Optimal Monetary Policy under Dollar Pricing*

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Abstract

The recent empirical evidence shows that most international prices are sticky in dollars. This paper studies the optimal monetary policy and the welfare implications of dollar pricing in a context of an open economy model with nominal rigidities and establishes the following results: 1) while monetary policy is less efficient and cannot implement the flexible-price allocation, inflation targeting remains robustly optimal for non-U.S. economies; 2) the implementation of this policy results in the "global monetary cycle" with other countries partially pegging their exchange rates to the dollar; 3) unilateral capital controls do not help to restore the efficient allocation; 4) optimal U.S. policy deviates from inflation targeting and partially internalizes the international spillovers on global economy; 5) the issuer of the dominant currency achieves a higher welfare than other economies, which can in turn benefit from forming a currency union such as the Eurozone.

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1 Introduction

What is the optimal monetary policy in an open economy? According to the standard Mundell-Fleming view, the central banks should focus on domestic targets such as price stability leaving the burden of external adjustment to freely floating exchange rates. Yet this prescription is rarely followed in practice with policymakers referring to international spillovers as a rationale for responding to foreign shocks and anchoring the exchange rate. Among other potential channels that might be responsible for this discrepancy between the conventional wisdom and actual policies, the recent literature has emphasized the asymmetric use of currencies in international trade with most import and export prices sticky in dollars (Gopinath 2016, Goldberg and Tille 2008). While a lot of progress has been made in understanding the positive consequencies of dollar currency pricing (DCP) (see Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Møller 2018), much less is known about its implications for the optimal policy and the welfare.[1] These questions are, however, at the heart of recent policy debates: Should countries peg exchange rates or let them float (Friedman 1953)? Can capital controls insulates economies from foreign spillovers? Are there gains from cooperation and/or forming a currency union (Mundell 1961)? Should the Fed be concerned about international spillovers of its policy (Bernanke 2017, Obstfeld 2019)? Does the U.S. enjoy an "exorbitant privilege" from DCP (Eichengreen 2011)?

We answer these questions in a context of a generalized version of a conventional sticky-price open economy model by Gali and Monacelli (2005) augmented with a more realistic structure of the international price system. In particular, firms use producer currency pricing (PCP) in domestic market and DCP in exports. In addition, we assume that only a fraction of imports is used for final consumption, while the rest is used as inputs in production. This is consistent with the fact that intermediates account for most of the international trade (see e.g. Johnson and Noguera 2012) and even final goods contain significant local distribution margin (Burstein, Neves, and Rebelo 2003). Combined together, these two assumptions allow the model to simultaneously reproduce a high pass-through of U.S. exchange rate into import prices at the doc level and a low pass-through at the retail level (Auer, Burstein, and Lein 2018) and, as we show, have important implications for the optimal policy. We keep the rest of the model quite general — allowing for flexible (in)complete asset markets, a rich set of shocks, arbitrary functional form of utility and production, and either Rotemberg or Calvo price friction — and solve for the optimal non-cooperative policy in a sequential game with the U.S. acting as a Stackelberg leader.

Our central finding is that even though the actual allocation depends on the particular assumptions, such as for example completeness of asset markets and the degree of price stickiness, the optimal monetary policy in non-U.S. economies can be summarized with one "sufficient statistic", namely, price stability, which is robust to all details of the environment, is independent from the values of model parameters, and is directly measurable in the data. Though similar to the case of closed economy and open economy under PCP, this result is arguably much more surprising in a context of DCP when, in con-

[1] The literature review below discusses in detail a few important exceptions and contrasts them with our analysis.

trast to the former two cases, the monetary policy *cannot* implement the first-best allocation. Indeed, because domestic and export prices are sticky in different currencies, the planner faces a trade-off between internal and external margins: domestic price stabilization eliminates distortions in production of local goods for domestic consumption, but implies inefficient terms-of-trade and trade balances. The result that the optimal policy focuses exclusively on the internal margin and neglects the external margin seems to be counterintuitive and inconsistent with a standard second-best logic. The reason for this "corner solution" is that conditional on the optimal value of marginal costs in local currency, the prices set by exporters are *constrained efficient*, i.e. a planner who has additional instrument to manipulate the markups of exporters, but is subject to the same price-adjustment friction as firms, would decide to not intervene. Given that domestic price stabilization ensures efficiency of both internal margin and the marginal costs of exporters in local currency, the external margin is constrained efficient and there is no room for further improvement. As a result, the policy is also time consistent. Remarkably, this result holds for both state-dependent (Rotemberg) and time-dependent (Calvo) price setting as well as for the non-CES (Kimball) demand.

In spite of the fact that the optimal non-U.S. policy can be formulated in terms of domestic target, it is generically outward-looking, responds asymmetrically to U.S. shocks and, under plausible assumptions, partially pegs exchange rate to the dollar. In particular, the optimal policy stabilizes all prices that are sticky in local currency, which includes retail prices of imported goods and prices of producers that use foreign intermediates. Given that all international prices are sticky in dollars, an appreciation of U.S. exchange rate increases import prices in other economies and through both channels puts upward pressure on local prices. To keep them constant, other countries have to tighten their monetary policy giving rise to a "Global Monetary Cycle" — a situation when monetary stance is correlated across countries even if exogenous shocks are purely idiosyncratic. Moreover, the pass-through of U.S. monetary shocks into local interest rates is higher for more open countries with a larger share of DCP in imports, rationalizing the recent empirical findings from Zhang (2018). This "leaning against the wind", however, is optimal more broadly in response to any local and U.S. shocks (except for productivity ones) and effectively introduces a partial peg to the dollar. Thus, the widespread "fear of floating" and the anchoring of exchange rates to the dollar in the data (Calvo and Reinhart 2002, Ilzetzki, Reinhart, and Rogoff 2018) can be due not only to the global financial cycle (Rey 2013), but also to the dominant status of the dollar in international trade. This analogy between the two sources of international spillovers applies also to the Trilemma-Dilemma debate: dollar pricing implies that a floating exchange rate cannot fully insulate countries from foreign spillovers and worsens the trade-off faced by the policymakers, which however, does not mean that the planner should give up on exchange rates all together as they still constitute an important margin of adjustment (see Gourinchas 2018, Kalemli-Ozcan 2019).

Motivated by the recent arguments in favor of macroprudential instruments to deal with international spillovers (see e.g. Blanchard 2017), we next study the optimal mix of monetary policy and capital controls and find, surprisingly, that the latter do *not* insulate economies from foreign spillovers and are

set to zero in a non-cooperative equilibrium. Again, this outcome seems to be inconsistent with the second-best intuition, which prescribes intervening in asset markets to improve the allocation in goods markets: as shown forcefully by Farhi and Werning (2016), the laissez-faire risk sharing is generically inefficient when monetary policy cannot implement the first-best allocation. This discrepancy comes from the fact that there are two sources of the aggregate demand externality in our setup — local and foreign ones. On the one hand, the optimal monetary policy ensures that domestic demand is efficient and fully eliminates the local externality. On the other hand, the foreign demand for exported goods is in general suboptimal due to dollar pricing, but cannot be changed by a small open economy via capital controls. As a result, under the optimal monetary policy and arbitrary structure of asset markets, the private risk sharing is constrained efficient leaving no room for macroprudential policies. This shows that it is not just the number of available instruments, but also the nature of the international spillovers that is important for the optimal policy: while capital controls might be efficient in curbing financial spillovers (see e.g. Jeanne and Korinek 2010, Bianchi 2011), they are unlikely to help with the spillovers arising from DCP. At the same time, we show that the optimal *cooperative* policy, which might be of greater interest for the IMF, generically involves non-zero capital controls. Importantly, the latter are used to redistribute global demand not to the countries with output gap as their monetary policy ensures efficiency of the local demand, but instead, to the countries with an inefficiently low demand for imported goods.

In contrast, the state-contingent export tariffs combined with production subsidies can be quite efficient in restoring the first-best allocation. Intuitively, as log as the tariffs are imposed on top of the export prices, the planner can implement the optimal terms of trade, while production subsidy ensures that firms keep their dollar-invoiced prices constant saving on price-adjustment costs. Note that because the Lerner symmetry does not hold under DCP (Barbiero, Farhi, Gopinath, and Itskhoki 2019), the export tariff is crucial and cannot be substituted for other border taxes or the VAT. Importantly, the optimal monetary policy remains robust and still targets domestic prices.

We then show that the optimal U.S. policy internalizes its global spillovers in the international goods and asset markets, deviates from a pure inflation targeting, and can be expressed in terms of intuitive sufficient statistics. On the one hand, the fact that both domestic and export prices are set in producer currency allows U.S. monetary policy to achieve the efficient level of both domestic and foreign demand. As a result, from the ex-post perspective, the Fed does not need to worry about the negative international spillovers of its policy and can focus exclusively on targeting domestic inflation. This policy is, however, suboptimal from the ex-ante perspective. First, even if the U.S. is a small economy, because of DCP, its monetary policy affects the global stochastic discount factor and hence, the asset prices at which the country can borrow and save. This gives rise to the "dynamic terms-of-trade motive" (Costinot, Lorenzoni, and Werning 2014) and contributes to U.S. "exorbitant privilege" (Gourinchas and Rey 2007): to borrow cheaply and earn high interest on savings, U.S. policy stimulates world economy in states of the world, in which it runs a negative current account, and depresses world

economy in states of the world, in which its current account is positive. Second, the prices preset by foreign exporters depend on the expectations about U.S. monetary shocks. By partially stabilizing the dollar value of marginal costs of world producers, the optimal policy dampens their markups and makes foreign goods cheaper for the U.S. and other economies (*cf.* Devereux and Engel 1998).

In spite of the fact that the latter motive pushes the U.S. closer to the optimal *cooperative* policy, a full-scale coordination is not in U.S. self-interest. Indeed, a global planner would use U.S. monetary tools solely to stabilize the average dollar prices of world exporters. While beneficial for other countries and the global economy, such policy decreases the welfare of the U.S., which therefore has no incentives to coordinate its policy with other economies. Our analysis also reveals a new source of gains from forming a currency union such as the Eurozone. While the standard critique — that a member of a currency area loses an independent monetary policy and cannot use it to stabilize the economy — still applies in our setting, forming a currency union can boost the welfare if it promotes the status of the common currency. In particular, we show that a country with imports and exports invoiced in its currency (e.g. the Eurozone) achieve a higher level of welfare than other non-U.S. countries.

Finally, we argue that relative to other countries, the U.S. is likely to benefit from the dominant status of its currency in global trade. The welfare analysis is complicated by the fact that there are two sources of differences across countries — the asymmetric transmission of shocks and differences in the optimal monetary policy — which can work in the opposite directions. We show, however, that the differences in the optimal policy are of the second-order importance for the welfare when the home bias is strong enough. The U.S. is then better off than any other economy because of a higher efficiency of its monetary policy, which can stimulate both local and foreign demand. In a globalized economy with firms likely to coordinate on a single currency, this naturally leads to a competition between nations for the status of the dominant-currency provider.

We consider several extensions of the baseline model and demonstrate robustness of our results to the alternative assumptions about strategic interactions across countries, heterogeneous firms, and endogenous choice of invoicing currencies. In particular, while most of our analysis focuses on a subgame perfect equilibrium of the sequential game with the U.S. moving first and other economies choosing their policy afterwards, the same equilibrium would arise in a simultaneous-move game with all countries forming their strategies in terms of domestic inflation. This is because the optimal policy of non-U.S. economies always stabilizes local prices and hence, the policy of the U.S. is invariant to the timing assumption if it takes domestic inflation of other countries as given. Perhaps more importantly, following Mukhin (2018), we extend the model to allow for endogenous currency choice. Because of nominal rigidities, the prices deviate from profit-maximizing values and firms optimally choose the currency of invoicing to replicate the latter as close as possible. Given that monetary policy stabilizes prices of domestic suppliers in local currency, these firms unambiguously choose PCP. In contrast, strategic complementarities in currency choice between exporters from different countries that arise via input-output linkages and the Kimball demand imply that exporters are more likely to coordinate

on one dominant currency. Thus, the optimal policy from the baseline model can be sustained as an equilibrium outcome with firms choosing the currency of invoicing simultaneously with the planners making policy decisions. Remarkably, however, our results do not change even when the policymakers internalize the effect of their policies on firms' currency choice: similarly to their price setting, the invoicing decisions of exporters are also constrained efficient and cannot be improved upon by the planner.

Related literature This paper contributes to a vast literature on the optimal policy in New-Keynesian open-economy models. The seminal papers by Obstfeld and Rogoff (1995), Clarida, Gali, and Gertler (2001, 2002), Gali and Monacelli (2005) and their extensions by De Paoli (2009) and Dmitriev and Hoddenbagh (2014) solve for the monetary policy under PCP formalizing the Friedman (1953) argument in favor of floating exchange rates. Motivated by the fact that import prices are sticky in consumer currency at the retail level, Devereux and Engel (2003, 2007) and Engel (2011) contrast this analysis with the case of LCP and show the desirability of the peg in this setting. The gains from monetary cooperation are the focus of Obstfeld and Rogoff (2002), Benigno and Benigno (2003, 2006), Sutherland (2004) and Corsetti and Pesenti (2005), while Corsetti, Dedola, and Leduc (2010, 2018) make important contribution on the optimal policy under incomplete markets. Our main departure from this literature is that we assume a more realistic structure of the international price system with most of global trade invoiced in one dominant currency.

Our work is most closely related to Corsetti and Pesenti (2007), Devereux, Shi, and Xu (2007), Goldberg and Tille (2009), and Casas, Díez, Gopinath, and Gourinchas (2017) who solve for the optimal monetary policy under DCP. Despite the apparent similarity, there are three key differences that distinguish our paper from this literature (see Table A1 for detailed comparison). First, in contrast to these papers, we allow for incomplete pass-through of border prices into retail prices, which is pronounced in the data and, as we show, has important implications for the optimal policy. Second, although our central result about the optimality of price stabilization in non-U.S. economies resembles the findings of the previous literature, the intuition is qualitatively different. Indeed, the papers mentioned above make strong assumptions about fully sticky prices, log-linear preferences and/or complete asset markets, under which the terms of trade are exogenous to monetary policy and, unsurprisingly, are not targeted by a planner. In contrast to these knife-edge cases, we show that it is *generically* optimal for monetary authorities to stabilize domestic prices even when the terms of trade are endogenous to the policy. Finally, several results of the paper are completely new to the literature. This includes the suboptimality of capital controls, the robustness of optimal policy to endogenous currency choice, and the new motives of U.S. monetary policy that are absent in special cases analyzed in the previous models. Our analysis is complementary to Mukhin (2018), who however, focuses exclusively on the *cooperative* policy and hence, cannot speak to the strategic interactions between countries that arise under DCP.

This paper is also related to the recent literature on the role of fiscal instruments when the monetary

policy alone is not sufficient to implement the optimal allocation. Our analysis of optimal fiscal policy follows Adao, Correia, and Teles (2009), Farhi, Gopinath, and Itskhoki (2014) and Chen, Devereux, Xu, and Shi (2018) who show how the effects of exchange rate depreciation can be replicated with taxes and tariffs. Lorenzoni (2008), Jeanne and Korinek (2010), Bianchi (2011) and Dávila and Korinek (2017) study the implications of the pecuniary externality for the macroprudential policy, while Farhi and Werning (2012, 2013, 2016, 2017) and Fornaro and Romei (2019) derive the optimal capital controls in the presence of aggregate demand externality. We complement this literature with the analysis of optimal capital controls under DCP.

Finally, our results go beyond the normative analysis as they also have important positive implications. The optimality of partial peg to the dollar under DCP complements other explanations for the "fear of floating" by Calvo and Reinhart (2002), Rey (2013), Fanelli (2017) and Hassan, Mertens, and Zhang (2019). It also generates additional complementarities between the dominant status of the dollar as an international currency of invoicing, funding, anchoring, and reserves as discussed by Gopinath and Stein (2017), Mukhin (2018), Maggiori, Neiman, and Schreger (2018), and Gourinchas (2019).

2 Environment

This section describes the baseline setup, which builds on a conventional sticky-price open economy model by Gali and Monacelli (2005) that has been extensively used in the recent normative literature (see e.g. Farhi and Werning 2012, 2013). We augment this model with two additional ingredients to allow for a more realistic structure of the international price system. First, all international prices are sticky in dollars rather than in the currency of producer or buyer, while domestic products are invoiced in local currency. While an extreme assumption, the empirical literature shows that it provides a good first-order approximation to the real world (Gopinath 2016). For simplicity, we take the currency of invoicing as given for now and show that our results remain true under endogenous currency choice in Section 3.5. Second, only a fraction of imports is used for final consumption, while the rest is used as intermediate goods in production. This is consistent with the fact that intermediates account for most of the international trade (see e.g. Johnson and Noguera 2012) and even final goods contain significant local distribution margin (Burstein, Neves, and Rebelo 2003). Combined together, these two assumptions allow the model to simultaneously reproduce a high pass-through of U.S. exchange rate into import prices at the doc and a low pass-through at the retail level (Auer, Burstein, and Lein 2018) and have important implications for the optimal policy.

The rest of the model is standard. Time is discrete and the horizon is infinite. The world consists of a continuum of symmetric small open economies $i \in [0, 1]$. To disentangle the role of the dominant currency, we assume that the U.S. is a small economy indexed by $i = 0$ and is symmetric to other countries in all respects except for the use of the dollar in international trade. Each country is populated by identical households that consume local and foreign products, supply labor and make savings

decisions. Monopolistic firms produce domestic and export goods and are subject to price-adjustment frictions. We generalize the model of Gali and Monacelli (2005) in several dimensions to demonstrate robustness of our results — allowing for flexible preferences and technologies, complete and incomplete asset markets, and a rich set of exogenous shocks — yet try to keep the model tractable enough to convey the main mechanisms.

2.1 Households

A representative household in country i has preferences over consumption C_{it} and labor N_{it}:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t U\big(C_{it}, N_{it}, \xi_{it}\big),$$

where ξ_{it} stays for both intra- and inter-temporal preference shocks and $U(\cdot)$ satisfied the standard regularity conditions. Consumption is an aggregator of home and foreign goods:

$$C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{it}^{*\,\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where $1-\gamma$ reflects the home bias in consumption that can arise due to trade costs or preferences for locally produced goods. The import bundle aggregates products from all other countries

$$C_{it}^{*} = \left(\int C_{jit}^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

while each bilateral trade flow includes a continuum of unique products $\omega \in [0,1]$:

$$C_{jit} = \left(\int C_{jit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}\omega\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Thus, following the previous literature, we allow for a nested CES structure to distinguish the "macro elasticity" of substitution θ between home and foreign goods and a "micro elasticity" $\varepsilon > 1$ governing the substitution between varieties of foreign goods (as e.g. in Feenstra, Luck, Obstfeld, and Russ 2018).[2] These functional forms — with constant elasticities θ and ε and symmetric import shares across countries — are standard and used here for tractability reasons, but are not essential for our results and are all relaxed in the extension of Section 3.5.

[2] To simplify notation, we assume the same elasticity of substitution ε between varieties produced within each country and across foreign economies.

Each period t, households face the flow budget constraint:

$$P_{it}C_{it} + \mathcal{E}_{it}\sum_{h\in H_t}\mathcal{Q}_t^h B_{it+1}^h + \frac{\mathcal{B}_{it+1}^i}{R_{it}} = W_{it}N_{it} + \Pi_{it}^f + T_{it} + \mathcal{E}_{it}\sum_{h\in H_{t-1}}(\mathcal{Q}_t^h + D_t^h)B_{it}^h + \mathcal{B}_{it}^i,$$

where consumer prices index P_{it} and nominal wages W_{it} are in local currency, H_t is the set of internationally traded assets in period t with prices $\mathcal{Q}_t^h$ and payouts D_t expressed in dollars, and $\mathcal{E}_{it}$ is the nominal exchange rate of country i against the dollar. Households receive transfers T_{it} from the government and profits Π_{it}^f from local firms. In addition to globally traded securities B_{it}^h, they can also invest in local government bonds $\mathcal{B}_{it}^i$ with the nominal rate of return R_{it}.

Households choose consumption, labor and asset portfolio to maximize expected utility subject to the budget constraint. The resulting optimal labor supply is given by

$$-\frac{U_{Nit}}{U_{Cit}} = \frac{W_{it}}{P_{it}}, \tag{1}$$

where U_C and U_N are respectively the marginal utilities of consumption and work. As usual, the intertemporal Euler equation for nominal bonds is:

$$\mathbb{E}_t\Theta_{it,t+1}R_{it} = 1, \quad \text{where } \Theta_{it,t+\tau} \equiv \beta^\tau \frac{U_{Cit+\tau}}{U_{Cit}}\frac{P_{it}}{P_{it+\tau}} \tag{2}$$

is the nominal stochastic discount factor. The optimal portfolio of households is characterized by the set of no-arbitrage conditions:

$$\mathbb{E}_t\Theta_{it,t+1}\frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}}\frac{\mathcal{Q}_{t+1}^h + D_{t+1}^h}{\mathcal{Q}_t^h} = 1, \ \ \forall h \in H_t. \tag{3}$$

Moving to goods market, the optimal demand for domestic and foreign products is

$$C_{iit}(\omega) = (1-\gamma)\left(\frac{P_{iit}(\omega)}{P_{iit}}\right)^{-\varepsilon}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}C_{it}, \qquad C_{jit}(\omega) = \gamma\left(\frac{P_{jit}(\omega)}{\mathcal{E}_{it}P_t^*}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}C_{it},$$

where $P_{iit} = \left(\int P_{iit}(\omega)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}$ is the price index for home goods in local currency and $P_t^* = \left(\int P_{it}^{*1-\varepsilon}\mathrm{d}i\right)^{\frac{1}{1-\varepsilon}}$ is the import price index in dollars, which according to the DCP assumption is the same for all countries and is the aggregate of dollar export prices of all economies P_{it}^*. The consumer price index can then be written as

$$P_{it} = \left[(1-\gamma)P_{iit}^{1-\theta} + \gamma(\mathcal{E}_{it}P_t^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{4}$$

2.2 Firms

Production In each country i, there is a continuum of firms, each producing a unique variety ω from labor L_{it} and a bundle of intermediate goods X_{it} via production function

$$Y_{it} = A_{it}F(L_{it}, X_{it}).$$

Though not essential for our results, we assume that technology $F(\cdot)$ is constant returns to scale, which in particular, allows us to abstract from the issue whether different markets are served by the same or different firms and simplifies the aggregation. Producers use a roundabout production with the same bundle of intermediates as in final consumption. While it is a hardly realistic assumption that import share γ is the same for consumers, domestic producers and exporters, we start with this symmetric case as it implies that the relevant price indices are the same for all agents and greatly simplifies the notation. Importantly, however, our main results remain true when we relax the assumption in Section 3.5.

Solving the firm's cost minimization problem, we get the relative demand for inputs:

$$\frac{X_{it}}{L_{it}} = g\left(\frac{W_{it}}{P_{it}}\right) = g\left(\frac{-U_{Nit}}{U_{Cit}}\right), \tag{5}$$

where function $g(\cdot)$ is implicitly defined by $\frac{F_L(1,g(z))}{F_X(1,g(z))} \equiv z$ with $g'(\cdot) > 0$ and the last equality uses the labor supply condition. The resulting real marginal costs of production are

$$\frac{MC_{it}}{P_{it}} = \frac{h\left(\frac{W_{it}}{P_{it}}\right)}{A_{it}} = \frac{h\left(\frac{-U_{Nit}}{U_{Cit}}\right)}{A_{it}},$$

where we define $h(z) \equiv \frac{1}{F_X(1,g(z))}$.

Domestic prices Firms are monopolistic competitors, are subject to a price-adjustment friction, and set local prices in domestic currency and export prices in dollars. While there is little consensus in the literature about the best way of modelling sticky prices, it is encouraging that our results hold for both time-dependent and state-dependent models of nominal rigidities. Therefore, in our baseline specification, we adopt the Rotemberg (1982) model with labor adjustment costs, which results in a tractable *non-linear* New-Keynesian Phillips curve.[3] Appendix **??** then extends the results to a setup with the Calvo (1983) friction.

In domestic market, firms set prices in local currency to maximize expected profits net of price-

[3]The previous literature has explored several alternative assumptions about adjustment costs in the Rotemberg model (*cf.* e.g. Schmitt-Grohé and Uribe 2004, Faia and Monacelli 2008, Kaplan, Moll, and Violante 2018). Our results hold independently whether the costs are set in labor units or product units and whether they are scaled by firms' output or not.

adjustment costs:

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(P_t - \tau_i MC_{it}\right) \left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{iit} - (1-\gamma)\frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it} \right],$$

where $Y_{iit} \equiv C_{iit} + X_{iit}$ is the local demand shifter, menu costs are normalized by the size of domestic market $1-\gamma$, τ_i is the time-invariant production subsidy to local sellers[4], and firms are allowed to choose any initial price in period $t = 0$ without paying adjustment costs, i.e. they can freely choose P_{-1}. Taking the first-order conditions and imposing the symmetry across firms allows to express the domestic Phillips curve as

$$\pi_{iit} \left(\pi_{iit} + 1\right) W_{it} = -\kappa \left(P_{iit} - \frac{\varepsilon \tau_i}{\varepsilon - 1} MC_{it} \right) \frac{Y_{iit}}{1-\gamma} + \beta \mathbb{E}_t \Theta_{it,t+1} \pi_{iit+1} \left(\pi_{iit+1} + 1\right) W_{it+1}, \tag{6}$$

where $\kappa \equiv \frac{\varepsilon - 1}{\varphi} > 0$ and $\pi_{iit} \equiv \frac{P_{iit}}{P_{iit-1}} - 1$ is the PPI inflation rate (with $\pi_{ii0} = 0$). As usual, the dynamic price setting equation can be thought as an error-correction model: if prices were flexible, the firms would charge a constant markup over marginal costs, i.e. $P_{iit} = \frac{\varepsilon \tau_i}{\varepsilon - 1} MC_{it}$ in all states of the world. In the presence of nominal rigidities, the price deviates from the optimal level: when it is too high, the bracket on the right-hand side is positive and, other things equal, firms decrease their prices, i.e. $\pi_{iit} < 0$. The speed of adjustment is higher when prices are more flexible, i.e. κ is larger, and when deviations from the optimal price are associated with larger sales Y_{iit}. As standard in the New-Keynesian literature, the Phillips curve is forward looking with firms trying to smooth marginal costs $\pi_{iit} \left(\pi_{iit} + 1\right) W_{it}$ of price adjustment across periods.

Export prices We assume that exporters set a uniform dollar price for all markets of destination:[5]

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\mathcal{E}_{it} P_t - \tau_i^* MC_{it}\right) \left(\frac{P_t}{P_{it}^*}\right)^{-\varepsilon} Y_{it}^* - \gamma \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it} \right],$$

where $Y_{it}^* \equiv \int \left(C_{ijt} + X_{ijt}\right) \mathrm{d}j$ is the global demand shifter for exported goods, τ_i^* denotes the time-invariant production subsidy to exporters, and firms can freely choose P_{-1}. The first-order optimality condition can be rewritten as an export Phillips curve:

$$\pi_{it}^* \left(\pi_{it}^* + 1\right) W_{it} = -\kappa \left(\mathcal{E}_{it} P_{it}^* - \frac{\varepsilon \tau_i^*}{\varepsilon - 1} MC_{it} \right) \frac{Y_{it}^*}{\gamma} + \beta \mathbb{E}_t \Theta_{it,t+1} \pi_{it+1}^* \left(\pi_{it+1}^* + 1\right) W_{it+1}, \tag{7}$$

[4] Although we assume that the subsidy applies only to *production* costs, our analysis would still go through if the government also subsidized the price-adjustment costs.

[5] The assumption that the price is the same for all foreign buyers is consistent with the empirical evidence from Cavallo, Neiman, and Rigobon (2014, 2015), which shows that the law of one price holds well across markets with the same currency.

where $\pi^*_{it} \equiv \frac{P^*_{it}}{P^*_{it-1}} - 1$ is inflation rate for the dollar export price index (with $\pi^*_{i0} = 0$). Equation (7) is a direct counterpart of (7) with inflation going up when dollar export price P^*_{it} is below the optimal flexible-price level $\frac{\varepsilon\tau^*_i}{\varepsilon-1}\frac{MC_{it}}{\mathcal{E}_{it}}$ and the speed of adjustment is increasing in the global demand shifter Y^*_{it}.

2.3 Market clearing

The goods market clears when total production equals the sum of local and foreign demand for final and intermediate goods:

$$A_{it}F(L_{it}, X_{it}) = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it} + X_{it}) + \gamma\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}\int\left(\frac{\mathcal{E}_{jt}P^*_t}{P_{jt}}\right)^{-\theta}(C_{jt} + X_{jt})\,\mathrm{d}j, \quad (8)$$

where we have used the symmetry across producers within a country. The labor market clearing condition aggregates the labor used in production and price adjustment:

$$N_{it} = L_{it} + \frac{\varphi}{2}(1-\gamma)\pi^2_{iit} + \frac{\varphi}{2}\gamma\pi^{*2}_{it}, \quad (9)$$

We assume that the government balances its budget in every period by returning the net income from labor subsidy and export tax via transfers to the households, which is also without loss of generality since the Ricardian equivalence holds in the model. Combing the budget constraint of households with firms' profits, we arrive at the country's budget constraint:

$$\begin{aligned}&\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}}(\mathcal{Q}^h_t + D^h_t)B^h_{it} \\ &\quad = \gamma\left[P^*_{it}\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}\int\left(\frac{\mathcal{E}_{jt}P^*_t}{P_{jt}}\right)^{-\theta}(C_{jt} + X_{jt})\,\mathrm{d}j - P^*_t\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta}(C_{it} + X_{it})\right] + \psi_{it}.\end{aligned} \quad (10)$$

where the right-hand side is the dollar value of country's net exports and the left-hand side combines the change in the net foreign asset position with the valuation effect. Notice that we allow for an idiosyncratic wealth shock ψ_{it} with a zero global mean $\int \psi_{it}\mathrm{d}i = 0$. This shock is meant to capture several additional (unmodelled) sources of volatility that previous literature has found as important drivers of business cycles and exchange rates in an open economies: e.g. fluctuations in commodity prices (as in Casas, Díez, Gopinath, and Gourinchas 2017), terms-of-trade shocks (Mendoza 1995), taste shocks for foreign versus domestic goods (Pavlova and Rigobon 2007) as well a redistribution of wealth arising from financial shocks (Lustig and Verdelhan 2019). Importantly, the shock ensures that the model can potentially be made consistent with the exchange rate disconnect in the data (see Itskhoki and Mukhin 2019a).

Finally, given that households earn the profits of local firms, we can assume without loss of gener-

ality that internationally traded assets are in zero net supply:

$$\int B^h_{it+1} \mathrm{d}i = 0, \quad \forall h \in H_t, \qquad \mathcal{B}^i_{it} = 0. \tag{11}$$

2.4 Equilibrium

As usual, for given monetary policies, the equilibrium is such that households maximize expected utility subject to the sequence of budget constraints, firms maximize expected profits, the government's budget constraint is satisfied, and the markets clear. Following the primal approach, we let the planners choose the optimal allocation subject to the relevant equilibrium conditions. In particular, substitute out nominal interest rates R_{it} using the Euler equation (2) and nominal wages W_{it} using the labor supply condition (1), while keep other prices that are either subject to adjustment frictions (P_{iit}, P^*_{it}) or are not directly chosen by an individual country ($\mathcal{Q}^h_t$).[6] The next lemma summarizes restrictions imposed by households' and firms' optimality conditions and the resource constraints.

Lemma 1 (Implementability) *The allocation* $\{C_{it}, N_{it}, L_{it}, X_{it}, B^h_{it}\}$ *and prices* $\{\pi_{it}, \pi_{iit}, \pi^*_{it}, \mathcal{E}_{it}, \mathcal{Q}^h_t\}$ *constitute part of the equilibrium if and only if equations* (3) – (11) *hold.*

While the lemma describes the set of all implementable equilibria, to solve for the optimal *non-cooperative* policy, one needs to specify further the strategic interactions between policymakers. Henceforth, we adopt the following notion of equilibria:

Definition *We are looking for a SPNE in the following game: (i) each country chooses a state-contingent plan of CPI inflation* $\{\pi_{it}\}$*, (ii) the U.S. moves first and other economies move simultaneously after that, (iii) all countries have full commitment.*

Thus, given the special role of the dollar in global economy, we assume that the U.S. is a Stackelberg leader, which internalizes the effect of its decisions on other economies when choosing the optimal monetary policy. In contrast, each non-U.S. economy takes the policy of the U.S. and other countries as given. We assume that strategies are formed in terms of inflation, i.e. for each history of exogenous shocks and inflation rates of other countries, a planner chooses its best response. While there are several restrictions imposed by this definition, the next lemma shows that the same allocation arises as an equilibrium outcome in a much larger class of games.

Lemma 2 (Equilibrium) *The equilibrium remains the same in each of the following cases:*

1. *instead of* π_{it}*, countries choose* $C_{it}, N_{it}, L_{it}, X_{it}, \pi_{iit}, \pi^*_{it}$*, or any their combination,*
2. *countries move simultaneously and form their strategies in terms of domestic inflation* π_{iit}*,*
3. *non-U.S. economies lack commitment and choose the optimal discretionary policy.*

[6] See Section 3.2 for the discussion of the interest rate policy that implements the optimal allocation.

The lemma effectively relaxes all three assumptions embodied in our benchmark definition of equilibrium (see Appendix **??** for the proof). First, the equilibrium remains the same independently from whether countries form their strategies in terms of CPI/PPI inflation, aggregate consumption, output gap, etc.[7] Intuitively, this is the case because non-U.S. countries are small and take *all* foreign variables as given. Second, the baseline sequential game can be reformulated as a particular simultaneous move game without any first-mover advantage on the U.S. side. This result follows directly from Proposition 1 below, which shows that the optimal non-U.S. policy can be formulated in terms of a simple rule $\pi_{iit} = 0$. Given this best response of other economies, the problem of the U.S. becomes invariant to the timing assumption. Finally, we also show below that non-U.S. optimal policy is time consistent and therefore, the equilibrium would not change if countries other than the U.S. lacked commitment.

In sum, while the equilibrium allocation does depend on the assumption about strategic interactions across countries, Lemma 2 shows that there is a rich class of games, which — largely due to the particular form of the optimal policy in our model — result in the same equilibrium outcome, supporting the robustness of our analysis.

2.5 First-best allocation

As emphasized above, our setup described above is fairly general in many dimensions and can be generalized even further without altering the results. In contrast, the next two assumption are crucial for the analysis: combined together, they ensure that the monetary policy is not used as a second-best instrument to eliminate distortions in the economy other than the ones associated with nominal rigidities and allows us to focus exclusively on the constraints imposed by the structure of the international price system.

A 1 *Time-invariant production subsidy* $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, $\tau_i^* = 0$ *eliminates the monopolistic distortion and the terms-of-trade externality. There are no exogenous shocks to markups.*

As is well known, the standard New-Keynesian open economy models have two types of distortions on top of sticky prices: monopolistic markups and the terms-of-trade externality (see Corsetti and Pesenti 2001, Benigno and Benigno 2003, Faia and Monacelli 2008). The former increases domestic prices, lowers the real wage and labor supply, and results in suboptimal production and consumption. The later implies that the country should exploit its market power in international goods market setting a higher price for its goods abroad than at home. A destination-specific production subsidy from Assumption 1 eliminates both distortions offsetting the markups in domestic prices and, at the same

[7] The only important restriction that we impose on the set of strategies is that policies are formulated in terms of domestic objectives. In particular, to avoid pathological cases, countries are not allowed to set policies that they cannot directly implement, e.g. in terms of foreign output. While we exclude bilateral exchange rates from Lemma 2 for the same reason, the equilibrium would be the same if non-U.S. economies were choosing $\mathcal{E}_{it}$ and the U.S. had some domestic objective.

time, maintaining the monopolistic markups in export prices.[8] While most of the sticky-price normative papers employ a time-invariant production subsidy, this instrument is usually applied uniformly across *all* firms, which does not allow to simultaneously eliminate both externalities. Relative to this standard approach, assuming more sophisticated instrument(s) comes at little cost in terms of realism, but significantly improves the tractability of the analysis.[9] In particular, it allows us to disentangle new motives of monetary policy associated with DCP from the standard and well-understood inflationary bias and terms-of-trade externality. For the same reason, we also abstract from markup shocks: while arguably important in practice (see e.g. Chari, Kehoe, and McGrattan 2007, Smets and Wouters 2007), the markup shocks have been widely studied in the previous normative literature (see e.g. Clarida, Gali, and Gertler 1999).

A2 *The payoffs of internationally traded assets D_t^h, $h \in H_t$ and wealth shocks ψ_{it} are independent from monetary policies of individual countries.*

This assumption aims to exclude the additional motive for the monetary policy that arises under incomplete markets: by manipulating the payoffs, the policy can "complete" — at least partially — the span of the assets, improving the risk sharing between countries.[10] The simplest example of such policy would be to inflate away the nominal debt in bad states of the world when borrowing is done in local currency (Engel and Park 2018, Ottonello and Perez 2019). Moreover, changing the real returns of the assets also allows the monetary authorities to manipulate the risk-premium charged by foreign investors, which translates into different equilibrium allocations (Itskhoki and Mukhin 2019b). Both channels have been recently studied from the perspective of the optimal policy (see Fanelli 2017, Hassan, Mertens, and Zhang 2019) and are not the focus of this paper. Assumption 2 excludes such motives for monetary interventions, while still allowing for a rich set of possible asset market structures.[11]

In what follows, we assume that conditions A1-A2 are satisfied. The next lemma describes the case of flexible prices, which provides an important benchmark for our analysis.

Lemma 3 (Flexible prices) *The flexible-price equilibrium $\varphi = 0$ with arbitrary monetary policies implements the first-best allocation, i.e. solves the problem of a non-cooperative social planner.*

This result is based on two properties of the model. First, due to assumption A2, the monetary policy can affect equilibrium allocation only through nominal rigidities and hence, becomes neutral when

[8] There are, of course, several alternative instruments that can sustain the same allocation, e.g. a uniform production subsidy and all firms combined with an export tax. In fact, two instruments would be necessary if demand elasticities and hence, optimal markups were different across domestic and foreign markets. Notice also that one can also use import tariffs instead of export taxes as the Lerner (1936) symmetry holds in the model.

[9] From empirical perspective, it might be easier for the government to subsidize local firms than exporters because of the WTO rules. From theoretical perspective, if the planner could optimally choose both monetary policy and time-invariant subsidies, condition A1 would arise as an endogenous outcome.

[10] A similar motive also arises in a closed economy under distortionary taxation (see Chari, Christiano, and Kehoe 1991).

[11] Notice, in particular, that although we abstract form borrowing constraints to keep analysis more tractable, the results would remain unchanged if one allows for constraints that are not associated with pecuniary externalities.

prices are flexible. Second, assumption A1 eliminates all flexible-price distortions in the economy, which implies that a decentralized equilibrium is efficient and a non-cooperative social planner who is subject to resource constraints cannot improve upon it. An important corollary of Lemma 3 is that in a world with all prices set in producer currency (PCP), the monetary policy can and will implement the first-best non-cooperative allocation.[12] Indeed, given one price in each country that is sticky, the monetary policy can adjust nominal variables in such way that this price stays constant and does not need to adjust. The relative prices and allocations then coincide with the ones under flexible prices and deliver the first best. This result formalizes the famous argument in favor of floating exchange rates by Friedman (1953). With this benchmark in mind, we next study how the optimal policy changes when international prices are sticky in dollars.

3 Non-U.S. Policy

This section describes the optimal policy in a representative non-U.S. economy. We start with the central result that the monetary authorities should target domestic prices and then study several corollaries of such policy: the arising peg to the dollar, a global monetary cycle and the redundancy of the capital controls. The section concludes with several extensions of the baseline model that demonstrate robustness of the results.

3.1 Optimal policy

The planner's problem in a representative non-U.S. economy i is

$$\max_{\{C_{it},N_{it},L_{it},X_{it},B^h_{it},P_{it},P_{iit},P^*_{it},\mathcal{E}_{it}\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it},N_{it},\xi_{it}\right)$$
$$\text{s.t.} \qquad (3)-(10).$$

where all foreign variables are taken as given and independent from domestic policy. Solving this problem, we establish the following result:

Proposition 1 (Non-U.S. policy) *The optimal monetary policy in a non-U.S. economy stabilizes prices of domestic producers:*

$$\pi_{iit}=0. \tag{12}$$

The resulting allocation is generically not the first best.

We emphasize three important features of this result before describing the intuition behind it. First, the proposition shows that the optimal monetary policy — no matter how complicated is the imple-

[12] Despite the first-mover advantage of the U.S., the allocation is symmetric across countries in this case.

menting it interest rate rule — can be summarized in terms of a simple policy target, namely, a zero domestic inflation. This "sufficient statistic" is directly measurable in the data and does not depend on any parameters of the model, including the openness of the economy or the inter-/intra-temporal elasticities of substitution.[13] Second, while formally the policy-relevant price index P_{iit} corresponds to the PPI rather than the CPI, the interpretation becomes more ambiguous once we take into account that most imported consumer goods can be considered as intermediate inputs of wholesalers and retailers. Indeed, more generally, the optimal policy stabilizes the average price of goods that are sticky in domestic currency, which includes the retail prices of foreign products (see Section 3.5 for details).

Finally, notice while it is well known that the optimal monetary policy should stabilize prices in a closed economy or a standard open economy with PCP, it is more surprising that the same policy remains optimal in a fairly general class of DCP models. Indeed, in contrast to the former two cases, the monetary policy can no longer implement the first-best allocation when import and export prices are sticky in dollars and exchange rate devaluations have only limited effect on the terms of trade: the "divine coincidence" does not hold and targeting inflation does not close the output gap.[14] A standard second-best logic suggests in this case, that monetary instruments should allow for (distortionary) domestic inflation to alleviate the terms-of-trade distortion and diminish the output gap. Proposition 1 shows, however, that this is not the case. To understand the intuition behind this surprising result, we start with two special cases considered in the previous literature and then contrast them with the general case.

Special case #1: following the early literature on the optimal monetary policy under DCP (Corsetti and Pesenti 2007, Devereux, Shi, and Xu 2007, Goldberg and Tille 2009), consider the case with prices fully sticky in the currency of invoicing $\varphi \to \infty$. Because of logarithmic preferences, no intermediates and no wealth shocks in these models, the risk sharing is independent from completeness of asset markets and country's trade is balanced in every state of the world:

$$NX_{it} \equiv \gamma \left[P^*_{it} \left(\frac{P^*_{it}}{P^*_t} \right)^{-\varepsilon} \int \left(\frac{\mathcal{E}_{jt} P^*_t}{P_{jt}} \right)^{-\theta} C_{jt} \mathrm{d}j - P^*_t \left(\frac{\mathcal{E}_{it} P^*_t}{P_{it}} \right)^{-\theta} C_{it} \right] = 0.$$

The inspection of this equality reveals the key property of the planner's problem in this limiting case: country's terms of trade, exports, and imports are independent from monetary policy. Indeed, starting from the left, the export price P^*_{it} is constant in dollars, meaning that depreciation of local exchange rate has no effect on the relative price of exports to competing products abroad $\frac{P^*_{it}}{P^*_t}$. With both export prices and quantities exogenous to monetary policy, the trade balance implies that the dollar value of

[13] It is also this simple form of the optimal policy that allows us to solve the planner's problem in a full non-linear stochastic environment without using the second-order approximations, which are usually employed in such cases.

[14] In fact, the monetary policy looses any stabilizing effect as economy becomes fully open $\gamma \to 1$ and there are no prices left that are sticky in domestic currency.

imports is also independent of the planner's policy. But import prices P_t^* are constant in dollars as well, so the quantity imported — the value divided by price — is exogenous too. Notice that this is true despite expenditure switching between domestic and foreign goods generated by exchange rate depreciation: in equilibrium, such policy only changes production and consumption of local goods, but leaves the volume of imports unchanged.

Being unable to affect the external margin, the planner focuses on local production and consumption. As in a closed economy, the optimal policy then prescribes stabilizing the marginal costs of local producers, so that they do not need to adjust prices. Just as in the case of PCP described by Gali and Monacelli (2005), the monetary policy takes producer price index P_{iit} that is sticky in local currency as a numeraire and adjusts other flexible prices — the nominal wage and the exchange rate — to replicate the relative prices from the flexible-price equilibrium. The difference from the PCP case, however, is that export price is different from the domestic one and does not adjust in response to monetary policy.

Special case #2: consider next a setup from Casas, Díez, Gopinath, and Gourinchas (2017) with gradually adjusting prices, which in addition assumes complete asset markets, the Cole and Obstfeld (1991) log-linear preferences $U = \log C - L$, $\theta = 1$, and no intermediate goods in production $F_X = 0$.[15] The key insight of the paper is that combined together, these assumptions ensure that country's export prices and hence, terms of trade are exogenous to monetary policy. To see this, consider the dollar value of exporters' marginal costs:

$$\frac{MC_{it}}{\mathcal{E}_{it}} = \frac{W_{it}}{A_{it}\mathcal{E}_{it}} = \frac{P_{it}C_{it}}{A_{it}\Lambda_i \frac{P_{it}C_{it}}{P_{0t}C_{0t}}} = \frac{P_{0t}C_{0t}}{A_{it}\Lambda_i},$$

where the first equality uses the fact that labor is the only input, the second equality follows from the labor supply and the optimal risk-sharing under complete markets, and Λ_i is a constant determined from the country's budget constraint. Given that both productivity shocks A_{it} and foreign variables $P_{0t}C_{0t}$ are exogenous to the local the planner, the dollar marginal costs of exporters evolve independently from the monetary policy. Intuitively, monetary shocks increase nominal wages and depreciate the nominal exchange rate by the same amount, leaving the dollar value of marginal costs unchanged. This, in turn, implies that dollar export prices and the terms of trade are also exogenous from the planner's perspective. As a result, similarly to the case of fully sticky prices, the optimal policy focuses on domestic margin and targets domestic inflation π_{iit}.

General case: though insightful, the two cases discussed above are clearly quite special — the monetary policy stabilizes domestic prices because it *cannot* affect the terms of trade. Generically, away from these two knife-edge case, the terms of trade are endogenous to monetary policy and yet, according to

[15] While they also assume a unit elasticity of substitution between goods from different countries and Calvo price setting, the results remain the same in our setting with an optimal export tax and Rotemberg pricing.

Proposition 1, the planner *finds it optimal* to target exclusively domestic prices. To see the intuition, consider again the case of PCP: a depreciation of exchange rate automatically — without any adjustment of export prices in the currency of invoicing — makes exported goods cheaper in the currency of destination and generates expenditure switching towards them. This is no longer the case under DCP as the local monetary policy cannot affect the exchange rate between the currency of invoicing (dollar) and the currency of destination. The only way to affect export prices for the planner is to change the dollar value of firms' marginal costs and make them adjust their prices, which is costly under sticky prices. Thus, there is a non-trivial trade-off facing the policymaker: in which direction should it distort domestic margin to improve the external one? For example, if there is a positive productivity shock, should it respond aggressively and raise the terms of trade at the expense of higher price-adjustment costs or should it instead respond mildly to avoid these costs at the expense of more suboptimal terms of trade? It turns out that neither is optimal and the policy should not sacrifice domestic margin at all.

More formally, one can strengthen Proposition 1 by showing that under the optimal monetary policy that targets $\pi_{iit} = 0$, the planner who is allowed to choose export prices π^*_{it} subject to the same adjustment costs as private firms, would set them at the same level as in a decentralized equilibrium. In other words, by targeting producer prices that are sticky in local currency, the optimal monetary policy ensures the optimality of relative prices of domestic products to labor and imported goods. This implies, in turn, that firms' marginal costs, which are a combination of wages, import prices and domestic prices, are also optimal. Given that *conditional* on their marginal costs, exporters set prices in a socially optimal way, the economy achieves a constrained efficient outcome and there is no incentives for the planner to deviate from domestic price stabilization.

The robustness of the intuition can be seen clearly from the extensions of the model. As Appendix **??** makes clear, the same logic applies under Calvo friction despite no explicit costs of price adjustment in this case. The optimal price is trivially the same from private and social perspectives for firms that cannot change their prices in a given period, while all adjusting firms set optimal prices conditional on production costs. It is also easy to extend the result to the setup with idiosyncratic productivity shocks and other forms of heterogeneity across firms. To see the limits of this result, consider a counterexample — a Calvo model with domestic varieties aggregated into a separate CES bundle with elasticity $\eta \neq \varepsilon$ before being exported to other countries. In this case, adjusting exporters set suboptimal price from the social perspective as they do not internalize the externality on non-adjusting firms: indeed, by lowering the prices of the former, the planner can decrease the export price index P^*_{it} and stimulate demand for products with non-adjusting prices when it is too low otherwise. As a result, Proposition 1 does not hold in this case, even though it is still likely to provide a good approximation to the optimal policy. However, the recent empirical evidence casts doubt that the price dispersion within product categories is an important source of inflation costs even in a closed economy (see Nakamura, Steinsson, Sun, and Villar 2018), to say nothing about the dispersion of export prices.

Time consistency Because the optimal policy is independent from the initial foreign asset positions B^h_{i0} and preset prices P_{ii-1}, it is time consistent: if the planner could revise its policy in the future, it would optimally choose not to do so. This result stems from assumptions A1-A2. For example, with some foreign debt denominated in local currency, the government has an incentives to inflate it away. Similarly, there is a well-known inflationary bias when output is inefficiently low due to monopolistic markups. Finally, the price-level stabilization and inflation targeting are no longer equivalent in the presence of markup shocks.

Corollary 1.1 (Time consistency) *The optimal non-U.S. monetary policy is time consistent.*

As emphasized in Lemma 2, the important implication of this result is that the global equilibrium is independent from whether we assume that non-U.S. economies choose their policy under commitment or discretion. On the other hand, the commitment of the U.S. planner is crucial as we discuss below.

3.2 Global monetary cycle

While the previous section shows that the optimal policy targets domestic inflation, this does not necessarily mean that the policy is purely "inward-looking" and does not respond to foreign shocks. Indeed, it seems intuitive that given the effect of the U.S. policy on foreign trade, consumption, and output, it might be optimal for other countries to use their monetary policy to "lean against the wind" and counteract these negative spillovers. To check this hypothesis, we next discuss the *implementation* of the optimal policy, i.e. the dynamics of the nominal interest rates set by the central bank that support the equilibrium allocation.[16]

Although given the generality of our baseline model, it is impossible to characterize the equilibrium allocation and interest rates in closed form, there are important robust predictions that can be inferred from a subset of equilibrium conditions. In particular, we iterate forward the Euler equation (2) for local bonds to back out the nominal interest rates:

$$\frac{U_{Cit}}{P_{it}} = \beta R_{it} \mathbb{E}_t \frac{U_{Cit+1}}{P_{it+1}} = \lim_{T\to\infty} \beta^T \mathbb{E}_t \left(\prod_{\tau=0}^{T-1} R_{it+\tau} \right) \frac{U_{Cit+T}}{P_{it+T}}.$$

Assuming stationarity of real variables and using the optimal monetary policy, one can show that the long-run value of $\lim_{T\to\infty} \frac{U_{Cit+T}}{P_{it+T}}$ is constant and independent from shocks. This implies that $\frac{U_{Cit}}{P_{it}}$ is equal to the expected present value of future interest rates — the characteristic of the monetary policy we focus on henceforth.

[16] We focus in this section on the *equilibrium* behavior of interest rates, which is not enough to uniquely pin down the equilibrium. As in other New-Keynesian models, one needs to go beyond that and specify the policies *off equilibrium* to resolve the indeterminacy, which might also require the use of complementary monetary instruments such as money supply (see Atkeson, Chari, and Kehoe 2010).

Recall next that the marginal costs of producers are a function of nominal wages and prices of intermediates. Using the labor supply condition and imposing the optimal policy $\pi_{iit} = 0$, we get

$$MC_{it} = \frac{m(W_{it}, P_{it})}{A_{it}} = \frac{m(\frac{-U_{Nit}}{U_{Cit}/P_{it}}, P_{it})}{A_{it}} = \text{const},$$

where $m(\cdot)$ is an increasing function homogeneous of degree one. This implies that the response of (average) interest rates — summarized by U_{Cit}/P_{it} — to local and foreign shocks depends on A_{it}, P_{it}, and U_{Nit}. It follows immediately from this expression that despite the fact that the optimal policy targets domestic inflation π_{iit}, it is *generically outward-looking*. In particular, there are two channels through which foreign shocks affect domestic monetary policy. First, consumer price index (4) is proportional to $\mathcal{E}_{it}$. Therefore, as long as imported goods are used as intermediates in production or have to go through wholesale and retail sectors before reaching consumers and the prices of final goods are sticky in local currency, the stabilization of domestic prices requires that the policy partially offsets the fluctuations in the nominal exchange rate and hence, reacts to foreign shocks. Second, the movements in exchange rates affect country's exports and domestic production changing demand for labor and requires the response of monetary policy to stabilize nominal wages. Both channels are, however, absent in the previous normative literature that focused on a knife-edge case with no intermediates $F_X = 0$ and infinite Frisch elasticity $U_{NN} = 0$ resulting in a purely inward-looking policy (see Corsetti and Pesenti 2007, Goldberg and Tille 2009, Casas, Díez, Gopinath, and Gourinchas 2017).

To make further progress in characterizing the optimal policy, we next focus on a case of linear disutility from labor (see Hansen 1985, Rogerson 1988), so that monetary response to foreign shocks is exclusively due to the presence of imported intermediates. While this significantly simplifies the analysis, we note that the second channel does not necessarily work in the opposite direction or change our analytical results under realistic values of parameters (see Section **??**).

Corollary 1.2 (Global monetary cycle) *Assume that $U_{NN} = 0$ and $F_X > 0$. If country's exchange rate depreciates against the dollar due to some U.S. shock or local non-productivity shock, then the optimal policy responds by increasing the nominal interest rates.*

Thus, even if exogenous shocks are completely uncorrelated across countries, the synchronized response of individual economies to U.S. shocks gives rise to a *"Global Monetary Cycle"*. Indeed, because import prices are sticky in dollars, it is the bilateral exchange rate against the U.S. that matters most for the policymakers and generates a highly asymmetric response to shocks originated in different countries. As a result, in spite of a trivial share of the U.S. in global production, consumption and trade, its monetary stance affects the policy in all other economies generating a common global component. The country-specific loadings on this global factor, in turn, increase in the share of DCP in imports and the openness of the economy. Importantly, this normative prediction of the model is strongly supported by the recent empirical evidence by Zhang (2018) who shows that the use of dollar in imports explains the

cross-sectional heterogeneity in pass-through of U.S. monetary shocks into foreign policy rates. Our theoretical results suggest that this pattern holds not only conditional on U.S. monetary shock, but also unconditionally for any U.S. or local shock except for A_{it}.

Moreover, Corollary 1.2 also suggests that it is optimal for non-U.S. economies to have *a partial peg to the dollar*. Indeed, so far, we have taken the equilibrium exchange rates as given as their dynamics depends on other details of the model — the structure of the international asset markets in the first place. Impose next a relatively mild assumption that the exchange rate appreciates in response to a positive interest rate shock, as it is the case for example under complete markets in our model.[17] Then given an exogenous shock that depreciates domestic currency relative the dollar, the optimal monetary policy raises interest rates, which partially offsets the depreciation of local currency and smooths movements in the bilateral exchange rate against the dollar.[18] Thus, our results suggest that the wide use of the dollar in the international trade contributes to the fact that the U.S. retains its dominant position in the global monetary system despite the end of the Bretton Woods System. In particular, the model is consistent with the fact that most countries in the world experience a "fear of floating" and use the dollar as an anchor currency in their monetary policy (Calvo and Reinhart 2002). It also helps to rationalize the fact that emerging economies like Argentina, Brazil, and Turkey with a high ratio of imports-to-GDP and almost all imported goods invoiced in dollars are more sensitive to U.S. shocks and have a stronger peg to the dollar (Ilzetzki, Reinhart, and Rogoff 2018).

Thus, the implications of DCP are similar and highly complementary to the international *financial* spillovers of U.S. monetary policy, which are the focus of a growing "Global Financial Cycle" literature (see e.g. Rey 2013, Aoki, Benigno, and Kiyotaki 2016, Giovanni, Kalemli-Ozcan, Ulu, and Baskaya 2017). Just as in our model, the trade-off facing the policymakers in non-U.S. economies is worsened by the U.S. spillovers: the free floating exchange rate does not fully insulate countries from negative foreign spillovers and does not allow to achieve the efficient allocation, transforming the Trilemma into a "Dilemma". At the same time, both in our setup and in the case of financial spillovers, this does not mean that the planner should give up on exchange rates all together as they still constitute an important margin of the stabilization mechanism (Gourinchas 2018, Kalemli-Ozcan 2019): e.g. as in the case of PCP, it is optimal to fully adjust exchange rates in response to local productivity shocks, even though the resulting expenditure switching under DCP is dampened and cannot restore efficient trade balance.[19] In contrast to the literature on the global financial cycle, however, our results are *not* driven by the frictions in the international asset markets — as they remain true even when the markets are complete — and instead are solely due to the dominance of the dollar as the invoicing currency in global trade.

[17] While this assumption is in line with most empirical evidence (see e.g. Eichenbaum and Evans 1995), there can be important exceptions as e.g. the effect of U.S. forward guidance during the Great Recession (Stavrakeva and Tang 2019).

[18] The mechanism is consistent with the fact that central banks often mention volatile import prices as a rationale for pegging exchange rate.

[19] That said, the optimal peg to the dollar can be quite tight when most volatility in exchange rates are driven by non-productivity shocks.

To take the stock, we show that the optimal policy actively responds U.S. shocks and partially pegs exchange rate to the dollar. These results hold both conditional on U.S. monetary shock, but also unconditionally, i.e. for most local and U.S. shocks. The described optimal policy under DCP is much closer to the one observed in the data than the normative predictions of standard open-economy models.

3.3 Capital controls

The inability of exchange rates to shield economies from the negative U.S. spillovers under DCP and the failure of the Trilemma — at least in its naive formulation — raise a question whether additional restrictions on capital mobility can be used to improve the allocation and increase country's welfare. Indeed, a modern consensus among both policymakers and scholars is that "[the use of capital controls by emerging economies] allows advanced economies to use monetary policy to increase domestic demand, while shielding emerging economies of the undesirable exchange rate effects" (Blanchard 2017). In other words, the U.S. can focus on its domestic objectives when setting the monetary policy, while other countries can insulate their economies from the arising spillovers by augmenting the optimal monetary policy with the appropriate capital controls. And even though most often this argument is made in a context of international financial spillovers, it might be equally important for the spillovers from DCP in global trade.

To study this question, we allow the planner to choose a time-varying asset-specific tax τ^h_{it} on foreign positions of households, so that no-arbitrage conditions (3) describing individual portfolio choice become

$$\mathbb{E}_t \Theta_{it,t+1} \frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}} \frac{\mathcal{Q}^h_{t+1} + D^h_{t+1}}{(1-\tau^h_{it+1})\mathcal{Q}^h_t} = 1.$$

Following the primal approach, we therefore exclude these equilibrium conditions from the planner's problem and allow the government to choose directly the international asset positions $\{B^h_{it}\}$ of the country in addition to the monetary policy:

$$\begin{aligned} \max_{\{C_{it},N_{it},L_{it},X_{it},B^h_{it},P_{it},P_{iit},P^*_{it},\mathcal{E}_{it}\}_t} \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{it}, N_{it}, \xi_{it}\right) \\ \text{s.t.} \quad & (4) - (10). \end{aligned}$$

Solving for the optimal allocation and then using household no-arbitrage conditions to back out the optimal taxes on capital flows, we obtain the following result:

Corollary 1.3 (Capital controls) *Given the optimal monetary policy, capital controls do not insulate other economies from U.S. spillovers and are not used by the planner, i.e.* $\tau^h_{it} = 0$ *under the optimal policy.*

Thus, relaxing the planner's problem by augmenting the monetary instrument with the macro-

prudential tools does not change the optimal policy or the equilibrium allocation in our model as the planner optimally chooses not to intervene into asset markets.[20] At a first glance, this may look as a surprising result: after all, the general principle of the second-best policy is that it is usually optimal to mitigate distortions in one market by distorting other margins in the economy. In our model, this would correspond to distorting intertemporal portfolio decisions of households to improve the allocation in the static goods markets. Indeed, as shown by Farhi and Werning (2016), the laissez-faire risk sharing is *generically* inefficient when monetary policy cannot implement the first-best allocation. Yet, Corollary 1.3 shows that despite suboptimal allocation under the optimal monetary policy, capital controls are redundant and are not used by the planner in our setting independently from the completeness of asset markets. The international spillovers arising from DCP are therefore very different from the ones arising from the financial frictions and cannot be eliminated with macroprudential policy.

To see the intuition behind this result, we map it into the general setup of Farhi and Werning (2016). For the sake of clarity, assume no intermediate goods in production and complete asset markets. There are three goods produced in each country — local consumption good, exported good, and labor. Adopting their notation, let $C_{iit}(I_{it}, \{P_t\})$ and $C^*_{it}(I_{it}, \{P_t\})$ denote household demand respectively for local and export goods as functions of income I_{it} and a vector of prices $\{P_t\}$). Define wedges for two goods against labor as

$$1 - \bar{\tau}_{iit} \equiv \frac{W_{it}}{P_{iit}} \frac{1}{A_{it}}, \qquad 1 - \bar{\tau}^*_{it} \equiv \frac{\varepsilon}{\varepsilon - 1} \frac{W_{it}}{\mathcal{E}_{it} P^*_{it}} \frac{1}{A_{it}}.$$

Clearly, $\bar{\tau}_{iit} = \bar{\tau}^*_{it} = 0$ at the first-best allocation, while positive (negative) values of wedges correspond to a deficit (excess) of demand for this good. With a slight abuse of notation, let τ^h_{it} denote the tax imposed on the asset that pays one unit conditional on a given history h and zero otherwise. Its optimal value is then shown to be

$$\tau^h_{it} = P_{iit} C_{I,iit} \bar{\tau}_{iit} + \mathcal{E}_{it} P^*_{it} C^*_{I,it} \bar{\tau}^*_{it},$$

where $C_I \equiv \frac{\partial C}{\partial I}$ is the marginal propensity to consume for a given product. Intuitively, the reason the risk sharing is generically inefficient under sticky prices is the aggregate demand externality: when making portfolio decisions, individual agents do not internalize the fact that a wealth transfer changes the aggregate demand C_I for depressed/overheated goods and mitigates wedges $\bar{\tau}$ in goods market. Therefore, the planner subsidizes the purchase of assets $\tau^h_{it} > 0$ that pay in the states of the world with inefficiently low demand.

In our setting, however, the optimal monetary policy fully eliminates the wedge for domestic goods by stabilizing the marginal costs of local producers, i.e. $\bar{\tau}_{iit} = 0$. The equilibrium allocation is still not the first-best because the wedge for export goods remains open $\bar{\tau}^*_{it} \neq 0$, but local households do not consume these goods $C^*_{I,it} = 0$ and hence, a redistribution of wealth across states has no effect on

[20] In contrast to the "approximate efficiency" of the risk sharing in Fanelli (2017), the laissez-faire portfolio choice is *exactly* optimal in our setting.

the export output gap. As a result, the main theorem from Farhi and Werning (2016) that taxes τ_{it}^h are generically non-zero does not hold in our model: while it is true that any perturbation of preferences that makes $C_{I,it}^* > 0$ would result in non-trivial financial taxes, a zero demand for export goods is not a knife-edge case, but rather is true *by definition*. Thus, even though the optimal monetary policy cannot implement the first-best allocation under dollar pricing — as in the settings with the zero lower bound or fixed nominal exchange rates (see e.g. Farhi and Werning 2017) — in contrast to these cases, it is powerful enough to eliminate the aggregate demand externality and close the gap between private and social value of insurance.

To be clear, our result should not be interpreted as an argument against the macroprudential policy, which might be important to offset financial spillovers and the local pecuniary externalities (see e.g. Bianchi 2011, Jeanne and Korinek 2010). Instead, it shows that capital controls are not a panacea and cannot be used unilaterally to insulate the country from spillovers arising under DCP.

Cooperative policy We next contrast the optimal capital controls under the non-cooperative policy with the cooperative case, which is arguably of greater interest for the international organizations such as the International Monetary Fund. Since the focus is on the implications of DCP, assume that asset markets are complete and hence, there are no pecuniary externalities that would make international risk sharing constrained inefficient (as in Stiglitz 1982, Geanakoplos and Polemarchakis 1986). The optimal monetary policy still stabilizes domestic prices, but cannot implement the first-best allocation because of suboptimal export prices.[21] One can show then that the optimal cooperative capital controls are still zero in our baseline model. This is, however, an artifact of the assumption that all countries have the same basket of imported goods: a redistribution of wealth across countries does not change the global demand for individual products. Once we allow for non-homothetic preferences for foreign goods, i.e. demand shifters γ_{ji} of country i for goods from country j differ across i, this result does not hold any longer and non-zero capital controls are generically optimal from global perspective. In particular, it can be shown that the global planner redistributes wealth towards countries that consume more of depressed goods. Though a straightforward extension of the result from Farhi and Werning (2016) to a setting with multiple monetary policies, it shows that the optimal cooperative macroprudential policy under DCP should redistribute not to countries with output gaps as the optimal local demand can be ensured by the monetary policy, but instead to foreign countries that import more from them.

3.4 Fiscal policy

Given the limited efficiency of monetary policy and capital controls, a natural question is which other tools can be used to restore the first-best allocation? Indeed, as argued by the recent literature, a

[21] Given that U.S. monetary policy cannot simultaneously stabilize all export prices, the allocation is not the first-best no matter what exactly the U.S. policy is.

sufficiently rich set of fiscal instruments can always replicate the effects of a given monetary policy (Adao, Correia, and Teles 2009) and implement the optimal allocation (Correia, Nicolini, and Teles 2008). Most closely related to our setting, Chen, Devereux, Xu, and Shi (2018) show how the state-contingent taxes can be used to restore the efficiency in the global economy when international prices are sticky in the currency of destination (LCP). To answer the question, we augment the planner with two additional time-varying fiscal instruments: the production subsidy to exporters τ_{it}^* and an export tax levied *on top* of firms' prices at the doc τ_{it}^E. While the two instruments would be largely isomorphic under flexible prices, their effects are quite different in the presence of nominal rigidities: e.g. in the limit of fully sticky prices, an unexpected change in τ_{it}^* has no effect on export dollar prices, while τ_{it}^E allows the planner to have direct control over export prices.

Corollary 1.4 (Fiscal policy) *The non-cooperative first-best allocation can be implemented by (i) monetary policy stabilizing domestic prices $\pi_{iit} = 0$, (ii) production subsidy to exporters τ_{it}^* stabilizing their dollar prices $\pi_{it}^* = 0$, and (iii) export tax stabilizing destination prices in domestic currency $\tau_{it}^E \mathcal{E}_{it} = 1$.*

Thus, a mix of monetary policy with two fiscal instruments is sufficient to replicate the efficient allocation. Intuitively, this is because sticky prices generate three types of distortions relative to the flexible price equilibrium, which according to Lemma 3, is efficient. The first one comes from the suboptimal production and consumption of domestic goods and, as explained above, is fully eliminated by the monetary policy. The second distortion is due to price-adjustment costs of exporters. The only way to avoid these costs for the economy is to ensure that the marginal costs of exporters are constant in dollars and hence, there is no need for firms to change their prices. This goal can be achieved by using a time-varying production subsidy to exporters τ_{it}^*. Having export prices constant in dollars, however, cannot be optimal because of the suboptimal terms of trade. To eliminate this last distortion, the planner uses the export tax τ_{it}^E to target the prices faced by consumers in foreign markets of destination. In particular, since domestic prices are stable in local currency and export prices are constant in dollars, this last instrument is necessary to eliminate deviations from the law of one price.

While it is hardly surprising that the efficient allocation can be restored using the sufficient number of fiscal instruments, it is interesting that the optimal policy is actually quite simple and can be characterized in terms of three directly observable targets: domestic prices and two measures of export prices. No other details of the models, including the values of any parameters, are relevant for the policymaker given these three sufficient statistic, although the resulting allocation and the particular values of taxes and interest rates implementing this allocation are sensitive to the details of the model. Thus, similarly to the fiscal devaluations in Farhi, Gopinath, and Itskhoki (2014), our policies are also "robust", but in terms of objectives rather than implementation.

Clearly, this is just one set of policies that are sufficient to implement the optimal allocation, while as usual, there exist alternative instruments that can achieve the same goal. For example, one can use production subsidy to local firms τ_{it} to stabilize their marginal costs and allow the monetary policy

to target instead other distortions. That said, the export tax τ_{it}^E is crucial for the implementation of the efficient allocation and cannot be substituted with other instruments. In particular, the Lerner symmetry does not hold under DCP, as pointed out by Barbiero, Farhi, Gopinath, and Itskhoki (2019), and hence, one cannot replace the export tax with import tariffs.

We conclude this section by emphasizing that in spite of the benefits and relative simplicity of the optimal fiscal policy, it is still uncommon in modern world to have state-dependent production subsidies and export taxes. The relative flexibility of monetary policy, on the other hand, makes it a first instrument to be used against exogenous shocks. Interestingly, however, as our results show, the optimal monetary policy is the same no mater whether it is the only policy tool or if it is complemented with export taxes and subsidies.

3.5 Extensions

To keep equilibrium conditions tractable and directly comparable with the ones from the previous literature, the baseline model assumes that all firms and consumers in an economy share the same simple CES aggregator of home and foreign products. This section relaxes the assumption. The goal is threefold. First, this setup is arguably more realistic and therefore, provides a natural laboratory for our numerical exercise in Section **??**. Second, it clarifies several aspects of the sufficient statistic result from above: e.g. whether the policy should stabilize the costs of local firms or exporters. Finally, the additional ingredients allow us to endogeneize currency choice and show that our normative results are not subject to Lucas (1976) critique.

Setup In particular, we relax three parametric assumptions from the baseline model. First, the home bias is allowed to be different for consumers, local producers and exporters:

$$
\begin{aligned}
C_{it} &= \left[(1-\gamma_c)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma_c^{\frac{1}{\theta}} C_{it}^{*\,\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \\
X_{it}^d &= \left[(1-\gamma_d)^{\frac{1}{\theta}} X_{iit}^{\frac{\theta-1}{\theta}} + \gamma_d^{\frac{1}{\theta}} X_{it}^{*\,\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \\
X_{it}^e &= \left[(1-\gamma_e)^{\frac{1}{\theta}} X_{iit}^{\frac{\theta-1}{\theta}} + \gamma_e^{\frac{1}{\theta}} X_{it}^{*\,\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},
\end{aligned}
$$

where X_{it}^d and X_{it}^e are respectively the bundles of intermediates for domestic firms and exporters. The heterogenous γ's allow the model to capture, among other things, the fact that exporting firms are also the largest importers and that consumers might not have direct access to foreign goods, but rather have to buy them from local retailers.

Second, we allow for heterogenous production functions used by local firms and exporters

$$
Y_{iit} = A_{iit} F(L_{iit}, X_{it}^d) \quad \text{and} \quad Y_{it}^* = A_{it}^* G(L_{it}^*, X_{it}^e),
$$

where $F(\cdot)$ and $G(\cdot)$ are both constant returns to scale. Thus, the labor intensity and productivity shocks might differ across two types of firms. Finally, instead of the CES bundle, we assume that both local and foreign varieties are combined via the Kimball (1995) aggregator, e.g. the bundles of local and foreign consumption products C_{iit} and C^*_{it} are defined by

$$\int \Upsilon\left(\frac{C_{iit}(\omega)}{C_{iit}}\right) \mathrm{d}\omega = 1, \quad \text{and} \quad \int\int \Upsilon\left(\frac{C_{jit}(\omega)}{C^*_{it}}\right) \mathrm{d}\omega \mathrm{d}j = 1,$$

where $\Upsilon(1) = \Upsilon'(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$, and the bundles of intermediates X_{iit} and X^*_{it} are defined symmetrically. Appendix **??** discusses the resulting price index and the optimal price setting. The key difference from the baseline model introduced by the Kimball aggregator is that the prices of individual firms now depend not only on their marginal costs, but also on prices of competitors. This, in turn, implies that the model can reproduce pricing-to-market and the incomplete pass-through that have been well documented in the data even for the long horizons when the prices adjust fully (for a survey of the literature see Burstein and Gopinath 2012).

Optimal policy The extended version of the model is significantly more complex than the baseline setup, yet Lemmas 1-3 still apply. In particular, under assumptions A1-A2, the flexible-price equilibrium is efficient from the perspective of an individual country. Notice that this is true despite the incomplete pass-through of productivity and exchange rate shocks into prices. The reason is that import prices are taken as given by a small open economy, while foreign demand function is effectively the same for individual exporters and the planner, which implies that export prices P^*_{it} are efficient as well. As a result, the optimal policy under sticky prices remains the same as in the baseline model:

Lemma 4 (Robustness) *All results from Proposition 1 and Corollaries 1.1-1.3 hold in this general setup.*

Importantly, the lemma shows not only the robustness of our results, but also clarifies the key feature of the optimal policy: it is the marginal costs of *local* sellers that the monetary authorities should stabilize. On the one hand, that might include the prices of most imported goods if they have to go through the wholesale and retail sectors before reaching the households. On the other hand, this target does not include foreign intermediates used by local exporters and does not directly depend on the productivity shocks in the export sector. Thus, the important corollary of Lemma 4 is that the optimal policy can vary across countries not only because of different openness and the share of DCP (as discussed in Section 3.2), but also because of heterogeneous input-output structure of production. In particular, economies with a higher share of consumption goods in imports sold by local retailers need to respond more strongly to U.S. shocks and have a tighter peg to the dollar than countries in which exporters account for most of imported goods.

Currency choice So far, we have treated the currency of invoicing as a primitive of the model following most of the previous normative literature. One might, however, be concerned that our results

are subject to Lucas critique and can change dramatically once the firms are allowed to make *optimally* the invoicing decisions. In what follows, we argue that this is not the case and both DCP and the optimal policy described above can be sustained as an equilibrium outcome. To this end, we augment the extended version of the model with the endogenous currency choice from Mukhin (2018), which in turn builds on the seminal contributions of Engel (2006) and Gopinath, Itskhoki, and Rigobon (2010).[22] For tractability, consider a one-period version of the model — extending results to a multi-period framework is straightforward (see the papers cited above) — so that we can skip the time subscript t. In addition, as is standard in the literature, we use the first- and second-order approximations to the equilibrium conditions to solve for the currency choice (see Appendix **??** for details).

For concreteness, consider the price setting of exporter from country i, while the derivations for domestic firms are symmetric. Let $\bar{p}_i^*$ denote the export price preset by the firm before the realization of shocks, $\tilde{p}_i^*$ is the price the firm would set in a given state of the world if prices were flexible, and p_i^* is the price the firm actually sets given the adjustment costs and the preset price. All prices are expressed in dollars and are the log deviations from the steady-state values. The optimal flexible price maximizes firm's profits in a given state of the world and, to the first-order approximation, can be written as

$$\begin{aligned}\tilde{p}_i^* &= (1-\alpha)(mc_i - e_i) + \alpha p^* \\ &= (1-\alpha)\Big[(1-\phi_e)(w_i - e_i) + \phi_e(1-\gamma_e)(p_{ii} - e_i) + \phi_e\gamma_e p^* - a_i^*\Big] + \alpha p^*,\end{aligned}$$

where α depends on the curvature of the Kimball demand and ϕ_e is the (steady-state) share of intermediates in marginal costs of exporters. Thus, in contrast to the CES case, which corresponds to $\alpha = 0$, the Kimball demand generates strategic complementarities in price setting: the optimal price depends not only on firm's marginal costs $mc_i - e_i$ (converted into dollars), but also on the prices of competitors p^*. The marginal costs, in turn, depend on wages, prices of local and foreign intermediates and productivity shocks.

The profit maximization after the realization of shocks, implies that the optimal ex-post price can be expressed as a weighted average of the preset and the flexible prices:

$$p_i^* = \lambda \bar{p}_i^* + (1-\lambda)\tilde{p}_i^*,$$

where λ is increasing in adjustment costs parameter φ. Given that firm's profits attain maximum at $\tilde{p}_i^*$, to the second-order of approximation, the losses due to sticky prices are proportional to the deviation of the ex-post price from the flexible price: $\left(p_i^* - \tilde{p}_i^*\right)^2 = \lambda^2\left(\bar{p}_i^* - \tilde{p}_i^*\right)^2$. Therefore, the ex-ante objective

[22] See also closely related models by Corsetti and Pesenti (2002), Devereux, Engel, and Storgaard (2004), Bacchetta and van Wincoop (2005), Goldberg and Tille (2008), Cravino (2014). The two appealing features of these models are that they do not require any additional frictions on top of sticky prices and are supported by empirical studies. For the alternative models of endogenous currency choice see Gopinath and Stein (2017), Drenik, Kirpalani, and Perez (2018), Chahrour and Valchev (2017), Rey (2001).

of the firm is to choose the currency of invoicing k and the preset price $\bar{p}_i^k$ in that currency to minimize

$$\mathbb{E}\left(\bar{p}_i^* - \tilde{p}_i^*\right)^2 = \mathbb{E}\left(\bar{p}_i^k - e_k - \tilde{p}_i^*\right)^2 = \mathbb{E}\left(\tilde{p}_i^* + e_k\right)^2,$$

where the last equality uses the fact that shocks have zero mean and hence, $\bar{p}_i^k = 0$ for any currency k. Intuitively, because it is costly to adjust prices after the realization of shocks, the firm looks for currency k, in which its optimal price $\tilde{p}_i^* + e_k$ is most stable and requires the least adjustment ex post.[23] It follows that exporters choose DCP when the optimal price has lower volatility in dollars than in any other currency, i.e. $\mathbb{E}\left(\tilde{p}_i^*\right)^2 \leq \mathbb{E}\left(\tilde{p}_i^* + e_k\right)^2$ for $\forall k$.[24] Similarly, domestic firms set prices in local currency if $\mathbb{E}\left(\tilde{p}_{ii}\right)^2 \leq \mathbb{E}\left(\tilde{p}_{ii} - e_i + e_k\right)^2$ for $\forall k$.

Given that the currency choice depends on the properties of nominal wages and exchange rates, there are two-way interactions between the monetary policy and firms' invoicing decisions. This implies that we need to specify the strategic interactions between the two agents and in particular, whether the planner internalizes the effect of monetary policy on firms' currency choice or rather takes it as given. It turns out, however, that in either case, one can establish the following result:

Lemma 5 (Endogenous currency choice) *Assume that α and $\phi_e\gamma_e$ are sufficiently high. Then there exists an equilibrium, in which firms choose local currency in domestic market and dollars in foreign markets, while the optimal policy is described by Lemma 4.*

Consider first the case when invoicing is taken as given by the monetary authorities, i.e. we are looking for a Nash equilibrium in a game where planner and firms move simultaneously. Conjecture as in the baseline model that producers use local currency domestically and dollars abroad, so that the optimal policy is given by Lemma 4. Since domestic prices are fully stable in local currency $p_{ii} = 0$, firms unambiguously choose home currency in domestic market as it allows them to fully avoid price-adjustment costs. In contrast, exporters face a non-trivial trade-off: the wages and prices of local intermediates are relatively more stable in producer currency, while the prices of imported inputs and competing products are more stable in foreign currencies, in which exporters from other countries set their prices. High values of α and $\phi_e\gamma_e$ ensure that these strategic complementarities across exporters from different countries dominate and it is optimal for them to coordinate on a single currency to synchronize movements in their prices. This allows to sustain a DCP equilibrium in international trade, confirming the initial conjecture.[25]

[23] For example, if the optimal price is \$100 in all states of the world, setting the price in dollars is clearly optimal as no ex-post adjustments are required in this case, while setting the price in euros is suboptimal as the ex-post price would deviate from the optimal level due to movements in exchange rates.

[24] The optimal currency choice condition can also be rewritten as $-\frac{\mathbb{E}\tilde{p}_i^* e_k}{\mathbb{E}e_k^2} \leq \frac{1}{2}$, i.e. firms choose DCP when the pass-through of any exchange rate $-e_k$ into the optimal dollar price $\tilde{p}_i^*$ is below one half.

[25] As usual, strong strategic complementarities can potentially lead to multiple equilibria with different currencies used as a dominant one. The dollar is, however, more likely to play this role given the stabilizing policy of the U.S. (see Section 4.1).

Perhaps more surprisingly, Lemma 5 holds even when the planner acts as a Stackelberg leader and commits to monetary policy before firms make their invoicing decisions. To see where this result is coming from, consider three invoicing margins: domestic, import and export ones. Producer currency pricing in the domestic market increases the efficiency of the monetary policy to stabilize the economy, so the planner has no incentives to distort this margin. The currency choice of importers, on the other hand, is almost exogenous to the planner: note that no characteristics of the destination economy enter the optimal price $\tilde{p}_i^*$. The only way, in which a planner can affect the invoicing decisions of importers is by making its exchange rate so stable that firms choose its currency as a dominant one. At the same time, the DCP equilibrium might still survive especially if the U.S. has a large-economy advantage.[26] Finally, the most interesting margin is the currency choice of exporters as it is the one that both depends on local monetary policy and distorts the allocation relative to the flexible-price equilibrium. One can show, however, that exporters' invoicing decisions are efficient from the perspective of individual economy and the planner would not change them even it were allowed to do so (in addition to setting the monetary policy). Thus, this result strengthens the intuition behind Proposition 1: conditional on monetary policy that stabilizes p_{ii}, both prices and currency choice of exporters are socially optimal, which — combined with the fact that targeting p_{ii} is the optimal response of monetary policy given DCP — implies that the planner has no incentives to deviate from this equilibrium.

4 U.S. Policy

The previous section describes the optimal policy of non-U.S. economies for an arbitrary monetary policy of the U.S. To characterize the subgame-perfect equilibrium, we next apply the backward induction and use the best responses of other countries to solve the U.S. planner's problem.[27] In contrast to other economies, the optimal policy of the U.S. balances several objectives and cannot be summarized with a simple target rule. We characterize analytically the motives of the optimal policy and its welfare implications under complete asset markets and provide numerical results for the alternative cases.

[26] See Mukhin (2018) for details. Note that in contrast to that paper, we assume the Kimball aggregator at the level of internationally traded goods and do not include local products in the market of destination to simplify the analysis.

[27] Notice, however, that none of our results below are driven by the timing assumption as the same equilibrium would arise in a simultaneous-move game with all countries choosing π_{iit} (Lemma 2). Indeed, in both sequential and simultaneous games, the U.S. takes the strategies of other economies $\pi_{iit} = 0$ as given and hence, chooses the same optimal policy.

4.1 Optimal policy

As a Stackelberg leader, the U.S. planner maximizes the welfare over all prices and quantities in global economy taking as given the optimal policy in other countries:

$$\max_{\{C_{jt},N_{jt},L_{jt},X_{jt},B^h_{jt},P_{jt},P_{jjt},P^*_{jt},\mathcal{E}_{jt}\}_{j,t}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it},N_{it},\xi_{it}\right)$$
$$\text{s.t.} \qquad (3)-(12).$$

This problem is fundamentally different from the problem of other economies discussed in Section 3. The non-U.S. countries take global prices and quantities as given and their monetary policy can only affect local margins. As a result, the equilibrium with flexible domestic and export prices gives the upper bound on the welfare they can achieve in the presence of nominal frictions. In contrast, because of the dollar invoicing in international trade, the U.S. monetary policy can affect the relative prices outside of its economy and can potentially implement allocations and welfare that are unattainable under flexible prices. This can be perhaps best seen in the extreme case with no home bias $\gamma \to 1$ when all prices in global economy are sticky in dollars and hence, non-U.S. monetary policy becomes completely inefficient, while the U.S. policy determines allocations in all countries.

Because of the global effects, the optimal U.S. policy generically deviates from inflation targeting and pursues other objectives as well. As a result, in contrast to optimal policy in other economies, it cannot be summarized with a simple targeting rule and depends on the details of the setup. To make progress in understanding the motives of U.S. policy, we adopt a different approach than in Section 3 and impose more structure in the model:

A 3 *Domestic and export prices are fully rigid $\varphi \to \infty$, and the asset markets are complete.*

While in general case, the optimal policy is implicitly determined by a large system of dynamic equations, assumption A3 provides a lot of tractability by allowing for a state-by-state analysis. It is worth emphasizing, however, that the same motives that arise in this "static" case are equally applicable in the general dynamic setup and are studied numerically in Section **??**. Combining the optimality conditions and taking the first-order approximation around the symmetric steady state, one can decompose the optimal policy in a given state into three separate motives.[28] Intuitively, U.S. problem can be partitioned into three blocks — the static market clearing conditions, the intertemporal risk-sharing, and the ex-ante price-setting — each of which generate a separate motive for the optimal policy. The next

[28] An alternative and widely used way to characterize the motives of the optimal policy is to focus on the terms in a quadratic approximation of the objective function (see e.g. Engel 2011, Corsetti, Dedola, and Leduc 2010). We do this for a special parametrization of the model in Appendix **??**. At the same time, the presence of a "gap" in the loss function does not necessarily imply that the optimal policy should target it (see Casas, Díez, Gopinath, and Gourinchas 2017 for a counterexample).

proposition expresses the policy rule in terms of simple sufficient statistics, which even if not directly observable in the data, provide important intuition for the motives of the U.S. policy.

Proposition 2 (U.S. policy) *Under assumptions A1-A3, the optimal policy rule for U.S. monetary policy balances three motives and to the first-order of approximation is given by*

$$\Gamma \cdot mc_{it} - \gamma\epsilon \cdot nx_{it} + \gamma\Xi \cdot \int mc^*_{jt} \mathrm{d}j = 0, \tag{13}$$

where mc_{it} is the log-deviation of marginal costs, $mc^*_{it} \equiv mc_{it} - e_{it}$ is the dollar value of marginal costs, nx_{it} is the net export, ϵ is the steady-state elasticity of global SDF with respect to U.S. monetary policy, and $\Gamma, \Xi > 0$ are steady-state constants — all defined in the appendix.

Motive #1: The first term in (13) corresponds to the marginal costs of local producers and constitutes the *price-stabilization motive* of the monetary policy. Notice that this motive dominates when international trade is small, i.e. $\gamma \approx 0$, and all other terms drop out. While price stability constitutes an important objective for all economies under DCP, the implications of this policy for the U.S. are largely different than for other countries. First, in contrast to other economies, in which inflation targeting stabilizes only internal margin, but leaves exports at the suboptimal level, the monetary policy of the U.S. simultaneously affects domestic and foreign demand for its products. Indeed, because of PCP of U.S. exporters, a depreciation of the dollar decreases their prices in the currency of destination and boosts country's exports.[29] Interestingly, this stimulative effect holds even when other countries peg their exchange rates to the dollar. Intuitively, to prevent the appreciation of their currencies in response to monetary easing in the U.S., other economies follow a similar stimulative policy raising demand for both local and imported goods, including the ones exported from the U.S.

The second key difference between the policy in the U.S. and other countries is that its implementation leads to asymmetric response to foreign shocks. According to Corollary 1.2, the movements in U.S. exchange rate affect the prices of imported intermediates in all other countries and require intervention of local monetary policy to keep domestic inflation at zero level. At the same time, because of dollar pricing, U.S. import prices are unresponsive to movements in foreign exchange rates making U.S. monetary policy fully inward-looking and responding only to local shocks.

Motive #2: The fact that the U.S. monetary policy affects production and consumption in other countries implies that it also controls global stochastic discount factor and hence, the prices of Arrow-Debreu securities. This in turn, gives rise to the second term in policy rule (13), which corresponds to the *"dynamic terms-of-trade manipulation" motive* as defined by Costinot, Lorenzoni, and Werning (2014). As they show in a context of a flexible-price model with one internationally traded good, a large economy

[29] As a result, the price-stabilization motive remains relevant even in the limit with no home bias $\gamma \to 1$.

faces a downward slopping demand from the rest of the world: because of the consumption smoothing motive, countries are willing to pay more for the assets with high returns in states of the world when their consumption is low (see also Hassan 2013). For this reason, a large country can act as a monopolist and extract additional rents by manipulating its current account relative to the laissez-faire equilibrium. Our analysis shows that the same motive is also relevant for the monetary policy of an infinitely *small* economy if it is the issuer of the dominant currency.

Consider for example the case when a stimulative monetary policy in the U.S. decreases consumer prices worldwide boosting consumption and decreasing the global SDF, i.e. $\epsilon < 0$. According to equation (13), the optimal policy then overstimulates the economy — i.e. generates an inflationary pressure by setting marginal costs above the price level — in the states of the world, in which the U.S. borrows from the rest of the world $nx_{it} < 0$. This lowers the SDF and allows the country to borrow more cheaply. As long as U.S. trade with other countries depends on foreign shocks, the optimal policy will respond to them and is outward-looking. Symmetrically, the U.S. has incentives to contract the economy and raise the global SDF to enjoy higher interest rates in the states of the world when country runs a positive current account $nx_{it} > 0$.[30] Thus, the effect of Fed's policy on international asset markets under DCP can potentially contribute to the U.S. "exorbitant privilege" (Gourinchas and Rey 2007).

Note that this motive is new to the literature as the previous models focused on log preferences, which make trade balanced $nx_{it} = 0$ in every state of the world (see Corsetti and Pesenti 2007, Goldberg and Tille 2009). The dynamic terms-of-trade motive also drops out under financial autarky when there are internationally traded assets, but is relevant under any other form of (in)complete markets and does not rely on the simplifying assumption of complete markets.

Motive #3: The final term in the optimal policy rule (13) comes from the price-setting constraint of exporters from other countries. Intuitively, in contrast to other countries, the U.S. can affect its terms of trade not only through export prices, but also by changing the prices of foreign exporters. As a result, the optimal markups of U.S. exporters do not eliminate the *terms-of-trade motive* of the monetary policy. Indeed, if these prices were preset at an exogenously given level, the U.S. could inflate them away and buy foreign goods at an arbitrary low prices. Of course, this does not happen under rational expectations when foreign prices cannot deviate too much from the average marginal costs, imposing an important constraint on the global real effects of U.S. monetary policy. The optimal policy, however, aims to relax this constraint by partially stabilizing the dollar prices of global exporters.[31] This shifts the U.S. policy in the direction of international cooperation (see below) and makes it respond to global shocks outside the U.S.

[30] Our analysis focuses on the approximation around the symmetric steady-state with zero net foreign asset positions. If in contrast one assumes that the U.S. runs a negative trade balance in the steady-state, the optimal policy rule has an additional term with the time-varying stochastic discount factor.

[31] This motive was first described in a context of LCP model by Devereux and Engel (1998) and is also present in DCP models of Devereux, Shi, and Xu (2007), Corsetti and Pesenti (2007), Goldberg and Tille (2009).

To what extent should the Fed be concerned about the global spillovers of its policy? Proposition 2 provides new insights to this classical question. On the one hand, the optimal *ex-post* policy in a given state of the world focuses exclusively on the first motive — price stability. Independently from the sign and size of the spillovers on other economies, such policy allows the U.S. to achieve the optimal level of both local and foreign demand. In this sense, the Fed does not need to worry about global effects of its policy and can focus exclusively on domestic objectives, consistent with the view of several U.S. policymakers (see e.g. Bernanke 2017). On the other hand, as the two additional motives in policy rule (13) make clear, ignoring the spillovers is suboptimal from the *ex-ante* perspective as they can backfire through both international financial and goods markets making the U.S. borrowing and buying foreign goods more expensive. It is therefore in the Fed interests to pay attention to the global effects of its monetary policy, even though the relative weight of these motives is proportional to the openness of the U.S. economy γ and is likely to be small in practice.

Time inconsistency The ex-ante nature of the two motives for U.S. monetary policy described above implies that commitment is crucial for the implementation of the optimal policy.

Corollary 2.1 (Time inconsistency) *The optimal U.S. monetary policy is time inconsistent.*

In contrast to the case with full commitment, a discretionary planner maximizes the welfare separately in each state of the world taking all preset prices and transfers between countries as given. As a result, the policymaker cannot manipulate the import prices or the prices of internationally traded securities and deviates from the optimal policy with commitment.

4.2 Gains from DCP

The previous sections show that both the international transmission of shocks and the optimal monetary policy are different for the U.S. than for other economies. The fundamental question that remains open, however, is whether these asymmetries translate into differences in countries' welfare? Does the U.S. actually benefit from the dominance of the dollar? Answering this question is less straightforward than one could expect. In particular, while targeting the global export prices — the last term in U.S. policy (13) — increases the absolute welfare of the U.S., it also decreases its welfare relative to other countries: stable international prices benefit all economies, but come at the expense of domestic price stabilization for the U.S. This makes welfare analysis quite challenging. The important insight of this section is that much progress can be done in answering the question by focusing on the case when openness of economies γ is relatively low. This limit is clearly relevant for the U.S., but we also use it for other economies to preserve the symmetry between countries, so that DCP remains the only source of differences in welfare. We confirm the accuracy of this approximation in numerical section below.

Corollary 2.2 (Welfare) *Assume complete asset markets and sufficiently low openness of economies γ. Then U.S. welfare is higher than in other economies.*

While we relegate the full proof of this result to the appendix, it is instructive to discuss the key steps. Let $\mathcal{W}_i(\Omega_i, \Omega_{-i}, \gamma)$ denote the equilibrium welfare of country i as a function of its policy Ω_i and policies of other countries Ω_{-i}, and the openness of economies γ. Note that both policy and welfare are the same for the U.S. as for other economies in the autarky limit, so that the ranking of the welfare in the neighbourhood $\gamma \gtrless 0$ depends on the first derivative $\frac{\mathrm{d}\mathcal{W}_i}{\mathrm{d}\gamma} |_{\gamma=0}$. The envelope condition implies that we can ignore the partial derivative with respect to country's own policy $\frac{\partial \mathcal{W}_i}{\partial \Omega_i} = 0$. Moreover, the welfare is also independent from foreign policy in the autarky limit, so that $\frac{\partial \mathcal{W}_i}{\partial \Omega_{-i}} = 0$. Thus, one only needs to take a partial derivative with respect to γ and evaluate it in the closed-economy limit with monetary policy targeting domestic inflation in both economies.

Intuitively, there are two sources of differences in welfare between countries: the asymmetric transmission of shocks for a given policy and the differences in the optimal policies. The fact that we only need to look at the *partial* derivative of the welfare with respect to γ shows that the latter channel is of the second order and is dominated by the differences in the spillovers when economies are close enough to the autarky limit. A higher welfare of the U.S. then follows directly from the fact that its monetary policy is more efficient than the policy of other countries: in contrast to non-U.S. economies that cannot stabilize their external margin, domestic price targeting in the U.S. also eliminates the costs of export price adjustment and closes the output gap. In this sense, the U.S. benefits from DCP: the dominant-currency provider is better off than any other economy. It follows that if complementarities in price setting are strong enough — as they arguably are in the modern globalized world — making firms coordinate on one currency, a competition among countries can emerge for the status of the issuer of the dominant currency (*cf.* Farhi and Maggiori 2017).

4.3 Gains from cooperation

Are there welfare gains from international cooperation under DCP? Just as in the case of flexible prices or prices sticky in producer currency, such cooperation can be Pareto improving as it precludes countries from the "beggar-thy-neighbour" policies exploiting the terms-of-trade externality (see Corsetti and Pesenti 2001). In particular, if the global planner is free to choose both monetary policy and production subsidies to firms, it would eliminate monopolistic markups of both domestic producers and exporters, i.e. $\tau_i^* = \tau_i = \frac{\varepsilon - 1}{\varepsilon}$. Such policy unambiguously benefits all countries, including the U.S.

What distinguishes the case of DCP, however, are the externalities that arise from the asymmetric spillovers of U.S. *monetary* policy on other economies. U.S. welfare has an infinitely small weight in the objective function of the global planner, but its monetary policy affects all international prices. It follows that the optimal cooperative solution is to use U.S. monetary tools to bring global export prices closer to the optimal level and respond only to global shocks rather than focus on the idiosyncratic

shocks in the U.S.[32] More formally, the optimal policy stabilizes the markups of exporters around the world weighted by their sales shares, which as in the ideal price index, are proportional to $P_{it}^{*1-\varepsilon}$, or equivalently, to $S_{it}^{\varepsilon-1}$. At the same time, the optimal policy in other countries does not change relative to the non-cooperative case and still targets local prices.

Corollary 2.3 (Cooperative policy) *Assume complete asset markets and* $\tau_i^* = \tau_i = \frac{\varepsilon-1}{\varepsilon}$. *Then the optimal cooperative policy stabilizes domestic prices in non-U.S. economies* $\pi_{iit} = 0$ *and uses U.S. monetary policy to stabilize in dollars a weighted average of world export prices, i.e.* $\int \frac{MC_{it}}{\mathcal{E}_{it}P_{it}^*} S_{it}^{\varepsilon-1} \mathrm{d}i = 0$.

It follows, there is a conflict of interests between countries. The non-U.S. countries unambiguously win from cooperation because of the elimination of both terms-of-trade externality and the negative spillovers from the U.S. In contrast, the U.S. benefits from lower markups of foreign exporters, but loses from changes in monetary policy: the cooperation requires the U.S. to sacrifice domestic objectives for the world economy while leaving the monetary policies of other economies unchanged. Therefore, in contrast to the quid pro quo under PCP, the U.S. might be not interested in joining other countries in international cooperation under DCP. One special case when countries' interests are, however, perfectly aligned is the response to global shocks: the U.S. does not have to choose between local and global objectives and the stabilization of marginal costs in all countries achieves the optimal allocation. This prediction is consistent with the high level of cooperation between central bankers around the world during the global financial crisis of 2008–2009. The model also explains why it is much harder to sustain the global cooperation when the crisis ends in the U.S., while other countries have not fully recovered.

4.4 Currency union

Our model also has important implications for the optimal currency area (Mundell 1961). While there is much debate about the costs of having a common currency, the benefits of currency unions are less well understood. The new insight that emerges from Corollary 2.2 is that forming a currency union such as the Eurozone can improve the welfare of its members if it helps to promote the common currency in the international trade.[33] Indeed, exporters within the union are more likely to use the local currency instead of the dollar as the currencies of producer and buyer coincide in this case (see Mukhin 2018).[34] Moreover, because of strategic complementarities in currency choice, the trade flows between the union and other countries are more likely to switch to the common currency as well. Thus, if the euro manages to replace the dollar at the global stage, the Eurozone monetary policy becomes more efficient in generating the optimal expenditure switching towards the exported goods. These

[32] This is, of course, an extreme result that is due the small size of the U.S. Under a more realistic assumption that the U.S. accounts for a significant fraction of global GDP, the policy would target a weighted average of local and global shocks.

[33] See Chari, Dovis, and Kehoe (2013) for other benefits of a currency union arising from a higher level of commitment.

[34] Cavallo, Neiman, and Rigobon (2014) provide supporting evidence that the deviations from the law of one price are smaller for countries with a common currency.

new gains from trading with countries outside of the union can potentially outweigh the costs of less efficient stabilization within the union. A more empirically relevant question, however, is whether the union gains from a common currency when it is used for invoicing of the Eurozone's imports and exports, but does not replace the dollar as a vehicle currency in trade between third countries.[35]

Corollary 2.4 (Eurozone) *Assume that the Eurozone is a small economy with imports and exports invoiced in euros and all other international trade is in dollars. Then under complete asset markets, the welfare of the Eurozone is higher than for other non-U.S. economies.*

5 Conclusion

This paper characterizes the optimal non-cooperative monetary policy of the U.S. and other economies in a world with international prices sticky in dollars. We show that although targeting domestic inflation remains optimal for non-U.S. economies, such policy cannot implement the efficient allocation, generates a partial peg to the dollar and gives rise to the global monetary cycle. Individual countries cannot unilaterally improve the allocation using capital controls, which are only employed by a global planner. The optimal U.S. policy, on the other hand, deviates from price stabilization to extract rents in the goods and asset markets. The U.S. is likely to benefit from the dominant status of its currency relative to other economies, while international cooperation leads to asymmetric gains across countries and might be not in U.S. self-interest.

[35] Gopinath (2016) shows that the share of the Euro in global trade is close to the trade share of the Eurozone.

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A Appendix

A.1 Additional Figures and Table

Table A1: Comparison to the literature

	DSX	CP	GT	CDGG	EM
Environment:					
# of countries	two		three	SOE	continuum
preferences	log-linear				general
intermediates	no				yes
asset markets	complete				arbitrary
prices	fully sticky			Calvo	Rotemberg/Calvo
terms-of-trade	exogenous to MP				endogenous
currency choice	rationalized	exogenous			endogenous
Non-U.S. policy:					
optimal target	price stabilization				
allocation	inefficient				
implementation	inward-looking				outward-looking
exchange rates	floating				partial peg
capital contols	–				inefficient
trade policy	–				efficient
U.S. policy motives:					
import prices	yes			–	yes
dynamic ToT	no			–	yes
welfare effects	negative	–	ambiguous	–	ambiguous
cooperative policy	yes			–	yes

Note: DSX stands for Devereux, Shi, and Xu (2007), CP for Corsetti and Pesenti (2007), GT for Goldberg and Tille (2009), CDGG for Casas, Díez, Gopinath, and Gourinchas (2017), EM for this paper.

A.2 Equilibrium Conditions

A.2.1 Demand block

Households choose C_{it} and N_{it} in each state h to maximize (2.1)

$$U_i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - N_{it} \right]$$

subject to the ex-ante budget constraint (**??**)

$$\sum_{t,h} \frac{Z_t^h}{\mathcal{E}_{it}} \left[P_{it} C_{it} - W_{it} N_{it} - \Pi_{it} - T_{it} \right] = 0,$$

where we have plugged in the ex-post budget constraint (**??**) . The optimality conditions imply the labor supply condition

$$\frac{W_{it}}{P_{it}} = C_{it}^{\sigma}, \tag{A1}$$

and the equilibrium price of an Arrow-Debreu security Z_t^h,

$$Z_t^h = \mathcal{E}_{it}\lambda_i^{-1} Pr_{ht}\Theta_{it}, \tag{A2}$$

which has to be equal to the nominal stochastic discount factor $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$ adjusted for the nominal exchange rate $\mathcal{E}_{it}$, for the state- and time-invariant Lagrange multiplier λ_i, and for the probability of state h, Pr_{ht}.

Since prices of the Arrow-Debreu securities have to be the same in all countries, households' optimality conditions imply the perfect risk-sharing,

$$\frac{\mathcal{E}_{it}\lambda_i^{-1}}{P_{it}C_{it}^{\sigma}} = \frac{\mathcal{E}_{jt}\lambda_j^{-1}}{P_{jt}C_{jt}^{\sigma}}.$$

Multiply both sides by the import price index P_t^*, use the definition of the real exchange rate (**??**),

$$\lambda_j^{-\frac{1}{\sigma}} C_{it} = \left(\frac{Q_{it}}{\lambda_i}\right)^{\frac{1}{\sigma}} Q_{jt}^{-\frac{1}{\sigma}} C_{jt},$$

integrate it over all countries j, and define a constant $\Lambda_i \equiv \lambda_i \left(\int \lambda_j^{-\frac{1}{\sigma}} \mathrm{d}j\right)^{\sigma}$ to arrive at the risk-sharing (**??**)

$$C_{it} = \left(\frac{Q_{it}}{\Lambda_i}\right)^{\frac{1}{\sigma}} \int Q_{jt}^{-\frac{1}{\sigma}} C_{jt} \mathrm{d}j.$$

To derive (**??**), start with the price index (4)

$$P_{it}^{1-\theta} = (1-\gamma) P_{iit}^{1-\theta} + \gamma \int P_{jit}^{1-\theta} \mathrm{d}j,$$

divide everything by $(\mathcal{E}_{it}P_t^*)^{1-\theta}$, and rewrite it as

$$Q_{it}^{\theta-1} = (1-\gamma)\left(\frac{P_{iit}}{\mathcal{E}_{it}P_{it}^*}\frac{P_{it}^*}{P_t^*}\right)^{1-\theta} + \gamma \frac{\int P_{jt}^{*1-\theta}\mathrm{d}j}{P_t^{*1-\theta}},$$

where we have used the definition of the real exchange rate Q_{it} from (**??**) and converted export prices to dollars, $P_{jit}/\mathcal{E}_{it} = P_{jt}^*$. Then, use the definition of the import price index, $P_t^* = \left(\int P_{jt}^{*1-\theta}\mathrm{d}j\right)^{\frac{1}{1-\theta}}$, and the definitions of the terms of trade S_{it} and the deviation of prices Φ_{it} from (**??**) to arrive at (**??**),

$$Q_{it}^{\theta-1} = \gamma + (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta-1}.$$

Next, note that the definitions of prices (??) imply that

$$\frac{P_{iit}}{P_{it}} = \frac{Q_{it}}{\Phi_{it} S_{it}}. \tag{A3}$$

Use this condition to rewrite the market clearing condition (8) as

$$\begin{aligned} Y_{it} &= (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma \int \left(\frac{P_{ijt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j \\ &= (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta} Q_{it}^{-\theta}(C_{it}+X_{it}) + \gamma \int \left(\frac{P^*_{it}}{P^*_t}\frac{P^*_t \mathcal{E}_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j \\ &= (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta} Q_{it}^{-\theta}(C_{it}+X_{it}) + \gamma S_{it}^{\theta} \int Q_{jt}^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j, \end{aligned}$$

which turns to (??) after we plug in the production function (2.2).

To find firm's demand for intermediate inputs, recall that with the Cobb-Douglas production function (2.2), firms spend share α on intermediate goods and share $1-\alpha$ on labor,

$$\frac{P_{it}X_{it}}{W_{it}L_{it}} = \frac{\alpha}{1-\alpha}.$$

Plug in the labor supply condition (A1) and arrive at (5).

To derive the country's ex-post budget constraint (??), combine the households' budget constraint (??)

$$P_{it}C_{it} = W_{it}N_{it} + \Pi_{it} + T_{it} + \mathcal{E}_{it}B^h_{it},$$

firms' domestic and export profits (2.2) and (2.2)

$$\Pi_{it} = \left(P_{iit} - \tau_i MC_{it}\right) Y^D_{it} + \left(\mathcal{E}_{it}\Psi_i^{-1}P^*_{it} - \tau_i MC_{it}\right) Y^E_{it} - \frac{\varphi}{2}\tau_{Ri}\left((1-\gamma)\,\Pi^2_{iit} + \gamma \Pi^{*2}_{it}\right) W_{it},$$

and the government budget constraint

$$T_{it} = (\tau_i - 1)\, MC_{it} Y_{it} + (\Psi_i - 1)\,\mathcal{E}_{it}\Psi_i^{-1} P^*_{it} Y^E_{it} + (\tau_{Ri} - 1)\,\frac{\varphi}{2}\left((1-\gamma)\,\Pi^2_{iit} + \gamma \Pi^{*2}_{it}\right) W_{it},$$

to get

$$P_{it}C_{it} = W_{it}N_{it} + \mathcal{E}_{it}B^h_{it} + \left(P_{iit} - MC_{it}\right) Y^D_{it} + \left(\mathcal{E}_{it}P^*_{it} - MC_{it}\right) Y^E_{it} - \frac{\varphi}{2}\left((1-\gamma)\,\Pi^2_{iit} + \gamma \Pi^{*2}_{it}\right) W_{it}.$$

Note that the firms' expenditures are equal to wages and purchases of intermediate goods,

$$MC_{it}\left(Y^D_{it} + Y^E_{it}\right) = W_{it}L_{it} + P_{it}X_{it},$$

and use the labor market balance (9)

$$N_{it} = L_{it} + \frac{\varphi}{2}(1-\gamma)\,\Pi_{iit}^2 + \frac{\varphi}{2}\gamma\Pi_{it}^{*2}$$

to simplify the budget constraint to

$$P_{it}\left(C_{it} + X_{it}\right) = \mathcal{E}_{it}B_{it}^h + P_{iit}Y_{it}^D + \mathcal{E}_{it}P_{it}^*Y_{it}^E.$$

Plug in the households' and firms' demand for goods, and note that domestic revenues of firms are equal to households' and firms' expenditures on domestic goods, so that we are left with exports minus imports plus international transfers,

$$0 = \gamma\mathcal{E}_{it}P_{it}^*\int\left(\frac{P_{ijt}}{P_{jt}}\right)^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j - \gamma\int P_{jit}\left(\frac{P_{jit}}{P_{it}}\right)^{-\theta}\mathrm{d}j\left(C_{it}+X_{it}\right) + \mathcal{E}_{it}B_{it}^h.$$

Divide everything by $\mathcal{E}_{it}$ to express all payments in dollars, convert export prices to dollars $P_{jit} = \mathcal{E}_{it}P_{jt}^*$, use the definition of the import price index $P_t^{*1-\theta} \equiv \int P_{jt}^{*1-\theta}\mathrm{d}j$ to get

$$0 = \gamma P_t^*\left(\frac{P_{it}^*}{P_t^*}\right)^{1-\theta}\int\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j - \gamma\mathcal{E}_{it}^{-\theta}P_t^{*1-\theta}P_{it}^{\theta}\left(C_{it}+X_{it}\right) + B_{it}^h,$$

and finally use the definitions of the real exchange rate Q_{it} and the terms of trade S_{it} from (**??**) to arrive at (**??**),

$$0 = \gamma P_t^*\left[S_{it}^{\theta-1}\int Q_{jt}^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right] + B_{it}^h.$$

If instead we started from the households' ex-ante budget constraint (**??**), $\sum_{t,h} Z_t^h B_{it}^h = 0$, we would arrive at

$$\sum_{t,h} Z_t^h N X_{it} = 0,$$

and with Z_t^h plugged in from (A2) we would get

$$\mathbb{E}\sum_t \frac{\mathcal{E}_{it}P_t^*}{P_{it}C_{it}^{\sigma}}\beta^t\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right] = 0.$$

Use the definition of the real exchange rate Q_{it} from (**??**), the risk sharing (**??**), and arrive at (**??**)

$$\mathbb{E}\sum_t \beta^t\bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right] = 0.$$

Proof of Lemma ?? Consider the net exports (**??**). Plug in the risk sharing (**??**), use $\alpha = 0$ and $\theta = 1$ to express it as

$$NX_{it} = \gamma P_t^* \left[C_t^* - Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_i^{-\frac{1}{\sigma}} \bar{C}_t \right].$$

Further, recall the definition of the global demand shifter $C_t^* \equiv \int Q_{jt}^{-\theta} \left(C_{jt} + X_{jt} \right) \mathrm{d}j$. Plug in the risk sharing sharing (**??**) and $\alpha = 0$ to get

$$C_t^* \equiv \int Q_{jt}^{\frac{1}{\sigma}-\theta} \Lambda_j^{-\frac{1}{\sigma}} \bar{C}_t \mathrm{d}j.$$

Under $\gamma = 1$, the real exchange rate constraint (**??**) implies that $Q_{it} = 1$. Under $\sigma\theta = 1$, neither expression from above depends on the real exchange rate. Combine the two expressions under either $\gamma = 1$ or $\sigma\theta = 1$,

$$NX_{it} = \gamma P_t^* \bar{C}_t \left[\int \Lambda_j^{-\frac{1}{\sigma}} \mathrm{d}j - \Lambda_i^{-\frac{1}{\sigma}} \right].$$

Note that the sign of NX_{it} is state- and time-invariant, and thus it has to be the case that $NX_{it} = 0$ for all t and h. ■

Finally, the world trade should be balanced at all times t, $\int NX_{it} \mathrm{d}i = 0$. Plug in the expression for NX_{it} from (**??**)

$$\int \gamma P_t^* \left[S_{it}^{\theta-1} \int Q_{jt}^{-\theta} \left(C_{jt} + X_{jt} \right) \mathrm{d}j - Q_{it}^{-\theta} \left(C_{it} + X_{it} \right) \right] \mathrm{d}i = 0,$$

rearrange the order of integration, and arrive at

$$\int S_{it}^{\theta-1} \mathrm{d}i = 1. \tag{A4}$$

Note that this condition follows immediately from the import price index definition, $P_t^* = \left(\int P_{jt}^{*1-\theta} \mathrm{d}j \right)^{\frac{1}{1-\theta}}$, and the definition of the terms of trade, (**??**).

A.2.2 Price-setting block

Domestic Phillips curve To derive the domestic Phillips curve, take the FOC from the firm's problem (2.2),

$$\begin{aligned} 0 = \Theta_{it} \left(1-\varepsilon\right) \left(1 - \frac{\varepsilon}{\varepsilon - 1} \tau_i \frac{MC_{it}}{P_t}\right) \left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{it}^D - \left(1-\gamma\right) \varphi \tau_{Ri} \left(\frac{P_t}{P_{t-1}} - 1\right) \frac{1}{P_{t-1}} W_{it} \Theta_{it} \\ + \left(1-\gamma\right) \varphi \tau_{Ri} \mathbb{E} \left(\frac{P_{t+1}}{P_t} - 1\right) \frac{P_{t+1}}{P_t^2} W_{it+1} \Theta_{it+1}. \end{aligned}$$

Use the value of the production subsidy $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, symmetry across all domestic firms $P_t = P_{iit}$, and the definition of inflation rate $\Pi_{iit} \equiv P_{iit}/P_{iit-1} - 1$ to rewrite it as

$$\Pi_{iit}\left(\Pi_{iit}+1\right)W_{it}\Theta_{it} = \frac{\Theta_{it}}{1-\gamma}\frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(1-\frac{MC_{it}}{P_{iit}}\right)P_{iit}Y_{it}^{D} + \mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right)W_{it+1}\Theta_{it+1}.$$

Denote $\kappa \equiv \frac{\varepsilon-1}{\varphi\tau_{Ri}}$, plug in the SDF $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$, the local demand shifter $Y_{it}^{D} = (1-\gamma)\left(P_{iit}/P_{it}\right)^{-\theta}\left(C_{it}+X_{it}\right)$ from (8), the nominal wage $W_{it} = P_{it}C_{it}^{\sigma}$ from the labor supply condition (A1), and the marginal costs $MC_{it} = P_{it}^{\alpha}W_{it}^{1-\alpha}/A_{it}$

$$\Pi_{iit}\left(\Pi_{iit}+1\right) = -\kappa\left(1-\frac{P_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}P_{iit}}\right)\frac{P_{iit}}{P_{it}C_{it}^{\sigma}}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right) + \beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right)$$

Use definitions of various prices from (**??**) and (A3) to arrive at (**??**)

$$\Pi_{iit}(\Pi_{iit}+1) = -\kappa\left(1-\frac{\Phi_{it}S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}(C_{it}+X_{it})C_{it}^{-\sigma} + \beta\mathbb{E}_t\Pi_{iit+1}(\Pi_{iit+1}+1).$$

The optimal choice of the initial price at $t=0$, P_{-1}, leads to $P_0 = P_{-1}$ or, the same, $\Pi_{ii0} = 0$.

Export Phillips curve Similarly, start at the firm's problem (2.2), and take the FOC

$$\begin{aligned}0 = {}& \Theta_{it}\left(1-\varepsilon\right)\left(\mathcal{E}_{it}-\frac{\varepsilon}{\varepsilon-1}\tau_i\frac{MC_{it}}{P_t^*}\right)\left(\frac{\Psi_i P_t^*}{P_{it}^*}\right)^{-\varepsilon}Y_{it}^{E} - \gamma\varphi\tau_{Ri}\left(\frac{P_t^*}{P_{t-1}^*}-1\right)\frac{1}{P_{t-1}^*}W_{it}\Theta_{it}\\ &+\gamma\varphi\tau_{Ri}\mathbb{E}\left(\frac{P_{t+1}^*}{P_t^*}-1\right)\frac{P_{t+1}^*}{P_t^{*2}}W_{it+1}\Theta_{it+1}.\end{aligned}$$

Use the value of the production subsidy $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, symmetry across all firms $\Psi_i P_t^* = P_{it}^*$, and the definition of inflation rate $\Pi_{it}^* \equiv P_{it}^*/P_{it-1}^* - 1$ to rewrite it as

$$\Pi_{it}^*\left(\Pi_{it}^*+1\right)W_{it}\Theta_{it} = \frac{\Theta_{it}}{\gamma\Psi_i}\frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(\mathcal{E}_{it}-\Psi_i\frac{MC_{it}}{P_{it}^*}\right)P_{it}^*Y_{it}^{E} + \mathbb{E}\Pi_{it+1}^*\left(\Pi_{it+1}^*+1\right)W_{it+1}\Theta_{it+1}.$$

Denote $\kappa \equiv \frac{\varepsilon-1}{\varphi\tau_{Ri}}$, plug in the SDF $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$, the export demand shifter $Y_{it}^{E} = \gamma S_{it}^{\theta}C_t^*$ from (**??**), the nominal wage $W_{it} = P_{it}C_{it}^{\sigma}$ from the labor supply condition (A1), and the marginal costs $MC_{it} = P_{it}^{\alpha}W_{it}^{1-\alpha}/A_{it}$

$$\Pi_{it}^*\left(\Pi_{it}^*+1\right) = -\frac{\kappa}{\Psi}\left(1-\Psi_i\frac{P_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}\mathcal{E}_{it}P_{it}^*}\right)\frac{\mathcal{E}_{it}P_{it}^*}{P_{it}C_{it}^{\sigma}}S_{it}^{\theta}C_t^* + \beta\mathbb{E}\Pi_{it+1}^*\left(\Pi_{it+1}^*+1\right).$$

Use definitions of various prices from (**??**) to arrive at (**??**)

$$\Pi^*_{it}\left(\Pi^*_{it}+1\right)=-\frac{\kappa}{\Psi}\left(1-\frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma}+\beta\mathbb{E}\Pi^*_{it+1}\left(\Pi^*_{it+1}+1\right).$$

Again, the optimal choice of the initial price leads to $\Pi^*_{i0}=0$.

Phillips curve for the U.S. Note that the problem (**??**) differs from the problem of a non-U.S. domestic producer (2.2) in two ways: there is the global demand shifter Y_{it} instead of the domestic Y^D_{it}, and Rotemberg costs are not multiplied by $(1-\gamma)$. Then, following the same steps as before, one can derive the following optimality condition

$$\Pi_{iit}\left(\Pi_{iit}+1\right)=-\kappa\left(1-\frac{P_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}P_{iit}}\right)\frac{P_{iit}}{P_{it}C_{it}^{\sigma}}Y_{it}+\beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right).$$

Let's plug in the demand shifter Y_{it} from (**??**), use definitions of the real exchange rate and the terms of trade from (**??**) , keep in mind that for the U.S. $\Phi_{it}=\Psi_i$,

$$\Pi_{iit}(\Pi_{iit}+1)=-\frac{\kappa}{\Psi_i}\left(1-\frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)\left[(1-\gamma)\,\Psi_i^{\theta}Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)+\gamma C_t^*\right]Q_{it}S_{it}^{\theta-1}C_{it}^{-\sigma}$$
$$+\beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right),$$

and the optimal choice of the initial price leads to $\Pi_{ii0}=0$.

Ex-ante price setting Finally, each of the Phillips curves above, or the ex-post price setting conditions, should be complemented by the corresponding ex-ante price setting condition, which reflects the optimal choice of prices at time -1, P_{-1}.

First, consider the non-U.S. domestic price setting, where the optimal choice of P_{-1} implies $\Pi_{ii0}=0$. Plug it in the ex-post price setting and iterate forward to get the ex-ante price setting condition

$$0=\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Phi_{it}S_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}Q_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(C_{it}+X_{it}\right)C_{it}^{-\sigma}. \tag{A5}$$

Similary, the ex-ante non-U.S. export price setting and the U.S. price setting conditions are

$$0=\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma}, \tag{A6}$$

$$0=\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)\left((1-\gamma)\,\Psi_i^{\theta}Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)+\gamma C_t^*\right)Q_{it}S_{it}^{\theta-1}C_{it}^{-\sigma}. \tag{A7}$$

A.2.3 Equilibrium

Proof of Lemma 1 Let's list all equilibrium conditions that for given policies constitute an equilibrium. These are (??)–(??) (where the price setting conditions (??), (??), (??) implicitly contain their ex-ante versions (A5), (A6), (A7) as well), the labor supply condition (A1), the global trade balance (A4), 3 definitions of inflation rates Π_{iit}, Π^*_{it}, Π_{00t}, and the definition of the import price index.

Note that the latter condition, $P_t^{*1-\theta} \equiv \int P_{jt}^{*1-\theta} \mathrm{d}j$, is equivalent to the terms of trade normalization (A4), and thus we can drop it.

Implicitly, we have already used the labor supply condition (A1) to get rid of nominal wages W_{it} in all other equilibrium conditions. Similarly, let's use definitions of Q_{it}, Φ_{it}, and S_{it} to get rid of P_{iit}, P^*_{it}, and P_{it}. All of the conditions (??)–(??) already do not contain P_{iit}, P^*_{it}, P_{it}. Definition of Q_{it} is the only condition that contains P_{it} and thus it could be dropped.

For the non-U.S. countries, the definition of Φ_{it} is the only condition with $\mathcal{E}_{it}$, and thus could be dropped as well. Let's plug in S_{it} to the export inflation

$$\Pi^*_{it} = \frac{P^*_{it}}{P^*_{it-1}} - 1 = \frac{P^*_t}{P^*_{t-1}} \frac{S_{it-1}}{S_{it}} - 1.$$

Then the definition of S_{it} could be dropped as it is the only condition with P^*_{it}. The definition of domestic inflation $\Pi_{iit} = P_{iit}/P_{iit-1} - 1$ could be dropped as it is the only condition with P_{iit}.

For the U.S., plug in $\Phi_{0t} = \Psi_0 = P^*_{0t}/P_{00t}$ and the definition of S_{0t} to the definition of inflation

$$\Pi_{00t} = \frac{P_{00t}}{P_{00t-1}} - 1 = \frac{P^*_t}{P^*_{t-1}} \frac{S_{0t-1}}{S_{0t}} - 1.$$

Then these definitions of Φ_{0t} and S_{0t} could be dropped since no other conditions contain either P_{00t} or P^*_{0t}.

Finally, by combining definitions of Π^*_{it} and Π_{00t}, we can get rid of P^*_t and get

$$\Pi^*_{it} = (\Pi_{00t} + 1) \frac{S_{0t}}{S_{0t-1}} \frac{S_{it-1}}{S_{it}} - 1. \tag{A8}$$

In the end, we can reduce the system of equilibrium varaibles to the allocation $\{C_{it}, L_{it}, X_{it}, \Lambda_i, B^h_{it}\}$ and prices $\{Q_{it}, S_{it}, \Phi_{it}, \Pi_{iit}, \Pi^*_{it}\}$. Then, all equilibrium conditions are represented by equations (??)–(??), and definitions of various prices, which result in the global terms of trade normalization (A4) and the dynamic equation for inflation rates and terms of trade (A8). ■

A.3 Static Model

In a static model, there is no inflation, and thus the labor market clearing (9) collapses to $N_{it} = L_{it}$.

A.3.1 Generalization

In this section, we generalize our framework.

Consumers maximize

$$U_i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(C_{it}, L_{it}\right)$$

instead of (2.1), subject to

$$\sum_{t,h} \frac{Z_t^h}{\mathcal{E}_{it}} \left[P_{it} C_{it} - W_{it} L_{it} - \Pi_{it} - T_{it}\right] = 0.$$

Then the labor supply condition (A1) changes to

$$-\frac{u_L\left(C_{it}, L_{it}\right)}{u_C\left(C_{it}, L_{it}\right)} = \frac{W_{it}}{P_{it}}, \tag{A9}$$

and the risk-sharing (A2) becomes

$$Q_{it} u_C\left(C_{it}, L_{it}\right) = \lambda_i \beta^{-t} \frac{Z_t^h P_t^*}{Pr_{ht}}. \tag{A10}$$

The firms' production function is

$$Y_{it} = A_{it} F\left(L_{it}, X_{it}\right) \tag{A11}$$

instead of (2.2), where function F exhibits constant returns to scale. A cost minimization problem

$$\begin{aligned} &\min_{L_{it}, X_{it}} W_{it} L_{it} + P_{it} X_{it} \\ &\text{s.t.} \quad A_{it} F\left(L_{it}, X_{it}\right) = Y_{it}, \end{aligned}$$

yields the following optimality condition

$$\frac{W_{it}}{P_{it}} = \frac{F_L\left(L_{it}, X_{it}\right)}{F_X\left(L_{it}, X_{it}\right)}. \tag{A12}$$

Next, express the total costs as

$$TC_{it} \equiv W_{it} L_{it} + P_{it} X_{it} = P_{it} \left(\frac{W_{it}}{P_{it}} L_{it} + X_{it}\right),$$

plug in the optimality condition (A12),

$$TC_{it} = \frac{P_{it}}{F_X\left(L_{it}, X_{it}\right)}\left(F_L\left(L_{it}, X_{it}\right) L_{it} + F_X\left(L_{it}, X_{it}\right) X_{it}\right) = \frac{P_{it} Y_{it}}{A_{it} F_X\left(L_{it}, X_{it}\right)}, \tag{A13}$$

where the last equality is due to Euler's theorem. Then, the marginal costs are just $MC_{it} \equiv P_{it}/\left(A_{it} F_X\left(L_{it}, X_{it}\right)\right)$.

Finally, for conveniency of notation, plug in the labor supply condition (A9) into the firm's optimality condition (A12), and express its solution as $X_{it} = X\left(C_{it}, L_{it}\right)$, which we will use instead of condition (5).

A.3.2 Efficient allocation

To solve for the efficient allocation in one country, we allow the planner to choose all quantities in this country directly. However, the planner has to take international prices as given and respect the country's budget constraint as well as the foreign demand for the domestic goods. Thus, the social planner's problem can be written as

$$\begin{aligned}
&\max_{C_{it}, L_{it}, X_{it}, S_{it}, \{C_{jit}, X_{jit}\}_j} u\left(C_{it}, L_{it}\right) \\
\text{s.t.} \quad & A_{it} F\left(L_{it}, X_{it}\right) = C_{iit} + X_{iit} + \gamma S_{it}^{\theta} C_t^*, \\
& P_t^* \gamma S_{it}^{\theta-1} C_t^* - \int P_{jt}^*\left(C_{jit} + X_{jit}\right) \mathrm{d}j + B_{it}^h = 0, \\
& C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} \int C_{jit}^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
& X_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} X_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} \int X_{jit}^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}.
\end{aligned}$$

Here the planner can choose any export price in dollars by choosing S_{it}, but all import prices in dollars, P_t^* and P_{jt}^*, are taken as given. Plug in the last two conditions to get rid of C_{it} and X_{it}, and denote the Lagrange multiplier for the market clearing as η_{it} and for the budget constraint as ρ_{it}. Then the FOCs are

$$\begin{aligned}
0 &= u_C\left(C_{it}, L_{it}\right) \frac{\mathrm{d}C_{it}}{\mathrm{d}C_{iit}} - \eta_{it} \\
0 &= u_C\left(C_{it}, L_{it}\right) \frac{\mathrm{d}C_{it}}{\mathrm{d}C_{jit}} - \rho_{it} P_{jt}^* \\
0 &= u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it} F_L\left(L_{it}, X_{it}\right) \\
0 &= \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) \frac{\mathrm{d}X_{it}}{\mathrm{d}X_{iit}} - \eta_{it}
\end{aligned}$$

$$0 = \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) \frac{\mathrm{d}X_{it}}{\mathrm{d}X_{jit}} - \rho_{it} P_{jt}^*$$

$$0 = -\eta_{it}\gamma\theta S_{it}^{\theta-1} C_t^* + \rho_{it}\gamma P_t^* \left(\theta - 1\right) S_{it}^{\theta-2} C_t^*$$

Use the FOC with respect to L_{it} to substitute for η_{it} in all other conditions,

$$\eta_{it} = \frac{-u_L\left(C_{it}, L_{it}\right)}{A_{it} F_L\left(L_{it}, X_{it}\right)}.$$

Similarly, use the FOC with respect to S_{it} to substitute for ρ_{it},

$$\rho_{it} = \frac{-u_L\left(C_{it}, L_{it}\right)}{A_{it} F_L\left(L_{it}, X_{it}\right) P_t^*} \frac{\theta}{\theta - 1} S_{it}.$$

Divide the first FOC by the second (and the fourth by the fifth) to find the relative demand for domestic and foreign varieties,

$$\frac{C_{iit}}{C_{jit}} = \frac{X_{iit}}{X_{jit}} = \frac{1-\gamma}{\gamma} \left(\frac{\theta}{\theta - 1} S_{it} \frac{P_{jt}^*}{P_t^*}\right)^{\theta}. \tag{A14}$$

Next, combine the first two FOCs and use the consumption aggregator to derive

$$\left(\frac{-u_L\left(C_{it}, L_{it}\right)}{u_C\left(C_{it}, L_{it}\right) A_{it} F_L\left(L_{it}, X_{it}\right)}\right)^{\theta-1} = (1-\gamma) + \gamma \left(\frac{\theta}{\theta - 1} S_{it}\right)^{1-\theta}.$$

Similarly, combine the the fourth and the fifth FOCs and use the intermediates aggregator to get

$$\left(A_{it} F_X\left(L_{it}, X_{it}\right)\right)^{1-\theta} = (1-\gamma) + \gamma \left(\frac{\theta}{\theta - 1} S_{it}\right)^{1-\theta}, \tag{A15}$$

and therefore the two conditions together imply

$$\frac{-u_L\left(C_{it}, L_{it}\right)}{u_C\left(C_{it}, L_{it}\right)} = \frac{F_L\left(L_{it}, X_{it}\right)}{F_X\left(L_{it}, X_{it}\right)}. \tag{A16}$$

Lemma A4 *Assume that prices are flexible. Then the optimal non-cooperative allocation can be implemented with* $\tau_i = \frac{\varepsilon - 1}{\varepsilon}$ *and* $\Psi_i = \frac{\theta}{\theta - 1}$.

Proof of Lemma A4 The labor supply condition (A9) and the firms' optimality condition (A12) together yield the last of the planner's conditions, (A16).

With CES demand, as in (8), we get

$$\frac{C_{iit}}{C_{jit}} = \frac{X_{iit}}{X_{jit}} = \frac{1-\gamma}{\gamma} \left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} \left(\frac{P_{jt}^*}{P_t^*} \frac{\mathcal{E}_{it} P_t^*}{P_{it}}\right)^{\theta},$$

and with definition of prices (??) it becomes

$$\frac{C_{iit}}{C_{jit}} = \frac{X_{iit}}{X_{jit}} = \frac{1-\gamma}{\gamma}\left(\Phi_{it} S_{it} \frac{P^*_{jt}}{P^*_t}\right)^{\theta}. \tag{A17}$$

Compare this condition with the planner's (A14), and the two coincide when $\Phi_{it} = \Psi_i = \frac{\theta}{\theta-1}$.

Under flexible prices, the price setting conditions (??) and (??) collapse to[36]

$$A_{it} F_X\left(L_{it}, X_{it}\right) = \frac{S_{it}\Phi_{it}}{Q_{it}}, \qquad A_{it} F_X\left(L_{it}, X_{it}\right) = \frac{S_{it}\Psi_i}{Q_{it}}.$$

The two together once again imply $\Phi_{it} = \Psi_i = \frac{\theta}{\theta-1}$, but combing the latter one with the price index constraint (??) yields

$$\left((1-\gamma) + \gamma\left(\Psi_i S_{it}\right)^{1-\theta}\right)^{\frac{1}{\theta-1}} = \frac{1}{A_{it} F_X\left(L_{it}, X_{it}\right)},$$

which is the same condition as planner's (A15). ■

A.3.3 Non-U.S. policy

Proof of Proposition 1 The policy problem is

$$\begin{aligned}
&\max_{C_{it}, L_{it}, X_{it}, \Phi_{it}, Q_{it}} u\left(C_{it}, L_{it}\right) \\
\text{s.t.} \quad & A_{it} F\left(L_{it}, X_{it}\right) = (1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta}\left(C_{it} + X_{it}\right) + \gamma S_{it}^{\theta} C^*_t, \\
& \gamma P^*_t\left[S_{it}^{\theta-1} C^*_t - Q_{it}^{-\theta}\left(C_{it} + X_{it}\right)\right] + B^h_{it} = 0, \\
& X_{it} = X\left(C_{it}, L_{it}\right), \\
& Q_{it}^{\theta-1} = \gamma + (1-\gamma)\left(\Phi_{it} S_{it}\right)^{\theta-1}.
\end{aligned}$$

Plug in the last two conditions to get rid of X_{it} and Q_{it}, and denote the Lagrange multiplier for the market clearing as η_{it} and for the budget constraint as ρ_{it}. Then the FOCs are

$$\begin{aligned}
0 = {} & u_C\left(C_{it}, L_{it}\right) + \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) X_C\left(C_{it}, L_{it}\right) \\
& - \eta_{it}(1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta}\left(1 + X_C\left(C_{it}, L_{it}\right)\right) - \rho_{it}\gamma P^*_t Q_{it}^{-\theta}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)
\end{aligned}$$

$$\begin{aligned}
0 = {} & u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right) X_L\left(C_{it}, L_{it}\right)\right) \\
& - \eta_{it}(1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} X_L\left(C_{it}, L_{it}\right) - \rho_{it}\gamma P^*_t Q_{it}^{-\theta} X_L\left(C_{it}, L_{it}\right)
\end{aligned}$$

[36] Recall that flexible prices lead to the same condition as stabilization of domestic prices, that is $MC_{it}/P_{iit} = 1$. And (A13) implies $MC_{it} \equiv P_{it}/\left(A_{it} F_X\left(L_{it}, X_{it}\right)\right)$. For the export prices, the corresponding condition is $\Psi_i MC_{it} = \mathcal{E}_{it} P^*_{it}$.

$$0 = -\eta_{it}(1-\gamma)\left(-\theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1}\Phi_{it}^{2\theta-2} + Q_{it}^{-\theta}\theta\Phi_{it}^{\theta-1}\right) S_{it}^{\theta}(C_{it}+X_{it}) + \rho_{it}\gamma P_t^*\theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1}\Phi_{it}^{\theta-2}(C_{it}+X_{it})$$

Use the real exchange rate constraint (**??**) to simplify the last FOC to

$$\rho_{it}P_t^* = \eta_{it}S_{it}\Phi_{it}.$$

Plug in ρ_{it} from this condition to the first two FOCs, and similarly simplify them to

$$0 = u_L(C_{it}, L_{it}) + \eta_{it}A_{it}\left(F_L(L_{it}, X_{it}) + F_X(L_{it}, X_{it}) X_L(C_{it}, L_{it})\right) - \eta_{it}S_{it}\Phi_{it}Q_{it}^{-1}X_L(C_{it}, L_{it}),$$

$$0 = u_C(C_{it}, L_{it}) + \eta_{it}A_{it}F_X(L_{it}, X_{it}) X_C(C_{it}, L_{it}) - \eta_{it}S_{it}\Phi_{it}Q_{it}^{-1}(1 + X_C(C_{it}, L_{it})).$$

Divide one by another, use the labor supply (A9), the firms' optimality condition (A12), and rearrange to get

$$0 = \left(A_{it}F_X(L_{it}, X_{it}) - S_{it}\Phi_{it}Q_{it}^{-1}\right)\left[\frac{W_{it}}{P_{it}}(1 + X_C(C_{it}, L_{it})) + X_L(C_{it}, L_{it})\right].$$

Under well-behaved $u(C_{it}, L_{it})$ and $F_X(L_{it}, X_{it})$, both $X_C(C_{it}, L_{it})$ and $X_L(C_{it}, L_{it})$ are non-negative, and thus it has to be the case that

$$A_{it}F_X(L_{it}, X_{it}) = \frac{S_{it}\Phi_{it}}{Q_{it}}. \tag{A18}$$

Plug in (A3) and note that due to (A13) it collapses to the stabilization of domestic prices, $MC_{it}/P_{iit} = 1$. This is the first part of the Proposition.

For the second part, note that the firm's optimality condition (A12) and its constraint (A11), could be jointly solved for L_{it} and X_{it} as functions of A_{it}, W_{it}, P_{it}, and Y_{it} only. Then, the marginal costs from (A13) could be expressed as a function of just two prices and productivity, in particular denote it as $MC_{it} = G(W_{it}, P_{it})/A_{it}$. Then, with linear disutility from labor, $u_L(C_{it}, L_{it}) = -1$, and separable utility function, $u_{CL}(C_{it}, L_{it}) = 0$, the stabilization of domestic prices could be expressed as

$$G\left(P_{it}u_C(C_{it})^{-1}, P_{it}\right)/A_{it} = P_{iit},$$

where we have used the labor supply condition (A9).

Next, combine the risk-sharing (A10) for a non-U.S. country and for the U.S. (using the definition of the real exchange rate from (**??**)),

$$\mathcal{E}_{it} = \frac{P_{it}u_C(C_{it})^{-1}}{P_{0t}u_C(C_{0t})^{-1}}\frac{\lambda_i}{\lambda_0}.$$

Then, rewrite the price index (4) as

$$P_{it}^{1-\theta} = (1-\gamma) P_{iit}^{1-\theta} + \gamma \mathcal{E}_{it}^{1-\theta} P_t^{*1-\theta}.$$

Finally, consider a system of these three conditions, where P_{iit} and P_t^* are fixed. Suppose that the U.S. depreciates and $P_{0t} u_C \left(C_{0t}\right)^{-1}$ rises. All else equal, this will lead (through a nominal exchange rate $\mathcal{E}_{it}$ appreciation) to lower price index P_{it} and lower marignal costs for a non-U.S. country. Then, to stabilize domestic prices, the non-U.S. policymaker has to increase its nominal demand, $P_{it} u_C \left(C_{it}\right)^{-1}$. Since this nominal demand enters the marignal costs through wages as well as through the price index, an increase in $P_{it} u_C \left(C_{it}\right)^{-1}$ will be smaller than an increase in $P_{0t} u_C \left(C_{0t}\right)^{-1}$. This constitutes a partial peg to the dollar and gives rise to a global monetary cycle.

For the last part of the Proposition, look at the planner's optimality conditions. (A16) is true as it depends only on the labor supply (A9) and the firms' optimality condition (A12). As with flexible prices, the CES demand leads to (A17), which coincide with (A14) only under $\Phi_{it} = \Psi_i = \frac{\theta}{\theta-1}$. Finally, plug in the price index constraint (**??**) to the domestic price stabilization condition (A18) to get

$$\left(A_{it} F_X \left(L_{it}, X_{it}\right)\right)^{1-\theta} = (1-\gamma) + \gamma \left(\Phi_{it} S_{it}\right)^{1-\theta},$$

which is almost the same as the planner's (A15). However, the planner's terms of trade S_{it} move with shocks and $\Phi_{it} = \Psi_i$ is constant, while in the current condition S_{it} is fixed and Φ_{it} responds to shocks. ■

A.3.4 U.S. policy

Proof of Proposition 2 To set up the U.S. policy problem, start with the U.S. market clearing (**??**), the demand for intermediates (5), and the budget constraint (**??**). Note that this system depends on the global demand C_t^*,

$$C_t^* \equiv \int Q_{jt}^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j, \tag{A19}$$

which the U.S. does not take as given. To find how the global demand C_t^* depends on the non-U.S. policy, consider the set of equilibrium conditions for each non-U.S. country j,

$$Q_{jt}^{\theta-1} = \gamma + (1-\gamma) \left(\Phi_{jt} S_{jt}\right)^{\theta-1}$$

$$A_{jt} F \left(L_{jt}, X_{jt}\right) = (1-\gamma) Q_{jt}^{-\theta} \Phi_{jt}^{\theta} S_{jt}^{\theta} \left(C_{jt} + X_{jt}\right) + \gamma S_{jt}^{\theta} C_t^*$$

$$\gamma P_t^* \left[S_{jt}^{\theta-1} C_t^* - Q_{jt}^{-\theta} \left(C_{jt} + X_{jt}\right)\right] + B_{jt}^h = 0$$

$$X_{jt} = X \left(C_{jt}, L_{jt}\right)$$

$$A_{jt}F_X\left(L_{jt}, X_{jt}\right) = \frac{S_{jt}\Phi_{jt}}{Q_{jt}}$$

These conditions are the same as the constraints of the non-U.S. policy problem, except that we have added the optimal policy condition (A18).

This system of the non-U.S. equilibrium conditions contains 5 equations and 5 non-U.S. variables (Q_{jt}, Φ_{jt}, L_{jt}, X_{jt}, C_{jt}). One can solve this system and find all non-U.S. variables as functions of predetermined variables $\left\{S_{jt}, B^h_{jt}, P^*_t\right\}$, shocks $\{A_{jt}\}$, and the global demand C^*_t. Plug these solutions back to the definition of C^*_t, (A19), and get an additional constraint for the U.S. policy problem.

However, this additional constraint will always be satisfied by construction. To see that, integrate the budget constraint (**??**) over all non-U.S. countries j,

$$\gamma P^*_t\left[\int S^{\theta-1}_{jt}\mathrm{d}j C^*_t - \int Q^{-\theta}_{jt}\left(C_{jt}+X_{jt}\right)\mathrm{d}j\right] + \int B^h_{jt}\mathrm{d}j = 0.$$

Note that the sum of all international transfers has to be equal to zero, $\int B^h_{jt}\mathrm{d}j = 0$, and the global terms of trade normalization (A4) implies $\int S^{\theta-1}_{jt}\mathrm{d}j = 1$. Then this condition collapses to (A19).

Thus, there are no additional constraints for the U.S., and its policy problem could be written as

$$\begin{aligned}
&\max_{C_{it},L_{it},X_{it},C^*_t} u\left(C_{it}, L_{it}\right)\\
\text{s.t.}\quad & A_{it}F\left(L_{it}, X_{it}\right) = (1-\gamma)\, Q^{-\theta}_{it}\Psi^{\theta}_i S^{\theta}_{it}\left(C_{it}+X_{it}\right) + \gamma S^{\theta}_{it}C^*_t,\\
&\gamma P^*_t\left[S^{\theta-1}_{it}C^*_t - Q^{-\theta}_{it}\left(C_{it}+X_{it}\right)\right] + B^h_{it} = 0,\\
&X_{it} = X\left(C_{it}, L_{it}\right).
\end{aligned}$$

This proves the first part of the Proposition.

As before, denote the first two Lagrange multipliers as η_{it} and ρ_{it}, and plug in the last condition to get rid of X_{it}. The resulting FOCs are:

$$\begin{aligned}
0 = {} & u_C\left(C_{it}, L_{it}\right) + \eta_{it}A_{it}F_X\left(L_{it}, X_{it}\right)X_C\left(C_{it}, L_{it}\right)\\
& - \eta_{it}\left(1-\gamma\right)Q^{-\theta}_{it}\Psi^{\theta}_i S^{\theta}_{it}\left(1+X_C\left(C_{it}, L_{it}\right)\right) - \rho_{it}\gamma P^*_t Q^{-\theta}_{it}\left(1+X_C\left(C_{it}, L_{it}\right)\right)
\end{aligned}$$

$$\begin{aligned}
0 = {} & u_L\left(C_{it}, L_{it}\right) + \eta_{it}A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right)X_L\left(C_{it}, L_{it}\right)\right)\\
& - \eta_{it}\left(1-\gamma\right)Q^{-\theta}_{it}\Psi^{\theta}_i S^{\theta}_{it}X_L\left(C_{it}, L_{it}\right) - \rho_{it}\gamma P^*_t Q^{-\theta}_{it}X_L\left(C_{it}, L_{it}\right)
\end{aligned}$$

$$0 = -\eta_{it}\gamma S^{\theta}_{it} + \rho_{it}\gamma P^*_t S^{\theta-1}_{it}$$

Use the last FOC to substitute for ρ_{it}, and use the price index constraint (**??**) along with $\Psi_i = 1$ to

rewrite the first two FOCs as

$$0 = u_C\left(C_{it}, L_{it}\right) + \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) X_C\left(C_{it}, L_{it}\right) - \eta_{it} S_{it} Q_{it}^{-1}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)$$

$$0 = u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right) X_L\left(C_{it}, L_{it}\right)\right) - \eta_{it} S_{it} Q_{it}^{-1} X_L\left(C_{it}, L_{it}\right)$$

Divide one by another, use the labor supply (A9), the firms' optimality condition (A12), and rearrange to get

$$0 = \left(A_{it} F_X\left(L_{it}, X_{it}\right) - S_{it} Q_{it}^{-1}\right)\left(X_L\left(C_{it}, L_{it}\right) + \frac{W_{it}}{P_{it}}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)\right)$$

As before, under well-behaved $u\left(C_{it}, L_{it}\right)$ and $F_X\left(L_{it}, X_{it}\right)$, both $X_C\left(C_{it}, L_{it}\right)$ and $X_L\left(C_{it}, L_{it}\right)$ are non-negative, and thus it has to be the case that

$$A_{it} F_X\left(L_{it}, X_{it}\right) = \frac{S_{it}}{Q_{it}}$$

Plug in (A3) and note that due to (A13) it collapses to the stabilization of domestic prices, $MC_{it}/P_{iit} = 1$. This is the second part of the Proposition.

To show that this allocation does not coincide with the efficient allocation, it's enough to note that in the efficient allocation $\Phi_{it} = \frac{\theta}{\theta - 1}$, while for the U.S. $\Phi_{it} = 1$. ■

A.3.5 Capital controls

Proof of Proposition ?? The policy problem can be rewritten as

$$\begin{aligned}
&\max_{C_{it}, L_{it}, X_{it}, \Phi_{it}, Q_{it}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(C_{it}, L_{it}\right) \\
\text{s.t.} \quad & A_{it} F\left(L_{it}, X_{it}\right) = \left(1 - \gamma\right) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta}\left(C_{it} + X_{it}\right) + \gamma S_{it}^{\theta} C_t^*, \\
& \sum_{t,h} Z_t^h \left[S_{it}^{\theta - 1} C_t^* - Q_{it}^{-\theta}\left(C_{it} + X_{it}\right)\right] = 0, \\
& X_{it} = X\left(C_{it}, L_{it}\right), \\
& Q_{it}^{\theta - 1} = \gamma + \left(1 - \gamma\right)\left(\Phi_{it} S_{it}\right)^{\theta - 1}.
\end{aligned}$$

where we expressed the budget constraint through the prices of Arrow-Debreu securities, as in (**??**). $\left\{Z_t^h\right\}$ are taken as given by the non-U.S. policymaker.

As before, denote the Lagrange multipliers η_{it} and ρ_i (ρ_i now is state- and time-invariant), and plug

in the last two constraints. Then the FOCs are:

$$0 = u_C\left(C_{it}, L_{it}\right) + \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) X_C\left(C_{it}, L_{it}\right)$$
$$- \eta_{it}\left(1-\gamma\right) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta}\left(1 + X_C\left(C_{it}, L_{it}\right)\right) - \rho_i \frac{Z_t^h}{\beta^t P r_{ht}} Q_{it}^{-\theta}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)$$

$$0 = u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right) X_L\left(C_{it}, L_{it}\right)\right)$$
$$- \eta_{it}\left(1-\gamma\right) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} X_L\left(C_{it}, L_{it}\right) - \rho_i \frac{Z_t^h}{\beta^t P r_{ht}} Q_{it}^{-\theta} X_L\left(C_{it}, L_{it}\right)$$

$$0 = -\eta_{it}\left(1-\gamma\right)\left(-\theta Q_{it}^{1-2\theta}\left(1-\gamma\right) S_{it}^{\theta-1} \Phi_{it}^{2\theta-2} + Q_{it}^{-\theta} \theta \Phi_{it}^{\theta-1}\right) S_{it}^{\theta}\left(C_{it} + X_{it}\right)$$
$$+ \rho_i \frac{Z_t^h}{\beta^t P r_{ht}} \theta Q_{it}^{1-2\theta}\left(1-\gamma\right) S_{it}^{\theta-1} \Phi_{it}^{\theta-2}\left(C_{it} + X_{it}\right)$$

Use the price index constraint (**??**) to simplify the last FOC to

$$\rho_i \frac{Z_t^h}{\beta^t P r_{ht}} = \eta_{it} \gamma \Phi_{it} S_{it}.$$

Plug in ρ_i from this condition to the first two FOCs, and similarly simplify them to

$$0 = u_C\left(C_{it}, L_{it}\right) + \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) X_C\left(C_{it}, L_{it}\right) - \eta_{it} \Phi_{it} S_{it} Q_{it}^{-1}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)$$

$$0 = u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right) X_L\left(C_{it}, L_{it}\right)\right) - \eta_{it} \Phi_{it} S_{it} Q_{it}^{-1} X_L\left(C_{it}, L_{it}\right)$$

Note that they are exactly the same as in the non-U.S problem without capital controls (Appendix A.3.3), and thus they also lead to the domestic price stabilization,

$$A_{it} F_X\left(L_{it}, X_{it}\right) = \frac{S_{it} \Phi_{it}}{Q_{it}}.$$

Next, plug in this domestic price stabilization to the first FOC, and use the third FOC to go back from η_{it} to ρ_i,

$$Q_{it} u_C\left(C_{it}, L_{it}\right) = \rho_i \beta^{-t} \frac{Z_t^h}{\gamma P r_{ht}}.$$

Compare this with the private risk-sharing (A10), and note that they are the same (P_t^* is state-invariant).

Thus, the non-U.S. monetary policy still stabilizes domestic prices, and the capital controls achieve the same allocation of transfers across states as the private risk-sharing. ■

A.3.6 Gains from cooperation

Proof of Proposition ?? The global policy problem is

$$\max_{\{C_{it},L_{it},X_{it},\Phi_{it},Q_{it}\}_i} \int u\left(C_{it},L_{it}\right)\mathrm{d}i$$

$$\text{s.t.} \quad A_{it}F\left(L_{it},X_{it}\right) = \left(1-\gamma\right)Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\left(C_{it}+X_{it}\right) + \gamma S_{it}^{\theta}\int Q_{jt}^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j,$$

$$\gamma P_t^*\left[S_{it}^{\theta-1}\int Q_{jt}^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right] + B_{it}^h = 0,$$

$$X_{it} = X\left(C_{it},L_{it}\right),$$

$$Q_{it}^{\theta-1} = \gamma + \left(1-\gamma\right)\left(\Phi_{it}S_{it}\right)^{\theta-1},$$

$$\Phi_{0t} = 1.$$

Here we have a continuum of constraints, each corresponding to a different country. All terms of trade $\{S_{it}\}_i$ and transfers $\left\{B_{it}^h\right\}_i$ are taken as given, and they have to satisfy global balances $\int S_{jt}^{\theta-1}\mathrm{d}j = 1$ and $\int B_{jt}^h\mathrm{d}j = 0$. P_t^* is taken as given as well.

As before, plug in the last three constraints, and denote the remaining Lagrange multipliers as η_{it} and ρ_{it}. Then the FOCs are:

$$\begin{aligned}
0 = {} & u_C\left(C_{it},L_{it}\right) + \eta_{it}A_{it}F_X\left(L_{it},X_{it}\right)X_C\left(C_{it},L_{it}\right) \\
& - \eta_{it}\left(1-\gamma\right)Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right) - \int \eta_{jt}S_{jt}^{\theta}\mathrm{d}j\gamma Q_{it}^{-\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right) \\
& + \int \rho_{jt}S_{jt}^{\theta-1}\mathrm{d}j\gamma P_t^*Q_{it}^{-\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right) - \rho_{it}\gamma P_t^*Q_{it}^{-\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right)
\end{aligned}$$

$$\begin{aligned}
0 = {} & u_L\left(C_{it},L_{it}\right) + \eta_{it}A_{it}\left(F_L\left(L_{it},X_{it}\right) + F_X\left(L_{it},X_{it}\right)X_L\left(C_{it},L_{it}\right)\right) \\
& - \eta_{it}\left(1-\gamma\right)Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}X_L\left(C_{it},L_{it}\right) - \int \eta_{jt}S_{jt}^{\theta}\mathrm{d}j\gamma Q_{it}^{-\theta}X_L\left(C_{it},L_{it}\right) \\
& + \int \rho_{jt}S_{jt}^{\theta-1}\mathrm{d}j\gamma P_t^*Q_{it}^{-\theta}X_L\left(C_{it},L_{it}\right) - \rho_{it}\gamma P_t^*Q_{it}^{-\theta}X_L\left(C_{it},L_{it}\right)
\end{aligned}$$

$$\begin{aligned}
0 = {} & -\eta_{it}\left(1-\gamma\right)\left(-\theta Q_{it}^{1-2\theta}\left(1-\gamma\right)S_{it}^{\theta-1}\Phi_{it}^{2\theta-2} + Q_{it}^{-\theta}\theta\Phi_{it}^{\theta-1}\right)S_{it}^{\theta}\left(C_{it}+X_{it}\right) \\
& + \int \eta_{jt}S_{jt}^{\theta}\mathrm{d}j\gamma\theta Q_{it}^{1-2\theta}\left(1-\gamma\right)S_{it}^{\theta-1}\Phi_{it}^{\theta-2}\left(C_{it}+X_{it}\right) \\
& - \int \rho_{jt}S_{jt}^{\theta-1}\mathrm{d}j\gamma P_t^*\theta Q_{it}^{1-2\theta}\left(1-\gamma\right)S_{it}^{\theta-1}\Phi_{it}^{\theta-2}\left(C_{it}+X_{it}\right) + \rho_{it}\gamma P_t^*\theta Q_{it}^{1-2\theta}\left(1-\gamma\right)S_{it}^{\theta-1}\Phi_{it}^{\theta-2}\left(C_{it}+X_{it}\right)
\end{aligned}$$

Use the real exchange rate constraint (**??**) to simplify the last FOC (which holds for all i except for the

U.S.) to

$$\rho_{it}P_t^* = \eta_{it}\Phi_{it}S_{it} - \int \eta_{jt}S_{jt}^{\theta}\mathrm{d}j + \int \rho_{jt}P_t^* S_{jt}^{\theta-1}\mathrm{d}j$$

Plug in ρ_{it} from this condition to the first two FOCs, and similarly simplify them to

$$0 = u_C\left(C_{it}, L_{it}\right) + \eta_{it}A_{it}F_X\left(L_{it}, X_{it}\right)X_C\left(C_{it}, L_{it}\right) - \eta_{it}\Phi_{it}S_{it}Q_{it}^{-1}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)$$

$$0 = u_L\left(C_{it}, L_{it}\right) + \eta_{it}A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right)X_L\left(C_{it}, L_{it}\right)\right) - \eta_{it}\Phi_{it}S_{it}Q_{it}^{-1}X_L\left(C_{it}, L_{it}\right)$$

Note that they are exactly the same as in the non-U.S. non-cooperative problem (Appendix A.3.3), and thus they also lead to the domestic price stabilization in the non-U.S. countries,

$$A_{it}F_X\left(L_{it}, X_{it}\right) = \frac{S_{it}\Phi_{it}}{Q_{it}}.$$

For the U.S., there is no third FOC. Then, there are always two values of Lagrange multipliers, η_{0t} and ρ_{0t}, to support any allocation in the U.S.

Next, plug in the domestic price stabilization to the first FOC,

$$0 = u_C\left(C_{it}, L_{it}\right) - \eta_{it}\Phi_{it}S_{it}Q_{it}^{-1},$$

Plug this in the third FOC, multiply by $S_{it}^{\theta-1}$, integrate over i, and use the terms of trade normalization $\int S_{jt}^{\theta-1}\mathrm{d}j = 1$,

$$0 = \int Q_{jt}u_C\left(C_{jt}, L_{jt}\right)S_{jt}^{\theta-1}\mathrm{d}j - \int \frac{Q_{jt}u_C\left(C_{jt}, L_{jt}\right)}{\Phi_{jt}}S_{jt}^{\theta-1}\mathrm{d}j.$$

Use the risk-sharing across countries (A10), $Q_{it}u_C\left(C_{it}, L_{it}\right)\lambda_i^{-1} = Q_{jt}u_C\left(C_{jt}, L_{jt}\right)\lambda_j^{-1}$, and the ex-ante symmetry across all non-U.S. countries, which implies $\lambda_i = \lambda_j$ and $S_{jt} = 1$. Then

$$1 = \int \frac{1}{\Phi_{jt}}\mathrm{d}j.$$

Finally, plug in the domestic marginal cost stabilization one more time, and use definitions of various prices (**??**) along with the marginal cost definition $MC_{it} \equiv P_{it}/\left(A_{it}F_X\left(L_{it}, X_{it}\right)\right)$,

$$1 = \int \frac{S_{jt}}{Q_{jt}A_{jt}F_X\left(L_{jt}, X_{jt}\right)}\mathrm{d}j = \int \frac{MC_{jt}}{\mathcal{E}_{jt}P_{jt}^*}\mathrm{d}j.$$

■

A.4 Dynamic Model

A.4.1 Non-U.S. policy

Proof of Proposition ?? The non-U.S. policy problem with capital controls could be written as

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma}-N_{it}\right]$$

$$\text{s.t.}\quad \frac{A_{it}}{1-\alpha}C_{it}^{\sigma\alpha}L_{it}=(1-\gamma)\,Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\left(C_{it}+\frac{\alpha}{1-\alpha}C_{it}^{\sigma}L_{it}\right)+\gamma S_{it}^{\theta}C_t^*,$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^*-Q_{it}^{-\theta}\left(C_{it}+\frac{\alpha}{1-\alpha}C_{it}^{\sigma}L_{it}\right)\right]=0,$$

$$N_{it}=L_{it}+\frac{\varphi}{2}(1-\gamma)\,\Pi_{iit}^2+\frac{\varphi}{2}\gamma\Pi_{it}^{*2},$$

$$C_{it}=\left(\frac{Q_{it}}{\Lambda_{it}}\right)^{\frac{1}{\sigma}}\bar{C}_t,$$

$$Q_{it}^{\theta-1}=(1-\gamma)\,(\Phi_{it}S_{it})^{\theta-1}+\gamma,$$

$$\Pi_{iit}\,(\Pi_{iit}+1)=\beta\mathbb{E}_t\Pi_{iit+1}\,(\Pi_{iit+1}+1)+\frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(1-\frac{\Phi_{it}S_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(C_{it}^{1-\sigma}+\frac{\alpha}{1-\alpha}L_{it}\right),$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Phi_{it}S_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(C_{it}^{1-\sigma}+\frac{\alpha}{1-\alpha}L_{it}\right)=0,$$

$$\Pi_{it}^*\,(\Pi_{it}^*+1)=\beta\mathbb{E}_t\Pi_{it+1}^*\left(\Pi_{it+1}^*+1\right)+\frac{1-\varepsilon}{\varphi\tau_{Ri}}\Psi_i^{-1}\left(1-\frac{\Psi_iS_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma},$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Psi_iS_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma}=0,$$

$$Q_{it}=\frac{\mathcal{E}_{it}P_t^*}{P_{it}},\quad S_{it}=\frac{P_t^*}{P_{it}^*},\quad \Phi_{it}=\frac{\mathcal{E}_{it}P_{it}^*}{P_{iit}},$$

$$\Pi_{iit}=\frac{P_{iit}}{P_{iit-1}}-1,\quad \Pi_{it}^*=\frac{P_{it}^*}{P_{it-1}^*}-1.$$

Policymaker chooses C_{it}, N_{it}, L_{it}, Q_{it}, Φ_{it}, S_{it}, Π_{iit}, Π_{it}^*, Λ_{it}, $\mathcal{E}_{it}$, P_{it}, P_{it}^*, P_{iit} state-by-state. The presence of capital controls is expressed in the fact that Λ_{it} can change state-by-state and across time (but subject to the country's budget constraint). C_t^*, $\bar{C}_t$, and P_t^* are taken as given.

Here we included the market clearing (**??**) (with demand for intermediates (5) plugged in), the ex-ante budget constraint (**??**), the labor balance (9), the risk-sharing (**??**), the price index constraint (**??**), the ex-post and the ex-ante domestic price setting (**??**), the ex-post and the ex-ante export price setting (**??**), as well as definitions of various prices (**??**) and of inflation rates. Recall that in order to get the ex-ante domestic price setting constraint, we write down the ex-post constraint (**??**), iterate it forward until the infinity, and then impose $\Pi_{ii0}=0$. The ex-ante export price setting constraint is similar.

Let's use the primal approach. Use $S_{it} = \frac{P_t^*}{P_{it}^*}$ to plug in P_{it}^* in all other constraints, and drop this constraint. P_{it} enters only in the definition of Q_{it}, and thus we can drop both this variable and this constraint. Similarly, $\mathcal{E}_{it}$ enters only the definition of Φ_{it}, and we again drop this constraint. The, P_{iit} enters only the definition of Π_{iit}, which because of that could be dropped. Finally, the last two lines of our policy problem collapse just to

$$\Pi_{it}^* = \frac{P_t^*}{P_{t-1}^*}\frac{S_{it-1}}{S_{it}} - 1, \tag{A20}$$

where the policymaker takes the path of P_t^* as given. (Note that this constraint is equivalent to (A8).) Then, let's plug in the risk-sharing instead of C_{it}, and the labor balance instead of N_{it}, and arrive at the final policy problem

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{Q_{it}^{\frac{1}{\sigma}-1}\Lambda_{it}^{1-\frac{1}{\sigma}}\bar{C}_t^{1-\sigma}}{1-\sigma} - L_{it} - \frac{\varphi}{2}(1-\gamma)\Pi_{iit}^2 - \frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right]$$

$$\text{s.t.}\quad \frac{A_{it}}{1-\alpha}Q_{it}^{\alpha}\Lambda_{it}^{-\alpha}\bar{C}_t^{\sigma\alpha}L_{it} = (1-\gamma)\Phi_{it}^{\theta}S_{it}^{\theta}Q_{it}^{\frac{1}{\sigma}-\theta}\Lambda_{it}^{-\frac{1}{\sigma}}\bar{C}_t + \frac{\alpha}{1-\alpha}(1-\gamma)Q_{it}^{1-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\Lambda_{it}^{-1}\bar{C}_t^{\sigma}L_{it} + \gamma S_{it}^{\theta}C_t^*,$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{\frac{1}{\sigma}-\theta}\Lambda_{it}^{-\frac{1}{\sigma}}\bar{C}_t - \frac{\alpha}{1-\alpha}Q_{it}^{1-\theta}\Lambda_{it}^{-1}\bar{C}_t^{\sigma}L_{it}\right] = 0,$$

$$Q_{it}^{\theta-1} = (1-\gamma)(\Phi_{it}S_{it})^{\theta-1} + \gamma,$$

$$\Pi_{iit}(\Pi_{iit}+1) = \beta\mathbb{E}_t\Pi_{iit+1}(\Pi_{iit+1}+1) + \frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(1 - \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}}Q_{it}^{-\alpha}\Lambda_{it}^{\alpha-1}\Phi_{it}S_{it}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(Q_{it}^{\frac{1}{\sigma}-1}\Lambda_{it}^{1-\frac{1}{\sigma}}\bar{C}_t^{1-\sigma} + \frac{\alpha}{1-\alpha}L_{it}\right),$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1 - \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}}Q_{it}^{-\alpha}\Lambda_{it}^{\alpha-1}\Phi_{it}S_{it}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(Q_{it}^{\frac{1}{\sigma}-1}\Lambda_{it}^{1-\frac{1}{\sigma}}\bar{C}_t^{1-\sigma} + \frac{\alpha}{1-\alpha}L_{it}\right) = 0,$$

$$\Pi_{it}^*(\Pi_{it}^*+1) = \beta\mathbb{E}_t\Pi_{it+1}^*\left(\Pi_{it+1}^*+1\right) + \frac{1-\varepsilon}{\varphi\tau_{Ri}}\Psi_i^{-1}\left(1 - \Psi_i S_{it}Q_{it}^{-\alpha}\Lambda_{it}^{\alpha-1}\frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}}\right)\Lambda_{it}S_{it}^{\theta-1}C_t^*\bar{C}_t^{-\sigma},$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1 - \Psi_i S_{it}Q_{it}^{-\alpha}\Lambda_{it}^{\alpha-1}\frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}}\right)\Lambda_{it}S_{it}^{\theta-1}C_t^*\bar{C}_t^{-\sigma} = 0,$$

$$\Pi_{it}^* = \frac{P_t^*}{P_{t-1}^*}\frac{S_{it-1}}{S_{it}} - 1.$$

Plug in the price index (**??**) to get rid of Q_{it}, and denote the Lagrange multipliers for the remaining constraints as η_t, ρ, μ_t, ν, μ_t^*, ν^*, ζ_t. Then the FOCs are:

- with respect to Φ_{it}

$$
\begin{aligned}
0 = & \frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} + \eta_t \frac{A_{it}}{1-\alpha} \alpha Q_{it}^{\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} L_{it} \\
& - \eta_t (1-\gamma) \theta \Phi_{it}^{\theta-1} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t - \eta_t (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} \left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \theta \Phi_{it}^{\theta-1} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) (1-\theta) Q_{it}^{1-\theta} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \rho \bar{C}_t^{-\sigma} \left[\left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t + \frac{\alpha}{1-\alpha} (1-\theta) Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \right] (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \\
& + \mu_t [...] + \nu [...] + \mu_t^* [...] + \nu^* [...]
\end{aligned}
$$

- with respect to S_{it}

$$
\begin{aligned}
0 = & \frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} + \eta_t \frac{A_{it}}{1-\alpha} \alpha Q_{it}^{\alpha} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} L_{it} \\
& - \eta_t (1-\gamma) \Phi_{it}^{\theta} \theta S_{it}^{\theta-1} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t - \eta_t (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} \left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} \theta S_{it}^{\theta-1} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) (1-\theta) Q_{it}^{1-\theta} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \eta_t \gamma \theta S_{it}^{\theta-1} C_t^* + \rho \bar{C}_t^{-\sigma} (\theta - 1) S_{it}^{\theta-2} C_t^* \\
& - \rho \bar{C}_t^{-\sigma} \left[\left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t + \frac{\alpha}{1-\alpha} (1-\theta) Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \right] (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \\
& + \zeta_t \frac{S_{it-1}}{S_{it}^2} \frac{P_t^*}{P_{t-1}^*} - \beta \mathbb{E}_t \zeta_{t+1} \frac{1}{S_{it+1}} \frac{P_{t+1}^*}{P_t^*} + \mu_t [...] + \nu [...] + \mu_t^* [...] + \nu^* [...]
\end{aligned}
$$

- with respect to L_{it}

$$
\begin{aligned}
0 = & -1 + \eta_t \frac{A_{it}}{1-\alpha} Q_{it}^{\alpha} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} \\
& - \rho \bar{C}_t^{-\sigma} \frac{\alpha}{1-\alpha} Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} + \mu_t [...] + \nu [...]
\end{aligned}
$$

- with respect to Λ_{it}

$$0 = -\frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} - \alpha\eta_t \frac{A_{it}}{1-\alpha} Q_{it}^{\alpha} \Lambda_{it}^{-\alpha-1} \bar{C}_t^{\sigma\alpha} L_{it} + \eta_t \frac{1}{\sigma} (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}-1} \bar{C}_t$$
$$+ \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-2} \bar{C}_t^{\sigma} L_{it} + \rho \bar{C}_t^{-\sigma} \left[\frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}-1} \bar{C}_t + \frac{\alpha}{1-\alpha} Q_{it}^{1-\theta} \Lambda_{it}^{-2} \bar{C}_t^{\sigma} L_{it} \right]$$
$$+ \mu_t [...] + \nu [...] + \mu_t^* [...] + \nu^* [...]$$

- with respect to Π_{iit}

$$0 = -\varphi (1-\gamma) \Pi_{iit} + \mu_t [...]$$

- with respect to Π_{it}^*

$$0 = -\varphi\gamma \Pi_{it}^* + \zeta_t + \mu_t^* [...]$$

Now let's guess and verify that the optimal policy stabilizes domestic prices, $\Pi_{iit} = 0$, and thus the ex-post domestic price setting has to hold state-by-state,

$$1 = \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}} Q_{it}^{-\alpha} \Lambda_{it}^{\alpha-1} \Phi_{it} S_{it},$$

that no capital controls are used $\Lambda_{it} = 1$,[37] and that all 4 price setting constraints do not bind, $\mu_t = \nu = \mu_t^* = \nu^* = 0$.

Then the FOC with respect to Π_{it}^* implies $\zeta_t = \varphi\gamma\Pi_{it}^*$. The FOC with respect to L_{it} can be reduced to

$$\rho \bar{C}_t^{-\sigma} \frac{\alpha}{1-\alpha} = -\bar{C}_t^{-\sigma} Q_{it}^{\theta-1} + \eta_t \left((1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} + \gamma \frac{1}{1-\alpha} \Phi_{it} S_{it} \right).$$

The FOC with respect to Λ_{it} can be used to find η_t, $\eta_t \Phi_{it} S_{it} = \bar{C}_t^{-\sigma}$, and then the FOC with respect to L_{it} determines ρ, $\rho = \gamma$. The FOC with respect to Φ_{it} can be shown to be satisfied. The last remaining FOC, the one with repsect to S_{it}, can be then reduced to

$$\Pi_{it}^* (\Pi_{it}^* + 1) = \beta \mathbb{E}_t \Pi_{it+1}^* \left(\Pi_{it+1}^* + 1 \right) + \frac{\theta - 1}{\varphi} \left(\frac{\theta}{\theta - 1} \Phi_{it}^{-1} - 1 \right) S_{it}^{\theta-1} \bar{C}_t^{-\sigma} C_t^*.$$

Note that under the domestic price stabilization, the export price setting becomes

$$\Pi_{it}^* (\Pi_{it}^* + 1) = \beta \mathbb{E}_t \Pi_{it+1}^* \left(\Pi_{it+1}^* + 1 \right) + \frac{1-\varepsilon}{\varphi \tau_{Ri}} \Psi_i^{-1} \left(1 - \Psi_i \Phi_{it}^{-1} \right) S_{it}^{\theta-1} \bar{C}_t^{-\sigma} C_t^*,$$

[37] No capital controls implies $\Lambda_{it} = const$, and then the ex-ante symmetry of the non-U.S. countries implies $\Lambda_{it} = 1$.

and the two coincide under the appropriate level of the price-adjustment subsidy, $\tau_{Ri} = \frac{\varepsilon-1}{\theta}$.

Thus, we have shown that if this policy (domestic price stabilization and no capital controls) is feasible, that is it satisfies all the constraints, then it is optimal, that is it satisfies all the FOCs. ■

A.4.2 U.S. policy

Proof of Proposition ?? To derive the welfare loss function, let's start with the utility function (2.1), the market clearing (**??**), the ex-ante budget constraint (**??**), and the price index constraint (**??**), rewritten as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - (1-\gamma)\,A_{it}^{-1}\Psi_i^{\theta}S_{it}^{\theta}Q_{it}^{-\theta}C_{it} - \gamma A_{it}^{-1}S_{it}^{\theta}C_t^* - \frac{\varphi}{2}\Pi_{iit}^2\right],$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}C_{it}\right] = 0,$$

$$Q_{it}^{\theta-1} = (1-\gamma)\left(\Psi_i S_{it}\right)^{\theta-1} + \gamma.$$

Let's take second-order approximations around the non-stochastic steady state to all three conditions,

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\left[\omega\Psi c_{it} - \theta\,(1-\omega)\,s_{it} + \theta\,(1-\omega)\,q_{it} - \omega c_t^* - \omega\theta s_{it} + \frac{1}{2}\,(1-\omega+\omega\Psi)\,(1-\sigma)\,c_{it}^2\right.$$
$$\left. -\frac{1}{2}\,(1-\omega)\,(c_{it} - a_{it} + \theta s_{it} - \theta q_{it})^2 - \frac{1}{2}\omega\,(c_t^* - a_{it} + \theta s_{it})^2 - L^{-1}\frac{\varphi}{2}\Pi_{iit}^2 + a_{it} + \mathcal{O}\left(\epsilon^3\right)\right],$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[(\theta-1)\,s_{it} + c_t^* + \theta q_{it} - c_{it} + \frac{1}{2}\,((\theta-1)\,s_{it} + c_t^* - \sigma\bar{c}_t)^2 - \frac{1}{2}\,(-\theta q_{it} + c_{it} - \sigma\bar{c}_t)^2\right] = \mathcal{O}\left(\epsilon^3\right),$$

$$q_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}s_{it} + \frac{1}{2}\,(\theta-1)\,\frac{(1-\omega)\,\omega\Psi}{(1-\omega+\omega\Psi)^2}s_{it}^2 + \mathcal{O}\left(\epsilon^3\right),$$

where L is the steady state-level of L_{it}, ω is the steady-state measure of openness

$$\omega \equiv \frac{\gamma A_i^{-1}S_i^{\theta}\bar{C}}{L} = \frac{\gamma}{(1-\gamma)\,\Psi_i^{\theta} + \gamma},$$

and "small-letter" variables are log-deviations of "big-letter" variables. In the price index constraint, we have used its first-order version to get rid of q_{it}^2-term.

Use the budget constraint to express the first-order consumption, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_{it}$, as a function of other

variables, and plug it in the welfare along with the price index constraint to get

$$\begin{aligned}U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\Big[&\omega\left(\Psi-1\right)c_t^* - L^{-1}\frac{\varphi}{2}\Pi_{iit}^2\\ &-\frac{1}{2}\left(1-\omega+\omega\Psi\right)\sigma\left(c_{it}-\frac{\omega\Psi}{1-\omega+\omega\Psi}\bar{c}_t-\frac{1-\omega}{1-\omega+\omega\Psi}\theta a_{it}\right)^2\\ &-\frac{1}{2}\theta\omega\left(1+\frac{\left(1-\omega\right)\Psi}{1-\omega+\omega\Psi}\right)\left(s_{it}+\sigma\bar{c}_t-a_{it}\right)^2\\ &+\frac{1}{2}\omega\left(\Psi-1\right)c_t^{*2}+\frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2\\ &-\omega\Psi\sigma c_t^*\bar{c}_t+\omega c_t^* a_{it}-\omega\bar{c}_t a_{it}+\frac{1}{2}\left(\theta-1\right)a_{it}^2+\mathcal{O}\left(\epsilon^3\right)\Big].\end{aligned}$$

Note that under flexible prices, output is equal to θa_{it}, $y_{it}^f = \theta a_{it}$, and thus we define output gap as

$$\tilde{y}_{it} \equiv y_{it} - \theta a_{it}. \tag{A21}$$

Similarly, if we consider a world equilibrium with sticky prices and DCP, and then allow all prices in one non-U.S. country to be flexible, then in that particular country $s_{it}^f = -\sigma\bar{c}_t + a_{it}$ and $c_{it}^f = \frac{\omega\Psi}{1-\omega+\omega\Psi}\bar{c}_t + \frac{1-\omega}{1-\omega+\omega\Psi}\theta a_{it}$, and thus we adopt the following definitions of gaps,

$$\tilde{s}_{it} \equiv s_{it} + \sigma\bar{c}_t - a_{it}, \quad \tilde{c}_{it} \equiv c_{it} - \frac{\omega\Psi}{1-\omega+\omega\Psi}\bar{c}_t - \frac{1-\omega}{1-\omega+\omega\Psi}\theta a_{it}. \tag{A22}$$

Next, let's take the first-order approximation to the risk-sharing (**??**),

$$q_{it} - \sigma c_{it} = -\sigma\bar{c}_t + \mathcal{O}\left(\epsilon^2\right),$$

and to the market clearing (**??**),

$$A_{it}L_{it} = Y_{it} = \left(1-\gamma\right)Q_{it}^{-\theta}\Psi_i^{\theta}S_{it}^{\theta}C_{it} + \gamma S_{it}^{\theta}C_t^*,$$

$$y_{it} = -\left(1-\omega\right)\theta q_{it} + \left(1-\omega\right)c_{it} + \theta s_{it} + \omega c_t^* + \mathcal{O}\left(\epsilon^2\right).$$

Together with the first-order approximation to the price index constraint, these 3 conditions imply that

$$\tilde{s}_{it} = \sigma\tilde{y}_{it} + \mathcal{O}\left(\epsilon^2\right), \quad \tilde{c}_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}\tilde{y}_{it} + \mathcal{O}\left(\epsilon^2\right). \tag{A23}$$

Then we can rewrite the welfare loss function as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\left[\omega\left(\Psi-1\right)c_t^* - \frac{1}{2}\sigma\tilde{y}_{it}^2 - L^{-1}\frac{\varphi}{2}\Pi_{iit}^2\right.$$
$$+\frac{1}{2}\omega\left(\Psi-1\right)c_t^{*2} + \frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2$$
$$\left.-\omega\Psi\sigma c_t^*\bar{c}_t + \omega c_t^* a_{it} - \omega\bar{c}_t a_{it} + \frac{1}{2}\left(\theta-1\right)a_{it}^2 + \mathcal{O}\left(\epsilon^3\right)\right].$$

Next, recall the definition of the global demand C_t^*,

$$C_t^* \equiv \int Q_{jt}^{-\theta} C_{jt}\mathrm{d}j,$$

and plug in the risk-sharing (**??**) to get

$$C_t^* = \bar{C}_t,$$

where we have used the ex-ante symmetry of the non-U.S. countries, $\Lambda_i = 1$, and the Faia-Monacelli parametrization $\sigma\theta = 1$. Recall the global terms of trade normalization (A4), $\int S_{jt}^{\theta-1}\mathrm{d}j = 1$. Finally, recall the ex-ante export price setting of the non-U.S. countries (**??**)

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\Psi_i S_{it}\frac{\bar{C}_t^{\sigma}}{A_{it}}\right)S_{it}^{\theta-1}C_t^*\bar{C}_t^{-\sigma} = 0,$$

where again we have used the ex-ante symmetry of the non-U.S. countries, $\Lambda_i = 1$, but have not used their optimal policy.

Now let's take second-order approximations to the last two conditions,

$$\int\left[s_{jt} + \frac{1}{2}\left(\theta-1\right)s_{jt}^2\right]\mathrm{d}j = \mathcal{O}\left(\epsilon^3\right),$$

$$\mathcal{O}\left(\epsilon^3\right) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-\sigma\bar{c}_t - \int s_{jt}\mathrm{d}j + \frac{1}{2}\sigma^2\bar{c}_t^2 + \frac{1}{2}\left(1-2\theta\right)\int s_{jt}^2\mathrm{d}j - \frac{1}{2}\int a_{jt}^2\mathrm{d}j\right.$$
$$\left.-\sigma\bar{c}_t c_t^* - \int c_t^* s_{jt}\mathrm{d}j - \int\sigma\left(\theta-1\right)s_{jt}\bar{c}_t\mathrm{d}j + \int c_t^* a_{jt}\mathrm{d}j + \theta\int s_{jt}a_{jt}\mathrm{d}j\right].$$

Use the first condition to substitute for $\int s_{jt}\mathrm{d}j$ and the global demand constraint $\bar{c}_t = c_t^*$ to reduce the price setting to

$$\mathcal{O}\left(\epsilon^3\right) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-c_t^* - \frac{1}{2}\theta^2\int\left(s_{jt}+\sigma\bar{c}_t - a_{jt}\right)^2\mathrm{d}j + \frac{1}{2}\theta\left(\theta-1\right)\int a_{jt}^2\mathrm{d}j + \frac{1}{2}\left(\sigma-1\right)\bar{c}_t^2\right].$$

Finally, express the linear global demand, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_t^*$, as a function of other variables, and plug it in the welfare to arrive at

$$U_i = -\mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{L}{2}\left[\sigma\tilde{y}_{it}^2 + L^{-1}\varphi\Pi_{iit}^2 + \omega\frac{\theta^2}{\theta-1}\int \tilde{s}_{jt}^2 \mathrm{d}j + t.i.p. + \mathcal{O}\left(\epsilon^3\right)\right].$$

This is the U.S. welfare loss function (**??**).

Now we can formulate the linear-quadratic policy problem for the U.S. One way to do this is by rewriting the loss function through the terms of trade gap instead of the output gap (remember from (A23) that these gaps are proportional to each other). Ultimately, the policy problem can be expressed as

$$\min_{\left\{\tilde{s}_{it},\tilde{s}_{jt},\Pi_{iit},\Pi_{jt}^*\right\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{L}{2}\left[\theta\tilde{s}_{it}^2 + L^{-1}\varphi\Pi_{iit}^2 + \omega\frac{\theta^2}{\theta-1}\int \tilde{s}_{jt}^2 \mathrm{d}j + t.i.p. + \mathcal{O}\left(\epsilon^3\right)\right]$$

$$\Pi_{iit} = \beta\mathbb{E}_t\Pi_{iit+1} + \frac{\varepsilon-1}{\tau_{Ri}\varphi}\left[(1-\gamma)+\gamma\Psi_i^{-\theta}\right]A_i^{\theta-1}\tilde{s}_{it} + \mathcal{O}\left(\epsilon^2\right),$$

$$\Pi_{jt}^* = \beta\mathbb{E}_t\Pi_{jt+1}^* + \frac{\theta-1}{\varphi}\Psi_i^{1-\theta}A_i^{\theta-1}\tilde{s}_{jt} + \mathcal{O}\left(\epsilon^2\right),$$

$$\Pi_{iit} - \Pi_{jt}^* = \tilde{s}_{it-1} - \tilde{s}_{it} - \tilde{s}_{jt-1} + \tilde{s}_{jt} + a_{it-1} - a_{it} - a_{jt-1} + a_{jt} + \mathcal{O}\left(\epsilon^2\right).$$

All ex-ante constraints have dropped out since, ultimately, all "small-letter" variables are linear functions of shocks, and shocks are zero mean. Out of the ex-post constraints, we have used the price index constraint (**??**) to get rid of q_{it}, the risk-sharing (**??**) to get rid of c_{it}, the global terms of trade normalization (A4) to get rid of the global demand $\bar{c}_t$. Then all we are left with is the ex-post price setting for the U.S. firms (**??**), the ex-post export price setting for the non-U.S. countries (**??**), and the dynamic constraint (A8), which links the inflation definitions with the terms of trade dynamics.

Note that this problem does not depend on the optimal policy of the non-U.S. countries. This is due to the fact that the global demand constraint, $C_t^* \equiv \int Q_{jt}^{-\theta}C_{jt}\mathrm{d}j = \int Q_{jt}^{\frac{1}{\sigma}-\theta}\mathrm{d}j\bar{C}_t$, has collapsed to just $C_t^* = \bar{C}_t$. Then, similar to the logic from the static model (Appendix A.3.4), this constraint will follow by construction from the budget constraints of non-U.S. countries and global balances. Even though the U.S. problem also depends on $\bar{C}_t$ through the risk-sharing (**??**) (and thus, in general, $\bar{C}_t$ depends not only on C_t^*, but also on Q_{jt}, which, in turn, are determined by the non-U.S. policies), under the Faia-Monacelli parametrization $\sigma\theta = 1$, $\bar{C}_t$ is equivalent to C_t^*, and thus can be freely chosen by the U.S.

Finally, it is straightforward to see that the U.S. can not simultaneously close both the domestic and the global gaps, $\tilde{s}_{it}^2$ and $\int \tilde{s}_{jt}^2\mathrm{d}j$, thus the optimal policy will balance between the two and depend on foreign shocks a_{jt}. ■

A.4.3 Gains from DCP

Proof of Proposition ?? First, let's derive the non-U.S. welfare loss function (**??**).

Once again, let's start with the utility function (2.1), the market clearing (**??**), the ex-ante budget constraint (**??**), and the price index constraint (**??**), rewritten as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - (1-\gamma)\,A_{it}^{-1}\Phi_{it}^{\theta}S_{it}^{\theta}Q_{it}^{-\theta}C_{it} - \gamma A_{it}^{-1}S_{it}^{\theta}C_t^* - \frac{\varphi}{2}(1-\gamma)\,\Pi_{iit}^2 - \frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right],$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}C_{it}\right] = 0,$$

$$Q_{it}^{\theta-1} = (1-\gamma)\,(\Phi_{it}S_{it})^{\theta-1} + \gamma.$$

The key difference from the U.S. is that now we have state- and time-dependent Φ_{it} instead of a constant Ψ_i and that we have two inflation rates instead of one. Let's take the second-order approximations around the non-stochastic steady state to all three conditions,

$$\begin{aligned}U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\,[&\omega\Psi c_{it} - \theta(1-\omega)\,s_{it} + \theta(1-\omega)\,(q_{it}-\phi_{it}) - \omega c_t^* - \omega\theta s_{it}\\ &+\frac{1}{2}(1-\omega+\omega\Psi)(1-\sigma)\,c_{it}^2 - \frac{1}{2}(1-\omega)\,(c_{it}-a_{it}+\theta s_{it}-\theta(q_{it}-\phi_{it}))^2\\ &-\frac{1}{2}\omega\,(c_t^*-a_{it}+\theta s_{it})^2 - L^{-1}\frac{\varphi}{2}(1-\gamma)\,\Pi_{iit}^2 - L^{-1}\frac{\varphi}{2}\gamma\Pi_{it}^{*2} + a_{it} + \mathcal{O}\left(\epsilon^3\right)\Big],\end{aligned}$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[(\theta-1)\,s_{it} + c_t^* + \theta q_{it} - c_{it} + \frac{1}{2}\left((\theta-1)\,s_{it} + c_t^* - \sigma\bar{c}_t\right)^2 - \frac{1}{2}\left(-\theta q_{it} + c_{it} - \sigma\bar{c}_t\right)^2\right] = \mathcal{O}\left(\epsilon^3\right),$$

$$q_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}\,(s_{it}+\phi_{it}) + \frac{1}{2}(\theta-1)\,\frac{(1-\omega)\,\omega\Psi}{(1-\omega+\omega\Psi)^2}\,(s_{it}+\phi_{it})^2 + \mathcal{O}\left(\epsilon^3\right),$$

where L is again the steady state-level of L_{it}, ω is the steady-state measure of openness.

Use the budget constraint to express the first-order consumption, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_{it}$, as a function of other variables, and plug it in the welfare along with the price index constraint to get

$$\begin{aligned}U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\,\Big[&\omega\,(\Psi-1)\,c_t^* - L^{-1}\frac{\varphi}{2}(1-\gamma)\,\Pi_{iit}^2 - L^{-1}\frac{\varphi}{2}\gamma\Pi_{it}^{*2}\\ &-\frac{1}{2}(1-\omega+\omega\Psi)\,\sigma\tilde{c}_{it}^2 - \frac{1}{2}\theta\frac{\omega\Psi}{1-\omega}(1-\omega+\omega\Psi)\,\tilde{q}_{it}^2 - \frac{1}{2}\theta\omega\tilde{s}_{it}^2\\ &+\frac{1}{2}\omega\,(\Psi-1)\,c_t^{*2} + \frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2 - \omega\Psi\sigma c_t^*\bar{c}_t + \omega c_t^* a_{it} - \omega\bar{c}_t a_{it} + \frac{1}{2}(\theta-1)\,a_{it}^2 + \mathcal{O}\left(\epsilon^3\right)\Big].\end{aligned}$$

Here the consumption and terms of trade gaps, $\tilde{c}_{it}$ and $\tilde{s}_{it}$, are defined in (A22), and the real exchange

rate gap is the deviation of q_{it} from its flexible price value,

$$\tilde{q}_{it} \equiv q_{it} + \sigma \frac{1-\omega}{1-\omega+\omega\Psi}\bar{c}_t - \frac{1-\omega}{1-\omega+\omega\Psi}a_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}\left(\tilde{s}_{it} + \tilde{\phi}_{it}\right), \tag{A24}$$

where the last equality follows from the first-order approximation to the price index constraint (**??**). The law of one price deviation gap is just equal to its level, $\tilde{\phi}_{it} \equiv \phi_{it}$, since the law of one price holds in the flexible price equilibrium.

Using these definitions, and the first-order approximations to the market clearing (**??**) and to the risk-sharing (**??**), one can show that

$$\tilde{y}_{it} = \theta\left(1-\omega\right)\tilde{\phi}_{it} + \theta\tilde{s}_{it} + \mathcal{O}\left(\epsilon^2\right), \quad \tilde{q}_{it} = \sigma\tilde{c}_{it} + \mathcal{O}\left(\epsilon^2\right), \tag{A25}$$

instead of (A23), where the output gap is still defined in (A21). Then, the welfare can be rewritten as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\left[\omega\left(\Psi-1\right)c_t^* - \frac{1}{2}\sigma\tilde{y}_{it}^2 - \frac{1}{2}\omega\left(1-\omega\right)\theta\tilde{\phi}_{it}^2 - L^{-1}\frac{\varphi}{2}\left(1-\gamma\right)\Pi_{iit}^2 - L^{-1}\frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right.$$
$$\left. + \frac{1}{2}\omega\left(\Psi-1\right)c_t^{*2} + \frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2 - \omega\Psi\sigma c_t^*\bar{c}_t + \omega c_t^* a_{it} - \omega\bar{c}_t a_{it} + \frac{1}{2}\left(\theta-1\right)a_{it}^2 + \mathcal{O}\left(\epsilon^3\right)\right].$$

After that, once again, one can repeat the same steps as for the U.S. welfare loss function (Appendix A.4.2), that is use second-order approximations to the global terms of trade normalization (A4), to the ex-ante export price setting of the non-U.S. countries (**??**), and use the global demand constraint $C_t^* = \bar{C}_t$ to arrive at the non-U.S. welfare loss function (**??**)

$$U_i = -\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{L}{2}\left[\sigma\tilde{y}_{it}^2 + \omega\left(1-\omega\right)\theta\tilde{\phi}_{it}^2 + L^{-1}\varphi\left(\left(1-\gamma\right)\Pi_{iit}^2 + \gamma\Pi_{it}^{*2}\right) + \omega\frac{\theta^2}{\theta-1}\int\tilde{s}_{jt}^2\mathrm{d}j + t.i.p. + \mathcal{O}\left(\epsilon^3\right)\right].$$

Now, let's show that under the domestic price stabilization in all countries, the U.S. welfare is always higher.

First, note that under the domestic price stabilization in the U.S., $\Pi_{00t} = 0$ by definition, and then the ex-post price setting (**??**) implies $\tilde{s}_{0t} = 0$, which then from (A23) leads to $\tilde{y}_{0t} = 0$. Thus, under this policy, the U.S. can close all of its domestic gaps.

Second, under the domestic price stabilization in the non-U.S. countries, once again $\Pi_{jjt} = 0$ by definition, but the ex-post domestic price setting (**??**) then implies $\tilde{\phi}_{jt} + \tilde{s}_{jt} = 0$, which then from (A24) leads to $\tilde{q}_{jt} = 0$. Still, as it can be seen from (A25), $\tilde{y}_{jt} \neq 0$, and both $\tilde{\phi}_{jt} \neq 0$ and $\tilde{s}_{jt} \neq 0$. It can be shown from the ex-post export price setting (**??**) that $\tilde{s}_{jt} \neq 0$ leads to $\Pi_{jt}^{*2} \neq 0$.

Importantly, it can be shown that both the U.S. loss function (**??**) and the non-U.S. loss function (**??**) have the same "terms independent of policy", "$t.i.p.$" Then, under symmetric distribution of shocks in

all countries, we can take a difference, and arrive at

$$U_0 - U_j = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{L}{2}\left[\sigma\tilde{y}_{jt}^2 + \omega\left(1-\omega\right)\theta\tilde{\phi}_{jt}^2 + L^{-1}\varphi\gamma\Pi_{jt}^{*2} + \mathcal{O}\left(\epsilon^3\right)\right],$$

which then implies that $U_0 > U_j$.

Finally, the U.S. policy under commitment balances between closing the domestic gaps, $\tilde{y}_{0t}^2$ and Π_{00t}^2, and the global gap $\int \tilde{s}_{it}^2 \mathrm{d}i$. In doing so, the U.S. improves the expected welfare of all countries. In particular, all countries get to benefit from lower $\mathbb{E}\int \tilde{s}_{0t}^2 \mathrm{d}j$. Note also that for a non-U.S. country it can be shown that

$$\sigma\tilde{y}_{jt}^2 + \omega\left(1-\omega\right)\theta\tilde{\phi}_{jt}^2 = \theta\left(1-\omega\right)\left(\tilde{s}_{jt} + \tilde{\phi}_{jt}\right)^2 + \theta\omega\tilde{s}_{jt}^2 + \mathcal{O}\left(\epsilon^3\right),$$

and the optimal non-U.S. policy leads to $\tilde{s}_{jt} + \tilde{\phi}_{jt} = 0$. Then, the only "open" gap for a non-U.S. country is the terms of trade gap $\tilde{s}_{jt}^2$ and associated with it export inflation Π_{jt}^{*2}. As long as the U.S. policy helps to reduce the expected global terms of trade gap, $\mathbb{E}\int \tilde{s}_{it}^2 \mathrm{d}i = \int \mathbb{E}\tilde{s}_{it}^2 \mathrm{d}i = \mathbb{E}\tilde{s}_{it}^2$ (where the last equality follows from the ex-ante symmetry of the non-U.S. countries), it can only help to close the non-U.S. terms of trade gap. Thus, the U.S. policy under commitment increases the U.S. welfare by less than it increases the non-U.S. welfare (relative to the domestic price stabilization). Then it's possible that under optimal policy the U.S. welfare could be smaller than the non-U.S. welfare. ■

A.4.4 Currency union

Preliminaries First, let's derive all the equilibrium conditions for the Eurozone problem.

Most importantly, the import price index for the Eurozone, which we denote as $\tilde{P}_{it}^*$, is different from the global import price index P_t^*. While the latter is set and is sticky in dollars, the former is set and is sticky in euros. Then, the Eurozone monetary policy is going to cause the law of one price deviation for the bundle of import goods, which we denote as $\Phi_{it} = \mathcal{E}_{it}P_t^*/\tilde{P}_{it}^*$. Then we define the real exchange rate for the Eurozone as $Q_{it} \equiv \tilde{P}_{it}^*/P_{it}$ and the terms of trade as $S_{it} \equiv \tilde{P}_{it}^*/\left(\Psi_i P_{iit}\right)$ (here we have used the law of one price between domestic and export goods for the Eurozone, $\Psi_i P_{iit} = \mathcal{E}_{it}P_{it}^*$). Then the price index (4) becomes

$$Q_{it}^{\theta-1} = \gamma + \left(1-\gamma\right)\left(\Psi_i S_{it}\right)^{\theta-1}.$$

The risk-sharing changes to

$$C_{it} = \Phi_{it}^{\frac{1}{\sigma}} Q_{it}^{\frac{1}{\sigma}} \Lambda_i^{-\frac{1}{\sigma}} \bar{C}_t,$$

the ex-ante budget constraint becomes

$$\mathbb{E}\sum_t \beta^t \bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}\Phi_{it}^{\theta-1}C_t^* - \Phi_{it}^{\frac{1}{\sigma}-1}Q_{it}^{\frac{1}{\sigma}-\theta}\Lambda_i^{-\frac{1}{\sigma}}\bar{C}_t\right] = 0.$$

The market clearing is

$$Y_{it} = A_{it}L_{it} = (1-\gamma)\,\Psi_i^{\theta} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_i^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}} \bar{C}_t + \gamma S_{it}^{\theta} \Phi_{it}^{\theta} C_t^*.$$

Next, the individual firm's problem in the Eurozone becomes very similar to the problem of U.S. firms (**??**), since both domestic and export prices are set in producer currency,

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{it} \left[(P_t - \tau_i MC_{it}) \left(\frac{P_t}{P_{iit}} \right)^{-\varepsilon} Y_{it} - \frac{\varphi}{2} \tau_{Ri} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 W_{it} \right].$$

Taking the optimality conditions leads to the ex-post price setting

$$\begin{aligned} \Pi_{iit} \left(\Pi_{iit} + 1 \right) = & -\frac{\kappa}{\Psi_i} \left(1 - \frac{\Psi_i S_{it}}{Q_{it}} \frac{C_{it}^{\sigma}}{A_{it}} \right) \left((1-\gamma)\,\Psi_i^{\theta} Q_{it}^{-\theta} C_{it} + \gamma \Phi_{it}^{\theta} C_t^* \right) Q_{it} S_{it}^{\theta-1} C_{it}^{-\sigma} \\ & + \beta \mathbb{E} \Pi_{iit+1} \left(\Pi_{iit+1} + 1 \right), \end{aligned}$$

and the corresponding ex-ante price setting $\Pi_{ii0} = 0$.

Moreover, now the non-U.S. firms that export to the Eurozone set their prices in euros. An individual firm's problem is

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{jt} \left[\left(\frac{\mathcal{E}_{jt}}{\mathcal{E}_{it}} P_t - \tau_j MC_{jt} \right) \left(\frac{\Psi_j P_t}{P_{jit}} \right)^{-\varepsilon} \left(\frac{P_{jit}}{\tilde{P}_{it}^*} \right)^{-\theta} Q_{it}^{-\theta} C_{it} - \gamma \frac{\varphi}{2} \tau_{Rj} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 W_{jt} \right],$$

where P_t is set in currency i, the euro, and the bilateral nominal exchange rate, $\mathcal{E}_{jt}/\mathcal{E}_{it}$, converts it to the producer currency j. Next, let's define the special terms of trade for a non-U.S. country j, $S_{jit} \equiv \mathcal{E}_{it} P_t^* / P_{jit}$, which reflect the price of j's imports relative to its export to the Eurozone, i. Then, taking the optimality conditions leads to the following ex-post price setting

$$\Pi_{jit} \left(\Pi_{jit} + 1 \right) = \beta \mathbb{E} \Pi_{jit+1} \left(\Pi_{jit+1} + 1 \right) - \frac{\kappa}{\Psi_j} \left(1 - \Psi_j S_{jit} \frac{\bar{C}_t^{\sigma}}{A_{jt}} \right) \Phi_{it}^{-\theta} S_{jit}^{\theta-1} Q_{it}^{-\theta} C_{it} \bar{C}_t^{-\sigma},$$

and the corresponding ex-ante price setting $\Pi_{ji0} = 0$, where the inflation rate is defined as $\Pi_{jit} \equiv P_{jit}/P_{jit-1} - 1$. Note that the key difference between this price setting and the non-U.S. export price setting (**??**) is that the former depends on the Eurozone demand specifically, while the latter does not.

Proof of Proposition ?? Now we can set up the Eurozone policy problem,

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it} - \frac{\varphi}{2} \Pi_{iit}^2 \right]$$

$$\text{s.t.} \quad A_{it}L_{it} = (1-\gamma)\,\Psi_i^{\theta} S_{it}^{\theta} \Lambda_i^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}} \bar{C}_t + \gamma S_{it}^{\theta} \Phi_{it}^{\theta} C_t^*,$$

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \bar{C}_t^{-\sigma} \left[S_{it}^{\theta-1} \Phi_{it}^{\theta-1} C_t^* - \Lambda_i^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}-1} \bar{C}_t \right] = 0,$$

$$C_{it} = Q_{it}^{\frac{1}{\sigma}} \Lambda_{it}^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}} \bar{C}_t,$$

$$Q_{it}^{\theta-1} = (1-\gamma)\left(\Psi_i S_{it}\right)^{\theta-1} + \gamma,$$

$$\Pi_{iit}\left(\Pi_{iit}+1\right) = \beta \mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right) - \frac{\kappa}{\Psi_i}\left(1 - \frac{\Psi_i S_{it}}{Q_{it}} \frac{C_{it}^{\sigma}}{A_{it}}\right)\left((1-\gamma)\,\Psi_i^{\theta} Q_{it}^{-\theta} C_{it} + \gamma \Phi_{it}^{\theta} C_t^*\right) \frac{Q_{it} S_{it}^{\theta-1}}{C_{it}^{\sigma}},$$

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\Psi_i S_{it}}{Q_{it}} \frac{C_{it}^{\sigma}}{A_{it}}\right)\left((1-\gamma)\,\Psi_i^{\theta} Q_{it}^{-\theta} C_{it} + \gamma \Phi_{it}^{\theta} C_t^*\right) \frac{Q_{it} S_{it}^{\theta-1}}{C_{it}^{\sigma}} = 0,$$

$$\Pi_{jit}\left(\Pi_{jit}+1\right) = \beta \mathbb{E}\Pi_{jit+1}\left(\Pi_{jit+1}+1\right) - \frac{\kappa}{\Psi_j}\left(1 - \Psi_j S_{jit} \frac{\bar{C}_t^{\sigma}}{A_{jt}}\right) \Phi_{it}^{-\theta} S_{jit}^{\theta-1} Q_{it}^{-\theta} C_{it} \bar{C}_t^{-\sigma},$$

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \left(1 - \Psi_j S_{jit} \frac{\bar{C}_t^{\sigma}}{A_{jt}}\right) \Phi_{it}^{-\theta} S_{jit}^{\theta-1} Q_{it}^{-\theta} C_{it} \bar{C}_t^{-\sigma} = 0,$$

$$Q_{it} = \frac{\tilde{P}_{it}^*}{P_{it}}, \quad S_{it} = \frac{\tilde{P}_{it}^*}{\Psi_i P_{iit}}, \quad \Phi_{it} = \frac{\mathcal{E}_{it} P_t^*}{\tilde{P}_{it}^*}, \quad S_{jit} = \frac{\mathcal{E}_{it} P_t^*}{P_{jit}},$$

$$\Pi_{iit} = \frac{P_{iit}}{P_{iit-1}} - 1, \quad \Pi_{jit} = \frac{P_{jit}}{P_{jit-1}} - 1,$$

$$\Psi_i = \frac{\mathcal{E}_{it} P_{it}^*}{P_{iit}}, \quad C_t^* = \bar{C}_t,$$

$$\tilde{P}_{it}^* = \left(\int \left(P_{jit}\right)^{1-\theta} \mathrm{d}j \right)^{\frac{1}{1-\theta}}.$$

Policymaker chooses C_{it}, L_{it}, Q_{it}, Φ_{it}, S_{it}, Π_{iit}, Λ_i, $\mathcal{E}_{it}$, P_{it}, P_{it}^*, P_{iit}, $\tilde{P}_{it}^*$, P_{jit}, Π_{jit} state-by-state, while C_t^*, $\bar{C}_t$, and P_t^* are taken as given.

Here we included the market clearing, the ex-ante budget constraint, the risk-sharing, the price index constraint, the ex-post and the ex-ante domestic price setting, the ex-post and the ex-ante import price setting, as well as definitions of various prices and of inflation rates, the law of one price, the global demand constraint, and the definition of the import price index.

Crucially, the Eurozone can not affect global demand C_t^* or $\bar{C}_t$. However, note that in this policy problem, the global demand often enters multiplicatively with the law of one price deviation, Φ_{it}, which the Eurozone can affect. In fact, let's denote the "effective" global demand as $Z_{it} \equiv \Phi_{it}^{\theta} C_t^*$ and adjusted terms of trade as $\tilde{S}_{jit} \equiv \tilde{P}_{it}^*/P_{jit}$. Then, following the primal approach, we can ultimately rewrite this

policy problem as

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{Q_{it}^{\frac{1}{\sigma}-1}\Lambda_i^{1-\frac{1}{\sigma}}Z_{it}^{1-\sigma}}{1-\sigma}-(1-\gamma)\,A_{it}^{-1}\Psi_i^{\theta}S_{it}^{\theta}\Lambda_i^{-\frac{1}{\sigma}}Z_{it}-\gamma A_{it}^{-1}S_{it}^{\theta}Z_{it}-\frac{\varphi}{2}\Pi_{iit}^2\right]$$

$$\text{s.t.}\quad \mathbb{E}\sum_{t=0}^{\infty}\beta^t Z_{it}^{1-\sigma}\left[S_{it}^{\theta-1}-\Lambda_i^{-\frac{1}{\sigma}}\right]=0,$$

$$Q_{it}^{\theta-1}=(1-\gamma)\left(\Psi_i S_{it}\right)^{\theta-1}+\gamma,$$

$$\Pi_{iit}\left(\Pi_{iit}+1\right)=-\kappa\left(1-\Psi_i\frac{S_{it}Z_{it}^{\sigma}}{\Lambda_{it}A_{it}}\right)\left[(1-\gamma)\,\Psi_i^{\theta-1}S_{it}^{\theta-1}\Lambda_i^{1-\frac{1}{\sigma}}Z_{it}^{1-\sigma}+\gamma\Psi_i^{-1}\Lambda_i S_{it}^{\theta-1}Z_{it}^{1-\sigma}\right]$$
$$+\beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right),$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\Psi_i\frac{S_{it}Z_{it}^{\sigma}}{\Lambda_{it}A_{it}}\right)\left[(1-\gamma)\,\Psi_i^{\theta-1}S_{it}^{\theta-1}\Lambda_i^{1-\frac{1}{\sigma}}Z_{it}^{1-\sigma}+\gamma\Psi_i^{-1}\Lambda_i S_{it}^{\theta-1}Z_{it}^{1-\sigma}\right]=0,$$

$$\Pi_{jit}\left(\Pi_{jit}+1\right)=\beta\mathbb{E}\Pi_{jit+1}\left(\Pi_{jit+1}+1\right)-\frac{\kappa}{\Psi_j}\left(1-\frac{\Psi_j Z_{it}^{\sigma}}{A_{jt}}\tilde{S}_{jit}\right)\Lambda_i^{-\frac{1}{\sigma}}Z_{it}^{1-\sigma}\tilde{S}_{jit}^{\theta-1},$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Psi_j Z_{it}^{\sigma}}{A_{jt}}\tilde{S}_{jit}\right)Z_{it}^{1-\sigma}\tilde{S}_{jit}^{\theta-1}=0,$$

$$\Pi_{iit}=\frac{S_{it-1}}{S_{it}}\frac{\tilde{P}_{it}^{*}}{\tilde{P}_{it-1}^{*}}-1,\quad \Pi_{jit}=\frac{\tilde{S}_{jit-1}}{\tilde{S}_{jit}}\frac{\tilde{P}_{it}^{*}}{\tilde{P}_{it-1}^{*}}-1,$$

$$\int\tilde{S}_{jit}^{\theta-1}\mathrm{d}j=1.$$

Note that this policy problem is almost identical to the U.S. policy problem. The global demand C_t^* or $\bar{C}_t$ does not appear in this problem at all once we have introduced the "effective" global demand Z_{it}. So, it turns out, that the Eurozone can achieve the same outcomes as the U.S. by manipulating the law of one price deviation Φ_{it} instead of manipulating the global demand C_t^*.

The only difference between this problem and the U.S. problem is that the import price setting in the former depends on Λ_i, while the import price setting in the latter did not. However, it can be shown that this constant is of the second order, $\Lambda_i=\mathcal{O}\left(\epsilon^2\right)$, and thus one can repeat all the steps to derive the welfare loss function. Crucially, one needs to take the second-order approximation to the non-U.S. export price setting (**??**), and not to the Eurozone import price setting, and so this difference in two problems will not affect the welfare loss function derivation. Then, once the linear-quadratic problem is set up, we need to take only the first-order approximation to the import price setting, and thus Λ_i will not show up there. Thus, the linear-quadratic problems of the U.S. and the Eurozone are isomorphic to each other. It then follows that the two countries achieve the same welfare up to the second order. ■