Skewness in the macroeconomy: Three facts point to one model

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Abstract

We establish three facts about volatility and skewness in the economy: (1) sector and aggregate output growth is negatively skewed, with the magnitude increasing with the level of aggregation; (2) the cross-sectional variance of output growth is countercyclical and the cross-sectional skewness is procyclical; (3) when a sector shrinks, it subsequently covaries more with other sectors. Those facts can all be generated by a multisector equilibrium model with the key feature that production inputs are gross complements. Three alternative models that have been proposed to generate skewness and stochastic volatility are unable to match all three facts even qualitatively.

1 Introduction

A defining feature of the business cycle is the existence of recessions as distinct episodes. Rather than simply experiencing symmetric random fluctuations up and down in output, unemployment, and other measures of the state of the economy, there are sharp declines and relatively smooth expansions: growth in real activity is skewed left.[1] At the same time, many measures of volatility, both in the aggregate and the cross-section, are countercyclical. As a mathematical matter there is a mechanical link between skewness and countercyclical volatility – high volatility in bad times leads to a long left tail of outcomes – but few models capture that feature of the economy.

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[1]See Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), and Berger, Dew-Becker, and Giglio (2019), among others.

It is also not just aggregate output that is skewed left: sector- and firm-level activity display the same behavior. Similarly, while aggregate output features countercyclical volatility, the variance of the *cross-sectional* distribution is also countercyclical (and, in addition, the skewness is procyclical).

This paper begins by systematically documenting a set of key facts about aggregate and cross-sectional variance and skewness:

1. Economic activity is skewed left, with negative skewness increasing with the level of aggregation;

2. The variance of the cross-sectional distribution of activity is countercyclical, while the skewness is procyclical (and the behavior is the same for the part of sector activity orthogonal to the aggregate)

3. When activity in sector i declines, sector i becomes more central in the covariance matrix of sector activity, in the sense that elements of the covariance matrix in its row and column rise compared to other elements.

While left skewness in economic activity is well known, this is the first paper to show that it systematically increases with the level of aggregation, which will be a key fact helping distinguish models. The cyclicality of cross-sectional variance and skewness is also studied by Salgado et al. (2018), though the extension to sector-specific residuals is novel to this paper and again useful in distinguishing models. Finally, the time-series variation in sector covariances is entirely novel to this paper.

We show that the three facts are naturally generated in an aggregative model – where individual sectors of the economy are explicitly modeled and aggregate to produce total GDP (e.g. Long and Plosser (1983)) – and in which production at the sector and aggregate level is complementary in inputs – the elasticity of substitution is less than 1. To build intuition, consider an extreme case where aggregate output is Leontief in a range of inputs (consistent with the estimates of Atalay (2017)), and that production of the inputs depends only on sector-specific productivity shocks. Then the distribution of output is the distribution of the minimum of the technology shocks, which is in general skewed left. That minimum – the productivity of the worst-performing sector – becomes a common component that affects all sectors. When the sectors share a skewed common shock but also have idiosyncratic unskewed shocks, aggregate output is more skewed than sector output.

The cross-sectional facts are then generated simply because when the realized distribution of productivity shocks is by chance wider in a particular period, the minimum will tend to be lower (that is, the sample variance and minimum of a set of random draws are negatively correlated). So cross-sectional volatility is countercyclical. Cross-sectional skewness is

procyclical for the same reason.

We include the third fact precisely because it directly addresses the key mechanism in the model. In a model with complementarities, when the supply of one input shrinks, that input becomes more important, in the sense that output becomes relatively more sensitive to it and less sensitive to other inputs. In the Leontief case, in fact, the level of aggregate output depends only on the supply of the most constrained input. Because the sectors that shrink become more relevant, they also covary more strongly with any other sector that buys their output (including final production). This also implies that increases in dispersion are associated with higher future aggregate volatility, which we also show holds empirically.

A single simple idea, then, that production features complementarities, explains aggregate skewness, the time-series patterns of cross-sectional volatility and skewness, and the response of the cross-sector covariance matrix to shocks. Another way to put it is that the model is overidentified relative to the three empirical facts. A single parameter – the elasticity of substitution across inputs – determines the signs of the time-series skewness of output, the cyclicality of cross-sectional variance, and the relationship between a sector's centrality and its past shocks.

As noted above, there are other models that have been proposed to match some of the facts that we study. What we find, though, is that none of the alternative models we examine can match all of the facts simultaneously, and those that are closest to success are not as parsimonious, in having a single parameter determine all three empirical patterns. We study three basic specifications:

a. Skewed aggregate shocks (either fundamental shocks or perhaps skewness in a universal input, such as finance, as in Brunnermeier and Sannikov (2014)) with differential loadings across sectors;

b. Stochastic volatility and skewness in aggregate and idiosyncratic shocks (Bloom (2009));

c. Concave responses to shocks (Ilut, Kehrig, and Schneider (2018)).

Each of the models under suitable restrictions, is able to generate a subset of the facts that we document, but they all fail on at least one. The comparison with the alternative models also helps illustrate precisely why the aggregative model is able to fit the data. In brief, fact 1 (skewness increases with aggregation) implies the existence of a skewed common component in output, while facts 2 and 3 imply that the common component is endogenous to sector-level shocks. Explicit aggregation is therefore critical because it creates that endogeneity.

The results in this paper have important implications for how to think about volatility

and skewness in the economy.[2] A large literature studies stochastic volatility (and, more recently, stochastic skewness), often implicitly assuming that because the dispersion in observed distributions changes over time, that means that there are "uncertainty shocks." For example, Bloom et al. (2018) use time variation in a number of cross-sectional sample moments (e.g. the realized cross-sectional standard deviation of sales growth) to quantify the cyclicality of uncertainty. The results here show that time-variation in moments – and cyclicality – need not have anything to do with uncertainty. Rather, increases in the dispersion of realized shocks may simply be associated with declines in output because of strong concavity in the production function.

The model and empirical results also demonstrate that the economy in an important sense does not have a single fixed network structure – in terms of the covariance of output across sectors – and in fact that the variation is important in causing asymmetries in outcomes. Baqaee and Farhi (2019) study this point extensively, and our results provide further empirical support. The reason that output is skewed left and stochastically volatile is precisely because a single sector – or set of sectors – will occasionally receive a negative shock and become (approximately) a limiting factor in production. It is exactly that change in the network – that the negatively shocked industry becomes central – that creates a recession, negative skewness, and volatility. So while it is common for studies of economic networks to use snapshots of the inter-sector trade structure from the input-output tables, such an analysis can miss important variation in linkages.

Finally, and perhaps most importantly, the analysis helps understand why there are business cycles. Again, a defining characteristic of business cycles is that they feature short and sharp contractions. Hypotheses like skewness in aggregate shocks or concavity in firm responses to shocks have been proposed to match that fact, but they turn out to fail on other dimensions. The necessary concavity arises naturally when there is complementarity in production.

The paper is related to a number of active literatures. First, as described above, is work on time-variation in time-series and cross-sectional moments of output. Second is an emerging literature on production networks with complementarities in production. Atalay (2017) and Atalay, Drautzburg, and Wang (2018) estimate the elasticity of substitution using data on sector production and input-output linkages, finding extremely strong complementarities (up to the point of a Leontief production function). Baqaee and Farhi (2019) theoretically

[2] For work on unconditional skewness, see Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), and Berger, Dew-Becker, and Giglio (2019). For recent work on time-varying volatility in the real economy, see Justiniano and Primiceri (2008), Clark and Ravazzolo (2015), and Schorfheide, Song, and Yaron (2018). work on time-varying cross-sectional moments includes Guvenen, Ozkan, and Song (2014), Salgado et al. (2018), and Dew-Becker and Giglio (2019).

study models with complementarities and show in a calibration that they can generate a realistic level of skewness at the aggregate level, in addition to studying how covariances and the network structure change over time in general production frameworks. Carvalho et al. (2016) study the economy of Japan following the 2011 earthquake in a network model with non-unitary elasticities of substitution.

The remainder of the paper is organized as follows. Section 2 documents the three empirical facts. Section 3 presents a simple and stylized model with complementarities and formally shows that it can match the three empirical facts. In section 4 we examine the predictions of three other models of economic skewness. Section 5 examines a model with complementarities in a more quantitatively realistic calibration, and section 6 concludes.

2 Empirical facts

This section establishes three facts about variance, covariance, and skewness in the economy:

1. Economic activity is skewed left, with negative skewness increasing with the level of aggregation.

2. The variance of the cross-sectional distribution of activity is countercyclical, while the skewness is procyclical.

3. When activity in sector i declines, sector i becomes more central in the covariance matrix of sector activity, in the sense that elements of the covariance matrix in its row and column rise compared to other elements.

2.1 Data

We focus on measures of activity that have data at monthly frequency and are measured at a high level of sectoral detail. The two-main series are industrial production (from the Federal Reserve), which is measured to the four-digit NAICS level in manufacturing industries, and employment (from the Current Employment Survey of the BLS), which is measured at up to the six-digit NAICS level and covers the entire economy. For industrial production, we follow Foerster, Sarte, and Watson (2011) and study data since 1972. For employment, the sample with detailed NAICS coverage begins in 1990.

Some of the results below also use data on stock returns. We obtain daily sector-level returns from Kenneth French's website. Unless otherwise specified, we use the set of 49 industries. The return on the overall market is the CRSP value-weighted portfolio, and the risk-free rate is measured using the 30-day Treasury bill rate (all results are for returns in excess of the risk-free rate).

2.2 Time-series skewness

The fact that aggregate output and employment are skewed left has been established in previous work.[3] Not surprisingly, sector-level growth rates are also generally skewed left. We now show, though, that they are less negatively skewed than aggregate growth rates, and that the magnitude of the negative skewness increases with the level of aggregation.[4]

Table 1 reports the average level of skewness at different levels of aggregation for industrial production growth, employment growth, and stock returns. That is, for each sector i, we calculate the coefficient of skewness for the time series of its, say, industrial production growth. The table then reports the mean and median of those skewness coefficients at each level of aggregation. For industrial production and employment that corresponds to different levels of NAICS detail, while for stock returns it corresponds to more versus less detailed industry portfolios. Standard errors are reported in brackets.[5]

The top panel reports results for industrial production. The skewness of total industrial production growth is -1.18. At the two-digit level – just three sectors: durable and non-durable manufacturing and mining – average skewness is -0.96. At the three- and four-digit levels, where there are 43 and 81 total sectors, respectively, skewness falls to -0.55 then -0.45. Finally, at the five-digit level skewness is only -0.41. A similar pattern holds for medians and using quarterly instead of monthly growth rates, and in both cases the differences between the skewness at the various levels of aggregation are themselves statistically significant. In other words, at every level, as aggregation increases, skewness becomes more negative.

Panel B of table 1 shows that similar results hold for employment growth. As the level of aggregation increases, skewness gets monotonically more negative.

Finally, panel C of table 1 reports similar results for aggregate and sector-level stock returns. In both daily and monthly returns, the pattern is the same as those for employment and industrial production: skewness is negative on average at all levels of aggregation, and it becomes more negative as the level of aggregation increases. The magnitudes of the skewness coefficients are also similar to those observed for industrial production and employment.

[3] Berger, Dew-Becker, and Giglio (2019), show that growth rates of employment, capacity utilization, industrial production, GDP, durable and non-durable consumption, and residential and nonresidential investment are all skewed left. Furthermore, returns on the S&P 500 are skewed left, as is their option-implied distribution. Morley and Piger (2012) provide a much more extensive analysis, focusing on asymmetry in the output gap – that is, on skewness in levels, rather than growth rates – and finding similar results – while Sichel (1993) provides an earlier analysis distinguishing asymmetry in levels from growth rates. See also references therein for the literature on business cycle asymmetry.

[4] Ilut, Kehrig, and Schneider (2018) examine employment and find skewness at both the firm and aggregate levels, but, as discussed below, do not emphasize the difference between the two.

[5] The standard errors are calculated with a blockwise jackknife that clusters by date. Specifically, each jackknife replication removes 50 consecutive months of data from the sample – the same 50 months for all sectors – and we iterate over all possible starting months for the excluded dates.

So far we have discussed skewness of the raw data. It is also interesting, though, to look at skewness of sector-specific growth rates. We do that by examining residuals from a time-series regression of each sector's growth rate on aggregate growth. Specifically, if $x_{i,t}$ is growth in sector i in month t and $\bar{x}_t$ is growth in the aggregate in month t, we estimate the regression

$$x_{i,t} = a_i + b_i \bar{x}_t + \nu_{i,t}, \tag{1}$$

and table 1 reports skewness for the residuals $\nu_{i,t}$ in the column labeled "residuals".

The results are very different from what is observed for the raw growth rates. The residuals are still negatively skewed, but by much less than the raw growth rates, and with surprising stability across levels of aggregation. In other words, most of the negative skewness appears to come from each sector's exposure to a negatively skewed common factor ($\bar{x}$) – the sector-specific component contributes little, and does not affect the pattern across levels of aggregation. While that is natural from the perspective of some models, it will turn out to rule out others.

To summarize, then, we have two basic results regarding time series skewness of raw growth rates: measures of real activity are skewed to the left, and the magnitude of that skewness increases with the level of aggregation. Skewness of economy-wide measures is generally three times larger than skewness at the five-digit sector level. After controlling for the aggregate component, though, there is little skewness remaining in the residuals, $\nu_{i,t}$, and it does not increase with aggregation.

2.3 Realized cross-sectional variance and skewness

Our second empirical fact is that cross-sectional variance is countercyclical and cross-sectional skewness is procyclical.[6] In each month t, we calculate three statistics in the cross-sections of monthly growth rates of industrial production and employment (using the set of 117 sectors for IP from Foerster, Sarte, and Watson (2011) and four-digit NAICS industries for employment): variance, skewness, and Kelley skewness (the sum of the 90th and 10th percentiles minus twice the median).

It is important to emphasize that these are *realized* sample moments – they do not measure a conditional distribution. That is, the conditional probability density from which

[6] A similar fact is well known from the literature individual earnings (Storesletten, Telmer, and Yaron (2004); Guvenen, Ozkan, and Song (2014)). Salgado, Guvenen, and Bloom (2018) provide evidence both within and across countries on the cyclicality of the variance and skewness of cross-sectional outcomes. We contribute to that literature by providing novel evidence on the cyclicality of skewness in the cross-section of sectoral industrial production and employment growth, including looking at the sector-specific component, as above.

the sector growth rates are drawn could be constant over time. We are simply measuring sample moments – which are random variables – and examining their contemporaneous relationship with the state of the business cycle.[7]

Table 2 reports results from regressions of three cross-sectional statistics on an NBER recession indicator and aggregate employment growth as two different measures of the state of the business cycle (in separate regressions). Across the 12 different regressions, in all cases cross-sectional variance is countercyclical and cross-sectional skewness is procyclical. Similar results hold at alternative levels of aggregation for employment and industrial production). The R^2s range between 1 and 36 percent – they are lowest for the skewness coefficient, which is relatively sensitive to outliers and thus noisier, and highest for Kelley skewness, which is more robust to outliers than either of the two sample moments.

As in table 2, we also estimate the regressions using residuals from a regression of sector growth rates on aggregate growth (the $\nu_{i,t}$ from above). Contrary to table 1, in this case the results are largely unchanged looking at the cross-sectional distribution of residuals compared to raw growth rates. While the point estimates of cyclicality for the residuals are generally somewhat smaller than for raw growth rates, they are still mostly significantly different from zero and are within the 95-percent confidence bands for the cyclicality of the raw growth rates. So both raw growth rates and residuals have similar cyclicality for cross-sectional moments. Unlike what is observed in table 1, then, the behavior of the cross-sectional moments is not driven purely by the behavior of a common component; rather, it appears similar in both raw growth rates and residuals purged of the aggregate component.

Similar results also hold for stock returns. Using the sample of 49 industry portfolios, we calculate cross-sectional variance and skewness on each day and average those statistics within each month. We then regress the realized monthly cross-sectional variance and skewness on four leads, the current value, and 12 lags of market returns. Figure 1 plots the estimated coefficients from those regressions (confidence intervals are constructed using Newey–West standard errors with 24 lags).

The coefficients on the leading values of stock returns are close to zero and not statistically significant. These coefficients can be thought of as a placebo test. If market returns are close to unpredictable, then the coefficients on the leading values of market returns should be close

[7]The distinction between the conditional distribution and a realized moment is emphasized by Berger, Dew-Becker, and Giglio (2019). As an example, consider some mean-zero shock ε with a constant conditional distribution. One could look at the realization of ε_t^2 – essentially a sample variance – and ask how it correlates with other variables. For example, one might find that ε_t^2 is negatively correlated with ε_t. That does not mean that the conditional variance of ε_t (i.e. uncertainty) is countercyclical. Rather, it is just an indication that the distribution of ε_t is left skewed. So nothing in this subsection should be interpreted as measuring variation in conditional distributions. In the example in this footnote, doing so would require *forecasting* ε_t^2, not just looking at contemporaneous correlations.

to zero.

The coefficients on the current and lagged values of returns, on the other hand, are generally statistically significant, positive for cross-sectional variance and negative for skewness. So the figure again shows that cross-sectional variance is countercyclical and cross-sectional skewness is procyclical, where cyclicality here is measured based on covariance with market returns.

2.4 Conditional covariances

The final set of facts relates most directly to the key mechanisms in the structural model. This section shows that when a sector contracts, it can be thought of as becoming more central in the economy, in the sense that it becomes more volatile and covariance links to it become stronger relative to covariances between other sectors.

2.4.1 Empirical method

Define Σ_t to be the (unobservable) conditional covariance matrix of sector-level growth rates on date t. Define $\Sigma_{t,i}$ to be the sum of the i'th column of Σ_t, excluding the i,i element. $\Sigma_{t,i}$ is the average of the covariances of sector i with all other sectors (or, equivalently, the covariance of growth in sector i with average growth in all other sectors). When we say that sector i covaries more strongly with other sectors, we mean $\Sigma_{t,i}$ rises.

The goal in this section is to estimate a relationship of the form

$$\Sigma_{t,i} = a_{i,1} + b_1(L)\,\varepsilon_{i,t-1} + c_{1,t} + \tilde{\eta}^1_{i,t} \tag{2}$$

where $\varepsilon_{i,t}$ measures the innovation to the level of activity in sector i on date t, b_1 is a polynomial in the lag operator, L ($L^j x_{i,t} = x_{i,t-j}$), and $\tilde{\eta}^1_{i,t}$ is a residual. The problem is that $\Sigma_{t,i}$ is not observable. We therefore proxy for it with a simple date-t product, similar to the literature on heteroskedasticity and feasible GLS.

More specifically, define $\varepsilon_{i,t}$ to be the residual from an AR(p) model for growth in activity in sector i. We then proxy for $\Sigma_{t,i}$ with $\sum_{j\neq i}\varepsilon_{i,t}\varepsilon_{j,t}$. That product is a single-observation sample moment, with the property that $E\left[\sum_{j\neq i}\varepsilon_{i,t}\varepsilon_{j,t}\right] = \Sigma_{t,i}$. The regression that we actually estimate is then

$$\sum_{j\neq i}\varepsilon_{i,t}\varepsilon_{j,t} = a_{i,1} + b_1(L)\,\varepsilon_{i,t-1} + c_{1,t} + \eta^1_{i,t} \tag{3}$$

$\eta^1_{i,t}$ captures both the true residual, $\tilde{\eta}^1_{i,t}$, and also the measurement error in the dependent

variable, $E\left[\sum_{j\neq i}\varepsilon_{i,t}\varepsilon_{j,t}\right]-\Sigma_{t,i}$. The errors are therefore in general non-Gaussian. For example, if the ε are Gaussian, then the measurement error is related to a chi-square.

Because there may be common components across sectors in the innovations, $\varepsilon_{i,t}$, we include time fixed effects, $c_{1,t}$, in the estimation and cluster the standard errors by date. Similarly, some sectors will covary with others more strongly on average, so the constant $a_{i,1}$ is allowed to vary across sectors (i.e. the estimation includes sector fixed effects).

The inclusion of time fixed effects means that changes in $\Sigma_{t,i}$ must be interpreted as changes relative to $\Sigma_{t,j}$ for $j\neq i$. Specifically, the time fixed effect can be interpreted as taking out the mean across sectors of $\Sigma_{t,i}$. Since the date-t mean of $\Sigma_{t,i}$ is equal to the mean of all pairwise covariances, a positive value for $b_1(L)$ means that a positive shock to sector i raises its covariances *relative to those between other sectors.*

Because $\varepsilon_{i,t}$ appears on both sides of the regression, we include only lagged values on the right-hand side. Specifically, the specification of $b_1(L)$ is

$$b_1(L)=b_{1,1}+b_{1,2}L+b_{1,3}L^2 \tag{4}$$

Unlike the regressions in the previous section, these are all forecasting regressions – they predict moments of the joint distribution of the $\{\varepsilon_{i,t}\}$ (collected over i) conditional on information available on date $t-1$.

2.4.2 Results

The top-left panel of table 3 reports results for the regressions for $\Sigma_{t,i}$ (i.e. the $b_{1,j}$ coefficients) using the same set of sectors as table 2. For both industrial production and employment, an increase in activity in a given sector is followed by a decline in that sector's realized covariance with other sectors. The coefficients at all lags are negative in both cases. The coefficients are generally not individually significant, but their sum is in each case. To get a sense of the magnitude, the mean of the left-hand side variable in the regressions is 7.1×10^{-5}, while the standard deviation after controlling for the time and sector dummies is 5.3×10^{-4}. For $\varepsilon_{i,t}$, the standard deviation after controlling for the dummies is 4.0×10^{-2}. A coefficient of -3.6×10^{-4} means that a unit standard deviation increase in $\varepsilon_{i,t}$ is associated with a relative increase in the covariance of three percent of its standard deviation, or 20 percent of its mean. **[Need to update these numbers]**

2.4.3 Alternative using stock returns

The analysis above proxies for an expectation with a single realization (i.e. proxying for $\Sigma_{t,i} = E\left[\varepsilon_{i,t}\sum_{j\neq i}\varepsilon_{j,t}\right]$ with $\varepsilon_{i,t}\sum_{j\neq i}\varepsilon_{j,t}$). While that may yield a consistent estimator, it would be expected to be noisy. There is little choice in the matter, though, because industrial production and employment are only measured at the monthly frequency. A better situation would be if we had high-frequency observations, at a frequency much higher than that at which $\Sigma_{t,i}$ changes. In that case, $\Sigma_{t,i}$ could be measured based on many observations, instead of just one. Stock returns have high-frequency data, allowing for such a strategy.

Specifically, in each month we construct $\hat{\Sigma}_t$ as the sample covariance matrix of daily sector-level stock returns in month t. $\hat{\Sigma}_{t,i}$ is then, as before, the average of the elements of $\hat{\Sigma}_t$ in column i, excluding element i,i.

We then estimate the counterparts to the regressions the previous section:

$$\hat{\Sigma}_{t,i} = a_{1,i} + b_1(L)\, r_{i,t-1} + c_{1,t} + u_{1,i,t} \tag{5}$$

$$\hat{\sigma}^2_{i,t} = a_{2,i} + b_2(L)\, r_{i,t-1} + c_{2,t} + u_{2,i,t} \tag{6}$$

$$\log\hat{\sigma}^2_{i,t} = a_{3,i} + b_3(L)\, r_{i,t-1} + c_{3,t} + u_{3,i,t} \tag{7}$$

As before, the lag polynomials take the form $b_n(L) = \sum_{j=0}^{N} b_{n,j}L^j$

Again, the difference between the analysis here and in the previous section is that we are able to use high-frequency data on stuck returns to estimate the conditional covariances more accurately, instead of just relying on a single observation in each month. On the other hand, there are two potential drawbacks to this method. First, we have only half as many sectors for stock returns as for IP, and only one fifth as many as for employment. Second, stock returns are typically more difficult to model theoretically than output or employment, so the link to the underlying theory will end up weaker.

2.4.4 Results

Table 4 replicates table 3, with the regressions involving the return covariances. The coefficients are again all negative. They are more precisely estimated, and are generally individually statistically significant. So for returns, using high-frequency data giving more precise estimates of the covariance matrix, we obtain the same results as with employment and industrial production.

[Talk about the economic magnitude, similar to above]

3 Aggregative model with complementarities

This section presents our benchmark multi-sector production model that can match the three facts set out above. Its key characteristic, as we will show formally, is that there is effectively a common shock to output in all sectors, and that rather than being some exogenous process, it is endogenous to the sectoral shocks themselves. That is, it is specifically the explicit aggregation that causes the model to match the data.

In order to obtain analytic results, the model is highly stylized, with very strong forms of symmetry that yield simple solutions. The model is still rich enough to qualitatively match the facts above, but a quantitatively accurate model would require substantial enrichment. Section 5 examines numerical results in a more quantitatively realistic setting.

3.1 Economic structure

Output is produced in sector i according to the function

$$Y_i = z_i N_i^{1-\alpha} \left(\sum_j a_j x_{i,j}^{\gamma} \right)^{\alpha/\gamma} \tag{8}$$

where z_i is sector i's productivity, N_i is sector i's use of labor, $x_{i,j}$ is sector i's use of input j, $1/(1-\gamma)$ is the elasticity of substitution (so $\gamma < 1$), and a_j a coefficient that determines the relative importance of good j in production.

The underlying shocks are symmetrically distributed in the sense that

$$\log z_i \sim N\left(0, \sigma_z^2\right) \tag{9}$$

Aggregate consumption is

$$C = \left(\sum_j a_j x_{C,j}^{\gamma} \right)^{1/\gamma} \tag{10}$$

where $x_{C,j}$ is consumption of good j. The parameters of the final good production function are the same as for the individual sectors except that no labor is used ($\alpha = 1$) and productivity is constant (in other words, 10 can also be a feature of preferences, rather than a physical production function). There is no investment and the economy is closed, so gross domestic product is equal to C.

The resource constraint for each sector is

$$\sum_i x_{i,j} + x_{C,j} = Y_j \tag{11}$$

There are a number of major restrictions in this setup that allow for tractability. First, the assumption that production takes the constant elasticity form, though standard, is obviously highly restrictive. Second, and more specific to this model, the coefficients determining the importance of each input, a_j, are the same across sectors. While this assumption will yield tractability, it is obviously counterfactual. Third, the elasticities of substitution are the same in all sectors. Again, this assumption is made for analytic tractability.

There are at least three ways labor can be modeled: the aggregate supply can be fixed and it can adjust endogenously across sectors (e.g. Long and Plosser (1983)); it can be fixed within each sector (Baqaee and Farhi (2019)); or the wage can be fixed and the quantity allowed to vary freely (e.g. Blanchard and Gali (2010). We focus on the case where N_i is fixed, as in Baqaee and Farhi (2019), (and we simply normalize it to 1), but we also examine results in the other two cases. **[Need to do this.]**

The model can be solved either by defining prices for the various goods and maximizing profits in each sector or by finding the social planner's optimum.

3.2 Solution

The appendix shows that the model's solution for sector and aggregate output is (up to constant factors)

$$Y_i \propto \left(n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\frac{\gamma}{1-\alpha\gamma}} \right)^{\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}} a_i^{\frac{\alpha}{1-\alpha\gamma}} z_i^{\frac{1}{1-\alpha\gamma}} \tag{12}$$

$$C \propto \left(n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\frac{\gamma}{1-\alpha\gamma}} \right)^{\frac{1-\alpha\gamma}{\gamma}\frac{\alpha}{1-\alpha}} \tag{13}$$

$$\gamma = (\zeta - 1)/\zeta \tag{14}$$

$$\alpha = \mu \tag{15}$$

$$\zeta = 1/(1-\gamma) \tag{16}$$

$$1-\zeta = \frac{-\gamma}{1-\gamma} \tag{17}$$

$$Y \propto \left(\sum \exp\left(\frac{\zeta-1}{1-(1-\zeta)(1-\mu)} z_i\right)\right)^{\left(\frac{\zeta-1}{1-(1-\zeta)(1-\mu)}\right)^{-1}\frac{1}{1-\mu}} \tag{18}$$

$$= \sum \exp\left(\frac{\gamma}{1-\alpha\gamma} z_i\right)^{-\left(\frac{1-\gamma\alpha}{\gamma}\right)\frac{1}{(1-\alpha)}} \tag{19}$$

$$\frac{\gamma}{1-\gamma\alpha} \tag{20}$$

$$\left(\sum \exp\left(\frac{\gamma}{1-\gamma\alpha} z_i\right)\right)^{\frac{1-\gamma\alpha}{\gamma}\frac{1}{1-\alpha}} \tag{21}$$

Aggregate output is a CES aggregate over the sector productivities, with exponent $\gamma/(1-\alpha\gamma)$. Since $(1-\alpha\gamma) > 0$ for $\gamma < 1$, the sign of that exponent depends just on the sign of γ. When the inputs are gross complements, $\gamma < 0$, aggregate output is a complementary function of the sector productivities.

Sector-level output can be decomposed into two terms. The first is a non-negative power of aggregate output (positive for $\gamma < 1$). The second is a positive power of z_i. So each sector's output has in essence a common component and a sector-specific component. In that respect, it is similar to many reduced-form statistical specifications. Unlike here, though, those models almost always specify the common component as being independent of (or at least orthogonal to) the sector-specific shocks. In what follows, the endogeneity of the common component to the sector shocks will become clear.

3.3 Implications for observables

We now examine the model's ability to match the three empirical facts developed above.

The empirical analysis is entirely in terms of growth rates due to the fact that measures of real activity are non-stationary (and not even cointegrated across sectors). The model is in general very difficult to analyze in terms of growth rates, however. In linear Gaussian models, the distinction between levels and growth rates is nearly immaterial, so typical theoretical analyses have little trouble with the distinction. Ilut, Kehrig, and Schneider (2018), who also study a nonlinear model, simply specify the entire structure in terms of growth rates from the beginning, but that does not work in the present setting because aggregation happens in terms of levels.

We therefore take the following approach. For the first two facts – regarding time-series skewness and the cyclicality of cross-sectional moments – we analyze the model purely in terms of levels. Appendix D.1 shows that in a particular limiting case of an autoregressive time series with fat-tailed shocks, the distribution of growth rates is tightly linked to the distribution of the levels – they can actually have the same moments.

To evaluate whether the theoretical results derived in terms of levels are actually revealing for growth rates, we examine both a simple numerical example of the model in appendix D.2, and also a more serious calibration in section 5, which confirm that the analytic results that we obtain for the model in levels do in fact also apply in terms of growth rates.

Finally, for the third fact – time-variation in conditional covariances – we are able to derive results in a continuous-time version of the model, which we also show hold in the calibration.

3.3.1 Unconditional skewness

The log of aggregate output is

$$\log C = \frac{1-\alpha\gamma}{\gamma}\frac{\alpha}{1-\alpha}\log\left(n^{-1}\sum_j a_j^{\frac{1}{1-\alpha\gamma}}\exp\left(\frac{\gamma}{1-\alpha\gamma}\varepsilon_j\right)\right) \tag{22}$$

The appendix shows that the skewness of $\log C$ is an increasing function of γ, and that its sign is the same as that of γ. So when $\gamma < 0$ – when the inputs are gross complements – aggregate output is skewed left. When inputs are gross substitutes, on the other hand, aggregate output is skewed right. So in order for this model to generate negative skewness, γ must be negative.

The log of sector output is

$$\log Y_i = \frac{1-\gamma}{1-\alpha\gamma}\log C + \frac{1}{1-\alpha\gamma}\varepsilon_i + \text{constants} \tag{23}$$

Because sector output is equal to aggregate output plus a Normally distributed shock, it inherits the skewness of aggregate output, but with a smaller magnitude. The model therefore matches the result, when $\gamma < 0$, that output is skewed left, with the magnitude increasing as the level of aggregation increases.

In addition to the results on skewness of overall growth rates, table 1 also reports skewness for residuals, showing that once the aggregate component is controlled for, sector-level growth rates are substantially less skewed and that there is no increase in skewness with aggregation. Here, the residual component of sector output, after controlling for aggregate output, is

exactly $\frac{1}{1-\alpha\gamma}\varepsilon_i$. We have assumed that there is no skewness in the ε_i, so there is then no skewness in the residuals. That is more extreme than what we find empirically, but it is qualitatively consistent in that skewness is greater for total output, Y_i, than the residuals, ε_i.

3.3.2 Realized cross-sectional variance and skewness

The model generates countercyclical volatility and procyclical skewness essentially because aggregate output is a concave function of the shocks ε_i. All else equal, an increase in the dispersion in the ε_i yields a decline in output.

The appendix proves the following result:

Proposition 1 *As the number of sectors grows large, and with $a_j = 1$ for all j, the coefficient in a regression of cross-sectional variance on log output is*

$$\frac{\operatorname{cov}\left(n^{-1}\sum_i\left(\log Y_i - n^{-1}\sum_j \log Y_j\right)^2, \log C\right)}{\operatorname{var}(\log C)} \rightarrow \sigma_\varepsilon^4\left(\frac{\gamma}{1-\alpha\gamma}\right)^3\left(\exp\left(\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\sigma^2\right)-1\right)^{-1} \quad (24)$$

$$\approx \sigma_\varepsilon^2\left(\frac{\gamma}{1-\alpha\gamma}\right) \quad (25)$$

and the coefficient in a regression of cross-sectional skewness and log output is

$$\operatorname{cov}\left(\frac{n^{-1}\sum_i\left(\log Y_i - n^{-1}\sum_j \log Y_j\right)^3}{\left[n^{-1}\sum_i\left(\log Y_i - n^{-1}\sum_j \log Y_j\right)^2\right]^{3/2}}, \log C\right) \rightarrow \sigma_\varepsilon^3\left(\frac{\gamma}{1-\alpha\gamma}\right)^4\left(\exp\left(\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\sigma^2\right)-1\right)^{-1} \quad (26)$$

$$\approx \sigma_\varepsilon\left(\frac{\gamma}{1-\alpha\gamma}\right)^2 \quad (27)$$

When $\gamma < 0$, the first coefficient is negative and the second positive.

[This can probably be extended to allow more general a_j using the Lyapunov CLT. Specifically, I think the effect is to multiply the two coefficients by the factor $\left(n^{-1}\sum_{j=1}^n a_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^2\left(n^{-1}\sum_{j=1}^n a_j^{2\frac{\gamma}{1-\alpha\gamma}}\right)$ I'm not 100 percent sure, though, and my math is a bit informal. See the appendix]

The proposition shows formally that in this model, when $\gamma < 0$ so that inputs are complements, cross-sectional skewness is procyclical while cross-sectional variance is countercyclical.

A simple way to see why is to note that the Taylor series for $\log C$ in terms of $\frac{\gamma}{1-\alpha\gamma}$ is

$$\log C = \frac{\alpha}{1-\alpha} \sum_{j=1}^{\infty} \frac{1}{j!} \left(\frac{\gamma}{1-\alpha\gamma} \right)^{j-1} K_{\varepsilon}^{(j)}(0) \tag{28}$$

where $K_{\varepsilon}^{(j)}$ is the jth derivative of the empirical cumulant generating function of $\{\varepsilon\}$.[8] So output is linear in the empirical cumulants of the set of realized productivity shocks, $\{\varepsilon\}$. When $\gamma < 0$, the coefficients on the even cumulants are negative, while the coefficients on the odd cumulants are positive. So an increase in the cross-sectional variance, $K_{\varepsilon}^{(2)}(0)$, holding all other cumulants fixed, mechanically reduces output. Similarly, an increase in the third central moment, $K_{\varepsilon}^{(3)}(0)$, all else equal, increases output.

[Provisional: When the a_j can differ from 1, the signs of the coefficients are unchanged. Variation in the relative importance of the sectors just affects the magnitude of the coefficients. Holding the mean of $a_j^{\frac{\gamma}{1-\alpha\gamma}}$ fixed, an increase in the variance of $a_j^{\frac{\gamma}{1-\alpha\gamma}}$ – the variance of the weights on the technology shocks – reduces the regression coefficient. That is, cross-sectional variance and skewness move less for a given change in output.]

Conversely, when $\gamma > 0$, the proposition above shows that cross-sectional variance becomes procyclical, while skewness remains procyclical. In terms of the cumulants, that is because the sign of the coefficients on the odd cumulants is always positive, while the sign of the coefficients on the negative cumulants is equal to the sign of γ. So in this model, the sign of the cyclicality of cross-sectional variance identifies the sign of γ.

As with the empirical results, a key observation is that the cyclicality of cross-sectional variance and skewness – whether they are positive or negative – has nothing to do with changes in conditional distributions or "time-varying uncertainty". The cross-sectional distribution is a random variable that is correlated with output. All the fundamental shocks have constant volatility, and there is no sense in which there are uncertainty shocks here.

Finally, recall that table 2 also shows that the cross-sectional moments of the residuals in sector growth rates, after controlling for the aggregate component, display the same cyclicality as the cross-sectional moments of overall sector growth. The model generates the same result. The reason is simple: the cross-sectional distribution of the sector growth rates depends entirely on the cross-sectional distribution of the sector residuals, ε_i. The cross-sectional moments are in fact exactly the same for total growth rates and residuals, so the cyclicality is exactly the same. The model thus matches the findings in table 2 both for overall growth rates and for residuals.

[8] $K_{\varepsilon}^{(j)}(t) \equiv \frac{d^j}{dt^j} \log\left(n^{-1} \sum_i \exp(t\varepsilon_i)\right)$

There are two notable features of the results in this section. First, they continue to rely on $\gamma < 0$, showing that the model is overidentified. Second, they rely on the fact that the common component is a function of the sector shocks. If aggregate output were independent of the sector-specific shocks – i.e. if the sector shocks in some sense "washed out" – then proposition 1 would not hold. The explicit aggregation in the model is therefore critical to its ability to match the data.

3.3.3 Conditional covariances

The model description above does not say anything about the time series dependence of the levels of productivity, z_i. In order to analyze the conditional covariances we examined empirically, we consider here a version of the model in which the z_i follow geometric diffusions with constant volatility (i.e. continuous-time Gaussian ARMA processes). We then define the instantaneous covariance of growth rates between any pair of sectors to be

$$E_t \left[d \log Y_{i,t} d \log Y_{k,t} \right] \tag{29}$$

The total covariance of sector i with all other sectors is then $\sum_{k \neq i} E \left[d \log Y_i d \log Y_k \right]$.

Proposition 2 *Sector i's covariance with other sectors increases relative to covariances between other sectors when z_i falls for $\gamma < 0$ (the result is reversed when $\gamma > 0$). Specifically,*

$$\frac{d}{dz_i} \left[\begin{array}{c} \sum_{k \neq i} E \left[d \log Y_i d \log Y_k \right] \\ - \sum_{k \neq j} E \left[d \log Y_j d \log Y_k \right] \end{array} \right] \propto \frac{\gamma \left(1 - \gamma \right)}{\left(1 - \alpha \gamma \right)^3} \tag{30}$$

for all pairs of distinct i and j, where the constant of proportionality is positive. The sign of that derivative is equal to the sign of γ.

Intuitively, proposition 2 follows from the fact that when a sector receives a negative shock, it becomes relatively more important in determining variation in aggregate output when $\gamma < 0$. The importance of a sector i is quantified by

$$\frac{n^{-1} a_i^{\gamma/(1-\alpha\gamma)} z_i^{\gamma/(1-\alpha\gamma)}}{n^{-1} \sum_j a_j^{\gamma/(1-\alpha\gamma)} z_j^{\gamma/(1-\alpha\gamma)}} \tag{31}$$

The sign of the derivative of that ratio with respect to z_i is the sign of γ. Since all covariation between sectors comes through variation in aggregate output, the relative increase in the importance of the given sector increases its covariances with all other sectors and reduces the covariances of other sectors with each other.

Proposition 2 formalizes the idea that under complementarity, when a sector receives a negative shock it becomes more central. That result is not universal – there are production networks for which it can fail to hold. However, it is natural in the baseline symmetric case, and it is apparently a reasonable description of the data. As with proposition 1, proposition 2 follows from the fact that the common component of output is endogenous to the sector-specific shocks, again emphasizing the importance of explicit aggregation.

3.3.4 Overidentification

Nearly every result derived in this section depends on the parameter γ. Specifically, the sign of the skewness of output growth, the sign of the cyclicality of the cross-sectional variance of output growth (both total growth and residuals with respect to aggregate output), and the sign of the response of a sector's covariances to its past growth rate all depend on the sign of γ. In that sense, the model is overidentified. All three of the empirical results (and a fourth involving the residual growth rates) depend on a single parameter.

That is the formal sense in which the model is overidentified or parsimonious. γ cannot be chosen completely freely. As soon as it is selected to match one moment, that predicts the signs of three others. That parsimony will not be shared by all of the alternatives below.

4 Alternative models of skewness

This section examines alternative models of skewness. We examine relatively stylized forms of models meant to capture different potential economic mechanisms that could be driving aggregate or cross-sectional skewness. The analysis shows that none of the alternatives are able to match the three sets of facts about volatility and skewness presented above, which the model with complementarity is able to match.

More specifically we examine three general classes of models:

1. Skewed aggregate shocks.
2. Sector output is a concave function of underlying shocks.
3. Time-varying distributions for aggregate and sectoral shocks.

While these models are all appealing for various reasons, we will show that in baseline cases, without significant enrichment, they do not generate the three facts presented above. There is always a reduced-form model that can match any pattern of facts we may observe at the aggregate and sector levels. The claim here cannot be that no other model, or no modification of the models below, can match the empirical facts. Rather, it is simply that

among a set of prominent specifications, the model with complementarity is unique in its parsimony and qualitative fit to the data.

4.1 Skewed aggregate shocks

Consider a simple empirical model for sector output, $Y_{i,t}$, which could be generated by a number of different structural models:[9]

$$Y_{i,t} = b_i \varepsilon_t + \mu_{i,t} \tag{32}$$

where b_i is sector i's loading on the aggregate shock ε_t, and $\mu_{i,t}$ is a sector specific shock. Suppose ε_t is skewed to the left, while $\mu_{i,t}$ has a symmetrical distribution and is independent of ε_t, and both shocks have zero mean. This model seems natural in light of the result in table 1 that the time-series skewness comes from the common component, rather than sector-specific residuals.

We now examine the model's ability to match the three empirical facts.

4.1.1 Increasing negative skewness with aggregation

In the limit as the number of sectors grows large, if they aggregate with weight $1/n$, we have, informally,

$$Y_t = n^{-1} \sum_{i=1}^{n} Y_{i,t} \to \left(n^{-1} \sum_{i=1}^{n} b_i \right) \varepsilon_t \tag{33}$$

So both Y_t and $Y_{i,t}$ are negatively skewed. Furthermore, it follows from the fact that $\mu_{i,t}$ has a symmetric distribution and is independent of ε_t that Y_t has a more negative skewness coefficient than $Y_{i,t}$.[10] In addition, the the $\mu_{i,t}$ are the residuals of sector output after

[9] One possibility is that there are fundamental shocks to the economy, e.g. technology or policy shocks, that are skewed left and can be represented by ε_t (e.g. rare disasters models (Rietz (1988)**?**, Barro (2006)**?**) or smaller business-cycle frequency shocks as in Berger, Dew-Becker, and Giglio (2019)). Another micro-foundation would be that some input to production used by all sectors, e.g. the output of the financial sector (financial intermediation) is skewed to the left. For example, the financial sector might face occasionally binding constraints, causing its ability to provide funding to the rest of the economy to face occasional crashes (e.g. Brunnermeier and Sannikov (2014), Kocherlakota (2000)).

[10] Specifically, $\text{var}\left(b_i \varepsilon_t + \mu_{i,t}\right) > \text{var}\left(b_i \varepsilon_t\right)$. Furthermore,

$$\begin{aligned} E\left[\left(b_i \varepsilon_t + \mu_{i,t}\right)^3\right] &= E\left[\left(b_i \varepsilon_t + \mu_{i,t}\right)^3\right] && (34) \\ &= E\left[\left(b_i^3 \varepsilon_t^3 + \mu_{i,t}^3 + 3 b_i \varepsilon_t \mu_{i,t}^2 + 3 b_i^2 \varepsilon_t^2 \mu_{i,t}\right)\right] && (35) \\ &= E\left[b_i^3 \varepsilon_t^3\right] && (36) \end{aligned}$$

which immediately then implies that the skewness coefficient of $b_i \varepsilon_t + \mu_{i,t}$ is smaller than that of $b_i \varepsilon_t$.

controlling for the common component, the residuals are unskewed. This model can thus generate the first fact, both for overall growth rates and for residuals.

4.1.2 Countercyclical cross-sectional variance and procyclical cross-sectional skewness

On any date, the cross-sectional variance and skewness of sector output is

$$\begin{aligned} \text{var}_t\left(Y_{i,t}\right) &= \varepsilon_t^2 \,\text{var}\left(b_i\right) + \text{var}_t\left(\mu_{i,t}\right) && (37) \\ skew_t\left(Y_{i,t}\right) &= skew_t\left(b_i\varepsilon_t + \mu_{i,t}\right) && (38) \end{aligned}$$

When the number of sectors is large, the cross-sectional variance of $\mu_{i,t}$, $\text{var}_t\left(\mu_{i,t}\right)$, converges to a constant, while the term $\varepsilon_t^2 \,\text{var}\left(b_i\right)$ will vary over time as long as $\text{var}\left(b_i\right) > 0$. That is, as long as there is variation across sectors in loadings on the aggregate shock. Moreover, $\varepsilon_t^2 \,\text{var}\left(b_i\right)$ is countercyclical in the sense that

$$corr\left(\varepsilon_t^2 \,\text{var}\left(b_i\right), Y_t\right) < 0 \tag{39}$$

because ε_t is skewed left (since $E\left[\varepsilon_t^3\right] < 0 \Rightarrow corr\left(\varepsilon_t, \varepsilon_t^2\right) < 0$).

The cross-sectional skewness of sector output depends on the skewness of $b_i\varepsilon_t$. If b_i is skewed to the right, then the cross-sectional skewness is procyclical. That is

$$skew\left(b_i\right) > 0 \Rightarrow corr\left(skew_t\left(Y_{i,t}\right), Y_t\right) > 0 \tag{40}$$

So this model can generate countercyclical volatility and procyclical skewness as long as b_i differs across sectors and its distribution is skewed to the right.

Recall, though, that the empirical results in table 2 apply not just to sector growth rates, but also to residuals from regressions of sector growth rates on aggregate growth. In this case, that regression (see equation (1)) identifies $\mu_{i,t}$. If ε_t and $\mu_{i,t}$ are independent, then the cross-sectional moments of the $\mu_{i,t}$ are acyclical.

In other words, then, a model with skewed aggregate shocks can generate countercyclical volatility and procyclical skewness (with $skew\left(b_i\right) > 0$)) in sector output, $Y_{i,t}$, but it cannot replicate the empirical result that the cross-sectional distribution of residuals from regressions of sector output on aggregate output has the same cyclical properties. So at best the model only get half-way to matching our second empirical fact.

The failure of this model to match the cyclicality of the cross-sectional distribution of the residuals is instructive. The basic structure of this model is superficially similar to the

equilibrium of the aggregative model – it is nearly a reduced form representation. There is a critical difference, however: the common component here is exogenous, and independent of the sector-specific shocks, whereas in the aggregative model, the common component is endogenous to the sector-specific shocks.

4.1.3 Sector covariances rise following negative shocks

Under this model, the covariance between any pair of sectors is

$$\text{cov}\left(Y_{i,t}, Y_{j,t}\right) = b_i b_j \text{ var}\left(\varepsilon_t\right) + \text{cov}\left(\mu_{i,t}, \mu_{j,t}\right) \tag{41}$$

Nothing about the model therefore implies that when a sector shrinks its covariance with other sectors should rise relative to their covariances with each other. Obviously the model could be augmented so that the covariances of the idiosyncratic shocks, or the loadings b, change over time, but there is nothing fundamental about the hypothesis that aggregate shocks are skewed to the right that would require such a scenario.

The failure of the model on this dimension is again a result of the independence of the aggregate and sector-specific components.

4.1.4 Summary

A model with negatively skewed aggregate shocks naturally generates aggregate skewness. It can potentially generate time-varying cross-sectional moments, under specific conditions on the loadings on the aggregate shock, but it does not naturally generate time-varying moments in the idiosyncratic component of sector output ($\mu_{i,t}$), nor does it generate time-varying covariances. The model's ability to fit the facts on time-series skewness is due to the assumption of a skewed common shock. Its failure on the other two dimensions is due to the common shock being independent of the sector shocks.

4.2 Sector output is a concave function of symmetric shocks

Ilut, Kehrig, and Schneider (2018) study aggregate and firm-level skewness and argue that it can be generated by a model in which firms have concave responses to economic shocks, such that firm output or employment takes the form

$$Y_{i,t} = f\left(\varepsilon_t + \mu_{i,t}\right) \tag{42}$$

where ε_t is again an aggregate shock and $\mu_{i,t}$ is an idiosyncratic shock.[11] The shocks are mean-zero and independent with symmetrical distributions. The function f is assumed to be smooth, strictly increasing, and strictly concave.

4.2.1 Unconditional skewness

It is straightforward to show that sector (or firm) output, $Y_{i,t}$, is skewed left, simply due to the concavity of f. However, skewness in this model decreases with the level of aggregation. To see why, consider a second-order approximation to sector output,

$$Y_{i,t} \approx f(0) + f'(0)\left(\varepsilon_t + \mu_{i,t}\right) + \frac{1}{2}f''(0)\left(\varepsilon_t + \mu_{i,t}\right)^2 \tag{43}$$

As before, we model aggregate output as the sum of sector outputs and assume that the number of sectors is large, which implies

$$\begin{aligned} Y_t &\approx f(0) + f'(0)\,\varepsilon_t + \frac{1}{2}f''(0)\,\varepsilon_t^2 + f'(0)\,n^{-1}\sum_{i=1}^{n}\mu_{i,t} + \frac{1}{2}f''(0)\,2\varepsilon_t n^{-1}\sum_{i=1}^{n}\mu_{i,t} + \frac{1}{2}f''(0)\,n^{-1}\sum_{i=1}^{n}\mu_{i,t}^2 \qquad (44)\\ &= f(0) + f'(0)\,\varepsilon_t + \frac{1}{2}f''(0)\left(\varepsilon_t^2 + \operatorname{var}\left(\mu_{i,t}\right)\right) \qquad (45)\end{aligned}$$

The quadratic term in $Y_{i,t}$ is $\frac{1}{2}f''(0)\left(\varepsilon_t + \mu_{i,t}\right)^2$, while in Y_t it is $\frac{1}{2}f''(0)\left(\varepsilon_t^2 + \operatorname{var}\left(\mu_{i,t}\right)\right)$. The skewness of $Y_{i,t}$ is larger than the skewness of Y_t essentially because there is more variability in what is being squared at the sector than at the aggregate level – $\varepsilon_t + \mu_{i,t}$ instead of just ε_t. Appendix XX derives this formally. The same result appears in Ilut, Kehrig, and Schneider's (2018) calibration (see table 9).

4.2.2 Cyclicality of cross-sectional volatility and skewness

This model naturally generates countercyclical volatility. Consider a simple linear approximation to sector output around different levels of the aggregate shock,

$$Y_{i,t} \approx f(\varepsilon_t) + f'(\varepsilon_t)\,\mu_{i,t} \tag{46}$$

The cross-sectional variance is then

$$\operatorname{var}_t\left(Y_{i,t}\right) \approx f'(\varepsilon_t)^2 \operatorname{var}\left(\mu_{i,t}\right) \tag{47}$$

[11] Their model is a convenient reduced-form representation of a general phenomenon that appears in more structural models. They point, for example, to George and Kuhn (2014), Senga (2016), and Ferraro (2018).

By assumption, $f'(\varepsilon_t)$ is strictly positive and it strictly increases as ε_t declines. So cross-sectional variance is naturally countercyclical.

Time-varying cross-sectional skewness does not arise as naturally here. Using a second-order approximation to sector output,

$$Y_{i,t} \approx f(\varepsilon_t) + f'(\varepsilon_t)\mu_{i,t} + \frac{1}{2}f''(\varepsilon_t)\mu_{i,t}^2 \tag{48}$$

The appendix shows that cross-sectional skewness then depends on the value of $f'(\varepsilon_t)/f''(\varepsilon_t)$. The number may rise or fall with ε_t – the assumptions so far do not constrain it.[12]

Interestingly, the model also generates countercyclicality for the variance of the sector-specific component of output, as in the data. In the first-order approximation above, the sector-specific component, after removing aggregate output, is $f'(\varepsilon_t)\mu_{i,t}$. The cross-sectional variance of those residuals is then the same as the cross-sectional variance of output itself, and thus has the same cyclicality.

4.2.3 Sector covariances following negative shocks

Since all covariance across sectors comes through the ε_t term, consider the following approximation

$$Y_{i,t} \approx f(\mu_{i,t}) + f'(\mu_{i,t})\varepsilon_t \tag{49}$$

$$\operatorname{cov}(Y_{i,t}, Y_{j,t}) \approx f'(\mu_{i,t}) f'(\mu_{j,t}) \operatorname{var}(\varepsilon_t) \tag{50}$$

When a sector receives a negative shock, $f'(\mu_{i,t})$ rises. That raises the covariance of sector i's output with all other sectors, without having any effect on covariances between other sectors. In that sense, sector i becomes more central. This result is not noted by Ilut, Kehrig, and Schneider (2018), but it represents an additional empirical fact that the model matches.

4.2.4 Summary

The model of concave decision rules generates negative skewness, and can match the cross-sectional facts, but it fails to generate the result that skewness increases with the level of aggregation. Instead, it robustly implies that skewness is smaller at the aggregate than at the firm or sector level. Intuitively, the key difference between this model and the model of

[12] Proposition 3 of Ilut, Kehrig, and Schneider (2018) gives a proof of the same result without requiring an approximation.

complementarity in production is where skewness arises. With concave decision rules, the skewness arises at the firm or sector level, whereas in the aggregative model it is fundamentally created by aggregation, so it is inevitably stronger at higher levels of aggregation, consistent with the data.

4.3 Idiosyncratic and aggregate shocks with time-varying distributions

A large literature studies models in which shock volatilities change over time. There can be changes in volatility at the aggregate level (Gourio (2012)), idiosyncratic level (Christiano, Motto, and Rostagno (2014)), or both (Bloom (2009)). Such a model, suitably enriched, can potentially also generate skewness. To see why, consider the following specification,

$$
\begin{aligned}
Y_t &= \varepsilon_t - k\sigma_{t-1}^2 && (51)\\
Y_{i,t} &= \varepsilon_t + \mu_{i,t} - k\sigma_{t-1}^2 && (52)\\
\varepsilon_t &\sim N\left(0, \sigma_{t-1}^2\right) && (53)\\
\mu_{i,t} &\sim N\left(0, m\sigma_{t-1}^2\right) && (54)
\end{aligned}
$$

where Y_t, $Y_{i,t}$, ε_t, and $\mu_{i,t}$ continue to represent aggregate and sector output and innovations, k is a coefficient determining how output responds to variation in cross-sectional volatility relates to the level of output (as in the models cited above), and m determines the relative volatility of the two shocks.

4.3.1 Unconditional skewness

Aggregate and sectoral output are naturally skewed left because there is a mechanical relationship between variance and the level of output – the variance is higher when output is low. The appendix derives this result formally. Furthermore, skewness is greater at the aggregate than at the sector level because sector output has the additional (symmetric) component $\mu_{i,t}$, which drives its skewness towards zero.

The residual component of sector output is $\mu_{i,t}$, which is, by assumption, unskewed, roughly consistent with the data.

4.3.2 Time-varying cross-sectional moments

Cross-sectional variance is time-varying by assumption, since the distribution of $\mu_{i,t}$ has variance σ^2_{t-1}. Cross-sectional variance is also countercyclical for $k > 0$. This result holds both for total sector output and for the sector specific component, $\mu_{i,t}$. This again emphasizes that the key feature of the model to match the cyclicality of cross-sectional moments is that the common component of output cannot be independent of the sector-level residuals.

Nothing in the model as it is written here implies anything about cross-sectional skewness, though – cross-sectional skewness is identically zero.

The model could be extended to allow skewness to also change over time, so that sector and aggregate output are

$$Y_t = \varepsilon_t - k\sigma^2_{t-1} + p\kappa_{t-1} \tag{55}$$

$$Y_{i,t} = \varepsilon_t + \mu_{i,t} - k\sigma^2_{t-1} + p\kappa_{t-1} \tag{56}$$

where κ_{t-1} determines the skewness of the distribution of $\mu_{i,t}$ (that is, $\mu_{i,t}$ is no longer Normally distributed). Then, for $p > 0$, skewness, both for sector output and sector-specific residuals, would be procyclical.[13]

It is worth noting here, though, that for each of these facts, there is a different parameter in the model. Aggregate skewness is determined by the parameter k. Cyclicality of cross-sectional volatility depends on k and m. Cross-sectional skewness would then further require a new skewness process, κ_t, along with a parameter p. This is all in stark contrast to the aggregative model, where the empirical facts all ultimately depend on the sign of just one parameter (γ, the elasticity of substitution).

4.3.3 Sector covariances following negative shocks

The covariance of output between sectors is

$$\text{cov}\left(Y_{i,t}, Y_{j,t}\right) = \text{var}\left(\varepsilon_t\right) = \sigma^2_{t-1} \tag{57}$$

In other words, the covariances are all identical. They change over time due to σ^2_t, and they are all countercyclical, but there is no variation across sectors. Certainly nothing in the model requires that when a firm receives a negative shock it will covary more strongly with other sectors, unlike the aggregative model or the one with concave decision rules. In both

[13] An alternative device that would generate time-varying cross-sectional skewness would be as in the model of skewed aggregate shocks, if sectors have different loadings on the aggregate shock, b_i. Then, as before, if b_i is positively skewed, cross-sectional skewness is procyclical.

of those cases, the source of the increased covariance is that a sector loads more heavily on the common component following a negative sector-specific shock. This again illustrates the value of the common component being endogenous to the sector shocks, unlike here, where it is purely exogenous.

4.3.4 Summary

This model naturally generates unconditional negative skewness that increases in magnitude with the level of aggregation, and it generates countercyclical cross-sectional volatility. Without augmentation, though, it does not deliver time-varying cross-sectional skewness, and it has no prediction for differences in covariances across sectors. Moreover, this is the least parsimonious of the models studied here, in the sense that each of the moments it fits is determined by a different free parameter.

4.4 Implications

There is obviously no single correct model of the economy, but some models can fit a set of facts more parsimoniously than others. There are two overall conclusions we draw from the analysis of this section. First, while there are a few models that generate skewness in the economy, none of those that we study here match all three of the empirical facts documented in section 2. Second, these models all require a number of auxiliary assumptions in order to match more than one of the facts that we document – with aggregate skewed shocks one must assume skewed sector loadings on the aggregate shocks (thereby just assuming skewness ex ante), with concave decision rules there is a restriction on the shape of the decision function to match cross-sectional variance, and with stochastic volatility one must assume multiple exogenous processes with specific parameter values.

All of these alternative models have valuable insights, but the power of the aggregative model is that a single idea – that output is produced through a production function that is strongly concave in individual inputs, so that the common component is endogenous to sector shocks – is able to fit a wide range of empirical results.

5 Quantitative illustration

The analysis of the models so far has been purely qualitative. It is apparent that the complementary aggregative model can qualitatively match the features of the data presented in section 2. However, the version of the model was highly stylized; more general setups (e.g.

with different production parameters across sectors) are not in general solvable and are extremely difficult to analyze by hand.

This section briefly asks whether a quantitative version of that model – which allows for much more general parameter choices than in our theoretical analysis – can broadly match the actual numbers reported in section 2. The model is still necessarily somewhat stylized, as we will detail further below, so we are essentially looking for agreement within an order of magnitude – does the model come at least broadly match the quantitative features of the data? We leave the formidable task of full estimation of a large-scale multisector model to future work.

5.1 Model and calibration

The model we simulate follows that of Baqaee and Farhi (2019) closely (drawing on their posted replication files with only minor modifications). The structure of the economy is a general CES setup, where production at the sector level, Y_i, relative to its steady-state, $\bar{Y}_i$, is

$$\frac{Y_i}{\bar{Y}_i} = A_i \left(\omega_{iL} + (1 - \omega_{iL}) \left(\frac{\hat{X}_i}{\bar{X}_i} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{58}$$

where A_i is a technology shock, ω_{iL} determines the importance of labor in production (where labor input is assumed to be fixed and normalized to 1) and $\hat{X}_i$ measures the use of intermediates, with

$$\frac{\hat{X}_i}{\bar{X}_i} = \left(\sum_{j=1}^{N} \omega_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{59}$$

where variables with bars again represent steady-state values. Finally, aggregate output (consumption) is

$$\frac{C}{\bar{C}} = \left(\sum_{i=1}^{N} \omega_{0i} \left(\frac{c_i}{\bar{c}} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{60}$$

The model is both more general than that studied above and at the same time still highly restricted. The specification here allows for different elasticities of substitution at the aggregate and sector levels, for an arbitrary elasticity between inputs and labor, and for arbitrary weights ($\omega_\cdot$) on the inputs in each sector. At the same time, all sectors have the same elasticity parameters, θ and ε, labor cannot be adjusted (which can be thought of as essentially a short-run restriction), and the production weights, $\omega_\cdot$, are fixed over time. In reality, all of those assumptions are likely violated.

Our calibration follows that of Baqaee and Farhi (2019) for the most part. The expenditure share parameters, $\omega_{\cdot}$, are chosen to match the 1982 input-output table. TFP shocks are calibrated to match the relative variance of TFP by industry. Their overall scale is chosen to match the volatility of industrial production growth (and the moments we will examine will be matched to the IP data). We draw the shocks from a t distribution with four degrees of freedom, which generates fat tails and helps match observed skewness. The shocks are assumed to be uncorrelated across sectors. We set the autocorrelation of sector productivity to 0.85 (at the monthly frequency) to match the dynamics of sector-level IP growth.

The elasticities of substitution are set to $(\sigma, \theta, \varepsilon) = (0.9, 0.5, 0.1)$. Aggregate output is thus relatively substitutable across sectors – close to a log-linear production function. The strong complementarity arises in production at the sector level, where the assumption is that the mix of material inputs is not amenable to adjustment. These values are similar to those estimated empirically by Atalay (2017).

5.2 Results

Table 5 reports moments of the model corresponding to the results from tables 1–4 along with the associated empirical results for industrial production. The top section shows that, as in the data, skewness is higher at the aggregate than the sector level. The magnitudes are smaller by about a factor of two, but the ratio of aggregate to sector skewness is highly similar in the model and the data.

The second section examines the cyclicality of cross-sectional variance and skewness. NBER recessions in the model are defined as periods when aggregate output is in the bottom 15 percent of the unconditional distribution, to replicate the empirical frequency of recessions. For variance and skewness, the sign of the cyclicality matches what is observed in the data, though in three of four cases the magnitude of the coefficient is much smaller than what is observed empirically – by about a factor of 10. For Kelley skewness, the model generates essentially zero cyclicality (the coefficients have the wrong sign, but using a bootstrap on the simulations, even with 10,000 months of data, the t-statistics are close to zero).

The third panel examines the ability of cross-sectional standard deviation to predict volatility. The sign is correct, but the coefficient is again far smaller than what is observed empirically. These two results thus suggest that the model fares quantitatively relatively poorly in matching cross-sectional moments.

Finally, the bottom panel examines the results from table 4 on conditional variances and covariances. In this case, the model comes very close – within about a factor of 2 – in three of four cases, and gets the sign of the relationship right in all four cases. So the

quantitative model performs fairly well in matching conditional moments, along with the time-series skewness results in the top panel.

Overall, table 5 shows that a richer version of the model, which is designed to be closer to quantitative realism than the highly restricted setup analyzed theoretically above, is able to broadly match the empirical behavior of the economy documented in tables 1–4. That said, there are still large failures, in particular in matching the behavior of cross-sectional moments, that future work needs to address.

6 Conclusion

This paper develops a set of facts regarding aggregate and sectoral variances, covariances, and skewness. We then examine a model in which production displays complementarity across inputs and show that it can match all three facts: skewness is negative and increases in magnitude with the level of aggregation, cross-sectional variance is countercyclical and cross-sectional skewness is procyclical, and a decline in output in a sector is associated with an increase in the sector's relative covariance with other sectors. The idea of complementarity, advanced most recently by Baqaee and Farhi (2019), is powerful in understanding both the aggregate and cross-sectional behavior of the economy.

That idea has important implications for how to think about skewness and time-varying volatility. The model implies that second and third moments change over time and are cyclical. In the past, it has sometimes been argued that the observed cyclicality of those moments implies that there are exogenous shocks to *uncertainty*, and that uncertainty then has negative effects on the economy. The model advanced here, though, is one in which changes in volatility are a result of fundamental productivity shocks and have no independent effect on the level of output.

A second important implication, which is supported by our empirical contributions, is that the centrality of sectors changes over time. In some models, recessions have common causes, e.g. technology shocks. Here, though, every episode is different. When a sector receives a negative shock, it becomes relatively more important. So in a period where oil stocks are low, shocks to the oil sector become a major driving force (e.g. Hamilton (2003), Kilian (2008)), whereas in periods when the financial sector is highly constrained, financial shocks become most relevant (e.g. Brunnermeier and Sannikov (2014). A key insight of this paper is that complementarity means that the aggregate effects of shocks change in important ways over time, those changes can be measured from the covariances of sector growth rates, and many models fail to match them.

References

Atalay, Enghin, "How important are sectoral shocks?," *American Economic Journal: Macroeconomics*, 2017, *9* (4), 254–80.

___ , Thorsten Drautzburg, and Zhenting Wang, "Accounting for the sources of macroeconomic tail risks," *Economics Letters*, 2018, *165*, 65–69.

Baqaee, David Rezza and Emmanuel Farhi, "The Macroeconomic Impact of Microeconomic Shocks: Beyond HultenŠs Theorem," 2019. Forthcoming, Econometrica.

Berger, David, Ian Dew-Becker, and Stefano Giglio, "Uncertainty shocks as second-moment news shocks," 2019. Forthcoming, Review of Economic Studies.

Blanchard, Olivier and Jordi Gali, "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 1–30.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009, *77(3)*, 623–685.

___ , Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry, "Really uncertain business cycles," *Econometrica*, 2018, *86* (3), 1031–1065.

Brunnermeier, Markus K and Yuliy Sannikov, "A macroeconomic model with a financial sector," *American Economic Review*, 2014, *104* (2), 379–421.

Carvalho, Vasco M, Makoto Nirei, Yukiko Saito, and Alireza Tahbaz-Salehi, "Supply chain disruptions: Evidence from the great east japan earthquake," 2016. Working paper.

Christiano, Lawrence J., Roberto Motto, and Massimio Rostagno, "Risk Shocks," *American Economic Review*, 2014, *104(1)*, 27–65.

Clark, Todd E and Francesco Ravazzolo, "Macroeconomic forecasting performance under alternative specifications of time-varying volatility," *Journal of Applied Econometrics*, 2015, *30* (4), 551–575.

Dew-Becker, Ian and Stefano Giglio, "Measuring the effects of cross-sectional uncertainty: evidence from implied volatilities, 1980–2017," 2019. Working paper.

Ferraro, Domenico, "The asymmetric cyclical behavior of the us labor market," *Review of Economic Dynamics*, 2018, *30*, 145–162.

Foerster, Andrew T, Pierre-Daniel G Sarte, and Mark W Watson, "Sectoral versus aggregate shocks: A structural factor analysis of industrial production," *Journal of Political Economy*, 2011, *119* (1), 1–38.

George, Chacko and Florian Kuhn, "Aggregate Implications of Capacity Constraints under Demand Shocks," 2014. Working paper.

Gourio, Francois, "Disaster Risk and Business Cycles," *American Economic Review*, 2012, *102(6)*, 2734–2766.

Guvenen, Fatih, Serdar Ozkan, and Jae Song, "The Nature of Countercyclical Income Risk," *Journal of Political Economy*, 2014, *122* (3), 621–660.

Hamilton, James D, "What is an oil shock?," *Journal of econometrics*, 2003, *113* (2), 363–398.

Ilut, Cosmin, Matthias Kehrig, and Martin Schneider, "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News," *Journal of Political Economy*, 2018, *126* (5), 2011–2071.

Jr, John B Long and Charles I Plosser, "Real business cycles," *Journal of political Economy*, 1983, *91* (1), 39–69.

Justiniano, Alejandro and Giorgio E Primiceri, "The time-varying volatility of macroeconomic fluctuations," *American Economic Review*, *98* (3), 604–41.

Kilian, Lutz, "The economic effects of energy price shocks," *Journal of Economic Literature*, 2008, *46* (4), 871–909.

Kocherlakota, Narayana, "Creating business cycles through credit constraints," *Federal Reserve Bank of Minneapolis Quarterly Review*, 2000, *24* (3), 2–10.

McKay, Alisdair and Ricardo Reis, "The brevity and violence of contractions and expansions," *Journal of Monetary Economics*, 2008, *55* (4), 738–751.

Morley, James and Jeremy Piger, "The asymmetric business cycle," *Review of Economics and Statistics*, 2012, *94* (1), 208–221.

Salgado, Sergio, Fatih Guvenen, and Nicholas Bloom, "Skewed Business Cycles," 2018. Working paper.

Schorfheide, Frank, Dongho Song, and Amir Yaron, "Identifying Long-run Risks: A Bayesian Mixed-Frequency Approach," *Econometrica*, 2018, *86* (2), 617–654.

Senga, Tatsuro, "A New Look at Uncertainty Shocks: Imperfect Information and Misallocation," 2016. Working paper.

Sichel, Daniel E, "Business cycle asymmetry: a deeper look," *Economic inquiry*, 1993, *31* (2), 224–236.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, 2004, *112* (3), 695–717.

A Solution of network model

The structure is

$$C = \left(n^{-1} \sum_i a_i x_{C,i}^{\gamma} \right)^{1/\gamma} \tag{61}$$

$$Y_i = z_i \left(n^{-1} \sum_j a_j x_{i,j}^{\gamma} \right)^{\alpha/\gamma} \tag{62}$$

$$Y_i = x_{C,i} + n^{-1} \sum_j x_{j,i} \tag{63}$$

The first equation is the production function for the consumption sector, the second for the intermediate sectors, and the third the resource constraint.

Now we guess that every sector uses the same mix of inputs, which means that $x_{i,j} = \bar{x}_i x_{C,j}$ for some $\bar{x}_i$ (this is simple to prove). The resource constraint and sector production functions become

$$Y_i = x_{C,i} \left(1 + n^{-1} \sum_j \bar{x}_j \right) \tag{64}$$

$$Y_i = z_i \bar{x}_i^{\alpha} \left(n^{-1} \sum_j a_j x_{C,j}^{\gamma} \right)^{\alpha/\gamma} = z_i \bar{x}_i^{\alpha} C^{\alpha} \tag{65}$$

The aggregate optimization problem and FOC are

$$\max \left(n^{-1} \sum_i a_i x_{C,i}^{\gamma} \right)^{1/\gamma} - \sum_i x_{C,i} p_i \tag{66}$$

$$p_i = C^{1-\gamma} n^{-1} a_i x_{C,i}^{\gamma-1} \tag{67}$$

The sector optimization problem and FOC are

$$\max p_i z_i \bar{x}_i^{\alpha} \left(n^{-1} \sum_j a_j x_{C,j}^{\gamma} \right)^{\alpha/\gamma} - \bar{x}_i \sum_j p_j x_{C,j} \tag{68}$$

$$\sum_i p_j x_{C,j} = \alpha p_i z_i \bar{x}_i^{\alpha-1} C \tag{69}$$

A.1 Proportionality results and solution

Equations (67) and (69), respectively, imply

$$x_{C,i} \propto (p_i/a_i)^{\frac{1}{\gamma-1}} \tag{70}$$

$$\bar{x}_i \propto (p_i z_i)^{\frac{1}{1-\alpha}} \tag{71}$$

where the factors of proportionality do not depend on any i-indexed variables. The resource constraint combined with the production function implies

$$x_{C,i} \propto z_i \bar{x}_i^{\alpha} \tag{72}$$

which, combined with the above, yields

$$a_i^{\frac{1-\alpha}{1-\alpha\gamma}} z_i^{\frac{\gamma-1}{1-\alpha\gamma}} \propto p_i \tag{73}$$

So there exist $\bar{X}$ and $\bar{X}_C$ such that

$$\bar{x}_i = \bar{X} a_i^{\frac{1}{1-\alpha\gamma}} z_i^{\gamma \frac{1}{1-\alpha\gamma}} \tag{74}$$

$$x_{C,i} = \bar{X}_C a_i^{\alpha \frac{1}{1-\alpha\gamma}} z_i^{\frac{1}{1-\alpha\gamma}} \tag{75}$$

Inserting those into the resource constraint, we have

$$\bar{X}_C a_i^{\alpha\frac{1}{1-\alpha\gamma}} z_i^{\frac{1}{1-\alpha\gamma}} \left(1 + \bar{X} n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\gamma\frac{1}{1-\alpha\gamma}}\right) = z_i \bar{X}^\alpha a_i^{\frac{\alpha}{1-\alpha\gamma}} z_i^{\gamma\frac{\alpha}{1-\alpha\gamma}} \bar{X}_C^\alpha \left(n^{-1} \sum_j a_j a_j^{\alpha\frac{1}{1-\alpha\gamma}} z_j^{\frac{1}{1-\alpha\gamma}\gamma}\right)^{\alpha/\gamma} \quad (76)$$

$$\bar{X}_C^{1-\alpha} + \bar{X}_C^{1-\alpha} \bar{X} \left(n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\frac{\gamma}{1-\alpha\gamma}}\right) = \bar{X}^\alpha \left(n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\alpha/\gamma} \quad (77)$$

The FOC for $\bar{x}_i$ becomes

$$\bar{X}_C \bar{X} = \left(n^{-1}\alpha\right)^{\frac{1}{1-\alpha}} \left(n^{-1} \sum_i a_i^{\frac{1}{1-\alpha\gamma}} z_i^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{(\alpha-\gamma)}{1-\alpha}/\gamma} \quad (78)$$

Inserting that into (77) yields

$$\frac{(n^{-1}\alpha)}{(1-n^{-1}\alpha)} \left(n^{-1} \sum_i a_i^{\frac{1}{1-\alpha\gamma}} z_i^{\frac{\gamma}{1-\alpha\gamma}}\right)^{-1} = \bar{X} \quad (79)$$

which then implies

$$\bar{X}_C = \left(1 - n^{-1}\alpha\right) \left(n^{-1}\alpha\right)^{\frac{\alpha}{1-\alpha}} \left(n^{-1} \sum_i a_i^{\frac{1}{1-\alpha\gamma}} z_i^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}} \quad (80)$$

Finally, then, aggregate output is

$$C = \left(1 - n^{-1}\alpha\right) \left(n^{-1}\alpha\right)^{\frac{\alpha}{1-\alpha}} \left(n^{-1} \sum_i a_i^{\frac{1}{1-\alpha\gamma}} z_i^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{1-\alpha\gamma}{\gamma}\frac{1}{1-\alpha}} \quad (81)$$

A.2 Results

Aggregate output is (up to a factor of proportionality)

$$C \propto \left(n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{1-\alpha\gamma}{\gamma}\frac{1}{1-\alpha}} \quad (82)$$

Sector output is

$$Y_i \propto \left(n^{-1} \sum_j a_j^{\frac{1}{1-\alpha\gamma}} z_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^{\frac{1-\gamma}{1-\alpha}\frac{\alpha}{\gamma}} a_i^{\frac{\alpha}{1-\alpha\gamma}} z_i^{\frac{1}{1-\alpha\gamma}} \quad (83)$$

B Proposition 1

Define

$$\kappa_t \equiv n^{-1} \sum \varepsilon^t \tag{84}$$

B.1 Lemma 1

The MGF is

$$M_\varepsilon(t) = n^{-1} \sum \exp(t\varepsilon) \tag{85}$$

By the CLT, this converges to

$$\sqrt{n}\left(M_\varepsilon(t) - M(t)\right) \to N\left(0, \operatorname{var}\left(\exp(t\varepsilon)\right)\right) \tag{86}$$

For a normal, we have

$$\begin{aligned} \operatorname{var}\left(\exp(t\varepsilon)\right) &= E\left[\exp(2t\varepsilon)\right] - E\left[\exp(t\varepsilon)\right]^2 & (87)\\ &= \exp\left(2t^2\sigma^2\right) - \exp\left(t^2\sigma^2\right) & (88) \end{aligned}$$

Now the log of that is

$$\begin{aligned} t^{-1}\log M_\varepsilon(t) &\approx t^{-1}\log M(t) + \frac{M_\varepsilon(t) - M(t)}{M(t)} & (89)\\ \sqrt{n}\left(\log M_\varepsilon(t) - \log M(t)\right) &\to N\left(0, \frac{\exp(2t^2\sigma^2) - \exp(t^2\sigma^2)}{M(t)^2}\right) & (90)\\ &= N\left(0, \exp\left(t^2\sigma^2\right) - 1\right) & (91) \end{aligned}$$

So this gives the asymptotic distribution of the cumulant generating function.

$$M(t) = \exp\left(\frac{t^2}{2}\sigma^2\right) \tag{92}$$

B.2 Some preliminary results

B.2.1 Sample variance approximation

The sample variance is

$$\text{var} = n^{-1} \sum \varepsilon^2 - \left(n^{-1} \sum \varepsilon\right)^2 \tag{93}$$

This can be approximated in terms of κ_1 and κ_2 using the derivatives

$$\frac{\partial \operatorname{var}}{\partial \kappa_1} = -2\bar{\kappa}_1 = 0 \tag{94}$$

$$\frac{\partial \operatorname{var}}{\partial \kappa_2} = 1 \tag{95}$$

where bars denote unconditional expectations. This approximation will be used for the delta method.

B.2.2 Sample skewness approximation

$$skew = \frac{n^{-1}\sum(\varepsilon-\kappa_1)^3}{(\kappa_2-\kappa_1^2)^{3/2}} \tag{96}$$

$$= \frac{n^{-1}\sum(\varepsilon^3-3\varepsilon^2\kappa_1+3\varepsilon\kappa_1^2-\kappa_1^3)}{(\kappa_2-\kappa_1^2)^{3/2}} \tag{97}$$

$$= \frac{\kappa_3-3\kappa_2\kappa_1+2\kappa_1^3}{(\kappa_2-\kappa_1^2)^{3/2}} \tag{98}$$

$$\frac{\partial skew}{\partial \kappa_1} = \frac{(\kappa_2-\kappa_1^2)^{3/2}(-3\kappa_2+6\kappa_1^2)-(\kappa_3-3\kappa_2\kappa_1+2\kappa_1^3)(-2\kappa_1)}{(\kappa_2-\kappa_1^2)^3} = -3\sigma^{-1} \tag{99}$$

$$\frac{\partial skew}{\partial \kappa_2} = \frac{(\kappa_2-\kappa_1^2)^{3/2}(-3\kappa_1)-(\kappa_3-3\kappa_2\kappa_1+2\kappa_1^3)}{(\kappa_2-\kappa_1^2)^3} = 0 \tag{100}$$

$$\frac{\partial skew}{\partial \kappa_3} = \frac{1}{(\kappa_2-\kappa_1^2)^{3/2}} = \sigma^{-3} \tag{101}$$

B.2.3 Covariances between the empirical MGF and moments

The covariance of ε, ε^2, and ε^3 with $\exp(t\varepsilon)$ are, respectively

$$E[\varepsilon\exp(t\varepsilon)]-E[\varepsilon]E[\exp(t\varepsilon)] = \sigma^2 t\exp\left(\frac{t^2}{2}\sigma^2\right) \tag{102}$$

$$E\left[\varepsilon^2\exp(t\varepsilon)\right]-E\left[\varepsilon^2\right]E[\exp(t\varepsilon)] = \sigma^2\exp\left(\frac{t^2}{2}\sigma^2\right)\left(\sigma^2t^2+1\right)-\sigma^2\exp\left(\frac{t^2}{2}\sigma^2\right) \tag{103}$$

$$= \sigma^4t^2\exp\left(\frac{t^2}{2}\sigma^2\right) \tag{104}$$

$$E\left[\varepsilon^3 \exp\left(t\varepsilon\right)\right] - E\left[\varepsilon^3\right] E\left[\exp\left(t\varepsilon\right)\right] = \sigma^4 t \exp\left(\frac{t^2}{2}\sigma^2\right)\left(\sigma^2 t^2 + 3\right) \tag{105}$$

B.2.4 CLT and covariances

Applying the CLT to κ_1, κ_2, κ_3, and $n^{-1}\sum \exp\left(t\varepsilon\right)$, we have

$$\sqrt{n}\left(\begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ n^{-1}\sum \exp\left(t\varepsilon\right) \end{bmatrix} - \begin{bmatrix} 0 \\ \sigma^2 \\ 0 \\ \exp\left(\frac{t^2}{2}\sigma^2\right) \end{bmatrix}\right) \rightarrow N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & 0 & 3\sigma^4 & \sigma^2 t \exp\left(\frac{t^2}{2}\sigma^2\right) \\ & 2\sigma^4 & 0 & \sigma^4 t^2 \exp\left(\frac{t^2}{2}\sigma^2\right) \\ & & 15\sigma^6 & \sigma^4 t \exp\left(\frac{t^2}{2}\sigma^2\right)\left(\sigma^2 t^2 + 3\right) \\ & & & \exp\left(2t^2\sigma^2\right) - \exp\left(t^2\sigma^2\right) \end{bmatrix}\right) \tag{106}$$

We then have

$$\begin{aligned} \sqrt{n}\operatorname{cov}\left(skew, \log\left(n^{-1}\sum \exp\left(t\varepsilon\right)\right)\right) &= \frac{1}{\exp\left(\frac{t^2}{2}\sigma^2\right)} \begin{pmatrix} -3\sigma^{-1}\sigma^2 t \exp\left(\frac{t^2}{2}\sigma^2\right) \\ +\sigma^{-3}\sigma^4 t \exp\left(\frac{t^2}{2}\sigma^2\right)\left(\sigma^2 t^2 + 3\right) \end{pmatrix} & (107) \\ &= \sigma^3 t^3 & (108) \end{aligned}$$

and

$$\begin{aligned} \operatorname{cov}\left(\operatorname{var}, \log\left(n^{-1}\sum \exp\left(t\varepsilon\right)\right)\right) &= \frac{1}{\exp\left(\frac{t^2}{2}\sigma^2\right)}\sigma^4 t^2 \exp\left(\frac{t^2}{2}\sigma^2\right) & (109) \\ &= \sigma^4 t^2 & (110) \end{aligned}$$

B.3 Final result

$$\begin{aligned} \sqrt{n}\operatorname{cov}\left(skew, C\right) &= \frac{1-\alpha\gamma}{\gamma}\operatorname{cov}\left(skew, \log\left(n^{-1}\sum \exp\left(\frac{\gamma}{1-\alpha\gamma}\varepsilon\right)\right)\right) & (111) \\ &= \sigma^3\left(\frac{\gamma}{1-\alpha\gamma}\right)^2 & (112) \end{aligned}$$

$$\sqrt{n}\operatorname{cov}\left(\operatorname{var}, C\right) = \sigma^4\left(\frac{\gamma}{1-\alpha\gamma}\right) \tag{113}$$

the variance of output is

$$\sqrt{n}\operatorname{var}\left(C\right) = \left(\frac{1-\alpha\gamma}{\gamma}\right)^2\left(\exp\left(\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\sigma^2\right) - 1\right) \tag{114}$$

The regression coefficients in the main text then follow immediately as ratios of the covariances with output to the variance of output.

B.4 Allowing for $a_j \neq 1$

We now need to work with

$$n^{-1}\sum_{j=1}^{n} a_j^t \exp(t\varepsilon) \tag{115}$$

By the CLT, this converges to [Is this correct?]

$$\sqrt{n}\left(M_\varepsilon(t) - M(t)\right) \to N\left(0, \left(n^{-1}\sum_{j=1}^{n} a_j^{2t}\right)\operatorname{var}\left(\exp(t\varepsilon)\right)\right) \tag{116}$$

For a normal, we have

$$\begin{aligned}
\operatorname{var}\left(\exp(t\varepsilon)\right) &= E\left[\exp(2t\varepsilon)\right] - E\left[\exp(t\varepsilon)\right]^2 && (117)\\
&= \exp\left(2t^2\sigma^2\right) - \exp\left(t^2\sigma^2\right) && (118)
\end{aligned}$$

Now the log of that is

$$\begin{aligned}
\sqrt{n}\left(\log M_{\varepsilon,a}(t) - \log M_a(t)\right) &\to N\left(0, \left(n^{-1}\sum_{j=1}^{n} a_j^{2t}\right)\frac{\exp(2t^2\sigma^2) - \exp(t^2\sigma^2)}{M_a(t)^2}\right) && (119)\\
&= N\left(0, \frac{\left(n^{-1}\sum_{j=1}^{n} a_j^{2t}\right)}{\left(n^{-1}\sum_{j=1}^{n} a_j^{t}\right)^2}\left[\exp\left(t^2\sigma^2\right) - 1\right]\right) && (120)
\end{aligned}$$

So this gives the asymptotic distribution of the cumulant generating function.

$$\begin{aligned}
M(t) &= E\left[n^{-1}\sum_{j=1}^{n} a_j^t \exp(t\varepsilon)\right] && (121)\\
&= \left(n^{-1}\sum_{j=1}^{n} a_j^t\right)\exp\left(\frac{t^2}{2}\sigma^2\right) && (122)
\end{aligned}$$

$$\begin{aligned}
\operatorname{cov}(\operatorname{var}, CGF) &= \frac{1}{M(t)}\sigma^4 t^2 \exp\left(\frac{t^2}{2}\sigma^2\right) && (123)\\
&= \sigma^4 t^2 && (124)
\end{aligned}$$

B.5 Results

$$\sqrt{n}\begin{bmatrix}\kappa_1\\ \kappa_2\\ \kappa_3\\ \exp(t\varepsilon)\end{bmatrix} \to N\left(\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}\sigma^2 & 0 & 3\sigma^4 & \sigma^2 t\exp\left(\frac{t^2}{2}\sigma^2\right)\\ & 2\sigma^4 & 0 & \sigma^4 t^2\exp\left(\frac{t^2}{2}\sigma^2\right)\\ & & 15\sigma^6 & \sigma^4 t\exp\left(\frac{t^2}{2}\sigma^2\right)(\sigma^2t^2+3)\\ & & & \exp(2t^2\sigma^2)-\exp(t^2\sigma^2)\end{bmatrix}\right) \tag{125}$$

$$\text{var} \approx \kappa_2 \tag{126}$$

$$\begin{aligned}\sqrt{n}\,\text{cov}(skew, C) &= \frac{1-\alpha\gamma}{\gamma}\,\text{cov}\left(skew, CGF\left(\frac{\gamma}{1-\alpha\gamma}\right)\right) &(127)\\ &= \sigma^3\left(\frac{\gamma}{1-\alpha\gamma}\right)^2 &(128)\end{aligned}$$

$$\sqrt{n}\,\text{cov}(\text{var}, C) = \sigma^4\left(\frac{\gamma}{1-\alpha\gamma}\right) \tag{129}$$

the variance of output is

$$\sqrt{n}\,\text{var}(C) = \left(\frac{1-\alpha\gamma}{\gamma}\right)^2 \frac{\left(n^{-1}\sum_{j=1}^n a_j^{2\frac{\gamma}{1-\alpha\gamma}}\right)}{\left(n^{-1}\sum_{j=1}^n a_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^2}\left[\exp\left(\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\sigma^2\right)-1\right] \tag{130}$$

So then

$$\begin{aligned}\frac{\sqrt{n}\,\text{cov}(\text{var}, C)}{\sqrt{n}\,\text{var}(C)} &= \sigma^4\left(\frac{\gamma}{1-\alpha\gamma}\right)^3 \frac{\left(n^{-1}\sum_{j=1}^n a_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^2}{\left(n^{-1}\sum_{j=1}^n a_j^{2\frac{\gamma}{1-\alpha\gamma}}\right)}\left[\exp\left(\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\sigma^2\right)-1\right]^{-1} &(131)\\ &\approx \sigma^2\left(\frac{\gamma}{1-\alpha\gamma}\right)\frac{\left(n^{-1}\sum_{j=1}^n a_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^2}{\left(n^{-1}\sum_{j=1}^n a_j^{2\frac{\gamma}{1-\alpha\gamma}}\right)} &(132)\end{aligned}$$

$$\begin{aligned}\frac{\sqrt{n}\,\text{cov}(skew, C)}{\sqrt{n}\,\text{var}(C)} &= \sigma^3\left(\frac{\gamma}{1-\alpha\gamma}\right)^4 \frac{\left(n^{-1}\sum_{j=1}^n a_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^2}{\left(n^{-1}\sum_{j=1}^n a_j^{2\frac{\gamma}{1-\alpha\gamma}}\right)}\left[\exp\left(\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\sigma^2\right)-1\right]^{-1} &(133)\\ &\approx \sigma\left(\frac{\gamma}{1-\alpha\gamma}\right)^2\frac{\left(n^{-1}\sum_{j=1}^n a_j^{\frac{\gamma}{1-\alpha\gamma}}\right)^2}{\left(n^{-1}\sum_{j=1}^n a_j^{2\frac{\gamma}{1-\alpha\gamma}}\right)} &(134)\end{aligned}$$

C Proposition 2

Suppose the z's follow geometric Brownian motions. The instantaneous covariance of output growth between any pair of sectors is

$$E\left[d\log Y_i d\log Y_j\right] = E\left[\frac{1-\gamma}{1-\alpha\gamma}d\log C + \frac{1}{1-\alpha\gamma}d\log z_i, \frac{1-\gamma}{1-\alpha\gamma}d\log C + \frac{1}{1-\alpha\gamma}d\log z_j\right] \quad (135)$$

$$= \left(\frac{1-\gamma}{(1-\alpha\gamma)^2}\right) E\left[(1-\gamma)\left(d\log C\right)^2 + \left(d\log z_i + d\log z_j\right) d\log C\right] \quad (136)$$

Using the solution for $\log C$ and Ito's lemma,

$$\log C = \frac{1-\alpha\gamma}{\gamma}\frac{\alpha}{1-\alpha}\log\left(\frac{1}{n}\sum_j a_j^{\gamma/(1-\alpha\gamma)} z_j^{\gamma/(1-\alpha\gamma)}\right) \quad (137)$$

$$d\log C = ...dt + \frac{\alpha}{1-\alpha}\frac{\sum_k n^{-1} a_k^{\gamma/(1-\alpha\gamma)} z_k^{\gamma/(1-\alpha\gamma)} dB_k}{n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)} z_j^{\gamma/(1-\alpha\gamma)}} \quad (138)$$

We then have

$$E\left[\left(d\log C\right)\left(d\log z_k\right)\right] = \frac{\alpha}{1-\alpha}\frac{n^{-1} a_k^{\gamma/(1-\alpha\gamma)} z_k^{\gamma/(1-\alpha\gamma)}}{n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)} z_j^{\gamma/(1-\alpha\gamma)}}\sigma^2 dt \quad (139)$$

and

$$E\left[d\log Y_i d\log Y_k\right] = \left(\frac{1-\gamma}{(1-\alpha\gamma)^2}\right)\frac{\alpha}{1-\alpha}\frac{n^{-1} a_k^{\gamma/(1-\alpha\gamma)} z_k^{\gamma/(1-\alpha\gamma)} + n^{-1} a_i^{\gamma/(1-\alpha\gamma)} z_i^{\gamma/(1-\alpha\gamma)}}{n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)} z_j^{\gamma/(1-\alpha\gamma)}}\sigma^2 dt \quad (140)$$

Now consider the so of the covariances of sector i's growth rate with those of all other sectors:

$$\sum_{k\neq i} E\left[d\log Y_i d\log Y_k\right] = (n-2)\left(\frac{1-\gamma}{(1-\alpha\gamma)^2}\right)\frac{\alpha}{1-\alpha}\frac{n^{-1} a_i^{\gamma/(1-\alpha\gamma)} z_i^{\gamma/(1-\alpha\gamma)}}{n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)} z_j^{\gamma/(1-\alpha\gamma)}}\sigma^2 dt \quad (141)$$

$$+\text{terms independent of } i \quad (142)$$

The terms independent of i are terms shared by all sectors. Since we are only concerned about changes in i's covariances relative to those of other sectors, we can ignore those terms.

Now consider a derivative with respect to z_i,

$$\begin{aligned}\frac{d}{dz_i}\left[\frac{n^{-1}a_i^{\gamma/(1-\alpha\gamma)}z_i^{\gamma/(1-\alpha\gamma)}}{n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)}z_j^{\gamma/(1-\alpha\gamma)}}\right] &= \frac{\frac{\gamma}{1-\alpha\gamma}\left(n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)}z_j^{\gamma/(1-\alpha\gamma)}\right)n^{-1}a_i^{\gamma/(1-\alpha\gamma)}z_i^{\gamma/(1-\alpha\gamma)} - \left(n^{-1}a_i^{\gamma/(1-\alpha}\right.}{\left(n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)}z_j^{\gamma/(1-\alpha\gamma)}\right)^2} \\ &= \frac{\gamma}{1-\alpha\gamma}\left(n^{-1}a_i^{\gamma/(1-\alpha\gamma)}z_i^{\gamma/(1-\alpha\gamma)}\right)\frac{\left(n^{-1}\sum_{j\neq i} a_j^{\gamma/(1-\alpha\gamma)}z_j^{\gamma/(1-\alpha\gamma)}\right)}{\left(n^{-1}\sum_j a_j^{\gamma/(1-\alpha\gamma)}z_j^{\gamma/(1-\alpha\gamma)}\right)^2}\end{aligned}$$

$$\frac{d}{dz_i}\sum_{k\neq i} E\left[d\log Y_i d\log Y_k\right] \propto \gamma\left(1-\gamma\right) \tag{145}$$

so the sign of this derivative is equal to the sign of γ. When $\gamma < 0$, if a sector receives a negative productivity shock, its output falls relative to all other sectors, and its covariance with all other sectors rises, relative to the covariances of other sectors.

D Results in levels and growth rates

This section examines the implications of studying the aggregative model in terms of levels versus growth rates. The first section shows that in a particular limit of an AR(1) model with a type of fat-tailed shocks, the model has equivalent implications in levels and growth rates. The second section examines a numerical example using the analytically solvable version of the aggregative model. It shows that the assumption of fat-tailed shocks is supported by the data, and that the qualitative results derived analytically in terms of levels also hold numerically in terms of growth rates.

D.1 Limiting AR(1) case

This section provides a heuristic derivation of a limiting version of a continuous-time non-linear model. We consider a simple setting in which there is a scalar productivity process, x_t, and nonlinear production function, f.

x_t follows an Ornstein-Uhlenbeck type process, where the innovations are pure jumps, rather than Brownian increments,

$$dx_t = -bx_t + \varepsilon_t dN_t \tag{146}$$

where N_t is a Poisson counting process with jump intensity λ (i.e. in any interval of length s, $N_t - N_{t-s}$ is Poisson distributed with intensity λs) and ε_t is a symmetrically distributed

shock.

Denote discretely sampled growth rates by $\Delta_s x_t \equiv x_t - x_{t-s}$, where s determines the length of a period. In the absence of a jump between times $t-s$ and t (i.e. $N_t = N_{t-s}$), we have

$$\Delta_s x_t < -bsx_{t-s} \tag{147}$$

Now consider a limit in which $s \to 0$, $b \to \infty$, and $bs \to 0$. That is, b is of a smaller order than s^{-1}. In this limit, the probability that a jump occurs with $x_t \neq 0$ approaches zero, since any time x_t jumps it returns to zero infinitely fast (but continuously).

Suppose a jump occurs at a time j in the interval $(t-s, t]$. By the assumption that $bs \to 0$, $x_t = x_{t-s} + \varepsilon_j$ and $\Delta_s x_t = \varepsilon_j$ almost surely. So then $\Delta_s f(x_t)$ converges in distribution to a Bernoulli mixture,

$$\Delta_s f(x_t) \to \begin{cases} 0 \text{ with prob. } 1-\lambda s \\ f(\varepsilon_t) \text{ with prob } \lambda s \end{cases} \tag{148}$$

Statements about the unconditional distribution of $f(\varepsilon)$ are then statements about growth rates of $f(x)$ mixed with zeros. The moment generating function of $\Delta_s f(x_t)$ is

$$M_{\Delta_s f}(t) = E\left[\exp\left(t\Delta_s f(x)\right)\right] = \lambda s M_{f(\varepsilon)}(t) + (1-\lambda s) \tag{149}$$

So all the moments of $\Delta_s f(x)$ are equal to λs times the moments of $f(\varepsilon)$. The variance therefore goes to zero as $s \to 0$, but variance per unit time converges to a constant (λ times the variance of $f(\varepsilon)$).

Intuitively, the idea behind this result is that the jumps explore the distribution of $f(\varepsilon)$, while the return of $f(x)$ back to $f(0)$ happens through increments that are effectively equal to zero (through the assumption that $bs \to 0$). This is obviously a special limiting case, but it illustrates a more general idea, which is that when shocks are fat tailed, the distribution of the growth rates becomes similar to the distribution of the levels.

The main text analyzes the behavior of the model in levels – e.g. its time-series skewness. Specifically, recall that aggregate and sector output are nonlinear functions of underlying productivity, which is just assumed to have a symmetrical unconditional distribution – the results hold for any symmetrical distribution. So that distribution could be the distribution of ε here. This section shows that if productivity x follows the process (146), then the moments of $\Delta_s f(x)$ are tightly linked to those of $f(\varepsilon)$. That is, analyzing the model in levels, with shocks ε, is equivalent to deriving results for its differences, $\Delta_s f(x)$ (subject to the various assumptions and limits in this section).

D.2 Numerical example

Recall that in the model, the log output of sector i on date t is affine in aggregate output and sector i's log productivity. So the dynamics of log productivity can be estimated by studying the residuals from regressions of each sector's output on aggregate output.

We estimate that regression empirically using data on industrial production. Denote the residuals from that regression by $x_{i,t}$ (which, again, under the model proxy for sector i's productivity). We then estimate an AR(1) model for $x_{i,t}$ sector-by-sector. The average autoregressive coefficient is -0.16 (indicating that the corresponding coefficient in levels is 0.84), while the average standard deviation of the innovations is 0.025. More importantly, the skewness of the innovations to $x_{i,t}$ is, on average across all sectors, -0.16, while the kurtosis is on average 5.73.

The high average degree of kurtosis is consistent with the model in the previous subsection in which the innovations have fat tails. The innovations are asymmetrically distributed, but the level of skewness is relatively small compared to what is observed for industrial production itself (as in table 1 and discussed in the main text).

We simulate the model setting $\alpha = 0.15$, the elasticity of substitution to 1/100, and the standard deviation and persistence to match the empirical values discussed above. The shocks are drawn from a t distribution with 5 degrees of freedom to approximately match the empirical degree of kurtosis.

The table below reports the time series skewness at the aggregate and sector levels for both the level of output and its growth rate, along with the correlation of cross-sectional standard deviation and skewness with aggregate output, again both in levels and growth rates. That is, we are calculating here the statistics reported in tables 1 and 2.

	Levels	Differences
skew(y)	-0.25	-1.55
skew(y_i)	-0.21	-1.24
Corr(skew$_t$(y_i),y_t)	0.19	0.37
Corr(stdev$_t$(y_i),y_t)	-0.27	-0.13

The results using levels and differences are the same qualitatively: skewness is more negative at the aggregate than at the sector level in both cases, and the cyclicality of the cross-sectional moments is the same in both cases. They are not the same quanitatively, and the relative magnitudes depend on the exact parameter choices. Nevertheless, this example shows that in at least a simple case, the analytic results that we are able to obtain for the model in levels also seem to be a good guide for its behavior in terms of first differences. The calibration in section 5 further emphasizes this point.

E Results on model of concave responses

Suppose sector and aggregate output are

$$Y_{i,t} = f\left(\varepsilon_t + \mu_{i,t}\right) \tag{150}$$

$$Y_t = \int_i Y_{i,t} \tag{151}$$

We can approximate these, assuming symmetric fundamental shocks, as

$$Y_{i,t} \approx f(0) + f'(0)\left(\varepsilon_t + \mu_{i,t}\right) + \frac{1}{2}f''(0)\left(\varepsilon_t + \mu_{i,t}\right)^2 \tag{152}$$

$$Y_t \approx f(0) + f'(0)\varepsilon_t + \frac{1}{2}f''(0)\varepsilon_t^2 + f'(0)\int_i \mu_{i,t} + \frac{1}{2}f''(0)\,2\varepsilon_t \int_i \mu_{i,t} + \frac{1}{2}f''(0)\int_i \mu_{i,t}^2 \tag{153}$$

$$= f(0) + f'(0)\varepsilon_t + \frac{1}{2}f''(0)\left(\varepsilon_t^2 + \sigma_\mu^2\right) \tag{154}$$

E.0.1 Lemma for skewness

First, a lemma. We are going to examine random variables of the form

$$x = \frac{a\sigma^2}{2}\left(\frac{\varepsilon}{\sigma}\right)^2 + b\sigma\frac{\varepsilon}{\sigma} + c \tag{155}$$

This has the form of a non-central χ^2. Specifically,

$$x = \frac{a}{2}\sigma^2\left(\frac{\varepsilon}{\sigma_\varepsilon} + \frac{b}{a\sigma}\right)^2 - \frac{1}{2}\frac{b^2}{a} + c \tag{156}$$

$$\sim -\frac{1}{2}\frac{b^2}{a} + c + \frac{a}{2}\sigma^2\chi^2(1,\lambda) \tag{157}$$

where $\chi^2(1,\lambda)$ is a non-central χ^2 with one degree of freedom and noncentrality parameter λ, where in this case $\lambda = \left(\frac{b}{a\sigma}\right)^2$. The skewness of x is then

$$skew(x) = sign\left(\frac{a}{2}\sigma^2\right)2^{3/2}\frac{k+3\lambda^2}{\left(k+2\lambda^2\right)^{3/2}} \tag{158}$$

$$\frac{dskew(x)}{d\lambda} = -sign\left(\frac{a}{2}\sigma^2\right)\frac{3\lambda}{(1+2\lambda)^{5/2}} \tag{159}$$

Finally,

$$\frac{dskew\left(x\right)}{d\sigma} = \frac{dskew\left(x\right)}{d\lambda}\frac{d\lambda}{d\sigma} \tag{160}$$

$$= -2\frac{dskew\left(x\right)}{d\lambda}\left(\frac{b}{a}\right)^{2}\sigma^{-3} \tag{161}$$

which implies

$$sign\left(\frac{dskew\left(x\right)}{d\sigma}\right) = -sign\left(\frac{dskew\left(x\right)}{d\lambda}\right) \tag{162}$$

$$sign\left(\frac{dskew\left(x\right)}{d\sigma}\right) = sign\left(a\right) \tag{163}$$

E.1 Aggregate and sector skewness

Both aggregate and sector output take the form

$$x = \frac{a\sigma^{2}}{2}\left(\frac{\varepsilon}{\sigma}\right)^{2} + b\sigma_{\varepsilon}\frac{\varepsilon}{\sigma} \tag{164}$$

from above. Specifically, for aggregate output

$$Y_{t} = \frac{1}{2}\sigma_{\varepsilon}^{2}f''\left(0\right)\frac{\varepsilon_{t}^{2}}{\sigma_{\varepsilon}^{2}} + f'\left(0\right)\sigma_{\varepsilon}\frac{\varepsilon_{t}}{\sigma_{\varepsilon}} + f\left(0\right) + \frac{1}{2}f''\left(0\right)\sigma_{\mu}^{2} \tag{165}$$

$$a = f''\left(0\right), b = f'\left(0\right), \text{ and } \sigma^{2} = \sigma_{\varepsilon}^{2} \tag{166}$$

and for sector output

$$Y_{i,t} = \frac{1}{2}f''\left(0\right)\left(\sigma_{\varepsilon}^{2} + \sigma_{\mu}^{2}\right)\frac{\left(\varepsilon_{t} + \mu_{i,t}\right)^{2}}{\left(\sigma_{\varepsilon}^{2} + \sigma_{\mu}^{2}\right)} + f'\left(0\right)\sqrt{\sigma_{\varepsilon}^{2} + \sigma_{\mu}^{2}}\frac{\left(\varepsilon_{t} + \mu_{i,t}\right)}{\sqrt{\sigma_{\varepsilon}^{2} + \sigma_{\mu}^{2}}} + f\left(0\right) \tag{167}$$

$$a = f''\left(0\right), b = f'\left(0\right), \text{ and } \sigma^{2} = \sigma_{\varepsilon}^{2} + \sigma_{\mu}^{2} \tag{168}$$

so they have the same form except for differences in the variance – σ_{ε}^{2} versus $\sigma_{\varepsilon}^{2} + \sigma_{\mu}^{2}$. We can therefore apply the result from above, Since the σ^{2} for sector output is greater than the σ^{2} for aggregate output, and since $sign\left(a\right) < 0$, $skew\left(Y_{i,t}\right) < skew\left(Y_{t}\right)$. That is, sector skewness is more negative than aggregate skewness. Intuitively, the curvature in f has greater relevance when the shocks are larger – for very small shocks, f is effectively linear and induces no asymmetry in the distribution. So in general this model predicts aggregate skewness is smaller than sector skewness.

F Skewness in the model with stochastic volatility

Suppose sector and aggregate output are

$$\begin{aligned} Y_t &= \varepsilon_t && (169)\\ Y_{i,t} &= \varepsilon_t + \mu_{i,t} && (170) \end{aligned}$$

and $\mu_{i,t}$ has the characteristics

$$\begin{aligned} corr\left(\text{var}\left(\mu_{i,t}\right), \varepsilon_t\right) &< 0 && (171)\\ corr\left(skew\left(\mu_{i,t}\right), \varepsilon_t\right) &> 0 && (172) \end{aligned}$$

This model trivially generates the facts about cyclicality of skewness and variance. It has no prediction for either aggregate skewness or changes in covariances following sector-level shocks.

The model could be extended as follows (I think of this is a reduced-form version of Nick Bloom's models):

$$\begin{aligned} Y_t &= \varepsilon_t - k\sigma_{t-1}^2 && (173)\\ Y_{i,t} &= \varepsilon_t + \mu_{i,t} - k\sigma_{t-1}^2 && (174)\\ \varepsilon_t &\sim N\left(0, \sigma_{t-1}^2\right) && (175)\\ \mu_{i,t} &\sim N\left(0, m\sigma_{t-1}^2\right) && (176) \end{aligned}$$

Think of this as a setup where sector and aggregate uncertainty move stochastically and directly affect the level of aggregate output.

The unconditional distribution of aggregate output will be skewed left. The proof is:

$$\begin{aligned} E\left[Y_t^3\right] &= E\left[\left(\varepsilon_t - \widehat{k\sigma_{t-1}^2}\right)^3\right] && (177)\\ &= E\left[\left(\varepsilon_t^2 - 2\widehat{k\sigma_{t-1}^2}\varepsilon_t + \widehat{k\sigma_{t-1}^2}^2\right)\left(\varepsilon_t - \widehat{k\sigma_{t-1}^2}\right)\right] && (178)\\ &= E\left[\left(\varepsilon_t^3 - 2\widehat{k\sigma_{t-1}^2}\varepsilon_t^2 + \left(\widehat{k\sigma_{t-1}^2}\right)^2\varepsilon_t\right) - \left(\widehat{k\sigma_{t-1}^2}\varepsilon_t^2 - 2\left(\widehat{k\sigma_{t-1}^2}\right)^2\varepsilon_t + \left(\widehat{k\sigma_{t-1}^2}\right)^3\right)\right] && (179) \end{aligned}$$

$$= E\left[-3k\widehat{\sigma^2_{t-1}}\varepsilon_t^2 - \left(k\widehat{\sigma^2_{t-1}}\right)^3\right] \quad (180)$$

$$= E\left[-3k\widehat{\sigma^2_{t-1}}\sigma^2_{t-1}\right] \quad (181)$$

$$= -3k \operatorname{var}\left(\sigma^2_{t-1}\right) \quad (182)$$

Sector-level output will also be skewed left. I'm not sure whether by less or more.

G Simulations of the conditional covariance estimation

To check the validity of the coefficient estimates and associated t-statistics reported above, this section examines results from estimating the same regressions in two sets of simulations.

The first simulation constructs a dataset of the same size as the empirical sample but eliminating all cross-sectional and time series. Specifically, it redraws a set $\{\tilde{\varepsilon}_{i,t}\}$ randomly from the full set of $\{\varepsilon_{j,s}\}$ (for all j, s). The simulated $\tilde{\varepsilon}_{i,t}$ are therefore by construction i.i.d. draws from the observed empirical distribution. We would then expect the estimated coefficients in the four regressions, estimated on the $\tilde{\varepsilon}$, to all average to zero with t-statistics that are also mean zero with unit variance.

The second simulation holds the time series of $\varepsilon_{i,t}$ fixed for all i, but applies a random shift to each time series. That is, for a given simulation, a random integer, η_i, is drawn from the set of integers $\{1, ..., T\}$ for each sector. The simulated time series, $\tilde{\varepsilon}_{i,t}$ is then

$$\tilde{\varepsilon}_{i,t} = \varepsilon_{i,(t+s) \bmod T} \quad (183)$$

This simulation then retains the time series properties of the individual ε time series, but removes any cross-sectional dependence. We use it to check the validity of the first two regressions, since they measure cross-sectional dependence. In the regressions of $\tilde{\varepsilon}^2_{i,t}$ and $\log \tilde{\varepsilon}^2_{i,t}$ on lags of $\tilde{\varepsilon}_{i,t}$, we would expect nonzero coefficients since those regressions depend on the time-series properties of volatility *within* as opposed to *across* sectors.

Figures A1 and A2 plot histograms of coefficients and t-statistics for the three regressions in the two different simulation specifications. Figure A1 reports results for industrial production, while figure A2 plots results for employment. The top panels plot distributions of coefficients, and the bottom panels the t-statistics. The vertical line in each panel represents the empirical estimates, while "Sim 1" refers to the fully random simulation and "Sim 2" the simulation with random starting points for the time series (i.e. with no cross-sectional dependence).

[I think the first row here is a bit messed up – I need to rerun the simulations; the second

row is correct, though, because there's just a scaling problem in the coefficients – the t-stats are unaffected]

The first column reports results for $\Sigma_{t,i}$. In the top panel the empirical coefficient estimate is well outside the distribution from either simulation, and the t-statistic is in the tail of the distributions. Furthermore, the simulated t-statistics have means near zero and standard deviations of approximately 1 (0.007 and 1.007 for simulation 1 and 0.009 and 1.02 for simulation 2). These results show that the regressions explaining a sector's covariance are not driven by a mechanical issue and that the standard errors are accurate.

Columns 2 plots results for regressions of $\varepsilon_{i,t}^2$. Since this effect is driven by time series variation, we only use simulation 1 as a test, sine in simulation 2 the coefficients should not (and do not) center around zero. The results are again as expected – the empirical coefficients are well outside the sampling distributions under the i.i.d. simulation, and the simulated t-statistics have the correct distribution. Across all four panels, the results are similar for industrial production and employment.

Table 1. Means and medians of skewness of sector-level growth rates

Panel A: Industrial production

	Monthly growth rates		*Quarterly growth rates*		
Aggregation level	Raw	Residuals	Raw	Residuals	*# of sectors*
Aggregate	-1.183	N/A	-1.868	N/A	1
	[.135]		[.129]		
2-digit	-0.961	-0.224	-1.007	0.131	3
	[.142]	[.059]	[.103]	[.048]	
3-digit	-0.545	-0.207	-0.792	-0.054	43
	[.057]	[.033]	[.07]	[.036]	
4-digit	-0.454	-0.24	-0.589	-0.111	81
	[.057]	[.033]	[.07]	[.036]	
5-digit	-0.409	-0.21	-0.515	-0.125	62
	[.037]	[.024]	[.044]	[.017]	

Panel A: Employment

	Monthly growth rates		*Quarterly growth rates*		
Aggregation level	Raw	Residuals	Raw	Residuals	*# of sectors*
Aggregate	-1.65	N/A	-1.894	N/A	1
	[.311]		[.339]		
1-digit	-1.084	-0.2	-1.179	0.118	7
	[.192]	[.032]	[.224]	[.046]	
2-digit	-0.759	-0.168	-0.987	-0.138	18
	[.149]	[.054]	[.16]	[.057]	
3-digit	-0.641	-0.189	-0.771	-0.106	78
	[.097]	[.033]	[.125]	[.025]	
4-digit	-0.4	-0.107	-0.579	-0.081	237
	[.097]	[.033]	[.125]	[.025]	
5-digit	-0.337	-0.128	-0.467	-0.083	344
	[.054]	[.017]	[.081]	[.011]	

Panel C: Stock returns

	Daily returns		*Monthly returns*	
Aggregation level	Raw	Residuals	Raw	Residuals
Market	-0.53	N/A	-0.68	N/A
	[0.415]		[0.23]	
17 industries	-0.281	0.054	-0.479	-0.075
	[.294]	[.068]	[.17]	[.084]
30 industries	-0.248	0.086	-0.442	-0.031
	[.261]	[.073]	[.165]	[.089]
49 industries	-0.186	0.081	-0.414	-0.053
	[.219]	[.056]	[.155]	[.085]

Notes: Average time series skewness across sectors at various levels of aggregation. Jackknife standard errors (with a block length of 50 months) are reported in brackets. The "raw" columns are for the raw data, while the columns labeled "residuals" report skewness for residuals from regressions of sector on aggregate growth rates.

Table 2. Cyclicality of cross-sectional variance and skewness

Variance

	IP growth		IP growth residuals		Employment growth		Employment growth resids	
NBER rec. ind.	0.00182 ***		0.00173 ***		0.000065 **		4.4E-05 **	
	[0.00052]		[0.000048]		[0.000027]		[0.000020]	
Agg. empl. growth		-0.33 ***		-0.30 ***		-0.018 ***		-0.014 ***
		[0.11]		[0.10]		[0.005]		[0.003]
# of obs.	566	566	566		351	351		
R2	0.1	0.12			0.05	0.1		

Skewness

	IP growth		IP growth residuals		Employment growth		Employment growth resids	
NBER rec. ind.	-0.75 ***		-0.39 *		-0.89 ***		-0.79 ***	
	[0.24]		[0.23]		[0.24]		[0.27]	
Agg. empl. growth		79		11		250 **		213
		[50]		[50]		[100]		[100]
# of obs.	566	566			351	351		
R2	0.01	0.01			0.01	0.02		

Kelley skewness

	IP growth		IP growth residuals		Employment growth		Employment growth resids	
NBER rec. ind.	-0.0088 ***		0.00049		-0.0032 ***		-0.0010 ***	
	[0.0021]		[0.0013]		[0.0011]		[0.00029]	
Agg. empl. growth		1.46 ***		-0.22		0.78 **		0.21
		[0.31]		[0.21]		[0.13]		[0.05]
# of obs.	566	566			351	351		
R2	0.06	0.06			0.22	0.36		

Notes: Regressions of cross-sectional moments on NBER recession indicators and aggregate employment growth. Standard errors are calculated by Newey–West with 12 monthly lags. The columns labeled residuals use cross-sectional moments of residuals from regressions of sector growth rates on aggregate growth.

Table 3. Predicting volatility

	Employment	IP	Stock returns		
Dependent variable:	Log squared resid.	Log squared resid.	Log squared resid.	log VIX	log VIX
X-sectional sd(t-1)	0.23 *	0.36 ***	0.27 ***	0.047 ***	0.038 ***
	[0.13]	[0.10]	[0.09]	[0.016]	[0.015]
Market return(t-1)					-0.31 ***
					[0.07]
N	351	560	563	383	383

Notes: All regressions include three lags of the dependent variable. For employment and IP, the dependent variable is the log of the squared residual from an AR(4) model. In the third column it is the log squared return over the month. In the last two columns it is the log of the VIX (calculated from CME options prices in the period prior to the availability of CBOE data; see Berger, Dew-Becker, and Giglio (2019)). In the first four columns, the independent variable is the lagged cross-sectional standard deviation of industry growth rates. The lagged return on the stock market is included in the final column. Standard errors are reported in brackets.

Table A1. Predicting volatility with cross-sectional standard deviation of residuals

	Employment	IP	Stock returns		
Dependent variable:	Log squared resid.	Log squared resid.	Log squared resid.	log VIX	log VIX
X-sectional sd(t-1)	0.19	0.36 ***	0.29 ***	0.048 ***	0.041 ***
		[0.09]	[0.09]	[0.016]	[0.015]
Market return(t-1)					-0.32 ***
					[0.07]
N	351	560	563	383	383
R2					

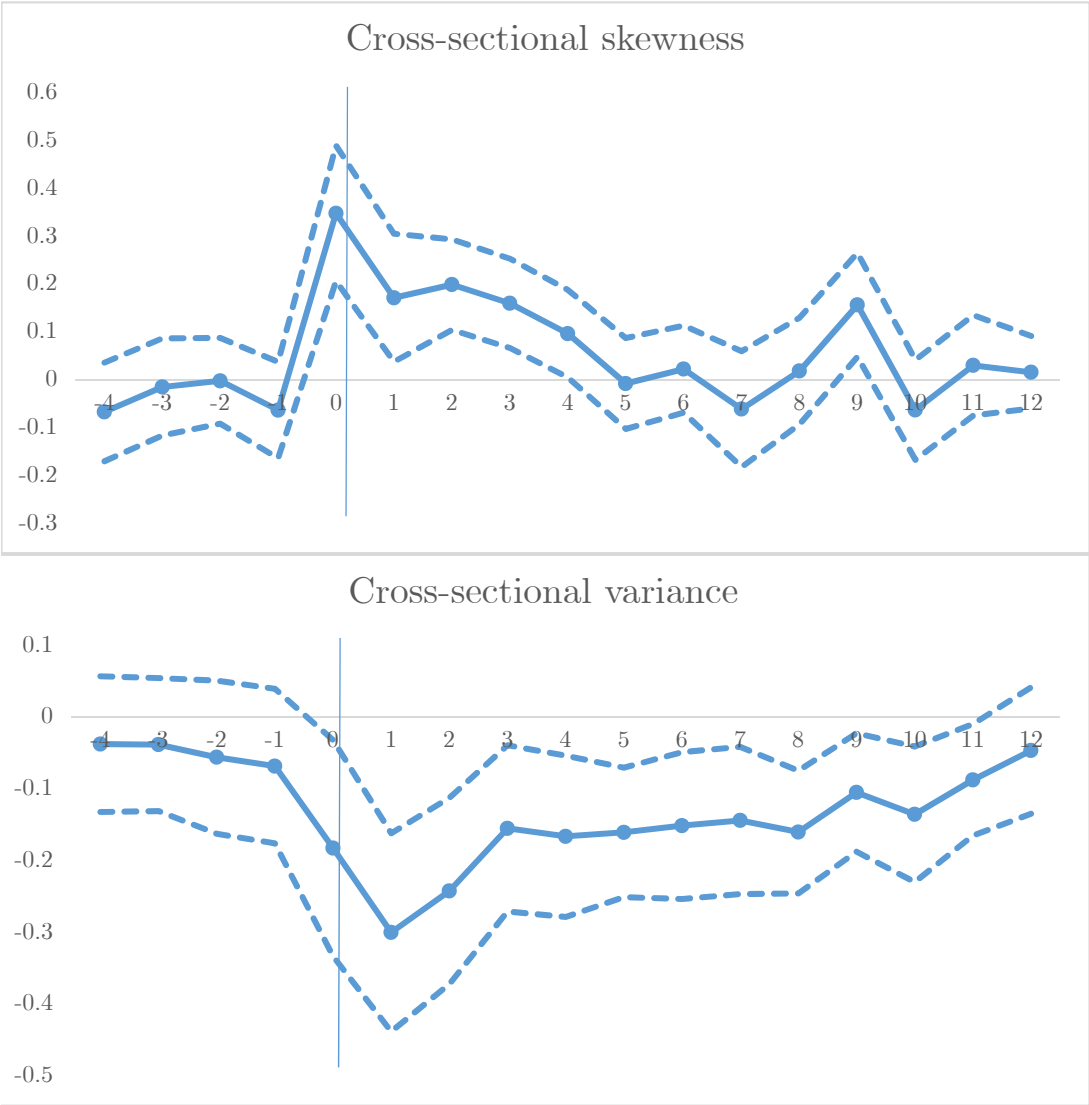
Notes: See table 4. The only difference here is that the cross-sectional standard deviation is on residual growth rates after controlling for aggregate growth.

Table 4. Conditional covariance regressions

Σ_i				β_i			
Activity measure:	IP	Empl	Returns	Activity measure:	IP	Empl	Returns
b_1	-3.6E-04	-8.2E-05 **	-0.14 ***		-0.00036 *	-1.9E-05	-0.058 **
	[2.4e-4]	[3.3e-5]	[0.05]		[0.00021]	[1.9e-5]	[0.024]
b_2	-1.9E-04	-2.0E-05	-0.11 **		-0.00023 *	-8.5E-07	-0.019
	[1.2e-4]	[1.9e-5]	[0.04]		[0.00013]	[1.2e-5]	[0.023]
b_3	-1.8E-04	-2.7E-05	-0.08		-0.00009	1.3E-05	0.002
	[1.1e-4]	[2.8e-5]	[0.06]		[0.00011]	[1.6e-5]	[0.022]
$b_1+b_2+b_3$	-7.3E-04 **	-1.3E-04 ***	-0.33 **		-0.00069 **	-7.0E-06	-0.075 *
	[3.3e-4]	[4.8e-5]	[0.13]		[0.00031]	[2.6e-5]	[0.04]
# of obs.	61,464	117,944	39,455		61,464		39,455
ε_i^2				$\log(\varepsilon_i^2)$			
Activity measure:	IP	Empl	Returns	Activity measure:	IP	Empl	Returns
b_1	0.012	-0.004	-0.65 ***	b_1	-1.24 ***	-1.64 **	-0.09 ***
	[0.008]	[0.005]	[0.20]		[0.28]	[0.77]	[0.03]
b_2	-0.007	-0.011 **	-0.48 ***	b_2	-1.21 ***	-2.00 ***	-0.04 *
	[0.004]	[0.005]	[0.18]		[0.25]	[0.72]	[0.02]
b_3	-0.004	-0.010 ***	-0.28	b_3	-0.81 ***	-2.00 ***	-0.01
	[0.004]	[0.004]	[0.24]		[0.24]	[0.63]	[0.03]
$b_1+b_2+b_3$	-0.023 *	-0.026 **	-1.41 ***	$b_1+b_2+b_3$	-3.27 ***	-5.64 ***	-0.15 ***
	[0.012]	[0.011]	[0.45]		[0.46]	[1.31]	[0.05]
# of obs.	61,464	117,944	39,455	# of obs.	61,464	117,944	39,455

Notes: Regressions of sector variance, covariance, and beta lagged sector growth rates. The b coefficients are those for lagged sector growth rates. Each regression includes three monthly lags of the dependent variable and sector and time fixed effects. Standard errors are reported in brackets and clustered by date.

Figure 1. Relationship between cross-sectional return distribution and market returns



Notes: The lines plot coefficients from regressions of the monthly cross-sectional variance and skewness of monthly industry stock returns on four leads and 12 lags market stock returns. The dots are point estimates and the dotted lines 95-percent confidence intervals. The x-axis reports lags (with negative values indicating leading values).