What Do We Learn From Cross-Sectional Empirical Estimates in Macroeconomics?

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Abstract

Recent empirical work uses variation across cities or regions to identify the effects of economic shocks of interest to macroeconomists. The interpretation of such estimates is complicated by the fact that they reflect both partial equilibrium and local general equilibrium effects of the shocks. We present a simple method for recovering estimates of partial equilibrium effects from these cross-sectional empirical estimates. We apply our method to recent estimates of housing wealth effects based on city-level variation. For this case, we derive conditions under which the partial equilibrium effect of changes in house prices on consumption is equal to the city-level estimate divided by an estimate of the local fiscal multiplier.

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1 Introduction

A growing literature uses variation across cities or regions to identify the effects of economic shocks of interest to macroeconomists. What exactly these estimates identify is often complicated by the fact that city- or region-level outcomes reflect both the partial equilibrium effect of the shock and the local general equilibrium response to that shock. In this paper we propose an approach by which applied researchers can isolate the partial equilibrium effect of the shock. The partial equilibrium effect has a clear theoretical interpretation and can be easily calculated in standard one-region macro models for calibration purposes. Our method therefore allows researchers to avoid formulating and solving a multi-region macro model to be able to compare their empirical results to analogous concepts in a model.

We apply this method to the analysis of the housing wealth effect. The US housing boom and bust in the early 2000s focused the attention of economists on the implications of home prices changes for consumer spending. Prominent recent papers that estimate this housing wealth effect use regional data to relate outcomes such as spending, car registrations, and employment to changes in home prices (e.g. Mian et al., 2013; Mian and Sufi, 2014). The appropriate interpretation of these empirical results is not straightforward. House prices are endogenous at the level of a city and a shock that changes home prices surely alters consumption through other channels such as through the equilibrium response of wages and interest rates. In particular, suppose an increase in home prices triggers some additional spending in a city through a housing wealth effect. The extra spending will raise incomes locally, which will lead to more spending setting in motion a consumption multiplier. For these reasons, it is not immediately clear what we can learn from the change in city-level consumption in response to a change in house prices in the city.

We show how existing empirical estimates of the housing wealth effect on consumption can be mapped into the partial-equilibrium effect of house prices on consumption. We start from the point of view that some prices are determined nationally and some are determined locally. For example, financial markets are highly integrated at a national level while labor markets are quite local in nature. We lay out the conditions under which time fixed effects included in empirical regression specifications absorb the variation in national prices. Turning to the local general equilibrium effects, we show how the local fiscal multiplier can be used to gauge the effect of local general equilibrium multipliers on the response of consumption to changes in house prices. Under certain assumptions that we discuss in more detail below, we show that one can remove local gen-

eral equilibrium effects from a city-level estimate of the housing wealth effect simply by dividing that estimate by an estimate of the local fiscal multiplier. The end result is an estimate of the partial equilibrium housing wealth effect and marginal propensity to consume out of housing wealth (MPCH) that corresponds to a change in home prices holding fixed wages other non-housing prices.

In recent complementary work, we estimate the housing wealth effect based on city-level variation in house prices and using retail employment as a proxy for local consumption (Guren et al., 2018). We estimate an elasticity of retail employment with respect to house prices of 0.053, which implies an MPCH of 2.4 cents on the dollar. Nakamura and Steinsson (2014) estimate the local fiscal multiplier to be approximately 1.5. Dividing our housing wealth effect estimate by the Nakamura and Steinsson's local fiscal multiplier estimate yields a partial equilibrium housing wealth effect estimate of $0.053/1.5 = 0.035$, which implies a partial equilbrium MPCH of 1.6 cents on the dollar.

The simple approach of dividing by the local fiscal multiplier makes sense if a change in house prices only directly stimulates the local economy through a housing wealth effect on consumption, which is then amplified by a consumption multiplier. An increase in house prices may however also stimulate the local economy by increasing construction activity. This is a separate channel from the standard consumption multiplier because the trigger is not consumption but residential investment. To assess the importance of this channel, we use the empirical research design that we developed in Guren et al. (2018) to estimate the response of construction employment to changes in house prices. We also show theoretically how to use such an estimate to strip out the effect of construction and get back to a partial equilibrium housing wealth effect on consumption. When we do this, we arrive at a partial-equilibrium housing wealth effect of 0.029 or an MPCH of 1.3 cents on the dollar.

We introduce our methods in the context of a static economy, but they extend to dynamic economies as well as we show in the context of our application. In a dynamic context, there is no single fiscal multiplier but an impulse response of output to a fiscal shock. For example, for the simple case the partial equilibrium effect of home prices on consumption is $F^{-1}E$ where E is a matrix where the (i, j) element gives the housing wealth effect of a price change at date j on consumption at date i and F is a matrix that gives the effect of a fiscal shock at date j on output at date i. Using simulations, we explore how well our static formula performs when the data are generated by a dynamic model. Under some conditions it holds exactly and in general it performs well.

[Add related literature here.]

The paper is organized as follows. Section 2 lays out the general method. Section 3 applies the method to the housing wealth effect. Section 4 provides a fully structural, multi-region macro model of the housing wealth, which we use to assess and motivate the identification assumptions used in Section 3. Section 5 shows how the method can be applied within the context of the full dynamic model. Section 6 conducts a Monte Carlo analysis of the fully structural model to evaluate the accuracy and robustness of the identification scheme used in Section 3.

2 The Method

We consider an economy composed of J regions that each produce and consume $\ell + L$ goods. The first ℓ goods are produced and consumed locally. The remaining L goods are traded in integrated markets. In total there are $J\ell + L$ goods. We normalize the price of the first good to 1. To fix ideas, the numeraire good could be final goods in the "home" region. Let p_j be the vector of local prices in region j and let P be the vector of national prices. Let x_j be a set of local shocks to region j and let X be a vector of aggregate shocks. Let the vector-valued functions $D(p_j, P, x_j, X)$ and $S(p_j, P, x_j, X)$ be the demand and supply curves for local goods in j. In the local equilibrium of region j we have $D(p_j, P, x_j, X) = S(p_j, P, x_j, X)$. If we assume linearity we can write

$$(D_p - S_p)\, p_j + (D_x - S_x)\, x_j + (D_P - S_P)\, P + (D_X - S_X)\, X = 0,$$

where D_p is the partial derivative of D with respect to p and so on. In this formulation the outcomes of the regions differ only because they experience different idiosyncratic shocks. In Section 4, we allow the economies to also differ in their housing supply curves.

We assume that the aggregate prices are insensitive to the idiosyncratic shocks. Using the implicit function theorem, the comparative statics of the effect of x_j on p_j are given by

$$\frac{dp_j}{dx_j} = -\left[D_p - S_p\right]^{-1} (D_x - S_x)\,. \tag{1}$$

Let y_j be the equilibrium quantity of local goods produced in j so that

$$y_j = D(p_j, P, x_j, X) = S(p_j, P, x_j, X).$$

Differentiating, we have

$$\frac{dy_j}{dx_j} = D_p \frac{dp_j}{dx_j} + D_x. \tag{2}$$

Suppose we observe some elements of $\frac{dy_j}{dx_j}$ and some elements of $\frac{dp_j}{dx_j}$, and we wish to identify some of the elements of D_p, S_p, D_x, and/or S_x. Our approach is to impose some structural assumptions on these unknown matrices and then manipulate (1) and (2) to solve for the unknowns in terms of the observed responses.

Notice that the aggregate shocks and prices play no role in the analysis. Neither do the matrices D_P, S_P, D_X, or S_X. These aggregate disturbances shift quantities and prices uniformly across regions and are absorbed by regression constants or time fixed effects. This outcome relies on two assumptions: linearity and equal incidence of the aggregate shocks and prices (i.e. D_P, S_P, D_X, and S_X are common across regions).

The classic approach to simultaneous equation models uses instruments that each appear in only one of the equations. An instrument in this sense implies a particular structure for D_x and or S_x.

3 Application to the Housing Wealth Effect

We now present two examples to illustrate the method presented in section 2. We start with a simple example that involves strong assumptions to facilitate the exposition before turning to successively richer examples.

3.1 Simple Example

We start by considering a simple, static model with three markets: a goods market, a housing market, and a labor market. As we discuss above, aggregate shocks and national prices do not affect the analysis. We, therefore, start with the behavior of a single region taking aggregate prices as given. Later in this section we incorporate multiple regions and trade linkages.

The economy produces goods using labor. The production function is $Y = N$, where Y is goods produced and N is labor supply. There is a labor supply function $N(w, p)$ that depends on the wage, w, and potentially also the price of housing, p. Both prices are expressed in terms of goods. Household demand for goods is given by $C(w, p)$. In addition to this private consumption demand,

goods are used for public consumption in amount g, where g is exogenous. The aggregate resource constraint is $Y = C(w,p) + g$. For housing, we specify an excess demand function $H(w,p,s)$, where s is a shock. Since we only use data on the price of housing and not on the quantity of housing that is traded, we do not need to specify housing supply and demand separately.

In this simple model there are no interest rates or taxes. We assume these variables are determined at the national level and do not differentially affect the regions so their consequences are absorbed by time fixed effects. We discuss these national prices in more detail in the context of the dynamic model presented in Section 4.

The equilibrium level of wages and house prices in this model is given by the solution to the following two equations:

$$\begin{aligned} C(w,p) + g &= N(w,p) \\ H(w,p,s) &= 0. \end{aligned}$$

Using the implicit function theorem as above, comparative statics for the price vector are

$$\frac{d(w,p)}{d(g,s)} = -\begin{pmatrix} C_w - N_w & C_p - N_p \\ H_w & H_p \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & H_s \end{pmatrix},$$

which we can rearrange as

$$\frac{d(w,p)}{d(g,s)} = \underbrace{\left[N_w - C_w + (C_p - N_p)\frac{H_w}{H_p}\right]^{-1}}_{\equiv M} \begin{pmatrix} 1 & \frac{H_s}{H_p}(N_p - C_p) \\ -\frac{H_w}{H_p} & \frac{H_s}{H_p}(C_w - N_w). \end{pmatrix} \tag{3}$$

Using (2), we then have

$$\frac{dY}{d(g,s)} = \begin{pmatrix} C_w & C_p \end{pmatrix} \frac{d(w,p)}{d(g,s)} + \begin{pmatrix} 1 & 0 \end{pmatrix}. \tag{4}$$

Our goal is to estimate C_p, the partial equilibrium effect of home prices on consumption. Suppose we observe an instrumental variables (IV) estimate of the housing wealth effect based on regional variation. In our notation, this is $dY/dp = (dY/ds)/(dp/ds)$, where s is the shock (instrument) used to estimate dY/dp. The trouble is that our IV estimate is the total derivative of consumption with respect to house prices, not the partial derivative C_p.

We make two additional structural assumptions to simplify the model: 1) there are no (short-

run) wealth effects on labor supply, $N_p = 0$; 2) house prices are independent of income, $H_w = 0$. The first of these assumptions we view as likely being a reasonable approximation to reality, while the second is less likely to be a good approximation. We will therefore relax the second of these assumptions below. But for now, we make these two assumptions. This simplifies equation (3) to

$$\frac{d(w,p)}{d(g,s)} = \underbrace{[N_w - C_w]^{-1}}_{\equiv M} \begin{pmatrix} 1 & -\frac{H_s}{H_p} C_p \\ 0 & \frac{H_s}{H_p} (C_w - N_w). \end{pmatrix}$$

Combining this equation with equation (4) then yields

$$\begin{aligned} \frac{dY}{dg} &= 1 + MC_w \\ \frac{dp}{ds} &= M \frac{H_s}{H_p} (C_w - N_w) \\ \frac{dY}{ds} &= -MC_p \frac{H_s}{H_p} N_w, \end{aligned}$$

which imply

$$\frac{dY/ds}{dp/ds} = C_p \frac{N_w}{N_w - C_w} \tag{5}$$

$$\frac{dY}{dg} = \frac{N_w}{N_w - C_w}, \tag{6}$$

where the second line uses the definition of M. Finally, dividing the first of these two equations by the second yields

$$C_p = \frac{(dY/ds)/(dp/ds)}{dY/dg} = \frac{dY/dp}{dY/dg}. \tag{7}$$

This shows that given the two simplifying assumptions we made above, the partial equilibrium effect of house prices on consumption, C_p, is equal to the cross-region IV estimate of the housing wealth effect, dY/dp, divided by the local fiscal multiplier, dY/dg. Looking back at equations (5) and (6), we see that the the partial equilibrium effect differs from the cross-region IV estimate by a local general equilibrium multiplier ($N_w/(N_w - C_w)$) and that the local fiscal multiplier provides an estimate of this local general equilibrium multiplier. An estimate of the local fiscal multiplier can therefore be used to convert a cross-region IV estimate of the housing wealth effect into an estimate of the partial equilibrium effect of house prices on consumption. Intuitively, an increase in home prices of one unit spurs an extra C_p of spending, which then triggers local adjustments in wages with accompanying consumption effects. These same local adjustments would occur if the

initial spending were due to a government spending shock.

In Guren et al. (2018) we estimate a marginal propensity to consume out of housing wealth (MPCH) of 2.4 cents on the dollar. This estimate corresponds to the total effect captured by $(dY/ds)/(dp/ds)$. Nakamura and Steinsson (2014) estimate a local fiscal multiplier of about 1.5.[1] Equation (7) implies that the partial equilibrium MPCH is 1.6 cents on the dollar.

3.2 Richer Example

We now relax two assumptions from the previous example. We allow residential investment to respond to home price changes and we allow housing demand to respond to income (i.e. we do not restrict $H_w = 0$). In this case, the equilibrium level of wages and house prices is given by the solution to the following two equations:

$$C(w,p) + I(w,p) + g = N(w,p)$$
$$H(w,p,s) = 0.$$

Relative to the previous example, we have now added demand for local goods coming from residential investment.

With this richer model, the conversion of the regional estimate of the housing wealth effect into the partial equilibrium effect of house prices on consumption will rely on us being able to observe several additional pieces of information. In particular, suppose we observe estimates of the response of house prices to government spending shocks, dp/dg, and the response of residential investment to a change in house prices, dI/dp, in addition to the estimates we assumed we had above.

As before, we assume that there is no (short-run) wealth effect on labor supply. In this case, we assume in addition that residential investment is independent of income, $I_w = 0$. (We could relax the $I_w = 0$ assumption if we observed dC/dg or dI/dg.) Under these assumptions, the partial-equilibrium effect of home prices on consumption is

$$C_p = \frac{E}{\frac{dY}{dg}(1 - EZ)} - \frac{dI}{dp} \tag{8}$$

where E is the housing wealth effect on total spending $E = \frac{(dC/ds)+(dI/ds)}{dp/ds}$ and $Z = \frac{dp/dg}{dY/dg}$ is the

[1] Nakamura and Steinsson find larger multipliers in regional data than in state data. As the analysis of the housing wealth effect is undertaken at the city (CBSA) level, it may be appropriate to use a fiscal multiplier somewhat below 1.5.

income effect on home prices. The derivation of equation 8 appears in Appendix A.

In equation (8), there are two considerations in addition to the fiscal multiplier intuition from the previous example. When house prices are affected by income, $H_w \neq 0$, a house price shock differs from the government spending shock because part of the response to the government spending shock comes through home prices. In computing the housing wealth effect we compute the change in spending relative to the observed total change in home prices. It is not appropriate to apply the full local fiscal multiplier to the initial spending, but rather we only want to apply the part of the local fiscal multiplier that operates through wages. Second, some part of the initial spending response to home prices comes from residential investment. We subtract this component to isolate the part coming from consumption.

We can now use these results to generate a new estimate of the partial equilibrium effect of house prices on consumption by plugging empirical estimates into equation (8). For this purpose, it is to rewrite equation (8) in terms of elasticities:

$$\frac{p}{C}C_p = \frac{\frac{p}{C}E}{\frac{dY}{dg}\left(1 - \frac{C}{Y}\frac{p}{C}E\frac{Y}{p}Z\right)} - \frac{I}{C}\frac{p}{I}\frac{dI}{dp}. \tag{9}$$

We have estimated the elasticity of construction employment to home prices to be 0.309. This estimate is based on an analogous specification to the full-sample housing wealth effect estimate we present in Column 1 of Table 1 of Guren et al. (2018) with construction and real estate employment as the outcome variable. We use this as our estimate of the elasticity of residential investment to home prices. Lamont and Stein (1999) provide estimates of the short-run income elasticity of house prices, which imply it is less than 0.8 and more likely near 0.3. We use 0.3 as our estimate, but our conclusions are little changed by using 0.8. Our estimate of the housing wealth effect as an elasticity is 0.053. Between 1975 and 2017, the average ratio of residential investment to personal consumption was 0.068. We use this for I/C. Over the same period, the C/Y ratio averaged 0.649. We will again use the Nakamura and Steinsson (2014) estimate of the local fiscal multiplier of 1.5. We can rewrite $(p/C)E$ as

$$\frac{p}{C}E = \frac{p}{C}\frac{dC/ds}{dp/ds} + \frac{I}{C}\frac{p}{I}\frac{dI/ds}{dp/ds} = 0.053 + 0.068 \times 0.309 = 0.074.$$

Plugging these numbers into equation (9) we get

$$\frac{p}{C}C_p = \frac{0.074}{1.5\left(1 - 0.649 \times 0.074 \times 0.3\right)} - 0.068 \times 0.309 = 0.029. \tag{10}$$

In terms of an MPCH, we divide by a ratio of housing wealth to consumption of 2.17 to arrive at 1.3 cents on the dollar.[2]

3.3 Trade Linkages

Now suppose there are two regions that trade goods with one another. Let ϕ be the expenditure share on local goods so $\phi > 1/2$ implies home bias in goods demand. For this example, we will assume the real exchange rate is fixed and equal to one. Building on the previous example we have the following system

$$\begin{aligned}
\phi\left[C(w,p) + I(w,p)\right] + (1-\phi)\left[C(w^*,p^*) + I(w^*,p^*)\right] + g &= N(w,p) = Y \\
(1-\phi)\left[C(w,p) + I(w,p)\right] + \phi\left[C(w^*,p^*) + I(w^*,p^*)\right] + g^* &= N(w^*,p^*) = Y^* \\
H(w,p,s) &= 0 \\
H(w^*,p^*,s^*) &= 0,
\end{aligned}$$

where a star denotes variables in the foreign region and variables without stars refer to the home region. If we linearize the system, take the difference across regions, and impose the structural assumptions from the previous subsection ($N_p = I_w = 0$) we have the following system of two equations

$$\begin{aligned}
(2\phi - 1)\left[C_w\hat{w} + \left(C_p + I_p\right)\hat{p}\right] + \hat{g} &= N_w\hat{w} = \hat{Y} \\
H_w\hat{w} + H_p\hat{p} + H_s\hat{s} &= 0,
\end{aligned}$$

where hats denote cross-region differences, e.g. $\hat{w} \equiv w - w^*$. This is the same as the linearized system we obtain in Section 3.2 except for two differences. First, the variables in the system are the cross-region differences. And second, some of the components of the first equation are scaled by $(2\phi - 1)$, which reflects the attenuation of cross-region differences in expenditure due to trade

[2] We take the value of owner-occupied housing from the flow of funds and divide it by total PCE less PCE on housing services and utilities and then take the average over 1975 to 2017 to arrive at a ratio of 2.17.

linkages. The main result of Section 3.2 , Equation (8), is now slightly modified

$$C_p = \frac{E}{\frac{dY}{dg}\left(1 - E(2\phi - 1)Z\right)} - \frac{dI}{dp} \tag{11}$$

The derivation of equation 8 that appears in Appendix A allows for trade linkages. The only difference here is that the adjustment to the fiscal multiplier for the effects of government spending on home prices is now attenuated due to trade linkages.

To apply the result to our empirical estimates we require a value of ϕ. We choose 0.69 following Nakamura and Steinsson (2014).[3] Adjusting for trade linkages has a quantitatively negligible effect on the implied partial equilibrium housing wealth effect. We arrive at an elasticity of 0.029 just as before.

4 Fully Structural Model

We now present a fully-microfounded, dynamic, macro model of multiple regions. After presenting the model we show how the method can be applied in the context of the full model. In subsequent sections we will use numeric solutions of this model to assess the accuracy of the formulae we derived using the simple examples we presented previously.

4.1 Model assumptions

Demographics There are two regions, "home" and "foreign." The population of the entire economy is 1 with a share n in the home region. All variables are expressed in per capita terms.

Preferences Households maximize

$$\sum_{t=0} \beta^t u(C_t, L_t, Q_t; \Omega_t),$$

where the arguments are consumption, labor supply, units of housing Q_t chosen at date t and held to date $t+1$, and Ω_t is an aggregate housing demand shock. The period utility function is given

[3] Nakamura and Steinsson estimate the local fiscal multiplier based on state-level data, while GMNS estimate the housing wealth effect on consumption at the CBSA level. The extent of trade linkages are likely larger for smaller geographic areas. So should we use a value of ϕ based on states or CBSAs? Incorporating trade linkages into our analysis is accounting for the fact that the local fiscal multiplier is attenuated by trade linkages so it makes sense to use a value of ϕ consistent with the geographic unit used to estimate the fiscal multiplier.

by

$$u(C, L, Q; \Omega) = \frac{1}{1-\sigma}\left[\left(C - \psi\frac{L^{1+\nu}}{1+\nu}\right)^{\kappa}(Q-\Omega_t)^{1-\kappa}\right]^{1-\sigma}.$$

Commodities and technology Consumption C_t is a bundle of final goods produced in home and foreign

$$C_t = \phi^{-\phi}(1-\phi)^{-(1-\phi)} C_{H,t}^{\phi} C_{F,t}^{1-\phi},$$

where $C_{F,t}$ is the consumption in home of the good produced in foreign.[4] We will use $*$ to denote foreign variables so $C_{H,t}^*$ is the consumption in foreign of the good produced in home and we assume

$$C_t^* = \phi^{*-\phi^*}(1-\phi^*)^{-(1-\phi^*)} C_{F,t}^{*\phi^*} C_{H,t}^{*(1-\phi^*)}.$$

$\phi > n$ and $\phi^* > 1-n$ capture the home-bias in demand for goods.

Each region produces a final good out of intermediate inputs. The production of the final good satisfies

$$Y_t = \left(\int_0^1 y_t(z)^{\frac{\eta-1}{\eta}} dz\right)^{\frac{\eta}{\eta-1}}$$

Each intermediate good is produced linearly out of labor $y_t(z) = L_t(z)$.

Housing supply The supply of housing satisfies:

$$Q_t = (1-\delta)Q_{t-1} + I_t^{\alpha} M_t^{1-\alpha} \tag{12}$$

where I_t is resources devoted to residential investment and M_t is units of construction permits sold by the federal government. Residential investment requires a mix of local and imported inputs like the one used for consumption

$$I_t = \phi^{-\phi}(1-\phi)^{-(1-\phi)} I_{H,t}^{\phi} I_{F,t}^{1-\phi}.$$

Markets The two regions share the same money, which serves as the numeraire. Intermediate goods markets are competitive and completely integrated across regions. Intermediate variety firms face Calvo price-setting frictions with probability of adjusting their price of $1-\chi$. The labor markets

[4] Including the term $\phi^{-\phi}(1-\phi)^{-(1-\phi)}$ in the definition of the bundle simplifies the expression for the price index.

are local to each region and competitive with real wages denoted w_t. Units of housing trade at relative price J_t. Households trade trade a nominal bond in quantity B_t at interest i_t. We consider two cases for asset markets. In the "incomplete markets" economy, there is only trade in bonds. In the "complete markets" economy, the regions also trade state-contingent assets in quantities A_t at prices $\Xi_{t,t+1}$.[5] Intermediate goods firms produce profits, which are rebated to the households in the region. We use D_t to denote the real profits received.

Government There is a government that purchases goods, sells construction permits, and sets monetary policy. Let G_t and G_t^* be per capita spending in home and foreign, respectively. The government buys local goods in each region. The exogenous process for G_t is

$$G_t = (1 - \rho_G)\bar{G} + \rho_G G_{t-1} + \epsilon_{G,t}. \tag{13}$$

There is a monetary policy rule that sets the nominal interest rate

$$1 + i_t = \frac{1}{\beta} + \varphi_\pi \left(\bar{\pi} - 1\right) \varphi_y \bar{Y}_t, \tag{14}$$

where $\bar{\pi} \equiv (\pi)^n (1 - \pi)^{1-n}$ and $\bar{Y}_t \equiv n\hat{Y}_t + (1-n)\hat{Y}_t^*$ is the average of log deviations of output from steady state.

The government sells construction permits according to the rule:

$$M_t = \bar{M} J_t^{\gamma} \tag{15}$$

The parameter γ reflects the elasticity of supply of vacant land. The government sets the price of a permit, P_t^M equal to the marginal product of land in the construction sector.

The government imposes lump sum taxes in nominal amounts $P_t T_t$ and $P_t^* T_t^*$. The national government budget constraint is

$$nP_{H,t}G_t + (1-n)P_{F,t}^* G_t^* = nP_t T_t + (1-n)P_t^* T_t^* + nP_t^M M_t + (1-n)P_t^{M*} M_t^*.$$

We assume that the government taxes each region equally in nominal terms.

[5] In the complete markets economy, the bond is redundant, but it can still be priced and this price will enter our monetary policy rule.

Market-clearing The market for home goods clears if

$$Y_t = \phi E_t^{\phi-1}\left(C_t + I_t\right) + \frac{1-n}{n}(1-\phi^*)E_t^{-\phi^*}\left(C_t^* + I_t^*\right) + G_t, \tag{16}$$

where $E_t \equiv P_{H,t}/P_{F,t}$ is the real exchange rate. This expression involves local and home expenditure on the bundles of home and foreign produced goods. The terms $\phi E_t^{\phi-1}$ and $(1-\phi^*)E_t^{-\phi^*}$ show that the cost-minimizing bundle depends on the degree of home bias and the real exchange rate. Similarly, the market for foreign goods clears if

$$Y_t^* = \frac{n}{1-n}(1-\phi)E_t^{\phi}\left(C_t + I_t\right) + \phi^* E_t^{1-\phi^*}\left(C_t^* + I_t^*\right) + G_t^*. \tag{17}$$

Bond market clearing requires

$$nB_t + (1-n)B_t^* = 0.$$

Decision rules. Define

$$\begin{aligned}
\tilde{C}_t &\equiv C_t - \psi\frac{L_t^{1+\nu}}{1+\nu} \\
\tilde{Q}_t &\equiv Q_t - \Omega_t \\
x_t &\equiv \frac{1-\kappa}{\kappa}\left(J_t - \mathbb{E}_t\left[J_{t+1}(1-\delta)\beta\frac{u_{C,t+1}}{u_{C,t}}\right]\right)^{-1}.
\end{aligned}$$

Utility maximization implies

$$\begin{aligned}
\tilde{Q}_t &= x_t\tilde{C}_t \\
\psi L_t^{\nu} &= w_t.
\end{aligned}$$

Under complete markets we have

$$\frac{\tilde{C}_t^{-\sigma}}{\tilde{C}_t^{*-\sigma}} = E_t^{\phi+\phi^*-1}\left(\frac{x_t^*}{x_t}\right)^{(1-\kappa)(1-\sigma)}$$

Under incomplete markets assuming certainty equivalence[6] so that the real return on bonds is treated as known $R_t \equiv (1+i_t)/\pi_{t+1}$ and future home prices are discounted at the safe interest rate,

[6]This case applies to the steady state, perfect foresight transitions, and first-order accurate solutions to stochastic economies.

we have

$$
\begin{aligned}
R_{t,t} &= 1 \\
R_{t,\tau} &\equiv R_{t,\tau-1} R_{\tau-1} \quad \forall \tau > t \\
X_{t,\tau} &\equiv \left[\beta^t R_{t,\tau} \left(\frac{x_\tau}{x_t} \right)^{(1-\kappa)(1-\sigma)} \right]^{1/\sigma} \\
\tilde{C}_t &= \kappa \frac{R_{t-1} B_{t-1} + Q_{t-1} J_t (1-\delta) + \sum_{\tau=t}^{\infty} R_{t,\tau}^{-1} \left[w_\tau L_\tau - \frac{1-\kappa}{\kappa x_\tau} \Omega_\tau - \psi \frac{L_\tau^{1+\nu}}{1+\nu} \right]}{\sum_{\tau=t}^{\infty} R_{t,\tau}^{-1} X_{t,\tau}}
\end{aligned} \tag{18}
$$

Furthermore we show in the appendix

$$I_t = \alpha^{\frac{1}{1-\alpha}} \bar{M} J_t^{\gamma + \frac{1}{1-\alpha}}$$

and the supply of new housing is

$$I^\alpha M_t^{1-\alpha} = (\alpha J_t)^{\frac{\alpha}{1-\alpha}} M_t.$$

More details. Appendix XX provides the full set of equations that characterize an equilibrium of the model.

5 Application in the Full Model

We now show how our method can be applied in the context of the full model. We consider perfect foresight transitions lasting $\mathcal{T}$ dates. We will assume that $1 - \phi^* = \frac{n}{1-n}(1-\phi)$. And we define $\Phi \equiv \phi + \phi^* - 1$. We will also assume that prices are rigid.

Starting with the market clearing conditions and take the difference across regions to arrive at

$$\hat{Y} = \Phi \left(\hat{C} + \hat{I} \right) + \hat{G},$$

where $\hat{Y}$ is a column vector of length $\mathcal{T}$ that gives values of $Y_t - Y_t^*$ for all t. $\hat{C}$, $\hat{I}$, and $\hat{G}$ are defined similarly. Linearize the consumption function (18) and take the difference across regions to arrive at

$$\hat{C} = \mathbf{C}_J \hat{J} + \mathbf{C}_W \hat{W} + \mathbf{C}_D \hat{D},$$

where $\mathbf{C}_J$ is a $\mathcal{T} \times \mathcal{T}$ matrix where the $[t, s]$ element gives the coefficient of the response of $C_{r,t}$ to $J_{r,s}$ and $\mathbf{C}_W$ and $\mathbf{C}_D$ are defined similarly. Here we have omitted the dependence of consumption on Ω and interest rates because these variables are common across regions and drop out when we take the difference. The linearized residential investment response is

$$\hat{I} = \mathbf{I}_{r,J}\hat{J}$$

Now use the linearized price functions, defined similarly to write

$$\hat{W} = \mathbf{W}\hat{Y}$$
$$\hat{D} = \mathbf{D}\hat{Y}.$$

Now combine all of this

$$\begin{aligned}\hat{Y} &= \Phi\left(\mathbf{C}_J\hat{J} + \mathbf{C}_W\hat{W} + \mathbf{C}_D\hat{D} + \mathbf{I}_{r,J}\hat{J}\right) + \hat{G} \\ \hat{Y} &= \Phi\mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_{r,J}\right)\hat{J} + \mathbf{M}\hat{G} \end{aligned} \tag{19}$$

where $\mathbf{m} \equiv \mathbf{C}_W\hat{W} + \mathbf{C}_D\hat{D}$ and $\mathbf{M} \equiv [I - \Phi\mathbf{m}]^{-1}$. The local expenditure response to home prices (including GE effects) is

$$\begin{aligned}\hat{C} + \hat{I} &= \left(\mathbf{C}_J + \mathbf{I}_{r,J}\right)\hat{J} + \mathbf{m}\hat{Y} \\ &= \mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_{r,J}\right)\hat{J} + \mathbf{m}\mathbf{M}\hat{G}\end{aligned} \tag{20}$$

where we have used the definition of $\mathbf{M}$ to note that $I + \mathbf{m}\Phi\mathbf{M} = \mathbf{M}$.

To get the fiscal multiplier we need to account for the endogenous response of home prices to shocks to G. To a first-order approximation, market clearing for housing requires

$$\mathbf{Q}_J J_r + \mathbf{Q}_i i + \mathbf{Q}_W W_r + \mathbf{Q}_D D_r + \mathbf{Q}_T T + \mathbf{Q}_\Omega \Omega = \mathbf{Q}^S_{r,J} J_r,$$

where $\mathbf{Q}^S_{r,J}$ determines the linearized supply curve. Rearranging and substituting we have

$$\left(\mathbf{Q}_J - \mathbf{Q}^S_{r,J}\right) J_r + \mathbf{Q}_W\mathbf{W}Y_r + \mathbf{Q}_D\mathbf{D}Y_r + \mathbf{Q}_i i + \mathbf{Q}_T T + \mathbf{Q}_\Omega \Omega = 0$$

$$\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)J_{r}=\left(\mathbf{Q}_{W}\mathbf{W}+\mathbf{Q}_{D}\mathbf{D}\right)Y_{r}+\mathbf{Q}_{i}i+\mathbf{Q}_{T}T_{r}+\mathbf{Q}_{\Omega}\hat{\Omega}$$

$$J_{r}=\mathbf{J}_{r,Y}Y_{r}+\underbrace{\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\mathbf{Q}_{i}}_{\equiv\mathbf{J}_{r,i}}i+\underbrace{\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\mathbf{Q}_{T}}_{\equiv\mathbf{J}_{r,T}}T+\underbrace{\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\mathbf{Q}_{\Omega}}_{\equiv\mathbf{J}_{r,\Omega}}\hat{\Omega},\tag{21}$$

where

$$\mathbf{J}_{r,Y}\equiv\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\left(\mathbf{Q}_{W}\mathbf{W}+\mathbf{Q}_{D}\mathbf{D}\right).$$

Putting the pieces together yields

$$J_{r}=\mathbf{J}_{r,Y}Y_{r}+\underbrace{\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\mathbf{Q}_{i}}_{\equiv\mathbf{J}_{r,i}}i+\underbrace{\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\mathbf{Q}_{T}}_{\equiv\mathbf{J}_{r,T}}T+\underbrace{\left(\mathbf{Q}_{r,J}^{S}-\mathbf{Q}_{J}\right)^{-1}\mathbf{Q}_{\Omega}}_{\equiv\mathbf{J}_{r,\Omega}}\hat{\Omega},\tag{22}$$

For the time being, we abstract from the heterogeneity in the housing supply curves. Differencing (22) across regions yields

$$\hat{J}=\mathbf{J}_{Y}\hat{Y}\tag{23}$$

We then have

$$\hat{Y}=\left[I-\Phi\mathbf{M}\left(\mathbf{C}_{J}+\mathbf{I}_{J}\right)\mathbf{J}_{Y}\right]^{-1}\mathbf{M}\hat{G}.\tag{24}$$

From equation (20), our IV estimate of home prices on expenditure is

$$E\equiv\mathbf{M}\left(\mathbf{C}_{J}+\mathbf{I}_{J}\right)$$

and from (24) the fiscal multiplier is

$$F\equiv\left[I-\Phi\mathbf{M}\left(\mathbf{C}_{J}+\mathbf{I}_{J}\right)\mathbf{J}_{Y}\right]^{-1}\mathbf{M}.$$

And the response of house prices to income is $Z\equiv\mathbf{J}_{Y}$. Rearranging we have

$$\mathbf{M}=\left[I-\Phi EZ\right]F$$

so

$$\mathbf{C}_J = F^{-1}\left[I - \Phi EZ\right]^{-1} E - \mathbf{I}_J.$$

This result is very similar to what we found in the static economy (see equation 8), however, here the components E, F, and so on are matrices rather scalars. To fix ideas, E is a $\mathcal{T} \times \mathcal{T}$ matrix in which the (i, j) element gives the response of output in period i to a change in home prices in period j. Similarly, the (i, j) element of F gives the response of output at period i to a change in government spending at date j. If these matrices have important off-diagonal elements, then the logic of our static examples is complicated by dynamic responses of the economy. On the other hand, if the matrices are close to diagonal, then the logic of the static economy goes through.

5.1 Empirical specifications and bias cancelation

At this stage, it is useful to consider what regression specifications are used in the cross-sectional analysis of housing wealth effects and fiscal multipliers. We will explain how heterogeneity in housing supply curves creates biases in the estimates of the housing wealth effect and investment response to home prices, but these two biases cancel when we apply our formula to recover the partial equilibrium housing wealth effect.

An estimate of the housing wealth effect might regress the change in consumption on the change in home prices and a constant or time fixed effect. We will assume that there are no dynamics so that the matrices are all diagonal.

The bias issues are easier to describe in terms of levels rather than cross-region differences. Note that $\hat{X} = X - X^* = 2\left(X - \bar{X}\right)$, where $\bar{X}$ is the cross-region mean so any equation that is linear in $\hat{X}$ will also be linear in $X - \bar{X}$. Rearrange (19)

$$\begin{aligned}\hat{Y} &= \Phi\mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_{r,J}\right)\hat{J} + \mathbf{M}\hat{G} \\ \left(Y_r - \bar{Y}\right) &= \Phi\mathbf{M}\left(\mathbf{C}_J + \mathbf{I}_{r,J}\right)\left(J_r - \bar{J}\right) + \mathbf{M}(G_r - \bar{G})\end{aligned}$$

Then rearranging the linearized consumption function yields

$$\begin{aligned}C_r = &\underbrace{\mathbf{M}\left(\mathbf{C}_J J_r + \mathbf{m}\Phi\bar{\mathbf{I}}_J\right)}_{\text{coef. of interest}} J_r + \underbrace{\mathbf{M}\left(\mathbf{I}_{r,J} - \bar{\mathbf{I}}_J\right)\left(J_r - \bar{J}\right) + \mathbf{m}\mathbf{M}G_r}_{\text{error}} \\ &\underbrace{-\mathbf{M}\left(\mathbf{C}_J J_r + \mathbf{m}\Phi\bar{\mathbf{I}}_J\right)\bar{J} - \mathbf{m}\mathbf{M}\bar{G} + \mathbf{m}\bar{Y} + \mathbf{C}_i i + \mathbf{C}_\Omega\Omega + \mathbf{C}_T T}_{\text{time fixed effect}}\end{aligned} \tag{25}$$

The key thing to note here is that changes in aggregate variables (i, Ω, T) are absorbed by the time fixed effect. The auxiliary regression of residential investment on home prices is

$$I_r = \underbrace{\bar{\mathbf{I}}_J}_{\text{coef. of interest}} J_r + \underbrace{\left(\mathbf{I}_{r,J} - \bar{\mathbf{I}}_{r,J}\right)\left(J_r - \bar{J}\right)}_{\text{error}} + \underbrace{\bar{I} + \bar{\mathbf{I}}_J \bar{J}}_{\text{time fixed effect}}. \tag{26}$$

Equations (25) and (26) show a potential source of bias in the housing wealth effect regression that has not been analyzed before: to the extent that cities differ in their housing supply elasticities they will differ in the response of residential investment to home prices and cities with larger price changes will have smaller elasticities of residential investment. This bias affects both equations. A benefit of the approach we put forward here is that the bias in the two equations cancels out and when we compute the partial equilibrium housing wealth effect. To see this, when we estimate equation (25) with time fixed effects we obtain a coefficient of interest of (see Appendix B)

$$\hat{\gamma}^C = \bar{\gamma}^C + \frac{E\left[\left(\gamma_r^C - \bar{\gamma}^C\right) J_{r,t} \tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]},$$

where $\gamma_r^C \equiv \mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\bar{\mathbf{I}}_{r,J}\right)$ and $\tilde{J}_{r,t} \equiv J_{r,t} - \bar{J}_t$. When we estimate (26) we obtain

$$\hat{\gamma}^I = \bar{\gamma}^I + \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t} \tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]},$$

where $\gamma_r^I \equiv \mathbf{I}_{r,J}$. Crucially, note that the regional variation in γ_r^C comes only from $\mathbf{I}_{r,J}$ so we have $\gamma_r^C - \bar{\gamma}^C = \mathbf{Mm}\left(\gamma_r^I - \bar{\gamma}^I\right)$.

To put the pieces together, we form $E \equiv \mathbf{M}\left(\mathbf{C}_J + \bar{\mathbf{I}}_{r,J}\right)$ by summing the coefficients of interest in in (25) and (26).[7] This gives

$$\begin{aligned}
\hat{E} &= \hat{\gamma}^C + \hat{\gamma}^I \\
&= \bar{\gamma}^C + \frac{E\left[\left(\gamma_r^C - \bar{\gamma}^C\right) J_{r,t} \tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]} + \bar{\gamma}^I + \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t} \tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]} \\
&= \bar{\gamma}^C + \bar{\gamma}^I + \mathbf{M}\frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t} \tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]}
\end{aligned}$$

[7] Recall $\mathbf{M} = 1 + \mathbf{Mm}$.

where we have again used $\mathbf{M} = 1 + \mathbf{Mm}$. Now form

$$\begin{aligned}
\hat{\mathbf{C}}_J &= \frac{\hat{E}}{\mathbf{M}} - \hat{\gamma}^I \\
&= \frac{\bar{\gamma}^C + \bar{\gamma}^I}{\mathbf{M}} + \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]} - \bar{\gamma}^I - \frac{E\left[\left(\gamma_r^I - \bar{\gamma}^I\right) J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}_{r,t}^2\right]} \\
&= \frac{\mathbf{M}\left(\mathbf{C}_J + \mathbf{m}\bar{\mathbf{I}}_{r,J}\right) + \bar{\mathbf{I}}_{r,J}}{\mathbf{M}} - \bar{\mathbf{I}}_{r,J} \\
&= \mathbf{C}_J.
\end{aligned}$$

In the second line the bias to the housing wealth effect E ends up canceling with the bias in the residential investment response. Underlying this result is the fact that heterogeneity in the housing supply curves across regions is the source of bias in both estimates.

6 Monte Carlo analysis

We proceed in two steps. In the first step, we specify a simple, static model with heterogeneous housing supply curves across regions to demonstrate that the formula holds exactly. In this context we can also validate the bias cancelation argument presented in Section 5.1. In the second step we use the full dynamic model. For a particular case of the model we show that the static formula holds exactly in the dynamic model. We then characterize how well the formula works in other cases.

6.1 Monte Carlo for static model

The model here is similar to the one discussed in Section 3.3 with the difference that here we have an aggregate shock to housing demand with heterogenous housing supply curves. The equations of the model are

$$\begin{aligned}
\phi\left[C(w,p) + I(w,p)\right] + (1-\phi)\left[C(w^*,p^*) + I^*(w^*,p^*)\right] + g &= N(w,p) = Y \\
(1-\phi)\left[C(w,p) + I(w,p)\right] + \phi\left[C(w^*,p^*) + I^*(w^*,p^*)\right] + g^* &= N(w^*,p^*) = Y^* \\
H(w,p,\Omega) &= I(w,p) \\
H(w^*,p^*,\Omega) &= I^*(w^*,p^*),
\end{aligned}$$

where $\partial I/\partial p > \partial I^*/\partial p > 0$ w.l.o.g. The common shock Ω increases housing demand ($\partial H/\partial \Omega > 0$), which will bid up house prices more in the foreign region than in the home region because residential investment is less elastic there.

For our first simulation we specify the home region as

$$\begin{aligned} C(w,p) &= 0.75 + 0.1w + 0.05p \\ I(w,p) &= 0.05 + 0.1p \\ N(w,p) &= 1 + 0.5w \\ H(w,p,\Omega) &= 0.05 + \Omega - p. \end{aligned}$$

The foreign region is the identical except that $I^*(w,p) = 0.02p$. We set $\rho = 0.7$ and the mean level of g to 0.2.

7 Conclusion

Cross-sectional empirical estimates have become part of the macroeconomist toolkit, but the appropriate interpretation of these estimates can be difficult as they often blend together partial-equilibrium responses with local general equilibrium effects. We have presented a method that allows the researcher to isolate the partial equilibrium effect, which we view as more useful as it is more easily compared to the predictions of standard one-region models.

We applied this method to compute the partial equilibrium housing wealth effect. The key step in the application is to use an estimate of the local fiscal multiplier to gauge the strength of the local general equilibrium effects. Accounting for local general equilibrium reduces the housing wealth effect almost by half.

A Proof for Sections 3.2 and 3.3

The proof for Section 3.2 is a special case of the proof for Section 3.3. Let $\Phi \equiv 1$ for Section 3.2 and let $\Phi \equiv 2\phi - 1$ for 3.3.

We proceed as in the first example to arrive at

$$\frac{d(w,p)}{d(g,s)} = \underbrace{\left[N_w - \Phi C_w + \Phi\left(C_p + I_p\right)\frac{H_w}{H_p}\right]^{-1}}_{\equiv M} \begin{pmatrix} 1 & -\Phi\frac{H_s}{H_p}\left(C_p + I_p\right) \\ -\frac{H_w}{H_p} & \frac{H_s}{H_p}\left(\Phi C_w - N_w\right). \end{pmatrix}$$

and

$$\frac{dY}{d(g,s)} = \Phi \begin{pmatrix} C_w & C_p + I_p \end{pmatrix} \frac{d(w,p)}{d(g,s)} + \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

The observables are

$$\frac{dY}{dg} = 1 + M\Phi\left[C_w - \left(C_p + I_p\right)\frac{H_w}{H_p}\right] \tag{27}$$

$$\frac{dp}{ds} = M\frac{H_s}{H_p}\left(\Phi C_w - N_w\right) \tag{28}$$

$$\frac{dC}{ds} = -C_w M\frac{H_s}{H_p}\Phi\left(C_p + I_p\right) + C_p M\frac{H_s}{H_p}\left(\Phi C_w - N_w\right) \tag{29}$$

$$\frac{dp}{dg} = -M\frac{H_w}{H_p} \tag{30}$$

$$\frac{dI}{ds} = I_p M\frac{H_s}{H_p}\left(\Phi C_w - N_w\right) \tag{31}$$

and the definition of M gives

$$M\left(N_w - \Phi C_w\right) = 1 - M\Phi\left(C_p + I_p\right)\frac{H_w}{H_p}. \tag{32}$$

Use (30) in (27) to arrive at

$$\frac{dY}{dg} = 1 + M\Phi C_w + \left(C_p + I_p\right)\Phi\frac{dp}{dg}. \tag{33}$$

Using (28), (29), and (31) we have

$$\begin{aligned} E &\equiv \frac{dC/ds + dI/ds}{dp/ds} \\ &= \frac{-C_w M \frac{H_s}{H_p} \Phi \left(C_p + I_p\right) + C_p M \frac{H_s}{H_p} \left(\Phi C_w - N_w\right) + I_p M \frac{H_s}{H_p} \left(\Phi C_w - N_w\right)}{M \frac{H_s}{H_p} \left(\Phi C_w - N_w\right)} \\ &= \frac{C_w M \Phi \left(C_p + I_p\right)}{M \left(N_w - \Phi C_w\right)} + \left(C_p + I_p\right). \end{aligned}$$

Now use (32) and then (30)

$$\begin{aligned} E &= \frac{C_w M \Phi \left(C_p + I_p\right)}{1 - M \Phi \left(C_p + I_p\right) \frac{H_w}{H_p}} + \left(C_p + I_p\right) \\ &= \frac{C_w M \Phi \left(C_p + I_p\right)}{1 + \Phi \left(C_p + I_p\right) \frac{dp}{dg}} + \left(C_p + I_p\right). \end{aligned}$$

Now use (33) to substitute for $C_w M \Phi$

$$E = \frac{\left[\frac{dY}{dg} - 1 - \Phi \left(C_p + I_p\right) \frac{dp}{dg}\right] \left(C_p + I_p\right)}{1 + \Phi \left(C_p + I_p\right) \frac{dp}{dg}} + \left(C_p + I_p\right)$$

and rearrange

$$\begin{aligned} E &= \frac{\frac{dY}{dg} \left(C_p + I_p\right)}{1 + \Phi \left(C_p + I_p\right) \frac{dp}{dg}} \\ E + E\Phi \left(C_p + I_p\right) \frac{dp}{dg} &= \left(C_p + I_p\right) \frac{dY}{dg} \\ E &= \left(C_p + I_p\right) \left(\frac{dY}{dg} - E\Phi \frac{dp}{dg}\right) \\ \frac{E}{\frac{dY}{dg} \left(1 - E\Phi \frac{dp/dg}{dY/dg}\right)} &= C_p + I_p. \end{aligned}$$

I_p can be calculated from (28) and (31) so we have

$$C_p = \frac{E}{\frac{dY}{dg} \left[1 - E\Phi \frac{dp/dg}{dY/dg}\right]} - \frac{dI/ds}{dp/ds}.$$

B Bias in estimating equations (25) and (26)

Consider the data generating process

$$y_{r,t} = f_t + \gamma_r J_{r,t} + \varepsilon_{r,t},$$

where t indexes time and r regions. Note that each region has its own γ_r. However, when we estimate the housing wealth effect we estimate a single γ. We do not recover the average γ across regions if, say, regions with larger γ's tend to have smaller fluctuations in home prices $J_{r,t}$, which is the implication of regions with more elastic housing supply having residential investment respond more to home prices but home prices fluctuate less.

We estimate with demeaned variables to eliminate f_t. Let $\tilde{y}_{r,t} = y_{r,t} - \bar{y}_t$. We then have

$$\begin{aligned}
\tilde{y}_{r,t} &= \gamma_r J_{r,t} - \bar{\gamma J}_t + \tilde{\varepsilon}_{r,t} \\
\tilde{y}_{r,t} &= \bar{\gamma} J_{r,t} - \bar{\gamma} J_{r,t} + \gamma_r J_{r,t} - \bar{\gamma}\bar{J}_t - cov_t + \tilde{\varepsilon}_{r,t} \\
\tilde{y}_{r,t} &= \bar{\gamma}\tilde{J}_{r,t} + (\gamma_r - \bar{\gamma})\, J_{r,t} - cov_t + \tilde{\varepsilon}_{r,t}
\end{aligned}$$

where $cov_t = E_r\left[(\gamma_r - \bar{\gamma})\left(J_{r,t} - \bar{J}_t\right)\right]$

We regress $\tilde{y}_{r,t} = \hat{\gamma}\tilde{J}_{r,t} + \nu_{r,t}$. The least squares moment condition is

$$E\left[\tilde{J}_{r,t}\left(\tilde{y}_{r,t} - \hat{\gamma}\tilde{J}_{r,t}\right)\right] = 0$$

Substituting in

$$\begin{aligned}
E\left[\tilde{J}_{r,t}\left((\bar{\gamma} - \hat{\gamma})\,\tilde{J}_{r,t} + (\gamma_r - \bar{\gamma})\, J_{r,t} - cov_t + \tilde{\varepsilon}_{r,t}\right)\right] &= 0 \\
(\bar{\gamma} - \hat{\gamma})\, E\left[\tilde{J}^2_{r,t}\right] + E\left[(\gamma_r - \bar{\gamma})\, J_{r,t}\tilde{J}_{r,t} - cov_t\tilde{J}_{r,t}\right] &= 0.
\end{aligned}$$

Note that cov_t has no variation over r and $\tilde{J}_{r,t}$ has no time-series variation. So we end up with

$$\hat{\gamma} = \bar{\gamma} + \frac{E\left[(\gamma_r - \bar{\gamma})\, J_{r,t}\tilde{J}_{r,t}\right]}{E\left[\tilde{J}^2_{r,t}\right]}$$

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