High Frequency Price Jumps and Price Discovery

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Abstract

This paper uses a novel time series representation of price series to examine intra-day price discovery in a state space framework. Using trade and quotation (TAQ) data and importance sampling methods this paper extracts the information content of high frequency price jumps. The preliminary results suggest that despite bring rare events which happen with a probability of 0.38 percent, jumps contribute 12.87 percent to price discovery and 36 percent to transitory mispricing.

1 Research Objectives

Since the seminal work of Clark (1973), there exists a long-established consensus that the evolution of stock prices is neither continuous nor normally distributed. In particular, high frequency stock prices can be characterized by continuous returns interspersed with extreme price moves known as *jumps*. Incorporating jumps in diffusion processes has become a common practice when modelling intra-day price characteristics (e.g. Andersen et al., 2010). This is further highlighted by the importance of jumps in option pricing (e.g. Merton, 1976; Kou, 2002), risk management (e.g. Duffie, Pan and Singleton 2001), systematic risk factors (e.g. Bollerslev, Li and Todorov, 2016) and idiosyncratic risk factors (Begin, Dorion and Gauthier 2017).

Despite the importance of jumps for asset pricing models which incorporate tail risk, the literature is largely silent on the importance of jumps for price discovery. In particular, the question of how much fundamental information and transitory mispricing is incorporated from jumps is left unaddressed. This paper proposes a state representation of jumps using importance sampling methods to

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model the non-Gaussian nature of jumps. By examining the contribution of jumps to price discovery this paper contributes towards understanding the market dynamics and associated liquidity risk of jumps. In particular, the informed component of jumps entails information risk and the transitory component is associated with profits from high frequency contrarian trading strategies. The interaction of these two may help explain the empirically observed disproportionate pricing of jump risk as compared to the corresponding continuous component.

2 Research Strategy

This paper models jump as a signal extraction model in a state space framework. The information content of jumps is extracted as the innovation to the permanent jump price, i.e. the increment to the martingale price. Whereas, the transitory component is the innovation to the stationary component of the jump price process. The likelihood function from the signal extraction model is numerically maximized using importance sample methods proposed in as series of papers by Durbin and Koopmans (1994, 1997 and 2000). Given the non-normality and high frequency characteristics of jumps, modeling price jumps in a signal extraction model is computationally intensive. However, it allows for the construction of realized estimators i.e. estimator of sample time paths of permanent and transitory jumps. These estimators are based on importance sampling methods and use all available information in past and future prices. By computing these realized estimators this paper will provide ex-post estimators of permanent and transitory jumps which would have important applications, such as, computing the adverse selection cost for liquidity supplier from jumps and realized markets betas which measure the jump co-movements in the transitory and permanent jumps. Therefore, these estimator would have important contribution in the market microstructure and asset pricing literature.

3 Computing Language

The computing language to be used is Python. The choice of Python is based on its ease of use with Message Passing Interface (MPI) for parallel computing. In addition, modules such as *numpy* and *numba* allow computation of arrays across multiple cores.

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