The Global Factor Structure of Exchange Rates *

Sofonias A. Korsaye† Fabio Trojani‡ Andrea Vedolin§

Abstract

We provide a model-free framework to study the global factor structure of exchange rates. To this end, we propose a new methodology to estimate model-free global stochastic discount factors (SDFs) pricing large cross-sections of international assets, such as stocks, bonds, and currencies, independently of the currency denomination and in the presence of trading frictions. We derive a unique mapping between the optimal portfolios of global investors trading in international markets with frictions and international SDFs, which allows us to recover global SDFs from asset return data alone. Trading frictions shrink portfolio weights of some assets to zero, leading to endogenously segmented markets and robust properties of international SDFs. From the cross-section of numéraire invariant SDFs, we extract global exchange rate factors and show that they are strongly related to dollar and carry. Using these factors, we obtain an excellent in- and out-of-sample fit of up to 80% for the cross-section of international asset returns, significantly improving upon the performance of benchmark factor models. Finally, we estimate the cost to obtain the portfolio home bias observed in the data and find it to be small.

Keywords: Stochastic discount factor, factor models, financial frictions, market segmentation, incomplete markets, home bias, regularization, lasso, elastic net

JEL Classification: F31, G15

This Version: November 2019

*We thank Alberto Quaini for helpful comments. All errors are ours.
†School of Economics and Management, University of Geneva and Swiss Finance Institute, sofonias.korsaye@unige.ch.
‡School of Economics and Management, University of Geneva and Swiss Finance Institute, fabio.trojani@unige.ch.
§Questrom School of Business, Boston University, NBER and CEPR, avedolin@bu.edu.

Canonical models in international finance, which match salient features of international asset returns, posit that stochastic discount factors (SDFs) are driven by both global and country-specific shocks.[1] The consensus view is that two global factors, dollar and carry, account for a significant fraction of the systematic variation of exchange rates, see, e.g., Verdelhan (2018). One common feature of this literature is that the nature of global risks is imposed ex ante into the construction of international SDFs, with global risks that enter all SDFs of all investors in all countries. In this paper, we take an agnostic view and develop a new model-free framework to study the nature and source of global exchange rate factors which emerge endogenously in a setting where investors face trading frictions.

One common assumption underlying the literature studying global factor structures of exchange rates is that markets are complete and agents can trade without frictions in global international markets. Under this assumption, the rate of appreciation of the exchange rate (X) is uniquely recovered from the ratio of the foreign (M_f) and the domestic SDF (M_d): $X = M_f/M_d$. This identity is called the asset market view of exchange rates (AMV) and implies that exchange rate shocks always reflect corresponding foreign or domestic SDF shocks. Validity of the AMV is convenient for studying global factor structures of exchange rates because of at least two reasons. First, it is well known that in order to address the volatility puzzle in exchange rates, international SDFs need to be almost perfectly correlated, which gives rise to a strong and parsimonious factor structure.[2] Second, if one assumes that the AMV holds, a factor that explains a significant part of the SDF variation in one currency is *by construction*, a global exchange rate factor that explains a significant part of the SDF variation in all other currencies. By triangular arbitrage, such SDF factors will explain bilateral exchange rate movements with respect to *any* currency numéraire. Hence, under the AMV, factors that price the cross-section of currencies in US dollars will also price the cross-section of currencies in any other currency denomination.

There are various important aspects to consider when relying on the AMV to understand global factor structures of exchange rates. First, as we deviate from the complete market assumption and allow markets to be incomplete, most domestic and foreign SDFs do not satisfy the AMV, see, e.g., Backus, Foresi, and Telmer (2001). Second, SDF properties, and hence, the factor structure of exchange rates depend crucially on the underlying assumptions about the menu of assets available to investors for trade, see, e.g., Sandulescu, Trojani, and Vedolin (2019). For example, as we increase the degree of segmentation across countries, SDFs become less correlated, and hence the factor structure weaker. In this case, the importance of global factors decreases whereas local factors may matter more. These issues, however, make the ensuing global factor structures dependent on

[1] For example, while exposure to country-specific risk factors can explain the negative coefficient in uncovered interest rate parity regressions, it cannot explain the cross-section of carry trade returns. Lustig, Roussanov, and Verdelhan (2011) find that the carry factor, a high minus low interest rate sorted currency portfolio, is a direct measure of a global common risk factor. This factor can explain two thirds of the cross-sectional variation of exchange rates. Lustig, Roussanov, and Verdelhan (2014) extend this work and find that US specific exposure to global risk, a dollar factor, is the main driver of currency return predictability. While the carry is linked to changes in global equity volatility, the dollar factor returns are correlated with the average growth rate of consumption across countries and the rate of US-specific component of industrial production growth.

[2] Recall from the AMV that (log) exchange rate changes are equal to the difference in (log) foreign and domestic SDFs. Taking variances on both sides and assuming exchange rate volatilities around 15% per year implies an almost perfect correlation for international SDFs; see, e.g., Brandt, Cochrane, and Santa-Clara (2006).

ad hoc assumptions about the degree of financial market integration, which in most cases may be economically hard to justify.

In this paper, we study global factor structures of exchange rates by means of a new model-free methodology, which allows SDFs to price large cross-sections of international assets, such as stocks, bonds, and currencies, in presence of real-world frictions and independent of the currency denomination. Incorporating frictions allows us to endogenize the optimal set of assets selected by global investors for trading, because barriers to trade induce optimal portfolio weights on some assets to shrink to zero. Hence, this produces endogenous international market segmentation without imposing ad hoc assumptions on the degree of international market segmentation. At the same time, sparse optimal portfolio weights produce more robust SDFs with improved pricing properties out-of-sample. With regard to the AMV, we theoretically identify market structures under which model-free SDFs satisfy the AMV and are numéraire invariant even in the presence of market frictions. This property makes them uniquely suited to estimate global exchange rate factors. From the cross-section of these model-free international SDFs, we then extract global SDF factors and show that they are directly related to dollar and carry, thus providing model-free support to the model-based evidence in Lustig, Roussanov, and Verdelhan (2011, 2014). We then price the cross-section of international asset returns with a global SDF constructed from the cross section of model-free SDFs and find that it substantially improves upon the performance of benchmark factor models of exchange rates, by explaining up to 80% of the variation of asset returns out-of-sample.

As it is well-known that standard methods to estimate SDFs using cross-sections of assets can lead to spurious estimates when the size of the cross-section increases, we rely on a new methodology for estimating model-free SDFs, which can endogenously produce sparse SDFs and sparse optimal portfolios. For instance, Kozak, Nagel, and Santosh (2019) shrink the coefficients of low variance principal components of characteristics-based factors using various penalization techniques from the machine learning literature. Different from this literature, we endogenously obtain sparse SDFs by imposing various forms of economically relevant financial market frictions.[3]

Our framework is general as it can incorporate various forms of frictions, such as proportional transaction costs, margin or collateral constraints, and short-sell constraints, while at the same time ensuring that model-free SDFs are consistent with the absence of arbitrage in asset markets with frictions. It is well-known that in frictionless and arbitrage-free markets, asset prices can be fully characterized by an SDF that only depends on asset returns, which at the same time prices all assets exactly, see, e.g., Ross (1978) and Hansen and Richard (1987), among many others. However, in the presence of frictions, linear SDF pricing implies non zero pricing errors that reflect the underlying structure of market frictions.[4] Earlier literature has often treated pricing errors as evidence of SDF misspecification, by expressing them in terms of the least squares distance between an SDF and the family of SDFs that price correctly all assets. We instead explicitly work under the assumption of arbitrage-free markets with frictions and the resulting pricing error structures. In this setting, we

[3]This approach is particularly appealing in the context of international financial markets, given the evidence of currency-biased portfolios of international investors; see, e.g., Maggiori, Neiman, and Schreger (2019).

[4]Non zero pricing errors may also emerge in frictionless arbitrage-free market settings under the notion of "asymptotic arbitrage" as in Ross (1976)' Arbitrage Pricing Theory (APT). Such settings are also compatible with our methodology.

characterize model-free SDFs that minimize established notions of dispersion, such as entropy or variance, in terms of the optimal portfolios of global investors constrained by market frictions, which allows us to recover global model-free SDFs from asset return data alone.

When relaxing the assumptions of complete and frictionless markets, violations of the AMV arise for the vast majority of international SDFs, which implies factor structures of exchange rates that are not exact. While this is expected, we theoretically show that there exist model-free SDF families which satisfy the AMV in international asset markets even in the presence of frictions as long as these frictions are symmetric. Market symmetry in frictions means that frictions are identical across currency denominations.[5] The family of model-free SDFs satisfying the AMV in symmetric international asset markets with frictions is the family of minimum entropy SDFs. We establish two powerful properties for these SDFs. First, we show that these SDFs are by construction numéraire-invariant, meaning that an optimal SDF pricing assets well in one currency also prices assets well in any other currency (after a simple multiplication with the exchange rate). Second, as long as markets are symmetric, optimal portfolios of global investors are identical leading to an exact factor structure of exchange rates. Therefore, in symmetric markets, the family of minimum entropy SDFs establish a benchmark to understand both the nature of global exchange rate risks and the implications of market friction asymmetries for the properties of these risks.

We study the global factor structure of exchange rates using a large cross-section of bonds and equities on developed countries, and we extract numéraire invariant minimum entropy SDFs under different market setting assumptions with varying transaction costs. When we assume that investors can trade internationally the full menu of assets in frictionless markets, we find that the ensuing SDFs are highly volatile (to the extent that they imply implausible Sharpe ratio bounds) and that they satisfy a virtually perfect single-factor structure with perfectly correlated SDFs. Moreover, the numéraire invariant optimal portfolio of global investors also implies positions in single assets that may be hard to maintain in practice without taking massively levered long and short positions. When instead we impose symmetric international market frictions, the SDF volatility decreases by no less than 30%. While the optimal portfolio of global investors is again independent of the currency denomination, it already implies a number of zero portfolio weights on some assets and segmentation arises endogenously. Similar to the frictionless case, international cross-country SDF correlations are still almost perfect, indicating once more a very strong factor structure. We also find that the key factor properties implied by minimum entropy SDF families in these settings are consistently generated also by other model-free SDF families, such as minimum variance SDF families, which theoretically do not satisfy the AMV. This evidence shows that in the data the AMV violations induced by these SDF families are not strong enough to alter in an economically essential way the resulting factor structure of model-free SDFs.

When we move from symmetric to asymmetric market settings, we find that SDF volatilities half relative to the symmetric frictions case and that the ensuing optimal portfolios of global investors become more sparse. While lower than under the symmetric market setting, minimum entropy SDF

[5]For instance, when we assume a symmetric market, we either impose no frictions at all or we impose the exact same friction across all countries. Asymmetric markets, on the other hand, imply differences in the friction structure of some countries, such as, e.g., lower trade barriers for investing in local assets than in foreign assets.

correlations are still very high and common factor structures very strong. The optimal portfolios of global investors in markets with frictions typically further shrink average international portfolio weights and reveal an interesting factor composition: Global investors always trade a carry, i.e., go long in high interest rate currencies and go short in low interest rate currencies, and they go long in US equity, while virtually completely ignoring long-term bonds. These findings connect to a larger literature in international finance documenting priced systematic dollar and carry risks. However, different from this literature our approach is completely model-free, it extracts SDFs directly from a broad cross-section of international asset returns (not only from a cross-section of exchange rates) and it allows us to interpret global factor compositions economically from the optimal portfolios of global investors.

From the cross-section of model-free international SDFs, we extract global and local SDF factors by estimating (i) the global factor as the first principal component in the cross-section of model-free SDFs and (ii) the local factors as the residuals of model-free local currency SDFs projected on the global factor. Inspection of the composition of the so obtained global factor confirms our previous results that global investors have the biggest exposures on carry and a long US equity position. Using local and global SDF factors we price the cross-section of international asset returns, including stock, bond, and currency returns, both in- and out-of-sample. The out-of-sample analysis is useful to control for overfitting effects, as we know, e.g., that in the market setting with no frictions in-sample model-free SDF pricing errors are zero by construction. Using rolling training periods of ten years for SDF estimation and one year rolling windows for validation, we find that the out-of-sample performance of model-free SDFs consistent with the presence of various forms of market frictions is excellent. Indeed, our out-of-sample global SDFs explain up to 80% of the variation in the cross-section of currencies and more than 80% of the variation in the cross-section of stock and bond returns. While local SDF components are significant as well in explaining the cross-section of international asset returns, we find that their economic relevance is dwarfed by the global SDF factor.

A large literature in international finance documents strong currency home bias with respect to stocks and bonds. For example, Maggiori, Neiman, and Schreger (2019) argue, that currency home bias may reflect a combination of financial frictions but also behavioral biases such as inattention that effectively segment financial markets. Since the latter cost are usually hard to quantify, we take our framework and ask how much asymmetry in frictions do we have to impose in order to match the almost perfect home bias that we observe in the data?[6] To this end, we take our asymmetric market friction setting and increase the cost to trade foreign assets. We find that in order to achieve almost perfect home bias, trading foreign assets needs to be around eight times more expensive than trading local assets. Given that the average bid ask spread is around 2bps, this implies that trading foreign assets entails a transaction cost of around 14bps in unobservable costs. With these costs, we find that nearly all local investors short the long-term bond and hold long positions in the local equity index. At the same time, we find that investors trade a carry independent of the currency denomination. For example, while US investors short long-term US bonds and buy US equity, they simultaneously short US short-term bonds and hold a long position in the Australian dollar. Overall, different from

[6] See Coeurdacier and Rey (2013) for a discussion of the implied costs.

earlier literature that has argued that transaction costs need to be unrealistically high to explain the home bias in the data, we conclude that these costs can be relatively small.

Literature Review: Our paper contributes to a growing literature in international finance studying global factors. The seminal work of Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2018) documents that two factors, carry and dollar, explain a significant share of the systematic variation in exchange rates. Panayotov (2019) studies global risk in an extended version of Lustig, Roussanov, and Verdelhan (2014), in which the US SDF has a larger exposure to global risk than all other international SDFs. After testing the model on numéraire invariant currency portfolios, the author finds evidence for a priced global risk factor linked to global equity indices. Maurer, Tô, and Tran (2019) extract two principal exchange rate components from the cross-section of all cross-currency returns related to dollar and carry, in order to construct country specific SDFs. These SDFs are shown to price international equity returns well in sample. Aloosh and Bekaert (2019) reduce the cross-section of currencies by means of currency baskets that measure the average appreciation of each currency against all other currencies. They then apply clustering techniques to the cross-section of currency baskets and identify two clusters, one related to the dollar and another related to the Euro. Lustig and Richmond (2019) model gravity in the cross-section of exchange rates and find factor structures related to physical, cultural, and institutional distances between countries. Jiang and Richmond (2019) link trade networks between countries to exchange rates comovement in order to explain the existence of the global dollar and carry factors.

Our paper is different from this literature along several dimensions. First, it relies on common SDF factors that are extracted from a family of model-free, numéraire invariant SDFs, which satisfy the AMV in arbitrage-free international asset markets with frictions. Second, using a new methodology for estimating model-free SDFs from large cross-sections of assets, we are able to extract global factors jointly from a cross-section of returns including international equities and bonds, in addition to exchange rate returns. Third, our factors are directly related to the optimal portfolios of global investors in international asset markets with frictions, which provide additional unique insights into the factor composition, in terms of the portfolio exposure of global investors to various international assets. Fourth, the generality of our methodology allows us to incorporate into our analysis several economically relevant forms of market frictions and endogenous market segmentation features, and to study their implications for global factor structures. In this context, we find that the first principal component of the cross-section of model-free SDFs, under varying assumptions on symmetric and asymmetric market frictions, is a linear combination of a carry and a dollar factor, in line with the findings of Lustig, Roussanov, and Verdelhan (2011, 2014). Finally, we show that our common factors jointly price cross-sections of returns including international equities and bonds, in addition to exchange rate returns, not just in- but also out-of-sample.

Our paper is related to another important recent literature which studies how to extract regularized SDFs from a large cross-section of asset returns. A common denominator of the various approaches in this literature is the introduction of various penalization devices that shrink the in-sample exposures of extracted SDFs to assets returns. These penalizations are designed to mitigate

in-sample overfitting and to improve the out-of-sample pricing performance. Moreover, depending on the chosen penalization structure, they can induce different degrees of sparsity in extracted SDFs, in order to reduce the number of relevant factors an SDF depends on. For example, Kozak, Nagel, and Santosh (2019) use an elastic net penalization to shrink the contributions of low-variance principal components of characteristics-based factors. This technique induces sparse linear SDFs, which depend on a small number of characteristics-based principal components, and is shown to produce a more robust out-of-sample pricing performance. Freyberger, Neuhierl, and Weber (2019) study the cross-section of expected stock returns, by means of an adaptive group lasso technique enclosing an additive nonparametric specification of nonlinear dependencies between expected returns and characteristics. Using this technique, they show that a small set of characteristics can explain the cross-section of out-of-sample expected returns. None of these papers makes use of penalizations directly motivated from corresponding transaction cost specifications.[7]

Model-free SDF in markets with frictions have been introduced earlier in the literature, by a small number of important contributions. He and Modest (1995) and Luttmer (1996) extend the Hansen and Jagannathan (1991) minimum variance SDF setting, by incorporating various specifications of sublinear transaction costs that give rise to generalized diagnostics for asset pricing models. Hansen, Heaton, and Luttmer (1995) provide the econometric tools for the evaluation of asset pricing models in such settings. A key theoretical finding in this literature is that the pricing functional sublinearity gives rise to SDFs with non zero pricing errors, which are tightly constrained by the given transaction cost structure. Korsaye, Quaini, and Trojani (2018) extend this theory to address minimum dispersion SDFs resulting from general convex pricing errors structures, which are uniquely characterized in terms of optimal portfolios of investors in markets with convex transaction costs.[8] We make use of this theory in order to identify financial market structures that deliver numéraire invariant model-free SDF families which satisfy the AMV. This result is key to identify parsimonious and insightful global exchange rate factors structures. Second, we exploit the theoretical relation between numéraire invariant model-free SDFs and the optimal portfolios of investors in markets with frictions, in order to study economic factor compositions in terms of the SDF exposures to international asset returns.

Our work is also related to the literature documenting that even small transaction costs can induce substantial market segmentation; see, e.g., Martin and Rey (2004) and Bhamra, Coeurdacier, and Guibaud (2014). For instance, Bhamra, Coeurdacier, and Guibaud (2014) show in a two country economy with transaction costs that any cost bearing on foreign equity holdings has two opposite effects on portfolios. A direct effect, which reduces cross-border holdings via lower expected returns on foreign assets, and an indirect effect, which reduces the substitutability between national assets because of a lower return correlation, thus increasing the willingness to diversify internationally. Since the latter effect is negligible, the authors show that a large home bias can arise also from small frictions. In contrast to these papers, our approach is model-free and does not require a

[7] DeMiguel, Martin-Utrera, Nogales, and Uppal (2019) extend a mean-variance portfolio problem with lasso penalization by an explicit additional penalization for characteristics-dependent proportional transaction costs. Their findings show that the resulting optimal portfolio depends on a larger number of characteristics, because combining characteristics reduces transaction costs involved with portfolio adjustments that often cancel out.

[8] These authors also develop the econometric theory for the empirical analysis of minimum dispersion SDFs with convex pricing errors and study the empirical properties of these SDFs in an APT context.

specification of a corresponding equilibrium economy. We directly estimate model-free international SDFs together with the associated optimal portfolios from the data, and find that even relatively small transaction costs on international asset holdings can substantially impact the portfolios of global investors, both by shrinking portfolio exposures and by implying zero investments on subsets of assets. In parallel, we also find that for the various specifications of market frictions we consider international global factor structures are in all cases quite parsimonious and similarly informative about the broad structure of global factors.

A last strand of literature related to our work studies exchanges rates in segmented markets. Chien, Lustig, and Naknoi (2019) show that limited stock market participation can reconcile in a structural equilibrium model highly correlated international SDFs with a low correlation in consumption growth. Gabaix and Maggiori (2015) study the disconnect puzzle and the violations of the uncovered interest rate parity (UIP), in an equilibrium setting where specialized financiers intermediate households' asset demands in segmented markets. Using a model-free approach, Sandulescu, Trojani, and Vedolin (2019) explore the implications of various forms of exogenous market segmentation for bilateral SDFs and the resulting deviations from the AMV, under the assumption of frictionless markets satisfying the law of one price. In contrast to these papers, we study the global factor structures implied by a cross-section of numéraire invariant model-free SDFs that jointly price international asset returns in each currency denomination. To this end, we make use of a new approach, estimating for a cross-section of countries model-free SDFs in large asset markets, under various specifications of symmetric and asymmetric global market frictions that give rise to endogenous segmentation and stock market participation patterns. For several relevant forms of frictions, our findings point to highly correlated SDFs and strong global factor structures in the cross-section of SDFs, giving rise to excellent pricing properties with respect to the cross-section of international out-of-sample equity, bond and currency returns.

Outline of the paper: The rest of the paper is organized as follows. Section 1 provides the theoretical framework for studying model-free SDFs in international financial markets with frictions. Section 2 presents our main empirical findings. Section 3 concludes. All proofs are gathered in a separate appendix.

1 Global Stochastic Discount Factors and Market Frictions

In this section, we develop a model-free framework that allows us to study international SDFs in the presence of various forms of frictions, such as bid-ask spreads, short-selling, margin, or collateral constraints. In this framework, we obtain a unique one-to-one relation between minimum dispersion SDFs and the optimal portfolio returns of a corresponding portfolio choice problem with penalized portfolio weights. This mapping between SDFs and optimal portfolio returns allows us to systematically pin down the optimal portfolios of global investors when there are barriers to trade. We then characterize international frictions structures that give rise to families of numéraire invariant model-free SDFs satisfying the AMV. We show that these SDF families are naturally suited to study global factor structures and detail their tight links to the cross-section of exchange rates.

1.1 Model-Free SDFs in the Presence of Frictions

To keep notation simple, for the time being, we drop all references to particular countries and we will relax this later on. We consider a two period incomplete market economy in which investors can trade a set of N basis assets with payoff vector $\boldsymbol{Z} := (Z_n)_{n=1,\dots,N}$ at time 1, and a corresponding price vector $\boldsymbol{P} := (P_n)_{n=1,\dots,N}$ at time 0. We assume that each payoff Z_n belongs to the space $L^q(\Omega, \mathcal{F}, \mathbb{P})$ of random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ with finite q-th moment, for some $1 < q \leq \infty$. We partition the set of assets into two subsets indexed by $D \subset \{1, \dots, N\}$, and $S := \{1, \dots, N\} \setminus D$, of size N_D and N_S, respectively, and we denote by $(\boldsymbol{X}_D, \boldsymbol{P}_D)$ and $(\boldsymbol{X}_S, \boldsymbol{P}_S)$ the payoff and price vectors of assets subject to market frictions and assets not subject to any market friction. All prices and payoffs are denominated in a common currency (or numéraire).

We model transaction costs in our economy using a closed and sublinear transaction cost function h that quantifies the costs associated with any portfolio in our economy, based on the same common numéraire for payoffs and prices.[9] Intuitively, sublinear transaction costs reflect the fact that the cost of a portfolio composed of different assets is lower or equal to the sum of the costs generated by trading each asset individually. This specification of transaction costs encompasses all transaction cost structures relevant for our analysis, such as, e.g., short selling constraints, bid-ask spreads, and proportional transaction costs, such as leverage constraints.

Following Harrison and Kreps (1979), we introduce a price system $(\mathcal{Y}, \pi)$, consisting of traded payoff space $\mathcal{Y}$ and a price functional π defined on $\mathcal{Y}$. We define $\mathcal{Y}$ as the set of portfolio payoffs that can be traded with finite transaction costs, out of the set of all portfolio payoffs generated by portfolios of payoffs in vector $\boldsymbol{Z}$:

$$\mathcal{Y} := \{Y = \theta' \boldsymbol{Z} : \ h(\boldsymbol{\theta}_D) < \infty, \ \boldsymbol{\theta} \in \mathbb{R}^N\}, \tag{1}$$

where $\boldsymbol{\theta}$ is the vector of portfolio weights of all assets and $\boldsymbol{\theta}_D$ the sub-vector of portfolio weights in assets subject to market frictions. The pricing functional in this economy with frictions is then simply given by the payoff minimum replicating cost when accounting for transaction costs:

$$\pi(Y) := \inf_{\boldsymbol{\theta} \in \mathbb{R}^N} \left\{ \boldsymbol{\theta}' \boldsymbol{P} + h(\boldsymbol{\theta}_D) \ : \ Y = \boldsymbol{\theta}' \boldsymbol{Z} \right\} . \tag{2}$$

We work under the assumption of an arbitrage-free price system $(\mathcal{Y}, \pi)$, where as usual, we define an arbitrage portfolio as a traded payoff $Y \geq 0$ such that $\pi(Y) \leq 0$ and $\mathbb{P}(Y > 0) > 0$.

The next proposition provides the foundation to our approach based on SDFs with non-zero pricing errors, by showing that the assumption of arbitrage-free markets is equivalent to the existence of such SDFs in our setting.

Proposition 1. *Price system $(\mathcal{Y}, \pi)$ defined in equations (1) and (2) is arbitrage-free if and only if there exists a strictly positive SDF $M \in L^p$ that satisfies the (unconditional) pricing conditions:*

$$\mathbb{E}[M \mathbf{R}_S] - \mathbf{1}_{N_S} = 0 \qquad \textit{and} \qquad \mathbb{E}[M \mathbf{R}_D] - \mathbf{1}_{N_D} \in C_h \, , \tag{3}$$

[9] A real valued function f on a real vector space V is called sublinear, if $f(x + y) \leq f(x) + f(y)$ for all $x, y \in V$ and $f(\lambda x) = \lambda f(x)$ for all $\lambda \geq 0$ and $x \in V$. f is called closed if its sublevel set $\{x \in dom(f) \ : \ f(x) \leq \alpha\}$ is closed for any $\alpha \in \mathbb{R}$.

with

$$C_h := \{\boldsymbol{y} \in \mathbb{R}^{N_D} \ : \ \boldsymbol{y}'\boldsymbol{w} \leq h(\boldsymbol{w}) \textit{ for all } \boldsymbol{w} \in \mathbb{R}^{N_D}\} \,, \tag{4}$$

and the gross return vectors $\boldsymbol{R}_S$, $\boldsymbol{R}_D$ associated with payoff vectors $\boldsymbol{Z}_S$, $\boldsymbol{Z}_D$.

Proposition 1 is akin to the well-known Fundamental Theorem of Asset Pricing and it characterizes by a standard no-arbitrage condition the existence of strictly positive SDFs in markets with frictions. According to pricing relation (3), such SDFs price exactly all assets not subject to trading frictions, while they imply non zero pricing errors constrained by the closed convex set C_h for assets subject to trading frictions.

Notice that proposition 1 extends earlier results in the literature for markets with pricing errors constrained by a convex cone; see, e.g., Cvitanić and Karatzas (1992), Hansen, Heaton, and Luttmer (1995), and Luttmer (1996), among others. Moreover, it also naturally contains the standard case of arbitrage-free markets with no frictions on any asset, which arises whenever $C_h = \{\mathbf{0}_{N_D}\}$, i.e., transaction costs on all assets are zero.

Proposition 1 constrains the set of relevant SDFs in our economy with frictions and it implies that the following set of non-negative SDFs contains at least one strictly positive SDF:

$$\mathcal{M}_h := \{M \in L^p \ : \ M \geq 0 \,, \ \mathbb{E}[M\boldsymbol{R}_S] - \mathbf{1}_{N_S} = 0 \text{ and } \mathbb{E}[M\boldsymbol{R}_D] - \mathbf{1}_{N_D} \in C_h\} \ . \tag{5}$$

Recall that since markets are incomplete, there exist many possible SDFs. In particular, set $\mathcal{M}_h$ can usually contain several non negative SDFs and we are interested in SDFs out of this set that minimize established notions of dispersion in the literature. To this end, we measure SDF dispersion by standard integral functionals of the form $\mathbb{E}[\phi(\cdot)] : L^p \to (-\infty, \infty]$, where ϕ is a closed and convex function. This setting contains several well-known minimum dispersion SDFs in the literature, e.g., Hansen and Jagannathan (1991) minimum variance SDFs, minimum Kullback-Leibler divergence, and minimum entropy SDFs in our arbitrage-free economy with frictions. We next define minimum dispersion SDFs in our economy with frictions.

Definition 1. Minimum dispersion SDFs. Given transaction cost function h, convex and closed dispersion function ϕ, and partition $S \cup F$ into assets subject and not subject to trading costs, a minimum dispersion SDF is the solution to following optimization problem:

$$\Pi_h := \inf_{M \in \mathcal{M}_h} \mathbb{E}[\phi(M)] \,. \tag{6}$$

By definition, a minimum dispersion SDF is the solution to an optimization problem of the form (6). Working directly with such optimization problems is difficult, as they are infinite-dimensional optimization problems that depend on unobservable random variables. Therefore, we follow Korsaye, Quaini, and Trojani (2018) and first characterize minimum dispersion SDFs via the solutions of a penalized portfolio problem in $\mathbb{R}^N$, in which a notion of portfolio payoff dispersion is minimized in presence of transaction costs. To this end, let:

$$\Delta_h := \min_{\theta \in \mathbb{R}^N} \left\{\mathbb{E}[\phi_+^*(-\boldsymbol{\theta}'\boldsymbol{R})] + \boldsymbol{\theta}'\mathbf{1}_N + h(\boldsymbol{\theta}_D)\right\} \ , \tag{7}$$

where ϕ_+^* is the convex conjugate of the restriction of ϕ to the nonnegative real line.[10] From portfolio problem (7), we obtain the following explicit characterization of minimum dispersion SDFs in our economy with frictions.

Proposition 2. *Given the convex and closed transaction cost function h, strictly convex and closed dispersion function ϕ, and arbitrage-free price system $(\mathcal{Y}, \pi)$, it follows:*

1. $\Pi_h = -\Delta_h$.
2. *If $\boldsymbol{\theta}_0$ is the solution to problem (7), then the unique solution M_0 to problem (6) is given by*

$$M_0 = (\phi_+^*)'(-\boldsymbol{\theta}_0' \boldsymbol{R}) \,, \tag{8}$$

where $(\phi_+^*)'(y)$ is the derivative of ϕ_+^* at y.

In Proposition 2, the unique SDF solving problem Π_h, as a function of the optimal portfolio payoff solving dual problem Δ_h, is completely determined by the shape of ϕ_+^*, via its derivative at the solution of problem (7). On the other hand, transaction cost function h impacts the structure of the optimal portfolio in the dual problem Δ_h. Importantly, with Proposition 2 we are now able to directly estimate model-free minimum dispersion SDFs in markets with frictions from asset return data alone, since from equation (8) the minimum dispersion SDF can be directly estimated from the return vector $\boldsymbol{R}$ using the optimal portfolio weight $\boldsymbol{\theta}_0$ from problem (7).

Korsaye, Quaini, and Trojani (2018) formally address the consistent estimation of parameter $\boldsymbol{\theta}_0$, which can be achieved with an extremum estimator that incorporates penalization term $h(\boldsymbol{\theta}_D)$ from equation (7). More specifically, given a stationary return process $\{\boldsymbol{R}_{t+1}\}_{t\in\mathbb{N}}$, the empirical version of (7) defines the following estimator:

$$\hat{\boldsymbol{\theta}} := \arg\min_{\hat{\boldsymbol{\theta}}\in\mathbb{R}^N} \left\{ \frac{1}{T}\sum_{t=1}^{T} \phi_+^*(-\boldsymbol{\theta}'\boldsymbol{R}_{t+1}) + \boldsymbol{\theta}'\mathbf{1}_N + h(\boldsymbol{\theta}_D) \right\} . \tag{9}$$

For the class of dispersion functions ϕ studied in this paper and for a large class of penalizations $h(\boldsymbol{\theta}_D)$, including virtually all penalizations in the machine learning literature and penalizations generated by market frictions as in this paper, estimator $\hat{\boldsymbol{\theta}}$ consistently estimates parameter $\boldsymbol{\theta}_0$.More specifically, from Proposition 2, a consistent estimator for minimum dispersion SDF M_0 is given by

$$\hat{M}_{t+1} := (\phi_+^*)'(-\hat{\boldsymbol{\theta}}'\boldsymbol{R}_{t+1}) \,. \tag{10}$$

We make use of this estimator to estimate international model-free SDFs in our empirical analysis of global exchange rate factors in Section 2.

1.2 Examples of Dispersion Metrics

Proposition 2 allows for a fairly general dispersion function ϕ in the specification of the SDF dispersion $\mathbb{E}[\phi(M)]$. However, three explicit choices of ϕ are particularly relevant for our analysis of exchange rate factor structures.

[10] Function ϕ_+ is defined by $\phi_+(y) = \phi(y)$ for $y \geq 0$ and by $\phi_+(y) = 0$ otherwise. Given a function f defined on $\mathbb{R}^m$, its convex conjugate is defined for $y \in \mathbb{R}^m$ by $f^*(y) := \sup_{w\in\mathbb{R}^m}\{w'y - f(w)\}$. The assumption that f is closed and convex insures a one to one relation between f and f^*.

1.2.1 Negative Entropy

The negative entropy SDF-dispersion is obtained for $\phi(x) = -\ln(x)$ when $x > 0$ and $\phi(x) = +\infty$ when $x \leq 0$. This corresponds to a convex conjugate portfolio dispersion function given by $\phi_+^*(y) = -\ln(-y)$ for $y < 0$ and $\phi_+^*(y) = +\infty$ otherwise. The minimum entropy SDF from Proposition 2 reads explicitly:

$$M_0 = -\frac{1}{\boldsymbol{\theta}_0' \boldsymbol{R}} \,. \tag{11}$$

The relevance of the negative entropy SDF-dispersion for our analysis comes from the fact that it defines a numéraire invariant optimization criterion in Definition 1 and hence a numéraire invariant minimum entropy SDF.

1.2.2 Variance

SDF variance corresponds to a dispersion function given by $\phi(x) = x^2/2$ for any $x \in \mathbb{R}$. This function is self-conjugate, which implies a portfolio dispersion function given by $\phi_+^*(y) = (y^+)^2/2$ for any $y \in \mathbb{R}$. The minimum variance SDF from Proposition 2 reads explicitly:

$$M_0 = (-\boldsymbol{\theta}_0' \boldsymbol{R})^+ \,. \tag{12}$$

The relevance of SDF-variance for our analysis comes from the fact that this SDF is piecewise linear in the optimal return $-\theta_0 \boldsymbol{R}$ implied by the solution of optimization problem (7). Hence, this SDF is tradable in settings where the zero lower bound is not attained, meaning we can replicate the SDF using basic linear assets such as stocks and bonds.

1.2.3 Kullback-Leibler dispersion

Kullback-Leibler SDF dispersion corresponds to a dispersion function $\phi(x) = x\ln(x)$ for $x > 0$ and $\phi(x) = +\infty$ for $x \leq 0$. This corresponds to a convex conjugate portfolio dispersion function given by $\phi_+^*(y) = e^y$ for any $y \in \mathbb{R}$. The minimum Kullback-Leibler dispersion SDF from Proposition 2 reads explicitly:

$$M_0 = e^{-\boldsymbol{\theta}_0' \boldsymbol{R}} \,. \tag{13}$$

The relevance of Kullback-Leibler SDF dispersion for our analysis comes from the fact that this SDF is strictly positive everywhere. The exponential in equation (13) is also convenient to obtain factor structures for log exchange rates that are naturally linear in the optimal returns solving optimization problem (7).

1.3 Examples of Market Frictions

While our setup can potentially incorporate a large variety of market frictions, we focus on three such frictions: short-selling constraints, bid-ask spreads, and proportional transaction costs, where the latter can equivalently be interpreted as leverage restrictions.

1.3.1 Short-Sell Constraints

Earlier literature explores the effect of short-sell constraints on SDFs; see Luttmer (1996) and Hansen, Heaton, and Luttmer (1995), among others. Incorporating such constraints into our setup is very straightforward, as it is sufficient to specify as a transaction cost function the characteristic function of the positive cone in $\mathbb{R}^{N_D}$:

$$h(\boldsymbol{\theta}_D) = \delta_{\mathbb{R}^{N_D}_+}(\boldsymbol{\theta}_D) := \begin{cases} 0 & \boldsymbol{\theta}_D \in \mathbb{R}^{N_D}_+ \\ \infty & \text{else} \end{cases} . \tag{14}$$

Under such a specification of transaction costs, set C_h in equation (4) of Proposition 1 is equal to the negative cone in $\mathbb{R}^{N_D}$ and all pricing errors on returns of assets subject to trading frictions are negative:

$$\mathbb{E}[M\boldsymbol{R}_D] - \mathbf{1}_{N_D} \leq 0 . \tag{15}$$

1.3.2 Proportional Transaction Costs

Proportional transaction costs are trading costs proportional to portfolio positions such as leverage constraints. We model such market frictions in a straightforward manner by applying portfolio weight norms. In particular, let $h(\cdot) := \lambda||\cdot||$, for some $\lambda \geq 0$ and a norm $||\cdot||$. Under such a specification of transaction costs, set C_h in equation (4) of Proposition 1 is given by:

$$C_h = \left\{ \boldsymbol{y} \in \mathbb{R}^{N_D} \ : \ ||\boldsymbol{y}||_* \leq \lambda \right\} , \tag{16}$$

where $||\cdot||_*$ is the dual norm of $||\cdot||$.[11] Hence, in the arbitrage-free economy with proportional transaction costs, the following pricing error inequality holds:

$$||\mathbb{E}[M\mathbf{R}_D] - \mathbf{1}_{N_D}||_* \leq \lambda . \tag{17}$$

One possible specification of proportional transaction costs relies on a l_1-norm, defined by $||\boldsymbol{y}||_1 := \sum_{i=1}^{N_D} |y_i|$. This specification gives rise to a lasso-type penalty in optimal portfolio problem (7) and to a bounded maximum pricing error in equation (17).[12] It is well-known from the machine learning literature that a lasso penalization produce sparse solutions in a corresponding optimization problem. In our setting, this feature leads to optimal portfolios and minimum dispersion SDFs that depend only on a strict subset of asset returns.

1.3.3 Bid-Ask Spreads

A natural way to incorporate bid-ask spreads in our framework is to consider long positions $\boldsymbol{\theta}^L$, when one buys an asset at ask price at time 0 and sells it at bid price at time 1, and short positions $\boldsymbol{\theta}^S$, when one buys an asset at bid price at time 0 and sells at ask price at time 1. This corresponds to a setting with (i) no short-selling constraints on long position $\boldsymbol{\theta}^L$ and (ii) no buying constraints on short

[11] The dual norm of norm $||\cdot||$ is defined by $||\boldsymbol{y}||_* = \sup_{\boldsymbol{w}}\{\boldsymbol{w}'\boldsymbol{y} : \ ||\boldsymbol{w}|| \leq 1\}$.

[12] If $||\cdot||$ is an l_p norm then $||\cdot||_*$ is an l_q norm, and viceversa, with p, q in $(1, \infty)$ and $1/p + 1/q = 1$. Moreover, l_1 and l_∞ norms are dual to each other. Finally, the l_2 norm is the only self-dual norm.

positions $\boldsymbol{\theta}^S$. Denoting by $\boldsymbol{\theta}_D = [\boldsymbol{\theta}_D^{L\prime}, \boldsymbol{\theta}_D^{S\prime}]'$ the extended portfolio vector of long and short position on each asset, we can easily incorporate these market frictions with following transaction cost function:

$$h(\boldsymbol{\theta}_D) := \delta_{\mathbb{R}_+^{N_D}}(\boldsymbol{\theta}_D^L) + \delta_{\mathbb{R}_-^{N_D}}(\boldsymbol{\theta}_D^S)\,. \tag{18}$$

Under this specification of transaction costs, set C_h in equation (4) of Proposition 1 is given by the Cartesian product of the positive cone in $\mathbb{R}^{N_D}$ and the negative cone in $\mathbb{R}^{N_D}$ ($C_h := \mathbb{R}_-^{N_D} \times \mathbb{R}_+^{N_D}$). Hence, pricing errors on long (short) positions are negative (positive):

$$\mathbb{E}[M\mathbf{R}_D^L] - \mathbf{1}_{N_D} \leq 0\,;\; \mathbb{E}[M\mathbf{R}_D^S] - \mathbf{1}_{N_D} \geq 0\,, \tag{19}$$

where $\mathbf{R}_D^L$ and $\mathbf{R}_D^S$ are the gross return vectors for long and short positions, respectively.

1.3.4 Combination of Frictions

It is straightforward to introduce different types of trading frictions for different subsets of assets. For example, one could introduce short-sell constraints for equities and at the same time impose bid-ask spreads on the carry trade. In such settings, set C_h in equation (4) of Proposition 1 for the global transaction cost constraints just becomes the Cartesian product of the individual sets reflecting the trading frictions applied to each subset of assets.

1.4 International SDFs

Our setting above can be naturally extended to account for the fact that global investors trade assets internationally in markets with possibly different currency denominations. To this end, consider an economy consisting of M countries, in which investors in each country can trade N assets, consisting of both local and foreign assets, denominated in the country's own currency. We denote with $\boldsymbol{R}^{(i)}$ the vector of gross returns denominated in country i's currency, i.e.,

$$\boldsymbol{R}^{(i)} := \{R_k^{(i)}\}_{k=1,\ldots,N}\,, \tag{20}$$

where $R_k^{(i)}$ is the gross return of asset k in country (currency) i. Denoting with $X_j^{(i)}$ the gross exchange rate return, with the exchange rate defined as the price in country i currency of one unit of country j's currency, it then follows:

$$\boldsymbol{R}^{(i)} = X_j^{(i)}\boldsymbol{R}^{(j)}. \tag{21}$$

We partition the assets available to an investor in country i into two groups: assets not subject to any market frictions, with gross return vector $\boldsymbol{R}_S^{(i)}$, and assets subject to possibly different types of market frictions, with return vector $\boldsymbol{R}_D^{(i)}$. Market frictions in market i are specified as $h^{(i)}(\boldsymbol{\theta}_D^{(i)})$, based on a sublinear transaction cost function $h^{(i)}$ applied to the sub-vector $\boldsymbol{\theta}_D^{(i)}$ of portfolio weights on assets subject to market frictions, within global portfolio vector $\boldsymbol{\theta}_D^{(i)}$ of country i.

We work under the assumption of arbitrage-free markets with respect to each currency denomination. This implies from Proposition 1 that the following sets of admissible SDFs in economies denominated in country i's currency ($i = 1, \ldots, M$) contain at least a strictly positive SDF:

$$\mathcal{M}_h^{(i)} := \left\{ M \in L^p \;:\; M \geq 0\,,\; \mathbb{E}[M\boldsymbol{R}_S^{(i)}] - \mathbf{1}_{N_S} = 0 \text{ and } \mathbb{E}[M\boldsymbol{R}_D^{(i)}] - \mathbf{1}_{N_D} \in C_h^{(i)} \right\}\,, \tag{22}$$

where $C_h^{(i)}$ is the pricing error constraint set for investors in country i, defined in equation (4). Model-free minimum dispersion SDFs are then defined by:

$$\Pi_h^{(i)} := \inf_{M \in \mathcal{M}_h^{(i)}} \mathbb{E}[\phi^{(i)}(M)] \,, \tag{23}$$

for corresponding closed and strictly convex dispersion functions $\phi^{(i)}$. Furthermore, Proposition 2 applies and minimum dispersion SDFs are uniquely pinned down as

$$M_0^{(i)} = (\phi_+^{(i)*})'(-\boldsymbol{\theta}_0^{(i)'} \boldsymbol{R}^{(i)}) \,,$$

where $\boldsymbol{\theta}_0^{(i)}$ presents the solution to the following optimal global portfolio problem for country i investors:

$$\Delta_h^{(i)} := \min_{\theta \in \mathbb{R}^{N_D}} \left\{ \mathbb{E}[\phi_+^{(i)*}(-\theta' \boldsymbol{R}^{(i)})] + \boldsymbol{\theta}' \mathbf{1}_N + h^{(i)}(\boldsymbol{\theta}_D) \right\} . \tag{24}$$

By construction, our setting allows us to consider potentially country-specific transaction cost specifications $h^{(i)}$ that can vary along different dimensions, such as the type of assets subject to transaction costs in each country or the type of transaction costs applied to each asset in a particular country. The resulting SDF can then be estimated from a stationary time series $\{\boldsymbol{R}_{t+1}^{(i)}\}_{t \in \mathbb{N}}$ of returns in currency denomination i, using the estimator provided in equation (10).

Country-specific transaction costs induce asymmetries in investors' behavior across countries. In contrast, identical specifications of transaction cost functions and identical definitions of assets subject to transaction costs in each country lead to identical optimal portfolios across countries or market symmetry. To clarify these concepts, we define market symmetry as follows:

Definition 2. An international economy consisting of M countries with sets of assets subject to transaction costs indexed by $\{D^{(1)}, \ldots, D^{(M)}\}$ and transaction cost specification $\{h^{(1)}, \ldots, h^{(M)}\}$ is called symmetric if:

$$D^{(1)} = \ldots = D^{(M)} \text{ and } h^{(1)} = \ldots = h^{(M)} \,. \tag{25}$$

As we show later on, assumptions about the symmetry of international markets have significant implications for the factor structure of exchange rates.

1.5 Asset Market View of Exchange Rates in the Presence of Frictions

A key assumption of many asset pricing models in international finance is the Asset Market View (AMV) of exchange rates, which stipulates that the exchange rate between domestic and foreign currencies is uniquely pinned down by the ratio of foreign and domestic SDFs. It is well-known that under the assumption of complete frictionless markets, the AMV holds with respect to a unique pair of international SDFs. This results holds also more generally in incomplete frictionless markets for the pair of minimum entropy domestic and foreign SDFs as long as markets are integrated, meaning that investors can trade all assets internationally via the exchange rate market, see Sandulescu, Trojani, and Vedolin (2019).

In a quest to describe the factor structure of exchange rates, the AMV comes in very handy. Intuitively, validity of the AMV directly implies that a cross section of (log) exchange rates is exactly

reproduced by a simple linear transformation of (log) international SDFs, meaning that factor structures in the cross-section of exchange rates are naturally explained by the factor structure in the cross section of SDFs. In addition, the fact that SDFs pairs which satisfy the AMV need to be strongly correlated to explain the low exchange rate volatilities makes such SDF factor structures potentially parsimonious and hence economically insightful.

Notice, however, that even under the assumption of the AMV, many exact exchange rate factor representations are possible. Intuitively, since optimal SDFs are uniquely linked to the optimal portfolios of international investors, various market frictions may impact these portfolios differentially. Unfortunately, this feature makes the estimated factor structures dependent on ad hoc assumptions on the set of common assets traded internationally.[13]

The theory developed in Section 1.1 allows us construct model-free minimum dispersion SDFs that (i) naturally endogenize the choice of the common assets traded internationally in markets with frictions and (ii) are sufficiently regularized by the imposed frictions to produce an accurate pricing performance not just in- but also out-of-sample. Therefore, this theory is a natural starting point to construct exchange rate factor structures with commonly traded international assets returns that are selected endogenously, based on economically plausible specifications for frictions. The missing link, in order to obtain exact exchange rate factor representations using minimum dispersion SDFs, is a result on the validity of the AMV in international asset markets with frictions. This result is provided next.

Proposition 3. *Consider an international economy consisting of M countries, which is symmetric in the sense of Definition 2. Moreover, let $\phi^{(1)}(\cdot) = \ldots = \phi^{(M)}(\cdot) = -\ln(\cdot)$ and denote by $M_0^{(1)}, \ldots, M_0^{(M)}$ the resulting family of minimum entropy SDFs solving problem (23). It then follows, for any $i, j \in \{1, \ldots, M\}$:*

$$X_j^{(i)} = M_0^{(j)} / M_0^{(i)} \,, \tag{26}$$

i.e., the AMV holds with respect to all ensuing minimum entropy SDFs.

In Proposition 3, the key condition for the validity of the AMV with respect to all minimum entropy SDF pairs in international asset markets with frictions, is that international frictions are symmetric. This condition also directly implies with Proposition 1 that pricing error constraints are as well *symmetric* across markets: $C_h^{(1)} = \ldots = C_h^{(M)}$. Another hidden implication of Proposition 3 is that the optimal portfolio returns are symmetric as well across markets, i.e.: $\boldsymbol{\theta}_0^{(1)} = \ldots = \boldsymbol{\theta}_0^{(M)}$, where $\boldsymbol{\theta}_0^{(i)}$ is the solution of problem (24) for market i under the conditions of Proposition 3. This feature is a consequence of the numéraire invariance of minimum entropy SDFs and it follows from the particular choice of the convex conjugate of dispersion function $\phi(\cdot) = -\ln(\cdot)$ in Proposition 2, which is detailed in Section 1.2.1 and implies for any $i, j \in \{1, \ldots, M\}$:

$$X_j^{(i)} = \frac{(\boldsymbol{\theta}_0^{(j)} \boldsymbol{R}^{(j)})^{-1}}{(\boldsymbol{\theta}_0^{(i)} \boldsymbol{R}^{(i)})^{-1}} = \frac{(\boldsymbol{\theta}_0^{(j)} \boldsymbol{R}^{(j)})^{-1}}{(\boldsymbol{\theta}_0^{(i)} \boldsymbol{R}^{(j)})^{-1}} X_j^{(i)} \,, \tag{27}$$

i.e., $\boldsymbol{\theta}_0^{(j)} \boldsymbol{R}^{(j)} = \boldsymbol{\theta}_0^{(i)} \boldsymbol{R}^{(j)}$ state by state, and hence $\boldsymbol{\theta}_0^{(i)} = \boldsymbol{\theta}_0^{(j)}$.

[13] Moreover, under the assumption of frictionless markets, estimation of model-free SDFs tends to produce over-fitted estimates and spurious results when the size of the cross-section of asset return increases.

One key implication of Proposition 3 for our analysis of the global factor structures of exchange rates is that the cross-section of $M \times (M-1)$ distinct exchange rates is always exactly described by a simple log linear transformation of the cross-section of M minimum entropy SDFs computed under symmetric international transaction cost structures.

1.6 Asset Market View and Exchange Rate Factors

We now address the relation between exchange rate factors and the AMV for our families of global minimum dispersion SDFs $\{M_0^{(i)}\}_{i=1,\ldots,M}$.

1.6.1 Asset Market View and Global Factors

In general, for an arbitrary choice of a family of minimum dispersion SDFs, the AMV is violated and deviations from it can be captured by a family of Backus, Foresi, and Telmer (2001) stochastic exchange rate wedges $\{\eta_j^{(i)}\}_{1\leq i,j\leq M}$, defined by:

$$X_j^{(i)} = \frac{M_0^{(j)}}{M_0^{(i)}} e^{\eta_j^{(i)}} \,. \tag{28}$$

After taking logs on both sides of this identity yields:

$$x_j^{(i)} = m_0^{(j)} - m_0^{(i)} + \eta_j^{(i)} \,, \tag{29}$$

where we denote with lower-case letters logs:, $x_j^{(i)} := \ln X_j^{(i)}$. It immediately follows that whenever a family of minimum dispersion SDFs satisfies the AMV, all wedges are zero and log exchange rate returns satisfy an exact linear model, in which exchange rate shocks are completely spanned by SDF shocks:

$$x_j^{(i)} = m_0^{(j)} - m_0^{(i)} \,. \tag{30}$$

From Proposition 3, this situation arises for all families of minimum entropy SDFs estimated under the assumption of market symmetry. Accordingly, the cross section of $M \times (M-1)$ log exchange rate returns is completely explained by a cross-section of M such log minimum entropy SDFs. Furthermore, these SDFs need to be highly correlated, in order to be consistent also with the low exchange rate volatilities in the data. Such a feature naturally induces a parsimonious factors structure in minimum entropy SDFs constructed under symmetric market frictions, which is uniquely suited to study global factors.

In the more general situation of asymmetric market frictions and/or SDF measures of dispersion different from entropy, deviations from the AMV arise and log exchange rates in linear model (29) are only approximately related to log minimum dispersion SDFs, because of the existence of non zero stochastic wedges. In this case, an exact representation of the cross-section of $M \times (M-1)$ exchange rates requires in principle information from the cross section of M minimum dispersion SDFs and all $M \times (M-1)$ stochastic wedges. In this representation, the factor structures in exchange rates and SDFs depend on both the degree of variability of the wedges and possible factor structures in the

cross-section of wedges that are not spanned by factors in the cross-sections of SDFs.[14] The last two wedge properties are endogenously determined by the degree of asymmetry in the market frictions assumed to construct minimum dispersion SDFs.

1.6.2 Asset Market View and Global Portfolios

Proposition 2 posits that global factors in the cross-section of minimum dispersion SDFs are directly linked to the corresponding portfolios of global investors in markets with frictions. In this setting, the numéraire invariance of the portfolios of minimum entropy investors in symmetric markets plays a particular role, implying that:

$$m_0^{(i)} = \ln \boldsymbol{\theta}_0' \boldsymbol{R}^{(i)} = \ln \boldsymbol{\theta}_0' \boldsymbol{R}^{(j)} + \ln X_j^{(i)} \,, \tag{31}$$

for each currency denomination $i \in \{1, \ldots, M\}$. In other words, the cross-section of these SDFs is exactly reproduced by the sum of the optimal return of a single global portfolio with respect to a reference currency j, and the cross-section of log exchange rate returns with respect to the reference currency. Therefore, the cross-section of these SDFs naturally contains a common factor $\ln \boldsymbol{\theta}_0' \boldsymbol{R}^{(j)}$ and additional potential factors reflecting common exchange rate factors not spanned by $\ln \boldsymbol{\theta}_0' \boldsymbol{R}^{(j)}$.

A second representation of minimum entropy SDFs in symmetric markets directly follows from identity (31), using weights $w_1, \ldots, w_M$ such that $\sum_{j=1}^M w_j = 1$:

$$m_0^{(i)} = \sum_{j=1}^M w_j \ln \boldsymbol{\theta}_0' \boldsymbol{R}^{(j)} + \sum_{j=1}^M w_j \ln X_j^{(i)} = \sum_{j=1}^M w_j \ln \boldsymbol{\theta}_0' \boldsymbol{R}^{(j)} + CB_i \,, \tag{32}$$

where $CB_i := \sum_{j=1, j\neq i}^M w_j \ln X_j^{(i)}$. Here, every minimum entropy SDF is written as the sum of (i) a common factor given by the average log return of the global optimal portfolio across currency denominations and (ii) a currency basket factor CB_i measuring the average appreciation of currency i relative to currencies $j \neq i$. In this representation, the cross-section of SDFs naturally contains a common factor, given by the average log return of the global optimal portfolio across currencies, and additional potential factors reflecting common currency basket factors not spanned by this average return. Our notion of currency baskets is related to Aloosh and Bekaert (2019) who use equal weighted baskets of all possible bilateral exchange rate with respect to a specific currency. Notice, however, that different from these authors, who focus exclusively on currency pricing in the cross-section, we price also stocks and bonds in the time-series, moreover, our SDFs are spanned by two distinct factors: the average log return and the common currency basket.

2 Empirical Analysis

Proposition 2 and in particular equation (10) allows us to estimate minimum dispersion international SDFs directly from returns data in the presence of trading frictions. In particular, we study two different market settings with varying transaction costs: symmetric and asymmetric markets and

[14] Since wedges need themselves to be consistent with the pricing constraints satisfied by minimum dispersion SDFs in each currency denomination, wedges and SDFs are typically correlated.

explore the impact of frictions on the properties of SDFs. Using SDFs denominated in different currencies, we then extract global and local factors to price cross-sections of currencies and international assets such as short- and long-term bonds and stocks.

2.1 Data

In our empirical analysis, we use monthly data between January 1988 and December 2015. We focus on the following developed markets: Australia, Canada, Euro Area, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom and United States.[15] We collect data on exchange rates, short- and long-term interest rates, and MSCI country equity indices' prices from Datastream. When we analyze a specific currency denomination, we treat the corresponding market as the domestic and all other markets as foreign. Hence, we do not consider bilateral trades but a global economy where global investors can trade all possible assets.

2.2 Market Settings

We study two different market settings, symmetric and asymmetric markets, and in each market impose different types of transaction costs.

SYMMETRIC MARKETS: For the first symmetric market setting, we assume that global investors can trade the full menu of assets and face no frictions. The second symmetric market setting arises when investors face proportional transaction costs. To this end, we assume that investors face no transaction costs when trading short-term bonds globally (i.e., investors can borrow and lend at the short-term interest rate without any frictions) but face transaction costs when trading long-term bonds and equity. In particular, we assume that transaction costs are proportional to their positions and in line with the size of bid-ask spreads. To this end, the proportional transaction cost parameter λ is chosen such that we have comparable pricing errors implied on the returns based on mid-prices by (i) SDFs in an economy with proportional transaction costs and by (ii) SDFs in an economy where market frictions are quantified by bid-ask spreads.

ASYMMETRIC MARKETS: In the first setting, we assume that investors face bid-ask spreads when buying and selling international assets. To this end, we use average bid-ask spreads for exchange rates directly available from Datastream which are in the order of 2bps. For the long-term bonds, we also assume average bid-ask spreads of 2bps in line with Adrian, Fleming, and Vogt (2017) for the US and Bank of International Settlement (2016) for Japan and Germany ten-year bonds. For equity indices, we assume a 6bps spread.[16] The second asymmetric market setting assumes again proportional transaction costs but now we assume that the local short-term bond can be traded without any frictions whereas all foreign short-term bonds, as well as long-term bonds and equities face proportional transaction costs which are consistent to the bid-ask spread.

[15] Before the introduction of the Euro, we take the Deutsche Mark in its place.

[16] It is in general impossible to know the exact bid and ask spread of assets. Luttmer (1996) uses bid and ask spreads of around 0.012% which corresponds to the tick size on the NYSE. Andersen, Bondarenko, Kyle, and Obizhaeva (2018) reports that the bid and ask spread on E-Mini Futures on the S&P500, one of top two most liquid exchange traded futures in the world, is around 0.25 index points.

2.3 Properties of Model-Free Global SDFs

As a first exercise, we study the properties of international SDFs, their comovement, and the corresponding optimal portfolio weights. Table 1 provides some summary statistics on SDFs for different currency denomination for the four market settings.

[Insert Table 1 and Figure 1 here.]

Panel A in Table 1 reports the summary statistics for SDFs for each currency denomination. We notice the very high volatility of each SDF, all over 100%. The fact that in the case where there are no frictions the SDFs are forced to price all assets exactly leads to the high volatilities which indicate that it may be hard for global investors to achieve such Sharpe ratios in international bond and equity markets. As we impose different trade barriers, SDF volatilities significantly decrease. For example, in the case where we have asymmetric proportional transaction costs (Panel D), we notice an almost 50% drop in the volatilities which now range between 60% and 70%.

The cross-country correlations in the four market settings are presented in the lower panels of Table 1. Not very surprisingly, the correlations are almost perfect in symmetric markets. The intuition for this is that when we assume that markets are symmetric, the AMV holds and "enforces" a high correlation among international SDFs, see Proposition 3. As we move to an asymmetric market setting where transaction costs vary among the different countries, we notice that the correlations are slightly lower but still all above 90%. This may be more surprising given that in this case, we have a violation of the AMV. This, however, indicates a strong factor structure of exchange rates even in the presence of market frictions.

Finally, Figure 1 plots the time-series of the average (across all currency denominations) SDF for the four market settings. While we notice a much higher volatility for the SDF in the no market friction case, there is high comovement among the different SDFs. They all increase during bad economic times such as recessions or during times of disruptions in financial markets such as the dot com bubble burst or the Lehman default.

2.4 Optimal Portfolio Weights

In order to get a better understanding of the optimal SDFs for different currencies, we now study optimal portfolios. Recall that our theoretical framework allows us to exactly identify optimal portfolio weights from agents' Euler equations. Figure 2 plots the optimal portfolio weights for symmetric markets with no frictions (left panel) and symmetric proportional transaction costs (right panel).

[Insert Figure 2 here.]

Recall that a direct consequence of Proposition 3 is that portfolio holdings have to be the same in symmetric markets. Therefore, we only plot the USD denominated portfolio weights. In the case when investors can trade without frictions, portfolio weights can be very large both long and short. The larger positions reveal that investors borrow in the lower interest rate currencies such as JPY, USD

and EUR and hold long positions in high interest rate countries such as NZD and AUD. Most positions in long-term bonds are short, with the exception of the EUR and JPY. Global investors hold large long positions in USD, AUD, and CHF equity indices. The large positions indicate that without taking large levered positions it may be hard to maintain this portfolio.

When we impose symmetric proportional transaction costs on investors, some weights on assets are zero, portfolio selection is sparse, and the size of the positions shrink, see the right panel in Figure 2. More specifically, most portfolio weights on the long-term bonds are zero. This is not very surprising given that currency risk premia at the long-end of the term structure are small, see, e.g., Lustig, Stathopolous, and Verdelhan (2019). Investors also drop most of the equity indices except for a long position in the USD and CHF and a small short position in the JPY. Interestingly, most of the wealth is held in short-term bonds. In particular, as in the no frictions case, global investors trade a "carry". Short positions are in typical funding currencies, whereas long positions are in investment currencies. Overall, we conclude that even small transaction costs which restrain the leverage that investors can take can have big impacts on the optimal portfolios held for global investors.

[Insert Figures 3 and 4 here.]

We can now study the asymmetric market settings. Figures 3 and 4 plot the cases where we assume asymmetric bid-ask spreads and proportional transaction costs. Because in asymmetric markets portfolio weights vary across the different currency denominations, we plot all currencies separately. As in the symmetric proportional transaction cost case, portfolios are more sparse and investors hold positions that resemble the carry and long equity. The most sparse portfolio is when investors face asymmetric proportional transaction costs as also reflected in the low SDF volatility in Table 1. Even though in asymmetric markets, portfolio weights are not enforced to be the same, positions look highly similar. As in the previous symmetric market cases, investors engage in carry trades: shorting the US dollar, EUR or CHF and go long in the NZD. In addition, investors trade long USD and CHF equity.

2.5 Factor Analysis

From the optimal SDFs, we can now extract global and local factors and price cross-sections of international assets both in- and out-of-sample. Recall that given the almost perfect correlation among international SDFs, we expect the SDF factor structure to be very parsimonious. Indeed, in line with earlier research on global factors, we expect to uncover at most two factors, see, e.g., Lustig, Roussanov, and Verdelhan (2011, 2014) or Engel, Mark, and West (2015). Notice, however, that in contrast to this literature, we do not extract PCs from the cross-section of currencies (or currency portfolios) directly but use SDFs which have been estimated from international equity and bond data.

To extract a global factor, we run a principal component analysis (PCA) on the SDFs denominated in all different currencies. Since the first PC explains almost all of the variation and the second PC does not add much in terms of cross-sectional fit, we focus only on the first PC and label it "global factor". To get a sense of the importance of local factors to explain the cross-section of currencies and international bonds and equities, we construct country level local factors from regressing the local

currency SDF on the global SDF and take the residual as our "local" SDF. Figure 5 plots the global SDFs together with the US local SDF for the various market settings.

[Insert Figure 5 here.]

Interestingly, we notice that the global SDFs spike during US specific bad events. For example, in all four market settings, we find that global SDFs exhibit a massive spike in August 2011 during the US downgrade from AAA to AA+ by S&P.

Before we delve into any cross-sectional analysis, it is informative to study the factor loadings of global factor on the asset returns themselves. This will provide us with some economic intuition as to what the global factors represent. Figure 6 plots the optimal factor loadings for the first PC for each market settings. We notice that for all four market settings, the highest asset loadings are observed for a "carry": large long position in the NZD and short positions in the USD and EUR. In addition, investors mostly hold long positions in CHF and USD equity. These findings corroborate earlier work by Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2018) who posit that two global factors, carry and dollar, price the cross-section of exchange rates well. Our findings complement theirs as our framework is model-free and we did not impose any structure on our SDFs other than no-arbitrage. In other words, even in settings where we allow investors to trade all possible assets, when we impose trade barriers such as transaction costs, the carry and US equity arise endogenously as the two dominating factors in global investors' SDFs.

[Insert Figure 6 here.]

Armed with these two factors, we now run two-step cross-sectional regressions both in- and out-of-sample for the cross-section of currencies and international stocks and bonds separately. Since in symmetric cases asset returns are matched perfectly by construction, we do not present the results here as the R^2 are 100%. We instead only show in-sample results for the asymmetric market settings.

2.5.1 In-Sample Pricing

Tables 2 to 5 present in-sample estimated factor premia for the cross-section of currencies and all assets (stocks, short- and long-term bonds). Almost all estimated premia are negative and highly statistically significant. In particular, we find that the global SDF alone explains between almost 80% (NZD) to nearly a 100% (GBP) of the cross-sectional variation of exchange rates. Again, recall that our SDFs are not estimated from currency returns. When pricing the cross-section of all assets, in-sample R^2 are even higher and almost 100% for all currency denominations. When comparing the local pricing factor to the global one, we notice that while estimated factor premia are all significant, the economic size is almost a factor 100 smaller compared to the global factor. Almost the identical picture emerges as we study the pricing for the SDFs with symmetric proportional transaction costs.

[Insert Tables 2 to 5 here.]

2.5.2 Out-Of-Sample Pricing

An economically more interesting question than in-sample pricing is whether our SDFs are able to price the cross-section of international assets out-of-sample. To this end, we use a ten-year training period for our SDFs and a rolling window of one year. We then evaluate the out-of-sample performance by calculating cross-sectional R^2s from out-of-sample returns and out-of-sample SDFs.

Tables 6 and 7 present the out-of-sample results for the no frictions case. The results are mixed for the cross-section of FX returns: While estimated factor premia are signficant in some of the specifications, the are insignificant in many specifications. The picture looks a bit better when pricing the cross-section of all assets, see Table 7. R^2 range between 60% for the NZD assets to 80% for the EUR denominated assets. In addition, the local factors are often not significant at all.

The dismal performance of the no friction SDFs is not very surprising given the extremely highly volatile nature of the SDFs documented in Table 1. As a comparison, we can gauge the estimated factor premia in the case where we impose proportional transaction costs in a symmetric market setting, see Table 8. In this case, we significantly improve on the cross-sectional pricing abilities of the local currency SDF as well as the global SDF. Except for the case when currencies returns are redenominated in either GBP or CAD, global factors are significant with R^2 ranging between 50% to over 80% for the EUR.

We can further improve upon the pricing performance by imposing asymmetric market settings as the ensuing SDFs feature more robust properties. For example, in the case where markets are asymmetric and global investors face proportional transaction costs, the global SDF is always significant except for the CAD denominated currency returns. R^2 are also high, ranging between 32% for the NZD and almost 90% for the EUR. Pricing the full cross-section of assets further improves R^2: Global and local SDFs explain between 70% to 80% of the cross-sectional variation of international stocks and bonds.

[Insert Tables 6 to 13 here.]

2.6 Home Bias in Global Portfolios

Proposition 3 posits that portfolios are identical across different currency denominations as long as markets are symmetric. This, however, is in strong contradiction to empirical evidence that documents a strong currency home bias in international portfolios. For example, French and Poterba (1991) and Lewis (1999), and more recently Camanho, Hau, and Rey (2019), show that the proportion of domestic stocks invested in portfolios exceeds their country's relative market capitalization in the world. This home bias phenomenon extends to bonds and is found to be even more pronounced, see, e.g., Maggiori, Neiman, and Schreger (2019). To rationalize home bias Coeurdacier and Rey (2013) review three explanations: (i) hedging motives in frictionless markets, (ii) asset trade costs in international financial markets (such as transaction costs, differences in tax treatments between national and foreign assets or differences in legal frameworks), and (iii) information frictions and behavioral biases.[17]

[17]Most studies that empirically explore the effect on asset trade costs conclude that the costs would need to be unrealistically high to explain the level of home bias observed in the data. For example, French and Poterba (1991) argue

While we can measure transaction costs such as bid-ask spreads, it is harder to quantify other types of asset trade costs such as the cost to trade via an intermediary or the role of international taxation. Intangible costs such as information frictions and behavioral biases are even harder to quantify, see, e.g., Coeurdacier and Rey (2013) for a discussion.[18] It is therefore natural to assume that the observed costs such as bid-ask spreads represent a lower bound to the true costs of trading foreign assets.

In the following, we can use our framework to estimate the unobservable cost of trading foreign assets such that portfolio holdings line up with the home bias observed in the data. To this end, we incrementally increase foreign transaction costs relative to local transaction costs and study its effect on optimal portfolio weights.

[Insert Figure 7 here.]

Figure 7 plots the optimal portfolio weights for each currency assuming that transaction costs on the foreign assets are eight times as big as on the local assets.[19] As we note, there is an almost perfect home bias in equities and long-term bonds for each currency. For nearly all currencies, local investors short the local long-term bond and have a long position in the local equity index. At the same time, we notice that across all currency denominations, investors trade the carry. For example, US investors short the US short-term bond while holding a long position in the Australian short-term bond. Australian global investors, on the other hand, short the Japanese Yen and buy New Zealand short-term bonds. This implies that independent of the foreign transaction cost, investors trade carry.

To get a sense of the implied cost to achieve home bias, recall that the average bid-ask spread is around 2bps in our data sample which implies that the "hidden" costs are around 14bps. As mentioned earlier, these costs can include differences in taxation, behavioral or informational costs, as well as intermediation costs. Overall, we conclude that even small transaction costs can lead to highly home bias portfolios.

3 Conclusions

Our paper makes the following contributions. First, we develop a theoretical model-free framework that allows us to identify global risk factors from a large cross-section of international assets such as stocks, bonds, and currencies when investors face barriers to trade. Intuitively, limiting the allocation of wealth that is invested into risky assets leads to sparse portfolios, endogenously segmented markets, and hence robust properties of international SDFs.

Our first theoretical contribution posits a unique mapping between optimal portfolio weights of global investors and stochastic discount factors in the presence of frictions. Our setting is very general and can incorporate various types of frictions or combinations thereof. Applying this result

in a mean-variance framework that these costs must be several hundred basis points. However, a different strand of the literature argues that if diversification benefits are small across countries, then these costs can be small and still explain home bias, see, e.g., Martin and Rey (2004) and Bhamra, Coeurdacier, and Guibaud (2014).

[18] Nieuwerburgh and Veldkamp (2009) study a model of home bias with informational frictions in international markets and link information asymmetry to earning forecasts, investors behavior, or pricing errors.

[19] To save space, we do not present the intermediate figures in the paper.

to international markets, our second theoretical contribution shows that we can always uniquely recover the exchange rate from the ratio of foreign and domestic SDFs even if there are frictions as long as markets are symmetric. This result is very useful when studying the global factor structure of exchange rates. Intuitively, the fact that the cross-section of (log) exchange rate changes are exactly reproduced by the cross-section of (log) SDFs, directly implies that the factor structure of exchange rates can be described by the factor structure of international SDFs. In addition because our SDFs are numéraire-invariant, it does not matter whether exchange rates are vis-à-vis one currency or another.

Second, we use this framework to estimate international SDFs from the cross-section of stocks, and short- and long-term bonds for developed countries distinguishing between two market settings: symmetric markets and asymmetric markets. In symmetric markets, we assume that investors either face no frictions or face proportional transaction costs, i.e., leverage constraints, on their optimal wealth allocation that is identical across countries. In asymmetric markets, on the other hand, investors face either bid-ask spreads or they face leverage constraints which are assumed to be less binding when investing in local compared to foreign assets.

When international agents face no barriers to trade, SDFs need to exactly price all assets which leads to excessively volatile SDFs exceeding 100%. Imposing market frictions significantly reduces this volatility to the extent that when we assume that investors are leverage constrained, SDF volatility is almost halved. Cross-country correlations are perfect in symmetric markets, because the AMV holds. However, we also find correlations to be nearly perfect when agents face frictions, the reason being that investors hold almost identical portfolios. A closer inspection of these portfolios reveal that global investors take their biggest exposures in the classical carry trade and long positions in USD and CHF equities. Long-term bonds, on the other hand, do not enter international investors' portfolios.

From the cross-section of country-level SDFs, we then extract a global and local SDF and assess their pricing ability for the cross-section of currencies and stocks and bonds. The highly volatile SDFs in the no frictions setting lead to instable properties and therefore worse out-of-sample pricing ability when pricing the cross-section of international assets. SDFs that incorporate frictions, however, exhibit superb out-of-sample R^2 of up to 80%.

Finally, we estimate the cost to achieve home biased portfolios as in the data. To this end, we incrementally increase transaction costs on foreign relative to local assets and find that when foreign assets are eight times as expensive to trade as local assets, local portfolios are nearly perfectly home biased in the long-term bond and equities. However, global investors trade carry independent of the transaction cost. Given that the average observable cost is around 2bps, we conclude that even small costs imply highly currency biased portfolios.

Figures

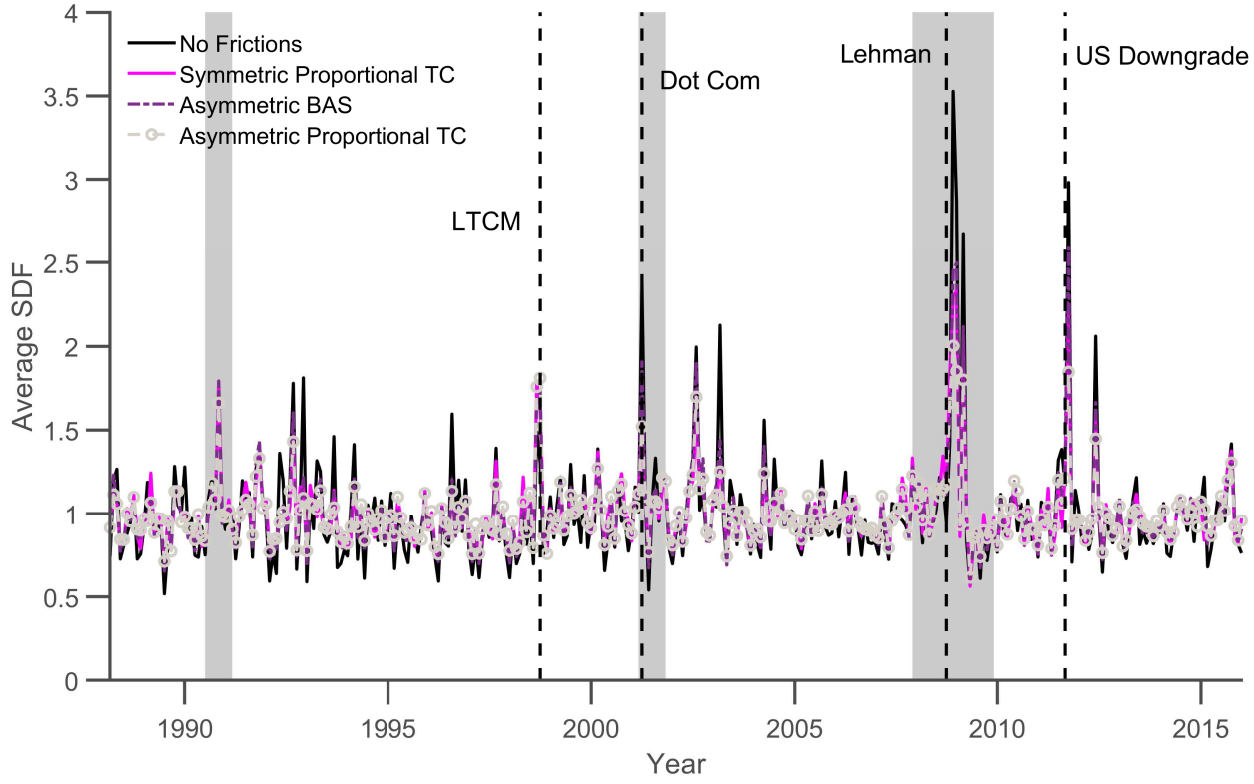


Figure 1. **Average minimum entropy SDF.** This figure plots the average SDF (across all currency denominations) for the SDF estimated with no frictions, with proportional transaction costs and for bid-ask spreads. Gray bars indicate recessions according to NBER. Data is monthly and runs from January 1988 to December 2015.

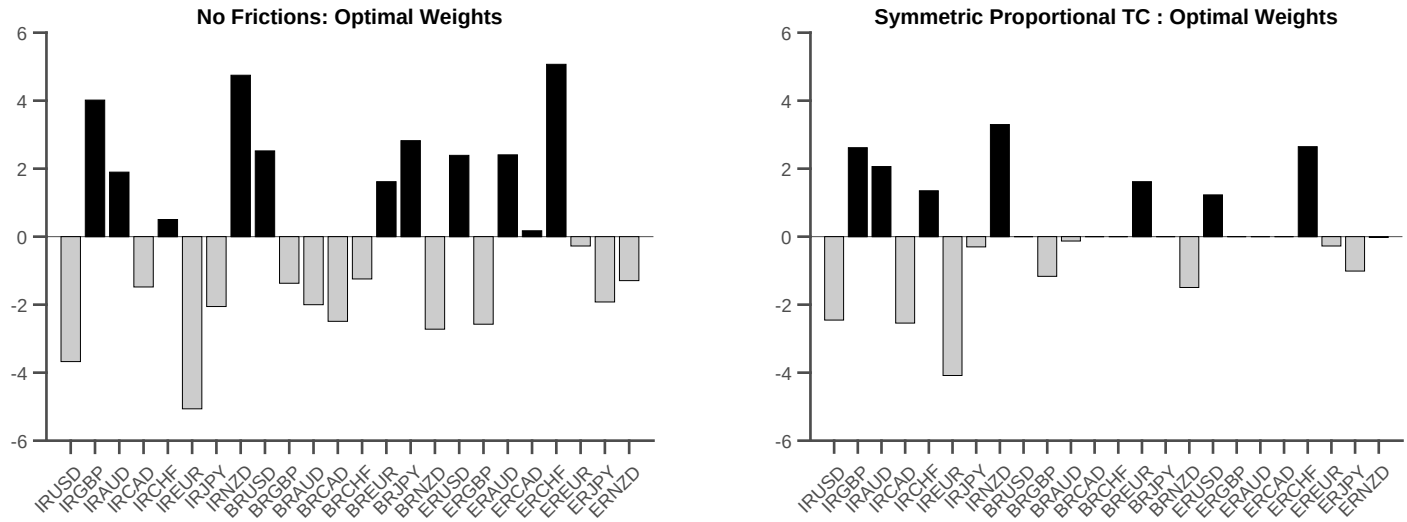


Figure 2. **Optimal Weights: Symmetric Markets.** The left panel plots the portfolio weights in each asset denominated in USD assuming that investors face no trade barriers. The right panel plots the portfolio weights in each asset denominated in USD assuming that investors face symmetric proportional transaction costs. IRXXX is the short-term bond for currency XXX, BRXXX is the long-term bond for currency XXX, ERXXX is the equity index for currency XXX. Data is monthly and runs from January 1988 to December 2015.

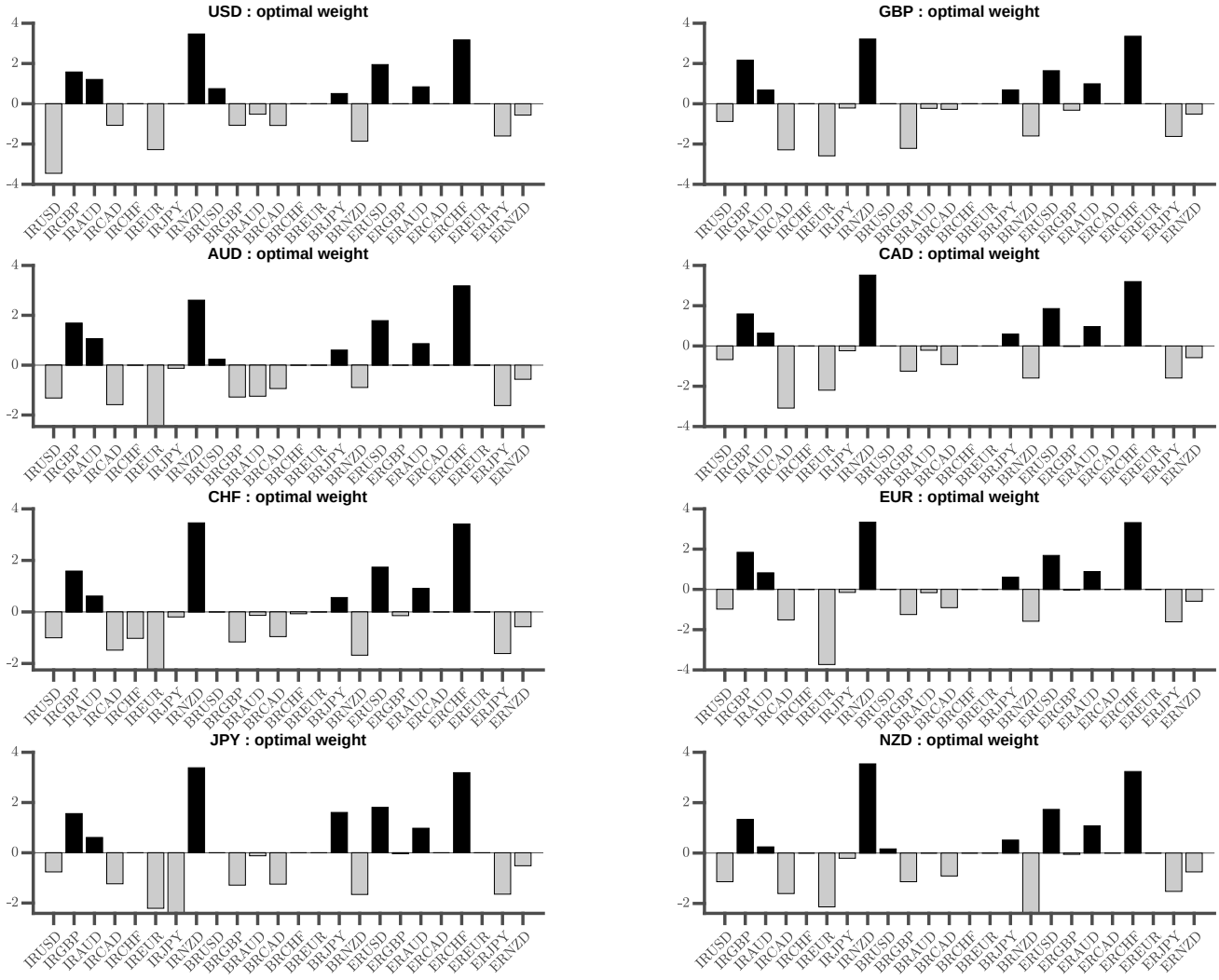


Figure 3. **Optimal Weights: Bid-Ask Spreads.** This figure plots the portfolio weights in each asset for all currency denominations assuming that investors face bid-ask spreads. IRXXX is the short-term bond for currency XXX, BRXXX is the long-term bond for currency XXX, ERXXX is the equity index for currency XXX. Data is monthly and runs from January 1988 to December 2015.

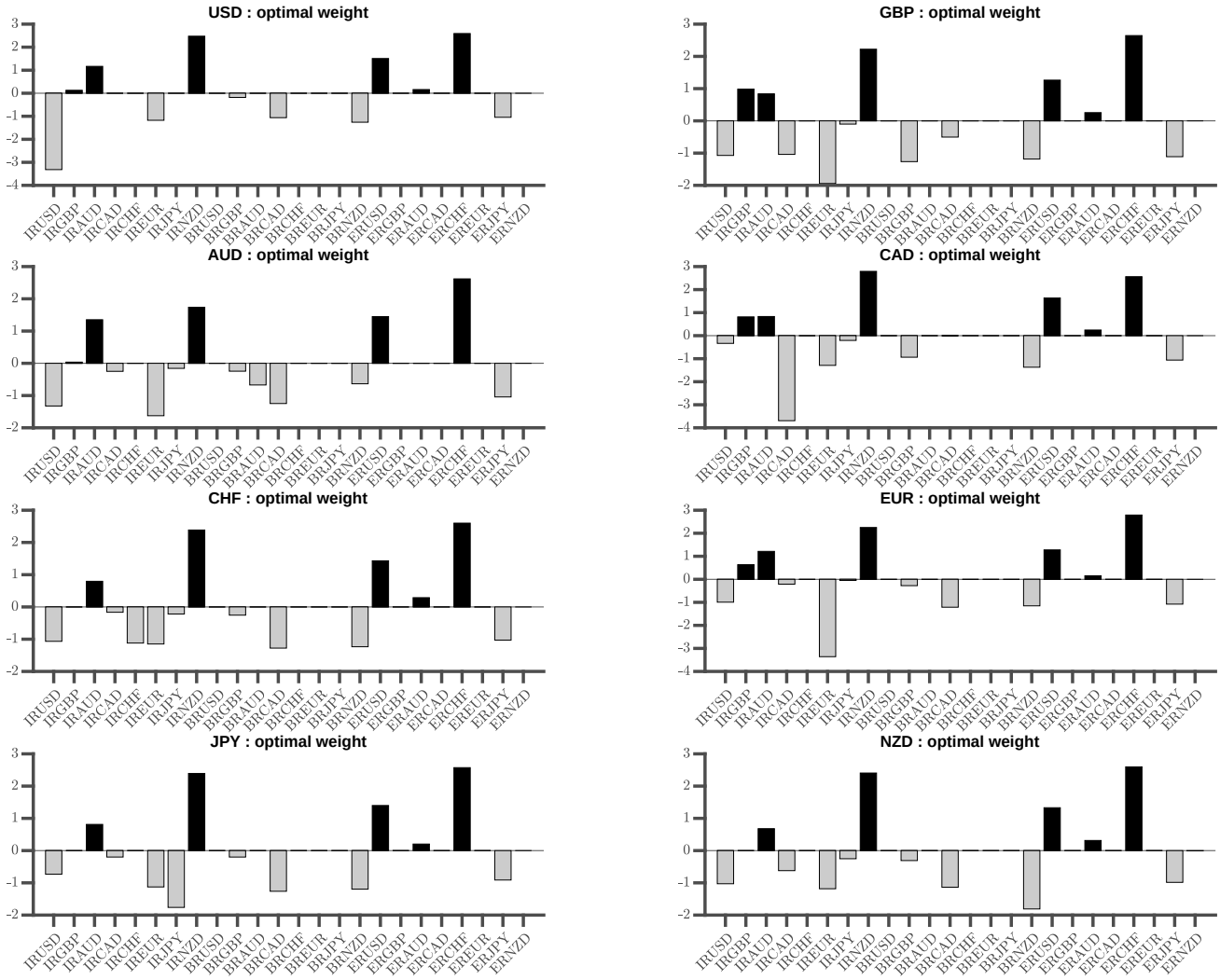


Figure 4. **Optimal Weights: Asymmetric Proportional Transaction Costs.** This figure plots the portfolio weights in each asset for all currency denominations assuming that investors face bid-ask spreads. IRXXX is the short-term bond for currency XXX, BRXXX is the long-term bond for currency XXX, ERXXX is the equity index for currency XXX. Data is monthly and runs from January 1988 to December 2015.

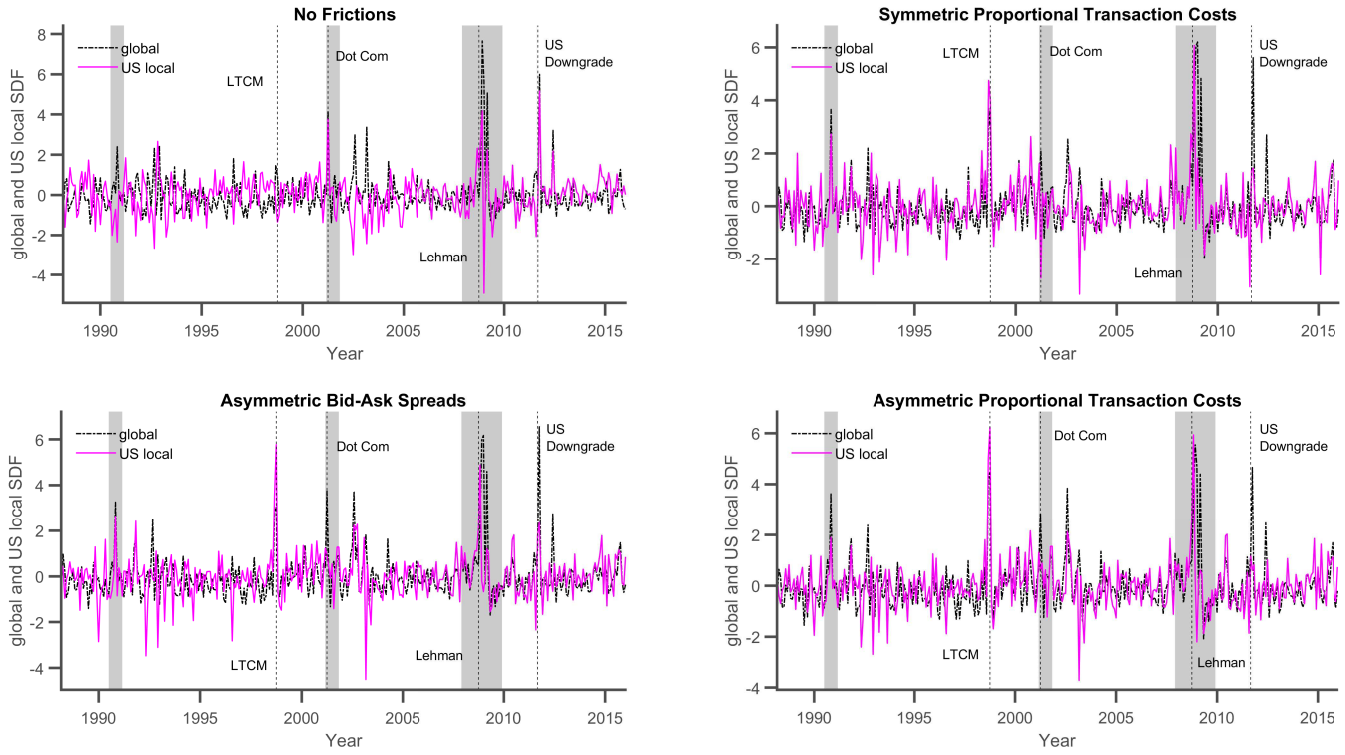


Figure 5. **Global and Local US SDFs.** This figure plots global and US local SDFs for the different market settings. All variables are standardized, meaning we de-mean and divide by the standard deviation. Gray bars indicate recessions according to NBER. Data is monthly and runs from January 1988 to December 2015.

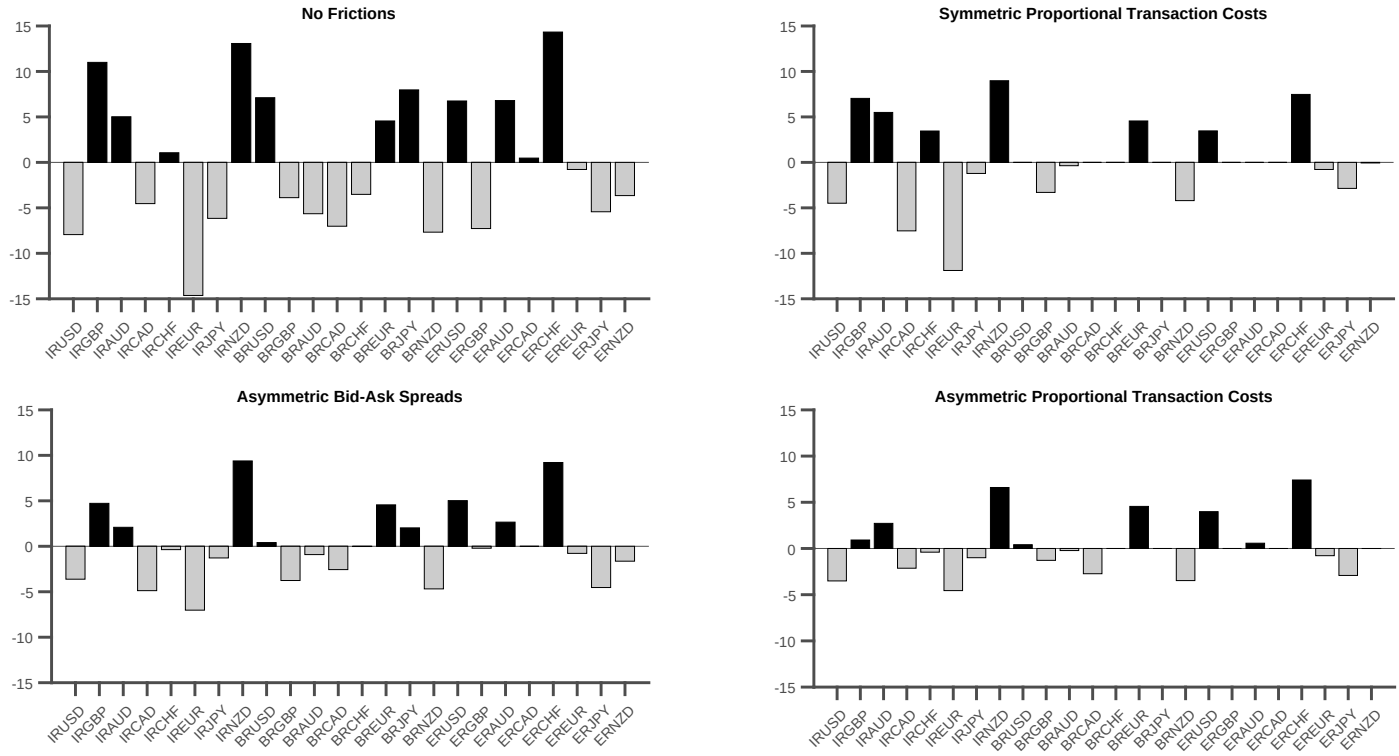


Figure 6. **PCA Weighted Optimal Weights.** This figure plots the first PCA weighted optimal portfolio weights for four different market settings. No frictions (upper left), symmetric proportional transaction costs (upper right), asymmetric bid-ask spreads (lower left) and asymmetric proportional transaction costs (lower right). Data is monthly and runs from January 1988 to December 2015.

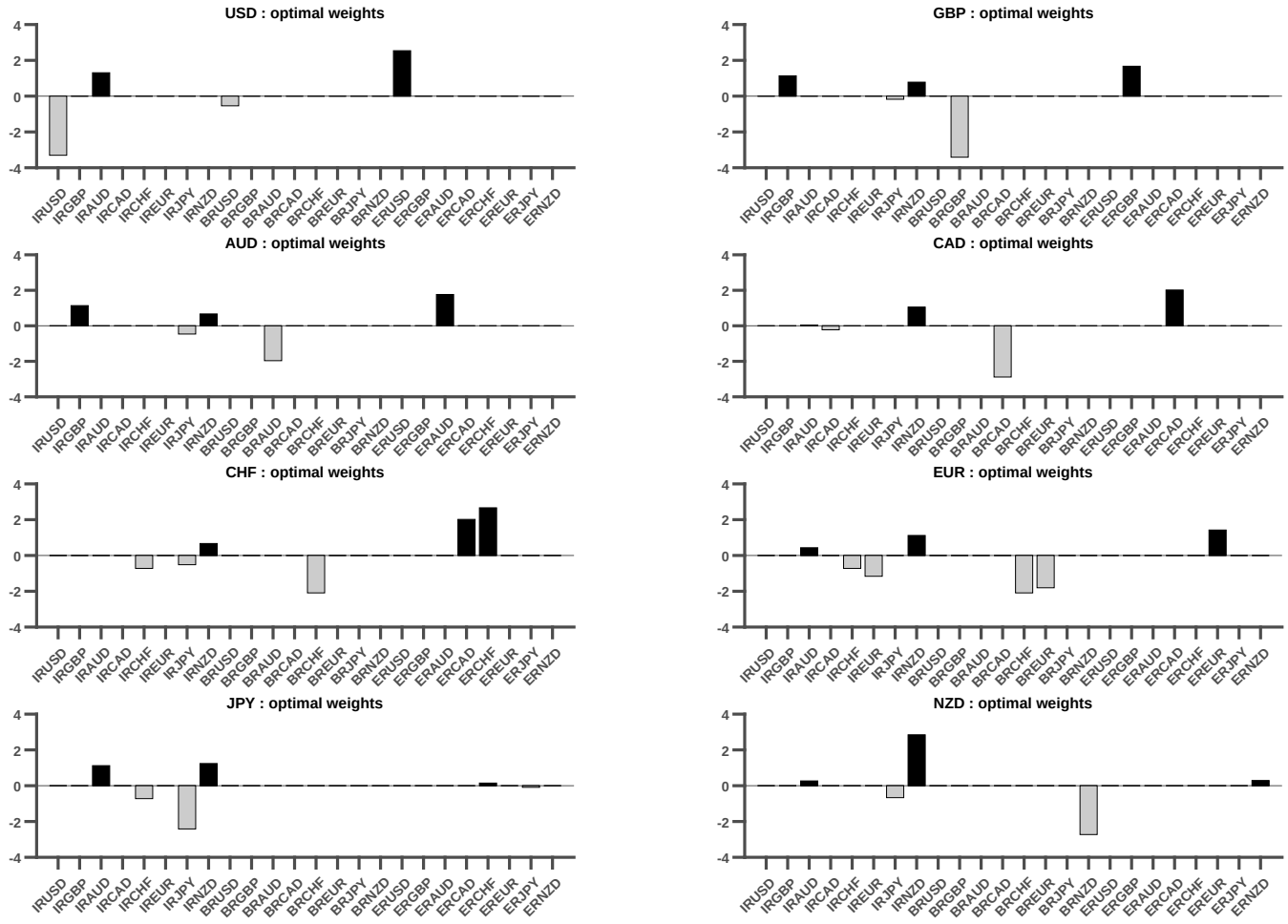


Figure 7. **Home Bias.** This figure plots the optimal portfolio weights in an asymmetric market setting where transaction costs on foreign assets are eight times larger than in local markets. Data is monthly and runs from January 1988 to December 2015.

Tables

Table 1. **Summary Statistics Global SDFs**

This table reports summary statistics for minimum entropy SDFs denominated in different currencies assuming for different market structures: no market frictions, symmetric transaction costs, bid-ask spreads, and asymmetric transaction costs. Data is monthly and runs from January 1988 to December 2015.

	Symmetric Markets								**Asymmetric Markets**							
	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD
	Panel A: No Frictions								*Panel C: Asymmetric Bid-Ask Spreads*							
mean	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.995	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.994
std	1.172	1.137	1.131	1.137	1.162	1.172	1.194	1.095	0.874	0.848	0.815	0.838	0.862	0.869	0.892	0.815
	Panel B: Symmetric Proportional Transaction Cost								*Panel D: Asymmetric Proportional Transaction Cost*							
mean	0.997	0.995	0.995	0.996	0.998	0.997	0.999	0.994	0.997	0.996	0.995	0.996	0.998	0.997	0.999	0.994
std	0.767	0.738	0.718	0.745	0.769	0.773	0.801	0.699	0.670	0.632	0.600	0.635	0.643	0.670	0.683	0.604
	SDF Correlations No Frictions								*SDF Correlation Asymmetric Bid-Ask Spreads*							
	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD	USD	GBP	AUD	CAD	CHF	EUR	JPY	NZD
USD	1.00								1.00							
GBP	1.00	1.00							0.98	1.00						
AUD	0.99	0.99	1.00						0.98	0.99	1.00					
CAD	1.00	0.99	0.99	1.00					0.98	0.98	0.98	1.00				
CHF	0.99	0.99	0.99	0.99	1.00				0.98	0.99	0.99	0.98	1.00			
EUR	0.99	1.00	1.00	0.99	1.00	1.00			0.97	0.99	0.99	0.97	1.00	1.00		
JPY	0.99	0.99	0.99	0.99	0.99	0.99	1.00		0.97	0.98	0.98	0.97	0.98	0.97	1.00	
NZD	0.99	0.99	0.98	0.99	0.98	0.99	0.99	1.00	0.98	0.98	0.97	0.99	0.97	0.97	0.97	1.00
	SDF Correlation Proportional Transaction Cost								*SDF Correlation Proportional Transaction Cost*							
USD	1.00								1.00							
GBP	0.99	1.00							0.97	1.00						
AUD	0.99	0.98	1.00						0.97	0.98	1.00					
CAD	0.99	0.99	0.98	1.00					0.94	0.95	0.94	1.00				
CHF	0.98	0.99	0.98	0.98	1.00				0.96	0.98	0.98	0.94	1.00			
EUR	0.99	0.99	0.99	0.98	1.00	1.00			0.94	0.98	0.97	0.92	0.98	1.00		
JPY	0.99	0.99	0.98	0.98	0.99	0.99	1.00		0.95	0.96	0.96	0.93	0.97	0.94	1.00	
NZD	0.99	0.98	0.97	0.99	0.97	0.97	0.98	1.00	0.96	0.97	0.97	0.97	0.96	0.95	0.96	1.00

Table 2. **Cross-Sectional Pricing FX: Bid-Ask Spreads In Sample**

This table reports estimated coefficients of two-step cross-sectional regressions of currency excess returns on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. Data runs from January 1988 to December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.068^{***}			98.783	0.000	-0.066^{***}			99.376	0.000
(-28.426)					(-30.970)				
	-0.200^{***}		92.406	0.000		-0.183^{***}		97.198	0.000
	(-8.930)					(-14.365)			
	-0.189^{***}	-0.001^{***}	99.121	0.000		-0.187^{***}	-0.001^{**}	99.578	0.000
	(-28.854)	(-8.274)				(-37.639)	(-5.933)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.059^{***}			98.635	0.000	-0.065^{***}			99.173	0.000
(-31.056)					(-30.106)				
	-0.128^{***}		78.705	0.000		-0.158^{**}		76.923	0.001
	(-9.242)					(-3.846)			
	-0.177^{***}	-0.001^{***}	98.590	0.000		-0.185^{***}	-0.001^{***}	99.276	0.000
	(-22.177)	(-7.327)				(-30.231)	(-16.895)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.065^{***}			98.541	0.000	-0.065^{***}			98.427	0.000
(-27.653)					(-29.426)				
	-0.228^{***}		94.089	0.000		-0.227^{***}		93.936	0.000
	(-10.881)					(-11.743)			
	-0.193^{***}	-0.001^{***}	99.406	0.000		-0.192^{***}	-0.001^{***}	99.262	0.000
	(-27.905)	(-8.790)				(-27.105)	(-8.051)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.067^{***}			97.590	0.000	-0.060^{***}			98.333	0.000
(-46.421)					(-32.102)				
	-0.238^{***}		96.402	0.000		-0.106^{***}		58.391	0.001
	(-28.699)					(-7.374)			
	-0.198^{***}	-0.001^{**}	98.850	0.000		-0.183^{***}	-0.002^{***}	98.961	0.000
	(-21.251)	(-4.713)				(-25.944)	(-11.937)		

Table 3. **Cross-Sectional Pricing All Assets: Bid-Ask Spreads In Sample**

This table reports estimated coefficients of two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. Data runs from January 1988 to December 2015.

USD					GBP				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.065^{***}			97.233	0.000	-0.063^{***}			97.542	0.000
(-34.771)					(-31.777)				
	-0.187^{***}		96.788	0.001		-0.177^{***}		97.041	0.000
	(-29.382)					(-28.164)			
	-0.179^{***}	-0.001^{**}	97.551	0.000		-0.176^{***}	-0.001^{*}	97.486	0.000
	(-31.766)	(-3.412)				(-31.002)	(-2.444)		
AUD					CAD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.058^{***}			97.207	0.000	-0.060^{***}			97.148	0.000
(-27.844)					(-32.020)				
	-0.154^{***}		92.977	0.001		-0.183^{***}		92.762	0.001
	(-17.618)					(-17.273)			
	-0.168^{***}	-0.001^{***}	97.085	0.000		-0.173^{***}	-0.001^{***}	97.284	0.000
	(-26.683)	(-5.452)				(-29.147)	(-7.208)		
CHF					EUR				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.062^{***}			97.319	0.000	-0.064^{***}			97.104	0.000
(-36.782)					(-36.968)				
	-0.196^{***}		96.563	0.001		-0.193^{***}		96.050	0.001
	(-29.153)					(-28.232)			
	-0.180^{***}	-0.001^{***}	97.733	0.000		-0.180^{***}	-0.001^{***}	97.457	0.000
	(-28.559)	(-4.188)				(-30.969)	(-4.372)		
JPY					NZD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.067^{***}			97.364	0.000	-0.059^{***}			97.555	0.000
(-50.268)					(-29.823)				
	-0.207^{***}		96.937	0.001		-0.143^{***}		82.828	0.001
	(-40.353)					(-10.854)			
	-0.186^{***}	-0.001^{***}	97.856	0.000		-0.169^{***}	-0.001^{***}	97.438	0.000
	(-29.146)	(-4.148)				(-29.103)	(-10.768)		

Table 4. **Cross-Sectional Pricing FX: Asymmetric Proportional Transaction Costs In Sample**

This table reports estimated coefficients of two-step cross-sectional regressions of currency excess returns on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. Data runs from January 1988 to December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.042^{***}			95.199	0.000	-0.041^{***}			96.847	0.000
(-14.590)					(-13.631)				
	-0.123^{***}		86.699	0.001		-0.115^{***}		98.207	0.000
	(-6.154)					(-18.145)			
	-0.117^{***}	-0.001^{**}	96.556	0.000		-0.115^{***}	-0.000	97.852	0.000
	(-14.452)	(-5.626)				(-16.554)	(-0.094)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.034^{***}			93.880	0.000	-0.041^{***}			95.713	0.000
(-14.755)					(-13.383)				
	-0.080^{***}		78.532	0.001		-0.086^{*}		67.699	0.001
	(-9.011)					(-2.829)			
	-0.104^{***}	-0.001^{*}	93.148	0.000		-0.115^{***}	-0.002^{***}	96.673	0.000
	(-10.103)	(-2.847)				(-13.901)	(-9.420)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.037^{***}			93.113	0.000	-0.040^{***}			93.295	0.000
(-13.137)					(-14.360)				
	-0.137^{***}		96.024	0.000		-0.142^{***}		91.673	0.001
	(-14.259)					(-9.477)			
	-0.123^{***}	-0.000	97.346	0.000		-0.121^{***}	-0.001^{**}	96.660	0.000
	(-13.304)	(-2.417)				(-12.876)	(-4.123)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.040^{***}			89.089	0.000	-0.035^{***}			92.577	0.000
(-23.181)					(-15.677)				
	-0.144^{***}		94.678	0.000		-0.066^{***}		55.157	0.001
	(-24.092)					(-6.883)			
	-0.124^{***}	-0.001	94.934	0.000		-0.115^{***}	-0.002^{**}	95.333	0.000
	(-10.661)	(-1.903)				(-12.085)	(-5.645)		

Table 5. **Cross-Sectional Pricing All Assets: Asymmetric Proportional Transaction Costs In Sample**

This table reports estimated coefficients of two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. Data runs from January 1988 to December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.041^{***}			90.738	0.001	-0.039^{***}			92.120	0.001
(-19.297)					(-17.720)				
	-0.119^{***}		92.715	0.001		-0.108^{***}		92.232	0.001
	(-19.242)					(-17.689)			
	-0.114^{***}	-0.001	92.804	0.001		-0.108^{***}	-0.000	91.902	0.001
	(-18.560)	(-2.060)				(-17.418)	(-0.556)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.037^{***}			92.933	0.001	-0.038^{***}			90.614	0.001
(-17.139)					(-17.985)				
	-0.099^{***}		89.301	0.001		-0.117^{***}		87.949	0.001
	(-13.929)					(-12.903)			
	-0.108^{***}	-0.001^{**}	92.600	0.001		-0.110^{***}	-0.002^{***}	92.442	0.001
	(-16.207)	(-3.114)				(-17.221)	(-4.935)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.037^{***}			90.654	0.001	-0.040^{***}			90.241	0.001
(-20.261)					(-20.613)				
	-0.120^{***}		94.567	0.001		-0.122^{***}		93.459	0.001
	(-24.391)					(-22.122)			
	-0.118^{***}	-0.000	94.220	0.001		-0.116^{***}	-0.001	93.453	0.001
	(-17.685)	(-0.565)				(-19.065)	(-1.916)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.040^{***}			89.713	0.001	-0.037^{***}			92.799	0.001
(-26.572)					(-16.951)				
	-0.127^{***}		94.013	0.001		-0.092^{***}		77.816	0.001
	(-30.284)					(-9.248)			
	-0.121^{***}	-0.001	93.531	0.001		-0.108^{***}	-0.001^{***}	92.563	0.001
	(-16.322)	(-1.066)				(-16.639)	(-6.321)		

Table 6. **Cross-Sectional Pricing FX: No Market Frictions OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of currency excess returns on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.312^{*}			63.02	0.001	-0.229			54.81	0.001
(-2.998)					(-1.954)				
	-0.874^{*}		61.95	0.001		-0.639		53.76	0.002
	(-2.853)					(-1.868)			
	-0.674^{*}	-0.003^{*}	63.15	0.001		-0.432	-0.004	50.89	0.001
	(-2.981)	(-2.706)				(-1.348)	(-1.723)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.246^{**}			58.14	0.001	-0.245			54.05	0.001
(-4.696)					(-2.257)				
	-0.674^{**}		55.42	0.001		-0.637		51.93	0.001
	(-4.578)					(-2.032)			
	-0.828^{*}	-0.002	63.91	0.001		-0.745^{*}	-0.002	44.06	0.001
	(-3.811)	(-0.970)				(-2.653)	(-1.698)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.167^{*}			32.27	0.001	-0.330^{**}			74.05	74.050
(-2.808)					(-5.545)				
	-0.496^{*}		33.88	0.001		-0.976^{**}		74.57	0.001
	(-2.875)					(-5.493)			
	-1.014^{*}	0.008	61.39	0.001		-0.900^{*}	-0.001	67.90	0.001
	(-3.362)	(-1.949)				(-2.975)	(-0.327)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.515^{*}			64.04	0.002	-0.165^{**}			20.64	0.001
(-3.274)					(-3.712)				
	-1.449^{*}		62.34	0.002		-0.427^{*}		18.07	0.001
	(-2.884)					(-3.672)			
	-0.711^{**}	-0.008^{***}	79.67	0.001		-0.554	-0.002	14.71	0.001
	(-4.554)	(-8.997)				(-1.726)	(-0.430)		

Table 7. **Cross-Sectional Pricing All Assets: No Market Frictions OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

USD					GBP				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.243^{***}			77.63	0.001	-0.230^{***}			74.53	0.001
(-9.722)					(-7.873)				
	-0.689^{***}		77.29	0.001		-0.649^{***}		74.15	0.001
	(-9.553)					(-7.745)			
	-0.573^{***}	-0.003^{***}	79.71	0.001		-0.616^{***}	-0.002^{*}	74.97	0.001
	(-9.151)	(-4.000)				(-7.790)	(-2.151)		
AUD					CAD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.177^{***}			74.47	0.001	-0.222^{***}			74.97	0.001
(-7.927)					(-8.619)				
	-0.500^{***}		74.43	0.001		-0.619^{***}		74.06	0.001
	(-7.957)					(-8.219)			
	-0.498^{***}	0.000	73.11	0.001		-0.598^{***}	-0.002^{**}	75.13	0.001
	(-7.652)	(-0.153)				(-9.798)	(-3.582)		
CHF					EUR				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.177^{***}			70.90	0.001	-0.235^{***}			81.29	0.001
(-10.976)					(-12.570)				
	-0.514^{***}		72.03	0.001		-0.678^{***}		80.97	0.001
	(-11.156)					(-12.250)			
	-0.607^{***}	0.003	78.11	0.001		-0.583^{***}	-0.002^{*}	79.96	0.001
	(-8.983)	(-1.809)				(-8.933)	(-2.312)		
JPY					NZD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.336^{***}			72.494	0.002	-0.169^{***}			64.42	0.001
(-8.560)					(-6.742)				
	-0.966^{***}		71.459	0.002		-0.452^{***}		61.71	0.001
	(-8.077)					(-6.449)			
	-0.475^{***}	-0.007^{***}	82.276	0.001		-0.509^{***}	-0.002	68.88	0.001
	(-6.263)	(-9.249)				(-7.100)	(-1.997)		

Table 8. **Cross-Sectional Pricing FX: Symmetric Proportional Transaction Costs OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.213^{**}			74.102	0.100	-0.160			61.214	0.138
(-4.494)					(-2.315)				
	-0.602^{**}		73.103	0.104		-0.437		59.010	0.143
	(-4.253)					(-2.131)			
	-0.520^{**}	-0.001	69.744	0.087		-0.386^{*}	-0.002^{*}	67.815	0.101
	(-4.155)	(-1.919)				(-2.648)	(-2.664)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.143^{**}			58.623	0.085	-0.174^{*}			63.290	0.116
(-5.219)					(-2.766)				
	-0.383^{**}		52.360	0.090		-0.432		58.535	0.128
	(-4.874)					(-2.300)			
	-0.504^{**}	-0.002	75.682	0.077		-0.540^{*}	-0.001	63.036	0.095
	(-5.333)	(-1.817)				(-3.690)	(-2.425)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.124^{*}			48.145	0.110	-0.205^{***}			81.987	0.073
(-3.527)					(-7.344)				
	-0.383^{*}		51.638	0.106		-0.615^{***}		82.378	0.075
	(-3.704)					(-7.204)			
	-0.684^{*}	0.003	71.537	0.089		-0.562^{*}	-0.000	77.830	0.080
	(-3.756)	(1.876)				(-3.784)	(-0.451)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.317^{**}			72.299	0.159	-0.124^{**}			35.556	0.093
(-5.349)					(-4.623)				
	-0.935^{**}		69.746	0.180		-0.300^{**}		27.700	0.097
	(-4.569)					(-4.381)			
	-0.408^{**}	-0.004^{***}	85.008	0.054		-0.488^{*}	-0.002	45.898	0.096
	(-4.485)	(-7.855)				(-2.596)	(-1.072)		

Table 9. **Cross-Sectional Pricing All: Symmetric Proportional Transaction Costs OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.148^{***}			79.216	0.122	-0.142^{***}			75.431	0.140
(-10.816)					(-8.208)				
	-0.418^{***}		78.898	0.124		-0.402^{***}		74.691	0.143
	(-10.600)					(-7.941)			
	-0.364^{***}	-0.002^{**}	78.847	0.103		-0.359^{***}	-0.002^{***}	78.755	0.111
	(-9.983)	(-3.311)				(-8.826)	(-3.949)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.109^{***}			74.926	0.103	-0.138^{***}			75.760	0.129
(-8.113)					(-8.850)				
	-0.306^{***}		74.491	0.103		-0.385^{***}		74.256	0.136
	(-8.080)					(-8.194)			
	-0.309^{***}	-0.000	73.736	0.105		-0.352^{***}	-0.001^{***}	76.217	0.107
	(-7.847)	(-0.362)				(-9.292)	(-3.837)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.115^{***}			75.007	0.105	-0.143^{***}			81.661	0.108
(-11.660)					(-13.076)				
	-0.337^{***}		76.309	0.104		-0.412^{***}		81.225	0.112
	(-11.857)					(-12.626)			
	-0.361^{***}	0.001	77.666	0.104		-0.363^{***}	-0.001^{*}	80.375	0.103
	(-8.910)	(0.847)				(-9.822)	(-2.245)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.202^{***}			75.808	0.192	-0.112^{***}			69.596	0.117
(-10.476)					(-7.316)				
	-0.585^{***}		74.484	0.204		-0.296^{***}		65.365	0.123
	(-9.718)					(-6.831)			
	-0.301^{***}	-0.004^{***}	84.212	0.094		-0.323^{***}	-0.001	70.257	0.118
	(-7.215)	(-9.126)				(-7.252)	(-1.692)		

Table 10. **Cross-Sectional Pricing FX: Bid-Ask Spreads OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

USD					GBP				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.258^{**}			77.646	0.089	-0.201^{*}			63.83	0.131
(-5.186)					(-2.543)				
	-0.722^{**}		75.278	0.098		-0.570^{*}		63.198	0.133
	(-4.622)					(-2.468)			
	-0.637^{**}	-0.004	76.959	0.074		-0.534	-0.002	56.405	0.142
	(-5.118)	(-2.312)				(-2.089)	(-0.521)		
AUD					CAD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.170^{**}			60.675	0.082	-0.218^{*}			66.435	0.111
(-5.472)					(-3.010)				
	-0.450^{**}		51.735	0.089		-0.519		59.643	0.126
	(-4.922)					(-2.350)			
	-0.607^{**}	-0.003	80.874	0.068		-0.690^{**}	-0.003^{*}	69.208	0.088
	(-6.290)	(-2.335)				(-4.166)	(-2.729)		
CHF					EUR				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.150^{*}			48.11	0.11	-0.250^{***}			83.542	0.07
(-3.523)					(-7.707)				
	-0.485^{**}		56.888	0.101		-0.736^{***}		84.111	0.07
	(-4.003)					(-7.731)			
	-0.842^{***}	0.007^{*}	90.711	0.054		-0.735^{*}	0	80.919	0.077
	(-7.660)	-4.011				(-3.574)	(-0.005)		
JPY					NZD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.380^{**}			72.736	0.16	-0.140^{**}			33.358	0.092
(-5.322)					(-4.723)				
	-1.118^{**}		70.661	0.176		-0.357^{**}		28.323	0.095
	(-4.703)					(-4.479)			
	-0.532^{**}	-0.009^{***}	85.467	0.058		-0.605^{*}	-0.005	57.088	0.084
	(-4.694)	(-7.112)				(-3.667)	(-1.662)		

Table 11. **Cross-Sectional Pricing All Assets: Bid-Ask Spreads OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.167^{***}			80.86	0.116	-0.163^{***}			76.725	0.135
(-11.512)					(-8.586)				
	-0.473^{***}		80.329	0.119		-0.456^{***}		76.702	0.135
	(-11.118)					(-8.608)			
	-0.398^{***}	-0.005^{**}	80.306	0.102		-0.457^{***}	-0.000	75.606	0.138
	(-8.966)	(-2.988)				(-8.305)	(-0.036)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.122^{***}			76.19	0.101	-0.157^{***}			77.73	0.123
(-8.343)					(-9.392)				
	-0.348^{***}		75.608	0.101		-0.437^{***}		75.813	0.132
	(-8.296)					(-8.551)			
	-0.347^{***}	-0.000	75.014	0.103		-0.404^{***}	-0.003^{***}	78.876	0.103
	(-8.106)	(-0.428)				(-9.875)	(-3.862)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.133^{***}			76.481	0.103	-0.164^{***}			82.423	0.107
(-11.975)					(-13.254)				
	-0.393^{***}		79.096	0.099		-0.469^{***}		81.931	0.11
	(-12.488)					(-12.803)			
	-0.429^{***}	0.002	82.191	0.097		-0.423^{***}	-0.005^{*}	81.701	0.099
	(-10.431)	1.354				(-11.182)	(-2.477)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.230^{***}			75.85	0.195	-0.128^{***}			71.871	0.112
(-10.264)					(-7.758)				
	-0.663^{***}		74.806	0.205		-0.344^{***}		69.136	0.116
	(-9.677)					(-7.368)			
	-0.399^{***}	-0.009^{***}	82.875	0.106		-0.372^{***}	-0.002	74.207	0.11
	(-8.178)	(-7.840)				(-7.998)	(-1.905)		

Table 12. **Cross-Sectional Pricing FX: Asymmetric Proportional Transaction Costs OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

		USD					GBP		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.196^{***}			80.515	0.079	-0.163^{*}			66.644	0.125
(-5.989)					(-2.777)				
	-0.562^{**}		78.689	0.086		-0.478^{*}		67.434	0.124
	(-5.381)					(-2.843)			
	-0.448^{**}	-0.005	77.362	0.073		-0.515	0.002	61.700	0.134
	(-4.186)	(-1.859)				(-2.488)	(0.377)		
		AUD					CAD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.133^{**}			62.856	0.080	-0.175^{*}			67.386	0.111
(-5.662)					(-3.004)				
	-0.341^{**}		50.690	0.089		-0.380		58.097	0.132
	(-4.952)					(-2.151)			
	-0.448^{**}	-0.002	73.946	0.076		-0.580^{**}	-0.005^{*}	74.657	0.078
	(-5.351)	(-1.792)				(-4.840)	(-3.464)		
		CHF					EUR		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.123^{**}			55.639	0.102	-0.196^{***}			84.731	0.067
(-3.932)					(-8.067)				
	-0.402^{**}		63.244	0.093		-0.578^{***}		86.161	0.065
	(-4.425)					(-8.329)			
	-0.702^{***}	0.005^{*}	90.428	0.054		-0.623^{**}	0.003	85.630	0.070
	(-7.062)	(3.564)				(-5.479)	(0.528)		
		JPY					NZD		
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.289^{***}			76.044	0.135	-0.114^{**}			38.577	0.086
(-6.451)					(-5.119)				
	-0.860^{**}		73.142	0.158		-0.289^{**}		32.788	0.091
	(-5.381)					(-4.752)			
	-0.491^{**}	-0.005^{**}	80.471	0.067		-0.507^{**}	-0.004	68.478	0.070
	(-5.031)	(-5.290)				(-4.775)	(-2.288)		

Table 13. **Cross-Sectional Pricing All: Asymmetric Proportional Transaction Costs OOS**

This table reports estimated coefficients of out-of-sample (OOS) two-step cross-sectional regressions of excess returns on currencies, equities and bonds on estimated factor loadings of the corresponding currency SDF, the global and local SDF. The global SDF is the first principal component calculated from the cross-section of all currency SDFs. The local SDF is from a project of the corresponding currency SDF on the global SDF. t-statistics are in parentheses. The OOS period starts in February 1998 and ends in December 2015.

USD					GBP				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.128^{***}			80.909	0.113	-0.126^{***}			76.839	0.133
(-11.844)					(-8.722)				
	-0.362^{***}		80.326	0.117		-0.352^{***}		77.079	0.131
	(-11.384)					(-8.885)			
	-0.279^{***}	-0.007^{**}	80.127	0.099		-0.344^{***}	0.001	75.736	0.133
	(-7.385)	(-3.108)				(-8.260)	(0.775)		
AUD					CAD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.095^{***}			75.764	0.102	-0.123^{***}			77.189	0.125
(-8.248)					(-9.181)				
	-0.269^{***}		75.144	0.102		-0.339^{***}		74.742	0.137
	(-8.216)					(-8.128)			
	-0.271^{***}	-0.000	74.499	0.104		-0.325^{***}	-0.004^{***}	78.143	0.103
	(-7.992)	(-0.419)				(-10.324)	(-4.225)		
CHF					EUR				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.103^{***}			77.634	0.102	-0.125^{***}			81.496	0.110
(-12.161)					(-12.859)				
	-0.308^{***}		79.390	0.100		-0.362^{***}		81.481	0.112
	(-12.424)					(-12.613)			
	-0.321^{***}	0.001	79.764	0.101		-0.319^{***}	-0.004	79.302	0.109
	(-9.294)	(0.548)				(-7.808)	(-1.468)		
JPY					NZD				
SDF	Global	Local	R^2	RMSE	SDF	Global	Local	R^2	RMSE
-0.175^{***}			76.644	0.185	-0.101^{***}			72.070	0.112
(-10.903)					(-7.744)				
	-0.508^{***}		75.381	0.198		-0.270^{***}		69.353	0.117
	(-10.106)					(-7.350)			
	-0.283^{***}	-0.006^{***}	79.996	0.116		-0.293^{***}	-0.001	74.353	0.110
	(-6.234)	(-6.549)				(-7.974)	(-1.912)		

References

ADRIAN, T., M. FLEMING, AND E. VOGT (2017): "An Index of Treasury Market Liquidity: 1991-2017," Federal Reserve Bank of New York Staff Report No. 827.

ALOOSH, A., AND G. BEKAERT (2019): "Currency Factors," Working Paper, Neoma Business School.

ANDERSEN, T. G., O. BONDARENKO, A. S. KYLE, AND A. A. OBIZHAEVA (2018): "Intraday Trading Invariance in the E-Mini S&P500 Futures Market," Working Paper, Northwestern University.

BACKUS, D. K., S. FORESI, AND C. I. TELMER (2001): "Affine term structure models and the forward premium anomaly," *Journal of Finance*, 56(1), 279–304.

BANK OF INTERNATIONAL SETTLEMENT (2016): "Fixed Income Market Liquidity," Committee on the Global Financial System Papers No 55.

BAUSCHKE, H. H., AND P. L. COMBETTES (2011): *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*. Springer.

BHAMRA, H. S., N. COEURDACIER, AND S. GUIBAUD (2014): "A Dynamic Equilibrium Model of Imperfectly Integrated Financial Markets," *Journal of Economic Theory*, 154, 490–542.

BORWEIN, J. M., AND A. S. LEWIS (1991): "Duality Relationships for Entropy-like Minimization Problems," *SIAM Journal on Control and Optimization*, 29(2), 325–338.

BORWEIN, J. M., AND A. S. LEWIS (1992): "Partially Finite Convex Programming, Part I: Quasi Relative Interiors and Duality Theory," *Mathematical Programming*, 57(1-3), 15–48.

BRANDT, M., J. H. COCHRANE, AND P. SANTA-CLARA (2006): "International risk sharing is better than you think, or exchange rates are too smooth," *Journal of Monetary Economics*, 53(4), 671–698.

CAMANHO, N., H. HAU, AND H. REY (2019): "Global Portfolio Rebalancing and Exchange Rates," NBER Working Paper No. 24320.

CHEN, Z. (2001): "Viable Costs and Equilibrium Prices in Frictional Securities Markets," *Annals of Economics and Finance*, 2(2), 297–323.

CHIEN, Y., H. LUSTIG, AND K. NAKNOI (2019): "Why Are Exchange Rates So Smooth? A Household Finance Explanation," *forthcoming, Journal of Monetary Economics*.

CLARK, S. A. (1993): "The Valuation Problem in Arbitrage Price Theory," *Journal of Mathematical Economics*, 22(5), 463–478.

COEURDACIER, N., AND H. REY (2013): "Home Bias in Open Economy Financial Macroeconomics," *Journal of Economic Literature*, 51, 63–115.

CVITANIĆ, J., AND I. KARATZAS (1992): "Convex Duality in Constrained Portfolio Optimization," *Annals of Applied Probability*, 2(4), 767–818.

DEMIGUEL, V., A. MARTIN-UTRERA, F. J. NOGALES, AND R. UPPAL (2019): "A Transaction Cost Perspective on the Multitude of Firm Characteristics," *forthcoming, Review of Financial Studies*.

ENGEL, C., N. C. MARK, AND K. D. WEST (2015): "Factor Model Forecasts of Exchange Rates," *Econometric Reviews*, 34, 32–55.

FRENCH, K. R., AND J. M. POTERBA (1991): "Investor Diversification and International Equity Markets," *American Economic Review*, 81(2), 222–226.

FREYBERGER, J., A. NEUHIERL, AND M. WEBER (2019): "Dissecting Characteristics Nonparametrically," *forthcoming, Review of Financial Studies*.

GABAIX, X., AND M. MAGGIORI (2015): "International Liquidity and Exchange Rate Dynamics," *Quarterly Journal of Economics*, 130(3), 1369–1420.

HANSEN, L. P., J. HEATON, AND E. G. LUTTMER (1995): "Econometric Evaluation of Asset Pricing Models," *Review of Financial Studies*, 8(2), 237–274.

HANSEN, L. P., AND R. JAGANNATHAN (1991): "Restrictions on intertemporal marginal rates of substitution implied by asset returns," *Journal of Political Economy*, 99(2), 225–262.

HANSEN, L. P., AND S. F. RICHARD (1987): "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models," *Econometrica*, 55, 587–613.

HARRISON, J. M., AND D. M. KREPS (1979): "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory*, 20(3), 381–408.

HE, H., AND D. MODEST (1995): "Market Frictions and Consumption-Based Asset Pricing," *Journal of Political Economy*, 103, 94–117.

HIRIART-URRUTY, J.-B., AND C. LEMARÉCHAL (2012): *Fundamentals of Convex Analysis*. Springer Science & Business Media.

JIANG, Z., AND R. J. RICHMOND (2019): "Origins of International Factor Structures," Working Paper, Northwestern University.

KORSAYE, S., A. QUAINI, AND F. TROJANI (2018): "Smart SDFs," Working Paper, University of Geneva.

KOZAK, S., S. NAGEL, AND S. SANTOSH (2019): "Shrinking the Cross Section," *forthcoming, Journal of Financial Economics*.

LEWIS, K. K. (1999): "Trying to Explain Home Bias in Equities and Consumption," *Journal of Economic Literature*, 37, 571–608.

LUSTIG, H., AND R. J. RICHMOND (2019): "Gravity in the Exchange Rate Factor Structure," *forthcoming, Review of Financial Studies*.

LUSTIG, H., N. ROUSSANOV, AND A. VERDELHAN (2011): "Common Risk Factors in Currency Markets," *Review of Financial Studies*, 11, 3731–3777.

——— (2014): "Countercylical Currency Risk Premia," *Journal of Financial Economics*, 111(3), 527–553.

LUSTIG, H., A. STATHOPOLOUS, AND A. VERDELHAN (2019): "The Term Structure of Currency Carry Trade Risk Premia," *forthcoming, American Economic Review*.

LUTTMER, E. G. (1996): "Asset Pricing in Economies with Frictions," *Econometrica*, 64(6), 1439–1467.

MAGGIORI, M., B. NEIMAN, AND J. SCHREGER (2019): "International Currencies and Capital Allocation," *forthcoming, Journal of Political Economy*.

MARTIN, P., AND H. REY (2004): "Financial Super-Markets: Size Matters for Asset Trade," *Journal of International Economics*, 64(2), 335–361.

MAURER, T. A., T.-D. TÔ, AND N.-K. TRAN (2019): "Pricing Risks Across Currency Denominations," *Management Science*, 65(11), 5308–5336.

NIEUWERBURGH, S. V., AND L. VELDKAMP (2009): "Information Immobility and the Home Bias Puzzle," *Journal of Finance*, 64(3), 1187–1215.

PANAYOTOV, G. (2019): "Global Risks in the Currency Market," Working Paper, Hong Kong University of Science and Technology.

ROSS, S. A. (1976): "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, 13, 341–360.

——— (1978): "A Simple Approach to the Valuation of Risky Streams," *Journal of Business*, 3, 453–476.

SANDULESCU, M., F. TROJANI, AND A. VEDOLIN (2019): "Model-Free International Stochastic Discount Factors," *forthcoming, Journal of Finance*.

VERDELHAN, A. (2018): "The Share of Systematic Variation in Bilateral Exchange Rates," *Journal of Finance*, 71(1), 375–418.

Appendix A Proofs

Proof of Proposition 1

Proof. The proof mainly follows Korsaye, Quaini, and Trojani (2018). Sublinearity of the transaction cost function h implies that of the price function, π, and that the set of payoffs, $\mathcal{Y}$, is a convex cone. Hence, by Chen (2001, Theorem 5) and Clark (1993, Theorem 6), the arbitrage-free condition is equivalent to the viability condition (Harrison and Kreps (1979)). By Chen (2001, Theorem 1), there exists a strictly positive, continuous linear functional ψ defined on L^q, such that $\psi|_{\mathcal{Y}} \leq \pi$. Moreover, by the Riesz Representation Theorem, there exists a strictly positive element, M, in the dual space of L^q, i.e. L^p, such that $\psi(Y) = \mathbb{E}[MY]$. Hence, we have that $\mathbb{E}[MY] \leq \pi(Y)$ for all Y in $\mathcal{Y}$. The definition of the price function, π implies that

$$\boldsymbol{\theta}_S'\mathbb{E}[M\boldsymbol{Z}_S] + \boldsymbol{\theta}_D'\mathbb{E}[M\boldsymbol{Z}_D] \leq \boldsymbol{\theta}_S'\boldsymbol{P}_S + \boldsymbol{\theta}_D'\boldsymbol{P}_D + h(\boldsymbol{\theta}_D) \tag{A.1}$$

for any $\boldsymbol{\theta} \in \mathbb{R}^N$. Hence, for any $\boldsymbol{\theta}_S \in \boldsymbol{R}^{N_S} : \boldsymbol{\theta}_S'\mathbb{E}[M\boldsymbol{Z}_S] \leq \boldsymbol{\theta}_S'\boldsymbol{P}_S$. Thus, $\mathbb{E}[M\boldsymbol{Z}_S] - \boldsymbol{P}_S = 0$. Similarly, inequality (A.1) implies that $\boldsymbol{\theta}_S'\mathbb{E}[M\boldsymbol{Z}_D] \leq \boldsymbol{\theta}_D'\boldsymbol{P}_D + h(\boldsymbol{\theta}_D)$ for all $\boldsymbol{\theta}_D \in \mathbb{R}^{N_D}$. This together with Bauschke and Combettes (2011, Prop. 13.10 (i)) implies that $h^*(\mathbb{E}[M\boldsymbol{Z}_D - \boldsymbol{P}_D]) \leq 0$, where h^* is the convex conjugate of the transaction cost function, h. Since h is closed and sublinear, by Hiriart-Urruty and Lemaréchal (2012, Theorem 3.1.1), it is a support function of C_h. The convex conjugate of a support function of C_h is given by an indicator function of the set C_h, i.e., $h^* = \delta_{C_h}$. This concludes the proof. □

Proof of Proposition 2

Proof. We define the linear operator $A : L_q \to \mathbb{R}^N$, with $A(M) := \mathbb{E}[M\boldsymbol{R}]$, then the primal problem (6) can be written as:

$$\Pi_h = \inf_{M \in L^p} \{\mathbb{E}[\phi_+(M)] \ : \ A(M) \in P\} \ , \tag{A.2}$$

where the closed convex set P is defined as $P := \{\mathbf{1}_{N_S}\} \times (\{\mathbf{1}_{N_D}\} + C_h)$ and ϕ_+ is the restriction to the nonnegative real line of ϕ. Under the no arbitrage, from Proposition 1 there exists a strictly positive element, $\bar{M}$, of $\mathcal{M}_h$. Hence, $A(\bar{M})$ is an element of the set $ri(A(\text{dom}(\phi_+))) \cap ri(P)$, where ri denotes the relative interior.[20] By Borwein and Lewis (1992, Theorem 4.2), the following duality relation then holds:

$$\Pi = -\min_{\boldsymbol{\theta} \in \mathbb{R}^N} \left\{\mathbb{E}[\phi_+^*(A^T\boldsymbol{\theta})] + \delta_P^*(-\boldsymbol{\theta})\right\} \ , \tag{A.3}$$

where A^T is the adjoint map relative to the linear operator$A$.[21] Since $\delta_P(A(M)) = \delta_{\{\mathbf{1}_{N_S}\}}(A(M)|_S) + \delta_{\{1_{N_D}\}+C_h}(A(M)|_D)$, its convex conjugate by Hiriart-Urruty and Lemaréchal (2012, Theorem 3.1.1) reads:

$$\delta_P^*(-\boldsymbol{\theta}) = -\boldsymbol{\theta}_S'\mathbf{1}_{N_S} - \boldsymbol{\theta}_D'\mathbf{1}_{N_D} + h(-\boldsymbol{\theta}_D) \ .$$

Hence, we obtain the first result. Now, under the condition that $-\boldsymbol{\theta}_0'\boldsymbol{R} \in \text{dom}(\phi_+^*)$, the link between the solutions of the primal and the dual problem becomes a direct result of Borwein and Lewis (1991, Theorem 4.8). This concludes the proof. □

[20] The relative interior of a set A is defined as the interior of the affine hull of set A.
[21] $A^T : \mathbb{R}^N \to L^p$ such that $\mathbb{E}[A^T(\theta)M] = \theta' A(M)$.

Proof of Proposition 3

Proof. Let $M_0^{(i)}$ be the domestic minimum entropy SDF. It then follows:

$$\begin{aligned} M_0^{(i)} &= \arg\min_{M\in\mathcal{M}_h^{(i)}} \{\mathbb{E}[-\ln M] : \mathbb{E}[M\boldsymbol{R}^{(i)} - \mathbf{1}] \in C\} \\ &= \arg\min_{M\in\mathcal{M}_h^{(i)}} \{\mathbb{E}[-\ln(M)] : \mathbb{E}[MX_j^{(i)}\boldsymbol{R}^{(j)} - \mathbf{1}] \in C\} \\ &= X_i^{(j)} \arg\min_{MX_j^{(i)}\in\mathcal{M}_h^{(j)}} \{\mathbb{E}[-\ln(MX_j^{(i)})] : \mathbb{E}[MX_j^{(i)}\boldsymbol{R}^{(j)} - \mathbf{1}] \in C\} \\ &= X_i^{(j)} M_0^{(j)} \,. \end{aligned}$$

This concludes the proof. □