Traffic in the City: The Impact of Infrastructure Improvements in the Presence of Endogenous Traffic Congestion

Treb Allen
Dartmouth and NBER

Costas Arkolakis
Yale and NBER

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1 Introduction

Recent developments in the spatial economics literature have provided powerful tools for enabling us to understand how improvements to the transportation infrastructure network affect the spatial distribution of economic activity (see e.g. Redding and Turner (2014); Redding and Rossi-Hansberg (2017) for excellent recent reviews). However, these frameworks abstract from the impact of the spatial distribution of economic activity on the efficacy of the infrastructure improvement itself. As anyone who has sat in a traffic jam during rush hour is aware, where people live and where people work has important implications for the cost of commuting across space.

In Allen and Arkolakis (2019), we develop a way of tractably incorporating the optimal routing choice of agents into a general equilibrium economic geography model of trade between cities.. In this proposal, weextend this framework to explicitly incorporate traffic congestion in a commuting model within a city. The framework provides a tractable way of characterizing the complicated feedback loop between traffic and economic activity and yields a "gravity" equation in both commuting and traffic flows, where the two are closely linked through the concept of commuter market access.

We propose to use this this framework to (1) assess how infrastructure improvements change the distribution of economic activity and equilibrium flows of traffic; and (2) characterize the optimal placement of new infrastructure projects (and/or the optimal improvement of existing infrastructure networks) in the presence of agglomeration and dispersion forces in the city.

Intellectual Merit The project contributes to three of the goals of the "Economics of Transportation in the 21st Century" project, namely (abridged): (1) "the returns to investments in transportation infrastructure"; (2) "the valuation of congestion"; and (3) "the impact of the transportation sector on the broader economy, including the internal structure of economic activity within urban areas."

We aim to contribute to a literature that uses computational models to evaluate infrastructure investment within the city in the presence of traffic congestion. Due to the difficulty of modeling traffic, prior work has focused primarily on laboratory-type infrastructure networks (e.g. De Palma, Kilani, and Lindsey (2005)), aggregate engineering estimates instead of full agent-based equilibrium setups (e.g. Geroliminis and Levinson (2009)), or based only a limited number of infrastructure links due to the computational limitations (Anas and Liu (2007)). Our model, in contrast, allows us to evaluate optimal infrastructure investment policies in large, realistic infrastructure networks, in full agent-based general equilibrium setup with agglomeration and dispersion externalities.

2 A Model of the City with Endogenous Traffic Congestion

In this section, we briefly describe how to incorporate endogenous traffic congestion into a model where agents interact across space. Given space constraints, we focus on the novel aspects of the model.

There is a measure $\bar{L}$ of individuals who populate a city composed of a finite number of city blocks $i \in \{1, ..., N\} \equiv \mathcal{N}$, where L_i is the measure of individuals residing in city block $i \in \mathcal{N}$. An individual residing in location i and working in location j receives an amenity u_i and uses her endowed unit of time for commuting and the production of a numeraire good, for which she earns a wage w_j.

Locations are organized on a weighted graph with an associated *infrastructure matrix* $\boldsymbol{T} = [t_{ij} \geq 1]$, where t_{ij} indicates the (endogenous) iceberg cost incurred from moving *directly* from i to j along a *link* (if no link between i and j exists, then $t_{ij} = \infty$). Moving from i to j requires taking a *route* r through the infrastructure matrix, where r is formally a sequence of locations beginning with location i and ending with location j, i.e. $r \equiv \{i = r_0, r_1, ..., r_K = j\}$, where K is the number of links crossed on the route, i.e. the *length* of route r. Let $\Re_{ij}$ denote the set of all the (countably infinite) possible routes from i to j.

2.1 A gravity equation for commuting

An individual $\nu \in [0, 1]$ residing in city block $i \in \mathcal{N}$ who works in city block $j \in \mathcal{N}$ and commuted via route r of length K to work receives a payoff $V_{ij,r}(\nu)$ that depends on (1) the wage in the workplace, w_j; (2) the amenity value of residence, u_i; (3) the time spent commuting; and (3) an idiosyncratic (Frechet distributed with shape parameter θ) route-, origin-, and destination-specific term:

$$V_{ij,r}(\nu) = \left(w_j u_i / \prod_{l=1}^{K} t_{r_{l-1},r_l} \right) \times \varepsilon_{ij,r}(\mu).$$

Individuals then choose where to live, work, and which route to take in order to maximize this value:

$$V(\nu) = \max_{i \in \mathcal{N}, j \in \mathcal{N}, r \in \Re_{ij}} V_{ij,r}(\nu). \tag{1}$$

The probability of an individual choosing to live in i and commute to j via route r, $L_{ij,r}$, is proportional to:[1]

$$L_{ij,r} \propto \prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta} \times u_i^{\theta} \times w_j^{\theta}$$

Similarly, the probability that an individual chooses to live in i and commute to j via any route, π_{ij}, can be calculated simply by summing across all routes, i.e.:

$$L_{ij} = \sum_{r \in \Re_{ij}} L_{ij,r} = \tau_{ij}^{-\theta} \times u_i^{\theta} \times w_j^{\theta}, \tag{2}$$

where $\tau_{ij} \equiv \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta} \right)^{\frac{1}{\theta}}$. As in the seminal work of Ahlfeldt, Redding, Sturm, and Wolf (2015), equation (2) implies that commuting flows exhibit gravity; here, however, the effective bilateral commuting cost τ_{ij} is a function of the underlying infrastructure matrix. As in Anderson and Van Wincoop (2003) and Tsivanidis (2018), we can re-write equation (2) to include "resident commuter market access" $\Phi_i^{RMA} \equiv \sum_j \tau_{ij}^{-\theta} \times w_j^{\theta}$ and "firm commuter market access" $\Phi_j^{FMA} \equiv \sum_i \tau_{ij}^{-\theta} u_i^{\theta}$ terms:

$$L_{ij} = \tau_{ij}^{-\theta} \times \frac{L_i^R}{\Phi_i^{RMA}} \times \frac{L_j^F}{\Phi_j^{FMA}}, \tag{3}$$

where $L_i^R \equiv \sum_j L_{ij}$ is the number of residents in location i and $L_j^F \equiv \sum_i L_{ij}$ is the number of workers in location j. This re-writing proves helpful in characterizing traffic flows below.

It turns out that the expression for τ_{ij} admits a particularly convenient analytical form. To see this, it is useful to characterize the *weighted adjacency matrix* $\mathbf{A} = \left[a_{ij} \equiv t_{ij}^{-\theta} \right]$. By summing over all routes

[1] Treating the city as a small open economy in a larger world with utility normalized to one ensures the proportionality is equal to one.

of length K, we can write τ_{ij} by explicitly summing across all locations that are traveled to first, second, etc. as follows:

$$\tau_{ij}^{-\theta} = \sum_{K=0}^{\infty} \left(\sum_{k_1=1}^{N} \sum_{k_2=1}^{N} ... \sum_{k_{K-1}=1}^{N} a_{i,k_1} \times a_{k_1,k_2} \times ... \times a_{k_{K-2},k_{K-1}} \times a_{k_{K-1},j} \right),$$

where k_n is the sub-index for the n^{th} location arrived at on a particular route. As long as the spectral radius of $\mathbf{A}$ is less than one, the geometric sum can be expressed as:[2]

$$\sum_{K=0}^{\infty} \mathbf{A}^K = (\mathbf{I} - \mathbf{A})^{-1} \equiv \mathbf{B},$$

where we call $\mathbf{B} = [b_{ij}]$ the *route cost matrix* (which is simply the Leontief inverse of the weighted adjacency matrix), so that the trade cost from i to j can be written as a simple function of the route cost matrix:

$$\tau_{ij} = b_{ij}^{-\frac{1}{\theta}}. \tag{4}$$

Equation (4) provides an analytical expression for the (endogenous) commuting costs as a function of the infrastructure matrix. Next, we proceed a step further and let the costs of travel over the infrastructure matrix to respond endogenously to agents' chosen commuting routes through traffic congestion.

2.2 A gravity equation for traffic and endogenous traffic congestion

To begin we characterize the expected number of times in which link kl is used when commuting from i to j, π_{ij}^{kl}. We show that it (also) obtains a particularly convenient functional form.

Note that for any route r of length K that travels through link kl at least once, so that there must exist some length $B \in [1, 2, ..., K-1]$ at which the route arrives at link kl. As a result, we can calculate π_{ij}^{kl} by explicitly enumerating all possible routes from i to k of length B and all possible routes from l to j of length $K - B - 1$ to obtain (after various ommitted matrix calculus calculations due to space constraints):

$$\pi_{ij}^{kl} = \frac{\tau_{ik}^{-\theta} t_{kl}^{-\theta} \tau_{lj}^{-\theta}}{\tau_{ij}^{-\theta}} \tag{5}$$

Intuitively, the more "out of the way" the transportation link kl is from the optimal route between i and j, the less frequently that route is used.

Expression (5) proves crucial in determining the total number of commuters using a given link in the infrastructure matrix (i.e. the "traffic"), which we denote Ξ_{kl}. To calculate Ξ_{kl}, we add the intensity within which link kl is used across across all origins and destinations, $\Xi_{kl} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij}^{kl} \times L_{ij}$. Combining equations (2) and (5) yields (with some straightforward algebra):

$$\Xi_{kl} = t_{kl}^{-\theta} \times \Phi_k^{FMA} \times \Phi_l^{RMA}. \tag{6}$$

To our knowledge, equation (6) is the first gravity equation for traffic in the infrastructure literature. Perhaps somewhat surprisingly, despite agents making commuting decisions over an arbitrary infrastructure network traffic over a link only depends on three things: (1) the cost of traveling over the link; (2) the firm market access at the beginning of the link; and (3) the residential market access at the end of the link. Intuitively, the greater the firm commuter market access, the greater the traffic going to the start of the link and the greater the resident commuter market access, the greater the traffic departing

[2]This is satisfied when $\sum_j t_{ij}^{-\theta} < 1$ which is necessarily the case if trade costs between connected locations are sufficiently large, the adjacency matrix is sufficiently sparse (i.e. many locations not directly connected), or the heterogeneity across routes is sufficiently small (i.e. θ is sufficiently large).

from the end of the link. The innovation of deriving a gravity equation for traffic is that it allows us to tractably incorporate traffic congestion into a commuting framework, which we turn to next.

Suppose that the direct cost of traveling over a particular link depends in part on the total traffic flowing over that link, i.e. there is traffic congestion. In particular, we assume that the direct cost of traveling over a link, t_{kl}, depends in part on the amount of traffic over that link Ξ_{kl} through the following simple functional form: $t_{kl} = \bar{t}_{kl}\Xi_{kl}^{\lambda}$,where $\lambda \geq 0$ governs the strength of traffic congestion and $\bar{\mathbf{T}} \equiv [\bar{t}_{kl}]$ is the *fundamental* infrastructure matrix. Combining this equation with the the gravity equation for Ξ_{kl} from equation (6) we immediately obtain:

$$t_{kl} = \bar{t}_{kl}^{\frac{1}{1+\theta\lambda}} \times \left(\Phi_k^{FMA}\right)^{\frac{\lambda}{1+\theta\lambda}} \times \left(\Phi_l^{RMA}\right)^{\frac{\lambda}{1+\theta\lambda}}. \tag{7}$$

Equation (7) provides an analytical relationship for the feedback loop from the commuting choices to the cost of travel along the infrastructure matrix (and back). Intuitively, the greater the commuter market access at the beginning and end of a link, the greater the traffic and hence the greater the cost of traversing that link, which in turn reduces the commuter market access.

2.3 Equilibrium

In turns out that solving for the full spatial equilibrium after accounting for endogenous traffic congestion is no more difficult than treating commuting costs as exogenous. To illustrate the procedure, we assume both the wages and amenity values are exogenous in the model; however, incorporating spillovers, introducing costly trade, or introducing land markets are all straightforward extensions. The starting point are the equilibrium conditions $L_i^R = \sum_j L_{ij}$ and $L_i^F = \sum_j L_{ji}$, which when combined with equation (4) and inverted result in:

$$L_i^R u_i^{-\theta} = \sum_j t_{ij}^{-\theta} L_j^R u_j^{-\theta} + v_i^{\theta} \tag{8}$$

$$L_i^F v_i^{-\theta} = \sum_j t_{ji}^{-\theta} L_i^F v_i^{-\theta} + u_i^{\theta} \tag{9}$$

Equations (8) and (9) are of independent interest, as they express the equilibrium as a function of the underlying infrastructure matrix $\{t_{ij}\}$ (rather than the endogenous commuting costs). However, we can do even better, substituting in equation (7) to capture the endogenous traffic costs to yield:

$$L_i^R u_i^{-\theta} \left(\frac{L_i^F}{v_i^{\theta}}\right)^{\frac{\lambda\theta}{1+\theta\lambda}} = \sum_j \bar{t}_{ij}^{-\frac{\theta}{1+\theta\lambda}} \left(L_j^R\right)^{\frac{1}{1+\theta\lambda}} u_j^{-\frac{\theta}{1+\theta\lambda}} + v_i^{\frac{\theta}{1+\theta\lambda}} \left(L_i^F\right)^{\frac{\lambda\theta}{1+\theta\lambda}} \tag{10}$$

$$L_i^F v_i^{-\theta} \left(\frac{L_i^R}{u_i^{\theta}}\right)^{\frac{\lambda\theta}{1+\theta\lambda}} = \sum_j \bar{t}_{ji}^{-\frac{\theta}{1+\theta\lambda}} \left(L_j^F\right)^{\frac{1}{1+\theta\lambda}} v_j^{-\frac{\theta}{1+\theta\lambda}} + u_i^{\frac{\theta}{1+\theta\lambda}} \left(L_i^R\right)^{\frac{\lambda}{1+\theta\lambda}} \tag{11}$$

Equations (10) and (11) provide an equilibrium system which allows us to determine the endogenous distribution of where people live taking into account the equilibrium the optimal commuting behavior of individuals and the endogenous traffic congestion that results. Moreover, these equations allows us to determine how changes to the exogenous infrastructure network $\{\bar{t}_{ij}\}$ (e.g. through new infrastructure improvements or through improving the existing infrastructure network) affects the overall spatial structure of the city. We plan to extend tools we have developed in prior work (Allen, Arkolakis, and Takahashi (2018); Allen, Arkolakis, and Li (2014)) to characterize existence and uniqueness of the spatial equilibrium in this system and develop efficient algorithms to compute its solutions. Finally, using estimates of transportation investment costs described below we can use the equilibrium equations (10) and (11) as constraint to maximize the objective function 1 and compute the optimal placement of new infrastructure projects.

3 Using the Model with Detailed Data

We plan to use the with detailed traffic data that cities or local governments collect at a large scale. For example, the government of Connecticut collects detailed traffic information on tens of thousands of highways and secondary road segments.[3] In addition, it is possible to consider various transportation modes based on a discrete choice model of transportation mode choice developed in Allen and Arkolakis (2014) that can be incorporated in this approach. Finally, for the purposes of obtaining more precise measures of costs of building and improving road infrastructures the authors plan to use detailed estimates arising from the HERS model developed by the Federal Highway Administration to provide reliable estimates of infrastructure costs (see Alam, Timothy, and Sissel (2005)).

4 Team and Financial Disclosure

The research team is composed of Treb Allen (Dartmouth and NBER) and Costas Arkolakis (Yale and NBER). We plan to hire a part-time research assistant for data collection and computer simulations. We have no financial ties to the transportation industry or other conflict of interest.

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[3] See, https://tminfo-dot.ct.gov/TMINFO/index. The website collects information on each segment of city roads and the number of vehicle using the roads at different time of the day. For the city of New Haven alone, the website contains traffic information for more than 400 road segments.