Indebted Demand*

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Abstract

We propose a theory of *indebted demand*, capturing the idea that large debt burdens by households and governments lower aggregate demand, and thus natural interest rates. At the core of the theory is the simple yet under-appreciated observation that borrowers and savers differ in their marginal propensities to save out of permanent income. Embedding this insight in a two-agent overlapping-generations model, we find that recent trends in income inequality and financial liberalization lead to indebted household demand, pushing down natural interest rates. Moreover, popular expansionary policies—such as accommodative monetary policy and deficit spending—generate a debt-financed short-run boom at the expense of indebted demand in the future. When demand is sufficiently indebted, the economy gets stuck in a debt-driven liquidity trap, or *debt trap*. We document that the behavior of our model is in line with several recent empirical facts and discuss policies to reduce demand indebtedness.

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1 Introduction

There has been a dramatic rise in debt levels and a decline in interest rates over the past 40 years in advanced economies. The average real interest rate dropped from about 6% in 1980 to less than zero in 2019 (Rachel and Summers 2019), and the average debt to GDP ratio almost doubled from 139% in 1980 to over 270% (Figure 1 below).

The broader implications of this environment have been a central point of discussion among policy-makers and academics. Monetary policy-makers, for example, worry that high debt levels may constrain their ability to tighten. A recent Wall Street Journal article cites monetary authorities worldwide in asserting that "borrowing helped pull countries out of recession but made it harder for policy makers to raise rates." Mark Carney, Governor of the Bank of England observed that "the sustainability of debt burdens depends on interest rates remaining low." Philip Lowe, Governor of the Reserve Bank of Australia has warned that "if interest rates were to rise . . . many consumers might have to severely curtail their spending to keep up their repayments."[1]

The high debt-low interest rate environment has also sparked a debate on fiscal policy, with wide-ranging views. Olivier Blanchard in his American Economic Association presidential address argued that "public debt may have no fiscal cost," (Blanchard 2019) whereas Kenneth Rogoff responded to Blanchard in a recent IMF conference that "I'm not sure [...] that we should just let debt drift up."

Two central research questions are raised by this discussion: How did this current situation of high debt and low interest rates come to be? And what are the implications of high debt levels and low interest rates for monetary and fiscal policy?

This study develops a new framework to tackle these difficult questions. The model introduces non-homothetic consumption-saving behavior (e.g., De Nardi 2004, Straub 2019) into an otherwise conventional, deterministic two-agent endowment economy. The assumption of non-homotheticity implies that the saver in the model saves a larger fraction of lifetime income than the borrower. This is not a new idea in economics. In fact, it is pervasive in the work of luminaries such as John Atkinson Hobson, Eugen von Böhm-Bawerk, Irving Fisher, and John Maynard Keynes, and empirically supported by recent work (e.g., Dynan, Skinner, and Zeldes 2004, Straub 2019, and Fagereng, Holm, Moll, and Natvik 2019). In the model, the wealthy lend to the rest of the population, which makes household debt an important financial asset in the portfolio of the wealthy.[2]

The assumption of non-homotheticity in our model generates the crucial property that large debt levels weigh negatively on aggregate demand: as borrowers reduce their spending to make debt payments to savers, the latter, having greater saving rates, only imperfectly offset the shortfall in borrowers' spending. We refer to a situation in which demand is depressed due to elevated debt

[1] See also Borio and White (2004), Koo (2008), Borio and Disyatat (2014), Lo and Rogoff (2015), Turner (2015) and Dalio (2018) for similar ideas.

[2] This implication of the model fits the data, as shown in Mian, Straub, and Sufi (2019); a substantial fraction of household debt in the United States reflects the top 10% of the wealth distribution lending to the bottom 90%.

levels as *indebted demand*.

The concept of indebted demand has broad implications for understanding what has led to the current high debt and low interest rate environment, and for evaluating what policies can potentially help advanced economies escape this equilibrium. An overarching theme of the model is that shifts or policies that boost demand today through debt accumulation necessarily reduce demand going forward by shifting resources from borrowers to savers; therefore, such shifts or policies actually contribute to persistently low interest rates.

The framework elucidates the channels through which advanced economies entered into the current high debt-low interest rate environment. A rise in income inequality, which has been a pervasive aspect of most advanced economies since the 1980s (Katz and Murphy 1992, Piketty and Saez 2003, Piketty 2014, Piketty, Saez, and Zucman 2017), naturally leads to a rise in household debt and a decline in interest rates in the model. Inequality shifts resources from borrowers to savers, pushing down interest rates due to savers' greater desire to save. Lower interest rates stimulate more debt, causing indebted demand—as debt is nothing other than an additional shift of resources in the form of debt service payments from borrowers to savers.

A similar logic implies that financial liberalization, which has also been a prominent feature of advanced economies since the 1980s, ultimately lowers interest rates. Financial liberalization increases the amount of debt taken on by borrowers, which redistributes resources to savers. For the goods markets to clear, such a redistribution requires interest rates to fall given that savers have a lower marginal propensity to consume out of these larger debt payments. The model explains why financial liberalization has been associated with falling interest rates, a robust empirical pattern that other models have difficulty generating, (e.g., Justiniano, Primiceri, and Tambalotti 2017).

More generally, the model offers a different perspective on the growth in the size of the financial sector since the 1980s. Traditional models in which the financial sector enables firms to borrow from households cannot explain the global rise in debt to GDP ratios, which has been concentrated in household and government debt. Furthermore, investment to GDP ratios and business productivity growth have actually fallen during this period. In contrast, the indebted demand model focuses on how secular forces such as rising inequality and financial liberalization can foster a large rise in households borrowing from other households. In both the model and the data, the household sector is crucial for understanding why debt levels have increased.

The concept of indebted demand also provides insight into discussions of monetary and fiscal policy. For example, deficit-financed fiscal policy in the model is associated with a short run rise in natural interest rates, which reverses into a reduction in interest rates in the long-run, as the government needs to raise taxes or cut spending in order to finance the greater government debt burden.[3] As long as some of the taxes are ultimately imposed on borrowers, deficit-financed government spending is similar to any policy which attempts to boost demand through debt accumulation. Ultimately, such a policy shifts resources from borrowers to savers, depressing aggregate

[3] As we discuss below, we find that a similar result holds up in the presence of spreads between government bond yields and the returns on other assets.

demand and therefore interest rates in the long run.

A similar argument applies to monetary policy, for which we extend our model to include nominal rigidities. Empirical evidence suggests that an important channel of accommodative monetary policy operates through an increase in debt accumulation (e.g., Bhutta and Keys 2016, Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao 2017, Beraja, Fuster, Hurst, and Vavra 2018, Di Maggio, Kermani, and Palmer 2019, Cloyne, Ferreira, and Surico 2019). This channel is also active in our model, boosting demand in the short-run. However, this boost reverses as monetary stimulus fades and debt needs to be serviced, beginning to drag on demand. Due to the presence of indebted demand, this drag can cause a persistent shift in natural interest rates after temporary monetary policy interventions. It is for this reason that monetary policy has *limited ammunition* in the model: successive monetary policy interventions build up debt levels, thereby lowering natural rates. This forces policy rates to keep falling with them to avoid a recession, thus approaching the effective lower bound.

When savers command sufficient resources in our economy, for instance due to high income inequality and large debt levels, the natural rate in our economy can be persistently below its effective lower bound. At that point, our economy is in a debt-driven liquidity trap, or *debt trap*, which is a well-defined stable steady state of our economy. The most striking aspect of this state is that it acts as a kind of "black hole". Simple, temporary policies, such as a one-time write-down in household debt only provide temporary cure before the economy falls right back into the trap. Moreover, policies that actually increase the long-run debt burden, such as financial liberalization or deficit spending, not only provide merely a temporary increase in aggregate demand, they may in fact deepen the trap further.

Literature. Our paper is part of a burgeoning literature on the causes of the recent fall in natural interest rates, referred to as "secular stagnation" by Summers (2014). Among the existing explanations are population aging (Eggertsson, Mehrotra, and Robbins 2019), income risk and income inequality (Auclert and Rognlie 2018, Straub 2019), the global saving glut (Bernanke 2005, Coeurdacier, Guibaud, and Jin 2015) and a shortage of safe assets (Caballero and Farhi 2017).[4] Our theory suggests a new force for reduced natural interest rates, namely indebted demand. It can act both as an amplifier of existing explanations—as we demonstrate for rising income inequality—or give rise to new explanations, as we demonstrate for financial liberalization, which is commonly thought to be a force against low interest rates.

The central element of our theory is the assumption of non-homothetic preferences, generating heterogeneous saving rates out of *permanent* income transfers.[5] As we mentioned above, such heterogeneity was an important aspect of many early studies of (non-optimizing) consumption behavior. Among the more recent papers in this tradition are Stiglitz (1969), Von Schlicht (1975), and Bourguignon (1981), who study the implications of such behavior on inequality. The earliest

[4]For an overview of multiple forces see Rachel and Summers (2019).

[5]This is not to be confused with heterogeneity in marginal propensities to consume, which even our homothetic benchmark economy below generates.

models of optimal consumption behavior that we know of and that allow for such preferences are Strotz (1956), Koopmans (1960) and Uzawa (1968). More recently, Carroll (2000), De Nardi (2004), and Benhabib, Bisin, and Luo (2019) argue that non-homothetic preferences are important to understand wealth inequality, and Straub (2019) studies their implications for a rise in income inequality.

Our implications for monetary policy are related to the debate around "leaning vs. cleaning" (Bernanke and Gertler 2001, Stein 2013, Svensson 2018) and to the nascent academic literature surrounding the idea that monetary policy might have limited ammunition. McKay and Wieland (2019) explore this idea in a model of durables spending, Caballero and Simsek (2019) in a model with asset price crashes.

The closest antecedents to our paper are Kumhof, Rancière, and Winant (2015) and Cairó and Sim (2018). Kumhof et al. (2015) study a two-agent endowment economy, where savers are more patient than borrowers and savers have non-homothetic preferences. They find that a rise in income inequality leads to greater debt levels and a greater likelihood of a financial crisis due to endogenous default, but no change in long-run interest rates. The driving force behind this result is the specific structure and heterogeneity of preferences. It generates a higher saving rate of savers out of labor income, compared to borrowers, but a lower saving rate out of financial income. This is why the model does not feature indebted demand: in fact, an increase in debt raises aggregate demand in the model and thus dampens the effects of income inequality. The model in Cairó and Sim (2018) builds on Kumhof et al. (2015) and studies implications for a richer set of shocks and for the conduct of monetary policy.

Finally, as a paper about household and government debt, it relates to a vast empirical and theoretical literature on the origins and consequences of high debt levels. Among the empirical papers, Schularick and Taylor (2012) document the well known "financial hockey stick" behavior of private debt; Mian and Sufi (2015), Jordà, Schularick, and Taylor (2016), Mian, Sufi, and Verner (2017) document that expansions in household debt predict weak future economic growth; Reinhart and Rogoff (2010) assess the consequences of large government debt. Among the theoretical papers, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017) study the effects of debt deleveraging on the economy. Our model emphasizes that even without deleveraging, debt reduces aggregate demand.

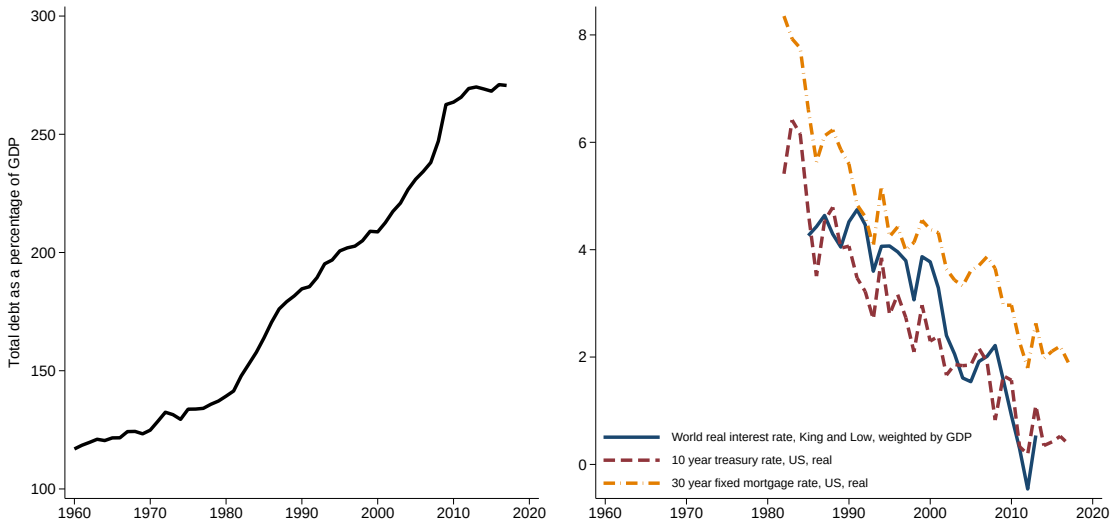
Layout. Section 2 presents motivating facts for our model in Section 3. Section 4 studies steady states and transitional dynamics in our model, introducing the concept of indebted demand. In Section 5, we feed trends in income inequality and financial liberalization into the model. Next, we study the implications of fiscal policy (Section 6) and monetary policy (Section 7), and what indebted demand means for an economy in a liquidity trap (Section 8). Section 9 provides several extensions, and Section 10 offers a simple "sufficient statistic" perspective on our theory. Our conclusion is in Section 11.

2 Motivating Facts

2.1 High debt, low interest rates, and credit-driven household demand

A defining feature of advanced economies over the past decade has been the simultaneous presence of historically high debt burdens and historically low interest rates. The left panel of Figure 1 plots the average debt to GDP ratio across 14 mostly advanced economies, where debt includes all borrowing by households, governments, and non-financial businesses. Both the high level of debt in recent years and its rapid ascent since 1980 are notable.

Figure 1: Debt and interest rates.

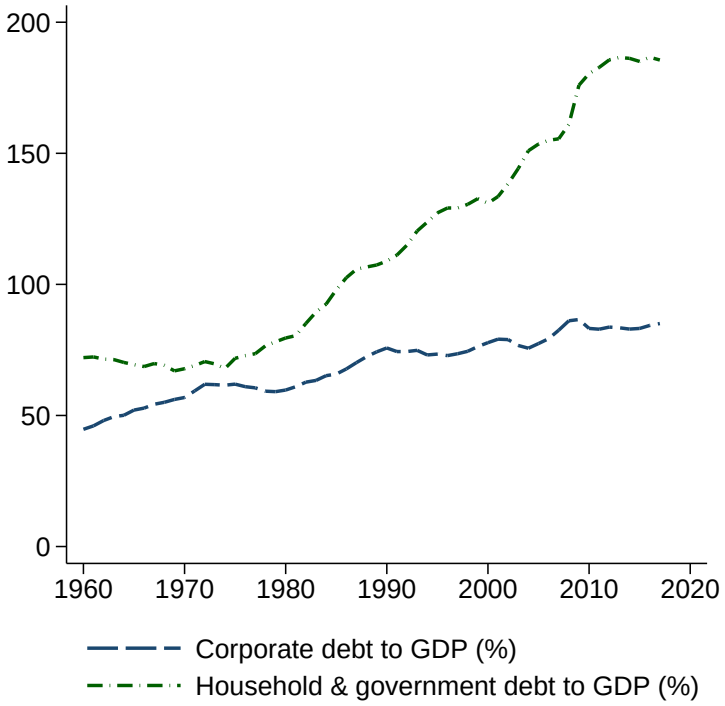


Left panel shows cross-country average total debt to GDP, weighted by real GDP in 1970. Total debt is the sum of credit to households, government and non-financial corporations. The countries in the sample are Australia, Canada, Finland, France, Germany, Italy, Japan, New Zealand, Norway, Portugal, Spain, Sweden, United States and United Kingdom. Data come from the IMF Global Debt Database, the Jorda-Schularick-Taylor Macrohistory Database and the New Zealand Treasury. Right panel shows evolution of real interest rates. The real interest rate data start in the early 1980s given that information on inflation expectations prior to 1980 is difficult to obtain. For more details on the data sources, see text.

The right panel shows the evolution of real interest rates from 1980 onward. The global interest rate is an estimate of the real rate of interest for 10-year government bonds from King and Low (2014). The right panel also shows the real interest rate on 10-year U.S. Treasuries, and an estimate of the real rate of interest for 30-year fixed rate mortgages in the United States. Measures of inflation expectations are taken from the Federal Reserve Bank of Cleveland (available since 1980). All three series show a substantial decline from 1980 onward.

Both the high level and rapid ascent of debt in advanced economies has been driven by borrowing by households and the government, as opposed to businesses. Figure 2 splits the total debt to GDP ratio into non-financial corporate debt and a second category that includes households and governments, and it shows that both the level and the rise in debt has been concentrated in the lat-

Figure 2: Households and governments drove the increase in debt.



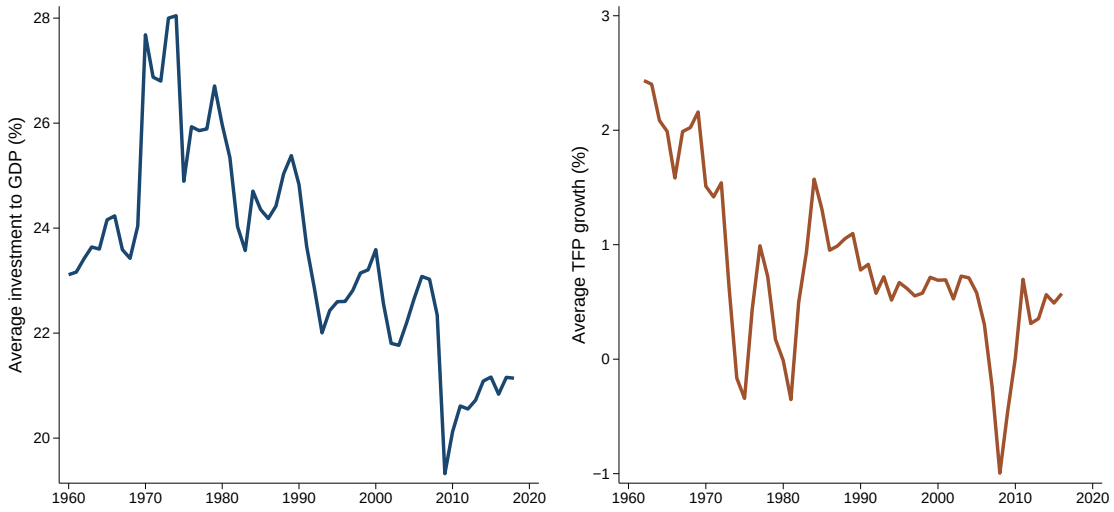
Series are cross-country average debt to GDP, for non-financial corporations and for households and governments together, weighted by real GDP in 1970. The countries in the sample are Australia, Canada, Finland, France, Germany, Italy, Japan, New Zealand, Norway, Portugal, Spain, Sweden, United States and United Kingdom. Data come from the IMF Global Debt Database, the Jorda-Schularick-Taylor Macrohistory Database and the New Zealand Treasury.

ter category. Household and government debt to GDP ratios were relatively constant before 1980, before beginning an impressive upward trend through recent years. However, borrowing by the non-financial corporate sector was modest, and therefore not responsible for the debt boom.

The fact that businesses have not been responsible for the rise in borrowing is consistent with the fact that investment to GDP ratios have actually declined during this period of rising debt and low interest rates. This is shown in the left panel of Figure 3, which plots the gross domestic investment to GDP ratio across the 14 countries in the sample. The right panel shows that productivity growth has been at best constant over this time period.

The rise in debt has not been associated with a traditional channel through which businesses and the government borrow from the household sector to boost investment and productivity growth. Instead, rising debt levels appear to have been used to finance personal consumption and government transfers. That is, it appears that the expansion of credit has been used to increase aggregate demand rather than supply. This is a key motivating fact behind the model of indebted demand below.

Figure 3: Secular decline of investment and TFP growth.



Left panel shows cross-country average gross capital formation as a percentage of GDP and the right panel shows cross-country average total factor productivity growth, where the averages are weighted by real GDP in 1970. The countries in the sample are Australia, Canada, Finland, France, Germany, Italy, Japan, New Zealand, Norway, Portugal, Spain, Sweden, United States and United Kingdom. Cross-country averages are constructed over 3-year moving averages at the country-level. Data come from the Penn World Tables and World Bank World Development Indicators.

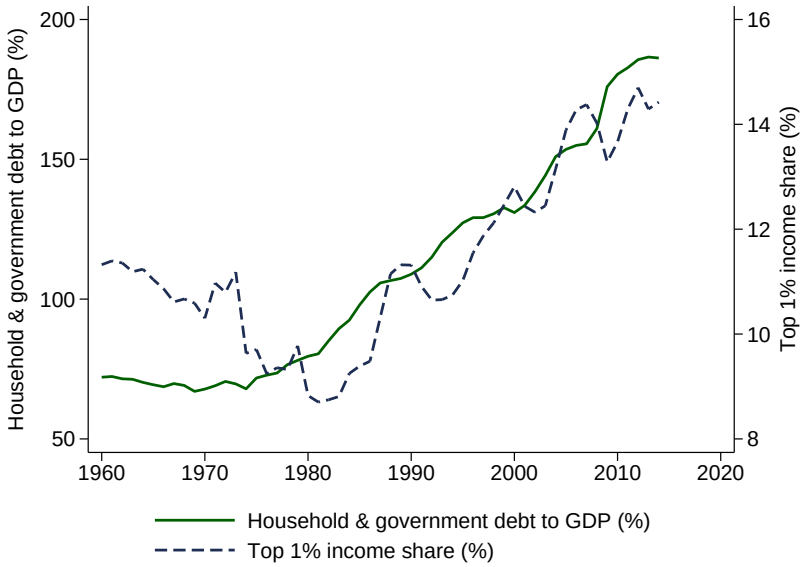
2.2 Inequality and financial liberalization

The model is also motivated by two trends that have emerged concurrently with growth in debt and the decline in interest rates: rising inequality and financial liberalization. The rise in the share of income earned by the top of the income distribution has been well-established in the literature (e.g., Piketty et al. 2017). What is perhaps less well known is that the rise in top income shares globally began in the 1980s at almost the exact same time as the rise in government and household borrowing, and the two patterns have been closely linked afterward. This is shown in Figure 4, which plots the rise in government and household borrowing from Figure 2 above, along with the top 1% share of income across 14 countries from the World Inequality Database.

This close association between the rise in inequality and the rise in household borrowing is explored in detail in a companion study, Mian et al. (2019). That study focuses on the United States, and it shows that the rise in income inequality has generated a "saving glut of the rich," which has financed a substantial rise in household debt by the non-rich.

Figures 5 and 6 come directly from Mian et al. (2019). Figure 5 shows the total saving of the top 1%, next 9%, and bottom 90% of the income distribution of the United States, scaled by national income. Since 1980, there has been a substantial increase in saving of the top 1%, and that saving has been linked to a substantial dis-saving of the bottom 90%. As shown in Mian et al. (2019), the rise in saving of the top 1% has not been transformed into investment or capital account surpluses; instead, it has been absorbed by a reduction in saving by the bottom 90%.

Figure 4: Rising inequality and rising debt.



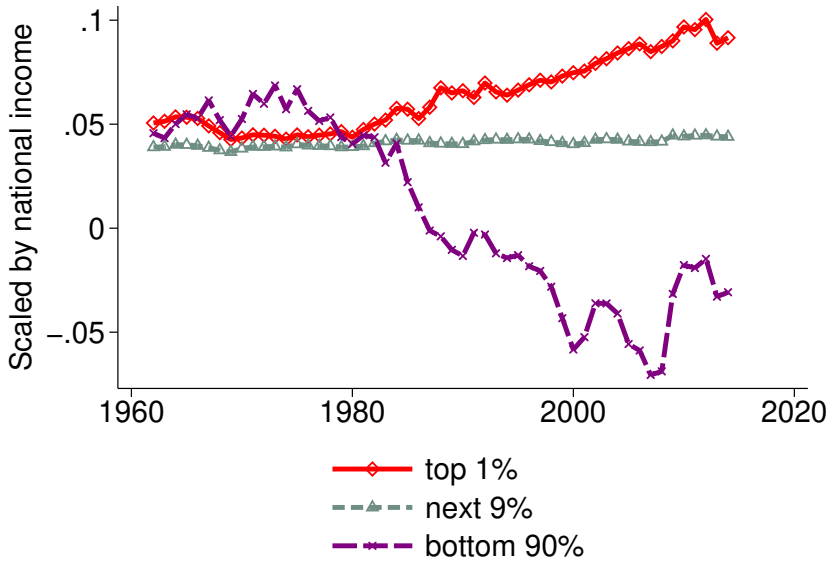
Series are cross-country averages, weighted by real GDP in 1970. The countries in the sample are Australia, Canada, Finland, France, Germany, Italy, Japan, New Zealand, Norway, Portugal, Spain, Sweden, United States and United Kingdom. Data come from the World Inequality Database, IMF Global Debt Database, the Jorda-Schularick-Taylor Macrohistory Database and the New Zealand Treasury.

Figure 6 focuses on the stock of net household debt across the wealth distribution. Net household debt is defined as gross household debt owed minus household debt held as a financial asset. As the figure shows, the top 1% of the wealth distribution experienced a significant decline in their net debt position from 1980 onward, which reflects the accumulation of a significant amount of household debt held a as a financial asset. In contrast, the bottom 90% experienced a large increase in their net debt position, as gross debt owed increased substantially but household debt held as a financial asset was relatively stable. To a large degree, the rich lend to the non-rich through the financial system.

In the United States, higher income inequality since 1980 has been associated with an increasing tendency of the rich to finance the borrowing of the non-rich. The strong correlation between rising top income shares and rising household debt in many advanced economies suggests that this could be a global phenomenon. These facts motivate us to build a model which focuses on lending by higher wealth households to lower wealth households.

There has also been a wave of financial liberalization and deregulation in advanced economies that started in the 1980s, perhaps best summarized by the first sentence in Rajan (2006): "In the last 30 years, financial systems around the world have undergone revolutionary change." The focus of this study in particular is on liberalization in household financial markets. Such liberalization began in the late 1970s and 1980s, and was focused on mortgage markets in particular. In a book focused primarily on the United States and the United Kingdom, Ball (1990) details aspects of what

Figure 5: The saving glut of the rich



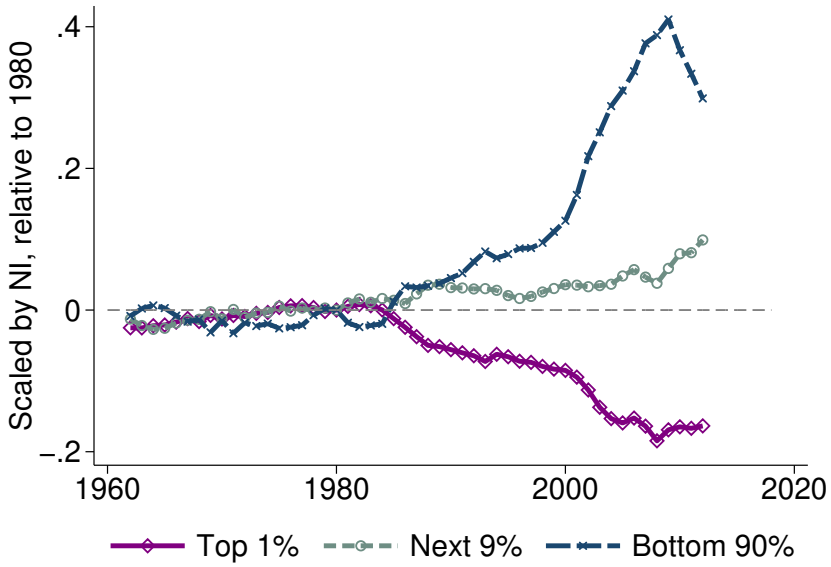
This figure comes directly from Mian et al. (2019). The figure focuses on the United States. The saving of the top 1% is defined to be the after-tax income of the top 1% of the income distribution minus personal consumption of the top 1% of the income distribution, scaled by national income. Net saving of the other two groups is similarly defined.

he calls the "mortgage finance revolution." This revolution entailed new institutions entering the market, the rise of securitization, and the loosening of constraints on the level of borrowing.

As an example, research by Bokhari, Torous, and Wheaton (2013) in the United States shows a large rise in the fraction of mortgages originated with an LTV ratio above 0.9 from 1986 to 1995. The fraction of mortgages originated with a debt-to-income ratio above 0.4 also increased substantially during this period. Research focused on the housing boom from 1998 to 2006 documents a substantial loosening of payment to income constraints (e.g., Greenwald 2018) and a loosening of credit standards more generally (e.g., Favilukis, Kohn, Ludvigson, and Van Nieuwerburgh 2012).

One tension in the literature noted by Justiniano et al. (2017) is that in most standard models, a loosening of such borrowing constraints should be associated with an increase in interest rates. This is obviously counter-factual, as noted above: since the 1980s, advanced economies have seen a loosening of borrowing constraints and a large decline in interest rates. Many models exploring how a loosening of borrowing constraints affected debt and house prices from 2000 to 2006 handle this tension by assuming some other exogenous force pushed down interest rates during this period (e.g., Favilukis, Ludvigson, and Van Nieuwerburgh 2017, Garriga, Kydland, and Šustek 2018, and Greenwald 2018). This tension is a key motivation behind the model developed in this study; in the model developed below, a loosening of borrowing constraints directly leads to both higher debt and lower interest rates without requiring some other force to be operative.

Figure 6: Rich households and the rise in household debt



This figure comes directly from Mian et al. (2019). The figure focuses on the United States. It shows net household borrowing by the U.S. household sector across the wealth distribution. It uses the debt and asset shares from Saez and Zucman (2016) to construct net household borrowing. Net household borrowing is defined as gross household borrowing minus household debt held as a financial asset. All series are scaled by national income, and the 1980 level is subtracted.

2.3 Saving across the income distribution

The final set of facts that motivate the model concern the idea that the rich save a larger fraction out of income, and out of permanent income in particular. This is an old idea in economics, showing up in the writings of John Atkinson Hobson, Eugen Bohm von Bawerk, Irving Fisher, and John Maynard Keynes among others. A recent body of research has resurrected this idea.

For the United States, Dynan et al. (2004) use panel data from the Survey of Consumer Finances to show that individuals in the top 20% of the income distribution have saving rates out of lifetime income that are substantially larger than the rest of the population. The saving rates for the top 1% and top 5% out of income are estimated to be particularly large, almost 5 times larger than at the median of the distribution (0.51 compared to 0.11 out of a dollar of income).

The recent study of Straub (2019) uses the Panel Study of Income Dynamics to estimate an elasticity of consumption to lifetime income, and it finds evidence of concavity in this relationship. The study finds estimates of this elasticity of 0.7, and the statistical analysis easily rejects the canonical benchmark model which assumes that the elasticity of consumption with respect to lifetime income across the lifetime income distribution is 1.

Fagereng et al. (2019) use administrative panel data from Norway to estimate saving rates out of income across the wealth distribution. The study finds substantially higher saving rates for wealthier households, with saving rates for the top 1% estimated to be almost double the saving

rates for the median of wealth distribution.

A key assumption of the model developed below will be non-homotheticity of saving behavior across the wealth distribution. This assumption is supported by a growing body of evidence from the empirical literature. Furthermore, as is shown below, the adoption of this empirically supported assumption into an otherwise standard model yields new insights that help explain the set of facts shown above.

3 Model

Motivated by the facts above, this section develops a model of indebted demand. The model is an infinite-horizon endowment economy, populated by two separate dynasties of agents trading real assets, or "Lucas trees". Each such asset produces one unit of the consumption good each instant. There are Y real assets in total, where we normalize $Y = 1$ for now.

The agents in the two dynasties share the same preferences and only differ by their endowments of the real asset. For reasons that will become clear below, we refer to the poorer ("non-rich") dynasty as *borrowers* $i = b$ and wealthier ("rich") dynasty as the *savers* $i = s$. At any point in time, there is a mass $\mu^b = 1 - \mu$ of borrowers and a mass $\mu^s = \mu$ of savers. We sometimes simply refer to all dynasties of type i as "agent" i.

The model is intentionally kept simple and tractable for now; several extensions can be found in Section 9 and Appendix C.

3.1 Preferences

We begin by setting up the agents' common preferences. An agent in dynasty $i \in \{b, s\}$ dies at rate $\delta > 0$ and discounts future utility at rate $\rho > 0$. At any date t, total consumption by dynasty i is c_t^i and total wealth by dynasty i is a_t^i. The average type-i agent therefore consumes c_t^i/μ^i and owns wealth a_t^i/μ^i, with a utility function given by

$$\int_0^\infty e^{-(\rho+\delta)t} \left\{ \log\left(c_t^i/\mu^i\right) + \frac{\delta}{\rho} v(a_t^i/\mu^i) \right\} dt \tag{1}$$

Utility is derived from two components: each instant, utility over flow per-capita consumption c_t^i/μ^i; and, arriving at rate δ, a warm-glow bequest motive captured by the function $v(a)/\rho$. We assume for now that upon death, the entire asset position of an agent is bequeathed to a single newborn offspring, ruling out any cross-dynasty mobility. The consolidated budget constraint of all agents of type i is therefore simply given by

$$c_t^i + \dot{a}_t^i \leq r_t a_t^i \tag{2}$$

where r_t is the endogenous flow interest rate at date t.

The function $v(a)$ represents a crucial aspect of this model. It characterizes the relationship

between wealth of a dynasty and its saving rate. To see this, consider the special case where $v(a) = \log a$. This choice of $v(a)$ makes the preferences in (1) homothetic: the borrower and saver dynasties would exhibit the exact same saving behavior, just scaled by their current wealth positions.[6]

This is no longer true as $v(a)$ deviates from $\log a$. To capture such deviations, we define $\eta^i(a)$ to be the marginal utility of v relative to log, that is,

$$\eta^i(a) \equiv a/\mu^i \cdot v'(a/\mu^i). \tag{3}$$

$\eta^i(a)$ is defined in per-capita terms and therefore depends on i. $\eta^i(a)$ plays an important role in the analysis, especially $\eta^s(a)$ which henceforth we also denote by $\eta(a)$. When $\eta^i(a)$ is constant, for instance $\eta^i(a) = 1$ when $v(a) = \log a$, utility is homothetic as marginal utility of bequests and marginal utility of consumption are proportional. When $\eta^i(a)$ is decreasing, the marginal utility of bequeathing assets decreases relatively more quickly than the marginal utility of consumption; in this case, wealthier agents save relatively less. When $\eta^i(a)$ is increasing, marginal bequest utility decays more slowly than that of consumption, implying that wealthier agents have a stronger desire to save. This latter, *non-homothetic* case is the most plausible case intuitively and best supported case empirically as mentioned in Section 2.3 above. This is the case focused on by the model.

3.2 Borrowing constraint

The two types of agents in the model maximize utility (1) subject to the budget constraint (2) and a borrowing constraint. To formulate the borrowing constraint, we separate type-i agents' wealth positions into two components: their *real assets* h^i_t and their *financial assets*, which if negative, we refer to as *debt* d^i_t, that is,

$$d^i_t = h^i_t - a^i_t \tag{4}$$

We assume for now that the agents' debt is adjustable-rate long-term debt which decays at some rate $\lambda > 0$.

Agents of type i own a total endowment of $\omega^i \in (0,1)$ of real assets (trees), where $\omega^s + \omega^b = 1$. Within this endowment, we assume that $\ell^i < \omega^i$ are *pledgeable real assets* (e.g. land, houses, businesses, etc) and $\omega^i - \ell^i$ are *non-pledgeable real assets* (e.g. human capital). Denoting

$$p_t \equiv \int_t^\infty e^{-\int_t^s r_u du} Y ds \tag{5}$$

the price of a single real asset (tree), type-i agents' total wealth in real assets is

$$h^i_t = p_t \omega^i \tag{6}$$

[6] In fact, given the normalization with $1/\rho$, $v(a_t) = \log a_t$ corresponds to an altruistic bequest motive in an equilibrium where $r_t = \rho$.

and type-i agents' pledgeable wealth is $p_t \ell^i$. Henceforth we assume that pledgeable wealth (per capita) is equal across agents, $\ell^b/(1-\mu) = \ell^s/\mu$, and denote $\ell \equiv \ell^b$, so that the only source of heterogeneity between the two agents are the endowments ω^i, or equivalently, the agents' real-asset earning shares. We assume that savers' per capita earnings exceed those of borrowers, $\omega^s/\mu^s > \omega^b/\mu^b$.

We impose the borrowing constraint

$$\dot{d}_t^i + \lambda d_t^i \leq \lambda p_t \ell \tag{7}$$

where, due to asset market clearing, $d_t^s + d_t^b = 0$.[7] We henceforth focus exclusively on the borrowers' total debt position $d_t \equiv d_t^b$, the key state variable for our analysis. d_t essentially captures how much borrowers have spent beyond earnings $\omega^b Y$ in the past, and how much of a debt burden borrowers need to service in the future.

According to borrowing constraint (7), new debt issuance $\dot{d}^i + \lambda d_t^i$ is bounded above by the value of pledgeable assets. As we emphasize below, most of our results do not rely on the specific constraint (7). In fact, we will often allow for a more general constraint of the form

$$\dot{d}_t^i + \lambda d_t^i \leq \lambda p_t \ell(\{r_s\}_{s \geq t}) \tag{8}$$

where $\ell = \ell(\{r_s\}_{s \geq t})$ is a general function of current and future interest rates. With slight abuse of notation, we denote by $\ell(r)$ the function $\ell(\{r_s\}_{s \geq t})$ in the case where rates are constant $r_s = r$ for all $s \geq t$. For example, in an economy with housing, $\ell(r) \propto \frac{r}{1+\varphi r}$ for some $\varphi > 0$. When borrowers are subject to uninsurable idiosyncratic income risk a la Bewley-Aiyagari, we show that, under mild conditions, they endogenously choose an average debt position of the form $p\ell(r)$ when $r_t = r$ in all periods. All examples share the characteristic that the constraint on debt $p\ell(r)$ is higher (or more relaxed) for a lower interest rate r.

3.3 Homothetic benchmark

Throughout the analysis, we compare the model to a *homothetic benchmark* model. This model is characterized by $\eta(a) = 1$, so that agents' preferences are indeed homothetic. Moreover, to avoid a continuum of steady state equilibria in the homothetic model, we allow the saver's discount factor to be different from, and smaller than, the borrower's discount factor, $\rho^s < \rho$. Heterogeneity of discount factors is not assumed in the non-homothetic model.

3.4 Equilibrium

We formally define equilibrium next.

[7] We multiply the right hand side by λ so that in a steady state, the constraint simplifies to $d^i \leq p\ell$. This is immaterial to our results.

Definition 1. Given initial debt $d_0 = d_0^b$ a *(competitive) equilibrium* of the model are sequences $\{c_t^i, a_t^i, d_t^i, h_t^i, p_t, r_t\}$ such that both agents choose $\{c_t^i, a_t^i\}$ to maximize utility (1) subject to the budget constraint (2) and the borrowing constraint (7); d_t^i is determined by (4); h_t^i is determined by (6); p_t is determined by (5); and financial markets clear at all times, that is, $d_t^s + d_t^b = 0$. The goods market clears by Walras' law.

A *steady state (equilibrium)* is an equilibrium in which $c_t^i, a_t^i, d_t^i, h_t^i$ and r_t are all constant.

A steady state with debt d is *stable* if there exists an $\epsilon > 0$ such that any equilibrium with initial debt $d_0 \in (d - \epsilon, d + \epsilon)$ has debt converge back to d, $d_t \to d$. All other steady states are *unstable*.

For illustrative purposes, we use the following parametrization of the non-homothetic and homothetic models throughout the paper. We interpret the saver as comprising the top 1% earning households of the economy, i.e. with a population share $\mu = 0.01$, and the borrower as the bottom 99%. We choose the saver's real (non-bond) earnings share ω^s to match a pre-tax income share of 20%. Subtracting the return to 150% household debt and government debt to GDP with a 4% interest rate, we arrive at $\omega^s = 14\%$. The discount factor ρ will roughly correspond to the borrower's discount factor and is set to a value of $\rho = 0.1$.

We directly calibrate $\eta(a) = v'(a/\mu)a/\mu$, letting it take the following simple functional form, $\eta(a) = 1 + \tilde{a}^{-1} \log\left(1 + e^{a-\tilde{a}}\right)$. From (3), this then determines $v(a)$ and $\eta^b(a)$. Given its functional form, $\eta(a)$ is strictly increasing with $\eta'(a) \in (0, 1)$ and admits the homothetic model as a special case if $\tilde{a} \to \infty$. The calibration uses the borrowing constraint (7). ℓ and $\tilde{a}$ are jointly pinned down to match a household-debt-to-GDP ratio of 80% and a steady-state interest rate $r = 4\%$, which corresponds to the average real return on wealth (rather than the real safe rate in the data). For the homothetic benchmark model, we achieve the interest rate target by assuming $\rho^s = 4\%$. The parameter λ has two roles. It determines the maturity of debt, and it governs the speed of the debt response to relaxations in the borrowing constraint. Until Section 9.6, we effectively assume the first role away by focusing exclusively on adjustable-rate debt, that is, debt has zero duration. Thus, we calibrate λ in line with its second role. To do so, we compare the impulse response of household debt over GDP to a monetary policy shock implied by our model to that commonly found to identified monetary policy shocks. In particular, we feed a 4-year 50 basis points interest rate cut (see (17)) into the Section 7 variant of our model and compare the household debt / GDP response at its peak (after 2 years) with the year-2 response found in Jordà, Schularick, and Taylor (2015). This procedure implies that $\lambda = 1$. Finally, we set $\delta = 0.1$. As we demonstrate in Section 10 below, this produces a reasonable local slope of the saving supply curve of around $dr/d\log a \approx -0.032$.

3.5 Discussion

What does $\eta(a)$ capture? The literature has pointed out numerous examples of why agents might care about their wealth beyond its value for financing their own consumption behavior. This includes bequests (De Nardi 2004), out-of-pocket medical expenses in old age (De Nardi,

French, Jones, and Gooptu 2011), utility over status (Cole, Mailath, and Postlewaite 1992), inter-vivos transfers (Straub 2019), and numerous other reasons that are documented in other papers in the literature (e.g. Carroll 2000, Dynan et al. 2004, Saez and Stantcheva 2018, Boar 2018). Moreover, the shape of $\eta(a)$ could also capture the idea that assets other than a given stock of liquid assets or human capital are illiquid and therefore being saved "by holding" (Fagereng et al. 2019). Due to its stylized nature, our model is based entirely on bequests. We suspect microfounded models of these other reasons for wealth accumulation among the rich behave similarly to ours.

Aggregate scale invariance. Our baseline non-homothetic model, with increasing $\eta(a)$, is not scale-invariant in aggregate. If aggregate output Y doubles, all agents are wealthier and thus, in line with a rising $\eta(a)$, would raise their savings by more than double. Taken at face value, this would generate rising saving rates in all growing economies, which seems counterfactual.

We believe that the key to understanding why a non-homothetic model, which breaks *individual scale invariance*, need not necessarily break *aggregate scale invariance* is that many of the motives for non-homothetic saving are relative to some economy-wide aggregates. For example, bequests are likely especially valued among the rich if they are large relative to the average wage or income in the economy, relative to the price of land, or relative to the average bequest. This suggests that $\eta(a)$ should really be thought of as a function of a relative to Y or aggregate wealth, i.e. $\eta(a/Y)$ or $\eta(a/(a^b + a^s))$. To incorporate this idea and reduce clutter in the formulas, we henceforth assume that η is of the form $\eta(a/Y)$ but output Y is normalized to 1, $Y = 1$.

4 Downward-sloping saving supply and Indebted Demand

We next characterize the equilibria in our model. We focus exclusively on equilibria in which debt is positive $d_t > 0$, that is, the borrower actually borrows and the saver actually saves.[8] Such equilibria always exist in our economy.

4.1 Saving supply curves

The saver's Euler equation is given by

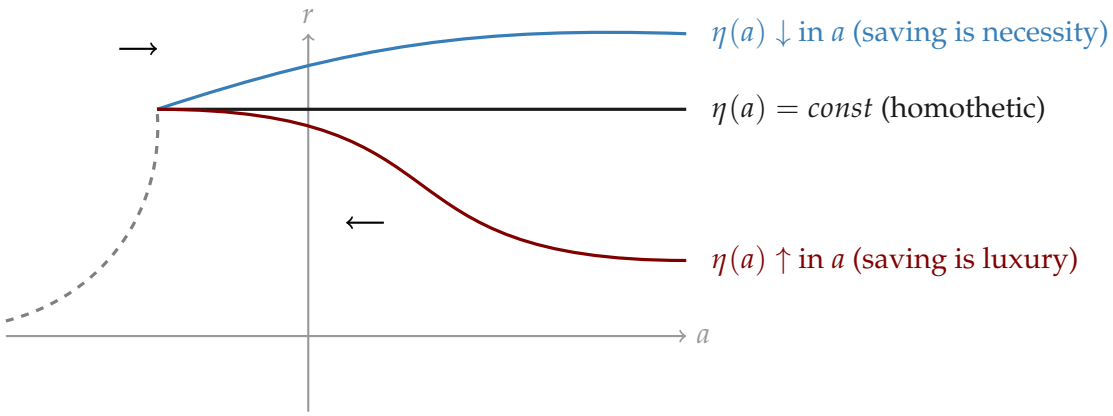
$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta \frac{c_t^s}{\rho a_t^s} \eta(a_t^s). \tag{9}$$

In a steady state, quantities and prices are constant, so that the budget constraint reads $c^s = ra^s$. Substituting this into the Euler equation (9), we find our first key steady state equilibrium condition

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(a^s)}. \tag{10}$$

[8] If we assumed away heterogeneity in per-capita real earnings ω^i/μ^i, "borrowers" and "savers" become entirely symmetric, so that for each equilibrium in which borrowers borrow and savers save, strictly speaking there would also exist one in which savers borrow and borrowers save. With a realistic gap in ω^i/μ^i, this possibility vanishes.

Figure 7: Long-run saving supply curves.



This equation can be understood as a long-run *saving supply curve*, describing the saving behavior of a possibly non-homothetic saver. Specifically, for each wealth position a^s, it describes the interest rate r that is necessary for a saver to find it optimal to keep his wealth constant at a^s.

The crucial object that determines the shape of the saving supply curve is the function $\eta(a)$, as illustrated in Figure 7. In the homothetic benchmark economy, where $\eta(a)$ is equal to 1 (or another constant), we recover the standard infinitely elastic long-run supply curve, $r = \rho$. When $\eta(a)$ falls in a, in which case saving is treated as a necessity by agents, the saving supply curve slopes *up*. Finally, and most importantly, when $\eta(a)$ rises in a and thus saving is treated as a luxury, the saving supply curve slopes *down*. This is the key property of our non-homothetic model. We summarize it in the following proposition.

Proposition 1. *The long-run saving supply curve* (10) *is downward sloping if and only if wealthier agents have a greater marginal propensity to save, that is, when $\eta(a)$ is increasing in a.*

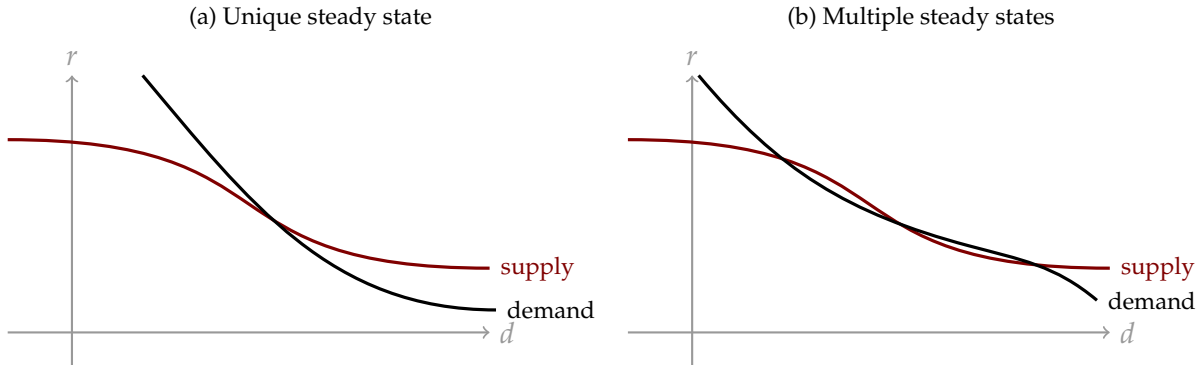
What is the intuition behind the negative slope? In a model in which wealthier agents save at higher rates, the higher an agent's wealth is, the *lower* must be the the return on wealth for the agent to be indifferent between saving and dis-saving.

To give an extreme example, consider the following stylized model of Bill Gates's saving behavior. Bill Gates consumes a fixed amount $c = \bar{c}$ and saves everything else, not caring about his wealth, perhaps so long as it does not shrink below some threshold. Above that threshold, Bill Gates's saving supply curve is nothing other than $r = \bar{c}/a$ and therefore slopes down in his wealth.

4.2 Steady state equilibria

Steady states are the intersections of saving supply curves with *debt demand* curves, as we characterize in the following proposition.

Figure 8: Steady state equilibria.



Proposition 2. *Any steady state with positive debt $d > 0$ corresponds to an intersection of a long-run saving supply curve*

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(\omega^s/r + d)} \tag{11}$$

with a long-run debt demand curve

$$d = \frac{\ell(r)}{r}. \tag{12}$$

Proposition 2 shows that the relevant saving supply curve is that of the saver, and that the relevant debt demand curve is given by the borrowing constraint of the borrower. We write both conditions in terms of the interest rate (return on wealth) r and debt d. Similar to models with discount rate heterogeneity, the borrower is up against the borrowing constraint in the steady state. As explained above, we focus on the natural case where the debt demand curve slopes down in r, that is, $\ell(r)/r$ is strictly decreasing in r, and where $\eta(a)$ is strictly increasing in a.

We illustrate the two curves and their intersections in Figure 8. As the two panels show, it might be the case that there is a single intersection, and thus a unique steady state equilibrium, or it might be the case that there are multiple intersections, and thus steady state multiplicity.

Multiple steady states. How can there be multiple steady states in this economy? Consider the high debt, low interest rate steady state in Figure 8 (b). Ceteris paribus, the high debt level leads to a large debt service burden for borrowers, and a corresponding permanent stream of debt service payments from borrowers to savers. If savers were adhering to the permanent income hypothesis (PIH), they would spend this additional income stream one-for-one. This would raise aggregate demand and hence the equilibrium interest rate sufficiently to incentivize borrowers to deleverage, which is why a high-debt, low-r equilibrium is impossible with PIH savers. In our model, however, savers do not satisfy the PIH. Instead, savers spend the additional income stream for debt service costs less than one-for-one. This causes there to be weaker demand, rationalizing a low equilibrium interest rate.

When are multiple steady states possible in this model? This crucially depends on two elasticities: $\epsilon_\eta \equiv \eta'(a)a/\eta(a)$ and $\epsilon_\ell \equiv \ell'(r)r/\ell(r)$. The first one, ϵ_η, governs the strength of non-

homotheticity in the model and thus the slope of the saving supply curve. The second one, ϵ_ℓ, captures how elastic borrowing constraints are to interest rates. When ϵ_ℓ is negative—as we show in Appendix C.2 can happen in models where borrowers keep buffer stocks—the debt demand curve can locally become sufficiently flat to allow for multiple steady states.

Indebted demand. At the core of this logic, and in fact at the core of many of the results in this paper, is that an increase in debt service costs, ceteris paribus, may lower aggregate demand, as we show in the following result.

Proposition 3 (Indebted demand)**.** *Starting from a steady state and holding r fixed, any permanent increase in debt service costs by dx moves aggregate spending on impact by*

$$dC = dc^s + dc^b = -\frac{\rho+\delta}{r}\frac{1}{2}\left(1 - \sqrt{1 - 4\left(1 - \frac{r}{\rho+\delta}\right)\epsilon_\eta}\right) dx \tag{13}$$

where $\epsilon_\eta \equiv \frac{\eta'(a)a}{\eta(a)}$ is a measure of the degree of non-homotheticity in preferences. In particular, aggregate spending falls, $dC < 0$, iff $\epsilon_\eta > 0$.

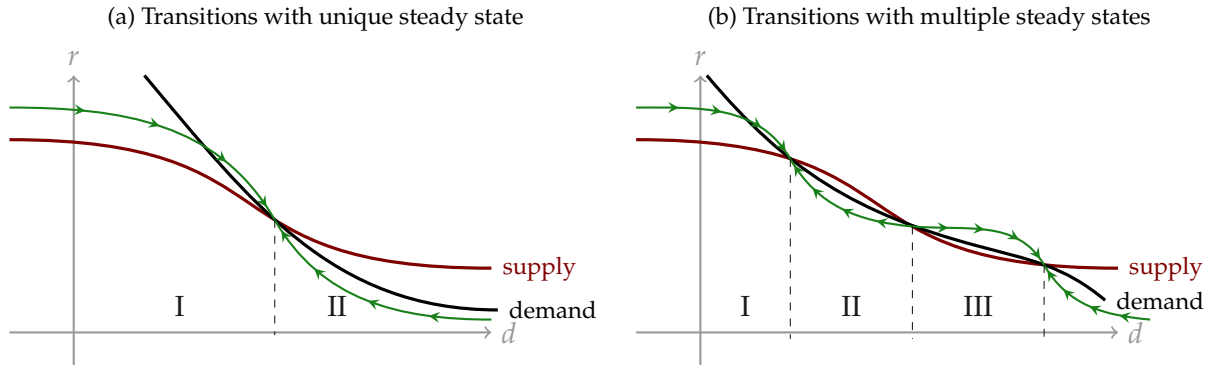
Proposition 3 highlights that any increase in debt service costs weighs down on aggregate demand, $dC < 0$, precisely if and only if $\epsilon_\eta > 0$, a phenomenon we henceforth call *indebted demand*. Why can demand be indebted in our model? The increase in debt service costs dx passes through to the borrower's spending one-for-one, $dc^b = -dx$. But, since savers have a greater saving propensity, their spending initially rises by less than the transfer, $dc^s < dx$. Thus, aggregate spending falls, $dC < 0$. For the goods market to clear, the equilibrium interest rate must therefore fall. As this mechanism only relies on heterogeneity in saving propensities out of a small permanent transfer dx, any model that generates such heterogeneity along the wealth distribution exhibits the property of indebted demand.

As a side remark, observe that our model predicts a positive consumption $dC > 0$ response to a reduction in debt service payments, $dx < 0$. Such a reduction could occur in reality when households refinance their mortgages to bring down the interest rate ("rate refi"). In homothetic models, as $\epsilon_\eta = 0$, there is no effect of "rate refis" on aggregate consumption (Greenwald 2018), which quantitatively limits their macroeconomic relevance (Berger, Milbradt, Tourre, and Vavra 2018). In non-homothetic models, such as ours, "rate refis" could instead have sizable consequences for aggregate consumption.

Steady states in the homothetic economy. In the homothetic economy, the interest rate in the unique steady state is necessarily pinned down by the saver's discount rate, $r = \rho^s$. The associated debt level is then $d = \ell(\rho^s)/\rho^s$.

Analytical example. The steady state conditions in Proposition 2 can be solved analytically in a simple special case, where $\eta(a)$ is a linear function in the relevant region of the state space and

Figure 9: Equilibrium transitions in the baseline model.



Note. Red: saving supply curve. Black: debt demand curve. Green: transitional dynamics.

$\ell(r) = \ell$ is constant. For example, assuming $\eta(a) = a$, there is a unique stable steady state in this region, with interest rate

$$r = \rho + \delta - \delta/\rho(\omega^s + \ell)$$

and associated debt level

$$d = \frac{\ell}{\rho + \delta - \delta/\rho(\omega^s + \ell)}.$$

4.3 Transitions

Having characterized the set of steady state equilibria in this economy, we now explore the entire set of equilibria, including the transitions along which the economy approaches the steady state(s). For this part, we focus on a simplified borrowing constraint, where $\ell(\{r_s\}_{s \geq t}) = \ell(p_t)$ only depends on current and future interest rates through the price of real assets p_t. We still require that $\ell(p_t)p_t$ increases in p_t, that is, the demand for debt is downward sloping in the interest rate. It turns out that our economy admits a unique equilibrium transition path for any given initial level of debt $d_0 > 0$, despite the possibility of multiple steady states. We verified this using phase diagrams, confirmed it in our numerical simulations, and provide an analytical local uniqueness & existence result in Appendix B.

Figure 9 illustrates the set of equilibria in the state space for two different positions of the saving supply and demand curves. In Panel (a), there is a single steady state. As can be seen, for each initial debt position d_0, there exists a unique transition path to the steady state. If d_0 is to the left of the steady state (region I), the borrower levers up and is able to front-load more consumption before hitting the borrowing constraint; if d_0 is to the right of the steady state (region II), the borrower has a desire to deleverage, pushing interest rates down. The magnitude of the decline in interest rates depends on the degree of non-homotheticity, as when there is more non-homotheticity, the saver spends less of the additional debt payments.

In Panel (b), the saver raises consumption by so little, that right at the middle steady state, interest rates fall sufficiently to help the borrower make his debt payments and still demand enough

for the goods market to clear. To the right of that steady state (region III), it is no longer just interest rates that adjust in order to clear the goods market. In fact, at first, the borrower increases his debt *even further* to finance his spending, moving away from the middle steady state. As the borrower approaches the borrowing constraint, however, and the speed at which new debt can be taken out slows, interest rates need to fall increasingly rapidly to keep the borrower's debt payments manageable. Ultimately, the borrower is at the debt limit and interest rates have fallen enough to make such high debt burdens relatively affordable for the borrower.

5 Inequality, Financial Liberalization, and Indebted Demand

The framework developed in the previous section may help understand the underlying factors that contributed to the simultaneous increase in debt and decline in interest rates that many advanced economies have experienced in the past 40 years. We explore this next.

5.1 Inequality

Long run. A rise in income inequality can be captured in the model as an increasing share ω^s of real earnings going to savers, and a corresponding fall in $\omega^b = 1 - \omega^s$. The following proposition characterizes the long-run implications of rising income inequality.

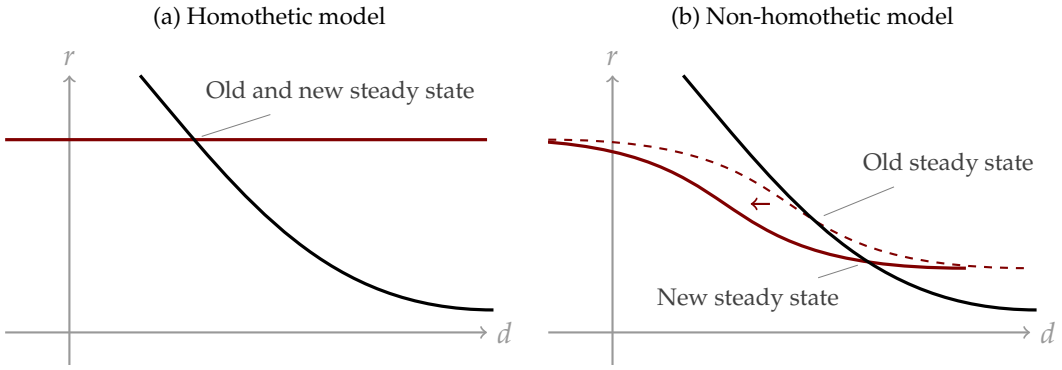
Proposition 4. *An increase in income inequality (greater ω^s) unambiguously reduces long-run equilibrium interest rates and raises household debt. In the homothetic model, long-run interest rates and household debt are unaffected by rising income inequality.*

The long-run implications of rising inequality are best understood in the context of our model's saving supply and debt demand curves. Figure 10 shows supply and demand diagrams for the homothetic economy in panel (a), and the non-homothetic economy in panel (b). In the homothetic case, the supply curve is pinned down by the discount factor and thus independent of inequality. The demand curve is also independent of inequality, and therefore the old and new steady states coincide.

In the non-homothetic economy, savers have a greater propensity to save. Thus, if they earn a greater share of income, total saving increases. This manifests itself in a shift of the saving supply curve (11) to the left. As Proposition 4 shows, and as is illustrated in Figure 10, the equilibrium interest rate falls and the amount of debt in the economy rises in response to the rise in inequality. The non-homothetic model thus helps rationalize the close empirical association between the rise in inequality and the simultaneous increase in debt and decline in interest rates across advanced economies (see Section 2).

Transition. This is confirmed numerically in Figure 11, which simulates the responses of a homothetic and a non-homothetic economy to a permanent increase in income inequality. Since

Figure 10: The effects of rising income inequality for long-run saving supply and debt demand.



this is a perfect-foresight transition, borrowers begin raising their debt levels already early on, in anticipation of lower interest rates in the future, which raises interest rates initially.

Interestingly, the transition shows a hump-shaped profile in the debt service ratio, which ultimately falls back to its pre-transition value. This demonstrates that the debt service ratio is a highly endogenous object, which can be low either when there is little debt (early in the transition), or, when there is high debt but interest rates are low (late in the transition).

5.2 Financial liberalization

As detailed in Section 2.2, another recent trend in advanced economies has been financial liberalization and deregulation. Several authors have pointed out that in standard homothetic models, if anything, this should have significantly increased interest rates, at odds with the recent experience of the United States and other advanced economies (e.g., Favilukis et al. 2017, Garriga et al. 2018, and Greenwald 2018). What is the effect of financial liberalization on debt and interest rates in the model developed here?

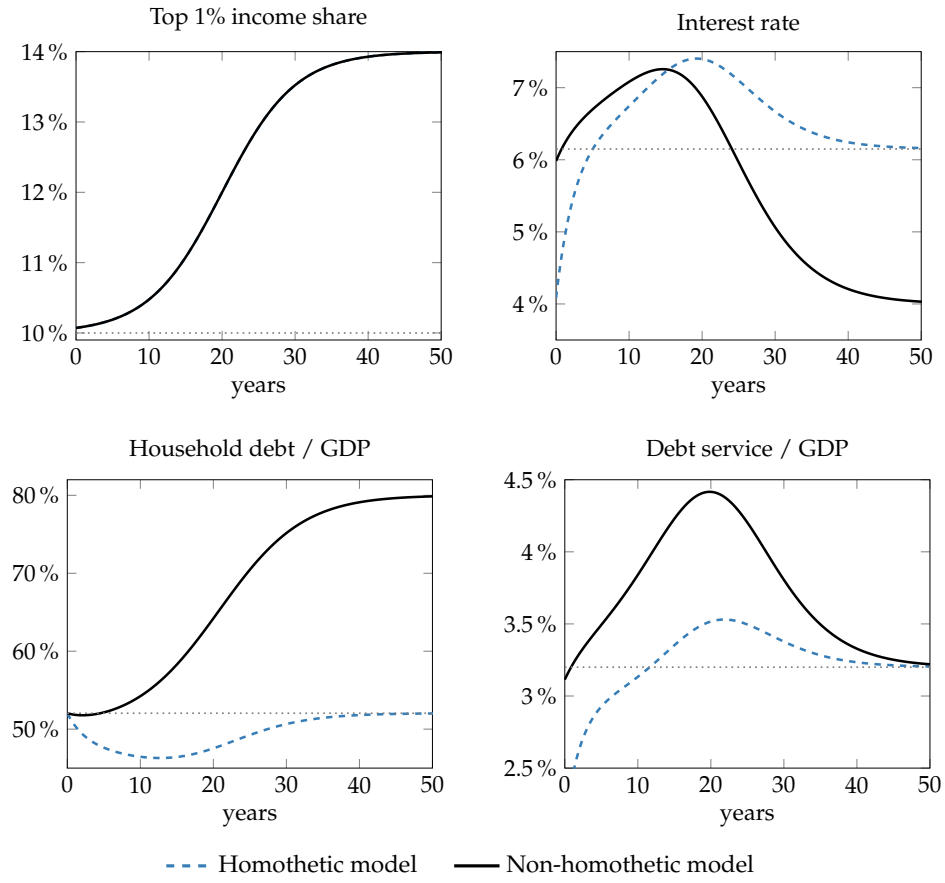
Financial liberalization is modeled as an increase in the pledgability $\ell(r)$ of real assets.[9] We find the following result.

Proposition 5. *Financial liberalization (greater $\ell(r)$) unambiguously reduces long-run equilibrium interest rates and increases household debt. By contrast, in the homothetic model, long-run interest rates are unaffected by financial liberalization and household debt rises by less.*

Figure 12 plots the implied shifts in the debt demand curve, as well as the qualitative transitional dynamics from the old steady state to the new one (green arrows). As can be seen, in both homothetic and non-homothetic models, the *short-run* saving supply curve is upward-sloping: the loosening of borrowing constraints initially *increases* interest rates, as household demand grows in response. In the long run, the saving supply curve is flat in the homothetic benchmark model, so that there is no long run effect of liberalization on interest rates.

[9] In our housing application in Section C.2.1 we show that an increase in the LTV ratio corresponds to an increase in $\ell(r)$.

Figure 11: Rising income inequality and debt.



Note. Plots show transitions from a steady state with $\omega^s = 0.10$ (where $r = 6.1\%$, $d = 55\%$, see dotted gray line) to our calibrated steady state with $\omega^s = 0.14$. The dashed blue line corresponds to the homothetic model with $\rho^s = 0.061$.

In the non-homothetic model, by contrast, the increased debt burden ultimately leads to a fall in equilibrium interest rates, which then again contributes to increasing debt further. Interestingly, this resolves the puzzle faced in the literature: the model shows that financial liberalization might only put upward pressure on interest rates in the short run, and it actually contributes to a declining interest rate in the long-run.

5.3 Persistent effects of temporary shocks

The idea of indebted demand can be sufficiently strong to imply that a *temporary* shock that leads to greater debt accumulation *permanently* shifts the equilibrium of the economy. This can happen in economies that admit multiple steady states.

To see how this works for the case of temporary financial liberalization, consider Figure 13. Before the shock, the economy is assumed to sit in the high-r, low-debt steady state. As the shock hits, raising ℓ, demand for borrowing expands and the black curve shifts out. Now, only a single steady state is left, and as the economy moves towards it, the level of debt rises.

Figure 12: The effects of financial liberalization for long-run saving supply and debt demand.

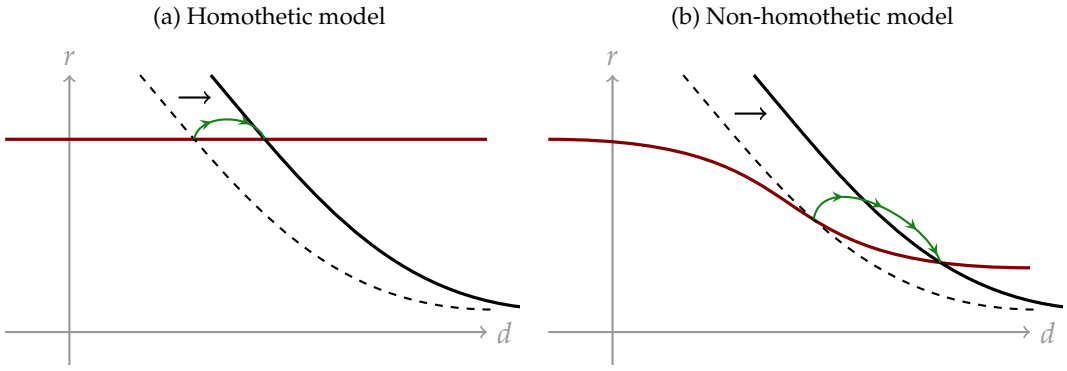
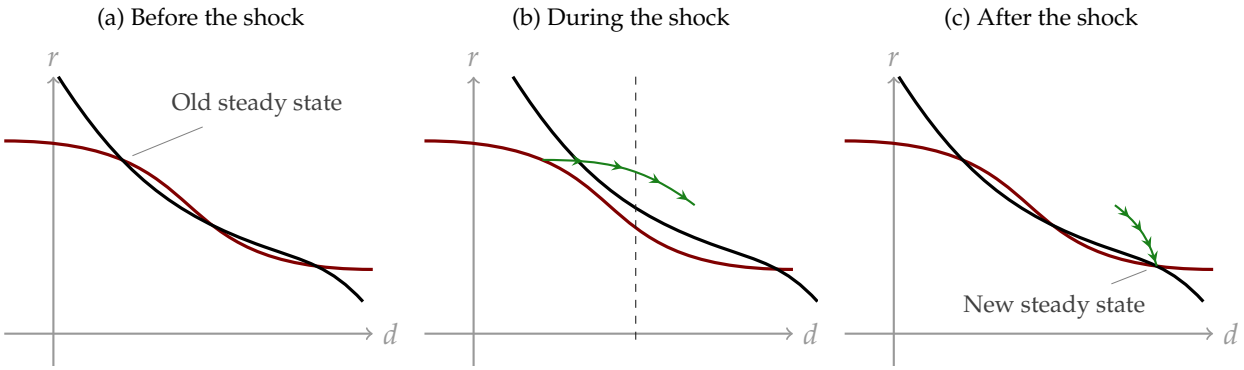


Figure 13: Permanent effects of temporary financial liberalization in an economy with multiple steady states



Once debt is sufficiently high that the economy crossed the gray dashed line, a reduction in ℓ back to its previous level does not suffice to bring the economy back to its previous steady state. Instead, debt is so high at that point, that the only way for the economy to generate enough demand to clear the goods market is for the interest rate to fall further, stimulating yet more debt. Thus, debt will remain permanently elevated, and interest rates permanently subdued.

While this effect relies on steady state multiplicity, the broader point here is that the model generates an important *asymmetry*: accumulating debt (in response to some shock) may be significantly faster than de-cumulating debt thereafter. This asymmetry is present even without steady state multiplicity.

6 Public deficits and indebted government demand

The previous section showed how private deficits lead to the accumulation of household debt, and thus indebted demand. A considerable portion of the recent increase in debt, however, has been public debt. This is shown in Figure 14, which shows the evolution of household debt and government debt separately for advanced economies. According to conventional wisdom, a rise in government debt exerts upward pressure on interest rates (e.g., Blanchard 1985, Aiyagari and McGrattan 1998).

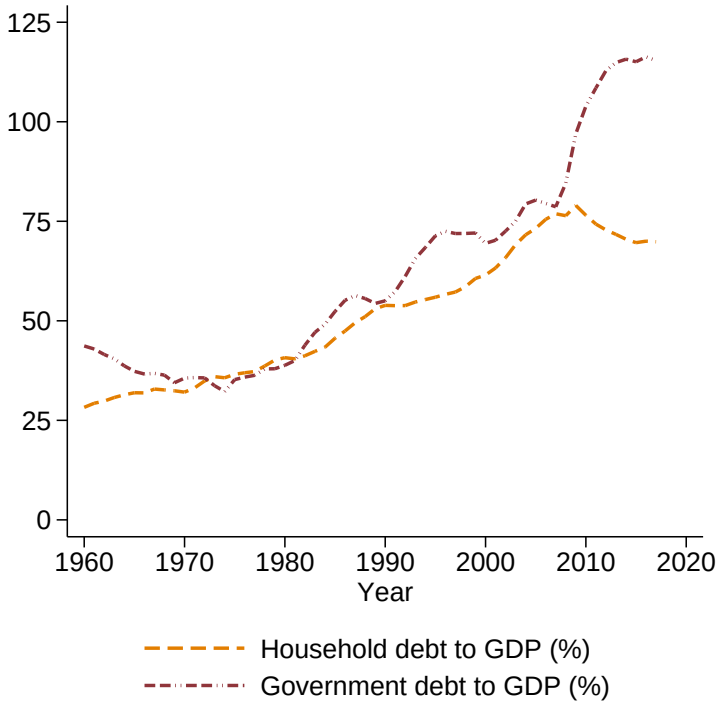
What are the implications of a rise in government debt in our non-homothetic model? This section focuses on this question in the context of the equilibrium introduced in Section 3.4, in which output is fixed at $Y = 1$, and therefore interest rates endogenously adjust to clear the goods market. Section 8 revisits fiscal policy in the presence of nominal rigidities and a binding zero-lower bound.

We consider fiscal policy in this section, as well as other policies in subsequent sections, mainly from a positive perspective, documenting its effects in our model without any notion of welfare. The reason for this choice is that there are several real-world considerations that are first-order for welfare but outside our model. For example, high debt levels and low interest rates are often associated with instability and risk-taking in the financial sector, and thus raise the likelihood of a financial crisis (e.g. Reinhart and Rogoff 2009, Schularick and Taylor 2012, Stein 2012). Low interest rates may also reduce growth (Liu, Mian, and Sufi 2019). One important dimension of welfare an extension of our model can speak to is the potential for a liquidity trap when the (natural) interest rate is sufficiently depressed. We discuss the welfare implications of our model in this context in Section 8.

6.1 Incorporating the government

We introduce a standard government sector into the economy. Specifically, the government is assumed to choose a debt position B_t, government spending G_t, and proportional income taxes τ_t^i

Figure 14: Debt by households and governments in advanced economies



Series are cross-country averages, weighted by real GDP in 1970. The countries in the sample are Australia, Canada, Finland, France, Germany, Italy, Japan, New Zealand, Norway, Portugal, Spain, Sweden, United States and United Kingdom. Data come from the IMF Global Debt Database, the Jorda-Schularick-Taylor Macrohistory Database and the New Zealand Treasury.

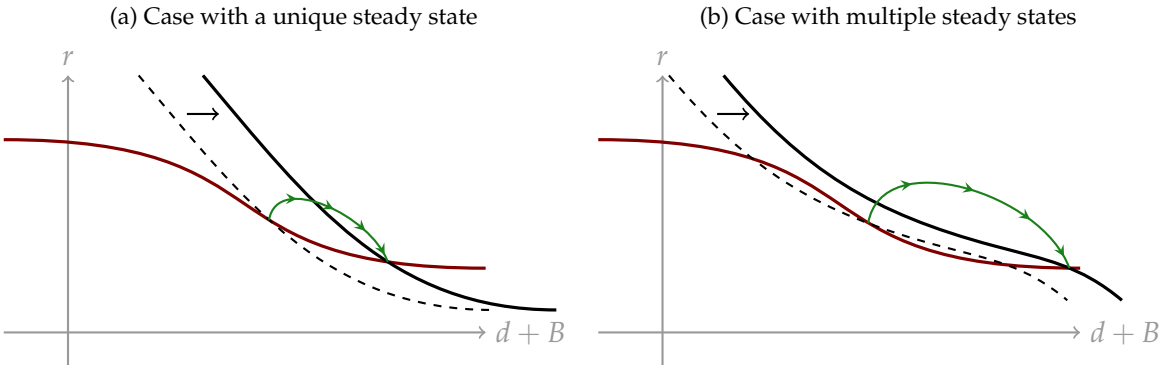
on agent i such that its flow budget constraint

$$G_t + r_t B_t \leq \dot{B}_t + \tau_t^s \omega^s + \tau_t^b \omega^b$$

is satisfied at all times t.[10] Ponzi schemes are ruled out by assuming that B_t is bounded above, uniformly in t. For simplicity, government spending is treated here as purchases of goods that are either wasted or—which is equivalent for the purposes of this current positive exercise—enter agents' utilities in an additively-separable form. Taxes are assumed to enter agents' real wealth in the natural way, $r_t h_t^i = (1 - \tau_t^i)\omega^i + \dot{h}^i$. Taking fiscal policy as given, the definition of a competitive equilibrium is unchanged from before, with the exception that the bond market clearing condition is now given by $d_t^b + d_t^s + B_t = 0$.

[10] An important question is whether government bonds in fact pay the same interest rate as other assets. To address this, we propose an extension in Section 9.3 that explicitly allows for a spread between government bond yields and the return on other wealth r_t.

Figure 15: Long-run effect of an increase in public debt B.



6.2 Long-run effects of fiscal policy

We begin by studying the long-run effects of fiscal policy, focusing on constant policies (G, B, τ^s, τ^b). In this case, the equilibrium conditions for steady state equilibria are given by

$$r = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(a)} \tag{14}$$

$$a = (1 - \tau^s)\frac{\omega^s}{r} + \frac{\ell(r)}{r} + B \tag{15}$$

Equations (14) and (15) characterize the long-run implications of fiscal policy. We are specifically interested in increases in B, financed by raising taxes τ^i on both agents or cutting expenditure G; as well as tax-financed increases in G. This yields the following result.

Proposition 6 (Long-run effects of fiscal policy on interest rates and debt.)**.** *In the long run,*

a) larger government debt ($B \uparrow$) depresses the interest rate ($r \downarrow$) and crowds in household debt ($d \uparrow$).

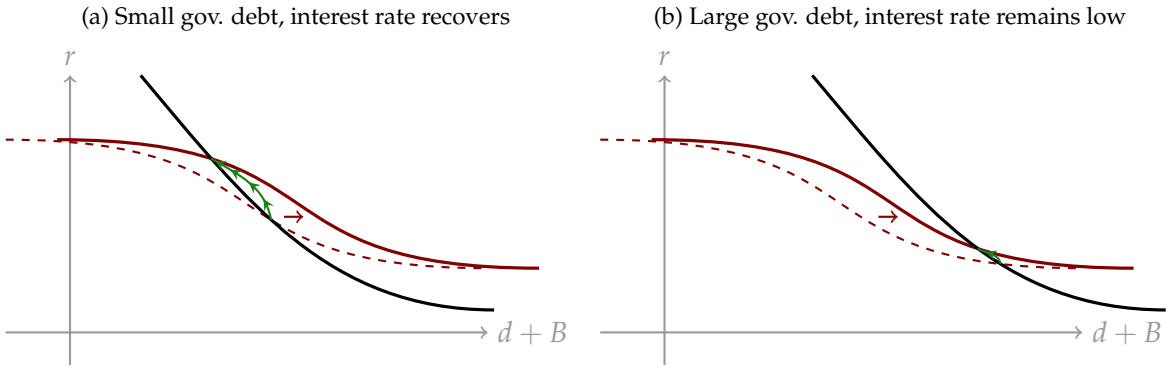
b) tax-financed government spending ($G \uparrow$) increases the interest rate ($r \uparrow$) and crowds out household debt ($d \downarrow$).

c) fiscal redistribution ($\tau^s \uparrow$, $\tau^b \downarrow$) increases the interest rate ($r \uparrow$) and crowds out household debt ($d \downarrow$).

With a homothetic saver, none of these policies have any effect on the long-run interest rate and on household debt.

An intuition for these results can be explained with the help of Figure 15. Consider the first policy in Proposition 6, and assume the greater debt level B is entirely paid for by a reduction in government expenditure G. As savers do not raise their consumption one-for-one with the increase in debt service payments by the government, aggregate demand would fall were it not for a reduction in interest rates. Graphically, the policy corresponds to an increase in the economy's total demand for debt, $d + B$, which shifts out to the right (Panel (a) in Figure 15). Notably, the reduction in interest rates will crowd-*in* household debt.

Figure 16: The lock-in effect of government debt.



Conversely, tax-financed government spending and fiscal redistribution reallocate resources from the saver to a "spender", which is either the government—in the case of government spending—or the borrower—in the case of redistribution. Such resource reallocation would raise aggregate demand were it not for an increase in interest rates.

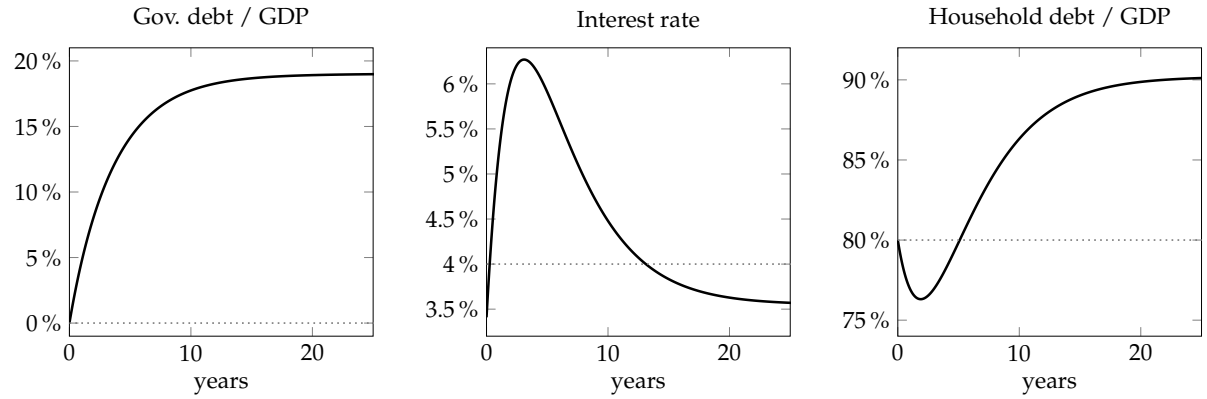
Proposition 6 and Figure 15 prescribe a very different role for fiscal policy in influencing interest rates than is typically assumed. What helps in the long-run is first and foremost redistribution between spenders and savers, not redistribution of taxes over time in the form of public deficits, which, paradoxically, *lowers* long-run interest rates even further as government demand becomes indebted.

Japanification and the "lock-in" effect of government debt. Fiscal policy might not only be relevant for shifting a given steady state, but also for the existence of steady states. For example, when an economy is in a high-debt low-r steady state, then any of these policies affect not only the interest rate associated with that steady state, but also the likelihood that another steady state—that is, one with low-debt and a higher interest rate—exists. Panel (b) of Figure 15 illustrates this for the case of the first policy. In this example, the increase in public debt is sufficiently large to rule out the existence of the low-debt steady state. The intuition is even more pronounced than before. Especially if interest rates are high—as in the low-debt steady state—any significant amounts of public debt constitute a drag on aggregate demand as long as their interest payments are partly covered by borrowers.

A similar effect appears in our economy even in case of a single steady state. We illustrate this in Figure 16. The greater government debt B is, the less the interest rate rises in response to greater τ^s, lower ω^s or lower ℓ. For large B, a same-sized increase in r would require a larger adjustment in government spending or taxes, reducing aggregate demand. This is why the interest rate response to the same changes in τ^s, ω^s or ℓ is smaller when B is large.

We interpret this finding as a sort of "lock-in" effect. High government debt "locks in" low equilibrium interest rates, a transition to higher rates becomes less likely. This effect is potentially an important factor in understanding the Japanese experience. Any material increase in Japanese

Figure 17: Deficit spending.



Note. Plot shows response of non-homothetic economy to temporary government spending shock (AR(1) spending path with $g_0 = 5.5\%$ of GDP and a half-life of 2 years). Fiscal rule: $\tau^i = r_\infty B$ where r_∞ is eventual steady state interest rate.

interest rates would burden the government with a significant debt service cost, to finance which a sizable fiscal adjustment would be necessary. That, however, would weigh on demand, making the interest rate increase unlikely in the first place.[11]

Fiscal policy in the analytical example. We can illustrate the effects of fiscal policy in the analytical example in Section 4.2. It is straightforward to obtain the steady state given a set of tax policies (G, B, τ^s, τ^b)

$$r = \frac{\rho + \delta - \frac{\delta}{\rho}\left((1-\tau^s)\omega^s + \ell\right)}{1 + \frac{\delta}{\rho}B} \qquad \text{and} \qquad d = \frac{\left(1 + \frac{\delta}{\rho}B\right)\ell}{\rho + \delta - \frac{\delta}{\rho}\left((1-\tau^s)\omega^s + \ell\right)}.$$

We see that greater redistribution and greater spending (both financed through greater τ^s) raises r and lowers d. Greater public debt B lowers r and crowds in d. Finally, greater B reduces the sensitivity of r to changes in τ^s, ω^s or ℓ.

6.3 Short run effects

Despite its novel long-run effects of government debt, the model predicts conventional short-run effects of debt-financed fiscal stimulus programs (whether through government spending or tax cuts). As before, in terms of saving supply and debt demand curves, this is due to an upward-sloping short-run saving supply curve. We illustrate this in Figure 17, which plots the dynamic response of the economy to temporary deficit-financed government spending. There is a short-run rise in the natural interest rate, lasting about as long as the fiscal stimulus itself. During this time, household debt is crowded out by higher interest rates. Afterward, however, the interest rate declines, falling below its original level and allowing debt to increase.

[11] This effect is amplified by the fact that in Japan, large scale asset purchases have shortened the duration of total government (incl. central bank) liabilities.

The opposite of the dynamics in Figure 17 would materialize in response to an austerity program, causing a short-term reduction in the natural rate but raising natural rates in the longer term.

In practice, this suggests a dilemma for economies that are currently stuck in a steady state with low interest rates and high public debt, but, for some reason outside our model, wish to raise rates going forward. If they expanded public debt even further, rates would rise in the short run, but subsequently fall again, even below their already undesirable previous levels. If they contracted public debt, rates will fall in the short-run—possibly below the effective lower bound, causing a recession—despite the prospect of greater rates in the long run.

A possible middle ground in such a scenario may be a gradual reduction in public debt, as long as economies are still able to do so without hitting the effective lower bound. This will allow those governments to save some of their "fiscal policy ammunition" for the future.

7 Monetary policy and the limited ammunition effect

In the previous section, we saw that deficit-financed fiscal stimulus reduces natural interest rates in our model in the long run. In the presence of an effective lower bound, this then implies that there is only a limited amount of fiscal stimulus "ammunition" that policy makers can use before interest rates hit the lower bound. In this section, we show a similar "limited ammunition" property for monetary policy, by which monetary policy not only affects current output, but also the natural interest rate going forward.

Introducing monetary policy. To do this, it is necessary to move away from an endowment economy, where output Y is fixed at 1 and interest rates endogenously adjust to clear goods markets. Instead, we now let output, henceforth denoted by $\hat{Y}$, adjust endogenously in response to monetary policy, that is, exogenous changes to interest rates $\{r_t\}$. We continue to denote potential output by $Y = 1$.

To allow output $\hat{Y}$ to be endogenous, we assume it is produced using labor, $\hat{Y} = N$, supplied by both types of agents. As in Werning (2015), total hours N are allocated across agents using a simple allocation rule, namely type-i households supply a total of $n^i = \omega^i N$ hours. We assume that prices are flexible and nominal wages are sticky.[12] This implies that real earnings by type i are $\omega^i \hat{Y}$, and thus that the income distribution is unaffected by aggregate output $\hat{Y}$. We call the allocation with $\hat{Y} = N = 1$ the *natural allocation*.

To ensure continued tractability of the model, we treat monetary policy as controlling the real rate directly.[13] Transmission of monetary policy then works in the standard way: monetary policy

[12] The precise formulation of wage stickiness is irrelevant for our purposes as we make the simplifying assumption below that monetary policy sets the real interest rate directly.

[13] One can easily relax this assumption by introducing a Taylor rule and a Phillips curve, without affecting the results in this section. Note that our economy is locally determinate under a real-rate rule.

changes the interest rate, which steers aggregate demand and thus output (and labor) in the economy. With exogenous interest rates $\{r_t\}$, the goods market now clears because $\{\hat{Y}_t\}$ is endogenous. We define as *natural interest rates* the sequence of real interest rates $\{r_t^n\}$ that achieves the potential (or natural) allocation, that is, it implements a path of aggregate demand at potential, $\hat{Y}_t = Y = 1$.

Monetary policy shocks. To gain the most intuition about the behavior of the model, we consider two types of monetary policy shocks, which hit the economy at a stable steady state with (c^b, c^s, r, d). The first type is a T-period long interest rate reduction, before a reversal back to the original interest rate

$$r_t = \begin{cases} \hat{r} & t \leq T \\ r & t > T \end{cases}. \tag{16}$$

The second type also starts with a T-period long interest rate reduction, but then reverses back to the path of *natural* interest rates

$$r_t = \begin{cases} \hat{r} & t \leq T \\ r_t^n & t > T \end{cases}, \tag{17}$$

ensuring that for any $t > T$ after the intervention $\hat{Y}_t = Y = 1$ in this case.

Debt and ammunition. We begin by studying monetary policy shocks of the first kind. In our model, they stimulate the economy via two separate channels. First, they relax borrowing constraints and encourage borrowers to use additional household debt for spending (*debt channel*). Second, through income and substitution effects, they provide incentives for savers to spend more (*saver channel*). To study the role of these channels for monetary transmission, we define the following present values

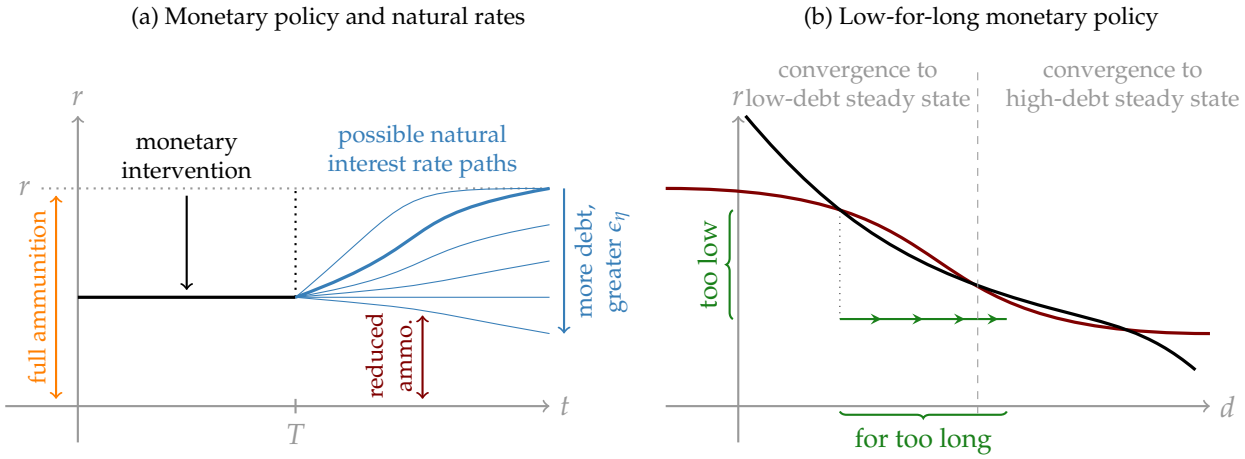
$$PV^\tau(\{c_t^i\}) = \int_0^\tau e^{-\int_0^t r_s ds} c_t^i dt - \int_0^\tau e^{-rt} c^i dt$$
$$PV^\tau(\{\hat{Y}_t\}) = \int_0^\tau e^{-\int_0^t r_s ds} \hat{Y}_t dt - \int_0^\tau e^{-rt} Y dt$$

The first is the increase in the present value of agent i's spending until period τ; the second is the increase in the present value of output until period τ. The next proposition shows that the two channels have asymmetric implications for the path of aggregate demand.

Proposition 7. *The τ-period present value of the output response to the monetary policy shock (16) is given by*

$$PV^\tau(\{\hat{Y}_t\}) = \underbrace{\frac{1}{\omega^s} PV^\tau(\{c_t^s\})}_{\text{saver channel}} + \underbrace{\frac{1}{\omega^s} e^{-\int_0^\tau r_s ds} (d_\tau - d)}_{\text{debt channel}}. \tag{18}$$

Figure 18: Debt limits the ammunition of monetary policy



In the long run ($\tau = \infty$), the present value of output is entirely determined by the saver channel,

$$PV^{\infty}(\{\hat{Y}_t\}) = \frac{1}{\omega^s} PV^{\infty}(\{c_t^s\}). \tag{19}$$

This implies that any output stimulus generated by debt accumulation necessarily weighs negatively on output going forward.

Proposition 7 shows that the two channels of monetary transmission have vastly different implications for the path of output. While the saver's consumption response to the interest rate change affects output permanently, the debt channel only has a temporary effect. In fact, as any additional debt taken out by borrowers eventually has to be serviced or even repaid, future demand is reduced by an active debt channel. Put differently, when monetary policy is used to stimulate the economy, any resulting increase in demand that is debt-financed does not sustainably raise demand and will contribute to reduced demand in the future.

One implication of the result in Proposition 7 is that if monetary policy is accommodative now, it endogenously limits its room to be accommodative in the future, as it also needs to ensure that the accumulated debt burden from past interventions does not cause a shortfall in demand. The accurate object summarizing the "room to be accommodative in the future" is the natural rate of interest r_t^n. Our next proposition studies the effect that monetary policy has on r_t^n.

Proposition 8. *To first order, a monetary policy shock as in (16) or (17) causes debt to rise and the natural rate to fall, $r_t^n < r$ for any t. For a given increase in debt, the natural rate falls by more (as measured by $\int_s^\infty e^{-r(t-s)} r_t^n dt$ for any s) if there is more non-homotheticity (as measured by the elasticity ϵ_η of η); and if interest rates are lower (lower $\hat{r}$) for longer (larger T).*
If the economy has another stable steady state with higher debt and lower interest rate, and if $\hat{r}$ is sufficiently low and T sufficiently large, the natural rate falls permanently and does not converge back to r.

Accommodative monetary policy systematically reduces natural interest rates in our model,

and thus endogenously limits the "ammunition" that is available to monetary policy in the future. This happens because in the presence of a greater debt burden, natural rates r_t^n cannot possibly be equal to r after $t = T$ as this would tighten borrowing constraints, and lead to the borrower severely contracting demand. Therefore, natural rates r_t^n are below r at least for some time after $t = T$ while the borrower deleverages (see Figure 18(a)).

This logic operates even absent non-homotheticity. However, in a homothetic model, the convergence process $r_t^n \to r$ is sped up significantly by the fact that the saver's consumption rises significantly due to the increase in the saver's permanent income, pushing the natural rate up, closer to r. In a non-homothetic model such as ours, an additional reason for a decline in r_t^n emerges—indebted demand—which leads to lower natural rates and a significantly reduced convergence rate back to r. In other words, non-homotheticity and indebted demand significantly aggravate the "limited ammunition" property of monetary policy (see Figure 18(a)). This can be sufficiently strong to permanently lower natural rates. Such behavior occurs when the economy exhibits multiple steady states, and the monetary intervention is "too low for too long", that is, $\hat{r}$ is sufficiently low and T sufficiently long (see Figure 18(b)).

Relationship to literature. A number of economists have recently emphasized how the effectiveness of monetary policy interventions can be reduced by past interventions (e.g. Eichenbaum, Rebelo, and Wong 2019, Berger et al. 2018, McKay and Wieland 2019). In this paper, we do not consider consecutive interventions. Instead, we focus on how much "ammunition" in terms of the natural interest rate, a single intervention costs. To give an analogy with the IS curve, we focus on the effect of monetary policy on the future level of the IS curve as opposed to its slope.

Practical implications for the conduct of monetary policy. Monetary policy can have long-lasting (if not permanent) effects on natural rates through the accumulation of debt. This should be taken into account when contemplating the force with which to respond to different kinds macroeconomic shocks. Temporary shocks to borrowers' ability to borrow for instance—e.g. during a financial crisis—can be met with aggressive monetary easing as debt is unlikely to rise in this context. However, when reacting to shocks that do not directly affect borrowers' demand for debt—e.g. negative shocks to business investment as during the 2001 recession—aggressive monetary policy could lead to significant and persistent increases in household debt, and therefore reduce monetary policy ammunition going forward.

When used in conjunction with macroprudential policies that are designed to keep debt in check, thereby dampening the debt channel, monetary policy can be used more aggressively. That way, the economy does not merely "pull forward" demand through debt, demand which it then lacks in the future.

The limited power of forward guidance. In times of "limited ammunition" due to reduced natural interest rates, one may wonder, whether the central bank can effectively use unconventional

policies such as forward guidance as a way to bring in new ammunition. To study the extent to which this is possible, we consider monetary policy shocks as in (17) for long periods T. We have the following result.

Proposition 9. *Consider the monetary policy shock* (17) *as* $T \to \infty$. *In the homothetic model, output explodes,* $\hat{Y}_t \to \infty$, *in every period* t. *In the non-homothetic model output converges to a well-defined finite limit,*

$$\hat{Y}_t \to \hat{Y}_t^{T=\infty} = (\omega^s)^{-1} \left(\hat{r} + \lambda + \lambda \ell(\hat{r})/\omega^s\right) (d_\infty - d_0)\, e^{-\lambda(1+\ell(\hat{r})/\omega^s)t} + \hat{Y}_\infty^{T=\infty} \tag{20}$$

where

$$\hat{Y}_\infty^{T=\infty} = Y \frac{\hat{r}}{\omega^s + \ell(\hat{r})} \cdot \eta^{-1} \left(\frac{\rho}{\hat{r}} (1 + \rho/\delta) - \rho/\delta\right), \qquad d_\infty = \ell(\hat{r}) \frac{\hat{Y}_\infty^{T=\infty}}{\hat{r}}.$$

In the homothetic economy, forward guidance is infinitely powerful, mitigating any concerns about "limited ammunition".[14] In the non-homothetic economy, by contrast, forward guidance faces decreasing returns in moving current output $\hat{Y}_0$ as the horizon T grows, so much so that even a permanent interest rate cut implies a finite response of $\hat{Y}_0$. It also has well-defined long-run effects (Proposition 9). Due to indebted demand, these long-run effects can even push output below potential. When the economy admits multiple stable steady states, $\lim_{t\to\infty} \hat{Y}_t^\infty$ can be below Y.

8 Debt trap

Thus far, we have implicitly assumed that the interest rate associated with our model's steady state(s) is above the effective lower bound (ELB). But when there is a sufficiently large debt burden in the economy, it might well be the case that this assumption is violated. This is what we consider next.

To do so, let $\underline{r} > 0$ be the (real) effective lower bound for our interest rate. It needs to be positive as we take r to be the real return on wealth in our model. To get a number for $\underline{r}$, we propose take an estimate of the real return on wealth during the ELB period—e.g. around 5% according to Farhi and Gourio (2018)—which then needs to be de-trended by productivity growth—e.g. 1.5%—to obtain $\underline{r}$. In this example, $\underline{r} = 3.5\%$.

The liquidity trap steady state. We focus on a steady state (r, d) with a natural interest rate r below the effective lower bound, $r < \underline{r}$, that is,

$$\underline{r} < \rho \frac{1 + \delta/\rho}{1 + \delta/\rho\, \eta\, (\omega^s/\underline{r} + d)} \tag{21}$$

[14] See the extensive literature on the forward guidance puzzle, e.g. Del Negro, Giannoni, and Patterson (2015), McKay, Nakamura, and Steinsson (2016), Werning (2015), Farhi and Werning (2019), Gabaix (2019), Angeletos and Lian (2018), Bilbiie (2019), Acharya and Dogra (2019).

Figure 19: Falling into the liquidity trap in response to greater income inequality.



Note. These plots simulate an increase in income inequality from $\omega^s = 0.14$ to $\omega^s = 0.15$ in the non-homothetic economy. The black line assumes an effective lower bound at $\underline{r} = 3.5\%$ (see text).

In this case, the economy gives rise to a stable liquidity trap steady state, which we henceforth also call *debt trap.*

Proposition 10. *In the presence of an effective lower bound with* (21)*, there exists a stable liquidity trap steady state ("debt trap"), in which output is reduced to*

$$\hat{Y} = Y \frac{\underline{r}}{\omega^s + \ell(\underline{r})} \cdot \eta^{-1} \left(\frac{\rho}{\underline{r}} (1 + \rho/\delta) - \rho/\delta \right) < Y \tag{22}$$

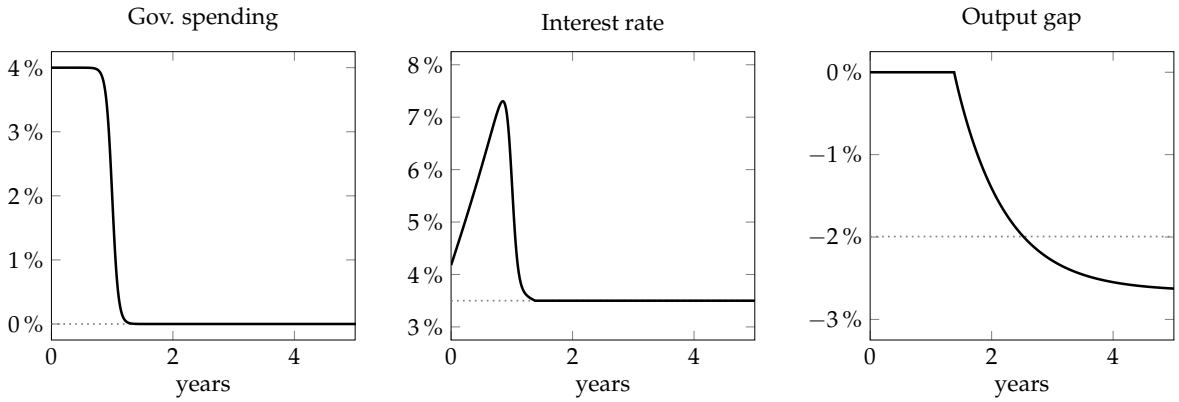
In the debt trap, household debt is high and output is permanently reduced due to indebted demand.[15] Thus, in our model, household debt is the key endogenous state variable that determines whether or not an economy is able to generate sufficient demand to avoid a liquidity trap. The presence of such an endogenous state variable sets our model apart from other recent papers modeling secular stagnation, e.g. Benigno and Fornaro (2019), Caballero and Farhi (2017), Eggertsson et al. (2019), Ravn and Sterk (2018). Moreover, the liquidity trap here is indeed a *trap*, meaning associated with a stable steady state, rather than a relatively brief episode driven by household deleveraging, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017).

Falling into the debt trap. Just like convergence to a steady state, our economy can fall into the debt trap. Figure 19 shows the dynamics in a simulation when in response to rising income inequality the only stable steady state has a natural rate below the effective lower bound $\underline{r}$. Thus, interest rates gradually fall until they hit the effective lower bound. At that point, a negative output gap (i.e. a recession) begins.

Interestingly, the debt trap drags the economy into the trap even before it would have otherwise crossed the lower bound (blue dashed line in Figure 19). The reason for this is that both

[15] This result would be amplified in the presence of a standard Phillips curve, as in that case, a reduction in output would endogenously lead to weaker inflation and a tighter effective lower bound $\underline{r}$.

Figure 20: Being pulled back into the liquidity trap after deficit spending.



Note. The plot shows the response of a non-homothetic economy to a temporary government spending shock (4% of GDP for one year). Fiscal rule: $\tau^i = r_\infty B$ where r_∞ is eventual steady state interest rate. The dotted line is the previous steady state.

agents anticipate a recession in the debt trap, and thus, in an attempt to smooth consumption, cut back on their spending already in advance. This, however, only accelerates the decline in the natural rate, pushing it below the effective lower bound $\underline{r}$ sooner.

Fiscal policy in the debt trap. Once an economy finds itself in the debt trap, how does it get back out again? We introduce the government exactly as in Section 6 and obtain the following result.

Proposition 11. *With proportional income taxes τ^s, τ^b on the saver and borrower, and a steady state level of public debt B, output in the debt trap steady state is given by*

$$\hat{Y} = Y \frac{\underline{r}}{(1-\tau^s)\omega^s + \ell(\underline{r})} \cdot \left[\eta^{-1} \left(\frac{\rho}{\underline{r}} (1 + \rho/\delta) - \rho/\delta \right) - B \right] \tag{23}$$

In particular: greater public debt lowers output in the steady state, greater redistribution through taxes raises output, and greater tax-financed government spending raises output.

This result is the mirror image of Proposition 6, except that at the effective lower bound, adjustments in the natural rate correspond to adjustments in aggregate demand and output. One of the most striking predictions is that deficit-financed fiscal policy—conventionally thought to be the best remedy against a liquidity trap—instead digs the economy even deeper into it. We illustrate this behavior in Figure 20. Temporarily, deficit spending raises demand, bringing output closer to potential, or even above potential. Eventually, however, as the public debt burden rises, along with the associated taxes to service it, demand falls again, and the economy finds itself back in the debt trap. To policymakers of the conventional view, running large deficits, the liquidity trap might thus indeed feel like a "trap", pulling the economy back into the trap after every round of deficit spending.

Redistributive tax policies and welfare. By contrast, Proposition 6 suggests that redistributive tax policies—greater τ^s—can raise output $\hat{Y}$, reducing the severity of the liquidity trap. What would be consequences of such a policy for the two agents? As long as the effective lower bound is still binding, steady state consumption and wealth are given by

$$c^s = \underline{r}\eta^{-1}\left(\frac{\rho}{\underline{r}}(1+\rho/\delta)-\rho/\delta\right)Y, \qquad a^s = \frac{c^s}{\underline{r}}, \qquad c^b = \hat{Y} - c^s. \tag{24}$$

The only object that is endogenous to the tax choice is $\hat{Y}$. Remarkably, greater redistribution therefore leaves steady-state c^s and a^s entirely unaffected, while boosting consumption of borrowers. The reason for this result is that the income loss of greater taxation of savers' incomes is exactly offset by rising overall incomes. This can happen in the liquidity trap due to aggregate demand externalities, as in Korinek and Simsek (2016) and Farhi and Werning (2016).

What are the implications for welfare? While (24) only holds across steady states, observe that any policy change that raises $\hat{Y}$ also relaxes the borrowing constraint and thus generates additional consumption for borrowers during the transition period. Moreover, one can show that consumption and wealth of savers are constant at the levels in (24) throughout the transition. Thus, our model implies that, in the liquidity trap, greater redistribution is Pareto-improving.

Financial regulation in the debt trap. A commonly prescribed medicine for economies with high debt burdens is macroprudential policy designed to bring down debt. As before, we think of such financial regulation as a reduction in ℓ. Equation (22) immediately implies that such a policy raises demand and output $\hat{Y}$ in the long run, mitigating the recession. However, during the period of deleveraging, the economy goes through a significant short-run bust. This emphasizes that debt is best reduced by reducing saving supply rather than the demand for debt.

Borrower bailouts. An alternative way to deal with a liquidity trap caused by high levels of debt is a bailout of borrowers. We consider two such bailout strategies. A *government-financed bailout of size* $\Delta > 0$ is an immediate increase in public debt B by Δ, and an immediate reduction in private debt d by the same amount Δ. We define a *debt jubilee of size* $\Delta > 0$ as an immediate reduction in both private debt d and saver's assets a^s by Δ. We assume that the public debt increase after a bailout is not entirely paid for by taxing the saver (or else it would be equivalent to a debt jubilee). The following result lays out the long-run implications of bailout and debt jubilee policies:

Proposition 12 (Bailouts and debt jubilee.)**.** *If the debt trap is the only steady state, a government-financed bailout lowers output in the long run, while a debt jubilee has no effect in the long run. If there are multiple steady states and the economy is in the low-r high-debt steady state, a debt jubilee can raise long-run output and interest rates if its size is sufficiently large.*

A debt jubilee amounts to a jump in the state variable d_t. With a unique stable steady state, this cannot have a long-run effect. In other words, borrowers would get themselves into debt

again. A government-financed bailout increases long-run public debt and therefore lowers long-run output, as in Proposition 11.

Things are somewhat more subtle with steady state multiplicity. Here, if Δ is sufficiently large, a debt jubilee, i.e. a jump in d_t, can put the economy on a permanently different trajectory, towards a different steady state. This also works for a government-financed bailout, with the caveat that when public debt is too large, another steady state, one with higher r and lower d, might cease to exist (see our discussion in Section 6.2).

9 Extensions

Our baseline model was intentionally kept simple. Next we discuss several extensions to our baseline model (see Appendix C for more). We begin with two generalizations of the supply side of our economy, introducing growth and investment.

9.1 Growth

To introduce growth into our model, we assume that each real asset (tree) now bears $Y_t = e^{gt}$ fruits, instead of $Y = 1$. For any quantity x (e.g. consumption, assets, asset prices) denote by $\hat{x}$ its de-trended version, that is, $\hat{x}_t \equiv e^{-gt}x_t$. As usual, the de-trended budget constraint of type i agents is then

$$\hat{c}_t^i + \dot{\hat{a}}_t^i \leq (r_t - g)\hat{a}_t^i.$$

Crucially, as we continue to assume that preferences are over wealth relative to income, $v(a_t^i/Y_t)$, as discussed in Section 3.5, we can express preferences (1) as function of de-trended variables,

$$\int_0^\infty e^{-(\rho+\delta)t}\left\{gt + \log\left(\hat{c}_t^i/\mu^i\right) + \frac{\delta}{\rho}v(\hat{a}_t^i/\mu^i)\right\}dt$$

where clearly the term gt is irrelevant for consumption choices. Finally, the de-trended price of a tree is

$$\hat{p}_t \equiv \int_t^\infty e^{g(s-t)-\int_t^s r_u du}ds.$$

Following exactly the same steps as before, we can derive two conditions that characterize the steady state(s) with growth. As it turns out, both equations are simply shifted versions of the conditions without growth. The saving supply and debt demand curves now read

$$r - g = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(\omega^s/(r-g) + d)} \qquad \text{and} \qquad d = \frac{\ell(r-g)}{r-g} \tag{25}$$

where we now write ℓ as a function of $r - g$ since common microfoundations (e.g. those in Appendix C.2) would precisely predict such a relationship. Equation (25) shows that growth is orthogonal to the steady states in our model. It merely shifts both curves vertically in parallel.

Similarly, one can show that growth does not change the dynamics of the economy either, after de-trending.

9.2 Investment

Having studied the effects of growth, we now turn to a different extension of the supply side of our economy, namely allowing for capital and investment. To do so, we allow output Y to be produced from three factors, capital K and both types of agents' labor supply L^i, and we write the net-of-depreciation production function as[16]

$$Y = F(K, L^b, L^s).$$

We assume that without loss that Y has constant returns to scale; otherwise we include a fixed factor owned by savers and/or borrowers. Thus, total income Y can be split up into income going to savers and income going to borrowers. As we assume that labor supplies L^i are fixed, the income shares only depend on the level of capital K, which itself is pinned down by the interest rate,

$$F_K = r.$$

Further, in our economy, only savers hold capital. The agents' income shares are then functions of the interest rate and given by

$$\omega^s(r) = \frac{F_K K}{F} + \frac{F_{L^s} L^s}{F} \qquad \text{and} \qquad \omega^b(r) = \frac{F_{L^b} L^b}{F} = 1 - \omega^s(r).$$

With these income shares, we can now characterize our economy's steady states as

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(\omega^s(r)/r + d)} \qquad \text{and} \qquad d = \frac{\ell(r)}{r}. \tag{26}$$

Crucially, $\omega^s(r)$ is now possibly a function of r. As we demonstrate in the following proposition, the shape of $\omega^s(r)$ depends on the (Allen) elasticity of substitution $\sigma \equiv F_{L^b} F_K / (F_{KL^b} F)$ between capital and borrowers' labor supply.

Proposition 13. *Let σ be the elasticity of substitution between capital and borrowers' labor supply in the economy with investment. Denote by $\omega^s(r)$ the savers' income share.*

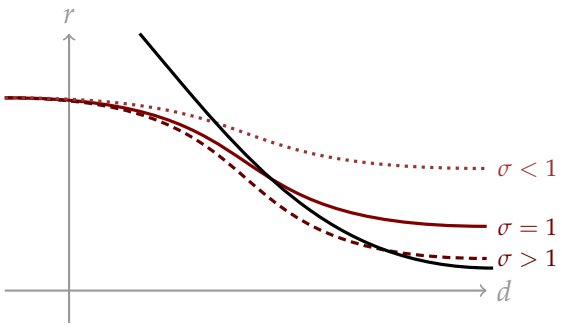
a) If $\sigma = 1$: $\omega^s(r)$ is independent of the interest rate r. All steady state(s) are identical to the economy without investment.

b) If $\sigma < 1$: $\omega^s(r)$ falls with lower r. This flattens the saving supply curve.

c) If $\sigma > 1$: $\omega^s(r)$ rises with lower r. This steepens in the saving supply curve.

[16] We assume the typical: F is strictly concave, satisfying Inada conditions.

Figure 21: Capital and indebted demand



Proposition 13 precisely characterizes the role of investment for the long-run economy. When an increase in capital leaves the income distribution unchanged, investment will have no effect on the long-run (case (a)). So to what extent capital matters depends on whether it crowds the share of income going to borrowers in or out. If capital is complementary to borrowers' labor supply, it reduces the extent of indebted demand (even if it can never fully undo it) as lower interest rates go hand in hand with greater capital and a more equitable income distribution (case (b)). If capital is substitutable with borrowers' labor supply—one may think of capital-skill complementarity and automation as in Krusell, Ohanian, Rios-Rull, and Violante (2000) and Autor, Levy, and Murnane (2003),—the opposite is the case. Lower interest rates endogenously lead to a more unequal income distribution, effectively steepening the saving supply curve and amplifying the problem of indebted demand (case (c)).[17] We illustrate the three cases in Figure 21.

This discussion focused on the long run. Investment contributes to demand in the short run as interest rates fall, irrespective of the structure of the production function F. For example, if F is Cobb-Douglas, and the economy sees a shift in income inequality, investment picks up initially, temporarily slowing the decline in r. As the investment boom recedes, however, the fall in r accelerates again, eventually falling to the exact same steady state level as would have occurred without investment (as in Section 5).

Observe that the recent US experience does not align well with this description of investment (see Figure 3). It did not seem that investment rose as interest rates fell. This is subject of a recent literature (Gutiérrez and Philippon 2017, Liu et al. 2019, Farhi and Gourio 2018, Eggertsson, Robbins, and Wolf 2018).

Which kind of debt causes indebted demand? Productive vs. unproductive debt. Investment may be funded with (corporate) debt, raising the question, which kind of debt actually causes indebted demand? In Section 4, we argued that, starting in some steady state, a one-time exogenous

[17] See Straub (2019) for a related point.

increase in debt by some dD, holding r fixed, causes a response of aggregate spending of

$$dC = -\frac{\rho+\delta}{2}\left(1-\sqrt{1-4\left(1-\frac{r}{\rho+\delta}\right)\epsilon_\eta}\right)dD \tag{27}$$

(see Proposition 13). The debt in this experiment is *unproductive*, that is, it is being used for consumption.

We now repeat this exercise with *productive* debt, that is, debt that is being used to raise the capital stock of the economy, $dK = dD$. Holding r and household debt d fixed, what are the implications for aggregate spending?

Proposition 14 (Indebted demand when debt is productive)**.** *Starting from a steady state and holding r and d, fixed, an exogenous increase in debt dD that raises the capital stock by the same amount affects aggregate spending by*

$$dC = -\frac{\rho+\delta}{2}\left(1-\sqrt{1-4\left(1-\frac{r}{\rho+\delta}\right)\epsilon_\eta\chi}\right)dD. \tag{28}$$

where $\chi \equiv \left(1-\sigma^{-1}\right)\omega^b - r\frac{rd}{Y} < 1$.

A first observation about Proposition 14 is that productive debt, (28), always causes strictly less indebted demand than unproductive debt, (27). To see this, note that $\chi = \left(1-\sigma^{-1}\right)\omega^b - r\frac{rd}{Y} < 1$. Thus, even if capital is perfectly substitutable with borrowers' labor supply, $\sigma = \infty$, productive debt does not cause as much indebted demand. The reason for this is that even if $\sigma = \infty$, capital raises aggregate output. A second observation is that the negative effect of dD on aggregate spending dC falls for lower values of σ, as one would expect given (13). In the case $\sigma = 1$, the effect is positive at first, $dC > 0$. Recall that (28) is the contemporary effect on spending—but once we allow household debt to increase with Y, the positive effect fades.

In sum, this suggests that productive debt always causes weaker indebted demand than unproductive debt. The degree to which it is weaker depends on the elasticity of substitution σ between the capital that is accumulated using debt and the borrowers' labor supply. This suggests that debt-financed productive investments made by firms or governments need not necessarily contribute to indebted demand.

9.3 Risk premia and convenience yields

A concern one might have with our fiscal policy results is that they were derived in a model in which government bonds pay the same return as any other asset, while this is clearly not the case in the data. What, for instance, if there is a spread between the overall return on wealth r and the return on government bonds r^B? How does this impact our predictions for fiscal policy?

We allow for such a spread by assuming that savers, who are the equilibrium holders of government bonds, derive an additional utility from holding government bonds, as in Krishnamurthy

and Vissing-Jorgensen (2012). In particular, since savers derive utility from both consumption and wealth, we assume the per-period utility function of a representative saver is

$$\log\left(\left(c_t^s + \xi B_t\right)/\mu^s\right) + \delta/\rho v\left(\left(a_t^s + \xi B_t/r\right)/\mu^s\right).$$

The saver's Euler equations now differ for bonds and other assets. In the steady state, we obtain

$$0 = r^B + \xi - \rho - \delta + \frac{\delta}{\rho} r\eta\left(a + \xi B/r\right) \tag{29}$$

for government bonds and

$$0 = r - \rho - \delta + \frac{\delta}{\rho} r\eta\left(a + \xi B/r\right) \tag{30}$$

for other assets. Subtracting (30) from (29) yields a steady-state government bond return of

$$r^B = r - \xi. \tag{31}$$

Due to the spread ξ in (31), r^B is always strictly lower than r. Thus, it might hit zero even as r is still positive. We distinguish two cases, depending on whether r^B is positive or zero.

Steady states without ZLB (positive r^B). The steady states in this economy turn out to be exactly the same as before, characterized by the intersections of the exact same two curves,

$$r = \rho\frac{1+\rho/\delta}{1+\rho/\delta\cdot\eta(a)} \qquad \text{and} \qquad a = (1-\tau^s)\frac{\omega^s}{r} + \frac{\ell(r)}{r} + B. \tag{32}$$

That is, an increase in government debt B shifts out the demand curve for borrowing, pushing down the long-run return on wealth r, and through (31), also the return on government debt.

The reason for this finding is simple. There is a positive spread in (31) since savers derive tangible benefits from investing in government bonds. These benefits, however, make savers effectively richer, offsetting the reduction in saver wealth due to lower interest income $(r-\xi)B$ from government bonds. Thus, when the benefits are included, the value of government bonds for savers is indeed B and not $r^B B/r$.

While the steady state curves (32) are unaffected, the spread in (31) obviously affects the government budget constraint. In particular, financing a steady-state public debt position B now only costs $(r-\xi)B$ instead of rB.[18] This becomes especially relevant in a liquidity trap.

Steady states with binding ZLB (zero r^B). We can use the model with government bond spreads to revisit the liquidity trap ("debt trap"). In Section 8 we assumed there to be an effective lower bound $\underline{r} > 0$ for the return on wealth r. In this economy, we can now explicitly impose a zero lower bound for the government bond yield r^B. Given (31), this is fully consistent with our previous

[18] Paradoxically, this might even raise the effect of B on steady state return on wealth r as τ^s adjusts by less in (32).

approach when $\underline{r} = \xi$.

Thus, as before, a debt trap steady state exists if the natural allocation would require a negative government bond yield $r^B < 0$, or equivalently, $r < \xi$. The result in Proposition 10 carries over to this economy, meaning output in the debt trap is given by

$$\hat{Y} = Y \frac{\underline{r}}{(1-\tau^s)\omega^s + \ell(\xi)} \cdot \left[\eta^{-1} \left(\frac{\rho}{\xi} (1 + \rho/\delta) - \rho/\delta \right) - B \right] \tag{33}$$

and therefore still decreasing in B.

There is, however, a new policy option on the table now, which was absent in Section 8. As governments pay zero interest rates on their bonds, $r^B = 0$, they can effectively run a Ponzi scheme: permanently raise government spending G by an amount equal to the shortfall in demand to restore the natural allocation, $\hat{Y} = Y$, financed by an ever-growing pile of government debt.

While this is a perfectly well-defined equilibrium in theory, it is risky in practice, not the least because a high public debt to GDP ratio might expose the government to rollover crises (unmodeled here) even if interest rates r^B are expected to remain low for long. There is also no exit strategy. As (33) highlights, the longer a government runs the Ponzi scheme, the more significant the output loss when exiting will be. In addition, as we demonstrate in Section 6.2, a recovery to greater equilibrium interest rates is more unlikely when government debt is large.

9.4 Intergenerational mobility

We have so far ignored mobility across saver and borrower dynasties. Yet, since current mobility levels in the US are low while income inequality is rising (Lee and Solon 2009, Chetty, Hendren, Kline, Saez, and Turner 2014), several policies have been proposed to promote intergenerational mobility. To study the effects of such policies on household debt and interest rates, we next extend our framework to include mobility.

To do so without jeopardizing the tractability of our model, we assume that with some probability $q > 0$, savers' offsprings turn into borrowers; and with probability $q\frac{\mu}{1-\mu}$, borrowers' offsprings turn into savers. A saver-turned-borrower immediately consumes his wealth down to the level of an average borrower $a^b/(1-\mu)$; and vice versa, a borrower-turned-saver immediately receives a transfer from all savers to achieve their average asset position a^s/μ.[19] Agents' preferences are unchanged.

This structure changes the saving supply curve to

$$r = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho\eta(a)} + \underbrace{q\gamma\delta \frac{\delta/\rho\eta(a)}{1 + \delta/\rho\eta(a)}}_{\text{contribution of mobility}} \tag{34}$$

where γ is a measure of steady-state income inequality, equal to 1 minus the ratio of per capita

[19] This may be interpreted as borrowers marrying into saver families.

incomes

$$\gamma = 1 - \frac{\left(\omega^b - \ell\right) / \left(1 - \mu\right)}{\left(\omega^s + \ell\right) / \mu}.$$

We see from (34) that the intergenerational mobility term increases in a. This makes the saving supply curve less downward-sloping, thus mitigating indebted demand. The effect of greater mobility (raising q) is particularly relevant when income inequality γ is high, i.e. close to 1, since in that case, it redistributes wealth within each generation in a similar fashion as redistributive taxation.

9.5 Open economy model

Many countries with large debt burdens are open economies. This begs the question whether our model can be generalized to an open economy setting, and if so, whether this generates new insights. This is what we briefly discuss next.

We assume that our two types of agents live in a small open economy (the "home" country), which trades a single good with the rest of the world. It has imperfect access to world financial markets: as in Gabaix and Maggiori (2015), a continuum of foreign-based financial intermediaries invests in the home country, earning a return spread equal to the domestic interest rate r minus the world interest rate r^*. Thus, home's net foreign asset position is given by

$$\text{nfa}_t = \frac{1}{\Gamma}\left[r^* - r_t\right]. \tag{35}$$

What do steady states look like in this economy? To understand this, it helps to view the rest of the world as another "borrower" in the economy. When the home interest rate r_t is lower than the world interest rate r^*, the rest of the world borrows an amount equal to nfa_t from the home economy. Thus, nfa_t expands total steady-state saving demand to $d_t + B_t + \text{nfa}_t$. Given (35), opening up the economy's financial account amounts to a counterclockwise twist around the point on the saving demand curve where $r = r^*$.

9.6 Longer duration debt

A recent literature highlighted that responses of economies to interest rate changes differ according to the type of debt contract agents hold (e.g. adjustable-rate vs. fixed-rate contracts), as well as the debt's maturity (Campbell 2013, Auclert 2019, Wong 2019). In this extension, we investigate the role of debt duration for indebted demand. We assume that all debt is fixed rate, and is being rolled over at Poisson rate λ. Since asset duration is irrelevant for the steady state(s), we focus on two transitional dynamics experiments: one of falling real interest rates and increasing debt levels (e.g. due to rising income inequality as in Figure 11) and one of rising real interest rates and falling debt levels (e.g. due to redistributive taxation). For both experiments, we ask whether fixed-rate debt (FR) would speed up the transition relative to adjustable-rate debt (AR), or slow it down.

To study the first experiment, when a shock suddenly lowers real interest rates relative to what they were expected to be, this does not affect the present value of AR debt, but pushes up the present value of FR debt. Formally, this means the state variable d jumps up on impact of the shock. Such a jump corresponds to a fast transition in practice. Since the present value increases without any actual increase in debt originations, the additional demand typically associated with strong credit growth is absent in this case.

To study the second experiment, consider a shock that pushes real interest rates up. The present value of AR debt is again unchanged, but the present value of FR debt *falls*, again speeding up the transition. Thus, in some sense AR debt makes it harder to leave a steady state with high levels of debt since any increase in interest rates leads to an immediate sharp fall in demand without favorable revaluation effect.

These discussions highlight that AR debt contracts slow down transitions into states with low r and high debt, and FR debt contracts speed up transitions away from such states. Fixed-rate contracts with automatic refinancing achieve both of these arguably favorable outcomes.[20] Policies that raise the share of refinancing among US fixed-rate mortgage owners are therefore beneficial from this perspective.

10 How indebted is demand in practice?

Measuring the slope of the saving supply curve. The key ingredient to our theory is the downward-sloping saving supply curve. This begs the question of whether one can get a sense of the magnitude of the slope of the saving supply curve. To investigate this, we employ a simple sufficient statistics approach.

Let $\mathcal{C}(r, a)$ be the steady state consumption of rich households in an economy (e.g. the top 5% or top 1%). The definition of the saving supply curve $r(a)$ as function of rich households' wealth requires that

$$\mathcal{C}(r(a), a) = r(a)a.$$

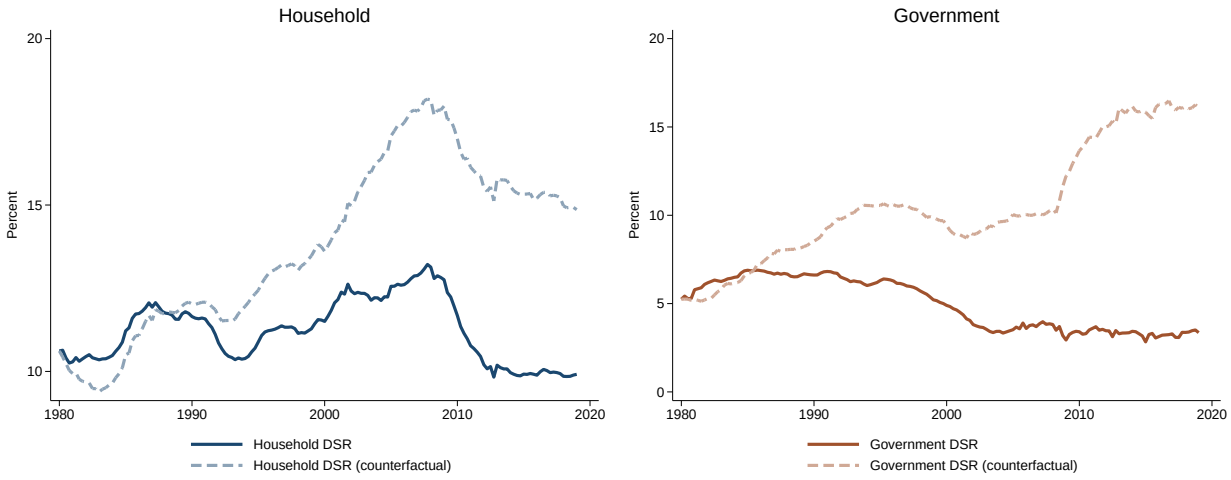
Total differentiation of this equation with respect to a allows us to isolate the local slope of the saving supply curve

$$\frac{dr}{d\log a} = \frac{MPC^{\text{cap. gains}} - r}{1 - \epsilon_r \frac{c}{a}}$$

where $\epsilon_r \equiv \frac{\partial \log c}{\partial r}$ is the rich's semi-elasticity of consumption with respect to a permanent shift in interest rates, and $MPC^{\text{cap. gains}}$ is the MPC out of wealth of the rich. To get a sense of magnitudes, we assume $\epsilon_r = 0$, which would be implied by log preferences. The $MPC^{\text{cap. gains}}$ should be interpreted as an MPC out of an increase in capital gains as most wealth gains for the very richest

[20] Lengthening the maturity of debt when debt is high, as in Campbell, Clara, and Cocco (2018), would further speed up transitions back to lower debt states as they would lengthen the duration of debt. Automatic refinancing is reminiscent of the idea to convert FR into AR contracts (Guren, Krishnamurthy, and McQuade 2017), albeit such a conversion would slow down transitions back to states with lower debt levels.

Figure 22: Hypothetical debt service costs had interest rates not declined.



See Appendix A for details.

households come in the form of capital gains. We use the estimate from Chodorow-Reich, Nenov, and Simsek (2019) for this MPC, $MPC^{\text{cap. gains}} = 0.028$, which is in the same ballpark as the estimates of Baker, Nagel, and Wurgler (2007) and Di Maggio, Kermani, and Majlesi (2019). Finally, recent estimates of the real return on wealth are on the order of 5-7% (Farhi and Gourio 2018). We take $r = 6\%$ for this exercise.

Together this simple calculation implies an estimate for the local slope of the saving supply curve

$$\frac{dr}{d\log a} \approx -0.032.$$

In words, this implies that if the richest households' wealth rises by 10%, the interest rate has to come down by 32 basis points. While this is certainly not a precise calculation, it gives a rough sense of the magnitudes that are at play in our model.

Implications for indebted demand in the US. A way to get a sense the extent of indebted demand in the US is to evaluate the hypothetical demand shortfall from high debt levels, had real interest rates *not* fallen since 1980. In the spirit of Proposition 3, we thus ask: if debt levels were as high as they are today, with interest rates still at 1980 levels, how much smaller would aggregate spending $dC + dG$ be?

To conduct this exercise, we need to know the hypothetical debt service costs that would have prevailed absent a fall in interest rates. We compute this for every year since 1980 in Figure 22, finding significant increases in hypothetical debt service costs, both for household and government debt. Together, they have amounted to 15% of household disposable income in recent years.

Thus, with the same $MPC^{\text{cap. gains}}$ estimate and r, the demand shortfall is in the ballpark of

$$dC \approx \underbrace{-15\%}_{\text{borrower debt service}} + \underbrace{\frac{MPC^{\text{cap. gains}}}{r} \cdot 15\%}_{\text{partial offset by savers}} = -8\%$$

This is a significant number, which would rationalize why the natural interest rate indeed had to decline significantly in order for debt burdens not to depress demand.

11 Conclusion

In this paper, we proposed a new theory connecting several recent secular trends: the increase in income inequality, financial liberalization, the decline in natural interest rates, and the rise in debt by households and governments.

The central element in our theory are non-homothetic preferences, which lead to richer households having greater saving rates out of a permanent income transfer. This gives rise to the idea of indebted demand: greater debt levels mean a greater transfer of income in the form of debt service payments from borrowers to savers, and thus depress demand.

We identified three main implications of indebted demand. First, secular economic shifts that raise debt levels (e.g. income inequality or financial liberalization) also lower natural interest rates, which then itself has an amplified effect on debt. Second, monetary and fiscal policy, to the extent that they involve household or government debt creation, can persistently reduce future natural interest rates. This means that there is only a limited number of such policy interventions that can be used before economies approach the effective lower bound. Finally, when the lower bound is binding, the economy is in a debt-driven liquidity trap with depressed output. In this "debt trap", debt-financed stimulus deepens the recession in the future, whereas redistribution of income mitigates it.

Loosely speaking, our results suggest that economies face a sort of "budget constraint for aggregate demand". They can stimulate aggregate demand through debt creation, but that reduces future demand (and thus natural interest rates). This logic suggests a new trade-off for debt-based stimulus policies. We vie an exploration of this trade-off in an optimal policy setting as a promising avenue for future research.

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Appendix

A Data and figures

Figure 22. Quarterly, seasonally adjusted data on disposable personal income (DPI) are obtained from the US Bureau of Economic Analysis. Levels and debt service ratios (DSR) for household mortgage debt and consumer credit debt are obtained from the Board of Governors of the Federal Reserve System. In these data, the overall DSR is by definition the sum of household mortgage and consumer credit DSRs, and debt service ratios are calculated as debt service payments as a percent of DPI. Following the methodology used by the Federal Reserve, we take the level of consumer credit debt for households and nonprofit organizations (LA153166000 in Financial Accounts of the United States) and the level of mortgage debt as total mortgages to households and nonprofit organizations (FL153165005 in Financial Accounts of the United States).

For each of household mortgages (HM) and consumer credit (CC), we assume that there is a constant amortization rate δ_d and a fixed interest rate $r_{d,t}$, which determine the debt service ratio, equal to:

$$DSR_{d,t} = \frac{D_{d,t} \times (r_{d,t} + \delta_d)}{DPI_t},$$

where d is a type of debt, DPI_t is disposable personal income and $D_{d,t}$ is the level of debt at time t. Note again that $DSR_{HM,t}$ and $DSR_{CC,t}$ sum to the total household debt service ratio $DSR_{HH,t}$.

We build a counterfactual series under the premise that $r_{d,t}$ remained constant over time, fixed at 1980 Q1 levels. We use this to recalculate the counterfactual DSRs for the two types of debt, as follows, with dots denoting counterfactual:

$$\ddot{DSR}_{d,t} = \frac{D_{d,t} \times (r_{d,1980Q1} + \delta_d)}{I_t}$$

Finally, we sum these to obtain $\ddot{DSR}_{HH,t} = \ddot{DSR}_{HM,t} + \ddot{DSR}_{CC,t}$. The left panel of Figure 22 plots $DSR_{HH,t}$ and $\ddot{DSR}_{HH,t}$.

For the government's debt service ratio, we conduct a similar analysis using total government debt $D_{G,t}$ and government interest payments $IP_{G,t}$. Government in this case refers solely to the federal government. We define the government debt service ratio $DSR_{G,t}$ to be:

$$DSR_{G,t} = \frac{IP_{G,t}}{DPI_t}.$$

We decompose the government debt service ratio in the same manner as we do for households:

$$DSR_{G,t} = \frac{D_{G,t} \times (r_{G,t} + \delta_G)}{DPI_t}$$

We build the government's counterfactual DSR series under the assumption that $r_{G,t}$ remained constant at 1980Q1 levels. Precisely,

$$D\ddot{S}R_{G,t} = \frac{D_{G,t} \times (r_{G,1980Q1} + \delta_G)}{DPI_t}$$

The right panel of Figure 22 plots $DSR_{G,t}$ and $D\ddot{S}R_{G,t}$.

B Proofs

Proof of Proposition 1. This proof is an immediate consequence of (10).

Proof of Proposition 2. Given that debt $d > 0$, savers cannot be up against their borrowing constraint. Thus, the Euler equation has to hold with equality, implying (11). Moreover, the Euler equation cannot hold with equality for borrowers, as their per capita wealth is much smaller than that of savers. Thus, their borrowing constraint is binding, implying (12)

Proof of Proposition 3. As borrowers are borrowing constrained, their consumption responds one for one, $dc^b = -dx$. To analyze the consumption response by savers define their consumption-wealth ratio $\chi_t \equiv c_t^s / a_t^s$. Expressing budget constraint and Euler equation in terms of (a_t^s, χ_t), we find

$$\frac{\dot{a}_t^s}{a_t^s} = r - \chi_t \qquad \text{and} \qquad \frac{\dot{\chi}_t}{\chi_t} = \chi_t - \rho - \delta + \frac{\delta}{\rho}\chi_t \eta(a_t^s).$$

To get the on-impact consumption response of savers, we need to solve the ODEs for a small change in wealth at date 0 away from the steady state, in which $\chi^* = r$ and $r = \rho + \delta - \delta/\rho r \eta(a^{s*})$. Let $\hat{a}, \hat{\chi}$ be the deviations (in levels, not logs) from the steady state. Defining $\epsilon_\eta \equiv \eta'(a^{s*})a^{s*}/\eta(a^{s*})$, we have

$$\dot{\hat{a}}_t = -a^{s*}\hat{\chi}_t \qquad \text{and} \qquad \dot{\hat{\chi}}_t = (\rho + \delta)\,\hat{\chi}_t + (\rho + \delta - r)\, r\epsilon_\eta \frac{\hat{a}_t}{a^{s*}}. \tag{36}$$

We guess and verify that $\hat{\chi} = -k\hat{a}/a^{s*}$ for $k > 0$. Using the equations in (36) we find a quadratic for k,

$$k^2 - (\rho + \delta)\, k + (\rho + \delta - r) r\epsilon_\eta = 0.$$

The only solution that leads to a positive consumption response to transfers is

$$k = \frac{\rho + \delta}{2}\left(1 - \sqrt{1 - 4\left(1 - \frac{r}{\rho + \delta}\right)\epsilon_\eta}\right).$$

Since $\hat{c}_t^s = a^{s*}\hat{\chi}_t + r\hat{a}_t = (r - k)\hat{a}_t$, and $\hat{a}_0 = dx/r$, we have

$$dc_0^s = dx - k/r dx$$

which together with dc_0^b implies (13).

Local uniqueness and existence result. Here we prove that locally around a stable steady state, i.e. for d_0 in a neighborhood of steady-state debt d^*, there is a unique equilibrium. To do this, we first collect the equations describing equilibrium when the borrower is up against a binding borrowing constraint. We focus on the special case with $\ell(p_t) = \ell = const$; the general case of a

stable steady state when $\ell(p_t)p_t$ increases in p_t follows analogously. The pricing equation for real assets is

$$r_t p_t = 1 + \dot{p}_t. \tag{37}$$

The debt level evolves as

$$\dot{d}_t = \lambda \ell p_t - \lambda d_t \tag{38}$$

which we can plug into the savers' collective budget constraint, we get the interest rate

$$r_t = \frac{1}{d_t}\left\{c_t^s + \lambda \ell p_t - \lambda d_t - \omega_t^s\right\}. \tag{39}$$

Finally, savers' Euler equation is given by

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta \frac{c_t^s}{\rho\left(\omega^s p_t + d_t\right)} \eta(\omega^s p_t + d_t). \tag{40}$$

Equations (37)–(40) describe a system of 3 ODEs in variables (p_t, d_t, c_t^s), after using (39) to substitute out the interest rate r_t. Denote by (p^*, d^*, c^{s*}) the respective steady state values of the three variables. We define two auxiliary variables

$$w \equiv \frac{\omega^s}{\ell} \qquad \text{and} \qquad \Delta \equiv \rho + \delta - r^*.$$

In the homothetic model, $\Delta = 0$. Using them, we find for the steady state variables

$$p^* = \frac{1}{r^*}, \quad d^* = \frac{\ell}{r^*}, \quad \frac{c^*}{d^*} = r^*(1+w), \quad \frac{p^*}{d^*} = \frac{1}{\ell}.$$

Denote by $\tilde{p}, \tilde{d}, \tilde{c}^s$ the log-linearized versions of p, d, c^s. Denote by $\tilde{r}$ the linearized version of r. Linearizing (37)–(40) we find

$$\begin{aligned} \tilde{r} &= -r^* \tilde{d} + \tilde{c} r^* (1+w) + \lambda \tilde{p} - \lambda \tilde{d} \\ \dot{\tilde{p}} &= \tilde{r} + r^* \tilde{p} \\ \dot{\tilde{d}} &= \lambda \tilde{p} - \lambda \tilde{d} \\ \dot{\tilde{c}} &= \tilde{r} + \Delta \left[\tilde{c} - \left(1 - \epsilon_\eta\right) \frac{w\tilde{p} + \tilde{d}}{w+1}\right] \end{aligned} \tag{41}$$

We guess and verify that the solution takes the form

$$\tilde{r} = -R\tilde{d}, \quad \tilde{p} = P\tilde{d}, \quad \tilde{c} = C\tilde{d}.$$

We also verify that $\dot{\tilde{d}}/\tilde{d} < 0$, which is necessary for an equilibrium in the neighborhood of the steady state. After some algebra, we arrive at a condition for P,

$$F(P) = 0$$

where

$$F(P) = \left[\Delta + \lambda\left(2-P\right) + r^*(1+w)\right]\left(\lambda(1-P)+r^*\right)P + \Delta P\lambda \\ - \lambda(\Delta + \lambda(1-P)) - r^*\lambda + r^*\Delta\left[\left(1-\epsilon_\eta\right)\left(1+Pw\right)-1\right].$$

As one can see easily, $F(0) < 0$, $F(1) > 0$, $F''(1) < 0$, $F'''(0) > 0$ (the coefficient on P^3 is positive). Together, they imply that F admits a unique root P below 1. This root is therefore unique in implying stability, $\dot{\tilde{d}}/\tilde{d} = \lambda(P-1) < 0$. It also implies that

$$R = P\lambda(1-P) + r^*P > 0, \tag{42}$$

so that the (natural) interest rate is lower if the economy has a higher debt level than steady state debt. Moreover, observe that F strictly declines in ϵ_η. Thus,

$$\frac{\partial P}{\partial \epsilon_\eta} > 0. \tag{43}$$

Proof of Proposition 4. The proof is an immediate consequence of Proposition 2.

Proof of Proposition 5. The proof is an immediate consequence of Proposition 2.

Proof of Proposition 6. This follows directly from (14) and (15).

Proof of Proposition 7. Savers' intertemporal budget constraint is given by

$$\int_0^\tau e^{-\int_0^t r_s ds} c_t^i dt = \omega^s \int_0^\tau e^{-\int_0^t r_s ds}\hat{Y}_t dt + d_0 - e^{-\int_0^\tau r_s ds} d_\tau \tag{44}$$

which implies (18) by subtracting from (44) its steady state analogue. (19) is a direct consequence of (18) and the fact that the steady state is stable, and so $d_\tau \to d$ as $\tau \to \infty$.[21]

Proof of Proposition 8. To start, we derive the system of equations characterizing the evolution of the economy in response to an arbitrary path of interest rates r_t. From the savers' budget constraint and Euler equation we have

$$\dot{a}_t^s = r_t a_t^s - c_t^s$$

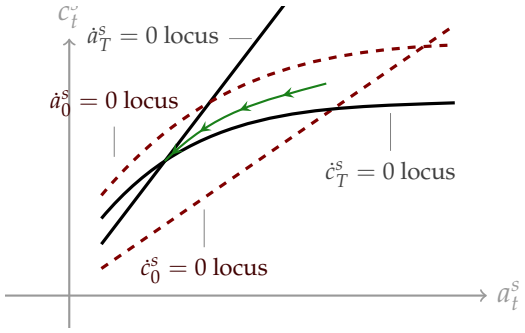
$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta\frac{c_t^s}{\rho a_t^s}\eta\left(a_t^s\right).$$

As one can easily see in a phase diagram (Figure 23), both equations are forward looking. Thus, under a monetary policy shock as in (16), c_t^s and a_t^s jump up on impact, converging back towards their steady state values, $c_t^s = c^s$ and $a_t^s = a^s$, which they reach at $t = T$. Noting that $a_t^s = \omega^s p_t + d_t$, we find that debt evolves as

$$\dot{d}_t = -\lambda(1+w^{-1})d_t + w^{-1}a_t^s \tag{45}$$

[21] Stability of a steady state for a fixed interest rate $\hat{r}$ follows from (45) below. For fixed $\hat{r}$, $\hat{a}_t^s$ is immediately at its steady state value (see discussion below and Figure 23).

Figure 23: Phase diagram for (c_t^s, a_t^s) under an exogenous interest rate.



implying that $d_t > d$ for all $t > 0$ ($w \equiv \omega^s / \ell$). Thus, under a monetary policy shock as in (16), debt strictly rises. An (first-order) increase in debt, as explained in (42), leads to a reduction in natural interest rates.

The increase in debt is greater for all t under monetary policy shock (17). The reason for this is that under (16), natural rates are still below steady state r after $t = T$. Thus, if $r_t = r_t^n$ is set after $t = T$, r_t is below what it would be under (16). But if r_t is lower, both a_t^s and c_t^s are greater everywhere (Figure 23), which by (45) must translate into a greater level of debt. Similarly, if we increase T or reduce $\hat{r}$, this further raises c_t^s and a_t^s in Figure 23, thus also increasing the path of debt d_t at all times.

Why do natural rates fall more with greater non-homotheticity? To prove this, we focus on a convenient measure of the path of natural rates, namely the present value, $R_s^n \equiv r \int_s^\infty e^{-r(t-s)} r_t^n dt$. We ask: how does $\tilde{R}_s^n \equiv R_s^n - r$ vary with the level of debt $\tilde{d}$? And how does that mapping depend on ϵ_η? By (41), we find that

$$\tilde{R}_t^n = -rP\tilde{d}_t.$$

Thus, natural interest rates always fall when $\tilde{d}_t > 0$, and more so the larger P is, e.g. when ϵ_η is greater (see (43)).

Finally, consider a situation with multiple steady states. Call $\underline{r}$ the interest rate of the alternative high-debt, low-interest-rate steady state, and $\underline{c}^s, \underline{a}^s, \underline{d}$ the associated values of c^s, a^s and $\underline{d}$. Based on Figure 23, setting $\hat{r} = \underline{r}$ forever ($T = \infty$) means $c_t^s = \underline{c}^s$ and $a_t^s = \underline{a}^s$ for any $t > 0$. Both variables jump immediately to their new values. Debt evolves as in (45), and converges to $\underline{d}$. Because the new steady state is stable, the economy does not converge back to its original values. Also, by continuity, there exists a threshold $\underline{T}$, such that any $T > \underline{T}$ will bring the economy sufficiently close to $\underline{d}$ that it will converge by itself (without any further stimulus) to the alternative steady state.

Proof of Proposition 9. As explained above, for fixed $\hat{r}$, c_t^s and a_t^s jump immediately to their new values c_∞^s, a_∞^s. Solving (45), we find

$$d_t = d_0 e^{-\lambda(1+w^{-1})t} + \frac{a_\infty^s}{\lambda(1+w^{-1})}\left(1 - e^{-\lambda(1+w^{-1})t}\right)$$

for the path of debt. This gives us the path of asset prices from $p_t = \frac{a_\infty^s - d_t}{\omega^s}$ and output from

$$\hat{Y} = r_\infty p_t - \dot{p}.$$

After some algebra, this yields (20).

Proof of Proposition 10. The expression for $\hat{Y}$ follows directly from inverting saving supply curve at $\underline{r}$,

$$\underline{r} = \rho \frac{1+\delta/\rho}{1+\delta/\rho\eta\left(\frac{\omega^s+\ell(\underline{r})}{r}\hat{Y}\right)}$$

to solve for $\hat{Y}$. The debt trap is stable since the economy, for a fixed interest rate $\underline{r}$, is described by constant c^s, a^s, and debt evolves as in (45), converging always back to the debt trap debt level.

Proof of Proposition 11.

$$\hat{dd}$$

(23) again follows directly from inverting the saving supply curve with fiscal policy at $\underline{r}$,

$$\underline{r} = \rho \frac{1+\delta/\rho}{1+\delta/\rho\eta\left(\frac{(1-\tau^s)\omega^s+\ell(\underline{r})}{r}\hat{Y}+B\right)}.$$

Proof of Proposition 12. A partial or full debt jubilee has no effect in the long run as the economy will always converge back to the unique debt trap steady state. A (government-financed) bailout lowers output in the long run as it increases government debt B (Proposition 11).

If there are multiple steady states, there is a threshold for debt d^* (equal to that of the second-largest-debt steady state), such that if the jubilee moves the economy to a debt level $d_0 \leq d^*$, long-run output and interest rates are greater than before.

Proof of Proposition 13. The result follows directly from (26) and the behavior of $\omega^s(r)$. Observe that

$$\frac{\partial \omega^s}{\partial K} = -\frac{\partial \omega^b}{\partial K} = \left(1-\sigma^{-1}\right)\omega^b \frac{F_K}{Y} \tag{46}$$

implying that ω^s increases with K (and falls with r) precisely iff $\sigma > 1$, and is constant if $\sigma = 1$. The slope of the saving supply curve is given by

$$\frac{\partial r}{\partial d} = -\frac{\delta\eta' r}{\rho+\delta\eta-\delta\eta'\frac{\omega^s(r)}{r}+\delta\eta'\omega^{s\prime}(r)}$$

where η, η' are evaluated at $\omega^s(r)/r + d$. Evidently, the larger (more positive) $\omega^{s\prime}(r)$ is, the flatter is the saving supply curve. Vice versa for negative $\omega^{s\prime}(r)$.

Proof of Proposition 14. To derive (28), observe that

$$\frac{\partial\left(a^s/Y\right)}{\partial D} = \frac{\partial}{\partial D}\left(\frac{\omega^s}{r}+\frac{rd}{Y}\right) = \left(1-\sigma^{-1}\right)\omega^b - \frac{rd}{Y}\frac{F_K}{Y} = \left(1-\sigma^{-1}\right)\omega^b - r\frac{rd}{Y} \equiv \chi$$

where we used (46). Following steps as in the proof of Proposition 3, we then find that the savers' consumption response to the increase in their assets dD is given by

$$dc_0^s = (r-k)dD$$

where now

$$k = \frac{\rho + \delta}{2}\left(1 - \sqrt{1 - 4\left(1 - \frac{r}{\rho + \delta}\right)\epsilon_\eta \chi}\right).$$

Thus, $dC = -kdD$.

C Model extensions

C.1 More general preferences

C.1.1 EIS different from one

Our model naturally generalizes to utility functions over consumption with an elasticity of intertemporal substitution (EIS) σ^{-1} different from one. Assume the preferences of a type-i agent are given by

$$\int_0^\infty e^{-(\rho+\delta)t}\left\{\frac{1}{1-\sigma}\left(c_t^i/\mu^i\right)^{1-\sigma} + \frac{\delta}{\rho^\sigma} v(a_t^i/\mu^i)\right\} dt.$$

Define $\eta(a) \equiv (a/\mu^s)^\sigma v'(a/\mu^s)$ so that the homothetic benchmark with $v'(a) \sim a^{-\sigma}$ continues to correspond to $\eta(a) = 1$. The Euler equation of savers is then given by

$$\sigma\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta\left(\frac{c_t^s}{\rho a_t^s}\right)^\sigma \eta(a_t^s)$$

which, at a steady state, reduces to

$$r = \rho + \delta - \delta\left(\frac{r}{\rho}\right)^\sigma \eta(a^s).$$

While not necessarily solvable in closed form for r, this equation still has a unique solution for r for any value of a^s, by the intermediate value theorem. Moreover, by the implicit function theorem, the slope of the implied saving supply curve is still negative.

C.1.2 Recursive preferences over consumption

Our preferences (1) involve a warm-glow utility over bequests. Does the negative slope of the saving supply curve hinge on bequests (or wealth more broadly) entering the utility function? We now argue that the answer is no. To do so, we give both agents a recursive utility function solely defined over consumption, as in Uzawa (1968) and Lucas and Stokey (1983). We thus assume preferences are given by

$$U^i = \int_0^\infty u(c_t^i/\mu^i)e^{-\Delta_t}dt$$

where $\dot{\Delta}_t = \rho\left(c_t^i/\mu^i\right)$, u is strictly increasing, continuously differentiable and concave, and ρ is continuously differentiable. If $\rho = const$, this corresponds to standard homothetic preferences, but in general, these preferences allow the discount factor to move with consumption.

After some math, we find that savers' steady state Euler equation is

$$r = \rho(ra^s/\mu^s).$$

This defines an implicit equation in r for any a^i. If ρ decreases, such that greater consumption

levels are associated with less impatience, the saving supply curve is downward-sloping, just as in the model of Section 3.

C.1.3 Preferences over relative wealth

As we mentioned in Section 3.5, an alternative way to set up the warm-glow bequest utility is to define it not relative to output Y, but relative to total wealth. In this formulation, preferences are given by

$$\int_0^\infty e^{-(\rho+\delta)t}\left\{\log\left(c_t^i/\mu^i\right)+\frac{\delta}{\rho}v\left(\frac{a_t^i/\mu^i}{A_t}\right)\right\}dt$$

where we let $A_t \equiv a_t^s + a_t^b$. Observe that $A = Y/r = 1/r$ in a steady state (Y is normalized to 1). Defining η as before, we then obtain the savers' Euler equation and saving supply curve

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta\frac{c_t^s}{\rho a_t^s}\eta\left(\frac{a_t^i}{A_t}\right) \qquad \Rightarrow \qquad r = \rho\frac{1+\delta/\rho}{1+\delta/\rho\eta\left(ra^s\right)}. \tag{47}$$

The only change in (47) relative to (10) is that there is now an additional "r" on the right hand side. Conceptually, this plays no role, however. When $\eta(a)$ is increasing, the right hand side falls in r, implying that there is a unique interest rate r for any a^s. Moreover, as the right hand side also falls in a^s, that interest rate must decline as a^s increases. Thus, we still have a negatively sloped saving supply curve.

C.2 Borrowing constraints nested

We provide three alternative microfoundations for a borrowing constraint that fits our general description in (8).

C.2.1 Housing as collateral asset

In Section 3, we derived a simple borrowing constraint based on the idea that agents can pledge an income stream ℓY (e.g. coming from land they own). We now show that one would obtain a similar borrowing constraint when borrowers purchase houses instead, and use them as collateral. To do so, assume there is a fixed mass μ^b of housing units, freely traded at price p_t^h, over which borrowers have additive preferences $\alpha \log h_t$ each period, $\alpha > 0$. We assume that the collateral constraint is $\dot{d} + \lambda d \leq \theta p_t^h h_t$, where θ is the loan-to-value (LTV) ratio. In steady state, one can show that this implies a market clearing house price

$$p^h(r) = \frac{\alpha\omega^b}{(1-\theta/\lambda)\rho + (1+\alpha)\theta r/\lambda}$$

This fits into our earlier framework by assuming that $\ell(r) = rp^h(r)$.

In this version of the model, agents in the borrower dynasty bequeath both the house and their debt position to their offsprings. In a more realistic model with a life-cycle, older agents would sell their house, pay down their debts and consume the proceeds before death. Younger agents would purchase houses, partly debt-financed. A larger house price (due to lower interest rates), would stimulate consumption of existing homeowners and sellers of houses, raising aggregate consumption of the borrower dynasty, even if younger agents now pay more for the same-sized house.

C.2.2 Bewley-Aiyagari model

For this extension, we model borrowers as in Achdou, Han, Lasry, Lions, and Moll (2017), that is, we deviate from homogeneous preferences across types of agents. We focus on a steady state with a constant interest rate r. Each borrower i maximizes utility

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c_t^i dt$$

subject to budget and borrowing constraint

$$\dot{d}_t^i = r d_t^i + c_t^i - y^b e_t^i \qquad \text{and} \qquad d_t^i \leq \bar{d} y^b$$

where $\bar{d} > 0$ and $e_t^i > 0$ is a random Markov process for productivity in continuous time; $y^b \equiv \omega^b Y / \mu^b$ is borrowers' average income. This specification implies that borrowers can borrow up to a fixed fraction (or multiple) of their average income. Following the logic in Achdou et al. (2017), we see that total debt taken out by borrowers, $d(r, y^b) \equiv \int d_t^i di$ in a steady state with rate r is continuous and approaches $-\infty$ as $r \uparrow \rho$ and approaches $\underline{d} y^b$ as $r \downarrow 0$. Moreover, observe that $d(r, y^b)$ scales in y^b due to homothetic preferences, see Straub (2019). Therefore, we can write total debt as

$$d(r, y^b) = d(r, 1) y^b = \underbrace{r d(r, 1) \omega^b / \mu^b}_{\equiv \ell(r)} \cdot \frac{Y}{r}$$

in line with our general borrowing constraint (8). Thus, this model generates a steady-state debt demand curve that is "mostly" downward-sloping, in the sense that $d(r, y^b)$ is $-\infty$ for $r \uparrow \rho$, rising to $\bar{d} y^b$ for $r \downarrow 0$. There could be non-monotonicity in between, although that is unlikely given the results in Achdou et al. (2017).

C.2.3 Simplified buffer stock model

We have found that a simplified and tractable version of the full Bewley-Aiyagari model is a useful way to understand its implications. Specifically, we assume that there is a small Poisson probability $v > 0$ that a borrower receives a one-time negative income shock of size $\varphi Y > 0$ (after which there are no more other shocks). We model the negative shock to be sufficiently large that it shows up in a borrower's asset position: when assets are a_t^b before the shock, they fall to $a_t^b - \varphi Y$ after the shock. As we make the shock probability very small, $v \to 0$, this setup converges to a model where the borrowing constraint is given by

$$\dot{d}_t^i + \lambda d_t^i \leq p_t \ell - \varphi Y$$

where the borrowing constraint is tightened by the potential income loss φY relative to (7). The right hand side can be rewritten in the form $\ell(\{r_s\}_{s \geq t})$.