Monetary Policy, Redistribution, and Risk Premia*

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Abstract

We study the transmission of monetary policy through risk premia in a heterogeneous agent New Keynesian environment. Heterogeneity in households' marginal propensity to take risk (MPR) summarizes differences in risk aversion, constraints, rules of thumb, background risk, and beliefs relevant for portfolio choice on the margin. An unexpected reduction in the nominal interest rate redistributes to households with high MPRs, lowering risk premia and amplifying the stimulus to the real economy. Quantitatively, this mechanism rationalizes the role of news about future excess returns in driving the stock market response to monetary policy shocks.

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1 Introduction

A growing literature finds that expansionary monetary policy lowers risk premia. This has been established for the equity premium in stock markets, the term premium in nominal bonds, and the external finance premium on risky corporate debt.[1] The basic New Keynesian framework as described by Woodford (2003) and Gali (2008) does not capture this aspect of monetary policy transmission. As noted by Kaplan and Violante (2018), this is equally true for an emerging body of heterogeneous agent New Keynesian models in which heterogeneity in the marginal propensity to consume enriches the transmission mechanism but still cannot explain the associated movements in risk premia.

This paper demonstrates that a New Keynesian model with heterogeneous households differing instead in risk-bearing capacity can quantitatively rationalize the observed effects of policy on risk premia, amplifying the transmission to the real economy. An expansionary monetary policy shock lowers the risk premium if it redistributes to households with a high marginal propensity to take risk (MPR), defined as the marginal propensity to save in capital relative to save overall. With underlying heterogeneity in risk aversion, portfolio constraints, rules of thumb, background risk, or beliefs, high MPR households borrow in the bond market from low MPR households to hold leveraged positions in capital. By generating unexpected inflation, raising profit income relative to labor income, and raising the price of capital, an expansionary monetary policy shock endogenously redistributes to high MPR households and thus lowers the market price of risk. In a calibration matching portfolio heterogeneity in the U.S. economy, this mechanism rationalizes the role of news about lower future excess returns in driving the empirically observed increase in the stock market. The stimulus to investment, consumption, and output is amplified relative to a counterfactual representative agent economy without heterogeneity in monetary policy exposures and MPRs.

Our baseline environment enriches an otherwise standard New Keynesian model with Epstein and Zin (1991) preferences and heterogeneity in risk aversion. Households consume, supply labor subject to adjustment costs in nominal wages, and choose a portfolio of nominal bonds and claims on capital. Production is subject to aggregate TFP shocks. Monetary policy follows a standard Taylor (1993) rule. Heterogeneity in risk aversion endogenously generates heterogeneity in MPRs and exposures to a monetary policy shock. Epstein-Zin preferences allow us to flexibly model this heterogeneity as distinct from households' intertemporal elasticities of substitution. We begin by analytically characterizing the effects of a monetary policy shock in a simple two-period version of this environment, providing an organizing framework for the quantitative analysis of the infinite horizon which follows.

[1] See Bernanke and Kuttner (2005), Hanson and Stein (2015), and Gertler and Karadi (2015), respectively.

An expansionary monetary policy shock lowers the risk premium by endogenously redistributing wealth to households with a high marginal propensity to save in capital relative to save overall — that is, a high MPR. Redistribution to high MPR households lowers the risk premium because of market clearing in asset markets: to the extent households on aggregate wish to increase their portfolio share in capital, its expected return must fall relative to that on bonds. An expansionary monetary policy shock redistributes across households by revaluing their initial balance sheets: it deflates nominal debt, raises the profits earned using capital, and raises the price of capital. More risk tolerant households hold leveraged positions in capital and have a higher MPR. Hence, an expansionary monetary policy shock will lower the risk premium by endogenously redistributing to these households.

These insights are robust to alternative sources of heterogeneity beyond risk aversion. We consider a richer environment in which households may also face portfolio constraints or follow rules-of-thumb as in models of limited participation or the financial accelerator; households may be subject to idiosyncratic background risk as in the large literatures on uninsured labor or entrepreneurial income risk; and households may have subjective beliefs regarding the value of capital. In this general environment, the MPR continues to summarize the relevant cross-sectional heterogeneity to evaluate the risk premium effects of redistributive shocks. Moreover, because each of these forms of heterogeneity imply that households holding more levered positions in capital will be the ones with a high MPR, they continue to imply that expansionary monetary policy will lower the risk premium through redistribution.

The reduction in the risk premium matters for the real economy depending on the endogenous reaction of the monetary feedback rule. Absent nominal rigidity, the investment effect of a shock lowering the risk premium depends critically on households' intertemporal elasticities of substitution, which in turn control the relative strength of income and substitution effects and thus the equilibrium response of the safe real interest rate to the shock.[2] With nominal rigidity, the response of the real interest rate also critically depends on the monetary policy rule. In our environment, this logic applies to the risk premium effects of the primitive monetary shock itself. Provided that the central bank's feedback rule limits the rise in the real interest rate following this shock, the decline in the risk premium will be reflected in lower required returns to capital, amplifying the stimulus to investment. The associated increase in household wealth, both due to an increase in labor income and an increase in the price of capital, amplifies the stimulus to consumption.

Accounting for the risk premium effects of monetary policy is important given empirical evidence implying that it may be a key component of the transmission mechanism. We refresh

[2]Intuitively, even if the risk premium falls, it could either be that the expected return to capital falls or rises (and thus investment rises or falls), in the latter case because the safe real interest rate rises sufficiently.

this point from Bernanke and Kuttner (2005) using the structural vector autoregression instrumental variables (SVAR-IV) approach in Gertler and Karadi (2015). Using monthly data from July 1979 through June 2012, we run a six-variable VAR including the excess return on the S&P 500 and the real return on a T-bill. Using Fed Funds surprises on FOMC days as an instrument, we find that a monetary policy shock resulting in a roughly $25bp$ reduction in the 1-year Treasury yield leads to an unexpected $2pp$ increase in the real S&P 500 return. Using a Campbell and Shiller (1988) decomposition and accounting for estimation uncertainty, $20\% - 100\%$ of this increase is driven by lower future excess returns, posing a strong challenge to existing New Keynesian frameworks where virtually all of the effect on the stock market operates instead through higher dividends or lower risk-free rates.

Extending the model to the infinite horizon, we investigate whether a calibration to the U.S. economy is capable of rationalizing these facts. We parameterize the model to match the cross-sectional heterogeneity in wealth, labor income, and financial portfolios in the Survey of Consumer Finances, together disciplining the model-implied exposures to a monetary policy shock and MPRs. We measure households' underlying positions in capital and nominal claims in particular by also accounting for the balance sheets of business and intermediaries in which they invest using the Financial Accounts of the United States. We use global solution methods to solve the model.[3] To make the computational burden tractable, we summarize the cross-sectional heterogeneity into three groups of households: two groups corresponding to the small fraction with high wealth relative to labor income, but differing in their risk tolerance and thus portfolio share in capital, and one group corresponding to the large fraction holding little wealth relative to labor income.

In response to an expansionary monetary policy shock, we find that the redistribution across households with heterogeneous MPRs can quantitatively explain the risk premium effects of monetary policy found in the data. Using the same Campbell-Shiller decomposition on model impulse responses as we used on the data, 33% of the return on equity in our baseline parameterization arises from news about lower future excess returns, consistent with estimates in the data unlike the 0% in a counterfactual representative agent economy. Consistent with the analytical results, the effect on the risk premium is amplified in parameterizations with a more persistent shock and thus larger debt deflation; higher stickiness and thus a larger increase in profit income relative to labor income; or higher investment adjustment costs and thus a larger increase in the price of capital.

[3]Our model does not provide a novel way to rationalize the level of the equity premium. We add a rare disaster to match this moment, following Barro (2006), but other approaches include adding long-run risk (Bansal and Yaron (2004)), habits (Campbell and Cochrane (1999)), idiosyncratic risk (Gomes and Michaelides (2008)), or limited participation and heterogeneous intertemporal elasticities of substitution (Guvenen (2009)). To obtain an accurate solution especially given the disaster, we use a global method.

The reduction in the risk premium through redistribution in turn amplifies the effect of policy on the real economy. In both our baseline economy and the counterfactual representative agent economy, we solve for monetary policy shocks which deliver the roughly $25bp$ decline in the 1-year Treasury yield which we estimate in the data. Given these shocks, our model amplifies the response of quantities by roughly 1.3 times: the peak investment, consumption, and output responses are roughly $70bp$, $20bp$, and $35bp$, while the counterparts in the representative agent economy are roughly $55bp$, $15bp$, and $25bp$. Relatedly, our results suggest state-dependent effects of monetary policy: when the relatively risk tolerant hold a larger fraction of total wealth (as in a bull market), the redistribution and thus risk premium effects of a monetary policy shock are larger, and hence so are its real effects.

Related literature Our paper contributes to the rapidly growing literature on heterogeneous agent New Keynesian (HANK) models by studying the transmission of monetary policy through risk premia. We build on Doepke and Schneider (2006) in our measurement of household portfolios, informing the heterogeneity in exposures to a monetary policy shock. The redistributive effects of monetary policy in our framework follow Auclert (2018). Relative to his work, we demonstrate that it is the covariance of these exposures with MPRs rather than MPCs which matters for policy transmission through risk premia rather than risk-free rates. Like Kaplan, Moll, and Violante (2018) and Luetticke (2018), we study a two-asset environment with bonds and capital. And like Alves, Kaplan, Moll, and Violante (2019) and Auclert, Rognlie, and Straub (2019), we study the effects of monetary policy shocks on asset prices. Unlike these models, in our framework assets differ in their exposure to aggregate risk rather than in their liquidity, allowing us to account for the important role of risk premia in driving the change in asset prices.

In doing so, we bring to the HANK literature many established insights from heterogeneous agent and intermediary-based asset pricing. The wealth distribution is a crucial determinant of the market price of risk as in other models with heterogeneous risk aversion (e.g., Garleanu and Panageas (2015)), segmented markets (e.g., He and Krishnamurthy (2013)), rules-of-thumb (e.g., Chien, Cole, and Lustig (2012)), background risk (e.g., Heaton and Lucas (1996)), or heterogeneous beliefs (e.g., Geanakoplos (2009)).[4] We build on this literature by focusing on the changes in wealth induced by a monetary policy shock in a production economy with nominal rigidities. In studying this question we follow Alvarez, Atkeson, and Kehoe (2009) and Drechsler, Savov, and Schnabl (2018), who study the effects of monetary policy on risk premia in an exchange economy with segmented markets and

[4]In recent work, Panageas (2019) and Toda and Walsh (2019) study the common structure and implications of these models. The latter paper in particular emphasizes portfolio heterogeneity as a summary statistic to evaluate the effects of redistribution with incomplete markets, as in our analysis.

in a model of banking, respectively.[5] We instead study these effects operating through the balance sheet revaluation of heterogeneous agents in a conventional New Keynesian setting.

Indeed, our paper most directly builds on prior work focused on risk premia in New Keynesian economies. We clarify the sense in which Bernanke, Gertler, and Gilchrist (1999) served as a seminal HANK model focused on heterogeneity in MPRs rather than MPCs.[6] As we demonstrate, however, heterogeneity in MPRs need not rely on market segmentation, justifying its relevance even in markets which may not be intermediated by specialists. In relating movements in the risk premium to the real economy, we make use of the insight in Caballero and Farhi (2018), Caballero and Simsek (2018), and Ilut and Schneider (2014) that an increase in the risk premium will induce a recession if the safe interest rate does not sufficiently fall in response.[7] We build especially on the first two papers, as well as Brunnermeier and Sannikov (2012, 2016), in emphasizing the effects of heterogeneity in asset valuations on risk premia. Relative to these papers, we explore the importance of such heterogeneity for monetary transmission in a calibration to the U.S. economy.

Outline The remainder of the paper is structured as follows. In section 2 we characterize our main insights in a two-period environment, characterizing the mechanisms through which a monetary easing will endogenously redistribute to high MPR households in a wide variety of settings. This provides an organizing framework for our quantitative analysis in the infinite horizon in section 3. Calibrated to the U.S. economy, the redistribution toward high MPR households rationalizes the empirical evidence on the effect of a monetary policy easing on the equity premium and amplifies the stimulus to the real economy.

2 Analytical insights in a two-period environment

We first characterize our main conceptual insights in a two-period environment allowing us to obtain simple analytical results. An expansionary monetary policy shock lowers the risk

[5]More recently, in complementary work Bhandari, Evans, and Golosov (2019) construct a segmented markets model in the spirit of Alvarez, Lucas, and Weber (2001) and Alvarez et al. (2009) in which monetary policy also has effects on risk premia. Chen and Phelan (2019) integrate the effects of monetary policy on risk premia in Drechsler et al. (2018) with the macroeconomic framework of Brunnermeier and Sannikov (2014) to study the effects of monetary policy on financial stability. Coimbra and Rey (2019) study the effects of changes in interest rates on risk premia and financial stability in a model with heterogeneous intermediaries.

[6]In Bernanke et al. (1999), households can only trade bonds while entrepreneurs can trade bonds and capital. In equilibrium, households have a zero MPR while entrepreneurs have a positive MPR. Changes in the distribution of net worth across these agents thus affects credit spreads and economic activity.

[7]While these authors make this point in the case of a time-varying price of risk (as in our model), a similar result obtains in the case of New Keynesian models with a time-varying quantity of risk as in Basu and Bundick (2017) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015) and in a neoclassical setting with money as in DiTella (2018).

premium on capital if it redistributes to households with relatively high MPRs. Heterogeneity in risk aversion induces a joint distribution of MPRs and monetary policy exposures such that an expansionary shock lowers the risk premium. A similar result obtains with heterogeneity in portfolio constraints, rules-of-thumb, background risk, or beliefs. The transmission of monetary policy to the real economy is amplified given the decline in the risk premium provided that the monetary feedback rule limits any rise in the real interest rate.

2.1 Baseline environment and equilibrium

There are two periods, 0 and 1. While we later relax a number of the specific features of this environment to demonstrate the generality of our results, this baseline environment is the one we will extend to the infinite horizon and study quantitatively in section 3 of the paper.

Households There is a unit measure of households indexed by $i \in [0,1]$, each comprising a continuum of members $j \in [0,1]$ supplying a differentiated variety of labor. There is full consumption insurance within each household. Household i has Epstein-Zin preferences over consumption in each period $\{c_0^i, c_1^i\}$ and labor supply $\{\ell_0^i(j)\}_{j=0}^1$

$$v_0^i = \left((1-\beta^i) \left(c_0^i \Phi^i \left(\int_0^1 \ell_0^i(j) dj \right) \right)^{1-1/\psi^i} + \beta^i \left(\mathbb{E}_0 \left[(c_1^i)^{1-\gamma^i} \right] \right)^{\frac{1-1/\psi^i}{1-\gamma^i}} \right)^{\frac{1}{1-1/\psi^i}} \tag{1}$$

with discount factor β^i, intertemporal elasticity of substitution ψ^i, relative risk aversion γ^i, and (dis)utility of labor given by the function $\Phi^i(\cdot)$. We assume for simplicity that households exogenously supply one unit of non-differentiated labor in period 1, though of course this assumption will be relaxed in the infinite horizon environment we study in the next section.

In addition to consuming and supplying labor, the household chooses its position in a nominal bond B_0^i and in capital k_0^i subject to the resource constraints

$$\begin{aligned} P_0 c_0^i + B_0^i + Q_0 k_0^i \leq\ & (1-\tau) \int_0^1 W_0(j) \ell_0^i(j) dj - \int_0^1 AC_0^W(j) dj\ + \\ & (1+i_{-1}) B_{-1}^i + (\Pi_0 + (1-\delta_0) Q_0) k_{-1}^i + T_0^i, \qquad (2) \\ P_1 c_1^i \leq\ & W_1 + (1+i_0) B_0^i + \Pi_1 k_0^i. \qquad (3) \end{aligned}$$

B_{-1}^i and k_{-1}^i are its endowments in these same assets. In terms of the economy's nominal unit of account ("dollars"),[8] the consumption good trades at P_t dollars at t, labor services for member j earn an after-tax wage $(1-\tau)W_0(j)$ dollars in period 0 and W_1 dollars in

[8]Following Woodford (2003), we model the economy at the cashless limit.

period 1, one dollar in bonds purchased at t yields $1 + i_t$ dollars at $t + 1$, and one unit of capital purchased for Q_t dollars at t yields a dividend Π_{t+1} plus non-depreciated value of capital $(1-\delta_{t+1})Q_{t+1}$ at $t+1$. We assume that capital fully depreciates after its use in period 1 ($\delta_1 = 1$), when the economy ends. Following Rotemberg (1982), in period 0 the household pays a quadratic cost of setting its wage for member j

$$AC_0^W(j) = \frac{\chi^W}{2} W_0 \ell_0 \left(\frac{W_0(j)}{W_{-1}} - 1 \right)^2$$

given some reference wage W_{-1} and the aggregate wage bill $W_0\ell_0$ defined below. We assume this is a cost paid to the government, rebated back to households through the i-specific government transfers T_0^i.

Supply-side A union representing each labor variety j across households chooses $W_0(j), \ell_0(j)$ to maximize the social welfare of union members subject to the allocation rule

$$\ell_0^i(j) = \ell^i(\ell_0(j)) := \int_0^1 \ell^i(\ell_0(j))di = \ell_0(j) \tag{4}$$

and Pareto weights $\{\mu^i\}$.[9] A representative labor packer purchases varieties supplied by each union and combines them to produce a CES aggregate with elasticity of substitution ϵ and which it can sell at price W_0, earning profits

$$W_0 \left[\int_0^1 \ell_0(j)^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_0(j)\ell_0(j)dj. \tag{5}$$

The representative producer hires ℓ_0 units of the labor aggregate in period 0 and ℓ_1 units of labor directly from households in period 1, and combines these with k_{t-1} units of capital rented from households each period t to produce the final good with TFP z_t. In period 0 the producer also uses $\left(\frac{k_0}{k_{-1}}\right)^{\chi^x} x_0$ units of the consumption good to produce x_0 new capital goods, where χ^x indexes the degree of adjustment costs and here we assume the representative producer takes k_0 as given. Taken together, the producer earns profits

$$\Pi_0 k_{-1} = P_0 z_0 \ell_0^{1-\alpha} k_{-1}^{\alpha} - W_0\ell_0 + Q_0 x_0 - P_0 \left(\frac{k_0}{k_{-1}} \right)^{\chi^x} x_0, \tag{6}$$

$$\Pi_1 k_0 = P_1 z_1 \ell_1^{1-\alpha} k_0^{\alpha} - W_1\ell_1. \tag{7}$$

[9] We demonstrate the robustness of our analytical results to the case of individually-supplied labor by each household in appendix B. In our quantitative analysis, we find the computation of the equilibrium more robust with a union because it reduces the dimension of the fixed point to be solved. Hence, to provide analytical insights closer to the model studied quantitatively, we assume the same union structure here.

Future TFP is uncertain in period 0, following

$$\log z_1 \sim N\left(\log \bar{z} - \frac{1}{2}\sigma^2, \sigma^2\right). \tag{8}$$

Policy The government sets $\tau = -\frac{1}{\epsilon-1}$ to undo the effects of monopolistic competition in the labor market. The government sets lump-sum transfers T_0^i according to

$$T_0^i = \int_0^1 AC_0^W(j)dj + \tau \int_0^1 W_0(j)\ell_0^i(j)dj, \tag{9}$$

reflecting the rebating of wage adjustment costs and the payroll tax. Finally, the government sets monetary policy $\{i_0, P_1\}$ by committing to a fixed $P_1 = \bar{P}_1$, eliminating inflation risk in the nominal bond, and setting i_0 according to the Taylor (1993) rule

$$1 + i_0 = (1 + \bar{i})\left(\frac{P_0}{P_{-1}}\right)^{\phi} m_0 \tag{10}$$

with reference price P_{-1}, where m_0 is the shock of interest. Note that in this two-period setting, the equilibrium can be locally unique even when $\phi \leq 1$, including the useful benchmark case where $\phi = -1$ and hence the real interest rate between periods 0 and 1

$$1 + r_1 \equiv (1 + i_0)\frac{P_0}{P_1} = \frac{(1 + \bar{i})P_{-1}}{\bar{P}_1} m_0,$$

so a shock to the nominal rate translates one-for-one into a shock to the real rate.[10]

Market clearing Market clearing in goods each period is

$$\int_0^1 c_0^i di + \left(\frac{k_0}{k_{-1}}\right)^{\chi^x} x_0 = z_0 \ell_0^{1-\alpha} k_{-1}^{\alpha}, \tag{11}$$

$$\int_0^1 c_1^i di = z_1 \ell_1^{1-\alpha} k_0^{\alpha}, \tag{12}$$

in labor is

$$\left[\int_0^1 \ell_0(j)^{(\epsilon-1)/\epsilon} dj\right]^{\epsilon/(\epsilon-1)} = \ell_0, \tag{13}$$

$$\int_0^1 \ell_1^i di = \ell_1, \tag{14}$$

[10] As in the infinite horizon model, between periods t and $t+1$ we denote i_t the nominal interest rate known in period t and r_{t+1} the realized real interest rate depending on the price level in period $t+1$.

in the capital rental market is

$$\int_0^1 k_{t-1}^i di = k_{t-1}, \;\; t \in \{0, 1\}, \tag{15}$$

in the capital claims market is

$$(1 - \delta_0) \int_0^1 k_{-1}^i di + x_0 = \int_0^1 k_0^i di, \tag{16}$$

and in bonds is

$$\int_0^1 B_{t-1}^i di = 0. \tag{17}$$

Equilibrium Given the state variables $\{W_{-1}, P_{-1}, \{B_{-1}^i, k_{-1}^i\}, i_{-1}, z_0, m_0\}$ and stochastic process for z_1 in (8), the definition of equilibrium is then standard:

Definition 1. *An equilibrium is a set of prices and policies such that: (i) each household i chooses $\{c_0^i, B_0^i, k_0^i, c_1^i\}$ to maximize (1) subject to (2)-(3), (ii) each union j chooses $\{W_0(j), \ell_0(j)\}$ to maximize the social welfare of its members subject to the allocation rule (4), (iii) the labor packer chooses $\{\ell_0(j)\}$ to maximize profits (5), (iv) the representative producer chooses $\{\ell_0, x_0\}$ and ℓ_1 to maximize profits (6)-(7), (v) the government sets $\{T_0^i\}$ according to (9) and $\{i_0, P_1\}$ according to $P_1 = \bar{P}_1$ and (10), and (vi) the goods, labor, capital, and bond markets clear according to (11)-(17).*

Since labor varieties and unions j are symmetric, $\ell_0(j) = \ell_0$ and we drop j going forward.

We will analytically study this economy around the point with zero aggregate risk and $m_0 = \bar{m}_0$: $\{\sigma = 0, z_1 = \bar{z}_1, m_0 = \bar{m}_0\}$. For any variable n, we denote $\bar{n}$ to be its value at the point of approximation, and $\hat{n}$ its log/level deviation from this point (except for σ, which is a perturbation parameter but will not be denoted as $\hat{\sigma}$). For expositional simplicity we do not treat z_0 as a perturbation parameter of interest, but we do so in appendix B. Like monetary policy shocks, TFP shocks redistribute across households, generating state-dependence in the risk premium and affecting the transmission of TFP shocks to economic activity.

2.2 Limiting portfolios and MPRs

To understand the effects of monetary policy shocks through risk premia, it will prove useful to first understand the determinants of households' portfolios in equilibrium as well as their marginal portfolio choices given an additional unit of income.

To do so, it is helpful to re-write households' micro-level optimization problem as

$$\begin{aligned} \max \left((1-\beta^i) \left(c_0^i \Phi^i(\ell_0^i) \right)^{1-1/\psi^i} + \beta^i \left(\mathbb{E}_0 \left[(c_1^i)^{1-\gamma^i} \right] \right)^{\frac{1-1/\psi^i}{1-\gamma^i}} \right)^{\frac{1}{1-1/\psi^i}} \quad s.t. \\ c_0^i + b_0^i + q_0 k_0^i = y_0^i(w_0 \ell_0^i, P_0, \pi_0, q_0), \\ c_1^i = w_1 + (1+r_1) b_0^i + \pi_1 k_0^i, \end{aligned} \tag{18}$$

where we have denominated in lower-case the real analogs to the nominal variables introduced earlier, we have made use of the definition of the real interest rate

$$1 + r_1 \equiv (1+i_0)\frac{P_0}{P_1},$$

and we have collected households' income in period 0 — which they take as exogenous along with $\{\ell_0^i, q_0, w_1, r_1, \pi_1\}$ — as a function of non-predetermined variables

$$y_0^i(w_0 \ell_0^i, P_0, \pi_0, q_0) \equiv w_0 \ell_0^i + \frac{1}{P_0}(1+i_{-1}) B_{-1}^i + (\pi_0 + (1-\delta_0) q_0) k_{-1}^i. \tag{19}$$

Defining the real savings of household i chosen in period 0 as

$$a_0^i \equiv b_0^i + q_0 k_0^i,$$

its equilibrium portfolio share in capital is given by $\frac{q_0 k_0^i}{a_0^i}$ and its policy functions imply the marginal propensities to consume, save in bonds, save in capital, and save overall

$$\left\{ \frac{\partial c_0^i}{\partial y_0^i}, \frac{\partial b_0^i}{\partial y_0^i}, \frac{\partial k_0^i}{\partial y_0^i}, \frac{\partial a_0^i}{\partial y_0^i} \right\},$$

where these partial derivatives hold fixed all prices and other variables which the household takes as given in (18): $\ell_0^i = \ell^i(\ell_0)$, which enters its utility, as well as q_0, r_1, and the probability distributions over w_1 and π_1. We then define a useful summary of the household's portfolio choice on the margin:

Definition 2. *Household i's* marginal propensity to take risk (MPR) *is given by*

$$mpr_0^i \equiv \frac{q_0 \frac{\partial k_0^i}{\partial y_0^i}}{\frac{\partial a_0^i}{\partial y_0^i}}.$$

The MPR summarizes the degree to which the household allocates the marginal dollar to capital instead of the bond. As we clarify in an extension where monetary policy allows

P_1 to be stochastic in appendix B, it need not be that the nominal bond is riskless in real terms. Nonetheless, we give the MPR its name because under any realistic calibration the payoff on capital is more risky than on bonds.

We can better understand the structural determinants of households' portfolios and MPRs by taking their limits as aggregate risk falls to zero. In doing so, we apply techniques developed by Devereux and Sutherland (2011) in the context of open-economy macroeconomics to the present heterogeneous agent New Keynesian environment and our particular statistics of interest. These authors take the difference across countries of a second-order approximation to optimal portfolio choice for each country, and then make use of the method of undetermined coefficients, to characterize cross-country portfolio shares as aggregate risk falls to zero. Analogous steps can be used to characterize households' portfolio shares in the present environment. Moreover, a second-order approximation to the *partial derivatives* of the first-order conditions of (18) with respect to y_0^i can be used to characterize the *marginal* portfolio responses to income as aggregate risk falls to zero. These steps lead to the first result of the paper, the proof of which (along with all other proofs) is in appendix A:

Proposition 1. *At the limit of zero aggregate risk, household i's portfolio share in capital is*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i} \right) \frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$

and its MPR is

$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i},$$

where

$$\gamma = \left[\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i} di \right]^{-1} \tag{20}$$

is the harmonic average of risk aversion weighted by households' period 1 consumption.

Intuitively, a household's portfolio share in capital and MPR is higher the less risk averse it is relative to the appropriately-defined average household in the economy. Even though we are asking how the individual household allocates wealth both in equilibrium and in response to a marginal dollar of income, the risk aversion of all other households is relevant because this controls the prices facing the household in general equilibrium.

Proposition 1 further clarifies two useful points regarding the MPR. First, it captures a dimension of heterogeneity in principle orthogonal to the marginal propensities to consume and save which have been emphasized in prior work: while in the limit of zero aggregate risk the latter are fully determined by households' attitudes towards consumption across *dates* (discount factors and intertemporal elasticities of substitution), MPRs are governed

by attitudes towards consumption across *states* (relative risk aversion). Second, the MPR may be quite distinct from observed portfolios because it captures portfolio allocation on the margin. Indeed, a household's portfolio share in capital depends not only on risk aversion but its motive to hedge labor income also subject to TFP shocks, captured by $\frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i}$ in (1). While this hedging motive matters for equilibrium portfolios, it is irrelevant on the margin.

2.3 Monetary policy, redistribution, and risk premia

The joint distribution of household portfolios and MPRs determines the effect of a monetary policy shock on the expected excess returns on capital, to which we now turn.

Let $1 + r_1^k$ denote the gross real returns on capital

$$1 + r_1^k \equiv \frac{\Pi_1}{Q_0}\frac{P_0}{P_1} = \frac{\pi_1}{q_0}.$$

Combining household i's first-order conditions with respect to bonds and capital yields

$$\mathbb{E}_0\left[\left(c_1^i\right)^{-\gamma^i}\left(r_1^k - r_1\right)\right] = 0.$$

Approximating this condition up to third order in the perturbation parameters, and using market clearing in bonds and capital, we obtain:

Proposition 2. *Up to third order in the perturbation parameters $\{\sigma, \hat{z}_1, \hat{m}_0\}$, the risk premium on capital is*

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \zeta_{m_0}\hat{m}_0\sigma^2 + o(||\cdot||^4), \tag{21}$$

where γ was defined in (20) and

$$\zeta_{m_0} = \gamma\int_0^1 \bar{\xi}_{m_0}^i\left(\overline{mpr}_0 - \overline{mpr}_0^i\right)di, \tag{22}$$

where $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[c_1^i/\int_0^1 c_1^{i'}di'\right]}}{dm_0}$ is the effect of a monetary shock on household i's consumption share in period 1, and $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'}di'}\overline{mpr}_0^i di = 1$ is the weighted average MPR in (1).

ζ_{m_0} summarizes the effect of a monetary policy shock on the risk premium. As is evident, in this simple two-period model a monetary policy shock affects the risk premium only through redistribution.[11] If monetary policy does not redistribute ($\bar{\xi}_{m_0}^i = 0$ for all i) or

[11] This is no longer the case in the infinite horizon where, for instance, monetary policy affects the future path of real interest rates and thus the intertemporal hedging demand for capital.

households have identical MPRs ($\overline{mpr}_0^i = \overline{mpr}_0 = 1$ for all i), the shock has no effect on the risk premium. Away from this case, redistributing wealth to households with relatively high MPRs will lower the risk premium. Intuitively, such redistribution raises the relative demand for capital versus bonds, lowering the required excess returns to clear asset markets.

The relevant measure of redistribution toward household i is the (endogenous) change in its future consumption share

$$\begin{aligned}\bar{\xi}_{m_0}^i &\equiv \frac{\overline{d\left[c_1^i / \int_0^1 c_1^{i'} di'\right]}}{dm_0}, \\ &= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'} \left[\overline{\frac{dc_1^i}{dm_0}} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}} di'\right]. \end{aligned} \tag{23}$$

Using standard tools from price theory, in appendix B we demonstrate that the redistribution following a monetary shock reflects heterogeneity in substitution effects, in the marginal propensity to save, and in changes in wealth. The latter in turn reflects the revaluation of households' financial wealth, the change in the households' non-traded labor income, and income effects induced by the change in the real interest rate. These heterogeneous exposures to a monetary shock have been previously exposited in the HANK literature, as by Auclert (2018). Proposition 2 implies that it is their covariance with MPRs rather than MPCs which matters for transmission through risk-premia rather than risk-free rates.

In a useful benchmark case, we can isolate the redistribution operating through the revaluation of households' financial wealth. This drives our quantitative results later in the paper and allows us to sign the effect of a monetary shock on the risk premium:

Proposition 3. *Suppose that households differ in risk aversion $\{\gamma^i\}$ and that their initial endowments in bonds and capital are identical to their choices in period 0 at the point of approximation ($\{B_{-1}^i = \bar{P}_0 \bar{b}_0^i,\ k_{-1}^i = \bar{k}_0^i\}$). Suppose households are otherwise fully identical, including in their other preference parameters and initial wealth. Then*

$$\bar{\xi}_{m_0}^i \propto -\frac{(1+i_{-1})B_{-1}^i}{\bar{P}_0} \frac{1}{\bar{P}_0} \overline{\frac{dP_0}{dm_0}} + (k_{-1}^i - k_{-1}) \left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right).$$

With zero wage inflation at the point of approximation ($W_{-1} = \bar{W}_0$) and for χ^W sufficiently big, it follows that $\zeta_{m_0} > 0$ and hence a lower nominal interest rate lowers the risk premium.

The first part of the claim characterizes the components of balance sheet revaluation following a monetary shock, the only source of redistribution when households only differ in risk aversion and initial endowments. If a lower nominal interest rate generates unexpected inflation ($\overline{\frac{dP_0}{dm_0}} < 0$), it will redistribute toward net debtors in the nominal bond. If it raises

short-run profits ($\overline{\frac{d\pi_0}{dm_0}} < 0$) or raises asset prices capitalizing the future stream of profits ($\overline{\frac{dq_0}{dm_0}} < 0$), it will redistribute those with disproportionate claims on capital. By Proposition 1, more risk tolerant households will choose to hold levered positions in capital; by the assumption in the claim, households' endowments are consistent with these portfolios as well.[12] Hence, provided that inflation, profits, and the price of capital respond as conjectured here, a monetary expansion will redistribute to the relatively risk tolerant.

The second part of the claim demonstrates that with sufficiently high nominal rigidity in wages, a lower nominal interest rate will indeed redistribute to the risk tolerant in this way.[13] By Proposition 1, the relatively risk tolerant also have relatively high MPRs. Hence, by Proposition 2, the risk premium on capital falls.

2.4 Generalizations to other sources of heterogeneity

The preceding results do not rely on heterogeneity in risk aversion alone. We demonstrate in this section that they generalize to environments with heterogeneity in portfolio constraints, rules-of-thumb, background risk, or beliefs.

Binding constraints or rules-of-thumb Suppose a measure of households in the set C are not at an interior optimum in their portfolio choice because of the additional constraint

$$q_0 k_0^i = \omega_0^i a_0^i,$$

reflecting either a binding leverage constraint or a rule-of-thumb in their portfolio allocation. When $\omega_0^i = 0$ in particular, this means the household cannot participate in the capital market. Such constraints are consistent with prior asset pricing models with segmented markets or rules-of-thumb such as Guvenen (2009), Chien et al. (2012), and He and Krishnamurthy (2013) as well as macro models of the financial accelerator such as Bernanke et al. (1999).

Background risk Suppose households are subject to idiosyncratic risk beyond the aggregate risk already described: their labor productivity and quality of capital together are

[12] In the infinite horizon, this is indeed the case when approximating around the deterministic steady-state.

[13] Intuitively, consider the effects of an increase in the nominal price level $\bar{P}_0$ induced by the monetary policy shock on aggregate labor $\bar{\ell}_0$. First, by lowering the real wage, the increase in $\bar{P}_0$ will stimulate $\bar{\ell}_0$ through the standard Keynesian labor demand channel. Second, by raising the real interest rate, putting downward pressure on investment and thus (through the resource constraint) upward pressure on consumption, the increase in $\bar{P}_0$ will lower $\bar{\ell}_0$ through the wealth effect on labor supply. A higher degree of nominal rigidity lowers the importance of labor supply relative to labor demand in determining equilibrium in the labor market, and hence the assumption of sufficiently high χ^W allows us to unambiguously sign the effects of a monetary expansion. The additional assumption that $W_{-1} = \bar{W}_0$ is only made to simplify the proof.

subject to a shock ϵ_1^i, where both are modeled as changes in the efficiency units of each factor in the production function. ϵ_1^i is *iid* across households, independent of the aggregate TFP shock z_1, and follows

$$\log \epsilon_1^i \sim N\left(-\frac{1}{2}\eta^i\sigma^2, \eta^i\sigma^2\right).$$

This environment captures features of the large literatures in macroeconomics and asset pricing with uninsurable labor income risk and/or entrepreneurial income risk.

Subjective beliefs Suppose households differ in their beliefs regarding TFP. In particular, household i believes

$$\log z_1 \sim N\left(\log \bar{z}_1 - \frac{1}{2}\varsigma^i\sigma^2, \varsigma^i\sigma^2\right)$$

even though the objective (true) probability distribution remains described by (8). We may label households with $\varsigma^i > 1$ "pessimists" and households with $\varsigma^i < 1$ "optimists". This environment captures features of the literature on belief disagreements, as in the work of Geanakoplos (2009), Simsek (2013), and Giglio, Maggiori, Stroebel, and Utkus (2019).

Robustness of the effects of monetary policy In appendix B, I generalize Propositions 1 and 2 to these richer environments. Households' MPRs remain the relevant statistics to evaluate the effects of redistribution on risk premia. A lower nominal interest rate will lower the risk premium if it redistributes wealth to households with relatively high MPRs.[14]

In each of these cases, the additional dimension of heterogeneity continues to imply that households with high MPRs also have high portfolio shares in capital: just as both are falling in risk aversion γ^i, they are falling in background risk η^i and pessimism ς^i and are rising in the leverage constraint or rule-of-thumb ω^i (if the household is constrained). It follows that, even in an environment where households can vary along *all* of these dimensions, more levered households have higher MPRs. The first part of Proposition 3 summarizing households' exposure to a monetary policy shock remains unchanged. Hence, an expansionary monetary policy shock will redistribute to high MPR households as in the case with heterogeneity in risk aversion alone, lowering the risk premium:

Proposition 4. *Suppose that households differ in risk aversion $\{\gamma^i\}$, being constrained and (among those that are) constraints $\{\omega^i\}$, the degree of background risk $\{\eta^i\}$, and beliefs $\{\varsigma^i\}$. Futher suppose that their initial endowments in bonds and capital are identical to their choices*

[14] It remains the case that the entire distribution of MPRs remains relevant even if some households face binding constraints or a rule-of-thumb. For instance, if wealth transfers to households who cannot trade capital and thus have $\overline{mpr}_0^i = 0$, in equilibrium the remaining households must be induced to hold a more levered position in capital and thus the risk premium must rise.

in period 0 at the point of approximation ($\{B^i_{-1} = \bar{P}_0\bar{b}^i_0,\ k^i_{-1} = \bar{k}^i_0\}$), and that households are otherwise identical. Then with zero wage inflation at the point of approximation ($W_{-1} = \bar{W}_0$) and for χ^W sufficiently big, $\zeta_{m_0} > 0$ and hence a lower nominal interest rate lowers the risk premium.

In this sense, while our quantitative analysis focuses on differences in risk aversion, we expect that our insights are robust to these other potential sources of heterogeneity generating the same distribution of MPRs and exposures to a monetary policy shock.

2.5 Monetary policy, redistribution, and the real economy

Before turning to our quantitative analysis in the infinite horizon, we conclude our analysis of this environment by asking how the risk premium effects of monetary policy matter for the transmission of policy to the real economy.

We focus on new capital k_0 and consumption $c_0 \equiv \int_0^1 c_0^i di$, which can be combined to understand the effect on output.[15] In terms of perturbation parameters, we have

$$\hat{k}_0 = \delta^{k_0}_{m_0}\hat{m}_0 + \frac{1}{2}\delta^{k_0}_{m_0^2}\hat{m}_0^2 + \frac{1}{6}\delta^{k_0}_{m_0^3}\hat{m}_0^3 + \frac{1}{2}\delta^{k_0}_{\sigma^2}\sigma^2 + \frac{1}{2}\delta^{k_0}_{m_0\sigma^2}\hat{m}_0\sigma^2 + o(||\cdot||^4), \tag{24}$$

$$\hat{c}_0 = \underbrace{\delta^{c_0}_{m_0}\hat{m}_0 + \frac{1}{2}\delta^{c_0}_{m_0^2}\hat{m}_0^2 + \frac{1}{6}\delta^{c_0}_{m_0^3}\hat{m}_0^3}_{\substack{\text{effects absent}\\ \text{aggregate risk}}} + \frac{1}{2}\delta^{c_0}_{\sigma^2}\sigma^2 + \underbrace{\frac{1}{2}\delta^{c_0}_{m_0\sigma^2}\hat{m}_0\sigma^2}_{\substack{\text{effect with}\\ \text{aggregate risk}}} + o(||\cdot||^4), \tag{25}$$

for some coefficients $\delta^{\cdot}_{\cdot}$. For each macroeconomic aggregate, the first three terms reflect the effects of a monetary policy shock even absent aggregate risk, and they are well understood. They reflect the standard channels through intertemporal substitution as well as heterogeneity in the marginal propensities to consume versus save. We instead focus on the additional effects of a monetary policy shock with aggregate risk, summarized in the terms $\delta^{k_0}_{m_0\sigma^2}$ and $\delta^{c_0}_{m_0\sigma^2}$. Equivalently, we seek to understand the quantity responses which accompany a change in the risk premium summarized by ζ_{m_0} in the previous results.

We begin with capital. Since the expected return on capital is

$$\mathbb{E}_0\left[1 + r^k_1\right] = \frac{\mathbb{E}_0\left[\pi_1\right]}{q_0} = \frac{\alpha\bar{z}_1 k_0^{\alpha-1}}{(k_0/k_{-1})^{\chi^x}},$$

the return on capital falls in the amount of new capital both because the marginal product of capital falls and the price of capital rises. Now consider the case where monetary policy redistributes to high MPR households and thus lowers the risk premium. Then to evaluate

[15] We work with new capital k_0 rather than investment x_0 only for expositional simplicity: $1+r^k_1$ is exactly log-linear in k_0 but not x_0, recalling that $k_0 = (1-\delta_0)k_{-1} + x_0$.

the effects on investment we must determine whether the required return on capital falls (and thus new capital rises) or the safe real interest rate simply rises (and thus new capital may remain unchanged or even fall).

Absent nominal rigidity, the equilibrium response of the real interest rate depends crucially on households' intertemporal elasticity of substitution. This reflects the presence of offsetting income and substitution effects in response to the shock. As described in Gourio (2012) and other papers in the literature on time-varying risk premia, in the case of a unitary intertemporal elasticity of substitution the real interest rate varies by exactly the amount to keep investment unchanged. This can explain what Cochrane (2017) calls the "macro-finance separation" in joint analyses of asset pricing and business cycles such as Tallarini Jr. (2000).

With nominal rigidity, Caballero and Farhi (2018) and Caballero and Simsek (2018) point out that the monetary policy rule also determines the extent to which the real interest rate fluctuates. These authors demonstrate, for instance, that a decrease in the risk premium will induce a boom if the nominal rate remains at the zero lower bound. In our environment, these insights apply to the risk premium effect of a primitive monetary policy shock itself. In the useful benchmark in which monetary policy targets a constant real interest rate subject to monetary policy shocks ($\phi = -1$), we obtain:

Proposition 5. *If monetary policy follows the rule (10) with $\phi = -1$, then in (24)*

$$\frac{1}{2}\delta^{k_0}_{m_0\sigma^2} = -\frac{1}{1-\alpha+\chi^x}\zeta_{m_0}.$$

Hence, if monetary policy lowers the risk premium by redistributing to high MPR households, as under the conditions in Proposition 4, it will amplify the stimulus to investment.

To understand the implications for consumption and thus output, it is most transparent to focus on the case with a unitary intertemporal elasticity of substitution $\psi^i = 1$ and identical discount factor $\beta^i = \beta$ for each household i. The former delivers a simple form for each household's optimal consumption policy solving (18):[16]

$$c_0^i = \underbrace{(1-\beta^i)}_{\text{MPC}}\left[\underbrace{y_0^i(w_0\ell_0^i, P_0, \pi_0, q_0)}_{\text{current income}} + \underbrace{\frac{1-\alpha}{\alpha}q_0k_0}_{\substack{\text{value of}\\ \text{future}\\ \text{labor income}}}\right],$$

where the first term in brackets was defined in (19) and the second term in brackets summarizes the value of future labor income because the returns to capital and labor are perfectly

[16] As demonstrated in the proof of the proposition which follows, the same consumption policy function is also obtained in the more general environment featuring portfolio constraints / rules-of-thumb, background risk, and subjective beliefs regarding aggregate TFP.

correlated. The additional assumption $\beta^i = \beta$ eliminates heterogeneity in marginal propensities to consume so that, together with goods market clearing (11), we can obtain a simple relationship between aggregate consumption and the value of the new capital stock

$$c_0 = \frac{1-\beta}{\beta}\frac{1}{\alpha}q_0 k_0 = \frac{1-\beta}{\beta}\frac{1}{\alpha}\left(\frac{k_0}{k_{-1}}\right)^{\chi^x} k_0.$$

We immediately obtain the following result:

Proposition 6. *If $\psi^i = 1$ and $\beta^i = \beta$ for all i, then in (25)*

$$\frac{1}{2}\delta^{c_0}_{m_0\sigma^2} = \frac{1}{2}\left(\chi^x + 1\right)\delta^{k_0}_{m_0\sigma^2}.$$

If monetary policy follows the rule (10) with $\phi = -1$, together with Proposition 5 this implies

$$\frac{1}{2}\delta^{c_0}_{m_0\sigma^2} = -\frac{\chi^x + 1}{1 - \alpha + \chi^x}\zeta_{m_0}.$$

Hence, if monetary policy lowers the risk premium by redistributing to high MPR households, it will also amplify the stimulus to consumption. We note from the derivation that this occurs for two reasons: first, higher investment raises aggregate demand and thus labor income in period 0 and raises the present value of future labor income in period 1; second, the lower risk premium is associated with a higher price of capital and thus higher household wealth in period 0. Importantly, the latter means that even as $\chi^x \to \infty$, while the response of investment in Proposition 5 falls to zero, the response of consumption does not.[17]

3 Quantitative relevance in the infinite horizon

We now evaluate the quantitative relevance of these insights in an extended infinite horizon setting. We first revisit the empirical evidence on the equity premium response to monetary policy shocks which poses a strong challenge to existing New Keynesian frameworks in which the equity premium barely moves. We then calibrate our model to match standard "macro" moments as well as novel "micro" moments from the Survey of Consumer Finances which discipline the cross-sectional heterogeneity in MPRs and exposures to monetary policy. In

[17]Recent empirical work supports the connection between the risk premium and real quantities underlying these results. Pflueger, Siriwardane, and Sunderam (2019) document that a decrease in perceived risk measured using the cross-section of stocks forecasts expansions in investment and output. While not directly isolating the change due to risk premia, a growing literature finds that an increase in asset prices raises consumption and output through wealth effects, as in the work of Mian, Rao, and Sufi (2013) for house prices and Chodorow-Reich, Nenov, and Simsek (2019) for the stock market.

response to an unexpected monetary easing in our model economy, wealth endogenously redistributes to relatively high MPR households, rationalizing the equity premium response found in the data and contributing to the stimulus in real activity.

3.1 Empirical effects of monetary policy shocks in U.S. data

The effects of an unexpected shock to monetary policy have been the subject of a large literature in empirical macroeconomics. In response to an unexpected loosening, this literature finds that the price level rises and production expands, consistent with workhorse New Keynesian models. But, as found in Bernanke and Kuttner (2005) and a number of subsequent papers using asset pricing data, the evidence further suggests that risk premia fall, posing a challenge to workhorse models where risk premia barely move.[18]

We refresh the findings in Bernanke and Kuttner (2005) using the structural vector autoregression instrumental variables (SVAR-IV) approach in Gertler and Karadi (2015). Using monthly data from July 1979 through June 2012, we first run a six-variable, six-lag VAR using the 1-year Treasury yield, CPI, industrial production, S&P 500 return relative to the 1-month T-bill, 1-month T-bill relative to the change in CPI, and smoothed dividend-price ratio on the S&P 500.[19,20] Over January 1991 through June 2012 we then instrument the residuals in the 1-year Treasury yield (the *monetary policy indicator*) with an external instrument: policy surprises constructed using the current Fed Funds futures contract on FOMC days aggregated to the month level from Gertler and Karadi (2015). The identification assumptions are that the exogenous variation in the monetary policy indicator in the VAR are due to the structural monetary shock and that the instrument is correlated with this structural shock but not the five others in the VAR. Under these assumptions, a first-stage regression of the monetary policy residual on the surprise, followed by a second-stage regression of all other residuals on the predicted residual, can be used to identify the effects of a monetary policy shock on all variables in the VAR. We summarize the results of the first stage in appendix C.1 in which we find that this instrument is indeed strong according to the threshold recommended by Stock, Wright, and Yogo (2002).

[18]This effect on risk premia may co-exist with the revelation of information due to the shock, a channel studied by Nakamura and Steinsson (2018) and others. The analysis of Jarocinski and Karadi (forthcoming) implies that by confounding "pure" monetary policy shocks with such information shocks in our analysis, our estimates may understate the increase in the stock market following a pure monetary easing.

[19]The series for the 1-year Treasury yield, CPI, and industrial production are taken from the dataset provided by Gertler and Karadi (2015). The remaining series are from CRSP.

[20]The smoothed dividend-price ratio is computed as the 3-month moving average of dividends paid in a month divided by the price of the stock at the end of the month, value-weighted over stocks in the S&P 500. We further linearly detrend this series given its secular decline over the sample period, consistent with the changes in corporate payout policy discussed in Bunn and Shiller (2014) and Jivraj and Shiller (2017).

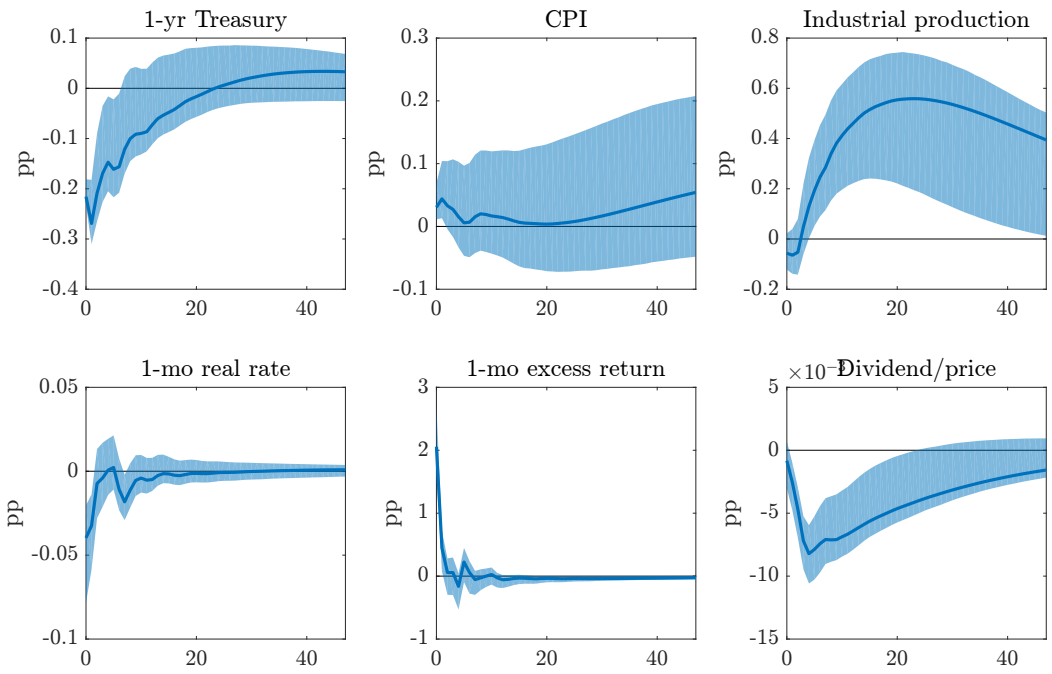


Figure 1: effects of 1 SD monetary shock

Notes: 90% confidence interval at each horizon is computed using the wild bootstrap (to account for uncertainty in the coefficients of the VAR) with 10,000 iterations, following Mertens and Ravn (2013) and Gertler and Karadi (2015).

We then plot the impulse responses to a negative monetary policy shock using this instrument in Figure 1. Since the structural monetary policy shock is not observed, its magnitude should be interpreted through the lens of the $22bp$ decrease in the 1-year yield on impact. Consistent with the wider literature, industrial production and the price level rise, and the real interest rate falls. Excess returns rise by $2pp$ on impact; given the comparatively tiny decline in the real interest rate, this means the real return on the stock market is also approximately $2pp$. Notably, excess returns are small and negative in the months which follow, consistent with a decline in the equity premium and the fall in the dividend/price ratio.

Following Bernanke and Kuttner (2005), we can more formally decompose the $2pp$ real return on the stock market into news about higher dividend growth, lower real risk-free discount rates, and lower future excess returns using a Campbell-Shiller decomposition:

$$\begin{aligned}(\text{real stock return})_t - \mathbb{E}_{t-1}[(\text{real stock return})_t] &= (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=0} \kappa^j \Delta(\text{dividends})_{t+j} \\ - (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1} \kappa^j (\text{real rate})_{t+j} &- (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1} \kappa^j (\text{excess return})_{t+j}, \end{aligned} \tag{26}$$

	pp	As share of effect on real stock return
Real stock return	2.02	
	[1.62,2.66]	
Dividend growth news	0.69	34%
	[-0.21, 1.54]	[-11%,71%]
− Future real rate news	0.15	7%
	[-0.14,0.41]	[-6%,21%]
− Future excess return news	1.18	58%
	[0.41,2.34]	[21%,106%]

Table 1: Campbell-Shiller decomposition following 1 SD monetary shock

Notes: decomposition in (26) uses $\kappa = 0.9962$ following Campbell and Ammer (1993). 90% confidence interval in brackets is computed using the wild bootstrap (to account for uncertainty in the coefficients of the VAR) with 10,000 iterations, following Mertens and Ravn (2013) and Gertler and Karadi (2015).

where $\kappa = \frac{1}{1+\frac{d}{p}}$ and $\frac{d}{p}$ is the steady-state dividend yield. Using the SVAR-IV to compute the revised expectations in real rates and excess returns given the monetary shock, we obtain the decomposition in Table 1.[21] 1.1*pp* (58%) of the initial return on the stock market is due to news about lower future excess returns, 0.1*pp* (7%) is due to news about lower future risk-free rates, and 0.7*pp* (34%) is due to news about higher dividend growth. Accounting for estimation uncertainty, we conclude that at least 21% and potentially all of the return on the stock market is due to news about lower future excess returns, validating the original message from Bernanke and Kuttner (2005) using the SVAR-IV approach.

The important role of the risk premium in explaining the return on the stock market is robust to details of the estimation approach. In appendix C.1 we modify the estimation approach along a number of dimensions. First, we change the number of lags used in the VAR, ranging from 4 months to 8 months. Second, we change the sample periods over which the VAR and/or first-stage is estimated. Third, we add variables to the VAR, such as the Gilchrist and Zakrajsek (2012) excess bond premium and other credit spreads used in Gertler and Karadi (2015) as well as the term spread and other variables known to predict excess stock market returns used in Bernanke and Kuttner (2005). Fourth, we change the instrument used for the monetary policy shock, using policy surprises constructed using the three-month ahead Fed Funds futures contract rather than the current contract. Across these cases we confirm the message of the baseline estimates above: in response to a monetary

[21] Following Bernanke and Kuttner (2005), our VAR enables us to compute $(\text{excess return})_t - \mathbb{E}_{t-1}[(\text{excess return})_t]$, $(\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=0} \rho^j (\text{real rate})_{t+j}$, and $(\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1} \rho^j (\text{excess return})_{t+j}$, and we assign to dividend growth the residual implied by (26). The alternative of directly including dividend growth on the S&P 500 in the VAR is made complicated by their strong seasonality in the data.

policy shock which reduces the 1-year Treasury yield by 17-23bp, real stock returns rise by 1.6-3.2pp, and news about future excess returns explains 35%-80% of this increase.

The dimensionality reduction offered by a VAR enables us to generate the long-horizon forecasts needed for the Campbell-Shiller decomposition, unlike a local projection. As noted by Stock and Watson (2018), we can test the assumption of invertibility implicit in the SVAR-IV both by assessing whether lagged values of the instrument have forecasting power when included in the VAR and by comparing the estimated impulse responses to those obtained using a local projection with instrumental variables (LP-IV). We show in appendix C.1 that both of these tests fail to reject the null hypothesis that invertibility in our application is satisfied. We further provide a visual comparison of the impulse responses over 4 years using the SVAR-IV and LP-IV. Our estimates using the SVAR-IV lie within the (quite large) confidence intervals obtained using the LP-IV at virtually every horizon.[22]

3.2 Infinite horizon environment

We now investigate whether redistribution can quantitatively rationalize the stock market response to a monetary policy shock and, if so, its implications for policy transmission to the real economy. We first extend the model to the infinite horizon, building on the environment from section 2.1. We describe the necessary changes when moving to the infinite horizon here and more fully describe the environment and our solution approach in appendix D.

3.2.1 Household preferences and constraints

Household i now maximizes a generalization of (1)

$$v_t^i = \left((1-\beta) \left(c_t^i \Phi \left(\int_0^1 \ell_t^i(j) dj \right) \right)^{1-1/\psi} + \beta \mathbb{E}_t \left[\left(v_{t+1}^i \right)^{1-\gamma^i} \right]^{\frac{1-1/\psi}{1-\gamma^i}} \right)^{\frac{1}{1-1/\psi}}, \tag{27}$$

with endogenous labor supply each period. We assume for simplicity that β, ψ, and $\Phi(\cdot)$ are now identical across types. We further assume for simplicity that the labor allocation rule and Pareto weights are symmetric: $\ell^i(\ell_t) = \ell_t$ and $\mu^i = 1$. We parameterize the disutility of

[22] Between the spectrum of the SVAR-IV and LP-IV is the approach of including the IV (and its lags) in the VAR and ordering it first as part of a recursive identification strategy. Plagborg-Moller and Wolf (2019) prove that this strategy is robust to non-invertibility, while estimation using a VAR still means that we can implement (26). While the estimated impulse responses using this approach are noisier than our baseline using the SVAR-IV, the point estimates imply that 69% of the increase in the stock market following a monetary shock is due to news about lower future excess returns. The recursive approach is closely related to the identification strategy used by Paul (2019) in recent work also finding that expansionary monetary policy raises the stock market in part by lowering future excess returns.

labor as in Shimer (2010),

$$\Phi(\ell_t^i) = \left(1 + (1/\psi - 1)\,\bar{\theta}\frac{(\ell_t^i)^{1+1/\theta}}{1+1/\theta}\right)^{\frac{1/\psi}{1-1/\psi}}, \tag{28}$$

where θ controls the Frisch elasticity of labor supply and $\bar{\theta}$ controls the disutility of labor. Finally, we assume households also face a lower bound on their position in capital

$$k_t^i \geq \underline{k} z_t \tag{29}$$

where z_t is productivity and is discussed further below. Such a constraint captures in a simple way the components of the economy's capital stock which households hold for reasons beyond their financial returns, most notably their primary residence and vehicles.

3.2.2 Aggregate productivity

In the infinite horizon we need to specify the dynamics of aggregate productivity. We assume that productivity z_t follows a unit root process

$$\log(z_t) = \log(z_{t-1}) + \sigma\epsilon_t^z + \varphi_t, \tag{30}$$

where ϵ_t^z is an *iid* shock from a standard Normal distribution, φ_t is a rare disaster equal to zero with probability $1-p_t$ and $\underline{\varphi} < 0$ with probability p_t, and p_t follows a two-state Markov process over $p - \frac{1}{2}\sigma^p$ and $p + \frac{1}{2}\sigma^p$ with probability of remaining in each state ρ^p. Following Barro (2006), we introduce the disaster to help match the level of the equity premium. Following Gourio (2012) and Wachter (2013), we introduce a time-varying probability of a disaster to help match the volatility of returns. We further assume that the disaster destroys capital and reduces the reference wage in households' wage adjustment costs in proportion to the decline in productivity. The first assumption implies that aggregate output is

$$y_t \equiv (z_t \ell_t)^{1-\alpha} \left(k_{t-1}\exp(\varphi_t)\right)^{\alpha}, \tag{31}$$

where productivity is now labor-augmenting and thus consistent with balanced growth.

3.2.3 Monetary and fiscal policy

Finally, monetary policy is now characterized by the Taylor Rule (10) each period

$$1 + i_t = (1 + \bar{i})\left(\frac{P_t}{P_{t-1}}\right)^{\phi} m_t, \tag{32}$$

where policy shocks follow an AR(1) process

$$\log m_t = \rho^m \log m_{t-1} + \sigma^m \epsilon_t^m \tag{33}$$

where ϵ_t^m is an *iid* shock from a standard Normal distribution.

Fiscal policy is characterized by a wage subsidy $\tau = -\frac{1}{\epsilon-1}$, government participation in the bond market B^g, and household-specific lump-sum transfers

$$T_t^i = \int_0^1 AC_t^W(j)dj + \tau \int_0^1 W_t(j)\ell_t^i(j)dj + i_{t-1}B^g - \mathfrak{T}_t^i. \tag{34}$$

Relative to (9) in the two-period environment, this specification of fiscal policy adds government debt $-B^g$ and a new source of taxes $\mathfrak{T}_t^i$.

Accounting for government debt $-B^g$ is important because empirically, the aggregate household sector has a positive net position in nominal claims, offset by the government and the rest of the world. We ignore the rest of the world for simplicity, so that the sum of nominal claims held by households and the government equals zero. Because we assume that marginal changes in government debt would be financed by lump-sum taxes and households face no constraints in the bond market, such changes would have no effect on the equilibrium allocation (in the sense that in equilibrium, households effectively choose their position in government debt accounting for their implicit position through the government, $B_t^i + B^g$). Hence, without loss of generality, we assume time-invariant government debt.

The taxes $\mathfrak{T}_t^i$ help us match the empirical distribution of wealth. They follow

$$\mathfrak{T}_t^i = \tau_t^i \left[(1 + i_{t-1})(B_{t-1}^i + B^g) + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i \exp(\varphi_t)\right] \tag{35}$$

where $\tau_t^i = \tau^i$ for all households except a positive measure, for whom $\tau_t^i = \tau_t$ ensures that $\int_0^1 \mathfrak{T}_t^{i'} di' = 0$.[23] For computational simplicity, we assume that households fully anticipate these taxes for all households excepts themselves, which they assume to be zero.

3.2.4 Equilibrium and model solution

The optimization problems of households and unions are naturally extended to incorporate the dynamics of the infinite horizon, and the optimization problem of producers are

[23] Since $\int_0^1 \mathfrak{T}_t^{i'} di' = 0$, it follows from (34) that

$$B^g + \int_0^1 T_t^{i'} di' = \int_0^1 AC_t^W(j)dj + \int_0^1 \tau \int_0^1 W_t(j)\ell_t^i(j)djdi' + (1 + i_{t-1})B^g,$$

so that the government's budget is indeed balanced.

unchanged. The definition of equilibrium then generalizes Definition 1.

We solve the model globally using numerical methods. While the perturbation approach used in the two-period environment remains feasible in the infinite horizon, we turn to this solution approach for two reasons. First, we find that household portfolios and MPRs solved analytically at the deterministic steady-state have non-trivial differences from their average values in simulations using a global solution.[24] Second, a perturbation solution is ill-suited to handle the addition of a disaster to our exogenous driving forces.

Given this global solution approach, we now limit the degree of heterogeneity across households to make the computational burden tractable. We divide the continuum of households into a finite number of groups within which households are perfectly symmetric. We choose three groups in our baseline parameterization, denoted $i \in \{a, b, c\}$ and where we now understand the index i to refer to groups and the representative household of each group. The fraction of households belonging to group i is denoted λ^i, where $\sum_i \lambda^i = 1$.

We solve a stationary transformation of the economy obtained by dividing all real variables except labor by z_t and nominal variables by $P_t z_t$. As is shown in appendix D, in the transformed economy we obtain a recursive representation of the equilibrium in which the aggregate state in period t is given by the monetary policy shock m_t, scaled aggregate capital $k_{t-1}/\exp(\sigma\epsilon_t^z)$, scaled prior period's real wage $w_{t-1}/\exp(\sigma\epsilon_t^z)$, and wealth shares $\{s_t^i\}$ of two groups as defined by

$$\begin{aligned} s_t^i &\equiv \lambda^i \frac{(1+i_{t-1})(B_{t-1}^i + B^g) + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i \exp(\varphi_t) - \mathfrak{T}_t^i}{(\Pi_t + (1-\delta)Q_t)k_{t-1}\exp(\varphi_t)}, \\ &= \lambda^i(1-\tau_t^i)\frac{(1+i_{t-1})(B_{t-1}^i + B^g) + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i \exp(\varphi_t)}{(\Pi_t + (1-\delta)Q_t)k_{t-1}\exp(\varphi_t)} \end{aligned} \tag{36}$$

where the second line uses (35). Productivity shocks inclusive of disasters only govern the transition across states, but do not separately enter the state space itself.

[24] We wish to emphasize, however, that the qualitative insights of section 2 continue to be relevant in the infinite horizon. For instance, setting $p = 0$ so that there is no disaster, we can prove the following analog of Proposition 2 in this infinite horizon environment around the deterministic steady-state (denominated without time subscripts):

$$\mathbb{E}_0 \hat{r}_1^k - \mathbb{E}_0 \hat{r}_1 = \Gamma\sigma^2 + \zeta_{m_0}\hat{m}_0\sigma^2 + o(||\cdot||^4),$$

where

$$\zeta_{m_0} = \tilde{\zeta}_{m_0} + \gamma \int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr} - \overline{mpr}^i\right) di$$

and $\tilde{\zeta}_{m_0}$ is the effect of a monetary shock on the risk premium in a counterfactual economy with $\gamma^i = \gamma$ for all i, given $\gamma = \left(\int_0^1 \frac{\bar{c}^i}{\int_0^1 \bar{c}^{i'} di'} \frac{1}{\gamma^i} di\right)^{-1}$, $\bar{\xi}_{m_0}^i = \frac{\overline{d\left[c_1^i / \int_0^1 c_1^{i'} di'\right]}}{dm_0}$, $\overline{mpr} = \int_0^1 \frac{\bar{c}^i}{\int_0^1 \bar{c}^{i'} di'} \overline{mpr}^i di$, and $\overline{mpr}^i \equiv \frac{\overline{q_0 \frac{\partial k_0^i}{dy_0^i}}}{\overline{\frac{\partial a_0^i}{dy_0^i}}}$. Hence, beyond the risk premium effect of monetary policy in an otherwise identical economy with homogenous risk aversion, it will lower the risk premium if it redistributes to households with relatively high MPRs.

We solve the model over a large grid of the aggregate states, making sure that the solution is robust to larger grid sizes and boundaries. When forming expectations over prices and policies, we use quadrature and linear interpolation over aggregate states, but (for households' value functions) interpolate using cubic splines over individual wealth. The stochastic equilibrium is determined through backward iteration, while dampening the updating of the price of capital and individuals' expectations over the dynamics of the aggregate states. The code is written in Fortran and parallelized using OpenMP, so that convergence can be achieved in less than twenty minutes on a modern computing system with eight cores. The computational algorithm is further described in appendix D.

3.3 Parameterization, first moments, and second moments

We now parameterize the model to match micro moments informing the heterogeneity across groups as well as macro moments regarding the business cycle and asset prices.

3.3.1 Micro moments: the distribution of wealth, labor income, and portfolios

We seek to match the distribution of wealth, labor income, and financial portfolios in U.S. data, giving us confidence in the model-implied MPRs and exposures to a monetary shock. We work with the 2016 Survey of Consumer Finances (SCF) and proceed in three steps.

First, we decompose each household's wealth (A^i) into claims on the economy's capital stock (Qk^i, in positive net supply) and nominal claims (B^i, in zero net supply accounting for the government and rest of the world).[25] We describe this procedure in detail in appendix C.2 and provide a broad overview here. We first add estimates of DB pension wealth for each household since this is the major component of household net worth which is excluded from the SCF.[26] We then proceed by line item to allocate how much household wealth is held in nominal claims versus claims on capital.[27] In the same spirit as Doepke and Schneider (2006), the key step in doing this is to account for the implicit leverage households have on capital through the leverage of firms and of financial intermediaries. In particular, if household i owns \$1 in equity in a firm which has net leverage

$$\frac{\text{assets net of bonds}}{\text{equity}} = lev^{firm},$$

[25]Consistent with the traded assets in our model, we do not distinguish between nominal claims having different duration. Extending the model to feature a richer set of traded assets would be valuable.

[26]We use the estimates of Sabelhaus and Volz (2019) described further in the appendix. We thank John Sabelhaus for generously sharing their estimates with us.

[27]An alternative approach to measuring households' portfolio composition is to relate their changes in wealth to changes in asset prices using panel data, as in the recent work of Gomes (2019).

		$\frac{A^i}{W\ell^i}$	
		$\geq p60$	$< p60$
$\frac{Qk^i}{A^i}$	$\geq p90$	**Group** a Share households: 4.0% $\sum_{i\in a} W\ell^i / \sum_i W\ell^i$: 3.7% $\sum_{i\in a} A^i / \sum_i A^i$: 22.1% $\sum_{i\in a} Qk^i / \sum_{i\in a} A^i$: 1.5	**Group** c Share households: 60.2% $\sum_{i\in c} W\ell^i / \sum_i W\ell^i$: 83.0% $\sum_{i\in c} A^i / \sum_i A^i$: 23.4% $\sum_{i\in c} Qk^i / \sum_{i\in c} A^i$: 1.1
	$< p90$	**Group** b Share households: 35.8% $\sum_{i\in b} W\ell^i / \sum_i W\ell^i$: 13.3% $\sum_{i\in b} A^i / \sum_i A^i$: 54.5% $\sum_{i\in b} Qk^i / \sum_{i\in b} A^i$: 0.7	

Table 2: heterogeneity in wealth to labor income and the capital portfolio share

Notes: observations are weighted by SCF sample weights.

then we assign the household

$$\{Qk^i = lev^{firm}, B^i = 1 - lev^{firm}\}.$$

If household i owns \$1 in equity in an intermediary which has net leverage

$$\frac{\text{assets net of bonds}}{\text{equity}} = lev^{inter}$$

and the intermediary's assets net of nominal claims are equity claims on the above firm, then we assign the household

$$\{Qk^i = lev^{inter} lev^{firm}, B^i = 1 - lev^{inter} lev^{firm}\}.$$

We use the Financial Accounts of the United States as well as analyses of hedge funds and private equity to inform these leverage assumptions. We outline the specific assumptions and present the resulting aggregate decomposition of household net worth in appendix C.2. We further demonstrate there that our assumptions on leverage are consistent with market clearing in nominal claims after accounting for the government and rest of the world.

Second, we stratify households by their wealth to labor income $\{\frac{A^i}{W\ell^i}\}$ and capital portfolio share $\{\frac{Qk^i}{A^i}\}$, defining subsamples mapping to our three groups.[28] We sort households on

[28]For each household we measure labor income as total wage and salary income for the previous calendar

these variables based on Proposition 1, which demonstrated that the capital portfolio share is informative about households' risk aversion and thus MPR only after properly accounting for their non-traded exposure to aggregate risk through labor income.[29] Group a corresponds to households with high wealth to labor income and a high capital portfolio share, group b corresponds to households with high wealth to labor income but a low capital portfolio share, and group c corresponds to households with low wealth to labor income. We define "high" wealth to labor income as households above the 60th percentile of households ordered by this measure, and a "high" capital portfolio share as households above the 90th percentile of households ordered by this measure.[30]

Third, we summarize the labor income, wealth, and financial portfolios of these three groups which we seek to match, provided in Table 2. Group a households constitute 4.0% of households, earn 3.7% of labor income, hold 22.1% of wealth, and have an aggregate capital portfolio share of 1.5. Group b households constitute 35.8% of households, earn only 13.3% of labor income, hold 54.5% of wealth, and have an aggregate capital portfolio share of 0.7. Finally, group c households constitute 60.2% of households, earn 83.0% of labor income, hold only 23.4% of wealth, and have an aggregate capital portfolio share of 1.1. To better understand the nature of households in each group, in Table 3 we first project an indicator for the household holding private business assets on households' group indicator. We find that households in group a are especially more likely to hold private business assets. We then project an indicator for the household head being older than 54 and out of the labor force, together capturing a retired household head, on households' group indicator. We find that households in group b are especially more likely to be retired.

3.3.2 Macro moments: business cycle dynamics and asset prices

We also calibrate the model to match standard macro moments regarding the business cycle and asset prices. In terms of the business cycle, we seek to match the volatilities of the growth rates of consumption, investment and hours worked. We use NIPA data on consumption of non-durables and services, investment in durables and capital, and hours worked, together

year as reported in the SCF summary extract.

[29] We sort households by a measure of their capital portfolio share after excluding from both the numerator and denominator assets and liabilities associated with the primary residence and vehicles, even though for each group we report and target the capital portfolio share accounting for all assets and liabilities. We sort households on the former measure since households' decisions regarding their primary residence and consumer durables may reflect considerations beyond the parsimonious model considered here.

[30] In appendix C.2, we vary these cutoffs. Raising the cutoff in the capital portfolio share in particular means that the wealth share of group a falls, but their capital portfolio share rises. In the context of our model, these changes have offsetting effects on the risk premium effects of a monetary policy shock. We choose a 90th percentile cutoff for our calibration to strike a balance between capturing the tail of levered households in the data and maintaining a large enough sample size to confidently estimate group moments.

	$1\{hbus^i = 1\}$	$1\{age^i > 54, lf^i = 0\}$
$1\{i \in a\}$	0.42	0.38
	(0.02)	(0.03)
$1\{i \in b\}$	0.05	0.55
	(0.01)	(0.01)
Observations	6,229	6,229
Adj R^2	0.06	0.37

Table 3: indicators for private business wealth or being retired on group indicators

Notes: observations are weighted by SCF sample weights and standard errors adjust for imputation and sampling variability following Pence (2015). Each specification includes a constant term (not shown), capturing the baseline probability of holding private business wealth or being retired among households in group c.

with the time series of the working age population provided by the BLS, to estimate quarterly per capita growth rates in those series over the sample period Q3 1979 to Q2 2012 (chosen to be consistent with our sample period for the VAR in section 3.1). In terms of asset prices, we seek to match the first moments of returns and second moments of the dividend price ratio (informative of second moments of expected returns). In particular, over July 1979 - June 2012 using the data from CRSP described earlier, we estimate the annualized average real interest rate and excess return on the S&P 500 as well as the standard deviation and year-over-year autocorrelation of the quarterly dividend price ratio. We will compare these moments to those implied by our model assuming that an equity claim (with return r^e) is a levered claim to capital with a debt to equity ratio of 0.6.[31]

3.3.3 Parameterization

A model period corresponds to one quarter. After setting a subset of parameters in accordance with the literature, we calibrate the remaining parameters to be consistent with the macro and micro moments described above. We note that all stochastic properties of the model are estimated using a simulation where no disasters are realized in sample.[32]

Externally set parameters A subset of model parameters summarized in Table 4 are set externally in accordance with the literature. Among the model's preference parameters, we set the IES to one. The Frisch elasticity of labor supply is set to $\theta = 0.8$, consistent with the micro evidence for aggregate hours in Chetty, Guren, Manoli, and Weber (2011). The

[31] This ratio is consistent with our estimate of leverage of 1.6 for the non-financial sector in appendix C.2, and is further comparable to the assumed leverage of equity claims in the literature (e.g., Barro (2006) and Garleanu and Panageas (2015)).

[32] We make this choice to maximize comparability between the model and data, since our data is all from the post-World War II period.

	Description	Value	Notes
ψ	IES	1	
θ	Frisch elasticity	0.8	Chetty et al. (2011)
λ^a	measure of a households	4%	labor income in SCF
λ^b	measure of b households	13%	labor income in SCF
α	1 - labor share	0.33	
δ	depreciation rate	2.5%	
ϵ	elast. of subs. across workers	20	
τ	undoes wage markup	-0.05	
χ^W	Rotemberg wage adj costs	200	$\approx \mathbb{P}(\text{adjust}) = 2$ qtrs
ϕ	Taylor coeff. on inflation	1.5	Taylor (1993)
σ^m	std. dev. MP shock	0.25%/4	
ρ^m	persistence MP shock	0	
p	disaster probability	0.5%	Barro (2006)
$\underline{\varphi}$	disaster shock	-15%	Nakamura et al. (2013)

Table 4: externally set parameters

three types have measure $\lambda^a = 4\%$, $\lambda^b = 13\%$ and $\lambda^c = 83\%$, consistent with the shares of labor income in our analysis of the SCF micro data in Table 2.

On the production side, we choose $\alpha = 0.33$ for the capital share of production and a quarterly depreciation rate of 2.5%, standard values in the literature. The disaster probability is set to $p = 0.5\%$, which follows Barro (2006) and implies that a disaster shock is expected to occur every 50 years. The depth of the disaster is set to $\underline{\varphi} = -15\%$, consistent with the long-run effects of a disaster estimated by Nakamura, Steinsson, Barro, and Ursua (2013) after accounting for the recovery after an initial disaster. We choose an elasticity of substitution across worker varieties $\epsilon = 20$ and Rotemberg wage adjustment costs of $\chi^W = 200$, which together imply a Calvo (1983)-equivalent frequency of wage adjustment around 2 quarters, a conservative choice at the lower bound of rigidity found in Grigsby, Hurst, and Yildirmaz (2019).[33] The Taylor coefficient on inflation is set to $\phi = 1.5$ and monetary policy shocks have a standard deviation of $\sigma^m = 0.25\%/4$ with zero persistence. Finally, we assume that the wage markup is perfectly offset by $\tau = -\frac{1}{\epsilon-1} = -\frac{1}{19}$.

[33] A quarterly probability of wage adjustment of 0.5 implies an annual probability of $1 - 0.5^4 = 0.94$, slightly higher than the annual probability of adjustment in wages plus bonuses in the ADP data studied by these authors, and substantially higher than other estimates of the adjustment probability which they obtain and survey in their Table 12.

	Description	Value	Moment	Target	Model
σ	std. dev. prod.	0.6%	$\sigma(\Delta \log c)$	0.5%	0.5%
χ^x	capital adj cost	12	$\sigma(\Delta \log x)$	2.1%	2.0%
β	discount factor	0.98	$\mathbb{E}r_{+1}$	1.4%	1.4%
$\gamma^b = \gamma^c$	RRA b, c	27	$\mathbb{E}\left[r^e_{+1} - r_{+1}\right]$	7.1%	7.3%
σ^p	variation dis. prob.	0.25%	$\sigma(d^e/p^e)$	0.2%	0.4%
ρ^p	persist. dis. prob.	0.95	$\rho(d^e/p^e, d^e_{-4}/p^e_{-4})$	0.8	0.6
γ^a	RRA a	11	k^a/a^a	1.5	2.5
$\underline{k}$	lower bound k^i	8.5	k^c/a^c	1.1	0.6
τ^a	controls tax on a	0.5%	$\lambda^a a^a / \sum_i \lambda^i a^i$	22.1%	22.7%
τ^c	controls tax on c	5.5%	$\lambda^c a^c / \sum_i \lambda^i c^i$	23.4%	20.5%
B^g	govt bonds	-1.1	$-\sum_i \lambda^i b^i / \sum_i \lambda^i a^i$	-3.6%	-3.6%
$\bar{\theta}$	ℓ disutility	0.88	ℓ	1	1.2

Table 5: targeted moments and calibrated parameters

Notes: targeted business cycle moments are from Q3/79-Q2/12 NIPA and targeted asset pricing moments are from 7/79-6/12 data underlying the VAR. The equity premium and second moments of the dividend price ratio in the model are calculated assuming a debt/equity ratio of 0.6 on a stock market claim. The first and second moments in the model are estimated over 50,000 quarters after a burn-in period of 5,000 quarters, with no disaster realizations in sample.

Calibrated parameters We calibrate the remaining parameters to target the macro and micro moments described above. While there is no one-to-one mapping between individual model parameters and those moments, Table 5 reports in each line a parameter choice, as well as a moment in model and data that this parameter is closely linked to.

The standard deviation of the productivity shock σ is set to 0.6%, which is a key determinant of the model's ability to match quarterly consumption growth volatility of 0.5%. The scale of the capital adjustment cost is set to $\chi^x = 12$ to dampen the volatility of investment growth in order to match the data. The discount factor is one key determinant of the expected real interest rate. Due to the precautionary savings motive, $\beta = 0.98$ is high enough to match the low annualized real rate observed in the data. Households' risk aversion $\{\gamma^i\}$ and the capital constraint $\underline{k}$ are jointly drivers of the risk premium in the economy and households' portfolio choices. γ^c and $\underline{k}$ are difficult to separately calibrate: for γ^c not too much lower than γ^a or γ^b, the relatively high ratio of labor income to wealth among group c households will mean that they will endogenously choose to hedge this exposure to productivity shocks by holding a lower position in capital, consistent with Proposition 1 in the analytical results. Their leverage is thus more likely to be endogenously constrained by (29). For parsimony we set $\gamma^c = \gamma^b$ and calibrate $\underline{k}$ to target the leverage of c households in the

Moment (ann.)	Data	Model
$\sigma(\Delta \log y)$	0.8%	0.9%
$\sigma(\Delta \log \ell)$	0.8%	0.9%
$\sigma\left(\mathbb{E}r_{+1}\right)$	1.2%	1.3%
$\sigma\left(\mathbb{E}\left[r^e_{+1} - r_{+1}\right]\right)$	5.7%	1.7%
$(\lambda^b \mathfrak{t}^b - \lambda^a \mathfrak{t}^a)/(w\ell + r^k k)$	-8.5%	-16.5%
$(\lambda^c \mathfrak{t}^c - \lambda^a \mathfrak{t}^a)/(w\ell + r^k k)$	3.5%	12.9%
$\sum^i \lambda^i mpr^i$	≈ 0.2	0.2
mpr^a		2.0
mpr^b		0.7
mpr^c		0.0

Table 6: untargeted macro and micro moments

Notes: business cycle moments are from Q3/79-Q2/12 NIPA and asset pricing moments are from 7/79-6/12 data underlying the VAR. The first and second moments in the model are estimated over 50,000 quarters after a burn-in period of 5,000 quarters, with no disaster realizations in sample.

data,[34] obtaining $\underline{k} = 8.5$ which is 26% of the average financial wealth of households in the model. The variation σ^p and persistence ρ^p in the disaster probability are chosen to target the standard deviation and year-over-year autocorrelation in the dividend price ratio. The tax rates τ^i are chosen to approximate the measured wealth shares of the three groups. We set B^g to clear the bond market, given the aggregate bond position of households relative to total wealth of 3.6% in the SCF data underlying Table 2.[35] Finally we set the disutility of labor to $\bar{\theta} = 0.88$, which targets a level of labor $\ell = 1.0$ as a normalization.

3.3.4 Untargeted moments

Table 6 reports the values of several untargeted moments and their empirical counterparts where available. In terms of macro moments, the model closely matches the quarterly volatilities of output growth and employment growth. It is consistent with the quarterly volatility of (annualized) expected real interest rates and matches the empirical pattern that this is

[34] We view this as a realistic description of the data, given that $\underline{k}$ is meant to capture components of the economy's capital stock which households hold for reasons beyond their financial returns. In the SCF data summarized in Table 2, 50% of the aggregate capital held by group c households is in their primary residence and vehicles, while the same ratio is only 26% for group b households and 8% for group a households.

[35] The aggregate household balance sheet summarized in appendix C.2 implies $\sum_i B^i / \sum_i A^i = 3,818/104,721 = 3.6\%$. Up to rounding error, this can also be inferred from the moments in Table 2 which imply $\sum_{i\in a} B^i / \sum_i A^i = \left(\sum_{i\in a} A^i / \sum_i A^i\right)\left(1 - \sum_{i\in a} Qk^i / \sum_{i\in a} A^i\right) = (0.221)(1-1.5) = -0.111$, and analogously $\sum_{i\in b} B^i / \sum_i A^i = 0.164$ and $\sum_{i\in c} B^i / \sum_i A^i = -0.023$. Combining, we obtain $\sum_i B^i / \sum A^i = 0.03$.

less than the quarterly volatility of (annualized) expected excess returns. Quantitatively, the model can account for roughly one third of the volatility in these expected excess returns.[36] We note that the empirical volatility is not directly observed and instead is implied by our VAR, which is on the high end of available estimates. Relative to the average of estimates surveyed in Duarte and Rosa (2015),[37] our model accounts for more than half of the volatility in expected excess returns, and relative to some estimates, it can account for all of it.

In terms of micro moments, we first compare the average lump-sum taxes on households with counterparts in the data. In the model, Table 6 reports $(\lambda^i \mathfrak{t}^i - \lambda^a \mathfrak{t}^a)/(w\ell + r^k k)$ for $i \in \{b, c\}$. The numerator summarizes the difference in aggregate taxes assessed on group i relative to group a, while the denominator summarizes average labor and interest income. We compare this to total federal taxes and transfers (inclusive of social security) for group i less group a in the SCF, where we use the NBER Taxsim program to estimate these taxes and transfers, divided by total income of households exclusive of taxes and transfers. We conclude that lump-sum taxes in the model are consistent with taxes paid in practice: relative to group a households, group b receives transfers and group c pays additional taxes.

We next discuss the model's predictions for MPRs. The model generates heterogeneity in quarterly MPRs consistent with Proposition 1 in the analytical results. Group a households are the most risk tolerant and have the highest MPR, borrowing \$1 for every \$1 of the marginal dollar in net worth to invest in capital. Group b and c households have higher levels of risk aversion and correspondingly lower MPRs. As noted above, group c households have a higher ratio of labor income to wealth and thus are (almost always) endogenously constrained by (29). Hence, on the margin their average MPR is essentially zero.

Quasi-experimental evidence is consistent with the average level of MPRs in our calibration. Weighting by the fraction of households — comparable to the average responses reported in the empirical literature — the average MPR in our model is 0.2. Using data on Norwegian lottery winners, Fagereng, Holm, and Natvik (2019b) estimate an average marginal propensity to save in risky assets relative to save overall of 0.14.[38] Using data on Swedish lottery winners, Briggs, Cesarini, Lindqvist, and Ostling (2015) estimate an analogous ratio of 0.15.[39] These imply an MPR of roughly 0.2 after accounting for reasonable

[36] We note that in the counterfactual representative agent economy described in the next subsection, the quarterly volatility of annualized expected excess returns is 1.4%. Hence, the endogenous redistribution of wealth across heterogeneous households amplifies the volatility of expected excess returns by roughly 1.2 times. While we emphasize monetary shocks in the main text given that they are our focus in this paper, they contribute little to aggregate fluctuations. Appendix E studies how productivity shocks and changes in disaster probabilities, the drivers of fluctuations in our model, redistribute wealth.

[37] See Table 7 in their paper.

[38] In their Table 4, the average marginal propensity to save in stocks, bonds, and mutual funds is 0.058 and the marginal propensity to save in these assets, deposits, or repay debt is 0.407. The ratio is thus 0.14.

[39] In their Table B.8, the average marginal propensity to save in risky assets is 0.085 and the marginal

estimates of the leverage of firms and intermediaries in which households invest.[40]

The evidence is further consistent with the pattern of heterogeneity in MPRs in our calibration, though it is less definitive on the degree of heterogeneity. MPRs rise with wealth per household in our calibration, recalling that wealth per household is highest among a households and then b households. This pattern is consistent with that found in the above studies using lottery winners.[41,42] While the range in estimated MPRs in these studies is smaller than that in our model, estimated MPRs based on lotteries may underestimate the relevant statistic for households in groups a and b of our model. Using the Panel Study of Income Dynamics, Brunnermeier and Nagel (2008) document significant inertia in financial portfolios, with a negative change in the risky share after receiving one dollar of cash or deposits but an increase in the risky share after receiving one dollar of unexpected returns on risky assets.[43] As lottery winnings are paid out as cash or riskless deposits, the estimated MPR may understate the MPR in response to dividends or capital gains, more relevant for the balance sheet revaluation among the wealthy (a and b households) emphasized in this paper. Among recipients of private business income (overrepresented in these households), the estimated MPR may particularly understate their true MPR because investment in private businesses is not included in the definition of (traded) risky assets.

3.4 Impulse responses to a monetary policy shock

We now simulate the effects of a negative shock to the nominal interest rate.

3.4.1 Model versus RANK

We first compare in Figures 2, 3, and 4 the impulse responses of the model with heterogeneity to the counterfactual representative agent New Keynesian (RANK) economy. In the latter, we set $\gamma^i = 20$ across all groups, equal to the harmonic mean of risk aversion in the model weighted by the average consumption share of each group, and we eliminate the lump-sum taxes $\mathfrak{T}_t^i$ by setting $\tau^i = 0$ for all groups.

propensity to save in these assets, safe assets, bank accounts, or repay debt is 0.58. The ratio is thus 0.15.

[40]Following section 3.3.1, we must account for firms' and intermediaries' net leverage to translate households' equity claims into capital claims and thus the MPR. Applying net leverage of 1.6 which we estimate in appendix C.2 yields an MPRs implied by these estimates of 0.22-0.24.

[41] See Table 8 of Fagereng et al. (2019b) and Figure 3 of Briggs et al. (2015).

[42]Further evidence of this point can be found in U.S. tax data. Hoopes, Langetieg, Nagel, Reck, Slemrod, and Stuart (2017) find that households with high taxable income, dividend income, and private business income are more likely to sell stocks following periods of market tumult. While this may reflect heterogeneous responses to prices rather than changes in income, it remains consistent with the patterns described here.

[43]These results are consistent with the more recent work of Fagereng, Holm, Moll, and Natvik (2019a) using Norwegian administrative data finding evidence that households "save by holding" on to nearly 100% of assets experiencing capital gains.

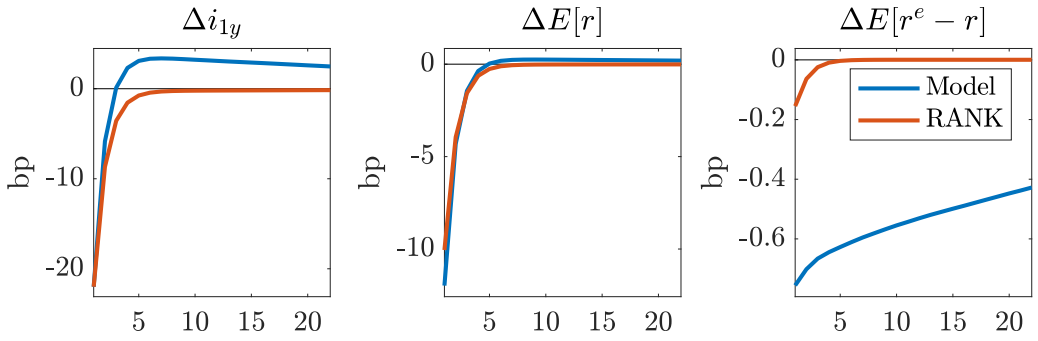


Figure 2: expected returns after negative monetary policy shock

Notes: all series are quarterly (non-annualized) measures, except for the 1-year nominal bond yield Δi_{1y}. Each impulse response is the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space with no disasters ever realized, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. *bp* denotes basis points (0.01%).

We choose the monetary shock ϵ_0^m in our model to match the 22*bp* reduction in the Treasury yield estimated in Figure 1. We obtain the yield on a 1-year nominal bond by computing, in each state, the price that each household would be willing to pay for the asset. We then set the price to that of the highest-valuation household. We re-calibrate the shock ϵ_0^m in the RANK economy to match the same 22*bp* reduction in the Treasury yield. This implies that the initial path of nominal interest rates in the RANK economy is comparable to that in our model, allowing us to partial out the initial difference in the endogenous response of monetary policy across environments.[44]

Figure 2 summarizes the effect of the monetary policy shock on key expected returns of interest. The first panel reports the change in the yield on the 1-year nominal bond due to the shock. The second and third panels depict the resulting change in the expected real interest rate and the expected excess returns on capital. The decline in the former reflects the monetary non-neutrality in our setting, while the decline in the latter demonstrates that the risk premium declines substantially in our model relative to the representative agent case. It is because of the additional stimulus induced by the fall in the risk premium that monetary policy endogenously tightens by more in our model relative to the RANK economy, evident in the higher 1-year nominal yields after the first quarter.

Figure 3 demonstrates that redistribution drives the decline in the risk premium in our model. The first panel of the first row demonstrates that realized excess returns on capital are substantially positive on impact, followed by small negative returns in the quarters which follow — consistent with the decline in the expected excess returns in the previous panel,

[44]In appendix E, we consider an alternative comparison with the same initial shock across environments.

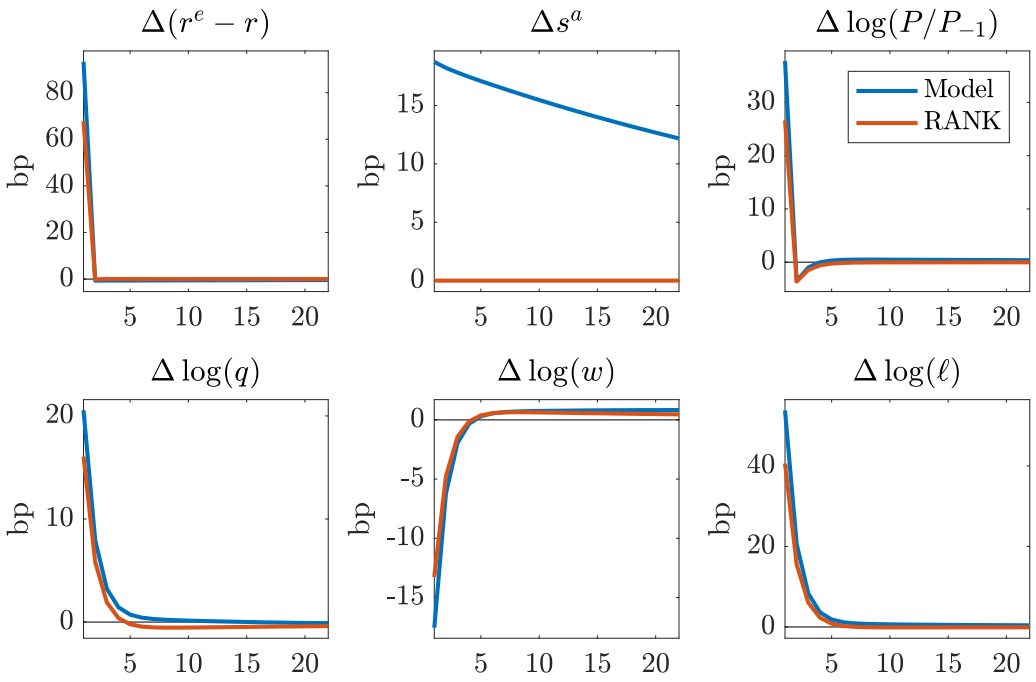


Figure 3: redistribution after negative monetary policy shock

Notes: all series are quarterly (non-annualized) measures. Each impulse response is the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space with no disasters ever realized, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. *bp* denotes basis points (0.01%).

and exactly the pattern estimated in the data in Figure 1. Through the lens of Proposition 7, the substantially positive excess returns on impact endogenously redistribute to the high MPR a households who hold levered claims on capital.[45] Indeed, the second panel in this row demonstrates that the financial wealth share of a households persistently rises after the shock. In part this results from unexpected inflation which lowers the realized real interest rate, shown in the third panel.[46] In part it also results from an increase in the price of capital, shown in the first panel of the second row. This reflects the increase in profits in the short-run because there are lower real wages and higher employment in this sticky wage environment, shown in the second and third panels of this row.

Figure 4 examines the consequences of this redistribution for the transmission of the policy shock to the real economy. The impact effects on investment and consumption are roughly 1.3 times larger versus the RANK economy. Moreover, the stimulus to investment

[45] This redistribution occurs essentially completely at the expense of b households.

[46] While the response of inflation is more immediate in the model than estimated in the data in Figure 1, we also note that much nominal debt in practice has longer duration than the one-period debt assumed in the model. Hence, we conjecture that enriching the model to feature more sluggish price inflation would not change the redistribution of wealth much if it were also enriched to feature longer duration nominal debt.

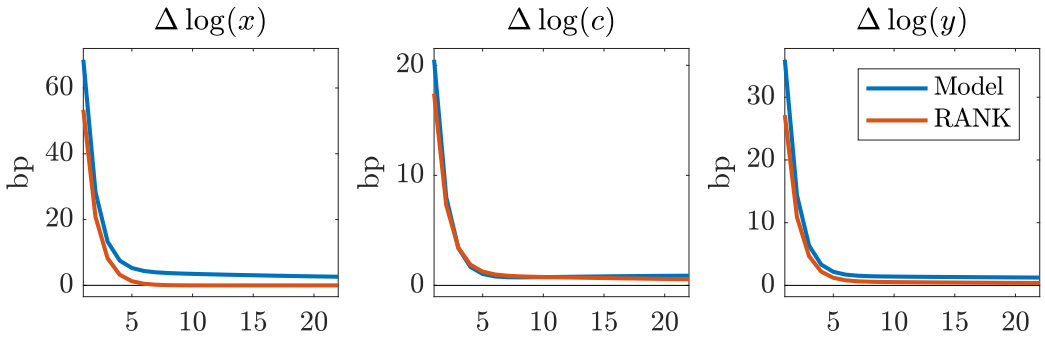


Figure 4: quantities after negative monetary policy shock

Notes: all series are quarterly (non-annualized) measures. Each impulse response is the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space with no disasters ever realized, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. *bp* denotes basis points (0.01%).

% Real stock return	Data ([90% CI])	Model	RANK
Dividend growth news	34% ([-11%,71%])	46%	60%
−Future real rate news	7% ([-6%,21%])	21%	40%
−Future excess return news	58% ([21%,106%])	33%	0%

Table 7: Campbell-Shiller decomposition of stock market return after monetary shock

Notes: estimates from data correspond to Table 1. Comparable estimates obtained in the model assuming a debt/equity ratio of 0.6 on a stock market claim.

in our model remains persistently higher than the RANK economy despite the endogenous tightening of monetary policy in subsequent quarters (because the risk premium falls by more than the risk-free rate rises). These patterns are consistent with our discussion of Propositions 5 and 6. As is demonstrated in the final panel of this figure, we see that they are inherited by the response of output as well.

Quantitatively, the price and quantity effects of the monetary policy shock in our model are consistent with the empirical estimates even though these were not targeted in the calibration. First, the impact effect on excess returns of 1*pp* is comparable to the 2*pp* increase estimated in Figure 1. Second and crucially, a Campbell-Shiller decomposition on the model impulse responses matches the role of news about lower future excess returns in driving the initial stock market return in the data. We summarize this decomposition in Table 7. The performance of our model contrasts starkly with the RANK economy, where essentially none of the transmission to the stock market operates though news about future excess returns. Third, the peak stimulus to output in the model of 0.4*pp* is comparable to the peak stimulus to industrial production estimated in Figure 1, giving us confidence in the

Row		Baseline	$\rho^m = 0.2$	$\chi^x = 0$	$\chi^W = 0$
1	$\Delta \log(P/P_{-1})$	38*bp*	46*bp*	50*bp*	49*bp*
2	$\Delta \log(q)$	20*bp*	22*bp*	0*bp*	1*bp*
3	$\Delta(r^e - r)$	94*bp*	110*bp*	82*bp*	80*bp*
4	Δs^a	19*bp*	22*bp*	17*bp*	17*bp*

Table 8: impact effects of negative monetary policy shock across parameterizations

Notes: for each parameterization, each row reports the average change in a particular variable on impact of the monetary policy shock starting at 1,000 different points drawn from the ergodic distribution of the state space with no disasters ever realized, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. All rows report quarterly (non-annualized) changes, and *bp* denotes basis points (0.01%).

model's predictions for transmission to the real economy.[47]

3.4.2 Inspecting the sources of redistribution

We seek to further understand the sources and implications of the redistribution in the model by varying a set of key parameters. Each column of Table 8 corresponds to a different parameterization, where in each column only a single parameter is changed from our baseline in the first column. The first three rows report the effect on inflation, the price of capital, and the realized excess return on equity in period 0. These three variables are central to understand the sources of redistribution in response to a monetary policy shock. The last row then reports the response in the wealth share of group a households.

The second column reports the results for an economy in which monetary policy shocks are persistent, setting $\rho^m = 0.2$, demonstrating the importance of redistribution through debt deflation. In that case, a monetary policy shock induces a stronger response of the inflation rate relative to the baseline, as can be seen in row 1. Through this debt deflation channel, the realized excess return on capital on impact rises in row 3 and the change in the wealth share of the levered household group a increases in row 4.

Eliminating the capital adjustment cost by setting $\chi^x = 0$, as reported in the third column, mitigates the redistribution through asset prices. In that case a monetary policy shock has no effect on the price of capital and therefore reduces the unexpected return on capital, as reported in rows 2 and 3, respectively. Redistribution is slightly smaller in row 4, but the effect is not especially large because of the countervailing effect of a larger inflation response in row 1: the smaller adjustment cost amplifies quantity responses in the capital

[47]The shape of the quantity responses differs from the hump-shapes typically estimated in the data, however. Adding features such as investment adjustment costs (as opposed to the present capital adjustment costs) could improve the model in this dimension, following Christiano, Eichenbaum, and Evans (2005).

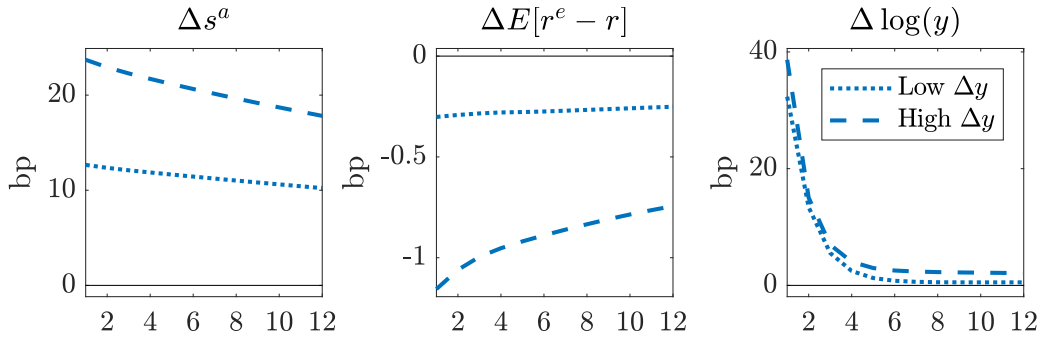


Figure 5: conditional impulse responses to negative monetary policy shock

Notes: all series are quarterly (non-annualized) measures. Each impulse response is the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space with no disasters ever realized, conditional on the impact effect on output being in the top or bottom quartile of its marginal distribution. *bp* denotes basis points (0.01%).

market, in turn amplifying the response in the labor market.

The fourth column eliminates nominal wage rigidity by setting $\chi^W = 0$, demonstrating the role of changes in profit income in inducing redistribution across households. When wage rigidity is zero, the decline in the real wage and the stimulus to employment is mitigated. It follows that the short-run increase in profits is mitigated. This not only reduces the increase in the current dividend but it also reduces the increase in the price of capital which capitalizes future dividends, evident in row 2. Redistribution to a households is thus mitigated in row 4, even though the redistribution through debt deflation is again amplified in row 1.

3.4.3 State-dependent effects of monetary policy

We conclude by noting that the redistribution across households with heterogeneous MPRs implies state-dependent effects of monetary policy.

In particular, the effects of monetary policy are larger when the wealth share of a households is high because the implied redistribution and thus effects on risk premia are larger. Figure 5 first depicts the average impulses responses to a monetary policy shock (in all cases resulting in a 22*bp* decline in the 1-year nominal yield) conditional on the impact effect on output being its top versus bottom quartile. As is evident, the periods in which the expansionary shock has a larger effect on output are also those in which the redistribution to a households and the resulting effect on risk premia are larger. Table 9 then characterizes conditional moments of these two sets in the state space. We see that the effect on output is high when the wealth share of a households is high. Notably, this is the case even though the leverage of a households is low. Evidently, the higher level of wealth among these households

$\Delta \log(y)$:	Bottom quartile	Top quartile
s^a	16.3%	30.8%
k^a/a^a	2.8	2.3
$\sum_{t=0}^{-3} [r_t^e - r_t]$	5.5%	9.1%
k/y	8.8	9.2

Table 9: first moments conditional on impact effect of monetary policy shock on output

Notes: moments computed using the ergodic distribution of the state space with no disasters ever realized, approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. Excess return assumes a debt/equity ratio of 0.6 on a stock market claim.

implies a larger redistribution of wealth induced by the policy shock, despite the fact that their leverage is lower. These states are consistent with bull markets — excess returns over the prior year have been especially high, and the economy's capital to output ratio is high — suggesting that monetary policy will be most potent during this part of the cycle.

4 Conclusion

In this paper we revisit the monetary transmission mechanism in a New Keynesian environment with heterogeneous propensities to bear risk. An expansionary monetary policy shock lowers the risk premium if it redistributes to households with high MPRs. Heterogeneity in risk aversion, portfolio constraints, rules of thumb, background risk, or beliefs induce a joint distribution of monetary policy exposures and MPRs such that an expansionary shock redistributes in this way. In a calibration matching micro-level heterogeneity in the U.S. economy, this mechanism quantitatively rationalizes the stock market effects of monetary policy which have eluded existing frameworks and amplifies its transmission to the real economy.

The framework of this paper can be further developed along a number of dimensions. First, it seems fruitful to synthesize our perspective emphasizing assets' exposure to aggregate risk with the existing HANK literature emphasizing asset liquidity: in such a setting, an investor's MPR out of liquid versus illiquid wealth will differ, likely a better match to the micro data. Second, while we have focused for concreteness on the equity premium, a natural question is the extent to which our insights can explain the broader effects of monetary policy across asset classes, as in the Treasury market or foreign exchange market. Third, while our analysis has focused on the conditional responses to monetary policy shocks, it would be useful to examine the model's implied comovements when featuring a richer set of business cycle shocks calibrated to the data. We leave these exciting questions for future work.

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Appendix For Online Publication

A Proofs

In this appendix we prove the results stated in section 2.

Proposition 1

Proof. We first characterize households' portfolio share in capital in the limit of zero aggregate risk. Optimal portfolio choice is

$$\mathbb{E}_0(c_1^i)^{-\gamma^i}(r_1^k - r_1) = 0.$$

Up to first-order, optimal portfolio choice yields

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 = o(||\cdot||^2).$$

It follows that given the first-order expansion in terms of the states

$$\begin{aligned}\hat{r}_1^k &= \hat{z}_1 + \delta_{m_0}^{r_1^k}\hat{m}_0 + o(||\cdot||^2),\\ \hat{r}_1 &= \delta_{m_0}^{r_1}\hat{m}_0 + o(||\cdot||^2),\end{aligned}$$

with coefficients $\delta_{\cdot}^{\cdot}$, we can conclude

$$\delta_{m_0}^{r_1^k} = \delta_{m_0}^{r_1}$$

by the method of undetermined coefficients.

Up to second-order, optimal portfolio choice yields

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\mathbb{E}_0(\hat{r}_1^k)^2 - \frac{1}{2}\hat{r}_1^2 = \gamma^i\mathbb{E}_0\hat{c}_1^i\left(\hat{r}_1^k - \hat{r}_1\right) + o(||\cdot||^3).$$

Using the above first-order approximations of $\hat{r}_1^k$ and $\hat{r}_1$ in terms of the underlying states, and the first-order approximation of $\hat{c}_1^i$

$$\hat{c}_1^i = \delta_{z_1}^{c_1^i}\hat{z}_1 + \delta_{m_0}^{c_1^i}\hat{m}_0 + o(||\cdot||^2),$$

it follows that optimal portfolio choice implies

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\delta_{z_1}^{c_1^i}\sigma^2 + o(||\cdot||^3). \tag{37}$$

Anticipating the result in Proposition 2 that

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + o(||\cdot||^3),$$

it follows that

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i}. \tag{38}$$

Approximating up to first order the period 1 resource constraint and equilibrium wages and profits

$$w_1 = (1-\alpha) z_1 k_0^{\alpha},$$
$$\pi_1 = \alpha z_1 k_0^{\alpha-1},$$

the method of undetermined coefficients implies

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}.$$

Substituting in (38) and re-arranging, we can conclude that

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \frac{\bar{\pi}_1 \bar{k}_0^i}{(1+\bar{r}_1)\bar{a}_0^i} = \frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$

where the first equality uses $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_1}$ absent aggregate risk.

We now characterize households' marginal responses to a unit of income in the limit of zero aggregate risk. Differentiating households' optimal portfolio choice condition yields

$$0 = \mathbb{E}_0^i m_{0,1}^i \frac{\gamma^i}{c_1^i}\left(r_1^k - r_1\right)\frac{\partial c_1^i}{\partial y_0^i}$$

where the household's stochastic discount factor between periods 0 and 1 is

$$m_{0,1}^i \equiv \frac{\beta^i}{1-\beta^i}(c_0^i)^{\frac{1}{\psi^i}}\Phi^i(l_0^i)^{1-\frac{1}{\psi^i}}(ce_0^i)^{\gamma^i-\frac{1}{\psi^i}}(c_1^i)^{-\gamma^i}.$$

Differentiating households' period 1 resource constraint yields

$$\frac{\partial c_1^i}{\partial y_0^i} = (1+r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \frac{\partial k_0^i}{\partial y_0^i}.$$

Combining the previous two equations yields

$$0 = \mathbb{E}_0 m_{0,1}^i \frac{\gamma^i}{c_1^i}(r_1^k - r_1)\left((1+r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \frac{\partial k_0^i}{\partial y_0^i}\right). \tag{39}$$

A second-order approximation then implies

$$\begin{aligned} 0 = &\left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\left(\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2\right) \\ &- \left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\frac{\gamma^i + 1}{\bar{c}_1^i}\left(\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right)\sigma^2 + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\sigma^2 + o(||\cdot||^3). \end{aligned} \tag{40}$$

Again anticipating the result in Proposition 2 that

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + o(||\cdot||^3)$$

and the above result that

$$\frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \frac{\gamma}{\gamma^i},$$

it follows from (40) that

$$\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i}\overline{\frac{\partial a_0^i}{\partial y_0^i}}$$

using $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_0}$ and $\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \overline{\frac{\partial a_0^i}{\partial y_0^i}}$. The expression for $\overline{mpr}_0^i \equiv \frac{\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}}}{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}$ then follows. □

Proposition 2

Proof. We first derive the result up to second order. Multiplying both sides of (37) by $\frac{\bar{c}_1^i}{\gamma^i}$, integrating over all households i, and making use of the market clearing conditions which imply that

$$\int_0^1 \bar{c}_1^i di = \int_0^1 \left(\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right) di,$$

we obtain

$$\mathbb{E}_0 \hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \left(\frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i}\right)^{-1} \sigma^2 + o(||\cdot||^3), \tag{41}$$

defining γ as in the claim.

We now derive the result up to third order. The third-order approximation of optimal

portfolio choice for household i is

$$\begin{aligned}\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\mathbb{E}_0(\hat{r}_1^k)^2 - \frac{1}{2}\hat{r}_1^2 \\ &= \gamma^i \mathbb{E}_0 \hat{c}_1^i \left(\hat{r}_1^k - \hat{r}_1\right) - \frac{1}{2}\left(\gamma^i\right)^2 \mathbb{E}_0\left(\hat{c}_1^i\right)^2 \left(\hat{r}_1^k - \hat{r}_1\right) \\ &\quad + \frac{1}{2}\gamma^i \mathbb{E}_0 \hat{c}_1^i \left((\hat{r}_1^k)^2 - \hat{r}_1^2\right) - \frac{1}{6}\left(\mathbb{E}_0(\hat{r}_1^k)^3 - \hat{r}_1^3\right) + o(||\cdot||^4). \quad (42)\end{aligned}$$

A second-order expansion of $\hat{r}_1^k$ and $\hat{r}_1$ in terms of the underlying states yields

$$\begin{aligned}\hat{r}_1^k &= \hat{z}_1 + \delta_{m_0}^{r_0}\hat{m}_0 + \frac{1}{2}\delta_{m_0^2}^{r_0}\hat{m}_0^2 + \left(-\frac{1}{2} + \gamma + \frac{1}{2}\delta_{\sigma^2}^{r_0}\right)\sigma^2, \\ \hat{r}_0 &= \delta_{m_0}^{r_0}\hat{m}_0 + \frac{1}{2}\delta_{m_0^2}^{r_0}\hat{m}_0^2 + \frac{1}{2}\delta_{\sigma^2}^{r_0}\sigma^2\end{aligned}$$

where we have already made use of the fact that, by the method of undetermined coefficients, (41) implies

$$\begin{aligned}\frac{1}{2}\delta_{m_0^2}^{r_1^k} &= \frac{1}{2}\delta_{m_0^2}^{r_1}, \\ \frac{1}{2}\delta_{\sigma^2}^{r_1^k} - \frac{1}{2}\delta_{\sigma^2}^{r_1} + \frac{1}{2} &= \gamma.\end{aligned}$$

A second-order expansion of $\hat{c}_1^i$ in terms of the underlying states yields

$$\hat{c}_1^i = \delta_{m_0}^{c_1^i}\hat{m}_0 + \delta_{z_1}^{c_1^i}\hat{z}_1 + \frac{1}{2}\delta_{m_0^2}^{c_1^i}\hat{m}_0^2 + \delta_{m_0 z_1}^{c_1^i}\hat{m}_0\hat{z}_1 + \frac{1}{2}\delta_{z_1^2}^{c_1^i}\hat{z}_1^2 + \frac{1}{2}\delta_{\sigma^2}^{c_1^i}\sigma^2 + o(||\cdot||^3).$$

Substituting these into (42) and collecting terms, we obtain

$$\begin{aligned}\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \\ \gamma^i\delta_{z_1}^{c_1^i}\sigma^2 + \left[\gamma^i\left(\delta_{m_0}^{c_1^i}\gamma + \delta_{m_0 z_1}^{c_1^i}\right) - \left(\gamma^i\right)^2\delta_{m_0}^{c_1^i}\delta_{z_1}^{c_1^i} + \gamma^i\delta_{z_1}^{c_1^i}\delta_{m_0}^{r_1} - \gamma\delta_{m_0}^{r_1}\right]\hat{m}_0\sigma^2 + o(||\cdot||^4).\end{aligned}$$

Making use of (38) substantially simplifies this to

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\delta_{z_1}^{c_1^i}\sigma^2 + \gamma^i\delta_{m_0 z_1}^{c_1^i}\hat{m}_0\sigma^2 + o(||\cdot||^4). \quad (43)$$

Again multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i}$, integrating over all households i, and making use of the

market clearing conditions, we obtain

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di}\left(\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di\right)\hat{m}_0\sigma^2 + o(||\cdot||^4).$$

Then, taking a second-order approximation of the period 1 resource constraint and equilibrium wages and profits, the method of undetermined coefficients implies

$$\bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} + \bar{c}_1^i \delta_{m_0}^{c_1^i}\delta_{z_1}^{c_1^i} = \alpha\bar{w}_1\delta_{m_0}^{k_0} + \bar{\pi}_1\delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0}. \tag{44}$$

It follows that

$$\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di = -\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i}\delta_{z_1}^{c_1^i} di + \bar{\pi}_1 \int_0^1 \delta_{m_0}^{k_0^i} di,$$

using

$$\int_0^1 \alpha\bar{w}_1\delta_{m_0}^{k_0} di = \alpha\bar{w}_1\delta_{m_0}^{k_0} = \alpha(1-\alpha)\bar{z}_1\bar{k}_0^\alpha\delta_{m_0}^{k_0},$$
$$\int_0^1 (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{m_0}^{k_0} di = (1-\alpha)\bar{\pi}_1\bar{k}_0\delta_{m_0}^{k_0} = \alpha(1-\alpha)\bar{z}_1\bar{k}_0^\alpha\delta_{m_0}^{k_0}$$

implied by market clearing and the definition of equilibrium wages and profits.[48] Since a first-order approximation to capital claims market clearing implies

$$\int_0^1 \delta_{m_0}^{k_0^i} di = \bar{k}_0\delta_{m_0}^{k_0},$$

it further follows that

$$\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di = -\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i}\delta_{z_1}^{c_1^i} di + \bar{\pi}_1\bar{k}_0\delta_{m_0}^{k_0}.$$

Moreover, since a first-order approximation to goods market clearing implies

$$\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} di = \alpha\bar{z}_1\bar{k}_0^\alpha\delta_{m_0}^{k_0}$$

[48]Note that we linearize rather than log-linearize with respect to $\{k_0^i, b_0^i, a_0^i\}$ since in principle these may be negative.

and $\bar{\pi}_1 = \alpha \bar{z}_1 \bar{k}_0^{\alpha-1}$, it further follows that

$$\begin{aligned}\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di &= -\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} di, \\ &= \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di, \\ &= \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right) di\end{aligned}$$

where the final line uses (38). Hence, we can conclude

$$\begin{aligned}\zeta_{m_0} &= \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left(\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di\right), \\ &= \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right) di.\end{aligned}$$

Recall from Proposition 1 that $\overline{mpr}_0^i \equiv \frac{\gamma}{\gamma^i}$. Since the definition of (20) implies

$$\int_0^1 \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right) di = \int_0^1 \left(\bar{c}_1^i \delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right) \left(1 - \frac{\gamma}{\gamma^i}\right) di$$

and

$$1 = \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i di \equiv \overline{mpr}_0,$$

and we further have

$$\begin{aligned}\overline{\frac{d\left[c_1^i / \int_0^1 c_1^{i'}\right]}{dm_0}} &= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'} \left(\overline{\frac{dc_1^i}{dm_0}} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}} di'\right), \\ &= \frac{1}{\int_0^1 \bar{c}_1^{i'} di'} \left(\bar{c}_1^i \delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right),\end{aligned}$$

we obtain the expression for ζ_{m_0} given in the claim. □

Proposition 3

Proof. Combining (23) with Proposition 7 and using

$$\begin{aligned}
\bar{c}_1^i &= \bar{c}_1, \\
\overline{\frac{\partial a_0^i}{\partial y_0^i}} &= \overline{\frac{\partial a_0}{\partial y_0}}, \\
\psi^i &= \psi, \\
\bar{\tau}^{\ell_0^i} &= \bar{\tau}^{\ell_0}, \\
\bar{\ell}_0^i &= \bar{\ell}_0, \\
\overline{\frac{d\ell_0^i}{dm_0}} &= \overline{\frac{d\ell_0}{dm_0}},
\end{aligned}$$

as assumed in the claim, we obtain

$$\bar{\xi}_{m_0}^i = \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})B_{-1}^i}{P_0}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k_{-1}^i - k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right].$$

By Proposition 1 and the assumptions in the claim,

$$\begin{aligned}
k_{-1}^i = \bar{k}_0^i &= \bar{a}_0\left[\frac{\bar{c}_1}{(1+\bar{r}_1)\bar{a}_0}\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0}\right], \\
&= \frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right) + \bar{k}_0, \\
B_{-1}^i = \bar{P}_0\bar{b}_0^i &= \bar{P}_0\left[\bar{a}_0 - \bar{k}_0^i\right], \\
&= -\bar{P}_0\frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right),
\end{aligned}$$

where

$$\gamma = \left[\int_0^1 \frac{1}{\gamma^i}di\right]^{-1}$$

and we use $\bar{q}_0 = 1$ following the assumption that $k_{-1} = \bar{k}_0$. It follows that

$$\begin{aligned}
-\frac{(1+i_{-1})B_{-1}^i}{P_0} &= (1+i_{-1})\frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right), \\
k_{-1}^i - k_{-1} &= \frac{\bar{k}_0}{\alpha}\left(\frac{\gamma}{\gamma^i} - 1\right).
\end{aligned}$$

Hence,

$$\begin{aligned}\zeta_{m_0} &= \gamma \int_0^1 \bar{\xi}^i_{m_0}(\overline{mpr}_0 - \overline{mpr}^i_0)di, \\ &= \gamma \int_0^1 \bar{\xi}^i_{m_0}\left(1 - \frac{\gamma}{\gamma^i}\right)di, \\ &\propto -(1+i_{-1})\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} - \overline{\frac{d\pi_0}{dm_0}} - (1-\delta_0)\overline{\frac{dq_0}{dm_0}} \end{aligned} \tag{45}$$

where the second line uses Proposition 1 and the third line uses the above results.

To sign (45), we now compute $\{\frac{dP_0}{dm_0}, \frac{d\pi_0}{dm_0}, \frac{dq_0}{dm_0}\}$ at the limit of $\sigma = 0$. Assuming these derivatives are continuous in σ, their values at the limit of $\sigma = 0$ will be equal to $\{\overline{\frac{dP_0}{dm_0}}, \overline{\frac{d\pi_0}{dm_0}}, \overline{\frac{dq_0}{dm_0}}\}$. The limiting Euler equation

$$\left(\bar{c}^i_0\right)^{-\frac{1}{\psi}} \Phi(\bar{\ell}_0)^{1-\frac{1}{\psi}} = \beta(1+\bar{r}_1)\left(\bar{c}^i_1\right)^{-\frac{1}{\psi}}$$

implies

$$-\frac{1}{\psi}\frac{1}{\bar{c}_0}\frac{d\bar{c}^i_0}{d\bar{m}_0} + \left(1 - \frac{1}{\psi}\right)\epsilon^{\Phi}_{\bar{\ell}_0}\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = \frac{1}{1+\bar{r}_1}\frac{d(1+\bar{r}_1)}{d\bar{m}_0} - \frac{1}{\psi}\frac{1}{\bar{c}_1}\frac{d\bar{c}^i_1}{d\bar{m}_0}, \tag{46}$$

where we write the elasticity of $\Phi(\ell_0)$ with respect to ℓ_0 evaluated at $\bar{\ell}_0$

$$\epsilon^{\Phi}_{\bar{\ell}_0} \equiv \frac{\Phi'(\bar{\ell}_0)\bar{\ell}_0}{\Phi(\bar{\ell}_0)}.$$

The union's limiting labor supply condition

$$\int_0^1 (\bar{v}^i_0)^{\frac{1}{\psi}}(\bar{c}^i_0)^{-\frac{1}{\psi}}\Phi(\bar{\ell}_0)^{1-\frac{1}{\psi}}\left[\frac{\bar{W}_0}{\bar{P}_0} + \bar{c}^i_0\frac{\Phi'(\bar{\ell}_0)}{\Phi(\bar{\ell}_0)} + \frac{\bar{W}_0}{\bar{P}_0}\frac{\chi^W}{\epsilon}\frac{\bar{W}_0}{W_{-1}}\left(\frac{\bar{W}_0}{W_{-1}} - 1\right)\right]di = 0$$

implies

$$\frac{1}{\bar{W}_0}\frac{d\bar{W}_0}{d\bar{m}_0} - \frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} - \frac{1}{\bar{c}_0}\left(\int_0^1 \frac{d\bar{c}^i_0}{d\bar{m}_0}di\right) - \left(\epsilon^{-\Phi'}_{\bar{\ell}_0} - \epsilon^{\Phi}_{\bar{\ell}_0}\right)\frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} + \frac{\chi^W}{\epsilon}\frac{1}{\bar{W}_0}\frac{d\bar{W}_0}{d\bar{m}_0} = 0. \tag{47}$$

where we have used the symmetry across households and $W_{-1} = \bar{W}_0$ at the point of approximation, and further defined the elasticity of the marginal disutility of labor

$$\epsilon^{-\Phi'}_{\bar{\ell}_0} \equiv \frac{-\Phi''(\bar{\ell}_0)\bar{\ell}_0}{-\Phi'(\bar{\ell}_0)}.$$

The limiting labor demand condition in period 0

$$\frac{\bar{W}_0}{\bar{P}_0} = (1-\alpha) z_0 \bar{\ell}_0^{-\alpha} k_{-1}^{\alpha},$$

implies

$$\frac{1}{\bar{W}_0}\frac{d\bar{W}_0}{d\bar{m}_0} - \frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = -\alpha \frac{1}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0}. \tag{48}$$

The limiting optimal investment condition

$$\bar{q}_0 = \left(\frac{\bar{k}_0}{k_{-1}}\right)^{\chi^x}$$

implies

$$\frac{d\bar{q}_0}{d\bar{m}_0} = \chi^x \frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} \tag{49}$$

where we have used $k_{-1} = \bar{k}_0$ at the point of approximation. The limiting goods market clearing condition in period 0

$$\int_0^1 \bar{c}_0^i di + \bar{q}_0 \left(\bar{k}_0 - (1-\delta_0)k_{-1}\right) = z_0 \bar{\ell}_0^{1-\alpha} k_{-1}^{\alpha}$$

implies

$$\int_0^1 \frac{d\bar{c}_0^i}{d\bar{m}_0} di + \frac{d\bar{k}_0}{d\bar{m}_0} + \delta \bar{k}_0 \frac{d\bar{q}_0}{d\bar{m}_0} = (1-\alpha) z_0 \bar{\ell}_0^{-\alpha} k_{-1}^{\alpha} \frac{d\bar{\ell}_0}{d\bar{m}_0}. \tag{50}$$

where we use $\bar{q}_0 = 1$ and $\bar{k}_0 = k_{-1}$ at the point of approximation. The limiting goods market clearing condition in period 1

$$\int_0^1 \bar{c}_1^i di = \bar{z}_1 \bar{k}_0^{\alpha}$$

implies

$$\int_0^1 \frac{d\bar{c}_1^i}{d\bar{m}_0} di = \alpha \bar{z}_1 \bar{k}_0^{\alpha-1} \frac{d\bar{k}_0}{d\bar{m}_0}. \tag{51}$$

The limiting definition of the returns

$$1 + \bar{r}_1 = \frac{\alpha \bar{z}_1 \bar{k}_0^{\alpha-1}}{\bar{q}_0}$$

implies

$$\frac{1}{1+\bar{r}_1}\frac{d(1+\bar{r}_1)}{d\bar{m}_0} = (\alpha - 1)\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} - \frac{d\bar{q}_0}{d\bar{m}_0} \tag{52}$$

where we again use $\bar{q}_0 = 1$ at the point of approximation. Finally, the limiting Fisher

equation together with the monetary policy rules (10) and $P_1 = \bar{P}_1$

$$1 + \bar{r}_1 = \frac{(1+\bar{i})}{(P_{-1})^{\phi}} \frac{(\bar{P}_0)^{1+\phi}}{\bar{P}_1} \bar{m}_0$$

implies

$$\frac{1}{1+\bar{r}_1} \frac{d(1+\bar{r}_1)}{d\bar{m}_0} = (1+\phi) \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0}. \tag{53}$$

Combining (49), (52), and (53) yields

$$\frac{1}{\bar{k}_0} \frac{d\bar{k}_0}{d\bar{m}_0} = -\frac{1}{1-\alpha+\chi^x} \left((1+\phi) \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0} \right). \tag{54}$$

Combining (46), (49), (50), (51), and (53) yields

$$\frac{1}{\bar{c}_0} \left(z_0 \bar{\ell}_0^{-\alpha} k_{-1}^{\alpha} (1-\alpha) \frac{d\bar{\ell}_0}{d\bar{m}_0} - (\delta_0 \chi^x + 1) \frac{d\bar{k}_0}{d\bar{m}_0} \right) + (1-\psi) \epsilon^{\Phi}_{\bar{\ell}_0} \frac{1}{\bar{\ell}_0} \frac{d\bar{\ell}_0}{d\bar{m}_0} =$$
$$- \psi \left((1+\phi) \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0} \right) + \alpha \frac{1}{\bar{k}_0} \frac{d\bar{k}_0}{d\bar{m}_0}.$$

Since by assumption $W_{-1} = \bar{W}_0$, it follows from the union's optimal labor supply and the representative producer's optimal labor demand that each household's labor wedge is zero at the point of approximation:

$$\bar{\tau}_0^{\ell_0^i} = \bar{\tau}_0^{\ell_0} = 1 - \frac{-\bar{c}_0 \Phi'(\bar{\ell}_0)/\Phi(\bar{\ell}_0)}{(1-\alpha) z_0 \bar{\ell}_0^{-\alpha} k_{-1}^{\alpha}} = 0.$$

Hence we can further simplify the above as

$$-\psi \epsilon^{\Phi}_{\bar{\ell}_0} \frac{1}{\bar{\ell}_0} \frac{d\bar{\ell}_0}{d\bar{m}_0} = -\psi \left((1+\phi) \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{m}_0} + \frac{1}{\bar{m}_0} \right) + \left(\alpha + (\delta_0 \chi^x + 1) \frac{\bar{k}_0}{\bar{c}_0} \right) \frac{1}{\bar{k}_0} \frac{d\bar{k}_0}{d\bar{m}_0}. \tag{55}$$

Combining (47), (48), (49), and (50) yields

$$\frac{\frac{\chi^W}{\epsilon}}{1+\frac{\chi^W}{\epsilon}} \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{m}_0} = \left(\alpha + \frac{1}{1+\frac{\chi^W}{\epsilon}} \left(\epsilon^{-\Phi'}_{\bar{\ell}_0} - 2\epsilon^{\Phi}_{\bar{\ell}_0} \right) \right) \frac{1}{\bar{\ell}_0} \frac{d\bar{\ell}_0}{d\bar{m}_0} - \frac{1}{1+\frac{\chi^W}{\epsilon}} (\delta_0 \chi^x + 1) \frac{1}{\bar{c}_0} \frac{d\bar{k}_0}{d\bar{m}_0}, \tag{56}$$

where we have again used the result that each household's labor wedge is zero at the point of approximation. Then (54)-(56) are 3 equations in the 3 unknowns $\{\frac{d\bar{\ell}_0}{d\bar{m}_0}, \frac{d\bar{k}_0}{d\bar{m}_0}, \frac{d\bar{P}_0}{d\bar{m}_0}\}$. Solving

this system yields

$$\frac{\bar{m}_0}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = -\frac{\left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)\frac{1}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{k_0}{c_0}}{1-\alpha+\chi^x}\right)(1+\phi)\left(\frac{\alpha\left(1+\frac{\epsilon}{\chi^W}\right)+\frac{\epsilon}{\chi^W}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0}-2\epsilon^{\Phi}_{\bar{\ell}_0}\right)}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}\right)},$$

$$\frac{\bar{m}_0}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = \frac{1}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}\times$$

$$\left[-\frac{\left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)\frac{\alpha\left(1+\frac{\epsilon}{\chi^W}\right)+\frac{\epsilon}{\chi^W}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0}-2\epsilon^{\Phi}_{\bar{\ell}_0}\right)}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\left(\frac{\alpha\left(1+\frac{\epsilon}{\chi^W}\right)+\frac{\epsilon}{\chi^W}\left(\epsilon^{-\Phi'}_{\bar{\ell}_0}-2\epsilon^{\Phi}_{\bar{\ell}_0}\right)}{1-\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1+\phi}{1-\alpha+\chi^x}}\right)}+\right.$$

$$\left.\frac{\epsilon}{\chi^W}(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}\frac{1}{1-\alpha+\chi^x}\right],$$

and then $\frac{\bar{m}_0}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0}$ implied by (54). For χ^W sufficiently large, each will be negative. Indeed, in the limit $\chi^W \to \infty$, these imply

$$\frac{\bar{m}_0}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} = -\frac{\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\alpha} < 0,$$

$$\frac{\bar{m}_0}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} = -\alpha\frac{\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\alpha} < 0,$$

$$\frac{\bar{m}_0}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} = -\frac{1}{1-\alpha+\chi^x}\frac{-\psi\epsilon^{\Phi}_{\bar{\ell}_0}}{-\psi\epsilon^{\Phi}_{\bar{\ell}_0} + \left(\psi + \frac{\alpha+(\delta_0\chi^x+1)\frac{\bar{k}_0}{\bar{c}_0}}{1-\alpha+\chi^x}\right)(1+\phi)\alpha} < 0.$$

Since

$$\frac{d\bar{\pi}_0}{d\bar{m}_0} = (1-\alpha)\frac{\bar{\pi}_0}{\bar{\ell}_0}\frac{d\bar{\ell}_0}{d\bar{m}_0} \propto \frac{d\bar{\ell}_0}{d\bar{m}_0}$$

and

$$\frac{d\bar{q}_0}{d\bar{m}_0} = \chi^x\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0} \propto \frac{d\bar{k}_0}{d\bar{m}_0},$$

it follows from (45) that $\zeta_{m_0} > 0$. □

Proposition 4

Proof. We first note that, in the general environment featuring portfolio constraints / rules-of-thumb, background risk, and subjective beliefs regarding aggregate TFP, households' limiting portfolios and MPRs are

$$\frac{\bar{q}_0\bar{k}_0^i}{\bar{a}_0^i} = \begin{cases} \omega_0^i \text{ for } i \in C, \\ \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i} \text{ for } i \notin C, \end{cases} \tag{57}$$

$$\overline{mpr}_0^i = \begin{cases} \omega_0^i \text{ for } i \in C, \\ \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} \text{ for } i \notin C, \end{cases} \tag{58}$$

where

$$\gamma = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\left(1 - \frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_0^i di}{\int_{i\notin C}\bar{c}_1^i di}\right). \tag{59}$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$, we obtain (21) with γ as in (59) and

$$\zeta_{m_0} = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_0^1 \bar{\xi}_{m_0}^i\left(\overline{mpr}_0 - \overline{mpr}_0^i\right)di, \tag{60}$$

where $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[(1+r_0)a_0^i/\int_0^1 c_1^{i'}di'\right]}}{dm_0}$ for $i \in C$ and $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[c_1^i/\int_0^1 c_1^{i'}di'\right]}}{dm_0}$ for $i \notin C$, and $\overline{mpr}_0 \equiv \int_{i\in C}\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di' + \int_{i'\notin C}\bar{c}_1^{i'}di'}\overline{mpr}_0^i di + \int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di' + \int_{i'\notin C}\bar{c}_1^{i'}di'}\overline{mpr}_0^i di = 1$. The proof of these results combines the proofs in Corollaries 1-3 and we do not repeat it here.

A household's limiting change in future consumption in response to a monetary policy shock $\frac{\overline{dc_1^i}}{dm_0}$ remains characterized by Proposition 7. Given the limiting period 1 budget constraint

$$\bar{c}_1^i = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^i, \tag{61}$$

a household's limiting change in $(1+r_1)a_0^i$ in response to a monetary policy shock is characterized by

$$\frac{\overline{d(1+r_1)a_0^i}}{dm_0} = \frac{\overline{dc_1^i}}{dm_0} - \frac{\overline{dw_1}}{dm_0}. \tag{62}$$

Then, under the assumptions that households are identical except for $\{\gamma^i, \omega_0^i, \eta^i, \varsigma^i\}$ and whether or not they are constrained, it follows that for unconstrained households ($i \notin C$)

$$\bar{\xi}_{m_0}^i = \frac{1}{\bar{c}_1}(1+\bar{r}_1)\frac{\overline{\partial a_0}}{\partial y_0}\left[-\frac{(1+i_{-1})}{P_0}B_{-1}^i\frac{1}{P_0}\frac{\overline{dP_0}}{dm_0} + (k_{-1}^i - k_{-1})\left(\frac{\overline{d\pi_0}}{dm_0} + (1-\delta_0)\frac{\overline{dq_0}}{dm_0}\right)\right].$$

as in the baseline case. By (57)

$$\begin{aligned} k^i_{-1} = \bar{k}^i_0 &= \bar{a}_0 \left[\frac{\bar{c}_1}{(1+\bar{r}_1)\bar{a}_0} \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0} \right], \\ &= \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right) + \bar{k}_0, \\ B^i_{-1} = \bar{P}_0 \bar{b}^i_0 &= \bar{P}_0 \left[\bar{a}_0 - \bar{k}^i_0 \right], \\ &= -\bar{P}_0 \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right), \end{aligned}$$

where we use $\bar{q}_0 = 1$ and, by (59),

$$\gamma = \left[\int_{i \notin C} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right]^{-1} \frac{\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}^i_0 \right] di}{\bar{c}_1}.$$

It follows that

$$\begin{aligned} -\frac{(1+i_{-1})B^i_{-1}}{P_0} &= (1+i_{-1}) \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right), \\ k^i_{-1} - k_{-1} &= \frac{\bar{k}_0}{\alpha} \left(\frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} - 1 \right). \end{aligned}$$

Hence,

$$\begin{aligned} &\left(\int_{i \notin C} \frac{\bar{c}^i_1}{\int_{i' \notin C} \bar{c}^{i'}_1 di'} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right)^{-1} \int_{i \notin C} \bar{\xi}^i_{m_0} (\overline{mpr}_0 - \overline{mpr}^i_0) di, \\ &= \left[\int_{i \notin C} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right]^{-1} \int_{i \notin C} \bar{\xi}^i_{m_0} \left(1 - \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} \right) di, \\ &= \left(\int_{i \notin C} \frac{1}{\gamma^i(1+\eta^i)\varsigma^i} di \right)^{-1} \left[-\frac{1}{\bar{c}_1}(1+\bar{r}_1) \overline{\frac{\partial a_0}{\partial y_0}} \frac{\bar{k}_0}{\alpha} \int_{i \notin C} \left(1 - \frac{\gamma}{\gamma^i(1+\eta^i)\varsigma^i} \right)^2 di \right] \times \\ &\quad \left[(1+i_{-1}) \bar{q}_0 \frac{1}{P_0} \overline{\frac{dP_0}{dm_0}} + \overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0) \overline{\frac{dq_0}{dm_0}} \right]. \end{aligned} \tag{63}$$

where the second equality uses (58) and the third equality uses the above results.

For constrained households ($i \in C$),

$$\begin{aligned}
\bar{\xi}^i_{m_0} &= \frac{1}{\bar{c}_1}\left[\overline{\frac{d(1+r_1)a^i_0}{dm_0}} - \frac{(1+\bar{r}_1)\bar{a}_0}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di\right], \\
&= \frac{1}{\bar{c}_1}\left[\overline{\frac{d(1+r_1)a^i_0}{dm_0}} - \int_0^1 \overline{\frac{dc^i_1}{dm_0}}di\right] + \frac{1}{\bar{c}_1}\frac{\bar{w}_1}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di, \\
&= \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})}{P_0}B^i_{-1}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k^i_{-1}-k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right] - \\
&\quad \frac{1}{\bar{c}_1}\overline{\frac{dw_1}{dm_0}} + \frac{1}{\bar{c}_1}\frac{\bar{w}_1}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di, \\
&= \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})}{P_0}B^i_{-1}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k^i_{-1}-k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right] + \\
&\quad \frac{\bar{w}_1}{\bar{c}_1}\left[\frac{1}{\bar{c}_1}\int_0^1 \overline{\frac{dc^i_1}{dm_0}}di - \frac{1}{\bar{w}_1}\overline{\frac{dw_1}{dm_0}}\right],
\end{aligned}$$

where the second equality uses (61) and the third equality uses (62) as well as Proposition 7. By (57)

$$\begin{aligned}
k^i_{-1} = \bar{k}^i_0 &= \frac{\bar{a}_0\omega^i_0}{\bar{q}_0}, \\
&= \bar{k}_0(\omega^i_0 - 1) + \bar{k}_0, \\
B^i_{-1} = \bar{P}_0\bar{b}^i_0 &= \bar{P}_0\bar{a}_0(1-\omega^i_0), \\
&= -\bar{P}_0\bar{k}_0\left(\omega^i_0 - 1\right).
\end{aligned}$$

It follows that

$$\begin{aligned}
-\frac{(1+i_{-1})B^i_{-1}}{\bar{P}_0} &= (1+i_{-1})\bar{k}_0\left(\omega^i_0 - 1\right), \\
k^i_{-1} - k_{-1} &= \bar{k}_0\left(\omega^i_0 - 1\right).
\end{aligned}$$

Hence,

$$\left(\int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_{i\in C}\bar{\xi}_{m_0}^i(\overline{mpr}_0-\overline{mpr}_0^i)di,$$
$$=\left(\int_{i\notin C}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_{i\in C}\bar{\xi}_{m_0}^i\left(1-\omega_0^i\right)di,$$
$$=\left(\int_{i\notin C}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\times$$
$$\left(\left[-\frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\bar{k}_0\int_{i\in C}\left(1-\omega_0^i\right)^2di\right]\left[(1+i_{-1})\bar{q}_0\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}}+\overline{\frac{d\pi_0}{dm_0}}+(1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right]+\right.$$
$$\left.\left(\int_{i\in C}(1-\omega_0^i)di\right)\frac{\bar{w}_1}{\bar{c}_1}\left[\frac{1}{\bar{c}_1}\int_0^1\overline{\frac{dc_1^i}{dm_0}}di-\frac{1}{\bar{w}_1}\overline{\frac{dw_1}{dm_0}}\right]\right). \tag{64}$$

Now note that the characterization of $\overline{\frac{dP_0}{dm_0}}$, $\overline{\frac{d\pi_0}{dm_0}}$, and $\overline{\frac{dq_0}{dm_0}}$ is unchanged from the proof of Proposition 3. Furthermore, since limiting goods market clearing in period 1

$$\int_0^1\bar{c}_1^i di=\bar{z}_1\bar{k}_0^\alpha$$

implies

$$\frac{1}{\bar{c}_1}\int_0^1\frac{d\bar{c}_1^i}{d\bar{m}_0}di=\alpha\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0}$$

while limiting labor demand in period 1

$$\bar{w}_1=(1-\alpha)\bar{z}_1\bar{k}_0^\alpha$$

implies

$$\frac{1}{\bar{w}_1}\frac{d\bar{w}_1}{d\bar{m}_0}=\alpha\frac{1}{\bar{k}_0}\frac{d\bar{k}_0}{d\bar{m}_0},$$

we have that

$$\left[\frac{1}{\bar{c}_1}\int_0^1\overline{\frac{dc_1^i}{dm_0}}di-\frac{1}{\bar{w}_1}\overline{\frac{dw_1}{dm_0}}\right]=0.$$

Hence, combining (63) and (64) in (60) implies

$$\begin{aligned}\zeta_{m_0} &= \left(\int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'}\frac{1}{\gamma^i(1+\eta^i)\varsigma^i}di\right)^{-1}\int_0^1\bar{\xi}_{m_0}^i(1-\overline{mpr}_0^i)di\\ &\propto -(1+i_{-1})\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}}-\overline{\frac{d\pi_0}{dm_0}}-(1-\delta_0)\overline{\frac{dq_0}{dm_0}},\\ &> 0\end{aligned}$$

for χ^W sufficiently large. □

Proposition 5

Proof. Recall that the monetary policy rule (10) and $P_1 = \bar{P}_1$ implies a real interest rate

$$1+r_1 = \frac{(1+\bar{i})}{P_{-1}^{\phi}}\frac{(P_0)^{1+\phi}}{\bar{P}_1}m_0,$$

which then implies the exact log-linear relationship

$$\hat{r}_1 = (1+\phi)\hat{P}_0 + \hat{m}_0.$$

When $\phi = -1$, it follows that

$$\hat{r}_1 = \hat{m}_0.$$

Given the expansion in state variables

$$\hat{r}_1^k = \delta_{m_0}^{r_1}\hat{m}_0 + \hat{z}_1 + \frac{1}{2}\delta_{\sigma^2}^{r_1^k}\sigma^2 + \frac{1}{2}\delta_{z_1^2}^{r_1^k}z_1^2 + \frac{1}{2}\delta_{m_0\sigma^2}^{r_1^k}\hat{m}_0\sigma^2 + \frac{1}{2}\delta_{m_0 z_1^2}^{r_1^k}\hat{m}_0 z_1^2 + o(||\cdot||^4),$$

it follows from Proposition 2 that

$$\frac{1}{2}\delta_{m_0\sigma^2}^{r_1^k} + \frac{1}{2}\delta_{m_0 z_1^2}^{r_1^k} = \zeta_{m_0}.$$

Now by the definition of the return on capital, we have the exact log-linear relationship

$$\hat{r}_1^k = \hat{z}_1 - (1-\alpha+\chi^x)\,\hat{k}_0$$

as derived in the main text. It follows by the method of undetermined coefficients

$$\frac{1}{2}\delta^{r_1^k}_{m_0\sigma^2} = -\left(1-\alpha+\chi^x\right)\frac{1}{2}\delta^{k_0}_{m_0\sigma^2},$$
$$\frac{1}{2}\delta^{r_1^k}_{m_0 z_1^2} = 0.$$

Hence, the above results imply

$$\frac{1}{2}\delta^{r_1^k}_{m_0\sigma^2} = -\frac{1}{1-\alpha+\chi^x}\zeta_{m_0},$$

proving the claim. □

Proposition 6

Proof. Given $\psi^i = 1$, each household's first order condition for saving in each asset given the problem (18) is

$$\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}(ce_0^i)^{\gamma^i-1}\mathbb{E}_0(c_1^i)^{-\gamma^i}(1+r_1^k), \tag{65}$$
$$\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}(ce_0^i)^{\gamma^i-1}\mathbb{E}_0(c_1^i)^{-\gamma^i}(1+r_1), \tag{66}$$

where $ce_0^i \equiv \left(\mathbb{E}_0\left[\left(c_1^i\right)^{1-\gamma^i}\right]\right)^{\frac{1}{1-\gamma^i}}$ is the certainty equivalent of future consumption. In equilibrium, recall that

$$w_1 = (1-\alpha)z_1 k_0^{\alpha},$$
$$1+r_1^k = \frac{\alpha z_1 k_0^{\alpha-1}}{q_0},$$

so the period 1 resource constraint can be written

$$c_1^i = \left(q_0 k_0^i + \frac{1-\alpha}{\alpha}q_0 k_0\right)(1+r_1^k) + b_0^i(1+r_1).$$

Hence, multiplying (65) by $\left(q_0 k_0^i + \frac{1-\alpha}{\alpha}q_0 k_0\right)$ and (66) by b_0^i, and then adding them, we obtain

$$\left(a_0^i + \frac{1-\alpha}{\alpha}q_0 k_0\right)\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}(ce_0^i)^{\gamma^i-1}\mathbb{E}_0(c_1^i)^{1-\gamma^i}$$

given total savings $a_0^i \equiv b_0^i + q_0 k_0^i$. Given the definition of the certainty equivalent, this simplifies to

$$\left(a_0^i + \frac{1-\alpha}{\alpha} q_0 k_0\right) \left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}. \tag{67}$$

Solving this for a_0^i and substituting this into the period 0 resource constraint implies

$$c_0^i = (1-\beta^i)\left[y_0^i(w_0 \ell_0^i, P_0, \pi_0, q_0) + \frac{1-\alpha}{\alpha} q_0 k_0\right].$$

As described in the main text, the additional assumption $\beta^i = \beta$, together with goods market clearing (11), implies that

$$c_0 = \frac{1-\beta}{\beta}\frac{1}{\alpha} q_0 k_0 = \frac{1-\beta}{\beta}\frac{1}{\alpha}\left(\frac{k_0}{k_{-1}}\right)^{\chi^x} k_0.$$

It follows that we have the exact log-linear relationship

$$\hat{c}_0 = (\chi^x + 1)\hat{k}_0.$$

The claim immediately follows.

We now demonstrate that the same result holds in the more general environment featuring portfolio constraints / rules-of-thumb, background risk, and subjective beliefs regarding aggregate TFP. In this environment, each household's problem (18) generalizes to

$$\begin{aligned} \max \Bigg((1-\beta^i)\left(c_0^i \Phi^i(\ell_0^i)\right)^{1-1/\psi^i} + \beta^i \left(\mathbb{E}_0^i\left[(c_1^i)^{1-\gamma^i}\right]\right)^{\frac{1-1/\psi^i}{1-\gamma^i}} \Bigg)^{\frac{1}{1-1/\psi^i}} \quad s.t. \\ c_0^i + b_0^i + q_0 k_0^i = y_0^i(w_0 \ell_0^i, P_0, \pi_0, q_0), \\ c_1^i = \epsilon_1^i w_1 + (1+r_1) b_0^i + \epsilon_1^i \pi_1 k_0^i, \end{aligned} \tag{68}$$

where $\mathbb{E}_0^i$ is the household's expectation given their subjective belief regarding TFP, ϵ_1^i is the household's idiosyncratic labor productivity and capital quality shock, and if $i \in C$ the household faces the additional constraint $q_0 k_0^i = \omega_0^i (b_0^i + q_0 k_0^i)$. For this claim we assume a particular form of the constraint among such agents, $\omega_0^i = 1$.

Then each household's first order condition for saving in each asset is

$$\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}(c e_0^i)^{\gamma^i - 1} \mathbb{E}_0^i (c_1^i)^{-\gamma^i}(1 + r_1^k) - \nu^i \left(\omega_0^i - 1\right), \tag{69}$$

$$\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}(c e_0^i)^{\gamma^i - 1} \mathbb{E}_0^i (c_1^i)^{-\gamma^i}(1 + r_1) - \nu^i \omega_0^i, \tag{70}$$

where ν^i is the (scaled) multiplier on the portfolio constraint, equaling zero for $i \notin C$, and $ce_0^i \equiv \left(\mathbb{E}_0^i\left[(c_1^i)^{1-\gamma^i}\right]\right)^{\frac{1}{1-\gamma^i}}$ is the certainty equivalent. The equilibrium relationship between w_1 and $1+r_1^k$ remains unchanged from above, so that the period 1 resource constraint can be written

$$c_1^i = \epsilon_1^i\left(q_0 k_0^i + \frac{1-\alpha}{\alpha} q_0 k_0\right)(1+r_1^k) + b_0^i(1+r_1).$$

Hence, multiplying (69) by $\left(q_0 k_0^i + \frac{1-\alpha}{\alpha} q_0 k_0\right)$ and (66) by b_0^i, and then adding them, we obtain

$$\left(a_0^i + \frac{1-\alpha}{\alpha} q_0 k_0\right)\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}(ce_0^i)^{\gamma^i - 1}\mathbb{E}_0^i(c_1^i)^{1-\gamma^i}$$

where we have used the assumption that for constrained households, $\omega_0^i = 1$. Given the definition of the certainty equivalent, this simplifies to

$$\left(a_0^i + \frac{1-\alpha}{\alpha} q_0 k_0\right)\left(c_0^i\right)^{-1} = \frac{\beta^i}{1-\beta^i}$$

as in (67). The remainder of the proof proceeds as in the baseline environment. □

B Additional analytical results

In this appendix we provide supplementary analytical results accompanying section 2.

B.1 Redistributive effects of a monetary policy shock

We first characterize the redistributive effects of a monetary policy shock.

As discussed in the main text, the relevant measure of redistribution to evaluate the effect on the risk premium is

$$\bar{\xi}_{m_0}^i = \frac{1}{\int_0^1 \bar{c}_1^{i'} di'}\left[\overline{\frac{dc_1^i}{dm_0}} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'}\int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}} di'\right].$$

Using standard tools from price theory, we can decompose each household's change in future consumption as follows:

Proposition 7. *A household's change in future consumption in response to a monetary*

policy shock is given by

$$\overline{\frac{dc_1^i}{dm_0}} = (1+\bar{r}_1)\underbrace{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}_{MPS}\Bigg[\underbrace{-\frac{(1+i_{-1})B_{-1}^i}{\bar{P}_0}\frac{1}{\bar{P}_0}\overline{\frac{dP_0}{dm_0}} + k_{-1}^i\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)}_{balance\ sheet\ revaluation}$$
$$+\underbrace{\left(\overline{\frac{dw_0\ell_0^i}{dm_0}} + \frac{1}{1+\bar{r}_1}\overline{\frac{dw_1}{dm_0}}\right)}_{change\ in\ non\text{-}traded\ income} + \underbrace{\bar{a}_0^i\frac{1}{1+\bar{r}_1}\overline{\frac{d(1+r_1)}{dm_0}}}_{income\ effect}$$
$$+\underbrace{\psi^i c_0^i\frac{1}{1+\bar{r}_1}\overline{\frac{d(1+r_1)}{dm_0}} + \left(\psi^i-1\right)\bar{w}_0\left(1-\bar{\tau}^{\ell_0^i}\right)\overline{\frac{d\ell_0^i}{dm_0}}}_{substitution\ effects}\Bigg]$$

given the labor wedge for household i $\bar{\tau}^{\ell_0^i} \equiv 1 - \frac{-\bar{c}_0^i\Phi^{i'}(\bar{\ell}_0^i)/\Phi^i(\bar{\ell}_0^i)}{(1-\alpha)\bar{z}_0(\bar{\ell}_0)^{-\alpha}k_{-1}^{\alpha}}$.

The resulting redistribution summarized in $\xi_{m_0}^i$ reflects heterogeneity in the marginal propensity to save; heterogeneity in changes in wealth; and heterogeneity in substitution effects. We further discuss the second here.[49]

First, a cut in the nominal interest rate will trigger a revaluation of household balance sheets, the only channel operative under the special conditions of Proposition 3 in the main text. Second, a cut in the nominal interest rate will change the net present value of non-traded labor income. If it lowers the real wage in the short run ($\overline{\frac{dw_0}{dm_0}} > 0$), the standard effect with sticky nominal wages rather than prices, it will redistribute to households supplying less labor. If it raises the quantity of labor demanded ($\overline{\frac{d\ell_0}{dm_0}} < 0$), it will redistribute to households whose labor demand is especially sensitive to the aggregate. Third, if a cut in the nominal interest rate lowers the equilibrium real interest rate ($\overline{\frac{d(1+r_0)}{dm_0}} > 0$), it will redistribute wealth away from net savers through a Slutsky income effect. These heterogeneous exposures to a monetary shock have been previously exposited in the literature on HANK models, as by Auclert (2018).[50] Our analysis demonstrates that it is their covariance with MPRs rather than MPCs which matters for transmission through risk-premia rather than risk-free rates.

[49] The role of the marginal propensity to save and substitution effect due to a change in the real interest rate is straightforward. The substitution effect due to a change in $\bar{\ell}_0^i$ reflects the non-separability of labor from consumption in period 0 when $\psi^i \neq 1$.

[50] We can re-arrange the terms in Proposition 7 to obtain a decomposition consistent with Auclert (2018). Since $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_1}$, we have that

$$\overline{\frac{dq_0}{dm_0}} = \frac{1}{1+\bar{r}_1}\overline{\frac{d\pi_1}{dm_0}} - \frac{\bar{q}_0}{1+\bar{r}_1}\overline{\frac{d(1+\bar{r}_1)}{dm_0}}.$$

It follows that we can equivalently write the sum of the balance sheet revaluation, change in non-traded

B.2 Generalizations to other sources of heterogeneity

We next demonstrate that Propositions 1 and 2 generalize to environments with heterogeneity in portfolio constraints, rules-of-thumb, background risk, or beliefs. We refer the reader to the description of these richer environments in section 2.4 of the main text.

Binding constraints or rules-of-thumb We have:

Corollary 1. *With binding constraints or rules-of-thumb, households' limiting portfolios and MPRs are*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \begin{cases} \omega_0^i \ \textit{for} \ i \in C, \\ \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i} \ \textit{for} \ i \notin C, \end{cases}$$

$$\overline{mpr}_0^i = \begin{cases} \omega_0^i \ \textit{for} \ i \in C, \\ \frac{\gamma}{\gamma^i} \ \textit{for} \ i \notin C, \end{cases}$$

where

$$\gamma = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\notin C} \bar{c}_1^{i'} di'} \frac{1}{\gamma^i} di\right)^{-1} \left(1 - \frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_0^i di}{\int_{i\notin C} \bar{c}_1^i di}\right).$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$, we obtain (21) with γ defined above and

$$\zeta_{m_0} = \left(\int_{i\notin C} \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i} di\right)^{-1} \int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i\right) di,$$

where $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[(1+r_0)a_0^i / \int_0^1 c_1^{i'} di'\right]}}{dm_0}$ *for* $i \in C$, $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[c_1^i / \int_0^1 c_1^{i'} di'\right]}}{dm_0}$ *for* $i \notin C$, *and*
$\overline{mpr}_0 \equiv \int_{i\in C} \frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'} di' + \int_{i'\notin C} \bar{c}_1^{i'} di'} \overline{mpr}_0^i di + \int_{i\notin C} \frac{\bar{c}_1^i}{\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'} di' + \int_{i'\notin C} \bar{c}_1^{i'} di'} \overline{mpr}_0^i di = 1.$

The risk premium γ now depends not only on the weighted average risk aversion of unconstrained households, but also on the leverage which these households must take in

income, and income effect as

$$\left[-\frac{(1+i_{-1})B_{-1}^i}{\bar{P}_0}\frac{1}{\bar{P}_0}\overline{\frac{dP_0}{dm_0}}\right] + \left[\left(\bar{b}_0^i + \bar{q}_0(\bar{k}_0^i - (1-\delta_0)k_{-1}^i)\right)\frac{1}{1+\bar{r}_1}\overline{\frac{d(1+r_1)}{dm_0}}\right] + \left[k_{-1}^i\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\frac{1}{1+\bar{r}_1}\overline{\frac{d\pi_1}{dm_0}}\right) + \left(\overline{\frac{dw_0\ell_0^i}{dm_0}} + \frac{1}{1+\bar{r}_1}\overline{\frac{dw_1}{dm_0}}\right)\right].$$

The first bracketed term corresponds to the Fisher channel, the second bracketed term to the interest rate exposure channel, and the third bracketed term to the earnings heterogeneity channel in Auclert (2018). In our decomposition, we find it convenient to explicitly account for the effect on the price of capital to aid the interpretation of our quantitative simulations.

aggregate to hold the economy's capital stock after accounting for the positions of constrained households. For this reason, the effect of a monetary policy shock on the risk premium ζ_{m_0} depends on the MPRs of constrained households. For instance, if wealth transfers to households who cannot trade capital and thus have $\overline{mpr}_0^i = 0$, in equilibrium the remaining households must be induced to hold a more levered position in capital and thus the risk premium must rise.

Background risk We have:[51]

Corollary 2. *With background risk, households' limiting portfolios and MPRs are*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i(1+\eta^i)} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$
$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i(1+\eta^i)}.$$

where

$$\gamma = \left(\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i(1+\eta^i)} di\right)^{-1}.$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$*, we obtain (21) with* γ *defined above and*

$$\zeta_{m_0} = \gamma \int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i\right) di,$$

where $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[c_1^i / \int_{i' \notin C} c_1^{i'} di'\right]}}{dm_0}$ *and* $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i di = 1$.

Corollary 2 is consistent with the empirical finding that households with greater background risk η^i uncorrelated with the stock market hold a lower portfolio share in stocks (Heaton and Lucas (2000)). It also implies that households with different amounts of background risk will have different MPRs. Redistribution across these households in turn will induce changes in the risk premium owing to their trades in asset markets.

Subjective beliefs We have:

[51] We continue to define $1 + r_1^k \equiv \frac{\pi_1}{q_0}$, a claim on capital aggregating over the idiosyncratic risk, even though each household i faces the set of asset returns $\{1 + r_0, 1 + r_1^{k,i} \equiv \frac{\pi_1 \epsilon_1^i}{q_0}\}$.

Corollary 3. *With subjective beliefs, households' limiting portfolios and MPRs are*

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i \varsigma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i},$$
$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i \varsigma^i}.$$

where

$$\gamma = \left(\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \frac{1}{\gamma^i \varsigma^i} di\right)^{-1}.$$

Up to third order in $\{\sigma, \hat{z}_1, \hat{m}_0\}$*, we obtain (21) with* γ *defined above and*

$$\zeta_{m_0} = \gamma \int_0^1 \bar{\xi}_{m_0}^i \left(\overline{mpr}_0 - \overline{mpr}_0^i\right) di,$$

where $\bar{\xi}_{m_0}^i \equiv \frac{\overline{d\left[c_1^i / \int c_1^{i'} di'\right]}}{dm_0}$ *and* $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}_0^i di = 1.$

Consistent with the empirical literature on belief disagreements, beliefs will be reflected in individual portfolio choice and marginal portfolio responses to shocks. Consistent with the theoretical literature exploring the consequences of such disagreements, these individual responses can then have important consequences for the macroeconomy through their effect on asset prices.

B.3 Robustness to features of economic environment

We next demonstrate the robustness of our analytical results to two features of the economic environment: the specification of labor supply, and inflation risk in the nominal bond.

B.3.1 Individually supplied labor

We first consider the case with individually-supplied labor rather than the union set-up assumed in the main text.

Modified environment and equilibrium We dispense with the index j and assume households directly supply distinct varieties of labor to the market at wages $\{W_0^i\}$. Household preferences thus can be written

$$v_0^i = \left((1-\beta^i)\left(c_0^i \Phi^i\left(\ell_0^i\right)\right)^{1-1/\psi^i} + \beta^i \left(\mathbb{E}_0\left[(c_1^i)^{1-\gamma^i}\right]\right)^{\frac{1-1/\psi^i}{1-\gamma^i}}\right)^{\frac{1}{1-1/\psi^i}}$$

and the resource constraints become

$$\begin{aligned} P_0 c_0^i + B_0^i + Q_0 k_0^i \leq\ & (1-\tau) W_0^i \ell_0^i - AC_0^{W,i} \ + \\ & (1+i_{-1}) B_{-1}^i + (\Pi_0 + (1-\delta_0) Q_0) k_{-1}^i + T_0^i, \\ P_1 c_1^i \leq\ & W_1 + (1+i_0) B_0^i + \Pi_1 k_0^i \end{aligned}$$

with adjustment costs

$$AC_0^{W,i} = \frac{\chi^W}{2} W_0 \ell_0 \left(\frac{W_0^i}{W_{-1}} - 1 \right)^2.$$

The labor packer directly hires labor from households and combines it using the CES aggregator, earning profits

$$W_0 \left[\int_0^1 (\ell_0^i)^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_0^i \ell_0^i di.$$

The notation in the government transfer condition (9) and labor market clearing condition (13) must be changed, and the equilibrium in Definition 1 is otherwise the same.

In equilibrium households will generically supply different amounts of labor and earn different wages solving

$$\begin{aligned} \frac{W_0^i}{P_0} + c_0^i \frac{\Phi^{i\prime}(\ell_0^i)}{\Phi^i(\ell_0^i)} + \frac{W_0}{P_0} \frac{\chi^W}{\epsilon} \frac{\ell_0}{\ell_0^i} \frac{W_0^i}{W_{-1}} \left(\frac{W_0^i}{W_{-1}} - 1 \right) = 0, \\ \ell_0^i = \left(\frac{W_0^i}{W_0} \right)^{-\epsilon} \ell_0, \end{aligned}$$

conditional on their choice of consumption c_0^i and the aggregates $\{W_0, P_0, \ell_0\}$. This contrasts with labor supply in the baseline economy, where households earn identical wages and the representative union's labor supply condition is

$$\begin{aligned} \int_0^1 \mu^i \left(v_0^i\right)^{\frac{1}{\psi^i}} \left(c_0^i\right)^{-\frac{1}{\psi^i}} \Phi^i(\ell^i(\ell_0^i))^{1-\frac{1}{\psi^i}} \ell^{i\prime}(\ell_0) \Bigg[& \frac{W_0}{P_0} \left(\frac{\ell^i(\ell_0)}{\ell_0 \ell^{i\prime}(\ell_0)} \frac{1}{1-\epsilon} - \frac{\epsilon}{1-\epsilon} \right) + c_0^i \frac{\Phi^{i\prime}(\ell^i(\ell_0))}{\Phi^i(\ell^i(\ell_0))} \\ & + \frac{W_0}{P_0} \frac{1}{\ell^{i\prime}(\ell_0)} \frac{\chi^W}{\epsilon} \frac{W_0}{W_{-1}} \left(\frac{W_0}{W_{-1}} - 1 \right) \Bigg] di = 0. \end{aligned}$$

given the allocation rule $\ell^i(\cdot)$ defined in (4) and Pareto weights $\{\mu^i\}$.

Robustness of results Now each household is characterized by a marginal propensity to work $\frac{\partial \ell_0^i}{\partial y_0^i}$ and marginal propensity to set its wage $\frac{\partial w_0^i}{\partial y_0^i}$ in addition to its marginal propensities to consume, save in bonds, save in capital, and save overall. Nonetheless, Proposition 1

remains unchanged.

The characterization of the risk premium up to third order in Proposition 2 is unchanged. However, the monetary policy exposures $\bar{\xi}^i_{m_0}$ now reflect households' alternative adjustment on the supply-side. In particular, Proposition 7 must be adjusted to reflect the fact that each household is no longer a price-taker in the labor market. However, when households are identical except for risk aversion and their portfolio shares, these changes are irrelevant for $\bar{\xi}^i_{m_0}$; that is, balance sheet revaluation remains the only source of redistribution. This is formalized in the following result:

Proposition 8. *Consider the case with individually-supplied labor by each household but assume the conditions of Proposition 3 hold. Then*

$$\bar{\xi}^i_{m_0} = \frac{1}{\bar{c}_1}(1+\bar{r}_1)\overline{\frac{\partial a_0}{\partial y_0}}\left[-\frac{(1+i_{-1})B^i_{-1}}{P_0}\frac{1}{P_0}\overline{\frac{dP_0}{dm_0}} + (k^i_{-1}-k_{-1})\left(\overline{\frac{d\pi_0}{dm_0}} + (1-\delta_0)\overline{\frac{dq_0}{dm_0}}\right)\right]$$

where $\bar{c}_1$ is the identical level of consumption across households and $\overline{\frac{\partial a_0}{\partial y_0}}$ the identical marginal propensity to save of households at the point of approximation.

It immediately follows that Proposition 3 remains unchanged. The generalizations to other forms of heterogeneity in Corollaries 1-3 are unchanged, and thus Proposition 4 is unchanged as well. Finally, the effects of a monetary policy shock on the real economy operating through the change in the risk premium in Propositions 5 and 6 are unchanged.

B.3.2 Inflation risk in the nominal bond

We next consider the case with inflation risk in the nominal bond.

Modified environment and equilibrium We now assume the monetary authority lets the future price level vary with TFP

$$P_1 = \bar{P}_1(z_1)^\iota.$$

The baseline environment featured $\iota = 0$. It is further straightforward to add another source of risk in P_1 corresponding to a distinct monetary policy shock, but we do not do that for expositional parsimony. Beyond this change to monetary policy, the definition of equilibrium in Definition 1 is otherwise unchanged.

In the baseline economy the realized real interest rate was given by

$$1 + r_1 \equiv (1+i_0)\frac{P_0}{P_1} = \frac{1+\bar{i}}{P^\phi_{-1}}\frac{P_0^{1+\phi}}{\bar{P}_1}m_0$$

and thus was known with certainty as of period 0. In contrast in the present economy the realized real interest rate given by

$$1 + r_1 \equiv (1 + i_0)\frac{P_0}{P_1} = \frac{1 + \bar{i}}{P_{-1}^{\phi}} \frac{P_0^{1+\phi}}{\bar{P}_1 (z_1)^{\iota}} m_0$$

is uncertain as of period 1.

Robustness of results Proposition 1 must be adjusted because households' limiting portfolios and MPRs are affected by the presence of inflation risk:

Proposition 9. *With inflation risk in the nominal bond,*

$$\begin{aligned}
\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} &= \frac{1}{1+\iota}\left[\left(\frac{\bar{c}_1^i}{(1+\bar{r}_1)\bar{a}_0^i}\right)\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1+\bar{r}_1)\bar{a}_0^i} + \iota\right], \\
\overline{mpr}_0^i &= \frac{1}{1+\iota}\left[\frac{\gamma}{\gamma^i} + \iota\right],
\end{aligned}$$

where γ remains characterized by (20).

Expected excess returns up to third order are also modified:

Proposition 10. *Up to third order in the perturbation parameters $\{\sigma, \hat{z}_1, \hat{m}_0\}$,*

$$\mathbb{E}_0 \hat{r}_1^k - \mathbb{E}_0 \hat{r}_1 + \frac{1}{2}\left(1 - \iota^2\right)\sigma^2 = \gamma\left(1 + \iota\right)\sigma^2 + \zeta_{m_0}\hat{m}_0\left(1 + \iota\right)^2\sigma^2 + o(||\cdot||^4),$$

where γ is defined in (20) and ζ_{m_0} is defined in (22).

Intuitively, as $\iota \to -1$, the real payoff to the nominal bond perfectly replicates that of capital, eliminating any excess returns on capital and the effect of a monetary policy shock on those expected excess returns. Aside from an appropriate re-scaling of each term, however, it remains the case that the distribution of monetary policy exposures and MPRs determines the effect of a monetary shock on the risk premium.

Proposition 7 characterizing the change in households' consumption in response to a monetary policy shock is unchanged. It follows that the balance sheet revaluation underlying Proposition 3 remains unchanged. The generalizations to other forms of heterogeneity in Corollaries 1-3 must be modified like the above results, but Proposition 4 remains unchanged. Finally, the effects of a monetary policy shock on the real economy operating through the change in the risk premium in Propositions 5 and 6 are unchanged.

B.4 Effect of TFP shocks

We finally demonstrate that our analytical insights extend beyond monetary policy shocks to any shock which redistributes across households in period 0 or (in expectation) period 1. Here we focus on a TFP shock in period 0, corresponding to our quantitative analysis of productivity shocks in appendix E. Formally, we treat $\hat{z}_0$ as another perturbation parameter of interest. For expositional simplicity, we assume $\hat{m}_0 = 0$, though it is straightforward to consider both TFP and monetary shocks since they simply enter additively.

We first obtain the analog of Proposition 2 for a TFP shock:

Proposition 11. *Up to third order in the perturbation parameters* $\{\sigma, \hat{z}_1, \hat{z}_0\}$*,*

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \zeta_{z_0}\hat{z}_0\sigma^2 + o(||\cdot||^4),$$

where γ *was defined in (20) and*

$$\zeta_{z_0} = \gamma \int_0^1 \bar{\xi}^i_{z_0} \left(\overline{mpr}_0 - \overline{mpr}^i_0\right) di,$$

where $\bar{\xi}^i_{z_0} \equiv \frac{\overline{d\left[c_1^i / \int_0^1 c_1^{i'} di'\right]}}{dz_0}$ *is the effect of a TFP shock on household* i*'s consumption share in period 1 and* $\overline{mpr}_0 \equiv \int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \overline{mpr}^i_0 = 1$ *is the weighted average MPR in (1).*

As is evident, ζ_{z_0} parallels ζ_{m_0} for a monetary shock. In this simple two-period environment, a TFP shock affects the risk premium only through redistribution. If a positive TFP shock redistributes wealth to households with high MPRs, it will lower the risk premium.

The change in consumption relevant to evaluate the redistributive effects of a TFP shock is analogous to Proposition 7:

Proposition 12. *A household's change in future consumption in response to a TFP shock is given by*

$$\begin{aligned}
\overline{\frac{dc_1^i}{dz_0}} = (1+\bar{r}_1) \underbrace{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}_{MPS} \Bigg[& \underbrace{- \frac{(1+i_{-1})B^i_{-1}}{\bar{P}_0} \frac{1}{\bar{P}_0} \overline{\frac{dP_0}{dz_0}} + k_{-1} \left(\overline{\frac{d\pi_0}{dz_0}} + (1-\delta_0) \overline{\frac{dq_0}{dz_0}} \right)}_{\text{balance sheet revaluation}} \\
& + \underbrace{\left(\overline{\frac{dw_0 \ell_0^i}{dz_0}} + \frac{1}{1+\bar{r}_1} \overline{\frac{dw_1}{dz_0}} \right)}_{\text{change in non-traded income}} + \underbrace{\bar{a}_0^i \frac{1}{1+\bar{r}_1} \overline{\frac{d(1+r_1)}{dz_0}}}_{\text{income effect}} \\
& + \underbrace{\psi^i c_0^i \frac{1}{1+\bar{r}_1} \overline{\frac{d(1+r_1)}{dz_0}} + \left(\psi^i - 1\right) \bar{w}_0 \left(1 - \bar{\tau}^{\ell_0^i}\right) \overline{\frac{d\ell_0^i}{dz_0}}}_{\text{substitution effects}} \Bigg]
\end{aligned}$$

given the steady-state labor wedge for household i $\bar{\tau}^{\ell^i}_0 \equiv 1 - \frac{-\bar{c}^i_0 \Phi^{i'}(\bar{\ell}^i_0)/\Phi^i(\bar{\ell}^i_0)}{(1-\alpha)\bar{z}_0(\bar{\ell}_0)^{-\alpha}k^{\alpha}_{-1}}$.

When households are symmetric in all respects except risk aversion and their portfolio shares, only the balance sheet revaluation channel will redistribute across them. However, a TFP shock can have different effects on prices, profits, and the price of capital than a monetary policy shock. For instance, for χ^W sufficiently large, as is assumed in Proposition 3, we can prove that $\overline{\frac{dP_0}{dz_0}} < 0$, $\overline{\frac{dq_0}{dz_0}} > 0$, and the sign of $\overline{\frac{d\pi_0}{dz_0}}$ depends on parameters. It follows that the effect of a positive TFP shock on the risk premium depends on parameters, because it both redistributes away from levered, high MPR households by raising the real value of their debt burden, redistributes toward these same households by raising the price of capital, and has an ambiguous effect on redistribution through short-run profits.

Corollaries 1-3 generalize as above to the case of TFP shocks, but again the risk premium effects of a TFP shock under the conditions of Proposition 4 are ambiguous. Whatever the sign of this risk premium response, we obtain the following analogs of Proposition 5 and 6:

Proposition 13. *If monetary policy follows the rule (10) with* $\phi = -1$*, then*

$$\frac{1}{2}\delta^{k_0}_{z_0\sigma^2} = -\frac{1}{1-\alpha+\chi^x}\zeta_{z_0},$$

given ζ_{z_0} *characterized in Proposition 11.*

Proposition 14. *If* $\psi^i = 1$ *and* $\beta^i = \beta$ *for all* i*, then*

$$\frac{1}{2}\delta^{c_0}_{m_0\sigma^2} = \frac{1}{2}\left(\chi^x + 1\right)\delta^{k_0}_{m_0\sigma^2}.$$

If monetary policy follows the rule (10) with $\phi = -1$*, this implies*

$$\frac{1}{2}\delta^{c_0}_{m_0\sigma^2} = -\frac{\chi^x + 1}{1-\alpha+\chi^x}\zeta_{m_0}.$$

Again, provided the monetary policy rule keeps the real interest rate constant, an increase (decrease) in the risk premium induced by the TFP shock will lower (raise) investment and consumption.

B.5 Proofs

Here we provide proofs of Proposition 7 and Corollaries 1-3. We exclude proofs of the other supplemental results in this appendix for brevity, but they are available on request.

Proposition 7

Proof. Assuming that $\frac{dc_1^i}{dm_0}$ is continuous in σ, it is equivalent to characterize $\frac{dc_1^i}{dm_0}$ and then evaluate its limit at the deterministic steady-state ($\sigma = 0$) or simply compute $\frac{d\bar{c}_1^i}{d\bar{m}_0}$ at this limit. It is expositionally simpler to do the latter, so we do that here.

Re-consider households' micro-level optimization problem (18) given $\sigma = 0$:

$$\begin{aligned}\max \left((1-\beta^i) \left(\bar{c}_0^i \Phi^i(\bar{\ell}_0^i) \right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\bar{c}_1^i \right)^{1-\frac{1}{\psi^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \quad s.t. \\ \bar{c}_0^i + \bar{a}_0^i = \bar{y}_0^i(\bar{w}_0 \bar{\ell}_0^i, \bar{P}_0, \bar{\pi}_0, \bar{q}_0), \\ \bar{c}_1^i = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^i,\end{aligned}$$

defining policy functions

$$\bar{c}_1^i(\bar{y}_0^i(\bar{w}_0 \bar{\ell}_0^i, \bar{P}_0, \bar{\pi}_0, \bar{q}_0), \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1),$$

where recall that

$$\bar{y}_0^i(\bar{w}_0 \bar{\ell}_0^i, \bar{P}_0, \bar{\pi}_0, \bar{q}_0) = \bar{w}_0 \bar{\ell}_0^i + \frac{1}{\bar{P}_0}(1+i_{-1}) B_{-1}^i + (\bar{\pi}_0 + (1-\delta_0)\bar{q}_0) k_{-1}^i.$$

It follows that

$$\begin{aligned}\frac{d\bar{c}_1^i}{d\bar{m}_0} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} \left[-\frac{1}{\bar{P}_0} B_{-1}^i \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{m}_0} + k_{-1}^i \left(\frac{d\bar{\pi}_0}{d\bar{m}_0} + (1-\delta_0) \frac{d\bar{q}_0}{d\bar{m}_0} \right) + \frac{d\bar{w}_0 \bar{\ell}_0^i}{dm_0} \right] \\ + \frac{\partial \bar{c}_1^i}{\partial \bar{\ell}_0^i} \frac{d\bar{\ell}_0^i}{d\bar{m}_0} + \frac{\partial \bar{c}_1^i}{\partial (1+\bar{r}_1)} \frac{d\bar{r}_1}{d\bar{m}_0} + \frac{\partial \bar{c}_1^i}{\partial \bar{w}_1} \frac{d\bar{w}_1}{d\bar{m}_0}, \quad (71)\end{aligned}$$

where each of the partial derivatives is evaluated with respect to the policy function above. We now characterize each of these partial derivatives in turn.

First note that it is clearly the case that

$$\frac{\partial \bar{c}_1^i}{\partial \bar{w}_1} = \frac{1}{1+\bar{r}_1} \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}.$$

Then define the expenditure minimization problem dual to the utility maximization problem

above

$$\min \bar{c}_0^{i,h} + \bar{a}_0^{i,h} \;\; s.t.$$
$$\left((1-\beta^i) \left(\bar{c}_0^{i,h} \Phi^i(\bar{\ell}_0^{i,h}) \right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\bar{c}_1^{i,h} \right)^{1-\frac{1}{\psi^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \geq \bar{u}^i,$$
$$\bar{c}_1^{i,h} = \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^{i,h},$$

where we use h superscripts to denote compensated (Hicksian) policies. Letting

$$\bar{e}_0^i(\bar{u}^i, \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1)$$

denote the level of period 0 expenditure solving this problem, duality implies

$$\bar{c}_1^i(\bar{e}_0^i(\bar{u}^i, \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1), \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1) = \bar{c}_1^{i,h}(\bar{u}^i, \bar{\ell}_0^i, 1+\bar{r}_1, \bar{w}_1).$$

This leads to Slutsky identities

$$\frac{\partial \bar{c}_1^i}{\partial \bar{\ell}_0^i} = \frac{\partial \bar{c}_1^{i,h}}{\partial \bar{\ell}_0^i} - \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} \frac{\partial \bar{e}_0^i}{\partial \bar{\ell}_0^i},$$
$$\frac{\partial \bar{c}_1^i}{\partial (1+\bar{r}_1)} = \frac{\partial \bar{c}_1^{i,h}}{\partial (1+\bar{r}_1)} - \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} \frac{\partial \bar{e}_0^i}{\partial (1+\bar{r}_1)}.$$

By the Envelope Theorem,

$$\frac{\partial \bar{e}_0^i}{\partial \bar{\ell}_0^i} = -\bar{c}_0^{i,h} \frac{\Phi^{i\prime}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)},$$
$$\frac{\partial \bar{e}_0^i}{\partial (1+\bar{r}_1)} = -\frac{1}{1+\bar{r}_1} \bar{a}_0^{i,h},$$

so that we may further write the above identities as

$$\frac{\partial \bar{c}_1^i}{\partial \bar{\ell}_0^i} = \frac{\partial \bar{c}_1^{i,h}}{\partial \bar{\ell}_0^i} + \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} \left(\bar{c}_0^{i,h} \frac{\Phi^{i\prime}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)} \right),$$
$$\frac{\partial \bar{c}_1^i}{\partial (1+\bar{r}_1)} = \frac{\partial \bar{c}_1^{i,h}}{\partial (1+\bar{r}_1)} + \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} \frac{1}{1+\bar{r}_1} \bar{a}_0^{i,h}.$$

Substituting the above results into (71), using $\bar{c}_0^{i,h} = \bar{c}_0^i$ and $\bar{a}_0^{i,h} = \bar{a}_0^i$ implied by duality, and

collecting terms, we obtain

$$\frac{d\bar{c}_1^i}{d\bar{m}_0} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\Bigg[-\frac{1}{\bar{P}_0} B_{-1}^i \frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{m}_0} + k_{-1}^i \left(\frac{d\bar{\pi}_0}{d\bar{m}_0} + (1-\delta_0)\frac{d\bar{q}_0}{d\bar{m}_0}\right) + \frac{d\bar{w}_0 \bar{\ell}_0^i}{d\bar{m}_0} + \frac{1}{1+\bar{r}_1}\frac{d\bar{w}_1}{d\bar{m}_0} + \frac{1}{1+\bar{r}_1}\bar{a}_0^i \frac{d\bar{r}_1}{d\bar{m}_0} + \bar{c}_0^i \frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\frac{d\bar{\ell}_0^i}{d\bar{m}_0}\Bigg] + \frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i}\frac{d\bar{l}_0^i}{d\bar{m}_0} + \frac{\partial \bar{c}_1^{i,h}}{\partial(1+\bar{r}_1)}\frac{d\bar{r}_1}{d\bar{m}_0} \quad (72)$$

We next characterize the compensated derivatives $\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i}$ and $\frac{\partial \bar{c}_1^{i,h}}{\partial(1+\bar{r}_1)}$. The compensated policies solve the system

$$\begin{aligned}
(1-\beta^i)(\Phi^i(\bar{l}_0^i))^{1-\frac{1}{\psi^i}}(\bar{c}_0^{i,h})^{-\frac{1}{\psi^i}} &= \beta^i(\bar{c}_1^{i,h})^{-\frac{1}{\psi^i}}(1+\bar{r}_1),\\
\left((1-\beta^i)\left(\bar{c}_0^{i,h}\Phi^i(\bar{\ell}_0^{i,h})\right)^{1-\frac{1}{\psi^i}} + \beta^i(\bar{c}_1^{i,h})^{1-\frac{1}{\psi^i}}\right)^{\frac{1}{1-\frac{1}{\psi^i}}} &= \bar{u}^i,\\
\bar{c}_1^{i,h} &= \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^{i,h}.
\end{aligned}$$

Straightforward differentiation of this system yields

$$\begin{aligned}
\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i} &= \frac{-\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^{i,h}}\frac{1}{1+\bar{r}_1} + \frac{1}{\psi^i}\frac{1}{\bar{c}_1^{i,h}}},\\
\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{r}_0} &= \frac{\frac{1}{1+\bar{r}_1}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^{i,h}}\frac{1}{1+\bar{r}_1} + \frac{1}{\psi^i}\frac{1}{\bar{c}_1^{i,h}}}.
\end{aligned}$$

Differentiating the system defining uncompensated policies

$$\begin{aligned}
(1-\beta^i)(\Phi^i(\bar{l}_0^i))^{1-\frac{1}{\psi^i}}(\bar{c}_0^{i,h})^{-\frac{1}{\psi^i}} &= \beta^i(\bar{c}_1^{i,h})^{-\frac{1}{\psi^i}}(1+\bar{r}_1),\\
\bar{c}_0^i + \bar{a}_0^i &= \bar{y}_0^i\\
\bar{c}_1^i &= \bar{w}_1 + (1+\bar{r}_1)\bar{a}_0^i,
\end{aligned}$$

implies that

$$\frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} = \frac{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^i}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^i}\frac{1}{1+\bar{r}_1} + \frac{1}{\psi^i}\frac{1}{\bar{c}_1^i}}.$$

Hence, making use of duality ($\bar{c}_0^i = \bar{c}_0^{i,h}$ and so on), we can more succinctly write

$$\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{l}_0^i} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\left(-\psi^i \bar{c}_0^i \frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\right),$$
$$\frac{\partial \bar{c}_1^{i,h}}{\partial \bar{r}_0} = \frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i}\left(\psi^i \bar{c}_0^i \frac{1}{1+\bar{r}_1}\right).$$

Combining the prior results and using

$$\frac{\partial \bar{c}_1^i}{\partial \bar{y}_0^i} = (1+\bar{r}_0)\frac{\partial \bar{a}_0^i}{\partial \bar{y}_0^i}$$

and the definition of the static labor wedge in this environment

$$\bar{\tau}^{\ell_0^i} = 1 - \frac{-\bar{c}_0^i \Phi^{i'}(\bar{l}_0^i)/\Phi^i(\bar{l}_0^i)}{\bar{w}_0}$$

yields the stated result in the claim. □

Corollary 1

Proof. First consider the case of a household i facing a binding leverage constraint or rule-of-thumb ($i \in C$). If the household maintains

$$q_0 k_0^i = \omega_0^i a_0^i$$

in response to a marginal change in income, clearly

$$q_0 \frac{\partial k_0^i}{\partial y_0^i} = \omega_0^i \frac{\partial a_0^i}{\partial y_0^i}$$

and so

$$mpr_0^i \equiv \frac{q_0 \frac{\partial k_0^i}{\partial y_0^i}}{\frac{\partial a_0^i}{\partial y_0^i}} = \omega_0^i.$$

Provided the household remains constrained in the limit of zero aggregate risk, it follows that

$$\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} = \omega_0^i,$$
$$\overline{mpr}_0^i = \omega_0^i.$$

Now consider a household i at an interior optimum in portfolio choice ($i \notin C$). Optimal portfolio choice remains

$$\mathbb{E}_0(c_1^i)^{-\gamma^i}(r_1^k - r_1) = 0.$$

As in the proof of Propositions 1 and 2, we successively consider higher-order approximations and repeatedly make use of the method of undetermined coefficients and market clearing.

The first- and second-order approximations imply (37) as in the proof of Proposition 1. As before,

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}.$$

Multiplying both sides of (37) by $\frac{\bar{c}_1^i}{\gamma^i}$ but now integrating *only* over households $i' \notin C$ and dividing by $\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di$ yields

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\mathbb{E}_0 \sigma^2 = \gamma \sigma^2 + o(||\cdot||^3)$$

for γ as defined in the claim, noting that

$$\frac{\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di}{\int_{i \notin C} \bar{c}_1^i di} = 1 - \frac{\int_{i \notin C} (1 + \bar{r}_1) \bar{b}_1^i di}{\int_{i \notin C} \bar{c}_1^i di}.$$

Moreover, by (37) we obtain (38) for $i \notin C$. It follows then that, as in the proof of Proposition 1, we obtain

$$\begin{aligned} \frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i} &= \left(\frac{\bar{c}_1^i}{(1 + \bar{r}_1)\bar{a}_0^i}\right) \frac{\gamma}{\gamma^i} - \frac{\bar{w}_1}{(1 + \bar{r}_1)\bar{a}_0^i}, \\ \overline{mpr}_0^i &= \frac{\gamma}{\gamma^i} \end{aligned}$$

for $i \notin C$.

A third-order approximation implies (42) as in the proof of Proposition 2. Using the same steps outlined therein yields (43). Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i}$ but now again integrating only over households $i \notin C$ and dividing by $\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di$ yields

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma \sigma^2 + \frac{\gamma}{\int_{i \notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right] di} \left(\int_{i \notin C} \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di\right) \hat{m}_0 \sigma^2 + o(||\cdot||^4).$$

By the period 1 resource constraint and equilibrium wages and profits, we again obtain (44).

Integrating again only over households $i \notin C$ yields

$$\begin{aligned}
\int_{i\notin C} \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di &= -\int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \int_{i\notin C} \left[\alpha \bar{w}_1 \delta_{m_0}^{k_0} + \bar{\pi}_1 \delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0}\right] di, \\
&= \int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di + \int_{i\notin C} \left[\alpha \bar{w}_1 \delta_{m_0}^{k_0} + \bar{\pi}_1 \delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0} - \bar{c}_1^i \delta_{m_0}^{c_1^i}\right] di, \\
&= \int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di + \\
&\quad \int_{i\notin C} \Big[\alpha \bar{w}_1 \delta_{m_0}^{k_0} + \bar{\pi}_1 \delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0} - \\
&\qquad \left(\alpha \bar{w}_1 \delta_{m_0}^{k_0} + (1+\bar{r}_1)\delta_{m_0}^{b_0^i} + \bar{b}_0^i \delta_{m_0}^{r_1} + \bar{\pi}_1 \delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0}\right)\Big] di, \\
&= \int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di - \int_{i\notin C} \left[(1+\bar{r}_1)\delta_{m_0}^{b_0^i} + \bar{b}_0^i \delta_{m_0}^{r_1}\right] di, \\
&= \int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di + \int_{i\in C} \left[(1+\bar{r}_1)\delta_{m_0}^{b_0^i} + \bar{b}_0^i \delta_{m_0}^{r_1}\right] di, \\
&= \int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di + \int_{i\in C} \left[(1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i \delta_{m_0}^{r_1}\right] \left(1 - \omega_0^i\right) di,
\end{aligned}$$

where the third equality substitutes in for $\bar{c}_1^i \delta_{m_0}^{c_1^i}$ implied by the period 1 resource constraint and equilibrium wages and profits; the fifth equality uses bond market clearing $\int_0^1 b_0^i di = 0$ both at the point of approximation and up to first order; and the final equality uses $b_0^i = (1-\omega_0^i)a_0^i$ both at the point of approximation and up to first order among constrained households. Using (38) and the expression for γ as defined in the claim, then note that

$$\begin{aligned}
\int_{i\notin C} \bar{c}_1^i \delta_{m_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di = & \\
& \int_{i\notin C} \left(\bar{c}_1^i \delta_{m_0}^{c_1^i} - \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^{i'} di'} \int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right) \left(1 - \frac{\gamma}{\gamma^i}\right) + \\
& \qquad \left(\int_0^1 \bar{c}_1^{i'} \delta_{m_0}^{c_1^{i'}} di'\right) \left(\frac{\int_{i'\notin C} \bar{c}_1^{i'} di'}{\int_0^1 \bar{c}_1^{i'} di'} - \frac{\int_{i'\notin C} \left[\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^{i'}\right] di'}{\int_0^1 \bar{c}_1^{i'} di'}\right).
\end{aligned}$$

Furthermore,

$$\int_{i\in C}\left[(1+\bar{r}_1)\delta^{a_0^i}_{m_0}+\bar{a}_0^i\delta^{r_1}_{m_0}\right]\left(1-\omega_0^i\right)di=$$
$$\int_{i\in C}\left[(1+\bar{r}_1)\delta^{a_0^i}_{m_0}+\bar{a}_0^i\delta^{r_1}_{m_0}-\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_0^1\bar{c}_1^{i'}di'}\int_0^1\bar{c}_1^{i'}\delta^{c_1^{i'}}_{m_0}di'\right]\left(1-\omega_0^i\right)+$$
$$\left(\int_0^1\bar{c}_1^{i'}\delta^{c_1^{i'}}_{m_0}di'\right)\left(\frac{(1+\bar{r}_1)\int_{i'\in C}\bar{a}_0^{i'}(1-\omega_0^{i'})di'}{\int_0^1\bar{c}_1^{i'}di'}\right).$$

Since bond market clearing implies

$$\frac{\int_{i'\notin C}\bar{c}_1^{i'}di'}{\int_0^1\bar{c}_1^{i'}di'}-\frac{\int_{i'\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^{i'}\right]di'}{\int_0^1\bar{c}_1^{i'}di'}+\frac{(1+\bar{r}_1)\int_{i'\in C}\bar{a}_0^{i'}(1-\omega_0^{i'})di'}{\int_0^1\bar{c}_1^{i'}di'}=0,$$

it follows that

$$\int_{i\notin C}\bar{c}_1^i\delta^{c_1^i}_{m_0}\left(1-\delta^{c_1^i}_{z_1}\right)di+\int_{i\in C}\left[(1+\bar{r}_1)\delta^{a_0^i}_{m_0}+\bar{a}_0^i\delta^{r_1}_{m_0}\right]\left(1-\omega_0^i\right)di=$$
$$\int_{i\notin C}\left(\bar{c}_1^i\delta^{c_1^i}_{m_0}-\frac{\bar{c}_1^i}{\int_0^1\bar{c}_1^{i'}di'}\int_0^1\bar{c}_1^{i'}\delta^{c_1^{i'}}_{m_0}di'\right)\left(1-\frac{\gamma}{\gamma^i}\right)+$$
$$\int_{i\in C}\left[(1+\bar{r}_1)\delta^{a_0^i}_{m_0}+\bar{a}_0^i\delta^{r_1}_{m_0}-\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_0^1\bar{c}_1^{i'}di'}\int_0^1\bar{c}_1^{i'}\delta^{c_1^{i'}}_{m_0}di'\right]\left(1-\omega_0^i\right).$$

Furthermore note that using the definition of γ given in the claim and bond market clearing,

$$1=\int_{i\notin C}\frac{\bar{c}_1^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'+\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di'}\overline{mpr}_0^i di+\int_{i\in C}\frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_{i'\notin C}\bar{c}_1^{i'}di'+\int_{i'\in C}(1+\bar{r}_1)\bar{a}_0^{i'}di'}\overline{mpr}_0^i di=\overline{mpr}_0.$$

Finally, since

$$\overline{\frac{d\left[c_1^i/\int_0^1c_1^{i'}\right]}{dm_0}}=\frac{1}{\int_0^1\bar{c}_1^{i'}di'}\left(\overline{\frac{dc_1^i}{dm_0}}-\frac{\bar{c}_1^i}{\int_0^1\bar{c}_1^{i'}di'}\int_0^1\overline{\frac{dc_1^{i'}}{dm_0}}di'\right),$$
$$=\frac{1}{\int_0^1\bar{c}_1^{i'}di'}\left(\bar{c}_1^i\delta^{c_1^i}_{m_0}-\frac{\bar{c}_1^i}{\int_0^1\bar{c}_1^{i'}di'}\int_0^1\bar{c}_1^{i'}\delta^{c_1^{i'}}_{m_0}di'\right),$$

and

$$\begin{aligned}\overline{\frac{d\left[(1+r_1)a_0^i/\int_0^1 c_1^{i'}\right]}{dm_0}} &= \frac{1}{\int_0^1 \bar{c}_1^{i'}di'}\left(\overline{\frac{d(1+r_1)a_0^i}{dm_0}} - \frac{(1+r_1)a_0^i}{\int_0^1 \bar{c}_1^{i'}di'}\int_0^1 \overline{\frac{dc_1^{i'}}{dm_0}}di'\right), \\ &= \frac{1}{\int_0^1 \bar{c}_1^{i'}di'}\left((1+\bar{r}_1)\delta_{m_0}^{a_0^i} + \bar{a}_0^i\delta_{m_0}^{r_1} - \frac{(1+\bar{r}_1)\bar{a}_0^i}{\int_0^1 \bar{c}_1^{i'}di'}\int_0^1 \bar{c}_1^{i'}\delta_{m_0}^{c_1^{i'}}di'\right),\end{aligned}$$

we can combine all of the previous results to write

$$\frac{\gamma}{\int_{i\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^i\right]di}\left(\int_{i\notin C}\bar{c}_1^i\delta_{m_0 z_1}^{c_1^i}di\right) = \gamma\frac{\int_0^1\bar{c}_1^i di}{\int_{i\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^i\right]di}\int_0^1\bar{\xi}_{m_0}^i\left(\overline{mpr}_0 - \overline{mpr}_0^i\right)di$$

for $\bar{\xi}_{m_0}^i = \overline{\frac{d\left[c_1^i/\int_0^1 c_1^{i'}\right]}{dm_0}}$ for $i\notin C$ and $\bar{\xi}_{m_0}^i = \overline{\frac{d\left[(1+r_1)a_0^i/\int_0^1 c_1^{i'}\right]}{dm_0}}$ for $i\in C$. Again noting that

$$\frac{\int_0^1\bar{c}_1^i di}{\int_{i\notin C}\left[\bar{w}_1+\bar{\pi}_1\bar{k}_0^i\right]di} = \left(1-\frac{\int_{i\notin C}(1+\bar{r}_1)\bar{b}_0^i di}{\int_{i\notin C}\bar{c}_1^i di}\right)^{-1}\frac{\int_0^1\bar{c}_1^i di}{\int_{i\notin C}\bar{c}_1^i di}.$$

yields the expression for ζ_{m_0} given in the claim. □

Corollary 2

Proof. The period 1 consumption of each household i is now

$$c_1^i = w_1\epsilon_1^i + (1+r_1)b_0^i + \pi_1\epsilon_1^i k_0^i,$$

where the real wage and real profits per unit of capital remain

$$\begin{aligned}w_1 &= (1-\alpha)z_1 k_0^\alpha, \\ \pi_1 &= \alpha z_1 k_0^{\alpha-1}\end{aligned}$$

since, by the law of large numbers, the aggregate efficiency units of labor supplied remains 1 and aggregate capital among households of type i remains k_0^i.[52] Define the capital return facing each household i

$$1+r_1^{k,i} \equiv \frac{\pi_1\epsilon_1^i}{q_0},$$

[52]Recall that we are assuming a double continuum of households now, where the continuum of households of type i are each subject to a distinct shock ϵ_1^i which is *iid* within and across i.

distinct from the return on capital aggregating over idiosyncratic risk

$$1 + r_1^k \equiv \frac{\pi_1}{q_0}.$$

Then household i's optimal portfolio choice is now

$$\mathbb{E}_0 \left(c_1^i\right)^{-\gamma^i} \left(r_1^{k,i} - r_1\right) = 0.$$

Using approximations up to first and second order as in the proof of Proposition 1 yields the analog to (37) in this environment,

$$\mathbb{E}_0 \hat{r}_1^{k,i} - \hat{r}_1 + \frac{1}{2}(1+\eta^i)\sigma^2 = \gamma^i \left(\delta_{z_1}^{c_1^i} + \eta^i \delta_{\epsilon_1^i}^{c_1^i}\right) \sigma^2 + o(||\cdot||^3). \tag{73}$$

Given the definitions of the idiosyncratic and aggregate capital returns,

$$\hat{r}_1^{k,i} = \hat{\epsilon}_1^i + \hat{r}_1^k.$$

By assumption,

$$\mathbb{E}_0 \hat{\epsilon}_1^i = -\frac{1}{2}\eta^i \sigma^2.$$

It follows that

$$\mathbb{E}_0 \hat{r}_1^{k,i} = -\frac{1}{2}\eta^i \sigma^2 + \mathbb{E}_0 \hat{r}_1^k,$$

so that (73) implies for the aggregate capital claim

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \left(\delta_{z_1}^{c_1^i} + \eta^i \delta_{\epsilon_1^i}^{c_1^i}\right) \sigma^2 + o(||\cdot||^3). \tag{74}$$

By the period 1 resource constraint and equilibrium wages and profits,

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}$$

as in the baseline environment and

$$\delta_{\epsilon_1^i}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \delta_{z_1}^{c_1^i}.$$

Hence, (74) implies

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \left(1 + \eta^i\right) \delta_{z_1}^{c_1^i} \sigma^2 + o(||\cdot||^3). \tag{75}$$

Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i(1+\eta^i)}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + o(||\cdot||^3)$$

for γ as defined in the claim. Furthermore, it follows from (75) that we generalize (38) to

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i(1+\eta^i)},$$

which implies

$$\frac{\bar{w}_1 + \bar{\pi}_1\bar{k}_0^i}{\bar{c}_1^i} = \frac{\gamma}{\gamma^i(1+\eta^i)}$$

and thus the expression for $\frac{\bar{q}_0\bar{k}_0^i}{\bar{a}_0^i}$ given in the claim.

Differentiating each household's optimality conditions and resource constraints generalizes (39) to

$$0 = \mathbb{E}_0 m_{0,1}^i \frac{\gamma^i}{c_1^i}\left(r_1^{k,i} - r_1\right)\left((1+r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1\epsilon_1^i\frac{\partial k_0^i}{\partial y_0^i}\right).$$

A second-order approximation then generalizes (40) to

$$\begin{aligned}0 = &\left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1\overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\left(\mathbb{E}_0\hat{r}_1^{k,i} - \hat{r}_1 + \frac{1}{2}(1+\eta^i)\sigma^2\right)\\ &- \left((1+\bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1\overline{\frac{\partial k_0^i}{\partial y_0^i}}\right)\frac{\gamma^i+1}{\bar{c}_1^i}\left(\bar{w}_1 + \bar{\pi}_1\bar{k}_0^i\right)(1+\eta^i)\sigma^2 + \bar{\pi}_1\overline{\frac{\partial k_0^i}{\partial y_0^i}}(1+\eta^i)\sigma^2 + o(||\cdot||^3).\end{aligned}$$

Using the above results, this implies

$$\bar{q}_0\overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i(1+\eta^i)}\overline{\frac{\partial a_0^i}{\partial y_0^i}},$$

from which the expression for $\overline{mpr}_0^i$ in the claim follows.

Finally, optimal portfolio choice up to third order, the above results, and steps analogous to those used in the proof of Proposition 2 yields the analog of (43)

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i\left(\delta_{z_1}^{c_1^i} + \delta_{\epsilon_1^i}^{c_1^i}\eta^i\right)\sigma^2 + \gamma^i\left(\delta_{m_0 z_1}^{c_1^i} + \delta_{m_0\epsilon_1^i}^{c_1^i}\eta^i\right)\hat{m}_0\sigma^2 + o(||\cdot||^4). \tag{76}$$

A second order expansion of the period 1 resource constraint implies

$$\begin{aligned}\bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} + \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{z_1}^{c_1^i} &= \alpha \bar{w}_1 \delta_{m_0}^{k_0} + \bar{\pi}_1 \delta_{m_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1 \bar{k}_0^i \delta_{m_0}^{k_0}, \\ &= \bar{c}_1^i \delta_{m_0 \epsilon_1^i}^{c_1^i} + \bar{c}_1^i \delta_{m_0}^{c_1^i} \delta_{\epsilon_1^i}^{c_1^i},\end{aligned}$$

from which we can conclude

$$\delta_{m_0 z_1}^{c_1^i} = \delta_{m_0 \epsilon_1^i}^{c_1^i}$$

since $\delta_{\epsilon_1^i}^{c_1^i} = \delta_{z_1}^{c_1^i}$ as argued above. It follows from (76) that

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \left(1+\eta^i\right) \delta_{z_1}^{c_1^i} \sigma^2 + \gamma^i \left(1+\eta^i\right) \delta_{m_0 z_1}^{c_1^i} \hat{m}_0 \sigma^2 + o(||\cdot||^4).$$

Then multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i(1+\eta^i)}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left(\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di \right) \hat{m}_0 \sigma^2 + o(||\cdot||^4).$$

Then following similar steps as in the proof of Proposition 2, using

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i(1+\eta^i)} = \overline{mpr}_0^i$$

implied by the above results, yields the expression for ζ_{m_0} given in the claim. □

Corollary 3

Proof. Denote with $\mathbb{E}_0^i$ the expectation under household i's subjective beliefs, and $\mathbb{E}_0$ that under the objective (true) probability distribution. Household i's optimal portfolio choice is then characterized by

$$\mathbb{E}_0^i \left(c_1^i\right)^{-\gamma^i} \left(r_1^k - r_1\right) = 0.$$

Using approximations up to first and second order as in the proof of Proposition 1 yields the analog to (37) in this environment,

$$\mathbb{E}_0^i \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\varsigma^i \sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \varsigma^i \sigma^2 + o(||\cdot||^3). \tag{77}$$

By the definition of returns,

$$\hat{r}_1^k = \hat{z}_1 + (\alpha - 1)\hat{k}_0 - \hat{q}_0$$

where there is no uncertainty over $\hat{k}_0$ or $\hat{q}_0$ as of period 0. Hence,

$$\mathbb{E}_0^i \hat{r}_1^k + \frac{1}{2}\varsigma^i \sigma^2 = \mathbb{E}_0 \hat{r}_1^k + \frac{1}{2}\sigma^2.$$

Hence, (77) implies

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \varsigma^i \sigma^2 + o(||\cdot||^3). \tag{78}$$

By the period 1 resource constraint and equilibrium wages and profits,

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}$$

as in the baseline environment. Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i \varsigma^i}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma \sigma^2 + o(||\cdot||^3)$$

for γ as defined in the claim. Furthermore, it follows from (78) that we generalize (38) to

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i \varsigma^i},$$

which implies

$$\frac{\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i} = \frac{\gamma}{\gamma^i \varsigma^i}$$

and thus the expression for $\frac{\bar{q}_0 \bar{k}_0^i}{\bar{a}_0^i}$ given in the claim.

Differentiating each household's optimality conditions and resource constraints generalizes (39) to

$$0 = \mathbb{E}_0^i m_{0,1}^i \frac{\gamma^i}{c_1^i} \left(r_1^k - r_1\right) \left((1 + r_1)\frac{\partial b_0^i}{\partial y_0^i} + \pi_1 \frac{\partial k_0^i}{\partial y_0^i}\right).$$

A second-order approximation then generalizes (40) to

$$\begin{aligned} 0 = &\left((1 + \bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right) \left(\mathbb{E}_0^i \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\varsigma^i \sigma^2\right) \\ &- \left((1 + \bar{r}_1)\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right) \frac{\gamma^i + 1}{\bar{c}_1^i} \left(\bar{w}_1 + \bar{\pi}_1 \bar{k}_0^i\right) \varsigma^i \sigma^2 + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}} \varsigma^i \sigma^2 + o(||\cdot||^3). \end{aligned}$$

Using the above results, this implies

$$\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i \varsigma^i} \overline{\frac{\partial a_0^i}{\partial y_0^i}},$$

from which the expression for $\overline{mpr}_0^i$ in the claim follows.

Finally, optimal portfolio choice up to third order, the above results, and steps analogous to those used in the proof of Proposition 2 yields the analog of (43)

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \varsigma^i \sigma^2 + \gamma^i \delta_{m_0 z_1}^{c_1^i} \varsigma^i \hat{m}_0 \sigma^2 + o(||\cdot||^4). \tag{79}$$

Multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i \varsigma^i}$, integrating over all households i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k - \hat{r}_1 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left(\int_0^1 \bar{c}_1^i \delta_{m_0 z_1}^{c_1^i} di \right) \hat{m}_0 \sigma^2 + o(||\cdot||^4).$$

Then following similar steps as in the proof of Proposition 2, using

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i \varsigma^i} = \overline{mpr}_0^i$$

implied by the above results, yields the expression for ζ_{m_0} given in the claim. □

C Empirical appendix

In this appendix we provide supplemental material for our empirical analyses provided in section 3.

C.1 The effect of monetary shocks

We first provide supplemental evidence on the empirical effects of a monetary policy shock studied in section 3.1.

C.1.1 Potential instruments and first stage of SVAR-IV

Here we evaluate potential instruments to use in the SVAR-IV procedure. The first stage using 5 possible measures of policy surprises constructed in Gertler and Karadi (2015) is summarized in Table 10: using the current Fed Funds futures contract; the 3-month ahead Fed Funds futures contract; the 2-quarters ahead 3-month Eurodollar contract; the 3-quarters

	1-year Treasury yield					
Current Fed Funds	0.84					0.25
	(0.22)					(0.43)
Expected Fed Funds, 3 mos ahead		1.15				1.26
		(0.28)				(0.46)
Expected 3-mo ED rate, 2 qtrs ahead			0.84			1.71
			(0.31)			(1.38)
Expected 3-mo ED rate, 3 qtrs ahead				0.68		-4.88
				(0.29)		(1.82)
Expected 3-mo ED rate, 4 qtrs ahead					0.71	3.03
					(0.31)	(1.05)
Observations	258	258	258	258	258	258
Adj R^2	0.04	0.06	0.03	0.02	0.02	0.07
F-statistic	14.40	16.30	7.62	5.27	5.03	6.36

Table 10: effects of monetary policy instruments on first-stage residuals of VAR

Notes: heteroskedasticity-robust standard errors given in parenthesis.

ahead 3-month Eurodollar contract; or the 4-quarters ahead 3-month Eurodollar contract. In all cases, the 1-year Treasury yield rises given a positive surprise, as would be expected. The effects are statistically significant at conventional levels, and for the first two instruments the F statistics above 10 exceed the threshold of a strong instrument recommended by Stock et al. (2002). Since shocks in our model are to the current nominal interest rate, to maximize comparability we focus on the current Fed Funds futures contract as our baseline instrument.

C.1.2 Robustness to details of estimation approach

Here we demonstrate that the broad messages of our baseline estimates are robust to the number of lags in the VAR, sample periods used in both stages of the SVAR-IV, variables included in the VAR, and instrument used.

Given a monetary policy shock, Table 11 summarizes the impact effect on the 1-year Treasury yield (the monetary policy indicator), the impact effect on the real S&P 500 return (implied by the real rate and excess return), and the share of the latter accounted for by news about future excess returns in the Campbell-Shiller decomposition (26). First, we find that the baseline results using 6 lags in the VAR are little affected if 4-8 lags are used instead. Second, we find that the results are broadly robust to using the same January 1991 - June 2012 period for both the VAR and IV regressions, or limiting the analysis of monetary policy shocks to the first half of the IV sample alone (January 1991 - September 2001). The expansionary monetary policy shock in fact lowers the stock market when using the second half of the IV sample alone (October 2001 - June 2012), but we note that the instrument

	1-year Treasury yield (*pp*)	Real stock return (*pp*)	Share future excess return news (%)
Baseline	-0.22	2.02	58%
Number of lags in VAR			
4	-0.21	1.93	52%
5	-0.22	1.88	54%
7	-0.23	1.94	62%
8	-0.23	1.99	55%
Sample periods			
VAR: 1/91-6/12, IV: 1/91-6/12	-0.14	1.52	36%
VAR: 7/79-6/12, IV: 1/91-9/01	-0.21	3.15	50%
VAR: 7/79-6/12, IV: 10/01-6/12	-0.17	-2.01	38%
Variable added to VAR			
Excess bond premium	-0.21	2.30	79%
Mortgage spread	-0.24	1.62	53%
3-month commercial paper spread	-0.19	2.26	66%
5-year Treasury rate	-0.17	1.64	77%
10-year Treasury rate	-0.17	1.60	75%
Term spread	-0.21	2.04	64%
Relative bill rate	-0.18	2.63	70%
Change in 3-month Treasury rate	-0.19	2.35	65%
3-month ahead FF as IV	-0.20	2.28	65%

Table 11: robustness of 1 SD monetary shock on returns and components

Notes: series for the Gilchrist and Zakrajsek (2012) excess bond premium, mortgage spread, 3-month commercial paper spread, 5-year Treasury rate, and 10-year Treasury rate are taken from the dataset provided by Gertler and Karadi (2015). The term spread (10-year Treasury rate less 1-month Treasury yield), relative bill rate (difference between the 3-month Treasury rate and its 12-month moving average), and change in the 3-month Treasury rate are constructed using CRSP.

is weak over this sub-sample (having a first-stage F statistic of 4.67, not shown). Third, we find that news about future excess returns tends to be, if anything, even more important when adding other variables included in the analyses of Bernanke and Kuttner (2005) and Gertler and Karadi (2015) on which we build. Finally, we find that similar results when using as the instrument the three-month ahead Fed Funds futures contract instead of the current contract, recalling that this was the other strong instrument indicated by Table 10.

C.1.3 Testing invertibility and comparing SVAR-IV and LP-IV

We now demonstrate that the assumption of invertibility used in the VAR is validated by statistical tests suggested in the literature. Relatedly, we demonstrate that our estimated impulse responses lie within the (quite large) confidence intervals obtained using an alternative

	1-yr Treasury	CPI	Industrial production	1-mo real rate	1-mo excess return	Dividend/ price
SW [2018] test	0.47	0.63	0.97	0.80	0.68	0.75
Granger causality test	0.07	0.15	0.88	0.12	0.44	0.93

Table 12: tests of invertibility assumed in the VAR

Notes: the first row is the bootstrapped p-value for the null hypothesis that the SVAR-IV and LP-IV impulse responses depicted in Figure 6 are the same 1, 13, 25, and 37 months after shock, using the test statistic provided in Stock and Watson (2018). We construct the variance matrix needed for this statistic using the 10,000 iterations of the wild bootstrap used to construct confidence intervals for our SVAR-IV estimates in the main text. The second row is the p-value for the null hypothesis that the coefficients on 6 lags of the instrument are jointly equal to zero when added to the VAR.

local projection instrumental variables approach (LP-IV) at virtually every horizon.

We implement the LP-IV by projecting each outcome variable h months ahead on the 1-year Treasury yield, instrumenting for the latter using the Fed Funds futures surprise also used in our baseline SVAR-IV. Following Stock and Watson (2018), to make this specification comparable with the SVAR-IV and further improve the precision of estimates, we include 6 lags of each of the variables included in the VAR as controls. Moreover, given the serial correlation of the instrument discussed in Ramey (2016) and Stock and Watson (2018), we include a lag of the instrument as an additional control. Figure 6 plots the estimates at each horizon $h \in \{0, \ldots, 47\}$, along with the point-wise 90% heteroskedasticity-robust confidence interval. These estimates are extremely noisy, but their confidence intervals contain our baseline estimates at virtually every horizon for each variable.

Stock and Watson (2018) discuss two tests of the invertibility assumption implicit in the SVAR-IV, the first of which formalizes this comparison of the SVAR-IV and LP-IV estimates. They propose a Hausman-type test statistic of the null hypothesis that invertibility is satisfied by comparing the impulse response at horizon h for a given variable under both approaches. The first row of Table 12 summarizes the p-value for this test in our setting jointly applied at horizons $h \in \{1, 13, 25, 37\}$ for each variable, demonstrating that we cannot reject the null at standard significance levels. They also recommend the use of the complementary Granger causality test discussed in Forni and Gambetti (2014): if invertibility is satisfied, lagged values of the instrument should not have predictive power given the variables included in the VAR. We include 6 lags of our instrument in the VAR and construct an F statistic associated with the null hypothesis that these coefficients are jointly zero for each variable in the VAR. We again cannot reject the null at standard significance levels.

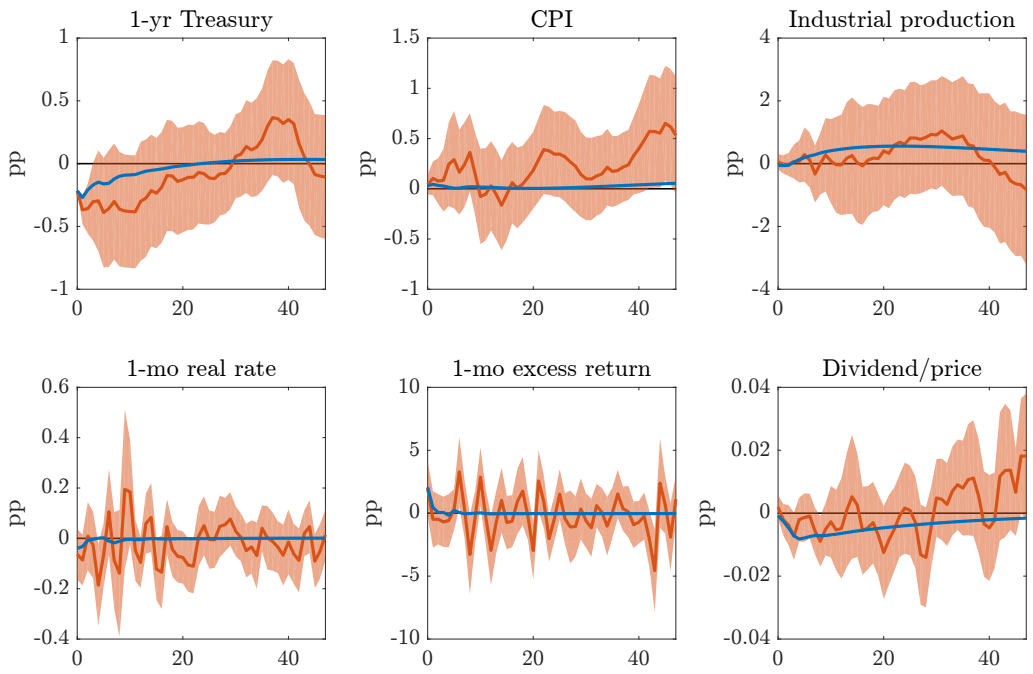


Figure 6: effects of monetary shock using SVAR-IV (blue) versus LP-IV (red)

Notes: heteroskedasticity-robust 90% confidence intervals at each horizon are computed for LP-IV. Since both the SVAR-IV and LP-IV approaches only identify impulse responses up to a normalization on the initial response of the monetary policy indicator (because the structural shock is not observed), for comparability we multiply all LP-IV estimates and confidence intervals by the same constant such that the initial point estimate for the 1-year Treasury is the same as in the SVAR-IV.

C.2 Micro moments from the SCF

We now provide supplemental details on our measurement of household portfolios using the 2016 SCF described in section 3.3.

C.2.1 Construction of household portfolios

We first describe how we construct household portfolios. We proceed in three steps.

First, to the SCF data we add an estimate of defined benefit (DB) pension wealth for each household, since this is the major component of household wealth which is not included in the SCF measure of net worth. We use the estimates of Sabelhaus and Volz (2019). Their approach involves (1) estimating the present discounted value of DB payments for households who report in the SCF that they are currently receiving a DB pension or expect to receive a known amount from a prior job, (2) subtracting the total DB pension wealth among these households from the total DB pension wealth in the Financial Accounts of the United States (FA) for 2016, and (3) allocating (based on age, earnings, and other characteristics) the

remaining DB pension wealth to households who expect to receive a DB pension based on their current job. Together with the other categories of wealth reported in the SCF, we have a comprehensive picture of U.S. household net worth.

Second, we proceed by line item to allocate how much household wealth is held in nominal claims versus claims on underlying capital. We assume the following line items reported in the SCF summary extract include traded claims on stocks:[53]

- *Savings accounts that may be invested in stocks, included in transaction accounts*: these include accounts classified as 529 or state-sponsored education accounts. We use the self-reported fraction of these accounts invested in stocks.
- *Stock mutual funds*: by definition, 100% of these accounts are invested in stocks.
- *Combination mutual funds*: we assume 50% of these accounts are invested in stocks.
- *Other mutual funds*: we assume 100% of these accounts are invested in stocks.
- *Directly held stocks*: by definition, 100% of these assets are invested in stocks.
- *Other managed assets*: we use the self-reported fraction of these accounts (which include annuities and trusts) invested in stocks.
- *Quasi-liquid retirement assets*: we use the self-reported fraction of these accounts (which include IRAs/Keoghs, account-type pensions in the current job, future pensions, and currently received account-type pensions) invested in stocks.

We assume *actively-managed businesses* and *non-actively-managed businesses* are also claims on underlying capital. Finally, we assume all other non-financial assets (*vehicles*, *primary residence*, *residential real estate excluding the primary residence*, *non-residential real estate*, and *other miscellaneous non-financial assets*) are direct claims on capital. We assume the remaining line items are purely nominal assets or liabilities.[54]

Third, we use the FA and supplementary sources to construct an estimate of leverage for the claims on capital described above. For traded stocks except those in *other mutual funds*, we assume that households hold a diversified portfolio of claims on the nonfinancial corporate sector (FA table S.5.a) and financial business sector (FA table S.6.a), net of the central bank (FA table S.61.a), government DB pension funds (FA tables L.119.b and L.120.b), all defined contribution (DC) pension funds (FA tables L.118.c, L.119.c, and L.120.c), and mutual funds

[53] Our approach here follows the construction of the EQUITY variable in the summary extract.

[54] We note in particular that we treat DB pension entitlements as a nominal asset of households, under the interpretation that households have a fixed claim on the pension sponsor which is then the residual claimant on the investment portfolio.

(FA table L.122).[55] Table 13 summarizes the balance sheet of this consolidated sector for 2016 using the Q2 2019 release of the FA. We compute net leverage for this sector by dividing its aggregate position in capital by net equity issued to other sectors. Capital is given by total assets net of nominal assets and equity assets. Net equity issued to other sectors is given by equity liabilities plus net worth net of equity assets. The resulting measure of leverage we use for traded stocks is $lev = 1.6$.

For private business wealth captured in *actively-managed businesses*, we assume that households hold a diversified portfolio of claims on the nonfinancial noncorporate sector (FA table S.4.a) and non-profit sector (FA table B.101.n). Table 13 also summarizes the balance sheet of this consolidated sector for 2016 using the Q2 2019 release of the FA. Computing leverage for this sector as described above, we obtain $lev = 1.1$.

An important limitation of the above measures of leverage is that they do not account for the leverage of hedge funds and private equity, since these entities are included within the household sector by the FA. To explore the implications of such leverage, in our baseline we assume that household wealth in *other mutual funds* is invested at hedge funds and *non-actively managed businesses* is invested in private equity buyout funds.[56] In their Table 2B, Ang, Gorovyy, and van Inwegen (2011) report that the average leverage of long-only hedge funds is 1.4. Assuming that these funds invest in a representative share of traded stocks as described above, this implies $lev = 1.4 \times 1.6 = 2.2$ for households' wealth in *other mutual funds*. In their Table 8, Axelson, Jenkinson, Stromberg, and Weisbach (2013) report that the average leverage of LBO transactions is 4.0. This implies $lev = 4.0$ for households' wealth in *non-actively managed businesses*.

Table 14 provides the resulting decomposition of aggregate net worth of U.S. households into nominal claims and claims on capital using these assumptions.

C.2.2 Consistency of assumptions with market clearing in nominal claims

We now demonstrate that the aggregate household balance sheet in Table 14 is roughly consistent with market clearing in the market for nominal claims after accounting for the

[55] We exclude the central bank and government DB pension funds because we model these as part of the government sector (the latter consistent with our interpretation of DB pensions in footnote 54). We exclude DC pension funds and mutual funds because we view these as pure pass-through entities whose assets have already been folded into that of households using our approach described so far.

[56] We map *other mutual funds* to hedge funds because the specific question asked in the SCF is: "Do you have any other mutual funds, ETFs, hedge funds, or REITs?" We map non-actively managed private business to LBOs because (i) non-actively managed private business wealth includes private equity (the specific question asked in the SCF is: "Do you (or anyone in your family living here) own or share ownership in any other businesses, business investments or other private equity that are not publicly traded and where you do NOT have an active management role?"), and (ii) assets under management in buyout funds comprises more than half of all assets under management in private equity globally (McKinsey (2018)).

		Nonfinancial corporates and financial businesses	Nonfinancial noncorporates and nonprofits
1	Assets	96,362	26,697
2	Non-financial assets	23,260	16,335
3	Monetary gold	0	
4	Currency and deposits	3,787	1,590
5	Debt securities	13,394	360
6	Loans	24,170	223
7	Money market fund shares	777	177
8	Corporate equities	6,134	1,088
9	Mutual fund shares	2,161	534
10	FDI in	6,412	
11	Other equity	4,551	10
12	Other assets	11,715	6,380
13	Liabilities	106,997	9,272
14	Currency and deposits	13,675	
15	Debt securities	19,510	217
16	Loans	6,766	5,392
17	Corporate equities	32,771	
18	FDI abroad	6,567	20
19	Other equity	4,597	
20	Insurance and pensions	10,807	
21	Other liabilities	12,305	3,643
22	Net worth	-10,634	17,426
23	Capital	23,260	18,649
24	Net equity	14,754	17,445
25	Leverage	1.6	1.1

Table 13: business net worth in FA

Notes: all values in $2016bn. Financial businesses exclude the central bank, government DB pension funds, all DC pension funds, and mutual funds. Missing values denote a line item not relevant for that sector. *Other equity* assets includes equity in GSEs, investment by holding companies, and stock in Federal Reserve banks. *Other assets* includes insurance and pensions and other accounts receivable. *Other equity* liabilities include equity in GSEs, equity in noncorporate business, and investment by parent companies. *Other liabilities* include money market mutual fund shares and other accounts payable. For nonprofits, we allocate the reported sum of corporate equities and mutual fund shares to each line item using the share of total household and nonprofit assets in each category in table S.3.a. Capital is given by (2)+(3). Net equity is given by (17)+(18)+(19)+(22)-(8)-0.67*(9)-(10)-(11). Leverage is given by capital divided by net equity.

		A	B	Qk
1	Transaction accounts	4,940	4,701	239
2	CDs	620	620	0
3	Stock mutual funds	5,939	-3,563	9,502
4	Tax-free bond mutual funds	1,329	1,329	0
5	Govt bond mutual funds	276	276	0
6	Other bond mutual funds	404	404	0
7	Combination mutual funds	757	151	605
8	Other mutual funds	1,011	-1,253	2,264
9	Savings bonds	104	104	0
10	Directly held stocks	5,742	-3,445	9,187
11	Directly held bonds	1,179	1,179	0
12	Cash value life insurance	914	914	0
13	Other managed assets	3,231	2,253	978
14	Quasi-liquid retirement assets	15,001	2,366	12,635
15	Other miscellaneous financial assets	659	659	0
16	DB pension assets	17,855	17,855	0
17	Vehicles	2,717	0	2,717
18	Primary residence	24,176	0	24,176
19	Residential real estate excl primary residence	6,301	0	6,301
20	Non-residential real estate	3,694	0	3,694
21	Actively-managed businesses	17,015	-1,701	18,716
22	Non-actively-managed businesses	2,332	-6,997	9,329
23	Other miscellaneous non-financial assets	559	0	559
24	Mortgage on primary residence	-8,310	-8,310	0
25	Mortgage excl primary residence	-1,128	-1,128	0
26	Other lines of credit	-127	-127	0
27	Credit card balance	-316	-316	0
28	Installment loans	-1,976	-1,976	0
29	Other debt	-176	-176	0
30	Total	104,721	3,818	100,903

Table 14: aggregate household net worth in SCF

Notes: all values in $2016bn. observations are weighted by SCF sample weights.

balance sheets of the government and rest of the world.

We proceed in three steps. First, we summarize the balance sheets of the government and rest of the world for 2016 using the Q2 2019 release of the FA. We construct the government balance sheet by adding that of the central bank (table S.61.a), federal government excluding DB funds (table S.7.a), state and local governments excluding DB funds (table S.8.a), federal DB funds (table L.119.b), and state and local DB funds (table L.120.b).[57] We obtain the rest of the world balance sheet in table S.9.a.

Second, we decompose the net worth of these sectors into nominal claims and claims on capital using assumptions consistent with those used for households in the SCF. We do so for each line item of the sector's assets and liabilities and then aggregate across them, analogous to our approach for the household sector. We map our assumptions for the SCF to the line items in the FA as follows:

- Corporate equities and FDI in/out are claims on capital with leverage 1.6, consistent with the net leverage of the nonfinancial corporate and financial business sectors estimated in Table 13.

- Mutual fund shares are a portfolio in which 67% is invested in traded stocks (and the remainder in bonds), where these stocks are again a claim on capital with leverage 1.6. The former is consistent with the portfolio share in corporate equities of the mutual fund sector (table L.224).

- Non-financial assets; monetary gold; and other equity claims (including equity in international organizations; equity in GSEs; equity under the public-private investment program; equity in Federal Reserve Banks; and investment by holding companies) are directly held claims on capital. We treat gold as capital because it is in positive net supply, even though it may not be directly used in production.

We assume all other assets and liabilities are nominal claims. The resulting decomposition of each sector's net worth is provided in Table 15. We note that while the rest of the world's non-financial assets and liabilities are not reported in the FA, this is not of primary interest here since these are claims on capital.

Third, we aggregate over the net position in nominal claims across sectors implied by Tables 14 and 15, accounting for the major discrepancies in measuring household wealth between the SCF and FA. Batty, Bricker, Briggs, Holmquist, McIntosh, Moore, Nielsen,

[57] By including the balance sheet of government DB funds with the government sector, we remain consistent with our treatment of DB pensions in footnote 54. We do not include government DC funds because these are pure pass-through entities whose assets will already have been reflected in the household balance sheets constructed in the prior subsection.

		Government			Rest of world		
		A	B	Qk	A	B	Qk
1	Assets	29,341	10,941	18,400	26,469	5,934	20,535
2	Non-financial assets	13,831	0	13,831	n/a	n/a	n/a
3	Monetary gold	11	0	11			
4	SDRs	49	49	0	48	48	0
5	Currency and deposits	1,038	1,038	0	1,372	1,372	0
6	Debt securities	8,363	8,363	0	10,661	10,661	0
7	Loans	1,774	1,774	0	918	918	0
8	Money market fund shares	83	83	0	93	93	0
9	Corporate equities	2,602	-1,561	4,163	5,783	-3,470	9,252
10	Mutual fund shares	306	-22	328	694	-50	744
11	FDI in				6,586	-3,952	10,538
12	Other equity	67	0	67			
13	Other assets	1,217	1,217	0	314	314	0
14	Liabilities	36,630	36,630	0	18,766	-2,995	21,760
15	SDR holdings	48	48	0	49	49	0
16	Currency and deposits	3,737	3,737	0	561	561	0
17	Debt securities	19,111	19,111	0	2,801	2,801	0
18	Loans	743	743	0	1,418	1,418	0
19	Corporate equities				7,146	-4,288	11,434
20	FDI abroad				6,412	-3,847	10,259
21	Other equity	30	30	0	62	-5	67
22	Insurance and pensions	11,667	11,667	0	248	248	0
23	Other liabilities	1,294	1,294	0	69	69	0
24	Net worth	-7,288	-25,688	18,400	7,704	8,929	-1,225

Table 15: government and rest of world net worth in FA

Notes: all values in $2016bn. Missing value denotes a line item not relevant for that sector, and n/a denotes a line item not available for that sector. *Other equity* assets include equity in international organizations, GSEs, and under the PPIP. *Other assets* includes insurance and pensions and other accounts receivable. *Other equity* liabilities include equity shares in Federal Reserve Banks, in international organizations, and investment by holding companies. *Other liabilities* includes other accounts payable.

		B
1	Households (SCF)	3,818
2	Reconciliation with FA	7,705
3	Time deposits and short-term investments	4,718
4	Net worth in private business	2,987
5	Households [=(1)+(2)]	11,523
6	Government (FA)	-25,688
7	Rest of world (FA)	8,929
8	Sum [=(5)+(6)+(7)]	-5,236

Table 16: net nominal claims by sector (source)

Notes: all values in $2016bn.

Reber, Shatto, Sommer, Sweeney, and Volz (2019) provide an exhaustive accounting of these discrepancies. Aside from the exclusion of DB pensions in the SCF (which we have already addressed), the two other largest sources of discrepancy are in time deposits and short-term investment assets and in wealth in noncorporate businesses. Following Batty et al. (2019), in Table 16 we adjust the aggregate household position in nominal claims to be consistent with the FA along each of these dimensions:

- We use the value of aggregate time deposits and short-term investment assets reported in the FA, instead of those reported in the SCF. The Q2 2019 FA reports for 2016 $8,864bn in such assets (table B.101.h), whereas Batty et al. (2019) cumulate such assets in the 2016 SCF and obtain $4,146bn. The difference is $4,718bn.

- We adjust households' wealth in *actively-managed businesses* and *non-actively-managed businesses* to reflect the average of the market value and cost basis, whereas our analysis in Table 14 used just the former. This follows the finding in Batty et al. (2019) that the average of the market value and cost basis in the SCF closely tracks the reported equity in noncorporate businesses (excluding real estate) in the FA. Applying our leverage assumptions outlined in the prior subsection to these revalued wealth positions and computing the difference in aggregate nominal claims for these line items versus those reported in Table 14 results in an adjustment of $2,987bn.

The resulting aggregate household position in nominal claims rises from $3,818bn to $11,523. Adding these to the position of the government sector and rest of the world computed using the FA, we obtain an aggregate net position in nominal claims of -$5,236bn. While such claims should be in zero net supply, as a fraction of gross positions, the discrepancy is not very big. For instance, as noted in Table 15, the gross liability of the government sector in nominal claims is $36,630bn, so that the discrepancy of $5,236bn is less than 15% of this

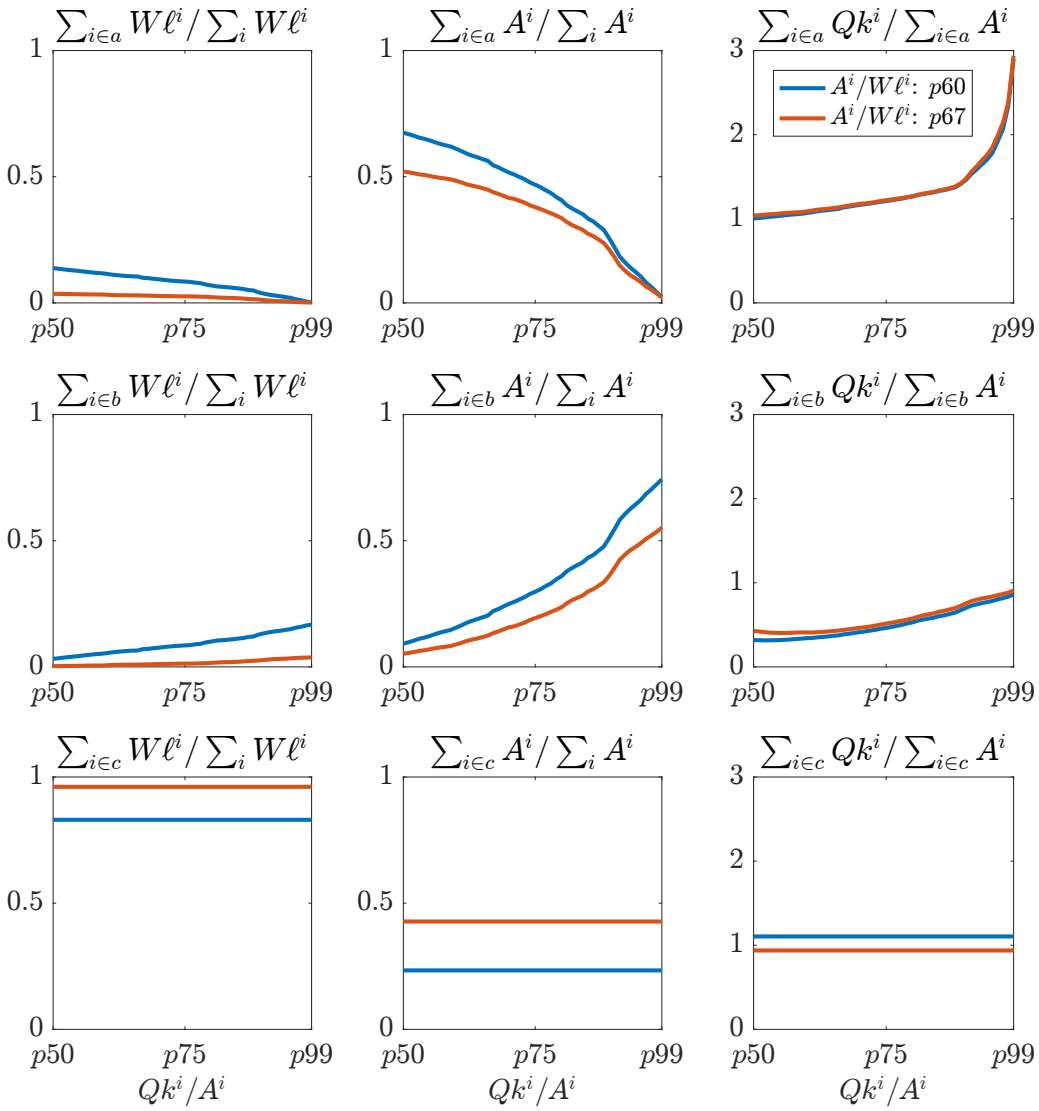


Figure 7: alternative cutoffs in wealth to labor income and the capital portfolio share

Notes: observations are weighted by SCF sample weights. Targeted moments in the calibration use a 60th percentile cutoff in wealth to labor income and 90th percentile cutoff in the capital portfolio share.

total. We conclude that our assumptions on household leverage are roughly consistent with market clearing in the market for nominal claims.

C.2.3 Sensitivity of targeted moments to cutoffs

We now characterize the sensitivity of our targeted moments in Table 2 to alternative cutoffs in the distributions of wealth to labor income and the capital portfolio share. Each row of Figure 7 summarizes the fraction of labor income, fraction of wealth, and ratio of aggregate

capital to wealth for each group. The first row presents moments for group a, the second for group b, and third for group c. On the x-axis of each panel we vary the cutoff in the capital portfolio share defining the groups (recalling that the cutoff used to construct targeted moments for the calibration is the 90th percentile). In each panel we plot moments with different cutoffs in the wealth to labor income ratio defining the groups (recalling that the cutoff used to construct targeted moments for the calibration is the 60th percentile).

A higher capital portfolio share cutoff naturally means that group a households are more levered, but conversely have a lower share of total wealth. In the context of our model, these effects have countervailing implications for the effect of monetary policy on the risk premium. Raising the wealth to labor income cutoff changes the shares of labor income and wealth for each group but leaves leverage little changed.

D Infinite horizon environment and solution

In this appendix we describe the infinite horizon environment studied and our computational algorithm used in section 3.

D.1 Environment and equilibrium

We first extend the environment described in section 2.1 to the infinite horizon. We closely follow the exposition in that section.

Households The unit measure of households is now organized into three groups $i \in \{a, b, c\}$ with measures $\{\lambda^i\}$ such that $\sum_i \lambda^i = 1$, where households are identical within groups. Each household continues to be comprised of a continuum of members $j \in [0, 1]$ supplying a differentiated variety of labor, and there remains full consumption insurance within households. The representative household i has Epstein-Zin preferences (27) with disutility of labor each period (28) following Shimer (2010). Each period, the household faces the resource constraint

$$\begin{aligned} P_t c_t^i + B_t^i + Q_t k_t^i \leq\ & (1-\tau)\int_0^1 W_t(j)\ell_t^i(j)dj - \int_0^1 AC_t^W(j)dj\ + \\ & (1+i_{t-1})B_{t-1}^i + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i \exp(\varphi_t) + T_t^i, \end{aligned} \tag{80}$$

where the Rotemberg (1982) cost of setting the wage for member j is

$$AC_t^W(j) = \frac{\chi^W}{2} W_t \ell_t \left(\frac{W_t(j)}{W_{t-1}\exp(\varphi_t)} - 1 \right)^2.$$

Households further face the capital constraint (29). In our calibration, this constraint will (almost always) only bind for c households.

Supply-side A union continues to represent each labor variety j across households. Each period, it chooses $W_t(j), \ell_t(j)$ to maximize the social welfare of union members subject to the allocation rule (4) and Pareto weights $\{\mu^i\}$. We now assume for simplicity that the allocation rule and Pareto weights are symmetric: $\ell^i(\ell_t) = \ell_t$ and $\mu^i = 1$. The labor packer combines varieties supplied by the union as in the two-period model, earning profits each period

$$W_t \left[\int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_t(j)\ell_t(j)dj. \tag{81}$$

The representative producer hires ℓ_t units of the labor aggregator in period t and combines it with $k_{t-1}\exp(\varphi_t)$ units of capital rented from households. It further uses $\left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x} x_t$ units of the consumption good to produce x_t new capital goods, where it again takes k_t as given. Taken together, it earns profits

$$\Pi_t k_{t-1}\exp(\varphi_t) = P_t\left(z_t\ell_t\right)^{1-\alpha}\left(k_{t-1}\exp(\varphi_t)\right)^{\alpha} - W_t\ell_t + Q_t x_t - P_t\left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x} x_t. \tag{82}$$

Productivity follows (30).

Policy The government follows a standard Taylor (1993) rule (32) where monetary policy shocks m_t follow (33). The government continues to set $\tau = -\frac{1}{\epsilon-1}$ and now sets household-specific lump-sum transfers T_t^i as in (34) given $\mathfrak{T}_t^i$ as in (35). We assume that $\tau_t^a = \tau^a$ and $\tau_t^c = \tau^c$ are constant, and τ_t^b ensures that $\sum_i \lambda^i \mathfrak{T}_t^i = 0$. As noted in the main text, we assume that households perfectly anticipate $\mathfrak{T}_t^i$ for all households except themselves.

Market clearing Market clearing in goods each period is now

$$\sum_i \lambda^i c_t^i + \left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x} x_t = \left(z_t\ell_t\right)^{1-\alpha}\left(k_{t-1}\exp(\varphi_t)\right)^{\alpha}, \tag{83}$$

in labor is

$$\left[\int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon}dj\right]^{\epsilon/(\epsilon-1)} = \ell_t, \tag{84}$$

in the capital rental market is

$$\sum_i \lambda^i k_{t-1}^i = k_{t-1}, \tag{85}$$

in the capital claims market is

$$(1-\delta)\sum_i \lambda^i k_{t-1}^i \exp(\varphi_t) + x_t = \sum_i \lambda^i k_t^i, \tag{86}$$

and in bonds is

$$\sum_i \lambda^i B_t^i + B^g = 0. \tag{87}$$

Equilibrium Given initial state variables $\{W_{-1}, \{B_{-1}^i, k_{-1}^i\}, i_{-1}, z_0, p_0, m_0\}$ and the stochastic processes (30)-(33), the definition of equilibrium naturally generalizes Definition 1:

Definition 3. *An equilibrium is a sequence of prices and policies such that: (i) each household i chooses $\{c_t^i, B_t^i, k_t^i\}$ to maximize (27) subject to (80) and (29), (ii) each union j chooses $\{W_t(j), \ell_t(j)\}$ to maximize the utilitarian social welfare of its members subject to the symmetric allocation rule $\{\ell_t^i(j) = \ell_t(j)\}$, (iii) the labor packer chooses $\{\ell_t(j)\}$ to maximize profits (81), (iv) the representative producer chooses $\{\ell_t, x_t\}$ to maximize profits (82), (v) the government sets $\{T_t^i\}$ and i_t according to (34) and (32), and (vi) the goods, labor, capital, and bond markets clear according to (83)-(87).*

Since labor varieties and unions j are symmetric, $\ell_t(j) = \ell_t$ and we again drop j going forward.

D.2 First-order conditions

We now outline households' and firms' optimality conditions.

Households Defining the realized real interest rate and real return on capital

$$\begin{aligned} 1 + r_{t+1} &\equiv (1+i_t)\frac{P_t}{P_{t+1}}, \\ 1 + r_{t+1}^k &=\equiv \frac{(\Pi_{t+1} + (1-\delta)Q_{t+1})\exp(\varphi_{t+1})}{Q_t}\frac{P_t}{P_{t+1}}, \end{aligned}$$

the representative household i's optimal consumption and savings decisions are characterized by

$$\begin{aligned} 1 &= \mathbb{E}_t m^i_{t,t+1}(1+r_{t+1}), \\ 1-\kappa^i_t &= \mathbb{E}_t m^i_{t,t+1}(1+r^k_{t+1}), \\ \kappa^i_t\left(k^i_t - \underline{k} z_t\right) &= 0, \\ k^i_t &\geq \underline{k} z_t, \\ \kappa^i_t &\geq 0, \end{aligned}$$

given the real stochastic discount factor

$$m^i_{t,t+1} = \beta\left(ce^i_t\right)^{\gamma^i - 1/\psi}\left(v^i_{t+1}\right)^{1/\psi-\gamma^i}\frac{\left(c^i_{t+1}\right)^{-\frac{1}{\psi}}\Phi(\ell_{t+1})^{1-\frac{1}{\psi}}}{\left(c^i_t\right)^{-\frac{1}{\psi}}\Phi(\ell_t)^{1-\frac{1}{\psi}}},$$

certainty equivalent $ce^i_t = \mathbb{E}_t\left[\left(v^i_{t+1}\right)^{1-\gamma^i}\right]^{\frac{1}{1-\gamma^i}}$, and multiplier on the constraint (29) $\kappa^i_t Q_t$.

Unions Defining the real wage $w_t \equiv \frac{W_t}{P_t}$, the representative union sets

$$\begin{aligned} \sum_i \lambda^i\left(v^i_t\right)^{\frac{1}{\psi}}\left(c^i_t\right)^{-\frac{1}{\psi}}\Bigg[& w_t + c^i_t\frac{\Phi'(\ell_t)}{\Phi(\ell_t)} \\ & + w_t\frac{\chi^W}{\epsilon}\left[\frac{w_t}{w_{t-1}\exp(\varphi_t)}\frac{P_t}{P_{t-1}}\left(\frac{w_t}{w_{t-1}}\frac{P_t}{P_{t-1}}-1\right)\right. \\ & \left. - \mathbb{E}_t m^i_{t,t+1}\left(\frac{w_{t+1}}{w_t\exp(\varphi_{t+1})}\right)^2\frac{P_{t+1}}{P_t}\frac{\ell_{t+1}}{\ell_t}\left(\frac{w_{t+1}}{w_t\exp(\varphi_{t+1})}\frac{P_{t+1}}{P_t}-1\right)\right]\Bigg] = 0. \end{aligned}$$

Producers Defining the real price of capital $q_t \equiv \frac{Q_t}{P_t}$, the representative producer follows

$$\begin{aligned} w_t &= (1-\alpha)z_t^{1-\alpha}\ell_t^{-\alpha}\left(k_{t-1}\exp(\varphi_t)\right)^{\alpha}, \\ q_t &= \left(\frac{k_t}{k_{t-1}\exp(\varphi_t)}\right)^{\chi^x}. \end{aligned}$$

D.3 Re-scaled economy

We now characterize the equilibrium conditions of an equivalent, stationary economy obtained by dividing households' resource constraints and market clearing conditions by the price level P_t, and further dividing these conditions as well as the first-order conditions in

the prior subsection by z_t. We denote real variables in lower-case (except for the nominal rate i_t) and further defined the re-scaled variables

$$\tilde{c}_t^i \equiv \frac{c_t^i}{z_t}, \;\; \tilde{ce}_t^i \equiv \frac{ce_t^i}{z_t}, \;\; \tilde{b}_t^i \equiv \frac{b_t^i + B^g/P_t}{z_t}, \;\; \tilde{k}_t^i \equiv \frac{k_t^i}{z_t}, \;\; \tilde{k}_t \equiv \frac{k_t}{z_t}, \;\; \tilde{w}_t \equiv \frac{w_t}{z_t}, \tag{88}$$

$$\tilde{m}_{t,t+1}^i \equiv m_{t,t+1}^i \left(\frac{z_{t+1}}{z_t}\right)^{-\gamma}, \tag{89}$$

$$\tilde{k}_{t-1}^i \equiv \frac{k_{t-1}^i}{\exp(\sigma\epsilon_t^z)}, \;\; \tilde{k}_{t-1} \equiv \frac{k_{t-1}}{\exp(\sigma\epsilon_t^z)}, \;\; \tilde{w}_{t-1} \equiv \frac{w_{t-1}}{\exp(\sigma\epsilon_t^z)}. \tag{90}$$

We note that $\tilde{b}_t^i$ is now interpreted as the productivity-adjusted bond position of household i accounting for the implicit position that household also has through the government.

Then the household's optimality conditions and constraints are equivalent to:

$$1 = \mathbb{E}_t \tilde{m}_{t,t+1}^i \exp\left(\gamma^i \left[\sigma\epsilon_{t+1}^z + \varphi_{t+1}\right]\right)(1 + r_{t+1}), \tag{91}$$

$$1 - \kappa_t = \mathbb{E}_t \tilde{m}_{t,t+1}^i \exp\left(\gamma^i \left[\sigma\epsilon_{t+1}^z + \varphi_{t+1}\right]\right)(1 + r_{t+1}^k), \tag{92}$$

$$\kappa_t^i \left(\tilde{k}_t^i - \underline{k}\right) = 0, \tag{93}$$

$$\tilde{k}_t^i \geq \underline{k}, \tag{94}$$

$$\kappa_t^i \geq 0, \tag{95}$$

$$\tilde{m}_{t,t+1}^i = \beta \left(\tilde{ce}_t^i\right)^{\gamma^i - 1/\psi} \left(\tilde{v}_{t+1}^i\right)^{1/\psi - \gamma^i} \frac{\left(\tilde{c}_{t+1}^i\right)^{-\frac{1}{\psi}} \Phi(\ell_{t+1})^{1-\frac{1}{\psi}}}{\left(\tilde{c}_t^i\right)^{-\frac{1}{\psi}} \Phi(\ell_t)^{1-\frac{1}{\psi}}}, \tag{96}$$

$$\tilde{ce}_t^i = \mathbb{E}_t \left[\exp\left(\left(1-\gamma^i\right)\left[\sigma\epsilon_{t+1}^z + \varphi_{t+1}\right]\right)\left(\tilde{v}_{t+1}^i\right)^{1-\gamma^i}\right]^{\frac{1}{1-\gamma^i}}, \tag{97}$$

$$\tilde{c}_t^i + \tilde{b}_t^i + q_t \tilde{k}_t^i = \tilde{w}_t \ell_t^i + \tilde{n}_{t-1}^i, \tag{98}$$

where financial wealth inclusive of taxes and transfers, accounting for government budget balance, is

$$\tilde{n}_{t-1}^i = \frac{1}{\lambda^i} s_t^i \left(\pi_t + (1-\delta) q_t\right) \tilde{k}_{t-1}. \tag{99}$$

The representative union's optimality condition is equivalent to:

$$\sum_i \lambda^i \left(\tilde{v}_t^i\right)^{\frac{1}{\psi}} \left(\tilde{c}_t^i\right)^{-\frac{1}{\psi}} \Bigg[\tilde{w}_t + \tilde{c}_t^i \frac{\Phi'(\ell_t)}{\Phi(\ell_t)} + \tilde{w}_t \frac{\chi^W}{\epsilon} \Bigg[\frac{\tilde{w}_t}{\tilde{w}_{t-1}} \frac{P_t}{P_{t-1}} \left(\frac{\tilde{w}_t}{\tilde{w}_{t-1}} \frac{P_t}{P_{t-1}} - 1\right) - \mathbb{E}_t \tilde{m}_{t,t+1}^i \exp\left(\gamma^i \left[\sigma \epsilon_{t+1}^z + \varphi_{t+1}\right]\right) \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^2 \frac{P_{t+1}}{P_t} \frac{\ell_{t+1}}{\ell_t} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \frac{P_{t+1}}{P_t} - 1\right) \Bigg]\Bigg] = 0. \tag{100}$$

The representative producer's optimality condition and flow of funds are equivalent to:

$$\tilde{w}_t = (1-\alpha)\ell_t^{-\alpha} \tilde{k}_{t-1}^{\alpha}, \tag{101}$$

$$q_t = \left(\frac{\tilde{k}_t}{\tilde{k}_{t-1}}\right)^{\chi^x}, \tag{102}$$

$$\pi_t \tilde{k}_{t-1} = \alpha \ell_t^{1-\alpha} \tilde{k}_{t-1}^{\alpha}. \tag{103}$$

The specifications of fiscal and monetary policy imply:

$$s_{t+1}^i = \lambda^i (1 - \tau_{t+1}^i) \frac{(1+r_{t+1})\tilde{b}_t + (\pi_{t+1} + (1-\delta)q_{t+1})\tilde{k}_t^i \exp(\varphi_{t+1})}{(\pi_{t+1} + (1-\delta)q_{t+1})\tilde{k}_t \exp(\varphi_{t+1})}, \tag{104}$$

$$1 + i_t = (1 + \bar{i}) \left(\frac{P_t}{P_{t-1}}\right)^{\phi} m_t. \tag{105}$$

The definitions of real returns remain:

$$1 + r_{t+1} \equiv (1 + i_t) \frac{P_t}{P_{t+1}}, \tag{106}$$

$$1 + r_{t+1}^k =\equiv \frac{(\pi_{t+1} + (1-\delta)q_{t+1}) \exp(\varphi_{t+1})}{q_t}. \tag{107}$$

The market clearing conditions are equivalent to:

$$\sum_i \lambda^i \tilde{c}_t^i + \left(\frac{\tilde{k}_t}{\tilde{k}_{t-1}}\right)^{\chi^x} \tilde{x}_t = \ell_t^{1-\alpha} \tilde{k}_{t-1}^{\alpha}, \tag{108}$$

$$\sum_i \lambda^i \tilde{k}_t^i = \tilde{k}_t, \tag{109}$$

$$(1-\delta) \sum_i \lambda^i \tilde{k}_{t-1}^i + \tilde{x}_t = \sum_i \lambda^i \tilde{k}_t^i, \tag{110}$$

$$\sum_i \lambda^i \tilde{b}_t^i = 0. \tag{111}$$

Finally, the evolution of exogenous state variables is:

$$\log m_{t+1} = \rho^m \log m_t + \sigma^m \epsilon_{t+1}^m. \tag{112}$$

After solving this transformed economy, we can simulate prices and quantities in the original economy by reversing the re-scaling in (88)-(90), where z_t follows (30) and P_t follows $P_t = P_{-1} \prod_{t'=0}^{t} \Pi_{t'}^P$ where $\Pi_{t'}^P \equiv \frac{P_{t'}}{P_{t'-1}}$.

D.4 Global solution algorithm

We now outline the computational algorithm used to solve the transformed economy.

Grids The model is solved over a discretized grid of aggregate states S. Each node is defined by the current monetary policy shock $m(S)$ and probability of disaster $p(S)$, the wealth shares $s^a(S)$ and $s^c(S)$ of groups a and c, the scaled capital chosen in the previous period $\tilde{k}_{-1}(S)$ as chosen in the previous period, as well as the scaled real wage $\tilde{w}_{-1}(S)$ set in the previous period. In the transformed, stationary economy, productivity shocks inclusive of disasters only govern the transition across states. The grid over states is given by a mesh grid over vectors of each state variable. In each dimension except the two-state disaster probability $p(S)$ we choose a vector length of at least five nodes, where the vector's upper and lower bound are iteratively updated to make sure that the state variables stay well within the chosen limits in ten simulations of 500 years each. We verify that the model solutions are robust to grid boundaries and size for the chosen values.

Expectations and interpolation When forming expectations, we use Gauss-Hermite quadrature for integration. Expectations over future states will typically not lie on the grid, and we use linear interpolation over aggregate states to find variable values for those states.

The value functions of the representative household in each group are solved over a vector of individual wealth inclusive of transfers n^i_{-1}, so that households can entertain a range of portfolio and savings choices when optimizing. We use cubic splines to interpolate over the idiosyncratic wealth levels, which also enables us to calculate value function derivatives.

Solution algorithm We look for a stationary solution to the model and use backward iteration until all equilibrium objects converge. We assume that convergence is satisfactory when relative period-to-period changes are smaller than 10^{-6}. For each state S, the solution objects are the price of capital $q(S)$, the nominal rate $i(S)$, the chosen real wage $w(S)$, the inflation rate $\Pi^P(S)$, labor supply $\ell(S)$, capital choices of each household group $k^i(S)$, real bond choices of each group $b^i(S)$, and the value functions of each group over a vector of wealth $v^i(n^i_{-1}, S)$.

The solution algorithm starts from an initial guess for $v^i(n^i_{-1}, S)$, $q(S)$, $i(S)$, $w(S)$, $\Pi^P(S)$, and $\ell(S)$ and proceeds as follows.

1. With this guess at hand we can solve each representative household's savings and portfolio choice problem (91)-(98) given its current wealth inclusive of transfers implied by (99), the interest rates implied by (103), (106), (107), and the evolution of state variables implied by (104), (109), and (112).

2. Observing the excess demand for bonds relative to the market clearing condition (111), we adjust $i(S)$ to lower the absolute value of the excess demand, returning to step 1, until excess bond demand relative to the aggregate capital stock is smaller than 10^{-12}.

3. The resulting choice of individual capital holdings, together with the market clearing condition (109), allows us to update the price of capital $q(S)$ according to (102).

4. We use the union's first-order condition (100) to update the wage choice $w(S)$, given labor demand for the representative producer (101).

5. Given the equilibrium nominal rate, we use the Taylor rule (105) to update $\Pi^P(S)$.

6. Finally, we update the value functions $v^i(n^i_{-1}, S)$ by solving the optimization problem (91)-(98) of all representative households for wealth away from the current state.

7. Using the updated equilibrium objects, we define new guesses $v^i(n^i_{-1}, S)$, $q(S)$, $i(S)$, $w(S)$, $\Pi^P(S)$, and $\ell(S)$ and return to step 1. For numerical stability we dampen the updating of most equilibrium objects.

At the conclusion of the algorithm, a policy function for $x(S)$ is implied by the capital accumulation condition (110), and goods market clearing (108) is satisfied by Walras' Law.

The solution code is written in Fortran and parallelized using OpenMP. Convergence can be achieved in less than twenty minutes on a modern computing system with eight cores.

E Additional quantitative results

In this appendix we provide supplementary quantitative results accompanying section 3.

E.1 Model versus RANK given the same initial shock

In the main text, we compared impulse responses to a monetary shock in our model to those of a counterfactual RANK economy where ϵ_0^m was re-calibrated to match the same initial $22bp$ decline in the 1-year nominal yield. Here we instead compare impulse responses to the exact same monetary shock ϵ_0^m. Figure 8 summarizes the variables of interest. While the risk premium response to the shock clearly differs across environments, we now see that the endogenous tightening of monetary policy in our model (evident in the 1-year nominal yields) largely eliminates any additional stimulus to quantities relative to RANK.

E.2 Impulse responses to a productivity shock

We now characterize the impulse responses to a productivity shock instead of a monetary shock. We compare in Figure 9 the impulse responses of the model with heterogeneity to a counterfactual economy with a representative agent, with the same primitive one standard deviation shock in productivity.

The first row again reports the change in the 1-year nominal bond yield, expected real returns, and expected excess returns. The first panel demonstrates that the central bank following a standard Taylor (1993) rule will cut the nominal interest rate in response to the price deflation induced by this shock. The second and third panels demonstrate that the expected real interest rate and the expected excess returns on capital decline following the shock. The decline in the former is a standard real business cycle response to the shock and also reflects the endogenous monetary easing in this New Keynesian setting. The decline in the latter demonstrates that productivity shocks induce a countercylical risk premium.

The second and third rows demonstrate that redistribution drives the decline in the risk premium following the shock. The first panel of the second row demonstrates that, as in the case of a negative monetary policy shock, realized excess returns on capital are substantially positive on impact and followed by small negative returns in the quarters which follow.

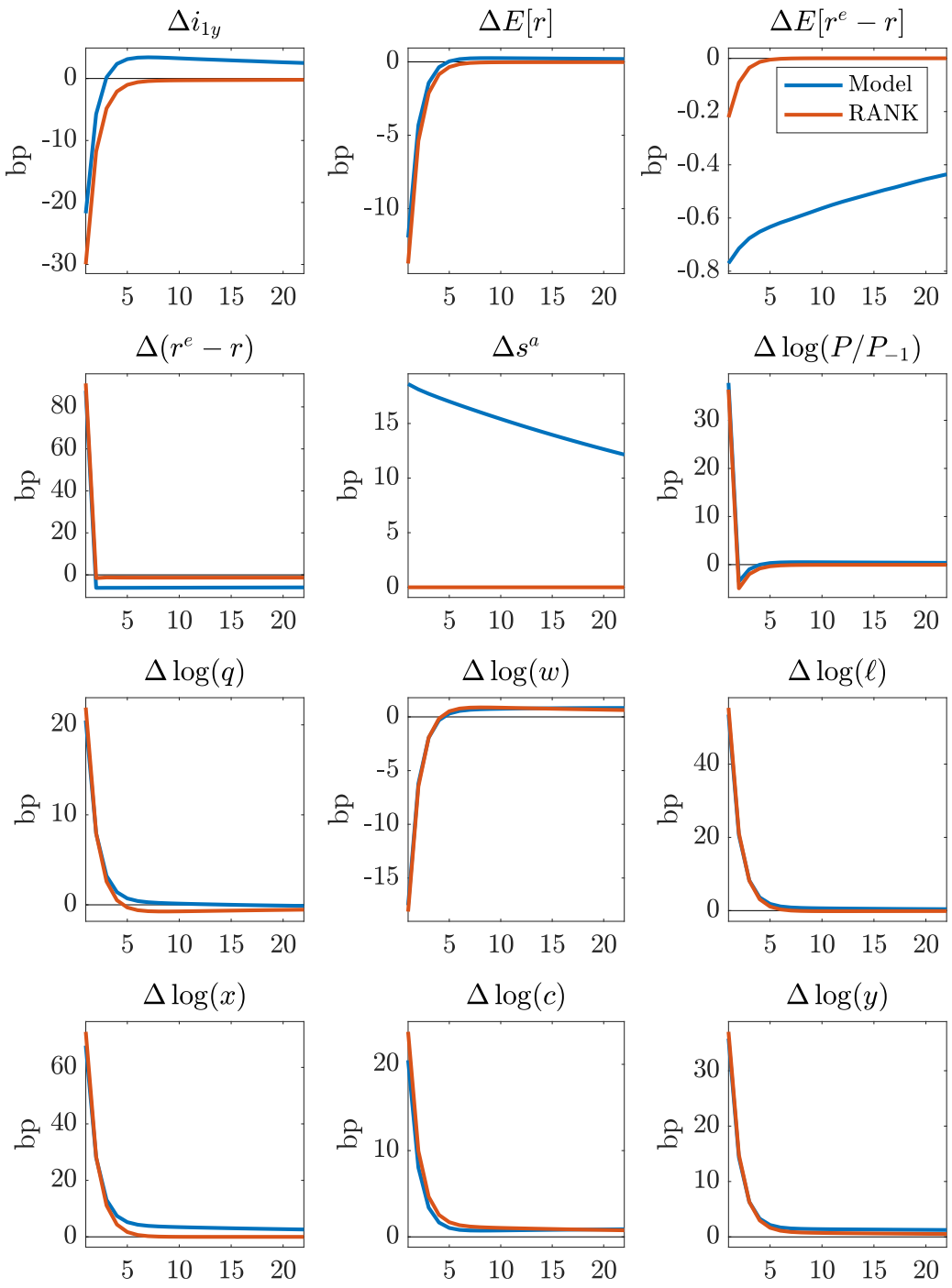


Figure 8: effects of negative monetary policy shock with same initial shock

Notes: all series are quarterly (non-annualized) measures, except for the 1-year nominal bond yield Δi_{1y}. Each impulse response is the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space with no disasters ever realized, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. *bp* denotes basis points (0.01%).

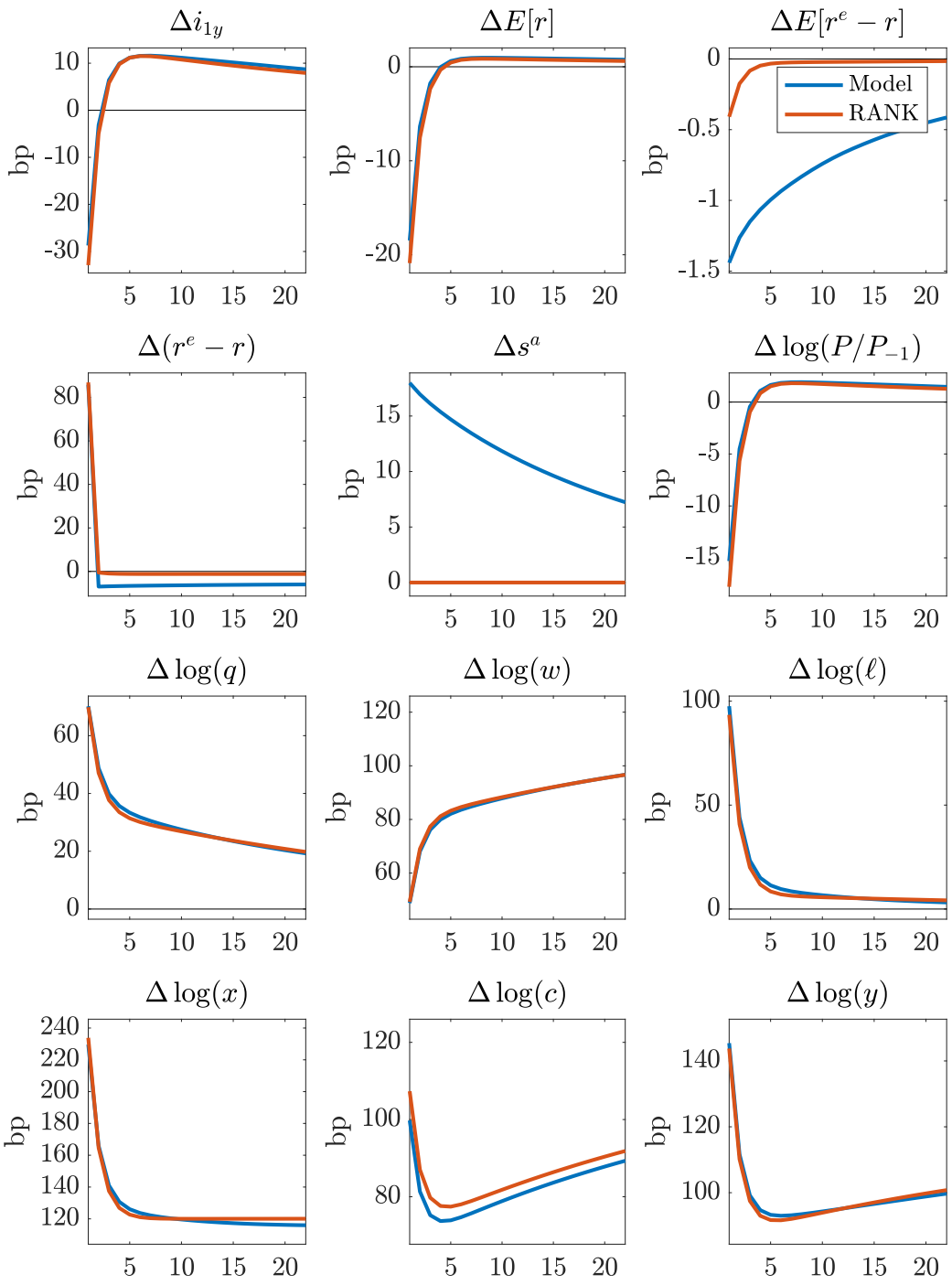


Figure 9: impulse responses to positive productivity shock

Notes: all series are quarterly (non-annualized) measures, except for the 1-year nominal bond yield Δi_{1y}. The impulse response for each series reports the average response (relative to no shock) starting at 1,000 different points drawn from the ergodic distribution of the state space, itself approximated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters. *bp* denotes basis points (0.01%).

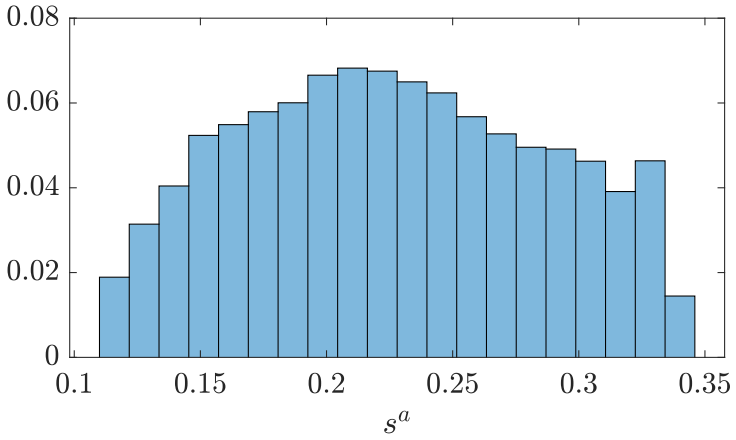


Figure 10: ergodic distribution of wealth share of a households

Notes: distribution estimated using a sample path over 50,000 quarters after a burn-in period of 5,000 quarters, with no disasters ever realized.

Through the lens of Proposition 12 characterized in appendix B, the substantially positive excess returns on impact endogenously redistribute to the high MPR a households who hold levered claims on capital, evidenced in the financial wealth share of a households in the second panel in this row. As described in appendix B, however, the mechanisms are more nuanced than in the case of a monetary shock. On the one hand, unexpected deflation raises the real interest rate, shown in the third panel. On the other hand, the increase in the price of capital raises the return on capital, shown in the first panel of the third row. The effect on profits reflects the competing effects of a higher real wage and higher employment. On balance, the return to capital increases and outweighs the higher realized real interest rate, and wealth redistributes to the high-MPR a households, lowering the risk premium.

The fourth row examines the consequences for real transmission. Comparing the responses in the model to the representative agent case, we find little difference in investment, consumption, or output. Consistent with the responses to the same monetary policy shock described in the last subsection, this is consistent with the endogenous tightening of monetary policy in the model in response to the stimulus from lower risk premia.

E.3 Ergodic distribution and sample path of s^a

As discussed in the main text, the changes in the wealth share of relatively risk tolerant a households (s^a) plays a key role in determining the effects of monetary shocks. Here we provide the ergodic distribution of this variable as well as a sample path, in both cases conditional on the assumption of no disasters actually being realized. Figure 10 depicts the

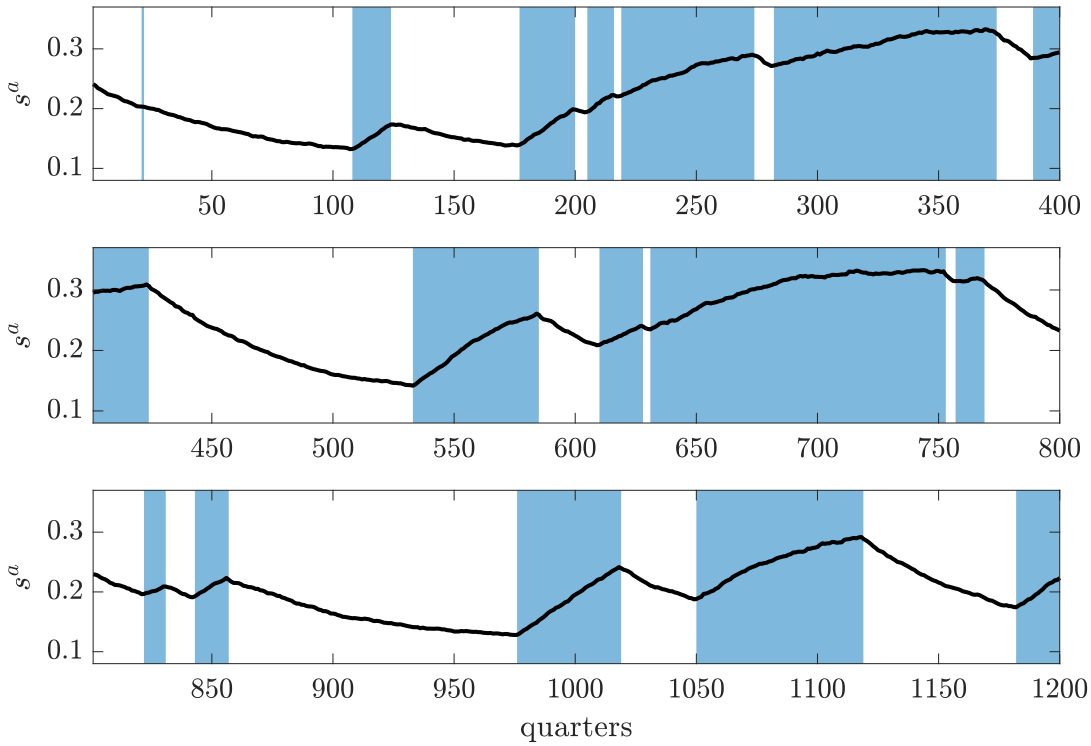


Figure 11: sample path of wealth share of a households

Notes: sample path shown after a burn-in period of 5,000 quarters, with no disasters ever realized. Highlighted areas denote quarters in which the disaster probability is at its higher level.

ergodic distribution over 50,000 quarters simulated after a burn-in period of 5,000 quarters. Figure 11 provides the sample path of the first 1,200 quarters after the burn-in period. In blue, we highlight quarters in which the disaster probability is at its higher level. Because these periods are associated with higher expected excess returns on capital (but no disasters are realized), they are associated with a rising wealth share of a households. The opposite is true when the disaster probability is at its lower level.