Mussa Puzzle Redux*

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PRELIMINARY AND INCOMPLETE

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Abstract

The Mussa (1986) puzzle — a sharp and simultaneous increase in the volatility of both nominal and real exchange rates after the end of the Bretton Woods System of pegged exchange rates in early 1970s — is commonly viewed as a central piece of evidence in favor of monetary non-neutrality. Indeed, a change in the *monetary* regime has caused a dramatic change in the equilibrium behavior of a *real* variable — the real exchange rate. The Mussa fact is further interpreted as direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this last conclusion is not supported by the data, as there was no simultaneous change in the properties of the other macro variables — neither nominal like inflation, nor real like consumption, output or net exports. We show that the extended set of Mussa facts equally falsifies both *conventional* flexible-price RBC models and sticky-price New Keynesian models. We present a resolution to this broader puzzle based on a model of segmented financial market — a particular type of financial friction by which the bulk of the *nominal* exchange rate risk is held by a small group of financial intermediaries and not shared smoothly throughout the economy. We argue that rather than discriminating between models with sticky versus flexible prices, or monetary versus productivity shocks, the Mussa puzzle provides sharp evidence in favor of models with monetary non-neutrality arising due to *financial market segmentation*. Sticky prices are neither necessary, nor sufficient for the qualitative success of the model, yet improve its quantitative fit on the margin.

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1 Introduction

The Mussa (1986) puzzle is the fact that the end of the Bretton Woods System and the change in the monetary policy regime in the early 1970s away from pegged towards floating exchange rates had naturally increased the volatility of the nominal exchange rates (by an order of magnitude), but had also instantaneously increased the volatility of the *real* exchange rate almost by the same proportion (see Figure 1). This fact is commonly viewed by economists as a central piece of evidence in favor of monetary non-neutrality, since a change in the monetary regime has caused a dramatic change in the equilibrium behavior of a real variable — the real exchange rate.[1] Indeed, in models with complete monetary neutrality, the property of the real exchange rate should not be affected by the change in the monetary rule, absent other contemporaneous changes.[2] However, the Mussa fact is further interpreted as the direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this last conclusion is not supported by the data and provide an alternative explanation to the puzzle.

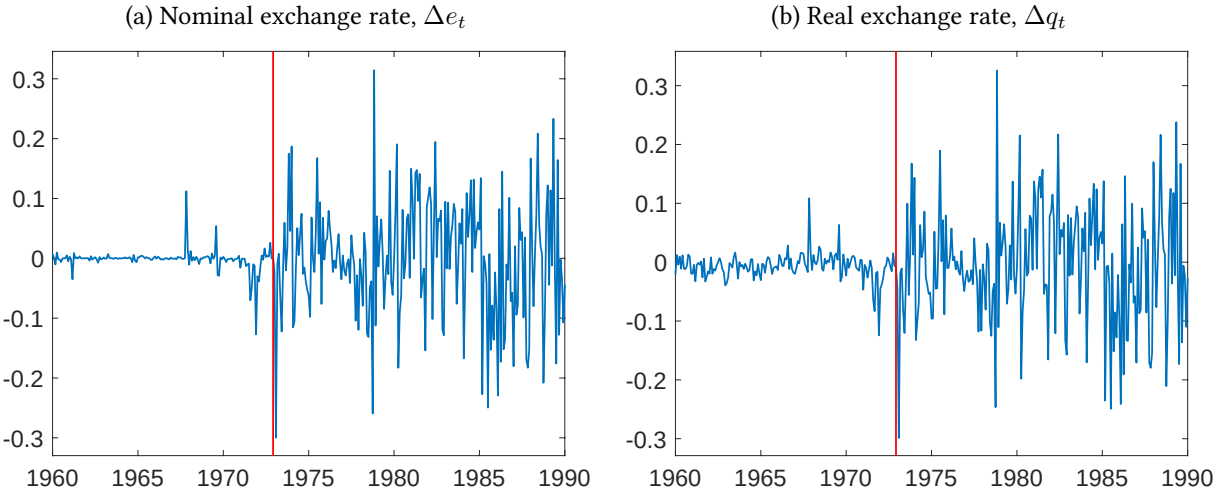


Figure 1: Nominal and real exchange rates, log changes

Note: US vs the rest of the world (defined as G7 countries except Canada plus Spain), monthly data from IFM IFS database. See Appendix Figure A1 for the comparison of volatilities and the correlation of the two exchange rate series over time.

We start by documenting empirically that while there was a change in the properties of the real exchange rate, there was no change in the properties of other macro variables — neither nominal like inflation, nor real like consumption, output or net exports (see Figure 2, which exhibits no evident structural break). One could interpret this as an extreme form of *neutrality*, where a major shift in the

[1] When Nakamura and Steinsson (2018, pp.69–70) surveyed "prominent macroeconomists [on what is the most convincing evidence for monetary nonneutrality], the three most common answers have been: the evidence presented in Friedman and Schwartz (1963) regarding the role of monetary policy in the severity of the Great Depression; the Volcker disinflation of the early 1980s and accompanying twin recession; and the sharp break in the volatility of the US real exchange rate accompanying the breakdown of the Bretton Woods System of fixed exchange rates in 1973." See also a textbook treatment of the Mussa puzzle in Uribe and Schmitt-Grohé (2017, Chapter 9.12) from the perspective of discriminating between flexible-price and sticky-price models.

[2] The argument here relies on the timing and the sharp discontinuity in the behavior of the exchange rates (see Figure 1), absent other immediate major changes in the environment.

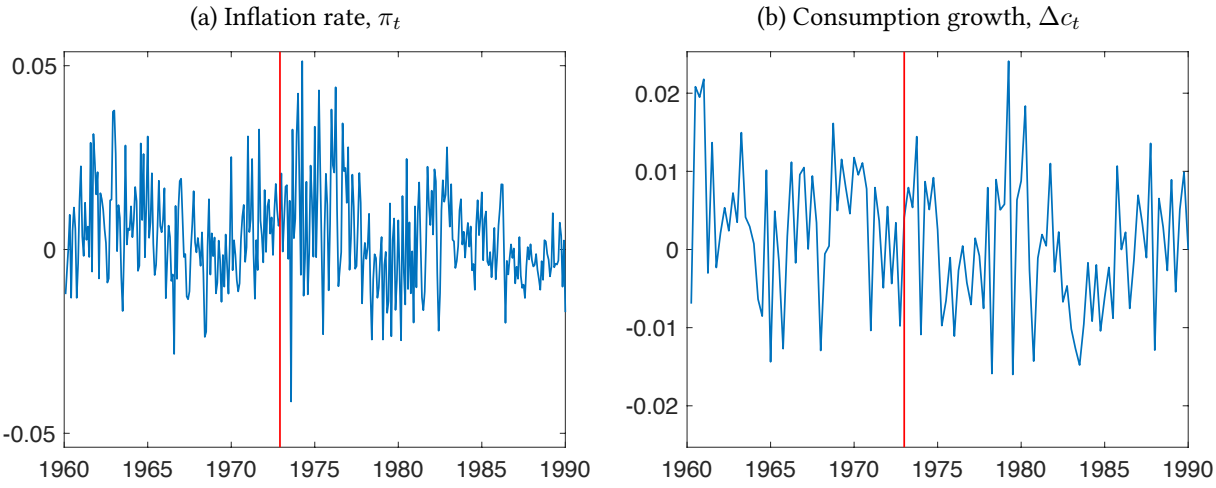


Figure 2: Inflation and consumption growth

Note: average inflation rates (monthly) and consumption growth rates (quarterly) for G7 countries except Canada plus Spain.

monetary regime, which increased the volatility of the nominal exchange rate by an order of magnitude, fails to affect the equilibrium properties of any macro variables, apart from the real exchange rate. In fact, this is a considerably more puzzling part of the larger set of "Mussa facts": while the lack of change in the volatility of nominal variables, like inflation, is inconsistent with models of monetary neutrality, the lack of change in the volatility of real variables, like consumption and output, is inconsistent with sticky-price models. Therefore, if we take the combined evidence, it does not seem to favor one type of models over the other, but rather rejects both types.

To provide immediate intuition for this logic, consider two equilibrium conditions. The first is simply the definition of the real exchange rate (in logs):

$$q_t = e_t + p_t^* - p_t, \tag{1}$$

where p_t and p_t^* are consumer price levels at home and abroad, and e_t and q_t are the nominal and real exchange rates respectively. In models with monetary neutrality (e.g., international RBC), a change to the monetary policy rule should *not* affect the process for q_t, and therefore (1) necessary implies that the volatility of $\pi_t - \pi_t^* \equiv \Delta p_t - \Delta p_t^*$ must change along with the volatility of Δe_t. In the data, the volatility of Δq_t and Δe_t increased simultaneously, while the volatility of $\pi_t - \pi_t^*$ remained stable and low (see Figure 3 and Table 1). This pattern can, however, be consistent with the conventional sticky-price models (see e.g. Monacelli 2004). This observation is at the core of the traditional interpretation of the Mussa puzzle, suggesting that sticky price models (NKOE) beat RBC models, and monetary policy must have real effects due to nominal rigidities.

This interpretation, however, misses the second half of the picture. Equilibrium dynamics in a general class of models satisfies the following equilibrium relationship between relative consumption (with the rest of the world) and the real exchange rate:

$$\sigma(c_t - c_t^*) = q_t + \zeta_t, \tag{2}$$

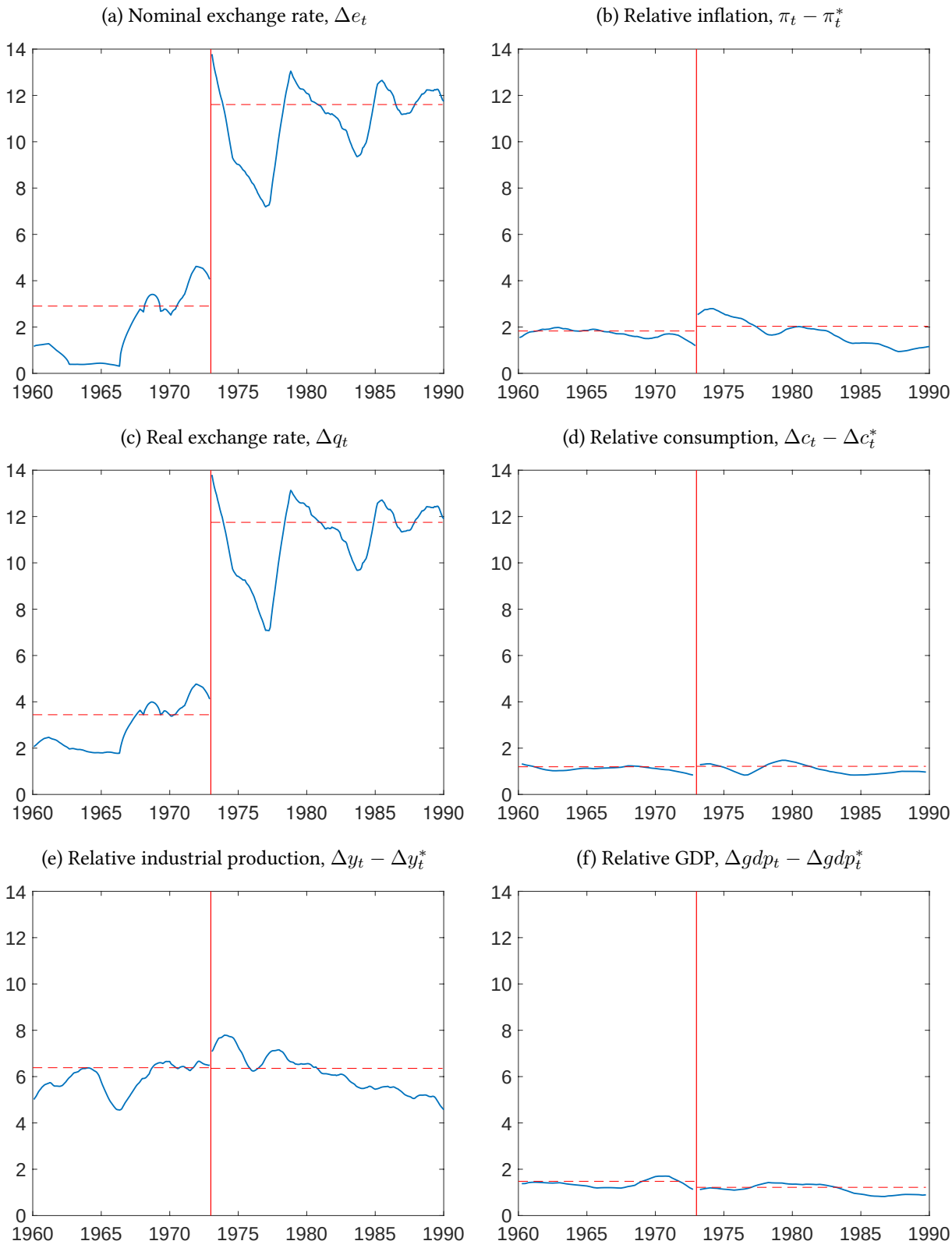


Figure 3: Volatility of macroeconomic variables over time

Note: All panels plot annualized standard deviations (of the log changes), estimated as triangular moving averages with a window over 18 months (or 10 quarters for quarterly data) before and after, treating 1973:01 as the end point for the two regimes; the dashed lines correspond to standard deviations measured over the entire subsamples (before and after 1973). US vs the rest of the world (as in Figure 1); monthly data from IFM IFS for panels a, b and e, and quarterly data from OECD in panels c, d and f. Appendix Figure A2 zooms in on the range of variation in panels b, d, e and f; Table 1 provides further details.

where $\sigma > 0$ and ζ_t can be interpreted as the equilibrium departure from the optimal international risk sharing.[3] Indeed, equation (20) with $\zeta_t \equiv 0$ corresponds to the classic Backus and Smith (1993) condition under separable utility with constant relative risk aversion σ. We show that equation (20) is considerably more general and emerges as an equilibrium relationship independently of asset market completeness and other features of the model. Furthermore, we show that in a large class of conventional models — including both IRBC and NKOE — the structural residual ζ_t is independent of the monetary policy regime. Therefore, a shift in the monetary policy regime, which changes dramatically the volatility of Δq_t, should necessarily change the volatility of $\Delta c_t - \Delta c_t^*$. In the data, however, the volatility of relative consumption growth, just like that of inflation, remained both stable and small (see Figure 3).

To summarize, the models of monetary neutrality are consistent with the observed lack of change in the volatility of consumption, but for the wrong reason — as they fail to predict the change in the volatility of the real exchange rate. In contrast, models with nominal rigidities can explain the changing behavior of the real exchange rate, but have the counterfactual implications for the missing change in the volatility of the real variables. Therefore, the extended Mussa facts falsify the conventional RBC and New Keynesian models alike.

We then present a new resolution to the Mussa puzzle, which is simultaneously consistent with all the empirical facts. In particular, we show that in a model with segmented financial markets and limits to arbitrage developed in Itskhoki and Mukhin (2019), shifts in monetary policy regime affect the volatility of both nominal and real exchange rates, *even* when prices are fully flexible. Intuitively, the unpredictable movements in *nominal* exchange rate are the main source of uncertainty for financial intermediaries, who as a result are less aggressive under free floating exchange rates in taking large currency positions to ensure that uncovered interest parity (UIP) holds. By consequence, the equilibrium UIP violations are larger under the floating exchange rate regime, consistent with the data (see Kollmann 2005). A pegged exchange rate, in contrast, decreases uncertainty and stimulates arbitrageurs to take larger positions. As a result, the real exchange rate is less sensitive to shocks in the financial market and has lower volatility. At the same time, the financial shocks do not constitute the main source of volatility in the other macro variables under *either* monetary regime, and thus the model is consistent with nearly no change in the macroeconomic volatility, apart from the exchange rates.[4]

This logic is summarized in the *modified UIP* condition, which holds in equilibrium of our economy with a segmented financial market:

$$\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}}{\omega \sigma_e^2} = \psi_t - b_{t+1}. \tag{3}$$

[3] Note the parallel between ζ_t and q_t, which can be viewed as defined by identifies (20) and (1) respectively: just like q_t is the departure from *parity* in the goods market (namely, the purchasing power parity), ζ_t can be viewed as the departure from "parity" in the financial market (namely, the optimal risk sharing).

[4] This additionally requires that economies are sufficiently closed to international trade — an important feature of the world as argued by Obstfeld and Rogoff (2001) — so that real exchange rate volatility does not translate into a large volatility increase in the price level, production and consumption. This is consistent with the *exchange rate disconnect* mechanism under the floating (Taylor rule) regime developed in Itskhoki and Mukhin (2019).

The left-hand side is the expected carry trade return scaled by the price of risk, where $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ is the quantity of risk and ω is the risk-aversion of the financiers intermediating the currency demand in the economy. The right hand side of (26) is the demand for foreign currency (bonds), with net foreign assets b_{t+1} representing the 'fundamental' demand due to international merchandise trade and ψ_t denoting the 'liquidity' (or noise-trader) demand. The departures from the UIP are necessary for the financiers to be willing to intermediate the currency demand in the market, and crucially the equilibrium extent of these departures is simultaneously determined with the volatility of the *nominal* exchange rate.

Equation (26) is the only unconventional equilibrium condition in an otherwise standard international DSGE model. A change in the monetary regime has a direct effect on the equilibrium in the financial market — via σ_e^2 in (26) — and this is what allows the model to be consistent with the umbrella of Mussa facts, independently of the presence of nominal rigidities and the source of the other shocks, as long as ψ_t is an important contributor to the exchange rate volatility under a floating regime.[5] Intuitively, as the government pledges to stabilize the volatility of the nominal exchange rate, this changes the incentives in the financial sector by encouraging risk-averse intermediaries to take on large currency exposure without requiring large UIP deviations.

We conclude that rather than discriminating between models with sticky versus flexible prices, and monetary versus productivity shocks, the Mussa puzzle provides a strong evidence in favor of models with monetary non-neutrality arising due to *financial market segmentation* — a particular type of financial friction by which the bulk of the *nominal exchange rate risk* is held by a small group of financial intermediaries and not shared smoothly throughout the economy. Sticky prices are neither necessary, nor sufficient ingredient for the qualitative success of this model. Nonetheless, realistic price and wage stickiness can improve the model's quantitative fit. Our analysis emphasizes that monetary non-neutrality is *not exclusive* to nominal rigidities in price setting, as changes in equilibrium properties of the nominal variables — such as the nominal exchange rate — can change the degree of financial market (in)completeness, and hence have real consequences for the real equilibrium outcomes.

Related literature Stockman (1988), Baxter and Stockman (1989), Flood and Rose (1995), Duarte (2003), Dedola and Leduc (2001), Duarte and Stockman (2002), Jeanne and Rose (2002), Kollmann (2005), Alvarez, Atkeson, and Kehoe (2007), Colacito and Croce (2013), Corsetti, Dedola, and Leduc (2008), Berka, Devereux, and Engel (2012), Devereux and Hnatkovska (2014), Kollmann, Frenkel-Levich...

[5] Itskhoki and Mukhin (2017) show that financial shocks ψ_t are essential for a successful model of exchange rate disconnect under a Taylor rule regime, and resolve a variety of exchange rate puzzles, including Meese-Rogoff, PPP, Backus-Smith and UIP puzzles. As we show below, their presence *per se* is not sufficient to resolve the Mussa puzzle. We further show the relationship between (20) and (26), and in particular how (26) implies the endogeneity of ζ_t to the exchange rate regime. Importantly, (26) does not rely on the assumption that an Euler equation holds for a representative consumer, or that there is any direct asset trades between home and foreign households.

2 Empirical Facts

Data We start by briefly describing the construction of our dataset, and provide further details in Appendix A.2. All monthly data (for nominal exchange rate, consumer prices and production index) come from the IFM IFS database, while all quarterly data (for GDP, consumption, imports and exports) are from the OECD database. All quantity variables (GDP, consumption, imports and exports) are real and seasonally-adjusted. Production index is also seasonally-adjusted, while nominal exchange rates and consumer price indexes are not. The *net export* variable is defined as the ratio of exports minus imports to the sum of exports and imports, in order to counter a mechanical increase in the volatility of net exports to GDP due to higher openness of the economies in later periods. All data are annualized to make volatilities (standard deviations) comparable across series.

There is ambiguity associated with identifying the exact end of the Bretton Woods System. In particular, during the Bretton Woods period, there are already large devaluations in the U.K. and Spain in November 1967, a devaluation in France and an appreciation in Germany in August–October 1969. While all countries officially allowed their exchange rates to float in February 1973, most of them were already adjusting their exchange rates since the "Nixon shock" in August 1971, which limited the direct convertibility of dollar to gold. Therefore, we label the period from 1960:01-1971:07 as "peg" and the period from 1973:01-1989:12 as "float", as used in tables and scatter plots below (which exclude the intermediate period 1971:08–1972:12).[6] The "regression discontinuity" graphs are done for two alternative break points — 1973:01 in the main figures and 1971:08 in the robustness figures in the appendix.

The rest of the world for the U.S. is constructed as a weighted average of percent changes in series across France, Germany, Italy, Japan, Spain and the U.K. (G7 countries except Canada plus Spain). Average GDP shares during the sample period are used to construct country weights.

Macroeconomic volatility Figure 3 displays the main empirical results of the paper. In the spirit of the regression discontinuity design (RDD), we estimate standard deviations of the variables using a rolling window that starts at 1973:01 and goes either forward or backward. In line with the seminal Mussa (1986) paper, the end of the Bretton Woods System is associated with a dramatic change in the volatility of both nominal and real exchange rates, from around 2% to 10% (more precisely, in units of annualized standard deviations of log changes). What makes this fact much more puzzling, however, is the absence of any comparable change in the volatility of the other variables — either nominal like inflation, or real like production and consumption.[7] Thus, while under the peg regime, the volatility of the real exchange rate is of the same order of magnitude as for the other macroeconomics variables, there is a clear "disconnect" between the real exchange rate and macroeconomic fundamentals in the floating regime. We emphasize the relative magnitudes of volatilities across different variables and regimes by keeping the same scale for standard deviations of all variables in Figure 3, while Appendix

[6] In Canada, the two exchange rate regimes occurred over different periods with free floating before 1962:06 and after 1970:05, and a peg in between. This is why we exclude Canada from the construction of the "rest of the world" in the figures.

[7] Figure 3 presents the *relative* volatilities of macroeconomic variables between the US and the rest of the world, as these are the relevant objects emphasized by the theory in Section 3. Table 1 also present the results for individual country variables across a range of countries, which exhibit similar patterns as the relative variables for the US vs the ROW.

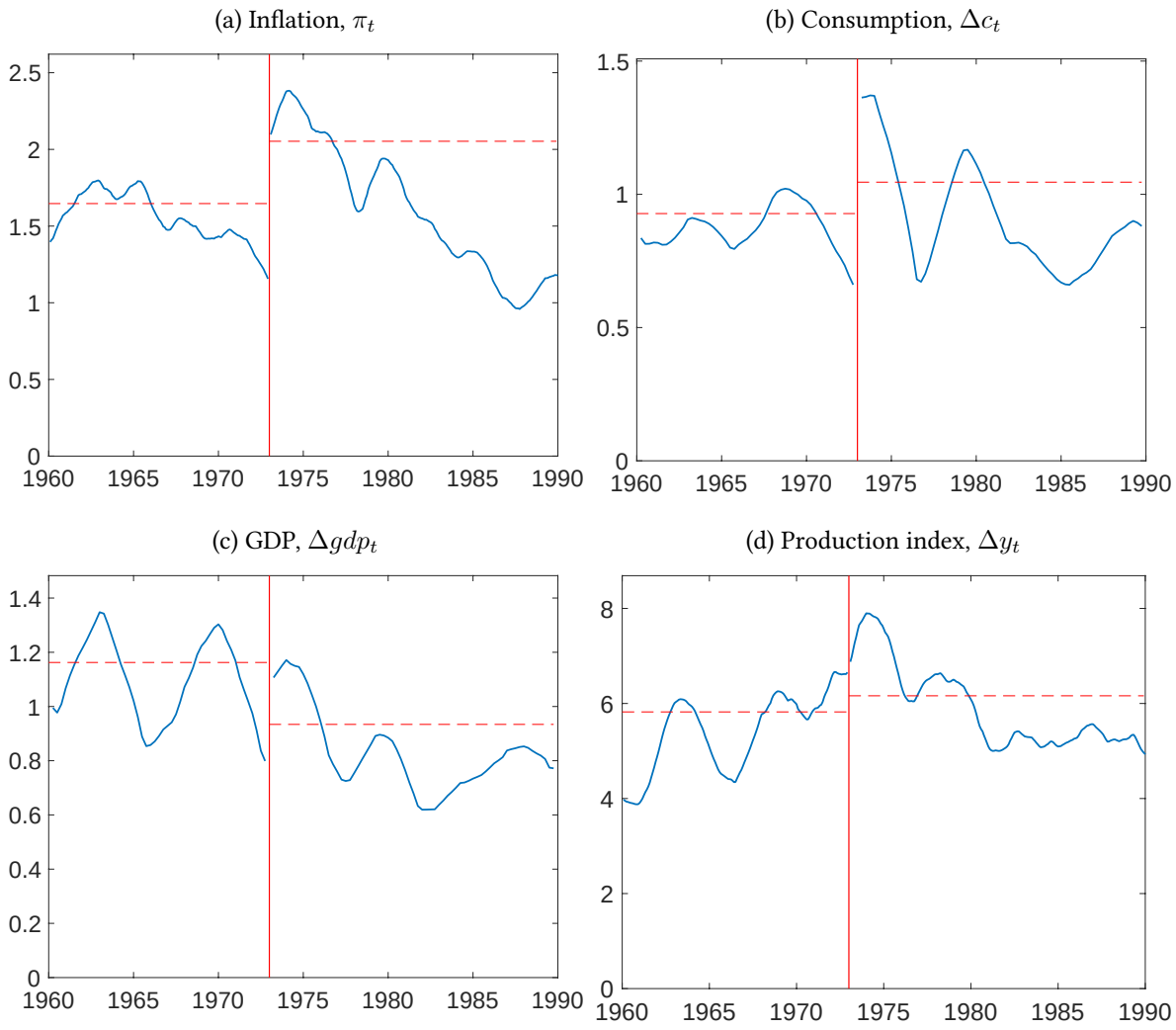


Figure 4: Macroeconomic volatility over time: country-level variables

Note: Trianglular moving averages of the standard deviations of macro variables, treating 1973:01 as the break point; average across countries (G7 except Canada plus Spain).

Figure A2 zooms in on the range of variation of individual macroeconomic variables to see (the lack of) the discontinuity in the behavior of inflation, consumption and output.[8]

The rest of the pictures and tables expend on this finding and provide some additional details. Figure A3 provides a robustness check using 1971:08 as the alternative break point. There is no evidence of changing volatility for macroeconomic variables in this case either. The missing change in the volatility is true not only for the cross-country differences of variables, but also for the fundamentals at the country level. In particular, from Figure 4, there is almost no differences in the volatilities of macro variables.

We now unpack the rest of the world (RoW) into separate countries and show that the main results hold in the panel as well. Table 1 summarizes the standard deviation of various variables for each coun-

[8]Note a slight increase in the volatility of consumer price inflation in the brief period after the break up of the Bretton Woods System, which quickly comes back down so that the average relative inflation volatility before and after 1973 is about the same. This increase in the volatility of inflation in the second half of 1970s is likely a response to the two large oil price shocks. There is also a slight increase in the volatility of consumption briefly after 1973, due to the 1974 recession in Japan.

Table 1: Empirical moments: standard deviations

	Δe_t			Δq_t			$\pi_t - \pi_t^*$			$\Delta c_t - \Delta c_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	0.8	4.4	5.7*	1.5	4.7	3.0*	1.3	1.4	1.1	0.8	1.1	0.9
France	3.4	11.8	3.5*	3.7	11.8	3.2*	1.3	1.3	1.0	1.2	0.9	0.7*
Germany	2.4	12.3	5.0*	2.7	12.5	4.7*	1.4	1.3	0.9	1.3	1.2	0.9
Italy	0.5	10.4	18.8*	1.5	10.4	6.9*	1.4	1.9	1.3*	1.0	1.1	1.0
Japan	0.8	11.7	13.8*	2.7	11.9	4.4*	2.7	2.8	1.0	1.1	1.3	1.2
Spain	4.4	10.8	2.5*	4.7	10.8	2.3*	2.7	2.6	0.9	1.2	1.0	0.8
U.K.	4.1	11.5	2.8*	4.4	12.0	2.7*	1.7	2.5	1.5*	1.4	1.5	1.1
RoW	1.2	9.8	8.0*	1.8	9.9	5.6*	1.3	1.4	1.1	0.9	0.9	1.0
	$\Delta gdp_t - \Delta gdp_t^*$			$\Delta y_t - \Delta y_t^*$			Δnx_t			$\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.0	1.0	1.0	3.8	4.9	1.3	1.7	1.6	0.9	2.4	4.5	1.9*
France	1.2	1.0	0.8	5.3	5.6	1.1	1.5	1.4	0.9	4.4	12.2	2.7*
Germany	1.8	1.2	0.7*	6.7	6.0	0.9	1.8	1.7	0.9	3.9	13.7	3.5*
Italy	1.5	1.3	0.8	8.1	9.7	1.2	2.5	2.2	0.9	2.8	11.4	4.1*
Japan	1.5	1.3	0.8	5.5	5.0	0.9	2.4	2.2	0.9	2.8	13.1	4.7*
Spain	1.6	1.2	0.7*	10.1	10.4	1.0	5.4	2.1	0.4*	5.8	11.4	2.0*
U.K.	1.4	1.4	0.9	3.9	6.0	1.5*	2.2	1.9	0.9	5.2	11.8	2.2*
RoW	1.1	1.0	0.8	3.9	3.5	0.9	1.1	1.0	0.9	2.5	10.7	4.3*
	π_t			Δc_t			Δgdp_t			Δy_t		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.3	1.4	1.1	0.8	0.9	1.1	0.9	0.9	1.0	4.1	5.1	1.2
France	1.1	1.3	1.2*	0.9	0.8	0.9	0.9	0.6	0.6*	4.2	5.4	1.3*
Germany	1.2	1.1	0.9	1.0	1.0	1.0	1.5	1.0	0.7	6.2	5.7	0.9
Italy	1.0	2.1	2.0*	0.7	0.8	1.2	1.3	1.0	0.8	7.5	9.5	1.3*
Japan	2.6	2.9	1.1	1.0	1.3	1.3	1.1	1.1	0.9	4.6	4.9	1.1
Spain	2.5	2.5	1.0	1.0	0.7	0.7	1.4	0.7	0.5*	10.1	10.1	1.0
U.K.	1.6	2.6	1.6*	1.2	1.4	1.2	1.0	1.3	1.2	3.5	5.9	1.7*
U.S.	0.9	1.3	1.5*	0.7	0.8	1.1	0.9	1.0	1.1	2.9	2.9	1.0
Average												

Note: "peg" corresponds to the period from 1960:01-1971:07 (except for Canada where it is from 1962:04-1970:01); "float" is from 1973:08-1989:12. The RoW corresponds to France, Germany, Italy, Japan, Spain, and the U.K., aggregated using 2010 GDP as weights. nx_t is the ratio of export minus imports to the sum of imports and exports. We use $\sigma = 2$ in panel eight. * indicates significance of the difference between peg and float at the 5% level (`robvar` test in Stata).

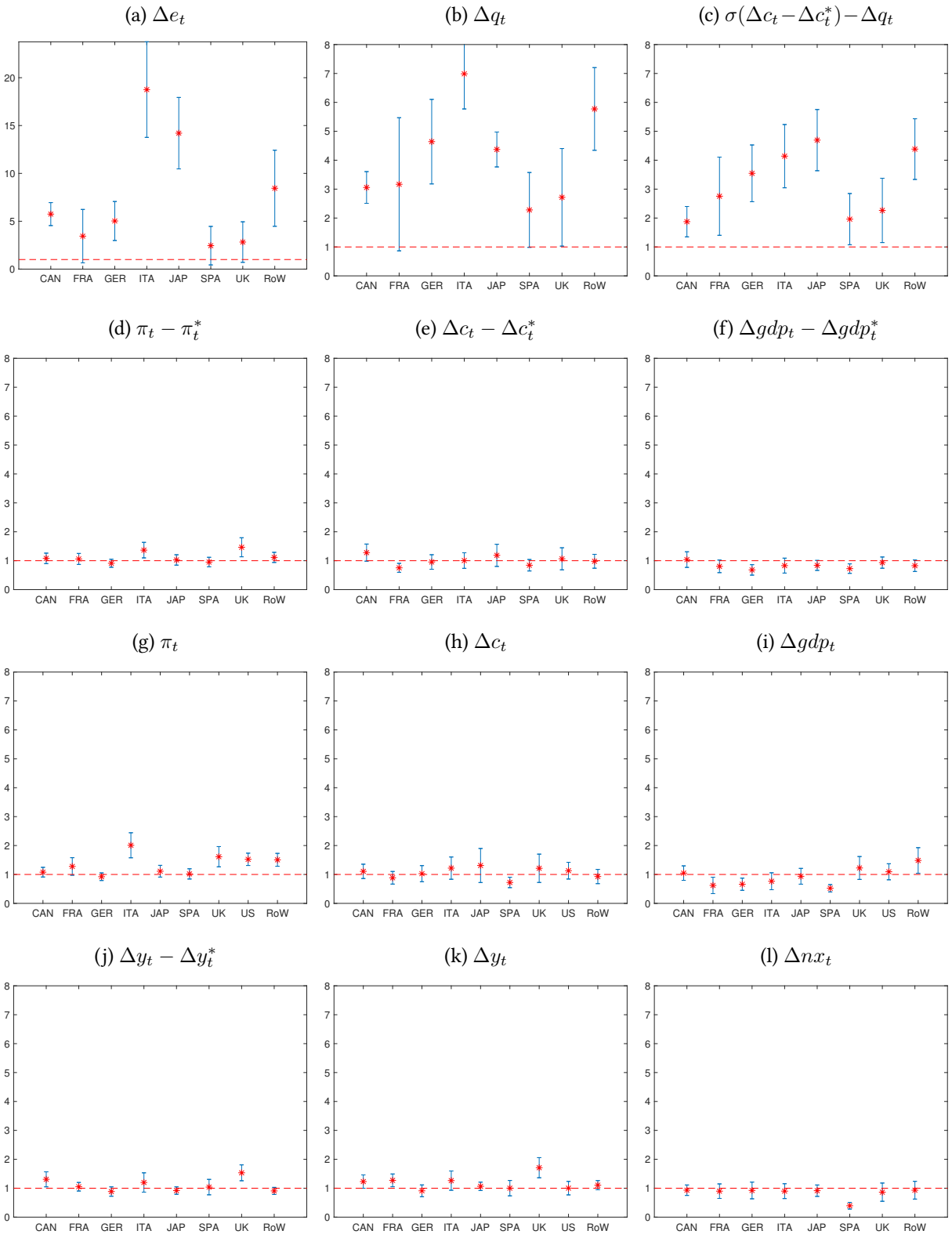


Figure 5: Volatility ratio float/peg, across variables and countries

Note: plots show the ratios of standard deviations under floating and peg regimes across individual countries with 90% confidence intervals estimated using Newey-West (HAC) standard errors. y-axis has the same scale for all plots except Δe_t.

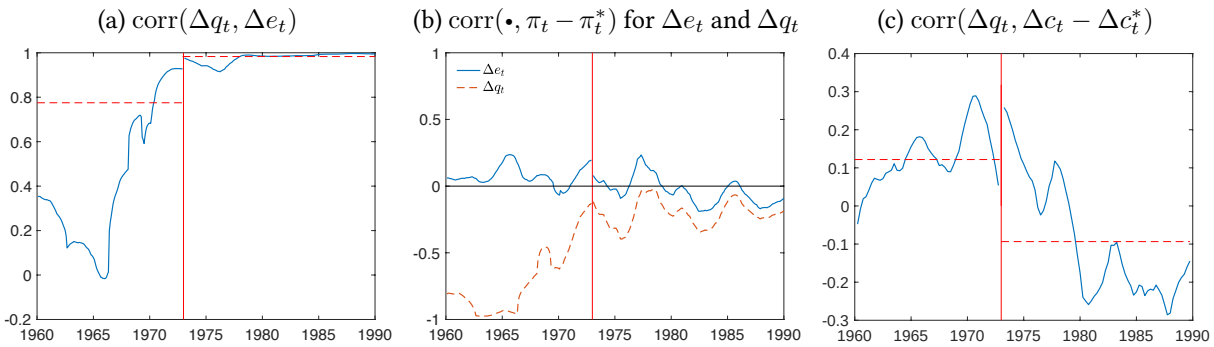


Figure 6: Correlations of exchange rates over time

Note: Triangular moving average correlations, treating 1973:01 as the end point for the two regimes.

try in our sample across the two monetary regimes, as well as provides a formal test of the equality of the variances of variables under the two regimes. We confirm that the change in the volatility of the exchange rates was large and highly significant in every country, while changes in the volatility of the other variables were small and generally insignificant. Note that rather than emphasizing the lack of any change in other variables, we emphasize the difference in the order of magnitude. Table 1 shows that while nominal and real exchange rate volatility increased on average by about 8 and 6 times respectively, the volatility of the other variables changed in different direction across countries and by an order of magnitude of about 10% — a stark difference. We further illustrate this point in Figure 5 which plots the ratio of the volatilities under the two regimes for each variable and country, on a common scale for comparability, with a Newey-West (HAC) robust 90% confidence interval (see also Appendix Figures A5 and A6).

Perhaps most surprisingly, there is no increase in the volatility of net exports, despite a large increase in the volatility of the real exchange rate (see Figures A2(e) and 5(l) and Table 1). Systematic data on terms of trade is not available for this period, however, in the FRED data we see that the volatility of the US terms of trade increased only twofold, while the volatility of the US real exchange rate increased six times. Our estimates for the other countries suggest an even smaller increase, if any, in the volatility of their terms of trade. Therefore, we conjecture that the lack of the increased volatility in net exports is in part due to a much muted response of the behavior of the terms of trade and in part due to muted response of net exports themselves to international relative prices.

Correlations Figure 6 plots the correlations between the nominal and real exchange rates, as well as their correlations with macro variables — relative inflation and relative consumption growth — over time, calculated as triangular moving average with a break point at 1973:01. The first two panels identify two clear shifts with the break of the fixed exchange rate regime. In particular, the correlation between nominal and real exchange rates is positive but not very strong during the peg, where nominal exchange rates barely moved before 1967, and it becomes virtually perfect (0.98) after the early 1970s. The latter correlation is around 0.7 and does not change significantly with the end of the Bretton Woods System. In contrast, the real exchange rate is tightly correlated with the relative inflation during the peg, yet it quickly becomes nearly orthogonal with relative inflation as soon as nominal exchange rates begin to

Table 2: Empirical moments: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Canada	0.77	0.92	0.03	−0.07	0.01	0.05	0.31	0.47	0.40	0.25	0.28	0.57
France	0.96	0.99	0.05	−0.08	0.23	0.12	0.09	0.30	−0.24	0.29	0.51	0.48
Germany	0.87	0.99	0.04	−0.19	−0.06	0.00	−0.01	0.28	−0.11	0.11	0.57	0.58
Italy	0.54	0.97	0.07	−0.13	0.02	−0.01	0.04	0.17	−0.18	0.13	0.64	0.45
Japan	0.76	0.98	0.21	−0.00	0.03	0.21	−0.08	0.24	0.11	0.23	0.70	0.71
Spain	0.83	0.96	−0.09	−0.18	−0.06	0.16	0.05	0.09	−0.06	0.05	0.56	0.63
U.K.	0.94	0.96	0.09	−0.10	−0.39	−0.16	−0.11	0.30	−0.02	0.22	0.59	0.71
RoW	0.80	0.98	0.05	−0.19	−0.20	0.21	−0.03	0.39	−0.11	0.31	0.68	0.72

Note: see notes to Table 1. Cross-country correlation are with the U.S. as the foreign counterpart indicated with a star.

float. At the same time, the nominal exchange rate is orthogonal with the relative inflation both before and after the 1973. While the pattern of correlation during the peg is mechanical — since $\Delta e_t \approx 0$ and thus $\Delta q_t \approx -(\pi_t - \pi_t^*)$ — the comovement of variables during the float is rather puzzling, as nominal depreciations if anything are negatively correlated with relative domestic inflation.

The last panel of Figure 6 shows that the correlation between real exchange rate and relative consumption growth, while somewhat positive under the peg, has become noticeably negative under the float, suggesting that the Backus-Smith puzzle became more pronounced with floating exchange rates.[9] While the correlations are not very large, this pattern is observed robustly across the countries, as we document in Table 2.

Table 2 reports correlations between various variables under the two exchange rate regimes, while Appendix Figure A4 plots the evolution of these correlation over time. Table 2 identifies another robust correlation pattern — between consumption and GDP growth, which is stable around 0.7 and does not change at all with the end of the Bretton Woods System. The other correlations, including that between RER and net exports, as well as for GDP (consumption) growth between countries, are generally not very strong and not particularly stable over time, suggesting only weak patterns of change across the two monetary regimes. The correlation between the real exchange rate and net exports switches from negative under the peg to positive under the float, but in both cases it is close to zero for most countries in our sample.

Finally, both GDP growth and consumption growth are uncorrelated or mildly negatively correlated across countries (with the exception of Canada) before 1970s and since then become sizably positively correlated, especially the GDP growth rates — a surprising pattern emphasized by Kollmann (2005). Figure A4 reveals that this is, however, largely driven by the high correlation of growth rates across countries in the late 1970s, a period of large global oil price shocks.

[9] This observation is consistent with the findings in Colacito and Croce (2013) that both Backus-Smith and UIP conditions held better under the pegged exchange rates, as well as in Devereux and Hnatkovska (2014) that the Backus-Smith condition holds better across regions within countries, in contrast with its cross-country violations. Another pattern emphasized by Berka, Devereux, and Engel (2012) is a substantially greater role of the non-tradable (Balassa-Samuelson) component in the RER variation under a nominal peg.

3 Theoretical Framework

We describe the general theoretical framework, which we use in Sections 4–6, where we consider various special cases. We build on a standard New Keynesian open-economy model (NKOE) with capital, intermediate goods, pricing to market, productivity and monetary shocks, wage and price stickiness, with border prices sticky in either producer or local currency. The model features home bias in consumption with additional exogenous shocks to home bias and international risk sharing. We allow for various degrees of financial market (in)completeness, as well as a model of segmented financial markets.

There are two mostly symmetric countries — home (Europe) and foreign (US, denoted with a $*$). Each country has its nominal unit of account in which the local prices are quoted: for example, the home wage rate is W_t euros and the foreign wage rate is W_t^* dollars. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, hence an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the euro (the home currency). The monetary policy is conducted according to a conventional Taylor rule targeting inflation or nominal exchange rate — depending on the monetary regime. In particular, the foreign country (US) always targets inflation, while the home country (Europe) switches from an exchange rate peg ('peg') to an inflation targeting ('float').

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right), \tag{4}$$

where σ is the relative risk aversion parameter and φ is the inverse Frisch elasticity of labor supply. The flow budget constraint is given by:

$$P_t C_t + P_t Z_t + \sum_{j \in J_t} \Theta_t^j B_{t+1}^j \leq W_t L_t + R_t^K K_t + \sum_{j \in J_{t-1}} e^{-\zeta_t^j} D_t^j B_t^j + \Pi_t + T_t, \tag{5}$$

where P_t is the consumer price index and W_t is the nominal wage rate, Π_t are profits of home firms, T_t are lump-sum transfers from the government, and B_{t+1}^j is quantity of asset $j \in J_t$ purchased at time t at price Θ_t^j and paying out a state-contingent dividend D_{t+1}^j at $t+1$ taxed at rate ζ_t^j (which we interpret as a Chari, Kehoe, and McGrattan 2007 wedge).[10] Finally, R_t^K is the nominal rental rate of capital, Z_t is the gross investment into the domestic capital stock K_t, which accumulates according to a standard rule with depreciation δ and quadratic capital adjustment costs with parameter κ.[11]

The foreign households are symmetric, having access to a set J_t^* of state contingent assets with dividends taxed at country-specific tax rate ζ_t^{j*}. The assets $j \in J_t \cap J_t^*$ can be purchased by households of both countries at a common price Θ_t^j in units of home currency, or equivalently $\Theta_t^j / \mathcal{E}_t$ in units of foreign currency.

[10] When set J_t contains a home-currency risk-free bond B_{t+1}^f, its price is one over gross nominal interest rate $\Theta_t^f = 1/R_t$ and it pays out one unit of home currency in every state of the world next period, $D_{t+1}^f \equiv 1$; when it contains a foreign-currency risk-free bond B_{t+1}^{f*}, its price is $\mathcal{E}_t / R_t^*$ and its dividend is $D_{t+1}^{f*} = \mathcal{E}_{t+1}$.

[11] Specifically, $K_{t+1} = (1-\delta) K_t + \left[Z_t - \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t} \right]$, where the term in brackets is investment net of adjustment costs.

Expenditure and demand The domestic households allocate their within-period consumption expenditure between home and foreign varieties of the goods, $P_t C_t = \int_0^1 \left[P_{Ht}(i)C_{Ht}(i)+P_{Ft}(i)C_{Ft}(i)\right]\mathrm{d}i$ to maximize the CES consumption aggregator:

$$C_t = \left[\int_0^1 \left(\left[(1-\gamma)e^{-\gamma\xi_t}\right]^{1/\theta} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + \left[\gamma e^{(1-\gamma)\xi_t}\right]^{1/\theta} C_{Ft}(i)^{\frac{\theta-1}{\theta}}\right)\mathrm{d}i\right]^{\frac{\theta}{\theta-1}}, \tag{6}$$

with parameter $\gamma \in [0, 1/2)$ capturing the level of the *home bias*, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see Obstfeld and Rogoff 2001) and ξ_t denoting the relative demand shock for the foreign good or other sources of time-varying home bias (see Pavlova and Rigobon 2007).[12] In the quantitative model of Section 6, we extend the analysis from CES to Kimball (1995) demand system to allow for *pricing to market*, as in Itskhoki and Mukhin (2019). The solution to the optimal expenditure allocation results in the conventional constant-elasticity demand schedules:

$$C_{Ht}(i) = (1-\gamma)e^{-\gamma\xi_t}\left(\frac{P_{Ht}(i)}{P_t}\right)^{-\theta} C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma e^{(1-\gamma)\xi_t}\left(\frac{P_{Ft}(j)}{P_t}\right)^{-\theta} C_t, \tag{7}$$

where the price index is given by $P_t = \left[\int_0^1 \left((1-\gamma)e^{-\gamma\xi_t}P_{Ht}(i)^{1-\theta} + \gamma e^{(1-\gamma)\xi_t}P_{Ft}(i)^{1-\theta}\right)\mathrm{d}i\right]^{1/(1-\theta)}$.

The expenditure allocation of the foreign households is symmetrically given by:

$$C^*_{Ht}(i) = \gamma e^{(1-\gamma)\xi^*_t}\left(\frac{P^*_{Ht}(i)}{P^*_t}\right)^{-\theta} C^*_t \qquad \text{and} \qquad C^*_{Ft}(j) = (1-\gamma)e^{-\gamma\xi^*_t}\left(\frac{P^*_{Ft}(j)}{P^*_t}\right)^{-\theta} C^*_t, \tag{8}$$

where ξ^*_t is the foreign demand shock for home goods, $P^*_{Ht}(i)$ and $P^*_{Ft}(j)$ are the foreign-currency prices of the home and foreign goods in the foreign market, and P^*_t is the foreign price level. The *real exchange rate* is the relative consumer price level in the two countries:

$$\mathcal{Q}_t \equiv P^*_t \mathcal{E}_t / P_t, \tag{9}$$

with an increase in $\mathcal{Q}_t$ corresponding to a real depreciation, that is a decrease in the relative price of the home consumption basket (note that (1) is the log version of (9)).

Production and profits Home output is produced by a given pool of identical firms (hence we omit indicator i) according to a Cobb-Douglas technology in labor L_t, capital K_t and intermediate inputs X_t:

$$Y_t = \left(e^{a_t} K_t^{\vartheta} L_t^{1-\vartheta}\right)^{1-\phi} X_t^{\phi}, \tag{10}$$

where a_t is the aggregate productivity shock, and ϑ and ϕ determine the input expenditure shares.

[12] The particular way in which we introduce the foreign-good demand shock ξ_t in (6) ensures that changes in ξ_t shift the relative demand between home and foreign goods without having a first-order effect on the price level. The aggregate implications of the model do not dependent on whether the home bias emerges on the extensive margin due to non-tradable goods or on the intensive margin due to trade costs and preferences, and therefore we do not explicitly introduce the non-tradables.

Intermediates X_t, as well as investment goods Z_t, are the same bundle of home and foreign varieties as the final consumption bundle (6), and hence their price index is also given by P_t. As a result, the marginal costs of the firms are given by $MC_t = \varpi\left[e^{-a_t}(R_t^K)^{\vartheta}W_t^{1-\vartheta}\right]^{1-\phi}P_t^{\phi}$, where ϖ is a constant, and therefore firm i profits (in home currency) from serving both home and foreign markets are given by:

$$\Pi_t(i) = (P_{Ht}(i) - MC_t)Y_{Ht}(i) + (P^*_{Ht}(i)\mathcal{E}_t - MC_t)Y^*_{Ht}(i), \tag{11}$$

where $P_{Ht}(i)$ and $P^*_{Ht}(i)$ are the home and foreign market prices charged by the firm, by convention expressed in home and foreign currency respectively. The supply to the home-market is split between home demand for consumption, intermediate and investment goods:

$$Y_{Ht}(i) = C_{Ht}(i) + X_{Ht}(i) + Z_{Ht}(i), \tag{12}$$

which all satisfy the demand schedules analogous to (7), and similarly in the foreign market. The aggregate profits of the domestic firms, $\Pi_t = \int_0^1 \Pi_t(i)\mathrm{d}i$, are distributed to the domestic households, and we assume no entry or exit of firms, focusing on the medium-run dynamics.

Wage and price setting In the IRBC version of the model, wages and prices are flexible. In particular, the equilibrium wage rate clears the labor market by equalizing the labor demand of the profit-maximizing firms with the optimal labor supply of the households. The prices are set by the monopolistically competitive firms as a markup over the marginal cost MC_t.[13]

In the NKOE versions of the model, the wages and prices are adjusted infrequently à la Calvo with a constant per-period non-adjustment hazard rate λ_w and λ_p, respectively. We adopt conventional sticky-wage and price formulations, as described in e.g. Galí (2008). For border prices, we allow for either producer currency pricing, in which case the *law of one price* holds under CES demand, or the alternative regime of local currency pricing, which results in the short-run local price stability and the violation of the law of one price. Under wage and price stickiness, the quantities are demand-determined: specifically, labor supply must satisfy labor demand given the preset wage rate, as well as the supply of goods must satisfy the demand given prices.

Financial sector The financial sector features additionally financial intermediaries and noise traders, who participate in *currency carry trades* by taking zero-capital positions in home and foreign-currency bonds. For concreteness, we assume they return the earned profits and losses to foreign households along with firm profits, Π^*_t. Whenever home and foreign households can trade some assets directly, that is $J_t \cap J^*_t \neq \emptyset$ for all t, the presence of financial intermediaries and noise traders does not materially affect the allocations in the economy.[14] All assets j are in zero net supply, and for $j \in J_t \cap J^*_t$, we have

[13] Under the CES demand (6), the markup is constant and equal to $\theta/(\theta-1)$, which for concreteness we assume is offset by a price subsidy financed with a lump-sum tax on the firms. Under the Kimball demand in the quantitative section, the optimal markup is variable, decreasing with the relative price of the firm.

[14] In particular, their presence leaves the risk-sharing conditions between home and foreign households unchanged, exerting only a general-equilibrium effect through the change in the composition of assets held by the households and the resulting effect on the equilibrium consumption paths, which is second order.

$B^j_{t+1} + B^{j*}_{t+1} + D^j_{t+1} + N^j_{t+1} = 0$ given a common home-currency price Θ^j_t, where D^j_{t+1} and N^j_{t+1} are the positions taken by the intermediaries and the noise traders respectively.

When the financial market is *segmented*, and the home households cannot trade assets directly with the foreign households, that is $J_t \cap J^*_t = \emptyset$, the presence of noise traders and financial intermediaries has important effects on equilibrium allocations, and noise trader shocks affect international risk sharing. We study this case in detail in Section 5, where in particular we describe the behavior of both the intermediaries and the noise traders.

Government The fiscal authority is passive, collecting exogenous taxes ζ^j_t on financial dividends and returning them lump-sum to the households:

$$T_t = \sum\nolimits_{j \in J_{t-1}} \left(1 - e^{-\zeta^j_t}\right) D^j_t B^j_t. \tag{13}$$

The monetary policy is implemented by means of a Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m)\left[\phi_\pi \pi_t + \phi_e (e_t - \bar{e})\right] + \sigma_m \varepsilon^m_t, \tag{14}$$

where $i_t = \log R_t$ is the log nominal interest rate, $\pi_t = \Delta \log P_t$ is the inflation rate, $\varepsilon^m_t \sim iid(0,1)$ is the monetary policy shock with volatility parameter $\sigma_m \geq 0$, and the parameter ρ_m characterizes the persistence of the monetary policy rule. The coefficients $\phi_\pi > 1$ and ϕ_e are the Taylor rule parameters which weigh the two nominal objectives of the monetary policy — inflation and exchange rate stabilization. We assume that the foreign country (the US) only has the inflation objective, so that $\phi^*_e = 0$, while the home country changes ϕ_e depending on the monetary policy regime, namely float with $\phi_e = 0$ or peg with $\phi_e \gg 0$. We study the differential behavior of the macro variables across the two monetary regimes of the home country.

4 Conventional Models: Falsification

We consider here a special case of the general modeling framework from Section 3, which allows us to prove a sharp theoretical result — a robust equilibrium relationship between relative consumption and the real exchange rate — which is falsified by the data on the end of the Bretton Woods. In particular, we specialize the model to the case of the Cole and Obstfeld (1991) parameter restriction, which is ubiquitously used in international economics (see e.g. Galí and Monacelli 2005, Heathcote and Perri 2013). Importantly, the result that we prove does not depend on the supply side of the economy — and in particular on the specific nature of price and wage stickiness (e.g., LCP or PCP) and shocks — and holds for a rather general structure of the international asset market. While the Cole-Obstfeld case is clearly special, the sharp analytical result that we establish in this section holds true approximately in a much richer quantitative environment and guides our quantitative approach in Section 6.

4.1 Equilibrium conditions

The result of this section relies on two equilibrium conditions — the home country intertemporal budget constraint and an international risk sharing condition between home and foreign households. The home country budget constraint derives from substituting firm profits (11) and government transfers (13) into the household budget constraint (5):

$$\mathcal{B}_{t+1} - \mathcal{R}_t \mathcal{B}_t = NX_t, \tag{15}$$

where the right-hand side is net exports $NX_t = \mathcal{E}_t \int_0^1 P^*_{Ht}(i) Y^*_{Ht}(i) \mathrm{d}i - \int_0^1 P_{Ft}(i) Y_{Ft}(i) \mathrm{d}i$, with $Y^*_{Ht}(i)$ and $Y_{Ft}(i)$ corresponding to the supply of home good to the foreign market and foreign good to the home market respectively. The left hand-side of (15) is the evolution of home net foreign assets $\mathcal{B}_{t+1} \equiv \sum_{j \in J_t} \Theta^j_t B^j_{t+1}$ with the cumulative realized return $\mathcal{R}_t \equiv \frac{\sum_{j \in J_{t-1}} D^j_t B^j_t}{\sum_{j \in J_{t-1}} \Theta^j_{t-1} B^j_t}$. The foreign budget constraint is redundant by Walras Law.

If the set of assets traded between home and foreign households is not empty, that is $J_t \cap J^*_t \neq \emptyset$, the international risk sharing conditions derive from the Euler equations of the home and foreign households, which can be combined to yield:

$$\mathbb{E}_t \left\{ \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} - \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}} e^{\tilde{\zeta}^j_{t+1}} \right] e^{r^j_{t+1}} \right\} = 0 \qquad \forall j \in J_t \cap J^*_t, \tag{16}$$

where $\tilde{\zeta}^j_{t+1} \equiv \zeta^j_{t+1} - \zeta^{j*}_{t+1}$ denotes the relative wedge across countries and $r^j_{t+1} \equiv \log \left(\frac{e^{-\zeta^j_{t+1}} D^j_{t+1}}{\Theta^j_t P_{t+1}/P_t} \right)$ denotes the log after-tax real return on asset j at home.

To summarize, condition (15) simply states that when home runs a trade surplus, it accumulates net foreign assets. Conditions (16) state that both home and foreign households consider as fair the equilibrium return r^j_{t+1} of all assets j that they can mutually trade. This allows the households to synchronize their stochastic discount factors in the best way possible given the available set of internationally-traded assets. Together, conditions (15) and (16) characterize equilibrium allocations under a variety of asset market structures that span from financial autarky to complete international asset markets, depending on the richness of the set $J_t \cap J^*_t$.

The special case of the model We now specialize the model environment with the following set of assumptions:

Assumption 1 $\vartheta = \phi = 0$, *that is the model features no capital and no intermediate goods in production.*

Assumption 2 (Cole-Obstfeld) $\sigma = \theta = 1$, *that is the model features unitary relative risk aversion (log utility of consumption) and elasticity of substitution between goods (Cobb-Douglas preference aggregator).*

Assumption 1 simplifies the analysis by dropping two dynamic state variable (home and foreign capital) and, together with Assumption 2, allows to make the results in this section independent from the production side of the economy, and in particular the nature of price and wage stickiness. We

maintain Assumption 1 in the next section as well, but relax it in our quantitative analysis in Section 6. We relax the Cole-Obstfeld assumption in both following Sections 5 and 6.

Under Assumption 1, we have $Y^*_{Ht} = C^*_{Ht}$ and $Y_{Ft} = C_{Ft}$, and using demand (7)–(8) and additionally imposing Assumption 2, we can write net exports in (15) as:[15]

$$NX_t = \gamma e^{(1-\gamma)\xi_t} P_t \left[e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{Q}_t C^*_t - C_t \right], \tag{17}$$

where $\tilde{\xi}_t \equiv \xi_t - \xi^*_t$ denotes the relative preference shock for foreign goods. Note that (17) together with (16) provide two conditions which link together relative consumption C_t/C^*_t with real exchange rate $\mathcal{Q}_t$, and as a result allow us to characterize their joint behavior independently of the supply side of the economy and the monetary policy regime.

Implicitly, we are making an additional assumption that the set of internationally traded assets is neither complete, nor empty, that is $J_t \cap J^*_t \neq \emptyset$, and for concreteness at least one risk-free foreign-currency (dollar) nominal bond is available for both home and foreign households. This assumption, however, is without loss of generality, as our main result in this section holds trivially under both financial autarky ($J_t \cap J^*_t = \emptyset$) and under complete asset markets (see below). Importantly, note that in the presence of financial wedges $\tilde{\zeta}^j_{t+1}$ and/or home bias shocks $\tilde{\xi}_t$, the equibrium in the Cole-Obstfeld case is not equivalent to the complete market allocation (see e.g. Pavlova and Rigobon 2007).

4.2 Equilibrium relationship

We now derive the stable equilibrium relationship that holds between relative consumption and the real exchange rate under Assumptions 1–2 and independently from the nature of sticky prices and from the monetary policy regime. We use (15)–(17) to derive the following two dynamic equations in log-deviation terms (denoted with respective lower case letters):

$$\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\} = \hat{\psi}_t, \tag{18}$$

$$\beta b_{t+1} - b_t = \gamma\left[\hat{\theta} q_t - (c_t - c^*_t) - (1-\gamma)\hat{\xi}_t\right], \tag{19}$$

and where under the Cole-Obstfeld Assumption 2 the two parameters $\sigma = \hat{\theta} = 1$. These two conditions can be viewed as either log-linearization of (15)–(17) around a symmetric steady state, in which case $\hat{\psi}_t \equiv -\mathbb{E}_t\tilde{\zeta}^j_{t+1}$ and $\hat{\xi}_t \equiv \tilde{\xi}_t$, or more generally as the exact equations with $\hat{\psi}_t$ and $\hat{\xi}_t$ capturing the full residual term. Since we do not impose any statistical properties on the co-evolution of $\{\hat{\psi}_t, \hat{\xi}_t\}_t$, this latter interpretation is without loss of generality.

Under Assumption 2, we solve the system (18)–(19) forward, for a general stochastic path of $\{\hat{\psi}_t, \hat{\xi}_t\}_t$

[15] Without the Cole-Obstfeld assumption, net exports additionally depends on terms of trade $\mathcal{S}_t$:

$$NX_t = \gamma e^{(1-\gamma)\xi_t} (\mathcal{E}_t P^*_{Ft})^{1-\theta} P^\theta_t \left[e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{S}^{\theta-1}_t \mathcal{Q}^\theta_t C^*_t - C_t \right],$$

where $\mathcal{S}_t \equiv P_{Ft}/(P^*_{Ht}\mathcal{E}_t)$. As a result, outside the PCP case, changes in the monetary policy regime would affect the transmission of exchange rate shock via the terms of trade.

and imposing a No-Ponzi-Game Condition (NPGC) on net foreign assets. This results in:[16]

Lemma 1 *Under Assumptions 1–2, the equilibrium relationship between relative consumption and the real exchange rate satisfies:*

$$\sigma(c_t - c_t^*) - q_t = \Xi_t, \tag{20}$$

where the random variable $\Xi_t \equiv \frac{1-\beta}{\gamma} b_t + \sum_{j=0}^{\infty} \beta^j \left[\beta \mathbb{E}_t \hat{\psi}_{t+j} - (1-\beta)(1-\gamma)\mathbb{E}_t \hat{\xi}_{t+j}\right]$ *with the state variable* b_t *evolving according to* (19).

Note again that the coefficient in (20) is $\sigma = 1$, but we leave it in as the result of Lemma 1 generalizes this way beyond the Cole-Obstfeld case. The equilibrium relationship (20) expresses the gap between relative consumption and the real exchange rate as a random variable that depends on the state variable b_t and the expected path of exogenous shocks to international risk sharing and international trade in goods. Note also that a version of (20) also holds in the limiting cases of complete markets and financial autarky.[17]

We next define what we mean by conventional business cycle models and prove our main result for such models. We then provide a discussion of the definition and the result.

Definition 1 (Conventional Models) *Conventional models are defined by the property that a change in the monetary regime does not change the stochastic path of the exogenous shocks* $\{\hat{\psi}_t, \hat{\xi}_t\}_t$.

Proposition 1 *Under Assumptions 1 and 2, in conventional models, the statistical properties of* $\sigma(\Delta c_t - \Delta c_t) - \Delta q_t$ *do not change with a change in the monetary policy rule and the exchange rate regime.*

This proposition follows from Definition 1 and Lemma 1, after combining (20) with (19) to solve out the process for the evolution of the state variable Δb_{t+1} as a function of exogenous shocks.[18]

Proposition 1 suggests that, in a class of models labeled *conventional*, the statistical properties of a particular linear combination of relative consumption growth and real exchange rate changes should not have a break point with the sharp change in the monetary policy regime from a peg to a float. This is very clearly falsified by the data from the end of the Bretton Woods, as we show in Section 2, in particular in Table 1, Figure 5c and Figure 7. The data suggests a dramatic increase in the volatility of $\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ after the end of the Bretton Woods, falsifying the models nested by the proposition.

Interpretation While the Cole-Obstfeld case is very special, it offers a sharp analytical insight that extends quantitatively to a much richer model of Section 6, which in particular relaxes both Assumptions 1 and 2. The advantage of the Cole-Obstfeld case is in that we do not have to fully specify the

[16] Sovling (18) forward, we have $\sigma(c_t - c_t) - q_t = \sum_{j=0}^{\infty} \mathbb{E}_t \hat{\psi}_{t+j} + v_t^{\infty}$, where $v_t^{\infty} \equiv \lim_{j\to\infty} \mathbb{E}_t \left\{\sigma(c_{t+j} - c_{t+j}^*) - q_{t+j}\right\}$. Also solve (19) forward, imposing NPGC $\lim_{j\to\infty} \beta^j b_{t+j} = 0$, to obtain $\gamma \sum_{j=0}^{\infty} \beta^j \left[(c_{t+j} - c_{t+j}^*) - \hat{\theta} q_{t+j} + (1-\gamma)\hat{\xi}_{t+j}\right] = b_t$. Combine the two together, under the assumption $\sigma = \hat{\theta} = 1$, to solve for v_t^{∞} and arrive at (20).

[17] Under financial autarky, $b_t \equiv 0$, and therefore $nx_t = \gamma\left[\hat{\theta} q_t - (c_t - c_t^*) - (1-\gamma)\hat{\xi}_t\right] = 0$. Under complete markets, we instead have $\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} = \tilde{\zeta}_{t+1}^j$. These relationships replace (20) in the two corresponding limiting cases.

[18] Substituting (20) into (19), we obtain $\frac{1}{\gamma}\Delta b_{t+1} = -(1-\gamma)\hat{\xi}_t + (1-\gamma)\frac{1-\beta}{\beta} \sum_{j=1}^{\infty} \beta^j \mathbb{E}_t \hat{\xi}_{t+j} - \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{\psi}_{t+j}$.

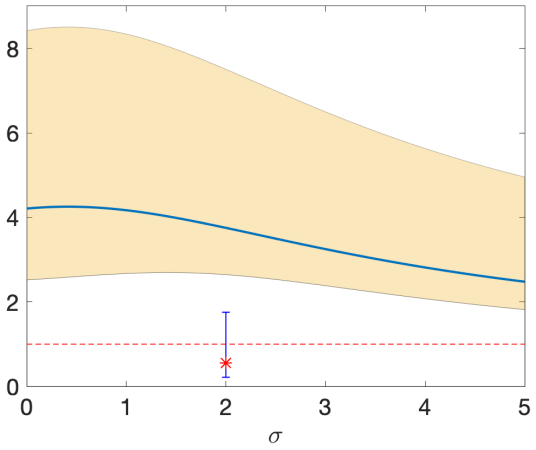


Figure 7: Ratio of $\mathrm{std}\big(\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t\big)$ after/during the Bretton Woods System

Note: $\mathrm{std}\big(\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t\big)$ is computed for 1960–72 and 1973–89 for the RoW vs the U.S. for different value of σ. The dashed red line at 1 illustrates the prediction of Proposition 1 for the ratio of the standard deviation across the regimes. The red asterisk (and the simulated 90% blue confidence interval) correspond to the calibrated quantitative model, which is not nested as a special case of Proposition 1 (relaxes both Assumptions 1 and 2).

supply side of the economy and the particular nature of shocks that are the key drivers of exchange rates and macro variables.[19]

While Proposition 1 equally falsifies flexible-price models and sticky-price models with both producer and local currency pricing, the patterns of consumption and real exchange rate impulse responses to shocks differ vastly across these specifications, as well as change differentially with the monetary regime (for illustration see Appendix Figure A7). What Proposition 1 emphasizes instead is that a particular linear combination of relative consumption and the real exchange rate has stable impulse responses to shocks independently of the monetary regime and the details of the supply side. This, in turn, implies that if a switch in the monetary regime affects the equilibrium properties of the real exchange rate, it must also affect the equilibrium properties of relative consumption, at least as long as we stay in the class of conventional models.

In light of Proposition 1, and its quantitative robustness outside the special Cole-Obstfeld case, a necessary property of a model to resolve the Mussa puzzle is to have the processes for $\hat{\psi}_t$ and/or $\hat{\xi}_t$ to change with the monetary regime. This makes the model 'unconventional' according to Definition 1. Standard models with incomplete asset markets and/or a failure of the law of one price in the goods market can feature such effects. For example, a change in the monetary regime in general changes the patterns of risk premia in incomplete asset markets.[20] However, in standard business cycle models such effects are quantitatively small, and hence the assumption of conventional models in Definition 1 still provides an accurate approximation, as we discuss further in Section 6. In the next section, we build

[19] In appendix, we discuss alternative special cases under which a similar analytical result is obtained. For example, a version of Proposition 1 holds outside of the Cole-Obstfeld case in the limit of $\beta \to 1$ (i.e., perfect patience and no intertemporal discounting).

[20] Indeed, with incomplete asset markets, switches in monetary policy change the spanning of states by nominal assets even in the absence of nominal rigidities, and create nominal non-neutrality through this alternative channel (endogenous $\hat{\psi}_t$). Furthermore, outside the Cole-Obstfeld case and under local currency price stickiness, switches in monetary regime change the transmission of exchange rate shocks to terms of trade (via differential law of one price deviations) and hence to net exports (endogenous $\hat{\xi}_t$). We evaluate these mechanisms in Section 6 and find them to be quantitatively negligible in standard models.

an 'unconventional' model with a segmented financial market, which magnifies the effect of shifts in monetary policy on the risk premium in foreign-currency bonds and correspondingly the extent of the UIP deviations. In other words, we endogenize the path of $\hat{\psi}_t$ to the exchange rate regime, and show that such a model can successfully capture the Mussa facts, without requiring any additional ingredients.

5 An Alternative Model of Non-neutrality

We now present an alternative explanation to the broad set of Mussa facts documented in Section 2. Specifically, we propose a model with monetary non-neutrality emerging due to financial market segmentation, rather than as a result of goods-market price stickiness. We maintain the general modeling environment of Section 3, but to emphasize the point assume away nominal rigities (set $\lambda_p = \lambda_w = 0$), which we turn back on in Section 6. For analytical tractability, we maintain Assumption 1 from Section 4, but relax the Cole-Obstfeld Assumption 2 and allow for general parameter values. The only new special feature is the modeling of the international financial market, as we describe next.

5.1 Segmented financial market

There are three types of agents participating in the financial market: households, noise traders and professional intermediaries. The home and foreign households trade local-currency bonds only. In particular, the home households demand at time t a quantity B_{t+1} of the home-currency bonds. Similarly, foreign households demand a quantity B^*_{t+1} of the foreign-currency bonds. Therefore, in the notation of Section 3, $|J_t| = |J^*_t| = 1$ and $J_t \cap J^*_t = \emptyset$. Both B_{t+1} and B^*_{t+1} can take positive or negative values, depending on whether the households save or borrow respectively. Finally, for concreteness, we assume that these bonds pay out $D_{t+1} = 1$ euro and $D^*_{t+1} = 1$ dollar at period $t+1$, and hence their period t prices are $\Theta_t = 1/R_t$ euros and $\Theta^*_t = 1/R^*_t$ dollars, respectively. Note that we assume away the exogenous wedges, $\zeta_{t+1} = \zeta^*_{t+1} \equiv 0$, as the risk-sharing wedge emerges endogenously in equilibrium.

The trades of the households are intermediated by risk-averse intermediaries, or market makers. There are m symmetric intermediaries, who adopt a zero-capital *carry trade* strategy, that is take a long position in the foreign-currency bonds and a short position of equal value in the home-currency bonds, or vice versa. The return on this carry trade is therefore:

$$\tilde{R}^*_{t+1} = R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{21}$$

per one dollar invested in the foreign-currency bond and $\mathcal{E}_t$ euros sold of the home-currency bonds at time t. We denote the size of individual positions by d^*_{t+1}, which may take positive or negative values, and assume that intermediaries maximize the CARA utility of the real return on their investment in units of the foreign consumption good:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} \right) \right\}, \tag{22}$$

where $\omega \geq 0$ is the risk aversion parameter.[21] In aggregate, all m intermediaries invest $\frac{D^*_{t+1}}{R^*_t} = m\frac{d^*_{t+1}}{R^*_t}$ dollars in foreign-currency bonds, as one dollar at time t affords a quantity R^*_t of dollar bonds D^*_{t+1}, each paying out one dollar at $t+1$. The intermediaries also take an offsetting position of $\frac{D_{t+1}}{R_t} = -\mathcal{E}_t\frac{D^*_{t+1}}{R^*_t}$ euros in home-currency bonds, resulting in a zero-capital portfolio at time t.

Finally, in addition to the household *fundamental* demand for currency (bonds), the financial market features a source of *liquidity* currency demand from (a measure) n symmetric noise traders, which evolves independently of the expected return of currency and the other macroeconomic fundamentals.[22] Like intermediaries, noise traders take a zero-capital position long in the foreign currency and short equal value in the home currency, or vice versa if they have an excess demand for the home currency. The overall position of the noise traders is

$$\frac{N^*_{t+1}}{R^*_t} = n\left(e^{\psi_t} - 1\right) \tag{23}$$

dollars invested in the foreign-currency bonds and respectively $\frac{N_{t+1}}{R_t} = -\mathcal{E}_t\frac{N^*_{t+1}}{R^*_t}$ euros sold of the home-currency bonds. We refer to the noise trader shock ψ_t as the *financial shock*, and assume it follows an AR(1) process:

$$\psi_t = \rho_\psi\psi_{t-1} + \sigma_\psi\varepsilon_t, \qquad \varepsilon_t \sim iid(0,1), \tag{24}$$

where $\rho_\psi \in [0,1]$ is the persistence of the financial shock and $\sigma_\psi \geq 0$ is its volatility.

Both currency bonds are in zero net supply, and therefore financial market clearing requires that the positions of the households, noise traders and intermediaries balance out:

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \qquad \text{and} \qquad B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0. \tag{25}$$

As both noise traders and intermediaries hold zero-capital positions, financial market clearing (25) implies a balanced position for the home and foreign households combined, $\frac{B_{t+1}}{R_t} + \mathcal{E}_t\frac{B^*_{t+1}}{R^*_t} = 0$. In other words, the financial market merely intermediates the intertemporal borrowing between home and foreign households, without them directly trading any assets. Lastly, the trades of intermediaries and noise traders result in income (or losses), which we assume are returned to the foreign households at the end of each trading period, as a lump-sum payment together with the dividends of the foreign firms.[23]

In equilibrium, the intermediaries absorb the demand for home and foreign currency of both households and noise traders. If intermediaries were risk neutral, $\omega = 0$, they would do so without a risk premium, resulting in the *uncovered interest parity* (UIP), or equivalently a zero expected real return

[21] CARA utility provides tractability, as it results in portfolio choice that does not depend on the level of wealth of the intermediaries (see Appendix), thus avoiding the need to carry it as an additional state variable; the tradeoff of working with CARA-utility, however, is that intermediaries need to be short-lived, maximizing the one-period return on their investment.

[22] Apart from exogenous liquidity needs, the noise trader currency demand can emerge from biased expectations about the exchange rate, $\mathbb{E}^n_t\mathcal{E}_{t+1} \neq \mathbb{E}_t\mathcal{E}_{t+1}$, as in Jeanne and Rose (2002).

[23] This generates an additional income of $\tilde{R}^*_{t+1}(D^*_{t+1} + N^*_{t+1})/R^*_t$ dollars for the foreign households. As a result of this transfer, the foreign country budget constraint becomes the same as the home country budget constraint (15), with $\mathcal{B}_{t+1} = B_{t+1}/R_t$ and $\mathcal{R}_t\mathcal{B}_t = B_t$, despite foreign households facing a generally different rate of return $R^*_t \neq R_t$. See Appendix for details, where we also show that this transfer is second order, and hence does not affect the first-order dynamics of the equilibrium system.

on the carry trade, $\mathbb{E}_t\{\tilde{R}^*_{t+1}/P^*_{t+1}\} = 0$. However, risk-averse intermediaries are not willing to take a risky carry trade without an appropriate compensation, resulting in equilibrium risk premia and deviations from the UIP. We characterize the equilibrium in the financial market in (see proof in appendix):

Lemma 2 *The equilibrium condition in the financial market, log-linearized around a symmetric steady state with $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{\mathcal{Q}} = 1$ and a finite nonzero $\omega\sigma_e^2/m$, is given by:*

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \tag{26}$$

where $i_t - i_t^* \equiv \log(R_t/R_t^*)$, $b_{t+1} \equiv B_{t+1}/\bar{Y}$, and the coefficients $\chi_1 \equiv \frac{n}{\beta}\frac{\omega\sigma_e^2}{m}$ and $\chi_2 \equiv \bar{Y}\frac{\omega\sigma_e^2}{m}$, with $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ denoting the conditional volatility of the nominal exchange rate.

The equilibrium condition (26) is the *modified UIP* in our model with imperfect financial intermediation, where the right-hand side corresponds to the departures from the UIP. Condition (26) arises from the combination of the financial market clearing (25) with the solution to the portfolio choice problem of the intermediaries (22), as we formally show in Appendix. Intuitively, the optimal portfolio of the intermediaries D^*_{t+1} is proportional to the expected log return on the carry trade, $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, scaled by the risk absorption capacity of the intermediary sector, $\omega\sigma_e^2/m$, i.e. the product of their effective risk aversion (ω/m, the price of risk) and the volatility of the carry trade return (σ_e^2, the exchange rate risk). As $\omega\sigma_e^2/m \to 0$, the risk absorption capacity of the intermediaries increases, and the UIP deviations disappear in the limit as $\chi_1, \chi_2 \to 0$. With $\omega\sigma_e^2/m > 0$, the UIP deviations remain first order and hence affect the first-order equilibrium dynamics. Finally, note that both $\psi_t > 0$ and $b_{t+1} < 0$ correspond to the excess demand for the foreign-currency bond — by noise traders and households, respectively — and hence result in a negative expected return on the foreign currency bond.

Discussion Condition (26) characterizes equilibrium in the financial market. It can be combined with the household Euler equations to obtain the analog of the international risk-sharing condition (18) with

$$\hat{\psi}_t \equiv \chi_1 \psi_t - \chi_2 b_{t+1}, \tag{27}$$

endogenizing the risk-sharing wedge.[24] When the monetary policy affects the equilibrium volatility of the nominal exchange rate, σ_e^2, it changes the volatility of the risk-sharing wedge, $\hat{\psi}_t$, via its effects on χ_1 and χ_2. Despite fully flexible prices, monetary policy is non-neutral in our model because of financial frictions: the completeness of asset markets is endogenous to monetary policy, which determines the real returns on nominal bonds and, hence, affects the risk sharing between agents. Indeed, a shift to an exchange rate peg stabilizes the nominal exchange rate and encourages financial intermediaries to take larger positions against both households and noise traders, reducing the extent of the equilibrium UIP deviations and resulting in greater risk sharing between home and foreign households.[25]

[24] The log-linearized home household Euler equation is $i_t = \mathbb{E}_t\{\sigma\Delta c_{t+1} + \Delta p_{t+1}\}$, and similarly for the foreign household.

[25] Note that, despite the changing properties of the UIP violations with the monetary regime, the *covered interest parity* (CIP) holds in the model independently of the monetary regime. Indeed, any CIP violations would result in arbitrage profit opportunities, and the intermediaries would be willing to take unbounded asset positions in order to exploit them. This

Thus, the 'unconventional' feature of the model is that monetary policy affects equilibrium risk premia and allocations in the financial market, and the financial market segmentation magnifies these effects. Indeed, for financial intermediaries who earn carry trade returns, the relevant measure of risk premium is the volatility of the *nominal* exchange rate, rather than its covariance with aggregate consumption, as would be the case in a conventional model. In the data, while the volatility of the nominal exchange rate changed dramatically after the end of the Bretton Woods, the covariance of consumption with the nominal exchange rate remained negligibly small, emphasizing again the quantitative challenge for conventional models.

5.2 Mussa puzzle resolution

We now study the properties of the general equilibrium dynamics in the model with a segmented financial market under alternative monetary policy regimes. For simplicity, we consider the limiting case in which the monetary authorities have the ability to fully stabilize prices or the nominal exchange rate, depending on the regime. That is, the foreign country always chooses $\pi_t^* \equiv 0$, while the home country adopts either a peg with $\Delta e_t \equiv 0$ or a float with $\pi_t \equiv 0$. Therefore, under a peg $\sigma_e^2 = 0$ and hence $\chi_1 = \chi_2 = 0$ in (26), while under a float $\sigma_e^2 > 0$ and hence $\chi_1, \chi_2 > 0$. Furthermore, we consider the case with two types of shocks — the noise trader shock ψ_t given by (24) and country-specific productivity shocks (a_t, a_t^*), which also follow AR(1) processes with persistence ρ_a and standard deviation of innovations σ_a. For simplicity, we assume a common persistence parameter, that is $\rho_a = \rho_\psi = \rho \in [0, 1]$.

The equilibrium system consists of two dynamics equations, still given by the risk-sharing condition (18) and the intertemporal budget constraint (19), where in the former the overall risk-sharing wedge $\hat{\psi}_t$ satisfies (27), while in the latter $\hat{\xi}_t \equiv 0$. Outside the Cole-Obstfeld case, the parameter $\hat{\theta} \equiv \frac{2\theta(1-\gamma)-1}{1-2\gamma}$.[26] The additional equilibrium condition characterizes market clearing in the labor and product markets, and can be written as:

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t, \tag{28}$$

where $\kappa_a \equiv \frac{1+\varphi}{\sigma+(1-2\gamma)\varphi}$ and $\kappa_q \equiv \frac{2}{1-2\gamma}\frac{2\theta(1-\gamma)\varphi+1}{\sigma+(1-2\gamma)\varphi}$. Relative consumption increases in the relative home productivity and decreases with the real exchange rate depreciation (increase in q_t), both of which are required to clear the goods market.

Substituting (28) into both dynamic equations, (18) and (19), results in an autonomous dynamic system in (q_t, b_t):

$$-(1+\gamma\sigma\kappa_q)\mathbb{E}_t\Delta q_{t+1} = \chi_1\psi_t + \sigma(1-\rho)\kappa_a\tilde{a}_t - \chi_2 b_{t+1}, \tag{29}$$

$$\beta b_{t+1} - b_t = \gamma\big[(\hat{\theta}+\gamma\kappa_q)q_t - \kappa_a\tilde{a}_t\big], \tag{30}$$

pattern of changing UIP violations and *un*changing CIP across a shift from peg to float is in line with the empirical evidence from the end of the Bretton Woods (see e.g. Frenkel and Levich 1975), as we further discuss below.

[26] Recall that in general, when $\theta \neq 1$, NX_t increases in $\mathcal{S}_t^{\theta-1}\mathcal{Q}_t^\theta$, and in a model without law of one price violations, log-linearization yields $s_t = \frac{1}{1-2\gamma}q_t$. Therefore, nx_t increases with $\hat{\theta}q_t$, where $\hat{\theta} = \theta + \frac{1}{1-2\gamma}(\theta-1)$.

where $\tilde{a}_t \equiv a_t - a_t^*$ is the relative productivity shock, and in deriving (29) we used the fact that $\tilde{a}_t$ follows an AR(1) with persistence ρ. Recall that χ_1 and χ_2 in (29) are now endogenous to the monetary regime. Solving this dynamic system, we characterize the equilibrium process for the real exchange rate (see proof in appendix):

Lemma 3 *The equilibrium always exists under both floating and peg regimes and the real exchange rate follows ARMA(2,1) process:*

$$(1-\delta L)q_t = \frac{1}{1+\gamma\sigma\kappa_q}\frac{\beta\delta}{1-\beta\rho\delta}\Bigg[(1-\beta^{-1}L)\chi_1\psi_t + \left(\frac{1}{\beta\delta}\frac{1-\beta\delta}{1+\frac{\sigma\hat{\theta}-1}{1+\gamma\sigma\kappa_q}}(1-\rho\delta L)+(1-\rho)(1-\beta^{-1}L)\right)\sigma\kappa_a\tilde{a}_t\Bigg], \tag{31}$$

where δ is the eigenvalue of the dynamic system (29)–(30), such that $\delta \in (0,1]$, and $\delta < 1$ when $\chi_2 > 0$ (under the float) and $\delta = 1$ when $\chi_2 = 0$ (under the peg).

In line with the empirical evidence, the real exchange rate exhibits near-random-walk properties under the float when $\beta\rho \approx 1$: the autocorrelation of changes in exchange rate, $\mathrm{corr}(\Delta e_t, \Delta e_{t-1})$, is arbitrary close to zero and the contribution of the predictable component $\mathbb{E}_t\Delta e_{t+j}$ to the variance of Δe_{t+j} is negligible. Intuitively, persistent shocks with $\rho \approx 1$ generate prolonged depreciation or appreciation in the future, which under incomplete asset markets, have to be compensated by a jump in exchange rate on impact to ensure that the country's budget constraint holds. These unexpected initial jumps dominate in exchange rate dynamics in the limit $\beta\rho \to 1$ (see Itskhoki and Mukhin 2019).

While the ARMA process (31) nests both monetary regimes, there is an important difference between the two. Under the peg, the monetary policy fully eliminates exchange rate risk associated with the carry trade, allowing arbitrageurs to take large positions that fully offset the demand of the noise traders, i.e. $\chi_1 = 0$ and shocks ψ_t have no effect on the real exchange rate. In contrast, under floating regime, $\chi_1 > 0$, arbitrageurs are reluctant to take large gross positions and noise-trader shocks ψ_t result in UIP deviations. Thus, given that shocks ψ_t matter under the float, but not under the peg, the model can generate arbitrary large differences in the volatility of the real exchange rate across two regimes. In addition, because the nominal exchange rate is by assumption fully stable under the peg and mirrors dynamics of the real exchange rate under the float, the model also predicts a discontinuous change in the volatility of the *nominal* exchange rate. Notice, however, that the real exchange rate responds to productivity shocks under both regimes, consistent with the non-negligible volatility of q_t (relative to that of e_t) under the peg (see Figure A1). In the limiting case of $\rho \approx 1$, the volatility of the real exchange rate that is due to productivity shocks is independent from monetary policy and we obtain the following result:

Proposition 2 *A change in the monetary policy rule from peg to float results in a sharp increase in the volatility of both nominal and real exchange rates — arbitrary large when $\rho \approx 1$ — with the change in the behavior of the other real macro variables vanishingly small when $\gamma \approx 0$.*

The first part of the proposition rationalizes the classical Mussa fact about the discontinuity in the volatility of exchange rates across float and peg regimes, while the second part shows that the model is also consistent with the additional facts about no change in the volatility of other macro aggregates. The intuition for the latter can be clearly seen from the market clearing condition (28), which characterizes the equilibrium relationship between relative consumption, real exchange rate and relative productivity in the two countries. With flexible prices, the only channel through which monetary policy affects relative consumption is through the volatility of the real exchange rate. This effect is, however, proportional to the openness of the economy — in the autarky limit with $\gamma \approx 0$ movements in relative prices across countries are irrelevant for allocation as domestic consumption are determined solely by local productivity shocks. The same logic applies to output and all other real outcomes. As a result, large changes in the volatility of the real exchange rate are consistent with only minor changes in the volatility of the real macro variables, if the equilibrium behavior of the latter is mostly driven by the productivity shocks, even in the presence of substantial exchange rate volatility. In particular, this is the case when countries are sufficiently home biased in the goods market.

The case of the nominal macro variables is somewhat more involved. Consider the consumer price inflation, π_t. The volatility of prices changes from zero under the float to a positive number which is proportional to the volatility of the productivity shock. Under the peg, when $\rho \to 1$, the volatility of inflation (and hence the real exchange rate, since $\Delta q_t = -\pi_t$) is given by:

$$\mathrm{std}(\pi_t) = \mathrm{std}(\Delta q_t) = \frac{\kappa_a}{\hat{\theta} + \gamma\kappa_q}\mathrm{std}(\Delta\tilde{a}_t).$$

Therefore, $\mathrm{std}(\pi_t)$ is arbitrary smaller than $\mathrm{std}(\Delta q_t)$ under the float, and it is equal to $\mathrm{std}(\Delta q_t)$ under the peg, consistent with empirical patterns in Figure 3. The next section relaxes the extreme assumption of full price/nominal exchange rate stabilization under the two regimes, and shows that under conventional Taylor rule the volatility of inflation changes little across the two regimes.

We interpret Proposition 2 as an 'order-of-magnitude' result, which illustrates that the model is capable — in certain limiting cases — to generate *arbitrary large* changes in the real and nominal exchange rate volatility with *vanishingly small* changes in the macro volatility, as the exchange rate regime switches from a peg to a float. In Section 6, we explore the quantitative properties of a calibrated model away from these limiting cases, which nonetheless provide useful intuition for the quantitative results.

5.3 Additional evidence

So far, our analysis has focused mostly on the volatilities of exchange rates and macro variables across the two monetary regimes. This choice of moments is driven mostly by robust discontinuities, or the lack thereof, around the end of the Bretton Woods in the data. As Table 2 makes clear, the patterns are less obvious for other second moments, namely the correlations. Nevertheless, the change in the empirical correlations is an important overidentification test of the theoretical mechanism and is consistent with the predictions of the model with segmented financial market.

In particular, the key prediction of our model is that financial shocks are central to the exchange

rate dynamics under the floating regime, and become significantly less important under the peg. It follows that the main drivers of the real exchange rate under the peg are 'fundamental' shocks, such as productivity shocks. Given a conventional transmission of these shocks, the model predicts that most exchange rate puzzles that emerge under a floating regime should disappear under a peg. This is true in particular for the forward premium puzzle (Fama 1984), the Backus-Smith puzzle (Backus and Smith 1993, Kollmann 1995), and the Balassa-Samuelson effect (Balassa 1964, Samuelson 1964).[27]

Proposition 3 *A change in the monetary policy rule from peg to float results in the emergence of (a) the forward premium puzzle, (b) the Backus-Smith puzzle, and (c) a weaker Balassa-Samuelson effect.*

Consider first the forward premium puzzle. Clearly, this anomaly cannot emerge when risk premium is zero and therefore, one would expect smaller deviations from the UIP under the peg. The empirical evidence is largely consistent with this prediction of the model: using historical data for the U.K and the U.S., Colacito and Croce (2013) show that the estimated UIP coefficient was close to one during most of the Bretton-Woods periods and became negative afterwards. Note also that the model is consistent with the CIP holding well in the data both before and after the end of the Bretton Woods, as documented in Frenkel and Levich (1975).

Similarly, the model predicts that the Backus-Smith condition should hold — at least conditionally in expected terms — in the absence of risk-premium shocks in the financial markets. Table 2 shows that the correlation between the real exchange rate and the relative consumption is indeed higher under the peg than under the float for every single country in our sample, and flips the sign to negative under the float in all cases but one. This is also consistent with the findings of Colacito and Croce (2013) based on longer historical series for the U.S. and the U.K. In addition, Devereux and Hnatkovska (2014) provide empirical evidence using the alternative quasi-experiment, namely the formation of the Eurozone. In particular, they show that the Backus-Smith risk-sharing condition holds much better for members of the currency union than for both the same countries before the formation of the Eurozone and between the countries with different currencies.

Finally, in a straightforward extension of the baseline model with two sectors, the real exchange rates should appreciate according to the Balassa-Samuelson effect when country's productivity in the tradable sector goes up relative to the productivity in the non-tradable sector. While true under either monetary regime, this correlation is harder to identify under the floating regime because of the relatively small overall contribution of productivity shocks in the exchange rate dynamics. Again, we find empirical evidence in line with the model's predictions: while the Balassa-Samuelson effect has almost no explanatory power under the float (Engel 1999), the recent literature has shown that the effect holds surprisingly well for the Eurozone countries with a fixed exchange rate (Berka, Devereux, and Engel 2012, 2018).

[27] Note that the Meese and Rogoff (1983) puzzle trivially disappears under the peg given that nominal exchange rate becomes perfectly predictable. The case of the PPP puzzle — the surprising combination of high volatility and high persistence of the real exchange rate (Rogoff 1996) — is more involved: in line with the empirical evidence, while the peg decreases the volatility of the real exchange rate, it does not necessarily change its persistence (see Lemma 3).

6 Quantitative Exploration

This section shows that both negative results from Propositions 1 and positive results from Proposition 2 and 3 remain robust in a quantitative version of the model. We compare three classes of models – without UIP shocks, with exogenous UIP shocks, and finally with UIP shocks endogenous to the monetary policy regime, consistent with the model of the segmented financial market in Section 5. We show that *only* the latter class of models is consistent with the umbrella of Mussa facts documented in Section 2, while nominal rigidities are neither necessary, nor sufficient to explain the puzzle.

The quantitative version of the model is from Itskhoki and Mukhin (2019) and is described in Appendix. In particular, we use the general modeling framework of Section 3 with capital investment with adjustment costs, intermediate goods, Calvo sticky wages and LCP sticky prices, additionally augmenting the model pricing-to-market due to Kimball demand. The monetary policy in the model is conducted according to a conventional Taylor rule with a weight on the nominal exchange rate, as in (14). The change in the exchange rate regime corresponds to a change in this weight that the monetary authority puts on the nominal exchange rate in its Taylor rule.

6.1 Calibration

For most parameters we use standard values from the literature (see Appendix Tables A1–A2). In particular, we calibrate the openness of the economy to $\gamma = 0.035$ to match the average import-to-GDP ratio of 0.07 for the U.S. for the periode from 1960–1990. Following a common practice in the literature (see Schmitt-Grohé and Uribe 2003), and consistent with our own model of the financial sector, we make sure that the model is stationary by assuming a coefficient $\chi_2 = 0.001$ in front of b_{t+1} in the UIP condition (26) under a floating regime, allowing it to change with the monetary policy in the model with endogenous risk premium. We consider three versions of each model: a flexible-price model with productivity shocks (IRBC), a sticky-price model with productivity shocks (NKOE-1), and a sticky-price model with monetary shocks (NKOE-2). In all specifications, we also allow for the financial shock ψ_t and taste shocks ξ_t between foreign and home goods. The baseline specifications with sticky prices assume local currency pricing (LCP), but results are also similar under the dominant currency pricing (DCP).

In all specifications of the model, the correlation of productivity (monetary) shocks across countries is calibrated to match $\mathrm{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the floating regime (see Table A2). We target the volatility of the nominal exchange rate $\mathrm{std}(\Delta e_t)$ under the float to calibrate the volatility of the shocks. The relative volatility of productivity (monetary) and trade shocks is calibrated to match respectively $\mathrm{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$ under floating regime and $\mathrm{corr}(\Delta q_t, \Delta nx_t) = -0.1$ under the peg.[28] The costs of capital adjustment are calibrated to match $\frac{\mathrm{std}(\Delta z_t)}{\mathrm{std}(\Delta gdp_t)} = 2.5$ under the floating regime, where z_t is real investment expenditure. The weight of NER in the Taylor rule of Foreign country is always zero, while the weight for Home country is zero under the floating regime and is calibrated under the peg to match an eight-fold decrease in $\mathrm{std}(\Delta e_t)$. We keep all other coefficients

[28] In the version of the model with exogenous risk-premium shocks, we use the same calibration as the one with endogenous financial shocks.

constant across regimes. In the model with the endogenous UIP shocks, we change the coefficients χ_1 and χ_2 in the UIP condition (26) in proportion with the change in σ_e^2 across the two monetary regimes.[29]

6.2 Results

Table 3 reports the volatilities of exchange rates and macro variables under the two monetary regimes for the alternative versions of the model. Consider first the three models (IRBC, NKOE-1 and 2) without the UIP shock ψ_t. Calibrated to match the volatility of nominal exchange rates, all three specifications also feature a high volatility in the real exchange rate. This, however, comes at a cost: in the absence of financial shocks, the models require large productivity or monetary shocks to generate volatile exchange rates, which in turn results in counterfactually volatile macro variables. Indeed, the volatility of consumption and GDP is 5-10 times higher than in the data and is of the same order of magnitude as of the exchange rates. Thus, consistent with the findings in Itskhoki and Mukhin (2019), the models without financial shocks are inconsistent with the exchange rate disconnect under the floating regime.

Setting aside the inability of this class of models to generate the disconnect in volatilities between the macro variables and the exchange rates, note that the NKOE-2 version of this model with LCP border price stickiness cannot be immediately falsified by the ratio of volatilities in consumption and GDP. While the levels of volatility are always too high relative to the data, their ratios across the two exchange rate regimes are close to unity, as in the data. Nonetheless, focusing on the equilibrium relationship between consumption and the real exchange rate, $\sigma(c_t - c_t^*) - q_t$, allows to falsify the NKOE-2 model in this class, as this ratio remains stable in the model, while it changes dramatically in the data. This confirms the usefulness of our theoretical Propostion 1 even for the quantitative models that do not satisfy Assumptions 1–2 which were used to prove the exact theoretical result. The reason why this version of the model fails is that it predicts a counterfactually high positive correlation between $\sigma(c_t - c_t^*)$ and q_t under the float — the Backus-Smith puzzle.[30]

The next three specifications of the model allow for the exogenous UIP shocks with the same volatility under the two monetary regimes. All three specifications are now much more successful in addressing the disconnect puzzle — under the floating regime, the volatility of exchange rates is an order of magnitude higher than the volatility of consumption, GDP and inflation. At the same time, the models struggle to match the Mussa facts. Expectedly, the flexible-price IRBC model produces no change in the behaviour of real variables across monetary regimes and is inconsistent with the original Mussa puzzle about structural break in the dynamics of the real exchange rate in 1973. The specifications with nominal rigidities, on the other hand, perform much better in terms of the exchange rate moments — the sluggish adjustment of prices ensures that the real exchange rate follows closely the nominal one and is about 5 times more volatile under the float relative to the peg. The sticky-price models, however, have counterfactual predictions for other macro variables. In contrast to almost no differ-

[29] All moments are calculated by simulating the model for $T = 100,000$ quarters – all standard errors are below 0.01 in this case. The results remain almost the same if one uses medians from several simulations of $T = 120$ quarters except for the autocorrelations and the UIP coefficient, which are severely biased downwards in small samples.

[30] This also illustrates why the simple partial-equilibrium logic of low exchange rate pass-through under LCP is insufficient for our analysis, which relies on the full general equilibrium structure of the model, and in particular on the properties of the equilibrium international risk sharing.

Table 3: Quantitative models

Panel A: standard deviations

	Δq_t			π_t			Δc_t			Δgdp_t			$\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Data	1.8	9.9	5.6	1.6	2.2	1.3	0.9	1.0	1.1	1.1	0.9	0.8	2.5	10.7	4.3
Models w/o UIP shock:															
IRBC	15.4	15.4	1.0	12.7	3.2	0.2	9.1	9.1	1.0	15.0	15.0	1.0	9.3	9.3	1.0
NKOE-1	4.2	12.8	3.0	3.1	1.8	0.6	7.1	6.8	1.0	17.7	11.7	0.7	11.3	10.8	1.0
NKOE-2	1.5	11.5	7.4	1.3	1.3	1.0	5.0	5.2	1.0	8.1	8.4	1.0	2.0	2.0	1.0
Models w/ exogenous UIP shock:															
IRBC	11.0	11.0	1.0	10.2	0.9	0.1	1.8	1.8	1.0	2.5	2.5	1.0	13.6	13.6	1.0
NKOE-1	2.2	11.9	5.3	1.4	0.4	0.3	5.8	1.3	0.2	14.5	2.1	0.1	14.0	13.2	0.9
NKOE-2	2.1	11.8	5.7	1.3	0.3	0.2	5.8	1.1	0.2	8.6	1.8	0.2	13.9	13.1	0.9
Models w/ endogenous UIP shock:															
IRBC	3.0	11.0	3.6	1.4	0.9	0.7	1.6	1.8	1.1	2.5	2.5	1.0	1.5	13.6	9.1
NKOE-1	1.7	11.9	6.9	0.4	0.4	1.0	1.1	1.3	1.1	1.9	2.1	1.1	1.2	13.2	11
NKOE-2	1.4	11.8	8.2	0.2	0.3	1.5	0.9	1.1	1.2	1.5	1.8	1.2	0.5	13.1	27

Panel B: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$		β^{UIP}	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Data	0.80	0.98	0.05	−0.40	−0.10	0.20	−0.03	0.30	−0.11	0.31	0.68	0.72	0.9	−1.0
Models w/o UIP shock:														
IRBC	0.86	0.99	0.91	0.91	−0.10	−0.10	0.30	0.30	0.34	0.34	0.99	0.99	0.8	0.9
NKOE-1	0.67	0.99	0.28	0.70	−0.10	−0.49	0.38	0.31	0.65	0.41	0.91	0.97	0.3	1.0
NKOE-2	0.96	0.99	0.49	0.99	−0.10	0.05	0.95	0.30	0.97	0.33	1.00	1.00	1.0	1.0
Models w/ exogenous UIP shock:														
IRBC	0.86	0.99	−0.40	−0.40	0.93	0.93	0.30	0.30	0.15	0.15	0.88	0.88	0.0	−1.3
NKOE-1	0.81	1.00	−0.88	−0.40	0.89	0.93	0.60	0.30	−0.06	0.32	0.99	0.84	−0.1	−1.6
NKOE-2	0.82	1.00	−0.89	−0.40	0.92	0.97	0.51	0.30	−0.10	0.26	1.00	0.79	−0.1	−2.2
Models w/ endogenous UIP shock:														
IRBC	0.98	1.00	0.92	−0.40	−0.10	0.93	0.30	0.30	0.39	0.16	0.99	0.88	1.0	−1.4
NKOE-1	0.98	1.00	0.84	−0.40	−0.10	0.93	0.44	0.30	0.54	0.32	0.96	0.84	1.0	−1.6
NKOE-2	1.00	1.00	0.94	−0.40	−0.10	0.97	0.66	0.30	0.70	0.26	0.99	0.79	1.0	−2.3

Note: see the text and notes to Appendix Table A2.

ences in the volatilities of inflation, consumption and GDP across two monetary regimes in the data, the models feature a 5-10 times increase in standard deviations of macro variables. This generalizes the insight from Proposition 1 by showing that models with exogenous UIP shocks are inconsistent with the broader set of Mussa puzzles not only in the special limiting cases, but also in a realistically calibrated quantitative models.

Finally, the last three rows of Table 3 allow for an endogenous UIP shock, as in the model of the segmented financial market in Section 5. Under the float, the model specifications are naturally isomorphic to the ones with the exogenous UIP shocks, and produce a high volatility in exchange rates relative to the other macro variables, consistent with exchange rate disconnect. In contrast to the previous three specifications, however, the models also match well the data under the peg — the volatility of the real exchange rate is 4-to-8 times lower than under the float and the volatility of other macro variables changes only modestly by about 10%, as is the case in the data. Importantly, the results are

Table 4: Variance decomposition

	peg			float		
	ψ	ξ	a or m	ψ	ξ	a or m
Real exchange rate, $\mathrm{var}(\Delta q_t)$						
IRBC	1	23	76	92	3	5
NKOE-1	1	22	77	97	1	2
NKOE-2	1	4	95	97	1	2
Consumption, $\mathrm{var}(\Delta c_t)$						
IRBC	0	1	99	15	1	84
NKOE-1	0	1	99	10	0	90
NKOE-2	0	1	99	13	0	87

Note: variance decomposition of RER and consumption in model specifications with the endogenous UIP shock.

similar across specifications and do not depend strongly on the type of shocks (productivity vs monetary) or the presence of nominal rigidities. In this sense, nominal rigidities are neither necessary, nor sufficient to match the Mussa puzzle. On the margin, however, sticky prices do slightly improve the fit of the model.

To see the intuition behind these results, consider the variance decomposition in the last set of model specifications with endogenous UIP shocks. From Table 4, more than 90% of real exchange rate volatility is driven by financial shocks. A switch in monetary policy to the peg eliminates most of the carry trade risk and almost fully eliminates financial shocks, which results in a drastic fall in the volatility of the real exchange rate. At the same time, the dynamics of other variables does not change much for two reasons. On the one hand, because of low openness of the economies, financial shocks account only for a modest share of volatility in macro variables even under the float (namely, 10–15% of consumption volatility). As a result, the decreasing importance of the UIP shocks under the peg has only minor implications for macro aggregates. Nonetheless, a change in the monetary policy *per se* could significantly change the behavior of inflation (under flexible prices) and real variables (under sticky prices). This does not happen, however, due to small changes in the *equilibrium* monetary policy. Indeed, with financial shocks gone, the policy does not need to change much to ensure a stable nominal exchange rate. In other words, the government commitment to peg already goes a long way towards stabilizing exchange rates and does not require large interventions on the equilibrium path.

Lastly, the lower panel of Table 3 complements the analysis with several correlations. Consistent with Proposition 3 and the empirical evidence from Section 2, it shows that the UIP puzzle and the Backus-Smith puzzle are much more pronounced under the floating regime, while the moments are closer to the predictions of standard IRBC and NKOE models under the peg regime. Indeed, both anomalies are due to financial shocks and disappear when monetary policy eliminates risk premium in the international financial markets. At the same time, most other business cycle correlations — e.g. between consumption and output within countries and between home and foreign GDP across countries — do not change much, which is in line with empirical evidence in Section 2.

7 Conclusion

Our analysis shows that the Mussa puzzle offers a powerful test of alternative models of the exchange rates. In particular, the data rejects both *conventional* IRBC and NKOE models, in which the transmission of the monetary policy is exclusively via the aggregate demand in the goods market. The Mussa facts are also inconsistent with models of currency risk premia, which are exogenous to the monetary policy regime. This includes models of convenience yield with bonds in the utility (e.g. Valchev 2016), as well as models with *complete* asset markets, where risk premia are amplified by means of habits (e.g. Verdelhan 2010), long-run risk (e.g. Colacito and Croce 2011) or rare disasters (e.g. Farhi and Gabaix 2016). While augmenting these models with nominal rigidities can potentially generate risk premia that are endogenous to monetary policy (see e.g. Caballero, Farhi, and Gourinchas 2015), it is unlikely that a switch from a peg to a float in such environments would change only the forward premium without affecting the other macro variables. Perhaps, a more promising avenue for future research is to extend these models to environments with incomplete and segmented asset markets, as in Jeanne and Rose (2002), Alvarez, Atkeson, and Kehoe (2009) and/or Gabaix and Maggiori (2015). Yet another possible source of monetary non-neutrality are informational frictions and expectational errors (e.g. Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006). In this class of models, a nominal peg anchors the expectations of agents and eliminates the deviations from the UIP, additionally implying a lower turnover of assets when expectations are more closely aligned across agents.

We develop a model of incomplete segmented markets, in which monetary non-neutrality arises due to the effects of the monetary policy on risk premia and allocations in the financial market. With incomplete asset markets, switches in monetary policy change the spanning of states by nominal assets even in the absence of nominal rigidities, and create nominal non-neutrality through this alternative channel. In particular, we show how in a segmented asset market, a switch from a peg to a float changes the currency risk premia, which directly depend on the *nominal* exchange rate volatility, rather than the covariance of the nominal exchange rate with aggregate consumption. This leads a change in the monetary regime to result in a sharp change in the volatility of the real exchange rate, yet without significantly affecting the behavior of the other macro variables, which are largely insensitive to the currency risk premia.

This paper considers a major shift from a floating to a fixed exchange rate, emphasizing the transmission of this policy change through the financial market. It is intriguing to study, both theoretically and empirically, this transmissions channel for more ubiquitous types of monetary policy shocks (see e.g. Alvarez, Atkeson, and Kehoe 2007). Furthermore, our model emphasizes an important tradeoff for monetary policy associated with the two transmission channels — one conventional via the goods market and the other unconventional via the financial market. In particular, a floating exchange rate regime improves allocations in an open economy in response to conventional productivity shocks, yet it possibly results in excessive exchange rate volatility in response to financial shocks, which reduces the extent of international risk sharing to suboptimal levels. Another interesting question then whether more conventional types of monetary policy shocks also feature such tradeoffs.

A Appendix

A.1 Additional Tables and Figures

Table A1: Calibrated parametrers

β	discount factor	0.99
σ	inverse of the IES	2
γ	openness of economy	0.035
φ	inverse of Frisch elasticity	1
ϕ	intermediate share in production	0.5
ϑ	capital share	0.3
δ	capital depreciation rate	0.02
θ	elasticity of substitution between H and F goods	1.5
ϵ	elasticity of substitution between different types of labor	4
λ_w	Calvo parameter for wages	0.85
λ_p	Calvo parameter for prices	0.75
ρ	autocorrelation of shocks	0.97
ρ_r	Taylor rule: persistence of interest rates	0.95
ϕ_π	Taylor rule: reaction to inflation	2.15

Table A2: Estimated parameters

	σ_ψ	σ_ξ	σ_a	σ_m	ρ_{a,a^*}	κ	ϕ_e
Models w/o UIP shock:							
IRBC	0.00	13.8	8.1	–	0.28	11	13.0
NKOE-1	0.00	5.71	10.6	–	0.30	7	1.8
NKOE-2	0.00	4.38	–	0.77	0.30	22	5.3
Models w/ exogenous UIP shock:							
IRBC	0.61	3.37	1.41	–	0.30	15	14.5
NKOE-1	0.59	2.80	1.01	–	0.35	7.5	3.7
NKOE-2	0.59	1.23	–	0.15	0.42	20	3.6
Models w/ endogenous UIP shock:							
IRBC	0.61	3.37	1.41	–	0.30	15	0.25
NKOE-1	0.59	2.80	1.01	–	0.35	7.5	0.03
NKOE-2	0.59	1.23	–	0.15	0.42	20	0.08

Note: in all calibrations, shocks are normalized to obtain $\mathrm{std}(\Delta e_t) = 12\%$. Parameter ϕ_e in the Taylor rule is calibrated to generate eightfold reduction in $\mathrm{std}(\Delta e_t)$ from float to peg. Relative volatilities of shocks are calibrated to match $\mathrm{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$ under the float and $\mathrm{corr}(\Delta q_t, \Delta nx_t) = -0.1$ under the peg. The cross-country correlation of productivity/monetary shocks matches $\mathrm{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the float. Capital adjustment parameter ensures that $\frac{\mathrm{std}(\Delta z_t)}{\mathrm{std}(\Delta gdp_t)} = 2.5$ under the float. The moments are calculated by simulating the model for $T = 100,000$ quarters.

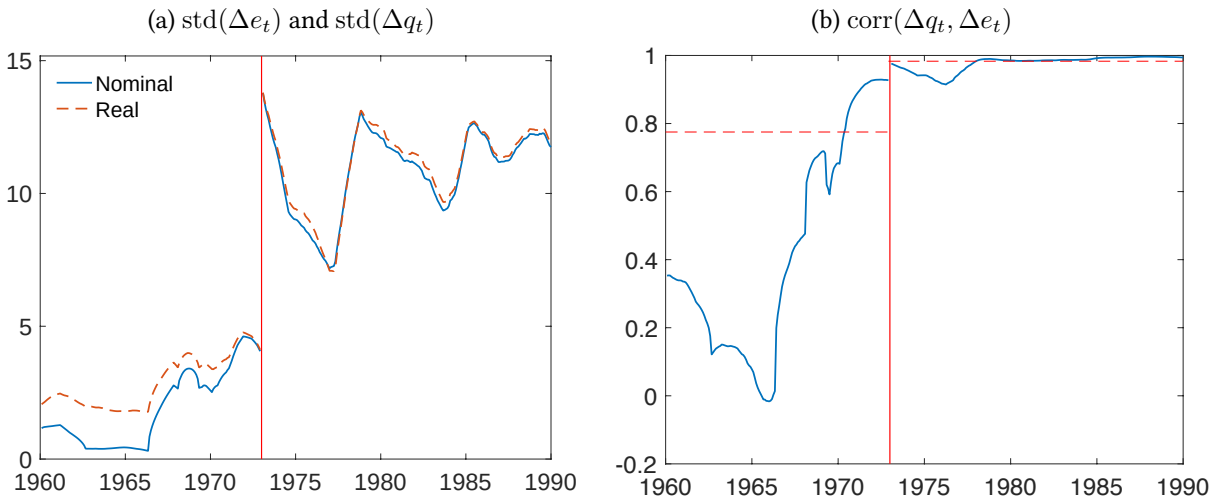


Figure A1: Volatility and correlation of nominal and real exchange rates over time

Note: The left panel plots standard deviations and the right panel plots correlation of Δe_t and Δq_t over time. Series from Figure 1; triangular moving averages with a window over 18 months before and after the date, treating 1973:01 as the end point for the two regimes (see Figure 3 and Appendix Figure A4 for other variables).

A.2 Data

Additional details for Section 2:

1. CPI data for Canada in 1960 is from OECD, but is downloaded from FRED and made consistent with the rest of the series.

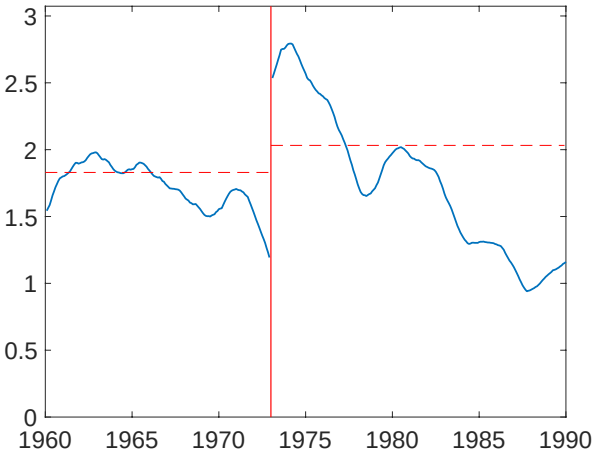
2. **Outliers**:

   (a) civil unrests in France in May–June 1968 led to a more than 20% fall in production; France also had abnormally volatile production index during the whole 1960s;

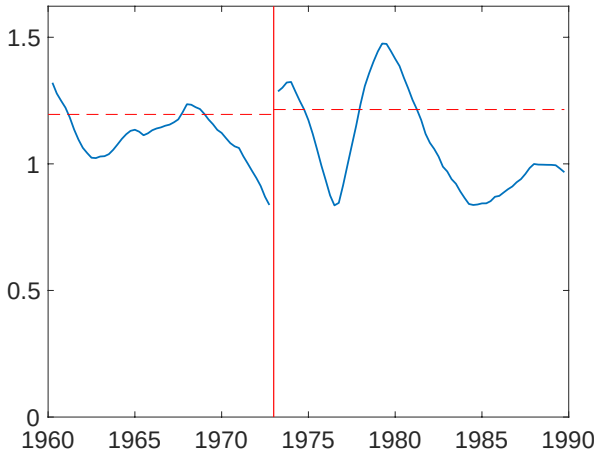
   (b) earthquake and tsunami in Japan in March–April 2011 led to 17% fall in production. Since these observations are required for aggregation across non-U.S. countries, I replaced them with extrapolations using the values before and after the episodes.

   (c) The same applies to Germany production index in 1984:06, which constitutes a clear measurement error, and Spain production index in 1960, which is missing.

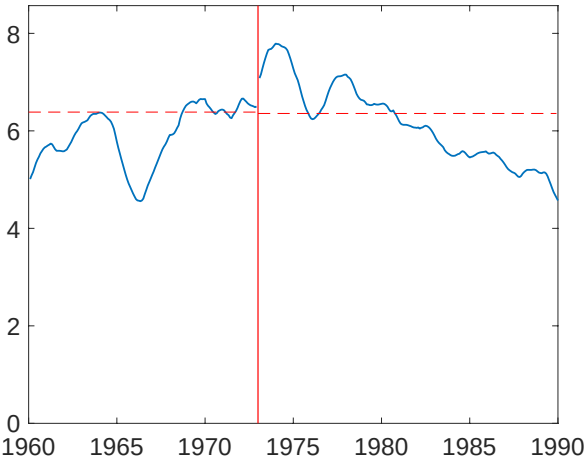
(a) Relative inflation, $\pi_t - \pi_t^*$



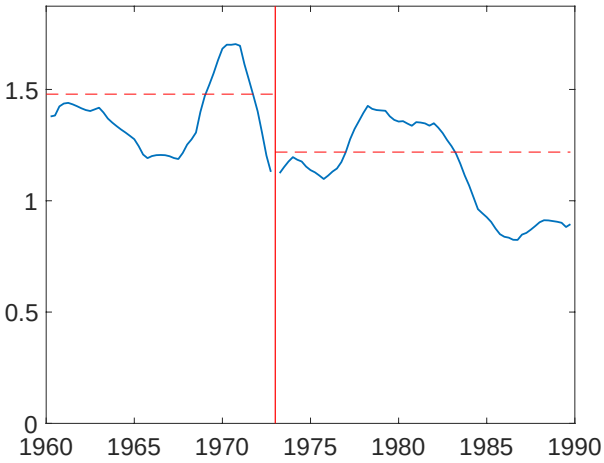
(b) Relative consumption, $\Delta c_t - \Delta c_t^*$



(c) Relative industrial production index, $\Delta y_t - \Delta y_t^*$



(d) Relative GDP, $\Delta gdp_t - \Delta gdp_t^*$



(e) Net exports, nx_t

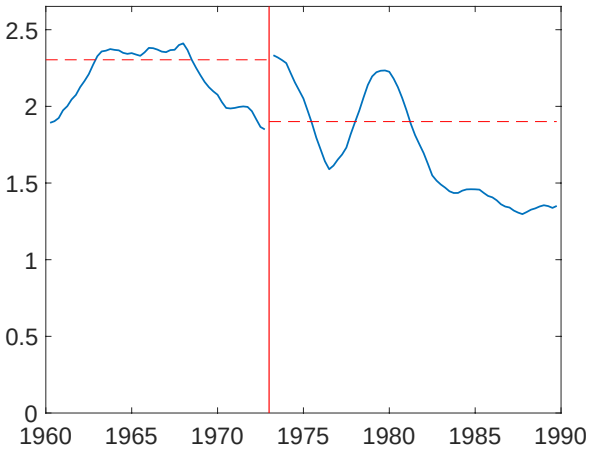


Figure A2: Volatility of macroeconomic variables over time

Note: The top four panels zoom in on the panels b, d, e and f in Figure 3. The bottom panel plots the standard deviations of nx_t defined as the ratio of export minus imports to the sum of exports and imports, for the US against the rest of the world.

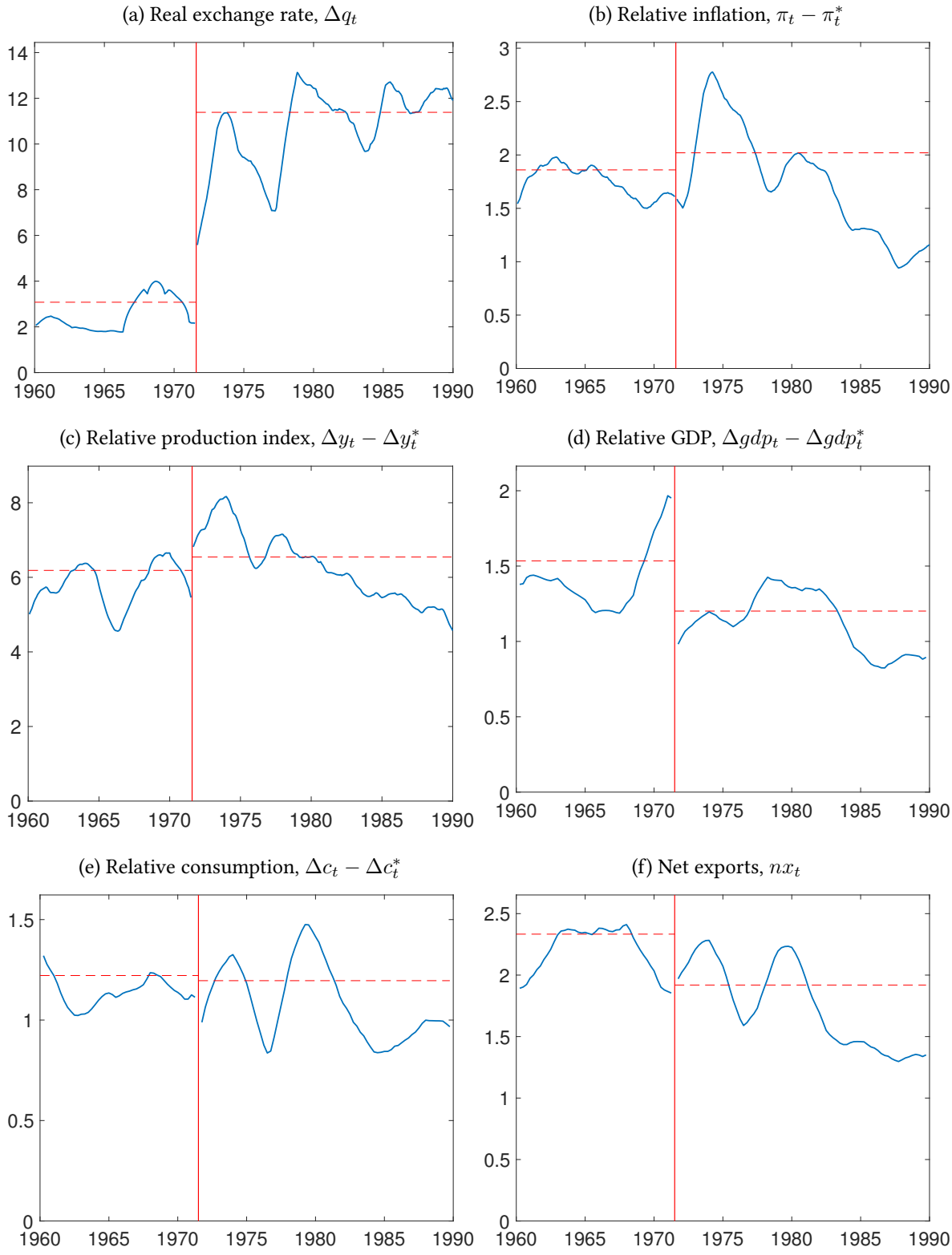


Figure A3: Volatility over time: alternative break at 1971:08

Note: Like Appendix Figure A2, but with an alternative break date in 1971:08 (1971:Q3). DIFFERENT DEFINITION

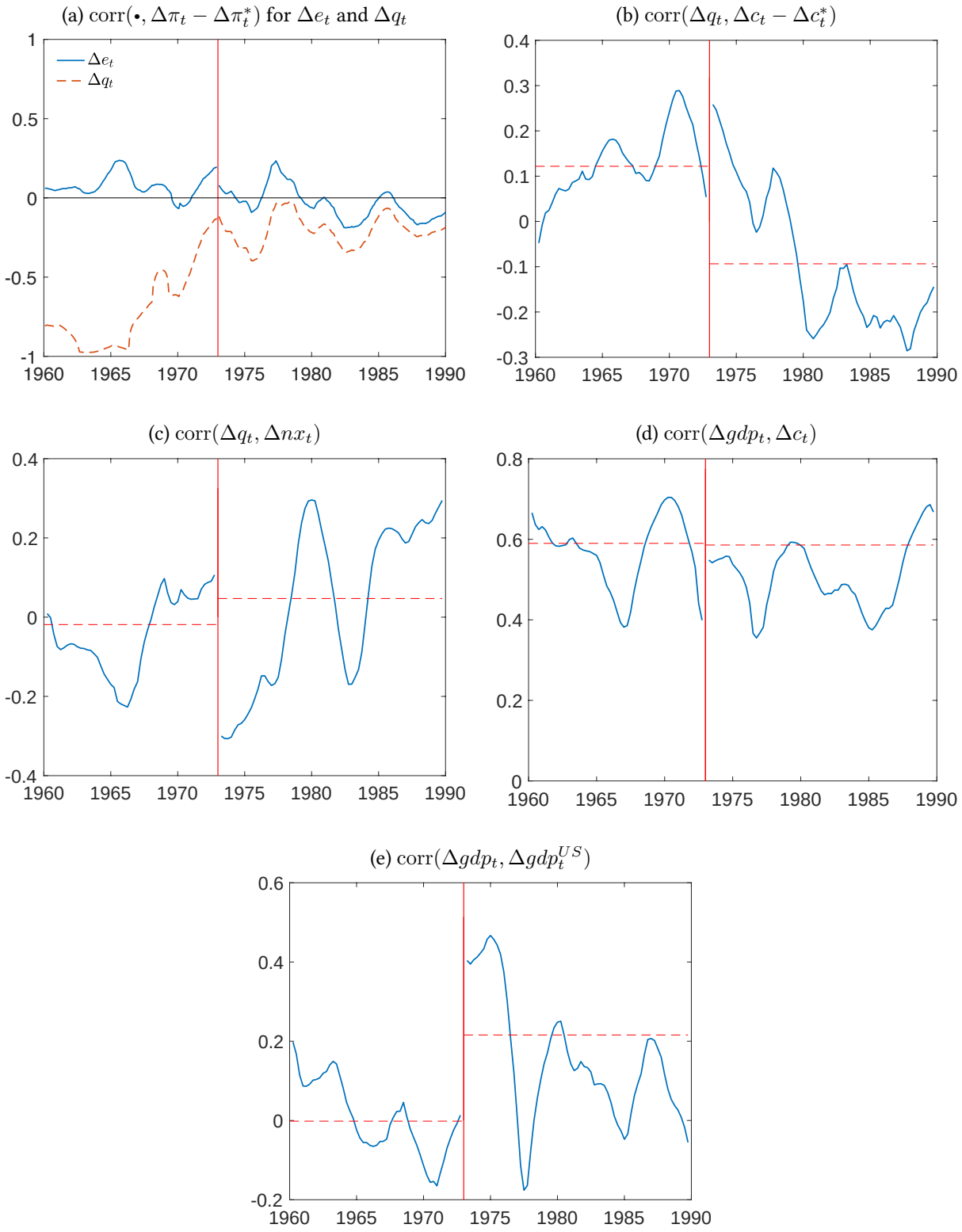
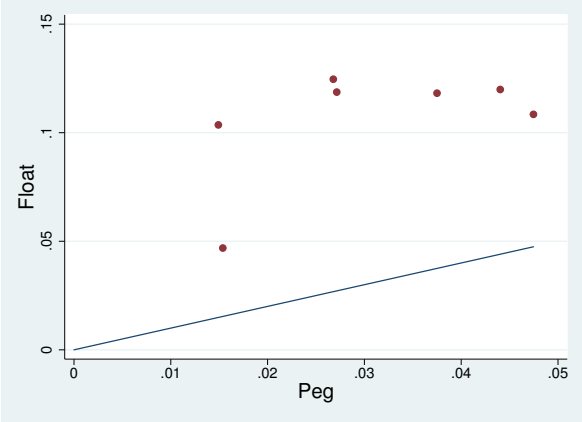


Figure A4: Correlations over time

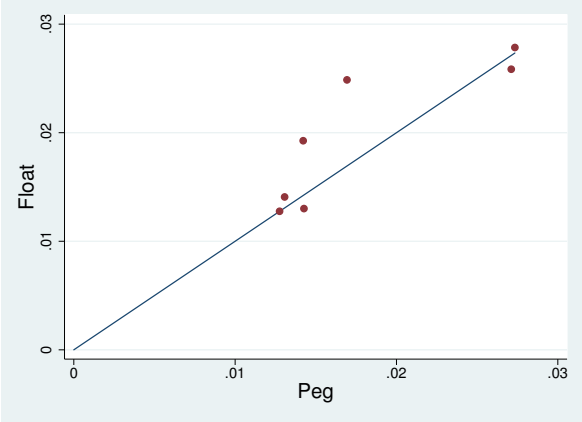
Note: the pictures shows the correlations for the RoW estimated separately before and after January 1973. The rolling window is up to 5 years for quarterly series – shorter at the corners – with linearly decreasing weights.

Figure A5: Scatter plots: volatility before and after the end of the Bretton-Woods System

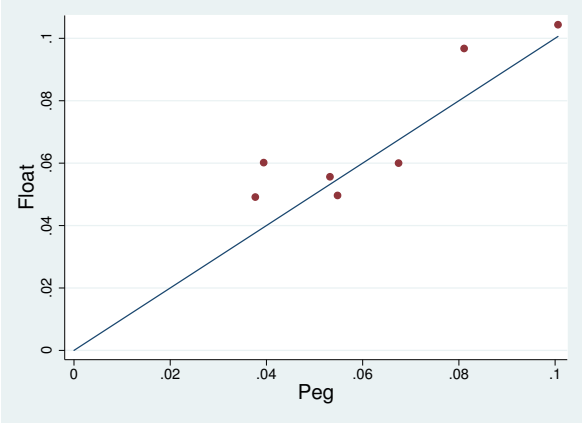
(a) RER



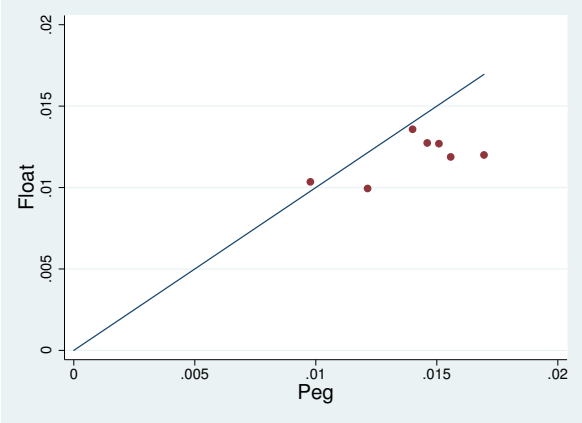
(b) inflation



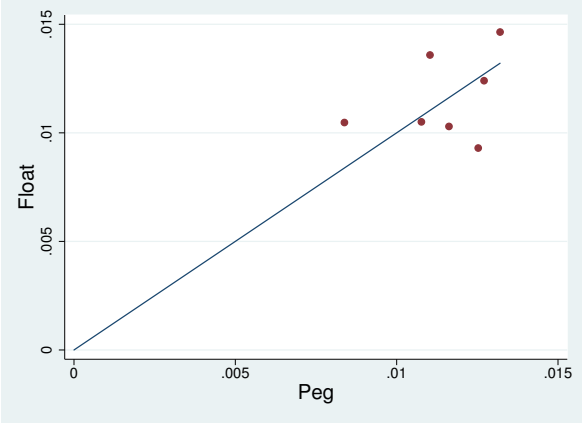
(c) production index



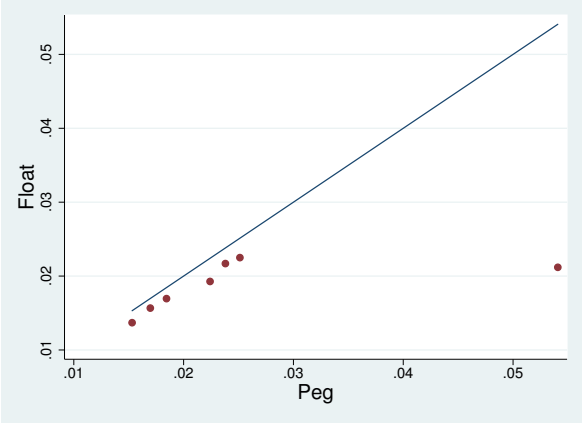
(d) GDP



(e) consumption



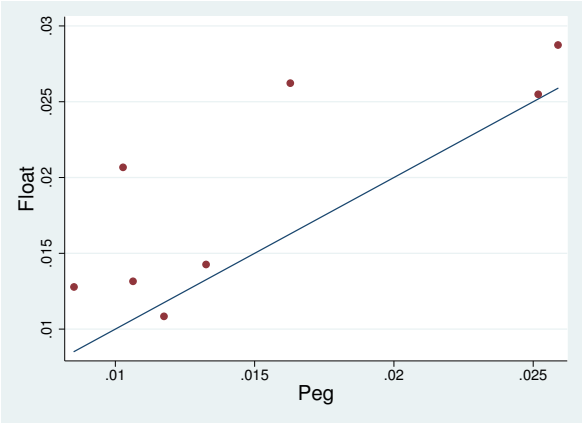
(f) net exports



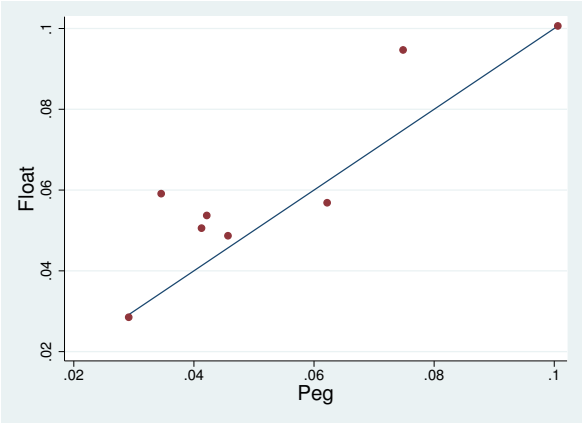
Note: the plot shows standard deviations of different variables for 1960-71:07 vs. 1973-1989 across individual countries. Canada is the outlier in terms of float-RER volatility and Spain is the outlier in terms of peg-NX volatility.

Figure A6: Scatter plots: country-level instead of cross-differences

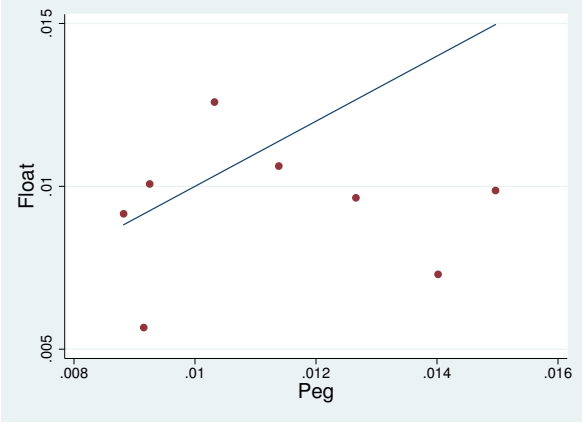
(a) inflation



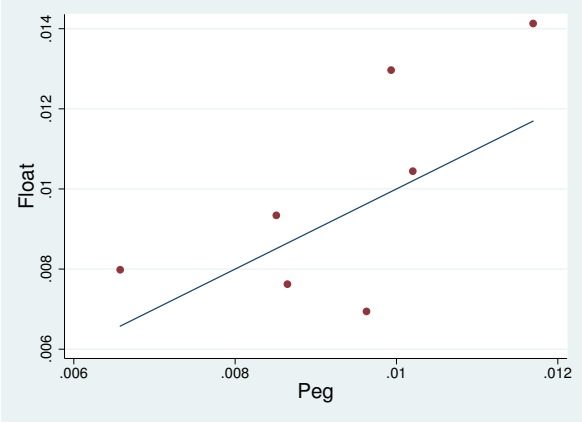
(b) production index



(c) GDP



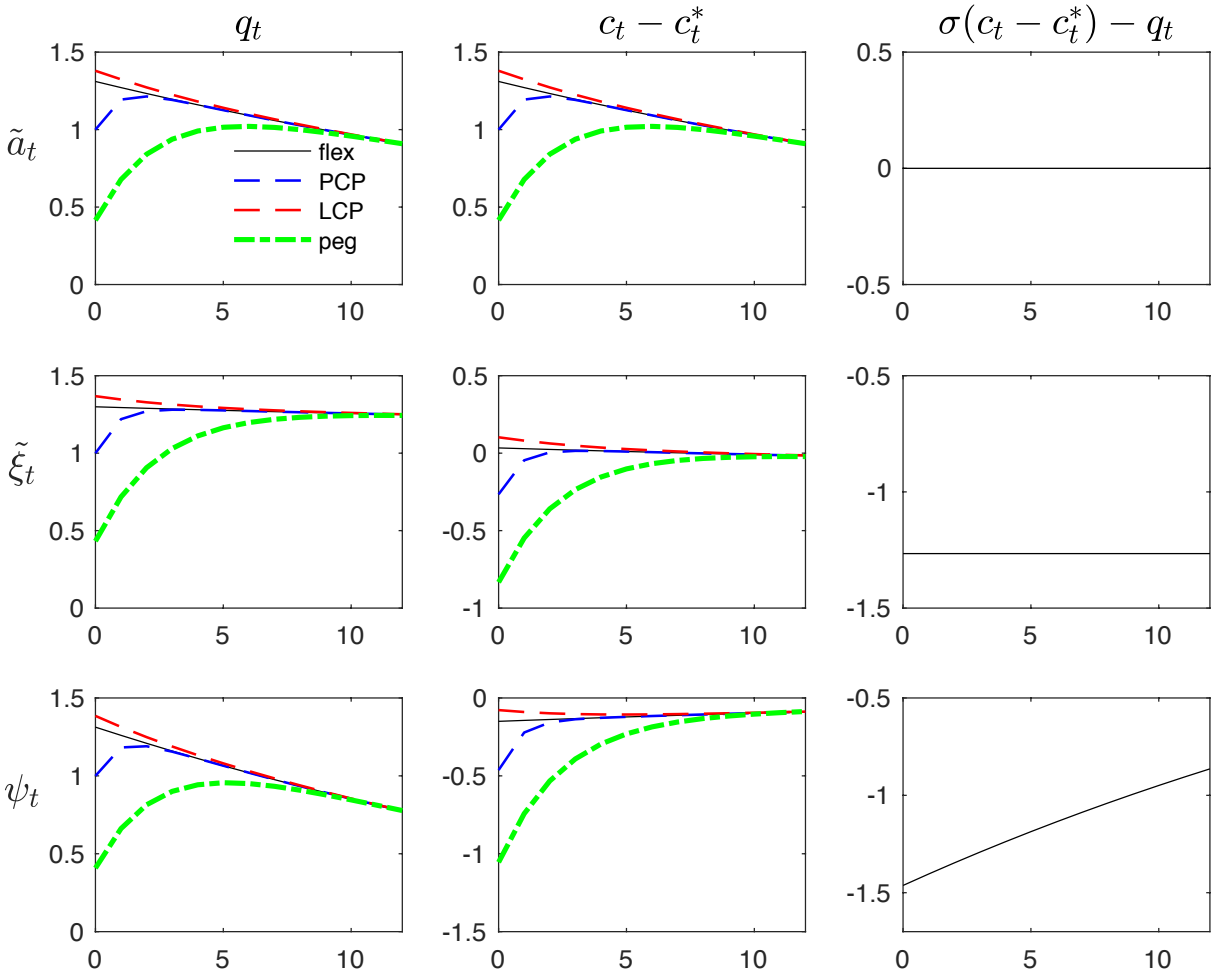
(d) consumption



Note: the plot shows standard deviations of different variables for 1960-71:07 vs. 1973-1989 across individual countries. Canada is the outlier in terms of float-RER volatility and Spain is the outlier in terms of peg-NX volatility.

The scatter plots from Figures A5 show that the volatility of the RER is higher under the float than under the peg for every single country in our sample. Interestingly, the floating regime results in almost equal volatility of exchange rates across countries except for Canada, which retained partial peg to the dollar during 1970-80s. At the same time, the countries concentrate tightly along the 45-degree line for other macroeconomic variables indicating small changes in their volatilities across the regimes.(The only exception is Spain with an abnormally high volatility of net exports in 1960s.) Interestingly, there is more variation for country-level series instead of the cross-country differences (see Figure A6), but again we find no systematic differences for fundamentals between two regimes.

Figure A7: IRFs to various shocks under alternative regimes



Note: Impulse response of (1) q_t, (2) $c_t - c_t^*$, (3) $\sigma(c_t - c_t^*) - q_t$ by columns, to shocks (1) $\tilde{a}_t$, (2) $\tilde{\xi}_t$, (3) exogenous $\tilde{\psi}_t$ by rows under (1) flexible prices (independent of monetary regime), (2) peg (independently of PCP or LCP), (3) PCP-float and (4) LPC-float. 'Conventional' models under Cole-Obstfeld parameter restriction (Assumption 2). Note that the impulse responses of q_t and $c_t - c_t^*$ change with both the supply side (flex prices vs PCP vs LCP) and the monetary policy regime (peg vs float), however, the IRF of $\sigma(c_t - c_t^*) - q_t$ does not depend on these details of equilibrium environment, and hence the unconditional statistical properties of $\sigma(c_t - c_t^*) - q_t$ also do not depend on the monetary regime, illustrating Proposition 1.

A.3 Derivations

A.3.1 Equilibrium system, steady state, log-linearization

A lot of the derivations here build on Itskhoki and Mukhin (2019)...

A.3.2 Segmented financial market

Proof of Lemma 2 The proof of the lemma follows two steps. First, it characterizes the solution to the portfolio problem (22) of the intermediaries. Second, it combines this solution with the financial market clearing (25) to derive the equilibrium condition (26).

(a) Portfolio choice: *The solution to the portfolio choice problem* (22) *when the time periods are short is given by:*

$$\frac{d^*_{t+1}}{P^*_t} = -\frac{i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}, \tag{A1}$$

where $i_t - i^*_t \equiv \log(R_t/R^*_t)$, $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ *and* $\sigma_{e\pi^*} = \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$.

Proof: The proof follows Campbell and Viceira (2002, Chapter 3 and Appendix 2.1.1). Consider the objective in the intermediary problem (22) and rewrite it as:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \left(1 - e^{x^*_{t+1}}\right) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t} \right) \right\}, \tag{A2}$$

where we used the definition of $\tilde{R}^*_{t+1}$ in (21) and the following algebraic manipulation:

$$\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} = \frac{\tilde{R}^*_{t+1}/R^*_t}{P^*_{t+1}/P^*_t} \frac{d^*_{t+1}}{P^*_t} = \frac{1 - \frac{R_{t+1}}{R^*_t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}}{e^{\pi^*_{t+1}}} \frac{d^*_{t+1}}{P^*_t} = \left(1 - e^{x^*_{t+1}}\right) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t}$$

and defined the log Carry trade return and foreign inflation rate as

$$x^*_{t+1} \equiv i_t - i^*_t - \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta \log \mathcal{E}_{t+1} \qquad \text{and} \qquad \pi^*_{t+1} \equiv \Delta \log P^*_{t+1}.$$

When time periods are short, (x^*_{t+1}, π^*_{t+1}) correspond to the increments of a vector normal diffusion process $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ with time-varying drift $\boldsymbol{\mu}_t$ and time-invariant conditional variance matrix $\boldsymbol{\sigma}$:

$$\begin{pmatrix} \mathrm{d}\mathcal{X}^*_t \\ \mathrm{d}\mathcal{P}^*_t \end{pmatrix} = \boldsymbol{\mu}_t \mathrm{d}t + \boldsymbol{\sigma} \mathrm{d}B_t, \tag{A3}$$

where B_t is a standard two-dimensional Brownian motion. Indeed, as we show below, in equilibrium x^*_{t+1} and π^*_{t+1} follow stationary linear stochastic processes (ARMAs) with correlated innovations, and therefore

$$(x^*_{t+1}, \pi^*_{t+1}) \,\big|\, \mathcal{I}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\sigma}^2),$$

where $\mathcal{I}_t$ is the information set at time t, and the drift and variance matrix are given by:

$$\boldsymbol{\mu}_t = \mathbb{E}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} \\ \mathbb{E}_t \pi^*_{t+1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\sigma}^2 = \mathrm{var}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} \sigma_e^2 & -\sigma_{e\pi^*} \\ -\sigma_{e\pi^*} & \sigma^2_{\pi^*} \end{pmatrix},$$

where $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$, $\sigma_{\pi^*}^2 \equiv \text{var}_t(\Delta p_{t+1}^*)$ and $\sigma_{e\pi^*} \equiv \text{cov}_t(\Delta e_{t+1}, \Delta p_{t+1}^*)$ are time-invariant (annualized) conditional second moments. Following Campbell and Viceira (2002), we treat (x_{t+1}^*, π_{t+1}^*) as discrete-interval differences of the continuous process, $(\mathcal{X}_{t+1}^* - \mathcal{X}_t^*, \mathcal{P}_{t+1}^* - \mathcal{P}_t^*)$. With short time periods, the solution to (A2) is equivalent to

$$\max_{d^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \big(1 - e^{\mathrm{d}\mathcal{X}_t^*}\big) e^{-\mathrm{d}\mathcal{P}_t^*} \frac{d^*}{P_t^*} \right) \right\}, \tag{A4}$$

where $(\mathrm{d}\mathcal{X}_t^*, \mathrm{d}\mathcal{P}_t^*)$ follow (A3). Using Ito's Lemma, we rewrite the objective as:

$$\begin{aligned}
&\mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \Big(-\mathrm{d}\mathcal{X}_t^* - \tfrac{1}{2}(\mathrm{d}\mathcal{X}_t^*)^2 \Big) \Big(1 - \mathrm{d}\mathcal{P}_t^* + \tfrac{1}{2}(\mathrm{d}\mathcal{P}_t^*)^2\Big) \frac{d^*}{P_t^*} \right) \right\} \\
&= \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \Big(-\mathrm{d}\mathcal{X}_t^* - \tfrac{1}{2}(\mathrm{d}\mathcal{X}_t^*)^2 + \mathrm{d}\mathcal{X}_t^* \mathrm{d}\mathcal{P}_t^* \Big) \frac{d^*}{P_t^*} \right) \right\} \\
&= -\frac{1}{\omega} \exp\left(\left[\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}\big) \frac{d^*}{P_t^*} + \frac{\omega^2 \sigma_e^2}{2} \left(\frac{d^*}{P_t^*} \right)^2 \right] \mathrm{d}t \right),
\end{aligned}$$

where the last line uses the facts that $(\mathrm{d}\mathcal{X}_t^*)^2 = \sigma_e^2 \mathrm{d}t$ and $\mathrm{d}\mathcal{X}_t^* \mathrm{d}\mathcal{P}_t^* = -\sigma_{e\pi^*}\mathrm{d}t$, as well as the property of the expectation of an exponent of a normally distributed random variable; $\mu_{1,t}$ denotes the first component of the drift vector $\boldsymbol{\mu}_t$. Therefore, maximization in (A4) is equivalent to:

$$\max_{d^*} \left\{ -\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}\big) \frac{d^*}{P_t^*} - \tfrac{1}{2}\omega^2 \sigma_e^2 \left(\frac{d^*}{P_t^*} \right)^2 \right\} \quad \text{w/solution} \quad \frac{d^*}{P_t^*} = -\frac{\mu_{1,t} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}.$$

This is the portfolio choice equation (A1), which obtains under CARA utility in the limit of short time periods, but note is also equivalent to the exact solution under mean-variance preferences. The extra terms in the numerator correspond to Jensen's Inequality corrections to the expected real log return on the carry trade. ■

(b) Equilibrium condition: To derive the modified UIP condition (26), we combine the portfolio choice solution (A1) with the market clearing condition (25) and the noise-trader currency demand (23) to obtain:

$$B_{t+1}^* + R_t^* n\big(e^{\psi_t} - 1\big) - m P_t^* \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2} = 0. \tag{A5}$$

The market clearing conditions in (25) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1}+N_{t+1}}{R_t} = -\mathcal{E}_t \frac{D_{t+1}^*+N_{t+1}^*}{R_t^*}$, results in the equilibrium balance between home and foreign household asset positions, $\frac{B_{t+1}}{R_t} = -\mathcal{E}_t \frac{B_{t+1}^*}{R_t^*}$. Therefore, we can rewrite (A5) as:

$$\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2 / m} = \frac{R_t^*}{P_t^*} n\big(e^{\psi_t} - 1\big) - \frac{R_t^*}{R_t} \frac{Y_t}{\mathcal{Q}_t} \frac{B_{t+1}}{P_t Y_t},$$

where we normalized net foreign assets by nominal output $P_t Y_t$ and used the definition of the

real exchange rate $\mathcal{Q}_t$ in (1). We next log-linearize this equilibrium condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{B} = \bar{B}^* = 0$, $\mathcal{Q} = 1$, and $\bar{P} = \bar{P}^* = 1$ and some $\bar{Y}$. As shocks become small, the (co)variances σ_e^2 and $\sigma_{e\pi^*}$ become second order and drop out from the log-linearization. We adopt the asymptotics in which as σ_e^2 shrinks ω/m increases proportionally leaving the risk premium term $\omega\sigma_e^2/m$ constant, finite and nonzero in the limit.[31] As a result, the log-linearized equilibrium condition is:

$$\frac{1}{\omega\sigma_e^2/m}\Big(i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1}\Big) = \frac{n}{\beta}\psi_t - \bar{Y}b_{t+1}, \tag{A6}$$

where $b_{t+1} = \frac{1}{\bar{P}\bar{Y}}B_{t+1} = -\frac{1}{\bar{P}\bar{Y}}B^*_{t+1}$. This corresponds to the modified UIP condition (26) in Lemma 2, which completes the proof of the lemma. ■

Income and losses in the financial market Consider the income and losses of the non-household participants in the financial market — the intermediaries and the noise traders:

$$\frac{D^*_{t+1} + N^*_{t+1}}{R^*_t}\tilde{R}^*_{t+1} = \Big(md^*_{t+1} + R^*_t n(e^{\psi_t} - 1)\Big)\left(1 - e^{x_{t+1}}\right),$$

where we used the definition of $\tilde{R}^*_{t+1}$ in (21) and the log Carry trade return $x_{t+1} \equiv i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \log(R_t/R_t^*) - \Delta\log\mathcal{E}_{t+1}$. Using the same steps as in the proof of Lemma 2, we can approximate this income as:

$$\left(-m\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2} + \frac{n}{\beta}\psi_t\right)\left(-x_{t+1}\right) = m\left(\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2} - \frac{n}{\beta m}\psi_t\right)x_{t+1} = -\bar{Y}b_{t+1}x_{t+1},$$

where the last equality uses (A6). Therefore, while the UIP deviations (realized x_{t+1} and expected $\mathbb{E}_t x_{t+1}$) are first order, the income and losses in the financial markets are only second order, as $B_{t+1} = \bar{P}\bar{Y}b_{t+1}$ is first order around $\bar{B} = 0$. Intuitively, the income and losses in the financial market are equal to the realized UIP deviation times the gross portfolio position — while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

[31] Note that σ_e^2/m is the quantity of risk per intermediary and ω is their aversion to risk; alternatively, ω/m can be viewed as the effective risk aversion of the whole sector of intermediaries who jointly hold all exchange rate risk. Our approach follows Hansen and Sargent (2011) and Hansen and Miao (2018), who consider the continuous-time limit in the models with ambiguity aversion. The economic rationale of this asymptotics is not that second moments are zero and effective risk aversion ω/m is infinite, but rather that risk premia terms, which are proportional to $\omega\sigma_e^2/m$, are finite and nonzero. Indeed, the first-order dynamics of the equilibrium system results in well-defined second moments of the variables, including σ_e^2, as in Devereux and Sutherland (2011) and Tille and van Wincoop (2010); an important difference of our solution concept is that it allows for a non-zero first-order component of the return differential, namely a non-zero expected Carry trade return. We characterize the equilibrium σ_e^2 below.

A.3.3 Real exchange rate

Equilibrium exchange rate dynamics (Proof of Lemma 3) Rewrite the equilibrium dynamic system (29)–(30) in matrix form:

$$\begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbb{E}_t q_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\beta \end{pmatrix} \begin{pmatrix} q_t \\ \hat{b}_t \end{pmatrix} - \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix},$$

where for brevity we made the following substitution of variables:

$$\hat{b}_t \equiv \frac{\beta}{\gamma(\hat{\theta}+\gamma\kappa_a)} b_t, \quad \hat{a}_t \equiv \frac{\kappa_a}{\hat{\theta}+\gamma\kappa_a}\tilde{a}_t, \quad \hat{\chi}_1 \equiv \frac{\chi_1}{1+\gamma\sigma\kappa_q}, \quad \hat{\chi}_2 \equiv \frac{\gamma(\hat{\theta}+\gamma\kappa_a)}{\beta(1+\gamma\sigma\kappa_q)}\chi_2, \quad k \equiv \frac{\sigma(\hat{\theta}+\gamma\kappa_a)}{1+\gamma\sigma\kappa_q}. \tag{A7}$$

Diagonalizing the dynamic system, we have:

$$\mathbb{E}_t z_{t+1} = B z_t - C \begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix}, \quad \text{where} \quad B \equiv \begin{pmatrix} 1+\hat{\chi}_2 & \hat{\chi}_2/\beta \\ 1 & 1/\beta \end{pmatrix}, \quad C \equiv \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k+\hat{\chi}_2 \\ 0 & 1 \end{pmatrix},$$

and we denoted $z_t \equiv (q_t, \hat{b}_t)'$. The eigenvalues of B are:

$$\mu_{1,2} = \frac{(1+\hat{\chi}_2+1/\beta) \mp \sqrt{(1+\hat{\chi}_2+1/\beta)^2 - 4/\beta}}{2/\beta} \qquad \text{such that} \qquad 0 < \mu_1 \leq 1 < \frac{1}{\beta} \leq \mu_2,$$

and $\mu_1 + \mu_2 = 1 + \hat{\chi}_2 + 1/\beta$ and $\mu_1 \cdot \mu_2 = 1/\beta$. Note that when $\chi_2 = 0$, and hence $\hat{\chi}_2 = 0$, the two roots are simply $\mu_1 = 1$ and $\mu_2 = 1/\beta$. In the text, $\delta \equiv \mu_1$.

The left eigenvalue associated with $\mu_2 > 1$ is $v = (1, 1/\beta - \mu_1)$, such that $vB = \mu_2 v$. Therefore, we can pre-multiply the dynamic system by v and rearrange to obtain:

$$v z_t = \frac{1}{\mu_2}\mathbb{E}_t\{v z_{t+1}\} + \frac{1}{\mu_2}\hat{\chi}_1\psi_t + \left[\frac{(1-\rho)k+\hat{\chi}_2}{\mu_2} + \frac{1/\beta-\mu_1}{\mu_2}\right]\tilde{a}_t.$$

Using the facts that $\hat{\chi}_2 + 1/\beta - \mu_1 = \mu_2 - 1$ and $1/\mu_2 = \beta\mu_1$, we solve this dynamic equation forward to obtain the equilibrium cointegration relationship:

$$v z_t = q_t + (1/\beta - \mu_1)\hat{b}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{1-\beta\mu_1+\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}\hat{a}_t. \tag{A8}$$

Combining this with the second dynamic equation for $\hat{b}_{t+1}$, we solve for:

$$\hat{b}_{t+1} - \mu_1\hat{b}_t = \overbrace{q_t + \left(\tfrac{1}{\beta} - \mu_1\right)\hat{b}_t}^{=vz_t} - \hat{a}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t, \tag{A9}$$

Note that $\hat{b}_{t+1}$ in (A9) follows a stationary AR(2) with roots ρ and μ_1.

Finally, we apply lag operator $(1-\mu_1 L)$ to (A8) and use (A9) to solve for:

$$\begin{aligned}(1-\mu_1 L)q_t &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t\right] + (1-\mu_1 L)\hat{a}_t \\ &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k\hat{a}_t\right] + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\hat{a}_t, \qquad \text{(A10)}\end{aligned}$$

where L is the lag operator such that $Lq_t = q_{t-1}$. Therefore, equilibrium RER q_t follows a stationary ARMA(2,1) with autoregressive roots $\delta = \mu_1$ and ρ, as described in Lemma 3. Note that in the limit $\chi_2 \to 0$, $\delta = \mu_1 \to 1$, and this process becomes an ARIMA(1,1,1), which nonetheless has impulse responses that are arbitrary close to a stationary ARMA(2,1) with a large $\delta \lesssim 1$. ■

Equilibrium variance of the exchange rate Solution (A10) characterizes the behavior of q_t for given values of χ_1 and χ_2 (and hence μ_1, μ_2), which from (26) themselves depend on $\sigma_e^2 = \text{var}_t(\Delta e_{t+1})$. Under the peg, $\sigma_e^2 = 0$ and hence $\chi_1 = \chi_2 = 0$. Under the float, the monetary policy stabilizes inflation, ensuring $e_t = q_t$, we hence we have $\sigma_e^2 = \text{var}_t(\Delta q_{t+1})$, and we now solve for the equilibrium value of σ_e^2, and hence $(\chi_1, \chi_2, \mu_1, \mu_2)$.

Using (A10), we calculate $\sigma_e^2 = \text{var}_t(\Delta q_{t+1})$ for given χ_1 and χ_2:

$$\sigma_e^2 = \text{var}_t(\Delta q_{t+1}) = \left(\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\right)^2\sigma_\psi^2 + \left(\frac{\beta(1-\rho)\mu_1 k + (1-\beta\mu_1)}{1-\beta\rho\mu_1}\right)^2\sigma_a^2 = \frac{\hat{\chi}_1^2\sigma_\psi^2 + \big((1-\rho)k + (\mu_2-1)\big)^2\sigma_a^2}{(\mu_2-\rho)^2},$$

where the second line used the fact that $\beta\mu_1 = 1/\mu_2$. In addition, recall that:

$$\hat{\chi}_1 = \frac{n/\beta}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m}, \qquad \hat{\chi}_2 \equiv \frac{\gamma\lambda_q\bar{Y}/\beta}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m} \quad \text{and} \quad \mu_2 = \frac{(1+\beta\hat{\chi}_2+\beta) + \sqrt{(1+\beta\hat{\chi}_2+\beta)^2 - 4\beta}}{2}.$$

We therefore can rewrite the fixed point equation for $\sigma_e^2 > 0$ as follows:

$$F(x,\tilde{\omega}) = \big(\mu_2(\tilde{\omega}x) - \rho\big)^2 x - b(\tilde{\omega}x)^2 - c = 0, \qquad \text{(A11)}$$

where we used the following notation:

$$x \equiv \sigma_e^2 \geq 0, \qquad \tilde{\omega} = \frac{\omega}{m}, \qquad b \equiv \left(\frac{n/\beta}{1+\gamma\sigma\kappa_q}\right)^2\sigma_\psi^2, \qquad c \equiv \big((1-\rho)k + (\mu_2-1)\big)^2\sigma_a^2 \geq 0,$$

and $\mu_2(\cdot)$ is a function which gives the equilibrium values of μ_2 defined above as a function of $\tilde{\omega}\sigma_e^2$ for given values of the model parameters. Note that for any given $\tilde{\omega} > 0$:

$$\begin{aligned}&\lim_{x\to 0} F(x,\tilde{\omega}) = -c \leq 0, \\ &\lim_{x\to\infty}\frac{F(x,\tilde{\omega})}{x^3} = \lim_{x\to\infty}\left(\frac{\mu_2(\tilde{\omega}x)}{x}\right)^2 = \left(\frac{\beta\hat{\chi}_2^2}{\sigma_e^2}\right) = \left(\frac{\gamma\lambda_q\bar{Y}}{1+\gamma\sigma\kappa_q}\tilde{\omega}\right)^2 > 0.\end{aligned}$$

Therefore, by continuity at least one fixed-point $F(\sigma_e^2,\omega) = 0$ with $\sigma_e^2 \geq 0$ exists, and all such $\sigma_e^2 > 0$ whenever $c > 0$ (that is, when $\sigma_a > 0$). One can further show that when σ_a/σ_ψ is not too small, this

equilibrium is unique, which is in particular the case under our calibration.[32]

Finally, we consider the limit of log-linearization in Lemma 2, where $(\sigma_a, \sigma_\psi) = \sqrt{\xi} \cdot (\bar{\sigma}_a, \bar{\sigma}_\psi) = \mathcal{O}(\sqrt{\xi})$ as $\xi \to 0$, where $(\bar{\sigma}_a, \bar{\sigma}_\psi)$ are some fixed numbers. Then in (A11), $(b, c) = \mathcal{O}(\xi)$, as (b, c) are linear in $(\sigma_a^2, \sigma_\psi^2)$. This implies that for any given fixed point $(\bar{\sigma}_e^2, \bar{\omega})$, with $F(\bar{\sigma}_e^2, \bar{\omega}; \bar{\sigma}_a^2, \bar{\sigma}_\psi^2) = 0$, there exists a sequence of fixed points $F(\xi\bar{\sigma}_e^2, \bar{\omega}/\xi; \xi\bar{\sigma}_a^2, \xi\bar{\sigma}_\psi^2) = 0$ as $\xi \to 0$, for which $\sigma_e^2 = \xi\bar{\sigma}_e^2 = \mathcal{O}(\xi)$, $\tilde{\omega} = \bar{\omega}/\xi = \mathcal{O}(1/\xi)$ and $\tilde{\omega}\sigma_e^2 = \bar{\omega}\bar{\sigma}_e^2 = const.$ To verify this, one can simply divide (A11) by ξ and note that, for a given $\tilde{\omega}x$, $F(x, \tilde{\omega})$ is linear in (x, b, c), which means that the fixed point x scales with (b, c) provided that $\tilde{\omega}x$ stays constant. This confirms the conjecture used in the proof of Lemma 2. ■

[32] For $\sigma_a/\sigma_\psi \approx 0$, there typically exist three equilibria. In particular, when $\sigma_a = 0$, there always exists an equilibrium with $\sigma_e^2 = \chi_1 = 0$, in addition to two other potential equilibria with $\sigma_e^2 > 0$, which exist when σ_ψ is not too small (see Itskhoki and Mukhin 2017).

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