The Anatomy of Crises and Cycles in Mexico: An Endogenous Regime Switching Approach*

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Abstract

We propose a new approach to specifying and solving DSGE models with occasionally binding collateral constraints. The specification of the collateral constraint that we suggest assumes that the transition from the unconstrained to the constrained state is a stochastic rather than a deterministic function of the endogenous level of leverage. This formulation results in an endogenous regime-switching model that can be solved with perturbation methods. Next, using Bayesian full information methods, we estimate a model with an occasionally binding constraint in the spirit of Mendoza (AER 2010), with Mexican quarterly data since 1981, considering a comprehensive set of shocks as in Garcia-Cicco et. al. (AER 2010). We find that the estimated model fits the data well, characterizing both their second moments and the 1995 Tequila crisis period. It yields estimates of the regime probabilities that align well with the narrative of Mexico's financial crisis history. We also find that a cocktail of shocks that co-move in an adverse manner, rather than a sequence of large negative shocks with a negative correlation between the productivity and the interest rate shock as typically assumed in the extant literature, precedes large sudden stops episodes. Finally, we document that expenditure and impatience shocks drove Mexico's economy into the Tequila crisis, while productivity and interest shocks prevailed during that particular sudden stop episode.

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1 Introduction

The aftermath of the Global Financial Crisis of 2008-09 stimulated a large positive and normative literature that models financial crisis episodes as large economic dislocations associated with occasionally binding collateral constraints. These constraints amplify the effects of regular business cycle shocks and can provide a theoretical justification for policy interventions before and during crisis times. However, structural estimation of the friction parameters and the analysis of the shocks that historically might have driven the crisis events has been hampered by computational challenges. As a result, this class of models lacks a Smets and Wouters (2007) style evaluation, which is needed for implementation of the sound normative implications of dynamic, stochastic general equilibrium (DSGE) models. This paper performs such an evaluation on the well-known model of Mendoza (2010), considering an encompassing set of shocks previously considered in the literature, as in Garcia-Cicco et al. (2010). Thus, this paper bridges the gap between the literature on the econometric evaluation of DSGE models in the spirit of Smets and Wouters (2007) and the development and use of models with occasionally binding collateral constraints as, for instance, Bocola (2015) or Jermann and Quadrini (2012).

Following the seminal contribution of Mendoza (2010), models with occasionally binding collateral constraints have become the workhorse environment for the normative analysis of macroprudential policies, capital controls, as well as crisis resolution policies. Examples include Bianchi (2011), Benigno et al. (2013), Benigno et al. (2016), Jeanne and Korinek (2010) and Bianchi and Mendoza (2010) among many others. Korinek and Mendoza (2013) review this young literature and conclude by stating that an important future step for the new research program is the "development of numerical methods that combine the strengths of global solution methods in describing non-linear dynamics with the power of perturbation methods in dealing with a large number of variables so as to analyze sudden stops in even richer macroeconomic models." Our paper develops such an approach allowing us to empirically evaluate this class of models.

The paper makes four different contributions towards the formulation and estimation of models with occasionally binding constraints. First, we propose a new specification of a typical occasionally binding borrowing constraint (e.g. Kiyotaki and Moore, 1997) in which the transition from the unconstrained to the constrained state of the world is a stochastic function of the endogenous level of leverage in the model. Our model is such that, as leverage rises, the probability of the constraint actually binding increases, but there is no specific leverage level at which the constraint must bind. This specification results in an endogenous regime switching model. Our second contribution is to develop perturbation methods to solve the ensuing endogenous regime switching model rapidly and to higher orders, whereby an approximation at the second order is shown to be necessary to capture precautionary behaviors. Third, using the second-order solution of our model, we propose an algorithm to estimate its parameters with full-information Bayesian methods. Fourth and finally, we apply the framework to study business cycles and financial crises

in Mexico since 1981.

The model economy we use as a laboratory is that of Mendoza (2010). This is a small, open, production economy with an occasionally binding collateral constraint. Differently from Mendoza (2010), however, we consider a comprehensive and encompassing set of shocks, as in Garcia-Cicco et al. (2010). The model is then estimated from 1981 to 2016, using quarterly data for Mexico, showing that it fits the data well, producing second moments in line with the data, estimated crisis regime probabilities consistent with narratives of Mexico's currency crisis and business cycle history. The model, however, cannot generate financial crises followed by protracted economic stagnations, as it does not feature financial intermediation disruption or costly default in equilibrium. The empirical results also show that while interest rate and productivity shocks might have driven the 1995 Tequila crisis, expenditure and preference shocks might have lead the economy into the crisis during the preceding 4-8 quarters.

Our empirical results show that while all shocks play a role in driving the economy, they have varying relative importance for different features of the data. Variance decompositions show that regular fluctuations in real quantities are driven primarily by productivity, expenditure, and terms of trade shocks. Interest rate shocks, on the other hand, drive borrowing, the price of capital, and the current account balance dynamics. Critically, the world interest rate drives what we call the "borrowing cushion," defined as the distance between actual borrowing and the maximum value of debt consistent with the collateral constraint, which is a the state variable and determines the probability of switching from borrowing being unconstrained to constrained.

The core of the framework that we develop to evaluate models with occasionally binding constraints empirically is an endogenous regime switching model. In the endogenous regime switching model, there are two regimes: one in which the collateral constraint binds, and one in which the constraint is slack. The probability of transitioning from the unconstrained to the constrained regime is endogenous and determined by all economic variables in the economy. Likewise, the switch back from to the unconstrained regime depends on economic fundamentals. In our framework, the probability of switching to the crisis regime is a logistic function of the borrowing cushion. The borrowing cushion, in turn, is a function of the endogenous state variables in the model, the exogenous shocks, as well as all control variables. Importantly, agents in the economy know of this probability and how debt, output, and all other variables map into the probability of moving in and out of each regime, consistent with the rational expectations solution of the model. Our solution, therefore, ensures that decisions made when the constraint is slack fully internalize how those decisions affect the probability of moving into the constrained regime, as well how the economy will operate should that event occur.

The approach we develop captures the most salient features of an empirical model of financial crises. The model captures the non-linear nature of a financial crisis, as when the constraint binds the economy experience different dynamics and responses to to shocks. In addition, because we solve the regime switching model using a second-order approximation, we capture precautionary

behavior, which affects the decision rules as risk changes across regimes. As the transition probabilities from one regime to the other are endogenous, in principle, the model can characterize crisis episodes of varying duration. Since our solution method is perturbation-based, we can handle multiple state variables and shocks, and hence we are less constrained than current non-linear methods by the curse of dimensionality, allowing us to consider an encompassing set of crisis triggers. The speed of the perturbation solution method also means that we can use non-linear filters to calculate the likelihood function of the model for a full Bayesian estimation of the relevant parameters and use Bayesian statistics to evaluate the quantitative importance of different shocks and frictions that characterize the data.

In the literature on Markov-switching DSGE models (MSDSGE), this paper is most closely related to Foerster, Rubio-Ramirez, Waggoner, and Zha (2016), who developed perturbation methods for the solution of exogenous regime switching models working directly with the non-linear version of the model. This differs from the Markov-Switching Linear Rational Expectations (MSLRE) literature which starts with a system of linear rational expectations equations and imposes Markov Switching after linearization of the model (e.g. Leeper and Zha, 2003; Davig and Leeper, 2007; Farmer et al., 2011). Since our structural model has regime switching at its core, this is a very natural approach. It has the added benefit that the MSDSGE model can be solved to higher orders, whereas the MSLRE model is restricted to first order. Indeed, we find that a second-order solution is critical for capturing precutionary behavior.

The paper is also related to the literature that focuses on solving endogenous regime switching models. Davig and Leeper (2008), Davig et al. (2010), and Alpanda and Ueberfeldt (2016) all consider endogenous regime switching, but employ computationally costly global solution methods that prevent the possibility of a likelihood-based estimation. Lind (2014) develops a regime-switching perturbation approach for approximating non-linear models, but it requires repeatedly refining the points of approximation and hence it is not suitable for estimation purposes. Most closely related to our approach is the method developed by Barthlemy and Marx (2017), who consider a class of models with regime-dependent steady states, and Maih (2015) develops an approach for endogenous switching based on perturbation techniques that differs where the approximating point is regime dependent. In contrast, our generalization of the Foerster et al. (2016) perturbation approach uses a single point of approximation that is well suited for solving models in which the regime-dependency of the steady state may not be crucial given the relatively slow moving nature of state variables such as capital and debt.

Finally, the paper relates to Guerrieri and Iacoviello (2015) that develops a set of procedures for the solution of models with occasionally binding constraints, called OccBin. OccBin is a certainty equivalent solution method which rules out precautionary effects, which are critical feature of models with occasionally binding collateral constraints. A key feature of our approach is to allow for precautionary behavior effects.

The application of the methodology that we propose relates to the literature on emerging

market business cycles, including particularly, among others, Mendoza (1991), Neumeyer and Perri (2005), Mendoza (2010), and Garcia-Cicco et al. (2010). Encompassing most shocks previously considered in this literature, we include in our analysis productivity, preference, expenditure, interest rate, and terms of trade shocks. Relative to Mendoza (2010), we provide a Bayesian estimation of the model and consider a wider set of shocks, including expenditure and preference shocks, possibly capturing fiscal policy and private saving factors. While the implied model properties are similar, we find that the estimated parameters of the collateral constraint are significantly tighter than previously assumed. Relative to Garcia-Cicco et al. (2010), we evaluate empirically the relative importance of interest shocks in a fully non-linear estimated framework, finding that they are quantitatively important for certain features of the data, but not others. In particular, interest rate shocks matter most for financial variables over the business cycle, and during the 1995 Tequila crisis. Expenditure and preference shocks, however, seem to more important than other shocks in leading the economy into the crisis. Relative to Neumeyer and Perri (2005), we set up a framework that fits the data well without assuming any correlation between the productivity and the interest rate process. Nonetheless, we find that, while we can fit ergodic second moments of the data well with uncorrelated shocks, high and averse correlation configurations are associated with the simulated crisis dynamics in the model.

As Binning and Maih (2017) discuss, there are many possible applications of our approach to the specification of models with occasionally binding collateral constraints. For example, Bocola (2015) builds and estimates a model of sovereign debt default. The estimation procedure used is to first estimate the model outside of the crisis period, using a solution technique that assumes a crisis will not occur. Conditional on those parameter estimates, a crisis probability that is endogenously appended to the model. Our approach allows one to estimate model parameters fully incorporating the possibility of a crisis outside of the crisis period, and hence allowing for that crisis to be a function of the economic decisions. The methods here also apply to the literature on the zero-lower bound on interest rates such as for instance Aruoba, Cuba-Borda, and Schorfheide (2018), and Atkinson, Richter, and Throckmorton (2019).

The rest of the paper is organized as follows. Section 2 describes the model and discusses our formulation of the collateral constraint. Section 3 develops the perturbation solution methodology for endogenous regime switching models and describes our procedure for estimation using full information Bayesian Methods. Section 4 reports the empirical results. Section 5 concludes. Technical details are in the Appendix.

2 The Model

The model we set up and estimate is a small, open, production economy with an occasionally binding collateral constraint that is subject to a relatively large set of structural shocks. Encompassing all shocks considered in the extant literature except permanent productivity ones, the

model features temporary productivity, intertemporal preference, domestic expenditure, interest rate, and terms of trade shocks.[1] The restriction on access to international capital markets that we specify depends on the endogenous variables of the model, including borrowing, capital and its relative price, and leverage. Capital and debt choices respond to exogenous shocks, affecting leverage. Leverage, in turn, affects the probability of a binding collateral constraint. Because of the occasionally binding nature of the constraint, this framework can potentially account not only for normal business cycles, but also key aspects of financial crises in emerging market economies as in (Mendoza, 2010). In the rest of this section, we discuss the representative household-firm and the borrowing constraint, focusing on key model features. The formal definition of the equilibrium and a list with the full set of equilibrium conditions is reported in Appendix A.

2.1 Representative Household-Firm

There is a representative household-firm that maximizes the utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}, \tag{1}$$

with C_t denoting consumption, H_t the supply of labor, and d_t an exogenous and stochastic preference shock specified below, as in Garcia-Cicco et al. (2010). Households choose consumption, labor, capital (K_t), imported intermediate inputs (V_t) given their relative price P_t, and holdings of real one-period international bonds B_t. Negative values of B_t indicate that the household is borrowing from abroad. Thus, households face the following budget constraint:

$$C_t + I_t + E_t = Y_t - \phi r_t \left(W_t H_t + P_t V_t \right) - \frac{1}{(1+r_t)} B_t + B_{t-1}. \tag{2}$$

Here, Y_t denotes gross domestic product and is given by

$$Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t, \tag{3}$$

with A_t denoting an exogenous and stochastic temporary productivity shock, as in (Mendoza, 2010) and Garcia-Cicco et al. (2010). E_t is an exogenous and stochastic domestic expenditure process possibly interpreted as a fiscal shock as in Garcia-Cicco et al. (2010). The term $\phi r_t \left(W_t H_t + P_t V_t \right)$ describes a working capital constraint, stating that a fraction of the wage and intermediate good bill must be paid in advance of production with borrowed funds. The relative price of labor and capital are given by w_t and q_t, respectively, both of which are endogenous variables, but taken as given by the household-firm. Gross investment, I_t, is subject to adjustment

[1] We omit permanent technological shocks of the type analyzed by Aguiar and Gopinath (2007) both for consistency with Mendoza (2010) and because of the evidence reported by Garcia-Cicco et al. (2010), which suggests that these shocks are not quantitatively important in a framework with an financial frictions like ours.

costs as a function of net investment:

$$I_t = \delta K_{t-1} + (K_t - K_{t-1}) \left(1 + \frac{\iota}{2} \left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)\right). \tag{4}$$

Borrowing takes place in terms of one-period international bonds that pays the net interest rate r_t. The interest rate faced by borrowers is comprised of two exogenous components–one persistent and one transitory–plus an endogenous debt-elastic country-specific component. Thus, we specify the following interest rate process:

$$r_t = r_t^* + \psi \left(e^{\bar{B} - B_t} - 1\right) + \sigma_r \varepsilon_{r,t}, \tag{5}$$

where the persistent component, r_t^*, is given by

$$r_t^* = (1 - \rho_{r^*})\bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*} \varepsilon_{r^*,t}, \tag{6}$$

while $\varepsilon_{r^*,t}$ and $\varepsilon_{r,t}$ are i.i.d. $N(0,1)$ with σ_{r^*} and σ_r denoting parameters that control the variances of the two interest rate shocks.

A few remarks are in order here to highlight similarities and differences relative to other specifications of the interest rate process proposed in the extant literature. First, unlike previous studies, we do not assume any correlation between the innovations to the interest rate process and the productivity process spelled out below. Second, the endogenous interest rate component in equation (5) above serves the sole purpose of inducing independence of the model steady state from initial conditions, as in Schmitt-Grohe and Uribe (2003). In fact, in our model, as in as in Mendoza (2010), the endogenous component of the country spread is driven by the collateral constraint multiplier when the latter is binding. Because of this feature, the parameter ψ will be calibrated to a very small value and will not be estimated. Third and finally, note that the set of observable variables, unlike Garcia-Cicco et al. (2010), will include also the country interest rate constructed following Uribe and Yue (2006) and a measure of the Mexico's terms of trade. While $\varepsilon_{r^*,t}$ and $\varepsilon_{r,t}$ are not identified separately in equations (5) and (6), $\varepsilon_{r,t}$ will be identified in the data because assumed to have different persistence, and will turn out to be important to fit Mexico's current account balance and external debt dynamics.

The remaining processes, the preference shock d_t, the temporary productivity shock A_t, the shock to the relative price of intermediate goods P_t, and the domestic expenditure shock E_t, are governed by the following autoregressive processes:

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}, \tag{7}$$

$$\log A_t = (1 - \rho_A) A^* + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}, \tag{8}$$

$$\log P_t = (1-\rho_P)P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t}, \tag{9}$$

and

$$\log E_t = (1-\rho_E)E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t}, \tag{10}$$

where $\varepsilon_{.,t}$ are i.i.d. $N(0,1)$ innovations, and the $\sigma_{.,t}$ parameters control the size of their variances.

2.2 Borrowing Constraint

The collateral constraint limits total debt to a fraction of the market value of capital, thus limiting leverage in the economy. As in Mendoza (2010), Kiyotaki and Moore (1997), Aiyagari and Gertler (1999), and Kocherlakota (2000), among others, the collateral constraint does not derive from an optimal credit contract, but is imposed directly on the economy. Here, we follow Mendoza (2010) and include also working capital in the borrowing limit to pin down a well-behaved supply response of the economy during financial crises.

While our constraint is the same as in the quantitative financial friction literature above, we propose a new specification of its occasionally binding nature that is more tractable for estimation purposes and has appealing empirical properties. The main difference relative to the typical formulation in the literature is that we transform the deterministic relationship between leverage and the binding state of the borrowing constraint into a stochastic one. In the typical specification used in the literature, there is one specific leverage ratio associated with a binding constraint. In contrast, with our specification, increased leverage raises the *probability* of a binding constraint, but does not necessarily force the constraint to bind, consistent with how borrowing limits affect actual firm and household borrowing decisions.

The probabilities of switching from one state to the other depend on critical endogenous variables of the model. The probability of switching from the non-binding state to the binding state is a logistic function of the distance between actual borrowing and the borrowing limit equal to a fraction of the value of collateral. Therefore it is ultimately affected by all endogenous variables in the model. The probability of switching from the binding state back to the unconstrained one, instead, is a logistic function of the collateral constraint multiplier.

This formulation captures key findings of the empirical literature on financial crises, which document that the likelihood of a financial crisis increases with leverage, but high leverage does not necessarily lead to a financial crisis. Not having a unique level of leverage at which the collateral constraint binds, but rather a range of leverage levels that affect the likelihood of the constraint binding in a smooth manner adds an element of realism to the model. Empirical investigations into the anatomy of borrowing constraints find that when borrowers hit leverage limits, they adjust expenditures gradually by tapping into other sources of financing such as cash, precautionary credit lines, and asset sales. Thus, collateral constraints, in practice, do not bind at any particular leverage ratio in the real world, and representing them as stochastic functions

of leverage is a reasonable approximation for positive purposes.[2]

Most importantly, in our set up, agents know that higher leverage and borrowing levels increase the probability of switching to a constrained state, and vice-versa. This preserves the interaction in agents' behavior between the two states that gives rise to precautionary behavior, distinguishing this class of models from those in which financial frictions are always binding or are approximated with solution methods that eliminate these interactions across regimes.

Specifically, we model the occasionally binding nature of the borrowing constraint as an *endogenous regime switching* process. Thus, there is one regime in which the constraint binds and one in which it does not, where the regimes are denoted by $s_t \in \{0, 1\}$. When $s_t = 0$, the constraint does not bind and lenders finance all desired borrowing for debt and working capital loans:

$$\frac{1}{(1+r_t)} B_t - \phi \left(1 + r_t\right) \left(W_t H_t + P_t V_t\right). \tag{11}$$

In this regime, desired borrowing is pinned down by the presence of the debt elastic component of the interest rate in steady state, and the knowledge that higher debt will lead to a higher probability of crisis regime in the future outside the steady state defined below.

When $s_t = 1$, the constraint binds strictly, and total borrowing is equal to a fraction of the value of collateral, with

$$\frac{1}{(1+r_t)} B_t - \phi \left(1 + r_t\right) \left(W_t H_t + P_t V_t\right) = -\kappa q_t K_t, \tag{12}$$

where on the left hand side of this equation we have total debt and working capital loans, while on the right hand side there is the value of capital $q_t K_t$ times the leverage ratio κ. Thus, the constraint limits total borrowing, and hence consumption smoothing as well as the purchase of intermediate imported inputs for production purposes. The latter restricts output supply, causing the constraint to bind even tighter.[3] In the rest of this section, we discuss our proposed formulation of the slackness condition typically associated with an occasionally binding borrowing constraint and how we cast the model above in the form of an endogenous regime switching framework.

2.2.1 The Regime-Switching Slackness Condition

Denote the Lagrange multiplier associated with equation (11) as λ_t and define the "borrowing cushion" as the distance of the actual borrowing level from the debt limit:

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi \left(1 + r_t\right) \left(W_t H_t + P_t V_t\right) + \kappa_t q_t K_t. \tag{13}$$

[2] See, for instance, Campello et al. (2010) for survey information on the behavior of financially constrained firms and Ivashina and Scharfstein (2010) for loan level data showing that credit origination dropped during the crisis because firms drew down from pre-existing credit lines in order to satisfy their liquidity needs. Bank lending standards fluctuating over the cycle could also be consistent with a stochastic specification of the collateral constraint.

[3] Note here that, when the constraint binds, as the quantity and value of capital fluctuates, the amount of borrowing will also fluctuate.

Thus, when the borrowing cushion is small, total borrowing is high relative to the value of collateral, meaning the leverage ratio is high. The critical step for mapping the model above into an endogenous regime-switching framework is to modify the typical slackness condition from models with strict inequality constraints ($B_t^* \lambda_t = 0$), so that the two variables, B_t^* and $\lambda_t = 0$, are zero only in the relevant regime. Thus, we must have that in the unconstrained regime ($s_t = 0$) the constraint does not bind so $\lambda_t = 0$, and in the constrained regime ($s_t = 1$) so that the borrowing constraint binds, and hence the borrowing cushion $B_t^* = 0$.

To accomplish this mapping, and to be consistent with the literature on regime-switching DSGE models in which *parameters* are the model objects that switch across regimes, define two auxiliary regime-dependent parameters, $\varphi(s_t)$ and $\nu(s_t)$, such that $\varphi(0) = \nu(0) = 0$, and $\varphi(1) = \nu(1) = 1$. [4] Next, we introduce the *regime-switching slackness condition*, written as

$$\varphi(s_t) B_{ss}^* + \nu(s_t)(B_t^* - B_{ss}^*) = (1 - \varphi(s_t))\lambda_{ss} + (1 - \nu(s_t))(\lambda_t - \lambda_{ss}), \tag{14}$$

where B_{ss}^* and λ_{ss} are the steady state borrowing cushion and collateral constraint multiplier, respectively, defined more precisely in Section 3 below. It is now easy to see that equation (14) implies that, when $s_t = 0$, $\lambda_t = 0$, while when $s_t = 1$ $B_t^* = 0$. Yet, equation (14) remains continuously differentiable for any value of B_t^*, as no inequality constraint is imposed on it.

Technically, this formulation of the slackness condition "preserves" information in our perturbation approximation, since, at first order, both variables are constant in the respective regimes. The use of the regime-dependent auxiliary switching parameters $\varphi(s_t)$ and $\nu(s_t)$ follows from the Partition Principle of Foerster et al. (2016), which separates parameters based upon whether they affect the steady state or not. Intuitively, $\varphi(s_t)$ captures the fact that the level of the economy changes across regimes, while $\nu(s_t)$ captures the fact that the dynamic responses differ across regimes.

2.2.2 Modelling Endogenous Regime Switching

To model the transition from one regime to the other, we rely on logistic functions, which are tractable and parsimoniously parameterized.[5] Specifically, assume that the transition between regimes is a logistic function of endogenous variables determined in equilibrium. In the non-binding regime, the transition into a binding regime is assumed to depend on the borrowing cushion, B_t^*:

$$\Pr(s_{t+1} = 1 | s_t = 0, B_t^*) = \frac{\exp(-\gamma_0 B_t^*)}{1 + \exp(-\gamma_0 B_t^*)}. \tag{15}$$

[4] While in our model the auxiliary parameters coincide with the regime-switching indicator variable s_t, in more general settings they may not. For this reason, in the following section, we provide a general formulation of the endogenous regime switching that is applicable to setups possibly different than the one associated with our specific application. See, for example, the different applications discussed by Binning and Maih (2017).

[5] Davig et al. (2010) and Bi and Traum (2014) use it to study fiscal policy, while Kumhof et al. (2015) use a logistic fuction to model theoretically the transition to a default regime. In Appendix B, we provide a possible structural interpretation of our formulation in terms of a stochastic monitoring technology shock.

Thus, the likelihood that the constraint binds in the following period depends on the size of the borrowing cushion in the current period. The parameter γ_0 controls the steepness of the logistic function, determining the sensitivity of the probability of switching regime to the size of the borrowing cushion. For small values of γ_0, the cushion has a small impact on the probability of a switch to the binding regime. For larger values of this parameter, the probability of a switch to the crisis regimes increases more rapidly toward 1, as B_t^* crosses the borrowing threshold defined by the collateral constraint. Note here that, for certain draws from the logistic function, the borrowing cushion could become negative and the economy could temporarily remain in the non-binding regime, but long spells with a negative borrowing cushion are unlikely.

Similarly, when the constraint is binding, the transition probability is a logistic function of the Lagrange multiplier, λ_t, according to

$$\Pr\left(s_{t+1}=0|s_t=1,\lambda_t\right)=\frac{\exp\left(-\gamma_1\lambda_t\right)}{1+\exp\left(-\gamma_1\lambda_t\right)}. \tag{16}$$

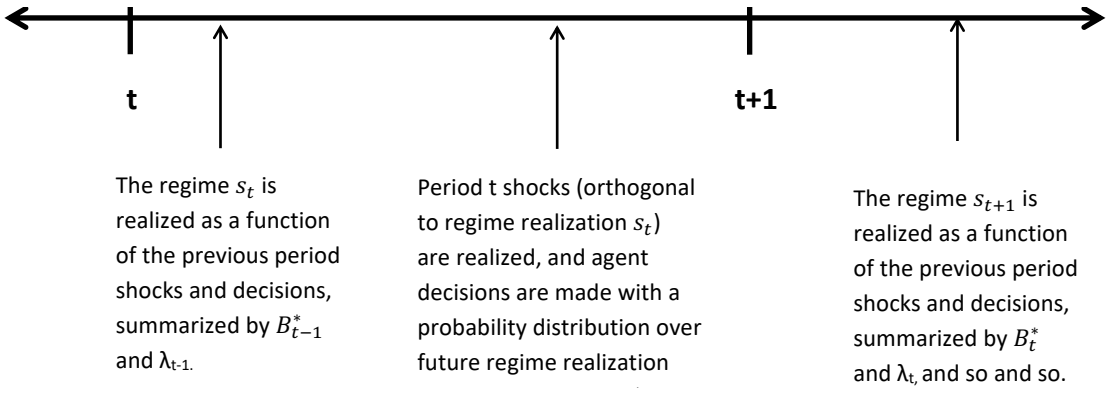
The probability of switching back from a constrained to an unconstrained regime, therefore, depends on the shadow value of the economy's desired borrowing relative to the limit set by the collateral constraint. A large positive multiplier implies that, when the constraint binds tightly, the probability of staying in the binding regime is higher. As in the case of a switch from the constrained to constrained regime, the parameter γ_1 affects the sensitivity of this probability to the value of the multiplier. In particular, in the binding regime, it is possible that the desired level of borrowing is less than the level forced upon it, which would manifest itself with a negative collateral constraint multiplier.

Putting equations (15) and (16) together, the regime-switching model has an endogenous transition matrix, denoted $\mathbb{P}_t$, with elements given by $p_{s_t,s_{t+1},t}$, so

$$\mathbb{P}_t=\begin{bmatrix} p_{00,t} & p_{01,t} \\ p_{10,t} & p_{11,t} \end{bmatrix}=\begin{bmatrix} 1-\frac{\exp(-\gamma_0 B_t^*)}{1+\exp(-\gamma_0 B_t^*)} & \frac{\exp(-\gamma_0 B_t^*)}{1+\exp(-\gamma_0 B_t^*)} \\ \frac{\exp(-\gamma_1\lambda_t)}{1+\exp(-\gamma_1\lambda_t)} & 1-\frac{\exp(-\gamma_1\lambda_t)}{1+\exp(-\gamma_1\lambda_t)} \end{bmatrix}. \tag{17}$$

A few other remarks are useful to illustrate how our stochastic formulation of the borrowing constraint works and differs relative to a typical formulation of an occasionally binding collateral constraint such as Kiyotaki and Moore (1997) or Mendoza (2010). First, whether or not the constraint binds in a given period is determined *before* exogenous shocks are realized and economic decisions are made during that period. Figure 1 provides a graphical representation of the model timing and illustrates that, at the start of a given period t, the regime outcome s_t is drawn from the logistic distributions as a function of previous period borrowing cushion or value of the collateral constraint multiplier, B_{t-1}^* and λ_{t-1}. Next, exogenous shocks, which are orthogonal to the realization of the regime, are realized and agents take decisions during period t based on their knowledge of the regime outcome, s_t, as well as a probability distribution over the next period regime realization, s_{t+1}, as in equation (15) or (16). Finally, the regime realization for period $t+1$

Figure 1: Model Timing



is drawn based on exogenous shocks and agents' decisions that pin down B^*_t and λ_t, and so on.

Second, as already noted, an implication of our setup is that entry and exit of the economy from the binding regime might be temporarily associated with a counter intuitive value of the multiplier and the borrowing cushion. For instance, in the non-binding regime, the logistic function potentially allows the borrowing constraint to bind in the following period, even if the borrowing cushion in the current period is still positive. Conversely, negative values of the borrowing cushion in the non-binding regime are possible if the probability of a binding regime is elevated but such outcome is not realized. How likely these outcomes are depend on the parameter of the relevant logistic function, γ_0. The same logic applies to a probabilistic exit from the binding regime that depends on the multiplier λ_t and the relevant parameter of the logistic function γ_1. Despite positive values of the multiplier, the economy may end up coming out of the binding regime early. Conversely, the economy might be stuck in the constrained regime past the time when the collateral constrained multiplier turned negative. In fact, in this case, the economy may be "forced" to borrow the amount set by the constraint, which might be more than desired, until a non-binding realization of the regime is drawn. In our set up, therefore, the collateral constraint multiplier λ_t can take on negative values when the economy would like to borrow less than the value imposed by the collateral constraint, but it has not switched to the not constrained regime.

The third implication of our setup is that by making the probability of transitions from one regime to the other dependent upon endogenous variables, the model can generate transition probabilities that vary over time. In contrast, the exogenous Markov-switching setup used by Davig and Leeper (2007), Farmer et al. (2011), and Foerster et al. (2016), among others, has a constant probability of transitioning between regimes, which implies a flat hazard function that is independent of the structural shock realizations and the agent decisions. Thus, our endogenous-switching framework can in principle generate long- or short-lived binding regime episodes with non-constant hazard functions depending on the realization of shocks and agents' decisions: large or repeated contractionary shocks will tend to produce longer binding constraint periods than

smaller, infrequent shocks.

3 Solving and Estimating the Endogenous Switching Model

Having cast our model it in terms of an endogenous regime switching framework, this Section describes our solution method and our estimation procedure.

3.1 Model Solution

The model developed in the previous section can in principle be solved using global solution methods, as for example in Davig et al. (2010). However, such an approach would be time-consuming and would preclude model estimation. Instead, we solve the model using a perturbation approach, which allows for an accurate approximation that is fast enough to permit estimation. Perturbation generates Taylor-series expansions of the functions characterizing the solution of the set of equilibrium conditions. The competitive equilibrium of the endogenous regime switching version of the model is defined formally in Appendix A. We next discuss two important features of these approximations. First, how to define a steady state in this set up, which is the point around which the expansions are generated. Second, the importance of approximating to a second-order in our framework.

3.1.1 Defining the Steady State

Given the regime-switching slackness condition (14), defining a non-stochastic steady state of an endogenous regime-switching framework is challenging. A steady state in our framework can be defined as a state in which all shocks have ceased and the regime switching variables that affect the steady state take their *ergodic mean* values across regimes associated with the steady state transition matrix:

$$\mathbb{P}_{ss} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} & \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}. \tag{18}$$

Note here that, since this matrix also depends on the steady state level of the borrowing cushion and the multiplier, which in turn depend upon the ergodic means of the regime-switching variables, such steady state is the solution of a fixed point problem that describe in more detail in Appendix C.

More specifically, consider the model regime-specific parameters defined above and distinguish between $\varphi(s_t)$, which can affect the level behavior of the economy, and $\nu(s_t)$, which can affect only its dynamics with no effects on the steady state. Then denote with $\xi = [\xi_0, \xi_1]$ the ergodic vector of $\mathbb{P}_{ss}$. Next, apply the Partition Principle of Foerster et al. (2016) and write the ergodic

mean of the parameter that affects the steady state as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1). \tag{19}$$

Defining the steady state as the state in which the auxiliary parameter $\varphi(s_t)$ is at its ergodic mean value implies that the approximation point constructed lies between the steady state that would result in a model in which only the non-binding regime occurs, and one in which only the binding regime realizes. How close our approximation point is to each of these two other steady state concepts, therefore, will depend on the frequency of being in each of the two regimes. We note here that, since in our application we are modelling binding episodes with limited duration, the ergodic mean is a natural candidate as perturbation point. Given the nature of our application with slow-moving capital and debt state variables, the perturbation point will be in the area of the state space in which the economy operates most frequently. In fact, since the binding regime tends to be self-limiting–that is, being in the binding regime causes the economy to reduce leverage and hence switch back to the non-binding regime–the economy will rarely reach the area around the steady state of the "binding regime only" version of the model. [6]

3.1.2 On the Importance of Approximating at least to the Second-Order

Equipped with the steady state of the endogenous regime-switching economy, we then construct a second-order approximations to the decision rules by taking derivatives of the equilibrium conditions. We relegate details of these derivations to Appendix C, but here we provide a summary.

For each regime s_t, The solutions to our model take the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \qquad \mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \tag{20}$$

where $\mathbf{x}_t$ denotes predetermined variables, $\mathbf{y}_t$, non-predetermined variables, ε the set of shocks, and χ a perturbation parameter such that when $\chi = 1$ the fully stochastic model results and when $\chi = 0$ the model collapses to the non-stochastic steady state defined above. Using these functional forms, we can express the equilibrium conditions conditional on regime s_t as

$$\mathbb{F}_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = 0.$$

We then stack the regime-dependent conditions for $s_t = 0$ and $s_t = 1$, denoting the resulting system of equations with $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$, and successively differentiate with respect to $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$,

[6] Alternative methods for finding solutions to endogenous regime-switching models, such as Maih (2015) and Barthlemy and Marx (2017), advocate using the regime-dependent steady states as multiple approximation points. Such a strategy would also be ill-suited for our purposes because the binding regime steady state would also be a poor point of approximation given that are infrequent and usually of shorter duration than normal cycles of expansions and contractions.

evaluating at steady state. The systems

$$\mathbb{F}_{\mathbf{x}}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0, \qquad \mathbb{F}_{\varepsilon}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0, \qquad \mathbb{F}_{\chi}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0 \tag{21}$$

can then be solved for the unknown coefficients of the first-order Taylor expansion of the solutions (20). A second-order approximation can be found by taking the second derivatives of $\mathbb{F}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)$. In the end, we have matrices $H_{s_t}^{(1)}$ and $G_{s_t}^{(1)}$ characterizing the first-order coefficients, and $H_{s_t}^{(2)}$ and $G_{s_t}^{(2)}$ characterizing the second-order coefficients. Therefore, the approximate solutions are

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} \left(S_t \otimes S_t\right) \tag{22}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} \left(S_t \otimes S_t\right) \tag{23}$$

where $S_t = \begin{bmatrix} \left(\mathbf{x}_{t-1} - \mathbf{x}_{ss}\right)' & \varepsilon_t' & 1 \end{bmatrix}'$.

Our endogenous regime switching framework must be solved at least to the second order. If we were to use only a first-order approximation, our estimation would not capture precautionary behavior associated with rational expectations about the dependency of the probability of a regime change on the borrowing cushion and the multiplier. The following Proposition states this result formally.

Proposition 1 (**Irrelevance of First-Order**). *The first-order solution to the endogenous regime-switching model is identical to the first-order solution to an exogenous regime-switching model where the transition probabilities are given by the steady-state transition matrix* $\mathbb{P}_{ss}$.
Proof. *See Appendix C.*

This result illustrates that using a second-order approximation to the solution is necessary to characterize the model properties associated with the endogenous nature of the regime switching, including particularly precautionary behavior. This result is similar to the one stating that, in models with only one regime, first-order solutions are invariant to the size of shocks, second-order solutions captures precautionary behavior, and third-order solutions are needed to capture the effects of stochastic volatility (Fernandez-Villaverde et al., 2015). Unfortunately, however, the need to use a second-order approximation creates additional challenges for estimation purposes. We now turn to our proposed strategy to address them.

3.2 Estimating the Endogenous Switching Model

We estimate the model with a full information Bayesian procedure. The posterior distribution has no analytical solution so we use Markov-Chain Monte Carlo methods to sample from the posterior. A key obstacle in using this approach to sample from the posterior is the evaluation of the likelihood function. We face two difficulties here relative to linear DSGE models. The first

Table 1: Calibrated Parameters

Parameter	Value
Discount Factor	$\beta = 0.9798$
Risk Aversion	$\rho = 2$
Labor Supply	$\omega = 1.846$
Capital Share	$\eta = 0.3053$
Labor Share	$\alpha = 0.5927$
Depreciation Rate	$\delta = 0.02277$
Import Price Mean	$P^* = 1.028$
Expenditure Mean	$E^* = 0.2002$
Interest Rate Debt Elasticity	$\psi_r = 0.001$
Neutral Debt Level	$\bar{B} = -6.117$

is the nonlinearity due to the regime-switching model; the second is the need to approximate to the second-order the solution that governs the decision rules in each regime. Following Binning and Maih (2015), we use the Unscented Kalman Filter (UKF) to compute approximations to the evaluation of the likelihood function using Sigma Points.[7] Since the Metroplis-Hastings algorithm we use for sampling is a standard tool used in the literature, we omit a discussion of this step in our procedure. The details of the construction of the state space representation and the filtering steps for the evaluation of the likelihood are reported in the Technical Appendix.

Our objective is to estimate critical parameters governing the model's dynamics in both the binding and non-binding regime, as well as the parameters that govern the transitions between regimes on which we do not have strong a priori information. In order to make inference about these parameters using non-informative priors, we calibrate the subset of parameters on which we have good prior information from the extant literature; these parameters are listed in Table 1. We set these parameters largely following Mendoza (2010), based upon stylized facts from Mexico's National Accounts. The parameters that do not comes from Mendoza (2010) are: β, which we set to match the capital-to-output ratio; ψ_r, which we set to a very small value to eliminate the dependency of the steady state from initial conditions, while avoiding to have an impact on the the model dynamics (see Schmitt-Grohe and Uribe, 2003); and $\bar{B}$, which targets the debt-to-output ratio.

We then give uniform priors on the parameters to be estimated. The uniform distribution ranges are shown in Table 2. They impose sign restrictions and rule out values that generate implausible moments in model simulations. While possible in principle, we do not allow for regime switching in the shocks processes, because we want the collateral constraint to drive regime switching, rather than changes in the stochastic processes.[8]

[7] Binning and Maih (2015) provide a discussion of why the Sigma Point filters perform better than the Particle Filter in a regime switching context.

[8] Allowing for regime change in the shock processes may improve overall fit, but makes it harder to assess the model's ability to characterize financial crises as episodes in which the collateral constraint is binding.

Table 2: Estimated Parameters

Parameter	Prior	Posterior Mode
ι	Uniform(0,50)	12.5557
ϕ	Uniform(0,0.5)	0.4502
r^*	Uniform(0,0.03)	0.02
κ	Uniform(0,1)	0.1002
ρ_a	Uniform(0,1)	0.981
ρ_e	Uniform(0,1)	0.8881
ρ_p	Uniform(0,1)	0.9696
ρ_d	Uniform(0,1)	0.5567
ρ_w	Uniform(0,1)	0.8126
σ_a	Uniform(0,0.1)	0.0079
σ_e	Uniform(0,0.5)	0.0939
σ_p	Uniform(0,0.1)	0.0463
σ_d	Uniform(0,0.5)	0.0431
σ_r	Uniform(0,0.1)	0.0024
σ_w	Uniform(0,0.1)	0.0053
γ_0	Uniform(0,150)	135.194
γ_1	Uniform(0,150)	44.9981

The model is estimated with quarterly data for in gross domestic product (gross output less intermediate input payments), consumption growth, investment growth, and import price growth, as well as the current account-to-output ratio, and the real interest rate.[9] Following Uribe and Yue (2006), the real interest rate measure that we use is the nominal rate less inflation expectations, with expectations computed as forecasts from an autoregressive process. The fact that we have six data series and six structural shocks means that we do not need measurement errors in our estimation. However, measurement errors in the observation equation improves performance of the non-linear filtering algorithm and accounts for any actual measurement error in our data. To limit their impact on the inference we draw from the data, we limit their variance to 5% of the variance of the observable variables.

4 Empirical Results

Our empirical findings comprise three sets of results. The first includes the estimated parameters and various measures of model fit that are informative on the quality of the economic inference that we draw from the estimation exercise. The second set includes model moments that inform us about the ergodic properties of the model economy and the relative importance of different shocks for regular business cycles. This set of results will permit us to compare results and model properties with the extant literature on modeling business cycles in emerging markets. The third set of result pertain the model's ability to describe and interpret financial crises. In particular, we

[9] See Appendix D for details on variable definitions and data sources.

will look at how financial crisis episodes can be viewed through the lens of the model and which shocks are important to characterize specific historical episodes. We look at both a 'typical' crisis, as well as the anatomy of the 1995 "Tequila" crisis.

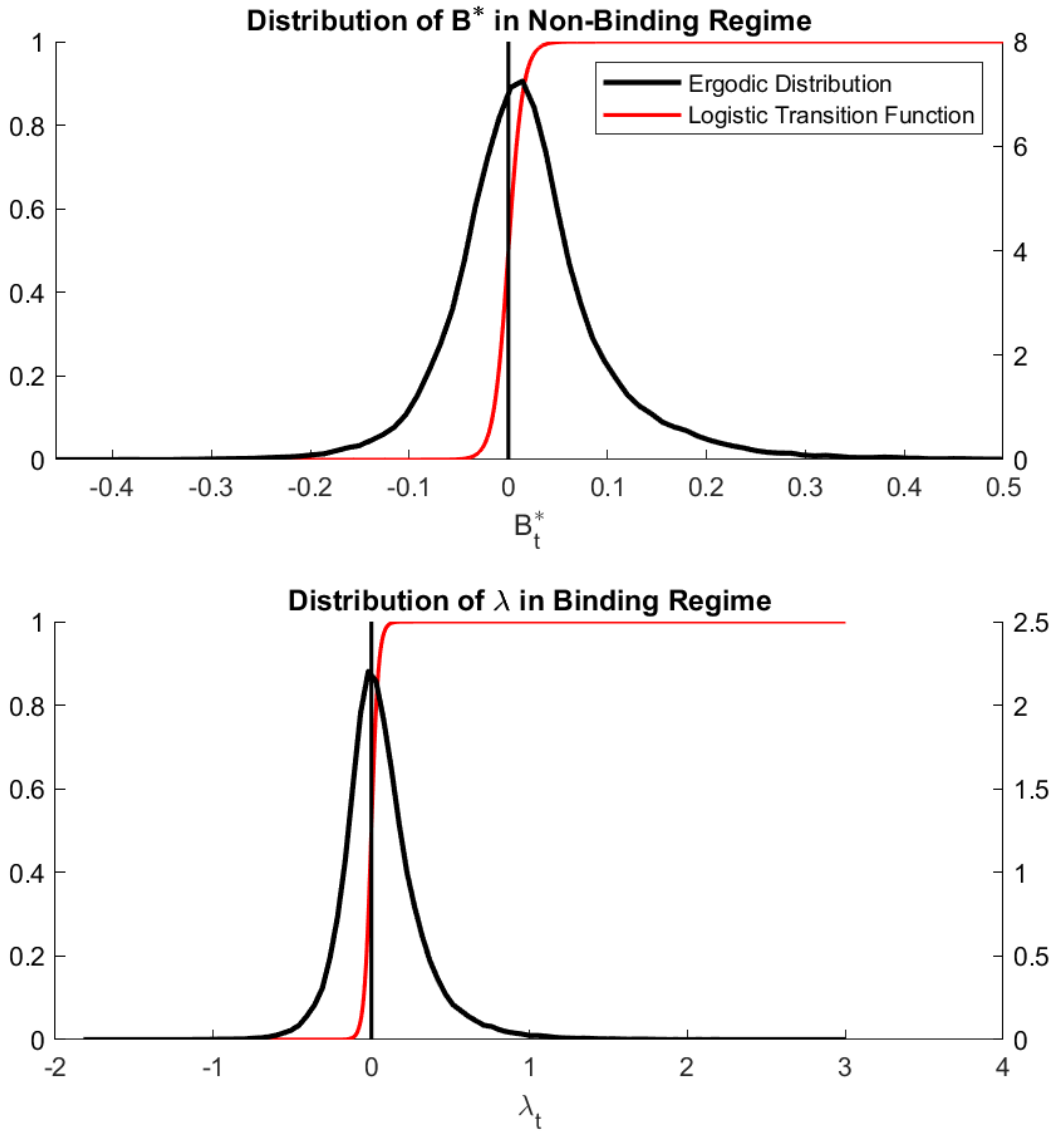
4.1 Estimated Parameters and Model Fit

Table 2 reports the mode of the posterior distribution of the estimated parameters. The parameters of the exogenous processes indicate that all shocks have a high degree of persistence, as it is typical in DSGE models, except the preference shock, d. The estimated mean interest rate, at 2% per quarter, is close to the value estimated by Mendoza (2010) and reflects that fact that our sample starts while US nominal interest rates were very high by historical standard.

The estimate of the investment adjustment cost parameter, ι, which does not affect the steady state and cannot be directly parameterized using National Accounts data and typically controls investment volatility, is 12.5. To provide context, this parameter was calibrated to the value of 2.75 by Mendoza (2010), although that calibration is at an annual frequency. The estimate for the working capital constraint parameter, at 44%, is an intermediate value between the very high 100% assumed by Neumeyer and Perri (2005) and the relatively low value of 25.79% set by Mendoza (2010). The estimated value of the haircut parameter in the borrowing constraint (κ), another parameter that cannot be pinned down from the data directly, is 0.099. This is significantly tighter than the benchmark value of 0.20 chosen by Mendoza (2010), who experimented with alternative values, ranging from 0.15 to 0.30. These are important differences since these parameters affect the ergodic moments of the economy and control amplification when the constraint binds, but cannot be calibrated from the data directly.

The estimated posterior modes of the logistic parameters in equation (15) and (16) are 135 and 45, respectively. These estimates have three implications. First, as they are significantly different from zero, thus suggesting that the data reject a model specification in which the transition probabilities are *exogenous*, which is in principle allowed for under the prior distribution. Second, they also suggests that the the probability of switching from the non-binding to the binding regime is less steep than the probability of exiting the binding regime, consistent with the notion that regular business cycles typically have longer duration than financial crises associated with sudden stops in capital flows. To illustrate this point, Figure 2 plots the complement of the probabilities in equation (15) and (16)—i.e., the probability of staying in the non-binding regime and the probability of staying in the binding regime given that the economy is in these regimes—evaluated at the posterior mode of γ_0 and γ_1, together with the estimated ergodic distributions of their arguments, the borrowing cushion, B^* and the the constraint multiplier λ. The figure shows that the ergodic distribution of the borrowing cushion is centered on a positive value, as the economy spends most of its time above the borrowing limit. As the borrowing cushion falls, the probability of staying in the non-binding regimes falls, and drops to zero for small negative values, with very little probability mass on large negative realizations. On the other hand, once

Figure 2: Logistic Functions and Ergodic Distributions of Their Arguments

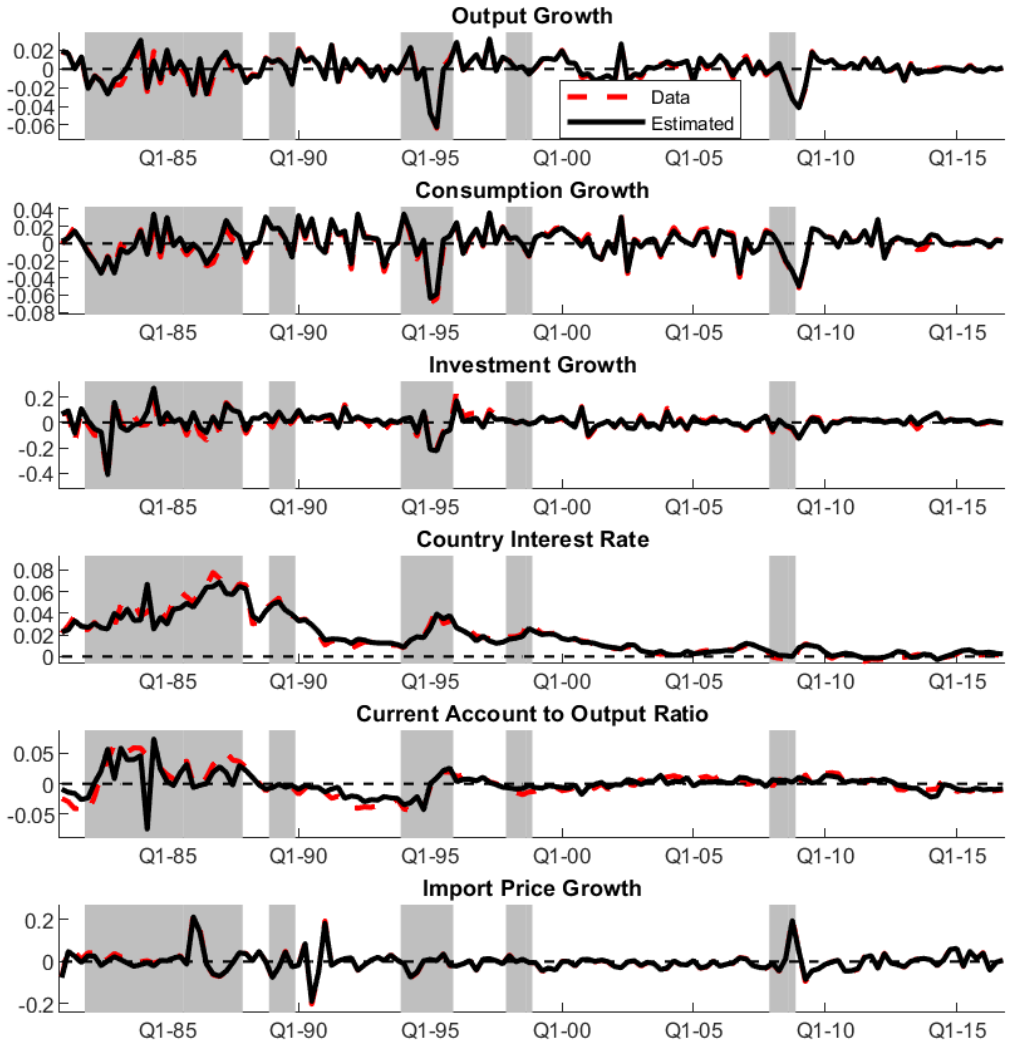


Note: The Figure plots the probability of staying in a regime given that the economy is in that regime together with the ergodic distribution of its argument.

the economy is in the binding regime, the ergodic distribution of the multiplier $\lambda = 0$ is centered on zero, with more probability mass on the right tail than the left tail. As λ approaches 0, the probability of staying in the binding regime falls much more sharply than in the case of the probability of staying in the non-binding regime because of the much smaller estimated value of γ_1.

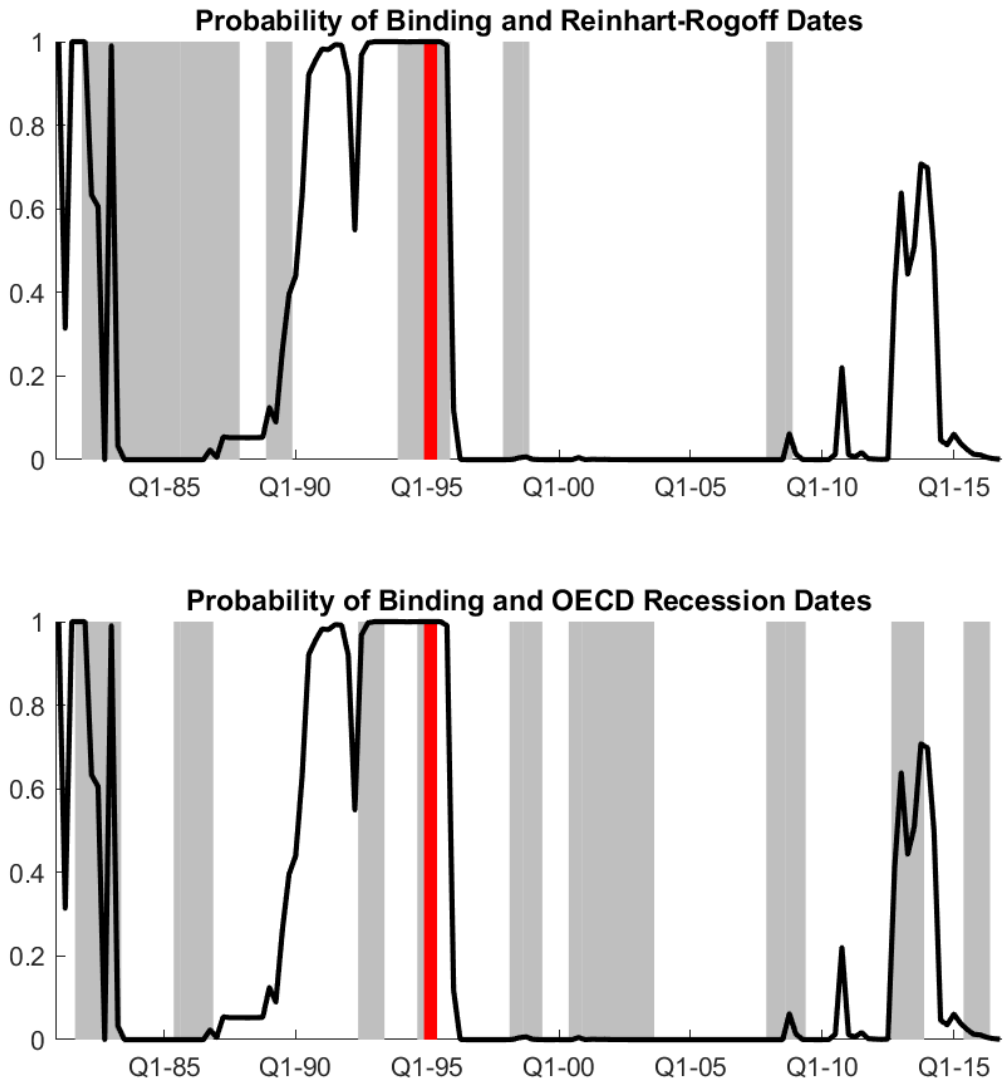
Model fit is summarized by Figure 3, which plots observable variables used in the estimation

Figure 3: Data and Model Estimates



together with the fitted values. The estimated processes largely follow the data, in part due to the relatively small measurement error used in the estimation procedure. Importantly, the model estimates track the data consistently throughout the sample, without major fit losses during times of currency crisis in Mexico according to the Reinhart and Rogoff (2009) classification. For example, during the "Tequila Crisis" of 1994-1995, the data showed large drops and rebounds in output, consumption, and investment growth, and a sharp increase in the current account to output ratio. The model tracks these movements as well as it does track regular fluctuations throughout the rest of the sample. If, by contrast, one were to observe a fit loss during these

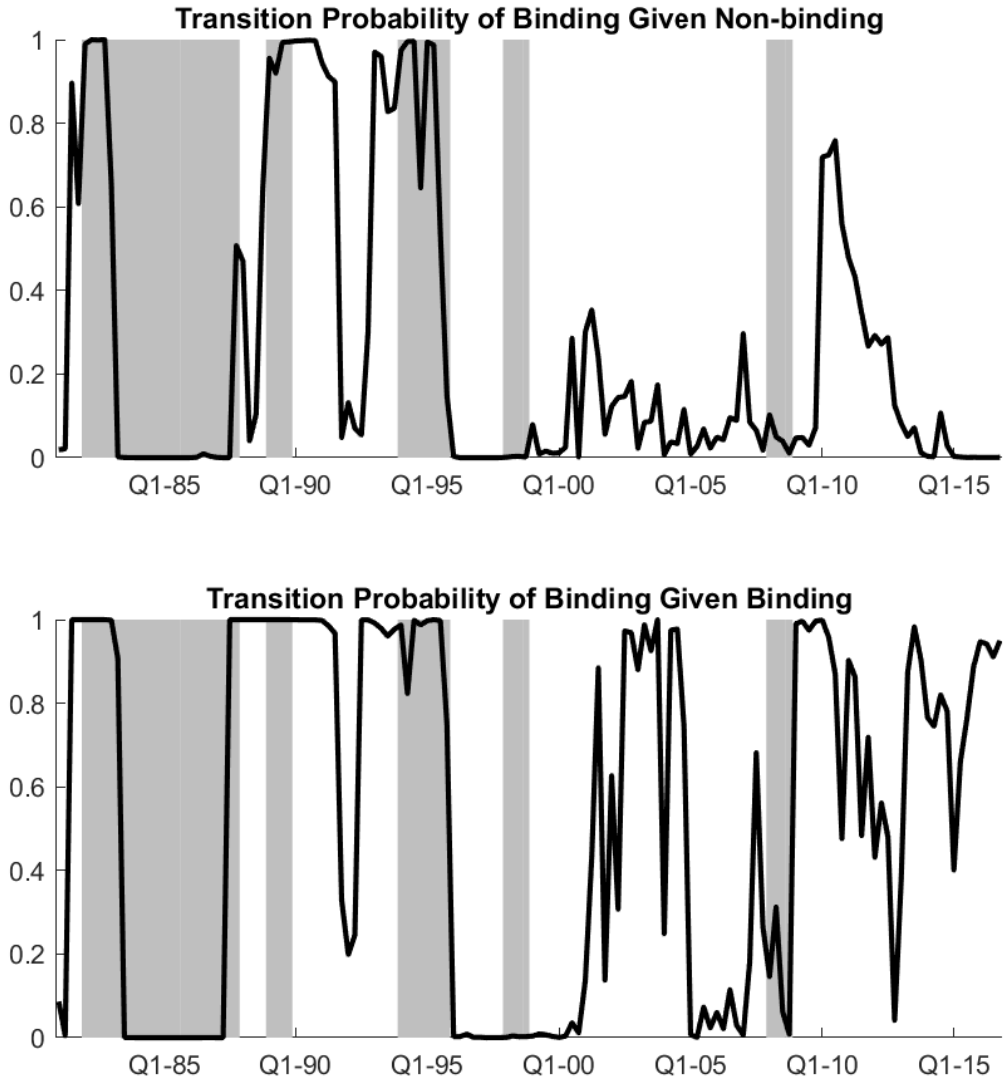
Figure 4: Smoothed Probability of Binding Regime



periods, it would suggest that our estimated model has finds it difficult to match the model dynamics during these episodes of critical interest in the empirical analysis.

The estimated model also provides a time-varying model-based estimate of the smoothed (i.e. based upon the full sample) probability of being in each regime. In the model, the regime is known by the household-firm, but the estimation procedure does not observe the regime and it must be inferred based on the information in the data. Figure 4 plots the estimated probability of being in the binding regime at each date against Reinhart and Rogoff (2009) currency crisis periods and the OECD-defined recession periods of Mexico. The red bar identifies periods in Mexico's history

Figure 5: Transition Probabilities



in which the definition of a financial crisis typically used in the empirical sudden stop literature is met.[10] The figure shows that the model provides a clear signal of when the economy is constrained, with estimates that are either near zero or one for most of the sample. The estimates indicate that Mexico had an elevated probability of being, or actually was in, the constrained regime for a number of years before the Tequila crisis in 1994-1995, and eventually entered an episode of large sudden stop in 1995:Q1 that lasted for two quarters. The model also signal a high probability of being in the binding regime earlier in the sample, during the debt crisis of the 1980s, but does not

[10]We provides this definition precisely, together with a more detailed discussion, below.

identify an crisis episode with protracted sluggish recovery as in the data. It also misses the 1998 and 2008 episodes associated with the Russian default and the LTCM collapse and the financial crisis in the United States. This result may reflect the fact that the latter two episodes were precipitated by shocks ultimately originating outside Mexico rather domestic ones. In addition, the fact that this probability does not align in the same way with OECD-defined recessions, as illustrated by the lower panel of Figure 4, correctly identifying most of these episodes, indicates that normal business cycle fluctuations are captured by the non-binding regime.

Figure 5 plots the filtered (or pseudo real-time) estimated transition probabilities to switch to the binding regime from the non-binding regime or to stay in the binding regime, based on equations (15) and (16), together with the Reinhart and Rogoff (2009) currency crisis narrative of Mexico. These probabilities provide the odds switching from one regime to the other as the model travels through the sample. Their behavior is driven by the estimated parameters γ_0 and γ_1 and the estimated values of B^* and λ. In the top panel, the figure plots the probability of switching into the binding regime given that the economy is in the nonbinding regime. In the bottom panel, the figure plots the probability to exit a binding regime. Both probabilities are time-varying, suggesting that a model with exogenous and constant switching probabilities would be misspecified. When the borrowing cushion is low, the model produces high odds of switching to the binding regime; when the multiplier is high the probability of staying in the binding regime is likewise high. Again, these probabilities align well with the Reinhart and Rogoff (2009) narrative of currency crisis in Mexico, as well as the estimates of the actual regime seen previously in Figure 4.

In summary, our results suggest that the values of the parameters critical for the specification of the borrowing constraint previously used in the literature might be too conservative, in the sense that the friction seems to be even tighter in the data than previously assumed. Also, our model fits not only the observable macroeconomic and financial data series, but also narratives of the business cycle and financial crisis history of Mexico. So we now turn to the analysis and discussion of the model implied business cycle and crisis properties.

4.2 Business Cycles Properties

Next, we report and discuss simulated model moments to characterize the estimated model's dynamics. These statistics are generated by simulating the model from the posterior mode estimates.[11] All statistics reported are unconditional, rather than conditional on a particular regime. In fact, when we compare simulated moments across regimes, we find that they are virtually identical for all variables except those related to the current account balance and the debt position, which are directly affect by the precautionary saving behavior.

[11]For these simulations, we generate 1,000 samples of 144 quarters long (the length of our data sample), after a burn-in period of 1,000 quarters. We then compute and report mean values across these 1,000 runs. We use a pruning method (Andreasen et al., 2018) to avoid explosive simulation paths.

Table 3: Simulated Second Moments: Data and Model

	Std. Dev.		Relative Std. Dev.		Correlations		Autocorrelation	
	Data	Model	Data	Model	Data	Model	Data	Model
Output Growth	1.41	3.02	1	1	1	1	0.15	-0.35
Consumption Growth	1.76	5.8	1.25	1.92	0.73	0.91	0.19	-0.53
Investment Growth	7.57	26.88	5.37	8.91	0.53	0.73	-0.1	-0.68
Country Interest Rate	1.92	0.91	1.36	0.3	-0.11	-0.04	0.95	0.73
Current Account to Output Ratio	2.2	3.51	1.56	1.16	-0.1	-0.63	0.89	-0.53
Import Price Growth	4.82	4.65	3.42	1.54	-0.27	-0.29	0.15	-0.01

The simulated second moments are reported in Table 3. The model closely matches the ranking of the relative volatilities of the observable variables, even though the model economy is much more volatile than the data in absolute terms, as in Mendoza (2010), except for the country interest rate that is significantly less volatile than in the data. Correlations with GDP have the correct sign and magnitude for all variables, except the current account balance as a share of GDP that is estimated to have a much lower correlation in a sample that extend to 2016 compared to what usually reported in the literature. The model captures the persistence of the country interest rate driven by the persistence of the process for the US real interest rate, but clearly misses the persistence of output, consumption, investment growth, and the current account balance.[12] Overall, the model broadly captures the distinguishing features of emerging market business cycles. Consumption is more volatile than output. The country interest rate and output growth are negatively correlated, and the current account balance is counter-cyclical, with persistence well below one. Note however that, unlike the extant literature, the model performance is not aided by assuming that the technology and interest rate shocks are negatively correlated.

Table 4 reports variance decompositions. Since the model is nonlinear, these decompositions are obtained by simulating the model using only one shock at the time, and normalizing so that they sum to 100.[13] The table illustrates that all shocks play a quantitatively sizable role in the model, even though different shocks matter more for different variables. Indeed, we will see below that the likelihood loss associated with turning off one shock at the time is broadly similar across shocks (see the top panel of Figure 8 below). A second important message from this table is that the estimated relative importance of different shocks is consistent with comparable estimation exercises conducted with linear models. In particular, we can see that output and consumption are driven by productivity and terms of trade shocks. Investment is significantly affected also by the intertemporal preference shock. In contrast, the financial variables of the model (the trade and current account balance, as well as the EFPD), are more driven by demand, preferences and

[12]Note here that the data moments, as well as the observable variables used in estimation, are in first difference. The gap between data and model persistence, therefor, is not comparable to that typically obtained in calibration exercises with HP filtering of the data and the simulated series.

[13]As a result, the decomposition ignores nonlinear interactions among the shocks that are present in the model when all shocks active; we ignore these nonlinearities for a more transparent comparison with linear models.

Table 4: Estimated Variance Decomposition (Unconditional)

Shocks Variables	TFP	Expend.	Terms of Trade	Preference	Temporary Int. Rate	Persistent Int. Rate
Y	52.4	11.29	27.36	3.648	1.072	4.23
C	48.39	14.75	24.72	4.897	1.84	5.401
I	31.92	23.48	17.99	12.45	3.911	10.25
H	46.46	13.49	24.36	7.052	2.592	6.047
V	27.1	7.87	55.88	4.113	1.512	3.527
W	46.46	13.49	24.36	7.052	2.592	6.047
Q	25.09	27.28	14.61	15.63	4.957	12.43
B^* (level)	10.31	28.46	7.656	22.44	7.038	24.1
B/Y (level)	32.94	22.91	16.78	9.128	3.016	15.23
CA/Y (level)	9.258	34.14	8.386	22.9	8.181	17.14
TB/Y (level)	8.771	31.32	7.783	21.01	8.797	22.33
$EFPD$ (level)	8.979	35.28	8.332	24.43	8.423	14.55
r (pp)	0	0	0	0	21.33	78.67
λ (level)	8.466	36.01	8.127	24.36	8.424	14.61
μ	48.43	16.66	24.19	4.697	1.129	4.898

Note: The variance decomposition is normalized so that it sums to 100 by row. All variables in log-change except when otherwise noted.

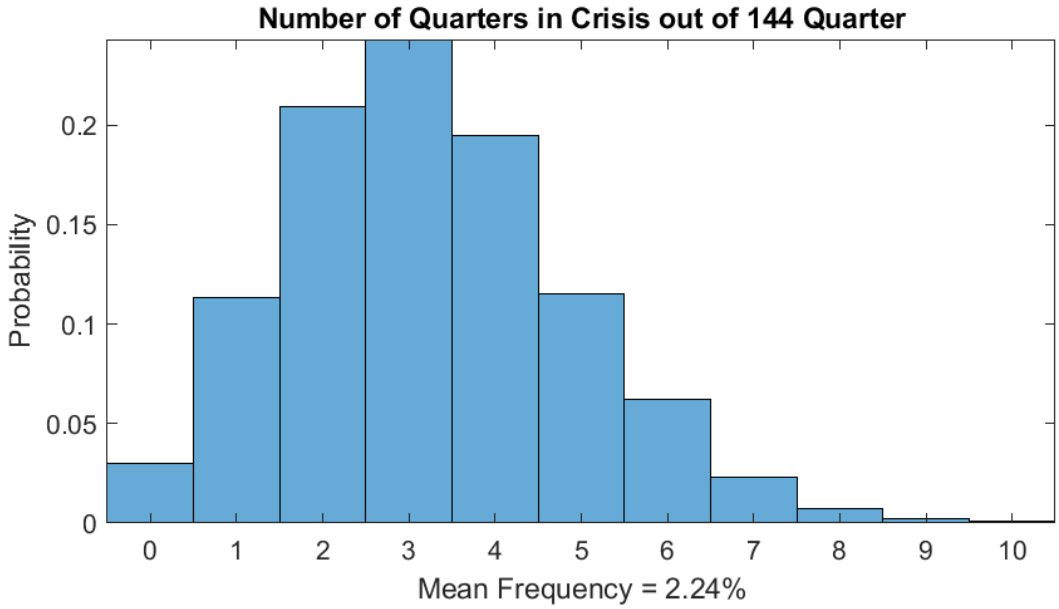
US interest rate shocks. The magnitude of the variance shares are broadly aligned with those estimated by Garcia-Cicco et al. (2010). Not surprisingly, our borrowing cushion variable, which is a function of the price of capital and debt, is largely driven by the expenditure shock and the persistent interest rate shock. We interpret these results as suggesting that both real and financial shocks matter for emerging market regular fluctuations.

4.3 Model-based Financial Crises Properties

The defining feature of the model we estimated is its ability to capture and describe the consequences of occasionally binding borrowing constraints and characterize episodes of financial crises. We now turn to empirical results on these episodes. Consistent with both the empirical literature on sudden stops and the quantitative literature on models with occasionally binding constraints, we define a financial crisis episode in our simulations an instance in which: (i) the economy is in a binding regime, with $\lambda > 0$, (ii) GDP growth is negative by more than one standard deviation and (iii) the TB/Y ratio increases by more than one standard deviation. As customary, this definition tries to avoid episodes of binding collateral constraint associated with small current account reversals and output drops.

Figure 6 reports statistics on the frequency and the duration of the crisis episodes as defined as above, generated by simulating 1,000 sample paths of 144 observations, like in our data sample. We then track crisis episodes that meet the definition above. The histogram shows the number of quarters in which the model meets the definition of a crisis provided relative to the total number

Figure 6: Financial Crises Statistics



of time periods in the sample draw (144). The figure says that the mean, across sample draws, of the unconditional crisis probability is 2.2%, which is close to typical estimates in the empirical literature on sudden stops, to which quantitative models with occasionally binding collateral constraints are often calibrated. Indeed, this probability is very close to the crisis probability in Mendoza (2010) or Benigno et al. (2013). However, the mean frequency masks some heterogeneity, with a mode frequency of 3 quarters out of 144, about 3 percent of the samples having no crises, and a number of samples having more than 6 quarters out of 144.

While the model can generate episodes with long sequences of *binding* collateral constraints, the average duration of crisis episodes as defined above is one quarter, as Mendoza (2010). Indeed, when we compute the same histogram as in the lower panel of Figure 6 without imposing the restriction that the binding constraint is associated with a large output drop and current account reversal, we find substantial heterogeneity in duration, with some samples with fewer than 30 quarters in the binding regime and some with more than 90. This means that, while the model can easily generate sample paths in which the economy spends long spells of time in the binding regime, most of them are not associated with persistent current account adjustment and output drops. In other words, the model does not generate crisis episodes characterized by repeated large current account reversals and output drops.

We next look at Sudden Stop dynamics through the lens of the model. Figure 7 reports the output from a long simulation of the model, again using parameters from the posterior mode. Each time the economy goes into a financial crisis as defined above, the simulated data is saved 5 quarters before the constraint binds and 5 quarters afterwards. We then plot the mean response and date the crisis as $t = 0$. Figure 7 show that the typical crisis episode is characterized by

shocks that comove in a distinctively averse manner: before the crisis, productivity falls; the price of intermediate input increases; expenditure and impatience rises, while the persistent portion of the real interest rate increases. Even though the structural shocks fed to the model are assumed to be orthogonal, and the estimated one are near orthogonal in the full sample, episodes characterized by a binding collateral constraint and large economic dislocation are preceded by shocks that are highly correlated. For instance, several of the correlations between the estimated shocks in our sample, during the 4 quarters before the Tequila crisis, have a correlation that is above $0.5 - 0.6$, and in some cases close to 1, in absolute value. Furthermore, the sequence of bad economic shocks persists well after the sudden stop, or does not reverts during the event. These results explain why linearized models of business cycles in emerging markets perform better at matching the simulated second moments when the productivity shock is assumed to be negatively correlated with the interest rate shocks, as extensively discussed by Neumeyer and Perri (2005).

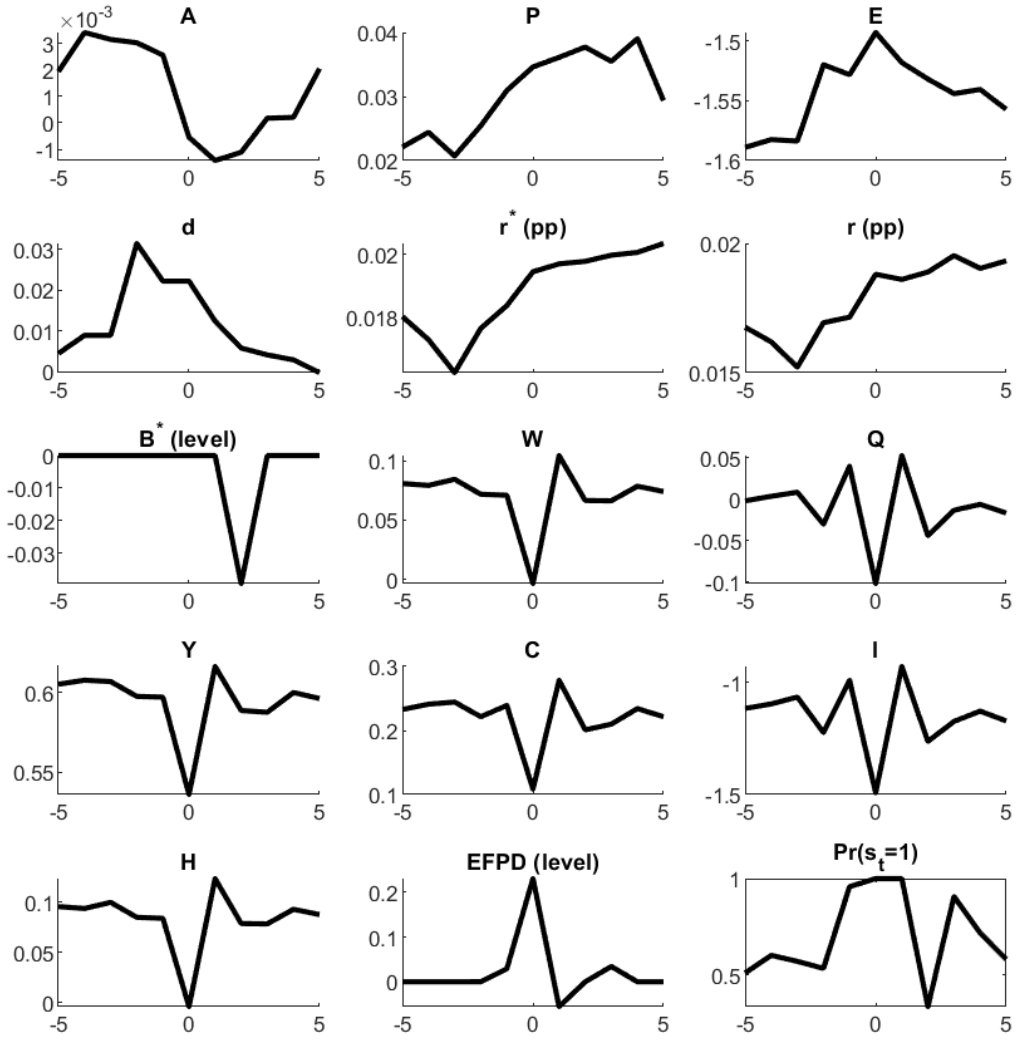
As a result of the persistent sequence of averse shocks, the economy heads into a crisis episode with a borrowing cushion that is zero for several quarters. Because of the exhausted borrowing capacity that drives the logistic function, the probability of switching to the constrained regime increases from .5 to 1, and eventually the economy switches to a binding regime one quarter before reaching the peack of the crisis. Once entered into the large sudden stop phase, output, consumption and investment collapse, and the trade balance reverts, swinging abruptly into surplus. Wages and the relative price of capital plummet, and the EFPD premium spikes.

Note here that, after the crisis occurs, economic activity rebounds sharply and quickly, as the model does not generate episodes with multiple large current account reversals and output drops. This result is not surprising, as we do not have financial intermediation or equilibrium default in the model, and suggests that the estimated estimated cannot generate episodes characterized by sluggish recoveries from crises, and is better suited to study V-shaped crisis as those experienced by emerging markets in 1990s. Indeed, as we saw earlier in Figure 4, the estimated model accurately can identify accurately the two-quarter long Tequila crisis episode, but fails to capture the persistency of the debt crisis of the 1980s.

So far, we have examined average model-based financial crisis episodes across simulations. These are useful for understating the economic dynamics of a crisis from the lens of our model. But one advantage of our framework is that we can also examine the historical shocks that drove a particular crisis episode and compared their relative importance with the same exercise over the full sample period.

As a counterfactual analysis, we calculate the likelihoods by turning off an individual shock at the time, inside the Unscented Kalman filter, and set all other remaining shocks to the filtered estimates obtained from the estimated parameter values. We consider three time windows: the full sample, the 4 quarters before the Tequila crisis episode, which according to the model starts in 1995:Q1 and ends in 1995:Q2. We then perform the following calculation: using the full sample likelihood as a benchmark, we counterfactually turn off one shock, and take the simple

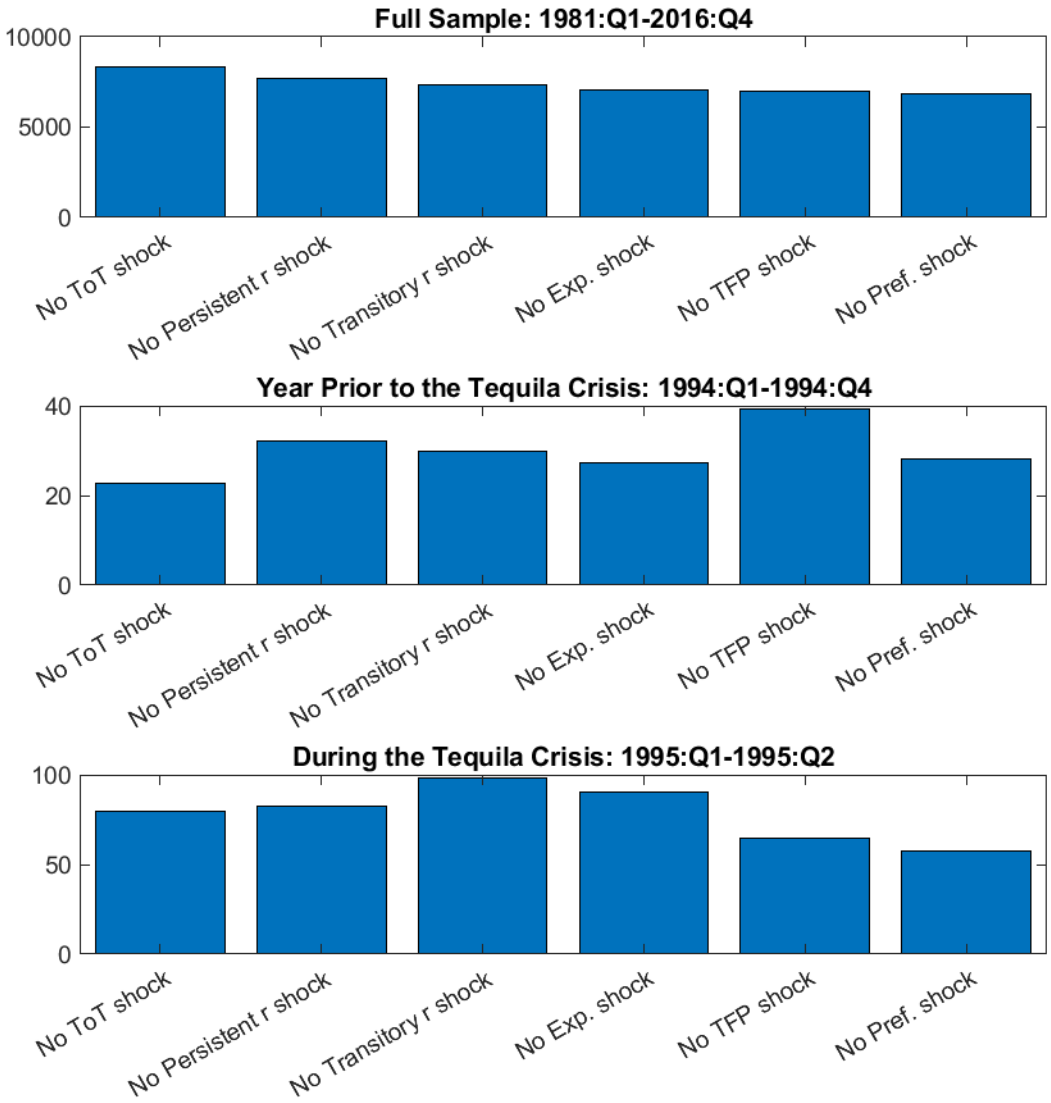
Figure 7: Dynamics of Crisis Episodes



difference between the benchmark and the resulting log-likelihood values. So, if the value of the log-likelihood in the full sample with all shocks is 100, and when we turn the productivity shock off over a particular time period the log likelihood drops to -90, the measure of contribution of the shock during that period will be 100-(-90) or 190. We then rank the shocks based on the the decreasing value of log-likelihood loss.

Figure 8 reports the results for the three sample periods we consider: the full sample, the run up to the 1995 Tequila crisis, and the two-quarter long crisis itself. As we can see, in the full sample, all shocks are contributing similarly, although one could argue that terms of trade

Figure 8: Counterfactuals



and persistent interest rate shocks are slightly more important. Before the crisis, shocks that increase aggregate demand seem to be the most important. In particular, the expenditure and the preference shocks seem to have had a larger role, arguably capturing the role of fiscal policy and other factors negatively affecting saving. In contrast, during the Tequila crisis, we can see that the two interest rate shocks and the productivity shock played a particularly large role.

Interestingly, when we conduct a similar exercise for the mean estimated crisis probability reported in Figure 6, we find results fully consistent with these likelihood based counterfactuals.

Table 5: Decomposition of the Crisis Frequency

Variable	Mean	StdDev
All Shocks	2.25	1.14
No Expenditure Shock	0.76	0.88
No Preference Shock	1.55	1.4
No TFP Shock	2.39	1.66
No Import Price Shock	2.42	1.64
No Transitory Interest Rate Shock	2.47	1.78

Note: The table reports the frequency of crisis episodes.

In fact, Table 5 provides a variance decomposition of the relative frequency with which crisis episodes realize across sample draws, by shutting down one shock at a time. As we can see, turning off the TFP shock, the terms of trade shock, and the transitory interest rate shock has almost no impact on the frequency of crises. Persistent interest rate shocks and preference shocks, and especially expenditure shocks, however, have a much larger impact on the crisis probability. This confirms that the latter shocks may play a relatively larger role in the theoretical model in driving the occurrence of crises.

When we look at the importance of different shocks for different variables in the run up to the Tequila crisis, we can confirm that expenditure shocks seem most important for understanding movement in the borrowing cushion. Preference shocks behaved in a similar fashion, driving debt movements in the second part of the crisis. World interest rate shocks drove the borrowing cushion, the current account, debt and interest rates, showing the importance of financial shocks during the crisis.

5 Conclusions

In this paper we propose a new approach to specifying, solving and estimating Dynamic Stochastic General Equilibrium (DSGE) models with occasionally binding collateral constraints. We first specify the occasionally binding constraint in such a way that the transition from the unconstrained to the constrained state of the world is a stochastic function of the endogenous variables in the model, which permits to cast the framework as an endogenous regime switching model. Next, we develop perturbation methods to solve this model and design an algorithm to estimate its parameters with full-information Bayesian methods. Finally, we apply the framework to quarterly Mexican data since 1981, finding that the model fit the data well, critical parameter estimates differ significantly to what previously used in the literature, and that aggregate demand shocks seems to matter more heading into crisis episodes, while productivity and interest rate shocks prevails during crisis episodes. All estimated shocks play an important role in the full sample. The estimated correlation of the shocks before the Tequila crisis appears particularly averse. The model does not produce persistent large financial crisis like the debt crisis of the 1980s, or

the Sub Prime crises in the United States. We regard the estimation of a model with financial intermediation or equilibrium default to generate persistent crisis as an important area of future research.

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Appendix A Competitive Equilibrium Definition

A competitive equilibrium of our economy is a sequence of quantities $\{K_t, B_t, C_t, H_t, V_t, I_t, A_t, E_t, \kappa_t, Y_t, \lambda_t, B_t^*\}$ and prices $\{P_t, r_t^*, r_t, q_t, w_t, \mu_t\}$ that, given the 5 exogenous processes in (8)-(6), satisfy the following first order conditions:

$$d_t \left(C_t - \frac{H_t^\omega}{\omega} \right)^{-\rho} = \mu_t, \tag{A.1}$$

$$(1 - \alpha - \eta) \frac{Y_t}{V_t} = P_t \left(1 + \phi r_t + \frac{\lambda_t}{\mu_t} \phi \left(1 + r_t \right) \right), \tag{A.2}$$

$$\alpha \frac{Y_t}{H_t} = \phi W_t \left(r_t + \frac{\lambda_t}{\mu_t} \left(1 + r_t \right) \right) + H_t^{\omega - 1}, \tag{A.3}$$

$$\mu_t = \lambda_t + \beta \left(1 + r_t \right) \mathbb{E}_t \mu_{t+1}, \tag{A.4}$$

$$\mathbb{E}_t \mu_{t+1} \beta \left(1 - \delta + \frac{\iota}{2} \left(\frac{K_{t+1}}{K_t} \right)^2 - \frac{\iota}{2} + \eta \frac{Y_{t+1}}{K_t} \right) = \mu_t \left(1 - \iota + \iota \left(\frac{K_t}{K_{t-1}} \right) \right) - \lambda_t \kappa_t q_t, \tag{A.5}$$

$$Y_t = A_t K_{t-1}^\eta H_t^\alpha V_t^{1 - \alpha - \eta}, \tag{A.6}$$

$$q_t = 1 + \iota \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right), \tag{A.7}$$

$$W_t = H_t^{\omega - 1}; \tag{A.8}$$

the budget constraint

$$C_t + I_t = Y_t + E_t - P_t V_t - \phi r_t \left(W_t H_t + P_t V_t \right) - \frac{1}{(1 + r_t)} B_t + B_{t-1}; \tag{A.9}$$

the capital accumulation euquation

$$I_t = \delta K_{t-1} + \left(K_t - K_{t-1} \right) \left(1 + \frac{\iota}{2} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right) \right); \tag{A.10}$$

the definition of a borrowing cushion

$$B_t^* = \frac{1}{(1 + r_t)} B_t - \phi \left(1 + r_t \right) \left(W_t H_t + P_t V_t \right) + \kappa_t q_t K_t; \tag{A.11}$$

the regime-switching slackness condition

$$\varphi\left(s_t\right) B_{ss}^* + \nu\left(s_t\right)\left(B_t^* - B_{ss}^*\right) + \left(1 - \varphi\left(s_t\right)\right)\lambda_{ss} + \left(1 - \nu\left(s_t\right)\right)\left(\lambda_t - \lambda_{ss}\right) = 0; \tag{A.12}$$

and finally the interest rate process

$$r_t = r_t^* + \psi\left(e^{\bar{B}-B_t} - 1\right) + \sigma_r \epsilon_{r,t}. \tag{A.13}$$

Appendix B Endogenous Switching and Stochastic Monitoring: A Possible Interpretation

On possible interpretation of our specification of the occasionally binding constraint is in terms of a stochastic monitoring technology. Specifically, one can model the transition from the non-binding regime at time t to the binding regime at time $t+1$ by considering a monitoring (or enforcement) shock in period $t+1$, ϵ_{t+1}^M. Accordingly, the realized regime in period $t+1$ can be seen as a function of the current period borrowing cushion B_t^* and a next period enforcement shock, according to

$$s_{t+1} = \Gamma(\epsilon_{t+1}^M | s_t = 0, B_t^*). \tag{B.1}$$

Given that the economy is in a non-binding regime in period t, the Γ function maps the borrowing cushion B_t^* and the monitoring shock ϵ_{t+1}^M into a realization of the regime in the next period, $s_{t+1} \in \{0, 1\}$. The monitoring shock reflects the notion that financial contracts are subject to random monitoring and enforcement, consistent with the evidence we referred to in the paper. This makes the dependence on the borrowing cushion a probabilistic, rather than deterministic, function. The enforcement shock, in other words, introduces an element of uncertainty in the mapping, that reflects the empirical evidence discussed above.

While the formulation of equation (B.1) is very general, we assume a logistic functional form for the probability of changing regimes. This permits us to drop ϵ_{t+1}^M from (B.1) and write the following probabilistic statement that depends only on the borrowing cushion:

$$\Pr\left(s_{t+1} = 1 | s_t = 0, B_t^*\right) = \frac{\exp\left(-\gamma_0 B_t^*\right)}{1 + \exp\left(-\gamma_0 B_t^*\right)}. \tag{B.2}$$

Similarly, when the constraint is binding, the Lagrange multiplier λ_t associated with the constraint is non-zero. We again could introduce a monitoring shock such that the next period regime depends on the multiplier and the monitoring shock,

$$s_{t+1} = \Gamma(\epsilon_{t+1}^M | s_t = 1, \lambda_t). \tag{B.3}$$

Appendix C Overview of the Solution Method

This Appendix provides details about two aspects of the solution method: (1) the definition of, and solution for, the steady state of the endogenous regime-switching economy; and (2) the perturbation method that generates second order Taylor expansions to the solution of the economy around the steady state.

C.1 Steady State Definition and Solution

The model has two features that make it challenging to define a steady state. First, as it is common in a regime-switching framework, some auxiliary or structural parameters may be switching. In the case of our application, there is only one auxiliary switching parameters that affect the steady state, $\varphi(s_t)$. Nonetheless, in principle, one could allow for regime switching also for the parameters of the exogenous processes, $a^*(s_t)$ and $p^*(s_t)$, or the structural parameter $\kappa^*(s_t)$, which affect the level of the economy directly, and will thus matter for steady state calculations.[14] Solution methods such as those proposed by Foerster et al. (2016) define the steady state by using the ergodic means of these parameters across regimes. However, in our case, the transition matrix P is endogenous, making the use of the ergodic distribution problematic, since it depends on economic variables that in turn depend on the ergodic means. The steady state solution method that we propose proceeds in two steps.

Step 1: Solve by Using a Steady State Transition Matrix. First, assume that the steady state transition matrix at iteration i, $P_{ss}^{(i)}$, is known. Next, let $\xi = [\xi_0, \xi_1]$ denote the ergodic vector of $P_{ss}^{(i)}$. Then, as explained in the paper, define the ergodic means of the switching parameters as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1).$$

Note here that this step provides us with $\varphi(ss)$, a_{ss}, E_{ss}, κ_{ss}, and p_{ss}. Next, assuming r_{ss} is known, the equilibrium conditions in Appendix A above can be solved for the steady state value of all the other variables, $(K_{ss}, B_{ss}, C_{ss}, H_{ss}, V_{ss}, I_{ss}, Y_{ss}, \lambda_{ss}, B_{ss}^*, r_{ss}^*, q_{ss}, w_{ss}, \mu_{ss})$. Then, r_{ss} must solve

$$\bar{\varphi} B_{ss}^* + (1 - \bar{\varphi}) \lambda_{ss} = 0. \tag{C.1}$$

Step 2: Update the Transition Matrix. Step 1 yields the variables B_{ss}^* and λ_{ss}, and hence have a new value of the transition matrix for iteration $i+1$:

$$P_{ss}^{(i+1)} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B_{ss}^*)}{1+\exp(-\gamma_0 B_{ss}^*)} & \frac{\exp(-\gamma_0 B_{ss}^*)}{1+\exp(-\gamma_0 B_{ss}^*)} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}, \tag{C.2}$$

[14] As it is well known, over finite periods of time, is statistically difficult to distinguish between unit root processes and processes with structural break of regime changes. Allowing for regime changes in the process for A_t, therefore, would be a way to accommodate permanent productivity shocks as in Aguiar and Gopinath (2007).

which can be checked against the guess in Step 1. Continue this iterative procedure until

$$\left\| P_{ss}^{(i+1)} - P_{ss}^{(i)} \right\| < tolerance.$$

C.2 Second order Approximation

To compute a second order approximation to the endogenous regime-switching model solution, we largely follow Foerster et al. (2016). The equilibrium conditions can be summarized as

$$\mathbb{E}_t f\left(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi\varepsilon_{t+1}, \varepsilon_t, \theta_{t+1}, \theta_t\right) = 0, \tag{C.3}$$

with variables grouped in predetermined, $\mathbf{x}_{t-1}$, and non-predetermined, $\mathbf{y}_t$, and shocks ε_t. Switching variables and parameters θ_t are partitioned into those that affect the steady state, in our case only $\theta_{1,t} = \varphi\left(s_t\right)$, and those that do not affect the steady state, $\theta_{2,t} = \nu\left(s_t\right)$.

Given the functional forms in (20) and

$$\theta_{1,t} = \bar{\theta}_1 + \chi\hat{\theta}_1\left(s_t\right), \; \theta_{2,t} = \theta_2\left(s_t\right) \tag{C.4}$$

for t and $t+1$, the transition probabilities satisfy

$$p_{s_t, s_{t+1}, t} = \pi_{s_t, s_{t+1}}\left(\mathbf{y}_t\right). \tag{C.5}$$

Using the latter in the equilibrium conditions and writing expectations explicitly conditional on $\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)$ and s_t, we have:

$$F_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) = \int \sum_{s'=0}^{1} \pi_{s_t, s'}\left(g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)\right) f\left(\begin{array}{c} g_{s_{t+1}}\left(h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \chi\varepsilon', \chi\right), \\ g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \\ h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \\ \mathbf{x}_{t-1}, \chi\varepsilon', \varepsilon_t, \\ \bar{\theta} + \chi\hat{\theta}\left(s'\right), \bar{\theta} + \chi\hat{\theta}\left(s_t\right) \end{array}\right) d\mu\varepsilon' = 0, \tag{C.6}$$

which, by stacking these conditions for each regime, produces

$$\mathbb{F}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) = \left[\begin{array}{c} F_{s_t=1}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) \\ F_{s_t=2}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) \end{array}\right]. \tag{C.7}$$

For perturbation, we finally take the stacked equilibrium conditions $\mathbb{F}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)$, and differentiate with respect to $\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)$. Prior to that, we set $\varepsilon_t = 0$ and $\chi = 0$, which implies a steady state given by

$$f\left(\mathbf{y}_{ss}, \mathbf{y}_{ss}, \mathbf{x}_{ss}, \mathbf{x}_{ss}, 0, 0, \bar{\theta}_{\mathbf{1}}, \theta_2(s'), \bar{\theta}_1, \theta_2(s)\right) = 0 \tag{C.8}$$

for all s', s.

In a typical regime-switching model, the first-order derivative with respect to $\mathbf{x}_{t-1}$ produces a complicated polynomial system denoted

$$\mathbb{F}_{\mathbf{x}}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0. \tag{C.9}$$

Often this system needs to be solved via Gröbner bases, which finds all possible solutions in order to check them for stability. In our case, with endogenous probabilities, the standard stability checks fail, so we will focus on finding a single solution and ignore the possibility of indeterminacy, a common simplification in the regime-switching literature with and without endogenous switching (e.g. Farmer et al., 2011; Foerster, 2015; Maih, 2015; Lind, 2014). In the literature that computes global solutions to non-regime switching occasionally binding constraint models (e.g. Benigno et al. (2013), Mendoza (2010)) there are no proofs of uniqueness, and the focus typically is also on computing a solution checking robustness to initial conditions. To find a solution to our model, we guess at a set of policy functions for regime $s_t = 1$, which collapses the equilibrium conditions $\mathbb{F}_{\mathbf{x}}\left(\mathbf{x}_{ss}, \mathbf{0}, 0; s_t = 0\right)$ into a fixed-regime eigenvalue problem, and solve for the policy functions for $s_t = 0$. Then, using this initial solution as guess, we solve for regime $s_t = 0$ under the fixed-regime eigenvalue problem, and iterate on this procedure to convergence. After solving the iterative eigenvalue problems, the other systems to solve are

$$\mathbb{F}_{\varepsilon}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0 \tag{C.10}$$

$$\mathbb{F}_{\chi}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0 \tag{C.11}$$

and second order systems of the form (can apply equality of cross-partials)

$$\mathbb{F}_{\mathbf{i},\mathbf{j}}\left(\mathbf{x}_{ss}, \mathbf{0}, 0\right) = 0,\ \mathbf{i}, \mathbf{j} \in \{\mathbf{x}, \varepsilon, \chi\}. \tag{C.12}$$

Appendix D Data Appendix

D.1 National Accounts

National accounts are from the National Statistic Office. The data series used in the analysis merge two set of official statistics by updating the level of the accounts based on 1993 constant prices with the quarterly rate of growth of the accounts based on 2008 constant prices (as the deflators to splice the accounts in levels were not available). The two set of accounts overlaps from 1993:Q1 to 2006:Q4. Over this period, the difference in annual rate of growth is less than 0.01 percent in absolute value for GDP, less than 0.05 percent for consumption, less than 2 percent for investment, and less than 1 and 3 percent for imports and exports, respectively. The correlations between the series are more than .9 for all series except investment that is .84. The differences, however are smaller the closer to the end of the sample. For this reason, we choose to update the

1993 accounts rather than backdate the 2008 ones. The specific data sources are as follows.

2006:Q1-2016:Q4 (Labeled 2008 accounts)–Supply and demand of goods and services. Original series (not seasonally adjusted). Constant prices, annual 2008 = 100 (Oferta y demanda de bienes y servicios. Series originales. A precios constantes 2008). Available from http://www3.inegi.org.mx/sistemas/tabuladosbasicos/tabdirecto.aspx.

1980:Q1-2006:Q4 (Labeled 1993 accounts)–Supply and demand of goods and services. Original Series (not seasonally adjusted). Constant prices, annual 1993 = 100. (We obtained these from Guadra Gabriel (2008), Hechos Estilizados del Ciclo Economico en Mexico, Banco de Mexico Working Papers No. 2008-14, Diciembre 2008).

The data are not seasonally adjusted and show a strong seasonal pattern. To seasonally adjust all series (assumed to be I(1) processes), we adjust the log-difference using the X-12 procedure with the additive option in Eviews. We then use the log of the first observation of the raw series (not seasonally adjusted) and cumulate the seasonally adjusted log-difference.

D.2 Interest rates

The country interest rate is calculated following Uribe and Yue (2006) as

$$r_t = r_t^* + EMBI_t \tag{D.1}$$

where r^* is the US real interest rate, and EMBI is Mexico sovereign spread.

We compute r^* as 3-month Treasury Constant Maturity Rate adjusted for CPI (annualized) quarterly inflation, using period average data. The source is FRED.

The EMBI spread is available only starting from 1993. In order to estimate the risk premium, we estimate the relationship between Mexico's nominal interest rates and the EMBI during the period over which the EMBI is observable, and then invert it. Specifically, we specify the following model:

$$i_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 EMBI_t \tag{D.2}$$

We then construct $\widehat{EMBI}_t$ by inverting the this regression.

We have monthly data for i_t and we take monthly average in each quarter. For CPI, we first convert monthly data into quarterly and then π_t is given by $\frac{4CPI_t}{CPI_{t-1}}$. There are three missing data points for the nominal interest rate: 1986 Aug, Sep and 1988 Nov. We take the average of the monthly data.

The results of the regression above are (t-statistics in parentheses and $R^2 = 0.883$):

$$i_t = \underset{(-0.42)}{-0.00346} + \underset{(4.46)}{0.397}\pi_t + \underset{(7.37)}{2.770} EMBI_t. \tag{D.3}$$