Resource Allocation across Fields: Proportionality, Demand Relativity, and Benchmarking

Marco Ottaviani
Bocconi University

July 6, 2019

Allocating Budget to Fields

Thorniest question in funding—how to assign resources across fields?

Two most common methods:

- **Fixed Budget:** Field-by-field budgets are set ex ante—**top down**
  fields have $\neq$ success rates depending on budget/applications
  E.g.: US Congress appropriations for NSF Directorates

- **Proportional**: Total budget allocated $\propto$ applications in each field
  success rate equalized across fields—**bottom up**
  E.g.: Canadian RFOs, European ERC, Marie-Curie Postdocs

Allocating Budget to Fields

Thorniest question in funding—how to assign resources across fields?

Two most common methods:

- **Fixed Budget:** Field-by-field budgets are set ex ante—**top down**
  fields have $\neq$ success rates depending on budget/applications
  E.g.: US Congress appropriations for NSF Directorates

- **Proportional**: Total budget allocated $\propto$ applications in each field
  success rate equalized across fields—**bottom up**
  E.g.: Canadian RFOs, European ERC, Marie-Curie Postdocs

At US National Institute of Health, NIH:

F. Budget assigned to NIH institutes/centers by Congress appropriations
P. Institutes' paylines equalize percentiled scores across study sections

Allocating Budget to Fields

Thorniest question in funding—how to assign resources across fields?

Two most common methods:

- **Fixed Budget:** Field-by-field budgets are set ex ante—**top down**
  fields have $\neq$ success rates depending on budget/applications
  E.g.: US Congress appropriations for NSF Directorates

- **Proportional**: Total budget allocated $\propto$ applications in each field
  success rate equalized across fields—**bottom up**
  E.g.: Canadian RFOs, European ERC, Marie-Curie Postdocs

At US National Institute of Health, NIH:

F. Budget assigned to NIH institutes/centers by Congress appropriations
P. Institutes' paylines equalize percentiled scores across study sections

Proportional Allocation across Fields

European Research Council ERC allocates budget (€13.1 b in 2014-20):

> "*. . .* **budget** *will be allocated* **to each panel**, **in proportion to the budgetary demand of its assigned proposals**."

- Proportional budget allocation used also by:
  - institutes of US National Institute for Health NIH: $39.2 b in 2019
  - Canadian research funding agencies: Can$4 b in 2018
  - EU Marie Skłodowska-Curie postdoc fellowships: €6.16 b in 2014-20

Impact of proportional allocation on application incentives?

Proportional Allocation across Fields

European Research Council ERC allocates budget (€13.1 b in 2014-20):

> "*. . .* **budget** *will be allocated* **to each panel**, **in proportion to the budgetary demand of its assigned proposals**."

- Proportional budget allocation used also by:
  - institutes of US National Institute for Health NIH: $39.2 b in 2019
  - Canadian research funding agencies: Can$4 b in 2018
  - EU Marie Skłodowska-Curie postdoc fellowships: €6.16 b in 2014-20

Impact of proportional allocation on application incentives?

- Analysis also relevant for:
  - proportional allocation of space in **academic journals** across fields
  - **student admissions** across majors/disciplines equalizing success rate

Proportional Allocation across Fields

European Research Council ERC allocates budget (€13.1 b in 2014-20):

> *". . .* **budget** *will be allocated* **to each panel**, **in proportion to the budgetary demand of its assigned proposals**."

- Proportional budget allocation used also by:
  - institutes of US National Institute for Health NIH: $39.2 b in 2019
  - Canadian research funding agencies: Can$4 b in 2018
  - EU Marie Skłodowska-Curie postdoc fellowships: €6.16 b in 2014-20

Impact of proportional allocation on application incentives?

- Analysis also relevant for:
  - proportional allocation of space in **academic journals** across fields
  - **student admissions** across majors/disciplines equalizing success rate

Paul Romer's *meta-idea*, supporting production and diffusion of other ideas

Proportional Accounting

- Total budget T assigned to panels $i = 1, 2, ..., N$ representing fields
- Grant applications in each panel evaluated by field experts

Proportional Accounting

- Total budget T assigned to panels $i = 1, 2, ..., N$ representing fields
- Grant applications in each panel evaluated by field experts
- Budget allocated to field i by formula

$$\frac{A_i}{\sum_{j=1}^{N} A_j} T$$

". . . budget is calculated as the **cumulative grant request of all proposals to the panel divided** *by the* **cumulative grant request of all proposals** *to the domain of the call,* **multiplied** *by the* **total indicative budget** *of the domain."*

Proportional Accounting

- Total budget T assigned to panels $i = 1, 2, ..., N$ representing fields
- Grant applications in each panel evaluated by field experts
- Budget allocated to field i by formula

$$\frac{A_i}{\sum_{j=1}^{N} A_j} T$$

". . . budget is calculated as the **cumulative grant request of all proposals to the panel divided** *by the* **cumulative grant request of all proposals** *to the domain of the call,* **multiplied** *by the* **total indicative budget** *of the domain."*

Concern that proportional allocation dis/favors certain fields...

Proportional Allocation Equalizes Success Rates

- Under proportional allocation, the *success rate* in field i is

$$\frac{\overbrace{\frac{A_i}{\sum_{j=1}^{N} A_j} T}^{\text{budget available for funding projects in field } i}}{\underbrace{A_i}_{\text{budget demanded by applications in field } i}} = \frac{T}{\sum_{j=1}^{N} A_j} =: p,$$

automatically equalized across fields

Proportional Allocation Equalizes Success Rates

- Under proportional allocation, the *success rate* in field i is

$$\frac{\overbrace{\frac{A_i}{\sum_{j=1}^{N} A_j} T}^{\text{budget available for funding projects in field } i}}{\underbrace{A_i}_{\text{budget demanded by applications in field } i}} = \frac{T}{\sum_{j=1}^{N} A_j} =: p,$$

  automatically equalized across fields

- NIH payline system with percentiled scores is proportional in disguise
  - NIH institutes assigned separate budgets by Congress appropriations
  - institutes award grants to applictions with percentiled score from 180 study sections in top $100 \times p\%$, set to exhaust budget

Proportional Allocation Equalizes Success Rates

- Under proportional allocation, the *success rate* in field i is

$$\frac{\overbrace{\frac{A_i}{\sum_{j=1}^{N} A_j} T}^{\text{budget available for funding projects in field } i}}{\underbrace{A_i}_{\text{budget demanded by applications in field } i}} = \frac{T}{\sum_{j=1}^{N} A_j} =: p,$$

  automatically equalized across fields

- NIH payline system with percentiled scores is proportional in disguise
  - NIH institutes assigned separate budgets by Congress appropriations
  - institutes award grants to applictions with percentiled score from 180 study sections in top $100 \times p\%$, set to exhaust budget

What's Next

Analyze model with costly applications in terms of **demand & supply**:

1. Characterize *partial equilibrium* in **SMALL panel**, GIVEN payline p
   - Panel experts [evaluator] select best $100 \times p\%$ among submissions

What's Next

Analyze model with costly applications in terms of **demand & supply**:

1. Characterize *partial equilibrium* in **SMALL panel**, GIVEN payline p
   - Panel experts [evaluator] select best $100 \times p\%$ among submissions

2. Determine **full equilibrium** with endogenous success rate, when:
   - success rate responds to applications in own field &
   - applications in other fields respond to success rate

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero
2. under realistic conditions, proportional allocation $\longrightarrow$ **MULTIPLE equilibria**

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero
2. under realistic conditions, proportional allocation $\longrightarrow$ **MULTIPLE equilibria**

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero
2. under realistic conditions, proportional allocation $\longrightarrow$ **MULTIPLE equilibria**
3. **applications** in all **stable** full equilibria in a field **increase in NOISE** of evaluation in that field—because of demand relativity

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero
2. under realistic conditions, proportional allocation $\longrightarrow$ **MULTIPLE equilibria**
3. **applications** in all **stable** full equilibria in a field **increase in NOISE** of evaluation in that field—because of demand relativity

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero
2. under realistic conditions, proportional allocation $\longrightarrow$ **MULTIPLE equilibria**
3. **applications** in all **stable** full equilibria in a field **increase in NOISE** of evaluation in that field—because of demand relativity
4. simple **DESIGN modifications** reduce perverse incentives

Preview of Insights

We head to show that:

1. with (sufficiently) precise evaluation, applications **UNRAVEL** to zero
2. under realistic conditions, proportional allocation $\longrightarrow$ **MULTIPLE equilibria**
3. **applications** in all **stable** full equilibria in a field **increase in NOISE** of evaluation in that field—because of demand relativity
4. simple **DESIGN modifications** reduce perverse incentives
5. but perverse incentives are enhanced by **STRATEGIC** *behavior by fields*

Single Field: Partial Equilibrium for Fixed Success Rate

- Field with continuum of risk-neutral potential applicants [=agents]
- Atomistic agents disregard their negligible impact on success rate p

- **Agent observes own merit** $\theta \sim G$ on $\left(\underline{\theta}, \bar{\theta}\right)$, possibly unbounded

Single Field: Partial Equilibrium for Fixed Success Rate

- Field with continuum of risk-neutral potential applicants [=agents]
- Atomistic agents disregard their negligible impact on success rate p

- **Agent observes own merit** $\theta \sim G$ on $\left(\underline{\theta}, \bar{\theta}\right)$, possibly unbounded
- To apply, agent must spend **application cost** $c > 0$
- If application is successful, agent obtains **benefit** $v > c$

Single Field: Partial Equilibrium for Fixed Success Rate

- Field with continuum of risk-neutral potential applicants [=agents]
- Atomistic agents disregard their negligible impact on success rate p

- **Agent observes own merit** $\theta \sim G$ on $\left(\underline{\theta}, \bar{\theta}\right)$, possibly unbounded
- To apply, agent must spend **application cost** $c > 0$
- If application is successful, agent obtains **benefit** $v > c$

- For every applicant, evaluator observes **signal** $x \sim F_{\theta,\sigma}$

$$F_{\theta,\sigma}(x) = F\left(\frac{x-\theta}{\sigma}\right) \tag{1}$$

with location θ (agent type) & scale σ (signal dispersion)

Single Field: Partial Equilibrium for Fixed Success Rate

- Field with continuum of risk-neutral potential applicants [=agents]
- Atomistic agents disregard their negligible impact on success rate p

- **Agent observes own merit** $\theta \sim G$ on $\left(\underline{\theta}, \bar{\theta}\right)$, possibly unbounded
- To apply, agent must spend **application cost** $c > 0$
- If application is successful, agent obtains **benefit** $v > c$

- For every applicant, evaluator observes **signal** $x \sim F_{\theta,\sigma}$

$$F_{\theta,\sigma}(x) = F\left(\frac{x-\theta}{\sigma}\right) \tag{1}$$

  with location θ (agent type) & scale σ (signal dispersion)
- Assume: G and F with continuous densities + f **logcav** [$\Leftrightarrow$MLRP]

Proportional Allocation: Timing

1. Each agent observes own type θ and decides whether to apply or not
2. Evaluator observes signal x & accepts best $100 \times p\%$ applicants

Proportional Allocation: Timing

1. Each agent observes own type θ and decides whether to apply or not
2. Evaluator observes signal x & accepts best $100 \times p\%$ applicants

All equilibria have a monotonic structure:

- Supply: by MLRP, $E[\theta|x]$ increases in signal, so evaluator accepts for

$$x \geq \hat{x}$$

Proportional Allocation: Timing

1. Each agent observes own type θ and decides whether to apply or not
2. Evaluator observes signal x & accepts best $100 \times p\%$ applicants

All equilibria have a monotonic structure:

- Supply: by MLRP, $E[\theta|x]$ increases in signal, so evaluator accepts for

$$x \geq \hat{x}$$

- Demand: higher θ agents are accepted w/ higher prob, thus apply for

$$\theta \geq \hat{\theta}$$

Demand

- Expecting acceptance standard $\hat{x}$, agents apply if

$$\underbrace{v\left[1 - F\left(\frac{\hat{x} - \theta}{\sigma}\right)\right]}_{\text{acceptance probability}} \geq c$$

Demand

- Expecting acceptance standard $\hat{x}$, agents apply if

$$\underbrace{v\left[1-F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right]}_{\text{acceptance probability}} \geq c$$

- Solving, indifferent **marginal applicant** is

$$\hat{\theta} = \hat{x} - \sigma F^{-1}(1-\gamma)$$

where $\gamma := c/v$ is cost-benefit ratio

- Application demand is

$$\begin{aligned} a^D(\hat{x}) &= 1 - G(\hat{\theta}) \\ &= 1 - G\left(\hat{x} - \sigma F^{-1}(1-\gamma)\right) \end{aligned}$$

Demand

- Expecting acceptance standard $\hat{x}$, agents apply if

$$\underbrace{v\left[1-F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right]}_{\text{acceptance probability}} \geq c$$

- Solving, indifferent **marginal applicant** is

$$\hat{\theta} = \hat{x} - \sigma F^{-1}(1-\gamma)$$

where $\gamma := c/v$ is cost-benefit ratio

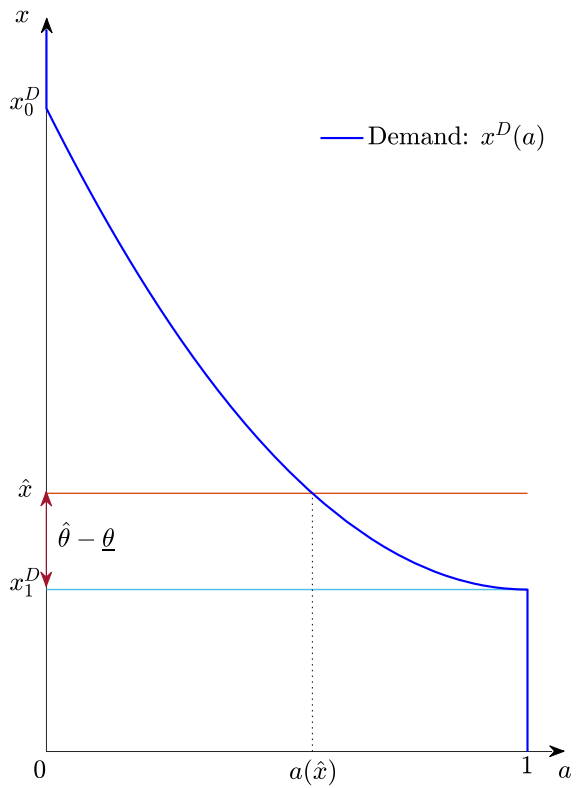
- Application demand is

$$\begin{aligned} a^D(\hat{x}) &= 1 - G(\hat{\theta}) \\ &= 1 - G\left(\hat{x} - \sigma F^{-1}(1-\gamma)\right) \end{aligned}$$

- Inverse demand is $\hat{x}^D(a) = \sigma F^{-1}(1-\gamma) + G^{-1}(1-a)$, **decreasing**

Application Demand

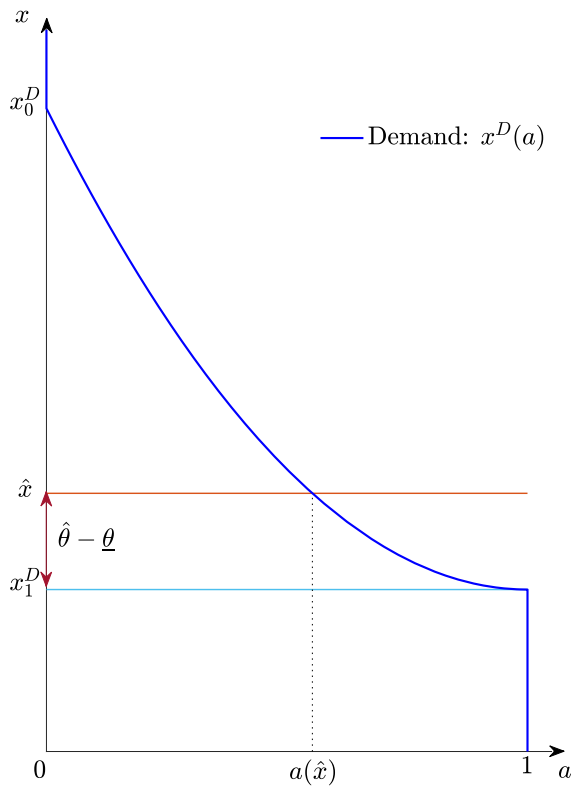
Vertical: acceptance **standard** $\hat{x}$ [like price]; horizontal: **demand** $a^D(\hat{x})$



- When $\hat{x} \uparrow$, win prob is lower for all applicants:

Application Demand

Vertical: acceptance **standard** $\hat{x}$ [like price]; horizontal: **demand** $a^D(\hat{x})$



- When $\hat{x} \uparrow$, win prob is lower for all applicants:
  - fewer agents apply, $a \downarrow$; marginal applicant $\hat{\theta} \uparrow$

Proportional Supply for Fixed Payline

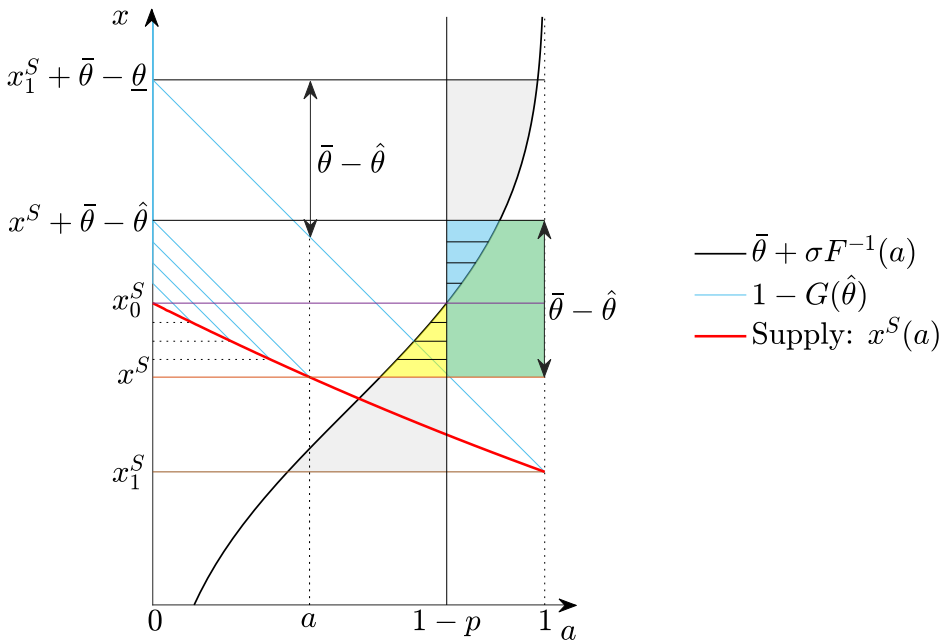
When top a agents apply, pa applications are accepted, so $\hat{x}^S(a)$ solves

$$\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1 - F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right] g(\theta) d\theta = pa$$

Proportional Supply for Fixed Payline

When top a agents apply, pa applications are accepted, so $\hat{x}^S(a)$ solves

$$\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1 - F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right] g(\theta) d\theta = pa$$

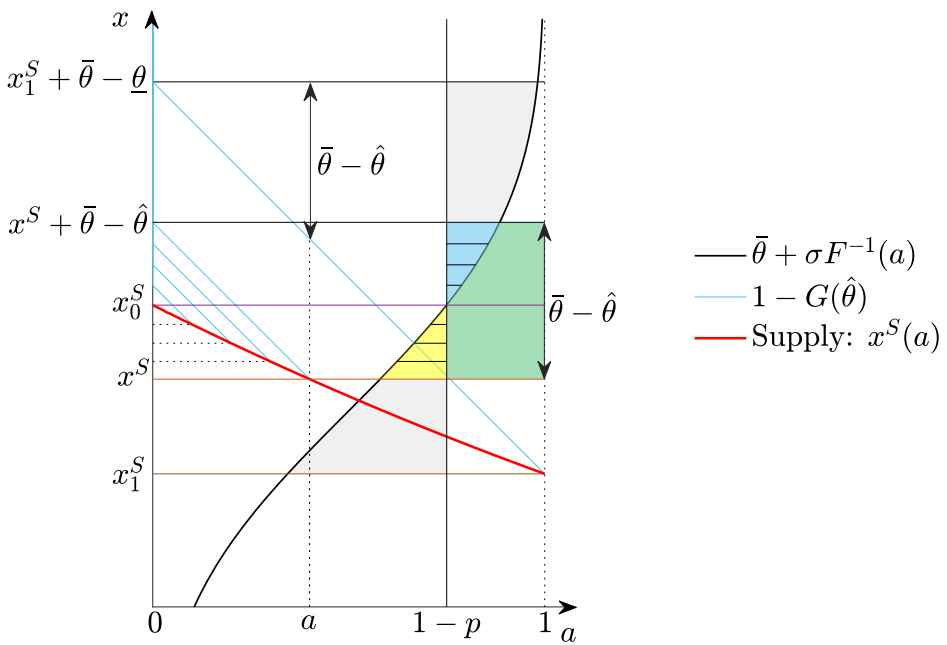


Intuitively, demand [MARGINAL] generates its own supply [AVERAGE]...

Proportional Supply for Fixed Payline

When top a agents apply, pa applications are accepted, so $\hat{x}^S(a)$ solves

$$\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1 - F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right] g(\theta) d\theta = pa$$

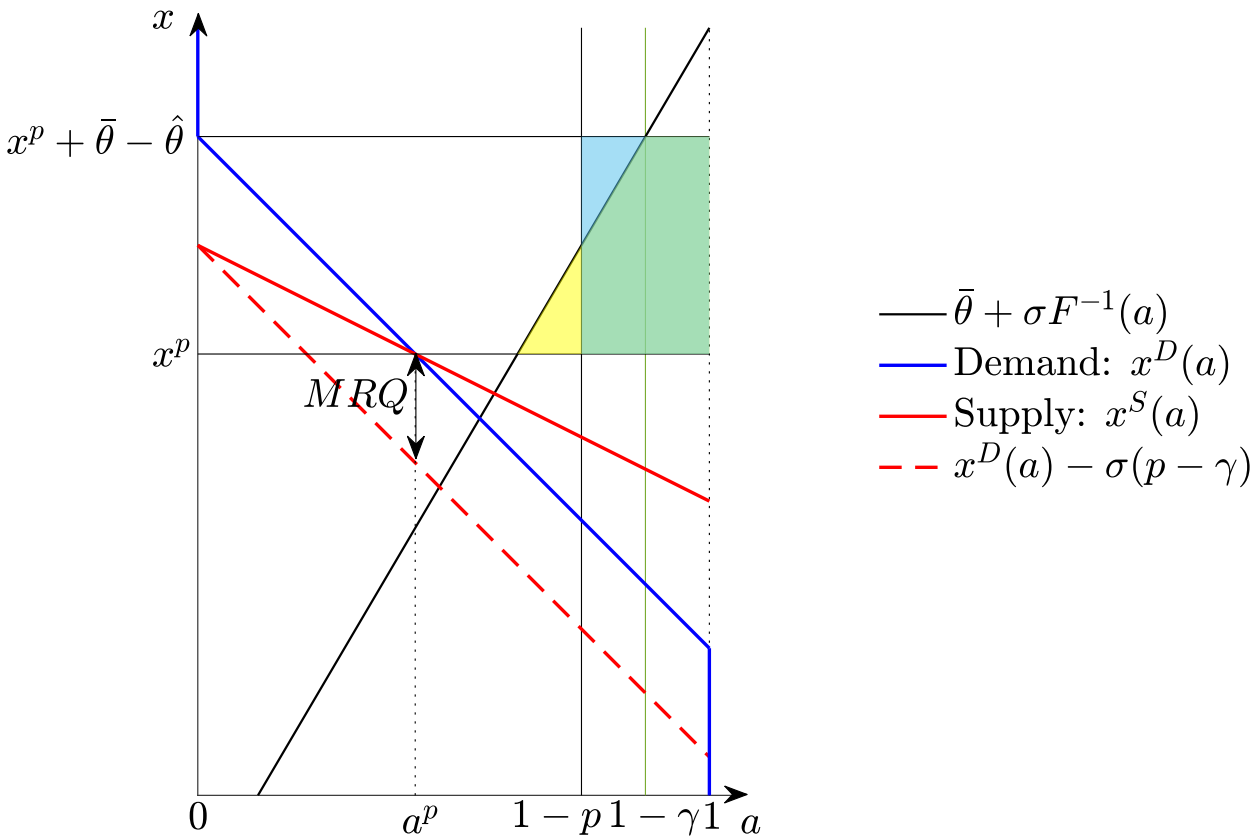


Intuitively, demand [MARGINAL] generates its own supply [AVERAGE]...

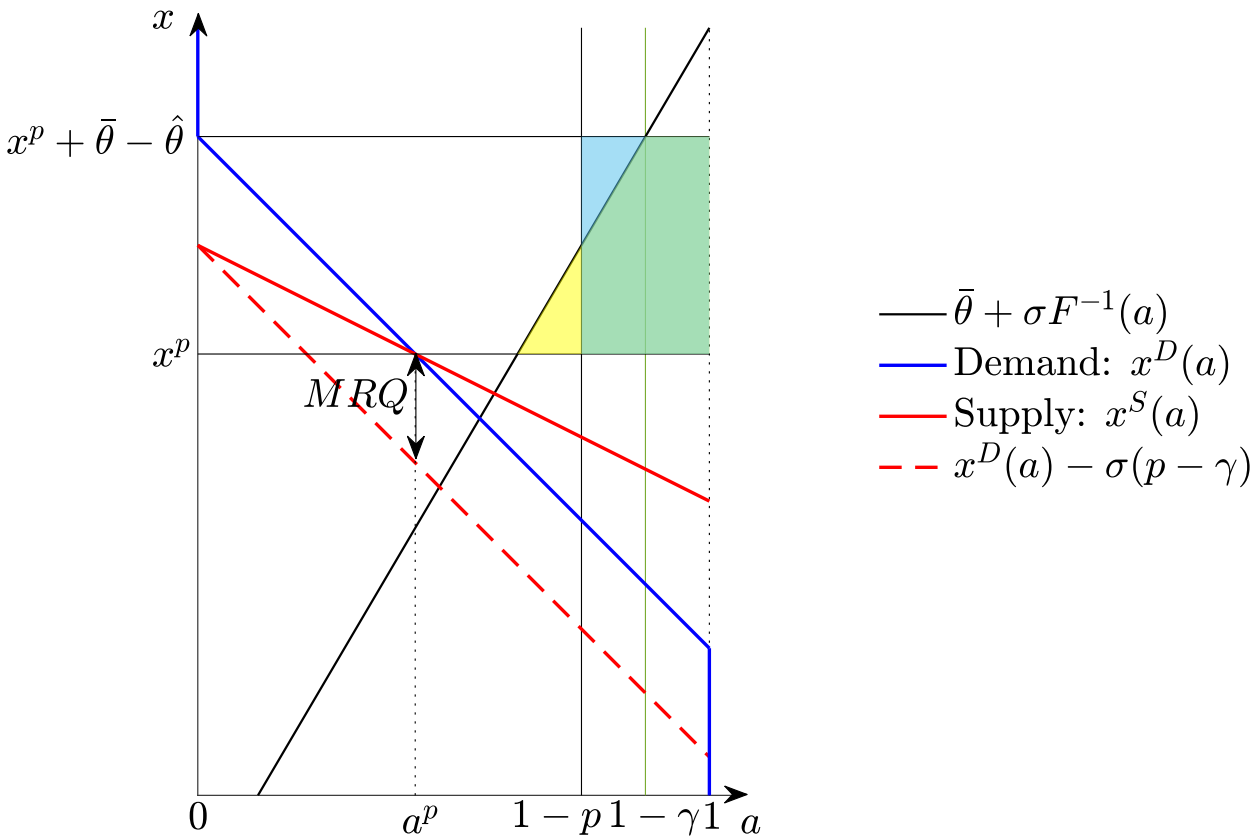
Proportional **supply** [in RED] curve **slopes down!**

- as $a \uparrow$, average applicant quality$\downarrow$, so $\hat{x} \downarrow$ to award constant fraction p

Partial Equilibrium for Fixed Payline



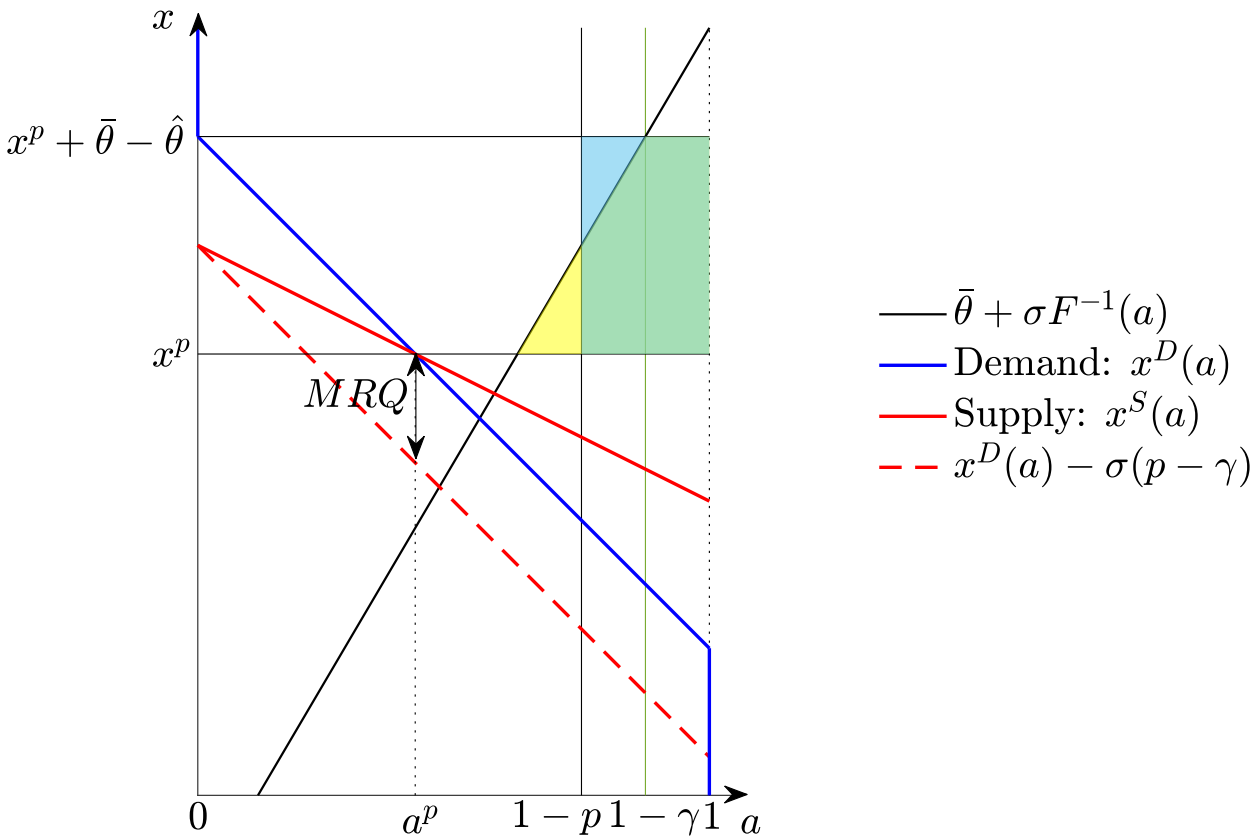
Partial Equilibrium for Fixed Payline



Both demand & supply slope down

- If **supply** flatter than **demand** $\Rightarrow$ equilibrium is unique & stable

Partial Equilibrium for Fixed Payline



Both demand & supply slope down

- If **supply** flatter than **demand** $\Rightarrow$ equilibrium is unique & stable

When does this hold? What happens more generally?

Demand and Supply with Uniform Signal

- Suppose $F(x) = 1/2 + x$ is uniform
- Inverse demand is

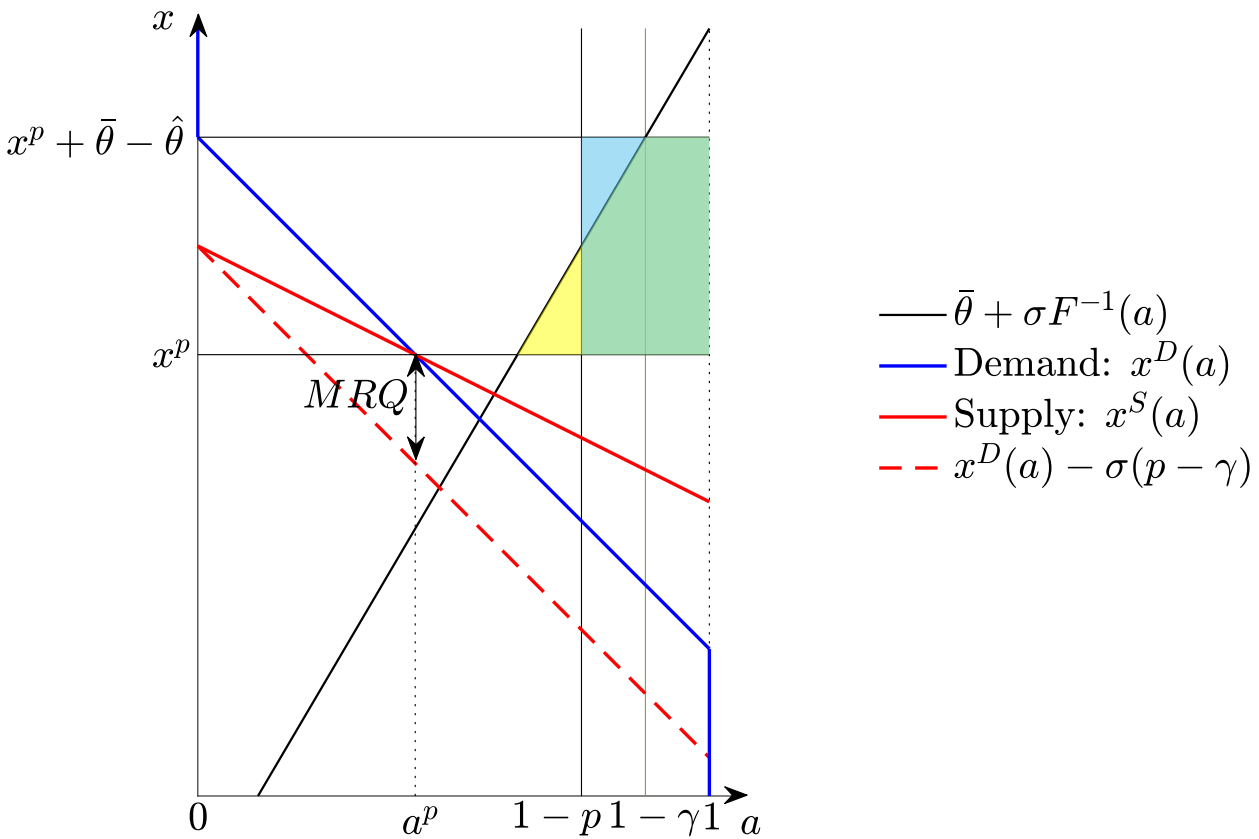
$$\hat{x}^D(a) = \sigma\left(\frac{1}{2} - \gamma\right) + G^{-1}(1-a),$$

- Supply has similar structure to demand

$$\hat{x}^S(a) = \sigma\left(\frac{1}{2} - p\right) + G^{-1}(1-a) + \underbrace{\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1-G(\theta)}{a} d\theta}_{MRQ(a) := E[\theta - G^{-1}(1-a) | \theta \geq G^{-1}(1-a)]},$$

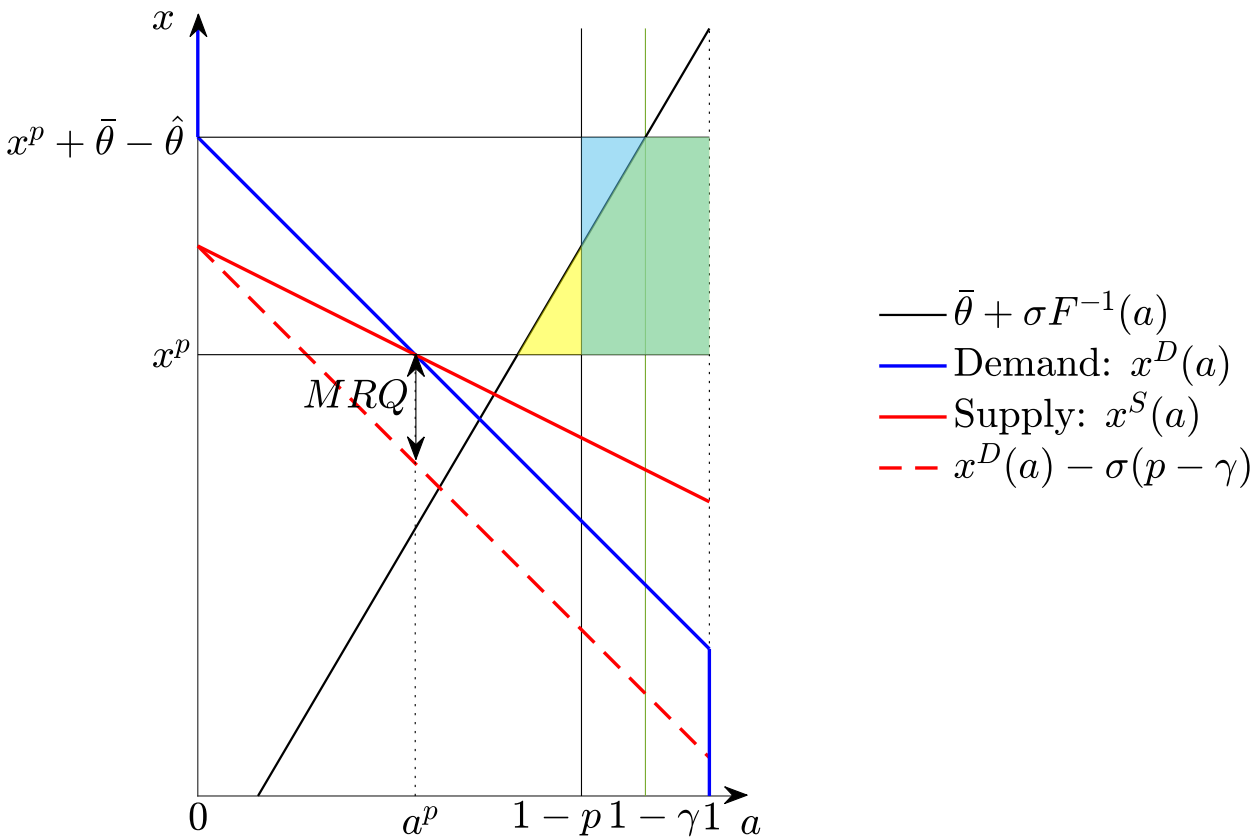
where $MRQ(a)$ is **mean residual quality** when top a applicants apply, with $MRQ(1) = E[\theta]$

$$\hat{x}^S(a) - \hat{x}^D(a) \underset{>}{\overset{<}{\lessgtr}} 0 \Leftrightarrow -\sigma(p-\gamma) + MRQ(a) \underset{>}{\overset{<}{\lessgtr}} 0$$



$g(\theta)$ logcav$\Rightarrow$ $MRQ(a) \Rightarrow$ equilibrium is unique & stable

$$\hat{x}^S(a) - \hat{x}^D(a) \underset{>}{\overset{<}{\lessgtr}} 0 \Leftrightarrow -\sigma(p-\gamma) + MRQ(a) \underset{>}{\overset{<}{\lessgtr}} 0$$



$g(\theta)$ logcav$\Rightarrow$ $MRQ(a) \Rightarrow$ equilibrium is unique & stable

Partial Equilibrium: General Characterization

Proposition

(a) If *the average mean residual life of the type distribution weighted by the signal distribution is lower than the Mills ratio of the type distribution*

$$\frac{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right)\left[1-G\left(t\right)\right] dt}{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right) g\left(t\right) dt} < \frac{1-G\left(\theta\right)}{g\left(\theta\right)}, \qquad \text{(KEY)}$$

partial equilibrium with fixed payline is **unique** & **stable**.

Partial Equilibrium: General Characterization

Proposition

(a) If *the average mean residual life of the type distribution weighted by the signal distribution is lower than the Mills ratio of the type distribution*

$$\frac{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right)[1-G(t)]\,dt}{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right) g(t)\,dt} < \frac{1-G(\theta)}{g(\theta)}, \qquad \text{(KEY)}$$

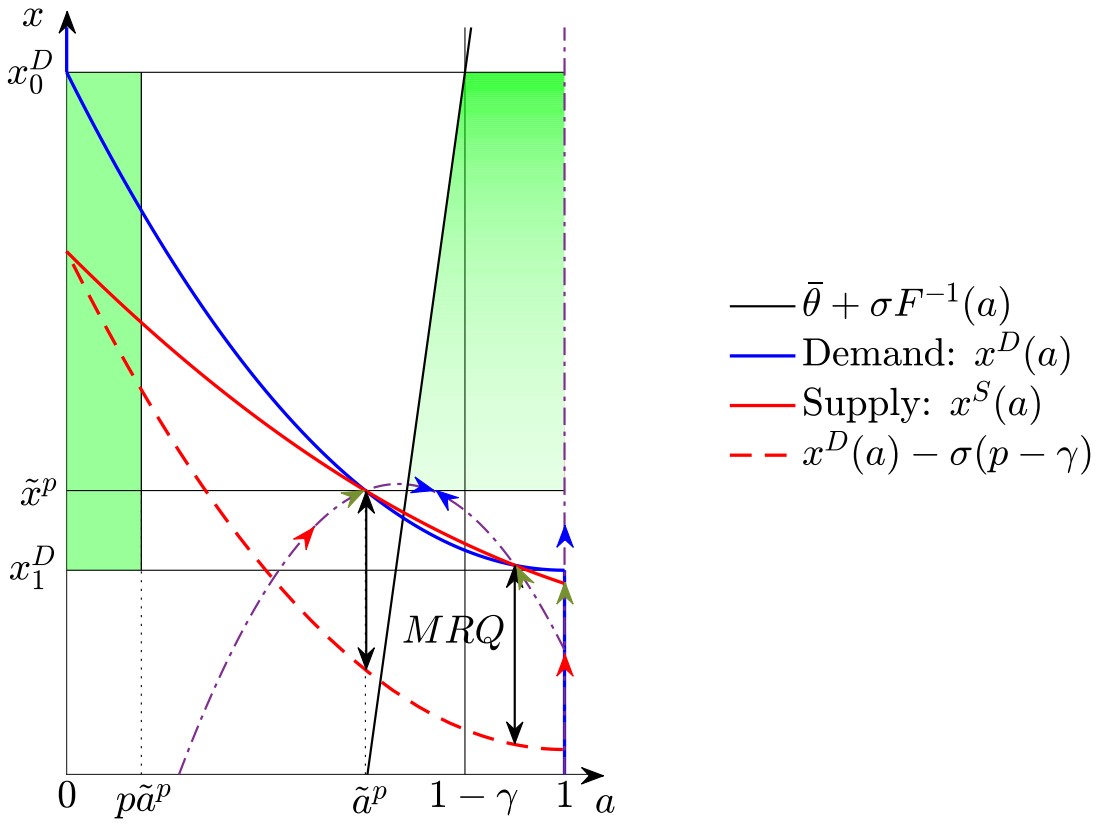
partial equilibrium with fixed payline is **unique** & **stable**.

(b) Sufficient conditions *for (KEY):*
(S1) increasing density $\frac{d}{d\theta} g(\theta) \geq 0$
OR
(S2) increasing hazard rate $\frac{d}{d\theta} \frac{g(\theta)}{1-G(\theta)} \geq 0$

Multiple Equilibria with Non-monotonic MRQ

If MRQ is not monotonic $\Rightarrow$ **multiple** equilibria for some parameters

If $\sigma < \frac{\lim_{\theta \to \bar{\theta}} MRL(\theta)}{(p-\gamma)}$ [e.g., $\lim_{\theta \to \bar{\theta}} MRL(\theta) = \infty$], $a^p = 0$ is a stable equilibrium



Example with $G(\theta) = \sqrt{\theta}$, steeply decreasing density

Comparative Statics wrt Dispersion

Proposition

In every stable (unstable) partial equilibrium, applications a^p increase (decrease) in signal dispersion σ

$$\frac{da^p}{d\sigma} \geq 0 \quad \left(\frac{da^p}{d\sigma} \leq 0\right),$$

strictly so when interior ($0 < a < 1$).

Unique Equilibrium Path as Dispersion Increases

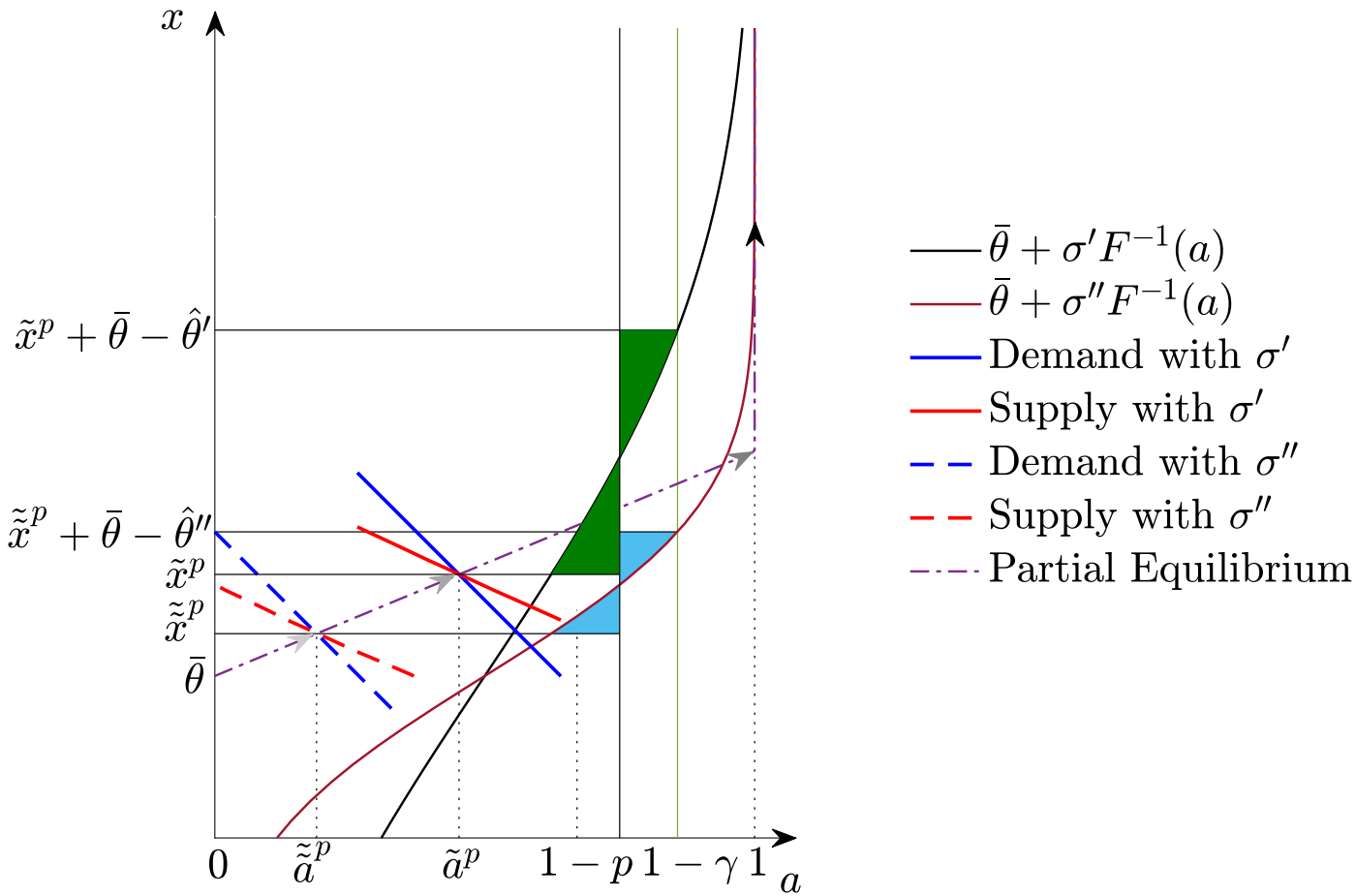


Figure: Partial equilibrium path as signal dispersion σ increases in example with G uniform and F normal.

Partial Equilibrium: Unraveling

- $\sigma \to \infty$, signal x is **completely uninformative** about θ
  - becomes a **lottery**, with everyone applying for $c < pv$

Partial Equilibrium: Unraveling

- $\sigma \to \infty$, signal x is **completely uninformative** about θ
  - becomes a **lottery**, with everyone applying for $c < pv$

- Now decrease $\sigma < \infty$
  - given cost $c > 0$, only top researchers self select into applying
  - within this self-selected pool, only top p applications win
  - as lower end applicants withdraw, bar is continuously raised

Partial Equilibrium: Unraveling

- $\sigma \to \infty$, signal x is **completely uninformative** about θ
  - becomes a **lottery**, with everyone applying for $c < pv$
- Now decrease $\sigma < \infty$
  - given cost $c > 0$, only top researchers self select into applying
  - within this self-selected pool, only top p applications win
  - as lower end applicants withdraw, bar is continuously raised
- $\sigma \to 0$, signal is **perfectly informative**
  - in limit, applicants only spend c if they are SURE to win
  - but only a FRACTION $p < 1$ of these applicants must win...
  - so, unique equilibrium ALWAYS **unravels**, with nobody applying!
- **Thick tail:** $HR \downarrow \Rightarrow MRQ \downarrow \Rightarrow$**unraveling for** $\sigma < \bar{\sigma}$ **for** $\bar{\sigma} > 0$

Akerlof on steroids...

Partial Equilibrium: Unraveling

- $\sigma \to \infty$, signal x is **completely uninformative** about θ
  - becomes a **lottery**, with everyone applying for $c < pv$
- Now decrease $\sigma < \infty$
  - given cost $c > 0$, only top researchers self select into applying
  - within this self-selected pool, only top p applications win
  - as lower end applicants withdraw, bar is continuously raised
- $\sigma \to 0$, signal is **perfectly informative**
  - in limit, applicants only spend c if they are SURE to win
  - but only a FRACTION $p < 1$ of these applicants must win...
  - so, unique equilibrium ALWAYS **unravels**, with nobody applying!
- **Thick tail:** $HR \downarrow \Rightarrow MRQ \downarrow \Rightarrow$**unraveling for** $\sigma < \bar{\sigma}$ **for** $\bar{\sigma} > 0$

Akerlof on steroids... or a cousin of Diamond paradox on dope?

Partial Equilibrium: Unraveling

- $\sigma \to \infty$, signal x is **completely uninformative** about θ
  - becomes a **lottery**, with everyone applying for $c < pv$
- Now decrease $\sigma < \infty$
  - given cost $c > 0$, only top researchers self select into applying
  - within this self-selected pool, only top p applications win
  - as lower end applicants withdraw, bar is continuously raised
- $\sigma \to 0$, signal is **perfectly informative**
  - in limit, applicants only spend c if they are SURE to win
  - but only a FRACTION $p < 1$ of these applicants must win...
  - so, unique equilibrium ALWAYS **unravels**, with nobody applying!
- **Thick tail:** $HR \downarrow \Rightarrow MRQ \downarrow \Rightarrow$**unraveling for** $\sigma < \bar{\sigma}$ **for** $\bar{\sigma} > 0$

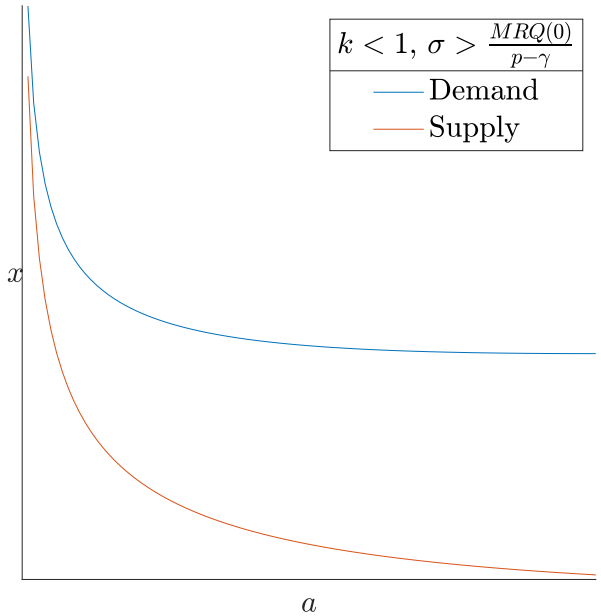
Akerlof on steroids... or a cousin of Diamond paradox on dope?

hold up logic—what is closest in literature?

Partial Equilibrium: Thick Tails

Suppose that $MRQ(a)$ **decreases in** a, 3 cases:

A. If $\sigma > \frac{MRQ(0)}{p-\gamma}$, $a^p = 1$ is unique (& stable) partial equilibrium

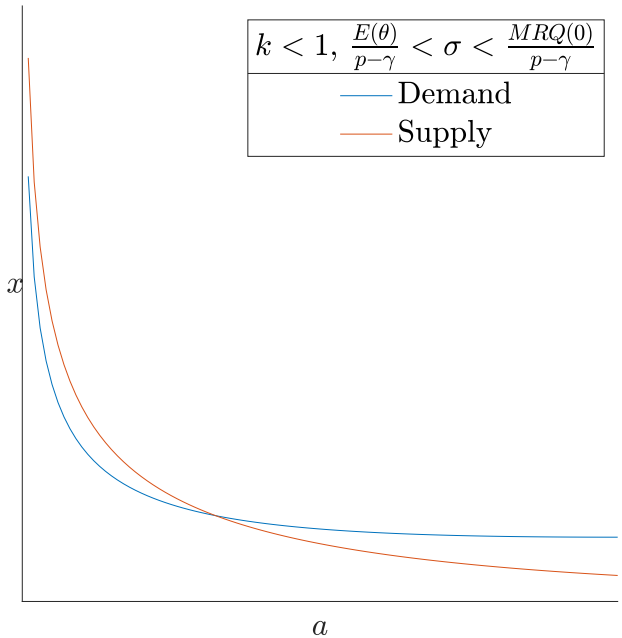


- supply is below demand at $a = 0$
- as $a \uparrow$, distance $\hat{x}^D(a) - \hat{x}^S(a) = \sigma(p - \gamma) - MRQ(a) \uparrow$
- supply always below$\Rightarrow$ $a^p = 1$

Partial Equilibrium: Thick Tails

Suppose that $MRQ(a)$ decreases in a

B. If $\sigma \in \left(\frac{E(\theta)}{p-\gamma}, \frac{MRQ(0)}{p-\gamma}\right) \Rightarrow$ 3 partial equilibria

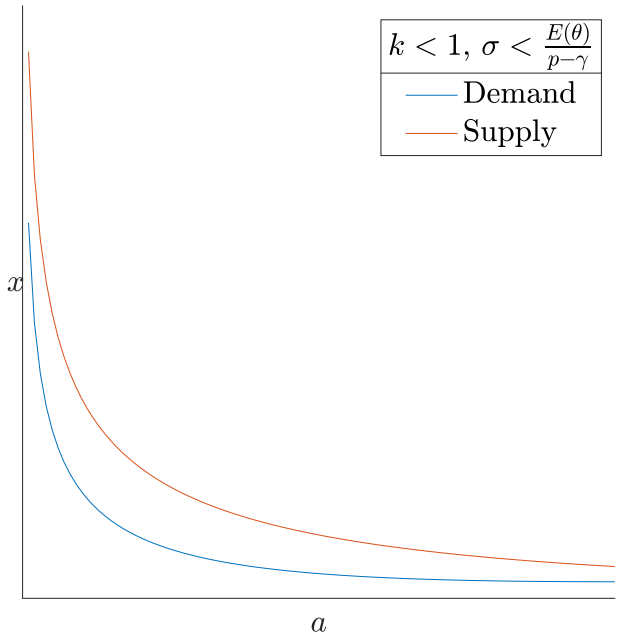


- supply is above demand at $a = 0 \Rightarrow a^p = 0$
- supply is below demand at $a = 1 \Rightarrow a^p = 1$
- supply crosses demand from above at unstable $a^p \in (0, 1)$

Partial Equilibrium: Thick Tails

Suppose that $MRQ(a)$ decreases in a

C. If $\sigma < \frac{E(\theta)}{p-\gamma} \Rightarrow a^p = 0$ is unique (& stable) partial equilibrium



- $\sigma < E(\theta) / (p - \gamma) \Rightarrow$ supply is above demand at $a = 1$
- as a decreases, distance $\hat{x}^S(a) - \hat{x}^D(a)$ increases
- only equilibrium with unraveling $a^p = 0$!

Endogenizing Payline

Field $i = 1, ..., N$ with $G_i, F_i, \gamma_i, \sigma_i$ requests funds $A_i := k_i n_i a_i$, where:

- n_i is #researchers
- k_i average \$ requested (typically at cap)
- a_i fraction of applicants in field

Endogenizing Payline

Field $i = 1, ..., N$ with $G_i, F_i, \gamma_i, \sigma_i$ requests funds $A_i := k_i n_i a_i$, where:

- n_i is #researchers
- k_i average \$ requested (typically at cap)
- a_i fraction of applicants in field

- Given T, $\langle p, (a_i, x_i)_{i=1}^N \rangle$ is a **full equilibrium** where demand equals supply in every field i and the budget condition holds

$$p = \frac{T}{\sum_{j=1}^N A_j},$$

endogenously determining the payline

Full Equilibrium: Characterization

Proposition

(a) *Under (KEY), full equilibrium is unique and stable;*

(b) *In every stable full equilibrium, applications in any field i* **(i)** *increase in the dispersion of evaluation in the same field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_i} \geq 0 \text{ (strictly when interior)}$$

Full Equilibrium: Characterization

Proposition

(a) *Under (KEY), full equilibrium is unique and stable;*

(b) *In every stable full equilibrium, applications in any field i* **(i)** *increase in the dispersion of evaluation in the same field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_i} \geq 0 \text{ (strictly when interior)}$$

(ii) *decrease in the dispersion of the evaluation in any other field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_j} \leq 0$$

Full Equilibrium: Characterization

Proposition

(a) *Under (KEY), full equilibrium is unique and stable;*
(b) *In every stable full equilibrium, applications in any field i* **(i)** *increase in the dispersion of evaluation in the same field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_i} \geq 0 \text{ (strictly when interior)}$$

(ii) *decrease in the dispersion of the evaluation in any other field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_j} \leq 0$$

(iii) *are less/more responsive to σ_i than with fixed/responsive payline*

$$\frac{da_i^p}{d\sigma_i} \geq \frac{da_i^{1,\ldots,N}}{d\sigma_i} \geq \frac{da_i^i}{d\sigma_i} \geq 0.$$

Equilibrium Paths as Dispersion Increases

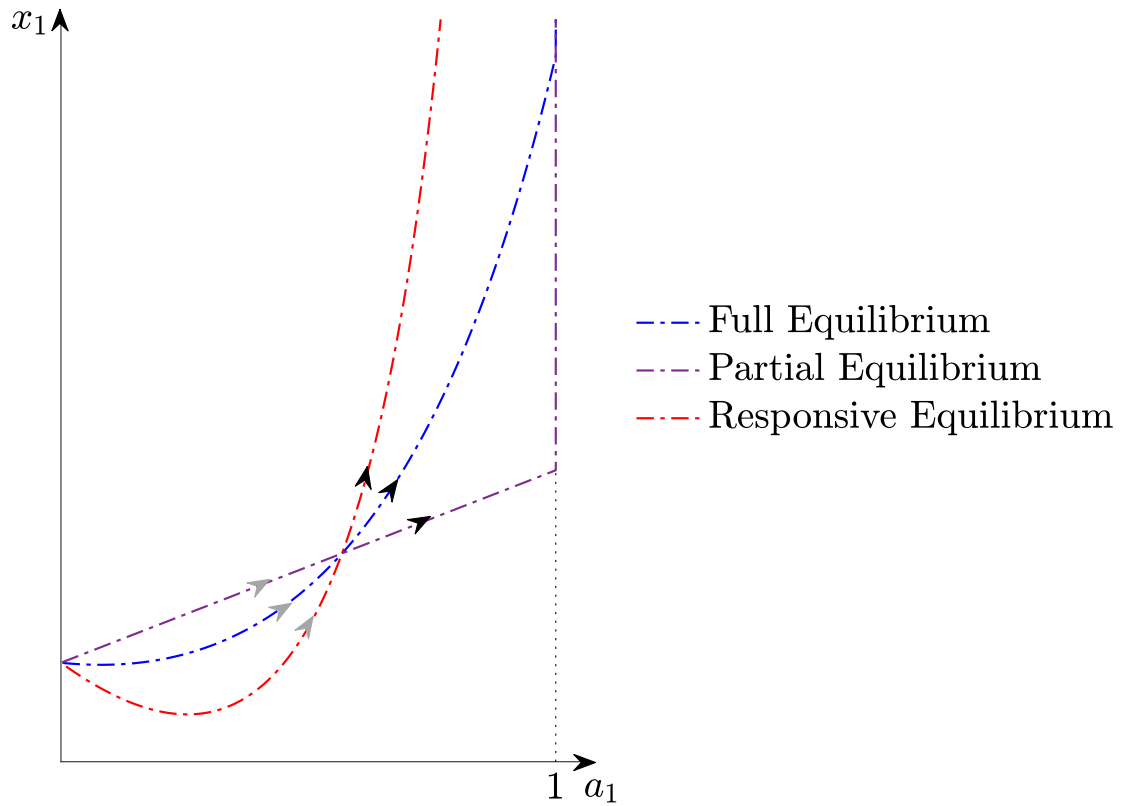


Figure: Example with G uniform and F uniform.

Stability Condition

$$\text{(KEY) } J^{ip} := \begin{vmatrix} \frac{\partial D_1}{\partial x_1} & \frac{\partial D_1}{\partial a_1} \\ \frac{\partial S_1}{\partial x_1} & \frac{\partial S_1^p}{\partial a_1} \end{vmatrix} < 0 \ \forall i \Rightarrow \text{full equilibrium is unique \& stable}$$

Stability Condition

(KEY) $J^{ip} := \begin{vmatrix} \frac{\partial D_1}{\partial x_1} & \frac{\partial D_1}{\partial a_1} \\ \frac{\partial S_1}{\partial x_1} & \frac{\partial S_1^p}{\partial a_1} \end{vmatrix} < 0 \ \forall i \Rightarrow$ full equilibrium is unique & stable

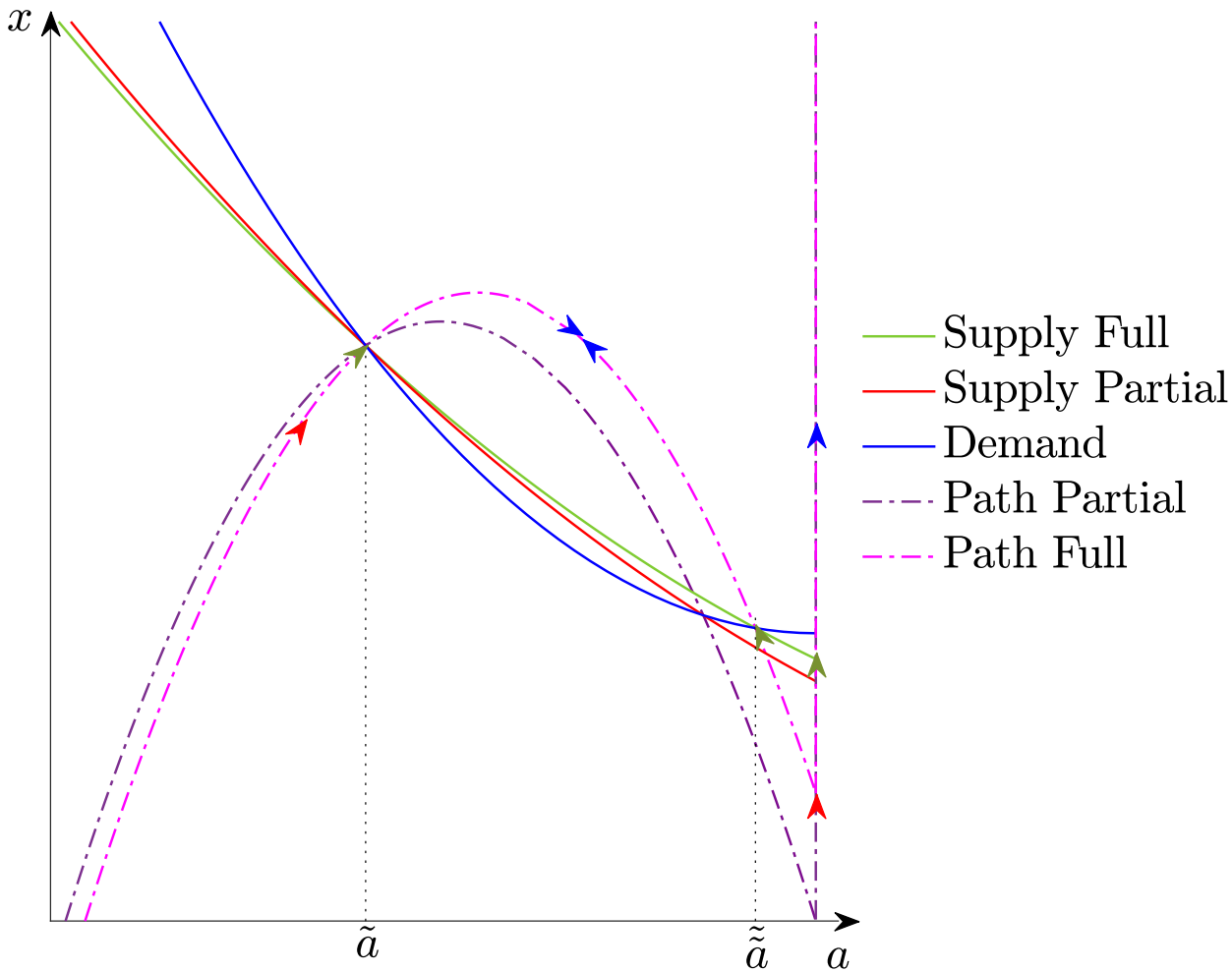
Proof: Jacobian determinant [with proportional allocation $\frac{\partial p_i}{\partial a_i} = \frac{\partial p}{\partial a}$]

$$\begin{vmatrix} \frac{\partial D_1}{\partial x_1} & \frac{\partial D_1}{\partial a_1} & 0 & 0 & 0 & 0 \\ \frac{\partial S_1}{\partial x_1} & \frac{\partial S_1^p}{\partial a_1} - \frac{\partial p_1}{\partial a_1} & 0 & -\frac{\partial p_i}{\partial a_i} & 0 & -\frac{\partial p_N}{\partial a_N} \\ & & \ddots & & & \\ 0 & 0 & \frac{\partial D_i}{\partial x_i} & \frac{\partial D_i}{\partial a_i} & 0 & 0 \\ 0 & -\frac{\partial p_1}{\partial a_1} & \frac{\partial S_i}{\partial x_i} & \frac{\partial S_i^p}{\partial a_i} - \frac{\partial p_i}{\partial a_i} & 0 & -\frac{\partial p_N}{\partial a_N} \\ & & & & \ddots & \\ 0 & 0 & 0 & 0 & \frac{\partial D_N}{\partial x_N} & \frac{\partial D_N}{\partial a_N} \\ 0 & -\frac{\partial p_1}{\partial a_1} & 0 & -\frac{\partial p_i}{\partial a_i} & \frac{\partial S_N}{\partial x_N} & \frac{\partial S_N^p}{\partial a_N} - \frac{\partial p_N}{\partial a_N} \end{vmatrix}$$

$= \prod_{i=1}^{N} \underbrace{J^{ip}} - \sum_{i=1}^{N} \overbrace{\frac{\partial p_i}{\partial a_i} \frac{\partial D_i}{\partial x_i}}^{-} \prod_{j \neq i} \underbrace{J^{jp}}^{-}$ alternates in sign

Multiple Full Equilibria

1 panel with $G(\theta) = \sqrt{\theta}$ & 12 other panels with $G(\theta) = \theta$



Impact of dispersion on applications is dampened in Full Equilibrium

Full Equilibria: Multiple Paths

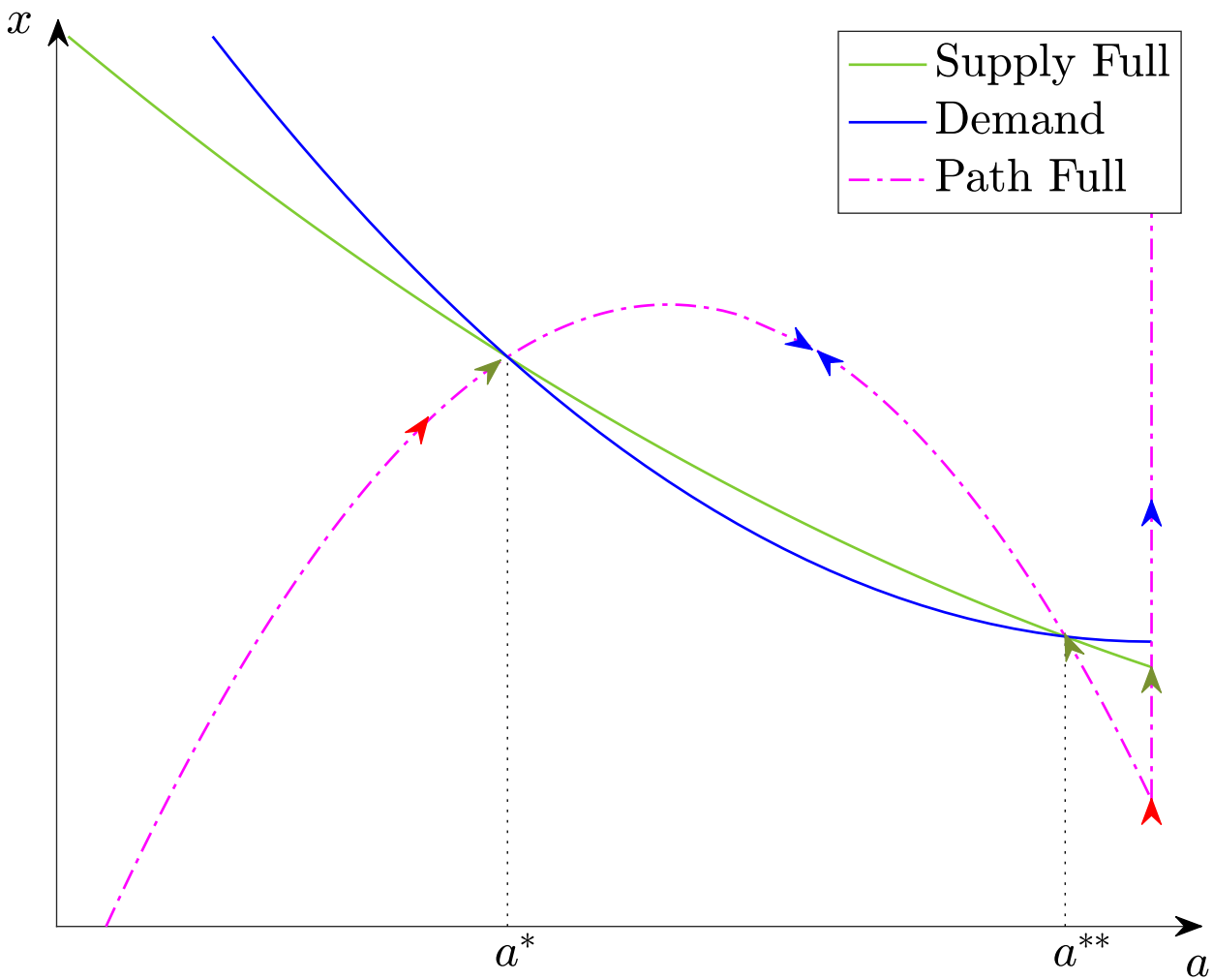
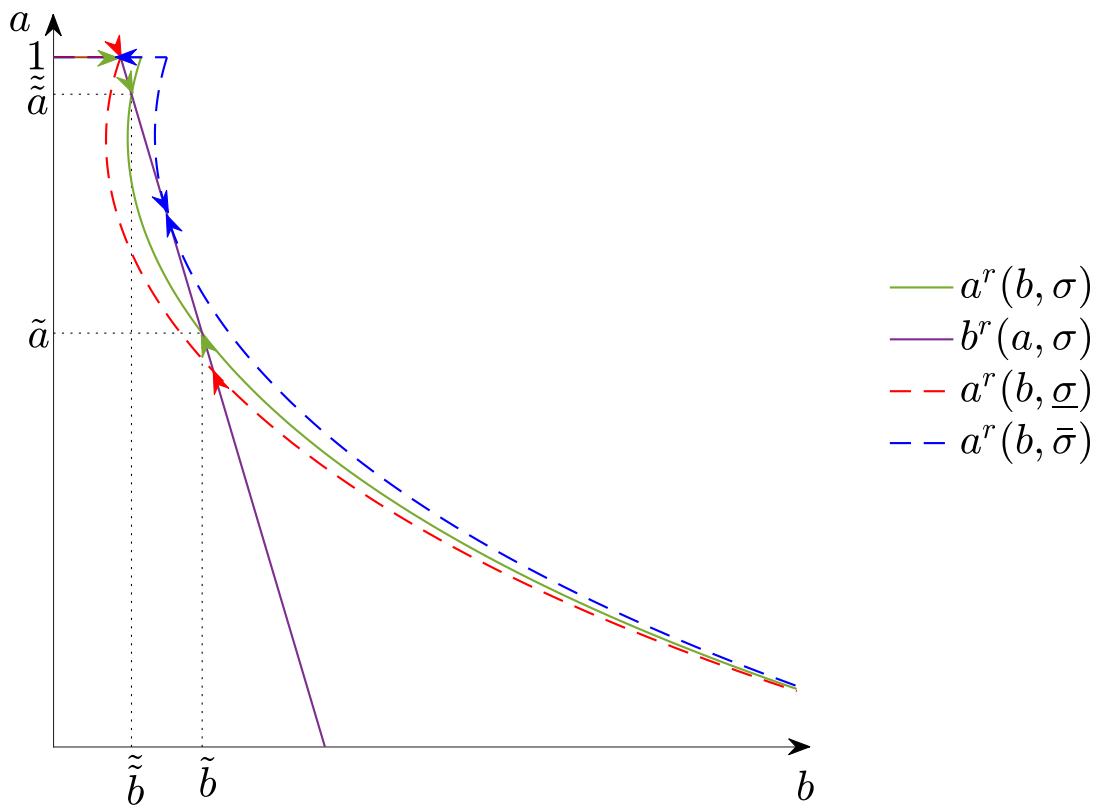


Figure: 13 Panels: 12 Uniformly distributed and 1 distributed with Power.

Partial Equilibrium Correspondences

Applications in field a_i depending on applications in other fields $a_{-i} = b$



FULL equilibrium at intersection of partial equilibrium correspondences

Full Equilbrium: Unraveling

Def: Field unravels when zero applications in all stable full equilibria

Full Equilbrium: Unraveling

Def: Field unravels when zero applications in all stable full equilibria

Asymptotic behavior of MRL $\approx$ inverse of hazard rate

$$\lim_{\theta \to \bar{\theta}} \int_{\theta}^{\bar{\theta}} \frac{1 - G(t)}{1 - G(\theta)} dt = \lim_{\theta \to \bar{\theta}} \frac{1 - G(\theta)}{g(\theta)}$$

Calabria and Pulcini (1987) & Bradley and Gupta (2003)

- If type support is bounded $\bar{\theta} < \infty$ or hazard rate is unbounded [e.g., Normal]

$$\lim_{\theta \to \bar{\theta}} \frac{g(\theta)}{1 - G(\theta)} = \infty \qquad \text{(UHR)}$$

  unraveling results only in a field with perfect info: $\sigma_i = 0$

Full Equilbrium: Unraveling

Def: Field unravels when zero applications in all stable full equilibria

Asymptotic behavior of MRL $\approx$ inverse of hazard rate

$$\lim_{\theta \to \bar{\theta}} \int_{\theta}^{\bar{\theta}} \frac{1 - G(t)}{1 - G(\theta)} dt = \lim_{\theta \to \bar{\theta}} \frac{1 - G(\theta)}{g(\theta)}$$

Calabria and Pulcini (1987) & Bradley and Gupta (2003)

- If type support is bounded $\bar{\theta} < \infty$ or hazard rate is unbounded [e.g., Normal]

$$\lim_{\theta \to \bar{\theta}} \frac{g(\theta)}{1 - G(\theta)} = \infty \qquad \text{(UHR)}$$

  unraveling results only in a field with perfect info: $\sigma_i = 0$

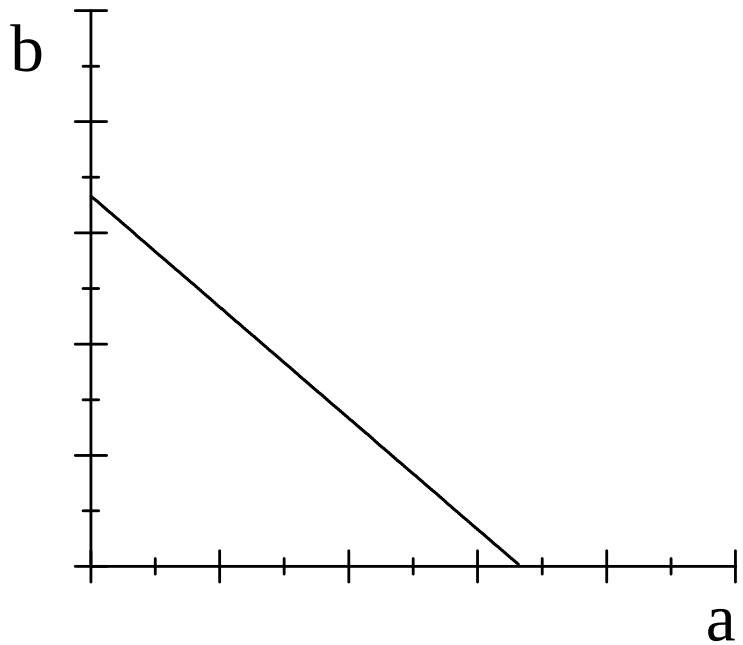
- If hazard rate is bounded [e.g., Pareto-Lomax], unraveling results for $\sigma_i < \underline{\sigma}_i$ with $\underline{\sigma}_i > 0$

Full Equilbrium: Perfect Information in All Fields?

- In field with perfect evaluation, $\sigma_i = 0$

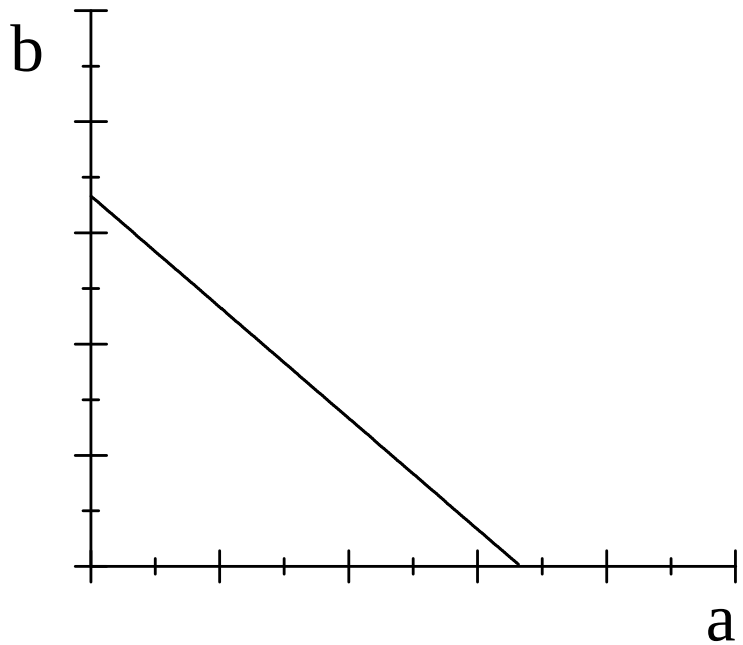
Full Equilbrium: Perfect Information in All Fields?

- In field with perfect evaluation, $\sigma_i = 0$
  - Demand $a_i^D(\hat{x}) = 1 - G(\hat{x})$
  - Supply $\hat{x}_i^S(a) = G^{-1}\left(1 - \frac{Ta}{a+b}\right)$ for given $b = a_{-i} = \sum_{j \neq i} a_j$
  - Partial equilibrium correspondence is $a_i^r(a_{-i}) = \max\langle T - a_{-i}, 0\rangle$



Full Equilbrium: Perfect Information in All Fields?

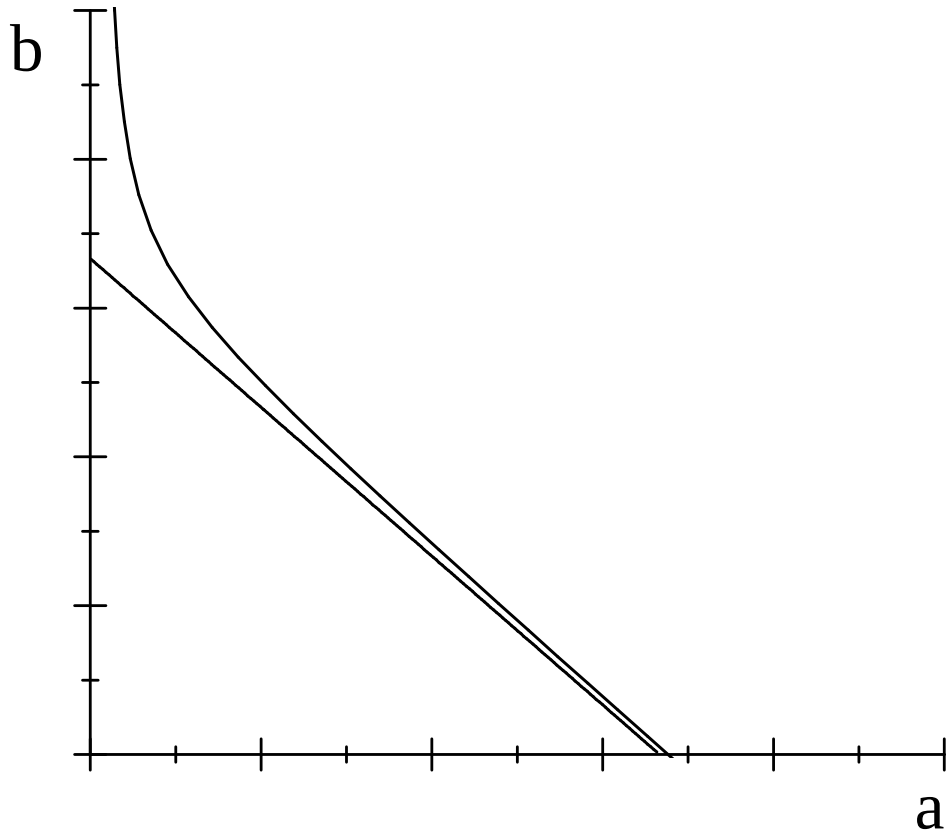
- In field with perfect evaluation, $\sigma_i = 0$
  - Demand $a_i^D(\hat{x}) = 1 - G(\hat{x})$
  - Supply $\hat{x}_i^S(a) = G^{-1}\left(1 - \frac{Ta}{a+b}\right)$ for given $b = a_{-i} = \sum_{j \neq i} a_j$
  - Partial equilibrium correspondence is $a_i^r(a_{-i}) = \max\langle T - a_{-i}, 0\rangle$



- **If** $\sigma_j = 0\ \forall j$, *any* **a** s.t. $\sum_{j=1}^{N} a_j = T$ is an equilibrium with $p = 1$
  - From $a_i^r(a_{-j}) = T - a_{-j} \Rightarrow a^r_{-i}(a_i) = \sum_{i \neq i} a_i^r(a_{-j}) = T - a_i$ QED

Full Equilbrium: Unraveling

- If there is at least one field j with $\sigma_j > 0$
  - then all fields i with $\sigma_i = 0$ unravel to $a_i = 0$!



Full Equilbrium: Unraveling with Exponential Types

The "Ace" Field Takes All!

- Suppose types are exponentially distributed, $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$
- Signal is also exponential in every field with σ_i

Full Equilbrium: Unraveling with Exponential Types

The "Ace" Field Takes All!

- Suppose types are exponentially distributed, $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$
- Signal is also exponential in every field with σ_i
- In full equilibrium all budget T **goes to field with highest index**

$$\iota_i = \frac{\sigma_i - \frac{1}{\alpha_i}}{\sigma_i \gamma_i} \tag{2}$$

Full Equilbrium: Unraveling with Exponential Types

The "Ace" Field Takes All!

- Suppose types are exponentially distributed, $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$
- Signal is also exponential in every field with σ_i
- In full equilibrium all budget T **goes to field with highest index**

$$\iota_i = \frac{\sigma_i - \frac{1}{\alpha_i}}{\sigma_i \gamma_i} \tag{2}$$

  - increasing in dispersion σ_i PERVERSE

Full Equilbrium: Unraveling with Exponential Types

The "Ace" Field Takes All!

- Suppose types are exponentially distributed, $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$
- Signal is also exponential in every field with σ_i
- In full equilibrium all budget T **goes to field with highest index**

$$\iota_i = \frac{\sigma_i - \frac{1}{\alpha_i}}{\sigma_i \gamma_i} \tag{2}$$

  - increasing in dispersion σ_i PERVERSE
  - decreasing in cost-benefit ratio γ_i VIRTUOUS

Full Equilbrium: Unraveling with Exponential Types

The "Ace" Field Takes All!

- Suppose types are exponentially distributed, $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$
- Signal is also exponential in every field with σ_i
- In full equilibrium all budget T **goes to field with highest index**

$$\iota_i = \frac{\sigma_i - \frac{1}{\alpha_i}}{\sigma_i \gamma_i} \tag{2}$$

  - increasing in dispersion σ_i PERVERSE
  - decreasing in cost-benefit ratio γ_i VIRTUOUS
  - decreasing in expected type $\frac{1}{\alpha_i}$ PERVERSE

Full Equilbrium: Unraveling with Exponential Types

The "Ace" Field Takes All!

- Suppose types are exponentially distributed, $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$
- Signal is also exponential in every field with σ_i
- In full equilibrium all budget T **goes to field with highest index**

$$\iota_i = \frac{\sigma_i - \frac{1}{\alpha_i}}{\sigma_i \gamma_i} \tag{2}$$

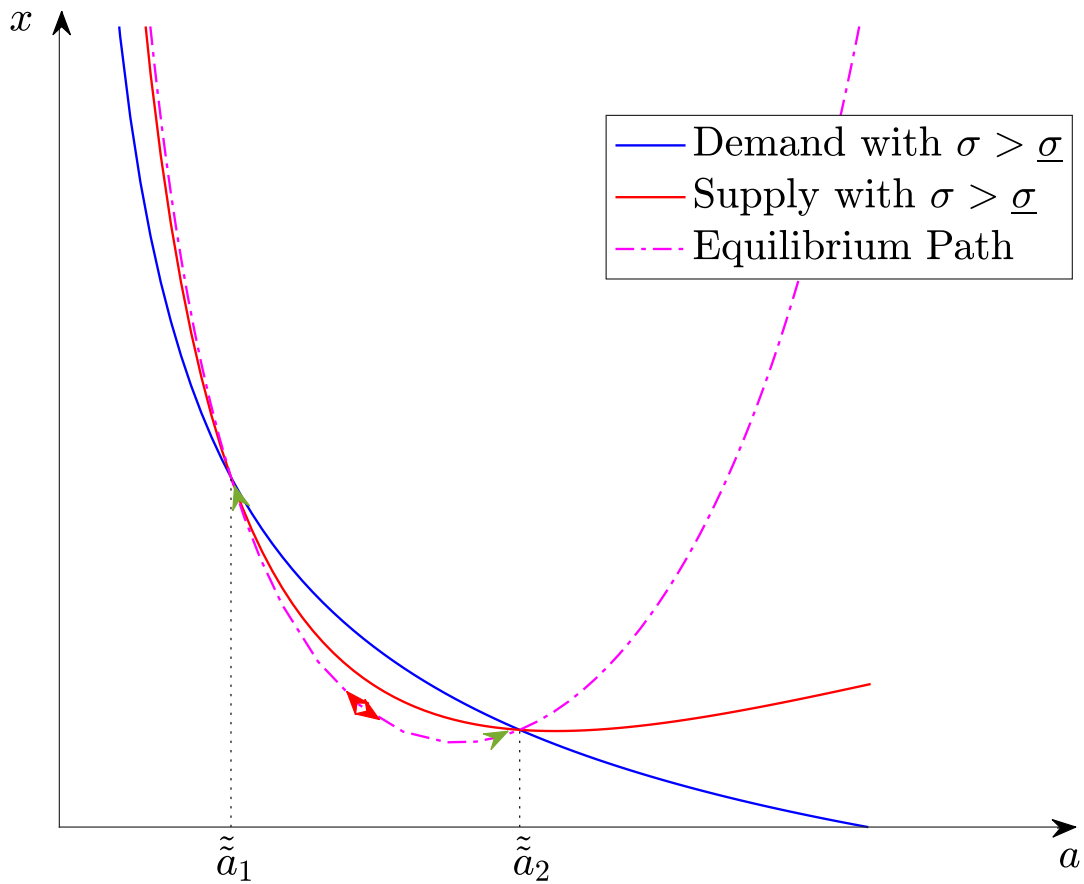
  - increasing in dispersion σ_i PERVERSE
  - decreasing in cost-benefit ratio γ_i VIRTUOUS
  - decreasing in expected type $\frac{1}{\alpha_i}$ PERVERSE
- Fields with *worse* agent pool (lower $1/\alpha_i$) generate *more* applications in equilibrium!
  - result holds more generally with positively skewed distributions

Heavy Tail Type Distribution: Multiplicity

Example: 1 panel with G Pareto-Lomax & 1 panel with G uniform

Heavy Tail Type Distribution: Multiplicity

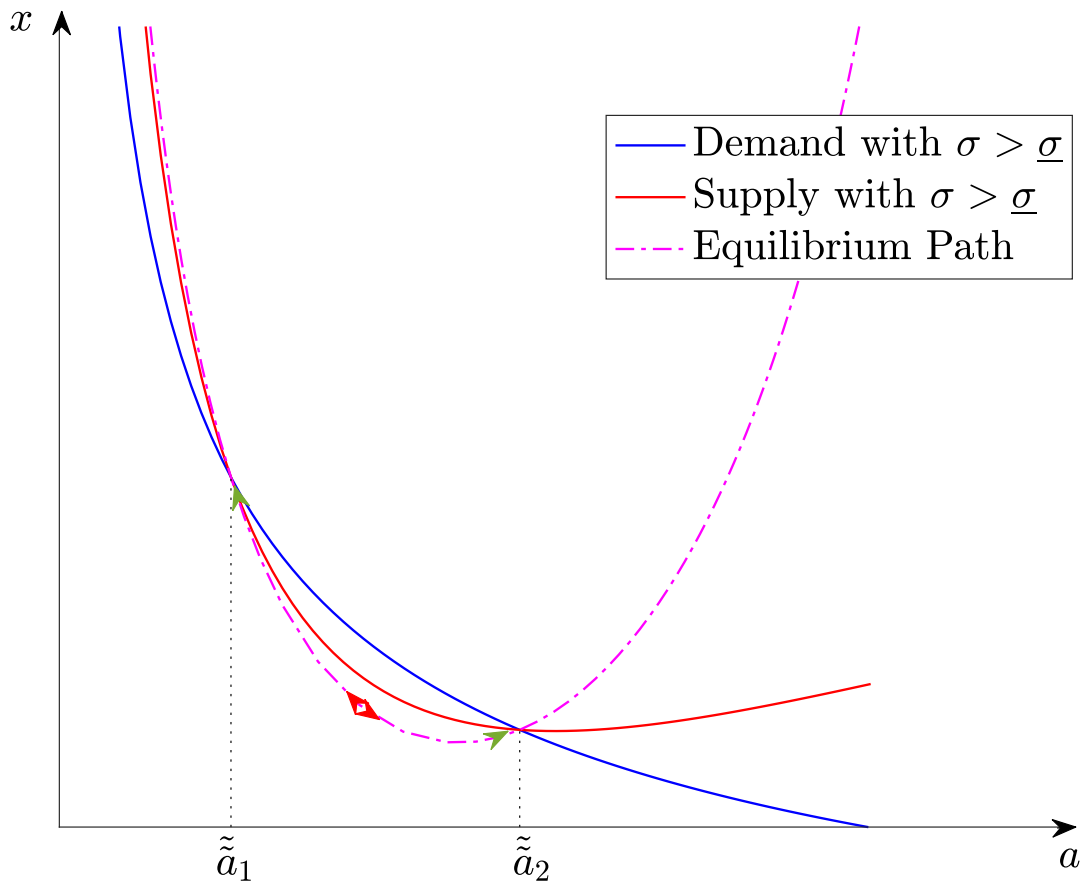
Example: 1 panel with G Pareto-Lomax & 1 panel with G uniform



$\sigma_a > \underline{\sigma}_a$: 1 stable unraveling eq + 2 interior eq: unstable & stable

Heavy Tail Type Distribution: Multiplicity

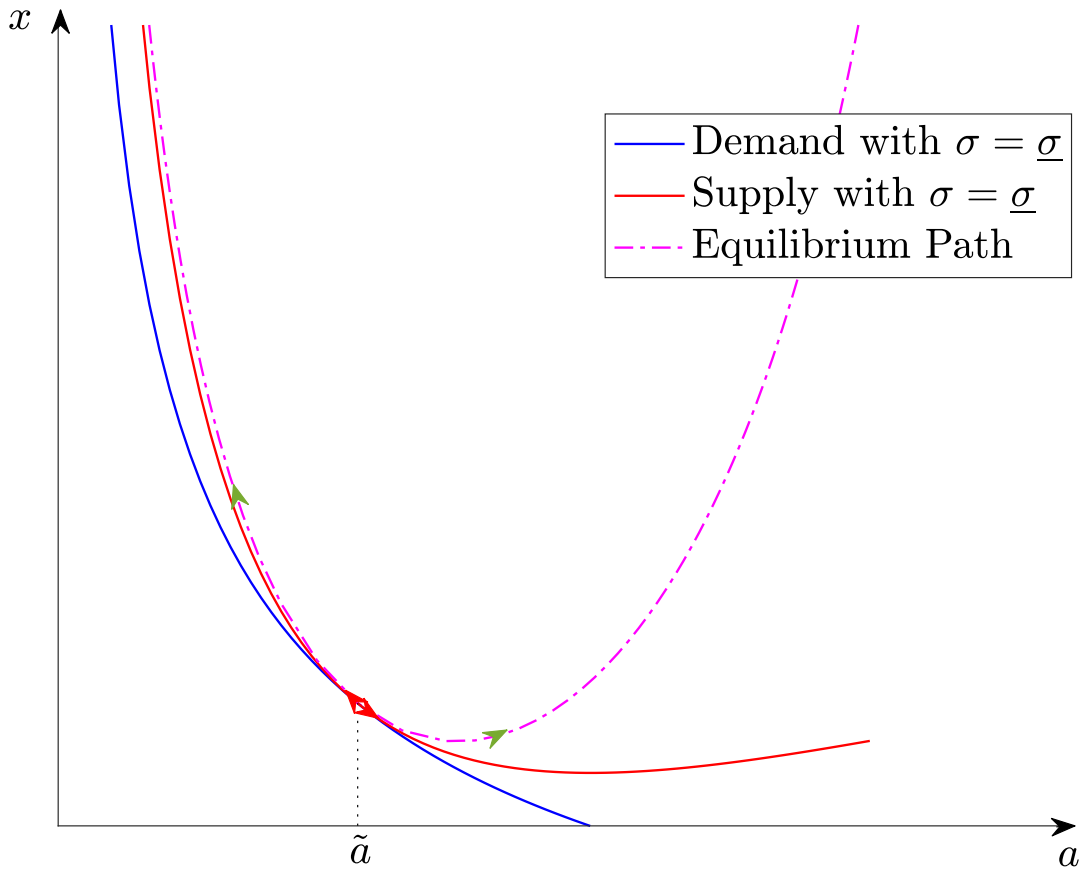
Example: 1 panel with G Pareto-Lomax & 1 panel with G uniform



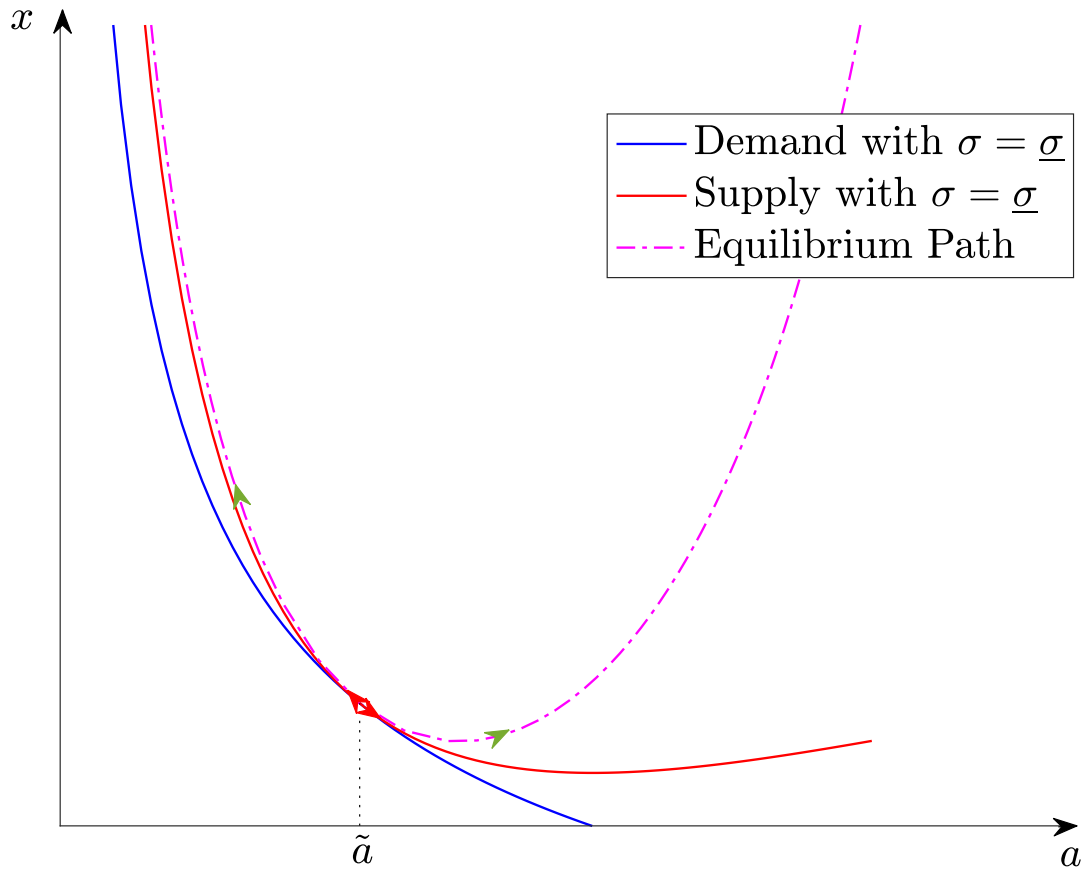
$\sigma_a > \underline{\sigma}_a$: 1 stable unraveling eq + 2 interior eq: unstable & stable

Full equilibrium path as σ increases

Heavy Tail Type Distribution: Tipping and Unraveling



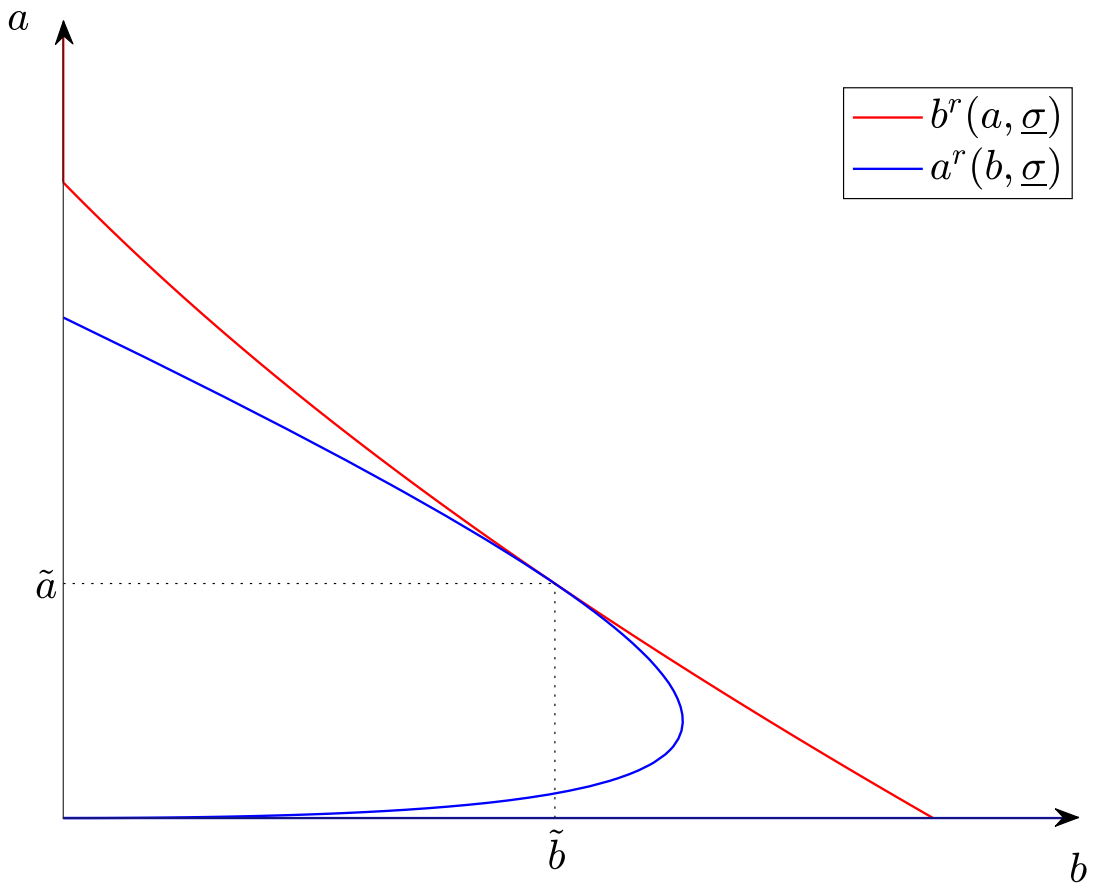
Heavy Tail Type Distribution: Tipping and Unraveling



For $\sigma_a < \underline{\sigma}_a$, with $\underline{\sigma}_a > 0$: ONE stable equilibria w/ unraveling!

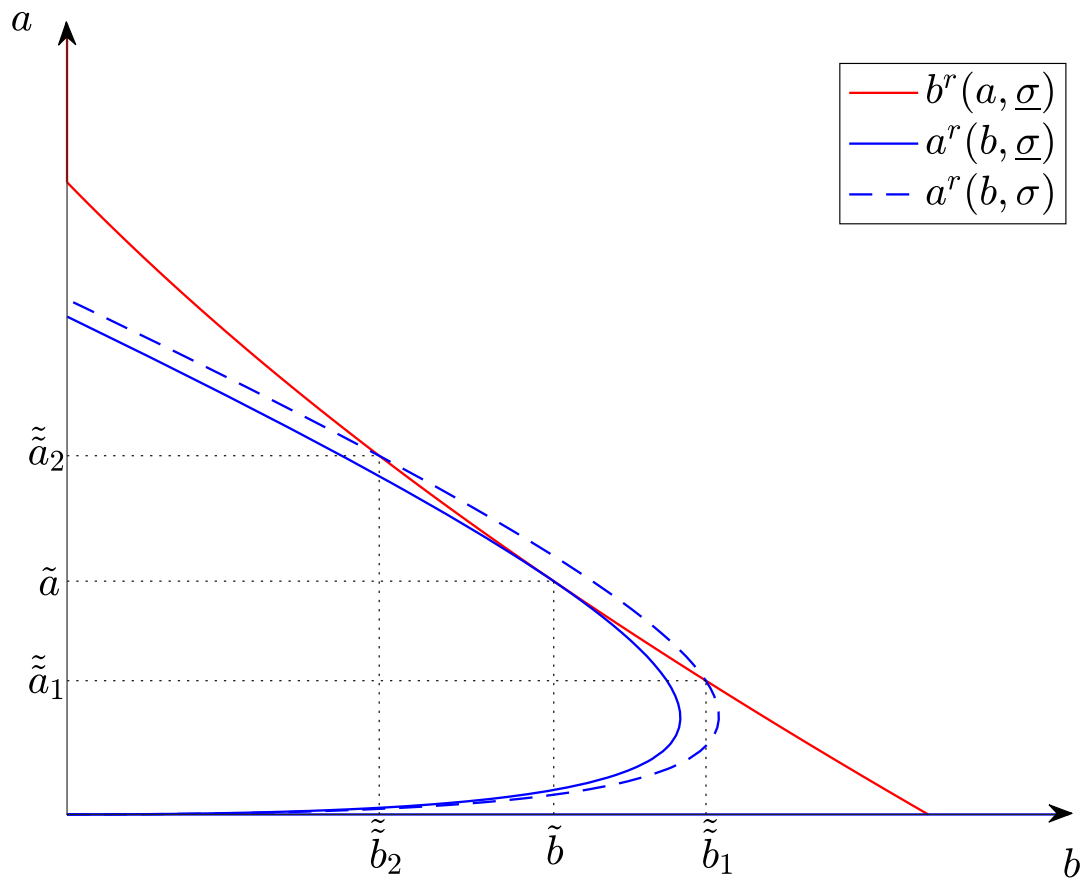
Pareto-Lomax: Partial Equilibrium Correspondences

Applications in field a_i depending on applications in other fielda $a_{-i} = b$



FULL equilibrium at intersection of partial equilibrium correspondences

Pareto-Lomax: Increase in Dispersion in Field a



Partial equilibrium corresp in field a as $\sigma_a \uparrow$ above $\underline{\sigma}_a$

Comparison of ERC Grants: 2008-14 v. 2014-20

Google scholar citations for Advanced Grant PIs (principal investigators)

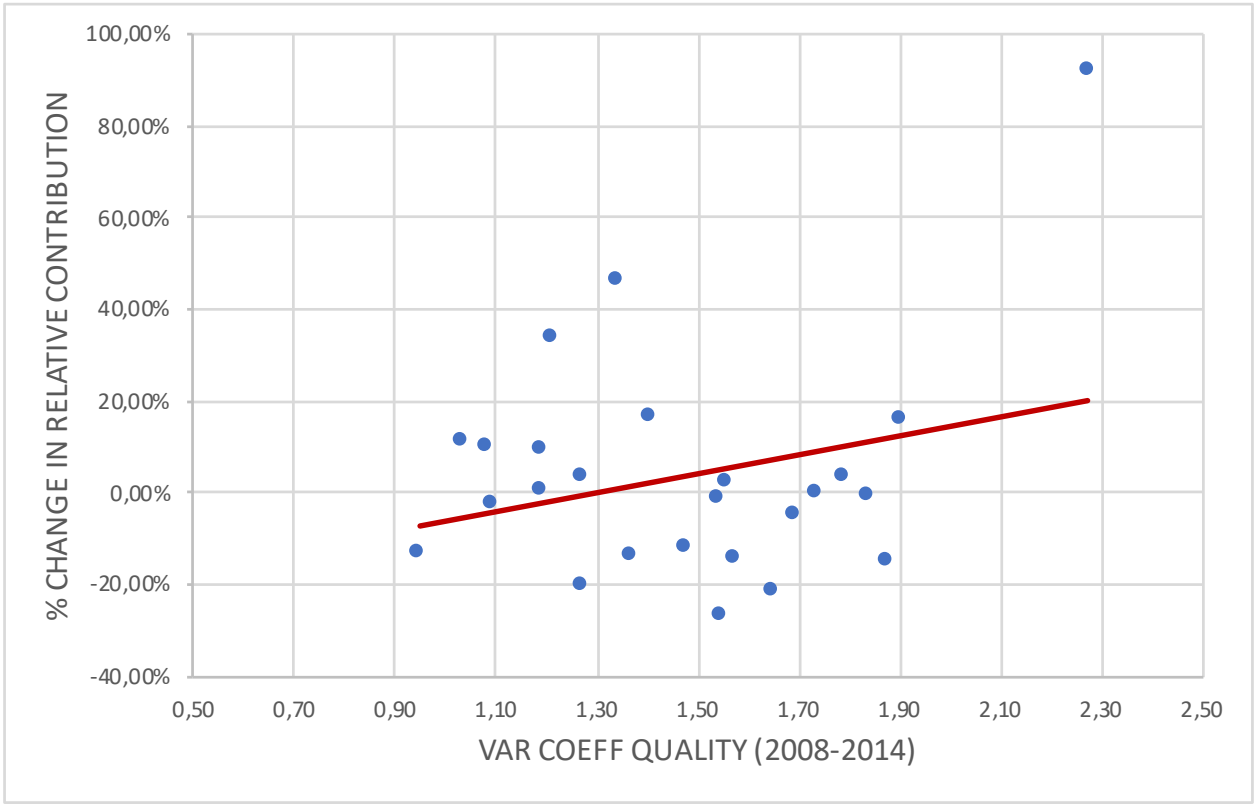
ERC Panel code (0)	Average citations 2008-14 (1)	**Variability in citations 2008-14 (2)**	Average citations 2015-20 (3)	Variability in citations 2015-20 (4)	´% **budget fraction (5)**
LS1	389.82	**1.65**	467.58	2.21	**–21.52%**
...	...	**...**	...	...	**...**
PE1	192.08	**1.55**	208.66	1.51	**–27.27%**
...	...	**...**	...	...	**...**
PE10	274.99	**1.03**	345.64	1.37	**11.1%**
SH1	418.12	**1.73**	654.4	1.29	**–0.31%**
SH2	260.58	**1.54**	413.79	1.88	**–1.59%**
SH3	220.79	**1.34**	290.04	1.43	**46.22%**
SH4	323.44	**1.19**	441.44	0.94	**0.15%**
SH5	35.8	**2.27**	109.66	2.89	**91.45%**
SH6	101.95	**1.9**	270.96	2.84	**15.74%**

Comparison of ERC Grants: 2008-14 v. 2014-20

Panels with **higher variability** in citations in **first period**:

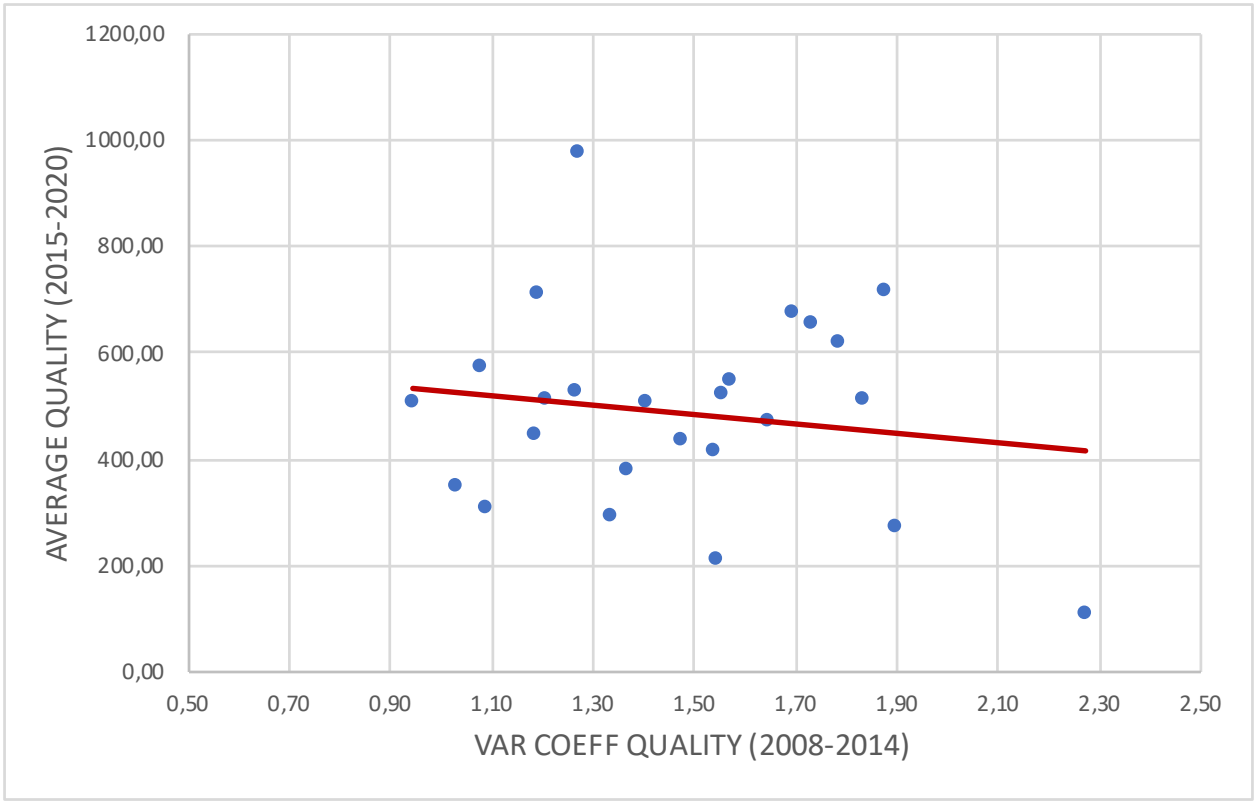
- in second period obtain **more funding**
- for PIs that tend to attract **less citations**

Funding Growth 08-14 to 14-20; CV of Citations 08-14



- Correlation 0.27 between:
  - coefficient of variation of citations in the first period (column 2)
  - growth in funding from first to second period (column 5)

Average Citations 14-20; CV of Citations 08-14



- Correlation -0.16 between:
  - coefficient of variation of citations in the first period (column 2)
  - average citations in the second period (column 3)

Inefficiency

- $N = 2$ fields otherwise identical fields with $\sigma_1 = 0$ & $\sigma_2 = \infty$
- Proportional allocation assigns all funds in field 2, randomly

Inefficiency

- $N = 2$ fields otherwise identical fields with $\sigma_1 = 0$ & $\sigma_2 = \infty$
- Proportional allocation assigns all funds in field 2, randomly
- In this admittedly simple scenario, **worst possible** system!

Optimizing Allocation Formula

Social welfare

$$W = \sum_{i=1}^{N} \int_{\hat{\theta}_i}^{\overline{\theta}} \{ \int_{\hat{x}_i}^{\overline{x}} [\underbrace{E\left(\theta | x; \theta \geq \hat{\theta}_i\right) - f_i}_{\text{evaluator}} + \underbrace{v_i}_{\text{agent}}] f(x|\theta)\, dx - \underbrace{c_i}_{\text{agent}} \} g(\theta)\, d\theta$$

expected net merit; benefit; cost

where

- $E\left(\theta | x; \theta \geq \hat{\theta}_i\right)$ incorporates self-selection into applying
- f_i is evaluator opportunity cost of funds for field i

Optimizing Allocation Formula

Social welfare

$$W = \sum_{i=1}^{N} \int_{\hat{\theta}_i}^{\overline{\theta}} \{ \int_{\hat{x}_i}^{\overline{x}} [\underbrace{E\left(\theta | x; \theta \geq \hat{\theta}_i\right) - f_i}_{\text{evaluator}} + \underbrace{v_i}_{\substack{\text{agent} \\ \text{benefit}}}] f(x|\theta)\, dx - \underbrace{c_i}_{\substack{\text{agent} \\ \text{cost}}} \} g(\theta)\, d\theta$$

expected net merit

where

- $E\left(\theta | x; \theta \geq \hat{\theta}_i\right)$ incorporates self-selection into applying
- f_i is evaluator opportunity cost of funds for field i

Some proportionality is desirable if $\frac{c_i}{v_i} \neq \frac{c_j}{v_j}$, less so if $\sigma_i \neq \sigma_j$

Adjusting the Allocation Formula

- Generalized proportional allocation: $\frac{g_i(A_i)}{\sum_{j=1}^{N} g_j(A_j)}$ eg $g_i(A_i) = b_i A_i^{\rho_i}$

$$\frac{\overbrace{\frac{g_i(A_i)}{\sum_{j=1}^{N} g_j(A_j)} T}^{\text{budget available for funding projects in field } i}}{\underbrace{A_i}_{\text{budget demanded by applications in field } i}} = \frac{T g_i(A_i)}{A_i \sum_{j=1}^{N} g_j(A_j)} =: p(A_i, A_j) \tag{3}$$

- Superproportional $\frac{\partial}{\partial A_i} \frac{g_i(A_i)}{A_i} \geq 0$ (eg $\rho_i \geq 1$)=>unraveling under KEY
  - actually, unraveling holds more broadly for $\sigma \leq \underline{\sigma}$ for $\underline{\sigma} > 0$
- Subproportional: $\rho_i < 1$ unraveling under KEY disappears
  - comparative statics robust

Power Allocation Formula

- Power allocation: $g_i(A_i) = A_i^\rho$

$$\frac{\overbrace{\frac{A_i^\rho}{\sum_{j=1}^N A_j^\rho} T}^{\text{budget available for funding projects in field } i}}{\underbrace{A_i}_{\text{budget demanded by applications in field } i}} = \frac{T A_i^{\rho-1}}{\sum_{j=1}^N A_j^\rho} =: p(A_i, A_j) \quad (4)$$

- $\frac{\partial a^{1,\ldots,N}}{\partial \sigma} > 0$ in all stable equilibria for all ρ, but less so for lower ρ

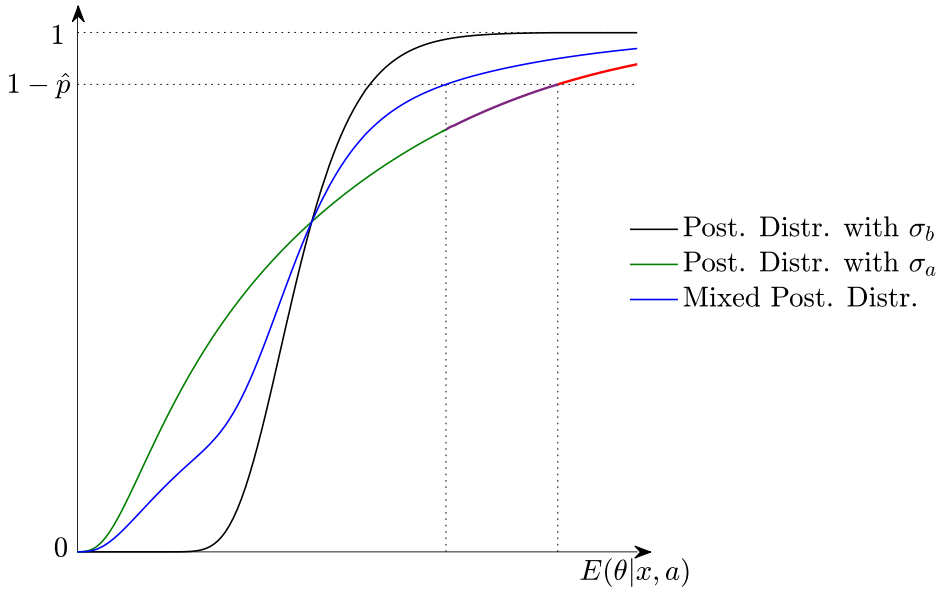
$$\frac{\partial a^{1,\ldots,N}}{\partial \sigma \partial \rho} > 0$$

NIH Benchmarking Improves Accuracy Incentives!

- Clever design tweak: NIH benchmarks scores against previous 2 cycles

NIH Benchmarking Improves Accuracy Incentives!

- Clever design tweak: NIH benchmarks scores against previous 2 cycles



- Distribution of **previous** scores $E[\theta|b,x]$, from normal signal with σ_a
- With $\sigma_a < \sigma_b$, **current** scores $E[\theta|a,x]$ are *MORE* dispersed
- Winners determined by crossing **mixture** distribution with payline p
  - now with improved acccuracy more applications win—extra violet!

Field Game

- Perverse incentives for fields, which face a collective action problem
- Scientific associations can coordinate field-level outcomes by
  - advertising availability of grants & supporting applications through
  - seed grant schemes, information sessions & seminars on grant writing

Field Game

- Perverse incentives for fields, which face a collective action problem
- Scientific associations can coordinate field-level outcomes by
  - advertising availability of grants & supporting applications through
  - seed grant schemes, information sessions & seminars on grant writing
- Field association i's maximizes total payoff of researchers in field

$$\max_{a_i} \quad \underbrace{v_i p\left(a_i, a_{-i}\right)}_{\text{inverse demand } P_i(a_i,a_{-i})=\frac{T}{a_i+a_{-i}}} a_i - \underbrace{c_i}_{\text{marginal cost}} a_i$$

- Proportional allocation rule induces a hyperbolic demand with

$$\varepsilon_i = \frac{da_i}{dP_i}\frac{P_i}{a_i} = -1 - \frac{a_i}{a_{-i}}$$

- Marshall's second law of demand holds $\partial\varepsilon_i/\partial a_i > 0 \Rightarrow$ concavity

Field Game: Equilibrium

- Lerner's formula gives best reply

$$\frac{P_i - c_i}{P_i} = \frac{p - \gamma_i}{\gamma_i} = \frac{1}{-\varepsilon} \Rightarrow a_i = \min \left\langle \sqrt{\frac{a_{-i} T}{\gamma_i}} - a_{-i}, 1 \right\rangle$$

Proposition

In unique interior equilibrium of the field game, applications in field i are

$$a_i^{(N)} = (N-1)\, T \frac{\sum_{j=1}^{N} \gamma_j - (N-1)\gamma_i}{\left(\sum_{j=1}^{N} \gamma_j\right)^2} \tag{5}$$

Equilibrium success rate (as well as payline) is $p = \frac{\sum_{j=1}^{N} \gamma_j}{N-1}$. If fields have identical $\gamma_i = \gamma$, equilibrium surplus in each field is vT/N^2 & total surplus is vT/N. In limit as $N \to \infty$, the success rate $p \to \gamma^+$ and surplus of each field as well as the total surplus of all fields converges to zero.

So What?

Bottom up vs. top down incentives in resource allocation across fields?

- We mostly stressed **cons** of proportional system:
  - perverse properties of principle of demand relativity
  - allocation orthogonal to social benefit

So What?

Bottom up vs. top down incentives in resource allocation across fields?

- We mostly stressed **cons** of proportional system:
  - perverse properties of principle of demand relativity
  - allocation orthogonal to social benefit
- But proportional system also has **pros**:
  - bottom up: quickly reacts to demand-side signals
  - hands off: robust to lobbying/meddling by politicians/administrators

Broader Relevance

- Academic **journals** subject to a similar, but informal, pressure to:
  - allocate space to different sub-fields in proportion to submissions

Broader Relevance

- Academic **journals** subject to a similar, but informal, pressure to:
  - allocate space to different sub-fields in proportion to submissions
- Similarly, **university admission** boards are tempted to:
  - admit students to different programs in proportion to applications, or
  - increase slots available in areas that attract more applications...

Literature

- No previous work on proportional allocation
- **Normative** theory of allocation of resources in research: Peirce (1867), Weisbrod (1963), Weinstein and Zeckhauser (1973)

Literature

- No previous work on proportional allocation
- **Normative** theory of allocation of resources in research: Peirce (1867), Weisbrod (1963), Weinstein and Zeckhauser (1973)
- **Positive** analyses:
  - Zuckerman and Merton (1971) notice lower acceptance rates at top academic journals in social sciences, less consensual than hard sciences
  - Wallner (2019) documents more inter-rater disagreement in social than physical & life sciences at ANVUR research assessment in Italy
  - Azoulay, Graff Zivin, Li, and Sampat (2019): funding->innovation

Literature: Contest Theory

- Contests focus on applicants' effort (Siegel, 2009), here disregarded
- Instead, we focus on noisy selection & allocation across fields

Literature: Contest Theory

- Contests focus on applicants' effort (Siegel, 2009), here disregarded
- Instead, we focus on noisy selection & allocation across fields

- Closest paper is Morgan, Sisak, and Várdy (2018):
  - analyze applicants incentives to choose field, here ignored
  - in model with fixed budget per field, here proportional

Literature: Research Subsidies

- Peirce (1867): normative theory of resource allocation across fields
- Arrow (1962): research invention is public good, underprovided
  - articulates role of public funding

Literature: Research Subsidies

- Peirce (1867): normative theory of resource allocation across fields
- Arrow (1962): research invention is public good, underprovided
  - articulates role of public funding
- Scotchmer (2004, Chapter 8):
  - incentives for repeated funding discipline and sort applicants

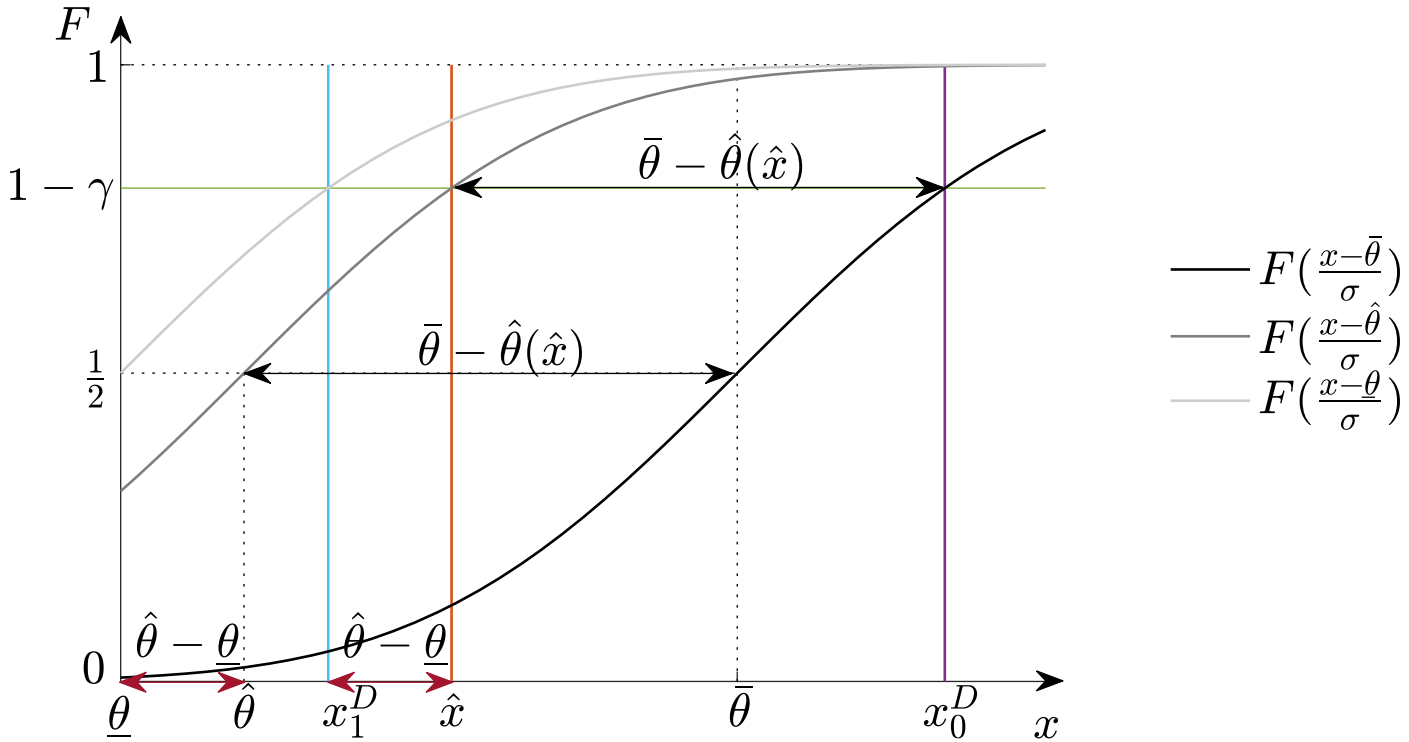
Literature: Research Subsidies

- Peirce (1867): normative theory of resource allocation across fields
- Arrow (1962): research invention is public good, underprovided
  - articulates role of public funding
- Scotchmer (2004, Chapter 8):
  - incentives for repeated funding discipline and sort applicants

- Gans and Murray (2012): different disclosure/openess requirements by funding scources for scientists: government, private firms' internal R&D, foundations & philanthropists
- Savage (1999): infuence process behind university earmarks v. merit-based public funding of research
- Cowen (2009): American arts funding as ingenious combination of direct subsidies & indirect schemes encouraging charitable giving

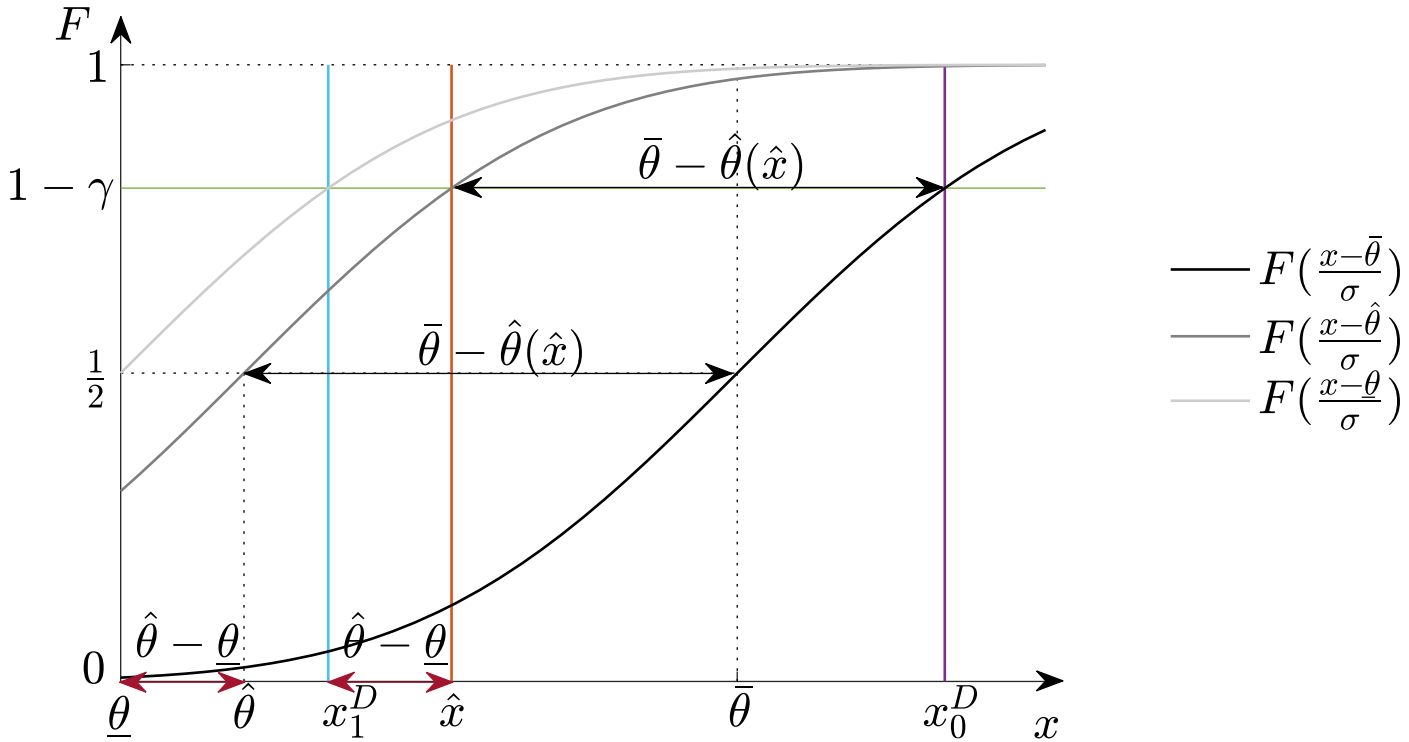
CONSTRUCTION: Signal Distributions for Different Types

- Signal distributions $F\left(\frac{x-\theta}{\sigma}\right)$ for $\theta = \underline{\theta}, \hat{\theta}, \bar{\theta}$, horizontally parallel



CONSTRUCTION: Signal Distributions for Different Types

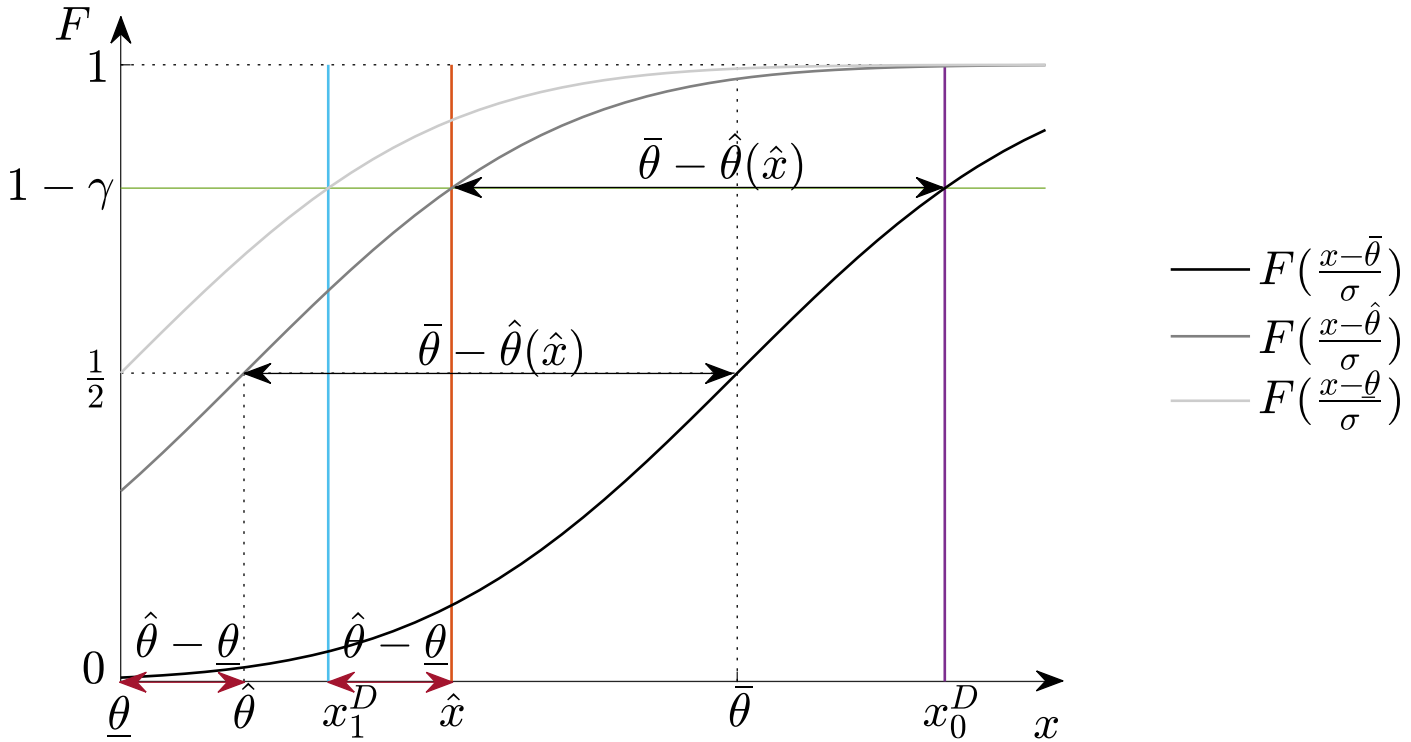
- Signal distributions $F\left(\frac{x-\theta}{\sigma}\right)$ for $\theta = \underline{\theta}, \hat{\theta}, \bar{\theta}$, horizontally parallel



- Given $\hat{x}$ RED, marginal $\theta = \hat{\theta}$ wins w/ probability $1 - F\left(\frac{\hat{x}-\hat{\theta}}{\sigma}\right) = \gamma$

CONSTRUCTION: Signal Distributions for Different Types

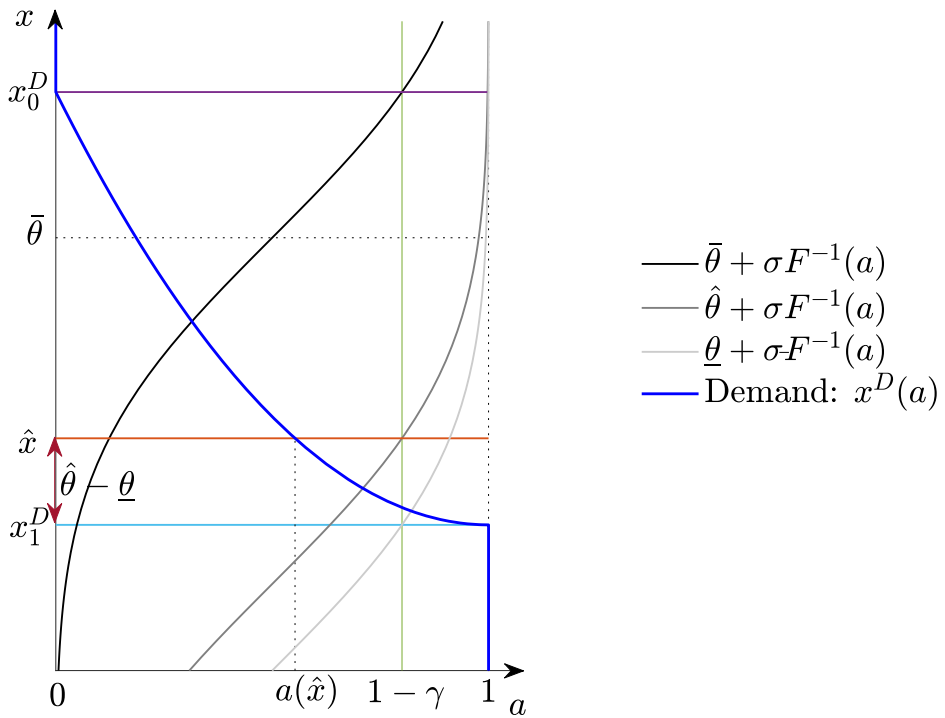
- Signal distributions $F\left(\frac{x-\theta}{\sigma}\right)$ for $\theta = \underline{\theta}, \hat{\theta}, \bar{\theta}$, horizontally parallel



- Given $\hat{x}$ RED, marginal $\theta = \hat{\theta}$ wins w/ probability $1 - F\left(\frac{\hat{x}-\hat{\theta}}{\sigma}\right) = \gamma$
- **Location property**: winning prob of type $\hat{\theta}$ at $\hat{x}$
  - can be read off distribution for $\bar{\theta}$ by shifting to right by $\bar{\theta} - \hat{\theta}$

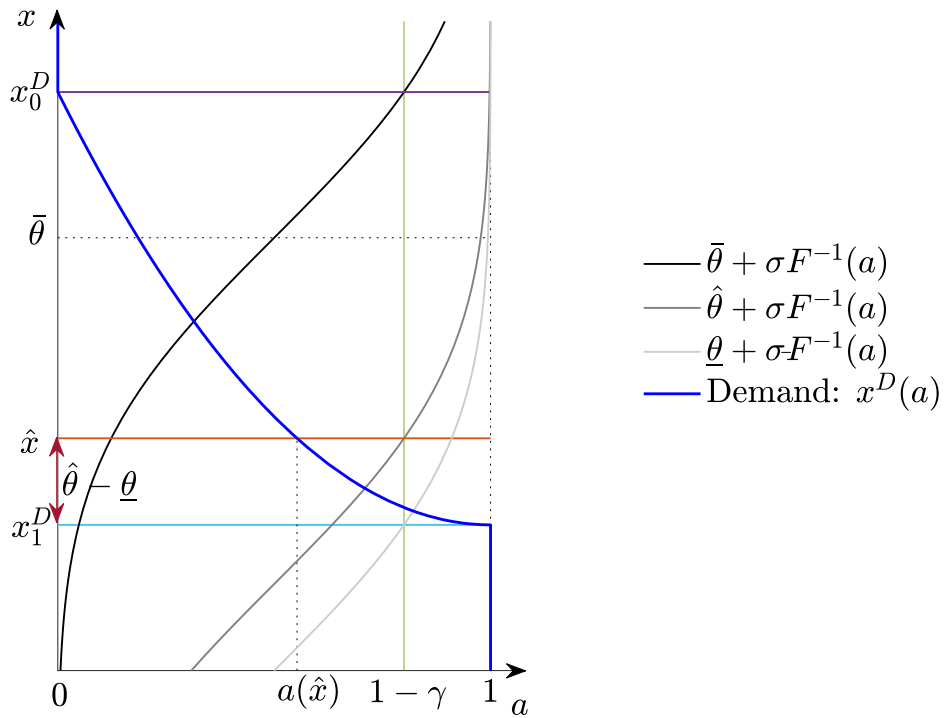
Demand: Construction

Vertical: acceptance **standard** $\hat{x}$ [like price]; horizontal: **demand** $a^D(\hat{x})$
Inverse demand is $\hat{x}^D(a) = \sigma F^{-1}(1-\gamma) + G^{-1}(1-a)$, **decreasing**



Demand: Construction

Vertical: acceptance **standard** $\hat{x}$ [like price]; horizontal: **demand** $a^D(\hat{x})$
Inverse demand is $\hat{x}^D(a) = \sigma F^{-1}(1-\gamma) + G^{-1}(1-a)$, **decreasing**



- Construction: BLACK upward sloping curve is quantile $F^{-1}_{\bar{\theta},\sigma}(y)$
  - at demand intercept $\hat{x}_0^D$, highest type $\bar{\theta}$ wins with prob γ—indifferent

When Demand DECREASES in Dispersion

- For given $\tilde{\sigma}$, at standard $\hat{x}(\tilde{\sigma})$ consider marginal type $\hat{\theta}(\tilde{\sigma})$
  - Demand increases iff win prob for $\hat{\theta}(\tilde{\sigma})$ increases in σ

$$\frac{\partial}{\partial \sigma}\left[1-F\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right)\right] = \frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma^2} f\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right) \gtreqless 0$$

$$\Leftrightarrow \quad \hat{x}(\tilde{\sigma}) \gtreqless \hat{\theta}(\tilde{\sigma})$$

When Demand DECREASES in Dispersion

- For given $\tilde{\sigma}$, at standard $\hat{x}(\tilde{\sigma})$ consider marginal type $\hat{\theta}(\tilde{\sigma})$
  - Demand increases iff win prob for $\hat{\theta}(\tilde{\sigma})$ increases in σ

$$\frac{\partial}{\partial \sigma}\left[1-F\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right)\right] = \frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma^2} f\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right) \gtreqless 0$$

$$\Leftrightarrow \quad \hat{x}(\tilde{\sigma}) \gtreqless \hat{\theta}(\tilde{\sigma})$$

Examples:

- If $F(x) = \exp(\lambda x)$ for $x \in (-\infty, 0) \Rightarrow F^{-1}(1-\gamma) < 0$ so $\partial a^D / \partial \sigma < 0$ **always**!
  - a^D *decreases* in σ if marginal applicant is a relatively high type
  - noisier/more dispersed signal discourages marginal applicant

When Demand DECREASES in Dispersion

- For given $\tilde{\sigma}$, at standard $\hat{x}(\tilde{\sigma})$ consider marginal type $\hat{\theta}(\tilde{\sigma})$
  - Demand increases iff win prob for $\hat{\theta}(\tilde{\sigma})$ increases in σ

$$\frac{\partial}{\partial \sigma}\left[1-F\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right)\right] = \frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma^2} f\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right) \gtreqless 0$$

$$\Leftrightarrow \quad \hat{x}(\tilde{\sigma}) \gtreqless \hat{\theta}(\tilde{\sigma})$$

Examples:

- If $F(x) = \exp(\lambda x)$ for $x \in (-\infty, 0) \Rightarrow F^{-1}(1-\gamma) < 0$ so $\partial a^D / \partial \sigma < 0$ **always**!
  - a^D *decreases* in σ if marginal applicant is a relatively high type
  - noisier/more dispersed signal discourages marginal applicant
- If signal is symmetric $F(0) = 1/2$, demand

  (i) $\frac{da^D}{d\sigma} \geq 0$ if marginal applicant wins with $\gamma \leq 1/2$, *tight contest*

  (ii) decreases $\frac{da^D}{d\sigma} \leq 0$ in a *loose contest* with $\gamma \geq 1/2$

When Demand DECREASES in Dispersion

- For given $\tilde{\sigma}$, at standard $\hat{x}(\tilde{\sigma})$ consider marginal type $\hat{\theta}(\tilde{\sigma})$
  - Demand increases iff win prob for $\hat{\theta}(\tilde{\sigma})$ increases in σ

$$\frac{\partial}{\partial \sigma}\left[1-F\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right)\right] = \frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma^2} f\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right) \gtreqless 0$$

$$\Leftrightarrow \quad \hat{x}(\tilde{\sigma}) \gtreqless \hat{\theta}(\tilde{\sigma})$$

Examples:

- If $F(x) = \exp(\lambda x)$ for $x \in (-\infty, 0) \Rightarrow F^{-1}(1-\gamma) < 0$ so $\partial a^D / \partial \sigma < 0$ **always**!
  - a^D *decreases* in σ if marginal applicant is a relatively high type
  - noisier/more dispersed signal discourages marginal applicant
- If signal is symmetric $F(0) = 1/2$, demand

  (i) $\frac{da^D}{d\sigma} \geq 0$ if marginal applicant wins with $\gamma \leq 1/2$, *tight contest*

  (ii) decreases $\frac{da^D}{d\sigma} \leq 0$ in a *loose contest* with $\gamma \geq 1/2$

Partial Equilibrium: Unraveling with Thin Tails

If $p < \gamma$, nobody applies—uninteresting

If $p > \gamma$, two sufficient conditions for $a^p = 0$ **UNRAVELING of partial equilibrium only for** $\sigma = 0$**:**

Partial Equilibrium: Unraveling with Thin Tails

If $p < \gamma$, nobody applies—uninteresting

If $p > \gamma$, two sufficient conditions for $a^p = 0$ **UNRAVELING of partial equilibrium only for** $\sigma = 0$**:**

1. $MRQ(a)$ increases in a

Partial Equilibrium: Unraveling with Thin Tails

If $p < \gamma$, nobody applies—uninteresting

If $p > \gamma$, two sufficient conditions for $a^p = 0$ **UNRAVELING of partial equilibrium only for** $\sigma = 0$**:**

1. $MRQ(a)$ increases in a
2. If (i) $\bar{\theta}$ bounded or (ii) UHR $\lim_{\theta \to \bar{\theta}} \frac{g(\theta)}{1-G(\theta)} = \infty$

[$\Rightarrow \lim_{a \to 0} MRQ(a) = 0$ Calabria and Pulcini (1987), Bradley and Gupta (2003) & Finkelstein (2014)]

Partial Equilibrium: Thick Tails

If $MRQ(a)$ decreases in a [from $MRQ(0)$ to $E(\theta)$]:

A. If $\sigma > \frac{MRQ(0)}{p-\gamma}$, $a^p = 1$ is unique (& stable) partial equilibrium

B. If $\sigma \in \left(\frac{E(\theta)}{p-\gamma}, \frac{MRQ(0)}{p-\gamma}\right) \Rightarrow a^p = 0$ & $a^1 = 1$ are stable eq [+ interior unstable eq]

C. If $\sigma < \frac{E(\theta)}{p-\gamma} \Rightarrow a^p = 0$ is unique (& stable) partial equilibrium

Impact of Noise on Demand?

Proposition

Demand increases/decreases in signal dispersion $\partial a^D / \partial \sigma \gtreqless 0$ *whenever the acceptance standard on the demand curve is above/below the marginal type,* $\hat{x} - \hat{\theta} = \sigma F^{-1}(1-\gamma) \gtreqless 0$.

Impact of Noise on Demand?

Proposition

Demand increases/decreases in signal dispersion $\partial a^D / \partial \sigma \gtreqless 0$ *whenever the acceptance standard on the demand curve is above/below the marginal type,* $\hat{x} - \hat{\theta} = \sigma F^{-1}(1-\gamma) \gtreqless 0$.

- $\frac{da^D}{d\sigma} \leq 0 \Leftrightarrow$win prob of marginal type $\gamma \geq 1 - F(0)$
  - $\frac{da^D}{d\sigma} \leq 0$ always$\Leftrightarrow F(0) = 1$ (signal is always below type)
  - when F is symmetric $F(0) = 1/2$, $\frac{da^D}{d\sigma} \leq 0 \Leftrightarrow \gamma \geq \frac{1}{2}$

Impact of Noise on Demand?

Proposition

Demand increases/decreases in signal dispersion $\partial a^D / \partial \sigma \gtreqless 0$ *whenever the acceptance standard on the demand curve is above/below the marginal type,* $\hat{x} - \hat{\theta} = \sigma F^{-1}(1-\gamma) \gtreqless 0$.

- $\frac{da^D}{d\sigma} \leq 0 \Leftrightarrow$win prob of marginal type $\gamma \geq 1 - F(0)$
  - $\frac{da^D}{d\sigma} \leq 0$ always$\Leftrightarrow F(0) = 1$ (signal is always below type)
  - when F is symmetric $F(0) = 1/2$, $\frac{da^D}{d\sigma} \leq 0 \Leftrightarrow \gamma \geq \frac{1}{2}$
- Intuitively, if $\hat{x} < \hat{\theta}$, for $\sigma = 0$ when $x \sim \delta_{\hat{\theta}}$ win prob for $\hat{\theta}$ is 1
  - now $\sigma \uparrow$, signal distribution rotates clockwise @ $\hat{\theta}$
  - given $\hat{x} < \hat{\theta}$, win prob $\downarrow$ & demand $\downarrow$

Impact of Noise on Demand?

Proposition

Demand increases/decreases in signal dispersion $\partial a^D / \partial \sigma \gtreqless 0$ *whenever the acceptance standard on the demand curve is above/below the marginal type,* $\hat{x} - \hat{\theta} = \sigma F^{-1}(1-\gamma) \gtreqless 0$.

- $\frac{da^D}{d\sigma} \leq 0 \Leftrightarrow$win prob of marginal type $\gamma \geq 1 - F(0)$
  - $\frac{da^D}{d\sigma} \leq 0$ always$\Leftrightarrow F(0) = 1$ (signal is always below type)
  - when F is symmetric $F(0) = 1/2$, $\frac{da^D}{d\sigma} \leq 0 \Leftrightarrow \gamma \geq \frac{1}{2}$
- Intuitively, if $\hat{x} < \hat{\theta}$, for $\sigma = 0$ when $x \sim \delta_{\hat{\theta}}$ win prob for $\hat{\theta}$ is 1
  - now $\sigma \uparrow$, signal distribution rotates clockwise @ $\hat{\theta}$
  - given $\hat{x} < \hat{\theta}$, win prob $\downarrow$ & demand $\downarrow$

We'll show: in all stable *equilibria* applications **always** increase in σ