Bargaining and Strategic Voting on Appellate Courts

Giri Parameswaran[a], Charles M. Cameron[b] , and Lewis A. Kornhauser[c]

[a]Haverford College, gparames@haverford.edu
[b]Princeton University, ccameron@princeton.edu
[c]New York University School of Law, lewis.kornhauser@nyu.edu

May 31, 2019

Abstract

We explore the properties of voting rules and procedures employed by appellate courts in the US. Our model features: (1) a two-stage decision-making process (first over case disposition, then over majority opinion content), (2) dispositional consistency (the new rule must yield the Court's indicated case disposition when applied to the instant case), (3) restricted bargaining entrée (only members of the winning dispositional coalition bargain over policy), (4) competitive offers (potentially many competitive majority opinions), and (5) absolute majority in joins (a majority of the court must endorse the rule in the majority opinion if it is to have precedential power). We show that the median judge is pivotal over case dispositions, although she (and others) may not vote sincerely. Strategic voting becomes more likely as the location of the case becomes more extreme, resulting in majority coalitions that give the appearance of less polarization on the court, than is truly the case. The equilibrium policy depends on the composition of the dispositional majority, and generically does not coincide with the ideal policy of the median judge either in the dispositional majority or the bench as a whole. Rather, opinions are drawn toward a weighted center of the dispositional majority but often reflect the preferences of the opinion author.

Key Words: Bargaining, Judicial Politics, Super-majority Rules, Strategic Voting, Appellate Courts.

JEL Codes: C78, H8, K40

1 Introduction

'Procedures plus preferences determine outcomes.' This insight has guided the new institutionalism in political science across a score of research fields: legislatures (Krehbiel (1998), Cox and McCubbins (2007)), executives (Moe and Howell (1999), Canes-Wrone (2010)), bureaucracies (Gailmard and Patty (2007), Hirsch (2016)), political parties (Snyder and Ting (2002), Bawn et al. (2012)), electoral systems, and more.

More problematic has been the application of the key institutionalist insight to apex appellate courts like the U.S. Supreme Court. Part of the challenge has been addressing the fraught question: What do judges want? But even more difficult has been coming to grips with the unique procedures employed by these bodies. The crux of the difficulty is that high appellate courts undertake two tasks simultaneously, not one. The first is common to all courts, namely, conflict resolution — determining a definitive winner in a legal dispute between two parties. In this regard, multi-member appellate courts somewhat resemble juries. The second task, undertaken in the context of a specific legal dispute, is policy making – addressing a hitherto unresolved issue in the law by articulating a new rule or doctrine to be applied in this and future cases. In this role, high appellate courts somewhat resemble legislatures.

The unique procedures employed by apex appellate courts largely derive from the simultaneous and intertwined completion of the two tasks. A prominent example is the tripartite voting rule employed on the U.S. Supreme Court. Here, each justice casts a vote of dissent, join, or concur (Wahlbeck, Spriggs and Maltzman (1999)). The first category indicates that the justice disagrees with the majorityâs resolution of the conflict between the two parties in the litigation and (consequently and necessarily) disagrees with the new rule devised by the majority to resolve the dispute. The second two distinctions both indicate agreement with the majority's resolution. But they indicate differences with respect to the new rule articulated in the court's majority opinion. A 'join' endorses the new rule so it indicates "I vote for the majority's resolution and also endorse the majority's new rule." A 'concur' (in starkest form) indicates agreement with the majority's resolution of the litigants' dispute, but withholds endorsement of the majority's new rule. So it indicates, "I vote for the majorityâs resolution but do not endorse the majorityâs new rule."[1]. On the U.S. Supreme Court, if a rule is to have precedential power it must attract five joins.

Needless to say, this tripartite voting rule is not used in any legislature since legislatures

[1]Some observers of the U.S. Supreme Court distinguish between "regular" concurrences and "special" concurrences. The former are effectively joins but offer minor cavils. The latter indicate genuine policy disagreement with the majority opinion. See Segal and Spaeth (2002).

do not resolve conflicts in legal disputes between two contending parties. Nor is it used by any jury since juries do not engage in policy creation. So, what are the properties of this distinctive voting rule? More generally, if procedures plus preferences yield outcomes, what are the implications of the basket of distinctive procedures employed on high appellate courts like the U.S. Supreme Court and state high courts? This question has sparked enormous empirical literatures investigating many of the Supreme Court's procedures, for example, case selection, opinion assignment, unstructured back-and-forth haggling over opinion content, the order of voting in conference, and more.

Not surprisingly, formal models of high courts have struggled to accommodate the peculiar procedures employed by these jury-legislature hybrids. Early models simply discarded the jury function of high courts, treating appellate courts as 'little legislatures' (Hammond, Bonneau and Sheehan (2005), Jacobi (2009)). Some models invoked the median voter theorem despite the absence of Condorcet-compatible procedures. Others, responding to the wide-spread observation that majority opinions often reflect the policy views of the opinion author, invoked the Romer and Rosenthal (1978) monopoly agenda setter model, as if, counter-factually, majority opinions were offered under a closed rule (Lax and Cameron (2007)). But these models did not explain the source or limits of the opinion author's monopoly agenda power; after all, any justice in the dispositional majority is free to offer a competing opinion and sometimes do. Another group of models discarded the legislative part of the hybrid, instead treating appellate courts as 'big juries' (big in the sense of singularly important) (Fischman (2011), Iaryczower and Shum (2012)). Despite the extremely interesting insights that follow from this approach, it jettisons much of what makes high appellate courts notable.

Fortunately, recent papers have made major strides in modeling, rather than ignoring, the distinctive features of apex appellate courts. Within the emerging procedural realism paradigm, Carrubba et al. (2012) stand outs as particularly innovative. The model in this paper breaks the tri-partite rule in twain, treating it as two sequential votes, the first on case disposition, the second on policy. It implicitly adopts an important constraint on policy, disposition consistency. In words, the announced rule must yield the Court's chosen case resolution when applied to the instant case. And, the model introduces what can be called *restricted bargaining entrée*: only members of the dispositional majority are allowed to engage in bargaining over the Court's soon-to-be-announced new rule. The model shows that bargaining entrée implies that court policy is often far from the median judge on the whole Court. Thus, the model reunites the two decisional spheres and demonstrates that they interact in a dramatic way. Consequently, ignoring either one is seriously misleading.

The theoretical results carry potentially revolutionary implications for the empirical study of high courts. For example, the particular model in Carrubba et al. (2012) identifies the median justice of the dispositional majority as decisive in controlling the content of the majority opinion and hence Supreme Court policy. This insight is itself revelatory; in addition, it implies that the same Court will produce distributions of policies depending on the exact make-up of the majority dispositional coalition. Thus, a 4-5 conservative case disposition will yield a policy quite different from that following a 5-4 liberal case disposition — a result impossible to derive in a little legislature or big jury model but one well in accord with common observation.

In this paper, we significantly extend the procedural realism approach to appellate courts by incorporating additional and arguably critical features of high court procedure. Following Carrubba et al. (2012), we treat the tripartite voting rule as two sequential votes, with entrée to policy bargaining conditional on aligning with the winning side in the first, dispositional, vote. We depart, however, by considering in depth strategic voting at the first stage. This is important because under restricted bargaining entrée voters face a strong incentive to be in the dispositional majority in order to influence subsequent policy. We show that the median of the entire court remains decisive for the dispositional vote, but the median's vote may not be sincere. We further identify the judges who are most and least inclined to engage in strategic dispositional voting, and the circumstances when they will be tempted to do so.

Second, we explicitly model the relatively unstructured policy bargaining within the dispositional majority. We do so by employing Banks-Duggan/Baron-Ferejohn sequential bargaining, a strong analytical tool for such situations (Baron and Ferejohn (1989),Banks and Duggan (2000)). Critically, we incorporate the absolute-majority-in-joins (AMJ) rule employed on the U.S. Supreme Court: five joins to the majority opinion are necessary if the opinion is to have precedential value. The AMJ rule means that the effective decisional threshold (voting quota) in the dispositional majority varies dramatically depending on the size of the dispositional majority, ranging from simple majority rule (when the disposition coalition is the whole Court) to unanimity (when the dispositional majority is a bare majority of the Court).

We show that bargaining under the AMJ rule has strong implications for policy outcomes. In particular, author influence re-emerges, but conditionally. More specifically, when the intensity of bargaining is low (as parameterized by the standard discount factor in sequential bargaining models) the designated majority opinion author has wide latitude to choose policy and opts for his policy ideal point. However, as the intensity of bargaining increases, left and

right ideological blocks form endogenously within the dispositional majority. The Court's policy is then driven to the effective center of the dispositional majority coalition, as if the two blocks were bargaining with each other. Generically, that central position does not correspond to the ideal point of the median justice in the dispositional majority but rather to the Nash Bargaining Solution between the two ideological blocks. Given measures of ideal points and bargaining intensity (e.g., case importance) that point is easy to calculate.

The model studied here, while closely tailored to the U.S. Supreme Court, has broader applicability. First, and most obviously, it applies to other appellate courts that use similar procedures, e.g., the U.S. Courts of Appeal and most state high courts. Second, it also applies to other decision making bodies that simultaneously produce case decisions and rules governing case decisions. Notable here are many independent regulatory commissions which (perhaps unsurprisingly) have adopted procedures similar to the U.S. Supreme Court in order to engage in the joint production of case dispositions and rules governing case dispositions. Third, albeit more distally, it has applicability to other settings with sequential decision-making and restricted bargaining entrée, that is, access to procedural advantages conditional on an initial vote. An example is the organization of legislatures (especially parliaments) at the beginning of terms. There, a leader is selected by pure majority vote (in Congress), or members must decide whether to join the government or sit on the cross-benches (in a parliamentary setting). Then members of the winning majority receive access to procedurally valuable resources like the control of committees, or participation in government. In broad terms, the incentives for strategic voting analyzed here will reoccur in settings like these.

The paper is organized in the following way. Section II presents the model. Section III examines policy bargaining within the dispositional majority. Section IV analyzes dispositional voting. Section V considers some extensions, and Section VI concludes. All proofs are presented in the Appendix.

2 The Model

2.1 Cases, Dispositions and Rules

There is a court consisting of n judges (where n is odd) that must decide a case. A *case* z encodes the details of an event that has occurred, for example, the level of care exercised by a manufacturer or the intrusiveness of a search by the police. Let $Z = [0, 1]$ be the case

space. A judicial *disposition* $d \in \{0,1\}$ of the case determines which party prevails in the dispute between the litigants.

Judges dispose of cases by applying a *legal rule*. A legal rule $r : Z \to \{0,1\}$ maps the set of possible cases into dispositions; it partitions the case space into cases that will be decided for the plaintiff, and cases that will be decided for the defendant. Let $X = 2^Z$ be the space of possible rules. We focus on an important class of legal rules, *cutpoint*-based doctrines, which take the form:

$$r(z;y) = \begin{cases} 1 & \text{if } z > y \\ 0 & \text{if } z < y \end{cases}$$

where y denotes the cutpoint. For example, in the context of negligence, the defendant is not liable if she exercised at least as much care as the cutpoint y.[2] Let X^C be the space of cut-point rules. We have $X^C = \{\{[0,y), [y,1]\} \mid y \in [0,1]\}$. It should be clear that rules live in an entirely different space to cases. The special structure of cut-point rules allows us to summarize them in terms of a threshold in case-space.

2.2 Decision Making by the Court

Decision-making by the justices occurs in two distinct stages. In the first stage, each judge casts a dispositional vote ($d^j \in \{0,1\}$), and the disposition of the case is determined by simple majority rule. The dispositional votes of each judge separate the judges into dispositional majority (denoted $M \subset \{1,..,n\}$) and minority coalitions. By construction, $|M| \geq \frac{n+1}{2}$.

In the second stage, the justices in the dispositional majority must agree upon a legal rule y that rationalizes the chosen disposition. Consistency requires that $y \leq z$ if $d = 1$ and $y \geq z$ otherwise.[3]

The judges in the dispositional majority bargain over the legal rule to be implemented. We formalize this by studying a bargaining framework á la Baron and Ferejohn (1989) and Banks and Duggan (2000). Initially, a judge from the dispositional majority is recognized to propose a policy y that is consistent with the majority's disposition. Upon seeing the proposal, each

[2]Other examples include allowable state restrictions on the provision of abortion services by medical set providers; state due process requirements for death sentences in capital crimes; the degree of procedural irregularities allowable during elections; the required degree of compactness in state electoral districts; and the allowable degree of intrusiveness of police searches. Many other examples of cutpoint rules may suggest themselves to the reader.

[3]For technical reasons, we require the weak inequality in both cases. We could make one of the inequalities strict by discretizing the policy space.

judge in the dispositional majority either votes to endorse the proposed opinion by 'joining' or declines to endorse the opinion by 'concurring'. To become the policy of the court, the proposal requires the assent of a majority *of the entire court*, not just the dispositional majority. Thus, in many cases, the dispositional majority will bargain under an effective super-majority rule.[4] If the proposal is accepted, it is implemented and the bargaining game ends. Else, the judges retire, and the process repeats itself in the following period, and this continues until a policy of the court emerges. Delay within the bargaining game is costly, and the judges share a common discount factor $\delta \in [0, 1)$.

In the first period of bargaining, we allow the identity of the proposing judge to be non-random, reflecting the current practice where the most senior judge in the dispositional majority determines who will write the opinion. However, in subsequent bargaining periods, we assume judges are randomly recognized with uniform probability, reflecting the equal right of every justice to counter-propose policies.[5]

2.3 Judicial Preferences

Following Carrubba et al. (2012) and Cameron and Kornhauser (2008), we assume that judges' preferences exhibit both *expressive* and *policy* components. Policy utility depends on the actual policy implemented by the dispositional majority, and stems from the judge's concern for how future cases will be decided. Expressive utility depends neither on the policy chosen, nor on the actual disposition of the case, but rather, on the judge's individual vote in the instant case.[6] Whereas policy preferences are consequentialist – they depend on actual outcomes – expressive preferences simply reflect the judge's desire to be seen to decide cases 'correctly', regardless of if or how their vote changes actual outcomes. As will become clear, absent an expressive component of utility, judges would never have an incentive to dissent. Rather than taking an *ad hoc* approach to specifying these preferences, we present a framework that makes sense of both components in a cohesive way. We begin by specifying the dispositional preferences of a given judge, and build both expressive and policy preferences from this.

[4]Intuitively, no judge in the dispositional minority will support the proposal, since doing so would require them to support a policy that is inconsistent with their dispositional vote.

[5]None of the results in the policy-making stage (Section 3) turn on the assumption of uniform recognition probabilities, and, as we show in Section 5, our analysis of policy-making can easily accommodate a more general recognition rule. However, the uniformity assumption does have implications for decision-making at the dispositional voting stage (Section 4). We discuss the implications of non-uniform recognition probabilities in section 5.

[6]Cameron and Kornhauser (2008) treats the utility of casting join vs concur votes as expressive; in contrast, here the value of such votes comes from the policy resulting from votes.

Suppose judge j has ideal threshold x^j, and that $0 \leq x^1 \leq .... \leq x^n \leq 1$, so that the judges are ordered by their ideal threshold. Judge j's *dispositional utility* is:

$$u_D(d^j; z, x^j) = \begin{cases} 0 & \text{if } d^j = r(z; x^j) \\ l(z - x^j) & \text{if } d^j \neq r(z; x^j) \end{cases}$$

where $l(\cdot)$ is a quasi-concave 'loss' function that satisfies $l(0) = 0$ and $l(\cdot) < 0$ otherwise (i.e. l has a single peak at 0). There is a cost to judges when the disposition is different to their ideal. The (strict) quasi-concavity of l implies that dispositional preferences satisfy the *increasing differences in dispositional values (IDID)* property (see Cameron, Kornhauser and Parameswaran (2019)), which entails that the cost of making 'incorrect decisions' becomes larger the further is the case from the threshold x^j. Intuitively, judges feel more strongly about 'incorrectly' deciding 'clear-cut' cases (those far from the threshold), than 'contestable' ones (those close to the boundary that separates acceptable and unacceptable conduct).[7]

The expressive component of a judge's utility is simply the dispositional utility associated with the outcome for which she votes. To construct policy utility, we must assess the implications for future decision-making of a given rule y. Suppose a case arises in the future and must be decided according to the chosen decision rule. The judge's *policy* utility is her expected per-period dispositional utility from having the rule implemented, given the distribution over cases that are likely to arise. Recall, $r(z, y)$ is the disposition that results from applying rule y to case z. We have:

$$u_P(y; x^j) = \int u_D(r(z, y); z, x^j) dF(z)$$

where cases are drawn from a continuous distribution $F(z)$ that admits a density $f(z)$.

The *IDID* property implies that policy utility $u_P(y; x)$ is strictly quasi-concave in y for every x, although it is not necessarily concave. Moreover, the *IDID* property implies that, whenever $x^i > x^j$, $\frac{\partial u_P(y;x^i)}{\partial y} > \frac{\partial u_P(y;x^j)}{\partial y}$, or equivalently, $\frac{\partial^2 u_P(y,x)}{\partial x \partial y} > 0$.[8] Hence, preferences exhibit the single-crossing property; the benefit from marginally increasing the policy y is monotone in the judges' ideal policies.

Example 1. Suppose cases are uniformly distributed on $[0, 1]$. In Table 1, we provide a

[7] Such preferences are commonly used in the judicial politics literature. For example, see Baker and Mezzetti (2012), Chen and Eraslan (2018), amongst others.

[8] To see this, note that, for any policy y and any ideal policy x, $\frac{\partial u_P(y;x)}{\partial y} = (-1)^{\mathbb{1}[y<x]} l(y - x)$. Then, $\frac{\partial^2 u_P(y,x)}{\partial x \partial y} = (-1)^{\mathbb{1}[y>x]} l'(y - x) > 0$, since $l'(z) > 0$ if $z < 0$ and $l'(z) < 0$ otherwise.

mapping between the dispositional loss function l and commonly used policy utility functions, including absolute value (tent-shaped) utility, quadratic utility, and bell-curve shaped (Gaussian density) utility. Bell-curve shaped policy utility will be shown to have some nice properties that we make use of in later examples. □

Dispositional Utility	Policy Utility
$l(z - x_i) = -1$	$u_i^P(y) = -\lvert y - x_i \rvert$
$l(z - x_i) = -\lvert z - x_i \rvert$	$u_i^P(y) = -\frac{1}{2}(y - x_i)^2$
$l(z - x_i) = -\lvert z - x_i \rvert e^{-\frac{1}{2}(z-x_i)^2}$	$u_i^P(y) = e^{-\frac{1}{2}(y-x_i)^2} - 1$

Table 1: Relationship between Dispositional and Policy Utility. The table shows the dispositional utility specification that induce commonly used policy utility functions including: (i) absolute value (tent-shaped) utility, (ii) quadratic utility, and (iii) bell-curve shaped (Gaussian density) utility.

During the bargaining game, the disagreement payoff to each judge is $u_P(D; x)$. We make the standard assumption that disagreement is worse for each judge than agreeing to any feasible policy (i.e. $u_P(D, x) \leq u_P(y, x)$ for all $y \in [0, 1]$).

Overall utility is the sum of policy and expressive components:

$$u_P(y; x^j) + \alpha u_D(d^j; z, x^j)$$

where $\alpha > 0$ denotes the relative importance of the expressive component of utility. Notice that policy utility depends on the actual chosen policy y, whereas expressive utility depends only upon the judge's dispositional vote.

Our formulation of judicial preferences can be further motivated in the following way: Consider a dynamic model in which the court confronts a single case in each future period, and suppose judges discount the future at rate $\rho \in (0, 1)$.[9] Take a given case z, and a rule y that decides the current and all future cases, each assumed to be an independent draw from distribution $F(z)$. Then, the expected lifetime utility of a judge having purely consequentialist preferences would be:

$$u_D(r(z; y); z, x^j) + \frac{\rho}{1 - \rho} u_P(y; x^j)$$

Setting $\alpha = \frac{1-\rho}{\rho}$, this expression almost exactly coincides with our formulation of judicial utility. (Our formulation differs only in that current period utility depends on the judge's

[9]To clarify, the discount factor δ captures the cost of delay in the bargaining phase of the court's deliberations in a single case. The discount factor ρ is reflects the judges' present bias, and the passage of time *between* cases.

dispositional vote, and not the actual disposition of the case.) Moreover, under this approach, α has a natural interpretation as the importance to utility of the current case relative to the future stream. As $\alpha \to 0$, the court becomes perfectly future (and thus, policy) oriented, whereas as $\alpha \to \infty$, the court ignores the future entirely, and thus only cares about the disposition of the current case.

We should note the role of the 'legal status quo' within the bargaining game, as this point has engendered some controversy among judicial scholars. We take the view that, although there is a prior legal policy, this policy effectively reverts to a null policy when the Court takes the case – policy is in limbo until the Court resolves the case. Indeed, our bargaining protocol requires that bargaining continue until a majority policy is agreed to. The only way for policy to revert to the status quo *ante* is for the Court to re-enact it anew in the majority opinion. In Section 5, we consider the alternative framework in which, if bargaining fails, policy reverts to the status quo *ante*. We show that our results continue to hold under this alternative formulation, and so the question of the 'legal status quo' is not crucial to our analysis.

Additionally, we note that our formulation implicitly assumes that the court can commit to implementing its chosen policy when deciding future cases; i.e. the announced policy is time-consistent and renegotiation-proof. In a recent paper (see Cameron, Kornhauser and Parameswaran (2019)), we showed that the *IDID* property was sufficient to sustain policy commitment in equilibrium, provided that judges were sufficiently concerned about the future. Rasmusen (1994) provides a similar analysis, although a different mechanism enforces commitment in his model. Beyond these, we know of no other models of collegial courts that address the problem of commitment. Recent legislative models of sequential policy making with evolving status quos determined by earlier rounds of policy-making are suggestive (Baron (1996), Kalandrakis (2010)) but we do not pursue this point any farther in this paper.

2.4 Strategies and Equilibrium

We analyze equilibrium in the policy-making stage and the dispositional-voting (adjudication) stage, in turn.

Given the repeated game structure of bargaining in the policy-making stage, strategies can be quite complex as they may be history dependent. We restrict attention to stationary strate-

gies, which require that players choose equivalent strategies in every structurally equivalent sub-game.

A strategy for judge j (in the dispositional majority) in the policy-making stage is a pair (y^j, A^j), where:

- $y^j(z, M, \delta)$ denotes the policy proposed by the judge, whenever she is recognized to make a proposal, given the case and the composition of the dispositional majority $M \subset \{1, .., n\}$.
- $A^j(z, M, \delta)$ denotes the set of proposals that the judge will accept, whenever she is in the dispositional majority.

The equilibrium concept is stationary sub-game perfection with weakly undominated strategies. Weak undominance requires that each judge vote for her more preferred option (regardless of whether her vote would sway the outcome or not). This rules out equilibria in which judges vote for less favored outcomes, sustained by the belief that their vote will be inconsequential to the dispositional outcome.

A strategy for judge j at the adjudication stage is a dispositional choice $d^j(z; \alpha, \delta) \in \{0, 1\}$ given a case z, anticipating the equilibrium rule that will be chosen in the policy-making stage. An adjudication (Nash) equilibrium is a pair (d, M) denoting the majority disposition and the composition of the dispositional majority, having the property that no judge could do better by switching her vote.

3 The Policy Stage

In this section we characterize behavior in the policy-making stage for a generic dispositional majority. In section 4, we find the optimal dispositional coalition, given the policy bargaining that is anticipated to follow.

We begin by characterizing equilibrium proposals when $\delta < 1$. As we will see, there will be a range of policies proposed in equilibrium, reflecting the agenda-setting prerogative of the opinion author. Accordingly, we distinguish our approach from median-voter-type models that predict a single equilibrium policy. We subsequently reconcile our approach with those appealing to median voter logic by taking the limit as the agenda-setter's power goes to zero. We show that, even in this scenario, the equilibrium policy will not generically coincide with the median judge's ideal.

3.1 Equilibrium Characterization

Let z be the case, and suppose the dispositional majority coalition $M \subseteq \{1, ..., n\}$ contains $m \in \{k, ..., n\}$ members, where $k = \frac{n+1}{2}$. Without confusion, we re-label the judges in the coalition, preserving the ordering of ideal policies, so that $M = \{1, ..., m\}$ with $x^1 \leq .... \leq x^m$. (Once the majority coalition has been determined, the preferences of the non-majority judges become inconsequential to policy-making, so we are free to disregard them, and focus on the m remaining judges.) Similarly, we are now free to focus solely on policy utility, since dispositional utility was determined at the time of the dispositional vote.

Recall that the policy must be consistent with the disposition of the court. If the majority disposition was 1, the majority must choose a policy in the interval $[0, z]$, whilst if the disposition was 0, it must choose a policy in the interval $[z, 1]$. Generically, the court's policy must be contained in $[\underline{x}, \overline{x}]$, where $\underline{x} \in \{0, z\}$ and $\overline{x} \in \{z, 1\}$.

The bargaining framework in this model is analogous to those studied by Banks and Duggan (2000), Cardona and Ponsati (2011) and Parameswaran and Murray (2018), although there are some important differences. Since those papers provide detailed expositions of the equilibrium characterization, we defer to them, and instead briefly provide an intuitive account of the equilibrium. Detailed proofs can be found in the Appendix.

Before presenting the formal proposition, we make note of two important details. First, each judge bases her decision to support a proposal or not by comparing the policy utility from the current proposal to her (discounted) expected policy utility from entertaining counter-proposals. The set of equilibrium counter-proposals, thus, establishes the opportunity cost of accepting a given proposal, which in turn establishes the set of proposals acceptable to each judge. Since each proposer seeks to build a winning coalition around their proposal, the anticipation of future counter-proposals disciplines each judge's decision about which policy to propose when they are recognized.

Second, because policy preferences satisfy the single-crossing property, in equilibrium, the policy coalitions that support and reject any proposal will both be connected.[10] We stress that this is an equilibrium phenomenon; the decision rule does not require that the 'join' and 'concur' coalitions be connected, but optimal behavior, nevertheless, ensures that they will be. Since the proposer only needs the support of $k = \frac{n+1}{2}$ judges, it suffices to either earn the

[10] To see this, note that, by the single-crossing property, for any two policies, y, y' with $y' > y$, if player i prefers y' to y, then every player j with $x^j > x^i$ will also prefer y' to y. Similarly, if judge i prefers y to y', then so will all judges j with $x^j < x^i$. The result follows immediately from this.

support of the left-most k judges $\{1, ..., k\}$ in the dispositional majority, or the right-most k judges $\{m-k+1, ..., m\}$, where judge $m-k+1$ is the k^{th} judge from the right. It follows that judges $\{m-k+1, ..., k\}$ must be in every equilibrium coalition. Moreover, judges $m-k+1$ and k are *decisive* in the sense that a proposal that loses their support cannot be winning. Following Compte and Jehiel (2010), we refer to these as the *left* and *right decisive judges*, respectively. If $m = n$, so that the join coalition need only be a simple majority of the dispositional coalition, then the left and right decisive judges will both coincide with the median judge. By contrast, for any $m < n$, $m-k+1 < k$, and so, generically, the decisive judges will be non-median players, with distinct preferences.

For notational convenience, we index the left and right decisive judges by l and r, so that $l = m-k+1$ and $r = k$. We have the following result, which is similar (though not identical) to results previously noted by Cho and Duggan (2003), Cardona and Ponsati (2011), Parameswaran and Murray (2018), amongst others:

Proposition 1. *For $\delta < 1$, the bargaining game admits a unique equilibrium. The equilibrium is in no-delay, and is characterized by a pair $\left(\underline{y}, \overline{y}\right)$, with $\underline{x} \leq \underline{y} < \overline{y} \leq \overline{x}$, such that:*

1. *When judge j is recognized, she will propose:* $y^j = \begin{cases} \underline{y} & x^j < \underline{y} \\ x^j & x^j \in \left[\underline{y}, \overline{y}\right] \\ \overline{y} & x^j > \overline{y} \end{cases}$
2. *The pair $\left(\underline{y}, \overline{y}\right)$ satisfies:*
   - $\underline{y} = \min\{y \geq \underline{x} | u_P(y; x^r) \geq (1-\delta) u_P(D, x^r) + \frac{\delta}{m} \sum_j u_P(y^j, x^r)\}$
   - $\overline{y} = \max\{y \leq \overline{x} | u_P(y; x^l) \geq (1-\delta) u_P(D, x^l) + \frac{\delta}{m} \sum_j u_P(y^j, x^l)\}$

Proposition 1 shows that our bargaining game admits a unique equilibrium which is characterized by an interval $[\underline{y}, \overline{y}]$ of 'socially acceptable' policies (i.e. which will receive the support of at least k agents). Naturally, in equilibrium, each judge will propose the socially acceptable policy closest to her ideal. Judges with 'moderate' preferences (those whose ideal policies lie within the interval) will be able to successfully implement their ideal rule in equilibrium, whilst judges with 'extreme' preferences must offer a compromise rule. All 'extreme left' judges will pool on the same proposal $\underline{y}$, whilst all 'extreme right' judges will pool on the same proposal $\overline{y}$. What constitutes 'moderate' and 'extreme' is itself determined in equilibrium, and depends on the discount factor δ, and the preferences of the left and right decisive

judges. $\overline{y}$ is the highest policy that the left decisive judge is willing to accept, given her continuation payoff. Similarly, $\underline{y}$ is the lowest policy that the right decisive judge is willing to accept, given her continuation utility. Any proposal in the region $\left[\underline{y}, \overline{y}\right]$ is equilibrium consistent.

To make intuitive sense of Proposition 1, let $E[y] = \sum_j p_j y^j$ be the expected policy that will be proposed (and accepted) in the continuation game. As we show in the Appendix, if $E[y]$ is proposed, it will receive unanimous support.[11] Take a judge j in the dispositional majority whose ideal policy lies below $E[y]$. Since delay is costly ($\delta < 1$), judge j can offer a policy slightly below $E[y]$ and still retain unanimous support. Decreasing the offer further, she will eventually lose the support of the right-most judge in the dispositional majority, then the second-most-right judge, and so on. Since it suffices to have the support of the right decisive judge, judge j will continue decreasing the offer until either she reaches her ideal policy, or the support of the right decisive judge would be lost. Hence, the lowest acceptable policy is pinned down by the preferences of the right decisive judge. A similar argument shows that the highest acceptable policy is determined by the left decisive judge.

Example 2. Consider a case $z = 0.45$, and suppose the disposition of the court is $d = 1$. To be consistent, the rule must satisfy $y \leq 0.45$. Suppose there are 6 judges in the majority (out of 9), with ideal policies $x^1 = 0$, $x^2 = 0.2$, $x^3 = 0.25$, $x^4 = 0.3$, $x^5 = 0.4$ and $x^6 = 0.6$. Judges 1,..,5 (and presumably the 3 dissenting judges) cast sincere dispositional votes, whilst judge 6 voted strategically. Suppose policy-making requires a majority of the entire bench, and so $k = 5$. The left and right decisive judges, then, are judges 2 and 5, respectively. Policy utility is given by $u_P(y, x) = -|y - x|$ and the common disagreement payoff is -1. Figure 1 depicts the set of socially acceptable policies for two values of δ.

In the first scenario ($\delta = \frac{21}{26}$), judges 2,3,4 and 5 are able to propose their ideal policies in equilibrium, whilst judge 1 must propose a compromise policy, which is the lowest policy acceptable to the right decisive judge (judge 5). It is infeasible for judge 6 to propose her ideal policy, and in equilibrium, she will propose the highest policy that is feasible $y = 0.45$. In fact, the left decisive judge would in principle be willing to accept policies up to $y \approx 0.5$, however any policy above $y = 0.45$ would be inconsistent with the disposition of the case, and thus infeasible. In the second scenario ($\delta = 0.95$), judges 3 and 4 are able to propose their ideal policies in equilibrium, whilst the remaining judges must propose compromise policies. □

[11] In typical bargaining games, this result follows from the concavity of players' preferences. In our model, preferences are not concave. However, the *IDID* property causes policy preferences to exhibit some concave-like features.

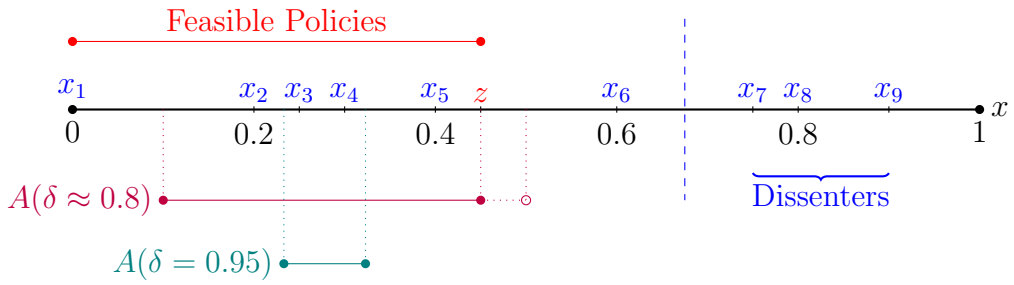


Figure 1: $A(\delta)$ represents the social acceptance set given a cost of delay δ. When $\delta = \frac{21}{26} \approx 0.8$, $A = [0.1, 0.45]$, whilst if $\delta = 0.95$, then $A = [0.2334, 0.3234]$. Notice that when $\delta \approx 0.8$, the consistency constraint (i.e $y \leq z$) is binding. The dotted extension to $A(\delta \approx 0.8)$ represents the additional policies that would be socially acceptable absent the consistency constraint.

The above example demonstrates the essential features of the equilibrium. There are a range of policies that are potentially proposed in equilibrium. 'Moderate' judges may propose their ideal policies, whereas 'extreme' judges (and, in particular, judges who vote insincerely) must propose compromise rules. Whether a judge is 'moderate' or 'extreme' depends on the preferences (and, in particular, the degree of patience) of the left and right decisive judges. Moreover, the social acceptance set may be constrained by the facts of the case itself; the consistency requirement may be binding.

3.2 Comparative Statics on δ

The discount rate δ parameterizes the cost of delay in bargaining, or (equivalently) the relative 'importance' of the legal issue. As Example 2 demonstrates, it is also a measure of the degree of agenda control that the proposer exerts. When $\delta = 0$, delay is exceedingly costly relative to the importance to each judge of implementing their ideal policy, that the non-proposing judges will accept any feasible policy. The proposer thus has complete control over the agenda and will propose the feasible policy closest to her ideal. As the following lemma shows, as $\delta \to 1$, the reverse becomes true; delay becomes costless relative to the importance of deciding the legal question correctly. The judges will bargain 'aggressively' over policy, such that, in equilibrium, the proposer loses control of the agenda entirely, and all judges will makes the same proposal.

Lemma 1. *In any equilibrium,* $\overline{y}(\delta) > \underline{y}(\delta)$ *whenever* $\delta < 1$. *Moreover,* $\lim_{\delta \to 1} \left(\overline{y}(\delta) - \underline{y}(\delta)\right) = 0$.

Lemma 1 states the familiar result that when delay is costly ($\delta < 1$), the proposer can pull the chosen policy somewhat towards her ideal. This results in equilibrium proposals that are dispersed around a mean. The lemma additionally shows that as $\delta \to 1$, the proposer's advantage disappears, and the best the proposer can do is to announce the expected policy. (See Predtetchinski (2011) and Parameswaran and Murray (2018).) The intuition is that, as $\delta \to 1$, delay becomes less costly, and so the decisive judges can both be more insistent that the equilibrium policy not be too far from the expected continuation policy. This causes the acceptance set to narrow. In the limit as delay becomes costless, the acceptance set collapses to a singleton, and so all judges make identical proposals. Thus, δ parameterizes the proposer's degree of agenda-control, and captures the extent to which policy outcomes depend on the particular whims of the judge chosen to author an opinion.

Taken together, Proposition 1 and Lemma 1 make strong predictions about the size and composition of the 'join' and 'concur' coalitions. When delta is low (as $\delta \to 0$), the cost of entertaining counter-proposals is sufficiently high that all judges will support the opinion of the court. The 'join' coalition will consist of all judges in the dispositional majority, and no judge will separately write a concurring opinion. By contrast, when delta is high (as $\delta \to 1$), judges become more demanding about the set of opinions which they will join. The size of the 'join' coalition will fall to a bare majority, consisting of either the left-most *or* right-most k judges. In either case, the 'concur' coalition will consist of judges from only one extreme (amongst those in the dispositional majority). Thus, regardless of the size of δ, an 'ends-against-the-middle' dynamic should never arise in which the 'join' coalition consists of relatively moderate judges, and extremists from both ends concur.

3.3 Limit Equilibria & 'Median Voter' Logic

As Proposition 1 makes clear, equilibrium policy-making by the Court is (generically) characterized by a menu of proposer-dependent policies. This feature arises from the fact of bargaining between the judges over policies, and requires that it is costly for judges to make (or entertain) counter-proposals. Our approach, thus, stands in contrast to many existing studies that predict a unique policy outcome, typically by appealing to median-voter logic. However, in the limit as $\delta \to 1$, equilibrium in our model is also characterized by a unique policy that is proposed by all judges. Taking the limit as counter-proposals become costless, thus, allows for fair comparisons between our model and those existing in the literature.

There is a tight connection between median-voter-type logic (or more generally, the equilibrium concept of the core) and the limit equilibria of our bargaining game. For example,

Cho and Duggan (2009) show that when agreement requires a simple majority of a committee, the limit equilibrium policy precisely coincides with the median committee member's ideal policy. The intuition is straight-forward: the logic of the median voter theorem is that whenever the proposed policy is other than the median voter's ideal, a majority coalition can be found that would replace it with something closer to the median voter's ideal. This is true in our bargaining game as well, except that, when delay is costly, some non-core policy might persist, simply because it is too costly to make the counter-proposal that replaces them. As delay become costless, this friction disappears, and so the outcome of bargaining should coincide with the median voter's ideal.

When agreement requires a super-majority, logic analogous to the median voter theorem predicts an equilibrium outcome in the core.[12] (Indeed, under simple majority rule (with an odd number of players), the core is uniquely the median voter's ideal policy; see Black (1948) and Downs (1957)). However, under super-majority rule, the core generically contains many policies. In fact, the core is precisely the interval of policies bounded by the ideal policies of the left and right decisive voters. This presents a problem since there are now a continuum of possible equilibrium policies. Parameswaran and Murray (2018) show that amongst this multiplicity, the limit equilibrium policy μ is focal – it is the one that is robust to introducing small costs to making counter-proposals. The bargaining limit can be thought of as a refinement that selects the most plausible core policy from amongst the multiplicity.[13]

Parameswaran and Murray (2018) provide an explicit characterization of this robust policy. Let $b_{i,i+1}$ be defined as follows:

$$b_{i,i+1} = \arg\max_{y} u_P(y, x^l)^i \cdot u_P(y, x^r)^{m-i}$$

To make sense of this policy, consider the following story: Suppose the judges in the dispositional majority separate into two connected factions $\{1, ..., i\}$ and $\{i+1, ..., m\}$ – which we dub the left and right factions. The factions contain i and $m-i$ judges, respectively. Suppose further that the factions behave as cohesive units and delegate decision making to the left and right decisive judges, respectively. (We can thus think of the left and right decisive judges as factional leaders.) To determine the equilibrium policy, the factional lead-

[12] The *core* is the set of policies for which there does not exist some other policy that is preferred to it by a winning coalition.

[13] To justify this refinement, they note the tight connection between the core and the bargaining protocol. When $\delta = 1$, there are a continuum of equilibria in the bargaining game, each associated with a particular policy within the core. However, for every $\delta < 1$, the bargaining game admits a unique equilibrium. Thus, our focus on the limit equilibrium is not *ad hoc*. Rather, we exploit the failure of lower-hemicontinuity of the bargaining equilibrium correspondence at $\delta = 1$. See Parameswaran and Murray (2018) for further details.

ers engage in asymmetric Nash Bargaining, with bargaining weights proportional to their faction's size. $b_{i,i+1}$ is the outcome that results from this procedure.[14]

Let μ denote the limit equilibrium policy. Parameswaran and Murray (2018) show that the limit equilibrium of the bargaining game is closely related to this asymmetric Nash Bargaining outcome. They show the following result:

Proposition 2. *Let* $i^* = \min\{i \,|x_i > b_{i,i+1}\}$. *Then:* $\mu = \min\{x_i^*, b_{i^*-1,i^*}\}$

Proposition 2 characterizes the common equilibrium policy that all judges will propose when the cost of delay becomes arbitrarily small. Depending on the arrangement of the judges' ideal policies, the limit proposal may either: (i) coincide with the ideal policy of one of the judges having ideal policies within the core, or (ii) be the solution to the asymmetric Nash Bargaining problem between the left and right decisive judges, with bargaining weights proportional to the size of each faction.

In strong contrast to existing results, our analysis shows that the equilibrium policy will generically not coincide with either the median judge on the bench[15] , nor the median judge in the dispositional majority. This should not be surprising. The logic of the median voter theorem is particular to decision making under simple majority rule. But, as we have argued, policy-making by the court often proceeds under an effective super-majority rule, and in such cases, there is no reason to privilege the median judge over the others.

Parameswaran and Murray (2018) describe the logic of this result in significant detail and we refer the interested reader to that paper. Here, we briefly provide intuition for the result. Conjecture that $\mu \in (x^{i-1}, x^i)$. Since $\underline{y} \to \mu$ and $\overline{y} \to \mu$, then for δ large enough, $x^{i-1} < \underline{y}(\delta) < \mu < \overline{y}(\delta) < x^i$. Hence, when δ is large, judges 1,...,$i-1$ will propose $\underline{y}(\delta)$ and judges $i, ..., m$ will propose $\overline{y}(\delta)$. It is as if the judges separate into cohesive factions, with all members of the same faction making the same equilibrium proposal. Furthermore, $\underline{y}(\delta)$ and $\overline{y}(\delta)$ were determined by the preferences of the right and left decisive judges, respectively. Hence, we can think of the left decisive judge as bargaining on behalf of the left faction, and the right decisive agent as bargaining on behalf of the right faction, with the bargaining weights being proportional to the size of their respective factions. Parameswaran

[14]Our notation emphasizes that the factions consist of judges $1, ..., i$ on the one hand, and judges $i+1, ..., m$. on the other.

[15]In fact, we establish in the following section that the robust policy will generically coincide with the ideal policy of the median judge *only* when the dispositional vote is unanimous.

and Murray (2018) show that, as $\delta \to 1$, the resulting policy coincides with the asymmetric Nash bargaining solution between the left and right decisive agents.[16]

Notice that the separation into factions was endogenous to the policy chosen in equilibrium. Hence, for the asymmetric Nash bargaining solution to indeed be equilibrium consistent, it must be that this solution lies in the interval (x^{i-1}, x^i) — so that players separate into the factions as conjectured. As the proof of Proposition 2 shows, there is a unique player i^* that determines the composition of factions, in equilibrium. There are two possibilities. For some arrangement of ideal policies, the factions $\{1, ..., i^* - 1\}$ and $\{i^*, ..., n\}$ induce a faction-size weighted Nash Bargaining solution that is equilibrium consistent — i.e. which falls in the required interval (x_{i^*-1}, x_{i^*}). Outside this range of parameters, the following problem arises: If i^* is conjectured to be in the left faction, then the location of the induced Nash Bargaining solution will cause i^* to want to defect to the right faction, and vice versa. Player i^* is *pivotal*. The only possibility is that the limit equilibrium coincides with i^*'s ideal policy, x_{i^*}.

The following example may help in building further intuition for the results in Proposition 2:

Example 3. Suppose $m = k = 5$, so that the dispositional majority is a bare majority of the Court. Then $l = 1$ and $r = 5$. Suppose policy preferences are bell-curve shaped: $u_P(y, x) = e^{-\frac{1}{2}(y-x)^2} - 1$, and let the disagreement payoff be $u_P(D, x) = -1$. Finally, normalize: $0 = x_1 \leq x_2 \leq ... \leq x_5 = 0.5$. The Nash bargaining solution when the left and right factions are $\{1\}$ and $\{2, 3, 4, 5\}$ is $b_{1,2} = 0.4$. Similarly, for the remaining factions, we

[16] Binmore, Rubinstein and Wolinsky (1986) establish a similar result in the Rubinstein two-person alternating offers bargaining setting.

have: $b_{2,3} = 0.3$, $b_{3,4} = 0.2$, and $b_{4,5} = 0.1$.[17] Notice that $b_{i,i+1}$ is decreasing in i. Then:

$$\mu = \begin{cases} b_{1,2} = 0.4 & x^2 > 0.4 \\ x^2 & 0.3 \leq x^2 \leq 0.4 \\ b_{2,3} = 0.3 & x^2 < 0.3 < x^3 \\ x^3 & 0.2 \leq x^3 \leq 0.3 \\ b_{3,4} = 0.2 & x^3 < 0.2 < x^4 \\ x^4 & 0.1 \leq x^4 \leq 0.2 \\ b_{4,5} = 0.1 & x^4 < 0.1 \end{cases}$$

Let us check the logic of the Proposition through the example. Suppose $x^3 = 0.15 < 0.25 = x^4$. Conjecture that $\mu \in (x^3, x^4)$. Then, per the logic in the previous paragraph, for δ large enough, it must be that judges 1 through 3 choose $\underline{y}(\delta)$ and judges 4 and 5 choose $\overline{y}(\delta)$. It follows that the equilibrium policy will be the asymmetric Nash Bargaining solution where the the left decisive voter has weight $\frac{3}{5}$ and the right decisive voter has weight $\frac{2}{5}$. This solution is $b_{3,4} = 0.2$, which indeed lies between $x^3 = 0.15$ and $x^4 = 0.25$, as conjectured.

By contrast, suppose that $x^4 = 0.18$, and conjecture again that $\mu \in (x^3, x^4)$. The same logic would lead us to conclude that $\mu = b_{3,4} = 0.2$. But this no longer satisfies the requirement that $x^3 < \mu < x^4$, since $x^4 = 0.18 < 0.2$. Our conjecture is not equilibrium consistent. We can similarly show that conjecturing $\mu \in (x^4, x^5)$ (or indeed, any other such interval) is not consistent. Under this arrangement of ideal policies, judge 4 is pivotal. Conjecturing that her ideal policy is below the limit expected policy causes us to conclude that it will, in fact, be above. Instead conjecturing that his ideal policy is above the limit expected policy causes us to conclude that it will be below. The only consistent alternative is that the limit policy coincides with x^4.

We note some features of the equilibrium mapping. First, for each judge between the decisive judges, there is some arrangement of ideal policies for which they are pivotal. For example, with 5-member majority coalition, the left and right decisive judges are judges 1 and 5, respectively. Then, it is possible that equilibrium policy reflects the ideal policies of any of judges 2, 3 and 4, as the cost of delay vanishes. In particular, the median judge in the

[17] The Nash bargaining solution is generically the weighted generalized mean of the ideal policies of the left and right decisive voters — there exists some function f such that, whatever the ideal policies of the decisive voters, the Nash bargaining solution satisfies: $f(b_{i,i+1}) = \frac{i}{m} f(x^l) + \frac{m-i}{m} f(x^r)$. In the special case of the preferences in this example, the Nash bargaining solution is simply the weighted arithmetic mean of the ideal policies. I.e. $f(x) = x$.

majority (judge 3) is not generically privileged. Additionally, there are arrangements of ideal policies under which no judge is pivotal, and the equilibrium policy is simply the solution to the asymmetric Nash Bargaining problem between the decisive judges. Note that, although the ideal policies of the remaining judges do not directly affect the equilibrium policy, they matter in so far as they determine the weights attributed to each faction.

Second, bargaining pushes the equilibrium policy towards the 'middle' of the core. In Example 3, the core is the interval $[0, 0.5]$. When the ideal policy of judge 3 (the median of the dispositional majority) is in the middle of this interval (i.e. $x^3 \in [0.2, 0.3]$), then the median of the majority is indeed pivotal. However, as the median's ideal policy becomes extreme, the equilibrium switches to some other less extreme policy. For example, if $x^3 > 0.3$, so that the median is further to the right, then equilibrium policy switches to a policy below the median's ideal. Initially it switches to $b_{2,3}$ —the policy that results from the judges dividing into factions $\{1, 2\}$ and $\{3, 4, 5\}$. However, this policy will cease to be equilibrium consistent if judge 2's ideal policy shifts too far to the right (i.e. if $x^2 > 0.3$). If so, then the equilibrium switches to judge 2's ideal policy, and if this too becomes extreme (i.e. if $x^2 > 0.4$), then the equilibrium shifts to the Nash bargaining solution associated with blocs $\{1\}$ and $\{2, .., 5\}$. Hence, bargaining exerts a moderating force that keeps the equilibrium closer to the middle of the core than would be the case under the median voter theorem.

In this paper, we do not take up the issue of nominations to the bench. However, in concluding this section, we briefly note the stark implications of Proposition 2 for the president's optimal nomination's choice. An important implication of the proposition is that equilibrium outcomes depend not only on the relative ordering of the ideal policies of the judges, but their absolute location in policy space. The president's nomination problem is, thus, not simply a 'move-the-median' game. The president could nominate two different judges, both occupying the same relative position in the ordering, but with different implications for the equilibrium policies chosen.

4 The Adjudication Stage

We now analyze optimal behavior in the adjudication stage, at which each judge must cast a dispositional vote.

4.1 First Round Assignment

In the first stage, each judge must cast a dispositional vote, taking into account the equilibrium policies that will result, given differently composed majority coalitions. This policy, in turn, depends on which judge is selected by the chief justice (or the most senior judge in the majority) to draft the initial proposal. For each majority coalition $M \subset \{1, ..., n\}$, let $s(M, d, z) \in \{1, .., n\}$ denote the judge who is selected to make the first proposal. We must have $s \in M$ — the selected judge must be in the majority coalition. Moreover, let $\gamma(M, d, z) = y^{s(M,d,z)}$ be the policy that the selected judge will propose in equilibrium.

The function s depends on the particular incentives faced by the chief (or most senior) judge. In a naive model, we might suppose that the chief is purely motivated to maximize her utility from the case. But this would imply that the chief judge always assigns the opinion to herself – an assumption at odds with the actual practice of recent chiefs. Indeed, the court has maintained a practice of trying to share the workload of opinion writing amongst its members. Such a policy might be rationalized by noting that opinion writing is costly, and so the chief makes her assignment choice taking into account the associated direct and opportunity costs. Other factors may also be at play. Given the many additional incentives that would need to be incorporated, it is clear that providing micro-foundations for the chief justice's selection is outside the scope of this paper.

Instead, we take a reduced form approach, taking the selection function s as given. We assume s satisfies the following:

Assumption 1. *Let $M, M' \subset \{1, ..., n\}$ be majority coalitions.*

1. *Suppose $j \notin M$. Then $s(M \cup \{j\}) \in \{s(M), j\}$.*
2. *Suppose for every $i \in M$, there exists $j \in M'$, such that $y^i(z, M, \delta) = y^j(z, M', \delta)$. Then $\gamma(M) = \gamma(M')$.*

Assumption 1 is in two parts. The first part is essentially a rationality condition that appeals to the independence of irrelevant alternatives (or Sen's condition α). It says that introducing a new member to the coalition shouldn't affect the chief's rankings amongst the previously available members. As we show (in the proof of Lemma 2), an implication of the first part of Assumption 1 is that, for any judge, joining the majority coalition cannot cause the equilibrium policy to become worse, *ceteris paribus*.

The second part states that, when confronted with two different coalitions that induce the same set of policy proposals, the chief should not make selections that cause different policies to be induced in the different instances. If replacing judge i in the coalition with judge j does not change the set of equilibrium proposals, then the chief should treat judges i and j as perfect substitutes for one another. Thus, the outcome induced when one is included in the coalition should be identical to the outcome when only the other is included.

Taken together, the two parts of Assumption 1 are intended to capture, in reduced form, structurally sound decision-making problem by the chief. (Of course, as $\delta \to 1$, the chief's selection becomes inconsequential, as all judges in a given coalition will propose the same policy.)

4.2 Optimal Dispositional Coalitions

Fix a case z. Let $M^0(z)$ and $M^1(z)$ denote the sets of judges who, if voting sincerely, would choose dispositions '0' and '1', respectively (i.e. $M^0(z) = \{i \,|\, z < x^i\}$ and $M^1(z) = \{i \,|\, z > x^i\}$). Let (d^*, M^*) denote the adjudication equilibrium, where $d^* \in \{0, 1\}$ denotes the disposition of the court, and M^* denotes the equilibrium majority coalition.

Lemma 2. *The majority coalition will include all judges who agree with the case disposition. Formally, if (d^*, M^*) is an adjudication (Nash) equilibrium, then $M^{d^*}(z) \subset M^*$.*

For a given equilibrium disposition d^*, Lemma 2 states that all judges who sincerely agree with the disposition of the case will be in the majority coalition. The intuition is straightforward: Being in the majority coalition is always beneficial on the policy-utility dimension in that it enables a judge to influence the equilibrium policy of the court, and pull the policy (weakly) closer to her ideal. When the judge agrees with the disposition, her expressive and policy motives are not in conflict. The benefit of voting to join the majority coalition comes at no cost to expressive utility. Thus, it is a (strictly) dominant strategy for all such judges to vote sincerely.

Judges who disagree with the disposition of the court face a more interesting trade-off. Voting strategically enables them to influence the equilibrium proposal, but incurs the expressively cost of voting insincerely. As we will see, the policy benefit of voting strategically (for each judge) depends on whether (and how many) other judges are also voting strategically. This gives rise to potentially multiple adjudication equilibria. To understand why, suppose the

disposition of the court is $d = 1$. Let M be a coalition consisting of at least k judges, with $M^1 \subset M$, and $\gamma(M)$ be the associated equilibrium policy proposal. Take two other judges, i and j, who, if they voted sincerely, would find themselves in the minority. It follows that $\gamma(M) \leq z < x^i \leq x^j$. We consider two scenarios that illustrate the sources of multiplicity.

First, suppose $\gamma(M) = \gamma(M \cup \{i\}) = \gamma(M \cup \{j\}) < \gamma(M \cup \{i,j\})$. In this scenario, adding one judge to coalition M has no effect on the equilibrium policy, whereas adding both judges does. If $\alpha > 0$, but is not too large, then the choices for judges i and j are strategic complements. Judges i and j face a coordination game; they either both want to vote sincerely, or both strategically. As long as the judges are coordinated, their choices are Nash equilibrium consistent.

Second, suppose $\gamma(M) < \gamma(M \cup \{i\}) = \gamma(M \cup \{j\}) = \gamma(M \cup \{i,j\})$. In this scenario, adding either judge to coalition M favorably affects the equilibrium policy offered during the policy stage. However, having added one judge, the marginal effect of the adding the second judge is zero. The judges choices are now strategic substitutes (for $\alpha > 0$ not too large). They are playing a game of chicken; each wants to vote strategically if and only if the other votes sincerely.

The following example illustrates both of these situations, and demonstrates the possibility of multiple Nash equilibria:

Example 4. Consider a case $z = 0.6$. Policy utility is bell-curve shaped: $u_P(y,x) = e^{-\frac{1}{2}(z-x)^2} - 1$, and the vector of ideal policies is: $(x^1, ..., x^9) = (0, 0.1, 0.3, 0.5, 0.5, 0.7, 0.8, 0.9, 1)$. The disagreement payoff is $u_P(D,x) = -1$, and $\delta = 1$ so that all judges make the same proposal.

Suppose the equilibrium disposition is $d^* = 1$. Judges 1-5 will always be in the majority. The equilibrium policies are $\gamma(M^1) = 0.3 = \gamma(M^1+1)$, $\gamma(M^1+2) \approx 0.414$, and $\gamma(M^1+3) = 0.5 = \gamma(M^1+4)$, where $\gamma(M^1+p)$ is the equilibrium offer when the majority coalition contains judges 1-5 (i.e. M^1) and any $p \in \{1,2,3,4\}$ of the remaining judges. (Judges in the sincere minority are perfect substitutes for one another, so it doesn't matter which of the judges join the dispositional majority; just how many.) The adjudication Nash equilibria (in which $d^* = 1$) are given Table 2:

□

As example 4 demonstrates there may be many Nash equilibria of the adjudication game. For ease of exposition, suppose α is very slightly positive, so that strategic voting is costly,

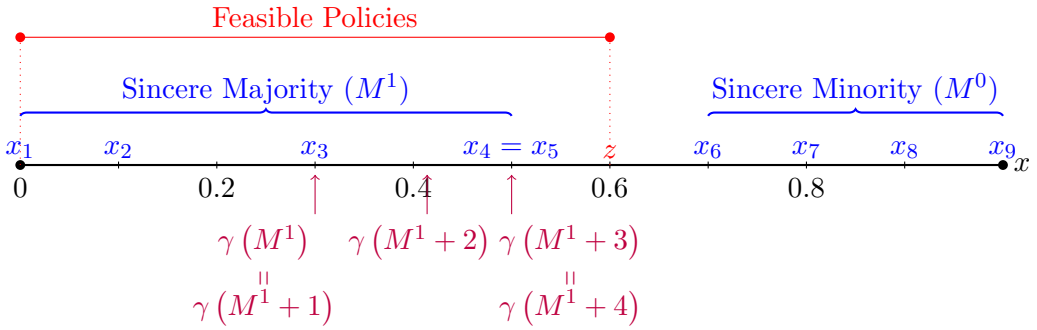


Figure 2: Equilibrium policies for differently sized dispositional coalitions. See Appendix for details.

Salience	$\alpha < 0.117$	$\alpha \in (0.11l7, 0.129)$	$\alpha \in (0.129, 0.273)$	$\alpha > 0.273$
# Equilibria	5	2	2	1
Sincere	M^1	M^1	M^1	M^1
Strategic	$M^1 \cup \{6,7,8\}$ $M^1 \cup \{6,7,9\}$ $M^1 \cup \{6,8,9\}$ $M^1 \cup \{7,8,9\}$	$M^1 \cup \{6,7,8\}$	$M^1 \cup \{6,7\}$	None

Table 2: Equilibrium coalitions that implement disposition $d^* = 1$. Equilibria with sincere voting (i.e. the dispositional coalition is M^1) always exist. For $\alpha < 0.273$, there are also equilibria in which judges in the sincere minority vote strategically. For $\alpha < 0.117$, there can be many such equilibria with strategic voting.

but this cost will be small relative to the policy gains whenever strategic voting causes the equilibrium policy to shift. Note that if all judges vote sincerely, then there is no benefit to any one of the judges in the minority in switching her vote and joining the majority; the equilibrium policy will be unchanged. Thus, sincere voting is an equilibrium (and this only becomes more true as *alpha* increases). However, if at least two judges voted strategically, then this would cause the resulting policy to shift, and for α sufficiently low, this joint deviation would be beneficial to each defecting judge. The fact that equilibria with strategic voting exist alongside the sincere equilibrium is a consequence of strategic complementarities.

Next, note that if three judges (in the sincere minority) vote strategically, there is no policy benefit to the fourth judge also joining the coalition. For α low enough, any dispositional majority containing judges 1-5 and three of the remaining four judges is equilibrium consistent. Here, the multiplicity is a consequence of strategic substitutes —equilibrium requires any three of the four judges is the sincere minority to vote strategically; it doesn't matter which ones.

Of course, as α increases, the cost of voting strategically increases, and so the possibility of sustaining various equilibria with strategic voting decreases. By the single-crossing property, the expressive cost of voting strategically becomes higher as judges' ideal policies become more extreme. Thus, as α increases, judge 9 is the first to cease voting strategically, then judge 8, and so on.

The first type of multiplicity (strategic complements) is perverse, in the following sense: Both judges i and j would prefer to coordinate on the equilibrium where they vote strategically. Nevertheless, the equilibrium where they both vote sincerely can be sustained by beliefs that that the other judge will vote sincerely. In the most extreme case, when α is sufficiently small, we can have an equilibrium in which all judges sincerely believe that the disposition should be 0, but all choose 1 believing that all others will do the same. On collegial courts, it is not unreasonable to assume that such beliefs can be dispelled by communication between the judges, and that a coalition of judges can conspire to jointly affect a favorable deviation.

To rule out perverse equilibria of this sort, we focus on equilibria that are coalition-proof (see Bernheim, Peleg and Whinston (1987)).[18] An equilibrium is coalition-proof if it is immune

[18]In standard bargaining models, the usual refinement is to require that strategies are *weakly dominant*. This, in effect, requires each player to vote as though they were pivotal. Such a refinement is reasonable in so far as the players' payoffs are constant under any scenario in which their vote is not pivotal. However, in our model, the agents will generically have a strict preference between their choices, even if their vote does not change the ultimate dispositional outcome. Hence, the weak-dominance criterion has no bite if applied to overall utilities, and is too strong if applied to dispositional outcomes alone.

to self-enforcing joint deviations by groups of players (i.e. some of the deviating players should not subsequently seek to deviate from their deviation). Whereas Nash equilibria need only survive unilateral deviations, Coalition-proof Nash equilibria (CPNE) must also survive joint deviations by stable coalitions. The notion of coalition-proofness thus refines the set of Nash Equilibria, by ruling out equilibria in which a subset of agents are trapped in a situation that is inferior, but from which they could jointly and stably escape. When strategic complementarity creates multiple equilibria, coalition-proofness selects the equilibrium that is 'most plausible', in the sense of ensuring that those complementarities are exploited as far as possible.

Lemma 3. *Let (d, M) and (d, M') both be adjudication (Nash) equilibria, and suppose $M \subset M'$. Then (d, M) is not coalition-proof.*

Lemma 3 shows that, whenever multiple equilibria arise (that implement the same dispositional outcome) because of strategic complementarities, the smaller coalitions cannot be coalition-proof. The CPNE refinement thus selects the adjudication equilibrium with the 'largest' coalitions, pushing the court towards greater (apparent) cohesion in its decision making.

Selecting a focal equilibrium from amongst the multiplicity that might arise when judges' choices are strategic substitutes is less straight-forward. Our approach is motivated by the following result:

Lemma 4. *Let (d, M) be an adjudication (Nash) equilibrium. There exists a connected coalition M' with $|M'| = |M|$ and such that (d, M') is also an adjudication (Nash) equilibrium coalition. Moreover, (d, M') can be sustained as an adjudication equilibrium over a (weakly) larger range of values of α than (d, M).*

To make sense of Lemma 4, first note that, by the single crossing property, the net benefit of voting strategically decreases as a judge's ideal policy moves further away from the equilibrium policy. (This is evident in Example 4, and by examining equations (A3) and (A4) in the Appendix.) Hence, more extreme judges should be less inclined to vote strategically than moderate judges. If a majority coalition is disconnected, then a relatively extreme judge finds it optimal to vote strategically, whilst a relatively moderate judge finds it optimal to vote sincerely. Lemma 4 formalizes the intuition, that if this true, the moderate judge must be willing to vote strategically if the extreme one does not. Moreover, the moderate judge should be more willing to do so, in the sense that he would continue to vote strategically even if the cost of insincerity (i.e. salience of expressive utility) somewhat increased.

Disconnected coalitions arise when judges' choices are strategic substitutes. For the remainder of this paper, we will focus attention on equilibria where both majority and minority coalitions are connected. Since for every equilibrium coalition that is disconnected, there is a corresponding connected coalition that induces the same disposition and the same equilibrium policy, limiting attention to connected coalitions is not substantively restrictive. Additionally, this equilibrium selection mechanism has two desirable features. First, as the second part of Lemma 4 tells us, connected coalitions are equilibrium consistent over a larger range of the salience parameter α than a corresponding disconnected coalition. It follows that connected coalitions are 'robust', in the sense that they are least brittle to perturbations in the salience of expressive utility. Second, as an empirical matter, coalitions appear for the most part to be connected.

We are now ready to characterize the main results in this section.

Proposition 3. *There exists a Connected Coalition-Proof Adjudcation Equilibrium (CCPAE). Moreover:*

1. *For any CCPAE (d, M):*
   - *If $d = 1$, then $M = \{1, ..., j_1\}$, where $j_1 \geq \frac{n+1}{2}$.*
   - *If $d = 0$, then $M = \{j_0, ...n\}$, where $j_0 \leq \frac{n+1}{2}$.*
2. *There exist at most two CCPAE. Moreover, if (d, M) and (d', M') are distinct CCPAE, then $d \neq d'$.*
3. *For each $z \in [0, 1]$, there exists $\alpha(z) \geq 0$, such that if $\alpha > \alpha(z)$, then the CCPAE is unique.*

Proposition 3 makes several claims. First, it shows that a connected coalition-proof adjudication equilibrium always exists. Although, it is trivial to show that a Nash equilibrium of the adjudication game exists, the Proposition shows that there is at least one that survives the refinements that we impose. In fact, as part 2 of Proposition shows, our refinements admit at most two equilibria – one for each dispositional outcome. Moreover, part 3 of the Proposition shows that when expressive utility becomes sufficiently salient, there can only be one adjudication equilibria that is connected and coalition-proof.

Parts 2 and 3 of Proposition 3 taken together imply the following: When α is sufficiently high, the expressive component of preferences disciplines sufficiently many judges from voting

strategically as to prevent one dispositional outcome or the other from arising in equilibrium. (In many case, but not always, the prevailing outcome will be the one that would arise if all judges cast their dispositional votes sincerely.) By contrast, when α is low enough, policy preferences dominate the decision-making of enough judges, that both outcomes can be sustained as equilibria. Judges are strongly motivated to be in the dispositional majority so that they can pull the equilibrium policy towards their ideal. The adjudication game resembles a dispositional coordination game. Most judges care less about which disposition prevails, than ensuring that they are part of the majority coalition. In particular, there will be equilibria in which a strong majority of judges favor one disposition, and yet the chosen disposition is the other.

An immediate corollary to Proposition 3 is that the court will (generically) be unanimous in any CCPAE when $\alpha = 0$. In practice on the Supreme Court, dissents by at least one judge are common, and 5-4 dispositional votes are not uncommon. Thus, we highlight the important role that expressive preferences play in describing behavior on the Court. Neither our model, nor any that is broadly similar, would be able to explain dissents if limited to judicial preferences that were purely consequentialist.[19]

Although, when α is sufficiently low, our refinement procedures may fail to select a unique equilibria, in practice, there might be other mechanisms that determine which of these equilibria prevail. For example, our modelling of the adjudication game assumed that dispositional voting occured simultaneously. The recent practice on the U.S. Supreme Court, by contrast, is for each justice to cast their vote in order of seniority. If there is a unique CCPAE, it shouldn't matter whether voting is simultaneous or sequential. By contrast, when there are multiple equilibria and voting is sequential, the order in which judges cast their votes may determine which equilibrium is selected.

Proposition 3 also describes the features of equilibrium coalitions. In any connected, coalition-proof adjudication equilibrium, the proposition shows that the majority coalition will contain all but (possibly) the most extreme right judges, if the disposition is '1', or all but (possibly) the most extreme left judges, if the disposition is '0'. An immediate implication of Proposition 3 is that the median judge will always be in the dispositional majority and so, the median justice is 'pivotal' over the case disposition. We stress, however, that whilst the

[19] A model in search of a consequentialist account would by necessity be dynamic, where the role of the dissent is to increase the likelihood of the current policy being over-turned in the future. Whilst we do not deny the merits of such an argument, we do note the many complexities such a model invites. For example, in any such model, by construction, judges will not be able to commit to implement currently chosen policies in the future. This would significantly dampen the import of policy-making today, and thus diminish the value of the dissent.

Salience	$\alpha < 0.118$	$\alpha \in (0.118, 0.585)$	$\alpha \in (0.585, 1.285)$	$\alpha > 1.285$
No. CCPAE	2	2	1	1
$d = 1$	$M = \{1, ..., 9\}$	$M = \{1, ..., 9\}$	None	$M = \{1, ..., 5\}$
$d = 0$	$M = \{1, ..., 9\}$	$M = \{5, ..., 9\}$	$M = \{5, ..., 9\}$	None
Potentially Strategic?	All	$\{5, ..., 9\}$	$\{5\}$	None

Table 3: CCPAE Equilibria in Example 5

median justice is pivotal, it need not follow that the disposition of the court coincides with the median judge's sincere assessment of the case; she may vote strategically. We illustrate this in Example 5, below.

A related implication of Proposition 3 is that the median judge will always be one of the decisive judges in the policy-making stage. However, unless the dispositional vote is unanimous, some other judge will also be decisive. To the extent that opinion-writers have agenda-setting power, the median judge may still be able to implement her ideal policy if she is assigned to write the opinion. However, as this agenda-setting privilege disappears (i.e. as $\delta \to 1$, and the equilibrium policy stems from 'median-voter-like' logic), the median voter's ideal policy will generically *not* be implemented. Instead, the equilibrium policy will either be to his left or right, depending on whether the majority coalition contains mostly leftist or rightist judges.

Example 5. Consider a case $z = 0.55$. Suppose again that policy preferences are bell-curve shaped: $u_P(y, x) = e^{-\frac{1}{2}(y-x)^2} - 1$, and let the disagreement payoff be $u_P(D, x) = -1$. Let the ideal policies be: $x^1 = x^2 = x^3 = x^4 = 0 < x^5 = 0.5 < 0.7 = x^6 = x^7 = x^8 = x^9$ (i.e. there is a relative extreme homogeneous left bloc of 4 judges, a relatively moderate right bloc of 4 judges, and a centrist median judge). The median judge's ideal disposition is $d^* = 1$. Again, for simplicity, suppose $\delta = 1$, so that all judges make the same proposal in equilibrium. Figure 3 illustrates this setup, and the equilibrium policies that will result, for each dispositional outcome, depending on whether the minority bloc votes strategically or not. The CCPAE are described in Table 3.

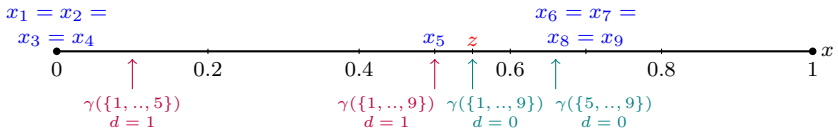


Figure 3: Equilibrium policies chosen for differently composed dispositional majorities.

It follows that $\alpha(z) = 0.5848$. We note that even when equilibria are unique and the median judge is dispositionally pivotal, the outcome need not comport with the median judge's ideal

disposition. There will be strategic voting unless $\alpha > 1.2848$. Moreover, as α increases, more extreme judges become less likely to vote strategically. Thus, the median judge potentially votes strategically over the largest range of α, whereas the left bloc of judges vote strategically over the smallest range of α. □

As Example 5 illustrates, when α is low, regardless of their actual preferences, there is a CCPAE in which all judges choose disposition $d = 1$, and a CCPAE where they all choose disposition $d = 0$. A similar result arises in Fischman (2008), although the mechanism is different. In Fischman's model, unanimity arises because it is costly to dissent (for example, because it would require the judge to expend resources writing a dissenting opinion). In our model, unanimity arises because the hedonic cost of voting insincerely is low relative to the policy gains.

4.3 Comparative Statics

4.3.1 Effect of Salience of Expressive Utility

As Example 5 demonstrates, the incentives for judges to vote strategically vary with the salience of expressive preferences α, and the distance of the case from each judge's ideal threshold. Intuitively, as expressive concerns become more salient, strategic voting becomes harder to sustain, and so the majority coalition shrinks in size. In fact, if expressive concerns are sufficiently large, then no judge will vote strategically, and equilibrium coalitions and case dispositions will reflect the sincere preferences of the judges.

Fix a case z. Let $d(z) \in \{0, 1\}$ denote the sincere disposition, which is the disposition that would prevail if all judges voted sincerely. We have $d(z) = \mathbf{1}[|M^1(z)| > |M^0(z)|]$. Similarly, let $M(z)$ denote the sincere majority coalition: $M(z) = M^{d(z)}(z)$. The above ideas are reflected in the following Lemma:

Lemma 5. *The following are true:*

1. *The size of equilibrium coalitions (with the same disposition) is decreasing in expressive concerns. (Formally, let (d, M) and (d', M') be CCPAE associated with salience levels α and α', with $\alpha > \alpha'$. If $d = d'$, then $M \subseteq M'$.)*
2. *When expressive concerns are sufficiently large, there is unique CCPAE characterized by sincere voting. (Formally, for a given case z, there exists $\overline{\alpha}(z) > 0$ s.t. for $\alpha > \overline{\alpha}(z)$ there is a unique CCPAE (d, M) with $d = d(z)$ and $M = M(z)$.)*

The features described in Lemma 5 are evident in Figure 4.3.2, below.

4.3.2 Effect of Case Location

One of the key insights of this paper is that rule-making by the court cannot be divorced from the specific facts of the case being adjudicated. (This stands in contrast to 'legislature-like' models of the judiciary, where the court purely focuses on choosing a policy, to which end the facts of the instant case are purely incidental.) Example 2 demonstrated that the case facts directly affected the set of feasible policies that the Court could implement, and that the consistency requirement might be binding.

The location of the case also affects the composition of the dispositional majority, and this will likely affect the equilibrium policy, even when the consistency constraint is non-binding. This occurs for two reasons. First, suppose all judges cast dispositional votes sincerely. Then, starting from the median judge's ideal threshold, as the case becomes more and more extreme, the number of judges who find themselves in the majority will increase. (This is rather obvious.)

Second, and more subtly, changing the case location can change the incentives for judges to vote strategically, and thus affect the composition of the dispositional majority. To see this, consider the example below, whose setup is identical to Example 5. We now two cases that would both result in the same dispositional majority if judges voted sincerely:

Example 6. Suppose that policy preferences and ideal policies are as in Example 5. Let $\alpha = 0.3$. Consider two cases: $z_1 = 0.1$ and $z_2 = 0.4$. In both scenarios, the case is located between the ideal policy of the left bloc and the median judge, so that the sincere disposition and sincere majority coalition would be $d(z_i) = 0$ and $M(z_i) = \{5, .., 9\}$. Both scenarios admit a unique CCPAE, with disposition $d = 0$.

- When $z = 0.1$, the majority coalition will be the entire bench $M = \{1, .., 9\}$, and the equilibrium policy will be $\gamma = 0.5$. There is strategic voting by the left bloc.
- When $z = 0.4$, the majority coalition will consist of a bare majority $M = \{5, .., 9\}$, and the equilibrium policy will be $\gamma = 0.66$. There is no strategic voting.

We stress that, in both scenarios, each judge would ideally decide both cases the same way. However, when z is close to the left bloc's threshold, the cost of strategic voting is lower,

and thus the judges are more inclined to vote strategically, to pull the ideal policy closer to their ideal. □

Although the set-up in the previous example is stark, it reflects a more general relationship between case location, the composition of dispositional majorities and the policy of the court. We see this relationship in Figure 4 below:

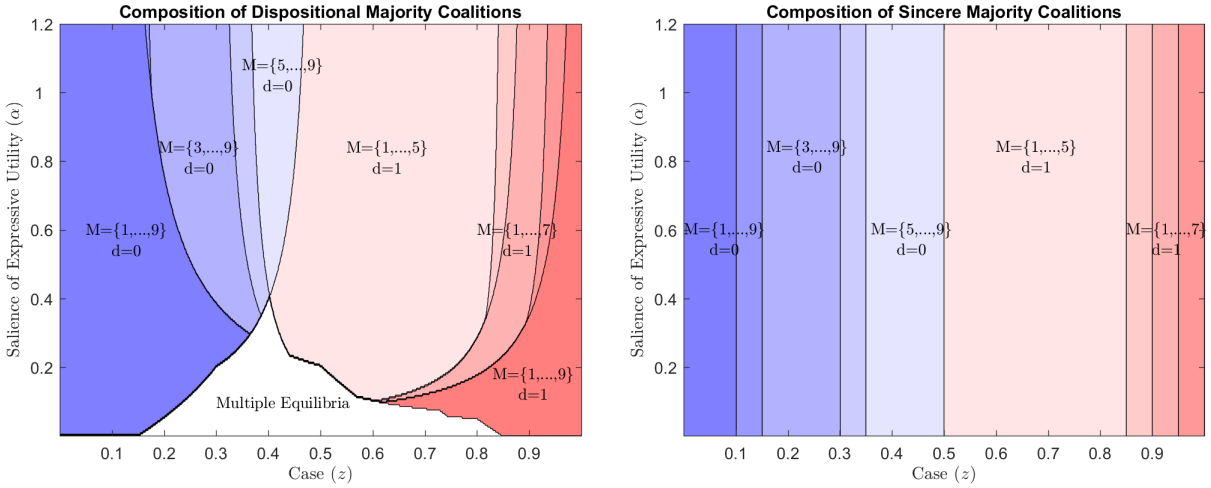


Figure 4: Impact of case location and the salience of the expressive component of utility on the composition of the dispositional majority, and the resulting policy. Policy preferences are bell-curve shaped: $u_P(y,x) = e^{-\frac{1}{2}(z-x)^2} - 1$, and the disagreement payoff be $u_P(D,x) = -1$. The vector of ideal policies be $(x^1,...,x^9) = (0.1.0.15, 0.3, 0.35, 0.5, 0.85, 0.9, 0.95, 1)$. As usual, $\delta = 1$, so that all judges make the same proposal in equilibrium. The left panel shows actual CCPAE dispositions and majority coalitions. The right panel shows dispositions and coalitions if the judges voted sincerely.

The left panel of Figure 4 shows how the equilibrium dispositions and coalitions vary as a function of case location and the salience of expressive utility. The blue and red areas represent regions where the majority disposition is $d = 1$, and $d = 0$, respectively. Darker regions indicate larger coalitions. The right panel represents the disposition and majority coalitions if judges voted sincerely. These regions would be vertical bands, since the outcome under sincere voting is independent of α.

One way to observe the extent of strategic voting is to see how 'sloped' or 'curved' the boundaries of the regions are. Naturally, as α becomes sufficiently large and sincere voting dominates, the lines demarking the coalitions become more vertical. Moreover, fixing any α, we notice that as the case becomes more extreme, equilibrium coalitions are more likely to be larger, and the likelihood of strategic voting increases. Indeed, since $x^9 = 1$, if judges

voted sincerely, judge 9 would always choose $d^9 = 0$. However, allowing for strategic voting, judge 9 potentially chooses $d = 1$ over a large range of cases, when α is low.

Tying the results from sections 3 and 4 together, then, yields the following insight. When the case is 'moderate' (in the sense of being close to the median judge's ideal threshold), then majority coalitions are likely to be smaller, and the resulting equilibrium policy is likely to be more extreme (in the sense of being farther from the median judge's ideal). As the case becomes more 'extreme' (i.e. farther from the median judge's threshold), then majority coalitions will become larger, and the resulting policy will likely be more moderate (i.e. closer to the median judge's ideal).

5 Extension

In this section, we consider the implications of relaxing some of the uniformity assumptions in the analysis.

5.1 Non-uniform Recognition Probabilities

In the baseline model, we assumed that, during the policy bargaining stage, each judge in the dispositional majority would be recognized with equal probability to make counter-proposals. The uniformity assumption is not at all crucial in the policy bargaining stage, and our results are robust to allowing that different judges to be recognized with the different probabilities (perhaps reflecting differences in judges areas of expertise or interest). Indeed, we prove the version of Proposition 1 with generic recognition probabilities in the Appendix. The only change is that, the bargaining weights used to define the Nash bargaining solution depend on the recognition probability weighted size of each faction – rather than simply the number of members in each faction. Our focus on uniform recognition probabilities is thus, in part, motivated to keep the model simple.

However, the assumption does play an important role in our analysis of dispositional voting. An implication of uniformity (along with Assumption 1) was that by joining the dispositional majority, a judge could pull the equilibrium policy (weakly) towards her ideal. If we entertained arbitrary recognition probabilities for every dispositional majority coalition, then by joining the majority coalition, a judge may cause the policy to become worse from their perspective, by skewing (counter)-proposal power towards judges whose ideal policies are far

from the contemplating judge's ideal. Thus, absent sufficient structure on the behavior of recognition probabilities across coalitions, we could neither conclude that coalitions would be connected, nor that the dispositional majority contains every judge who sincerely agrees with the outcome (i.e. Lemma 2 would no longer hold).[20]

5.2 Different Salience of Expressive Utility

In the baseline model, we also assumed that all judges trade-off the policy and expressive components of utility at the same rate α. Obviously, this assumption has no bearing on judges decisions during the policy bargaining phase. However, loosening this assumption will affect the nature of the judges' dispositional choices. In particular, there is no longer any guarantee that an equilibrium will exist in which the dispositional coalition is connected (i.e. Lemma 4 will not obtain). (Similarly, weakening part 2 of Assumption 2 will cause judges voting strategically to no longer be perfect substitutes for one another. This may result in disconnected coalitions.)

Although the implication is qualitatively similar to the effect of allowing non-uniform recognition probabilities, the mechanism is quite different. In that scenario, the result stemmed from the effect on equilibrium policy. The fact of changing recognition probabilities had the effect of potentially worsening policy outcomes for a given judge by virtue of his joining the dispositional majority. Here, policy outcomes are unaffected. The effect arises simply because judges are trading-off the same expressive losses and policy gains, differently. (Note importantly, that unlike the previous subsection, when α can vary, Lemma 2 will still obtain —all judges who sincerely agree with the outcome will definitely join the dispositional majority. The problem only arises for judges who disagree with the outcome, and therefore face a trade-off between policy and expressive utility.)

[20]Of course, the uniform structure that we impose is stronger than is necessary. For example, Lemma 2 would still obtain under the following structure: There are arbitrary recognition probabilities for the unanimous coalition $(p_1^N, ..., p_n^N)$ where $N = \{1, ..., n\}$. For any smaller coalition $C \subset N$, the recognition probabilities of non-member judges is reduced to 0, and the probabilities for all member judges is scaled proportionally i.e.

$$p_i^C = \begin{cases} \frac{p_i^N}{\sum_{j \in C} p_j^N} & i \in C \\ 0 & i \notin C \end{cases}$$

5.3 Reversion to Status Quo

A legislature may propose changes to a given law repeatedly, however, unless one of those proposals is accepted, it is understood that the existing law continues to be in effect. The same cannot be said of courts. As we argued in Section 2, the mere fact that the court agrees to hear a case signals to the community that the legal landscape is apt to change, even if the court fails to implement that change in deciding the instant case. Thus, our preferred model specification does not include a status quo policy and instead requires that the court, through the bargaining process, eventually settle on a new policy for the court.

Nevertheless, one might ask how our results would change if we instead assumed that failure to agree resulted in reversion to the status quo *ante*. The bargaining procedure would be amended as follows: in the event that a proposal is rejected, with probability δ a new proposer is selected and bargaining continues; however, with probability $1-\delta$, the bargaining terminates (exogenously), and the policy reverts to the status quo. This might represent the rare set of cases where no majority can be found to support any given opinion.

With this re-interpretation of the bargaining process, Proposition 1 (and all of the subsequent results) continue to hold true, replacing the disagreement utility with the utility of the status quo policy. Thus, our analysis is perfectly compatible with this alternative formulation.

Of course, reversion to a status quo imposes different costs on different judges, depending on where the status quo stands in relation to their ideal policy. As such, the equilibrium policies will be different, even if the essential structure of the equilibrium is unchanged. One can show (see Banks and Duggan (2006)) that, if the status quo lies outside the core (i.e. $y_{sq} \notin [x^l, x^r]$), then with $\delta < 1$, there will be a range of equilibrium policies that are proposed in equilibrium, and that the social acceptance set becomes narrower as the likelihood that bargaining fails gets smaller (i.e. δ becomes larger). Moreover, as $\delta \to 1$, equilibrium proposals converge to a unique policy, characterized by the asymmetric Nash bargaining solution, as in Proposition 2. The analysis from section 3 carries through exactly as described.

However, if the status quo lies within the core (i.e. $y_{sq} \in [x^l, x^r]$), then for any δ, the only policy that is equilibrium consistent is the status quo itself. (It turns out that, in this case, the status quo policy exactly coincides with the asymmetric Nash bargaining solution, by construction, so Proposition 2 continues to hold, albeit trivially.)

Although we do not take up the issue of *certiorari* decisions in this paper, this last point may shed some light on the issue. Since whenever the status quo lies within the core, the court

will fail to amend the existing rule, we should not expect the court to hear cases where such an outcome is likely to obtain. Moreover, since the core consists of the interval between the median judge's ideal, and the ideal policy of the other decisive judge (which, in the event of a unanimous dispositional vote, is also the median judge), it would be improvident for the court to grant *cert* on cases that where the status quo *ante* lies too close to the median judge's ideal policy.

Even when the status quo policy lies outside of the core (so that policies are chosen through a genuine process of bargaining), its location affects the policies that will be chosen in equilibrium. Interestingly, as the status quo policy becomes more extreme, the policy that is implemented is likely to be more moderate (in the sense of being closer to the 'middle' of the core), ceteris paribus. (See Parameswaran and Murray (2018).) Thus, policy-making by the court exhibits path dependence, with existing rules shaping the sorts of rules that courts can implement in the future.

6 Conclusion

In this paper, we presented a new model of judicial decision-making on multi-member appellate courts like the U.S. Supreme Court. Such courts undertake two interconnected tasks: they must render judgment, i.e. announce a disposition of the case, and formulate a legal rule that rationalizes the chosen disposition. In our model, these decisions are made sequentially, by majority rule. In the first stage, the judges cast dispositional votes, with a majority deciding the disposition of the case. The dispositional vote also determines the subset of judges who participate in the second stage that decides the legal rule governing the case. The policy of the court is determined by bargaining among the members of the dispositional majority, and requires endorsement by a majority of the full bench rather than a majority of the dispositional majority. The model treats the bargaining process as an infinite horizon game of sequential offers in which bargaining can be very intense or rather perfunctory, depending on parameter values. Judges are assumed to have preferences over both case dispositions and policy rules, and those preferences are required to be consistent with one another (in the sense that the latter may be derived from the former).

Our framework highlights several important features of the logic of decision making in institutions that employ joint case disposition-rule making procedures. In turn, that logic has strong implications for observed votes, dispositional and policy coalitions, and policy outcomes. Many are new to the literature on apex appellate courts and are testable empirically.

Several aspects of the logic of decision making stand out. First, in order to join the bargainers who select the policy, each judge may have an incentive to vote strategically in the first stage determination of the disposition. Moreover, the costs and benefits of voting strategically may vary along two dimensions of the nature of the case to be decided: (a) since the court's announced policy must be consistent with its disposition, there are limits on how much a strategically voting judge can 'moderate' the court's policy; (b) because dispositional preferences satisfy the property of increasing differences in dispositional value, the immediate cost of voting strategically depends on the location of the case relative to the rule cut-point. Judges may more profitably vote strategically if the case appears 'contestable' from their perspective, than if it appears clear cut.

We show that, despite incentives for strategic voting, in equilibrium the median judge is pivotal over case dispositions. Furthermore, we show that equilibrium dispositional coalitions are connected – meaning that the most extreme judges are the least likely to vote strategically. By contrast, moderate judges face strong incentives to vote contrary to their preferred outcome, in order to affect the court's policy outcome. In particular, although the median judge is dispositionally pivotal, she may nevertheless vote strategically; the majority disposition does not always coincide with the median judgeâs ideal.

Second, because rules require majority endorsement from the entire court, the dispositional majority faces an effective super-majority requirement if dispositional majorities are non-unanimous. We characterize the equilibria of unidimensional sequential spatial bargaining games for any (super)-majority condition. These equilibria generically depend upon the ideal policies of the agents in the dispositional majority as well as the location of the case. An analytic contribution of this paper is in characterizing the limit equilibria of such bargaining games as the discount parameter δ approaches 1, which we interpret as the limit as the cost of proposing counter-proposals becomes arbitrarily small – in other words, as bargaining becomes intense. We believe this situation is particularly interesting given the institutional setting of an apex court. We show that, in the limit, the dispositional majority endogenously separates into two factions. The announced rule is either the ideal policy of some pivotal judge (not necessarily the median of the dispositional majority), or the result of asymmetric Nash Bargaining between representative leaders of the factions, with bargaining weights proportional to factional size. Importantly, in the limit, the chosen policy will never coincide with the ideal policy of the median judge – and so whilst the median judge decides the disposition of the court, she does not determine the policy of the court. This result stands in contrast to both median voter results and median-of-the-majority results that have been proposed in the existing literature.

The model makes testable predictions in three areas: 1) the size and composition of dispositional voting coalitions, including strategic dispositional voting, 2) the size and composition of policy coalitions (join coalitions) formed around the majority opinion; and 3) the content (spatial location) of majority opinions. As some empirical implications are both novel and subtle, an extended discussion might better be undertaken in tandem with data. Some predictions, however, flow straight-forwardly from the basic logic of decision-making. For example, the model predicts that the extent of author influence over majority opinion content is inversely related to bargaining intensity. In other words, the majority opinions of cases that engender less intense bargaining will tend to be located near the ideal point of the opinion author, while cases that engender intense bargaining will tend to be driven toward a (weighted) center of the dispositional majority. The difference in the respective opinion locations can be substantial when dispositional majorities are large. As an example of a more subtle but easily testable prediction, the model indicates that policy coalitions (join coalitions within the dispositional majority) will be built from one side or other of the dispositional majority; they will not involve both-sides-against-the-middle voting. Finally, searching for evidence of strategic dispositional voting is apt to require empirical ingenuity but is far from impossible.

As mentioned in the Introduction, the model applies not only to the U.S. Supreme Court and state supreme courts but to many independent regulatory commissions. The model thus affords a new lens for understanding decision making on those bodies, which in turn present rich opportunities for new empirical analyses in light of the model's predictions.

7 Appendix

Proof of Proposition 1. The proof is similar to that in Parameswaran and Murray (2018). Since u_P is non-concave, we must first establish that equilibria must be in no-delay pure strategies. Let

$$\begin{aligned} v_P(F(y);F(x)) &= u_P\left(F^{-1}(F(y));F^{-1}(F(x))\right) = u_P(y,x) \\ &= -\left|\int_{F^{-1}(F(x))}^{F^{-1}(F(y))} l(z-x)dF(z)\right| \end{aligned}$$

be the policy utility after re-scaling the policy space. Notice that v_P is concave in $F(y)$:

$$\frac{\partial^2 v_P}{\partial F(y)^2} = -\left| l'\left(y-x\right)) \cdot \frac{1}{f(F(y))} \right| < 0$$

Now, take any (possibly mixed) profile of strategies in the continuation game. Let $\sigma(y,t)$ be the implied distribution over outcomes, where $\sigma(y,t)$ is the probability that policy y is agreed to at time t. Let $\Delta u_P(y,x) = u_P(y,x) - u_P(D,x)$ be the utility gain over disagreement of policy y for a judge with ideal policy x. Similarly, define $\Delta v_P(F(y),F(x))$. Let $\hat{y}$ be the policy defined by: $F(\hat{y}) = \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t F(y) dy$.

Then, the judge i's continuation payoff (over disagreement) if the current proposal is rejected is:

$$\begin{aligned}
\delta \Delta U(x^i) &= \delta \sum_{t=0}^{\infty} \int_{\underline{x}}^{\overline{x}} \sigma(y,t) \cdot \delta^t \Delta u_P(y,x^i) dy \\
&= \delta \left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \right) \cdot \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \frac{\sigma(F(y),t) \cdot \delta^t}{\left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \right)} \Delta v_P(F(y),F(x^i)) dy \\
&\leq \delta \left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \right) \cdot \Delta v_P(F(\hat{(y)}),F(x^i)) \\
&< \Delta u_P(\hat{y},x^i)
\end{aligned}$$

where we use the facts that v_P is concave, and that $\delta \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \leq \delta < 1$. Hence, there is a policy $\hat{y}$ that is strictly preferred by every judge to the continuation game. It is immediate, then, that there is a proposal for every judge that is socially acceptable and preferable to the continuation game. Moreover, since u_P is strictly quasi-concave, this policy is unique. Hence, every equilibrium must be in pure strategies and no-delay.

The acceptance set for any judge i is $A_i = \{y \in [\underline{x},\overline{x} \,|\, \Delta u_P(y,x^i) \geq \delta \Delta U(x^i)\}$. Since $u_P(y;x^i)$ is strictly quasi-concave in y, each individual acceptance set is an interval $A_i = [\underline{y}_i, \overline{y}_i]$. Let $C \subset \{1,...,m\}$ be any coalition containing at least k members. Then, the coalitional acceptance set $A_C = \cap_{i \in C} A_i$ is also an interval. Moreover, since each A_i (and thus each A_C) contains $\hat{y}$, the social acceptance set $A = \cup_C A_C$ must be an interval as well. Denote A be $\left[\underline{y}, \overline{y}\right]$.

Given this social acceptance set, the optimal offers for each agent are:

$$y_i = \begin{cases} \underline{y} & x^i \leq \underline{y} \\ x^i & x^i \in \left(\underline{y}, \overline{y}\right) \\ \overline{y} & x^i \geq \overline{y} \end{cases}$$

For notational convenience, we often denote $u_P(y, x^i)$ by $u_i(y)$. For any $x \in X$, let $P(x) = \sum_{x_i < x} p_i$. (The proof allows for p_i's to be different, although we typically focus on the case of $p_i = \frac{1}{m}$.) Then, given acceptance set $\left[\underline{y}, \overline{y}\right]$, the expected utility of each judge i is:

$$U_i\left(\underline{y}, \overline{y}\right) = P\left(\underline{y}\right) u_i\left(\underline{y}\right) + \sum_{j:\, x^j \in \left(\underline{y}, \overline{y}\right)} p_j u_i\left(x^j\right) + \left(1 - P\left(\overline{y}\right)\right) u_i\left(\overline{y}\right)$$

The remainder of the proof proceeds in two steps. First, we show that in any equilibrium, $\underline{y} = \underline{y}_r$ and $\overline{y} = \overline{y}_l$. Next, using this fact, we show that the equilibrium is a fixed point of a mapping, and that the mapping admits a unique fixed point. This suffices to prove uniqueness of the equilibrium.

Step 1. For any player i, suppose $u_i\left(\underline{y}\right) \leq (1-\delta)\,\underline{u} + \delta U_i\left(\underline{y}, \overline{y}\right)$ — i.e. that $\Delta u_i(y) < \delta \Delta U_i(\underline{y}, \overline{y})$. Since policy preferences satisfy the Spence-Mirlees condition, it must be that: $\Delta u_j\left(\underline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$. To see this, suppose not; i.e. suppose $u_j\left(\underline{y}\right) \geq (1-\delta)\,\underline{u} + \delta U_j\left(\underline{y}, \overline{y}\right)$. Then:

$$\Delta u_i(\underline{y}) - \Delta u_j(\underline{y}) < \delta[\Delta U_i\left(\underline{y}, \overline{y}\right) - \Delta U_j\left(\underline{y}, \overline{y}\right)]$$

Recall, that policy preferences satisfy the Spence-Mirlees condition, and so $x^i < x^j$ implies $\frac{\partial}{\partial y}(\Delta u_i - \Delta u_j) \leq 0$ (see footnote 8). Then:

$$\begin{aligned} \Delta U_i\left(\underline{y}, \overline{y}\right) - \Delta U_j\left(\underline{y}, \overline{y}\right) &= P\left(\underline{y}\right)\left[\Delta u_i\left(\underline{y}\right) - \Delta u_j\left(\underline{y}\right)\right] + \sum_{j:\, x^j \in \left(\underline{y}, \overline{y}\right)} p_j\left[\Delta u_i\left(x^j\right) - \Delta u_j\left(x^j\right)\right] \\ &\quad + \left(1 - P\left(\overline{y}\right)\right)\left[\Delta u_i\left(\overline{y}\right) - \Delta u_j\left(\overline{y}\right)\right] \\ &\leq \Delta u_i\left(\underline{y}\right) - \Delta u_j\left(\underline{y}\right) \end{aligned}$$

But by assumption, $u_i\left(\underline{y}\right) - u_j\left(\underline{y}\right) \leq \delta\left[U_i - U_j\right] < U_i - U_j$, which is a contradiction. Hence, $\Delta u_i\left(\underline{y}\right) \leq \delta \Delta U_i\left(\underline{y}, \overline{y}\right)$ implies that $\Delta u_j\left(\underline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$ whenever $x_j > x_i$. We can similarly show that $\Delta u_i\left(\overline{y}\right) \leq \delta \Delta U_i\left(\underline{y}, \overline{y}\right)$ implies $\Delta u_j\left(\overline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$ whenever $x_j < x_i$.

Suppose $\underline{y} < \underline{y}_r$, then any proposal $y \in \left[\underline{y}, \underline{y}_r\right)$ will be rejected by agent r and all agents

$j > r$. But since $r = k$, this implies that fewer than k agents will accept the proposal, which means it cannot be in the acceptance set. Hence $\underline{y} \geq \underline{y}_r$. Suppose $\underline{y} > \underline{y}_r$. Take any proposal $y \in \left(\underline{y}_r, \underline{y}\right)$. By construction $\Delta u_r(y) > \delta \Delta U\left[\underline{y}, \overline{y}\right]$, and so $u_j(y) > \delta \Delta U\left[\underline{y}, \overline{y}\right]$ for all agents $j < r$. But since $r = k$, this implies that at least k agents will accept proposal y. But this contradicts the assumption that y is outside the acceptance set. Hence $\underline{y} = \underline{y}_r$. We can similarly show that $\overline{y} = \overline{y}_l$.

Step 2. We now show that the equilibrium exists and is unique. For each i, define $\underline{\zeta}_i(z) = \min_{y \in X}\{y \leq x_i | \Delta u_i(y) \geq \delta \Delta U_i(y, z)\}$ and $\overline{\zeta}_i(z) = \max_{y \in X}\{y \geq x_i | \Delta u_i(y) \geq \delta \Delta U_i(z, y)\}$. Since u_i is continuous and X compact, then $\underline{\zeta}_i$ and $\overline{\zeta}_i$ are both continuous. Note also that:

$$\underline{\zeta}_j'(y) = \begin{cases} \frac{\delta(1-P(y))}{1-\delta P\left(\underline{\zeta}_j(y)\right)} \cdot \frac{u_j'(y)}{u_j'\left(\underline{\zeta}_j(y)\right)} & \underline{\zeta}_j(y) > \underline{x} \\ 0 & \underline{\zeta}_j(y) = \underline{x} \end{cases}$$

and:

$$\overline{\zeta}_i'(y) = \begin{cases} \frac{\delta P(y)}{1-\delta+\delta P\left(\overline{\zeta}_i(y)\right)} \cdot \frac{u_i'(y)}{u_i'\left(\overline{\zeta}_i(y)\right)} & \overline{\zeta}_i(y) < \overline{x} \\ 0 & \overline{\zeta}_i(y) = \overline{x} \end{cases}$$

By the previous step, we know that $\overline{y} = \overline{y}_l$ and $\underline{y} = \underline{y}_r$. Hence, $\overline{y} = \overline{\zeta}_l\left(\underline{y}\right)$ and $\underline{y} = \underline{\zeta}_r(\overline{y})$. Let $H(y) = \overline{\zeta}_l\left(\underline{\zeta}_r(y)\right)$. H is continuous since $\underline{\zeta}_r$ and $\overline{\zeta}_l$ are both continuous. It follows that if $\left[\underline{y}, \overline{y}\right]$ is an equilibrium acceptance set, then $\overline{y}$ is a fixed point of H, and $\underline{y} = \underline{\zeta}_r(\overline{y})$. Since X is compact and H is continuous and onto X, it follows by Brouwer's fixed point theorem that H admits a fixed point $\overline{y}$. Hence, an equilibrium of the bargaining exists.

To establish that H has a unique fixed point, it suffices to show that $H'(\overline{y}) < 1$ for any $\overline{y}$ that is a fixed point. (If there exist multiple fixed points, then $H' \geq 1$ for at least one fixed point.) By construction:

$$H'(\overline{y}) = \begin{cases} A(\overline{y}) \cdot \frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} \cdot \frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} & \underline{x} < \underline{y} \leq \overline{y} < \overline{x} \\ 0 & \underline{y} = \underline{x} \text{ or } \overline{y} = \overline{x} \end{cases}$$

where $\underline{y} = \underline{\zeta}_r(\overline{y}) < \min\{x_r, \overline{y}\}$, and $A(y) = \frac{\delta P\left(\underline{\zeta}_r(\overline{y})\right)}{1-\delta+\delta P(\overline{y})} \frac{\delta(1-P(\overline{y}))}{1-\delta P\left(\underline{\zeta}_r(\overline{y})\right)} \in (0, 1)$.

Suppose $H(\overline{y}) \geq 1$. Then at least one of $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})}\right| > 1$ or $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. There are several cases

to consider. First, suppose $\left|\frac{u_r'(\overline{y})}{u_r'(\underline{y})}\right| > 1$. Since $\underline{y} < \min\{x_r, \overline{y}\}$ then $u_r'(\underline{y}) > 0$. If $\underline{y} \leq \overline{y} \leq x_r$, then $0 \leq u_r'(\overline{y}) \leq u_r'(\underline{y})$, which contradicts $\left|\frac{u_r'(\overline{y})}{u_r'(\underline{y})}\right| > 1$. Hence $\underline{y} < x_r < \overline{y}$, and so $u_r'(\overline{y}) < 0$. Suppose additionally $x_l \leq \underline{y} < \overline{y}$. Then $u_l'(\underline{y}) < 0$ and $u_l'(\overline{y}) < 0$. Hence $\frac{u_r'(\overline{y})}{u_r'(\underline{y})} < -1$, and $\frac{u_l'(\underline{y})}{u_l'(\overline{y})} > 0$, and so $H < 0$, which cannot be. Hence $\underline{y} < x_l \leq x_r < \overline{y}$, and so:

$$\frac{u_l'(\underline{y})}{u_l'(\overline{y})} \cdot \frac{u_r'(\overline{y})}{u_r'(\underline{y})} = \frac{-l(\underline{y} - x_l)}{l(\overline{y} - x_l)} \cdot \frac{l(\overline{y} - x_r)}{-l(\underline{y} - x_r)} \leq 1$$

since $l(z)$ is weakly increasing for $z < 0$ and weakly decreasing for $z > 0$. Hence $H < 1$, which cannot be, and so $\left|\frac{u_r'(\overline{y})}{u_r'(\underline{y})}\right| \leq 1$.

Next, suppose that $\left|\frac{u_l'(\underline{y})}{u_l'(\overline{y})}\right| > 1$. Since $\overline{y} > \max\{x_l, \underline{y}\}$, then $u_l'(\overline{y}) < 0$. If $x_l \leq \underline{y} \leq \overline{y}$, then $u_l'(\overline{y}) \leq u_l(\underline{y}) \leq 0$, which contradicts that $\left|\frac{u_l'(\underline{y})}{u_l'(\overline{y})}\right| > 1$. Hence $\underline{y} < x_l < \overline{y}$, and so $u_l'(\underline{y}) > 0$. Suppose additionally that $\underline{y} < \overline{y} \leq x_r$. Then $u_r'(\underline{y}) > 0$ and $u_r'(\overline{y}) > 0$. Hence $\frac{u_r'(\overline{y})}{u_r'(\underline{y})} > 0$, and $\frac{u_l'(\underline{y})}{u_l'(\overline{y})} < -1$, and so $H < 0$, which cannot be. Hence $\underline{y} < x_l \leq x_r < \overline{y}$. But we know that this implies $H < 1$, which also cannot be. Hence our initial supposition was wrong; $H'(\overline{y}) \not\geq 1$. Hence, $H' < 1$ and so H admits a unique fixed point. □

Proof of Lemma 1 . Recall, the acceptance set is $A = [\underline{y_r}, \overline{y_l}]$, where $\underline{y_r} = \min\{y \geq \underline{x} \,|\, \Delta u_r(y) \geq \delta \Delta U_r(y, \overline{y_l})\}$, and $\overline{y_l} = \max\{x \leq \overline{x} \,|\, \Delta u_l(y) \geq \delta \Delta U_l(\underline{y_r}, y)\}$. Now, by construction $\Delta u_l(\underline{y_r}) \geq \Delta u_l(\overline{x_l})$, since l will accept $\underline{x_r}$. Then, since u is strictly quasi-concave, $\Delta u_l(y) > \Delta u_l(\overline{y_l})$ for all $y \in (\underline{y_r}, \overline{y_l})$. Similarly, $\Delta u_r(y) > \Delta u_r(\underline{y_r})$ for all $y \in (\underline{y_r}, \overline{y_l})$. Hence $\Delta U_l(\underline{y_r}, \overline{y_l}) > \Delta u_l(\overline{y_l})$ and $\Delta U_r(\underline{y_r}, \overline{y_l}) > \Delta u_r(\underline{y_r})$ whenever $\underline{y_r} < \overline{y_l}$.

Now, for every $\delta < 1$, $\frac{\Delta u_l(\overline{y_l})}{\Delta U_l(\underline{y_r}, \overline{y_l})} = \delta = \frac{\Delta u_r(y_l)}{\Delta \Delta U_r(\underline{y_r}, \overline{y_l})}$, and so as $\delta \to 1$, we need $\Delta U_l(\underline{y_r}, \overline{y_l}) - \Delta u_l(\overline{y_l}) \to 0$ and $\Delta U_r(\underline{y_r}, \overline{y_l}) - \Delta u_r(\underline{y_r}) \to 0$. But this requires $\overline{y_l} - \underline{y_r} \to 0$. Hence $A = [\underline{y_r}, \overline{y_l}] \to [\mu, \mu]$ as $\delta \to 1$. □

Proof of Proposition 2 . Take any $i \in \{1, ..., m\}$, and suppose $\mu \in (x^{i-1}, x^i)$. Then, by Lemma 1, there exists $\bar{\delta} < 1$ s.t. for $\delta > \bar{\delta}$, $x^{i-1} < \underline{y_r}(\delta) < \overline{y_l}(\delta) < x^i$. (For clarity, we make explicit the dependence of $\underline{y_r}$ and $\overline{y_l}$ on δ.) Then, by Proposition 1, all judges $j \in \{1, .., i-1\}$ will propose $\underline{y_r}$ and all judges $j \in \{i, .., n\}$ will propose $\overline{y_l}$. Again by

Proposition 1, this implies that:

$$\Delta u_r\left(\underline{y_r}\right) = \delta\left[\left(1-P_i\right)\Delta u_r\left(\underline{y_r}\right) + P_i \Delta u_r\left(\overline{y_l}\right)\right] \tag{1}$$

$$\Delta u_l\left(\overline{y_l}\right) = \delta\left[\left(1-P_i\right)\Delta u_l\left(\underline{y_r}\right) + P_i \Delta u_l\left(\overline{y_l}\right)\right] \tag{2}$$

where $P_i = \sum_{j \geq i} p_j$. By the implicit function theorem, this system of equations pins down $\underline{y_r}$ and $\overline{y_l}$ in terms of the model parameters.

Now, let $E[y] = (1-P_i)\,\underline{y_r} + P_i \overline{y_l}$. Note, by construction, that $\underline{y_r} < E[y] < \overline{y_l}$. Then $\overline{y_l} - E[y] = \frac{1-P_i}{P_i}\left(E[y] - \underline{y_r}\right)$. We affect the following change of variables: Let $\varepsilon = E[y] - \underline{y_r}$. Then, we have: $\underline{y_r} = E[y] - \varepsilon$ and $\overline{y_l} = E[y] + \frac{1-P_i}{P_i}\varepsilon$. Equations (1) and (2) become:

$$\left(1-\delta\left(1-P_i\right)\right)\Delta u_r\left(E[y]-\varepsilon\right) = \delta P_i \Delta u_r\left(E[y] + \frac{1-P_i}{P_i}\varepsilon\right) \tag{3}$$

$$\left(1-\delta P_i\right)\Delta u_l\left(E[y] + \frac{1-P_i}{P_i}\varepsilon\right) = \delta\left(1-P_i\right)\Delta u_l\left(E[y]-\varepsilon\right) \tag{4}$$

By the implicit function theorem, and since u is continuously differentiable, we have:

$$\begin{bmatrix} \left(1-\delta\left(1-P_i\right)\right)u_r'\left(\underline{y_r}\right) - \delta P_i u_r'\left(\overline{y_l}\right) & -\left(1-\delta\left(1-P_i\right)\right)u_r'\left(\underline{y_r}\right) - \delta\left(1-P_i\right)u_r'\left(\overline{y_l}\right) \\ \left(1-\delta P_i\right)u_l'\left(\overline{y_l}\right) - \delta\left(1-P_i\right)u_l'\left(\underline{y_r}\right) & \left(\frac{1-P_i}{P_i} - \delta\left(1-P_i\right)\right)u_l'\left(\overline{y_l}\right) + \delta\left(1-P_i\right)u_l'\left(\underline{y_r}\right) \end{bmatrix} \begin{pmatrix} \frac{\partial E[y]}{\partial \delta} \\ \frac{\partial \varepsilon}{\partial \delta} \end{pmatrix} =$$

$$\begin{pmatrix} \left(1-P_i\right)\Delta u_r\left(\underline{yr}\right) + P_i \Delta u_r\left(\overline{y_l}\right) \\ P_i \Delta u_l\left(\overline{y_l}\right) + \left(1-P_i\right)\Delta u_l\left(\underline{y_r}\right) \end{pmatrix}$$

Taking limits as $\delta \to 1$, we have:

$$\begin{bmatrix} 0 & -u_r'(\mu) \\ 0 & \frac{1-P_i}{P_i}u_l'(\mu) \end{bmatrix} \begin{pmatrix} \lim_{\delta\to 1}\frac{\partial E[y]}{\partial \delta} \\ \lim_{\delta\to 1}\frac{\partial \varepsilon}{\partial \delta} \end{pmatrix} = \begin{pmatrix} u_r(\mu) \\ u_l(\mu) \end{pmatrix}$$

These imply that:

$$\lim_{\delta\to 1}\frac{\partial \varepsilon}{\partial \delta} = -\frac{u_r(\mu)}{u_r'(\mu)} = \frac{P_i}{1-P_i}\frac{u_l(\mu)}{u_l'(\mu)}$$

The second equality provides an equation that uniquely defines the limit equilibrium.

Next, we note that equation defining μ_i coincides with the first order condition of the i^{th} Nash Bargaining problem. Recall, that problem was: $\max_{y\in X} u_P^l(y)^{1-P_i} u_P^r(y)^{P_i}$. Since utilities are concave, the maximizer must be the solution to the first order condition: $(1-P_i)\frac{u_{P,y}^l(b_{i-1,i})}{u_P^l(b_{i-1,i})} + P_i\frac{u_{P,y}^r(b_{i-1,i})}{u_P^r(b_{i-1,i})} = 0$. Re-arranging gives the desired result.

Notice that $b_{i-1,i}$ is increasing in P_i. (To see this, re-arrange the first order condition to give:

$\frac{u_l'(b_{i-1,i})}{u_r'(b_{i-1,i})} \cdot \frac{u_r(b_{i-1,i})}{u_l(b_{i-1,i})} = -\frac{P_i}{1-P_i}$. We know that $b \in \left[x^l, x^r\right]$. By single-peakedness, over this region we know that $u_P^l(b)$ is strictly decreasing in b and $u_r(b)$ is strictly increasing in b, and so $\frac{u_r(b)}{u_l(b)}$ is strictly decreasing in b. Similarly, by concavity (after transformation) of u, $u_l'(b)$ is decreasing in b and $u_r'(b)$ is increasing in b, and so $\frac{u_r'(b)}{u_l'(b)}$ is weakly decreasing in b. Hence, the left hand term is strictly decreasing in b. The right hand term is also strictly decreasing in P. Hence, as P increases, so must b.) Then, since P_i is decreasing in i, it follows that $b_{i-1,i}$ is decreasing is as well.

Since we conjectured $\mu \in (x^{i-1}, x^i)$, then the limit equilibrium policy coincides with i^{th} Nash Bargaining solution provided that $x^{i-1} < b_{i-1,i} < x^i$. Now, since x^i is increasing and $b_{i-1,i}$ is decreasing in i, then by the definition of i^*, $x^i < b_{i,i+1}$ for all $i < i^*$ and $x^i \geq b_{i,i+1}$ for all $i \geq i^*$. Moreover, for $i < i^*$, $x^{i-1} \leq x^i < b_{i,i+1} \leq b_{i-1,i}$, which is inconsistent. Similarly, for $i > i^*$, $b_{i-1,i} \leq x^{i-1} \leq x^i$, which is inconsistent. Hence, if $b_{i-1,i} \in (x^{i-1}, x^i)$, then $i = i^*$. Note however, that the converse need not be true. Setting $i = i^*$ gives two possibilities: (i) $x^{i^*-1} < b_{i^*-1,i^*} < x^{i^*}$, or (ii) $x^{i^*-1} \leq x^{i^*} \leq b_{i^*-1,i^*}$ (with at least one inequality strict). The former case is equilibrium consistent, and since the equilibrium is unique, we have $\mu = b_{i^*-1,i^*}$.

Suppose the latter case prevails. It follows that the limit equilibrium is not contained in any of the open intervals (x^{i-1}, x^i), and so $\mu \in \{x^1, ..., x^m\}$. (In fact, since $\underline{y_r} < x^r$ and $\overline{y_l} > x^l$ for all δ, and since $\lim_{\delta \to 1} \underline{y_r} = \mu = \lim_{\delta \to 1} \overline{y_l}$, then $x^l \leq \mu \leq x^r$, and so $\mu \in \left\{x^l, ..., x^r\right\}$.) Suppose $\mu = x^i$ for some $i \in \{l, ..., r\}$. Let $I = \{j | x^j = x^i\}$ and denote $I = \{i^-, ..., i^+\}$, where $i^- \leq j \leq i^+$ for all $j \in I$. (Obviously, I may be a singleton, in which case $i^- = i = i^+$.) Let $\Pi_i^- = \sum_{j<i^-} p_j$ and $\Pi_i^+ = \sum_{j>i^+} p_j$ and $\Pi_i = \sum_{j \in I} p_j$. Then, for δ sufficiently large, (1) becomes:

$$u_P^r(\underline{y}_r) = \delta \left[\Pi_i^- u_P^r(\underline{y}_r) + \Pi_i u_P^r(x^i) + \Pi_i^+ u_P^r(\overline{y}^l)\right]$$

Since $\underline{y}_r < x^i < \overline{y}^l$, there exists $\gamma \in (0,1)$ s.t. $x^i = \gamma \underline{y}^r + (1-\gamma)\overline{y}^l$. We can write (1) as:

$$\begin{aligned} u_P^r(\underline{y}^r) = {} & \delta \left[(\Pi_i^- + \Pi_i \gamma) u_P^r(\underline{y}^r) + (\Pi_i^+ + \Pi_i(1-\gamma)) u_P^r(\overline{y}_l)\right] \\ & + \delta \left[\Pi_i \gamma (u_P^r(\underline{y}_r) - u_P^r(x^i)) + \Pi_i (1-\gamma)(u_P^r(\overline{y}^l) - u_P^r(x^i))\right] \end{aligned} \tag{5}$$

Notice (5) is the sum of two terms, with the first term being analogous to the expression in (1), and the second term being a 'correction' term.

We repeat the procedure for equation (2), and then apply the change of basis method above, and take limits as $\delta \to 1$. Since $\underline{y}^r \to x^i$ and $\overline{y}^l \to x^i$, the 'correction' term in (5) goes to zero. It follows that $\mu = b(\rho^*)$, where $\rho^* = \Pi_i^+ + \Pi_i(1 - \lim_{\delta \to 1} \gamma(\delta))$. Now, there must

be some k s.t. $b_{k,k+1} < b(\rho^*) = x^i < b_{k-1,k}$. Moreover, it must be that $k \in I$, since $b_{i^+,i^++1} < b(\rho^*) < b_{i^- -2,i^- -1}$, by construction. But then, we can choose i appropriately s.t. $b_{i,i+1} < x^i < b_{i-1,i}$. But this requires $i = i^*$. □

Proof of Lemma 2. Let z be an arbitrary case. Suppose $d^* = 0$. (The other scenario is analogous.) Recall $M^0 = \{j | \, x^j > z\}$. Moreover, all feasible second stage policies must satisfy $y \geq z$. Suppose there is a j, such that $j \in M^0$ and $j \notin M^*$. Then

$$u^j \left(d^j = 1; d^{-j}\right) - u^j \left(d^j = 0; d^{-j}\right) > 0$$
$$\left[u_P(\gamma(M), x^j) - u_P\left(\gamma(M \cup \{j\}), x^j\right)\right] + \alpha l(z - x^j) > 0$$

By assumption 1, the term in square brackets is non-positive, since joining the coalition cannot make the policy worse from j's perspective. Moreover, the second term is negative by construction. Hence the LHS is negative, which is a contradiction. Hence $j \in M^*$. □

Derivation of Example 4. Begin with the equilibrium proposals. First, we show that for any bargaining weight $\phi \in [0, 1]$, the solution to the Nash bargaining problem $\max \left[u_l(y) - u_l(D)\right]^{\phi} \left[u_r(y) - u_r(D)\right]^{1-\phi}$ is simply $b(\phi) = \phi x_l + (1 - \phi)x_r$. To see this, note by the first order conditions, that the Nash bargaining solution satisfies:

$$\phi \frac{u_l'(y)}{u_l(y) - u_l(D)} + (1 - \phi) \frac{u_r'(y)}{u_r(y) - u_r(D)} = 0$$

Now, since $u_i(y) = e^{-\frac{1}{2}(y - x^i)^2} - 1$ and $u_i(D) = -1$, then $\frac{u_i'(y)}{u_i(y) - u_i(D)} = -(y - x^i)$. The result follows immediately.

Next, we find the equilibrium policy for each coalition size. Suppose none of the disagreeing judges join the coalition. Then $M = 1, .., 5$, and since $k = 5$, then $l = 1$ and $r = 5$. It is easily verified that $b_{2,3} = 0.2 <= x^3 = 0.3 \leq b_{3,4}$, and so by Proposition 2 judge 3 is pivotal. Now, suppose one of the disagreeing judges joins the coalition. No matter which one does, the decisive judges will be $l = 2$ and $r = 5$. Again, it is easily verified that judge 3 is pivotal. Suppose instead that two disagreeing judges join the coalition. Again, no matter which two these are, the decisive judges will be $l = 3$ and $r = 5$. We can verify that $x^3 = 0.3 < \frac{29}{70} = b_{3,4} < 0.5 = x^4$, and so by Proposition 2, no judges is pivotal, and the equilibrium policy is determined as if the judges break into two coalitions, with judges 1, 2 and 3 on the left, and judges 4, 5, and the two disagreeing judges, on the right. If

three disagreeing judges join, then $l = 4$ and $r = 5$. Since judges 4 and 5 share the same ideal policy bargaining is trivial, and their ideal policy is implemented. Similarly if all four disagreeing judges join, then $l = 5 = r$, and so judge 5 is pivotal. The median voter theorem applies.

Now, we check for equilibria. *** Do this *** □

Proof of Lemma 3. Suppose (d, M) and (d, M') are both adjudication (Nash) equilibria, with $M \subset M'$. Since M and M' are both equilibrium coalitions, it (generically) must be that $M' \geq M + 2$, where X denotes the cardinality of set X. (To see this, note that if $M' = M \cup \{i\}$ where $i \notin M$, then it must be that judge i is exactly indifferent between joining the majority coalition or not; otherwise, i would have a strictly improving unilateral deviation. This indifference is non-generic and requires an exact alignment of the case, the equilibrium policies chosen by the respective coalitions, and the salience parameter α.)

Note by Lemma 2 that $M^d(z) \subseteq M \subset M'$. WLOG, suppose $d = 1$. Then, by part 1 of Assumption 1, $\gamma(M) \leq \gamma(M' \setminus \{j\}) \leq \gamma(M')$ for every $j \in M' \setminus M$, since $M \subset M' \setminus \{j\}$. Moreover, for all $j \in M' \setminus M$, $\gamma(M) \leq \gamma(M') \leq z < x^j$. Now, since M' is a Nash equilibrium coalition, then $u_P(\gamma(M'), x^j) + \alpha l(z, x^j) \geq u_P(\gamma(M' \setminus \{j\}), x^j)$ for each $j \in M' \setminus M$, and given the above ordering, we know that $u_P(\gamma(M' \setminus \{j\}), x^j) \geq u_P(\gamma(M), x^j)$. Hence $u_P(\gamma(M'), x^j) + \alpha l(z, x^j) \geq u_P(\gamma(M), x^j)$ for all $j \in M' \setminus M$, and this inequality will generically be strict for some j. Hence, the joint deviation from M to M' is Pareto improvement within the deviating coalition.

We must also show that this deviation is stable. Suppose not. Then there exists a (strict) subcoalition $C \subset M' \setminus M$ that would deviate back to voting sincerely. It must be that C contains at least two judges, since otherwise it is a unilateral deviation, which cannot be, since M' is a Nash equilibrium coalition. (This implies that $M' \setminus M$ contains at least 3 judges.) Take some $k \in C$. By construction, $M' \setminus C \subset M' \setminus \{k\} \subset M'$, and so $\gamma(M' \setminus C) \leq \gamma(M' \setminus \{k\}) \leq \gamma(M')$. Since the deviation from the deviation is profitable, we have: $u_P(\gamma(M' \setminus C), x^k) > u_P(\gamma(M', x^k) + \alpha l(z, x^k) \geq u_P(\gamma(M' \setminus \{k\}, x^k)$, where the second inequality follows from the fact that M' is an equilibrium coalition. Hence $u_P(\gamma(M' \setminus C), x^k) > u_P(\gamma(M' \setminus \{k\}), x^k)$, which cannot be since $\gamma(M' \setminus C) \leq \gamma(M' \setminus \{k\}) < x^k$. Hence, the deviation is stable. □

Proof of Lemma 4. Let (d, M) be an adjudication (Nash) equilibrium, and suppose M is not connected. WLOG, suppose $d = 1$, so that, by Lemma 2, $M^1(z) \subset M$. Since M^1 is a connected coalition and M is disconnected, M must contain members of $M^0(z)$. Then there

exists $i < j$ with $i, j \in M^0(z)$, $i \notin M$ and $j \in M$. Then $z < x^i \leq x^j$. Let M' be identical to M except that judge j is replaced by judge i. By part 2 of Assumption 1, it must be that $\gamma(M) = \gamma(M')$. (To see this, note that replacing judge i with j causes the social acceptance set to be unchanged, and that both judges will make the same proposal ($\overline{y}$).) Since M is an equilibrium, it must be that:

$$u_P(\gamma(M), x^j) + \alpha l(z - x^j) \geq u_P(\gamma(M - \{j\}), x^j)$$

and:

$$u_P(\gamma(M \cup \{i\}), x^i) + \alpha l(z - x^i) < u_P(\gamma(M), x^i)$$

We seek to show that M' is also an equilibrium coalition. It suffices to show that:

$$u_P(\gamma(M'), x^i) + \alpha l(z - x^i) \geq u_P(\gamma(M' - \{i\}), x^i)$$

and:

$$u_P(\gamma(M' \cup \{j\}), x^j) + \alpha l(z - x^j) < u_P(\gamma(M'), x^j)$$

If $x^i = x^j$, it is trivial to do so, since i and j have identical preferences. Suppose $x^i < x^j$. Note that:

$$\begin{aligned}
&\left\{\left[u_P(\gamma(M), x^j) - u_P(\gamma(M - \{j\}), x^j)\right] + \alpha l(z - x^j)\right\} - \left\{\left[u_P(\gamma(M'), x^i) - u_P(\gamma(M' - \{i\}), x^i)\right] + \alpha l(z - x^i)\right\} \\
=&\left(\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x^j)dy + \alpha l(z - x^j)\right) - \left(\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x^i)dy + \alpha l(z - x^i)\right) \\
=&\int_{x^i}^{x^j} \frac{\partial}{\partial x}\left[\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x)dy + \alpha l(z - x)\right] dx \\
=&-\int_{x^i}^{x^j} \left[\int_{\gamma(M-\{j\})}^{\gamma(M)} l'(y - x)dy + \alpha l'(z - x)\right] dx \\
\leq&0
\end{aligned}$$

where the final line follows from the fact that $\gamma(M - \{j\}) < \gamma(M) \leq z < x^i < x^j$ and that, by the IDID property, $l'(y - x) > 0$ for all $y < x$. It follows that (7) implies (7). By a similar argument, we can show that (7) implies (7). Hence, M' is an equilibrium coalition as well.

Moreover, if $x^i < x^j$, then the inequality above is strict, and continues to be so for some $\alpha' > \alpha$ and even for some $\gamma(M') < \gamma(M)$. □

Proof of Proposition 3. Part (1) is straight-forward. Fix a case z. Suppose (d, M) is a

CCPAE. By Lemma 2, $M^d \in M$. Suppose $M^d \neq \emptyset$. Then, by the ordering over judges, $1 \in M^d$ if $d = 1$ and $n \in M^d$ if $d = 0$. Since M is connected and contains at least $k = \frac{n+1}{2}$ agents, then $\frac{n+1}{2} \in M$. Hence either $\{1, ..., \frac{n+1}{2}\} \subset M$ or $\{\frac{n+1}{2}, ..., n\} \subset M$. (If $M^d = \emptyset$, then the result follows provided that we rule out equilibria that relies upon a majority of judges voting strategically, but not those judges with the lowest cost of doing so.)

To show part (3), let (d, M) and (d', M') be distinct CCPAE, and suppose that $d = d'$. Then, by Lemma 2, $M^d(z) \subset M$ and $M^{d'}(z) \subset M'$. Since M and M' are connected, this implies (WLOG) that $M \subset M'$. But then, by Lemma 3, M cannot be coalition-proof, which is a contradiction. Hence, $d \neq d'$. Since distinct CCPAE must have distinct dispositions, and there are only two possible dispositional values, then there can be at most two CCPAE.

The existence of an adjudication (Nash) equilibrium follows by standard game theoretic results. We now establish the existence of a CCPAE. Let (d_0, M_0) be an adjudication (Nash) equilibrium, and suppose it is a candidate to be a CCPAE. By Lemma 3, we know that thereis no larger adjudication equilibrium with the same case disposition (i.e. there is no M' with $M_0 \subset M'$ s.t. (d_0, M') is an adjudication equilibrium. If (d_0, M_0) is not a CCPAE, then there must exist some other coalition C_0 and induced disposition d_0' s.t. all the members of $M_0 \cap C_0$ prefer to deviate from (d_0, M_0) to (d_0', C_0). Moreover, no subset of the deviators $M_0 \cap C_0$ should have a strict incentive to deviate from C_0. Immediately, this implies that (d_0', C_0) is an adjudication (Nash) equilibrium.

By construction, it cannot be that $d_0' = d_0$, since any smaller coalition inducing the same case disposition must be inferior for the deviating judges (by Lemma 3). Hence $d_0' = 1 - d_0$. Using the same logic as in Lemma 4, if C_0 is disconnected, we can always find some other coalition C_0' that is connected and which implies a strictly favorable deviation for the judges in $M_0 \cap C_0'$. Hence, it is WLOG to focus on deviations by connected coalitions. Hence (d_1, C_0) is a connected adjudication (Nash) equilibrium, where $d_1 = 1 - d_0$. Let (d_1, M_1) be the largest connected coalition that implements case disposition $d_1 = 1 - d_0$. Clearly $C_0 \subseteq M_1$. (d_1, M_1) is the only other candidate for a CCPAE. Suppose it is not. Then, by the same argument, there must be some connected $C_1 \subseteq M_0$, s.t. (d_0, C_1) is preferred by all judges in the deviating coalition $M_1 \cap C_1$, and this deviating coalition is stable.

Since each deviation flips the case disposition, and coalitions are connected, then the median judge must be a member of the deviating coalition in each case. WLOG, suppose $d_0 = 0$ and $d_1 = 1$. We have:

(6) $u_P(\gamma(C_0), x^{med}) + \mathbf{1}[z < x^{med}]l(z - x^{med}) > u_P(\gamma(M_0), x^{med}) + \mathbf{1}[z > x^{med}]l(z - x^{med})$

and

$$(7)\quad u_P(\gamma(C_1), x^{med}) + \mathbf{1}[z > x^{med}]l(z - x^{med}) > u_P(\gamma(M_1), x^{med}) + \mathbf{1}[z < x^{med}]l(z - x^{med})$$

Suppose $x^{med} < z$. By assumption 1, $\gamma(C_0) \leq \gamma(M_1) \leq z \leq \gamma(M_0) \leq \gamma(C_1)$. It cannot be that $x^{med} \leq \gamma(M_1)$, otherwise $u_P(\gamma(M_1), x^{med}) > u_P(\gamma(C_1), x^{med})$, which contradicts (6). Hence: $\gamma(C_0) \leq \gamma(M_1) < x^{med} < z \leq \gamma(M_0) \leq \gamma(C_1)$. But then, by the strict quasi-concavity of u_P, $u_P(\gamma(M_0), x^{med}) \geq u_P(\gamma(C_1), x^{med}) > u_P(\gamma(M_1), x^{med}) \geq u_P(\gamma(C_0), x^{med})$. But 6 implies that $u_P(\gamma(C_0), x^{med}) > u_P(\gamma(M_0), x^{med})$ (since $\mathbf{1}[x^{med} < z]$). We have a contradiction. By a symmetric argument, we can show that a contradiction arises in the scenario that $x^{med} > z$. Hence, it cannot be that both (d_0, M_0) and (d_1, M_1) are both *not* CCPAE. Existence is established.

Finally, we establish part (2). Fix some case z. For $j = \{1, .., \frac{n-1}{2}\}$, define:

$$\alpha_j(z) = \frac{u_P(\gamma(\{j+1, ..., n\}), x^j) - u_P(\gamma(\{j, ..., n\}), x^j)}{l(z - x^j)}$$

If $x^j < z$, so that j's ideal disposition is $d = 1$, then whenever $\alpha > \alpha_j(z)$, there cannot be an adjudication equilibrium in which j is the left-most judge who votes strategically. Similarly, for $j = \{\frac{n+3}{2}, ..., n\}$ define:

$$\alpha_j(z) = \frac{u_P(\gamma(\{1, , ..., j-1\}), x^j) - u_P(\gamma(\{1, ..., j\}), x^j)}{l(z - x^j)}$$

If $x^j > z$, so that j's ideal disposition $d = 0$, then whenever $\alpha > \alpha_j(z)$, there cannot be an adjudication equilibrium in which j is the right-most judge who votes strategically. Finally, define:

$$\alpha_{\frac{n+1}{2}}(z) = \frac{u_P(\gamma(\{1, ..., \frac{n+1}{2}\}), x^{med}) - u_P(\gamma(\{\frac{n+1}{2}, ..., n\}), x^{med})}{l(z - x^{med}})$$

Recall $M^1(z)$ and $M^0(z)$ are the coalitions that arise if judges vote sincerely. Since n is odd, one of these will be larger than the other. We refer to the larger coalition as the 'sincere majority coalition' and the smaller coalition as the 'sincere minority coalition'.

We consider two scenarios. First, suppose $|M^1(z) - M^0(z)| \geq 2$. This implies that if judges vote sincerely, the size of the majority and minority coalitions will differ by at least two. Then, for all $\alpha \geq 0$, there exists an adjudication (Nash) equilibrium in which all members of the sincere majority coalition vote sincerely. (To see why, note that if all judges in the

sincere majority coalition vote sincerely, then no judge is pivotal over the case disposition. The result is then an immediate consequence of Lemma 2. Note, of course, that judges in the sincere minority might nevertheless have an incentive to vote strategically.)

We show that, for α sufficiently large, there cannot be an adjudication (Nash) equilibrium which implements the opposite disposition. Suppose there is. By Lemma 4, we know that it suffices to focus on connected equilibria. Suppose $M^1(z) > M^0(z) + 1$, so that the sincere disposition is $d = 1$. The connected majority coalitions that implement the opposite disposition ($d = 0$) and satisfy Lemma 2 are of the form: $\{j, , , n\}$, where $j \in \{1, .., \frac{n+1}{2}\} \subseteq M^1(z)$. Define $\alpha(z) = \max\{\alpha_1, ..., \alpha_{\frac{n+1}{2}}\}$. By construction, if $\alpha > \alpha(z)$, then none of these coalitions is consistent with an adjudication equilibrium. Hence, if $\alpha > \alpha(z)$, there cannot be any adjudication equilibria that implement the sincere minority's preferred disposition. Hence, any adjudication equilibrium must implement the sincere majority's preferred disposition. By previous arguments, there is a unique CCPAE that achieves this.

Suppose instead that $M^0(z) > M^1(z) + 1$, so that the sincere disposition is $d = 0$. Then the result obtains by defining $\alpha(z) = \max\{\alpha_{\frac{n+1}{2}}, ..., n\}$.

Next, consider the scenario where $|M^1(z) - M^0(z)| = 1$, so that, if all judges vote sincerely, the median is pivotal. This scenario differs from the previous one only insofar as the median judge may have an incentive to vote strategically for α low enough, even if all other judges in the sincere majority vote sincerely. Again, first suppose that $x^{med} < z$, so that the sincere disposition is $d = 1$. Define:

$$\alpha(z) = \min\left\{\max\{\alpha_1, ..., \alpha_{\frac{n+1}{2}}\}, \max\{\alpha_{\frac{n+3}{2}}, ..., \alpha_n\}\right\}$$

Following the same logic, there is a unique equilibrium provided that $\alpha > \alpha(z)$. Supposing instead that $x^{med} > z$, then the result obtains by defining:

$$\alpha(z) = \min\left\{\max\{\alpha_1, ..., \alpha_{\frac{n-1}{2}}\}, \max\{\alpha_{\frac{n+1}{2}}, ..., \alpha_n\}\right\}$$

□

Proof of Lemma 5. [To do] □

References

Baker, Scott and Claudio Mezzetti. 2012. "A Theory of Rational Jurisprudence." *Journal of Political Economy* 120(3):513–551.

Banks, Jeffrey S. and John Duggan. 2000. "A Bargaining Model of Collective Choice." *The American Political Science Review* 94(1):pp. 73–88.

Banks, Jeffrey S and John Duggan. 2006. "A General Bargaining Model of Legislative Policy-making." *Quarterly Journal of Political Science* 1(1):49–85.

Baron, David P. 1996. "A dynamic theory of collective goods programs." *American Political Science Review* 90(2):316–330.

Baron, David P and John A Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83(04):1181–1206.

Bawn, Kathleen, Martin Cohen, David Karol, Seth Masket, Hans Noel and John Zaller. 2012. "A theory of political parties: Groups, policy demands and nominations in American politics." *Perspectives on Politics* 10(3):571–597.

Bernheim, B Douglas, Bezalel Peleg and Michael D Whinston. 1987. "Coalition-proof nash equilibria i. concepts." *Journal of Economic Theory* 42(1):1–12.

Binmore, Ken, Ariel Rubinstein and Asher Wolinsky. 1986. "The Nash Bargaining Solution in Economic Modelling." *The RAND Journal of Economics* pp. 176–188.

Black, D. 1948. "On the rationale of group decision-making." *The Journal of Political Economy* 56(1):23–34.

Cameron, Charles M and Lewis A Kornhauser. 2008. "Modeling Collegial Courts (3): Judicial Objectives, Opinion Content, Voting and Adjudication Equilibria.".

Cameron, Charles M, Lewis A Kornhauser and G Parameswaran. 2019. "Stare Decisis and Judicial Log-Rolls: A Gains-from-Trade Model." *RAND Journal of Economics (forthcoming)* .

Canes-Wrone, Brandice. 2010. *Who leads whom?: presidents, policy, and the public*. University of Chicago Press.

Cardona, Daniel and Clara Ponsati. 2011. "Uniqueness of stationary equilibria in bargaining one-dimensional policies under (super) majority rules." *Games and Economic Behavior* 73(1):65–75.

Carrubba, Cliff, Barry Friedman, Andrew D Martin and Georg Vanberg. 2012. "Who controls the content of Supreme Court opinions?" *American Journal of Political Science* 56(2):400–412.

Chen, Ying and Hulya Eraslan. 2018. "Learning While Setting Precedents.".

Cho, Seok-ju and John Duggan. 2003. "Uniqueness of stationary equilibria in a one-dimensional model of bargaining." *Journal of Economic Theory* 113(1):118–130.

Cho, Seok-ju and John Duggan. 2009. "Bargaining foundations of the median voter theorem." *Journal of Economic Theory* 144(2):851–868.

Compte, Olivier and Philippe Jehiel. 2010. "Bargaining and majority rules: A collective search perspective." *Journal of Political Economy* 118(2):189–221.

Cox, Gary W and Mathew D McCubbins. 2007. *Legislative leviathan: Party government in the House*. Cambridge University Press.

Downs, A. 1957. "An economic theory of political action in a democracy." *The Journal of Political Economy* 65(2):135–150.

Fischman, Joshua B. 2008. "Decision-making under a norm of consensus: A structural analysis of three-judge panels.".

Fischman, Joshua B. 2011. "Estimating Preferences of Circuit Judges: A Model of Consensus Voting." *The Journal of Law and Economics* 54(4):781–809.

Gailmard, Sean and John W Patty. 2007. "Slackers and zealots: Civil service, policy discretion, and bureaucratic expertise." *American Journal of Political Science* 51(4):873–889.

Hammond, Thomas H, Chris W Bonneau and Reginald S Sheehan. 2005. *Strategic behavior and policy choice on the US Supreme Court*. Stanford University Press.

Hirsch, Alexander V. 2016. "Experimentation and persuasion in political organizations." *American Political Science Review* 110(1):68–84.

Iaryczower, Matias and Matthew Shum. 2012. "The value of information in the court: Get it right, keep it tight." *The American Economic Review* 102(1):202–237.

Jacobi, Tonja. 2009. "Competing Models of Judicial Coalition Formation and Case Outcome Determination." *Journal of Legal Analysis* 1(2):411–458.

Kalandrakis, Tasos. 2010. "Minimum winning coalitions and endogenous status quo." *International Journal of Game Theory* 39(4):617–643.

Krehbiel, Keith. 1998. *Pivotal politics: A theory of US lawmaking*. University of Chicago Press.

Lax, Jeffrey R and Charles M Cameron. 2007. "Bargaining and Opinion Assignment on the US Supreme Court." *Journal of Law, Economics, and Organization* 23(2):276–302.

Moe, Terry M and William G Howell. 1999. "The presidential power of unilateral action." *The Journal of Law, Economics, and Organization* 15(1):132–179.

Parameswaran, Giri and Jacob Murray. 2018. "Limit Equilibrium of Unidimensional Bargaining Games: A Core Refinement.".

Predtetchinski, Arkadi. 2011. "One-dimensional bargaining." *Games and Economic Behavior* 72(2):526–543.

Rasmusen, Eric. 1994. "Judicial Legitimacy as a Repeated Game." *Journal of Law, Economics, and Organization* pp. 63–83.

Romer, T. and H. Rosenthal. 1978. "Political resource allocation, controlled agendas, and the status quo." *Public choice* 33(4):27–43.

Segal, Jeffrey A and Harold J Spaeth. 2002. *The Supreme Court and the attitudinal model revisited*. Cambridge University Press.

Snyder, James M and Michael M Ting. 2002. "An informational rationale for political parties." *American Journal of Political Science* pp. 90–110.

Wahlbeck, Paul J, James F Spriggs and Forrest Maltzman. 1999. "The politics of dissents and concurrences on the US Supreme Court." *American Politics Quarterly* 27(4):488–514.