Issuance Costs, Commitment, and Optimal Debt Dynamics*

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Abstract

The amount of tax benefits that can be extracted from leverage hinges critically upon the presence of issuance costs, and a firm's ability to commit to a conservative debt issuance policy. For each level of issuance costs (including zero), we identify the *global optimal* debt issuance policy. Shareholders can commit to an optimal policy because debtholders can credibly threaten to punish any deviation by forever pricing debt according to a *no-commitment* policy. In contrast, commitment to debt *repurchases* is not credible once realistic equity issuance costs are accounted for, providing one explanation for why firms are unable to issue risk-free debt.

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1 Introduction

A large and growing literature in corporate finance investigates the firm's optimal debt policy in a dynamic setting. Often, the focus has been on: i) the optimal level of debt, and ii) the optimal debt maturity. Much of this literature has studied these questions through the lens of the tradeoff between the cost of financial distress and the benefit of tax deductibility of interest payments. This paper demonstrates that the balance of this tradeoff hinges critically upon both the presence (or absence) of issuance costs, and the firm's ability to commit to a conservative debt issuance policy.

For reasons of tractability, much of the early theoretical literature restricted its focus to models in which the level of outstanding debt was assumed to be held constant over time.[1] Clearly, such a restriction is both empirically counterfactual and theoretically suboptimal, as firms which find their leverage fall over time due to increased profitability can (and do) increase net tax benefits by issuing additional debt. As this literature evolved, it began to investigate optimal *dynamic* capital structure policies. For example, focusing on firms that are subject to issuance costs, Goldstein, Ju and Leland (GJL, 2001) show that management can increase shareholder value by issuing additional debt whenever leverage ratios become sufficiently low, or equivalently, whenever inverse leverage ratios become sufficiently high. In their framework, the optimal debt issuance policy is characterized by a lower bankruptcy boundary and an upper debt-issuance boundary separated by a region of inaction.

More recently, in a *tour de force*, DeMarzo and He (DH, 2018) investigate the optimal debt-issuance policy for the case in which issuance costs are zero. Restricting their attention to equilibria with *no commitment*, they demonstrate that the optimal debt issuance policy is characterized by a "smooth" (i.e., locally deterministic) process ($G_t\, dt$) in which new debt is issued in all states of nature, even if the firm's current leverage is higher than its target leverage. Due to this aggressive issuance policy, DH report that such firms obtain no tax benefits to debt, regardless of the maturity of debt issued.

Although the scopes of the GJL and DH papers do not overlap—being partitioned by whether or not there are issuance costs, and whether or not management can commit to a particular debt policy—we note a crucial difference in their implications: The GJL framework predicts that tax benefits to debt are both positive and increasing as

[1] See, for example, Leland (1994), Leland and Toft (1996).

issuance costs go to zero. However, the DH framework predicts that there are no tax benefits to debt when issuance costs are identically zero. In this paper, we investigate the impact that issuance costs have on optimal debt policy for the cases with and without commitment, and the conditions under which commitment to those debt issuance policies that increase shareholder value is possible.

We begin by generalizing the framework of GJL[2] to finite-maturity, non-callable debt in the presence of issuance costs. We investigate two cases: (i) with and (ii) without commitment. For both cases, we specify the optimal policy in terms of a single state variable (inverse leverage (v)) and four parameters: a default boundary (v_b), a debt issuance boundary (v_u), a debt issue size parameter (γ), and an inverse maturity parameter (ξ). We first confirm the findings of GJL that these policies generate significant tax benefits to debt, and that these tax benefits increase as issuance costs drop. We then demonstrate that, for both cases, the optimal maturity is an increasing function of debt issuance costs. Importantly, for the case with commitment, we demonstrate that the identified policy for each maturity parameter ξ is *globally optimal* in that, compared to all other possible debt issuance policies (v_b, v_u, γ) consistent with limited liability,[3] our solution generates the highest values of equity across *all* values of $v \in (v_b, v_u)$.

We then investigate the limit economy in which debt issuance costs go to zero, focusing on the case with commitment. In this limit, the upper boundary reduces to a reflecting boundary in that only an infinitesimal amount of debt is issued in order to keep firm leverage inside the region of inaction. As the policy with commitment is a global optimum, it necessarily follows that the value of equity in every state of nature is higher than the value of equity associated with the DH policy. As such, we argue that the firm can credibly commit to following this optimal (i.e., conservative) debt issuance policy if debtholders threaten to punish any deviation by forever pricing debt according to the DH no-commitment policy.[4] Hence, our global-optimal policy can be characterized as the subgame perfect equilibrium policy, and hence the policy that a

[2]In contrast, GJL restrict their attention to infinite-maturity debt that the firm is contractually committed to repurchasing (i.e., calling) prior to any new debt issuance.

[3]Policies consistent with limited liability are those that generate positive equity values for all states of nature prior to default.

[4]Note that date-t purchasers of debt (who, in general, are of a different vintage than past or future debt purchasers) are not negatively impacted by the firm being in the no-commitment equilibrium at date-t, as debtholders always pay "fair price" for their claim at the issuance date. Therefore, the threat of debtholders to follow a "grim trigger" policy is credible.

manager would choose if her objective is to maximize shareholder value.

The economic intuition for why the global optimal policy generates higher equity valuations than the no-commitment policy is the following: in the no-commitment case, the optimal policy is identified by a continuum of myopic first order conditions which take the optimal policy of all other states of nature as given. Because these first order conditions are myopic, they do not fully account for the negative externalities that a larger-than-optimal debt issuance in one state of nature imposes on shareholder value in other states of nature, which in turn leads to a policy that issues too much debt compared to the global optimal policy. In contrast, the global optimal policy can be interpreted as being chosen by a social planner/manager who has the power to commit to a holistic debt issuance policy that maximizes shareholder value across *all* states of nature.[5]

Finally, we investigate the possibility that the lower boundary is not a default boundary, but rather a debt-repurchase (and hence, an equity-issuance) boundary. Under this scenario, debt is risk-free, as the debt policy implies that firms will maintain their leverage within a band such that the value of equity will always remain positive, and thus shareholders would never choose to default. We demonstrate that, once calibrated to realistic equity issuance costs, a firm cannot commit to such a policy, because shareholders would find it beneficial to deviate from this policy when the lower (i.e, debt-repurchase) boundary is reached.

Our paper builds upon the quickly evolving literature that examines optimal dynamic capital structure decisions of firms. There are only a few tractable frameworks in this literature, due to the difficulty of pricing the equity and debt claims in an economy in which their current prices depend on the future debt issuance policy of the firm.[6] Recent papers that investigate whether firms continue to issue debt in high leverage states, or whether they choose to repurchase outstanding debt in this situation, include Dangl and Zechner (2016), Admati et al (2018), and Xu (2018). Our paper also adds

[5]The role of time consistency of the manager's debt issuance policy and the debtholders' rational expectations of the manager's future actions is similar to the role of time consistency in other economic settings, such as the reliance on rules rather than discretion in policymakers' decisions (e.g., Kydland and Prescott (1977)), the durable good monopolist's problem (e.g., Coase (1972); Stokey (1981); Kahn (1986)), and the financing decisions of sovereign borrowers (e.g., Eaton and Gersovitz (1981); Bulow and Rogoff (1988)).

[6]See, for example, Fisher, Heinkel and Zechner (1989), Titman and Tsyplakov (2007), Stebulaev (2007), Dangl and Zechner (2016), Benzoni, Garlappi and Goldstein (2019).

to the literature that investigates optimal maturity structure.[7] Most of our models are calibrated to debt and equity issuance costs that are consistent with estimates from the empirical literature.[8] Finally, our findings of positive net tax benefits to debt is consistent with the empirical literature on this subject.[9]

The rest of the paper is as follows. In Section 2, we generalize GJL by investigating the optimal issuance and maturity policies for non-callable debt in the presence of issuance costs. For both cases, (i) with and (ii) without commitment, we demonstrate that this policy is characterized by a lower default boundary and an upper debt-issuance boundary separated by a region of inaction. Moreover, we demonstrate that our policy with commitment is the global optimal policy in that it generates higher equity values compared to any other policy (consistent with limited liability) across all states of nature. In Section 3, we identify the solution for the limiting case of vanishing issuance costs, and argue that, because our solution generates higher equity values than the *no commitment* solution of DH in all states of nature, the firm can credibly commit to this more conservative debt issuance policy. In Section 4, we investigate the possibility that the lower boundary is a debt-repurchase boundary, and show that, when calibrated to reasonable equity issuance costs, such a policy cannot be credibly committed to, implying that risk-free debt cannot be issued by firms. We conclude in Section 5. Proofs and additional special cases are investigated in Appendix A.

2 Optimal Debt Policy with Issuance Costs

In this section, we investigate the optimal debt issuance policy for a firm that is subject to a fixed cost each time it issues debt, the magnitude of which is controlled by the parameter β.[10] This section can be interpreted as a generalization of GJL (who focused on infinite maturity debt that must be repurchased/called prior to any additional debt issuance) to the case of finite maturity, noncallable debt.[11] We investigate two cases:

[7] See, for example, Leland and Toft (1996), Leland (1998), Brunnermeier and Yogo (2009), He and Xiong (2012), Chen, Xu and Yang (2012), Diamond and He (2014), He and Milbradt (2014), Abel (2016), DeMarzo, He and Tourre (2018).

[8] See, for example, Altınkılıç and Hansen (2000), Hennessy and Whited (2007), Titman and Tsyplakov (2007), and Gamba and Triantis (2008).

[9] See, for example, Binsbergen and Graham (2010), Blouin, Core and Guay (2010).

[10] The cost is "fixed" in that it is independent of the size of the debt issuance. However, for reasons of tractability, issuance costs are assumed to be linear in the size of the firm.

[11] More correctly, GJL combine proportional costs with the requirement that all outstanding debt be called prior to new debt issuance. This combination acts like a fixed cost in that optimal debt levels

with and without commitment. In the next section, we demonstrate that there is a well-defined limiting solution for the case $\beta \to 0$.

As in DeMarzo and He (2018), there are two state variables in this economy. The first is EBIT (Y_t), whose risk-neutral dynamics are exogenously specified:

$$\frac{dY}{Y} = \mu\, dt + \sigma\, dB^Q. \tag{1}$$

Assuming the parameter restriction ($r > \mu$), the present value (V_t) of the claim to EBIT can be determined from the risk-neutral expectation:

$$\begin{aligned} V_t &= \mathcal{E}_t^Q \left[\int_t^\infty ds\, e^{-r(s-t)}\, Y_s \right] \\ &= \frac{Y_t}{r-\mu}. \end{aligned} \tag{2}$$

Since the claim to EBIT is proportional to EBIT itself, it inherits the same dynamics:

$$\frac{dV}{V} = \mu\, dt + \sigma\, dB^Q. \tag{3}$$

As required to preclude arbitrage, these dynamics imply that the risk-neutral expected return on the claim to EBIT equals the risk free rate:

$$\frac{dV + Y\, dt}{V} = r\, dt + \sigma\, dB^Q. \tag{4}$$

Due to the linear relation between Y_t and V_t in equation (2), we are free to choose either as the exogenous state variable. We find it more convenient to choose V_t.

The second state variable is the face value of outstanding debt F_t, which is characterized by a constant amortization rate ξ and coupon rate c. The effective average maturity is $\frac{1}{\xi}$. During the interval $(t, t+dt)$, $\xi F_t\, dt$ units of debt mature. Therefore, over this interval, the sum of coupon and principal payments to debtholders is $(c+\xi)F_t\, dt$.

Here we conjecture (and later verify) that the optimal debt issuance policy can be characterized in terms of a single state variable $v_t = V_t/F_t$ that can be interpreted as inverse leverage.[12] There is a lower boundary v_b at which it is optimal for shareholders

jump by a discrete amount at new issuance dates.

[12]Note that V is not the sum of debt and equity values, but rather the claim to EBIT, which includes the claims to taxes, bankruptcy costs, and issuance costs. Still, we find it convenient to refer to $v = \frac{V}{F}$ as inverse leverage.

to default. We define the date of this event as τ_b. Therefore, we have the relation $v_b = V_{\tau_b}/F_{\tau_b}$. There is also an upper boundary v_u at which it is optimal to issue additional debt so that total outstanding debt scales by a factor γ (the size of which is chosen optimally). We define the date of this event as τ_u. Thus, at time τ_u, the level of outstanding debt jumps from F_{τ_u} to γF_{τ_u} ($\gamma > 1$), and shareholders pay a fixed cost of $\beta V_{\tau_u} = \beta v_u F_{\tau_u}$ (which follows from the relation $v_u = V_{\tau_u}/F_{\tau_u}$). Finally, it is optimal for shareholders to take no action when inverse leverage falls in the range $v \in (v_b, v_u)$.

There are two equivalent approaches that can be used to solve for asset prices. The first approach (which was used by GJL) takes advantage of the recursive nature of the framework, in which the firm at each debt issuance date "looks the same," except for some scaling factor. The second approach determines asset prices as solutions to ordinary differential equations (ODE's), subject to boundary conditions. The first approach provides more economic insights, and moreover naturally identifies the boundary conditions that are required inputs for the second method. The second approach, however, is more direct, so we use it here.

In order to price debt and equity, it is convenient to define period-0 as an interval of time that includes the present moment, and ends the first time either the lower or upper boundary is reached. We refer to the date that period-0 ends as $\tau = \min(\tau_b, \tau_u)$. Because there are no debt issuances (or repurchases) within period-0, it follows that the dynamics of F_t are driven only by its amortization rate:

$$dF_t = -\xi F_t \, dt, \tag{5}$$

with solution

$$F_s = F_t e^{-\xi(s-t)} \qquad \forall s \in (t, \tau). \tag{6}$$

Given the dynamics of V_t and F_t, it follows from Ito's lemma that the dynamics of $v_t = V_t/F_t$ follow:

$$\frac{dv_t}{v_t} = (\mu + \xi) \, dt + \sigma \, dB^Q. \tag{7}$$

The cash flows to equity (i.e., dividends), government (i.e., taxes) and outstanding debt (i.e., principal and interest payments) during the interval $(t, t+dt)$ are:

$$\begin{aligned} \text{Equity}: &\quad \left[Y_t - \overline{\pi}(Y_t - cF_t) - (c+\xi)F_t\right] dt \\ \text{Government}: &\quad \overline{\pi}(Y_t - cF_t)\, dt \\ \text{Debt}: &\quad (c+\xi)F_t\, dt. \end{aligned} \tag{8}$$

At default the claims are:

$$\begin{aligned} \text{Equity}: &\quad 0 \\ \text{Government}: &\quad (1-\alpha)\overline{\pi}V_b \\ \text{Debt}: &\quad (1-\alpha)(1-\overline{\pi})V_b \\ \text{Bankruptcy Costs}: &\quad \alpha V_b. \end{aligned} \tag{9}$$

2.1 Debt Valuation

Debt valuation can be determined via:

$$\begin{aligned} D(V_t, F_t) &= \mathcal{E}_t^Q \left[\int_t^\tau ds\, e^{-r(s-t)}\, (c+\xi)F_s\right] \\ &+\mathcal{E}_t^Q \left[e^{-r(\tau_u - t)}\, \mathbf{1}_{(\tau_u < \tau_b)}\, \left(\frac{1}{\gamma}\right) D(v_u F_{\tau_u}, \gamma F_{\tau_u})\right] + \mathcal{E}_t^Q \left[e^{-r(\tau_b - t)}\, \mathbf{1}_{(\tau_b < \tau_u)}\, (1-\alpha)(1-\overline{\pi})v_b F_{\tau_b}\right]. \end{aligned} \tag{10}$$

The first term captures the present value of cash flows prior to the firm reaching either the upper or lower boundary. The second term captures the fact that when the upper boundary is reached and the face value of debt jumps from F_{τ_u} to γF_{τ_u}, old bondholders have a claim to $\left(\frac{1}{\gamma}\right)$ of the total debt claim, as it is assumed that all debt is issued pari-passu. The third term captures recovery conditional upon default.

Equation (10) implies that $\left(e^{-rt}D(V_t, F_t) + \int_0^t ds\, e^{-rs} F_s\, (c+\xi)\right)$ is a Q-martingale, implying that:

$$0 = -rD + \mu V D_V + \frac{\sigma^2}{2} V^2 D_{VV} - \xi F D_F + (c+\xi)F, \tag{11}$$

subject to the boundary conditions

$$\begin{aligned} D(v_b F, F) &= (1-\alpha)(1-\overline{\pi})v_b F \\ D(v_u F, F) &= \left(\frac{1}{\gamma}\right) D(v_u F, \gamma F). \end{aligned} \tag{12}$$

Because both the cash flows to debtholders and state vector dynamics are linear in the state vector, it follows that the value of debt is homogeneous of degree one in the state vector. Thus, we look for a solution of the form:

$$D(V,F) = Fd\left(v = \frac{V}{F}\right). \tag{13}$$

Using the following relations:

$$\begin{aligned} D_V &= d_v \\ D_{VV} &= \left(\frac{1}{F}\right) d_{vv} \\ D_F &= d - v d_v, \end{aligned} \tag{14}$$

we can rewrite the ODE as:

$$0 = \frac{\sigma^2}{2} v^2 d_{vv} + v d_v (\mu + \xi) - d(r+\xi) + (c+\xi), \tag{15}$$

subject to the boundary conditions

$$d(v_b) = (1-\alpha)(1-\overline{\pi})v_b \tag{16}$$

$$d(v_u) = d(v_u/\gamma). \tag{17}$$

The intuition for the second boundary condition is that, in order to preclude an arbitrage opportunity, the price of debt per unit of face value must be the same just prior to (i.e., at date τ_u^-) and just after (i.e., at date τ_u^+) the debt issuance.

Defining the exponents

$$\begin{aligned} \phi &= \left(\frac{1}{\sigma^2}\right)\left[\left(\frac{\sigma^2}{2} - (\mu+\xi)\right) + \sqrt{\left(\frac{\sigma^2}{2} - (\mu+\xi)\right)^2 + 2(r+\xi)\sigma^2}\right] \\ \omega &= \left(\frac{1}{\sigma^2}\right)\left[\left(\frac{\sigma^2}{2} - (\mu+\xi)\right) - \sqrt{\left(\frac{\sigma^2}{2} - (\mu+\xi)\right)^2 + 2(r+\xi)\sigma^2}\right], \end{aligned} \tag{18}$$

the solution for the price of debt per unit face value can be expressed as:

$$d(v) = Mv^{\phi} + Nv^{\omega} + \frac{c+\xi}{r+\xi}, \tag{19}$$

where the constants (M, N) are uniquely determined by the boundary conditions given in equations (16, 17).

2.2 Equity valuation

The value of equity can be determined via the risk-neutral expectation:

$$\begin{aligned} E(V_t, F_t) &= \mathcal{E}_t^Q \left[\int_t^{\tau} ds\, e^{-r(s-t)} \left((1-\overline{\pi}) Y_s - (c(1-\overline{\pi}) + \xi) F_s \right) \right] \\ &+ \mathcal{E}_t^Q \left[e^{-r(\tau_u - t)} \mathbf{1}_{(\tau_u < \tau_b)} \left(E(v_u F_{\tau_u}, \gamma F_{\tau_u}) + D(v_u F_{\tau_u}, \gamma F_{\tau_u}) - D(v_u F_{\tau_u}, F_{\tau_u}) - \beta v_u F_{\tau_u} \right) \right]. \end{aligned} \tag{20}$$

The first term captures the present value of the claim to cash flows (i.e., dividends) throughout period-0, which ends at date $\tau = \min(\tau_b, \tau_u)$. The second term captures the fact that when the upper boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor of γ. In addition, shareholders receive as dividend the value of the new debt issuance (net of issuance costs). Because shareholders recover nothing in the event of default, there is no analogous third term as there was in the debt valuation equation.

Equation (20) implies that $\left(e^{-rt} E(V_t, F_t) + \int_0^t ds\, e^{-rs} \left[(1-\overline{\pi}) Y_s - (c(1-\overline{\pi}) + \xi) F_s \right] \right)$ is a Q-martingale, implying that:

$$0 = -rE + \mu V E_V + \frac{\sigma^2}{2} V^2 E_{VV} - \xi F E_F + V(r-\mu)(1-\overline{\pi}) - F\left(c(1-\overline{\pi}) + \xi\right), \tag{21}$$

where we have used the relation $Y = (r-\mu)V$.

The boundary conditions are:

$$\begin{aligned} E(v_b F, F) &= 0 \\ E(v_u F, F) &= E(v_u F, \gamma F) + D(v_u F, \gamma F) - D(v_u F, F) - \beta v_u F. \end{aligned} \tag{22}$$

The first boundary condition states that the value of equity is zero at the default boundary. The second boundary condition states that the value of equity at the upper boundary equals the sum of: i) the equity value after the debt issuance has taken place (and thus, $F \to \gamma F$), and ii) the value of the debt issuance, net of issuance costs.

As with the debt claim, because both the cash flows to equity, and state vector dynamics, are linear in the state vector, it follows that the claim to equity is homogeneous of degree one in the state vector. Thus, we look for a solution of the form:

$$E(V,F) \;=\; F\, e\!\left(v = \frac{V}{F}\right). \tag{23}$$

Using the relations:

$$\begin{aligned} E_{_V} &= e_{_v} \\ E_{_{VV}} &= \left(\frac{1}{F}\right) e_{_{vv}} \\ E_{_F} &= e - v e_{_v}, \end{aligned} \tag{24}$$

we can rewrite the ODE as:

$$0 \;=\; \frac{\sigma^2}{2} v^2 e_{_{vv}} + v e_{_v}\left(\mu + \xi\right) - e(r+\xi) + v(r-\mu)(1-\overline{\pi}) - \left(c(1-\overline{\pi}) + \xi\right), \tag{25}$$

subject to the boundary conditions

$$e(v_{_b}) \;=\; 0 \tag{26}$$

$$e(v_{_u}) \;=\; \gamma e(v_{_u}/\gamma) + (\gamma - 1) d(v_{_u}) - \beta v_{_u}, \tag{27}$$

where we have used the relation $d(v_{_u}) = d(v_{_u}/\gamma)$ in the last equation.

The solution is:

$$e(v) = A v^{\phi} + B v^{\omega} + (1-\overline{\pi}) v - \frac{\left(c(1-\overline{\pi}) + \xi\right)}{r+\xi}, \tag{28}$$

where the constants (A, B) are uniquely determined from the boundary conditions in equations (26), (27).

2.3 Tax benefits to debt

For a given policy $(v_{_b}, v_{_u}, \gamma, \xi)$, we estimate the tax benefits to debt by comparing the equity values of two firms, both of which "start" at date-0 with no debt (i.e., $F_{_0} = 0$). Hence, the date-0 state vector is $(V_{_0}, F_{_0} = 0)$. The first firm commits to never issuing debt in the future. Therefore, the value of equity is $(1-\overline{\pi})V_{_0}$. The second firm, for a

given (inverse) maturity parameter ξ, commits to issuing debt according to the optimal policy at all future dates. Moreover, at date-0, the firm issues sufficient debt to jump from $F_0 = 0$ to $F_{0^+} = \left(\frac{\gamma}{v_u}\right) V_0$. That is, the firm chooses to immediately jump to the "optimal post-issuance inverse leverage ratio" $\frac{V_0}{F_{0^+}} = \left(\frac{v_u}{\gamma}\right)$. The date-0 value of equity following this policy is:

$$E^*(V_0, F_0 = 0) \;=\; E(V_0, \gamma V_0/v_u) + D(V_0, \gamma V_0/v_u) - \beta V_0. \tag{29}$$

The intuition for this equation is that shareholder value equals the value of equity after the debt issuance, plus the value of the debt issuance (paid out as a dividend) net of issuance costs. Using the scaling feature inherent in equations (13), (23), the tax benefits to debt (TB), expressed as the ratio of equity valuations for the optimal issuance policy case vs. the no debt issuance case is:

$$\begin{aligned} TB &= \frac{E(V_0, \gamma V_0/v_u) + D(V_0, \gamma V_0/v_u) - \beta V_0}{(1-\overline{\pi})V_0} \\ &= \frac{\gamma\, e(v_u/\gamma) + \gamma\, d(v_u/\gamma) - \beta v_u}{(1-\overline{\pi})v_u}. \end{aligned} \tag{30}$$

2.4 Optimal debt policy with commitment

So far, we have determined the price of debt and equity for a *given* set of policy parameters (v_b, v_u, γ, ξ). In order to determine the *optimal* debt issuance and maturity policies for the case *with commitment*, we use the following approach. For a given set of parameters (μ, σ, r, $\overline{\pi}$, c, α, β), and for a given (inverse) maturity parameter ξ, we identify the optimal choices of (v_b, v_u, γ) via the first order conditions (FOC's):

$$\left.\frac{\partial e}{\partial v_b}\right|_{v=v_b} = 0 \tag{31}$$

$$\left.\frac{\partial e}{\partial v_u}\right|_{v=v_u} = 0 \tag{32}$$

$$\left.\frac{\partial e}{\partial \gamma}\right|_{v=v_u} = 0. \tag{33}$$

We then identify the value of ξ which leads to the greatest tax benefits to debt, as defined by equation (30). Hence, for each value of the issuance cost parameter β, we

identify the optimal debt policy parameters $\left(v_b^*(\beta), v_u^*(\beta), \gamma^*(\beta), \xi^*(\beta)\right)$.

Importantly, we emphasize that, although these FOC's have the appearance of being *myopic* in that they are applied only at the location at which action by the manager can be taken, this appearance is deceiving for two reasons. First, these FOC's do not determine the *no-commitment* solution because, in reality, there is not a single set of parameters (v_b, v_u, γ), but rather a set of parameters $(v_{b,i}, v_{u,i}, \gamma_i)$ for *each* period-i, which ends at (random) time τ_i. Now, due to the symmetry of our economy, ultimately, all optimally chosen parameters will equal each other across all periods. However, in the no-commitment equilibrium during period-0, for example, the manager only gets to choose whether or not to issue additional debt at each moment (i.e., whether or not to set $v_{u,0}$ equal to the current inverse leverage value v_t). That is, she has no control over the decision of the locations from $v_{u,i}$ for $i \in (1, \infty)$. Instead, the no-commitment optimal policy is identified by assuming that the manager during period-0 takes the controls $(v_{b,i}, v_{u,i}, \gamma_i)$ for $i \in (1, \infty)$ as exogenously given, and then imposes FOC's to determine the optimal values for $(v_{b,0}, v_{u,0}, \gamma_0)$. Only after these FOC's are taken do we set $(v_{b,0}, v_{u,0}, \gamma_0) = (v_{b,i}, v_{u,i}, \gamma_i)$. In contrast, the optimal policy *with commitment* first sets the period-0 policy parameters equal to the policy parameters for all periods, and then imposes the FOC's, as in equations (31) - (33). We investigate the no-commitment case in the next subsection.

The second reason that the appearance that the FOC's in equations (31) - (33) are myopic is misleading is that, for any choice of v_b, the same optimal values for (v_u, γ) are identified regardless of v_t. That is, the FOC's

$$\begin{aligned} \left.\frac{\partial e}{\partial v_u}\right|_{v=v_{any}, v_b} &= 0 \\ \left.\frac{\partial e}{\partial \gamma}\right|_{v=v_{any}, v_b} &= 0, \end{aligned} \tag{34}$$

lead to the same optimal values for (v_u^*, γ^*) regardless of choice v_{any}. In this sense, these policy choices are *time-consistent*. An intuitive explanation for why this result holds is the following: First, the coefficients (A, B) in equation (28) are uniquely determined by (v_b, v_u, γ), and these policy controls don't show up anywhere else in equation (28).

Second, we can show that, for *any* given choice of v_b, the conditions:

$$\begin{aligned} \frac{\partial A}{\partial v_u} &= 0 \\ \frac{\partial A}{\partial \gamma} &= 0 \end{aligned}$$

lead to the same optimal values for (v_u^*, γ^*) as obtained from equation (32) - (33).[13] This result is the first indication that the FOC's in equations (31) - (33) generate an equity valuation that is globally optimal.

A second indication that points to our equity valuation being globally optimal is to note that the FOC at v_b is equivalent to the so-called *smooth pasting* condition[14]

$$\left.\frac{\partial e}{\partial v}\right|_{v=v_b} = 0. \tag{35}$$

The reason this is important is the following: from the boundary condition in equation (26), we have $e(v_b) = 0$, which follows because equity receives zero in the event of bankruptcy. Therefore, in order for the equity valuation formula to satisfy the limited liability constraint, it is necessary that

$$\left.\frac{\partial e}{\partial v}\right|_{v=v_b} \geq 0. \tag{36}$$

Indeed, if this condition did not hold, then equity values would be negative for values of v slightly above v_b. Given the inequality constraint in equation (36), it is not surprising that the value of v_b^* for the globally optimal solution satisfies the equality constraint (as in equation (35)) rather than just the inequality constraint. Related, it can be shown that the value of v_b^* obtained from equation (35) is the smallest value of v_b consistent with limited liability. Therefore, it naturally follows that our policy generates the highest values of equity for values v_t just above v_b^*, since all other solutions for equity which satisfy limited liability will have an equity valuation of zero there.

[13]The same FOC's hold for the parameter B. This result is intuitive once it is realized that B can be expressed in terms of A and other parameters independent of (v_u, γ) when equation (28) is combined with the boundary condition in equation (26).

[14]It is worth noting that, as we show in Appendix A, the first order condition is not equivalent to the smooth pasting condition at the *upper* boundary. Intuitively, this is because the functional form of the exercise value, and not just the value itself, is impacted by the choice of the location of v_u.

2.5 Optimal debt policy: no commitment case

As noted in the previous subsection, the FOC's expressed in equation (31) - (33) do not correspond to the no-commitment optimal policy, because they imply, for example, that a manager at date-t in period-i who is deciding whether or not to issue additional debt at this time (and hence, deciding whether or not $v_t = v_{u,i}$), in fact has the power to commit to the location of the upper boundary for *all* future periods. In this subsection, we investigate the no-commitment case. Specifically, we set $(v_{b,i}, v_{u,i}, \gamma_i) = (v_b, v_u, \gamma)$ for $i \in (1, \infty)$, which we distinguish, for now, from the period-0 controls $(v_{b,0}, v_{u,0}, \gamma_0)$. For a given value of v_t, and given values for the controls for all periods, the period-0 value of debt is determined from first calculating the value of debt $d(v)$ for future periods:

$$d(v) = Mv^{\phi} + Nv^{\omega} + \frac{c+\xi}{r+\xi}, \tag{37}$$

where the constants (M, N) are uniquely determined from the boundary conditions

$$d(v_b) = (1-\alpha)(1-\overline{\pi})v_b \tag{38}$$

$$d(v_u) = d(v_u/\gamma). \tag{39}$$

We then determine the period-0, date-t value of debt from:

$$d_0(v_t) = M_0 v_t^{\phi} + N_0 v_t^{\omega} + \frac{c+\xi}{r+\xi}, \tag{40}$$

where the constants (M_0, N_0) are uniquely determined from the boundary conditions

$$d_0(v_{0b}) = (1-\alpha)(1-\overline{\pi})v_{0b} \tag{41}$$

$$d_0(v_{0u}) = d(v_{0u}/\gamma_0). \tag{42}$$

Note that the RHS of the equation (42) is expressed in terms of $d(\cdot)$, not $d_0(\cdot)$, which explains the necessity of introducing equation (37). This in turn captures the distinction between the debt policy controls during period-0, and the controls for all future periods.

Analogously, for a given value of v_t, and given values for the controls for all periods, the period-0, date-t value of equity is determined from first calculating the value of equity $e(v)$ for future periods:

$$e(v) = Av^{\phi} + Bv^{\omega} + (1-\overline{\pi})v - \frac{c(1-\overline{\pi})+\xi}{r+\xi}, \tag{43}$$

where the constants (A, B) are uniquely determined from the boundary conditions

$$\begin{aligned} e(v_b) &= 0 \quad (44) \\ e(v_u) &= \gamma e(v_u/\gamma) + \gamma d(v_u/\gamma) - d(v_u) - \beta v_u. \quad (45) \end{aligned}$$

We then determine the period-0, date-t value of equity from:

$$e_0(v_t) = A_0 v_t^{\phi} + B_0 v_t^{\omega} + (1-\overline{\pi})v_t - \frac{c(1-\overline{\pi})+\xi}{r+\xi}, \quad (46)$$

where the constants (A_0, B_0) are uniquely determined from the boundary conditions

$$\begin{aligned} e_0(v_{0b}) &= 0 \quad (47) \\ e_0(v_{0u}) &= \gamma_0 e(v_{0u}/\gamma_0) + \gamma_0 d(v_{0u}/\gamma_0) - d_0(v_{0u}) - \beta v_{0u}. \quad (48) \end{aligned}$$

Note that the RHS of the last equation uses both $d(\cdot)$ and $e(\cdot)$ in addition to $d_0(\cdot)$, justifying the need for identifying the values of debt and equity in future periods in order to determine the value of equity in the current period.

The equations above determine the value of period-0 debt and equity for a *given* set of controls (v_b, v_u, γ), $(v_{b,0}, v_{u,0}, \gamma_0)$. We identify the optimal values for these control under the no-commitment assumption via the set of first order conditions:

$$\begin{aligned} 0 &= \left.\frac{\partial e_0}{\partial v_{0u}}\right|_{(v=v_{0u}, v_u=v_{0u}, v_b=v_{0b}, \gamma=\gamma_0)} \\ 0 &= \left.\frac{\partial e_0}{\partial \gamma_0}\right|_{(v=v_{0u}, v_u=v_{0u}, v_b=v_{0b}, \gamma=\gamma_0)} \\ 0 &= \left.\frac{\partial e_0}{\partial v_{0b}}\right|_{(v=v_{0b}, v_u=v_{0u}, v_b=v_{0b}, \gamma=\gamma_0)}. \quad (49) \end{aligned}$$

2.6 Results for the cases with and without commitment

Here we investigate the model implications for the cases with and without commitment. In Figure 1, we investigate the tax benefits to debt as a function of debt maturity for the special case $\beta = 0.007$. This value for the issuance cost parameter is chosen because it reflects a fractional debt issuance cost of 2% for the case with commitment, consistent

with empirical estimations.[15] That is, β was chosen such that:

$$\begin{aligned} 0.02 &= \frac{\beta v_u F_{\tau_u}}{D(v_u F_{\tau_u}, \gamma F_{\tau_u}) - D(v_u F_{\tau_u}, F_{\tau_u})} \\ &= \frac{\beta v_u}{(\gamma - 1) d(v_u)}. \end{aligned} \tag{50}$$

As this figure demonstrates, for both the cases with and without commitment, there is a well-defined optimal maturity in which tax benefits to debt are optimized.[16] This result can be understood as follows. Issuing short-maturity debt has the advantage that the firm can choose relatively "high leverage" today (which creates a large tax shelter) while assuring that the firm's future debt obligations will be significantly lower. This way, even if future EBIT falls, the firm is more likely to remain solvent. However, short maturity debt has the disadvantage that, if future EBIT value remains steady, the firm will need to issue more debt in order to match the "optimal level" of tax deductions they had in the past. Hence, shorter maturity is associated with an increase in expected debt issuance costs. The existence of an optimal debt maturity for each value of β reflects a tradeoff between these two factors.

In Figure 2, for the special case $\beta = 0.007$, we report the value of equity $e(v)$ and debt $d(v)$ as a function of v for the case with commitment. As is intuitive, the value of debt is quite flat for large values of v (approaching its risk-free value $\left(\frac{c+\xi}{r+\xi}\right)$), but drops rather abruptly toward its recovery value (which is zero in this parametrization) as v approaches the default boundary v_b. In contrast, while equity demonstrates positive convexity (consistent with the smooth pasting condition) for values of v near the default boundary v_b, it is rather linear for larger values of v.

In Figure 3, we plot the *difference* between the values of the case with commitment and without commitment for equity and debt when the debt maturity $1/\xi$ is fixed at the optimal value under commitment. The reason we present the data in this manner is because the difference between the asset prices for these two models is about three orders of magnitude smaller than the levels of asset values themselves, implying that the curves basically lie on top of each other. Figure 3 shows that, for all values of v, the prices of both debt and equity are higher in the case with commitment.

[15] See, for example, Altınkılıç and Hansen (2000), Hennessy and Whited (2007), Titman and Tsyplakov (2007), and Gamba and Triantis (2008).

[16] Indeed, when $\beta = 0.007$ was calibrated, it was chosen consistent with this optimal maturity.

Recall that Figure 1 demonstrated the existence of an optimal maturity for the special case $\beta = 0.007$. In Figure 4, we present the optimal average maturity as a function of issuance costs β. In both cases with and without commitment, the optimal maturity is an increasing function of the issuance cost parameter β. This finding is consistent with the tradeoff mentioned in the previous paragraph. Specifically, as one lowers the issuance cost parameter β, the benefit to issuing shorter maturity debt mostly remains the same, but the expected costs associated with issuing shorter maturity debt decreases, leading to a lower optimal maturity. Interestingly, for all values of β, the optimal maturity for the case with commitment is slightly longer than the optimal maturity for the case without commitment.

In the next few figures, we report results associated with the optimal policy controls $\left(v_b^*(\beta), v_u^*(\beta), \gamma^*(\beta), \xi^*(\beta)\right)$ as a function of β. For example, in Figure 5, we present the level of tax benefits (estimated using equation (30)) as a function of the issuance cost parameter β. Two points are worth noting. First, in agreement with GJL (and intuition), the tax benefits to debt increase as issuance costs decrease. Second, the case with commitment generates higher tax benefits to debt than the case without commitment. As the case with commitment generates higher equity values in all states of nature, this result was to be expected.

In Figures 6 and 7, we present the optimal debt issuance size parameter $\gamma^*(\beta)$ and the optimal location of the upper boundary $v_u^*(\beta)$ as a function of the issuance cost parameter β. For both the cases of commitment and no commitment, both the optimal location of the upper boundary and the size of the debt issuance increase monotonically with issuance costs. Intuitively, this occurs because, when fixed issuance costs are high, it will be optimal to reduce the present value of these costs by making issuances less frequent. This can be accomplished in two ways. First, the location of v_u can be increased so that the inverse leverage of the firm (v_t) reaches the upper boundary v_u less often. Second, the size of the debt issuance can be increased so that, after each debt issuance date, the location of the firm's inverse leverage drops from $v_{\tau_u}^- = v_u$ to $v_{\tau_u}^+ = \left(\frac{v_u}{\gamma}\right)$. Thus, larger values of γ imply that, after a debt issuance, the firm's inverse leverage is further away from reissuing debt in the future. Also note that, in the limit $\beta \to 0$, we find $\gamma \to 1$, implying that the size of new debt issuances becomes infinitesimally small. That is, in the limit of vanishing issuance costs, the upper boundary acts like a reflecting

boundary.

In Figure 8, we present the optimal location of the lower boundary $v_b^*(\beta)$ as a function of the issuance cost parameter β. We see that, for a given β, the case with commitment is always associated with a lower default boundary compared to the case without commitment. Intuitively, this occurs because the value of equity is always higher for the case with commitment, making the option to maintain ownership more valuable.

3 Limiting Case: Vanishing Issuance Costs

Figures 4-8 suggest that, for the case with commitment, there is a well-defined limiting solution for debt and equity as the issuance cost parameter β approaches zero. Here, we identify this limit more formally. To begin, first note that both the ODE's (equations (15), (25)) and the lower boundary conditions (equations (16), (26)) for debt and equity are independent of the parameters (β, γ). Indeed, these parameters show up only in the upper boundary conditions (equations (17), (27)). Here we identify what these upper boundary conditions are for the case $\beta \to 0$.

Importantly, the figures in the previous section demonstrated that, as the issuance cost parameter $\beta \to 0$, the optimal debt issuance size parameter $\gamma \to 1$. Therefore, in order to identify the limiting case, we consider a Taylor series expansion around $\epsilon \equiv (\gamma - 1)$. The upper boundary condition for debt (i.e., equation (12)) can therefore be expressed as:

$$\begin{aligned} (1+\epsilon)D(v_{_u}F, F) &= D(v_{_u}F, F + \epsilon F) \\ &= D(v_{_u}F, F) + \epsilon F D_{_F}(V, F)\big|_{V=v_{_u}F}. \end{aligned} \tag{51}$$

Taking the limit $\epsilon \to 0$, the relation simplifies to

$$D(v_{_u}F, F) \overset{\epsilon \to 0}{=} F D_{_F}(V, F)\big|_{V=v_{_u}F}. \tag{52}$$

Applying equations (13) and (14) to equation (52), this boundary condition can also be expressed as:

$$d_{_v}(v_{_u}) = 0. \tag{53}$$

This equation can also be derived by taking a Taylor series expansion of equation (17). The form of this boundary condition is typical of reflecting boundaries. As noted previously, this equation can be interpreted as precluding an arbitrage opportunity in that

the price of debt per unit face value must be the same just prior to (i.e., at date τ_u^-) and just after (i.e., at date τ_u^+) the debt issuance.

A similar argument holds for the upper boundary condition on equity. Specifically, setting $\beta = 0$ in equation (22), and Taylor expanding about $\epsilon \equiv (\gamma - 1)$, this boundary condition for debt reduces to:

$$0 \quad = \quad (E_F(V,F) + D_F(V,F))\big|_{V=v_u F}. \tag{54}$$

Applying equations (14), (24) and (53) to equation (22), this boundary condition can also be expressed as:

$$v_u e_v(v_u) \quad = \quad e(v_u) + d(v_u). \tag{55}$$

This equation can also be obtained from equation (27) by setting $\beta = 0$, and then performing a Taylor series expansion on $\epsilon = (\gamma - 1)$.

It is worth noting that, using equation (24), we can rewrite equation (55) as the first order condition that characterizes the no-commitment equilibrium of DH:

$$-E_F(V,F)\big|_{V=v_u F} \quad = \quad d(v_u). \tag{56}$$

As noted previously, this condition captures the no-arbitrage condition that debt will be issued to the point at which the amount raised by issuing another dollar (in face value) of debt equals the marginal impact this issuance has on equity valuation. However, whereas this condition holds for *all* values of v in the no-commitment policy studied by DH, this condition holds at only the single point $v = v_u$ in our case with commitment.

In Figure 9, we present the value of debt $d(v)$ as a function of inverse leverage v for the case $\beta = 0$ and $\xi = 0.2$. For comparison, we also plot the value of debt for the no-commitment case of DH.[17] This figure shows that, for all state of nature v, the value of debt is higher in our model with commitment compared to the DH no-commitment equilibrium. Intuitively this occurs because our debt policy is more conservative, which leads to lower future default probabilities, and thus to higher debt valuations for all values of v. On this figure, we also plot $-E_F = (ve_v(v) - e(v))$. For our globally optimal solution, the value of debt is higher than the marginal value of equity for all values of v less than v_u. In contrast, as already mentioned, the condition $(-E_F(V,F) = d(v))$

[17] We choose $\xi = 0.2$ here to match the DH calibration.

holds for all values of v in the DH no-commitment equilibrium. Intuitively, what this is capturing is that a *myopic* manager would interpret $d(v) > -E_{_F}(v)$ as an arbitrage opportunity to issue additional debt. However, in this model with commitment, such a manager would not be accounting for the fact that any deviation from the optimal policy would have significant impact on the pricing of debt, and on the optimal debt issuance policies in other states of nature. Thus, the apparent arbitrage opportunity is in fact a mirage, and shareholder value is maximized by following the optimal policy that we have identified.

In Figure 10, we present the value of equity $e(v)$ as a function of inverse leverage v. For comparison, we also plot the value of equity for the DH no-commitment solution. As previously noted, the value of equity in our case with commitment is higher in all states of nature: $e(v) > e_{_{DH}}(v)\ \forall v$. Hence, there is no incentive for shareholders to ever deviate from the policy with commitment. Indeed, under the (credible) threat that any new debt issuances would be priced according to the no-commitment debt prices associated with the DH equilibrium if the firm were to ever deviate from following the debt issuance policy with commitment, combined with the proof of DH that the optimal deviation would be generated by an infinitesimal debt issuance, shareholders would gain zero cash benefit at the date of the deviation, and be left with an equity claim of lower value.

4 Equity Valuation with Debt Repurchase

In the previous sections, the lower boundary was specified to be a default boundary. In this section, however, we investigate whether management can credibly commit to the lower boundary being specified as a debt-repurchase/equity-issuance boundary. Specifically, here we consider the possibility that, in each period, there will exist a lower boundary $v_{_\ell}$ at which the firm will scale down the level of outstanding debt by a factor $\psi \leq 1$. The implication is that debt becomes risk-free, as inverse leverage always stays within the region $v_{_t} \in (v_{_\ell}, v_{_u})$ for which equity values remain positive. The value of

outstanding debt with coupon c and amortization rate ξ is therefore

$$\begin{aligned} D(V_t, F_t) &= \mathcal{E}_t^Q \left[\int_t^\infty ds\, e^{-r(s-t)} (c+\xi) F_s \right] \\ &= \left(\frac{c+\xi}{r+\xi} \right) F_t. \end{aligned} \tag{57}$$

In analogy with the previous section, we define τ_u as the time the upper boundary is reached, and τ_ℓ as the time the lower boundary is reached, and $\tau = \min(\tau_\ell, \tau_u)$ as the end of period-0.

Up to this point, we have assumed full tax loss offset in that, for those times in which EBIT fell below interest payments ($Y_t < cF_t$), our specification for the dividend in equation (8) implied that government pays the firm $\overline{\pi}(cF_t - Y_t)$. In reality, firms do not receive cash inflows from government, but rather only a tax loss carryforward, whose present value is significantly less than what this specification implies. Therefore, in this section, we investigate the situation in which the cash flow (i.e., dividends) to equity is specified as:

$$Div(Y_t, F_t) = \begin{cases} Y_t - \overline{\pi}(Y_t - cF_t) - (c+\xi)F_t & Y_t > cF_t \\ Y_t - (c+\xi)F_t & Y_t < cF_t \end{cases} \tag{58}$$

With a slight abuse of notation, we rewrite this formula in terms of our state vector (V_t, F_t):

$$Div(V_t, F_t) = \begin{cases} (1-\overline{\pi})(r-\mu)V_t - (c(1-\overline{\pi}) + \xi)\, F_t & V_t > \frac{cF_t}{r-\mu} \\ (r-\mu)V_t - (c+\xi)\, F_t & V_t < \frac{cF_t}{r-\mu} \end{cases} \tag{59}$$

As this specification implies no tax loss carry-forward, the actual situation falls between these two cases. However, as will be seen below, for the case of debt repurchase (which implies that debt is risk-free), this scenario is the more empirically relevant one.

The value of equity can be determined as the risk-neutral expectation:

$$\begin{aligned} E(V_t, F_t) &= \mathcal{E}_t^Q \left[\int_t^\tau ds\, e^{-r(s-t)}\, Div(Y_s, F_s) \right] \\ &+ \mathcal{E}_t^Q \left[e^{-r(\tau_u - t)}\, \mathbf{1}_{(\tau_u < \tau_\ell)} \left(E(v_u F_{\tau_u}, \gamma F_{\tau_u}) + (\gamma - 1) \left(\frac{c+\xi}{r+\xi} \right) F_{\tau_u} - \beta v_u F_{\tau_u} \right) \right] \\ &+ \mathcal{E}_t^Q \left[e^{-r(\tau_\ell - t)}\, \mathbf{1}_{(\tau_\ell < \tau_u)} \left(E(v_\ell F_{\tau_\ell}, \psi F_{\tau_\ell}) - (1-\psi) \left(\frac{c+\xi}{r+\xi} \right) F_{\tau_u} - \beta_E v_\ell F_{\tau_\ell} \right) \right]. \end{aligned} \tag{60}$$

The first term captures the present value of the claim to dividends throughout period-0, which ends at date $\tau = \min(\tau_{_\ell}, \tau_{_u})$. The second term captures the fact that when the upper boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor of $\gamma\,(\geq 1)$. In addition, shareholders receive as dividend the value of the new debt issuance (net of issuance costs), which, because debt is risk-free, has price per unit face value of $\left(\frac{c+\xi}{r+\xi}\right)$. The third term captures the fact that when the lower boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor of $\psi\,(\leq 1)$. In addition, shareholders pay for the repurchase of the (risk-free) debt, and an equity issuance cost controlled by the parameter $\beta_{_E}$.

Equation (60) implies that $\left[e^{-rt}\,E(V_{_t}, F_{_t}) + \int_0^t ds\, e^{-rs}\, Div(V_{_s}, F_{_s})\right]$ is a Q-martingale, implying that:

$$0 \;=\; -rE + \mu V E_{_V} + \frac{\sigma^2}{2} V^2 E_{_{VV}} - \xi F E_{_F} + Div(V, F). \tag{61}$$

As in the previous section, because both the cash flows to equity and state vector dynamics are linear in the state vector, the value of equity is homogeneous of degree one in the state vector. Thus, we look for a solution of the form:

$$E(V, F) \;=\; Fe(v = V/F). \tag{62}$$

This allows us to reexpress the PDE in equation (61) as the coupled ODE's:

$$0 \;=\; \tfrac{\sigma^2}{2} v^2 e_{_{vv}} + v e_{_v}\,(\mu + \xi) - e(r + \xi) + v(r - \mu) - (c + \xi) \qquad v < \frac{c}{r - \mu}, \tag{63}$$

$$0 \;=\; \tfrac{\sigma^2}{2} v^2 e_{_{vv}} + v e_{_v}\,(\mu + \xi) - e(r + \xi) + v(r - \mu)(1 - \overline{\pi}) - (c(1 - \overline{\pi}) + \xi) \quad v > \frac{c}{r - \mu}, \tag{64}$$

subject to the boundary conditions

$$e(v_{_\ell}) \;=\; \psi e(v_{_\ell}/\psi) - (1 - \psi)\left(\frac{c + \xi}{r + \xi}\right) - \beta_{_E} v_{_\ell} \tag{65}$$

$$e(v_{_u}) \;=\; \gamma e(v_{_u}/\gamma) + (\gamma - 1)\left(\frac{c + \xi}{r + \xi}\right) - \beta v_{_u}, \tag{66}$$

Under this specification, the value of equity is:

$$e(v) \;=\; \begin{cases} A_{_\uparrow} v^{\phi} + B_{_\uparrow} v^{\omega} + (1 - \overline{\pi})v - \frac{(c(1-\overline{\pi})+\xi)}{r+\xi} & v > \frac{c}{r-\mu} \\ A_{_\downarrow} v^{\phi} + B_{_\downarrow} v^{\omega} + v - \frac{c+\xi}{r+\xi} & v < \frac{c}{r-\mu} \end{cases} \tag{67}$$

where the constants $(A_{\uparrow}, A_{\downarrow}, B_{\uparrow}, B_{\downarrow})$ are uniquely determined from the boundary conditions in equations (65), and the value- and slope-matching conditions[18]

$$\begin{aligned} e\left(v \Rightarrow \left(\frac{c}{r-\mu}\right)^{+}\right) &= e\left(v \Rightarrow \left(\frac{c}{r-\mu}\right)^{-}\right) \\ e_{v}\left(v \Rightarrow \left(\frac{c}{r-\mu}\right)^{+}\right) &= e_{v}\left(v \Rightarrow \left(\frac{c}{r-\mu}\right)^{-}\right). \end{aligned} \tag{68}$$

We investigate this model under two different parameterizations: with and without issuance costs

4.1 Debt Repurchase in the Absence of Issuance Costs

For the case $\beta = 0$, $\beta_{E} = 0$, the value of equity has an analytic solution. Specifically, because the claim to bankruptcy costs is zero when debt is risk-free, and because government's claim to taxes can be chosen to be zero if v_{u} is chosen to guarantee that coupon payments always exceed EBIT, then the value of equity is equal to the value of the EBIT claim minus the value of debt. Hence, under the assumption $v_{u} \leq \frac{c}{r-\mu}$, (which implies the firm never pays any taxes because coupon payments always exceed EBIT), we have

$$e(v) = v - \left(\frac{c+\xi}{r+\xi}\right). \tag{69}$$

Now, for this equation to be compatible with limited liability, we must restrict the lower boundary v_{ℓ} such that:

$$\begin{aligned} 0 &\leq e(v_{\ell}) \\ &= v_{\ell} - \left(\frac{c+\xi}{r+\xi}\right), \end{aligned} \tag{70}$$

which implies:[19]

$$v_{\ell} \geq \left(\frac{c+\xi}{r+\xi}\right). \tag{71}$$

[18] These value- and slope-matching conditions assume that $v_{\ell} < \frac{c}{r-\mu}$ and $v_{u} > \frac{c}{r-\mu}$. If instead v_{ℓ} and v_{u} are both either above or both below $\frac{c}{r-\mu}$, then these additional conditions are not applicable.

[19] More generally, we impose the conditions:
i) $\frac{c+\xi}{r+\xi} \leq v_{\ell} \leq v_{u} \leq \frac{c}{r-\mu}$
ii) $v_{\ell} \leq \frac{v_{\ell}}{\psi} \leq v_{u}$
iii) $v_{\ell} \leq \frac{v_{u}}{\gamma} \leq v_{u}$

The question then becomes whether or not a firm can commit to following such a strategy, which we interpret as investigating whether the value of equity is higher in all states of nature v for the case with debt repurchase versus without. As shown in Figure 11, for the parametrization ($\psi = 1, \gamma = 1, \xi = 0.2, v_\ell = \frac{1}{2}\left(\frac{c+\xi}{r+\xi} + \frac{c}{r-\mu}\right), v_u = \frac{c}{r-\mu}$) this condition holds. We interpret this finding to imply that **if** equity issuance costs were zero, then a firm could credibly commit to issuing risk-free debt.

4.2 Debt Repurchase in the Presence of Issuance Costs

In this subsection, we investigate whether a firm can credibly commit to repurchasing debt at some lower threshold v_ℓ when realistic equity issuance costs are specified. Following the literature (e.g., Altınkılıç and Hansen (2000), Hennessy and Whited (2007), Titman and Tsyplakov (2007), Gamba and Triantis (2008)), we calibrate the model so that equity issuance costs are approximately 6%. We find that, for all possible calibrations, the value of equity at v_ℓ is lower for the model with debt repurchase than the value of equity in the global optimal solution reported in the previous section. We interpret this result as saying, with realistic equity issuance costs, shareholders cannot commit to repurchasing debt at v_ℓ, because they have financial incentive to deviate by not repurchasing the debt at v_ℓ. Therefore, potential debtholders, realizing this lack of incentive for shareholders to stick to the no-default policy, would not be willing to purchase the firm's debt at risk-free prices, implying that the debt-repurchase solution is not an equilibrium. We provide an example in Figure 12.

We emphasize that introducing equity issuance costs provides *one* mechanism for explaining why firms are unable to issue risk free debt. Many other mechanisms exist. For example, if the manager is more informed about the firm's asset value than creditors (e.g., Duffie and Lando (2001)), and the true asset value is sufficiently low, then a manager acting in the best interest of shareholders might not issue additional equity in order to pay for the debt repurchase, and would therefore deviate from the promise to repurchase debt.

5 Conclusion

Within the standard tradeoff paradigm, we investigate the optimal dynamic capital structure and optimal debt maturity policies of firms. We focus on two features: (i) the

presence (or absence) of issuance costs, and (ii) the ability of the firm to commit to a particular debt issuance policy. Our framework can be seen as a generalization of both Goldstein Ju and Leland (GJL, 2001) (who restrict their attention to infinite-maturity debt that the firm is contractually committed to repurchasing prior to issuing other debt), and DeMarzo and He (DH, 2018) (who restrict their attention to models of no-commitment in the absence of issuance cost). For a given level of issuance cost, the optimal policy is specified in terms of a single state variable (namely, inverse leverage) and four (optimally chosen) parameters: a lower default boundary $v_{_b}$, an upper debt-issuance boundary $v_{_u}$, a debt issuance size parameter γ, and an (inverse) maturity parameter ξ.

We investigate both the cases with and without commitment. We show that the optimal policy with commitment is a *global optimal* policy in that, for *any* given value of leverage, equity values are higher under this policy than any other policy consistent with limited liability. In contrast to DH, who report that firms are unable to extract any tax benefits to debt regardless of choice of debt maturity, we show that once realistic debt issuance costs are accounted for, there exists an interior optimal debt maturity, and that firms tax benefits to debt are available for firms following an optimal policy. Moreover, these properties hold regardless of whether or not the firm has the ability to commit to the global optimal issuance policy.

We then investigate our model in the limit of issuance costs going to zero. The optimal policy is characterized by debt of vanishing maturity, and infinitesimal debt issuance, implying that the upper boundary acts as a reflecting boundary. Because equity values are higher in every state of nature compared to equity values in the DH equilibrium, we argue that firms can credibly commit to not deviating from the global optimal policy under the (trigger) threat that any deviation would lead to new debt issuances being priced according to the DH no-commitment policy.

Finally, we investigate the possibility that the lower boundary is not a default boundary, but rather a debt repurchase boundary, which would imply that debt is risk-free. We show that, once such models are calibrated to realistic equity issuance cost, shareholder would not be able to credibly commit to following such a policy, because in those situations in which inverse leverage falls to $v_{_\ell}$, rather than repurchase debt as "promised," shareholders would prefer to deviate to the global optimal policy. As such,

our framework provides one explanation for why firms cannot commit to issuing risk-free debt.

Table 1: **Baseline Model Coefficients.** Below are the values of the model coefficients in the baseline calibration.

Parameter	Symbol	Value
Annual risk-free rate	r	0.04
Annual coupon rate	c	0.055
Annual asset drift	μ	0
Annual asset volatility	σ	0.22
Corporate tax rate	$\overline{\pi}$	0.2
Loss given default	α	1

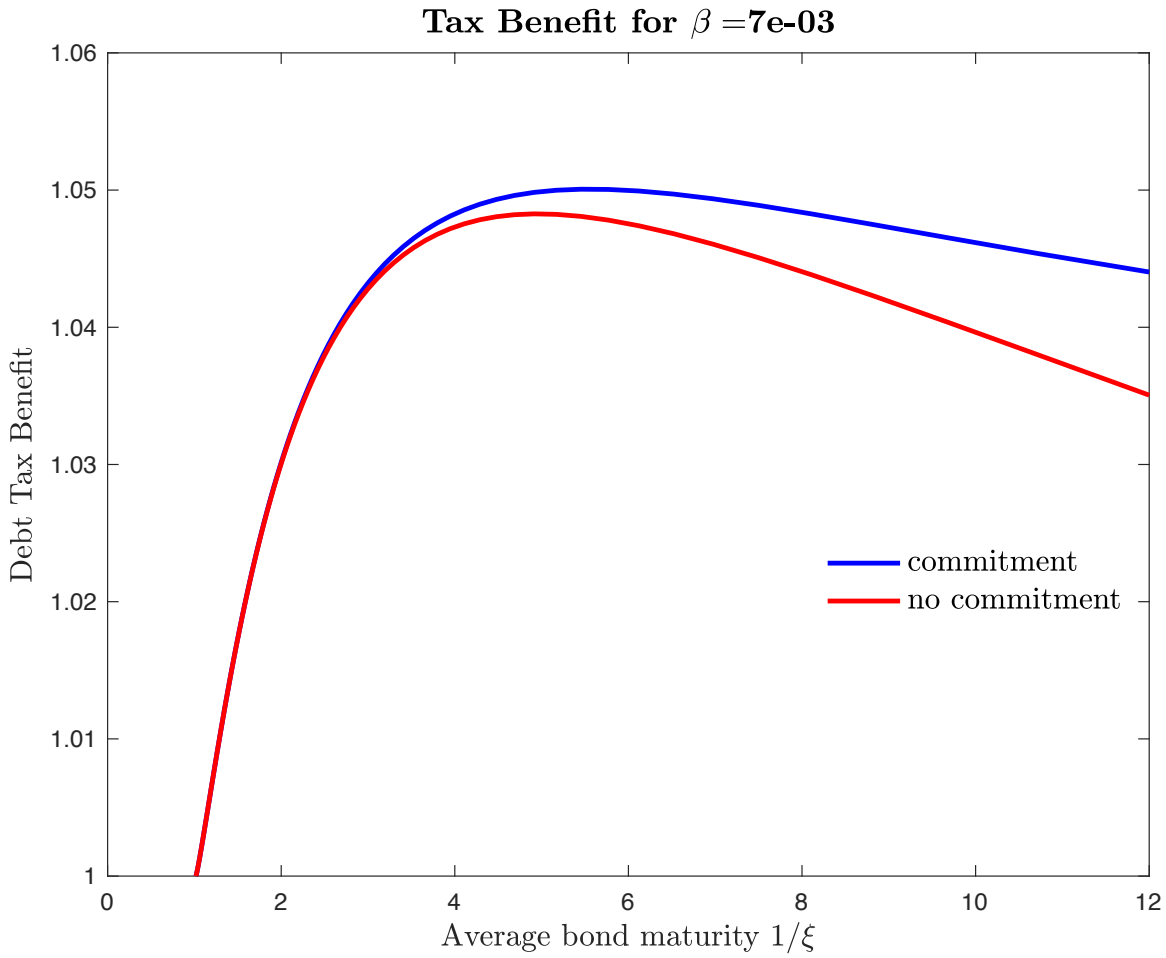


Figure 1: **Tax benefits to debt.** This figure presents the tax benefits to debt as a function of average debt maturity $(1/\xi)$ for two cases: i) with commitment, and ii) without commitment. The issuance cost parameter is set to $\beta = 0.007$, which is chosen to match a fractional debt issuance cost of 2%. The other parameter values are reported in Table 1.

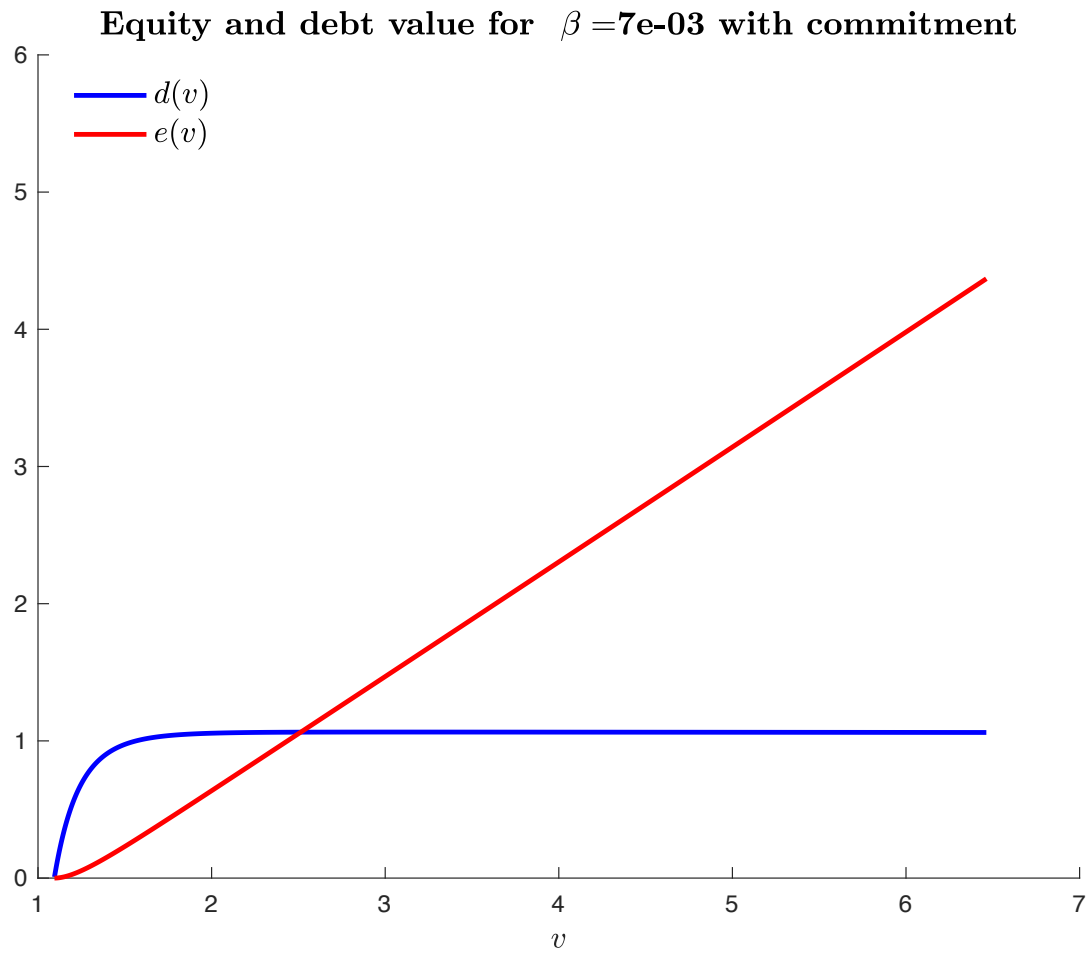


Figure 2: **Equity and debt valuations.** For the parameter choice $\beta = 0.007$, this figure presents the values of equity $e(v)$ and debt $d(v)$ as a function of inverse leverage v for the case with commitment. The other parameter values are reported in Table 1.

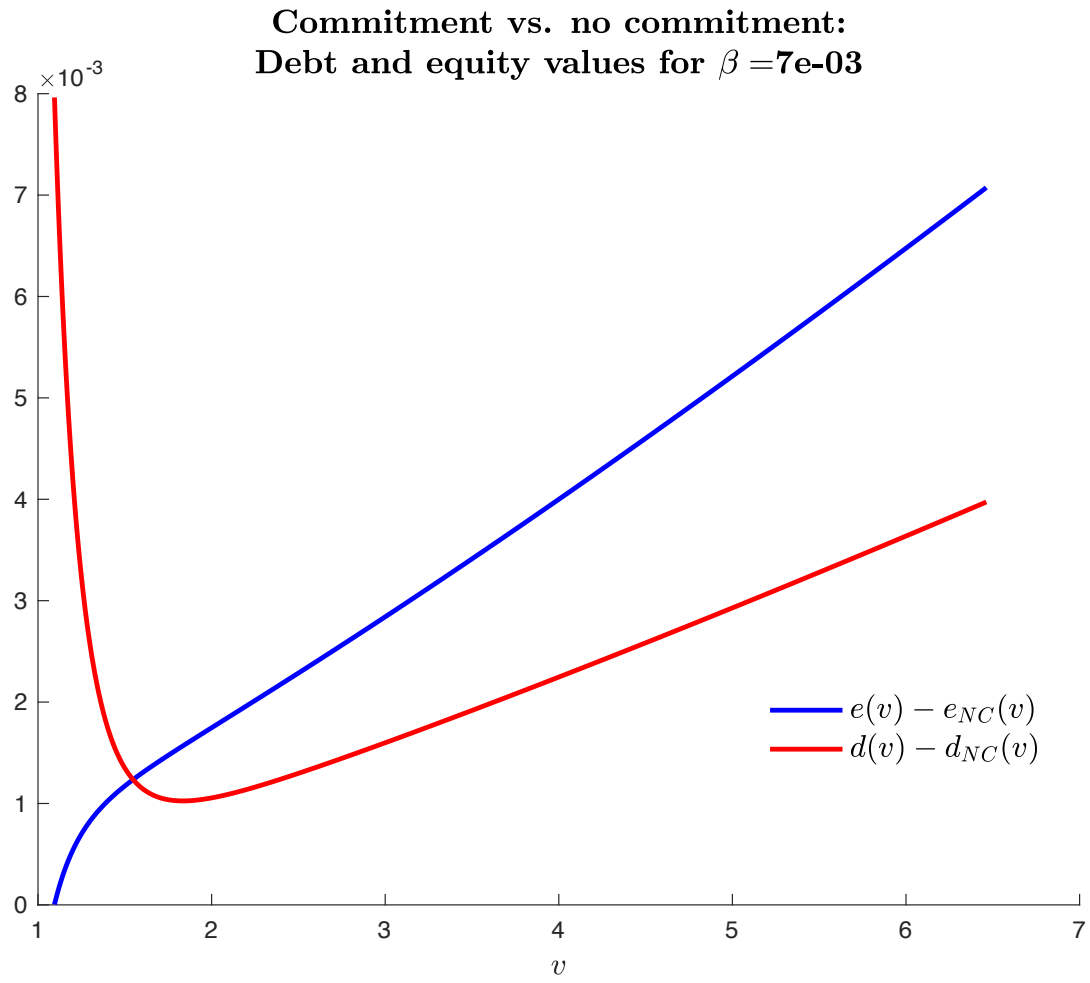


Figure 3: **Difference between asset values under commitment and no commitment.** For the parameter choice $\beta = 0.007$, this figure presents the difference in asset values for the cases of with and without commitment. Specifically, this figure presents the difference in equity values $(e(v) - e_{NC}(v))$ and debt values $(d(v) - d_{NC}(v))$. The other parameter values are reported in Table 1.

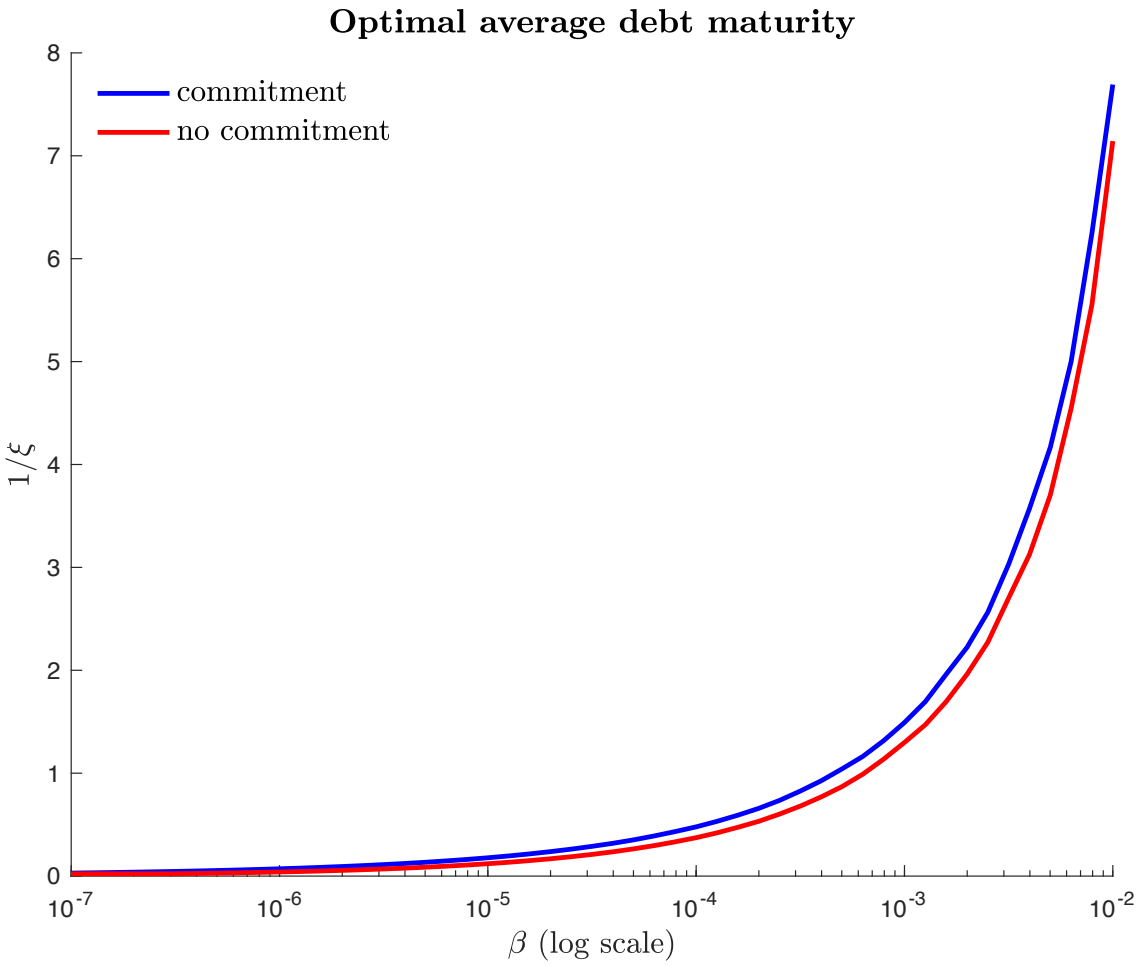


Figure 4: **Optimal maturity.** This figure presents the optimal debt maturity for both the cases of commitment and no-commitment as a function of β. The other parameter values are reported in Table 1.

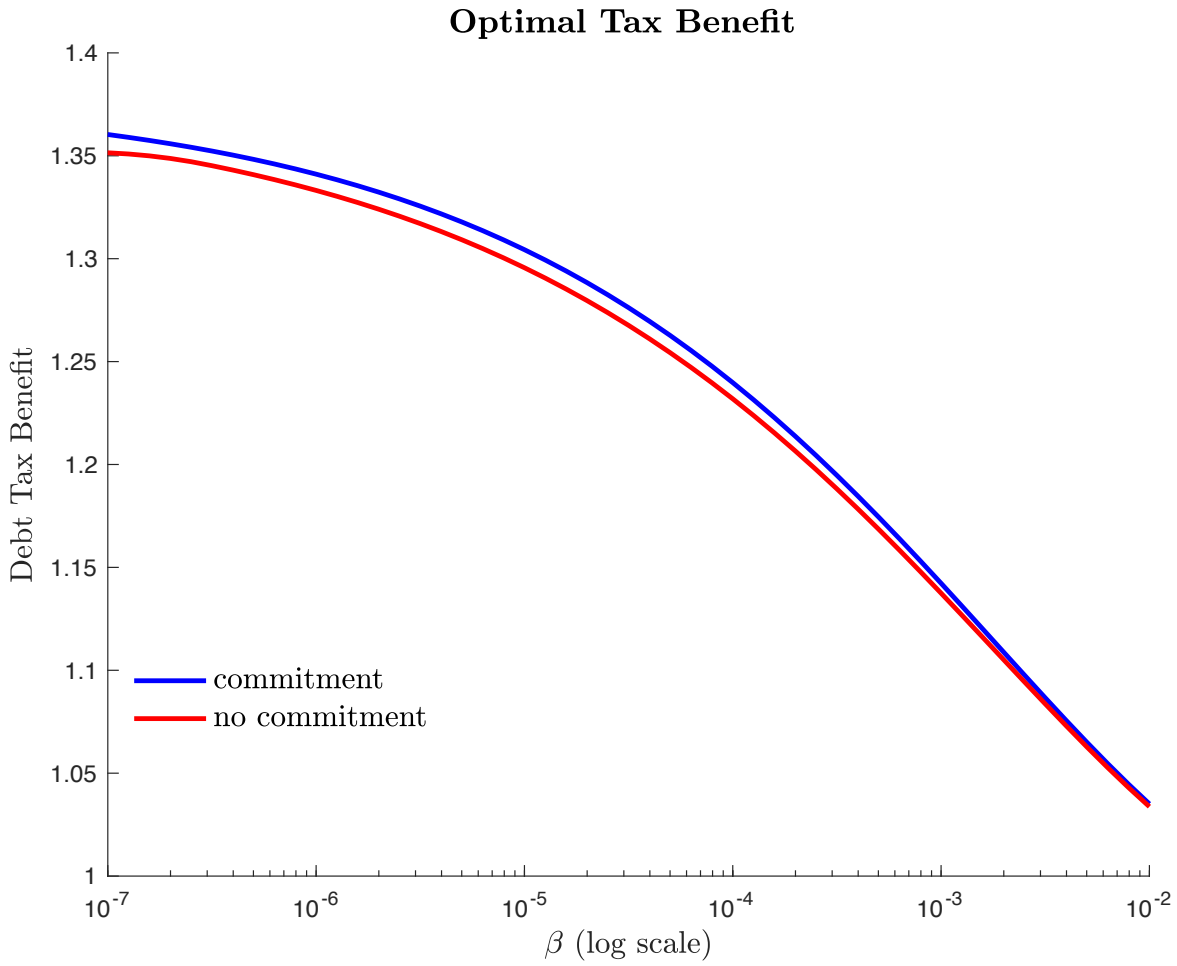


Figure 5: **Tax benefits to debt.** The figure shows the tax benefits to debt under the optimal policy parameters for both the cases of commitment and no-commitment as a function of β. The other parameter values are reported in Table 1.

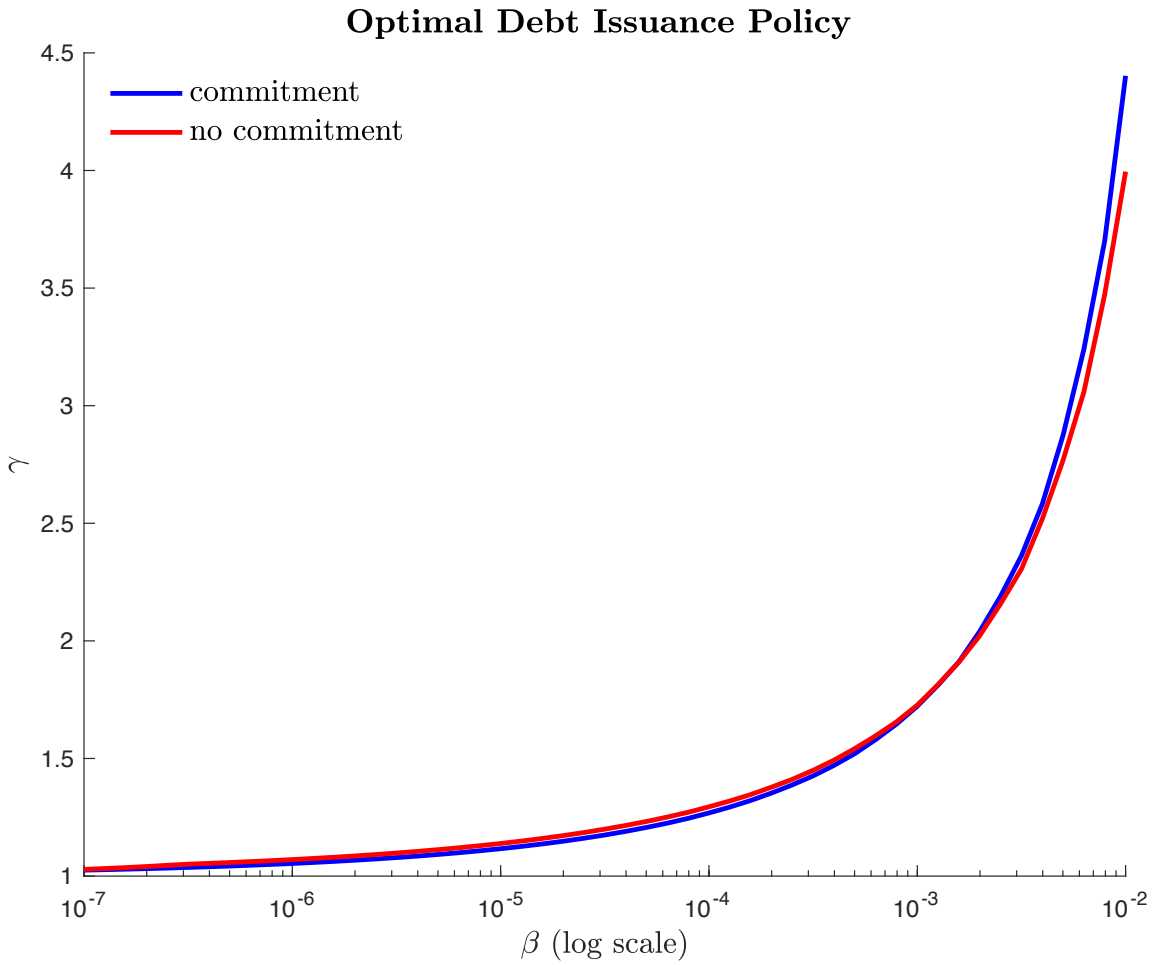


Figure 6: **Optimal debt issuance size.** The figure shows the optimal debt issuance parameter γ for both the cases of commitment and no-commitment as a function of β. The other parameter values are reported in Table 1.

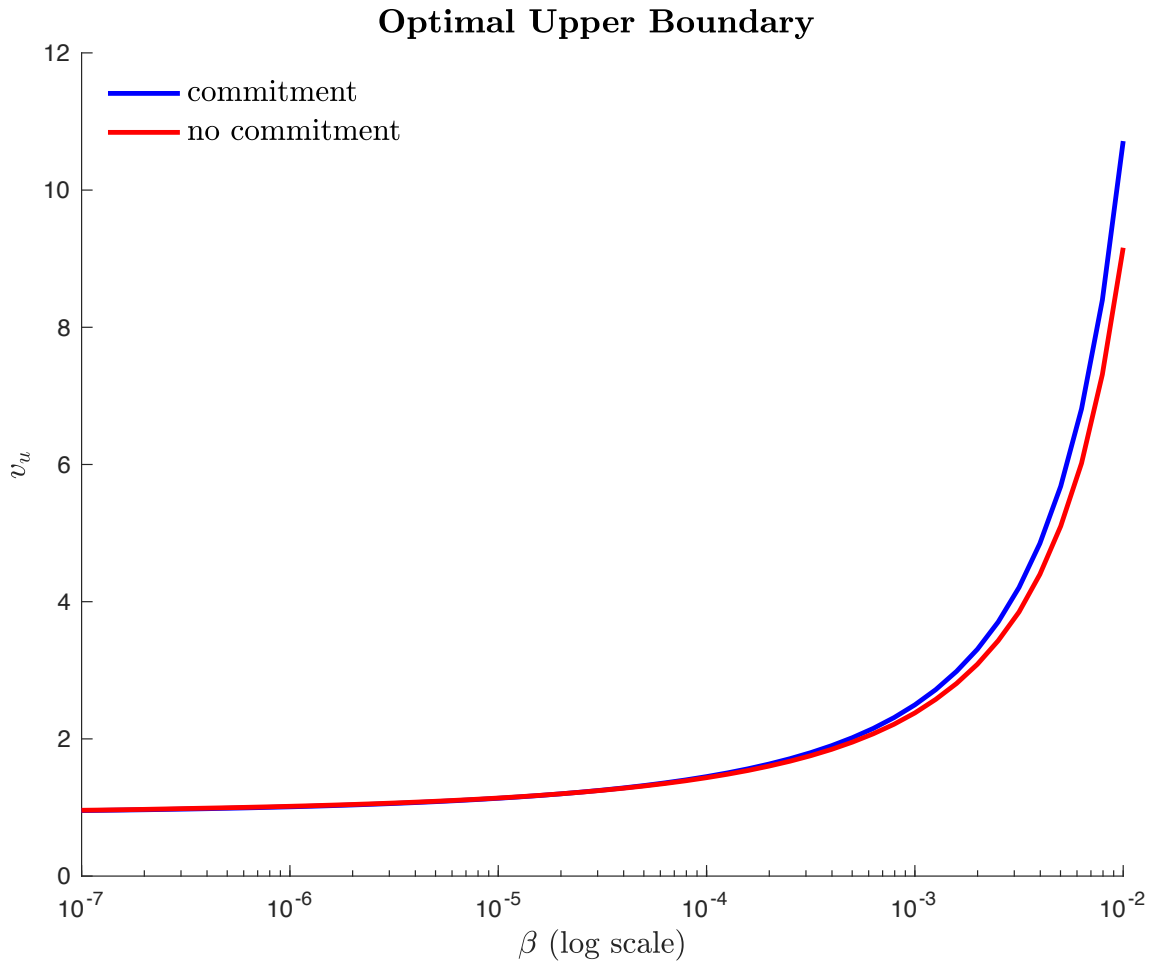


Figure 7: **Optimal debt issuance boundary.** The figure shows the location of the optimal debt issuance boundary v_u for both the cases of commitment and no-commitment as a function of β. The other parameter values are reported in Table 1.

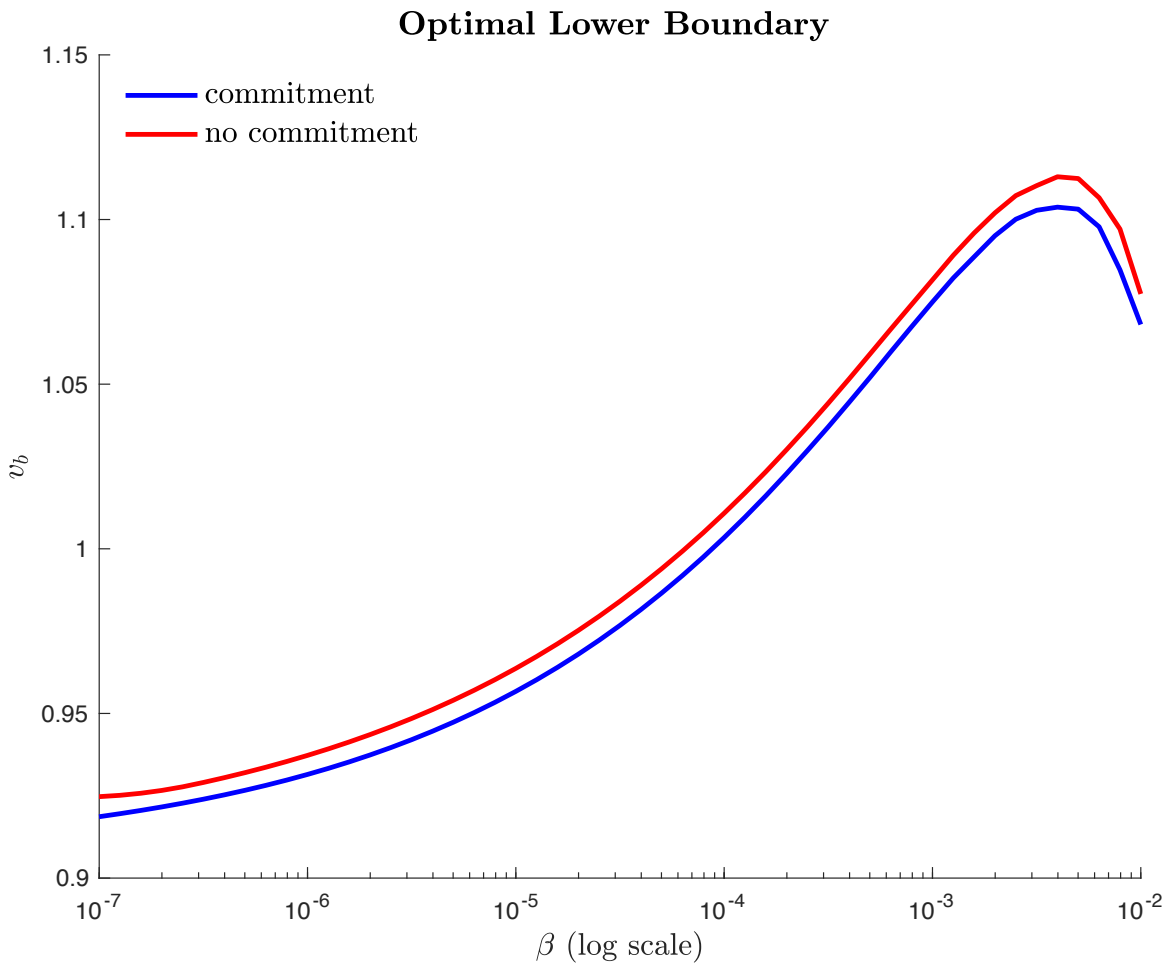


Figure 8: **Optimal default boundary.** The figure shows the location of the optimal default boundary v_b for both the cases of commitment and no-commitment as a function of β. The other parameter values are reported in Table 1.

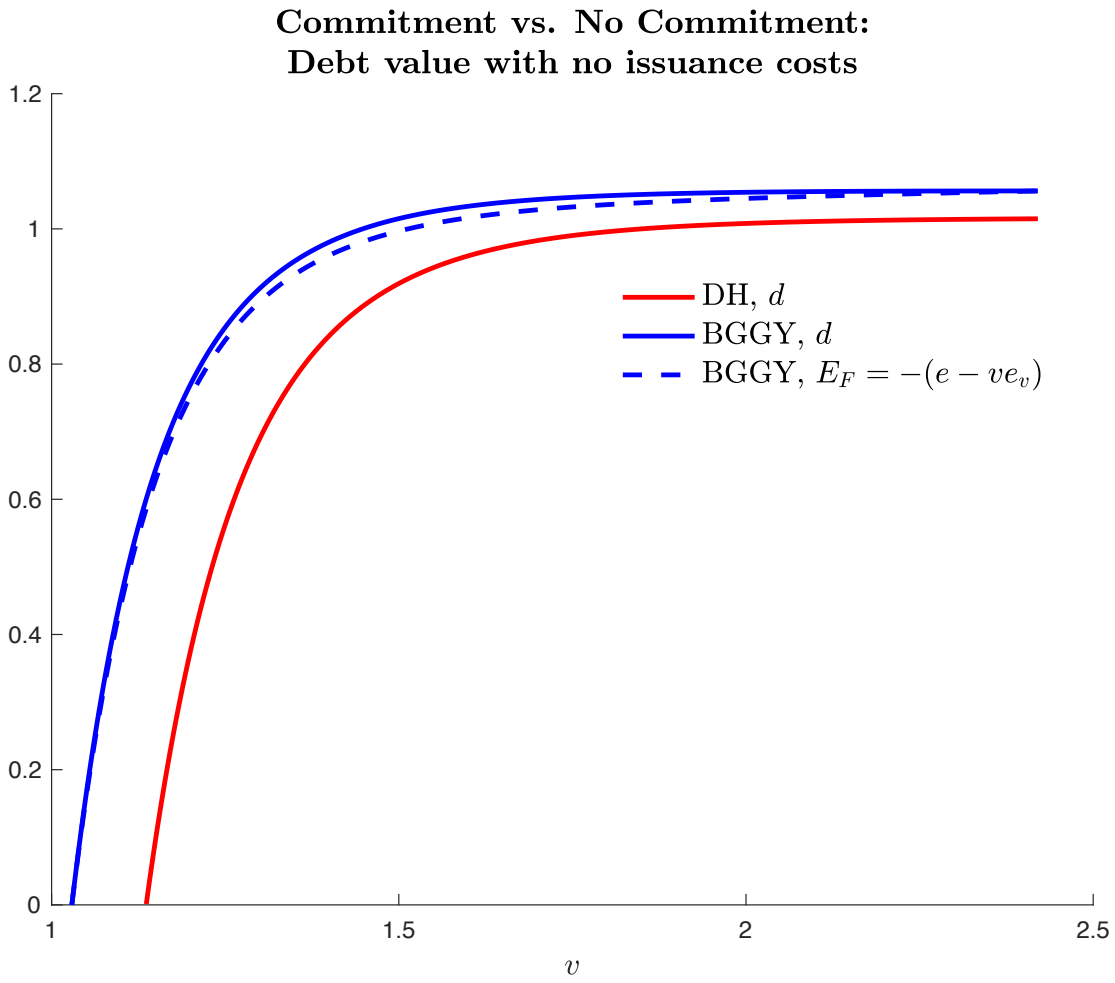


Figure 9: **Debt valuation in absence of issuance cost: commitment vs. no commitment.** This figure reports the value of the debt claim $d(v)$ for the case with commitment for values $v \in (v_b^*, v_u^*)$. This figure also reports the value of the equity claim $d_{DH}(v)$ for the no-commitment case studied by DeMarzo and He (2018). Note that their solution has no upper boundary, so our figure represents a truncated version of their function. The other parameter values are reported in Table 1.

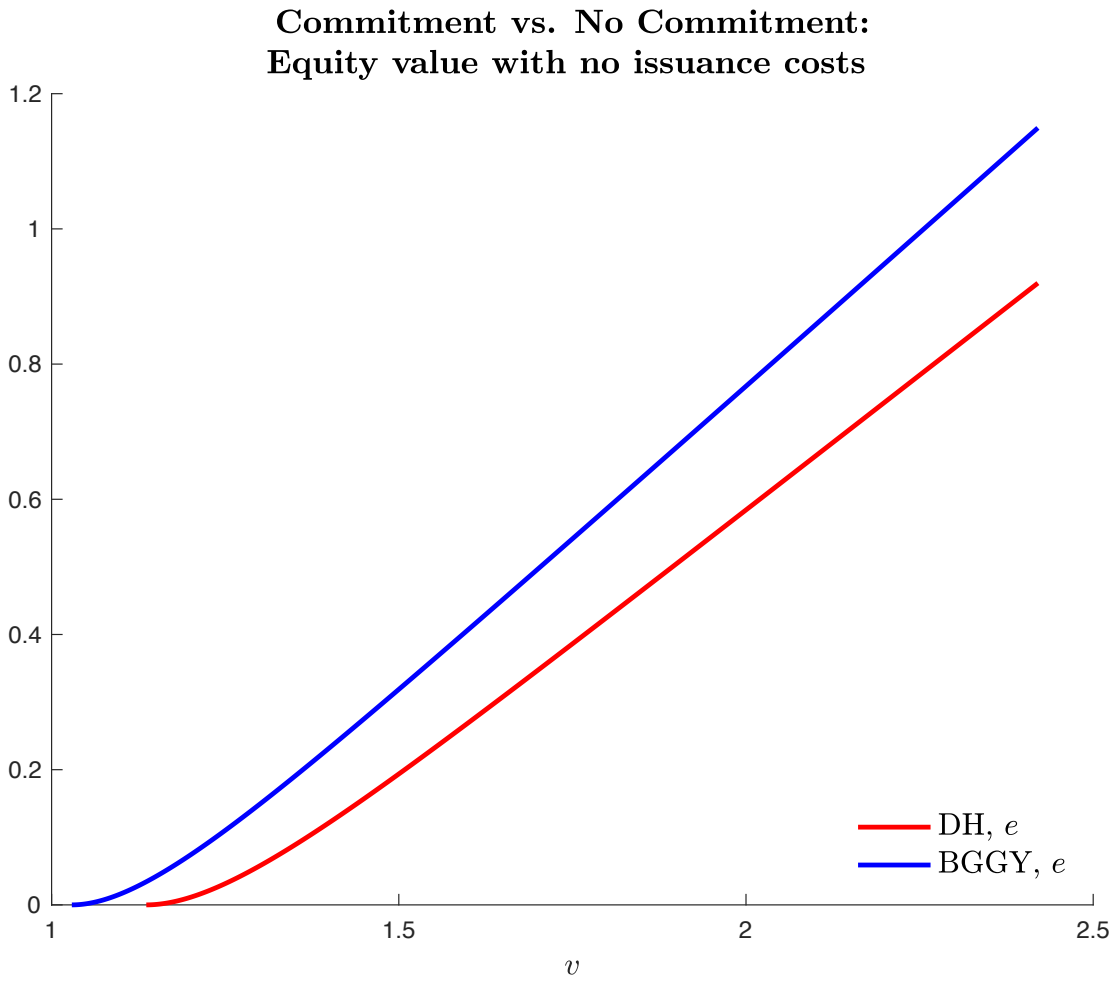


Figure 10: **Equity valuation in absence of issuance cost: commitment vs. no commitment.** This figure reports the value of the equity claim $e(v)$ for the case with commitment for values $v \in (v_b^*, v_u^*)$. This figure also reports the value of the equity claim $e_{DH}(v)$ for the no-commitment case studied by DeMarzo and He (2018). Note that their solution has no upper boundary, so our figure represents a truncated version of their function. The other parameter values are reported in Table 1.

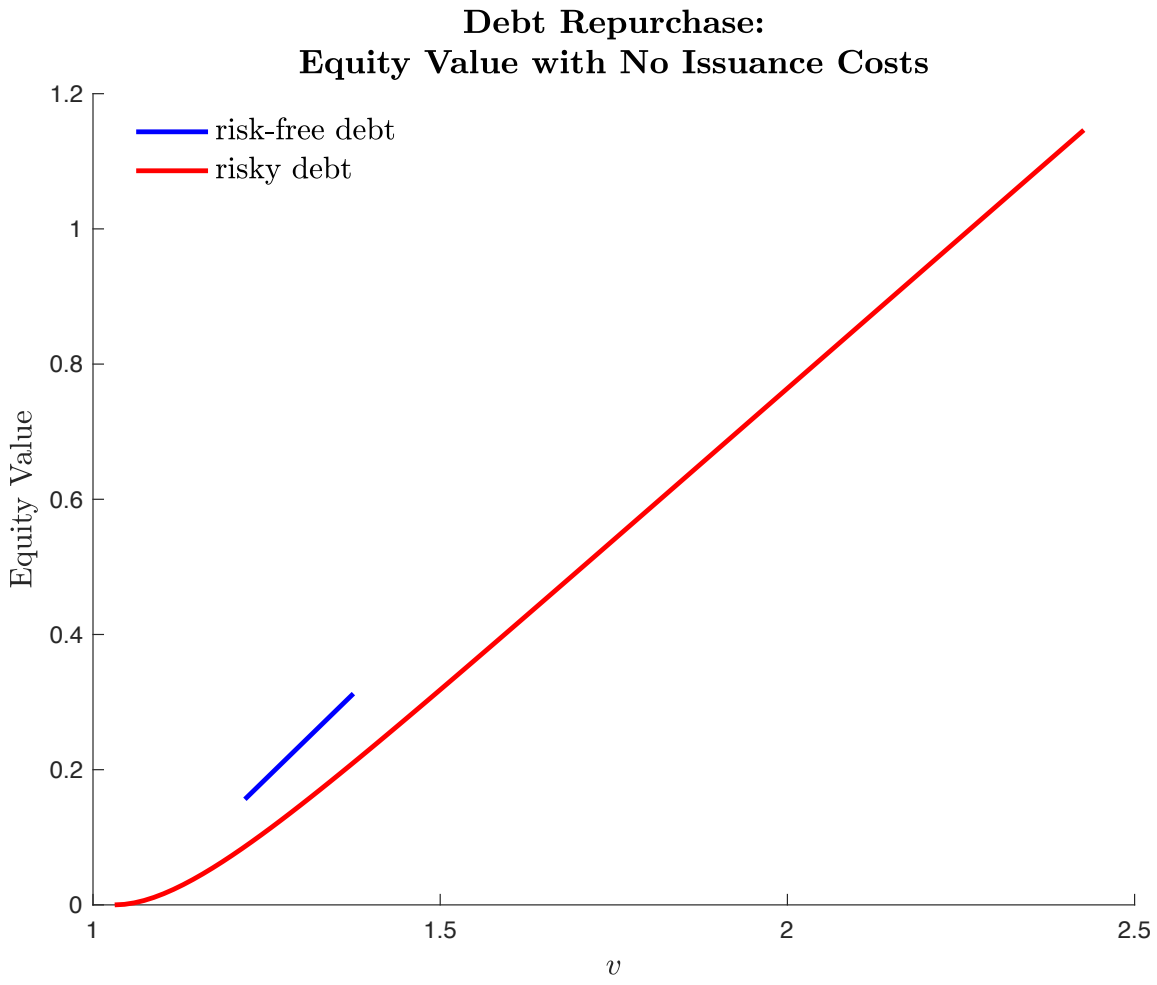


Figure 11: **Debt repurchase and zero issuance cost.** This figure shows the value of equity under both the risk-free and defaultable debt cases, where both equity and debt issuance costs are zero. Because the value of equity is higher in all states of nature v under the risk-free case than under the defaultable debt case, a firm could commit to issuing risk-free debt. For the risk-free debt, we use the following parameters: $\psi = 1$, $\gamma = 1$, $\xi = 0.2$, $v_\ell = \frac{1}{2}\left(\frac{c+\xi}{r+\xi} + \frac{c}{r-\mu}\right)$, $v_u = \frac{c}{r-\mu}$. The other parameter values are reported in Table 1.

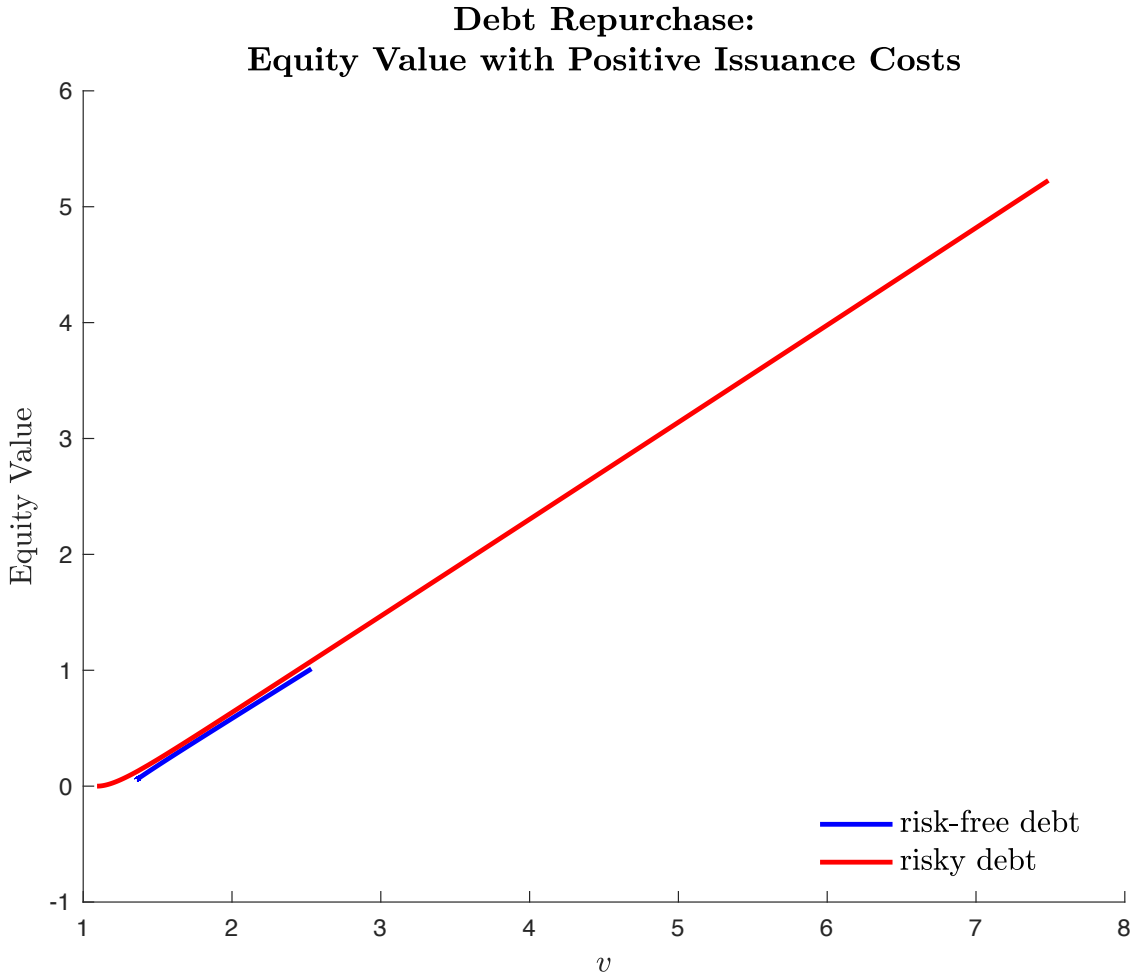


Figure 12: **Debt repurchase and positive issuance cost.** This figure shows the value of equity under both the risk-free and defaultable debt cases, where both equity and debt issuance costs are positive. Because the value of equity at v_ℓ is lower under the risk-free case, shareholders have incentive to deviate from their promise to repurchase debt at v_ℓ. The risk-free case is not an equilibrium because shareholders are not willing to repurchase debt at its risk-free price. The issuance cost parameter is set to $\beta = 0.007$, which is chosen to match a fractional debt issuance cost of 2% under the optimal maturity $\xi = 0.1789$. The equity issuance cost is set to $\beta_E = 0.001$, which is chosen to match a fractional equity issuance cost of 6%. For the risk-free debt, we choose the following parameters: $\psi = 0.9797$, $\gamma = 1.8299$, $v_\ell = 1.3611$, $v_u = 2.5335$. The other parameter values are reported in Table 1.

A Proof that smooth pasting condition and first order condition are not equivalent

Here we show that, for the case in which the exercise of an option at the boundary s^* generates a payoff $F(s^*)$ whose functional form is independent of s^*, then there are two equivalent approaches for identifying an optimal exercise boundary.[20] The first approach, which we refer to as the **FOC approach**, uses the relation

$$0 = \left.\frac{\partial V\left(s_t, s^*\right)}{\partial s^*}\right|_{s_t = s^*}, \tag{A.1}$$

where $\{s_t\}$ follows some stochastic process, and s^* is the optimal exercise boundary.

The second approach, which we refer to as the **Smooth Pasting approach**, uses the relation:

$$\left.\frac{\partial V\left(s_t, s^*\right)}{\partial s_t}\right|_{s_t = s^*} = \frac{\partial F\left(s^*\right)}{\partial s^*}, \tag{A.2}$$

where $F\left(\tilde{s}\right)$ is intrinsic value of the contingent claim if exercised at $\tilde{s}$, and therefore $F\left(s^*\right)$ is the intrinsic value of the contingent claim at the optimal exercise boundary.

Assuming $F\left(\tilde{s}\right) = V\left(\tilde{s}, \tilde{s}\right)$ is a differentiable function of $\tilde{s}$, we find:

$$\begin{aligned} \frac{dF\left(\tilde{s}\right)}{d\tilde{s}} &= \frac{\partial V\left(s, \tilde{s}\right)}{\partial s}\frac{\partial s}{\partial \tilde{s}}|_{s=\tilde{s}} + \frac{\partial V\left(s, \tilde{s}\right)}{\partial \tilde{s}}|_{s=\tilde{s}} \\ &= \frac{\partial V\left(s, \tilde{s}\right)}{\partial s}|_{s=\tilde{s}} + \frac{\partial V\left(s, \tilde{s}\right)}{\partial \tilde{s}}|_{s=\tilde{s}}, \end{aligned} \tag{A.3}$$

where the second line comes from $\frac{\partial s}{\partial \tilde{s}} = 1$ along $s = \tilde{s}$.

When $\tilde{s} = s^*$, we have from equation (A.1) that $\frac{\partial V(s,s^*)}{\partial s^*}|_{s=s^*} = 0$, which reduces equation (A.3) to:

$$\frac{dF\left(s^*\right)}{ds^*} = \frac{\partial V\left(s, s^*\right)}{\partial s}|_{s=s^*}, \tag{A.4}$$

which is the smooth pasting condition. Hence, the two approaches are equivalent when the intrinsic value of the contingent claim has an exogenously specified functional form $F(\tilde{s})$.

However, there are situations in which the functional form of the intrinsic/exercise value depends explicitly on s^*. This happens, for instance at the upper debt issuance boundary in our model. In this case, we have a second asset whose value is expressed as

[20] This proof can be found in footnote 3, p. 490 in Back (2017).

$F(s_t, s^*)$. The owner of the asset optimally chooses to exercise her option the first time $s_t = s^*$, at which time she exchanges her asset for this new asset. As such, we have the condition $V(\tilde{s}, \tilde{s}) = F(\tilde{s}, \tilde{s})$. Assuming both functions are differentiable, we find:

$$\frac{\partial F(s,\tilde{s})}{\partial s}\frac{\partial s}{\partial \tilde{s}}|_{s=\tilde{s}} + \frac{\partial F(s,\tilde{s})}{\partial \tilde{s}}|_{s=\tilde{s}} = \frac{\partial V(s,\tilde{s})}{\partial s}\frac{\partial s}{\partial \tilde{s}}|_{s=\tilde{s}} + \frac{\partial V(s,\tilde{s})}{\partial \tilde{s}}|_{s=\tilde{s}}. \quad (A.5)$$

Simplifying, we find

$$\frac{\partial F(s,\tilde{s})}{\partial s}|_{s=\tilde{s}} + \frac{\partial F(s,\tilde{s})}{\partial \tilde{s}}|_{s=\tilde{s}} = \frac{\partial V(s,\tilde{s})}{\partial s}|_{s=\tilde{s}} + \frac{\partial V(s,\tilde{s})}{\partial \tilde{s}}|_{s=\tilde{s}}. \quad (A.6)$$

Now when we impose the first order condition $\frac{\partial V(s,s^*)}{\partial s^*}|_{s=s^*} = 0$, this reduces equation (A.6) to:

$$\frac{\partial F(s,\tilde{s})}{\partial s}|_{s=\tilde{s}} + \frac{\partial F(s,\tilde{s})}{\partial \tilde{s}}|_{s=\tilde{s}} = \frac{\partial V(s,s^*)}{\partial s}|_{s=s^*}. \quad (A.7)$$

Hence, in these situations, the smooth pasting condition is not equivalent to the first order condition.

Here we demonstrate the non-equivalence of these two conditions by examining them at the upper boundary in our model. For the case without commitment, the value-matching condition at the upper boundary is:

$$E_0(V, V_{0u}, F)|_{V=V_{0u}} = [E(V, \gamma_0 F) + D(V, \gamma_0 F) - D_0(V, V_{0u}, F) - \beta V]|_{V=V_{0u}}.$$

Take the derivative and we can find

$$\begin{aligned} &(E_{0,V}(V, V_{0u}, F) + E_{0,V_{0u}}(V, V_{0u}, F))|_{V=V_{0u}} \\ &= [E_V(V, \gamma_0 F) + D_V(V, \gamma_0 F) - (D_{0,V}(V, V_{0u}, F) + D_{0,V_{0u}}(V, V_{0u}, F)) - \beta]|_{V=V_{0u}}. \end{aligned} \quad (A.8)$$

We impose the first order condition $E_{0,V_{0u}}(V, V_{0u}, F)|_{V=V_{0u}} = 0$. With the homogeneity, we can simplify it as

$$\begin{aligned} e_{0,v}(v, v_{0u})|_{v=v_{0u}} = {} & e_v\left(\frac{v}{\gamma_0}\right)|_{v=v_{0u}} + d_v\left(\frac{v}{\gamma_0}\right)|_{v=v_{0u}} - \\ & (d_{0,v}(v, v_{0u})|_{v=v_{0u}} + d_{0,v_{0u}}(v, v_{0u})|_{v=v_{0u}}) - \beta. \end{aligned} \quad (A.9)$$

The general form	Our model (no commitment)	Value
$\frac{\partial F(s,\tilde{s})}{\partial s}\mid_{s=\tilde{s}=s^*}$	$\left(e_v\left(\frac{v}{\gamma_0}\right)+d_v\left(\frac{v}{\gamma_0}\right)-d_{0,v}\left(v,v_{0u}\right)-\beta\right)\mid_{\mid v=v_{0u}}$	0.848742928103919
$\frac{\partial F(s,\tilde{s})}{\partial \tilde{s}}\mid_{s=\tilde{s}=s^*}$	$-d_{0,v_{0u}}\left(v,v_{0u}\right)\mid_{v=v_{0u}}$	-0.010360117704070
$\frac{\partial V(s,\tilde{s})}{\partial s}\mid_{s=\tilde{s}=s^*}$	$e_{0,v}\left(v,v_{0u}\right)\mid_{v=v_{0u}}$	0.838382810399239

Table A.1: Compare the FOC approach and the smooth pasting condition with no commitment, where $\beta = 7 * 10^{-3}$ and $\xi = 0.1789$.

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