Trademarks, Product Quality, and Welfare *

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Abstract

This paper analyzes the role of trademarks in the economy using a model where firms invest in product quality and build a customer base gradually through costly marketing under the threat of infringement and imitation by other firms. Firms can register trademarks at a cost to protect their customer base and lower the cost of informing consumers about their product. The model has predictions on which type of firms trademark, and when, as well as on firm growth patterns before and after trademarking. Firm-level data on trademark registrations are used to test these predictions and to calibrate the model's key parameters. Using the model, several experiments are conducted, including removal of trademark protection, reduction in firms' cost of informing consumers, and increase in the efficiency of quality provision. The effects on welfare, quality dispersion, concentration, and marketing expenditures are studied.

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1 Introduction

The last couple of decades have witnessed a surge in trademark activity—see Figure 1.[1] There has also been an acceleration in innovation during this period, as indicated by a rise in patenting.[2] These trends are closely related to underlying changes in the variety and quality of products available. By protecting brands and products from infringement and imitation, trademarks are critical for the provision of product variety and quality. Implications of trademarks for firm dynamics and the aggregate economy, however, are relatively less explored. In particular, the contribution of trademarks to quality dispersion, concentration, and welfare has not been quantified in a unified framework. How do trademarks alter firms' incentives to provide quality and engage in marketing activites? How much value do trademarks add to firms? To what extent do trademarks matter for firm dynamics, entry, exit, and concentration? What are the implications of trademark law on quality provision and welfare? This paper provides answers in a general equilibrium framework. A model is proposed to explore why firms trademark, which ones do, and when they do so in their life-cycles. It is then used to analyze the effects of trademark protection on firms and the economy. The model also highlights how trademark protection, informational frictions between firms and consumers, and the technology for generating quality interact. The consequences of technological progress in the production of quality and reduction in the informational barriers between firms and consumers are studied.

The model builds on the basic insight by Landes and Posner (1987, 1988, 2003) that trademarks serve two fundamental roles: (i) reduce informational frictions between consumers and firms, making it easier for firms to attract consumers, and (ii) protect against the erosion of a firm's customer base through imitation and infringement. These roles are incorporated in a framework where firms gradually build a customer base through costly dissemination of information to consumers about their products, similar to Dinlersoz and Yorukoglu (2012). Consumer information and product quality are central to the model. Each firm offers a product with a distinct quality level, where higher quality is more costly to produce. To generate sales, a firm has to engage in marketing activity to inform consumers about its product and quality. Without this activity, consumers have no information about the firm. There is otherwise no informational asymmetry about product quality between firms and consumers—consumers can tell a firm's product quality before they consume it, as long as they are informed about the firm and its product.

Firms can obtain trademarks at a cost. A trademark confers two benefits to firms in the model. First, it prevents the loss of a firm's customer base; unless a firm has a trademark, other firms can divert customers from that firm through imitation or infringement, leading to a decline in firm value. By protecting a firm's customer base, trademarks therefore encourage investment in quality and customer acquisition. Second, a trademark helps a firm acquire customers more easily through brand recognition or reputation: the trademark distinguishes the firm, reduces confusion among similar products, and generates reputation, all of which enable the firm to effectively spread information about its product at a lower cost. In other words, trademarks decrease the informational frictions between consumers and firms. This mechanism is introduced by allowing firms with a trademark to have a more efficient information dissemination and marketing technology—labelled

[1] See also Dinlersoz et al. (2018) and Graham et al. (2017) for trends in trademark activity.

[2] See, e.g., Kortum and Lerner (1998).

as "advertising technology". The protective role of trademarks encourages more advertising by firms, as firms with a trademark do not have to fear customer diversion.

The model generates endogenous quality dispersion due to informational frictions, even when all firms have the same efficiency in quality production. The model has predictions on which type of firms trademark, and when, as well as on firm growth patterns and advertising activities before and after trademarking. The model is calibrated to capture salient features of the U.S. economy and trademark registrations. Using the calibrated model, some experiments are conducted, including a removal of trademark protection altogether, a decline in the firms' cost of informing consumers, and a lower cost of quality provision. The effects on quality dispersion, concentration, investment in brand protection and the intensity of advertising are studied. The calibrated model also allows estimation of the added value from trademarks for each type of firm.

Firm-level data on trademark registrations is used to test the predictions of the model and to estimate the model's key parameters. This paper is the first to analyze the incidence and timing of the first trademark registration over the firm life-cycle for all firms (public and private) in the United States, and the relationship between registrations and firm outcomes. The data on firms and trademark registrations come from the trademark-firm database constructed by Dinlersoz et al. (2018). The database contains information on trademark registrations from the USPTO's Trademark Case File Database (TCFD).[3] Firm-level information is incorporated from the U.S. Census Bureau's Business Register (BR)—the administrative data for the universe of non-farm businesses in the United States. The key variables are employment, payroll, and revenues, as well as firm location and industry classification. The matched trademark-firm database also includes longitudinally-linked information from the BR, summarized in the U.S. Census Bureau's Longitudinal Business Database (LBD), which is used to track firms over their life-cycles. The trademark-firm database thus allows study of key events in a firm's life-cycle: firm birth and subsequent growth, first trademark registration, and firm death. In addition, this paper incorporates data on firm advertising expenditures available from the U.S. Census Bureau's economic censuses and surveys. This data is used to analyze the connection between marketing activities and trademarks—a key aspect of the model is the informational frictions between firms and consumers that can be reduced as a result of trademarks.

The rest of the paper proceeds as follows. Section 2 reviews the related literature. The model is introduced in Section 3, followed by the definition of equilibrium in Section 4. Section 5 describes the data. The results of the empirical analysis are presented in Section 6. Section 7 concludes.

2 Related Literature

2.1 Empirics

There is a growing empirical literature on firms' incentives to trademark and the association between trademarks and firm outcomes. Prior empirical work using U.S. data has been mainly limited to publicly-traded firms, with some exceptions that utilize relatively small surveys of private firms. Trademark-firm linked data for all firms in the United States, especially small and medium sized ones, has not been available

[3] See Graham et al. (2016) for a detailed analysis of the TFCD alone.

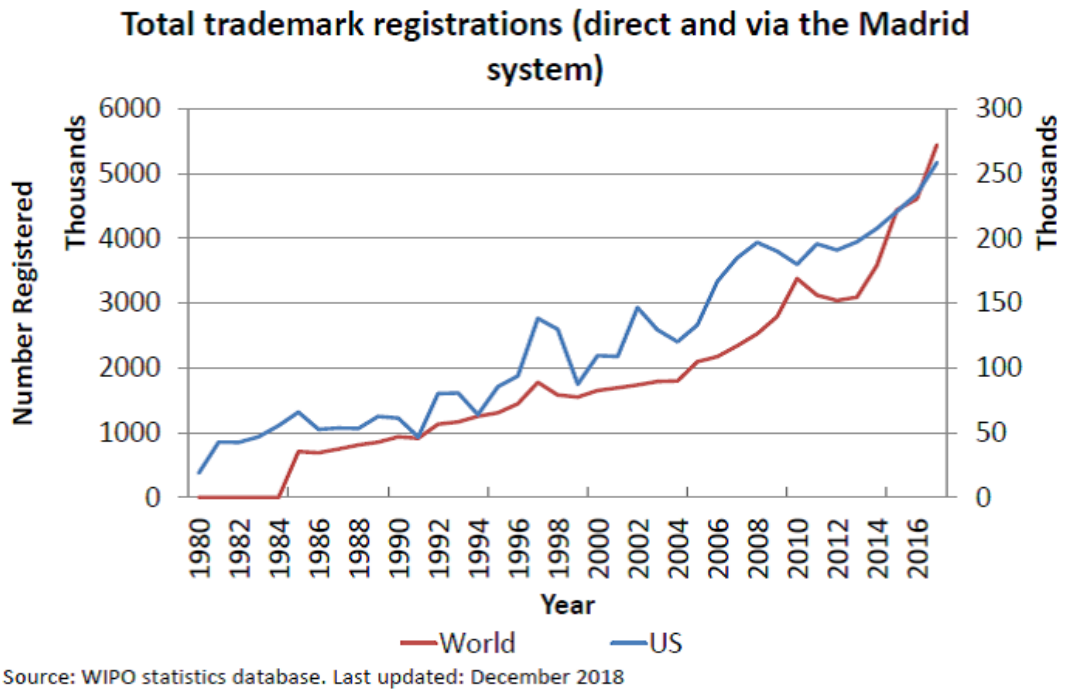


Figure 1: Trademark registrations

until recently. Dinlersoz et al. (2018) provide the first comprehensive link of the USPTO's Trademark Case File Database (TCFD) with the confidential firm-level data from the U.S. Census Bureau's Business Register (BR) and Longitudinal Business Database (LBD)—see Graham et al. (2016) for a comprehensive analysis of the TFCD alone. The linked dataset, also used in this paper, allows observation of trademark activity over the firm life-cycle for the universe of employer businesses (both public and private) in the United States. This paper is the first to utilize this dataset to study the role of trademarks in firm dynamics and the aggregate economy.

A large part of the empirical literature has developed around the data for other countries. Earlier research with European and Australian data has documented a number of empirical regularities on trademark activity. Schautshick and Greenhalgh's (2013) survey reveals some key facts across countries and over time. Notably, the surge in trademarks is not unique to United States. Similar rise in trademark applications have been observed since the mid-1970s in Australia and the United Kingdom, pointing to common trends across nations. At the country level, an increase in demand for product variety and quality appears to drive growth in trademark applications. Trademark applications are also found to be related to measures of product differentiation, quality and variety, as well as of marketing and innovation. Baroncelli, Fink, and Javorcik (2005) find evidence that, across counties, higher development is associated with a dominance of domestic brands at home and a stronger presence of these brands in foreign markets. Their analysis also suggests that trademark registrations are concentrated in R&D- and advertising-intensive industries, and registration intensity is also relatively high in business services—see also Millot (2011) for the case of Germany and France. For the United States, Graham et al. (2016) also note that service marks filings increased substantially in late 1990s, a likely

consequence of the dot.com boom of that era.

Some of the prior work utilizes firm-trademark matched datasets to examine the connection between firm characteristics and trademark filing or registration activity. Greenhalgh, Longland, and Bosworth (2003) find that trademark filing intensity is higher for smaller firms—within the set of large, publicly-traded firms. Greenhalgh and Rogers (2008) observe in the U.K. data that firm size is highly correlated with trademark registrations, and there are diminishing returns to firm size in trademark activity. Jensen and Webster (2006) also document a positive association between firm size and trademark activity in the case of Australian firms. There is also some evidence that, overall, trademarks have a positive effect on firm value. In a study covering several countries including the United States, Sandner and Block (2011) find that trademark registrations are positively associated with firm value, after controlling for patenting and R&D activity. Once again, this evidence is based on publicly-traded firms in the S&P's Compustat database in the case of United States.

Another strand of the empirical literature focuses on firm's motivations to register trademarks. Sandner (2009) provides a detailed investigation of trademark portfolios for large publicly-traded companies. His findings indicate that trademark portfolios are put together with an intent to protect brands. Firms with product introductions have to decide whether to extend their existing umbrella of trademarks to the new products, or register new trademarks. Overall, trademark registrations appear to be associated with both the creation of new brands and the expansion of existing brands to accommodate new products. Block, Fisch, Hahn, and Sandner (2015) use survey data from small and medium trademark applicants in internet and technology sectors and identify three distinct motivations for trademark registrations: protection, marketing, and exchange. Their cluster analysis to classify firms results in four types of firms: trademark skeptics, marketing-focused trademark users, marketing plus protection-focused trademark users, and trademark advocates. Based on data from Belgium, Flikkema, De Man, and Castaldi (2014) find that 60% of registered trademarks are associated with product or service innovations. These findings suggest, while there is significant heterogeneity across firms in the reasons behind trademark activity, product introduction, brand protection, and marketing are key considerations in firms, motivations to register trademarks.

Some recent work has also combined the USPTO trademark data with various other datasets in the U.S. to study trademark activity by firms. For instance, Aurora, Bei, and Cohen (2016) explore the incentives to use and protect trademarks by bringing together new survey data from The Division of Innovative Labor on firms' new product development activities and industry-level data from the NETS database. They find that trademarks help high quality firms maintain their customer base. Furthermore, trademarks protect a firm's first mover advantage. As a result, firms that have face more competition are more likely to file for trademarks. These findings are consistent with the structure and implications of the model proposed here.

There is also a growing body of work on the economic effects of trademark law in the United States. In recent work, Heath and Mace (2017) use the introduction of the 2006 Federal Trademark Dilution Act (FTDA) to argue that trademark registrations have economically significant effects for publicly-traded firms. Their analysis indicates that the FTDA raised treated firms' operating margins and firm values while inducing more trademark lawsuits, lower entry, and higher concentration in industries where FTDA had a larger bite. In addition, product quality and innovation declined for firms that were granted stronger trademark protection. Instead, such firms used their protected brands to expand into new product markets. One notable shortcoming

of the recent studies in the case of United States is that they do not provide a comprehensive analysis of the effects of trademark registration by all types of firms (public and private), and they do not explore the implications of trademarks on firm dynamics and the aggregate economy in a general equilibrium context. This paper fills this gap by offering both model-based and empirical analyses of trademark registrations.

2.2 Theory

The modeling approach in this paper is influenced by a number of key studies on firm quality, reputation, and trademarks. The model builds on the earlier insight by Landes and Posner (1987, 1988, 2003) that trademarks reduce consumer search costs and protect against the erosion of a firm's customer base through imitation and infringement. Economides (1988, 1998) also highlights the key functions of trademarks in the economy by emphasizing their role in the efficient provision of products with a wide range of variety and quality. Nevertheless, these earlier studies do not fully embed the key roles of trademarks in a general framework to analyze various aggregate implications of trademark law.

The process of gradual acquisition of a customer base in the model is related to recent work on the effects of customer acquisition processes on firm and industry dynamics, and aggregate economy (see, e.g., Dinlersoz and Yorukoglu (2012), Gourio and Rudanko (2014), Gilbukh and Roldan (2018)). The current paper differs in modeling the explicit role of trademarks in customer acquisition process. The analysis is also related to the literature on firm reputation (see e.g. Cabral (2005, 2012) and Cabral and Hortacsu (2012), and business name choice to signal quality (see, e.g. McDevitt (2011, 2012)). Trademark registrations are a key ingredient of firm's efforts to build a reputation among consumers.

The role of advertising technology in reducing informational frictions between consumers and firms is similar to that in Dinlersoz and Yorukoglu (2012).[4] The model emphasizes the informative role of advertising, similar to prior ground work, such as Butters (1977), Stegeman (1992), and Grossman and Shapiro (). Recent models have also incorporated advertising technology in models of trade (Arkolakis (2010)) and innovation (Cavenaile and Roldan (2018))—the latter embeds advertising's "persuasive" role in inducing perceived differences in quality.

The model is also related to prior work on firm product quality choice and its implications. There is a very large body of work on firm quality and price choice under imperfect consumer information about quality or prices, and under different types of competitive environments.[5] Some of this work also emphasizes the quality-signaling role of prices. For simplicity, the current model abstracts from informational asymmetry about product quality between firms and consumers. Consumers can tell firm's product quality before they consumer the product, once they become aware of the existence of the firm and its product through firms' advertising. Similar to Dinlersoz and Yorukoglu (2012), the only informational friction in the market is that not all consumers know about a firm's product and its quality unless they are informed by that firm through marketing activity. As a result, pricing by firms is akin to monopolistic competition pricing, owing to the

[4] See also Dinlersoz and Yorukoglu (2008) for an analysis of informative advertising in a competitive industry where firms differ in production efficiency.

[5] See, among many others, Chan and Leland (1982), Wolinsky (1983), Cooper and Ross (1984), Schwartz and Wilde (1985), Rogerson (1988), Bagwell and Riordan (1991), and Daughety and Reinganum (2007,2008).

fact that a consumer makes purchase decision based on the (known) qualities of the firms he is informed about. Consequently, firms do not compete on prices directly, and there is no signaling role for prices. Price dispersion in the model therefore reflects the dispersion in the underlying quality differences across firms. Quality dispersion itself stems from the consumers' lack of information on some firms and their products.

3 The model

Consider an economy consisting of different industries (products) which are indexed over the unit interval, $[0, 1]$. In each industry, there is a large number of firms offering the product at potentially different qualities to a large number of consumers. The population of firms and consumers are each represented by a continuum. A firm is *small* with respect to the rest of the industry. It has no influence on industry aggregates by itself and takes them as given. Time is discrete, denoted by $t \geq 1$.

The industries and products are totally symmetric. In each industry, the product can be produced at potentially different qualities by each firm. After paying a fixed entry cost κ, an entering firm learns its *efficiency level in product quality development,* e. This efficiency can be seen as a firm's capacity and ability to generate high quality products. After realizing its efficiency level, a firm can decide to exit, at no cost, without developing its product. If the firm decides to stay, it makes an irreversible product quality decision and incurs a product development cost. At the stage of product development, a higher product quality requires a larger one-time investment by the firm. After a firm's quality is set in its first period, it can not be changed throughout the firm's life. Higher quality products also require more resources at the production stage – the cost of such resources is incurred every period. The timing of events is given in the following Figure 1.

Each period, the firm, given the quality of its product, makes trademarking (if it is not already trademarked), marketing, pricing and employment decisions. The firm decides about how aggressively it will market its product in that period. Firms may target other firms' products in order to divert (confuse) their customers by making attributes of their product (name, appearance, marketing strategy **other than its quality**) look similar to the targeted firms' products. Through this diversion process firms may divert a δ fraction of targeted firms' customers. The only way an incumbent firm can protect itself and its customers from being targeted is to trademark its product.

Trademarking a product is costly. In order to trademark its product, a firm has to incur a one-time fixed cost of c_T. Each period, if the firm has not already trademarked its product, it decides whether to trademark it or not in that period.

In accordance with their optimal marketing decisions, firms then send out ads to consumers, who make their purchase decisions. Advertising technology here is assumed to include all marketing activity that enable informed consumers observe the firm's product quality perfectly.[6]

[6]Here for simplicity of the setup and notation, we assume that firms do not list their prices on their ads. We can easily show that even if firms included prices in their ads, in equilibrium, qualitative results wouldn't change, especially, firms would still follow a constant markup pricing policy. Since consumers anticipate that, if a as soon as a consumer observes a firm's quality, he/she can easily deduce the price he will face.

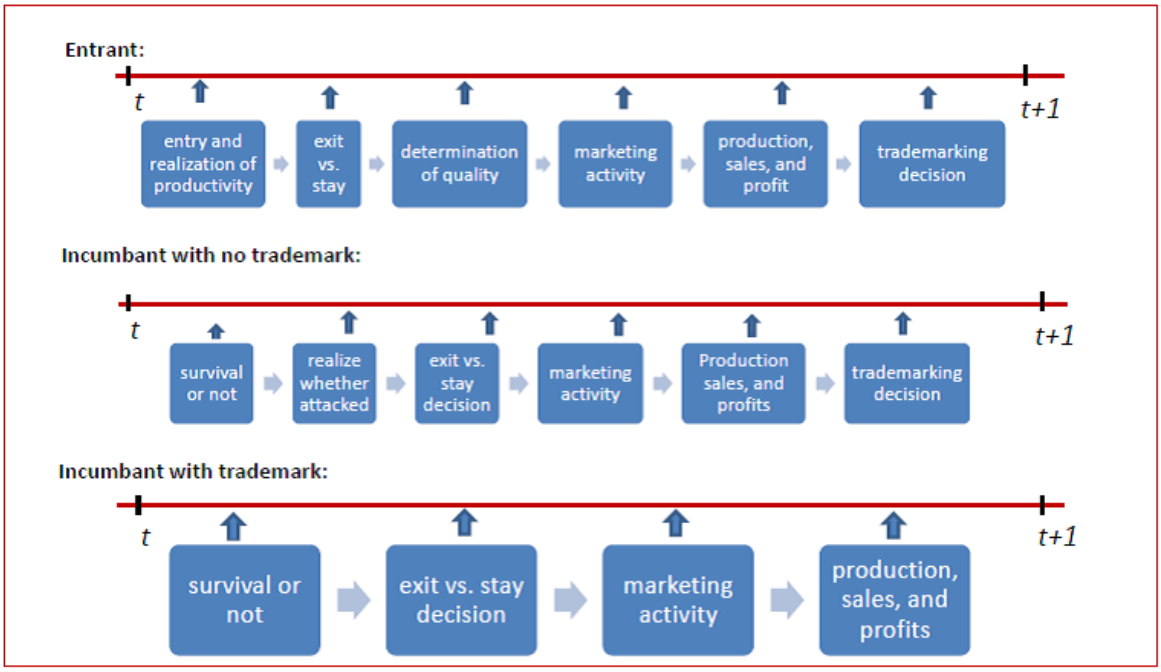


Figure 2: Timing of events

Firms and consumers have the same discount rate $\beta \in (0,1)$. Besides the possibility of endogenous exit, a constant fraction of the incumbent firms exit the industry exogenously every period. In the meantime, some consumers exit the market, while new ones enter at an exogenous rate λ.

3.1 Consumers

There is a unit mass of agents which recieve utility from consuming different variety of goods available at that point indexed between $[0,1]$. Each agent has one unit of labor which is exogenously supplied in the labor market. For each variety of goods, each period a fraction $\lambda \in (0,1)$ of the agents is replaced by new ones, agents start to and stop demanding each variety constantly, implying an expected demand cycle of $1/\lambda$ periods. The per period utility of agents is given by

$$U = \int_0^1 u(q_i, x_i)di$$

where $u(q_i, x_i)$ is the utility the agent receives from consuming x_i amount of the ith good of quality level q_i. Here $\frac{\partial u(q,x)}{\partial q}, \frac{\partial u(q,x)}{\partial x} > 0$, $\frac{\partial^2 u(q,x)}{\partial q^2}, \frac{\partial^2 u(q,x)}{\partial x^2} < 0$, $\lim_{x_i \to 0} \frac{\partial u(q,x)}{\partial x} = \infty$, and $u(q,0) = 0$. (We can use $u(q,x) = Bq^{\theta}x^{1-\theta}$, $1 \geq \varkappa > 0$.) The consumer likes higher quality but in a marginally diminishing manner.

3.1.1 Consumer information and memory

The information and memory setup of the model closely follows Dinlersoz and Yorukoglu (2012). For each variety, consumers receive ads randomly and independently. Within a period, no consumer receives more

than one ad from a firm, each ad is received by some consumer, and each consumer is equally likely to receive an ad. These assumptions are compatible with the fact that there is a very large number of consumers. Denote by $\Lambda_t(i)$ the probability that a consumer receives $i \geq 0$ integer ads related to a variety in period t.

Let $\mathbf{I}_t = \{I_t^i\}_0^1$ be the information flow to a consumer during period t. I_t^i contains information for the firms from which the consumer received an ad in period t on variety i, and for those firms visited in the past that provide the consumer with period t information about themselves. I_t^i is either empty ($\varnothing$), implying no new information, or consists of K integer elements. Each element is a vector of variables that describe the corresponding firm's attributes that help a consumer identify a firm and evaluate the surplus from patronizing it.

Because consumers can survive multiple periods, how a consumer stores and discards past information needs to be specified. There are several ways to specify consumer memory. At one extreme, a consumer recalls all past information (total recall). At the other extreme, a consumer forgets the past (no recall). Let $\mathbf{M}_t$ be a consumer's memory. No recall corresponds to $\mathbf{M}_t = \varnothing$, and total recall to $\mathbf{M}_t^{\overline{\tau}} = \bigcup_{j=t-\overline{\tau}}^{t-1} \mathbf{I}_t$, where $\overline{\tau} \geq 1$ is consumer age. Any arbitrary memory process can be represented as a subset of $\mathbf{M}_t^{\overline{\tau}}$. For instance, one can define a τ-period memory, $\mathbf{M}_t^{\tau} = \bigcup_{j=t-\tau}^{t-1} \{I_t^i\}_j$, as one that contains all the past information flow from period $t-\tau$ upto period $t-1$, where $\tau \in \{1, ..., \overline{\tau}\}$. Another example is a memory that contains only the firms from which a purchase was made, or the largest of such firms. A consumer's cumulative information in period t, $\mathbf{I}_t$, is the collection of his memory and current information flow, i.e. $\mathbf{I}_t = \mathbf{M}_t \cup \{I_t^i\}$ for $\tau \in \{0, 1, ..., \overline{\tau}\}$. A consumer makes his purchase decision based on $\mathbf{I}_t$.

Our baseline assumption about consumer information is a specific, but intuitive one: consumers do not recall any past ads or firms, except that if a consumer purchased from a firm in the previous period and that firm did not exit, the consumer possesses the current period payoff-relevant information about that firm. The baseline corresponds to a no recall memory ($\mathbf{M}_t = \varnothing$) and to an information flow I_t that contains the information in the new ads received plus the period-t information about the most-recently-visited firm (if any). The baseline cumulative information process is therefore $\mathbf{I}_t = \varnothing \cup \{I_t^i\} = \{I_t^i\}$.

To motivate the baseline, one might argue that a consumer becomes more familiar with the firm he purchased from in the previous period, compared to the firms he received an ad from, but chose not to visit. The most-recently-visited firm is also likely the freshest firm in consumer memory. The consumer may receive the current payoff-relevant information about this firm through means other than just random ads. Lists of previous customers are frequently used by firms to update their former customers directly. Current catalogs or coupons may be sent directly to previous customers' addresses. The cost of advertising defined below can include the cost of sending such updates to consumers who purchased from the firm in the previous period. The assumed information structure generates some persistence in a firm's customer base.

Next, consider the contents of I_t. A firm's product quality matters for consumer surplus and is a part of each element in I_t. In a dynamic environment where consumers can purchase from a firm for consecutive periods, other firm attributes that matter for firm survival are also pay-off relevant. Assumptions on consumer memory and information imply that the firm visited by a consumer in period t will be in I_{t+1}, unless that firm exits. To simplify, we assume that a firm's size is revealed in an ad. In practice, a firm may include proxies for its size in an ad. Firm size may also become common knowledge over time, especially for firms

that grow beyond a certain size.

3.1.2 Household's problem

Households have one unit of time endowment per period which they inelastically supply in the labor market. They start period t with some assets a_t, and memory of firm-information $\mathbf{M}_t$ from last period. For each variety, they receive ads from firms and given their memory from last period combine them to form their information set for the current period, $\mathbf{I}_t$. If $I_t^i = \varnothing$, the consumer does not purchase variety i in period t. Otherwise, the consumer purchases from the firm in his information set, I_t^i, that provides the highest *net surplus*.

Let the optimal lifetime discounted utility of a household which starts the period with firm-information memory $\mathbf{M}_t$ and assets a_t be $U(a_t, \mathbf{M}_t)$. The economic problem of the household reads,

$$U(a_t, \mathbf{M}_t) = \max_{a_{t+1}} \left\{ \underbrace{\max_{\{x_{ij,t}\}_{ij\in\mathbf{I}_t}} \left\{ \begin{array}{c} \int_0^1 u(q_{ij(i),t}, x_{ij(i),t})di \\ s.t. \quad j(i) = \arg\max_j \left\{ W_t^{\Gamma_{ij}}(q_{ij}, m_{ij}) \right\}_{j\in(M_t^i UI_t^i)} \end{array} \right\}}_{\text{first stage}} + \beta U(a_{t+1}, \mathbf{M}_{t+1}) \right\}$$

$$\underbrace{\begin{array}{c} \text{subject to } \int_0^1 \left\{ \sum_j p_{ij,t} x_{ij,t} \right\} di = w_t + a_t(1+r_t) - a_{t+1}, \\ x_{ij,t} \geq 0 \text{ for all } i,j; \ a_{t+1} \text{ given.} \end{array}}_{\text{second stage}}$$

s.t.

$$\int_0^1 \left\{ \sum_j p_{ij,t} x_{ij,t} \right\} di + a_{t+1} = w_t + a_t(1+r_t) \quad \text{for all } t.$$

$$M_{t+1}^i = \text{information } \left\{ \arg\max_j \left\{ W_t^{\Gamma_{ij}}(q_{ij}, m_{ij}) \right\}_{j\in(M_t^i UI_t^i)} \right\}$$

Here $x_{ij,t}$ denotes consumption of jth product of ith variety at time t. Each period, the households' utility maximization problem has two stages. In the first stage, given total expenditure, $\int_0^1 \left\{ \sum_j p_{ij,t} x_{ij,t} \right\} di$, for each variety the household chooses the product with maximum *net surplus* among all the products in his information set and determines optimal amount of consumption of that product given its price. In the second stage, the household picks the optimal amount of total expenditure (or given the budget constraint, equivalently, asset level, a_{t+1}) to be allocated to current period given the stream of interest rates and future income. Notice that given $W_t^{\Gamma_{ij}}(q_{ij}, m_{ij})$ the first stage of the household's problem is a static problem whereas the second stage is a dynamic one, although $W_t^{\Gamma_{ij}}(q_{ij}, m_{ij})$ has a rich dynamic problem in itself which will be discussed later. Dividing the household's problem into two stages simplifies the analysis of the household problem significantly. Clearly the state variables of the household's optimization problem at time t is $(a_t, \mathbf{M}_t)$, asset level of the household and the information the hosehold carries to next period. The information set available to the household at time t consists of the memory of past firms $\mathbf{M}_t$ that the agent carries over from last period and the new information he receives from firms for each variety in the current period, $\mathbf{I}_t$. The

memory of the household will carry to next period, $\mathbf{M}_{t+1}$, consists of the information for all the firms that the agent purchased products in the current period.

If $I_t^j = \varnothing$, the consumer does not purchase variety j in period t. Otherwise, the consumer purchases from the firm in his information set, $\mathbf{I}_t^j$, that provides the highest net surplus. Because a consumer can purchase from the most-recently-visited firm, consumers are heterogeneous with respect to their reservation values, which change over time randomly as new information is received.

Let $W_t^T(q,m)$ denote the discounted expected surplus from a trademark firm with product quality q, and that had $m \geq 0$ customers at the end of the period $t-1$. Similarly, let $W_t^N(q,m)$ represent discounted expected surplus from a non-trademark firm. We refer to m as the "size" of the firm at the beginning of period t. Let χ be the exogenous exit probability of a firm. Also, let $W_t^{\varnothing}$ be the discounted expected surplus of a consumer with no firm related information, i.e., with $\mathbf{I}_t^j = \varnothing$.

The discounted expected utility from being a customer of a trademark firm with quality level q and current period initial size m will read

$$W_t^T(q,m) = \max_x \left[u(q,x) - px\right] + \beta(1-\lambda) \tag{1}$$
$$\left\{ \begin{array}{c} \Lambda_{t+1}(0)\left[E\left[X^T(q,m')\right]W_{t+1}^T(q,m') + (1 - E\left[X^T(q,m')\right])W_{t+1}^{\varnothing}\right] \\ \sum_{i=1}^{\infty}\Lambda_{t+1}(i)\left[\begin{array}{c} E\left[X^T(q,m')\right]\int \max\{W, W_{t+1}^T(q,m')\}dZ_{t+1}^i(W) \\ +(1 - E\left[X^T(q,m')\right])\int W dZ_{t+1}^i(W) \end{array}\right] \end{array} \right\}.$$

In (1), Z_t^i is the period-t cumulative distribution function of the first-order statistic of net consumer surplus among $i > 0$ ads received by the consumer. Because ads from different firms arrive independently, one can write $Z_t^i(W) = [Z_t(W)]^i$, where $Z_t(W)$ is the probability that an ad is from a firm that offers a surplus of at most W. The expectations in (1) are taken with respect to all feasible next period m' for the firm conditional on survival in period $t+1$. Here, $X^T(q,m')$ denotes stay-exit decision; if the firm decides to stay $X^T(q,m') = 1$, otherwise $X^T(q,m') = 0$.

The first term on the right hand side of (1) is the current optimal net surplus from consuming the product. Given the price of the firm, $p = p(T,q,m)$, the agent chooses the optimal amount of consumption. The second term is the expected discounted future surplus if no ad is received next period. It has two components: the future surplus if the consumer's current firm stays in the market, and the future surplus if that firm exits, leaving the consumer with a surplus of

$$W_{t+1}^{\varnothing} = \beta(1-\lambda)\left\{\Lambda_{t+2}(0)W_{t+2}^{\varnothing} + \sum_{i=1}^{\infty}\Lambda_{t+2}(i)\int W dZ_{t+2}^i(W)\right\}. \tag{2}$$

In this case, the utility in the period $t+1$ is zero, and the following period's surplus depends on the number of ads received. If no ad is received, the surplus is $W_{t+2}^{\varnothing}$, otherwise the consumer chooses the firm that yields the highest surplus. The third and fourth terms in (1) together give the expected future surplus from receiving one or more ads. The third term applies when the consumer's current firm stays in the market. In this case, $\mathbf{I}_{t+1}^j$ has $i+1$ elements corresponding to i new ads and the new information about the most recently visited firm. Similarly, the fourth term represents the case where the current firm exits and the consumer has i new ads.

Similarly the discounted expected surplus from being a customer of a non-trademark firm with quality level q and size m will read

$$
\begin{aligned}
& W_t^N(q,m) && (3)\\
= \; & \max_x \left[u(q,x) - px\right] + \beta(1-\lambda) && (4)\\
& \left\{ \Lambda_{t+1}(0) \left(\begin{array}{c} E\left[X^N(q,m')\right]\left[(1-\phi(q)) + \phi(q)(1-\delta)\right] W_{t+1}^N(q,m') \\ +E\left[X^N(q,m')\right]\phi(q)\delta W_{t+1}^o + (1 - E\left[X(q,m')\right]) W_{t+1}^{\varnothing} \end{array} \right) \right. \\
& \left. + \sum_{i=1}^{\infty} \Lambda_{t+1}(i) \left(\begin{array}{c} E\left[X^N(q,m')\right]\left[\left[(1-\phi(q)) + \phi(q)(1-\delta)\right] E\left[\int \max\{W, W_{t+1}^N(q,m')\} dZ_{t+1}^i(W)\right]\right] \\ +E\left[X^N(q,m')\right]\left[\phi(q)\delta W_{t+1}^o(q,m')\right] + (1 - E\left[X^N(q,m')\right]) \int W dZ_{t+1}^i(W) \end{array} \right) \right\} && (5)
\end{aligned}
$$

where $p = p(T,q,m)$ is the price that the firm charges, and W_{t+1}^o denotes the total discounted expected consumer surplus from an average diverting firm. Also here,

$$E\left[W_{t+1}^N(q,m')\right] = I(q,m') E\left[W_{t+1}^T(q,m')\right] + (1 - I(q,m')) E\left[W_{t+1}^N(q,m')\right],$$

where $I(q,m')$ is an indicator function which is one if a firm with the state (q,m') chooses to trademark next period and zero otherwise.

Note that only non-tademark firms can divert customers among themselves. If all non-trademark firms have equal probability of diverting a customer from a type (q,m) firm then expected consumer surplus from being diverted will be,

$$E\left[W_{t+1}^o(q,m')\right] = \int_q W_{t+1}^N(q,m) \mu^N(q,m) d(N,q,m). \tag{6}$$

However, the assumption of equal probability of divertion of customers implies an important irrationality on the part of the consumers. Especially, why would a consumer be diluted by a lower surplus firm given that consumers have information on type of the firms, (q,m). Therefore, here, we will assume that only similar type firms can divert customers from each other. In this case the expected surplus of a customer diverted from a type (q,m) will be (q,m) again, therefore,

$$E\left[W_{t+1}^o(q,m')\right] = E\left[W_{t+1}^N(q,m')\right].$$

This assumption not only better maintains rationality on the part of consumers but also simplifies the consumer problem significantly. With this assumption consumer problem becomes

$$
\begin{aligned}
& W_t^N(q,m) && (7)\\
= \; & \max_x \left[u(q,x) - px\right] + \beta(1-\lambda) && (8)\\
& \left\{ \Lambda_{t+1}(0) \left(E\left[X^N(q,m')\right] W_{t+1}^N(q,m') + (1 - E\left[X(q,m')\right]) W_{t+1}^{\varnothing} \right) \right. \\
& \left. + \sum_{i=1}^{\infty} \Lambda_{t+1}(i) \left(\begin{array}{c} E\left[X^N(q,m')\right] E\left[\int \max\{W, W_{t+1}^N(q,m')\} dZ_{t+1}^i(W)\right] \\ +(1 - E\left[X^N(q,m')\right]) \int W dZ_{t+1}^i(W) \end{array} \right) \right\}, && (9)
\end{aligned}
$$

It is clear that $W_t^T(q,m), W_t^N(q,m) \geq W_t^{\varnothing} > 0$, i.e. being informed of some firm brings at least as much surplus as having no information.

The consumer chooses the firm in $\mathbf{I}_t^j$ that yields the highest expected surplus

$$\overline{W}_t(\mathbf{I}_t^j) = \begin{cases} \max\limits_{k=1,\ldots,K_j} W_t^{\Gamma_k}(q_k, m_k), & \text{if } \left|\mathbf{I}_t^j\right| = K_j \geq 1, \\ W_t^{\varnothing}, & \text{if } \mathbf{I}_t^j = \varnothing. \end{cases} \tag{10}$$

where $\Gamma_k = \{T, N\}$ if the kth firm is a trademark or a non-trademark firm, respectively.

It is important to emphasize that firm size matters for consumer surplus only through the probability of exit and expected time it will take to trademark for a non-trademark firm which may trademark in the future, which affects consumer's continuation surplus in (1). In other words, firm size will not matter for consumer surplus for a trademark firm.

Consider now the probability that a consumer purchases from firm (Γ, q, m), conditional on having information about the firm. Each such consumer can be of two types: one who patronized the firm in the previous period, and the other who did not. Let the probability of purchase for these two types of consumers be $z_t^\Gamma(q, m)$ and $\omega_t^\Gamma(q, m)$, respectively. Because the latter type may have purchased from a firm that provides higher surplus than firm (Γ, q, m), $z_t^\Gamma(q, m)$ weakly exceeds $\omega_t^\Gamma(q, m)$. In other words, firm (Γ, q, m) faces more demand from a previous-period customer than a consumer it reaches through current period advertising.

The function $z_t^\Gamma(q, m)$ can be more explicitly written as

$$z_t^\Gamma(q, m) = \sum_{i=0}^{\infty} \Lambda_t(i) Z_t^i(W_t^\Gamma(q, m)), \tag{11}$$

where the ith term is the probability of sale to a consumer who has received i ads from firms other than firm (Γ, q, m). Furthermore,

$$\omega_t^\Gamma(q, m) = [\eta_t + (1 - \eta_t)\psi_t^\Gamma(q, m)] z_t^\Gamma(q, m). \tag{12}$$

where η_t is the measure of consumers with no information from any other firms, and $\psi_t^\Gamma(q, m)$ is the probability that the firm a consumer purchased from in period $t-1$ provides more surplus than firm (Γ, q, m).

Lemma 1 $z_t^\Gamma(q, m) \geq \omega_t^\Gamma(q, m)$.

Proof of Lemma 1. The demand z_t is defined by (11), and the demand of a consumer who did not purchase from firm (q, m^o) in period $t-1$ is

$$\omega_t(q, m^o) = \sum_{i=0}^{\infty} \Omega_t(i) \widetilde{G}_t^i(W_t(q, m^o)), \tag{13}$$

where $\Omega_t(i)$ is the measure of consumers informed of $i \geq 0$ firms in addition to firm (q, m^o), and $\widetilde{G}_t^i$ is the probability that the consumer chooses firm (q, m^o). Let $\delta_t < 1$ denote the measure of consumers who purchased in period $t-1$ from a firm that exited at the beginning of period t. Let $\theta_t = \lambda + (1 - \lambda)\delta_t < 1$ be the sum of the mass of new consumers and the mass of surviving consumers whose firms exited at the beginning of period t. Label all of these consumers as 'unattached', since they have no information before they receive any

ads. Label the rest of the consumers as 'attached', as they are informed of their most-recently-visited firms before they receive any ads. Then,

$$\Omega_t(i) = \begin{cases} \theta_t \Psi_t(0) & \text{for } i = 0, \\ \theta_t \Psi_t(i) + (1-\theta_t)\Psi_t(i-1) & \text{for } i > 0. \end{cases} \tag{14}$$

In (14), the term $\theta_t \Psi_t(0)$ gives the mass of unattached consumers who receive zero ads. The term for $i > 0$ is the mass of unattached consumers who receive i ads, plus the mass of attached consumers who receive $(i-1)$ ads. Next, let $\alpha_t \equiv \alpha_t(q, m^o)$ be the probability that a consumer prefers firm (q, m^o) to the firm he purchased from in period $t-1$. Note that $G_t^0 = \widetilde{G}_t^0 = 1$. For $i \geq 1$, (14) implies

$$\widetilde{G}_t^i = \left(\frac{\theta_t \Psi_t(i)}{\Omega_t(i)} \right) G_t^i + \left(\frac{(1-\theta_t)\Psi_t(i-1)}{\Omega_t(i)} \right) G_t^{i-1} \alpha_t, \tag{15}$$

where the first term in parentheses is the probability that a consumer is unattached and has i new ads conditional on having information about i other firms, and the second term in parentheses is the probability that a consumer is attached and has $i-1$ new ads conditional on having information about i other firms. Using (13) and (15), one can then write

$$\omega_t(q, m^o) = \theta_t \sum_{i=0}^{\infty} \Psi_t(i) G_t^i + (1-\theta_t) \sum_{i=1}^{\infty} \Psi_t(i-1) G_t^{i-1} \alpha_t = [\theta_t + (1-\theta_t)\alpha_t] z_t(q, m^o). \tag{16}$$

Because $\alpha_t \leq 1$ and $\theta_t < 1$, it follows that $\omega_t(q, m^o) \leq z_t(q, m^o)$. Next, for $\alpha_t(q, m^o) < 1$, using (16) and differentiating with respect to price and rearranging yields

$$\varepsilon_{\omega_t}(q, m^o) = \frac{(1-\theta_t)\alpha_t(q, m^o)}{[\theta_t + (1-\theta_t)\alpha_t(q, m^o)]} \varepsilon_{\alpha_t}(q, m^o) + \varepsilon_{z_t}(q, m^o),$$

which implies $\varepsilon_{\omega_t}(q, m^o) > \varepsilon_{z_t}(q, m^o)$ because $\varepsilon_{\alpha_t}(q, m^o) > 0$ and $(1-\theta_t)\alpha_t(q, m^o) > 0$. For $\alpha_t(q, m^o) = 1$, $\varepsilon_{\omega_t}(q, m^o) = \varepsilon_{z_t}(q, m^o)$. Consequently, $\varepsilon_{\omega_t}(q, m^o) \geq \varepsilon_{z_t}(q, m^o)$. ■

It is clear that consumers always prefer to be a customer of any firm compared to not consuming that variety at a given point in time.

Lemma 2 *$W_t^{\Gamma}(q, m) > W_{t+1}^{\varnothing}$ for $\Gamma = \{T, N\}$.*

It can also be shown that consumers prefer to be a customer of a trademark firm keeping the quality and size of the firm fixed. There are two extra benefits of being a customer of a trademark firm. First, the exit probability of a trademark firm is lower; second, the probability of being targeted for a trademark firm is zero. However, note that in terms of customer preference there is no clear comparison between an untargeted-non-trademark firm and a targetted-non-trademark firm. For instance it may be preferable for a customer to be diverted from a low q and m non-trademark firm by a higher quality non-trademark firm.

Lemma 3 *$W_t^T(q, m) > W_t^N(q, m)$.*

Lemma 4 *$z_t^T(q, m) \geq z_t^N(q, m)$, and $\omega_t^T(q, m) \geq \omega_t^N(q, m)$.*

Since trademark firms do not endogenously exit (in stationary equilibrium, in other words, $X^T(q,m) > 1$), for customers, firm size does not matter if the firm is a trademark firm.

Lemma 5 *Let* $m_1, m_2 > 0$ *and* $m_1 \neq m_2$, $W_t^T(q,m_1) = W_t^T(q,m_2)$, $z_t^T(q,m_1) = z_t^T(q,m_2)$ *and* $\omega_t^T(q,m_1) = \omega_t^T(q,m_2)$.

Therefore, the probability that a consumer purchases from a firm of type (Γ, q, m), does not depend on the size of the firm if the firm is a trademark firm. However consumers strictly prefer larger ones among non-trademark firms.

Lemma 6 *Let* $m_1, m_2 > 0$ *and* $m_1 > m_2$, $W_t^N(q,m_1) > W_t^N(q,m_2)$.

Hence the probability that a consumer purchases from a firm of type (Γ, q, m) strictly increases in the size of the firm, if the firm is a non-trademark firm.

3.2 Firms

Firms pay a one-time fixed cost of $\kappa > 0$ upon entry. After paying the fixed entry cost, an entering firm learns its efficiency level, e, in product development. Product development efficiency, e, follows *c.d.f.* $H(e)$ with density function $h(e)$. Realizing its efficiency level, the firm can decide to exit without developing its product and without incurring any further costs. If it decides to stay, it makes an irreversible product quality decision and incurs a product development cost tied to its quality. At the stage of product development, a higher product quality requires a larger one-time investment by the firm. After a firm's quality is set in its first period, it can not be changed throughout the firm's life. Higher product quality involves higher cost. Let $C(e,q)$ be the cost function associated with developing a product of quality q for a firm with quality efficiency e, with $\frac{\partial C(e,q)}{\partial q} > 0$, $\frac{\partial^2 C(e,q)}{\partial q^2} > 0$, and $\frac{\partial C(e,q)}{\partial e} < 0$. (We can assume $C(e,q) = c\left(\frac{q}{e}\right)^\eta$ with $c > 0$, and $\eta > 1$.) Production technology of the firms is given by $F(n) = An$, $A > 0$.

3.2.1 Information technology

Firms disseminate information using a common advertising technology, Φ^Γ for $\Gamma = \{T, N\}$. Here $\Phi^\Gamma(a)$ is the total cost of reaching to and informing $a \in [0, \infty)$ customers through advertising.

Assumption 1 *(i)* Φ^Γ *is twice differentiable and* $\Phi^{\Gamma\prime} > 0$ *and* $\Phi^{\Gamma\prime\prime} > 0$ *on* $(0,\infty)$, *(ii)* $\Phi^\Gamma(0) = 0$, *and (iii)* $\lim_{a\to 0} \Phi^{\Gamma\prime}(a) = 0$.

Assumption 2 $\Phi^N(a) > \Phi^T(a)$ *for all* a.

Assumption 1(i) captures the basic empirical regularity of diminishing returns to advertising, as documented in Sutton (1991). From an individual firm's perspective, there is a very large pool of consumers and each ad reaches a distinct consumer. If the advertising technology exhibits constant or increasing returns to scale, some firms can reach an arbitrarily large number of consumers and grow without bound. Convexity in

advertising cost prevents such an outcome. As Grossman and Shapiro (1984) point out, increasing marginal cost of advertising can stem from media saturation, overlapping media, or the existence of different predispositions to view ads in the consumer population. As the amount of ads sent increases, it becomes increasingly costly for the firm to find an additional consumer who has not yet received an ad. In other words, deeper penetration to consumer population requires increasingly higher marginal cost of marketing.

Assumptions 1(ii) and 1(iii) are made for simplicity and ensure that all surviving firms advertise, preventing the possibility that a firm coasts for a while without advertising. A positive fixed cost of advertising or a positive marginal cost around zero can lead to no advertising for some firms in a period.

One benefit of trademarking a product is that firms can reach to new customers more easily if the product is trademarked. This is stated in Assumption 2. Here, we can further specify the relative productivity of information technology as $\Phi^T(a) = \varkappa\Phi^N(a)$, with $0 < \varkappa < 1$.

3.2.2 A firm's dynamic problem

Let (Γ, q, m) be a firm's *type*. The period profit of a type (Γ, q, m) firm with period starting size m, product quality q, if it charges price p and sends a ads, is

$$\Pi_t^T(p, n, a; q, m) = pD^T(p, a; q, m^T) - w_t n - \Phi^T(a) - \varphi(q) - f, \tag{17}$$

and

$$\Pi_t^N(p, n, a; q, m) = pD^N(p, a; q, m^N) - w_t n - \Phi^N(a) - f, \tag{18}$$

subject to $D^\Gamma(p, a; q, m^\Gamma) = F(q, n)$, where $D^\Gamma(p, a; q, m^\Gamma) = m^\Gamma(a; q, m)x^\Gamma(p, q)$. An individual customer's demand for the product with quality q and price p is given by $x^\Gamma(p, q)$. Per period fixed cost of operating the firm is given by f and w_t denotes the wage rate. Trademark firms incur costs to maintain their trademarks $\varphi(m_t^T)$, which includes the legal litigation costs. This cost is assumed to be a function of the quality of the firm's product q, and $\varphi(q) \geq 0$, $\varphi'(q) > 0$, and $\varphi''(q) < 0$. Here, m_t^Γ is the firm's end-of-period number of customers given by

$$m^T(a; q, m) = m(1 - \lambda)z^T(q, m) + a\omega^T(q, m), \tag{19}$$

A trademark firm that starts the period with m customers will loose λ fraction of those customers due to exogeneous customer turnover. The first term in (19) gives the number of loyal customers the firm will carry to current period and the second term gives the number of new customers the firm gains this period through current marketing activity. Similarly, current amount of customers of an untargeted nontrademark firm, a targeted nontrademark firm, and a firm which currently receives a trademark will respectively read

$$\begin{aligned} m^{NU}(a; q, m) &= m(1 - \lambda)z^{NU}(q, m) + a\omega^{NU}(q, m) + d(q), \\ m^{NT}(a; q, m) &= \left[m(1 - \lambda)z^{NT}(q, m) + a\omega^{NT}(q, m)\right](1 - \delta), \end{aligned} \tag{20}$$

and

$$m^{TR}(a; q, m) = \left[m(1 - \lambda)z^{TR}(q, m) + a\omega^{TR}(q, m)\right](1 - \xi\delta)$$

where d is the amount of customers diverted from non-trademark firms under attack. The law of motion (19,**??**,and 20) can also be viewed as capital accumulation subject to stochastic depreciation and investment. In this interpretation, m is the firm's capital stock at the beginning of period t. During period t, for a trademark firm, a fraction $\lambda + (1-\lambda)(1-z^T(q,m))$ of this stock depreciates, where λ is the industry-wide depreciation rate common to all firms and $(1-z^\Gamma(q,m))$ is the firm-specific depreciation rate. A fraction $\omega^T(q,m)$ of investment a turns into new capital, where $\omega^T(q,m)$ is the firm-specific investment success rate. For a targeted non-trademark firm depreciation of the stock amounts to $\lambda + (1-\lambda)(1-\delta)(1-z^{NT}(q,m))$. Notice that for a targeted nontrademark firm which gets a trademark in the current period the endogeneous depreciation is $\lambda + (1-\lambda)(1-\xi\delta)(1-z^{NT}(q,m))$. The total cost of investment is $\Phi^\Gamma(a)$.

It can be verified that Π_t is strictly increasing in q and m, strictly concave in a and p.

A non-trademark incumbent firm's being targeted or not by other firms follows an iid process with probability of being targeted is given by $\phi(q)$. Here, assume that $\phi(q) > 0, \phi'(q) > 0$, and $\phi''(q) < 0$.

At the beginning of a period, a type (T,q,m) firm has the value

$$V_t^T(q,m) = \max\left[\underbrace{0}_{\text{exit}}, \underbrace{\max_{p,n,a}\left[\Pi_t^T(p,a,n;q,m) + \beta(1-\chi)V_{t+1}^T(q,m')\right]}_{\text{stay}}\right]. \tag{21}$$

where $m' = m(1-\lambda)z^T(q,m) + a\omega^T(q,m)$. Here, first, the firm decides whether to stay in industry or not. If it decides to exit at this point, it does not incur the fixed operation cost f and its value is zero. If it stays, it has to make optimal marketing (a), employment (n), and pricing (p) decisions. Although this firm has already decided to stay, it may not be around next period with probability χ due to exogenous reasons out of its control. Similarly, a nontrademark firm of type (q,m) has the following optimization problem

$$V_t^N(q,m) = \max\left\{0, \max_a\left\{\phi(q)\max\left\{\begin{array}{c}\max_{p,n}\left[-c_T + \Pi_t^N(p,n,a;q,m^{TR}) + \beta(1-\chi)V_{t+1}^T(q,m^{TR})\right], \\ \max_{p,n}\left[\Pi_t^N(p,n,a;q,m^{NT}) + \beta(1-\chi)V_{t+1}^N(q,m^{NT})\right],\end{array}\right.\right. \tag{22}$$

$$\left.\left. +(1-\phi(q))\max\left\{\begin{array}{c}\max_{p,n}\left[-c_T + \Pi_t^N(p,n,a;q,m^{NU}) + \beta(1-\chi)V_{t+1}^T(q,m^{NU})\right], \\ \max_{p,n}\left[\Pi_t^N(p,n,a;q,m^{NU}) + \beta(1-\chi)V_{t+1}^N(q,m^{NU})\right],\end{array}\right\}\right\}\right\} \tag{23}$$

$$\tag{24}$$

where $m^{TR}(a;q,m) = \left[m(1-\lambda)z^{TR}(q,m) + a\omega^{TR}(q,m)\right](1-\xi\delta)$, $m^{NT}(a;q,m) = \left[m(1-\lambda)z^{NT}(q,m) + a\omega^{NT}(q,m)\right.$ $\left.\delta\right)$, and $m^{NU}(a;q,m) = m(1-\lambda)z^{NU}(q,m) + a\omega^{NU}(q,m) + d(q)$. In (22, the first term with the brackets is the value if the firm is targeted in the current period which will happen with probability $\phi(q)$. Similarly the second term with the brackets gives the firms value if it not targeted in current period. Whether it is currently targeted or not the firm will make its optimal stay-exit decision. If it stays, it has another important decision, it will decide whether it should trademark its product or not. The one-time-fixed cost of trademarking is denoted by c_T. If the firm decides to trademark its product, it continues as a trademark firm throughout its life without facing any problem of being targeted by other firms as long as it incurs the trademark maintanence cost. However if the firm decides to stay as a non-trademark firm, next period with probability $\phi(q)$, the firm will be targeted and it will loose δ fraction of its existing customers.

Entry Entry is unrestricted. Each period a mass of $M \geq 0$ of new firms enter the industry. After paying a fixed entry cost an entering firm makes an irreversible product quality decision. After a firm's quality is set in its first period, it can not be changed throughout the firm's life. A higher product quality has two types of costs associated with it. First, it requires a larger one-time investment by the firm by incurring the cost $C(e,q)$. Second, higher quality requires a higher variable cost of production which is incurred every period and incorporated in $F(q,n)$. An entrant starts off with zero customers but through customer diversion it steals customers from non-trademark incumbent firms. Let d denote the amount of customers that an entering firm steals from non-trademark incumbents in this way.

The optimal product quality decision of an entering firm with product development efficiency e will read,

$$q^*(e) = \arg\max_q \left[V^N(q, d(q)) - C(e,q)\right].$$

It is clear that an entering firm with efficiency level e will exit without incurring any product development cost if $\left[V^N(q^*(e), d(q)) - C(e, q^*(e))\right] < 0$.

Free entry implies

$$\int \max\left\{0, \left[V^N(q^*(e), d(q)) - C(e, q^*(e))\right]\right\} h(e)de \leq \kappa, \tag{25}$$

which holds with equality when $M_t > 0$.

The state of the industry in period t can be described by a measure μ_t over the set of firm types (Γ, q, m). For any Borel set $G \subset \{T, N\} \times \mathbb{R}^+ \times [\underline{q}, \infty)$, $\mu_t(G)$ gives the mass of firms with types in G. Given an initial measure of firms μ_o, μ_t results from entry, exit and changes in incumbents' sizes up to time t.

A consumer's demand function can now be related to the measure of firms. Let $B_t(\Gamma, q, m)$ denote the set of firm types that provide a consumer a surplus of at most $W_t^\Gamma(q,m)$. The probability that an ad comes from some firm with type in $B_t(\Gamma, q, m)$ is

$$Q_t(\Gamma, q, m; \mu_t) = \frac{1}{R_t}\left(\int_{B_t(\Gamma,q,m)} a_t(\Gamma, q, m) d\mu_t(\Gamma, q, m)\right), \tag{26}$$

where $a_t(\Gamma, q, m)$ is the number of ads sent by a type (Γ, q, m) firm, and

$$R_t = \int_{(\Gamma,q,m)} a_t(\Gamma, q, m) d\mu_t(\Gamma, q, m)$$

is the total number of ads in the economy. By noting that $Q_t(\Gamma, q, m; \mu_t) \equiv Z_t(W_t^\Gamma(q, m; \mu_t))$, the individual demand function in (11 and 12) can be expressed in terms of the measure of firms μ_t as $z_t^\Gamma(q, m; \mu_t) = \sum_{i=0}^\infty \Lambda_t(i) Q_t^i(q, m; \mu_t)$. Similarly, $\omega_t^\Gamma(q,m)$ can also be related to μ_t using (12).

Let $T(\Gamma, q, m)$ denote the optimal end of period trademark status of a firm which starts the period with state (Γ, q, m). Here, $T(\Gamma, q, m)$ is equal to T if the firm would have a trademark at the end of the period and Γ otherwise. $T(\Gamma, q, m)$ provides the evolution of trademark status of firms. Clearly $T(T, q, m) = T$.

Consider a firm of type (NU, q, m); $T(NU, q, m) = T$, if

$$\begin{aligned} & -c_T + \max_{p,n}\left[\Pi_t^{NU}(p, n; q, m^{NU}) + \beta(1-\chi)V_{t+1}^T(q, m^{NU})\right] \\ \geq & \max_{p,n}\left[\Pi_t^{NU}(p, n; q, m^{NU}) + \beta(1-\chi)V_{t+1}^N(q, m^{NU})\right], \end{aligned}$$

and $T(NU,q,m)=N$, otherwise. Similarly, a firm of type (NT,q,m); $T(NT,q,m)=T$, if

$$\begin{aligned} & -c_T+\max_{p,n}\left[\Pi_t^{TR}(p,n;q,m^{TR})+\beta(1-\chi)V_{t+1}^{T}(q,m^{TR})\right] \\ \geq\ & \max_{p,n,a}\left[\Pi_t^{NT}(p,n;q,m^{NT})+\beta(1-\chi)V_{t+1}^{N}(q,m^{NT})\right], \end{aligned}$$

and $T(NT,q,m)=N$, otherwise.

4 Stationary equilibrium

Let (*) identify variables and functions in equilibrium.

Definition 1 (Stationary equilibrium) *A stationary equilibrium consists of product quality, employment, pricing, advertising, trademarking, and stay-exit policies* p_f, $q^*(e)$, $n^*(\Gamma,q,m)$, $p^*(\Gamma,q,m)$, $a^*(\Gamma,q,m)$, $T^*(\Gamma,q,m)$, *and* $X^*(\Gamma,q,m)$, *a cumulative distribution of consumer surplus across ads* $Z^*(W)$, *a measure* μ^* *over firm types, and an entry mass* $M^*\geq 0$, *such that, in any given period of time,*

(i) Firms maximize their profits by adopting policies $q^*(e)$, $n^*(\Gamma,q,m)$, $p^*(\Gamma,q,m)$, $a^*(\Gamma,q,m)$, $T^*(\Gamma,q,m)$, *and* $X^*(\Gamma,q,m)$,

(ii) Consumers maximize their surplus,

(iii) M^* *satisfy the respective free entry condition (25),*

(iv) Z^* *is consistent with quality, pricing, and advertising policies,*

(v) μ^* *is consistent with conditions (i) through (v).*[7]

In the case of free advertising, i.e. $\Phi(a)=0$ for $a\geq 0$, every consumer has information about all firms. If there is a positive measure of firms, competition implies that firms offering the highest surplus make zero profit gross of the fixed cost, while all other firms make no sales. A firm's value gross of fixed cost is therefore zero. This polar case is ruled out by Assumption 1. The rest of the analysis focuses on equilibria with non-zero advertising cost where a positive mass of firms in the industry compete with each other. It can easily be shown that in stationary equilibrium there will be positive advertising. Now, we characterize the stationary equilibrium. Let $z^{\Gamma*}(q,m)$ and $\omega^{\Gamma*}(q,m)$ be the individual demand functions in equilibrium. The evolution of a firm's customer base is given by

$$m^{T*}(q,m)=m(1-\lambda)z^{T*}(q,m)+a^*(T,q,m)\omega^{T*}(q,m), \tag{27}$$

and

$$\begin{aligned} m^{TR*}(a;q,m) &= \left[m(1-\lambda)z^{TR*}(q,m)+a^*(TR,q,m)\omega^{TR*}(q,m)\right](1-\xi\delta) \\ m^{NU*}(q,m) &= m(1-\lambda)z^{NU*}(q,m)+a^*(NU,q,m)\omega^{NU*}(q,m)+d^*(q), \qquad (28) \\ m^{NT*}(q,m) &= \left[m(1-\lambda)z^{NT*}(q,m)+a^*(NT,q,m)\omega^{NT*}(q,m)\right](1-\delta), \qquad (29) \end{aligned}$$

[7] In other words, starting with the measure μ^* at the beginning of a period, firms' pricing and advertising policies, consumers' purchase decisions, entry, exit and changes in surviving firms' types together result in the same measure μ^* at the end of the period.

A firm's profit $\Pi^{\Gamma*}(q,m)$ is given by

$$\Pi^{T*}(q,m) = p^*(T,q,m)D^{T*}(q,m^T) - wn^*(T,q,m) - \Phi^T(a^*(T,q,m)) - \varphi(m^T) - f, \tag{30}$$

and

$$\Pi^{N*}(q,m) = p^*(N,q,m)D^{N*}(q,m^N) - wn^*(N,q,m) - \Phi^N(a^*(N,q,m)) - f, \tag{31}$$

subject to $D^{\Gamma*}(q,m) = F(q,n^*(\Gamma,q,m))$, where $D^{\Gamma*}(q,m) = m^{\Gamma*}(q,m)x^*(p^*(\Gamma,q,m),q)$.

A firm's value $V^{\Gamma*}(q,m)$ is

$$V^{T*}(q,m) = \max\left\{0, \Pi^{T*}(q,m^{T*}(q,m)) + \beta(1-\chi)V^{T*}(q,m^{T*}(q,m))\right\}. \tag{32}$$

(33)

$$V_t^{N*}(q,m) = \max\left\{0, \max_a\left\{\phi(q)\max\left\{\begin{array}{c}\max_{p,n}\left[-c_T + \Pi_t^{N*}(p,n,a;q,m^{TR}) + \beta(1-\chi)V_{t+1}^{T*}(q,m^{TR})\right], \\ \max_{p,n}\left[\Pi_t^{N*}(p,n,a;q,m^{NT}) + \beta(1-\chi)V_{t+1}^{N*}(q,m^{NT})\right],\end{array}\right\}\right. \tag{34}$$

$$\left. +(1-\phi(q))\max\left\{\begin{array}{c}\max_{p,n}\left[-c_T + \Pi_t^{N*}(p,n,a;q,m^{NU}) + \beta(1-\chi)V_{t+1}^{T*}(q,m^{NU})\right], \\ \max_{p,n}\left[\Pi_t^{N*}(p,n,a;q,m^{NU}) + \beta(1-\chi)V_{t+1}^{N*}(q,m^{NU})\right],\end{array}\right\}\right\}\right\} \tag{35}$$

Total amount of customers lost through diversion efforts should be equal to the total customers gained. Hence,

$$d^*(q,m) = \frac{\delta\phi(q)\int_{(N,m)} m^{N*}(q,m)d\mu^{N*}(q,m)}{(1-\phi(q))\int_{(N,m)} d\mu^{N*}(q,m)}.$$

Following the theory of stochastic dynamic programming in Stokey and Lucas (1993), it can be shown that there exists a unique continuous and bounded function $V^*(q,m)$ that satisfies (32).

Let $W^{\Gamma*}(q,m)$ be the surplus from visiting firm (Γ,q,m), and let $W^{\varnothing*}$ be the surplus of an uninformed consumer. W^* and $W^{\varnothing*}$ are the fixed-points of equations (1) and (2), respectively. Once again, the dynamic programming techniques in Stokey and Lucas (1993) ensure the existence and uniqueness of continuous and bounded functions W^* and $W^{\varnothing*}$. There exists then a unique bounded function $\overline{W}^*(\mathbf{I})$ that gives the maximum surplus associated with any cumulative information, $\mathbf{I}$. The individual demand function, firm value, and consumer surplus satisfy the following properties.

4.1 Characterizing the Stationary Equilibrium

Proposition 1 *(i) $z^{\Gamma*}(q,m^o)$ and $\omega^{\Gamma*}(q,m^o)$ are strictly increasing in q, (ii) $z^{\Gamma*}(q,m^o)$ and $\omega^{\Gamma*}(q,m^o)$ are strictly increasing in m^o, and (iii) $V^{\Gamma*}(q,m^o)$ is strictly increasing in q, and strictly increasing in m^o, and (iv) $V^{\Gamma*}(q,m^o)$ is strictly increasing in q, and strictly increasing in m^o.*

Proof of Proposition 1. Part (i). Using the definition (26), let $Q^*(q|m^o)$ denote the equilibrium probability that an ad comes from a firm with quality at most q, conditional on m^o. The cumulative distribution function, $1-Q^*(q|m^o)$, cannot have a mass point at any q in its support, otherwise any firm with quality

level q could increase its quality slightly and steal a positive mass of consumers from other firms charging q, leading to a discrete increase in profit. Also, $1-Q^*(q|m^o)$ cannot be flat over some interval $[q_1, q_2]$, otherwise any firm with quality level could decrease its quality to q_2 without a reduction in its probability of sale. Thus, $1-Q^*(q|m^o)$ is differentiable and strictly increasing in q over the interior of its support $[\underline{q}^*(m^o), \overline{q}^*(m^o)]$, where $\underline{q}^*(m^o)$ and $\overline{q}^*(m^o)$ are the minimum and maximum quality levels observed for firms with size m^o. Therefore, $\frac{\partial Q^*(q|m^o)}{\partial q} > 0$. Let $z^*(q|m^o) = \sum_{i=0}^{\infty} \Psi^*(i) \left[Q^*(q|m^o)\right]^i$ be the demand function for a consumer conditional on m^o. It follows that $\frac{\partial z^*(q|m^o)}{\partial q} = \sum_{i=0}^{\infty} \Psi^*(i) i Q^{*i-1} \frac{\partial Q^*}{\partial q} < 0$, which also implies $\frac{\partial z^*(q,m^o)}{\partial q} > 0$. Similar arguments apply to $\omega^*(q, m^o)$.

Parts (ii), (iii) and (iv). Note that $\frac{dV^*}{dq} = \frac{\partial V^*}{\partial q} = -m^* < 0$, by the envelope theorem. Next, observe that

$$\frac{\partial z^*}{\partial q} = \left(\sum_{i=0}^{\infty} \Psi^*(i) i G^{*i-1} \right) G^{*\prime} \frac{\partial W^*}{\partial q}. \tag{36}$$

Because $\frac{\partial z^*}{\partial q} > 0$ by Part (i) and $\left(\sum_{i=0}^{\infty} \Psi^*(i) i G^{*i-1}\right) G^{*\prime} > 0$, it follows that $\frac{\partial W^*}{\partial q} > 0$. Similar arguments establish $\frac{\partial \omega^*}{\partial q} > 0$. Next, we argue that z^*, V^*, and W^* must be strictly increasing in m^o. Differentiation of z^* yields

$$\frac{\partial z^*}{\partial m^o} = \left(\sum_{i=0}^{\infty} \Psi^*(i) i G^{*i-1} \right) G^{*\prime} \frac{\partial W^*}{\partial m^o}. \tag{37}$$

Thus, $\frac{\partial z^*}{\partial m^o}$ and $\frac{\partial W^*}{\partial m^o}$ have the same sign. Similarly, $\frac{\partial \omega^*}{\partial m^o}$ and $\frac{\partial W^*}{\partial m^o}$ have the same sign, and therefore so do $\frac{\partial \omega^*}{\partial m^o}$ and $\frac{\partial z^*}{\partial m^o}$. Furthermore, by the envelope theorem, $\frac{dV^*}{dm^o} = \frac{\partial V^*}{\partial m^o} = \frac{\partial m^*}{\partial m^o} \left((\nu - 1) \frac{q^\rho}{A} w + R^* \right)$.

The term in parentheses must be positive. Consequently, $\frac{dV^*}{dm^o}$ and $\frac{\partial m^*}{\partial m^o}$ have the same sign. Next, note that

$$\frac{\partial m^*}{\partial m^o} = (1-\lambda) z^* + \left(m^o (1-\lambda) \frac{\partial z^*}{\partial m^o} + a^* \frac{\partial \omega^*}{\partial m^o} \right).$$

To obtain a contradiction, suppose that $\frac{\partial z^*}{\partial m^o} \leq 0$ and $\frac{\partial \omega^*}{\partial m^o} \leq 0$. There are then two possibilities: $\frac{\partial m^*}{\partial m^o} \leq 0$ or $\frac{\partial m^*}{\partial m^o} > 0$. If $\frac{\partial m^*}{\partial m^o} \leq 0$, (**??**) implies $\frac{dV^*}{dm^o} \leq 0$. But then firm value is non-increasing in firm size, and a firm has no incentive to increase its size, because exit probability increases as firm size increases. Because the only way a firm can grow is to send ads, there is then no advertising. Therefore, $\frac{\partial z^*}{\partial m^o} \leq 0$, $\frac{\partial \omega^*}{\partial m^o} \leq 0$, and $\frac{\partial m^*}{\partial m^o} \leq 0$ cannot hold in an equilibrium with positive advertising.

If, on the other hand, $\frac{\partial m^*}{\partial m^o} > 0$, then, $\frac{dV^*}{dm^o} > 0$. Now, rewrite the consumer surplus in (1) in stationary equilibrium as

$$W^*(q, m^o) = U(p) + \beta(1-\lambda) \sum_{i=0}^{\infty} \Psi^*(i) \{ (1 - X^*(q, m^o)) W_1^*(q, m^1) + X^*(q, m^o) W_2^*] \},$$

where X^* is the probability of exit, (q, m^1) denotes the next period price and size, and

$$W_1^*(q, m^1) = \begin{cases} E\left[\int \max\{W, W^*(q, m^1)\} dG^{*i}(W) \right] & \text{if } i > 0 \\ E[W(q, m')] & \text{if } i = 0 \end{cases},$$

$$W_2^* = \begin{cases} E\left[\int W dG^{*i}(W) \right] & \text{if } i > 0 \\ W^{*\varnothing} & \text{if } i = 0 \end{cases}.$$

Then

$$\frac{\partial W^*}{\partial m^o} = \beta(1-\lambda)\sum_{i=0}^{\infty}\Psi^*(i)\left(\frac{\partial X^*}{\partial m^o}(W_2^* - W_1^*(q,m^1)) + (1-X^*)\frac{\partial W_1^*}{\partial m^1}\right), \tag{38}$$

and

$$\frac{\partial W_1^*}{\partial m^1} = \beta(1-\lambda)\sum_{i=0}^{\infty}\Psi^*(i)\left(\frac{\partial X^*}{\partial m^1}(W_2^* - W_1^*(q,m^2)) + (1-X^*)\frac{\partial W_1^*}{\partial m^2}\right)\frac{\partial m^1}{\partial m^o}, \tag{39}$$

where (q, m^2) is the two-period ahead price and size, and m^1 is the next period's demand. One can thus substitute $\frac{\partial W_1^*}{\partial m^1}$ in (38) using (39). Continuing with similar substitution for all future periods, one obtains

$$\frac{\partial W^*}{\partial m^o} = \sum_{t=1}^{\infty}\beta^t(1-\lambda)^t\left(\prod_{j=1}^{t-1}(1-X^*(q,m^t))\right)\frac{\partial X^*}{\partial m^t}\sum_{i=0}^{\infty}\Psi^*(i)(W_2^* - W_1^*(q,m^t))\left(\prod_{j=1}^{t-1}\frac{\partial m^j}{\partial m^{j-1}}\right), \tag{40}$$

where (q, m^t) is the $t-$ period ahead price and size, and m^j is the demand in period j. Because $\frac{dV^*}{dm^o} > 0$ and exit probability declines with firm value, exit probability also declines with firm size, i.e. $\frac{\partial X^*}{\partial m^t} < 0$ for all t. In addition, it was assumed that $\frac{\partial m^j}{\partial m^{j-1}} > 0$. Furthermore, $W_2^* - W_1^*(q,m^1) < 0$, by the definitions of W_1^* and W_2^*. Consequently, using (40), $\frac{\partial W^*}{\partial m^o} > 0$. But since $\frac{\partial z^*}{\partial m^o} \leq 0$, (37) implies $\frac{\partial W^*}{\partial m^o} \leq 0$, a contradiction. Therefore, $\frac{\partial z^*}{\partial m^o} \leq 0$ and $\frac{\partial \omega^*}{\partial m^o} \leq 0$ cannot hold. Thus, the only configuration consistent with positive advertising in equilibrium is $\frac{\partial z^*}{\partial m^o} > 0$, $\frac{\partial \omega^*}{\partial m^o} > 0$, $\frac{\partial V^*}{\partial m^o} > 0$ and $\frac{\partial W^*}{\partial m^o} > 0$. ■

Proposition 2 *Firms follow a constant mark-up pricing policy;* $p^*(\Gamma, q, m) = p^*(q) = \nu\frac{q^\rho}{A}w$, *with* $\nu > 1$.

Trademarking costs are typically not very high. However many firms choose not to trademark their products. This model is consistent with this empirical observation. In this model, even with an infinitesimal amount of trademarking cost, in equilibrium, some firms will not trademark. The following proposition asserts that some very low quality firms will never choose to trademark.

Proposition 3 *There exists a threshold quality level* q^o *such that firms with quality level below* q^o *will never trademark. Furthermore, for* $q > q^o$, *optimal trademarking decision of firms will be such that* $T^*(\Gamma, q, m)$ *is increasing in* q *and* m. *Furthermore* $T^*(NT, q, m) \geq T^*(NU, q, m)$.

Let $T^o(\Gamma, q, m)$ be a indicator function with

$$T^o(\Gamma, q, m) \;=\; \begin{cases} 1 & \text{if } T(y,q,m) = T,\ y \in \{NU, NT\}, \\ \\ 0 & \text{otherwise.} \end{cases}$$

The number of new trademarks issued in an industry at a given period will be given by

$$N^T = \int_{(\Gamma,q,m)} T^o(\Gamma, q, m)\mu(\Gamma, q, m)d(\Gamma, q, m).$$

Consider the state history of an age i trademark firm, $\Upsilon = \{(NU, q, m_1), (\Gamma_2, q, m_2), ..., (\Gamma_i, q, m_i)\}$, denote *age of trademarking* as $age^T = \min_j \{\Gamma_j = T, \text{ such that } (\Gamma_j, q, m_j) \in \Upsilon\}$.

Proposition 4 *(i)* $z^{S*}(q,m)$ *and* $\omega^{S*}(q,m)$ *are strictly increasing in* q*,(ii)* $V^{\Gamma*}(q,m)$ *is strictly increasing in* q *and* m *(iv)* $W^{\Gamma*}(q,m)$ *is strictly increasing in* q*.*

Consider next the pricing and advertising policies. Remember that firms do not put price information on their ads. Given its current customer base m, each firm chooses optimal price for the period. This clearly yields a constant markup pricing policy. Hence $p^*(\Gamma,q,m)=\nu\frac{q^\rho}{A}w$ where ν denotes the markup rate.

Assumption 1 ensures that firms advertise every period. The advertising policy, $a^*(\Gamma,m,c)$, is thus an interior maximizer defined implicitly by the first order condition

$$\frac{\partial m^{T*}}{\partial a}\left[\nu\frac{q^\rho}{A}w+\beta(1-\chi)(1-\lambda)\frac{\partial}{\partial m^{T*}}V^{T*}(q,m^{T*})\right]=\Phi^{T\prime}(a). \tag{41}$$

for trademark firms, and

$$\left\{\begin{array}{c}\phi(q)\left(I^{NT}(q,m)\left[\frac{\partial m^{TR}}{\partial a}\left[\frac{\partial\Pi^{TR}}{\partial m^{TR}}+\beta(1-\chi)(1-\lambda)\frac{\partial}{\partial m^{T*}}V^{T*}(q,m^{TR*})\right]\right]\right.\\ \left.+(1-I^{NT}(q,m))\left[\frac{\partial m^{NT}}{\partial a}\left[\frac{\partial\Pi^{NT}}{\partial m^{NT}}+\beta(1-\chi)(1-\lambda)\frac{\partial}{\partial m^{NT*}}V^{N*}(q,m^{NT*})\right]\right]\right\}\\ +(1-\phi(q))\left(I^{NU}(q,m)\left[\frac{\partial m^{NU}}{\partial a}\left[\frac{\partial\Pi^{NU}}{\partial m^{NU}}+\beta(1-\chi)(1-\lambda)\frac{\partial}{\partial m^{NU*}}V^{T*}(q,m^{NU*})\right]\right]\right.\\ \left.+(1-I^{NU}(q,m))\left[\frac{\partial m^{NU}}{\partial a}\left[\frac{\partial\Pi^{NU}}{\partial m^{NU}}+\beta(1-\chi)(1-\lambda)\frac{\partial}{\partial m^{T*}}V^{N*}(q,m^{NU*})\right]\right]\right\}\end{array}\right\}=\Phi^{T\prime}(a)$$

for nontrademark firms. Change in current period's advertising level affects the firm's continuation value. The second term in brackets in (41) pertains to the dynamic effects of the marginal ad. The advertising policy is an increasing function of a firm's efficiency and size, because the expected profit per consumer is higher for firms with larger clientele and higher efficiency.

Let the sales of a type (Γ,q,m) firm in any period be $r^{\Gamma*}(q,m)=m^{\Gamma*}(q,m)x^*(p^*(q),q)p^*(q)$. Hence, $r^{\Gamma*}(q,m)=m^{\Gamma*}(q,m)x^*(p^*(q),q)\nu\frac{q^\rho}{A}w$. The profit of the firm will be $\Pi^{\Gamma*}(q,m)=m^{\Gamma*}(q,m)x^*(p^*(q),q)(\nu-1)\frac{q^\rho}{A}w-f$.

Proposition 5 *There exists a unique advertising policy,* $a^*(\Gamma,q,m)$*, which is a strictly increasing function of quality,* q*;* $a^*(S,q,m)$ *does not depend on size,* m*. Also* $a^*(T,q,m)>a^*(N,q,m)$*. A firm's demand,* $m^{\Gamma*}(q,m)$*, and sales,* $r^{\Gamma*}(q,m)$*, are strictly increasing functions of quality,* q*, and size,* m*.*

Because a firm's end-of-period number of customers, $m^{\Gamma*}(q,m)$ is strictly increasing in previous-period size m, there is persistence in firm size.

Firms start unattacked at their entry. They accumulate customers as they age. Therefore in this model, in stationary equilibrium if there is entry, a firm will never exit unless it is attacked. Since by construction trademark firms can not be attacked, no trademark firm exits in equilibrium. The following proposition formally shows that in equilibrium only attacked firms exit.

Proposition 6 $X^{T*}(q,m)=X^{N*}(q,m)=1-\chi$ *for all* q,m*; there is no endogeneous exit in stationary equilibrium.*

Since there is only exogeneous exit in equilibrium, for consumers the sole important firm characteristic is its quality. Hence the consumer surplus functions simplfy significantly.

$$W_t^N(q) = \max_x [u(q,x) - px] + \beta(1-\lambda) \tag{42}$$
$$\left\{ \begin{array}{c} \Lambda_{t+1}(0)\left(\left[(1-\chi)E\left[W_{t+1}^N(q) \right] \right] + \chi W_{t+1}^{\varnothing} \right) \\ + \sum_{i=1}^{\infty} \Lambda_{t+1}(i) \left[(1-\chi)E\left[\int \max\{W, W_{t+1}^N(q)\} dZ_{t+1}^i(W) \right] + \chi \int W dZ_{t+1}^i(W) \right] \end{array} \right\},$$

and

$$W_t^T(q) = \max_x [u(q,x) - px] + \beta(1-\lambda) \tag{43}$$
$$\left\{ \begin{array}{c} \Lambda_{t+1}(0)\left[(1-\chi)W_{t+1}^T(q) + \chi W_{t+1}^{\varnothing} \right] \\ + \sum_{i=1}^{\infty} \Lambda_{t+1}(i) \left[(1-\chi) \int \max\{W, W_{t+1}^T(q)\} dZ_{t+1}^i(W) + \chi \int W dZ_{t+1}^i(W) \right] \end{array} \right\}.$$

As information technology improves and the advertising cost function becomes more efficient, i.e., $\Phi^{\Gamma\prime}(a) < \Phi^{\Gamma}(a)$ for all a, the sixth region in the geography of trademarking graph becomes wider. The firms threshold level of size for trademarking $\Gamma(q)$ decreases and the firms in the fifth region gets their trademark earlier. The following proposition formally shows this.

Proposition 7 *Expected age of trademarking at birth, $E\left[age^T(q)\right]$ is decreasing in q.*

Proposition 8 *Consider an improvement in information technology so that $\Phi^{\Gamma\prime}(a) < \Phi^{\Gamma}(a)$ for all a. After the improvement, expected age of trademarking decreases, $E\left[age^{T'}(q)\right] < E\left[age^T(q)\right]$ for all q. Similarly, an increase in trademarking cost, c_T, will increase the trademarking date. Let $c_T' > c_T$, then $E\left[age^T(q)\right]' \geq E\left[age^T(q)\right]$.*

Let the fraction of non-trademark firms that trademark at a given period be denoted by $fr(S,q,m) = \frac{\int T^o(y,q,m)\mu(y,q,m)d(y,q,m)}{\int \mu(y,q,m)d(y,q,m)}$, $y \in S = \{NU, NT\}$.

Proposition 9 *Probability of trademarking for an average non-trademark firm increases when it is targeted.*

Consider the size-quality plane in stationary equilibrium that is shown in Figure 3. It has six regions. In the first region, the efficiency of the firm in product development, e, is so low that the firm chooses not to enter the industry at all. In the second region, the firm enters the industry but the quality of the firm's product is so low that the firm does not trademark its product and it chooses to exit whenever it is attacked. Hence all the firms in region two are unattacked non-trademark (type NU) firms. There is no exit at the third region and firms in this region will never choose trademark their products. Their size will change from period to period as they wander between attacked and unattacked states. The third region will consist of all non-trademark firms but some of these firms may be attacked and others may be unattacked. Therefore it will include both type NU and type NT firms. In the fourth region the firms will get a trademark once they are attacked. As explained through a proposition earlier, getting a trademark will be a better option once a firm is attacked since trademark will break the spell of a customer diversion attack. Therefore in

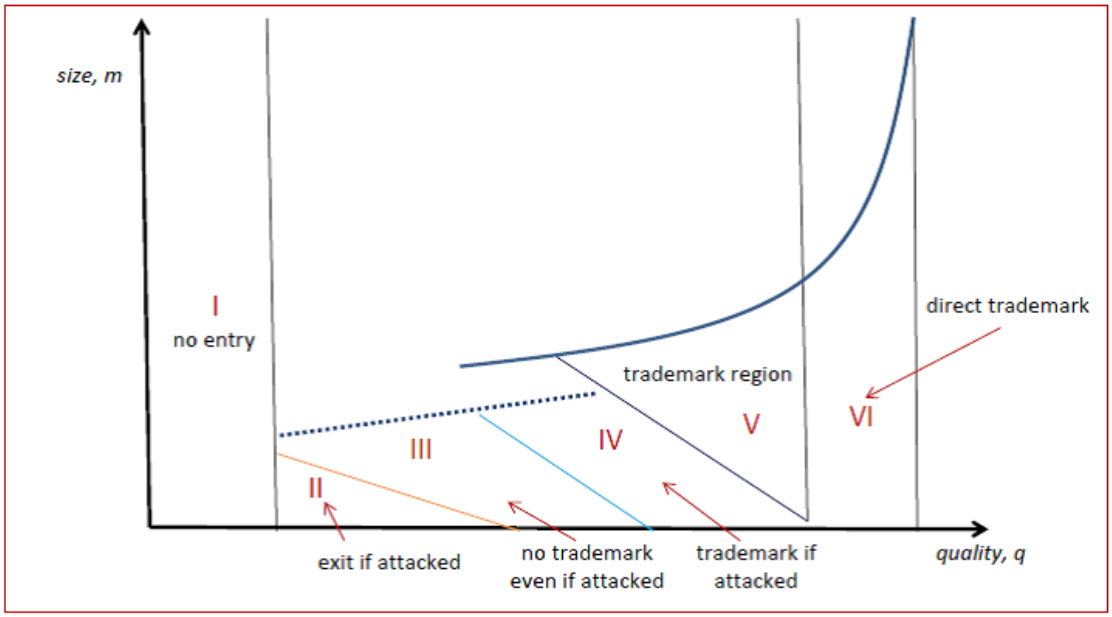


Figure 3: Geography of trademark decision; the size and quality plane in stationary equilibrium in economy with trademarks.

this region no attacked non-trademark firm will persist in that state. The firms in this region will be either unattacked non-trademark type or trademark type. The firms in the fifth region will all have trademarks. The unattacked non-trademark firms that become large enough to enter the fifth region will get a trademark right away. The last region, region six consists of very high quality firms which get trademark at birth upon entry. The solid curve in Figure 3 represents the maximum possible size of a trademark firm whereas the dashed curve represents the maximum size for a non-trademark firm in that region. In stationary equilibrium, the amount of new customers that a firm receives will just be equal to lost customers. No firm will choose to increase its size beyond the steady state given by these curves.

To contrast, let's consider the size-quality plane in no-trademark economy. Figure 4 plots the size-quality plane in stationary equilibrium in the economy with no trademarks. In stationary equilibrium the size-quality plane in the no-trademark economy will have three regions. In the first region, as in the trademark economy, a firm's efficiency in product development is so low that it will choose not to enter. Compared to stationary equilibrium of the trademark economy, however, ceteris paribus, first region in the no-trademark economy will be narrower. The firms in the second region will exit when they are attacked. Therefore this region will consist of unattacked firms. In the third region firms will never choose to exit. They will only exit exogenously. The solid curve in the graph represent the maximum size attainable for always unattacked firms in stationary equilibrium. Likewise, in stationary equilibrium no firms will be above the solid curve. The very lucky firm which was not attacked for a long period of time may bring up its size up to this level. The dashed curve represents the maximum attainable size for a firm which has been under attack for a very long period of time. Old enough firms will wander between these curves as their state changes between attacked and unattacked.

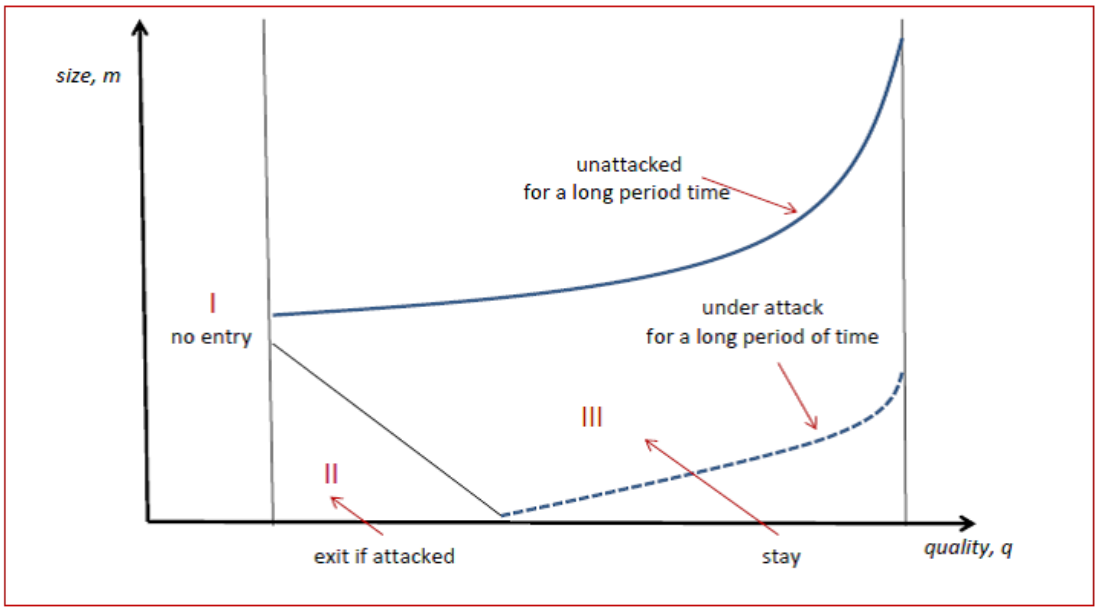


Figure 4: Equilibrium size-quality plane in no-trademark economy

Existence of a trademark system improves the welfare of agents in the economy significantly due to quality and marketing channels. Entering firms do not invest in quality development enough in no-trademark economy. Since they know that they will be attacked frequently and will loose their customers to other firms they neither invest in quality nor they invest in marketing of their products. The Figure 5 plots the equilibrium quality distribution of products in trademark and no-trademark economies. There is a strong contrast between trademark and no-trademark economies in three aspects. Products are of much lower quality in no-trademark economy. Because of lack of efficient marketing, consumers have much little information about products. Because of these two not enough new products are innovated.

Now let's consider how an improvement in information technology will affect the stationary equilibrium in an economy with trademarks. Consider that information technology improves so that the advertising cost function becomes $\Phi'(a) < \Phi(a)$ for all $a \geq 0$. As the Figure 6 shows an improvement in the information technology will affect the stationary equilibrium in a number of ways. First of all, the first and the sixth regions will be wider. A larger fraction of firms intending to enter and have paid the fixed cost κ will actually with product development efficiency lower than a level that is necessary for a profitable operation. With better information technology high quality firms will able attract much more customers easily and their maximum attainable size becomes much larger. Therefore, as seen in the figure, the solid curve becomes much steeper at high quality levels. This also implies that concentration will be more in the better information technology economy. As a result a larger fraction of firms will choose to trademark their products at birth to benefit from better information technology. The region two to five will likely to shrink with the advancement in information technology. The boundary lines between the regions II-V will also be steeper since quality rather than size will be more important in advanced information technology economy. This follows from the fact that a high quality small firm can much easily increase its size in with the help of better information technology.

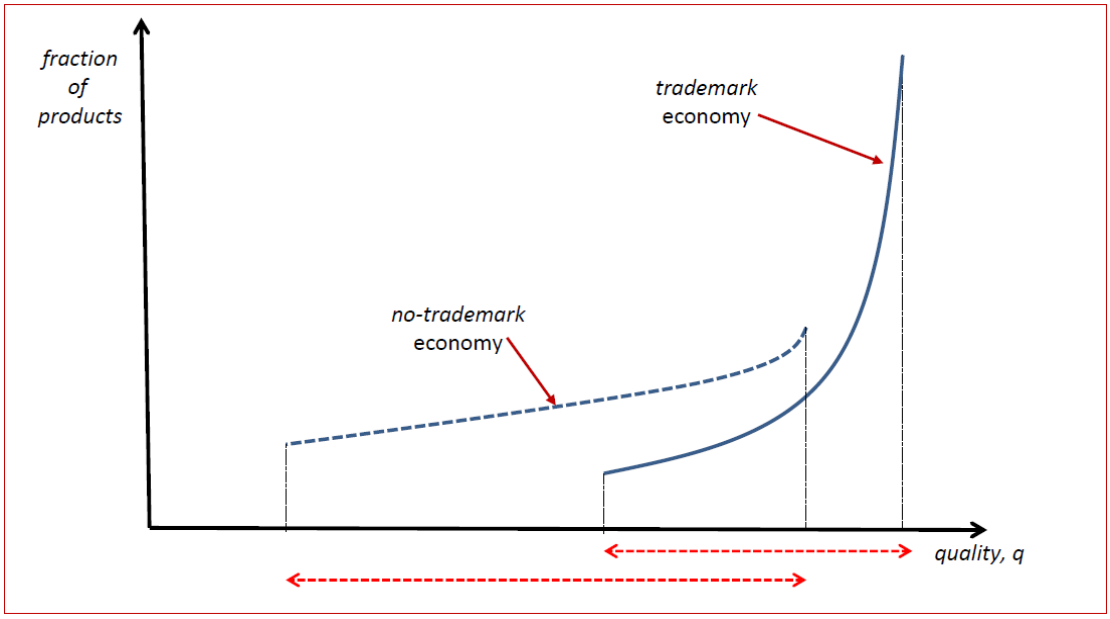


Figure 5: Quality distribution of products across quality levels in trademark and no-trademark economies

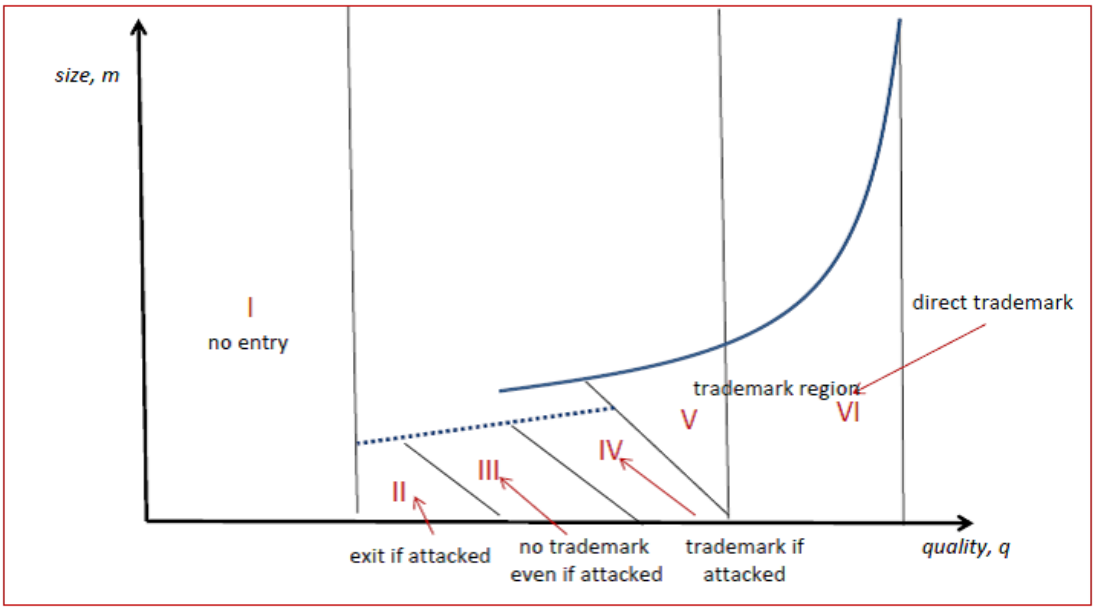


Figure 6: Stationary equilibrium size-quality plane in better information technology economy

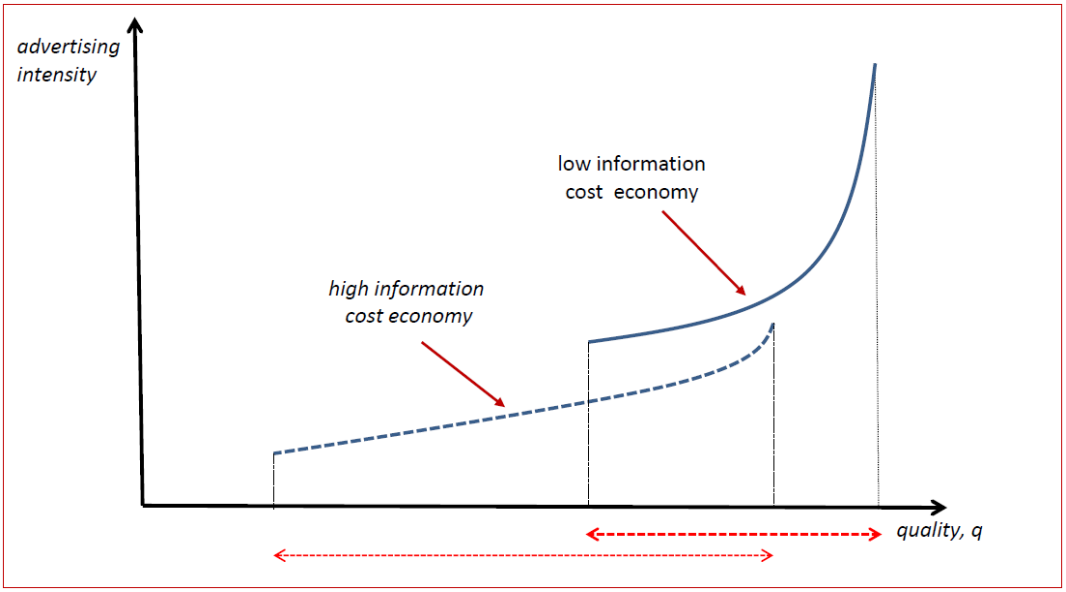


Figure 7: Advertising intensity of firms with different quality in stationary equilibrium in high low information cost economy

Let's call the economy with better information technology a *low information cost* economy for the rest of the analysis. The next figure Figure 7 plots the advertising intensity of firms with different quality levels in the high and low information cost economies. Quality range shifts up in the low information cost economy and becomes narrower. Advertising intensity increases for all firms but the top quality firms increase their advertising intensity drastically.

Since the advertising technology exhibits decreasing returns to scale, optimal advertising intensity of a firm falls as the firm's size increases. Figure 8 exhibits the optimal advertising intensity for different firm size in the high low information cost economies. Given firm size advertising intensity is higher in the low information cost economy.

The next figure, Figure 9 depicts the fraction of products with different quality levels in low and high information cost economies. In low information cost economy a larger fraction of products will be of high quality. The quality dispersion across products will, however, be lower as measured by $q_{\max} - q_{\min}$ and shown in the figure as the length of the arrow between lowest and highest quality products that coexist in stationary equilibrium. This result also suggests a sharp increase in concentration in low information cost economy.

Proposition 10 *Firm size inequality is higher among trademark firms.*

An important aspect of an economy with low information cost is higher firm concentration. This is because the quality distribution shifts right and a high quality firm could attract relatively more customers in a low information cost economy. The illustrative figure, Figure 10 plots the equilibrium concentration of firms across different quality levels in high and low information cost economies.

Proposition 11 *Let $ET(\Gamma, q, m)$ denote expected time it will take to trademark for a firm with state (Γ, q, m). $ET(\Gamma, q, m)$ is decreasing in q and m. Furthermore, $ET(NT, q, m) \leq ET(NU, q, m)$.*

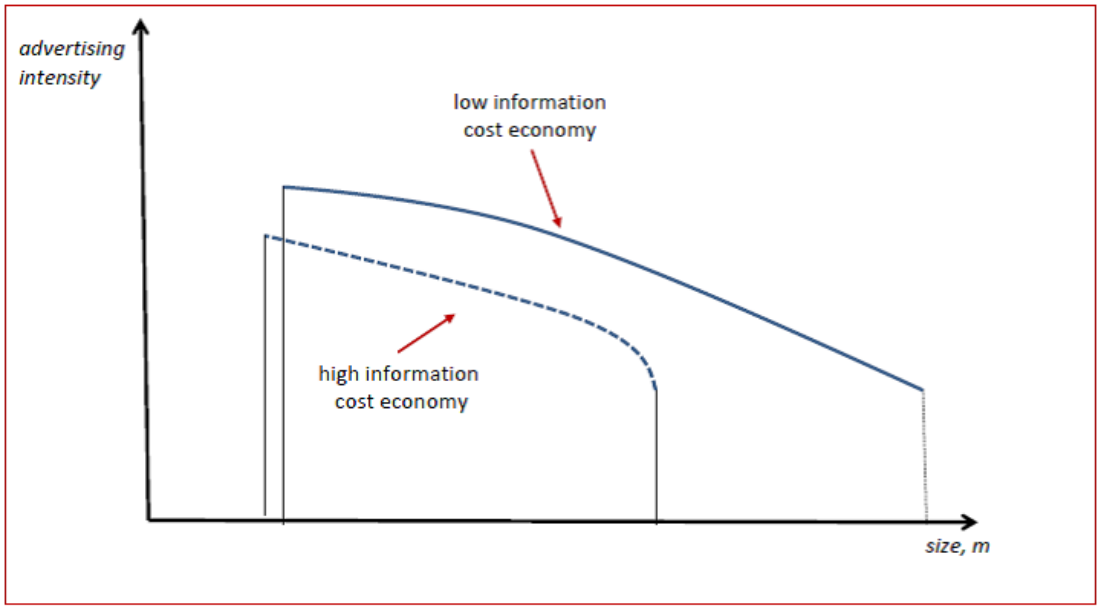


Figure 8: Advertising intensity of firms with different sizes in stationary equilibrium in high low information cost economy

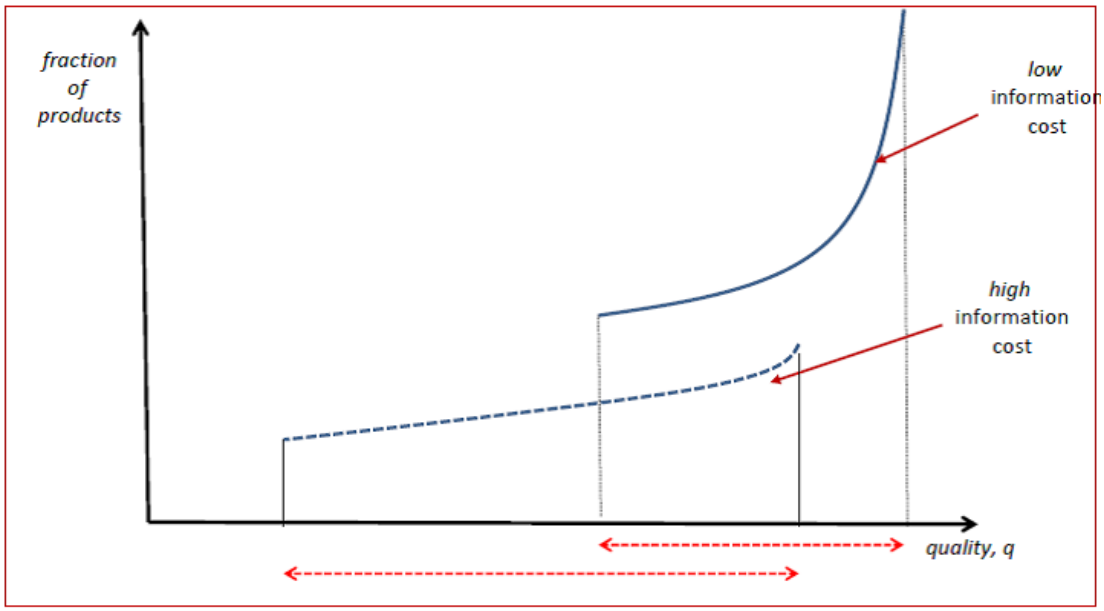


Figure 9: Fraction of products with different quality levels in low and high information cost economies

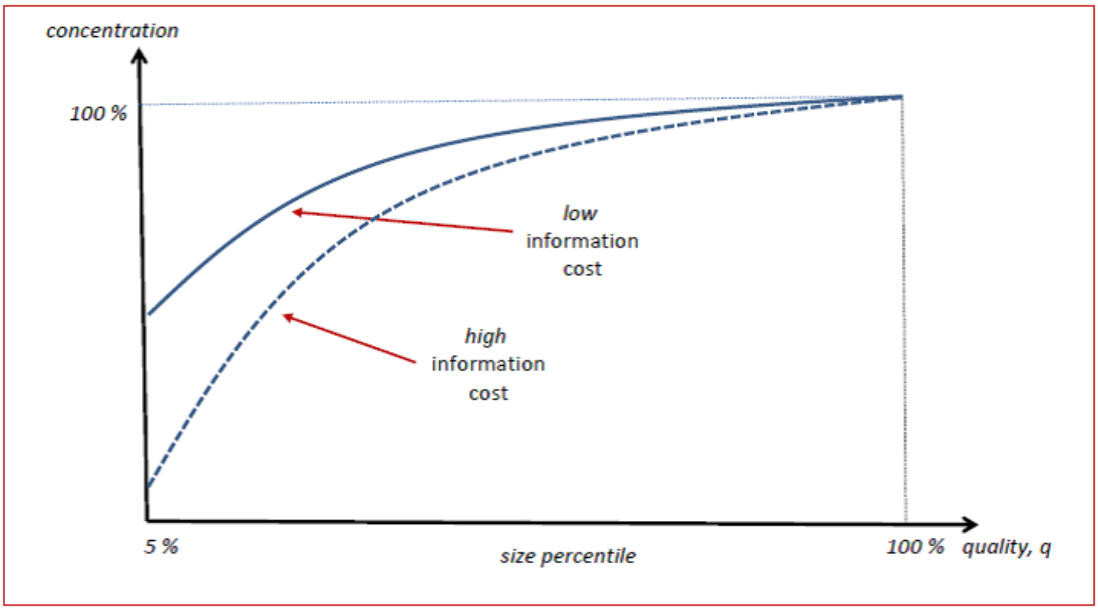


Figure 10: Equilibrium industry concentration in low and high information cost economies

When does a firm trademark its product? The geography of trademarking figure helps us understand the timing of trademarking across firms. The firms in the last region (region 6) have high enough quality that they choose to trademark their product at birth. The firms in region 5 trademark their product when they get large enough. However, the firms in region 4 trademark their products when they are attacked. Due to fact that some of the firms are forced to get a trademark when they are attacked, if relative size of a firm before, at the time of, and after receiving a trademark is traced, a check-shape evolution will be observed. Figure 11 plots the evolution of firm size around the trademark data in high and low information cost economies. The drop in relative size of the firm at the date of trademarking is because for some of the firms the trademarking date coincides with a first-time attack date where the firm looses some of its existing customers (region IV in Figure 3). Notice that the drop is smaller in the low information cost economy because among newly trademarking firms a lower fraction are those types that are recently attacked. This is because as discussed earlier the region IV is smaller in low information cost economy.

4.2 Simulation algorithm

We outline the simulation algorithm used to generate the comparative statics results. The baseline advertising technology, $\Phi(a) = \chi a^{\varphi}$, generates positive advertising for any $\varphi > 1$. To see this claim, note that Assumption 1(iii) is satisfied because $\lim_{a \to 0} \Phi'(a) = \Phi'(0) = \chi \varphi a^{\varphi - 1} = 0$ for $\varphi > 1$. Therefore, the marginal cost of advertising at $a = 0$, represented by the second term in the l.h.s. of (41), is zero. What remains to be shown is that the marginal return to advertising represented by the first term in the l.h.s. of (41) is positive, and thus exceeds the marginal cost at $a = 0$. Firms charge monopoly price and this yields positive revenue per ad for a firm, because, as long as advertising is not free, there is always a positive mass of consumers

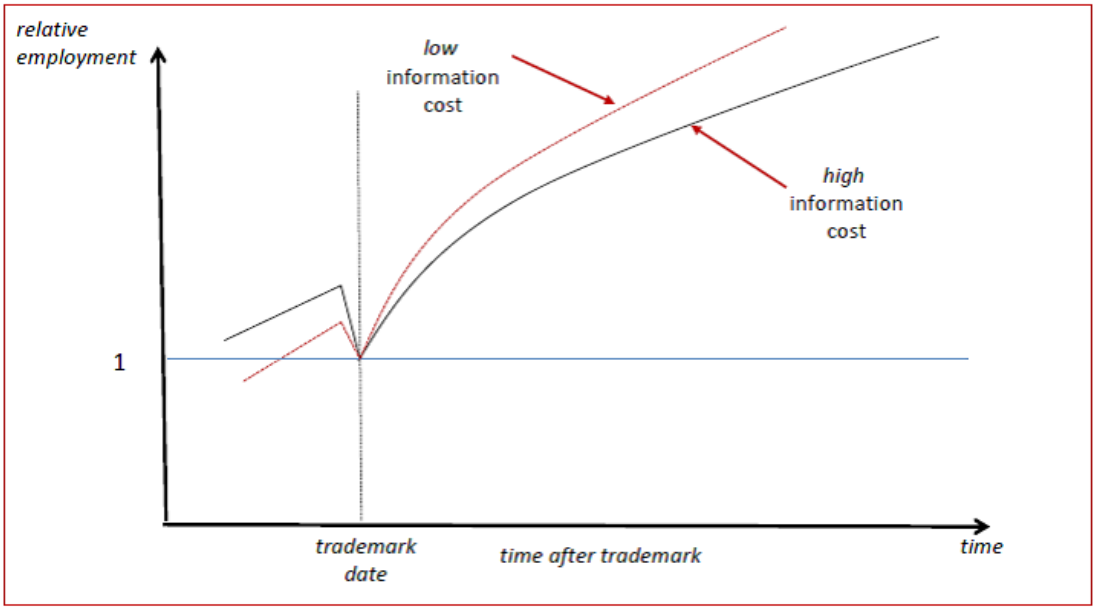


Figure 11: Evolution of firm size (relative employment) around the trademark date in high and low information cost economy

whose only ad is the ad they received from the firm. Formally, the revenue per ad for a type (q, m) firm when it charges its monopoly price $p(q, m)$ is

$$\omega^{\Gamma *}(q, m)(p^m - c + R^*) \geq \Omega^*(0)(p - c + R^*) > \Omega^*(0)(p - c),$$

where the first inequality follows from the fact that $\omega^*(q, m) \geq \Omega^*(0)$, and the second from $R^* \geq 0$. Consequently, there is positive advertising as long as the fixed cost is not too high and the industry is not empty, i.e. the entry cost is not too high. For the industry to be non-empty, there must exist some firm types which do not exit and the expected value of entry must not be lower than the entry cost. The conditions for a non-empty industry are checked as part of the simulation algorithm below.

The algorithm is based on Theorem 1 and is similar to value function iteration algorithms. The simulation setup requires a grid of firm types. We partition the support of the quality level distribution $U[1, 5]$ into a discrete set $\{q_1, ..., q_n\}$, where $q_1 = 1$ and $q_n = 5$. We also use a grid of firm size levels $\{m_1, ..., m_k\}$, where $m_1 = 0$ and $m_k = \overline{m}$, where $\overline{m}$ is taken sufficiently large to contain the upper bound on firm size. The algorithm has the following steps:

1. Start with initial guesses for the trademark status function, $\Gamma(q, m)$, the value function of firms, $V^\Gamma(q, m^o)$, the surplus function, $W^\Gamma(q, m^o)$, the measure of firms, $\mu^\Gamma(q, m^o)$, and the cumulative distribution of ads, $Q^\Gamma(q, m^o)$. We look for fixed points for these functions by simultaneously iterating on them.[8] We chose

[8] There are two alternative methods of iteration on these functions. One method is to construct separate loops of iteration for each function. The second method is simultaneous iteration of all or a subset of the functions under one single loop. The first method is more robust, but computationally slower. The second method is faster with potential problems of robustness. To deal with the robustness, small step sizes for updating functions may be used. We tried both methods, and given the dimensionality of the setup, we found that using the second method with small step sizes is easier for the model considered.

$V^{\Gamma}(q,m^o) \equiv V \geq 0$, $W^{\Gamma}(q,m^o) \equiv 0$, $W^{\varnothing} = 0$, and $\Delta\mu^{\Gamma}(q,m^o) = b \geq 0$ such that $\sum_{\Gamma}\sum_{m^o}\sum_{q}\Delta\mu^{\Gamma}(q,m^o) = \sum_{\Gamma}\sum_{m^o}\sum_{q} b = 1$, implying that $b = \frac{1}{nk}Q^{\Gamma}(q,m^o)$ is also set to be the $c.d.f.$ of a two-dimensional discrete uniform distribution over $\{q_1,...,q_l\} \times \{m_1,...,m_k\}$. Given $W^{\Gamma}(q,m^o)$ and $Q^{\Gamma}(q,m^o)$ compute the individual demand functions as

$$\begin{aligned} z^{\Gamma}(q,m^o) &= \sum_{i=0}^{T} \frac{e^{-A}A^i}{i!} Q^{\Gamma^i}(q,m^o), \\ \omega^{\Gamma}(q,m^o) &= Q_s^{\Gamma}(q,m^o) \sum_{i=0}^{T} \frac{e^{-A}A^i}{i!} Q^{\Gamma^i}(q,m^o) \end{aligned} \tag{44}$$

where $Q_s^{\Gamma}(q,m^o)$ is the probability that among all the sales that take place, the firm type that makes a randomly selected sale is not preferred to firm type (q,m^o). Since we do not have $Q_s^{\Gamma}(q,m^o)$ available initially, we start by assuming that $Q_s^{\Gamma}(q,m^o) = Q^{\Gamma}(q,m^o)$, which will be updated later in further iterations as a computed $Q_s^{\Gamma}(q,m^o)$ becomes available. The upper limit T in the summations (44) is set to a large enough number (we used $T = 20$) and the term $\frac{e^{-A}A^i}{i!}$ follows from a Poisson approximation to $\Psi(i)$.

2a. Compute $\frac{\Delta V^{\Gamma}(q,m_j^o)}{\Delta m_j^o} = \frac{V^{\Gamma}(q,m_j^o) - V^{\Gamma}(q,m_{j-1}^o)}{m_j^o - m_{j-1}^o}$ over a grid for q_{kn} and m_n^o. Solve the discrete-space version of the corresponding first order conditions (41), and find the optimal advertising levels $a_{i,j}$ for each firm type $k \times n$,given V and Q using,

$$a_{ij} = \left[\frac{\omega^{\Gamma}(q_{ij},m_i^o)\left[(p_{ij}-c_j) + \beta\sum_{\Gamma o}\frac{\Delta V^{\Gamma o}(m^*,c_k')}{\Delta m^*}\right]}{\varphi\chi} \right]^{\frac{1}{\varphi-1}}. \tag{45}$$

where $m^* = m_i^o(1-\lambda)z^{\Gamma}(q_{ij},m_i^o) + a_{ij}\omega^{\Gamma}(q_{ij},m_i^o)$

2b. Compute period sales and profits for all firm types

$$m_{ij}^* = m_i^o(1-\lambda)z^{\Gamma}(q_{ij},m_i^o) \ + a_{ij}\omega^{\Gamma}(q_{ij},m_i^o) \ ,$$

$$\Pi^{\Gamma}(q_{ij},m_i^o) \ = m_{ij}^*(p_{ij}-c_j) - \Phi^{\Gamma}(a_{ij}) - f.$$

Compute total number of ads

$$A = \sum_i \sum_j a_{ij}\Delta\mu(q_{ij},m_i^o) \ .$$

Obtain the updated value function of firms,

$$V^{\Gamma\prime}(q_i,m_j^o) \ = \max\{0, \Pi(q_i,m_j^o) + \beta\sum_{k=1}^{n} V^{\Gamma}(q_i,m_j^*)h(q_k)\}.$$

Compute the exit indicator for all firm types,

$$X^{\Gamma}(q_i,m_j^o) = \begin{cases} 0, & \text{if } \Pi^{\Gamma}(q_i,m_j^o) + \beta\sum_{k=1}^{n} V^{\Gamma}(q_i,m_j^*)h(q_k) \geq 0, \\ 1, & \text{otherwise.} \end{cases}$$

2c. Obtain the updated value functions of consumers,

$$W^{\varnothing\prime} = \frac{1}{(1-\beta(1-\lambda)e^{-A})} \sum_{i=1}^{T} \frac{e^{-A}A^i}{i!} \sum_k W_k \Delta G^i(W_k)$$

$$W^{T\prime}(q_i, m_j) = \max_x \left[u(q_i, x_{ij}) - p_{ij}x_i\right] + \beta(1-\lambda) \tag{46}$$
$$\left\{ \begin{array}{c} e^{-A}\left[X(T, q_i, m_j')\left[(1-\chi)\sum_k W^T(q_i, m_{ik}') + \chi W_{t+1}^{\varnothing}\right]\right] + (1 - X(\Gamma, q_i, m_{ik}'))W_{t+1}^{\varnothing} \\ \sum_{t=1}^{T} \frac{e^{-A}A^t}{t!}\left[X(T, q_i, m_j')\left[\begin{array}{c} (1-\chi)E\left[\int \max\{W, W_{t+1}^T(q, m')\} dZ_{t+1}^i(W)\right] \\ +\chi \int W dZ_{t+1}^i(W) \end{array} \right] + (1 - X(T, q_i, m_j'))W_{t+1}^{\varnothing} \right] \end{array} \right\}.$$

Similarly for $W^{NU\prime}(q_i, m_j)$ and $W^{NT\prime}(q_i, m_j)$

2d. Obtain the implied distribution of ads across firm types $Q^{\Gamma\prime}(q_i, m_j)$ using the optimal number of ads a_{ij} sent by each firm type and the measure of firms $\mu^{\Gamma}(q_i, m_j)$, i.e.

$$Q^{\Gamma\prime}(q_i, m_j) = \frac{1}{A} \sum_{(v,r) \text{ s.t. } W^{\Gamma}(p_{qr}, m_q) \leq W^{\Gamma}(q_i, m_j)} a_{vr} \Delta\mu(q_r, m_v^o).$$

Also, obtain the implied distribution of sales across firm types $Q_s^{\Gamma\prime}(q_i, m_j)$

$$S = \sum_i \sum_j m_{ij}^* \Delta\mu(q_i, m_j) \ ,$$

$$Q_s^{\Gamma\prime}(q_i, m_j) = \frac{1}{S} \sum_{(v,r) \text{ s.t. } W^{\Gamma}(p_{qr}, m_q) \leq W^{\Gamma}(q_i, m_j)} m_{vr}^* \Delta\mu(q_r, m_v^o).$$

3. Calculate the distances

$$\begin{aligned} d(Q, Q') &= \max_{i,j} |Q^{\Gamma}(q_i, m_j) - Q^{\Gamma\prime}(q_i, m_j)|, \\ d(V, V') &= \max_{i,j} |V^{\Gamma}(q_i, m_j) - V^{\Gamma\prime}(q_i, m_j)|, \\ d(W, W') &= \max_{i,j} |W^{\Gamma}(q_i, m_j) - W^{\Gamma\prime}(q_i, m_j)|, \\ d(\Gamma, \Gamma') &= \max_{i,j} |\Gamma(q_i, m_j) - \Gamma'(q_i, m_j)|. \end{aligned}$$

If $d(Q, Q') < \varepsilon$, $d(V, V') < \eta$, $d(W, W') < \sigma$, $d(\Gamma, \Gamma') < \upsilon$ for small $\varepsilon, \eta, \sigma, \nu > 0$, then go to step 4. Otherwise, set $V \equiv \alpha V + (1-\alpha)V'$, $W \equiv \alpha W + (1-\alpha)W'$, and $Q \equiv \alpha Q + (1-\alpha)Q'$ (with $\alpha = 0.1$) and go back to Step 2.

4. Obtain the entry mass M using the discrete version of the free entry condition,

$$M = \sup\{M : \sum_{i=1}^{n} V'(0, c_i)h(c_i) = \kappa\},$$

and compute the exit rule $c(m^o)$ point-wise as

$$c(m_i^o) = \max\{c_j : V'(m_i^o, c_j) > 0 \text{ and } V'(m_i^o, c_{j+1}) \leq 0\}.$$

5. Using M, and $c(m_i^o)$, and $\mu(m_i^o, c_j)$, obtain the updated measure

$$\mu'(m_i^o, c_j) = \sum_{m \leq m_i^o} \sum_{c' \leq \min\{c_j, c^o(m^o)\}} \mu(m, c') + M \sum_{c' \leq \min\{c_j, c^o(0)\}} h(c').$$

6. Calculate the distance

$$d(\mu, \mu') = \max_{(i,j)} |\mu(m_i^o, c_j) - \mu'(m_i^o, c_j)|.$$

If $d(\mu, \mu') < \xi$ for some small $\xi > 0$, then stop. If $M > 0$ and $c(m^o) > c_1$, then equilibrium entails positive entry and exit. Otherwise, set $\mu(m^o, c) = \alpha\mu(m^o, c) + (1 - \alpha)\mu'(m^o, c)$, $V \equiv \alpha V + (1 - \alpha)V'$, $W \equiv \alpha W + (1 - \alpha)W'$, $Q \equiv \alpha Q + (1 - \alpha)Q'$ (with $\alpha = 0.1$) and go back to Step 2.

5 Empirical Evidence

This section offers some evidence on the model's implications and tests some of its key predictions.

5.1 Data

The data on trademarks and firms used in this study come from the matched trademark-firm database constructed by Dinlersoz et al. (2018). The data on trademarks is available from the USPTO Trademark Case Files Dataset (TCFD) — see Graham, Hancock, Marco, and Myers (2016) for the details on TCFD. The TCFD contains information on USPTO trademarks applications and registrations for the period 1870-2015. The information on trademarks includes, but is not limited to, ownership, assignment, prosecution events, classification, and renewal history.

The trademark data is matched with the U.S. Census Bureau's Business Register (BR), which contains administrative data for the universe of non-farm businesses in the United States. Key firm-level variables are employment, payroll, and revenues, as well as geography and industry classification. Analysis of firm trademark activity requires longitudinal data to track firms over time and identify when in their life-cycle such activity occurs. The Longitudinal Business Database (LBD) provides a longitudinally linked version of the BR at the establishment level for the period 1976-2014. The LBD also enables identification of entry and exit of firms and their establishments. The TFCD and BR were merged using name and address matching techniques—for further details on data construction, see Dinlersoz et al. (2018). The trademark-firm dataset allows tracking of key events in a firm's life-cycle: firm birth, first trademark registration, and firm death.

Empirical analysis focuses on the event of first trademark registration by a firm. Trademark registrations are subject to a use requirement; the owner must use the mark on goods or in connection with services in order to establish and maintain trademark rights. While a firm can simply use common law trademark rights by using a mark in commerce, a trademark registration confers benefits beyond common law – most importantly

a national scope and prevention of the infringing goods or services, a key benefit of trademarks emphasized in the model.

Filing an application for a U.S. trademark registration costs a relatively small fee per class.[9] However, this fee does not reflect the underlying costs associated with owning and maintaining a trademark—for instance, the expected legal costs of defending a trademark against infringement can be very large. Not all applications for a trademark registration end up reaching the registration stage. Table 2 in Graham et al. (2016) indicates that as high as 31% of trademark applications are abandoned before registration. An application can become a registered trademark only if the associated good or service is already in commercial use, or if there is a bona fide intent to use the mark in commerce. The "intent-to-use applications" can only be registered after the owner uses the trademark in commerce and provides evidence supporting such use to USPTO. The USPTO determines whether the trademark in consideration is legally protectable and if there is a "likelihood of confusion" with a previously registered trademark. If the applied-for mark is registrable, the USPTO publishes the trademark for a limited opposition period, during which time third parties may file a formal opposition to the registration. However, oppositions rarely arise;less than 3% of published applications are opposed—see Graham et al., 2016. Unopposed applications for trademarks already in use are issued a U.S. trademark registration.[10] This paper's focus is on actual commercial activity associated with a trademark application. Material effects of a trademark application at the firm level are more likely observable when a trademark is actually registered. Therefore, the empirical analysis studies trademark registrations for which the presence of commercial activity is established.

The trademark-firm matched data is supplemented with data on advertising expenditures by firms, available from U.S. Census Bureau's Economic Census and Annual Survey of Manufacturers. The cost of advertising reported by firms does not necessarily fully correspond to the notion of information dissemination costs in the model. Advertising cost in the model includes all efforts by firms to inform consumers about their existence and products, some of which may not be captured by reported advertising expenses. In addition, the role of advertising in the model is purely informative. Some of the reported advertising costs, on the other hand, may also include spending associated with persuasive advertising. Nevertheles, the reported advertising expenditures can be used as a proxy for the model's advertising costs, as long as the reported expenditures across firms and over time reflect underlying changes in informative advertising.

5.2 Analysis

Empirical analysis focuses on the connection between firm level outcomes and the presence of a trademark registration by a firm. The analysis is motivated by key implications of the model.

[9] For most of the time period for which the matched data cover, the per class filing fee ranged from $175 to $375 for a paper filing and $325 to $335 for an electronic filing.

[10] A trademark registration can be renewed as long as the trademark is used on the associated goods or services. The owner must provide proof of continued use and pay fees six years after registration and at each ten-year renewal event to maintain the trademark. Otherwise, the registration is canceled. While trademark renewal activity is interesting on its own right, the model and empirical analysis do not study the trademark renewal process.

5.2.1 The effects of trademark registration: an event study

A key role of the trademarks in the model is to protect a firm's customer base from being eroded by infringement and imitation by other firms. This protection can help the firm maintain its customer base and expand it after the registration, compared with firms with no trademarks. However, there is also a selection effect associated with trademark registrations, as firms with higher quality of products benefit more from trademark protection. To assess the effects associated with trademark registration on firm growth, an event study is conducted where firms with trademark registration are matched with a control group to account for selection using observed firm characteristics.[11] Each firm with a trademark registration is matched with a control firm that has no trademark registration, but has similar employment, same age, same multi-unit status and same industry code in the year of the trademark registration as the treated firm.[12] The firms with trademark registration and their matched controls are then followed before and after the year of trademark registration to explore the evolution of firm employment.

Figure 12 shows the time profile for the average employment of firms with a trademark registration and their matched controls. For this figure, only the firms that register their first trademark at age one or later are considered in order to observe the path for average employment before registration. The analysis for firms that register their first trademark in their birth year is done separately—see Figure 13.

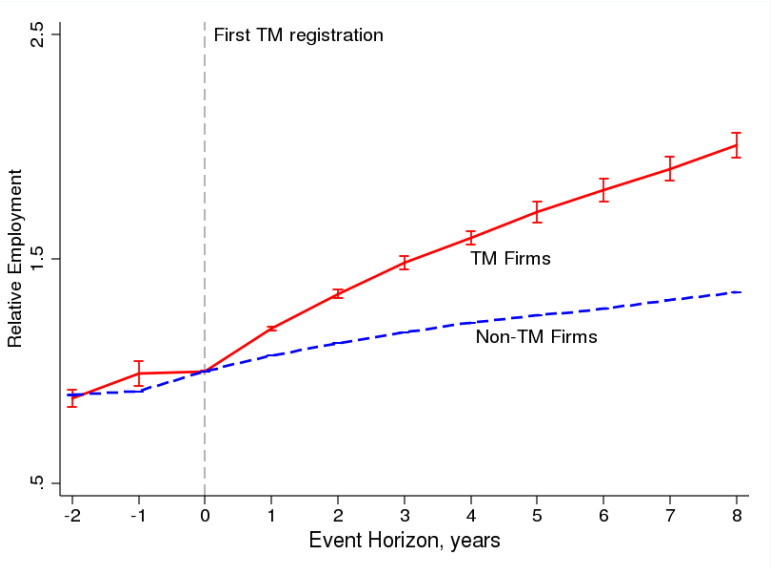


Figure 12: Evolution of firm employment before and after first trademark registration — firms with a trademark registration and their matched controls

The patterns in Figure 12 indicate that firms with a trademark registration exhibit similar trends in

[11] Selection may also result from unobserved characteristics of firms, or from observed characteristics that are not in the data available to the econometrician. For instance, product quality, which is unobserved, is a key driver of selection in the model.

[12] The detailed match criteria used are as follows: the same 4-digit NAICS industry for firms of size > 10 employees, the same 6-digit NAICS industry for firms of size <= 10 employees, same firm age, employment in the same size bin (110 different firm size bins were used), the same year as the registration year, and the same multi-unit status.

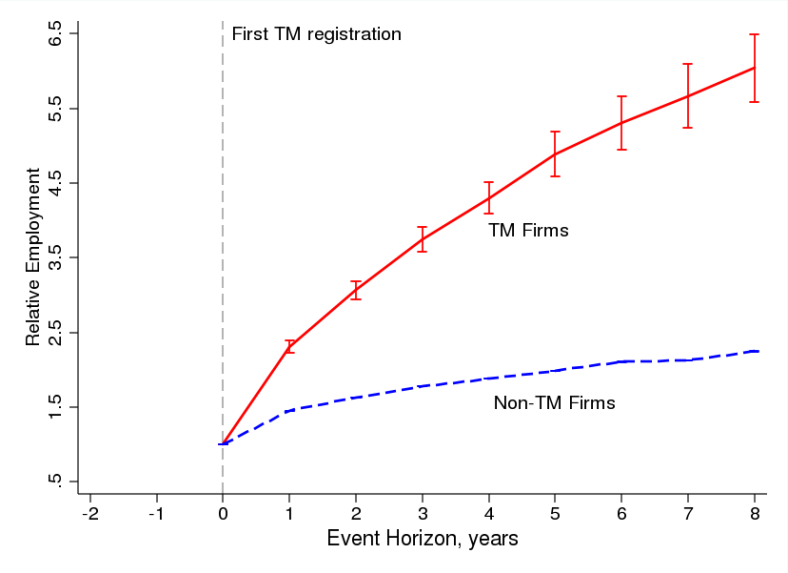


Figure 13: Evolution of firm employment before and after first trademark registration — firms with a trademark registration in their first year and their matched controls

average employment compared to the control group before trademark registration—indicating that matching strategy does a decent job of accounting for pre-registration trends. However, trends start to diverge after registration, as firms with a registration experience much higher growth in average employment.[13] A similar pattern is observed for firms that register a trademark in their first year, as shown in Figure 13—for these firms, there is no pre-registration period by definition. The patterns in Figures 12 and 13 suggest that firms tend to grow more after trademark registration, after selection based on observed characteristics is controlled for.

5.2.2 Trademark registration and firm survival

The model implies that firms with a registered trademark are less likely to exit (fail). This connection between firm survival and trademark registration arises due to two effects. One is selection: firms that take trademarks are of higher quality and hence they are more likely to survive and grow. The second effect is associated with the protection provided by trademarks. Once a firm registers a trademark, its customers cannot be diverted by other firms, preventing the erosion of the firm's customer base and increasing the likelihood of survival.

The model's implication on firm survival and trademark registration finds support in the data. Table 1 shows the exit rates for firms that register a trademark during their life-cycles versus those that do not, for all firms born during the period 1982-2012.[14] The likelihood of exit for firms with a trademark registration is nearly half of that for other firms, and the two rates are statistically different from each other.

[13] Note that selection due to firm failure over time operates alongside with the growth of surviving firms to generate the trends in Figures 12 and 13.

[14] A firm exit occurs in the Longitudinal Business Database (LBD) if all establishments of the firm cease to exist as entities with payroll.

Firm type:	Annual exit rate
Firms with no trademark registration	0.1106
	(0.0000)
Firms with a trademark registration	0.0544
	(0.0001)

Notes: Standard errors in parentheses.

Table 1. Exit probability by firm type

Part of the large difference in Table 1 is obviously driven by selection. To control for selection based on observables, a linear probability model is estimated where firm size (employment), age, and industry are controlled for, and the presence of trademark registration for a firm is indicated by a dummy variable that takes on a value of one if the firm ever registers a trademark during its life-span—see Table 2. The estimated coefficient for this dummy indicates that, after controlling for observables, a firm with a trademark registration is about 3.5 percentage points less likely to exit—a reduction of 32 percent from the average exit rate of firms with no trademark registration (11%, see Table 1). The results are similar when a probit or logit model is considered instead of a linear probability model.

	Annual exit indicator
Firm has a registered trademark	-0.0347^{***}
	(0.0001)
Employment (ln)	-0.0218^{***}
	(0.0000)

Notes: Standard errors clustered by industry in parentheses.

Regression includes firm age, year and 6-digit NAICS industry dummies.

Table 2. Likelihood of firm exit - Linear Probability Model

5.2.3 Initial firm size and the propensity of trademark registration

A prediction of the model is that firms with higher quality products are more likely to obtain a trademark, and they do so earlier in their life-cycles. These predictions are tested using initial size of a firm as a proxy for the quality of its products, which is unobserved. In the model, initial size of a firm is positively associated with the underlying firm-level quality.

Table 3 contains the results from three different approaches to test the connection between firm product quality and trademark registration likelihood. The first column shows the results from a regression analysis where the dependent variable is the number of years to first trademark registration (TM_DURATION) for all firms born between 1982 and 2012 that register a trademark during their life-cycle. The main independent variable is the initial firm employment—firm size in the year of birth. The controls include birth year dummies, a multi-unit indicator, and 6-digit NAICS industry dummies. The negative and statistically significant

coefficient associated with firm size indicates that firms that are born larger (i.e. of higher quality from the model's perspective) tend to register a trademark earlier, conditional on registering a trademark.

Columns two and three of Table 3 perform a similar exercise, but this time the dependent variable is the indicator of trademark registration within 2 years of birth (TM_LIKELIHOOD). Column two uses a linear probability model, and column three implements a Probit model. Both approaches indicate a statistically significant positive connection between initial firm size and the likelihood of trademark registration within two years of birth, conditional on registering a trademark. The Linear Probability Model (OLS) estimates suggest that a 1% increase in initial employment is associated with nearly 3 percentage points increase in the likelihood of a first trademark registration within 2 years of birth. Overall, Table 3 suggests that among firms that register a trademark, a larger size upon birth, which can be interpreted as higher quality, is associated with a higher likelihood of trademark registration soon after birth, or alternatively, a shorter duration between birth and first trademark registration.

	TM_DURATION	TM_LIKELIHOOD	
	OLS	Probit	OLS
Initial employment (ln)	-0.1604^{***}	0.0810^{***}	0.0287^{***}
	(0.0102)	(0.0034)	(0.0012)
N	400,000	400,000	400,000
R^2	0.21		0.12

Notes: Standard errors clustered by industry in parentheses. Models include year and 6-digit NAICS industry dummies. TM_DURATION is the number of years from firm birth to first trademark registration, conditional on registering a trademark. TM_LIKELIHOOD is an indicator of first trademark registration within 2 years of firm birth. N is rounded to prevent disclsoure of confidential information.

Table 3. Initial firm size and the duration until first trademark registration

5.2.4 Advertising and trademark registration

The model implies a positive connection between advertising expenditures, product quality, and the presence of a trademark. Firms with a higher quality have higher returns to advertising, as their ads are more likely to turn into sales. In addition, the marginal cost of advertising is lower at any level of advertising for firms that have trademarks, by assumption. Because firms with trademarks have their customer bases protected from stealing by other firms, marginal return to advertising is also larger for such firms. Therefore, firms with higher quality products and trademarks are expected to spend more on advertising.

Table 4 shows advertising intensity, defined as advertising expenditures as a percentage of value of shipments, for manufacturing firms that were born and observed in economic censuses during the period 1982-2012 (every five years). Firms are divided into two mutually exclusive groups: those that have a registered trademark during that period versus those that do not. Advertising intensity for firms with a trademark registration

are investigated both during the period before the first trademark registration, and the period after. Average advertising intensity for firms without a trademark registration is about 0.17%. In contrast, for firms with a registered trademark advertising intensity is 0.31%, nearly double the intensity of firms with no registration. Furthermore, advertising intensity increases from 0.22% to 0.40% after trademark registration for firms that register a trademark. Note also that the pre-registration advertising intensity for firms with a trademark exceeds that for firms with no registration. This observation is consistent with selection that arises in the model: firms that register trademarks tend to have higher quality products and are larger, implying that even before registration they advertise more compared with the firms that never register a trademark.

Firm type:	Mean advertising intensity (%)		
	Entire Firm Life-Cycle	Pre-Registration	Post-registration
Firms with no trademark registration	0.1732	–	–
	(0.0004)		
Firms with a trademark registration	0.3117	0.2254	0.4032
	(0.0022)	(0.0027)	(0.0034)

Notes: Standard errors in parentheses. Advertising intensity is the ratio of advertising costs to total value of shipments expressed as a percentage. The sample includes manufacturing firms in census years 1982-2012.

Table 4. Mean advertising intensity by firm type

The elasticity of advertising expenditures with respect to firm size is also strongly related to whether a firm has a registered trademark. For manufacturing firms that are observed in economic censuses 1982-2012, the following regression was run:

$$\begin{aligned}\ln(a_{it}+\sqrt{a_{it}^2+1}) &= \alpha_N+\beta_N \ln size_{it}+\alpha_{PRE}D_{PRE}+\beta_{PRE}D_{PRE}\ln size_{it}\\ &\quad +\beta_{POST}D_{POST}\ln size_{it}+\lambda_i+\tau_t+\sum_j \beta_j\gamma_{it}^j+\varepsilon_{it},\end{aligned}$$

where a_{it} is the cost of advertising as reported by the firms, $size_{it}$ is either employment or value of shipments, D_{PRE} is a dummy variable that indicates pre-registration period for a firm, D_{POST} is a dummy variable that indicates post-registration period, λ_i is a firm fixed effect, τ_i is a year fixed effect and γ_{it}^j is a dummy variable for firm age j at time t. The inverse hyperbolic sine transformation of a_{it} is used to account for cases when a firm reports zero advertising. The elasticity of advertising expenditures with respect to firm size is given approximately by β_N for firms with no trademark registration. Similarly, the coefficients β_{PRE} and β_{POST} measure the additional elasticity associated with firms with a trademark registration for the periods before and after registration, respectively. In other words, the elasticity of advertising expenditures for firms with a trademark registration is $\beta_N+\beta_{PRE}$ for the period before registration, and $\beta_N+\beta_{POST}$ for the period following registration (including the year of registration).

The results obtained from estimating the above regression are in Table 5. Regardless of how size is measured (employment or value of shipments), the estimated elasticity of advertising expenditures is higher for firms that have a trademark registration, both for the periods before and after registration. In particular, a

1% increase in value of shipments is associated with a 0.26% increase in advertising expenditures for firms with no trademark registration. The estimated elasticity is 0.37% ($= 0.21\% + 0.16\%$) for firms with a registration for the period before the registration. The estimated elasticity is even higher, 0.52% ($= 0.21\% + 0.31\%$) for firms with a registration for the period after the registration. Similar differences in the estimated elasticities are observed when employment is used as the measure of firm size. The patterns in Table 5 are consistent with the model in the sense that advertising expenditures increase with firm size, and does so more steeply for firms with trademark registration, both before and after registration.

	$\ln a_{it}$ (log advertising cost)	
size measure :	Value of Shipments	Employment
$\ln size_{it}$	0.2627^{***}	0.2174^{***}
	(0.0021)	(0.0021)
$D_{PRE} \ln size_{it}$	0.2087^{***}	0.1639^{***}
	(0.0111)	(0.0107)
$D_{POST} \ln size_{it}$	0.3883^{***}	0.3197^{***}
	(0.0120)	(0.0094)
N	$1,030,000$	$1,030,000$
R^2	0.54	0.53

Notes: Standard errors clustered by firm in parentheses. Regressions include firm fixed effects, firm age dummies, and year dummies. N is rounded to prevent disclosure of confidential information.

Table 5. The relationship between advertising expenditures and firm size

5.2.5 Trademarks and industry concentration

Table 6 investigates the connection between the intensity of trademark presence in an industry and industry concentration using regression analysis. For each year in the period 1982-2012, the dependent variable, industry concentration, is measured by the (log) Hirschman-Herfindahl Index (HHI) based on employment at the 6-digit NAICS industry level. The main independent variable, trademark intensity, is measured alternatively by the number of registered trademarks per firm and the total number of registered trademarks. The regressions control for average trademark age (as measured by the number of years from registration), the number of firms in the industry, and the average firm age within an industry, in addition to the industry fixed effects and year dummies.

In all specifications in Table 6, a positive and statistically significant association emerges between industry concentration and trademark intensity, whether it is measured by the number of trademarks per firm or the cumulative number of trademarks in the industry, though the former has a much stronger association. Note also that the average age of trademarks is also positively associated with industry concentration, while the

number of firms and average firm age are negatively associated with concentration. The positive relationship between average trademark age and industry concentration is consistent with first-mover advantages, whereas the negative association between average firm age and industry concentration may indicate the effects of new firm entry in older industries. The negative connection between the number of firms and industry concentration is in part driven by the definition of HHI—an increase in the number of firms with an equal share of industry employment for each firm implies lower concentration.

	$\ln HHI_{it}$ (industry concentration of employment)			
	I	II	III	IV
trademarks per firm (ln)	0.0850^{***}		0.0814^{***}	
	(0.0074)		(0.0073)	
total trademarks (ln)		0.0199^{***}		0.0278^{***}
		(0.0021)		(0.0021)
average trademark age (ln)			0.0123^{***}	0.0122^{***}
			(0.0017)	(0.0018)
number of firms (ln)			-0.0354^{***}	-0.0600^{***}
			(0.0031)	(0.0037)
average firm age (ln)			-0.0418^{***}	-0.0460^{***}
			(0.0075)	(0.0078)
N	32,000	32,000	32,000	32,000
R^2	0.12	0.05	0.16	0.13

Notes: Standard errors clustered by industry in parentheses. Regressions include 6-digit NAICS industry and year fixed effects. N is rounded to prevent disclosure of confidential information.

Table 6. The relationship between trademark intensity and industry concentration

6 Conclusion

A model of trademarks and firm dynamics is developed to study the aggregate implications of trademark protection in the U.S. economy. The model highlights the key role of trademarks in the provision of quality by reducing informational costs between firms and consumers, and protecting the erosion of a firm's customer base through imitation and infringement. The model generates endogenous quality and price dispersion. There is selection associated with trademark registrations. Trademarks are registered by firms with higher quality products and those whose products are infringed or imitated. There are also treatment effects. A firm that registers a trademark is less likely to exit and grows faster, compared to the case with no registration. The model also indicates that firms' marketing activities depend on its product quality and the presence of a trademark. Equilibrium entails a variety of firm types: firms that trademark immediately upon entry,

those that do so later, those that trademark only when an imitation or infringement of their product occurs, and those that never do (even when imitated or infringed). The model also suggests that higher trademark activity increases concentration among firms.

The model's implications are tested using new data on trademark registrations by all employer businesses in the United States. Many implications of the model find support. The model is calibrated using the key features of the U.S. economy and trademark activity. Several experiments are conducted based on the calibrated model. The analysis suggests that a reduction in informational frictions between firms and consumers leads to higher trademark activity and concentration, and a removal of trademark protection in the economy reduces welfare.

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