frac{(y_{b,0} - l_b(\lambda_b, c_{b,1}))R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda-\pi}{1-\pi}\right)\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
&- \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} \frac{2\varepsilon}{1-\pi}q(\lambda_b)\, u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)d\lambda_b.
\end{aligned}
$$

Let $V = 0$, which pins down θ^* and thus R^*. And since V is continuous in ε, we can take the limit of the equation above at $\varepsilon \to 0$:

$$\begin{aligned}\int_{\pi}^{1} v\left(R\left(\theta^*\right), \lambda_b\right) d\lambda_b = & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \left(u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R\left(\theta'\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right)\right) d\lambda_b \\ & + \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \left(u\left(\frac{\left(y_{b,0} - l_b\left(\lambda_b, c_{b,1}\right)\right) R\left(\theta'\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right)\right) d\lambda_b \quad \text{(C.1)} \\ & - \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} q\left(\lambda_b\right) u\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right) d\lambda_b .\end{aligned}$$

Multiplying both sides of (C.1) with $c_{b,1}$ and taking derivative with respect to $c_{b,1}$, we have the derivative expressed by

$$\begin{aligned} & c_{b,1} \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \left(-\frac{\lambda_b}{1-\lambda_b} u'\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R\left(\theta^*\right)}{1-\lambda_b}\right) - (1-\kappa\left(\lambda_b - \pi\right)) u'\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right)\right) d\lambda_b \\ + \; & c_{b,1} \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \left(-\frac{\frac{\partial l_b}{\partial c_{b,1}} R\left(\theta^*\right)}{1-\lambda_b} u'\left(\frac{\left(y_{b,0} - l_b\left(\lambda_b, c_{b,1}\right)\right) R\left(\theta^*\right)}{1-\lambda_b}\right) - (1-\kappa\left(\lambda_b - \pi\right)) u'\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right)\right) d\lambda_b \\ - \; & \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} \frac{1-\frac{\phi}{2}y_{b,0}^2}{\lambda_b} (1-\kappa\left(\lambda_b - \pi\right)) u'\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right) d\lambda_b \\ + \; & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \left(u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R\left(\theta^*\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right)\right) d\lambda_b \\ + \; & \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \left(u\left(\frac{\left(y_{b,0} - l_b\left(\lambda_b, c_{b,1}\right)\right) R\left(\theta^*\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right)\right) d\lambda_b \\ - \; & \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} q\left(\lambda_b\right) u\left(c_{b,1}(1-\kappa\left(\lambda_b - \pi\right))\right) d\lambda_b , \end{aligned}$$

where $\frac{\partial l_b}{\partial c_{b,1}} = \frac{\lambda_b}{1-\phi l_b}$ and $\frac{\partial q}{\partial c_{b,1}} = -\frac{1-\phi y_{b,1}^2}{\lambda_b c^2} < 0$. Note that it suffices to prove that

$$\begin{aligned} & c_{b,1} \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} -\frac{\lambda_b}{1-\lambda_b} u'\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b \\ + \; & c_{b,1} \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} -\frac{\frac{\partial l_b}{\partial c_{b,1}} R\left(\theta'\right)}{1-\lambda_b} u'\left(\frac{\left(y_{b,0} - l_b\left(\lambda_b, c_{b,1}\right)\right) R\left(\theta^*\right)}{1-\lambda_b}\right) d\lambda_b \\ + \; & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b \end{aligned}$$

$$+ \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} u\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b < 0\,. \tag{C.2}$$

Integrating by parts, we have the LHS of (C.2) re-expressed as:

$$\begin{aligned}
&-\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)\\
+&\quad\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\pi\left(1-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
+&\quad\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(x_{b,0}+y_{b,0}R\left(\theta'\right)-2\lambda_b c_{b,1}\right)u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b\\
+&\quad c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\\
+&\quad\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\left(c_{b,1}g_{\lambda_b}\left(c_{b,1},\lambda_b\right)+1\right)u\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b,
\end{aligned}\tag{C.3}$$

where $g\left(c_{b,1},\lambda_b\right)=\frac{(1-\lambda_b)\lambda_b}{1-c_{b,1}-\phi y_{b,0}l_b\left(\lambda_b,c_{b,1}\right)}$.

We first consider the first three terms of (C.3). Since $u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta^*)}{1-\lambda_b}\right)$ is decreasing in λ_b and $x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)<0$ for $c_{b,1}>1$, the sum of these three terms is less than:

$$\begin{aligned}
&-\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)\\
&+\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\pi\left(1-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
&+\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(x_{b,0}+y_{b,0}R\left(\theta'\right)-2c_{b,1}\frac{\frac{x_{b,0}}{c_{b,1}}+\pi}{2}\right)u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\lambda_b}\right)d\lambda_b\\
<&-\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)\\
&+\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\pi\left(1-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
&+\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\left(y_{b,0}R\left(\theta'\right)-c_{b,1}\pi\right)\left(\frac{x_{b,0}}{c_{b,1}}-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R\left(\theta'\right)}{1-\pi}\right)\\
<&-\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R\left(\theta'\right)}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{R(\theta')\, y_{b,0}x_{b,0}}{c_{b,1}\left(x_{b,0}-c_{b,1}+y_{b,0}R(\theta')\right)}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\\
&-\pi u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right)\\
<\;& -\frac{x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)-\pi u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right)<0\,.
\end{aligned}
$$

We then consider the last two terms of (C.3), which decrease in ϕ. When $\phi=0$, we have $l_b=\pi c_{b,1}-x_{b,0}$, and $g_{\lambda_b}(c_{b,1},\lambda_b)=\frac{1-2\lambda_b}{1-c_{b,1}}$. Then the sum of these two terms becomes

$$
\begin{aligned}
&c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}\left(1+\frac{c_{b,1}(1-2\lambda_b)}{1-c_{b,1}}\right)u\left(\frac{(1-\lambda_b c_{b,1})R(\theta')}{1-\lambda_b}\right)d\lambda_b\\
<\;& c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}\left(1+\frac{c_{b,1}\left(1-2\frac{1+x_{b,0}}{2c_{b,1}}\right)}{1-c_{b,1}}\right)u\left(\frac{(1-\lambda_b c_{b,1})R(\theta')}{1-\lambda_b}\right)d\lambda_b\\
=\;& c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}-\frac{x_{b,0}}{1-c_{b,1}}u\left(\frac{(1-\lambda_b c_{b,1})R(\theta')}{1-\lambda_b}\right)d\lambda_b\\
<\;& c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)-\frac{x_{b,0}}{1-c_{b,1}}\frac{1-x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\\
=\;& -\frac{x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)<0\,.
\end{aligned}
$$

In summary, V is decreasing in $c_{b,1}$ and thus $\frac{\partial\theta^*}{\partial c_{b,1}}>0$. That is, R^* is increasing in $c_{b,1}$.

To see why θ^* is decreasing in $x_{b,0}$, take the derivative of (C.1) with respect to $x_{b,0}$ and note that $y_{b,0}=1-x_{b,0}$, we have

$$
\begin{aligned}
&\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}-\frac{R(\theta')-1}{1-\lambda_b}u'\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right)d\lambda_b\\
&+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\frac{\left(\frac{1}{1-\phi l_b}-1\right)R(\theta')}{1-\lambda_b}u'\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R(\theta')}{1-\lambda_b}\right)d\lambda_b\\
=\;& \frac{R(\theta')-1}{R(\theta')-c_{b,1}-(R(\theta')-1)x_{b,0}}\left((1-\pi)u\left(\frac{x_{b,0}-c_{b,1}\pi+y_{b,0}R(\theta')}{1-\pi}\right)-\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,1}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right.\\
&\left.+\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right)d\lambda_b\right)
\end{aligned}
$$

$$+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \frac{\left(\frac{1}{1-\phi l_b}-1\right)R\left(\theta'\right)}{1-\lambda_b} u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b,c_{b,1}\right)\right)R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b.$$

Similar to above, we can prove that the first term is positive. Furthermore, note that $\phi l_b = 1 - \sqrt{1-2\phi\left(\lambda_b c_{b,1}-x\right)} < 1$, the second them is also positive. Following the same logic, V is increasing in $x_{b,0}$, and thus R^* is decreasing in $x_{b,0}$. ∎

Proof of Proposition 4. Let $C(l_f)$ be the amount of cash raised by prematurely liquidating l_f project at $t=1$. Assume that $C(0)=0$ and for any $l_f>0$, $0<C(l_f)<l_f$. Note that the parametric form of $C(l_f)$ in our baseline model satisfies these conditions. We consider two cases below.

First, consider $l_f>0$. Note that equations (2.9) and (2.10) give two ways to calculate NAV_1:

$$NAV_1(\lambda_f) = 1-l_f+C(l_f) \tag{C.4}$$

$$= \frac{x_{f,0}+C(l_f)}{\lambda_f}. \tag{C.5}$$

Solving (C.5) as an equation of λ_f yields:

$$\lambda_f = \frac{x_{f,0}+C(l_f)}{1-l_f+C(l_f)}. \tag{C.6}$$

Plugging (C.6) into the expression of NAV_2 (2.12):

$$\begin{aligned} NAV_2(\lambda_f) &= \frac{y_{f,0}-l_f}{1-\lambda_f}R \\ &= \frac{1-l_f+C(l_f)}{1-x_{f,0}-l_f}(y_{f,0}-l_f)R, \end{aligned}$$

which by (C.4) and the fact that $x_{f,0}+y_{f,0}=1$ immediately leads to

$$NAV_2(\lambda_f) = NAV_1(\lambda_f)R.$$

Hence, $NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R>1$ when $l_f>0$.

Then, consider $l_f=0$. In this case, $NAV_1(\lambda_f)=1$, and

$$\begin{aligned} NAV_2(\lambda_f) &= \frac{x_{f,0}-\lambda_f+y_{f,0}R}{1-\lambda_f} \\ &= 1+\frac{y_{f,0}}{1-\lambda_f}(R-1) \end{aligned} \tag{C.7}$$

$$= \quad NAV_1(\lambda_f) + \frac{y_{f,0}}{1-\lambda_f}(R-1)\,,$$

where (C.7) uses $x_{f,0} + y_{f,0} = 1$. This implies that $NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R > 1$ when $l_f = 0$. ∎

Proof of Proposition 5. Because ε_i is arbitrarily small, there is no fundamental uncertainty between $t = 1$ and 2. Thus, late households' problem reduces to:

$$\begin{cases} w_f = 0 & \text{if} \quad c_{f,1}(\lambda_f) < E[c_{f,2}(\lambda_f)|R]\,, \\ w_f \in (0, 1-\pi) & \text{if} \quad c_{f,1}(\lambda_f)(1-\kappa w_f) = E[c_{f,2}(\lambda_f)|R]\,, \\ w_f = 1-\pi & \text{if} \quad c_{f,1}(\lambda_f)(1-\kappa w_f)) > E[c_{f,2}(\lambda_f)|R]\,. \end{cases} \tag{C.8}$$

Note that (C.8) is reduced as a quadratic equation of w_f. Directly solving for w_f under the constraints yields the desired results. ∎

Proof of Proposition 6. This proof is directly built on the results of Proposition 5. There are three cases.

Case 1. When $x_{f,0} \geq \pi + w_f^*$, stored cash is sufficient to meet the redemption needs regardless of the flow-to-fundamental relationship. As a result, $l_f^* = 0$.

Case 2. When $R \geq 1$ and $x_{f,0} < \pi$ (note that $w_f^* = 0$ when $R \geq 1$), only early households redeem at $t = 1$ but stored cash is not sufficient to meet their redemption needs. Thus, solving (2.10) at $\lambda_f = \pi$, which is a quadratic function of l_f, yields the desired result.

Case 3. When $R < 1$ and $x_{f,0} < \pi + w_f^*$, all early households and some late households redeem at $t = 1$ and stored cash is not sufficient to meet their redemption needs. Thus, solving (2.10) at $\lambda_f = \pi + w_f^*$, which is a quadratic function of l_f, yields the desired result. ∎

Online Appendix

Not for publication

A Haircut Data Contruction

Haircut data construction follows the procedure developed in Bai, Krishnamurthy and Weymuller (2018). We first construct the panel of haircuts. Identify asset categories for banks and funds, and obtain haircut series for these categories over the sample period. See Table OA1 and Table OA2 for sources and details. Then, using the entire panel of haircuts, conduct principal component analysis across all asset classes and extract the first principal component, PC_t. Calculate haircut series $\{h_j\}_t$ for each asset class j as the predicted value from regressing the original haircut on PC_t, i.e. $h_{jt} = \alpha_j + \beta_j PC_t$.[47]

Next step is to determine per-dollar liquidation value of bank deposits and fund shares for a given outflow, $1 - H_t(\Omega_i, A_{it})$. We first calculate asset category shares for each bank and fund. Using the haircut panel, order the shares for bank (fund) i at time t from most to least liquid, represented as vector A_{it}. The first component of A_{it} is the share of the most liquid asset category that the bank (fund) i holds at time t, and the last component is the share of the most illiquid.

We finally calculate effective average haircut $H_t(\Omega_i, A_{it})$ for an outflow Ω_i. For bank i,

$$H_t(\Omega_i, A_{it}) = \begin{cases} 0 & \text{if } \Omega_i \leq \sum_{j=1}^{J} A_{ijt}(1 - h_{jt}) \\ \sum_{j=1}^{J} A_{ijt} h_{jt} & \text{otherwise (bank fails)} \end{cases} \tag{A.1}$$

whereas for fund i, we have,

$$H_t(\Omega_i, A_{it}) = \sum_{j=1}^{J_\Omega - 1} A_{ijt} h_{jt} + \widehat{A_{iJ_\Omega t}} h_{J_\Omega t} \tag{A.2}$$

where $\widehat{A_{iJ_\Omega t}}$ is the amount of the least liquid asset J_Ω being liquidated to meet withdrawal needs induced by outflow Ω_i.

[47] For consumer loans, only the latest period haircut value is available, so we allow the series to follow the loading of the family real estate loans series such that it ends at the observed latest value.

Table OA1: Haircuts by Asset Class

This table shows the average haircuts for the different asset classes over the sample period from 2010 to 2018. Cash holdings are assigned 0% haircut while fixed assets are assigned a 100% haircut. Securities haircut data is obtained from the Federal Reserve Bank of New York's published repo series. Haircuts for commercial loans, real estate loans and personal loans are from the Loan Syndications and Trading Association (LSTA), the Federal Home Loan Banks website and the Federal Reserve respectively.

Category	Mean
1. Treasuries & Agency Debentures	0.0200
2. Agency MBS & CMO	0.0216
3. Commercial Loans	0.0412
4. Money Market	0.0419
5. Municipal	0.0495
6. Corporate Bonds	0.0597
7. Equities	0.0767
8. Private ABS & CMO	0.0753
9. Consumer Loans	0.2850
10. Real Estate Loans (Family Housing)	0.2378
11. Real Estate Loans (Other)	0.3539
12. Cash	0
13. Fixed Assets	1

Table OA2: Asset Category Sources

This table shows the sources for banks and funds for each asset class used in the LPI calculation. Bank asset holdings are obtained using bank balance sheet data from call reports. The bank holdings variables all come from RCFD (for example, the corresponding cash variable is RCFD0010) except for real estate loans which also take variables from RCON. Mutual fund holdings data is obtained from the CRSP database, and fund cash holdings are taken from CRSP mutual funds summary data. For fund holdings, all asset classes except cash holdings are categorized directly from securities-level holdings using the mapping shown.

Category	Bank Source (RCFD)	Fund Holdings Mapping
1. Treasuries &	3531, 0213, 1287	US Government & Agency Bills, Bonds,
Agency Debentures	3532, 1290, 1293, 1295, 1298	Notes, Strips, Trust Certificates
2. Agency MBS & CMO	G301, G303, G305, G307, G313, G315, G317, G319	Agency MBS, TBA MBS, CMO,
	K143, K145, K151, K153, G379, G380, K197	Pass Through CTF, REMIC, ARM
3. Commercial Loans	F610, F614, F615, F616, K199, K210, F618	Syndicated Loans, Term Deposits,
	$(2122 - 3123) - 1975 - 1410$ if ≥ 0	Term Loans
4. Money Market		Money Market, CDs, Corporate Paper
5. Municipal	8499, 8497, 3533	Municipality Debt
6. Corporate Bonds	G386, 1738, 1741, 1743, 1746	Bonds, MTN, Foreign Gov'ts & Agencies
7. Equities	A511	Equities, Funds, Convertible bonds
8. Private ABS & CMO	G309, G311, G321, G323, K147, K149, K155, K157	ABS, CMO, CDO, CLO, Covered Bonds
9. Consumer Loans	1975	
10. Real Estate Loans (Family)	$1410 * (RCON3465/RCON3385)$	
11. Real Estate Loans (Other)	$1410 * (RCON3466/RCON3385)$	
12. Cash	0010	CRSP Mutual Funds summary Cash %
13. Fixed Assets	3541, 3543, total assets - sum of above variables	