The Anatomy of Crises and Cycles in Mexico: An Endogenous Regime-Switching Approach*

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Abstract

We perform quantitative analysis of a workhorse open economy model with occasionally binding constraints using structural estimation. First, we propose a new specification of an occasionally binding collateral constraint where the transition between the unconstrained and the constrained state is a stochastic function of the level of leverage and the borrowing multiplier. The specification maps into an endogenous regime-switching model. Second, we develop a general perturbation solution method to solve such a model, and show the importance of using a second-order approximation. Third, we estimate the model with Bayesian methods to fit Mexico's business cycle and financial crisis history since 1981. We identify 'peak crises' with sudden stop dynamics and 'near crises' when the economy is vulnerable. These episodes can have varying length and intensity. Sudden stops occur when a variety of adverse shocks hit an already prone economy. Different sets of shocks explain the three identified historical crisis episodes.

Keywords: Financial Crises, Endogenous Regime-Switching, Bayesian Estimation, Occasionally Binding Constraints, Mexico.
JEL Codes: G01, E3, F41, C11.

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1 Introduction

The Global Financial Crisis generated a renewed interest in understanding the causes and the dynamics associated with financial crises. In this context, dynamic stochastic general equilibrium (DSGE) models with occasionally binding frictions have proven successful as laboratories to study the anatomy of both business cycles and crises and to explore optimal policy responses to these dynamics. This success is because occasionally binding frictions are amplification mechanisms of regular business cycle dynamics. Structural estimation of these models is challenging, but would permits making inference, counterfactual policy analysis, and possibly forecasting.

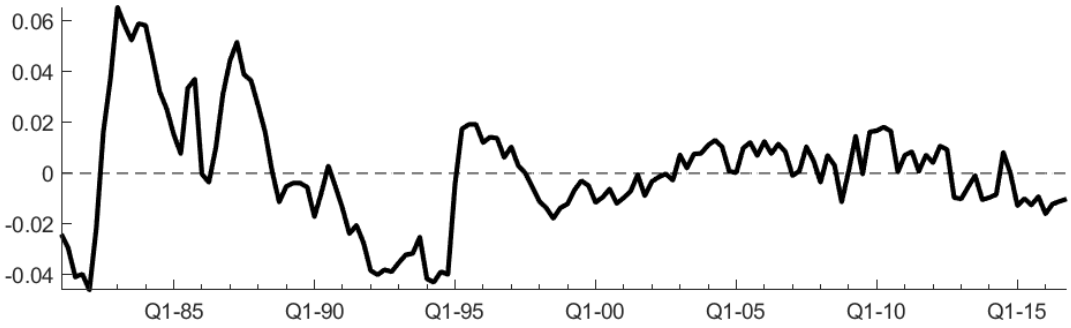
In this paper, we structurally estimate and quantitatively analyze a model with an occasionally binding borrowing constraint. We make three main contributions. First, we propose a new specification of the occasionally binding collateral constraint. Second, we device a perturbation solution approach suitable for solving models like ours in a way that permits likelihood-based estimation. Third, we focus on one particular type of crisis, the so-called sudden stop in international capital flows, and apply the proposed approach to the estimation of a medium-scale workhorse DSGE model of sudden stops, investigating sources and frictions of business cycles and sudden stop crises in Mexico since 1981.

As a first step, we propose a new formulation of occasionally binding constraint models. As in models with constraints written as inequalities, our set up has two regimes: one in which a given leverage ratio limits borrowing and amplifies regular shocks to explain financial crises as the economy cannot smooth consumption and expand production, and one in which access to financing is plentiful and the economy displays regular business cycles. In our specification, however, transitions between the two regimes occur in a stochastic rather than deterministic manner. Probabilities that depend on the borrowing capacity of the economy and the multiplier associated with the binding collateral constraint govern the transition between these regimes. This formulation maps the modeling of an occasionally binding constraint into an endogenous regime-switching model. The paper focuses on a particular financial friction, but the proposed specification has broader applicability to other occasionally binding constraints.

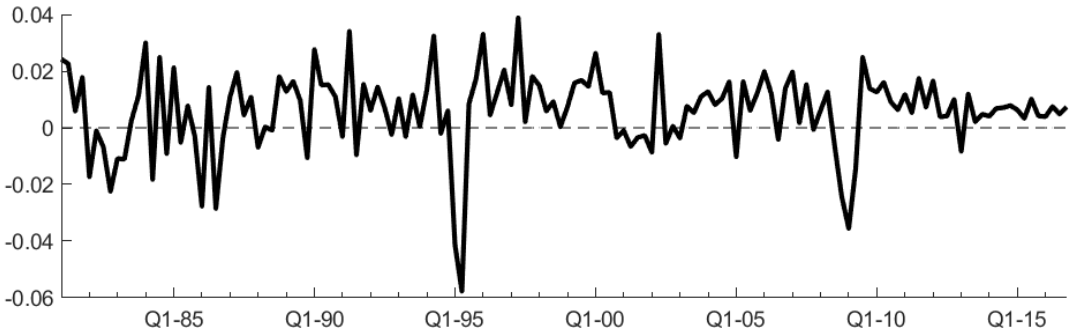
Next, we develop a perturbation-based solution method for solving our model. The perturbation method is fast enough to permit estimation, is readily scalable to

Figure 1: Current Account and Output in Mexico, 1981-2016

(a) Current Account to Output Ratio



(b) Quarterly Output Growth Rate



Note: Panel (a) plots the ratio of current account balance as a share of GDP. Panel (b) shows the quarterly log-deviation of real GDP. The sample period is 1981:Q1-2016:Q4. See Appendix for data sources.

models larger than the one we estimate in this paper, and displays typical levels of accuracy. We show that the proposed perturbation approach with a second-order approximation to the policy functions characterizing optimal behavior captures the effects of endogenous transition probabilities on precautionary behavior, and that these effects would be missed by linear approximations. As in our first contribution, the method applies beyond the scope of the paper, as it can be used with a wide range of endogenous regime-switching models.

Finally, we apply our borrowing constraint specification and solution method to the Bayesian estimation of a model that can characterizes both financial crises and business cycles in emerging market economies. Because our application focuses on sudden stops in capital flows, we estimate a medium-scale, workhorse open economy model. The model structure is the same as in Mendoza (2010), but we consider a

larger set of exogenous shocks as in Garcia-Cicco et al. (2010). The critical difference relative to the model in Mendoza (2010) is the specification of the collateral constraint, which as we said can be easily adapted to other model settings.

Figure 1 plots two relevant variables for our study: the current account balance as a share of GDP and quarterly GDP growth in deviation from sample mean. While the Figure provides evidence of fluctuations in both variables, it is equally apparent that Mexico had multiple episodes of large current account reversals and persistent output growth decline of varying size. Large current account reversals and output drops of varying persistence are the two main empirical features commonly associated with sudden stops in capital flows. In this paper, we focus on the challenge of fitting the model to the Mexican business cycle and the sudden stop crises that afflicted many other emerging markets since they started to open their capital account. The approach that we propose, however, is applicable much more generally to the formulation and estimation of models of occasionally binding constraints such as Schmitt-Grohe and Uribe (2018), Iacoviello (2005), and Liu et al. (2013).

Our estimated model fits Mexico's business cycles and sudden stop episodes well, without relying on large shocks to explain crisis periods. It produces business cycle statistics that match the second moments of the data. Most importantly, it can identify not only acute but short lived sudden stops periods (which we call peak crisis periods), as previously done in the literature with similar models, but also milder but more persistent sudden stop periods in which the economy is in the constrained regime without experiencing unusually large dynamics (which we call near crisis periods). Thus, as we will see later, our specification of the collateral constraint permits the identifying crisis episodes and estimating crisis dynamics of different duration and intensity. In particular, the estimated model identifies three peak crisis episodes: 1983:Q1, 1995:Q1-Q2, and 2009Q1-Q2, with near crisis dynamics surrounding these more acute phases. The estimated model also generates a peak crises frequency that matches the estimates in the sudden stop literature, and near crisis episodes that align well with an purely empirical notion of crisis in Mexico (Reinhart and Rogoff, 2009). Finally, we show that while different shocks explain different variables over the cycles, specific shocks matter for historical crisis dynamics.

Related Literature A few papers have already attempted estimation of models with occasionally binding constraints. Bocola (2016), in particular, builds and estimates a model of occasionally occurring debt and banking crises. Notably, estimation is accomplished while solving the model with global methods. This, however, is accomplished the cost of simplifying the estimation procedure to overcome speed limitations. In particular, the estimation procedure used is to first estimate the model outside the crisis period, using a solution technique that assumes a crisis will not occur, and then append the crisis in a second step.[1] Moreover, our approach is potentially scalable to larger and more complex models, while maintaining a satisfactory level of accuracy relative to global solution methods.

Our paper relates also to Guerrieri and Iacoviello (2015), who develop a set of procedures for the solution of models with occasionally binding constraints, called OccBin. OccBin is a certainty equivalent solution method which captures non-linearities but rules out precautionary effects, which are the critical manifestation of the linkages between states of the world in models with occasionally binding collateral constraints. A key feature of our approach is to preserve precautionary saving effects, as agents in the model adjust their behavior due to the presence of the constraint, even when the constraint does not bind and vice versa.

The constraint specification that we propose and the accompanying perturbation solution method also apply to the literature on the zero-lower bound on interest rates such as for instance Adam and Billi (2006), Adam and Billi (2007), Aruoba et al. (2018), and Atkinson et al. (2018). Estimation of such models is possible but either at higher computational costs that limit scalability (Gust et al., 2017), or restricting the to piecewise-linear approximations that do not fully capture precautionary behavior (Aruoba et al., 2020). Moreover, occasionally binding ZLB constraints are not comparable to the kind of model with endogenous collateral value that we estimate in this paper and used in the normative literature on macroprudential policies (Bianchi and Mendoza, 2018). Indeed, as it is well understood in the literature on the normative analysis of occasionally binding constraints, endogenous collateral valuations induce additional sources of computational complexity (Benigno et al., 2013, 2016).

In the literature on Markov-switching DSGE models, our paper expands upon the

[1] While this does not matter for the specific application in Bocola (2016), because of the nature of the constraints, it would be relevant for countries experiencing serial default and repeated banking crises. Our methodology could allows one to estimate this model in one-step, rather than with the two-step procedure adopted.

method developed by Foerster et al. (2016), who developed perturbation methods for the solution of exogenous regime-switching models. The perturbation approach allows for second- and higher-order approximations that go beyond the linearized models studied by Davig and Leeper (2007) and Farmer et al. (2011). In fact, we show that at least a second-order approximation is necessary in order to capture the effects of the endogenous switching.

The paper is also related to the literature that focuses on solving endogenous regime-switching models. Davig and Leeper (2008), Davig et al. (2010), and Alpanda and Ueberfeldt (2016) all consider endogenous regime-switching, but employ computationally costly global solution methods that prevent the possibility of a likelihood-based estimation. Lind (2014) develops a regime-switching perturbation approach for approximating non-linear models, but it requires repeatedly refining the points of approximation and hence it is not suitable for estimation purposes. Maih (2015) and Barthlemy and Marx (2017) develop perturbation methods for endogenous switching models, but employ a technique that approximates around regime-dependent steady states. In contrast, our generalization of the Foerster et al. (2016) perturbation approach uses a single point of approximation that is well suited for solving models in which the regime-dependency of the steady state is not be crucial because of the relatively slow moving nature of state variables such, as capital and debt, and the limited frequency and duration of financial crisis episodes.

We also contribute to the literature on estimating Markov-switching DSGE models (Bianchi, 2013; Bianchi and Ilut, 2017; Bianchi et al., 2018). Our algorithm differs in two key respects. First, our regime-switching transition matrix reflects the endogenous nature of the switching. Second, conditional on the regime, we have a second order solution, so we employ the Sigma Point Filter (Binning and Maih, 2015) to evaluate the likelihood function in place the modified Kalman filter in Bianchi (2013).

The application of the methodology that we propose relates to the literature on emerging market business cycles, including particularly, among others, Mendoza (1991), Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Mendoza (2010), Garcia-Cicco et al. (2010), Fernandez and Gulan (2015). Encompassing most shocks previously considered in this literature, we include in our analysis technology shocks, preference, expenditure, interest rate, and terms of trade shocks. Relative to Mendoza (2010), we provide a Bayesian estimation of the model and consider a wider set

of structural shocks. While the implied model properties are similar, we find that the estimated values of some of the parameters that are less easily calibrated to the stylized facts of the data differ. Relative to Garcia-Cicco et al. (2010), we evaluate empirically the relative importance of interest shocks in a fully non-linear estimated framework, with a more fully articulated specifications of the financial frictions driving amplifications, and finding that they are quantitatively important for certain features of the data, but not others. Relative to Neumeyer and Perri (2005), we set up a framework that fits the data well without assuming any correlation between the productivity and the interest rate process. Nonetheless, consistent with their main findings, and also with Fernandez and Gulan (2015) and Ates and Saffie (2016), we find that, while we can fit ergodic second moments of the data well with uncorrelated shocks, high and averse correlation configurations are associated with the simulated crisis dynamics in the model.

Finally, our paper relates to the now large literature on the Bayesian estimation of DSGE models (for example, Schorfheide, 2000; Otrok, 2001; Smets and Wouters, 2007). Our paper extends that successful approach to models with occasionally binding collateral constraints, which have become the benchmark for normative analysis of macro-prudential optimal policy (Bianchi and Mendoza, 2018; Benigno et al., 2013, 2016; Jeanne and Korinek, 2010). Welfare-base analysis of optimal macroprudential policies in model with occasionally binding constraints depends on calibrations assumptions. Structural estimation of these parameters within the same framework used for policy analysis disciplines the model specification, which in turn is critical for normative analysis.

The rest of the paper is organized as follows. Section 2 describes the model and discusses the proposed formulation of the collateral constraint. Section 3 presents our perturbation solution method for endogenous regime-switching models. Section 4 describes the full information Bayesian procedure we follow. Section 5 reports the empirical results. Section 6 concludes. All technical details are in Appendices.

2 The Model

The model we estimate is the medium scale, workhorse framework for the analysis of business cycles and sudden stop crisis in emerging market economies in Mendoza (2010), with a larger set of shocks as in (Garcia-Cicco, Pancrazi, and Uribe, 2010).

This is a small, open, production economy with an occasionally binding collateral constraint, subject to temporary productivity, intertemporal preference, expenditure, interest rate, and terms of trade shocks.[2] The collateral constraint that we specify depends on the endogenous variables of the model, including borrowing, capital and its relative price, and hence leverage. Capital and debt choices respond to exogenous shocks, affecting borrowing, which in turn affects the probability of a binding collateral constraint.

Due to the occasionally binding nature of the constraint, this framework can account not only for normal business cycles, but also key aspects of financial crises in both emerging markets advanced economies (e.g., Bianchi and Mendoza (2018)). Note however that, while our application focuses on one particular type of crisis, the so called sudden stop in capital flows, our framework is generally applicable to other macroeconomic models with financial frictions and crises (Kiyotaki and Moore, 1997; Iacoviello, 2005; Gertler and Karadi, 2011; Jermann and Quadrini, 2012; Liu et al., 2013; Gertler and Kiyotaki, 2015; Bocola, 2016; Schmitt-Grohe and Uribe, 2018).

In the rest of this section, we discuss the representative household-firm and the borrowing constraint. The formal definition of the equilibrium and the full set of equilibrium conditions is reported in Appendix A.

2.1 Preferences, Constraints, and Shock Processes

There is a representative household-firm that maximizes the following utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}, \tag{1}$$

where C_t denotes consumption, H_t the supply of labor, and d_t an exogenous and stochastic preference shock specified below. Households choose consumption, labor, capital (K_t), imported intermediate inputs (V_t) given an exogenous stochastic relative price, P_t also specified below, and holdings of real one-period international bonds, B_t. Negative values of B_t indicate borrowing from abroad. The households budget

[2] We omit permanent technology shocks of the type analyzed by Aguiar and Gopinath (2007) because these long-run components cannot be estimated precisely over samples periods of length comparable to ours. Garcia-Cicco et al. (2010) also find that the permanent technology shock is not quantitatively important in a framework with an financial frictions like ours.

constraint is:

$$C_t + I_t + E_t = Y_t - \phi r_t \left(W_t H_t + P_t V_t\right) - \frac{1}{(1+r_t)} B_t + B_{t-1}, \tag{2}$$

where Y_t is gross domestic product and is given by

$$Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t. \tag{3}$$

Here, A_t denotes an exogenous and stochastic temporary productivity shock. E_t is an exogenous and stochastic domestic expenditure process possibly interpreted as a fiscal or net export shock as in Garcia-Cicco et al. (2010). The term $\phi r_t \left(W_t H_t + P_t V_t\right)$ describes a working capital constraint, stating that a fraction of the wage and intermediate good bill must be paid in advance of production with borrowed funds. The relative price of labor and capital are given by W_t and q_t, respectively, both of which are endogenous market prices, but taken as given by the household-firm. Gross investment, I_t, is subject to adjustment costs as a function of net investment:

$$I_t = \delta K_{t-1} + \left(K_t - K_{t-1}\right)\left(1 + \frac{\iota}{2}\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)\right). \tag{4}$$

Household-firms can borrow in international markets issuing one-period bonds that pay a market or country net interest rate r_t. The country interest rate r_t has three components: an exogenous persistent component, an endogenous component that depends on the level of debt, and an exogenous transitory component. We will interpret the permanent exogenous component of r_t as capturing smooth but persistent movements in the world interest rate, and the temporary component as capturing larger and more volatile but not persistent changes in emerging markets bond spreads. The country interest rate is given by

$$r_t = r_t^* + \sigma_r \varepsilon_{r,t} + \psi\left(e^{\bar{B} - B_t} - 1\right), \tag{5}$$

where the persistent exogenous component, r_t^*, follows the process

$$r_t^* = (1 - \rho_{r^*})\bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*} \varepsilon_{r^*,t}, \tag{6}$$

while $\varepsilon_{r^*,t}$ and $\varepsilon_{r,t}$ are i.i.d. $N(0,1)$, with σ_{r^*} and σ_r denoting parameters that control

the variances of the two components.[3] As Mendoza (2010) notes, in addition, when the collateral constraint binds, the economy also faces an endogenous external financing premium on debt (EFPD), measured by the difference between the effective real interest rate, which corresponds to the intertemporal marginal rate of substitution in consumption, and market rate, r_t.[4] Because of this, the endogenous interest rate component of r_t in equation (5), $\psi_r \left(e^{\bar{B}-B_t} - 1\right)$, will serves the sole purpose of inducing independence of the model steady state from initial conditions, as in Schmitt-Grohe and Uribe (2003) and as the endogenous rate of time preference does in Mendoza (2010). Unlike Garcia-Cicco et al. (2010), therefore, we will not estimate the parameter controlling the debt-elasticity ψ_r, and we will calibrate it to a very small value. Note also that, unlike what is often assumed in the literature (for example, Neumeyer and Perri, 2005), we do not impose any correlation between the innovations to the interest rate process and the productivity process specified below.

The remaining exogenous processes, the preference shock d_t, the temporary productivity shock A_t, the shock to the relative price of intermediate goods P_t, and the domestic expenditure shock E_t, are specified as follows:

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}, \tag{7}$$

$$\log A_t = (1-\rho_A)A^* + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}, \tag{8}$$

$$\log P_t = (1-\rho_P)P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t}, \tag{9}$$

$$\log E_t = (1-\rho_E)E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t}, \tag{10}$$

where the starred variables and the $\rho_.$ coefficients denote the unconditional mean value and the persistence parameter of the processes, $\varepsilon_{.,t}$ are assumed i.i.d. $N(0,1)$ innovations, and the $\sigma_{.,t}$ parameters control the size of the process variances.

[3] Note here that while $\varepsilon_{r^*,t}$ and $\varepsilon_{r,t}$ are not identified separately in equations (5) and (A.19), $\varepsilon_{r,t}$ will be identified in the data because assumed to have different persistence, and will turn out to be important to fit Mexico's current account balance and external debt dynamics.

[4] Thus, $EFPD = \mathbb{E}_t[r_t^e - r_t] = \lambda_t / E_t[\mu_{t+1}]$, where $r_t^e = \mu_t / E_t[\mu_{t+1}]$ is the effective real interest rate, μ_t is the Lagrange multiplier on the budget constraint, and λ_t is the multiplier on the collateral constraint. The EFPD can be interpreted as the premium at which the economy choose debt amounts that satisfy the collateral constraint with equality in a credit market in which the constraint is not imposed directly.

2.2 Occasionally Binding Borrowing Constraint: An Endogenous Regime-Switching Specification

The central idea of this paper is to model the occasionally binding nature of a borrowing constraint as an *endogenous* regime-switching process. There is one regime in which the constraint binds and one in which it does not, where the regimes are denoted by $s_t \in \{0, 1\}$. In the regime denoted $s_t = 1$, the constraint binds strictly, and total borrowing is equal to a fraction of the value of collateral:

$$\frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t) = -\kappa q_t K_t. \tag{11}$$

The collateral constraint limits total debt to a fraction of the market value of capital, thus limiting leverage in the economy. On the left hand side of this equation we have total debt and working capital loans, while on the right hand side there is collateral, given by the value of capital, $q_t K_t$, times the leverage ratio κ. Thus, the constraint limits total borrowing, and hence consumption smoothing as well as the purchase of intermediate imported inputs for production purposes. Here, we follow Neumeyer and Perri (2005), Mendoza (2010), Fernandez and Gulan (2015), and Ates and Saffie (2016) among others and include working capital in total debt, which amplifies the supply response of the economy to shocks when the constraint binds.

In the regime denoted $s_t = 0$, the constraint does not bind and lenders finance all desired borrowing, including working capital loans, which is given by

$$\frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t). \tag{12}$$

In this regime, outside the steady state defined below, desired borrowing is pinned down by the knowledge that higher debt will lead to a higher probability of crisis regime in the future.

Our constraint is the same as in the benchmark mode of sudden stop crisis in Mendoza (2010). The critical difference is that we propose a new stochastic specification of its occasionally binding nature that eliminates non-differentiability and also has appealing empirical properties. In particular, the endogenous regime-switching specification transforms the *deterministic* relationship between leverage and the binding state of the borrowing constraint into a *stochastic* one. Typically, a deterministic

relationship implies that, for given values of the endogenous and exogenous states, there is one specific level of leverage associated with the binding state of the constraint. In contrast, in our specification, endogenously increasing leverage and shocks raise the *probability* of a binding state, but do not necessarily result in the constraint to bind.

Most importantly, in our set up, agents in the non-binding regime know that higher leverage and borrowing levels increase the probability of switching to a binding regime, and vice-versa. This preserves the interaction in agents' behavior between the two states that gives rise to precautionary behavior, distinguishing this class of models from those in which financial frictions are always binding or are approximated with solution methods that eliminate the interactions across regimes.

We assume that the probabilities of switching from one state to the other depends on the endogenous variables of the model. The probability of switching from the non-binding state to the binding state is a logistic function of the distance between actual borrowing and the borrowing limit equal to a fraction of the value of collateral. The probability of switching from the binding state back to the unconstrained one, instead, is a logistic function of the collateral constraint multiplier. Therefore the transitions are ultimately affected by all endogenous variables in the model, even though the parametric assumptions made impose restrictions relative to the probabilistic structure implicit in the stochastic solution of the conventional specification.

The stochastic specification of the occasionally nature of the the collateral constraint is not derived from an optimal contracting problem, but is imposed directly on the economy. Hence, it is not suitable for normative analysis of this class of models as in , as well as in many normative applications of this class of models as in the normative analyses of (Benigno et al., 2013, 2016; Jeanne and Korinek, 2010; Bianchi and Mendoza, 2018). Nonetheless, our probabilistic formulation captures the salient macroeconomic empirical finding that the likelihood of a financial crisis increases with leverage, but high leverage does not necessarily lead to a financial crisis.[5]

In addition, a growing body of microeconomic evidence indicates that a determin-

[5] Jorda et al. (2013) proxy financial leverage by the rate of change of private bank credit relative to GDP. In their database of 14 advanced countries from 1870 to 2008 there are 35 recessions associated with financial crises. Across these episodes, the change in leverage before a crisis is heterogeneous, with the standard deviation of financial leverage twice the mean across these episodes. This evidence suggests that while leverage matters, the exact level of leverage can vary considerably across financial crisis episodes.

istic constraint does not accurately capture lending and borrowing behaviors at the firm or bank level. For example, Chodorow-Reich and Falato (2017) and Greenwald (2019) among others, show that loan covenants are used to trigger renegotiation of credit lines as borrowers approach their borrowing limits, rather than simply being cut off from further borrowing.[6] Thus, in practice, collateral constraints do not seem to bind at any particular leverage ratio, and representing them as stochastic functions of leverage is a reasonable approximation for positive purposes.[7]

In the rest of this section, therefore, we discuss our proposed formulation of the slackness condition typically associated with an occasionally binding borrowing constraint and how this permits casting the occasionally binding constraint model in the form of an endogenous regime-switching framework. We then discuss the assumptions that we make to model the transition between regimes.

2.2.1 The Regime-Switching Slackness Condition

Denote the Lagrange multiplier associated with equation (11) as λ_t and define the "borrowing cushion" as the distance of the actual borrowing level from the debt limit:

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi \left(1+r_t\right)\left(W_t H_t + P_t V_t\right) + \kappa_t q_t K_t. \tag{13}$$

Thus, when the borrowing cushion is small, total borrowing is high relative to the value of collateral, meaning that the leverage ratio is high.

Now, the critical step is to modify the typical slackness condition from models with deterministic inequality constraints ($B_t^* \lambda_t = 0$), so that the two variables, B_t^* and $\lambda_t = 0$, are zero if the economy is in the relevant regime. So, we are seeking a representation in which when the economy is in the non-binding regime, then $\lambda_t = 0$, and when we are in the binding regime then the borrowing cushion $B_t^* = 0$.

To accomplish this, and to be consistent with the literature on regime-switching DSGE models in which *parameters* are the model objects that switch change states,

[6] Campello et al. (2010) provide survey information on the behavior of financially constrained firms and Ivashina and Scharfstein (2010) examine loan level data showing that credit origination dropped during the recent financial crisis because firms drew down from pre-existing credit lines in order to satisfy their liquidity needs. Bank lending standards fluctuating over the cycle could also be consistent with a stochastic specification of the collateral constraint.

[7] Exploring whether the our specification of the borrowing constraint may result from the solution of a limited enforcement problem with renegotiation, hidden liquidity, or random monitoring shocks is clearly beyond the scope of this paper.

we define two auxiliary regime-dependent parameters, $\varphi(s_t)$ and $\nu(s_t)$, such that $\varphi(0) = \nu(0) = 0$, and $\varphi(1) = \nu(1) = 1$.[8] Next, we introduce the following *regime-switching slackness condition*:

$$\varphi(s_t) B^*_{ss} + \nu(s_t)(B^*_t - B^*_{ss}) = (1 - \varphi(s_t))\lambda_{ss} + (1 - \nu(s_t))(\lambda_t - \lambda_{ss}), \tag{14}$$

where B^*_{ss} and λ_{ss} are the steady state borrowing cushion and collateral constraint multiplier, respectively, defined more precisely in Section 3 below. It is now easy to see that equation (14) implies that, as desired, when $s_t = 0$ then $\lambda_t = 0$, and when $s_t = 1$ then $B^*_t = 0$. Yet, given a regime s_t, equation (14) remains continuously differentiable for any value of B^*_t or λ_t, as no inequality constraint is imposed.[9]

2.2.2 Modelling Endogenous Regime-Switching

To model the transition from one regime to the other, we rely on logistic functions of endogenous variables determined in equilibrium that are tractable and parsimoniously parameterized.[10] Specifically, we assume that the transition from non-binding to binding regime depend on the borrowing cushion, B^*_t:

$$\Pr(s_{t+1} = 1 | s_t = 0, B^*_t) = \frac{\exp(-\gamma_0 B^*_t)}{1 + \exp(-\gamma_0 B^*_t)}. \tag{15}$$

Thus, the likelihood that the constraint binds in the following period depends on the size of the borrowing cushion in the current period. The parameter γ_0 controls the steepness of the logistic function, determining the sensitivity of the probability

[8] In our model these parameters coincide with the regime-switching indicator variable s_t, but in more general settings they may not. The notation provides a general formulation of the slackness condition that is applicable to setups possibly different than the one associated with our specific application. See, for example, the several applications discussed by Binning and Maih (2017).

[9] Technically, the regime-switching slackness condition "preserves" information in the perturbation approximation that we introduce in Section 3, since, at first order, both variables are constant in the respective regimes. The use of the regime-dependent switching parameters, $\varphi(s_t)$ and $\nu(s_t)$, follows from the Partition Principle of Foerster et al. (2016), which separates parameters based upon whether they affect the steady state or not. Intuitively, $\varphi(s_t)$ captures the *level* of the economy changing across regimes (e.g., capital is lower when the constraint binds), while $\nu(s_t)$ captures the dynamic responses differing across regimes (e.g., the response of investment to shocks changes when the constraint binds).

[10] Bocola (2016) and Kumhof et al. (2015) use a logistic function to model the transition to a default regime, and Davig et al. (2010) and Bi and Traum (2014) use it to study hitting a fiscal limit.

of switching regime to the size of the borrowing cushion. For small values of γ_0, the cushion has a small impact on the probability of a switch to the binding regime. For larger values of this parameter, the probability of a switch to the binding regime increases more rapidly toward 1. Note here that, for certain draws from the logistic function, the borrowing cushion could be negative and the economy could temporarily remain in the non-binding regime.

Similarly, when the constraint binds, the transition probability to the non-binding regime is a logistic function of the Lagrange multiplier, λ_t, according to

$$\Pr\left(s_{t+1}=0|s_t=1,\lambda_t\right)=\frac{\exp\left(-\gamma_1\lambda_t\right)}{1+\exp\left(-\gamma_1\lambda_t\right)}. \tag{16}$$

The probability of switching back from a constrained to an unconstrained regime, therefore, depends on the shadow value of the economy's desired borrowing relative to the limit set by the collateral constraint. As in the case of a switch from the constrained to constrained regime, the parameter γ_1 affects the sensitivity of this probability to the value of the multiplier, with larger values implying a greater sensitivity. A large positive multiplier implies that the constraint binds tightly, and the probability of exiting binding regime is lower. As the multiplier declines, this probability increases. Note again that in the binding regime, it is possible that the desired level of borrowing is less than the level forced upon it by a binding regime, which would manifest itself with a negative collateral constraint multiplier.[11]

Putting equations (15) and (16) together, the regime-switching model has an endogenous transition matrix

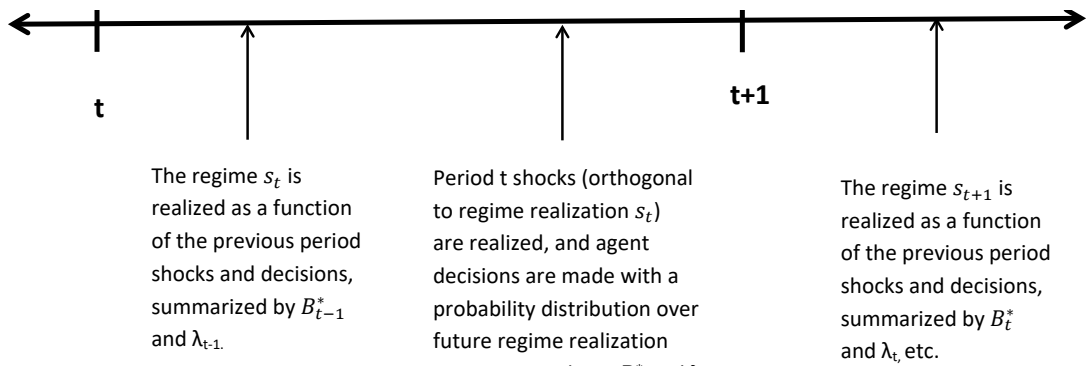
$$\mathbb{P}_t=\begin{bmatrix} 1-\frac{\exp(-\gamma_0 B_t^*)}{1+\exp(-\gamma_0 B_t^*)} & \frac{\exp(-\gamma_0 B_t^*)}{1+\exp(-\gamma_0 B_t^*)} \\ \frac{\exp(-\gamma_1\lambda_t)}{1+\exp(-\gamma_1\lambda_t)} & 1-\frac{\exp(-\gamma_1\lambda_t)}{1+\exp(-\gamma_1\lambda_t)} \end{bmatrix}. \tag{17}$$

A few remarks are useful to illustrate how our stochastic formulation of the borrowing constraint works and differs relative to a typical formulation of an occasionally binding collateral constraint (for example, Kiyotaki and Moore, 1997).

First, whether or not the constraint binds in a given period is determined *before*

[11] By construction, the transition probabilities equal 0.5 when their arguments are zero. In principle, one could relax this assumption by introducing a constant into the arguments of equations (15-16); however, preliminary estimates with this flexibility indicated these parameters were effectively zero, so for simplicity we omit them from the beginning.

Figure 2: Model Timing



exogenous shocks are realized and economic decisions are made during that period. Figure 2 provides a graphical representation of the model timing and illustrates that, at the start of a given period t, the regime outcome s_t is drawn from the logistic distributions as a function of previous period borrowing cushion or value of the collateral constraint multiplier, B^*_{t-1} and λ_{t-1}.[12] Next, exogenous shocks, which are orthogonal to the realization of the regime, are realized and agents take decisions during period t based on their knowledge of the regime outcome, s_t, as well as a probability distribution over the next period regime realization, s_{t+1}, as in equation (15) or (16). Finally, the regime realization for period $t+1$ is drawn based on exogenous shocks and agents' decisions that pin down B^*_t and λ_t, and so on.

Third, as we have already noted, an implication of our setup is that entry and exit of the economy from the binding regime might be temporarily associated with a negative values of the multiplier and the borrowing cushion. For instance, in the non-binding regime, the logistic function can allow the borrowing constraint to bind in the following period, even if the borrowing cushion in the current period is still positive. Conversely, negative values of the borrowing cushion in the non-binding regime are possible if the probability of a binding regime is elevated but such outcome is not realized. How likely these outcomes are depend on the parameter of the relevant logistic function, γ_0. The same logic applies to a probabilistic exit from the binding regime that depends on the multiplier λ_t and the parameter of the logistic function γ_1. Despite positive values of the multiplier, the economy may end up coming out of

[12] Timing restrictions are implicit also in the formulation of the working capital constraint that we borrow from the literature,

the binding regime early. Conversely, the economy might be stuck in the constrained regime past the time when the collateral constrained multiplier turned negative. In fact, in this case, the economy may be "forced" to borrow the amount set by the constraint, which might be more than desired, until a non-binding realization of the regime is drawn.

The third implication of our setup is that by making the probability of transition from one regime to the other dependent upon endogenous variables, the model can generate transition probabilities that vary over time. In contrast, the exogenous Markov-switching setup (Davig and Leeper, 2007; Farmer et al., 2011; Bianchi, 2013; Foerster et al., 2016) has a constant probability of transitioning between regimes, which implies a flat hazard function that is independent of the structural shock realizations and the agent decisions. Thus, our endogenous-switching framework can in principle generate long- or short-lived binding regime episodes with non-constant hazard functions depending on the realization of shocks and agents' decisions.

3 Solving the Endogenous Switching Model

Having cast our model it in terms of an endogenous regime-switching framework, this Section describes our solution method and our estimation procedure. The model developed in the previous section can in principle be solved using global solution methods, as for example in Davig et al. (2010). However, such an approach would be time-consuming on models with multiple state variables and many shocks like ours, possibly precluding likelihood-based estimation. Instead, we solve the model using a perturbation approach, which allows for an accurate approximation that is fast enough to permit estimation and potentially applicable to even larger set ups beyond our current application. The competitive equilibrium of the endogenous regime-switching version of the model is defined formally in Appendix A. Perturbation generates Taylor-series expansions of the functions characterizing the solution of the full set of equilibrium conditions around an approximation point. We now describe the approximation point and these expansions.

We relegate many of the derivations and details of the solution method to Appendix B. In what follows, however, we highlight two important features of our methodology. First, how to define a steady state in this setup, which is the point around which the expansions are generated. Second, the importance of approximat-

ing to a second-order in our framework.

3.1 Defining the Steady State

Given the regime-switching slackness condition (14), defining a non-stochastic steady state of an endogenous regime-switching framework is challenging. A steady state in our framework can be defined as a state in which all shocks have ceased and the regime-switching variables that affect the level of the economy ($\varphi(s_t)$) take the *ergodic mean* associated with the steady state transition matrix:

$$\mathbb{P}_{ss} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} & \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}. \tag{18}$$

Note here that, since this matrix also depends on the steady state level of the borrowing cushion and the multiplier, B^*_{ss} and λ_{ss}, which in turn depend upon the ergodic mean of the regime-switching parameters $\varphi(s_t)$ and $\nu(s_t)$, such steady state is the solution of a fixed point problem that is described in more detail in Appendix B.

More specifically, consider the model regime-specific parameters defined above and distinguish between $\varphi(s_t)$, which can affect the level behavior of the economy, and $\nu(s_t)$, which can affect only its dynamics with no effects on the steady state. Then denote with $\xi = [\xi_0, \xi_1]$ the ergodic vector of $\mathbb{P}_{ss}$. Next, apply the Partition Principle of Foerster et al. (2016), to focus only on parameters that affect the level of the economy, and write their ergodic mean as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1), \tag{19}$$

where $\bar{\varphi}$ thus denotes the ergodic mean value of the regime-switching variables–or parameters in a conventional approximation–that affects the steady state.

Defining the steady state as the state in which the auxiliary parameter $\varphi(s_t)$ is at its ergodic mean value implies that the approximation point constructed is a weighted average of the steady states of two separate models: a model in which only the non-binding regime occurs, and one in which only the binding regime occurs; the steady state does not explicitly capture behavior generated by changes in regimes. How close our approximation point is to each of these two other steady state concepts, therefore, will depend on the frequency of being in each of the two regimes. We note here that,

since in our application we are modelling binding episodes with limited duration, the ergodic mean is a natural candidate as perturbation point. Given the nature of our application with slow-moving capital and debt state variables, the perturbation point will be in the area of the state space in which the economy operates most frequently. In fact, since the binding regime tends to be self-limiting–that is, being in the binding regime causes the economy to reduce leverage and hence switch back to the non-binding regime–the economy will rarely reach the area around the steady state of the "binding regime only" version of the model.[13]

3.2 The Solution and Its Properties

Equipped with the steady state of the endogenous regime-switching economy, we then construct a second-order approximation to the policy functions by taking derivatives of the equilibrium conditions. We relegate details of these derivations to the Appendix B, but here we provide a summary.

For each regime s_t, the policy functions to our model take the form

$$\mathbf{x}_t = h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \qquad \mathbf{y}_t = g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \tag{20}$$

where $\mathbf{x}_t$ denotes predetermined variables, $\mathbf{y}_t$, non-predetermined variables, $\varepsilon_{\mathbf{t}}$ the set of shocks, and χ a perturbation parameter such that when $\chi = 1$ the fully stochastic model results and when $\chi = 0$ the model collapses to the non-stochastic steady state defined above. Using these functional forms, we can express the equilibrium conditions conditional on regime s_t as

$$\mathbb{F}_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) = 0. \tag{21}$$

We then stack the regime-dependent conditions for $s_t = 0$ and $s_t = 1$, denoting the resulting system of equations with $\mathbb{F}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)$, and successively differentiate with

[13] Alternative methods for finding solutions to endogenous regime-switching models, such as Maih (2015) and Barthlemy and Marx (2017), advocate using regime-dependent steady states as multiple approximation points. Such a strategy would not be suitable for our purposes because the binding regime steady state is a poor approximation point given that the state is infrequent and usually of shorter duration than normal cycles of expansions and contractions.

respect to $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$, evaluating at steady state. The systems

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \qquad \mathbb{F}_{\varepsilon}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \qquad \mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \tag{22}$$

can then be solved for the unknown coefficients of the first-order Taylor expansion of the policy functions in equation (20).

A second-order approximation can be found by taking the second derivatives of $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$. In the end, we have matrices $H_{s_t}^{(1)}$ and $G_{s_t}^{(1)}$ characterizing the first-order coefficients, and $H_{s_t}^{(2)}$ and $G_{s_t}^{(2)}$ characterizing the second-order coefficients. Therefore, the approximated policy functions are

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t) \tag{23}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t) \tag{24}$$

where $S_t = \begin{bmatrix} (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' & \varepsilon_t' & 1 \end{bmatrix}'$.

Our perturbation method produces a single approximated set of policy function, but cannot be used to guarantee that the solution is unique. This limitation is common to models of occasionally binding constraints that are solved globally with a numerical algorithm that converges but where additional solutions cannot be ruled out. (for example, Benigno et al., 2016). With endogenous regime-switching, we also lack conditions for ensuring stability of the full solution; instead, we check the mean-squared stability of the first-order approximation (Farmer et al., 2011; Foerster et al., 2016), and additionally check for explosive simulations.

Lastly, we tested for accuracy of our solution method in our current model as well the smaller model of Jermann and Quadrini (2012) in which we can more easily compare our perturbation method to with global solution methods. We find Euler equation errors for the model we use in this paper approximation errors on the order of \$1 per \$1,000 of consumption. This figure is in line with those found for perturbation in fixed probability regime-switching models (Foerster et al., 2016) as well as standard models without regime-switching (Aruoba et al., 2006). When we compare the perturbation method we propose with a standard global method on the endogenous regime-switching model or the version of the model with the traditional inequality constraint, we find that the solution methods produce similar moments for

key variables of interest, and the global and perturbation solutions of our endogenous regime-switching model produce similar Euler equation errors. See Appendix C for more details.

3.3 Approximation Order, Endogenous Switching and Precautionary Saving

Our endogenous regime-switching framework must be solved at least to the second order to capture the effects of endogenous probabilities, which include state-varying precautionary effects. If we were to use only a first-order approximation, our estimation would not capture precautionary behavior associated with rational expectations about the dependency of the probability of a regime change on the borrowing cushion and the multiplier. The following Proposition states this result formally.

Proposition 1 (**Irrelevance of Endogenous Switching in a First-Order Approximation**). *The first-order solution to the endogenous regime-switching model is identical to the first-order solution to an exogenous regime-switching model where the transition probabilities are given by the steady-state value of the time-varying transition matrix.*

Proof. *See Appendix C.*

The Proposition illustrates that using a second-order approximation to the solution is necessary to characterize the model properties associated with the endogenous nature of the regime-switching, including particularly particularly precautionary behavior. This result is similar to the one stating that, in models with only one regime, first-order solutions are invariant to the size of shocks, second-order solutions captures precautionary behavior, and third-order solutions are needed to capture the effects of stochastic volatility (Fernandez-Villaverde et al., 2015). Unfortunately, the need to use a second-order approximation along with regime-switching creates additional challenges for estimation purposes, and we now turn to our strategy to address them.

4 Estimating the Endogenous Switching Model

We estimate the model with a full information Bayesian procedure. The posterior distribution has no analytical solution so we use Markov-Chain Monte Carlo methods

to sample from the posterior. Since the Metroplis-Hastings algorithm we use for sampling is a standard tool used in the literature, we omit a discussion of this step in our procedure. The details of the construction of the state space representation and the filtering steps for the evaluation of the likelihood are reported in Appendix D.

A key obstacle in sampling from the posterior is the evaluation of the likelihood function. We face three difficulties here relative to linear DSGE models. The first is the nonlinearity due to the presence of multiple regimes. The second is the need to approximate to the second-order the solution that governs the decision rules in each regime. The third is the fact that the transition probabilities are endogenous. Bianchi (2013) develops an algorithm to address the first difficulty. Here we must use an alternative filter to deal with the second order solution plus endogenous probabilities in a tractable manner. We use the Unscented Kalman Filter (UKF) to compute approximations to the evaluation of the likelihood function using Sigma Points.[14] This is because, in our application, a switch in regime can lead to discarding a large number of simulated particles, lowering accuracy for a given number of particles and greatly increasing the computational cost of obtaining a given level of accuracy.

The model's posterior distribution is highly nonlinear, with many local modes due to the complexity of the model. To deal with this issue, we took the following steps: first, we estimated a version of the model without the occasionally binding collateral constraint and working capital, this step helped find approximate estimates for the exogenous processes and non-financial variables; second, conditional on our estimates from the first step, we performed a grid search over the remaining parameters (κ, ϕ, γ_0, and γ_1) to find high posterior regions; third, from the high posterior regions of this grid search, we used a mode-finding routine to find the posterior mode, which forms the basis for our empirical results; lastly, we sampled $500,000$ draws from the posterior using a random-walk Metropolis-Hastings algorithm to explore the parameter space around the mode and characterize credible sets for the estimates.

4.1 Observables, Data, and Measurement Errors

The model is estimated with quarterly data for GDP growth (gross output less intermediate input payments), consumption growth, investment growth, and intermediate

[14] See Binning and Maih (2015) for a discussion of the pros and cons of Sigma Point filter relative to the Particle Filter (for example, Fernandez-Villaverde and Rubio-Ramirez, 2007) in a regime-switching context, which include degeneracy issues.

Table 1: Calibrated Parameters

Parameter	Description	Value
β	Discount Factor	0.9798
ρ	Risk Aversion	2.0000
ω	Labor Supply	1.8460
η	Capital Share	0.3053
α	Labor Share	0.5927
δ	Depreciation Rate	0.0228
P^*	Mean Import Price	1.0280
E^*	Mean Expenditure	0.2002
ψ_r	Interest Rate Debt Elasticity	0.0010
$\bar{B}$	Neutral Debt Level	-6.1170

import price growth, as well as the current account-to-output ratio, and a measure of the country real interest rate. GDP, consumption, and investment are in quarterly, demeaned log differences.[15]

The fact that we have six data series and six structural shocks means that we do not need measurement errors in our estimation. However, measurement errors in the observation equation improves performance of the non-linear filtering algorithm and accounts for any actual measurement error in our data. To limit their impact on the inference we draw from the data, we limit their variance to 5% of the variance of the observable variables. This means that our model will fit the data relatively closely on average; how it performs across cycles and crises, and whether it relies on large shocks to fit the data, will be important for assessing model performance.

4.2 Calibrated Parameters and Prior Distributions

Our objective is to estimate critical parameters governing the model's dynamics in both the binding and non-binding regime, as well as the parameters that govern the transitions between regimes on which we do not have strong a priori information. In order to make inference about these parameters, we calibrate the subset of parameters on which we have good prior information–we discuss these two set of assumptions in turn, shortly.

[15]See Appendix F for details on variable definitions and data sources. The country interest rate is constructed, following Uribe and Yue (2006), and is the US 3-Month Treasury Bill minus ex post US CPI inflation rate plus Mexico's EMBI Spread.

Table 1 lists the parameters that we calibrate.[16] We set these parameters largely following Mendoza (2010), who calibrated them based upon stylized facts from Mexico's National Accounts. One parameter that does not come come from Mendoza (2010) is β, which we set to match the capital-to-output ratio and the debt-to-output ratio $\bar{B}$. Another important parameter that we calibrate is ψ_r, which is estimated in Garcia-Cicco et al. (2010). We set it to a very small value for the sole purpose of eliminating the dependency of the steady state from initial conditions, while avoiding to have an impact on the the model dynamics (see Schmitt-Grohe and Uribe, 2003).[17] Setting ψ_r very small allows us to evaluate the model's ability to match the behavior of the trade balance and the other key stylized facts of the data without introducing an alternative form of financial friction, in the form of a quantitatively important endogenous component of the interest rate in equations (5)-(A.19).

Table 2 below summarizes our assumptions on the prior distributions. We set two types of priors on the parameters to be estimated. The first type is priors on the parameters. They impose sign restrictions and put lower prior probability on parameter values that generate implausible moments in model simulations.[18] The second type of prior is on a model-implied object: the steady state transition probability of switching from the unconstrained to the constrained regime, given by the steady state value of equation (15), $\Pr(s_{t+1} = 1 | s_t = 0, B^*_{ss})$. We set this prior to be a Beta distribution with mean 0.25 and variance of 0.25. This prior puts lower probability mass on combinations of parameters that either generate extremely infrequent transitions to the constrained regime, or that imply the economy exits the unconstrained regime almost immediately.[19]

[16] See Appendix E for more details on the calibration and targets.

[17] Even though we have a borrowing constraint and precautionary savings, the presence of $\psi_r > 0$ serves the same purpose as endogenous discounting in Mendoza (2010). Recall here that our perturbation solution is constructed around a point between the steady state of the "non-binding only" model (which depends on ψ_r) and the "binding only" model.

[18] While possible in principle, we do not allow for regime-switching in the shocks processes, because we want the collateral constraint to drive regime-switching, rather than changes in the stochastic processes. Allowing for regime change in the shock processes might improve overall fit, but would make it harder to assess the model's ability to characterize financial crises as episodes in which the collateral constraint is binding.

[19] The use of priors on model-implied objects has been used by, for example, Otrok (2001) and Del Negro and Schorfheide (2008).

Table 2: Estimated Parameters

Par.	Description	Prior	Posterior			
			Mode	5%	50%	95%
ι	Capital Adj.	N(10,5)	12.703			
ϕ	Working Cap.	U(0,1)	0.7113			
r^*	Mean Int. Rate	N(0.0177,0.01)	0.0172			
κ	Leverage	U(0,1)	0.1727			
ρ_a	Autocor. TFP	B(0.6,0.2)	0.9796			
ρ_e	Autocor. Exp	B(0.6,0.2)	0.9111			
ρ_p	Autocor. Imp Price	B(0.6,0.2)	0.9711			
ρ_d	Autocor. Pref.	B(0.6,0.2)	0.9810			
ρ_{r^*}	Autocor. Persist. Int. Rate	B(0.6,0.2)	0.8929			
σ_a	SD TFP	IG(0.01,0.01)	0.0083			
σ_e	SD Exp.	IG(0.1,0.1)	0.1806			
σ_p	SD Imp. Price	IG(0.1,0.1)	0.0471			
σ_d	SD Pref.	IG(0.1,0.1)	0.1123			
σ_r	SD Trans. Int. Rate	IG(0.01,0.01)	0.0028			
σ_{r^*}	SD, Persist Int. Rate	IG(0.01,0.01)	0.0047			
γ_0	Logistic, Enter Binding	U(0,150)	13.552			
γ_1	Logistic, Exit Binding	U(0,150)	17.798			

Notes: Estimated parameters, with prior distribution and posterior moments. Priors are Normal, Uniform, Beta, or Inverse Gamma; prior distributions show mean and variance, except for uniform where lower and upper bounds are shown. Posterior distribution shows mode (used for model analysis), along with 5-th, 50-th, and 95-th percentiles from MCMC posterior draws.

5 Empirical Results

Our empirical findings comprise three sets of results. The first includes the estimated parameters and various measures of model fit. This set of results will help us characterize the tightness of the working capital and collateral constraints, how the model fits the data without using excessively large shocks to generate crises, and when Mexico appeared to be borrowing constrained and suffer crises. The second set includes model moments that inform us about the ergodic properties of the estimated model economy and the relative importance of different shocks for regular business cycles. The third set of results pertain to the model's ability to describe and interpret financial crises. In particular, we will look at how financial crisis episodes can be viewed through the lens of the model and which shocks are important to characterize specific historical episodes. We look at both a 'typical' crisis simulated by the model, as well as the anatomy of the debt crisis of the 1980s, the 1995 'Tequila' crisis, and the

spillover of the Global Financial Crisis (GFC) in 2008-2009.

5.1 Parameter Estimates, Model Fit, and Crisis Inference

5.1.1 Estimated Parameters

Table 2 reports the mode of the posterior distribution of the estimated parameters, together with the median, the 5th and the 95th percentile of the parameter distribution. The parameters of the exogenous processes indicate that all shocks have a high degree of persistence, but none is statistically close to 1. The estimated mean interest rate, slightly below 1.75% per quarter, is close to the value estimated by Mendoza (2010).

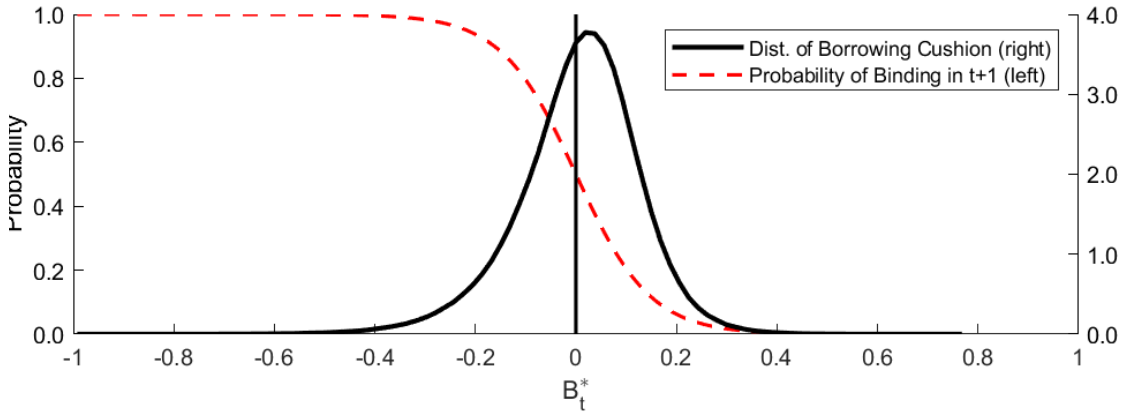
Importantly, the model provides precise estimates of critical parameters, namely the investment adjustment cost, working capital, and haircut parameter, and the parameters of the logistic function that help match the time series of the observable variables during both business cycles and financial crises. These parameters cannot be easily measured directly from stylized facts of the data–unlike, for example, capital or labor shares–but are nonetheless important for explaining economic behavior and the amplification of shocks.

The estimate of the investment adjustment cost parameter, ι, typically controls investment volatility and is 12.7. To provide context, this parameter was calibrated to the value of 2.75 by Mendoza (2010), although that calibration is at an annual frequency. The estimate for the working capital constraint parameter indicates that 71% of the wage and intermediate good bill needs to be paid in advance of production with borrowed funds; this estimate is substantially higher than the 25.79% value set by Mendoza (2010) but much lower than the 100% used by Neumeyer and Perri (2005) or the 125% used by Uribe and Yue (2006). The estimate is close to the 60% calculated by Ates and Saffie (2016) by using use interest payments and production costs from Chilean microeconomic data. The estimated value of the haircut parameter in the borrowing constraint (κ) is 0.17, indicating less than a fifth of the value of capital serves as collateral. The estimate is slightly tighter than the benchmark value of 0.20 chosen by Mendoza (2010), and on the low end of the 0.15 to 0.30 range of alternative values considered in that calibration.

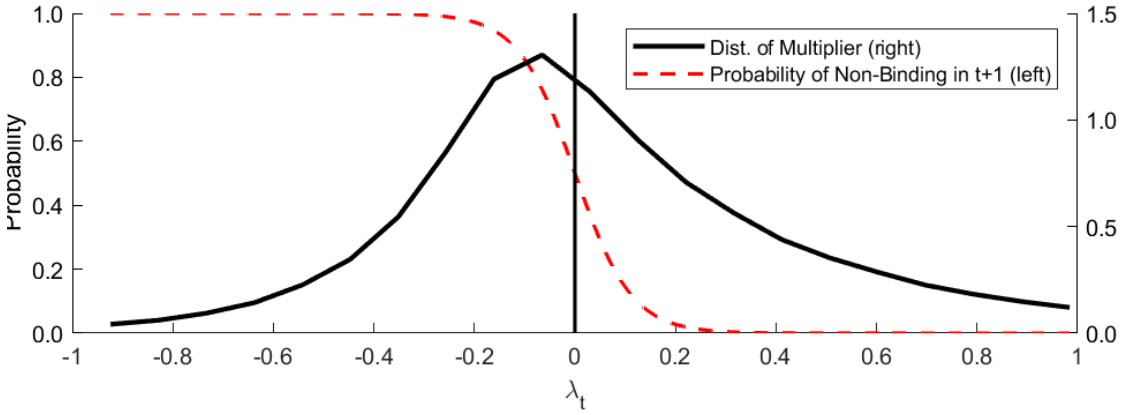
The estimated posterior modes of the logistic parameters in equation (15) and (16) are 13.6 and 17.8, respectively. These estimates have two implications. First,

Figure 3: Logistic Functions and Distributions of Their Arguments

(a) Borrowing Cushion and Transition Probability in Non-Binding Regime



(b) Multiplier and Transition Probability in Binding Regime



Note: The top panel shows the model-implied distribution of the borrowing cushion B^* in the non-binding regime, and the logistic transition function to the binding regime as in equation (15) implied by our estimates. The bottom panel shows the model-implied distribution of the multiplier, λ, in the binding regime, and the transition function to the non-binding regime as in equation (16) evaluated at the estimated value of corresponding parameter.

they are significantly different from zero, thus suggesting that the data reject a model specification in which the transition probabilities are *exogenous*, which is in principle allowed for under the prior distribution. Second, they also suggest that the probability of switching from the non-binding to the binding regime is flatter than the probability of exiting the binding regime, consistent with the notion that regular business cycles typically have longer duration than financial crises associated with sudden stops in

capital flows.

To illustrate this latter point, Figure 3 plots the implied probabilities from equation (15) and (16) evaluated at the posterior mode value of γ_0 and γ_1, together with the estimated ergodic distributions of their arguments, the borrowing cushion, B^* and the the constraint multiplier, λ. The figure shows that the ergodic distribution of the borrowing cushion is centered on a positive value, as the economy spends most of its time in the unconstrained regime, above the borrowing limit. As the borrowing cushion falls, the probability of switching to the binding regime increases, and gradually reaches 1 for small negative values, with very little probability mass on large negative realizations of the borrowing cushion.

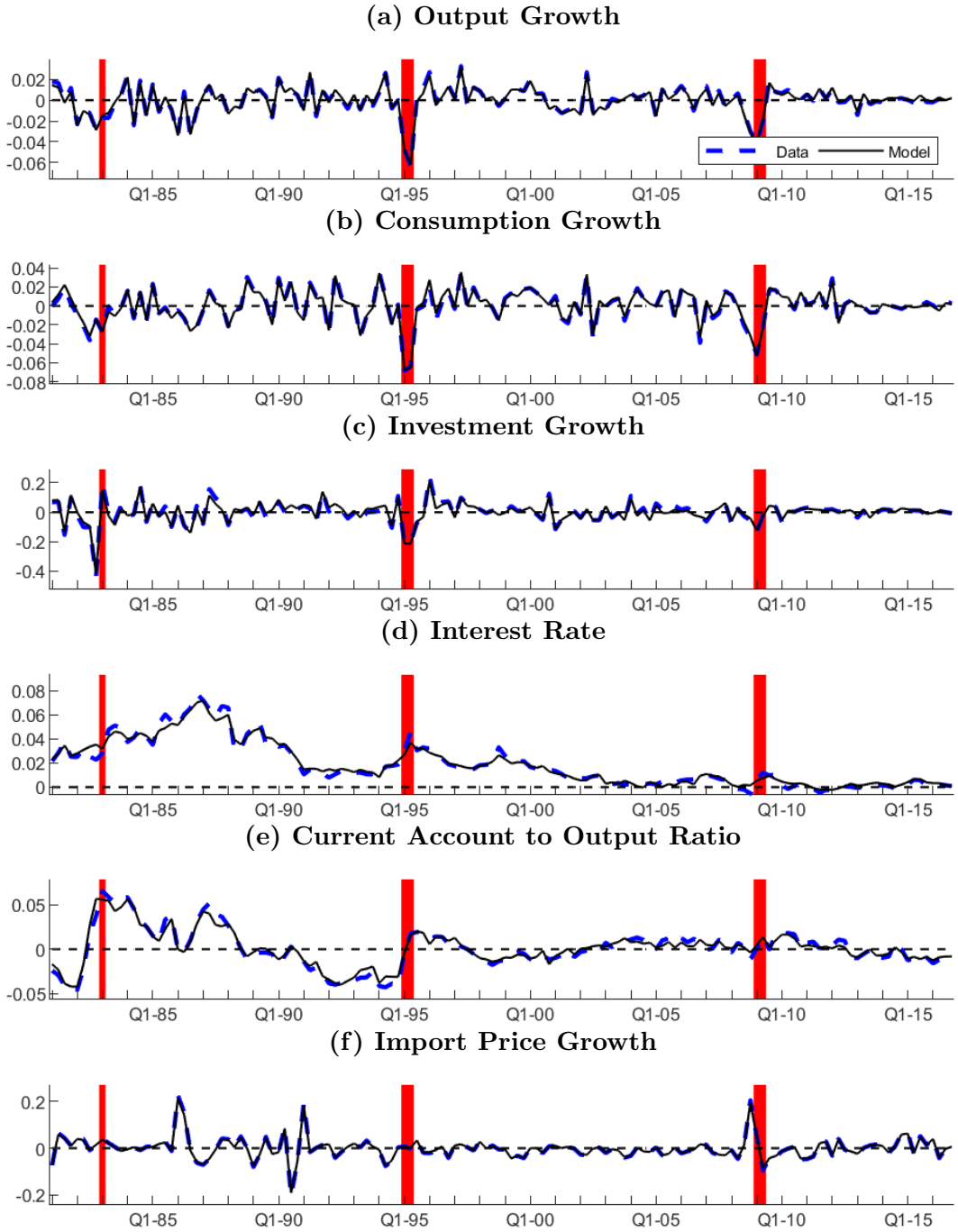
On the other hand, once the economy is in the binding regime, the ergodic distribution of the multiplier $\lambda = 0$ is centered on small negative values, with more probability mass on the right tail than the left tail. As λ approaches 0, the probability of switching to the non-binding regime increases and quickly reaches 1, with a mode on a small negative value. Nonetheless, the there is a significant probability mass for larger negative values. As we explained earlier, negative values of λ reflect instances in which, had the economy been in the non-binding regime, the borrowing cushion would be positive (as a result of the shock realizations and agent decisions as illustrated in Figure 2), but a switch to non-binding regime at has not been yet drawn.[20]

5.1.2 Model Fit

The model fit is summarized by Figure 4, which plots observable variables used in the estimation together with the fitted values. The Figure also includes model-identified peak crisis periods; we provide an definition shortly, but in brief they include a fall in output, an increase in the current account, and the economy in the binding regime. The fitted series largely follow the actual data, in part due to the relatively small role of measurement errors in the estimation procedure. Importantly, the model estimates track the data consistently throughout the sample, during both regular business cycle phases and crisis periods, without significant loss of fit during times of crisis. For example, during the 1995 "Tequila Crisis," the data show large drops and

[20]Sufficiently negative values of λ, approximately below -0.2, produce a nearly deterministic switch back to the binding regime. The ergodic distribution of λ in the binding regime (Figure 3b) implies that the probability of exiting that regime exceeds 99% about 1/4-th of the time.

Figure 4: Data and Model Estimates



Note: The figure plots observable variables used in estimation (dashed blue lines) and fitted values (i.e., model implied smoothed estimated series based upon the full sample, solid blue lines). Red bars indicate model-identified periods of crisis, see text for definition.

rebounds in output, consumption, and investment growth, and a very sharp reversal in the current account to output ratio. The model tracks these movements as well as it does track regular fluctuations throughout the rest of the sample. If, by contrast, one were to observe a loss of fit during crisis episodes, it would suggest that our estimated model finds it difficult to match the model dynamics during these episodes of critical interest in the empirical analysis.

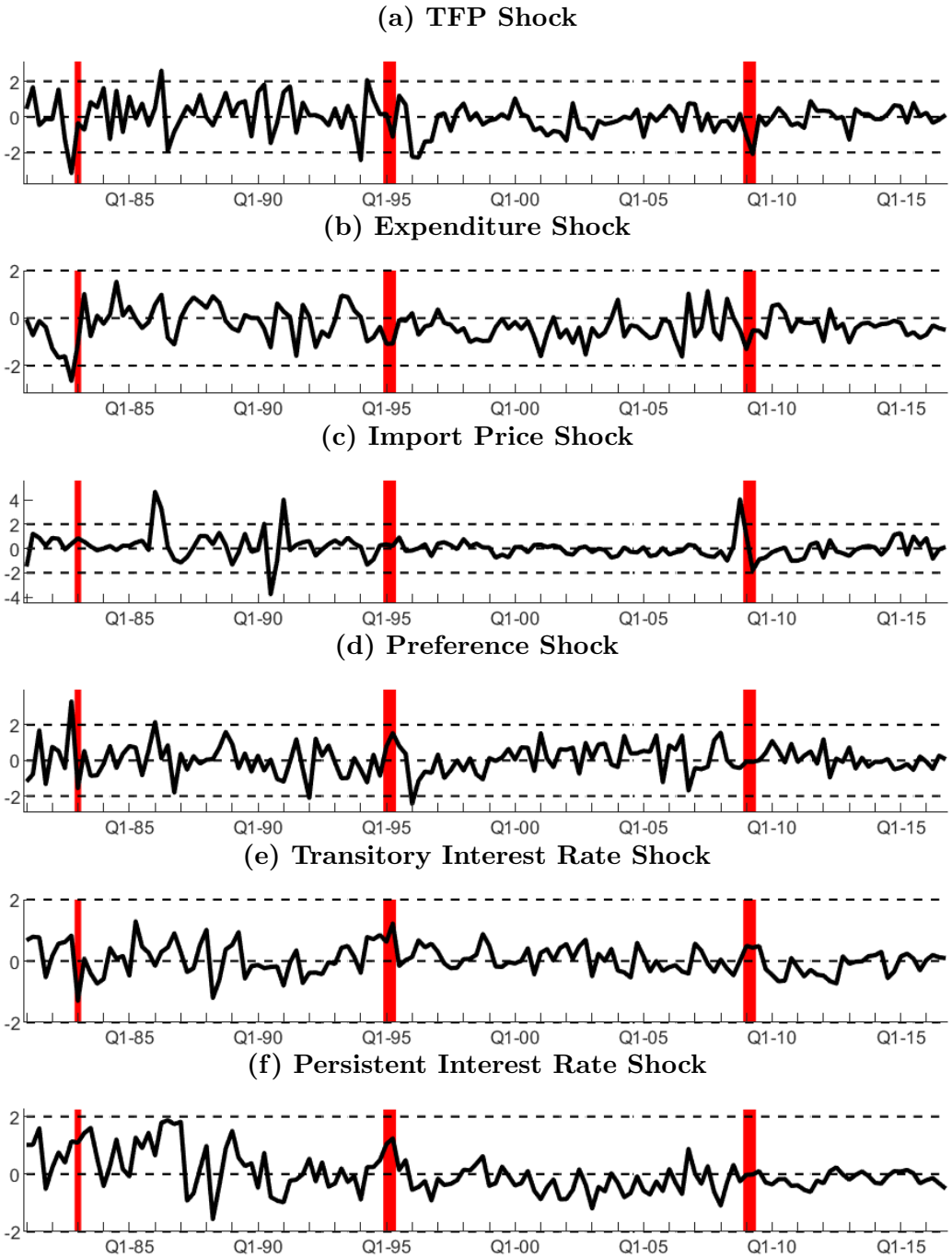
Figure 5 plots the estimated model implied shocks in standard deviation units, together with a two-standard deviations band. The figure shows that model fit is largely achieved without relying on unusually large shocks, especially during crisis times. Large shocks are needed to fit the import price process, including right before GFC, possibly due the large swings in oil prices during that period. Shocks slightly outside the two-standard divisions band also are estimated right before the 1982 debt crisis. However TFP, expenditure, and preference shocks are all well within the band during the that period.

5.1.3 Model Inference about Peak and Near Crises

The estimated model allows us to make inference on whether the economy is in the binding regime, and identify periods of crisis. In the model, the regime is known by the household-firm, but the estimation procedure does not observe the regime and it must be inferred based on the information in the data. The estimation results, therefore, can provide a time-varying estimate of the smoothed probability (i.e. based upon the full sample) of being in each regime. Figure 6 plots this estimated probability (solid black line), which gives an indication of when Mexico was borrowing constrained.

Using the information in Figure 6, we can identify model-consistent concepts of financial crises. First, we identify *near crisis* periods as those in which the model was in the binding regime with at least 90% probability. Second, we identify *peak crisis* periods in a way that uses concepts from the empirical literature on sudden stops (for example, Calvo et al., 2006) and the quantitative literature modeling sudden stops with occasionally binding constraints as in (for example, Mendoza, 2010). These peak crisis periods are show in Figure 6 shows *peak crisis* periods (vertical red bars), and occur when:(i) the smoothed estimate of the probability of being in the binding regime is at least 90%; (ii) the model estimate of output growth (Figure 4, Panel a) is negative by more than one standard deviation; (iii) the model estimate of the current

Figure 5: Model Estimated Shocks



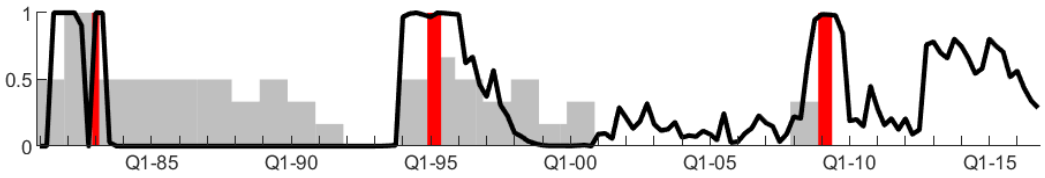
Notes: The figure plots the estimated model implied shocks, in standard deviation units, together with a two-standard deviations band (black dashed lines). Red bars indicate model-identified periods of crisis, see text for definition.

account ratio (Figure 4, Panel e) improves by more than one standard deviation.

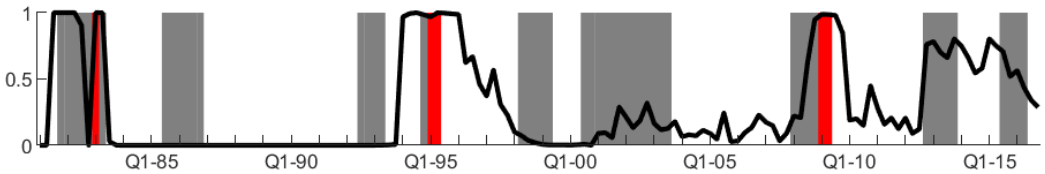
The model identifies three peak crisis episodes, each accompanied by near crisis periods. The three peak crisis episodes identified in Figure 6: 1983Q1 during the Debt Crisis, 1995Q1-Q2 after the onset of the Tequila Crisis, and 2009Q1-Q2 during the GFC. With each, the economy was in the binding regime prior to and after the peak crises, indicating a longer period of near crisis. In particular, the 1983 peak was preceded by several quarters of near crisis, the Tequila Crisis had a peak in the middle of a relatively longer period of near crisis, and the GFC appeared to have a relatively short-lived near crisis time.

Figure 6: Smoothed Probability of Binding Regime

(a) Probability of Binding and Reinhart-Rogoff Dates



(b) Probability of Binding and OECD Recession Dates



Notes: Black line shows model implied smoothed probability of being in the binding regime. Light gray regions in panel (a) indicates Reinhart and Rogoff (2009) crisis dates; crises are the tally index, and are plotted as fraction of possible values between 0 (no crisis) and 6 (most severe). Dark gray regions in panel (b) indicate OECD recession dates. Red bars indicate model-identified crises, see text for details.

Our model therefore identifies two kind of crisis periods: peak crisis episodes are short-lived periods in which the economy is in a binding state and there are large changes in output and the current account, whereas near crisis episodes are periods surrounding the peak crises in which the economy is in the binding regime. Near crisis episodes are characterized by milder but more persistent economic deviations from the normal phase of the business cycle typically associated with the run up and the recovery phase of a sudden stop crisis in the data.

Figure 6 also reports an alternative, purely empirical definition of crisis together with the the OECD dates of the business cycle for Mexico. The empirical notion

of crisis a normalized version of the *crisis tally* index of Reinhart and Rogoff (2009) (RR).[21] This empirical crisis indicator produces signals that are even more persistent than our purely model-based measure of the binding regime in place. Nonetheless, Figure 6 also illustrates that our notions of peak and near crisis align quite well with the RR index, a our measure of the binding regime tracks the RR tally index remarkably well around all three peak crises identified.

The model has a harder time tracking the very prolonged period of stress signalled by the tally index after the early 1980s Debt crisis, and to some extent also the post-Tequila crisis period. This is to be expected, however, as our model economy is not designed to capture stagnation, debt overhang, or financial intermediation disruptions that drive the classification of RR between 1983 and 1989 or in the mid-1990s and the dynamics in the data following the sudden stop crisis that triggered them.

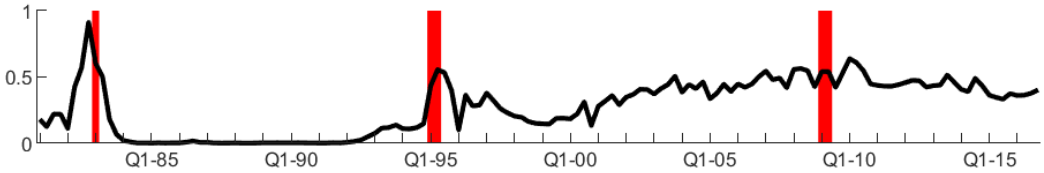
Nonetheless, the model does very well at tracking the consequences of the GFC and does not mistake ordinary recessions not associated with spikes in the tally index for near or peak crisis periods. Mexico OECD recessions are illustrated by the light dark shaded areas in the lower panel of Figure 6. As we can see the estimated probability of a binding regime is close to 0 in correspondence to OECD recessions before the Tequila crisis, during the US recession in 2001, and the Argentine crisis in 2000-2001. The estimated probability of a binding regime does not register stress also in correspondence of the 1998 Russian default and US Long-Term Capital Management debacle that affected only the currency and stock market, without triggering a sudden stop in Mexico. Overall, Figure 6 shows that our model provides an accurate signal of when the economy is likely to have experienced an actual crisis state or a period of fragility and vulnerability, without mistake regular recessions for financial crisis episodes.

Figure 7 plots the pseudo-real-time (i.e. filtered) estimated transition probabilities; panel (a) shows the probability of switching from the non-binding to the binding regime, while panel (b) shows the probability of switching from the binding to the non-binding regime. In other words, they plot the estimated counterpart of the proba-

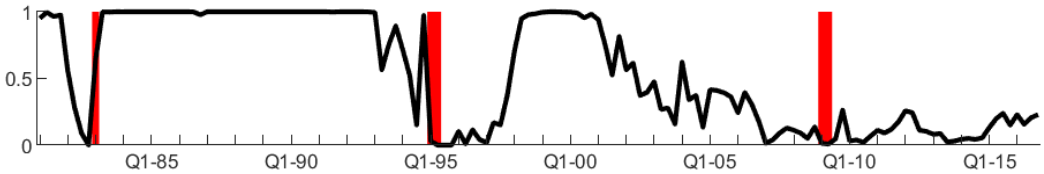
[21] The RR tally index ranges from 0 to 6, depending on whether a country-year observation is deemed to be in one or more of the following 6 varieties of crisis, assigning the value of one for each variety: Currency, Inflation, Stock Market, Sovereign Domestic or External Debt, and Banking Crisis. We follow their methodology to extend the index to cover our full sample. In Figure 6, the index is normalized to range between 0 and 1. See Chapter 1 of Reinhart and Rogoff (2009) for more details.

Figure 7: Transition Probabilities

(a) Transition Probability of Binding Given Non-Binding



(b) Transition Probability of Non-Binding Given Binding



Notes: Top panel shows model-implied filtered probability of transitioning to binding regime, conditional on being in the non-binding regime, in subsequent period. Bottom panel shows filtered probability of transitioning to non-binding regime, conditional on being in the binding regime, in subsequent period. Red bars indicate model-identified crises, see text for details.

bilities in equations (15) and (16) and the identified peak crisis periods defined above. These probabilities provide the odds of switching from one regime to the other as the model travels through the sample. Their behavior is driven by the estimated parameters γ_0 and γ_1 and the estimated values of B^* and λ. Both probabilities are time-varying and hence suggest that a model with exogenous and constant switching probabilities would be misspecified.

In summary, our results provide new estimated values of critical parameters needed for the specification of the borrowing constraint. We also found that our model fits the data well, not only the observable macroeconomic and financial data series used in estimation, but also external narratives of financial crisis history and the business cycle of the business cycle in Mexico. Finally, we showed that our model estimate of being in a crisis stacks up well against an external notion of crisis period, yielding longer lasting crisis episodes. We now turn to the analysis and discussion of the model implied business cycle and crisis properties.

5.2 Business Cycles Properties

In this section, we report and discuss simulated second moments to characterize the estimated model's dynamics and variance decompositions to identify key drivers of

the business cycle. These statistics are generated by simulating the model from the posterior mode estimates.[22] All statistics reported are unconditional, rather than conditional on a particular regime. In fact, as expected, when we compare simulated moments across regimes, we find that they are virtually identical for all variables except those related to the current account balance and the debt position, which are directly affected by the precautionary saving behavior.

Table 3: Simulated Second Moments: Data and Model

	Relative Std. Dev.		Correlations	
Data Series	Data	Model	Data	Model
Output Growth	1.00	1.00	1.00	1.00
Consumption Growth	1.25	1.92	0.73	0.98
Investment Growth	5.37	5.75	0.53	0.90
Trade Balance to Output Ratio	1.24	0.80	-0.20	-0.21
Country Interest Rate	1.36	0.15	-0.11	-0.03

Notes: The table compares second moments of the data, relative to output growth, with the same moment simulated from the model. To compute the simulated moments we draw 10,000 samples, each 144 quarters long as in the data, after a burn-in period of 1,000 quarters.

Table 3 compares data and simulated model second moments, reporting results for three variables used in estimation (output, consumption, investment and the country interest rate), and one critical trade variable, the trade balance ratio, not used in estimation. The model describes the business cycle moments quite well, matching the relative volatilities of output, consumption and investment. The volatility ranking is correct, with consumption significantly more volatile than output, which is a robust stylized fact of emerging market business cycles. The model underestimates the relative volatility or the trade balance ratio and, particularly, the country interest rate. The model implied comovements of all variables match the data counterparts remarkably well, again with the exception of the country interest rate, whose correlation is not estimated precisely in the model.[23] The trade balance, in particular, which

[22]For these simulations, we generate 10,000 samples of 144 quarters long (the length of our data sample), after a burn-in period of 1,000 quarters. We then compute and report mean values across these 1,000 runs. We use a pruning method (Andreasen et al., 2018) to avoid explosive simulation paths.

[23]Note that the model implied interest rate is net of the external financing premium on debt (EFPD) driven by the collateral multiplier. Also note that the model performance is not aided by the assumption that the technology and country interest rate shocks are negatively correlated as,

is not an observable variable used i estimation, is counter-cyclical as in the data, with a model-implied autocorrelation coefficient (not reported) well below one.

Table 4: Estimated Unconditional Variance Decomposition

Variables / Shocks	TFP	Expend.	Import Prices	Pref.	Temp. Int. Rate	Pers. Int. Rate
Output	33.2	17.2	15.7	25.4	2.5	6.0
Consumption	30.3	23.4	14.3	20.6	3.8	7.6
Investment	19.2	29.8	10.3	25.6	4.6	10.5
Trade Bal/Output	9.5	35.2	8.8	17.2	9.2	20.1
Interest Rate	0.0	0.0	0.0	0.0	21.1	78.9
Borrowing Cush.	10.6	32.3	9.9	21.3	9.9	16.0
Debt/Output	15.2	25.5	7.6	40.9	1.4	9.5
Multiplier	9.5	40.5	9.5	18.1	9.6	12.8

Note: The variance decomposition is normalized to sums to 100 by row, estimates may not equal 100 exactly due to rounding. Decomposition computed by removing each shock from full model to compute marginal impact of the shock. Computation uses simulation of 10,000 datasets of 144 quarters, allowing for regime changes.

Table 4 reports variance decompositions. Since the model is nonlinear, these decompositions are obtained by simulating the model using only one shock at the time, and normalizing so that they sum to 100.[24] The table illustrates that all shocks play a quantitatively sizable role in the model, even though different shocks matter more for different variables. Output and consumption are mostly driven by productivity, preference, expenditure, and terms of trade shocks, respectively. Investment is significantly affected by expenditure, preference, productivity, terms of trade, and persistent interest rate shocks. Expenditure and persistent interest rate shocks are the most important drivers of the trade balance, while the country interest rate is clearly driven by persistent interest rate shocks, and to a lesser extent by the temporary component of the cost of borrowing. Demand shocks (expenditure and preference) and interest rate shocks (permanent and temporary components) play a more important role than productivity and terms of trade shocks for financial variables and the multiplier.

for example, in Neumeyer and Perri (2005).

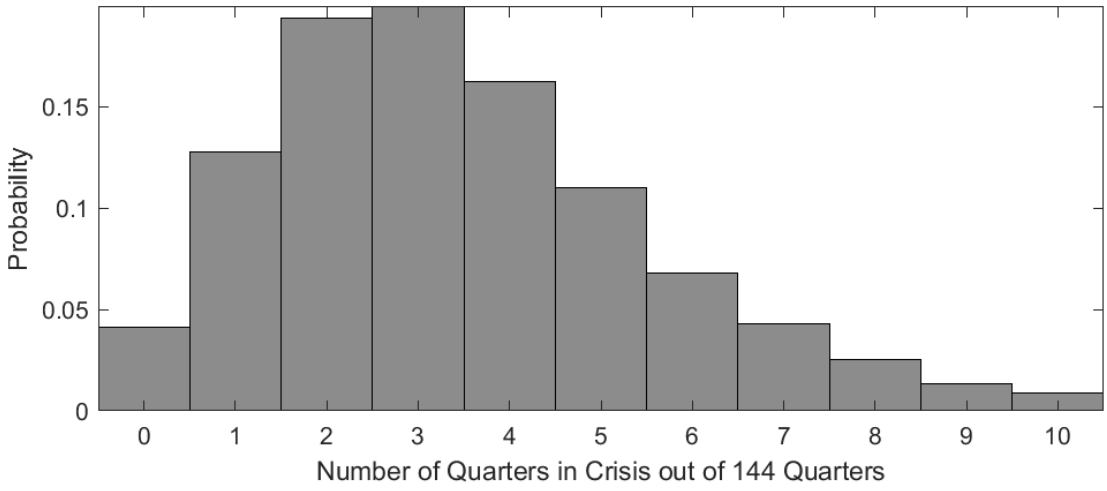
[24] In other words, for each variable, we obtain the variance under each shock in isolation, and then report this variance as a percentage of the sum of the variances over each shock in isolation. As a result, the decomposition ignores nonlinear interactions among the shocks that are present in the model when all shocks are active; we ignore these nonlinearities for a more transparent comparison with linear models.

While the magnitude of these variance shares are not directly comparable with those estimated by Garcia-Cicco et al. (2010), Fernandez and Gulan (2015), and Schmitt-Grohe and Uribe (2018), they suggest that both real and financial shocks matter for Mexico business cycles. In particular, we find lower share of productivity and interest rate shocks than Fernandez and Gulan (2015), although we consider also terms of trade and demand shocks. We also find a share of variance explained by terms of trade shocks that is very close to that estimated by Schmitt-Grohe and Uribe (2018), who use a structural vector autoregression model. The estimated share of variance explained by interest rates shocks, in general, is smaller than the one estimated by Garcia-Cicco et al. (2010), who use a different specification of the financial friction with a debt elastic country premium and a risk premium shock. However, they do not have an amplification from the financial accelerator (Fernandez and Gulan, 2015), or a borrowing constraint or a working capital as in Neumeyer and Perri (2005), Mendoza (2010), Fernandez and Gulan (2015), or Ates and Saffie (2016).

5.3 Model-based Financial Crises Properties

The defining feature of the estimated model is its ability to identify shocks and characterize dynamics not only over the regular business cycle, but also in periods of a particular type of financial crisis, the so-called sudden stop in capital flows. We now turn to empirical and simulation results on these episodes. As before we will focus both on peak and near crisis periods. Note however in the model simulations we, as the agents in the model, know in which regime the economy is without any uncertainty, and so we do not need to express the binding regime as a probabilistic statement, as we did from the econometrician's perspective in Figure 6. So here we define a model generated *peak* crisis as a period in which: (i) the economy *is* in the binding regime; (ii) output growth is negative by more than one standard deviation, and (iii) the current account to output ratio reverts by more than one standard deviation. We define *near* crisis as simply as episodes in which the economy is in the binding regime, and hence periods in which only the first of these three criterion holds. As before, the definition of peak crisis period avoids episodes of a binding regime associated with small current account reversals and output drops and focuses on episodes characterized by large economic dislocation.

Figure 8: Model Implied Peak Crisis Frequency



Notes: Histogram of the model-implied unconditional crisis probability based upon $10,000$ simulated datasets of 144 quarters in length, quantified as the number of quarters out of 144 in which the definition is met for each sample draw. See text for definition of crisis.

5.3.1 Model-based Crisis Episodes: Frequency and Dynamics

Figure 8 reports the relative frequency of peak crisis episodes, our model-based estimate of the unconditional probability of a sudden stop as defined in the empirical and quantitative literature. To construct this histogram, we generate 10,0000 sample paths of 144 quarters as in our data sample. For each path, we then count the number of quarters that meet the definition of peak crisis period. The figure then reports the number of peak crisis quarters. These simulations yield an average of 3.5 peak crisis quarters per sample, or 2.4% of the time. This probability is close to typical estimates in the empirical literature on sudden stops (Calvo et al., 2006), to which quantitative models with occasionally binding collateral constraints are usually calibrated. As the figure shows, however, not all sample draws are alike and there is some heterogeneity with a long right tail in the distribution. The mode frequency is 3 quarters out of 144, but about 4 % of the samples have no crises, and about 10% of the samples have 6 or more quarters in crisis.

However, as we noted above, while the economy rarely experiences what we called peak crises, it experiences near crisis periods much more frequently, with more muted economic consequences of a constrained borrowing capacity. Indeed, when we analyze

the model-based frequency of time spent in the binding regime–our definition near crisis period–we find a much higher frequency ranging from a minimum of 30 quarters to as many as 90 per sample draw. By comparison, for example, the RR tally index is zero in about 50 percent of the tines over our sample period. This result is also consistent with Figure 3, which showed the economy has a probability of switching to the constrained regime of about 50 % around the central tendency of the ergodic distribution of the borrowing cushion. In other words, while the model can generate sudden stops characterized by large and persistent current account adjustment and output drops as rare events, it can also account for near crisis periods of risk build up and sluggish recovery in which the economy is in either close to entering the binding regime or trying to exit it.

Next, we look through the lens of the model at the dynamics during the identified peak and near crisis quarters. Figure 9 reports the results of the same simulations of 10,000 sample draws of 144 quarters length using the posterior mode of the estimated parameters. Each time the economy enters a peak crisis period as defined above, we save the simulated data 5 quarters before and after the crisis episode. The Figure plots the median response across all episodes identified, with $t = 0$ indicating the peak crisis quarter.[25]

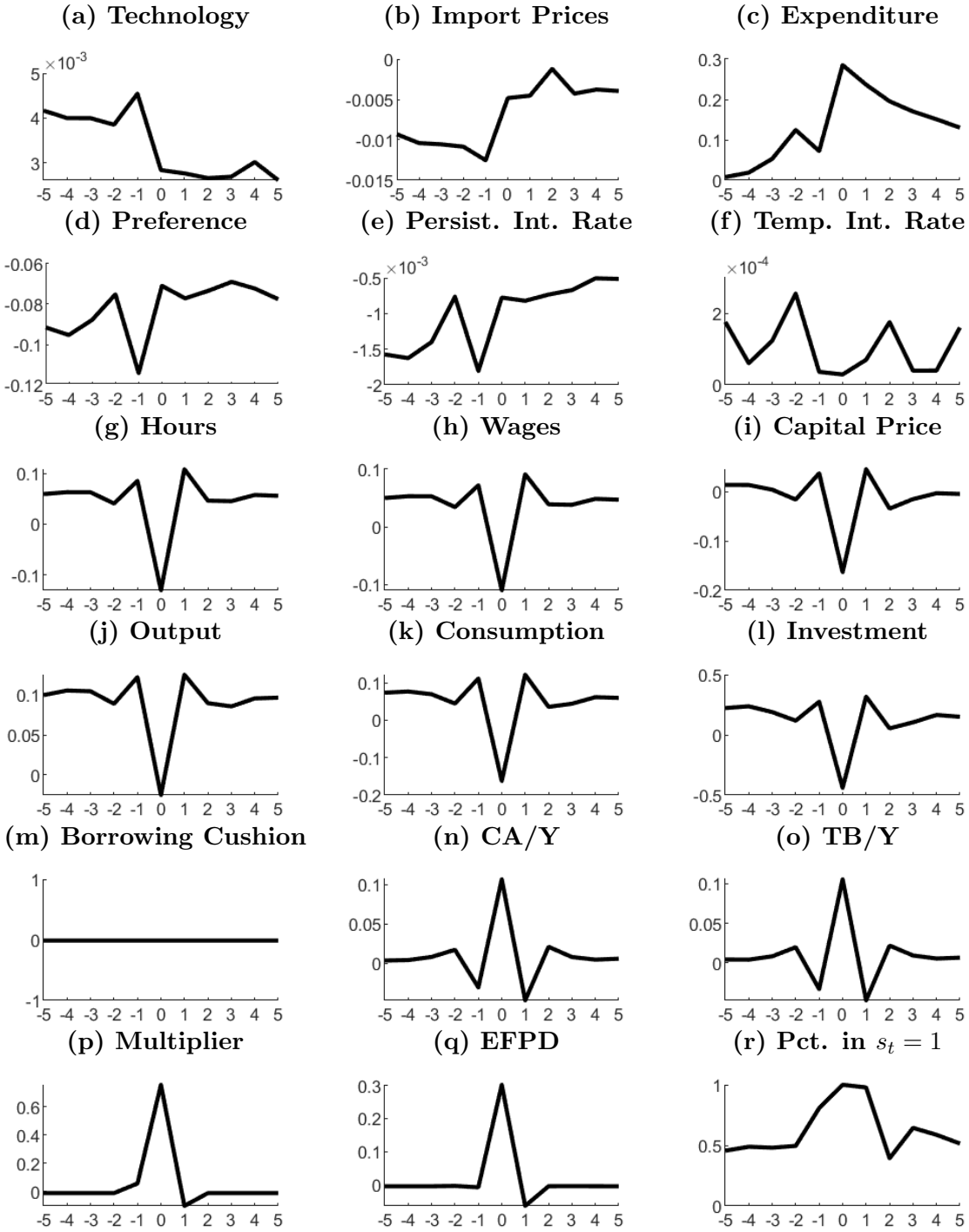
The typical crisis peak is precipitated by a sharp decline in productivity, an increase in the cost of intermediate inputs, combined with an expenditure increase and the persistent component of the country interest rate that also rises abruptly. At the peak of the crisis, output, consumption and investment collapse, and the trade balance and the current account revert, swinging abruptly from a deficit to a large surplus. Wages and the relative price of capital plummet. The EFPD premium spikes, driven by the positive value of the borrowing constraint multiplier.

Looking now at near crisis periods, it is highly likely that the economy is a binding regime, as at least 50 percent of the time the model in this regime the year before the peak of the crisis. After the peak, this percentage remains above the pre crisis-peak level hoovering around 60 percent or so.

In the year before the crisis, exogenous processes are also moving together in a

[25]We do not compare crisis dynamics to Mexican episodes in the data as we discussed model fit before and showed that the model tracks both cycles and crisis periods equally well–see Figure 4. Moreover, as we discuss below and highlight in Table 5, each of Mexico's crisis episodes exhibited distinct dynamics. Note, finally, that by showing the median episode, here, we are ensuring that tail events in the simulations are not driving these results.

Figure 9: Dynamics of Peak and Near Crisis Episodes



Notes: Model-implied crisis dynamics based upon crisis events in $10,000$ simulated datasets of 144 quarters in length. See text for definition of crisis. Crises occur at $t = 0$. Plotted dynamics in panels (a)-(q) are medians across all crises episodes, in log-deviations from steady state; panel (r) shows percent of simulations in the binding regime at each point in time.

distinct manner: productivity is on a downward trend and the persistent component of the country interest rate–capturing behavior or the cost of borrowing in global markets–is increasing. At the same time, the price of intermediate inputs is falling, expenditure is increasing (arguably consistent with a fiscal expansion), while impatience declines as evidenced by a negative realization of the preference process. This cocktail of shocks occurs while the borrowing cushion is already been exhausted, curtailing consumption smoothing possibilities and keeping the percentage of pre-peak binding regime realizations likelihood probability around 50 %.

After the crisis peak, the economy initially rebounds quickly, but the economic environment remains precarious. The sequence of bad economic shocks largely persists after the crisis peak: productivity remains depressed at a significantly lower level than before the crisis peak; the price of imports remains elevated; relative impatience continues; the persistent component of the interest rate increases further; and only expenditure falls gradually after the crisis peak. In the face these shocks, output, consumption, and investment growth initially overshoots, but then settles at a slightly lower growth rat than before the crisis peak.

During period of near crisis, the model does not generate sustained or large output declines and current account reversals, as we do not have financial intermediation or default in the model. This feature suggests that the framework we estimated is not suitable to generate episodes characterized by markedly sluggish recoveries from financial crises. Instead, it is better suited to study v-shaped episodes as Mexico's Tequila crisis, and those experienced by Asian countries and Russia toward the end of the 1990s.[26]

An important take-away from Figure 9 is that the typical model generated peak crisis period is characterized by shocks that comove in a distinctively averse manner, before, during and after the model identified peak sudden stop. This result is in line with the insight from calibrated models of the business cycles in emerging markets that can match the second moments of the data better when the productivity shock is assumed to be negatively correlated with the interest rate shocks.

[26]Indeed, as we saw earlier in Figure 4, the estimated model can identify accurately the two-quarter long Tequila crisis episode, but fails to capture the persistence of the debt crisis of the 1980s once we impose the criteria of a large output drop and sharp current account reversal used in the sudden stop literature (see Figure 6).

5.3.2 Anatomy of Mexico's Financial Crises

Thus far, we examined model-based crisis episodes by relying on simulations. These results characterized the dynamics through the lens of our model around a theoretical crisis peak. However, one key advantage of our framework is that we can also examine historical crises through the lens of the model, evaluating the relative importance of different shocks in driving each of the three crisis peaks that we identified before.

According to our crisis definition, Mexico experienced three crisis episodes since 1981: The debt crisis of the early 1980s, the Tequila crisis of 1994-1995, and an episode coinciding with the Global Financial Crisis that originated in the United States in 2008-09. In order to shed lights on the drivers of causes of these episodes, we conduct a decomposition that focuses on the relative importance of shocks in explaining the observables around each crisis peak. In particular, we recalculate the model likelihood at the posterior mode by considering a counterfactual where we set historical realizations of each shock to zero, while leaving all other shocks at their estimated values. We then compute the loss of model fit measured as the log-likelihood decline and then compare this statistic across shocks. We repeat the analysis for all six structural shocks in the model, and the three historical episodes above identified with the same criteria used in the simulations discussed before.[27] We then consider three time windows: the 4 quarters before the crisis episode, the peak crisis period, and the 4 quarters after the crisis as in Figure 9. The peak of the debt crisis of the 1980s is in 1983:Q1 and lasted one quarter, according to our estimates. The peak of the Tequila crisis is identified in 1995:Q1-Q2 as lasting two quarters, while the peak of the GFC is identified in 2009:Q1-Q2, also lasting two quarters.

Table 5 reports the results of such an historical decomposition by quantifying the likelihood loss associated with the muting of one structural shock at the time. We saw earlier that, in the full sample, all shocks play a role in the model. Table 5 confirms this finding by showing that all shocks contribute similarly to the likelihood (top raw). To evaluate the relative importance of each shock in a particular sub-sample period, we use the full sample values of the likelihood as a benchmark.

First examining the 1983 Debt crisis, expenditure and terms of trade shocks are the most important drivers in the run up of the crisis peak, with significant contributions

[27] Note that when we consider our counterfactuals with shocks set to zero, our inference about the observables and the regime in place will change, and the counterfactual economy may not be in a crisis or even in the binding regime.

Table 5: Anatomy of Mexico's Sudden Stops:

Time Period	TFP	Expend	Import Prices	Pref	Trans Int Rate	Persist Int Rate
Full Sample	16.3	17.4	16.4	16.5	16.7	16.7
1983 Debt Crisis						
4Q Prior	19.8	26.3	27.6	20.3	2.3	3.6
1983Q1	18.1	14.8	15.8	17.3	17.1	16.9
4Q After	18.4	17.9	18.1	9.0	18.1	18.5
1995 Tequila Crisis						
4Q Prior	17.3	16.9	16.2	14.9	17.4	17.3
1995Q1:Q2	13.4	26.9	16.1	13.2	15.0	15.4
4Q After	16.6	14.7	17.1	17.8	16.7	17.1
2009 Global Fin. Crisis						
4Q Prior	16.5	16.6	16.9	16.6	16.7	16.7
2009Q1:Q2	16.6	16.4	17.0	16.6	16.7	16.7
4Q After	16.3	17.4	16.0	17.2	16.5	16.6

Note: The table reports a likelihood-based measure of the relative importance of different shocks during historical peak and near crisis episodes. This measure is the estimated contribution of shocks to likelihood before, during, and after model-identified crisis episodes. Contributions calculated by difference of likelihood of full model with a restricted version that sets given shock to zero during period in question. Contributions normalized so rows sum to 100.

also from productivity and preference shocks.Interest rate shocks become important during and after the crisis, while preference shocks becomes much less important after the crisis. The productivity shock seemingly played a particularly large role throughout. The Tequila crisis is also preceded by a cocktail of equally important shocks, now including also interest rate shocks. In contrast, during the peak of crisis in 1995Q1:Q2, the expenditure shock clearly dominates all other driving forces, perhaps capturing the dramatic fall in domestic absorption that is underlying the current account improvement in the data. Lastly, the GFC episode looks similar to the rest of the sample, with roughly equal contributions from all the shocks before, during, and after the crisis.

These results highlight that, while each of these episodes had similar dynamics in that the economy was borrowing constrained, output fell significantly, and the current account increased markedly, each crisis is somewhat distinct in which shocks put the

economy in a vulnerable position, and which shocks drove the peak of the crisis, and the recovery.

6 Conclusions

In this paper we propose a new flexible and scaleable approach to specifying and solving Dynamic Stochastic General Equilibrium (DSGE) models with occasionally binding collateral constraints. We apply this new approach to the analysis of a particular type of crisis, the so-called sudden stop in capital flows that afflicted many emerging market economies since the early 1980s. We estimated a medium-scale workhorse model with occasionally binding collateral constraint with likelihood based methods.

The critical step to our approach is to specify the occasionally binding nature of the constraint stochastically so that the transition from the unconstrained to the constrained state of the world, and vice versa, can be mapped into an endogenous regime-switching model in which the transition probabilities depend on the state variables of the model and the collateral multiplier. We developed a perturbation method to solve the endogenous regime-switching model that is suitable for the application of standard non-linear Bayesian estimation procedures and methods. This permits estimating the model using full information methods, allowing us to obtain estimates of critical model parameters and conduct likelihood-based counterfactual experiments.

Finally, we apply the framework that we propose to the anatomy of Mexico's business cycle and crisis episodes since 1981, finding that the model fits the data well, critical parameter estimates differ from has been previously used in the literature, that different shocks matter for different variables and phases of the estimated dynamics around peak crisis periods. In particular, in the model simulations, we find that an averse cocktail of shocks typically drives the economy into the crisis rather than a coincidence of large shocks; this result helps explain why calibrated model of emerging market cycles perform better assuming that productivity and interest shocks are negatively correlated. The model estimates based on Mexican data also show that, while interest rate shocks are important drivers of the dynamics before the Tequila and the Global Financial crisis, they played a secondary role in the run up to the debt crisis of the 1980s.

We regard the estimation of larger models–including those with nominal or labor frictions and government intervention, those with permanent and temporary produc-

tivity shocks over longer sample periods, and those with financial intermediation or equilibrium default to generate persistent crisis–as important areas of future research.

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Appendix A Model and Competitive Equilibrium Definition

This Appendix shows full derivations for the model's equilibrium conditions and defines a competitive equilibrium. See the main text for a full motivation of the model.

A.1 Derivation of Equilibrium Conditions

The household-firm maximizes the utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}, \tag{A.1}$$

subject to

$$C_t + I_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t \left(W_t H_t + P_t V_t \right) - E_t - \frac{1}{(1+r_t)} B_t + B_{t-1} \tag{A.2}$$

where gross investment follows

$$I_t = \delta K_{t-1} + \left(K_t - K_{t-1} \right) \left(1 + \frac{\iota}{2} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right) \right) \tag{A.3}$$

When binding, the collateral constraint is given by

$$\frac{1}{(1+r_t)} B_t - \phi \left(1 + r_t \right) \left(W_t H_t + P_t V_t \right) = -\kappa q_t K_t \tag{A.4}$$

The first-order conditions of this problem are the following:

$$d_t \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{-\rho} = \mu_t \tag{A.5}$$

$$\left(1 - \alpha - \eta \right) A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{-\alpha-\eta} = P_t \left(1 + \phi r_t + \frac{\lambda_t}{\mu_t} \phi \left(1 + r_t \right) \right) \tag{A.6}$$

$$\alpha A_t K_{t-1}^{\eta} H_t^{\alpha-1} V_t^{1-\alpha-\eta} = \phi W_t \left(r_t + \frac{\lambda_t}{\mu_t} \left(1 + r_t \right) \right) + H_t^{\omega-1} \tag{A.7}$$

$$\mu_t = \lambda_t + \beta\left(1 + r_t\right)\mathbb{E}_t\mu_{t+1} \tag{A.8}$$

$$\mathbb{E}_t\mu_{t+1}\beta\left(\begin{array}{c} 1-\delta+\left(\frac{\iota}{2}\left(\frac{K_{t+1}}{K_t}\right)^2-\frac{\iota}{2}\right) \\ +\eta A_{t+1}K_t^{\eta-1}H_{t+1}^{\alpha}V_{t+1}^{1-\eta-\alpha} \end{array}\right) = \mu_t\left(1-\iota+\iota\left(\frac{K_t}{K_{t-1}}\right)\right) - \lambda_t\kappa q_t \tag{A.9}$$

Market optimal prices for capital and labor are

$$q_t = 1 + \iota\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) \tag{A.10}$$

$$W_t = H_t^{\omega-1} \tag{A.11}$$

The borrowing cushion is given by the amount of borrowing over the debt limit

$$B_t^* = \frac{1}{(1+r_t)}B_t - \phi\left(1+r_t\right)\left(W_tH_t + P_tV_t\right) + \kappa q_tK_t \tag{A.12}$$

and the regime-switching slackness condition is given by

$$\varphi\left(s_t\right)B_{ss}^* + \nu\left(s_t\right)\left(B_t^* - B_{ss}^*\right) = \left(1-\varphi\left(s_t\right)\right)\lambda_{ss} + \left(1-\nu\left(s_t\right)\right)\left(\lambda_t - \lambda_{ss}\right) \tag{A.13}$$

where λ_t is the multiplier on the international borrowing constraint. The interest rate has a debt elastic component

$$r_t = r_t^* + \psi_r\left(e^{\bar{B}-B_t} - 1\right) + \sigma_r\varepsilon_{r,t} \tag{A.14}$$

The exogenous processes are given by

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A\varepsilon_{A,t} \tag{A.15}$$

$$\log E_t = \left(1-\rho_E\right)\log E^* + \rho_E \log E_{t-1} + \sigma_E\varepsilon_{E,t} \tag{A.16}$$

$$\log P_t = \left(1-\rho_P\right)\log P^* + \rho_P \log P_{t-1} + \sigma_P\varepsilon_{P,t} \tag{A.17}$$

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t} \tag{A.18}$$

$$r_t^* = (1 - \rho_{r^*})\, \bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*} \varepsilon_{r^*,t} \tag{A.19}$$

A.2 Equilibrium Conditions

A competitive equilibrium of our economy is a sequence of quantities $\{K_t,\ B_t,\ C_t,\ H_t,\ V_t,\ I_t,\ A_t,\ E_t,\ B_t^*\}$ and prices $\{P_t,\ r_t^*,\ r_t,\ q_t,\ w_t,\ \mu_t,\ \lambda_t\}$ that, given the 5 exogenous processes (A.15)-(A.19), satisfy the first-order conditions for the representative household-firm (A.5)-(A.9), the market price equations (A.10)-(A.11), the market clearing conditions (A.2)-(A.3), the debt cushion definition (A.12), regime-switching slackness condition (A.13), and an equation for the interest rate (A.14).

We also have a number of auxiliary defitions given by a definition for GDP

$$Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t \tag{A.20}$$

for the debt-to-GDP ratio

$$\Phi_t^b = \frac{B_t}{Y_t} \tag{A.21}$$

and for the current account definition

$$\Phi_t^{ca} = \frac{B_t - B_{t-1}}{Y_t} \tag{A.22}$$

a definition for the trade-balance-to-GDP ratio

$$\Phi_t^{tb} = \frac{Y_t - E_t - C_t - I_t}{Y_t} \tag{A.23}$$

and a definition for the external financing premium on debt

$$EFPD_t = \frac{\lambda_t}{\beta \mathbb{E}_t \mu_{t+1}} \tag{A.24}$$

Appendix B Details of the Perturbation Solution Method

This Appendix provides details about two aspects of the solution method: (1) the definition of, and solution for, the steady state of the endogenous regime-switching economy; and (2) the perturbation method that generates second order Taylor expansions to the solution of the economy around the steady state.

B.1 Regime Switching Equilibrium

The 23 equilibrium conditions are written as

$$\mathbb{E}_t f\left(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi\varepsilon_{t+1}, \varepsilon_t, \theta_{t+1}, \theta_t\right) = 0. \tag{B.1}$$

We have 7 predetermined variables

$$\mathbf{x}_{t-1} = \left[K_{t-1}, B_{t-1}, A_{t-1}, P_{t-1}, E_{t-1}, d_{t-1}, r^*_{t-1}\right] \tag{B.2}$$

and 16 non-predetermined variables

$$\mathbf{y}_t = \left[C_t, H_t, V_t, I_t, k_t, r_t, q_t, W_t, \mu_t, \lambda_t, B^*_t, Y_t, \Phi^b_t, \Phi^{ca}_t, \Phi^{tb}_t, EFPD_t\right] \tag{B.3}$$

with 6 shocks

$$\varepsilon_t = \left[\varepsilon_{A,t}, \varepsilon_{E,t}, \varepsilon_{P,t}, \varepsilon_{d,t}, \varepsilon_{r,t}, \varepsilon_{r^*,t}\right] \tag{B.4}$$

and 2 switching variables

$$\theta_t = \left[\varphi\left(s_t\right), \nu\left(s_t\right)\right]. \tag{B.5}$$

These variables are partitioned into some that affect the steady state, $\theta_{1,t}$, and some that do not, $\theta_{2,t}$. The partition in this case is

$$\theta_{1,t} = \left[\varphi\left(s_t\right)\right] \qquad \theta_{2,t} = \left[\nu\left(s_t\right)\right] \tag{B.6}$$

For solving the model, the functional forms are

$$\theta_{1,t+1} = \bar{\theta}_1 + \chi\hat{\theta}_1\left(s_{t+1}\right), \qquad \theta_{1,t} = \bar{\theta}_1 + \chi\hat{\theta}_1\left(s_t\right) \tag{B.7}$$

$$\theta_{2,t+1} = \theta_2(s_{t+1}), \qquad \theta_{2,t} = \theta_2(s_t) \tag{B.8}$$

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \tag{B.9}$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \qquad \mathbf{y}_{t+1} = g_{s_{t+1}}(\mathbf{x}_t, \chi\varepsilon_{t+1}, \chi) \tag{B.10}$$

and

$$\mathbb{P}_{s_t, s_{t+1}, t} = \pi_{s_t, s_{t+1}}(\mathbf{y}_t). \tag{B.11}$$

Using these in the equilibrium conditions and being more explicit about the expectation operator, given $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ and s_t, then

$$F_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = \int \sum_{s'=0}^{1} \pi_{s_t, s'}(g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)) f \begin{pmatrix} g_{s_{t+1}}(h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \chi\varepsilon', \chi), \\ g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \\ h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \\ \mathbf{x}_{t-1}, \chi\varepsilon', \varepsilon_t, \\ \bar{\theta} + \chi\hat{\theta}(s'), \bar{\theta} + \chi\hat{\theta}(s_t) \end{pmatrix} d\mu\varepsilon' \tag{B.12}$$

Stacking these conditions for each regime produces

$$\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = \begin{bmatrix} F_{s_t=0}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \\ F_{s_t=1}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \end{bmatrix} = 0 \tag{B.13}$$

B.2 Steady State Definition and Solution

The model has two features that make it challenging to define a steady state. First, as it is common in a regime-switching framework, some auxiliary or structural parameters may be switching. In the case of our application, there is only one auxiliary switching parameters that affect the steady state, $\varphi(s_t)$. Nonetheless, in principle, one could allow for regime switching also for the parameters of the exogenous processes, $a^*(s_t)$ and $p^*(s_t)$, or the structural parameter $\kappa^*(s_t)$, which affect the level of the economy directly, and will thus matter for steady state calculations.[28] Solution

[28] As it is well known, over finite periods of time, is statistically difficult to distinguish between unit root processes and processes with structural break of regime changes. Allowing for regime changes in the process for A_t, therefore, would be a way to accommodate permanent productivity shocks as

methods such as those proposed by Foerster et al. (2016) define the steady state by using the ergodic means of these parameters across regimes. We set $\varepsilon_t = 0$ and $\chi = 0$, which implies a steady state given by

$$f\left(\mathbf{y}_{ss}, \mathbf{y}_{ss}, \mathbf{x}_{ss}, \mathbf{x}_{ss}, 0, 0, \bar{\theta}_{\mathbf{1}}, \theta_2(s'), \bar{\theta}_1, \theta_2(s)\right) = 0 \tag{B.14}$$

for all s', s.

In our case, the transition matrix P is endogenous, making the use of the ergodic distribution problematic, since it depends on economic variables that in turn depend on the ergodic means. The steady state solution method that we propose proceeds in two steps. In the first step, we assume the steady state transition matrix is known and solve for the steady state prices and quantities. In the second step, we use the steady state values of the borrowing cushion B^*_{ss} and multiplier λ_{ss} from Step 1 to update the steady state transition matrix. We iterate these two steps until convergence.

Step 1: Solve by Using a Steady State Transition Matrix. First, assume that the steady state transition matrix at iteration i, $P^{(i)}_{ss}$, is known. Next, let $\xi = [\xi_0, \xi_1]$ denote the ergodic vector of $P^{(i)}_{ss}$. Then, as explained in the paper, define the ergodic means of the switching parameters as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1).$$

The steady state of the economy depends on these ergodic means, we can partially solve for some of the steady state directly

$$A_{ss} = 1, d_{ss} = 1, E_{ss} = E^*, P_{ss} = P^*, q_{ss} = 1, r^*_{ss} = \bar{r}^* \tag{B.15}$$

Suppose we know r_{ss}. Then

$$\Omega_v \equiv \frac{A_{ss} K^{\eta}_{ss} H^{\alpha}_{ss} V^{1-\alpha-\eta}_{ss}}{P_{ss} V_{ss}} = \frac{1 + \phi r_{ss} + \phi(1 + r_{ss})(1 - \beta(1 + r_{ss}))}{1 - \alpha - \eta} \tag{B.16}$$

$$\Omega_h \equiv \frac{A_{ss} K^{\eta}_{ss} H^{\alpha}_{ss} V^{1-\alpha-\eta}_{ss}}{W_{ss} H_{ss}} = \frac{1 + \phi(r_{ss} + (1 + r_{ss})(1 - \beta(1 + r_{ss})))}{\alpha} \tag{B.17}$$

in Aguiar and Gopinath (2007).

$$\Omega_k \equiv \frac{A_{ss} K_{ss}^{\eta} H_{ss}^{\alpha} V_{ss}^{1-\alpha-\eta}}{K_{ss}} = \frac{1}{\eta}\left(\frac{1-\kappa\left(1-\beta\left(1+r_{ss}\right)\right)}{\beta} - 1 + \delta\right) \tag{B.18}$$

$$H_{ss} = \left(\frac{A_{ss}}{\Omega_k^{\eta}\Omega_h^{\alpha}\left(P_{ss}\Omega_v\right)^{1-\alpha-\eta}}\right)^{\frac{1}{\alpha(\omega-1)}} \tag{B.19}$$

$$V_{ss} \equiv \frac{\Omega_h}{P_{ss}\Omega_v} H_{ss}^{\omega} \tag{B.20}$$

$$K_{ss} = \frac{\Omega_h}{\Omega_k} H_{ss}^{\omega} \tag{B.21}$$

$$Y_{ss} = \Omega_h H_{ss}^{\omega} - P_{ss} V_{ss} \tag{B.22}$$

$$W_{ss} = H_{ss}^{\omega-1} \tag{B.23}$$

$$I_{ss} = \delta K_{ss} \tag{B.24}$$

$$k_{ss} = K_{ss} \tag{B.25}$$

$$B_{ss} = \bar{B} - \log\left(1 + \frac{r_{ss} - r^*}{\psi_r}\right) \tag{B.26}$$

$$C_{ss} = Y_{ss} - \phi r_{ss}\left(W_{ss}H_{ss} + P_{ss}V_{ss}\right) - E_{ss} + B_{ss}\left(1 - \frac{1}{\left(1+r_{ss}\right)}\right) - I_{ss} \tag{B.27}$$

$$\mu_{ss} = \left(C_{ss} - \frac{H_{ss}^{\omega}}{\omega}\right)^{-\rho} \tag{B.28}$$

$$\lambda_{ss} = \left(1 - \beta\left(1+r_{ss}\right)\right)\mu_{ss} \tag{B.29}$$

$$B_{ss}^{*} = \frac{1}{\left(1+r_{ss}\right)} B_{ss} - \phi\left(1+r_{ss}\right)\left(W_{ss}H_{ss} + P_{ss}V_{ss}\right) + \kappa K_{ss} \tag{B.30}$$

$$\Phi_{ss}^{b} = \frac{B_{ss}}{Y_{ss}} \tag{B.31}$$

$$\Phi_{ss}^{ca} = 0 \tag{B.32}$$

$$\Phi_{ss}^{tb} = \frac{Y_{ss} - E_{ss} - C_{ss} - I_{ss}}{Y_{ss}} \tag{B.33}$$

$$EFPD_{ss} = \frac{\lambda_{ss}}{\beta \mu_{ss}} \tag{B.34}$$

and then r_{ss} solves

$$\bar{\varphi} B_{ss}^{*} = (1 - \bar{\varphi})\, \lambda_{ss} \tag{B.35}$$

Step 2: Update the Transition Matrix. Step 1 yields the variables B_{ss}^{*} and λ_{ss}, and hence have a new value of the transition matrix for iteration $i+1$:

$$P_{ss}^{(i+1)} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B_{ss}^{*})}{1+\exp(-\gamma_0 B_{ss}^{*})} & \frac{\exp(-\gamma_0 B_{ss}^{*})}{1+\exp(-\gamma_0 B_{ss}^{*})} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}, \tag{B.36}$$

which can be checked against the guess in Step 1. Continue this iterative procedure until

$$\left\| P_{ss}^{(i+1)} - P_{ss}^{(i)} \right\| < tolerance.$$

B.3 Generating Approximations

To compute a second order approximation to the endogenous regime-switching model solution, we largely follow Foerster et al. (2016), but adapted to the case with endogenous probabilities.

We take the stacked equilibrium conditions $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$, and differentiate with respect to $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$. In general regime-switching models, the first-order derivative with respect to $\mathbf{x}_{t-1}$ produces a complicated polynomial system denoted

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0. \tag{B.37}$$

Often this system needs to be solved via Gröbner bases, which finds all possible solutions in order to check them for stability. In our case, with endogenous probabil-

ities, the standard stability checks fail, so we will focus on finding a single solution and ignore the possibility of indeterminacy, a common simplification in the regime-switching literature with and without endogenous switching (e.g. Farmer et al., 2011; Foerster, 2015; Maih, 2015; Lind, 2014). In the literature that computes global solutions to non-regime switching occasionally binding constraint models (e.g. Benigno et al. (2013), Mendoza (2010)) there are no proofs of uniqueness, and the focus typically is also on computing a solution checking robustness to initial conditions. To find a solution to our model, we guess at a set of policy functions for regime $s_t = 1$, which collapses the equilibrium conditions $\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0; s_t = 0)$ into a fixed-regime eigenvalue problem, and solve for the policy functions for $s_t = 0$. Then, using this initial solution as guess, we solve for regime $s_t = 0$ under the fixed-regime eigenvalue problem, and iterate on this procedure to convergence. After solving the iterative eigenvalue problems, the other systems to solve are

$$\mathbb{F}_{\varepsilon}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \tag{B.38}$$

$$\mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \tag{B.39}$$

and second order systems of the form (can apply equality of cross-partials)

$$\mathbb{F}_{\mathbf{i},\mathbf{j}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0,\ \mathbf{i}, \mathbf{j} \in \{\mathbf{x}, \varepsilon, \chi\}. \tag{B.40}$$

Recall the decision rules have the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \tag{B.41}$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \tag{B.42}$$

and so the second-order approximation takes the form

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H^{(1)}_{s_t} S_t + \frac{1}{2} H^{(2)}_{s_t} (S_t \otimes S_t) \tag{B.43}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G^{(1)}_{s_t} S_t + \frac{1}{2} G^{(2)}_{s_t} (S_t \otimes S_t) \tag{B.44}$$

where $S_t = \begin{bmatrix} (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' & \varepsilon_t' & 1 \end{bmatrix}'$.

B.4 Proposition 1: Irrelevance of First-Order

To show Proposition 1, we can take the first-order derivatives of (B.13) with respect to its arguments, evaluated at steady state. This differentiation produces

$$\mathbb{F}_{\mathbf{x},s_t}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=\begin{array}{c}\sum_{s'}\pi_{s_t,s',y}\left(\mathbf{y}_{ss}\right)g_{\mathbf{x},s_t}f_{ss}\left(s',s_t\right)\\ +\sum_{s'}\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)\left[\begin{array}{c}f_{\mathbf{y}_{t+1}}\left(s',s_t\right)g_{\mathbf{x},s'}h_{\mathbf{x},s_t}+f_{\mathbf{y}_t}\left(s',s_t\right)g_{\mathbf{x},s_t}\\ +f_{\mathbf{x}_t}\left(s',s_t\right)h_{\mathbf{x},s_t}+f_{\mathbf{x}_{t-1}}\left(s',s_t\right)\end{array}\right]\end{array} \tag{B.45}$$

$$\mathbb{F}_{\varepsilon,s_t}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=\begin{array}{c}\sum_{s'}\pi_{s_t,s',y}\left(\mathbf{y}_{ss}\right)g_{\varepsilon,s_t}f_{ss}\left(s',s_t\right)\\ +\sum_{s'}\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)\left[\begin{array}{c}f_{\mathbf{y}_{t+1}}\left(s',s_t\right)g_{\mathbf{x},s'}h_{\varepsilon,s_t}+f_{\mathbf{y}_t}\left(s',s_t\right)g_{\varepsilon,s_t}\\ +f_{\mathbf{x}_t}\left(s',s_t\right)h_{\varepsilon,s_t}+f_{\varepsilon_t}\left(s',s_t\right)\end{array}\right]\end{array} \tag{B.46}$$

and

$$\mathbb{F}_{\chi,s_t}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=\begin{array}{c}\sum_{s'}\pi_{s_t,s',y}\left(\mathbf{y}_{ss}\right)g_{\chi,s_t}f_{ss}\left(s',s_t\right)\\ +\sum_{s'}\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)\left[\begin{array}{c}f_{\mathbf{y}_{t+1}}\left(s',s_t\right)g_{\mathbf{x},s'}h_{\chi,s_t}+f_{\mathbf{y}_t}\left(s',s_t\right)g_{\chi,s_t}\\ +f_{\mathbf{x}_t}\left(s',s_t\right)h_{\chi,s_t}\\ +f_{\theta_{t+1}}\left(s',s_t\right)\hat{\theta}\left(s_{t+1}\right)+f_{\theta_t}\left(s',s_t\right)\hat{\theta}\left(s_t\right)\end{array}\right]\end{array}. \tag{B.47}$$

Note, by the definition of a steady state, $f_{ss}\left(s',s_t\right)=0$, and so the first term of each of these expressions equals zero. Hence, we are left with the formulas for a constant probability model from Foerster et al. (2016), with the probabilities given by $\mathbb{P}_{ss}=\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)$ as stated in the Proposition.

Appendix C Accuracy of the Solution

We assess accuracy by checking the Euler equation error, where

$$EEE_t=1-\frac{\lambda_t}{\mu_t}-\beta\left(1+r_t\right)\mathbb{E}_t\frac{\mu_{t+1}}{\mu_t} \tag{C.1}$$

Table C.1: Accuracy Check in Benigno et al. (2020)

Variable	Inequality Constraint	Endo Switch Global	Endo Switch Pert
Std Dev			
GDP	2.38	2.46	2.28
Hours	1.36	1.46	1.33
Autocorrelation			
GDP	0.94	0.94	0.94
Hours	0.77	0.76	0.77
Euler Eqn Errors ($\log_{10}$)	-10.47	-3.59	-3.41

and the policy functions can be denoted by

$$\lambda_t = \lambda_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right), \quad \mu_t = \mu_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right), \quad r_t = r_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right), \quad B_t^* = B_{s_t}^*\left(\mathbf{x}_{t-1}, \varepsilon_t\right) \tag{C.2}$$

and so, given $(\mathbf{x}_{t-1}, \varepsilon_t)$

$$EEE_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right) = 1 - \frac{\lambda_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right)}{\mu_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right)} - \frac{\beta\left(1 + r_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right)\right)}{\mu_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right)} \tag{C.3}$$

$$\times \sum_{s_{t+1}=0}^{1} p_{s_t, t+1}\left(\mathbf{x}_{t-1}, \varepsilon_t\right) \int_{\mathbb{R}^{\varepsilon}} \mu_{s_{t+1}}\left(\mathbf{x}_t, \varepsilon_{t+1}\right) \mu\left(\varepsilon_{t+1}\right) d\varepsilon_{t+1} \tag{C.4}$$

We simulate the model for 10,000 periods, after a 1,000 period burn-in to get sequences of s_t and $(\mathbf{x}_{t-1}, \varepsilon_t)$. For each period in the simulation, we draw 10,000 values of ε_{t+1} to compute the integral. We then average the absolute values across the 10,000 periods, and report the log base-10 of this average.

Our current model has too many shocks and variables to solve the model using global projection methods. To facilitate a comparison, in work-in-progress (Benigno et al., 2020), we solved a version of Jermann and Quadrini (2012) using three methods: first, the original inequality constraint; second, our endogenous regime switching formulation solved via global projection methods; and third, our endogenous regime switching formulation solved via our perturbation method.

Table C.1 highlights that the different solution methods produced nearly identical implications for the standard deviations and autocorrelations of GDP and hours, which are two key variables in that model. Further, the Euler equation errors all achieve reasonable levels of accuracy. The pure inequality constraint had the highest

accuracy; the endogenous switching model solved globally returned a reasonable value of accuracy of -3.6 in log-10 points (this value can be driven lower by improving the number of gridpoints in our global solution algorithm); and the perturbation solution returned -3.4 in log-10 points. The latter two numbers suggest an approximation error of about \$3.50 per \$10,000 of consumption.

Lastly, we also note that Binning and Maih (2017) have adapted our framework to additional contexts and found a high degree of accuracy.

Appendix D Estimation Procedure

D.1 State Space

For likelihood estimation, the state space representation is given as follows

$$\mathcal{X}_t = \mathcal{H}_{s_t}\left(\mathcal{X}_{t-1}, \varepsilon_t\right) \tag{D.1}$$

$$\mathcal{Y}_t = \mathcal{G}_{s_t}\left(\mathcal{X}_t, \mathcal{U}_t\right) \tag{D.2}$$

Recall the second-order approximation takes the form

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H^{(1)}_{s_t} S_t + \frac{1}{2} H^{(2)}_{s_t}\left(S_t \otimes S_t\right) \tag{D.3}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G^{(1)}_{s_t} S_t + \frac{1}{2} G^{(2)}_{s_t}\left(S_t \otimes S_t\right) \tag{D.4}$$

where $S_t = \begin{bmatrix} (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' & \varepsilon_t' & 1 \end{bmatrix}'$.

Therefore, we can define the state variable as

$$\mathcal{X}_t = \begin{bmatrix} \mathbf{x}_t' & \mathbf{x}_{t-1}' & \mathbf{y}_t' & \mathbf{y}_{t-1}' & \epsilon_t \end{bmatrix}'. \tag{D.5}$$

The nonlinear transition equations

$$\mathcal{X}_t = \mathcal{H}_{s_t}\left(\mathcal{X}_{t-1}, \varepsilon_t\right) \tag{D.6}$$

are given by

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{ss} + H^{(1)}_{s_t} S_t + \frac{1}{2} H^{(2)}_{s_t} \left(S_t \otimes S_t \right) \\ \mathbf{x}_{t-1} \\ \mathbf{y}_{ss} + G^{(1)}_{s_t} S_t + \frac{1}{2} G^{(2)}_{s_t} \left(S_t \otimes S_t \right) \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} \tag{D.7}$$

The observation equation

$$\mathcal{Y}_t = \mathcal{G}_{s_t} \left(\mathcal{X}_t, \mathcal{U}_t \right) \tag{D.8}$$

is given by

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ r_t \\ \Delta B_t / Y_t \\ \Delta P_t \end{bmatrix} = D \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} + \mathcal{U}_t \tag{D.9}$$

where D denotes a selection matrix of the form

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ r_t \\ \Delta B_t / Y_t \\ \Delta P_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1_{[y_t]} & -1_{[y_t]} & 0 \\ 0 & 0 & 1_{[c_t]} & -1_{[c_t]} & 0 \\ 0 & 0 & 1_{[i_t]} & -1_{[i_t]} & 0 \\ 0 & 0 & 1_{[r_t]} & 0 & 0 \\ 0 & 0 & 1_{[\Phi^{ca}_t]} & 0 & 0 \\ 1_{[P_t]} & -1_{[P_t]} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} + \mathcal{U}_t. \tag{D.10}$$

D.2 Filtering

The Unscented Kalman Filter (UKF) uses the unscented transformation to calculate the state mean and covariance matrix. It propagates the deterministically chosen sigma-points through nonlinear functions. The transformed points are used to calculate the mean and covariance matrix. As Julier and Uhlmann (1999) note, the key approximation taken to develop the UKF is that the prediction density and the filtering density are both Gaussian. The filtering and smoothing largely follow Binning and Maih (2015).

The filter starts by combining the state vector and exogenous disturbances into

a single vector $\mathcal{X}^a_{t-1} = [\mathcal{X}_{t-1}, \epsilon_t]'$ with the following mean and covariance matrix conditional on $Y_{1:t-1}$ and regime s_{t-1}:

$$\mathcal{X}^a_{t-1}(s_{t-1}) = \begin{bmatrix} \mathcal{X}_{t-1|t-1}(s_{t-1}) \\ 0_\epsilon \end{bmatrix} \tag{D.11}$$

$$P^a_{t-1}(s_{t-1}) = \begin{bmatrix} P^x_{t-1|t-1}(s_{t-1}) & 0 \\ 0 & I \end{bmatrix}. \tag{D.12}$$

The sigma-points $\mathcal{X}^a_{i,t-1}(s_{t-1})$ that consist of the sigma-points for state variables $\mathcal{X}^x_{i,t-1}(s_{t-1})$ and the sigma-points for exogenous shocks $\mathcal{X}^\varepsilon_{i,t-1}(s_{t-1})$ are chosen as follows:

$$\begin{aligned}
\mathcal{X}^a_{0,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) && \text{(D.13)} \\
\mathcal{X}^a_{0,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) && \text{(D.14)} \\
\mathcal{X}^a_{i,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) + (h\sqrt{P^a_{t-1}(s_{t-1})})_i \text{ for } i = 1 \dots L && \text{(D.15)} \\
\mathcal{X}^a_{i,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) - (h\sqrt{P^a_{t-1}(s_{t-1})})_{i-L} \text{ for } i = L+1 \dots 2L && \text{(D.16)}
\end{aligned}$$

where $h = \sqrt{3}$ and L denotes the number of state variables and exogenous shocks. The weights for the sigma-points are given by:

$$\begin{aligned}
w_0 &= \frac{h - L}{2h} && \text{(D.17)} \\
w_i &= \frac{1}{2h} \text{ for } i = 1 \dots 2L && \text{(D.18)}
\end{aligned}$$

The sigma-points and the assigned weights are used to calculate the expected mean and covariance by propagating sigma-points through transition equations and taking weighted average:

$$\mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) = H_{s_t}(\mathcal{X}^x_{i,t-1}(s_{t-1}), \mathcal{X}^\varepsilon_{i,t-1}(s_{t-1})) \tag{D.19}$$

$$\mathcal{X}_{t|t-1}(s_{t-1}, s_t) = \sum_{i=0}^{2L} w_i \mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) \tag{D.20}$$

$$P^x_{t|t-1}(s_{t-1}, s_t) = \sum_{i=0}^{2L} w_i \tilde{\mathcal{X}}_i \tilde{\mathcal{X}}_i^T \tag{D.21}$$

$$\mathcal{Y}_{t|t-1}(s_{t-1}, s_t) = D\mathcal{X}_{t|t-1}(s_{t-1}, s_t) \tag{D.22}$$

where $\tilde{\mathcal{X}}_i = \mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) - \mathcal{X}_{t|t-1}(s_{t-1}, s_t)$. By the above conditions, we get the Gaussian approximation predictive density
$p(\mathcal{X}_t|\mathcal{Y}_{1:t-1}, s_{t-1}, s_t) = N(\mathcal{X}_{t|t-1}(s_{t-1}, s_t), P^x_{t|t-1}(s_{t-1}, s_t))$. The predictions are then updated using the standard Kalman filter updating rule:

$$P^y_{t|t-1}(s_{t-1}, s_t) = DP^x_{t|t-1}(s_{t-1}, s_t)D^T + R \tag{D.23}$$

$$P^{xy}_{t|t-1}(s_{t-1}, s_t) = P^x_{t|t-1}(s_{t-1}, s_t)D^T \tag{D.24}$$

$$K_t(s_{t-1}, s_t) = P^{xy}_{t|t-1}(s_{t-1}, s_t)(P^y_{t|t-1}(s_{t-1}, s_t))^{-1} \tag{D.25}$$

$$\mathcal{X}_{t|t}(s_{t-1}, s_t) = \mathcal{X}_{t|t-1}(s_{t-1}, s_t) + K_t(s_{t-1}, s_t)(\mathcal{Y}_t - \mathcal{Y}_{t|t-1}(s_{t-1}, s_t)) \tag{D.26}$$

$$P^x_{t|t}(s_{t-1}, s_t) = P^x_{t|t-1}(s_{t-1}, s_t) - K_t(s_{t-1}, s_t)P^y_{t|t-1}(s_{t-1}, s_t)K_t^T(s_{t-1}, s_t) \tag{D.27}$$

The updating step gives $p(\mathcal{X}_t|\mathcal{Y}_{1:t}, s_{t-1}, s_t) = N(\mathcal{X}_{t|t}(s_{t-1}, s_t), P^x_{t|t}(s_{t-1}, s_t))$. As a by-product of the filter, we can get the density of $\mathcal{Y}_t$ conditional on $\mathcal{Y}_{1:t-1}$, s_t, and s_{t-1}

$$p(\mathcal{Y}_t|\mathcal{Y}_{1:t-1}, s_{t-1}, s_t; \theta) = N(\mathcal{Y}_{t|t-1}(s_{t-1}, s_t), P^y_{t|t-1}(s_{t-1}, s_t)) \tag{D.28}$$

Since the Unscented Kalman filter with regime switches creates a large number of nodes over each iteration where the filtered mean and covariance matrix need to be evaluated, we implement the following collapsing procedure suggested by Kim and Nelson (1999)

$$\mathcal{X}_{t|t}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:t})}\Big\{\sum_{i=1}^{M} \Pr(s_{t-1} = i, s_t = j|\mathcal{Y}_{1:t})\mathcal{X}_{t|t}(s_{t-1} = i, s_t = j)\Big\} \tag{D.29}$$

$$P^x_{t|t}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:t})}\Big\{\sum_{i=1}^{M}\Pr(s_{t-1} = i, s_t = j|\mathcal{Y}_{1:t})[P^x_{t|t}(s_{t-1} = i, s_t = j) \quad \text{(D.30)}$$

$$+(\mathcal{X}_{t|t}(s_t = j) - \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j))(\mathcal{X}_{t|t}(s_t = j) - \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j))^T]\Big\} \quad \text{(D.31)}$$

where $\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t})$ and $\Pr(s_t|\mathcal{Y}_{1:t})$ are obtained from the following standard Hamilton filter

$$\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1}) = \Pr(s_t|s_{t-1})\Pr(s_{t-1}|\mathcal{Y}_{1:t-1}) \quad \text{(D.32)}$$

$$\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t}) = \frac{p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1})\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1})}{\sum_{s_t}\sum_{s_{t-1}} p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1})\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1})} \quad \text{(D.33)}$$

$$\Pr(s_t|\mathcal{Y}_{1:t}) = \sum_{s_{t-1}}\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t}) \quad \text{(D.34)}$$

Finally, we can get the conditional marginal likelihood,

$$p(\mathcal{Y}_t|\mathcal{Y}_{1:t-1};\theta) = \sum_{s_t}\sum_{s_{t-1}} p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1})\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1}) \quad \text{(D.35)}$$

D.3 Smoothing

Once we run through the UKF for $t = 1, \ldots, T$, we can also get $\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T})$, $\Pr(s_t|\mathcal{Y}_{1:T})$, $x_{t|T}(s_t, s_T)$, and $P^x_{t|T}(s_t, s_T)$:

$$\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T}) = \frac{\Pr(s_{t+1}|\mathcal{Y}_{1:T})\Pr(s_t|\mathcal{Y}_{1:t})\Pr(s_{t+1}|s_t)}{\Pr(s_{t+1}|\mathcal{Y}_{1:t})} \quad \text{(D.36)}$$

$$\Pr(s_t|\mathcal{Y}_{1:T}) = \sum_{s_{t+1}}\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T}) \quad \text{(D.37)}$$

$$\mathcal{X}_{t|T}(s_t, s_{t+1}) = \mathcal{X}_{t|t}(s_t) + \tilde{K}_t(s_t, s_{t+1})(\mathcal{X}_{t+1|T}(s_{t+1}) - \mathcal{X}_{t+1|t}(s_t, s_{t+1})) \quad \text{(D.38)}$$

$$P^x_{t|T}(s_t, s_{t+1}) = P^x_{t|t}(s_t) - \tilde{K}_t(s_t, s_{t+1})(P^x_{t+1|T}(s_{t+1}) - P^x_{t+1|T}(s_t, s_{t+1}))\tilde{K}_t(s_t, s_{t+1})^T \quad \text{(D.39)}$$

Given the above smoothing algorithm, we implement the collapsing procedures

similar to those in the filtering steps:

$$\mathcal{X}_{t|T}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:T})}\Big\{\sum_{j=1}^{M} \Pr(s_t = i, s_{t+1} = j|\mathcal{Y}_{1:T})\mathcal{X}_{t|T}(s_t = i, s_{t+1} = j)\Big\} \tag{D.40}$$

$$P^x_{t|T}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:T})}\Big\{\sum_{j=1}^{M} \Pr(s_t = i, s_{t+1} = j|\mathcal{Y}_{1:T})[P^x_{t|T}(s_t = i, s_{t+1} = j) \tag{D.41}$$

$$+(\mathcal{X}_{t|T}(s_t = j) - \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j))(\mathcal{X}_{t|T}(s_t = j) - \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j))^T]\Big\} \tag{D.42}$$

Appendix E Calibrated Parameters

We largely follow Mendoza (2010), but adapted to the current framework. First, we start by calibrating certain parameters, using the steady state of the model in which there is no working capital constraint and the borrowing constraint does not bind. That is, $\phi = 0$ and $\bar{\varphi} = 0$, the latter implies $\lambda_{ss} = 0$. In addition, we get

$$\beta\,(1 + r_{ss}) = 1 \tag{E.1}$$

and ratios of

$$\Omega_v = \frac{1}{1-\alpha-\eta},\ \Omega_h = \frac{1}{\alpha},\ \Omega_k = \frac{1}{\eta}\left(\frac{1}{\beta} - 1 + \delta\right) \tag{E.2}$$

These imply factor payment ratios are

$$\frac{P_{ss}V_{ss}}{Y_{ss} + P_{ss}V_{ss}} = \frac{1}{\Omega_v} = 1 - \alpha - \eta \tag{E.3}$$

$$\frac{W_{ss}H_{ss}}{Y_{ss}} = \frac{1}{\Omega_h\left(1 - \frac{1}{\Omega_v}\right)} = \frac{\alpha}{\alpha+\eta} \tag{E.4}$$

$$\frac{\left(\frac{1}{\beta} - 1 + \delta\right)K_{ss}}{Y_{ss}} = \frac{\frac{1}{\beta} - 1 + \delta}{\Omega_k\left(1 - \frac{1}{\Omega_v}\right)} = \frac{\eta}{\alpha+\eta} \tag{E.5}$$

Using the National Accounts,

$$\left[\begin{array}{c} 1-\alpha-\eta=0.102 \\ \frac{\alpha}{\alpha+\eta}=0.66 \end{array}\right] \Longrightarrow \left[\begin{array}{c} \alpha=0.59268 \\ \eta=0.30532 \end{array}\right] \tag{E.6}$$

We set the depreciation rate to an annual value of 8.8 percent, so

$$(1-\delta)^4 = 1-0.088 \Longrightarrow \delta = 0.022766 \tag{E.7}$$

The capital-to-(annual) gross output ratio is 1.758, so in our case the capital-to-(quarterly) gross output ratio Ω_k^{-1} is set to be

$$\Omega_k^{-1} = \left(\frac{1}{\eta}\left(\frac{1}{\beta}-1+\delta\right)\right)^{-1} = 4 * 1.758 \Longrightarrow \beta = 0.97977 \tag{E.8}$$

Note that this implies an annualized real interest rate of

$$(1+r_{ss})^4 = \left(\frac{1}{\beta}\right)^4 = 1.0852, \tag{E.9}$$

which nearly replicates the number in Mendoza (2010) but using different discounting. From the resource constraint

$$\frac{C_{ss}}{Y_{ss}} + \frac{I_{ss}}{Y_{ss}} + \frac{E_{ss}}{Y_{ss}} = 1 + \left(1 - \frac{1}{1+r_{ss}}\right)\frac{B_{ss}}{Y_{ss}} \tag{E.10}$$

and interpreting E_{ss} as government spending

$$0.65 + 0.172 + 0.11 = 1 + (1-\beta)\frac{B_{ss}}{Y_{ss}} \Longrightarrow \frac{B_{ss}}{Y_{ss}} = -3.3605 \tag{E.11}$$

Note that this implies

$$\frac{B_{ss}}{4Y_{ss}} = -0.840127 \tag{E.12}$$

and we can get

$$Y_{ss} = \frac{\alpha+\eta}{\alpha}\left(\frac{1}{P_{ss}^{1-\alpha-\eta}\left(\frac{1}{\eta}\left(\frac{1}{\beta}-1+\delta\right)\right)^{\eta}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha-\eta}\right)^{1-\alpha-\eta}}\right)^{\frac{\omega}{\alpha(\omega-1)}} = 1.8202 \tag{E.13}$$

Which gives

$$E^* = \frac{E_{ss}}{Y_{ss}} Y_{ss} = 0.11 * 1.8202 = 0.20022 \tag{E.14}$$

as well as

$$B_{ss} = -0.840127 * 4 * 1.8202 = -6.11685 \tag{E.15}$$

then, conditional on r^* and ψ_r, $\bar{B}$ is pinned down via

$$\bar{B} = \log\left(1 + \frac{r_{ss} - r^*}{\psi_r}\right) + B_{ss}. \tag{E.16}$$

Appendix F Data Appendix

F.1 National Accounts

National accounts are from the National Statistic Office. The data series used in the analysis merge two set of official statistics by updating the level of the accounts based on 1993 constant prices with the quarterly rate of growth of the accounts based on 2008 at constant prices. This is necessary as the deflators to splice the accounts in levels were not available at the time of last download of the data (May 2017). The two set of accounts overlaps from 1993:Q1 to 2006:Q4. Over this period, the difference in annual rate of growth is less than 0.01 percent in absolute value for GDP, less than 0.05 percent for consumption, less than 2 percent for investment, and less than 1 and 3 percent for imports and exports, respectively. The correlations between the series are more than .9 for all series except investment that is .84, pointing to possibly larger measurement errors in this variable. The differences, however are smaller the closer to the end of the sample. For this reason, we choose to update the 1993 accounts rather than backdate the 2008 ones.

The specific sources of the data are as follows: **2006:Q1-2016:Q4** (Labeled 2008 accounts)–Supply and demand of goods and services. Original series (not seasonally adjusted). Constant prices, annual 2008 = 100 (Oferta y demanda de bienes y servicios. Series originales. A precios constantes 2008). Available from http://www3.inegi.org.mx/sistemas/tabuladosbasicos/tabdirecto.aspx.

1980:Q1-2006:Q4 (Labeled 1993 accounts)–Supply and demand of goods and services. Original Series (not seasonally adjusted). Constant prices, annual 1993 = 100. (We obtained these from Guadra Gabriel (2008), Hechos Estilizados del Ciclo

Economico en Mexico, Banco de Mexico Working Papers No. 2008-14, Diciembre 2008).

The net exports to GDP series, used to validate the model externally but not as observable variable in estimation, is calculated as exports minus real imports divided by real GDP. The current account as a percentage of GDP is from the balance of payment statistics, obtained the OECD Economic Outlook Database (Series MEX.CBGDPR.Q, OECD-EO-MEX-CBGDPR-Q).

As a proxy for the relative price of intermediate goods, entered as observable in estimation, we use a measure of Mexico's terms of trade

The data are not seasonally adjusted and show a strong seasonal pattern. To seasonally adjust all series (assumed to be I(1) processes), we adjust the log-difference using the X-12 procedure with the additive option in Eviews. We then use the log of the first observation of the raw series (not seasonally adjusted) and cumulate the seasonally adjusted log-difference.