Leisure-enhancing technological change*

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Abstract

This paper develops a theory of endogenous growth with leisure-enhancing innovations. When time allocation decisions depend on the menu of available leisure options, these leisure-enhancing innovations, carried out by two-sided platforms, may emerge in equilibrium. The model is highly tractable and can be solved analytically. There are four key results. First, the economy must be sufficiently developed for the leisure-enhancing innovations to be profitable. As it grows, the economy transitions endogenously from a balanced growth path with only productivity-enhancing technologies to one with leisure-enhancing innovations. Second, following the transition, hours worked decline, in line with the trend observed in the data. Third, the growth rate of the economy following the transition is permanently lower, consistent with recent experience in the United States and other advanced countries. However, GDP growth as measured by statistical agencies today does not account for the true value of leisure services and so exaggerates the extent of the slowdown. Finally, the theory highlights two new inefficiencies of the market equilibrium. Static inefficiency stems from the two-sided nature of the leisure market and suggests undersupply of leisure-enhancing services. Dynamic inefficiency goes the other way, emphasizing the adverse impact of leisure-enhancing innovations on future productivity.

Keywords: productivity, time allocation, digital economy, measurement

JEL Classification: O30, O31, O40, O41, O50.

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1 Introduction

In models of economic growth, technological change is an abstract generalization of a large and diverse set of innovations undertaken in the real world. What determines the success of these models is whether this generalization captures the essential features of the growth process. In this paper I distinguish between the familiar product- or process-innovations and inventions that are *leisure-enhancing*. I show that making this distinction helps us understand the salient patterns observed in the real world.

The defining difference between the two kinds of technologies is the way they are monetized. Improving a production process or introducing a new or higher-quality product tends to raise the profits of the innovator directly. Instead, leisure-enhancing innovations may sell below the cost of production (indeed, may be given away for free), bringing in little or no revenue. Instead, these innovations are used to capture peoples' time and attention, which are valuable and can ultimately be turned into profit. Some firms – such as platforms in the business of marketing, advertising and brand building – need consumers' time and attention in order to produce. Innovation efforts of these firms are thus focused on creating and improving services that complement leisure, in the hope that consumers recognize their attractiveness and spend more time enjoying them. This form of technological change makes leisure increasingly more attractive. The theoretical model developed in this paper traces out the implications of this phenomenon for the long-run equilibrium.

This paper attempts to make sense of three trends in the data. The first is that leisure-enhancing innovations have been around for a while and are now ubiquitous. The recently-constructed long-run data on the value of free content suggests that the leisure sector has been operating for some time (Figure 1).[1] TV channels, websites, social media and free newspapers are all products that many people use every day. Businesses involved in supplying these services are hugely profitable and very large. As of March 2018, the top six of the world's largest companies by market capitalization – Apple, Alphabet, Microsoft, Amazon, Facebook and Alibaba – are platforms whose business models, at least in part, rely on capturing peoples' time and attention. Appendix A provides further indicative evidence on the rise of leisure technologies.

The second observation which this work speaks to is the decline in average hours worked

[1]Measured at the cost of production, the value of free services has been relatively stable at about 1% of GDP over much of the past century in the US (see Figure 1). While not negligible, this share in GDP is certainly not large. The theory developed in this paper underscores that leisure-enhancing technological change may be of first-order importance even if the share of the sector is negligible in aggregate.

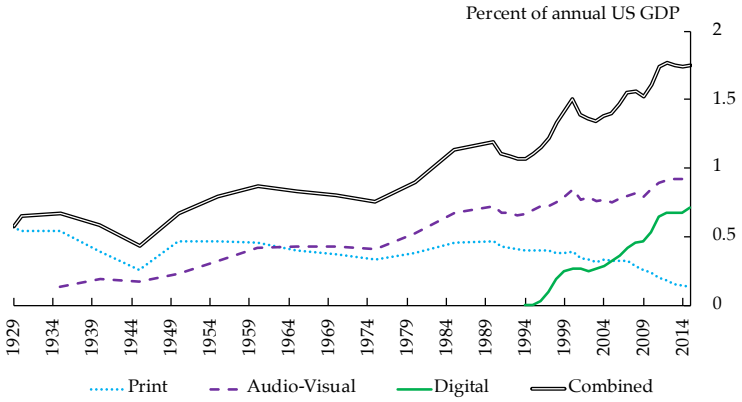


Figure 1: Free ad-supported consumer content in the United States

Source: Nakamura et al. (2017). The figure shows the ratio of free consumer content, measured by the costs of production, to GDP. Thus, for example, it does not capture a welfare measure of the value of Facebook, but only measure the cost of providing it.

and a corresponding rise in leisure hours. Figure 2 shows that, following the large falls in average hours worked at the start of the 20th century when the 5-day week became the norm, hours worked continued to decline also in the post-war period, by about 7 hours per week in the latter five decades of the last century.

Furthermore, there is suggestive evidence that changes in time allocation patterns may be linked to technology. Data that tracks how much time people spend on various electronic devices show a very rapid increase in recent years, largely due to widespread smartphone usage (Figure 3). This is true across many developed and developing economies.

The third trend that this paper speaks to is the slowdown in productivity growth. Despite seemingly rapid technological progress, productivity growth has been surprisingly weak across a wide range of advanced economies in recent decades (Figure 4) – what has come to be known as *productivity puzzle*.

To elucidate how these trends are related, I develop a simple general equilibrium model with demand for and supply of leisure services and advertising.

In the model, household leisure is defined as the utility generated from a range of activities, such as watching TV or browsing the web. To 'generate' leisure, households combine their time with *leisure services* (such as TV channels or web content). To isolate the novel economic forces at play, the theory focuses on the services that are *free* (available at zero price) and *non-rival* (equivalently, once produced, there is zero marginal cost of supplying

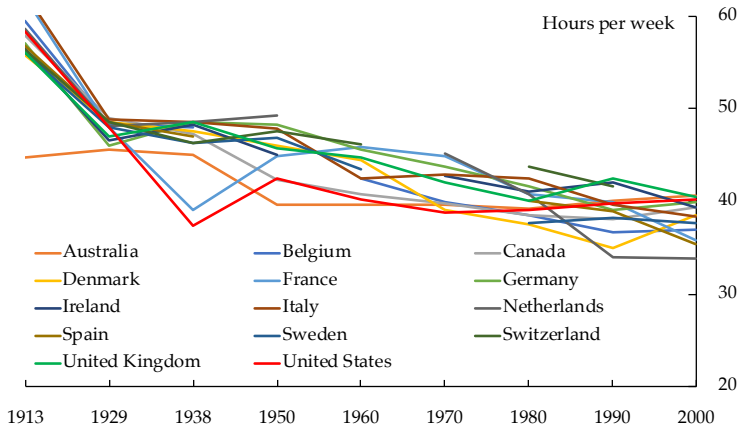


Figure 2: Trend growth rates of average hours worked in a range of advanced economies

Source: Huberman and Minns (2007). Note: Data refers to full-time production workers (male and female) in non-agricultural activities.

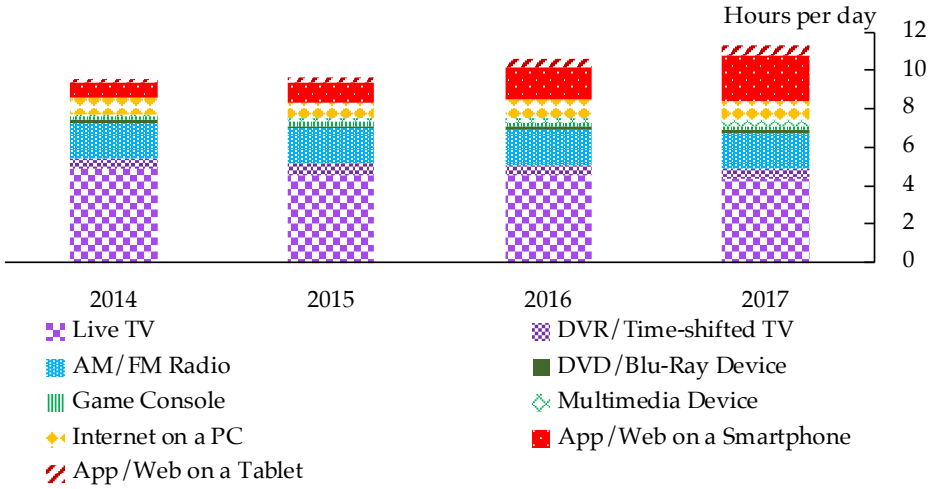


Figure 3: Average time spent on media consumption per adult in the US

Source: Nielsen. Note: Figures for representative samples of total US population (whether or not have the technology). Data on TV and internet usage, and the usage of TV-connected devices are based on 248,095 individuals in 2016 and similar sample sizes in previous years. Data on radio are based on a sample of around 400,000 individuals. There are approximately 9,000 smartphone and 1,300 tablet panelists in the U.S. across both iOS and Android smartphone devices.

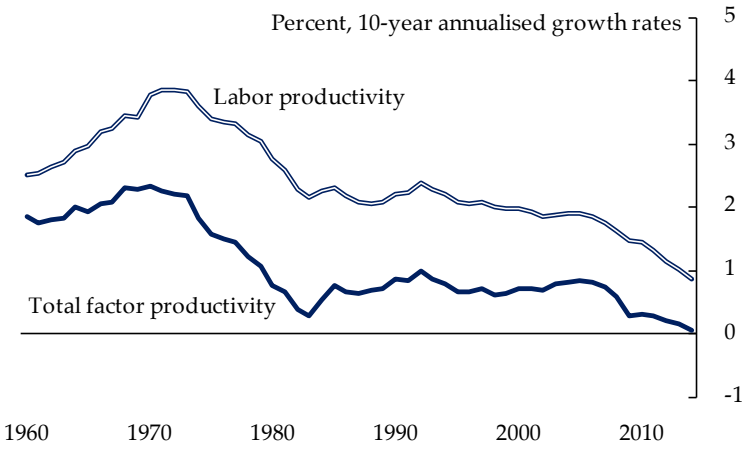


Figure 4: Trend growth rates of total factor productivity and labor productivity in advanced economies

Source: Adler et al. (2017). Based on data from Penn World Table 9.0; IMF World Economic Outlook. The figure shows purchasing power parity GDP-weighted average for the largest 20 advanced economies.

an extra user).[2] Households value variety in leisure options: the more leisure activities exist, the more attractive leisure is. On the firm side, households' time is an essential input to the production of marketing activities, which are supplied by a two-sided platform.[3] Demand for marketing services comes from producers of all other goods and services in the economy: each firm can shift its demand curve – sell more at any given price – by marketing its product.

Embedding these features in a dynamic setting with endogenous innovation and growth brings out three further insights.

First, leisure-enhancing technologies emerge endogenously in equilibrium, once the economy is sufficiently developed. This is driven by the non-homotheticity in household leisure choices: leisure technologies must be sufficiently well developed for households to spend positive time on leisure. The long-run equilibrium is characterized by what may be called a *segmented balanced growth path* (sBGP), with the initial segment that features no leisure technologies, followed by a transition once these technologies emerge.

[2]In reality, some leisure services may sell at positive prices but below the cost of production. These products can be thought of as a combination of the standard consumption good and leisure-enhancing products. The underlying economic forces are similar; and the stark split in the theory developed here underscores the novel aspects of incentives driving innovation.

[3]I assume there is only one platform. One way to think about this assumption is as approximating a sector with a high degree of concentration, perhaps due to fixed costs, economies of scale and network effects in production. To the first order, this should be a reasonably good approximation: in particular, Cournot competition with one dominant firm delivers behaviour arbitrarily close to monopoly solution. Still, future research may consider departing from this assumption by allowing for competition within the marketing sector, which may provide an extra impetus towards leisure-enhancing technological progress.

Second, in presence of leisure-enhancing innovations, hours worked decline and leisure hours rise along the sBGP. From a theoretical standpoint, the model provides a novel way to embed falling hours worked in the long-run equilibrium. Empirically, this result matches the trend in time use observed across countries (Aguiar and Hurst, 2007a).

Third, the growth rate of the traditional (non-leisure) economy declines with the introduction of leisure-enhancing technologies. This is because more leisure translates into a slower growth rate of the pool of resources that are devoted to generating new ideas and knowledge. At the same time, GDP as measured by the statistical agencies worldwide fails to capture the value of leisure services, and thus exaggerates the extent of the slowdown. Still, the growth rate of a measure of activity that incorporates the value of the leisure technologies is lower. Thus the associated decline in productivity is only partly due to mismeasurement.

The final set of results concerns the efficiency of the market equilibrium. The market equilibrium allocation involves two new inefficiencies. I show that in a simplified static setting there is undersupply of leisure technologies, as their production is governed solely by the prospective profitability of the marketing platform rather than on their true social value. In the dynamic model, there may be oversupply, because of the adverse implications of leisure decisions for the long-run growth rate.

This study is related to several strands of literature. First and foremost, it builds on and contributes to the extensive literature on endogenous growth (Romer (1990), Aghion and Howitt (1992), Jones (1995), Kortum (1997), Segerstrom (1998), Acemoglu and Guerrieri (2008), Ngai and Pissarides (2008), Aghion and Howitt (2009), Aghion et al. (2014)). More specifically, it focuses on the ability of endogenous growth models to generate balanced growth equilibrium in which hours worked decline over time, and in this sense it is closely related to the work of Boppart and Krusell (2016). The focus and the underlying economics differ substantially across the two papers, however: while Boppart and Krusell (2016) depart from the balanced growth preferences benchmark to develop the formulation consistent with declining hours and balanced growth (with the income effect stronger than the substitution effect), the theory in the present paper builds on balanced growth preferences and generates these outcomes endogenously in equilibrium.

The present paper draws on and contributes to the literature on the economics of time allocation. Starting from the seminal work of Becker (1965), economists have developed the theory of time allocation that is helpful in the analysis of the complementarities between consumption and leisure (recently summarized in Aguiar and Hurst (2016)). This paper is a first step in the direction of extending this literature to capture a salient features of the

modern economy, namely, the rising supply of free and non-rival leisure-enhancing services. The analysis of empirical evidence on time allocation patterns in recent years presented in Appendix A contributes to the literature that documents the changes in how people spend their time (with seminal contributions from Aguiar and Hurst (2007b) and Ramey and Francis (2009)).

The focus on free leisure-enhancing services brings this paper close to the studies by Terranova (2012), Wallsten (2013) and Boik et al. (2016), all of whom consider the time and attention allocation problem in the context of the rise of the internet and digital technologies. Unlike these papers, the present study analyses this problem in a fully specified macroeconomic model, which allows for the study of both positive and normative macro implications. The focus on macroeconomic implications of leisure technologies brings the paper close to Aguiar et al. (2017) who investigate how video games altered the supply of labor of young men in the United States. I cast the net more broadly, however, studying all leisure-enhancing technologies and highlighting the importance of the endogenous feedback mechanism between the innovations carried out by producers of these technologies and the resulting changes in the allocation of time and attention by consumers.

The present paper draws out implications for the growth rates of productivity and for measurement, contributing to a large and fast-growing literature on the productivity puzzle and on measurement of the modern economy (Aghion et al. (2017b)). In particular, the theory proposes a way to reconcile the two sides of the "productivity debate" by relating the rapid leisure-enhancing technological change to the lower rates of TFP growth.[4] The model developed here suggests that in some sense both sides of this debate are correct: rapid technological change in the leisure sector could well cause the growth rates of TFP to decline; thus the perception of rapid progress may well coincide with low observed TFP growth (Figure 4). Furthermore, the model allows for a clean analysis of different measures of activity, depending on whether and how one values the leisure services. It contributes to the literature on measurement challenges, particularly those that arise when goods and services are provided at zero prices (Bean (2016), Bridgman (2016), Syverson (2016), Brynjolfsson

[4] "Productivity optimists" build their case with anecdotes of groundbreaking technological innovations, speculating about their capacity to change routines and disrupt markets the way societies operate. They are not too concerned about the meagre productivity growth in the data, viewing them either as a transitory phase before the impending technological revolution (Brynjolfsson and McAfee (2014), Brynjolfsson et al. (2017b)) or as an artifact of the inability of the measurement techniques to keep up with the changing nature of the modern economy (Aghion et al., 2017a). The pessimists view the data as an early sign of persistent weakness, and speculate about economies' ability to generate long-run growth (Gordon (2016), Bloom et al. (2017b)).

et al. (2017a), Byrne et al. (2016), Nakamura et al. (2017), among others).

Finally, this paper builds on several literatures in industrial organization which study two-sided markets and economics of advertising. Classic references on the economics of platforms are Rochet and Tirole (2003) and Anderson and Renault (2006) who study the equilibrium pricing decisions in two-sided markets. More recently, Predrag Bejakovic (2018) provides a non-technical overview of the major forces at play, and highlights the rising importance of the platform business model in the modern economy. There is an extensive literature on the economics of advertising, going back to Marshall (1890) and Chamberlin (1933), and summarized in the IO Handbook Chapter by Bagwell (2007). More recent research has considered the effectiveness of advertising in the context of modern technologies (DellaVigna and Gentzkow (2010), Lewis and Reiley (2014), Lewis and Rao (2015)). The present paper contributes to these literatures by analyzing platforms in a general equilibrium setting and drawing out the implications within a canonical macroeconomic model of competition and growth.

The remainder of this paper is structured as follows. Section 2 focuses on the two novel ingredients introduced to model leisure-enhancing technological change. Section 3 outlines the model and defines the equilibrium, and Section 5 presents the main theorems characterizing the equilibrium. Section 6 illustrates the possible magnitudes of the various effects with a simple calibration. Section 5 discusses the welfare properties. Section 8 concludes with a discussion of areas for future work.

2 Two ingredients to model leisure technologies

2.1 Ingredient 1: household utility and leisure activities

In most macroeconomic models, leisure and leisure-time are one and the same thing: consider, for example, the simplest static maximization problem of a household with utility over consumption c and leisure l, unitary time endowment and wage w:

$$\max_{c,l} u(c,l) \text{ subject to } c = w \cdot (1-l),$$

This simplest formulation, which underlies most macro models, implies that leisure l is equivalent to leisure time.

Analysis of leisure technologies requires a more careful treatment of leisure. I develop a particularly tractable formulation in which households derive utility from a range of *leisure*

activities. Activities require not only household's time, but also leisure services. Furthermore, I focus on marketable, free leisure – I restrict attention to leisure activities that require commercial services provided at zero price to the consumer in the market equilibrium. Examples of these include TV and radio shows, social media platforms, apps, etc. Specifically, I assume that leisure is generated through:

$$l = \left(\int_0^M \underbrace{[\min\{h(j), m(j)\}]^{\frac{\nu-1}{\nu}}}_{\text{activity}(j)} dj \right)^{\frac{\nu}{\nu-1}}. \tag{1}$$

In this expression activities are indexed by j, and $h(j)$ is the time spent on activity j and $m(j)$ denotes leisure services required for this activity. There is a continuum of measure M of available leisure activities. Total leisure time is defined as the sum of time across all activities:

$$H_L := \int_0^M h(j)dj \tag{2}$$

Parameter ν is the elasticity of substitution across different activities, and is assumed to be greater than unity: $\nu > 1$.

Several assumptions are embodied in this formulation. First, the assumption that the elasticity ν is greater than 1 implies that different activities are imperfect substitutes: there is *love of variety* in preferences, in the sense that, all else equal, more varieties yield higher leisure utility. Second, each activity requires both time and leisure services, and only those. Thus the formulation assumes away any potential complementarities with consumption.[5] Finally, within each activity, there is no substitutability between time and free services: enjoying a favorite TV show or surfing the net requires leisure services and free time in fixed proportion (normalized to one, without the loss of generality). In other words, households cannot substitute away from their time and towards the leisure services within each activity.

To see the implications of this formulation, consider the optimal allocation of households' time across different activities. Recall that for each activity j, household can access any amount of services $m(j)$ free of charge. This implies that the availability of free services will never constrain households in their time allocation choices. Formally, because leisure

[5]Conceptually, there are several kinds of inputs that may feature in leisure production: (i) time; (ii) non-durable consumption expenditure (e.g. monthly broadband fee); (iii) durable consumption expenditure (e.g. TV or a laptop); (iv) free, non-excludable and non-rival services (e.g. radio station, web content, free apps). To simplify the analysis and to focus on the key and novel aspects of this problem, in what follows I maintain only the first and the last of these four groups of inputs. Extending the framework so that it encompasses (ii) and (iii) should not alter the key conclusions of this paper.

services are free and, because they are non-rival, from the perspective of a household they are available in an unlimited quantity, optimality implies that $m(j) \geq h(j) \forall j$. Consequently, we can simply write:

$$l = \left(\int_0^M h(j)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} dj. \tag{3}$$

Households choose how much time to devote to each individual activity in order to maximize production of leisure for any given amount of total leisure time. Because of the symmetry of this problem, the optimal choice is to spread free time evenly across the M available leisure options: $h(j) = h \ \forall j$ and therefore

$$H_L := \int_0^M h dj = Mh \Rightarrow h = \frac{H_L}{M}.$$

Plugging this into (3) we get the following Lemma:

Lemma 1. *Given (1), individuals' optimal allocation of time across available leisure varieties implies that the leisure production function is given by:*

$$l = H_L M^{\frac{1}{\nu-1}}. \tag{4}$$

Equation (4) underlines the difference between leisure utility and leisure time. Relative to the standard formulation where $l = H_L$, the model proposed here highlights the importance of technology for generating leisure utility. The state of leisure technology is summarized by the range of leisure activities M. Intuitively, the love of variety in leisure means that households attain higher utility from leisure time if more leisure options are available. A looser but more appropriate interpretation of M is that it is an index of overall advancement of leisure technology. Under some natural assumptions, better leisure technology – a higher M – will make households play more and work less, so that equilibrium leisure hours will increase as M goes up.

2.2 Ingredient 2: demand and supply of marketing activities

The second key ingredient in the model is that M is endogenously supplied by two-sided platform. These businesses are actively improving and expanding leisure technologies and through that capturing peoples' time, attention and data.

Why are peoples' time, attention and data valuable? This paper argues that their value comes from the fact that they are the inputs into production of *marketing activities* such as understanding customer habits, building a brand, creating awareness of the product, and advertising. These activities allow individual producers of goods and services to shift the demand curve for their product. An improved understanding of its consumers, greater awareness of the product among the public, or a successful advertising campaign mean that a firm may be able to sell more at a given price, or increase prices for given quantity sold, thus increasing its profit. Slightly more formally, the inverse demand curve for a product can be written as:

$$q = D(x) \cdot \beta(b)$$

where q is the price and x is quantity of the good, b is the marketing activity of the firm (perhaps relative to marketing activity of its competitors) and $D(\cdot)$ and $\beta(\cdot)$ are positive-valued functions such that $D' < 0$ and $\beta' \geq 0$. In this simple formulation, firms face a downward sloping demand curve $D(\cdot)$ which they can shift through engaging in a marketing activity. The standard model is recovered by setting $\beta(b) = 1$. Below I show how to incorporate this intuition in a standard model of monopolistic competition, and I demonstrate that in such setting firms choose a positive quantity of marketing activity in equilibrium.

Marketing services are supplied by two-sided platforms which use peoples' time and attention in the process. To produce, these entities must innovate and supply leisure-services. The platform is endowed with two technologies:

$$\begin{aligned} M &= F^M(L_M, X_M) \\ B &= F^B(H_L) \end{aligned} \tag{5}$$

where $F^M$and F^B are the production functions for leisure technologies and marketing activity, L_M and X_M are the labor and final good inputs in leisure innovation, respectively, and H_L is the leisure hours of the representative household.

3 Economic environment

The paper incorporates these two ingredients into a standard framework of monopolistic competition (Dixit and Stiglitz, 1977) with endogenous horizontal innovation as in Romer (1990) and Jones (1995). Figure 5 illustrates the basic structure of the static component of the model, with the "leisure sector", embodying the two ingredients discussed above,

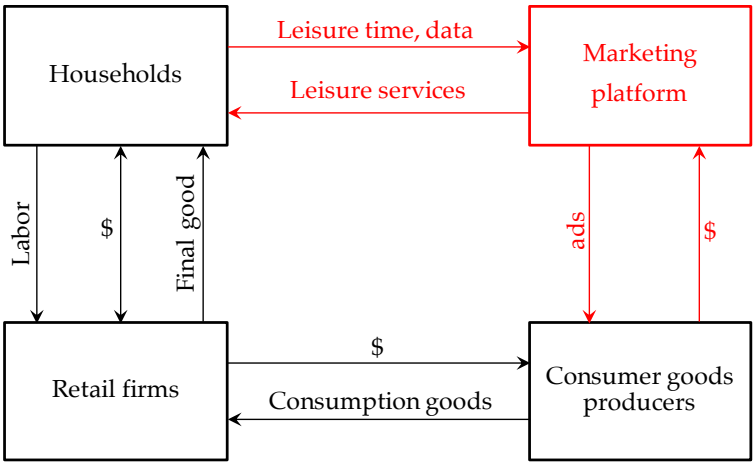


Figure 5: The simplified model structure

highlighted in red.

Time is continuous. For expositional clarity, in the main text I outline the model in stationary form and omit the time subscripts when this does not cause confusion.

3.1 Households

The economy is populated by two kinds of households: workers and capitalists. There is measure-N of both kinds of households, with population N growing at a constant rate n: $\frac{\dot{N}}{N} = n$. There is no heterogeneity within the two groups, so that the economy consists of a representative working household and a representative capitalist household. Any profits in the economy accrue to capitalists who consume those in a hand-to-mouth fashion. This simplifies the exposition without altering the economics of the model. The rest of the paper focuses on the decisions and behavior of working households.[6]

Within each period households allocate their time – endowment of which is normalized to 1 – between work and leisure. Their instantaneous utility is separable and takes the canonical form of balanced growth preferences, as in King et al. (1988) and (2002): consumption utility is log and leisure utility is a constant-elasticity power function. In addition, I allow for the possibility that there are network effects in leisure, external to individual choices, which arguably may be an important feature of some of the modern leisure services such as social

[6]Capitalists not work; they own all firms in the economy and finance hand-to-mouth consumption with the flow of corporate profits. This removes the profit flow from the working households budget constraint which allows for a clean analytical solution to the model without altering any of the economics.

media. The instantaneous utility function is:

$$u(c,l) = \log(c) + \frac{l^{1-\xi}}{1-\xi} \cdot \bar{H}_L^{\psi} \tag{6}$$

where l is leisure and $\bar{H}_L$ is the average time spent on leisure. The power function $\bar{H}_L^{\psi}$, $\psi \geq 0$ is the network effect.

In this economy, households solve the following problem:

$$\max_{\{c(t)\}_0^{\infty}\{h(j)\}_0^{M}} \int_0^{\infty} e^{-(\rho-n)t} u(c,l)dt \tag{7}$$

subject to:

$$\begin{aligned}
\dot{d} &= d(r-n) + w \cdot (1 - H_L) - c \\
H_L &= \int_0^M h(j)dj \\
l &= \left(\int_0^M [\min\{h(j), m(j)\}]^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} \\
0 &\leq \lim_{t\to\infty} \left[d \cdot \exp\left(-\int_0^t (r(s) - n)ds \right) \right]
\end{aligned} \tag{8}$$

where the instantaneous utility function is given by equation (6). Households maximize discounted stream of utility from per-capita consumption and leisure, with the effective discount rate $\rho - n$, subject to four constraints. The first is the flow budget constraint;[7] the second is the time allocation constraint, as in equation (2). The third is the leisure generating function (equation (1)) and the fourth is the standard no-Ponzi game condition.

3.2 Production technology

Differentiated goods $x(i)$, $i \in [0, A]$ are produced by a continuum of monopolistically competitive firms. Each firm has access to a constant returns technology that converts one unit

[7] The law of motion for total assets is $\dot{D} = rD + wH_W N - C$, where D are assets in zero net supply, C is total household consumption. c is consumption per capita $c = \frac{C}{N}$. Now defining per capita assets as $d = D/N$, dividing the law of motion by N, and noticing that $\dot{D} = \left(d\dot{N}\right) = \dot{d}N + d\dot{N}$ yields the law of motion of per capita assets.

of final output into one unit of differentiated good:

$$x(i) = X,$$

where X is the final good required to produce one unit of good i.

The competitive retail sector produces final consumption basket by combining the differentiated goods with the labour input L_Y. The only non-standard element here is the assumption that the share of each product in production is related to the *relative* marketing activity of each producer i. In particular, the final good is produced according to the following constant returns to scale aggregate production function:

$$Y = \int_0^A \left(\left(\frac{b(i)}{\bar{B}} \right)^{\chi \cdot \mathcal{O}} x(i) \right)^{\alpha} L_Y^{1-\alpha} di. \tag{9}$$

The novel term $\left(\frac{b(i)}{\bar{B}}\right)^{\chi \cdot \mathcal{O}}$ guides how the marketing activity works in the model economy. $b(i)$ is firm i's marketing activity and $\bar{B}$ is the average marketing expenditure across firms: $\bar{B} := \frac{1}{A}\int_0^A b(i)di$. Thus the fraction $\frac{b(i)}{\bar{B}}$ measures the relative advantage of firm i due to its marketing efforts as compared to its competitors.[8] Parameter $\chi \geq 0$ measures the sensitivity of demand to marketing activity: $\chi = 0$ corresponds to the case where individual firms view marketing as useless, and the model collapses to the standard monopolistic competition setting. The binary variable $\mathcal{O}$ takes value $\{0, 1\}$ and denotes the choice of the marketing platform whether or not to operate (discussed below).

The implication of equation (9) is that only by investing in marketing more than its competitors can a firm boost demand for its product: marketing is all about *relative* advantage. This is consistent with wealth of empirical evidence, summarized in Bagwell (2007): while marketing may have significant impact on any individual firm's sales, the effect disappears once the unit of observation is expanded to a sector or the macroeconomy.

The present model is silent on the channels through which marketing activities operate, and assumes they have no direct utility impact. The literature highlights three reasons why marketing may work: providing information, persuasion, and complementarity with consumption. In this paper I take it as given that marketing efforts shift relative demand, but

[8] The retail sector anticipates any shifts in relative demand due to firms' marketing efforts and demands more of the varieties that boasted higher advertising relative to competitors. The fact that it is the retail sector that "demands more" of a firm's product as a result of its advertising activity is inconsequential to the analysis: the model where consumers were choosing the products directly, and their relative taste for specific varieties was affected by ad expenditures, is isomorphic to the one presented here.

do not take a stance on the channels through which this operates. Correspondingly, I assume that these activities do not affect household's welfare directly: one possible interpretation is that the positive information effects broadly offset the negative utility impact associated with exposure to ads.

Each firm faces a demand curve for its product, and chooses its price, quantity and marketing spending $\{q(i),\ x(i),\ b(i)\}$ to maximize profits, taking as given the average level of marketing $\bar{B}$.

3.3 Leisure technology: the two-sided platform

The marketing platform simultaneously operates along two margins: it supplies marketing to businesses, and produces varieties of leisure services. The latter is necessary to operationalize the former, because peoples' time and attention are required inputs in the production of marketing. The platform maximizes profits:

$$\pi_B^* = \arg\max_{p_B} \int_0^A b(i) \cdot p_B di - \text{cost of producing} \int_0^A b(i) di\} \tag{10}$$

subject to technologies given by equation (5), firms' demand for marketing and households' supply of leisure time (both of which are derived in the next Section). It chooses to operate ($\mathcal{O} = 1$) if $\pi_B^* \geq 0$ or stays inactive ($\mathcal{O} = 0$) otherwise.

Embodied in (10) is the assumption that the platform charges a positive price only for the marketing activities and not for leisure services. This is a simplifying assumption, as in reality the decision how price the two sided of the market is endogenous. Classic papers in industrial organization by Rochet and Tirole (2003) and Armstrong (2006) and the recent book by Predrag Bejakovic (2018) suggest that the equilibrium pricing structures depend on the elasticities of demand on both sides of the market as well as on any potential externalities between two groups of customers. If one side of the market benefits substantially more than the other from the interaction, then the latter may receive the good or service for free as the platform tries to attract participation. In the case of leisure services, free provision appears to be prevalent (see Figure 1 in Section 1 and 13 in the Appendix), and I impose this outcome directly here.

3.4 R&D technology

New varieties of differentiated goods are invented by the R&D sector employing researchers. I follow the canonical semi-endogenous framework of Jones (1995) in which the ideas production function is given by:

$$\dot{A} = \zeta A^{\phi} L_A^{\lambda} \tag{11}$$

with $0 < \phi \leq 1,\ 0 < \lambda \leq 1$.

3.5 Decentralized equilibrium

Definition 1. The decentralized equilibrium is a set of paths of aggregate quantities: $\{c, C, X, X_B, H_W, H_L, L_A, L_Y, A, M, B\}_{t=0}^{\infty}$; micro-level quantities: $\{x(i), b(i), m(j), h(j)\}_{i\in A, j\in M; t=0}^{\infty}$; prices of each intermediate input and of marketing services: $\{q(i), p_B\}_{i\in A_t, t=0}^{\infty}$; interest rates and wages: $\{w, r\}_{t=0}^{\infty}$; marketing platform activity indicator: $\{\mathcal{O}\}_{t=0}^{\infty}$; such that:

- Working households maximize utility subject to the law of motion for per-capita assets, the leisure production technology, and the no-Ponzi game condition, taking as given all prices and the number of available leisure options;
- Capitalist households consume firms' profits every period;
- Retail sector firms maximize profits taking prices and marketing activity of each firm as given;
- Intermediate goods producers maximize profits taking as given the average level of marketing in the economy;
- The marketing platform maximizes profits subject to the marketing production technology, leisure varieties production technology, households' optimal time allocation decisions, and the demand curve for marketing services. It becomes active only if it can be profitable.
- The R&D producers maximize profits subject to the ideas production function.
- There is free entry to R&D activity.
- Labour market and goods market clear.

4 Conditions for balanced growth

This Section derives the conditions under which the economy admits a balanced growth path: a decentralized equilibrium in which the stock of knowledge, output, consumption and leisure varieties can grow at constant rates indefinitely. These conditions are more stringent than in a standard model, because of the central role played by households' leisure time, H_L, which is bounded by the $[0, 1]$ interval.

4.1 Household time allocation problem

Consider first the household time allocation problem. Equation (4) in Lemma (1) states that for a given amount of leisure time, leisure utility increases in M.

The optimal choice of leisure requires that the marginal rate of transformation is equal to the marginal rate of substitution between consumption and leisure:

$$\frac{1}{1-H_L} = H_L^{-\xi} M^{\frac{1-\xi}{\nu-1}} \bar{H}_L^{\psi}. \tag{12}$$

Because $\bar{H}_L = H_L$ in equilibrium, equation (12) pins down the equilibrium leisure hours as an increasing function of the state of leisure technology M. However, without further restrictions, optimality condition (12) is *not* consistent with balanced growth path along which M grows at a constant rate. Balanced growth requires that hours worked are constant or decline at a constant rate: we must have $\frac{\dot{H}_W}{H_W} \leq 0$.[9] Here we are particularly interested in the case where hours worked decline over time. Along such a path leisure hours increase at a *non-constant* rate, because $H_L + H_W = 1$.[10] This is inconsistent with condition (12). Additional assumption on the functional form of the utility function is needed to ensure balanced growth:

Proposition 1. Condition on preferences required for balanced growth. *Consider a dynamic environment in which household preferences are described by the instantaneous*

[9] Note that this is the only other possibility: for example, leisure hours rising at a constant rate would imply zero hours worked at finite horizon, which would yield zero consumption, an outcome clearly inconsistent with utility maximization.

[10] To see this mathematically, note that if $x(t)+y(t) = constant$, taking logs of both sides and differentiating the time constraint with respect to time yields: $\frac{1}{x+y}\left(\frac{dx(t)}{dt} + \frac{dy(t)}{dt}\right) = 0$. The expression on the left hand side can be rewritten as $\frac{x}{x+y}\frac{dx(t)}{dt}\frac{1}{x} + \frac{y}{x+y}\frac{dy(t)}{dt}\frac{1}{y} = \frac{x}{x+y}\gamma_x + \frac{y}{x+y}\gamma_y$ where γ denotes net growth rate of a variable, which itself is, in general, a function of time. Setting this equal to zero, the two growth rates satisfy $\gamma_y = -\frac{x}{y}\gamma_x$. This proves that if γ_x is constant, γ_y is not.

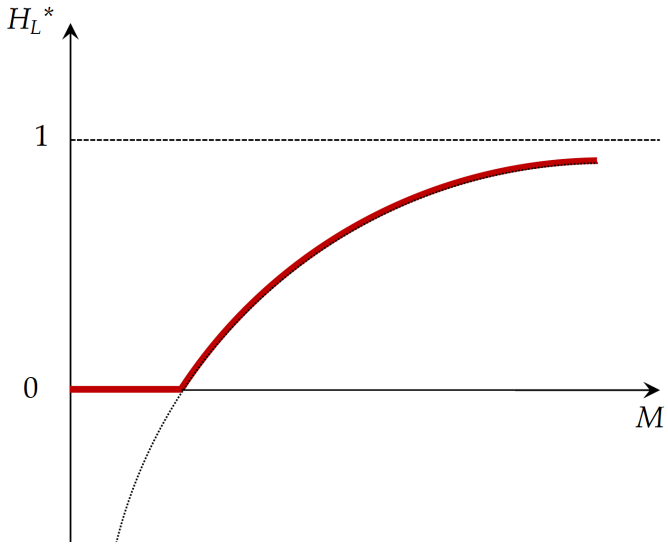


Figure 6: Optimal choice of leisure hours as a function of M

utility function in (6), in which the number of leisure varieties M grows at a constant rate in equilibrium. A balanced growth equilibrium with hours worked decreasing at a constant rate is consistent with these preferences if and only if $\psi = \xi$.

Proof. See Appendix (B). □

In the remainder of the paper I assume that the condition in 1 holds, and in particular that $\xi = \psi = 0$ so that utility is linear in l.[11] Under these assumptions, the optimal choice of leisure hours is given by:

$$H_L^* = \max\{0, 1 - M^{\frac{1-\xi}{1-\nu}}\}. \tag{13}$$

Equation (13) states that, for M sufficiently large, optimal leisure hours depend positively on the measure of available leisure options as long as $0 \leq \xi < 1$ (recall that $\nu > 1$). Figure (6) shows the optimal hours as a function of leisure technology in that relevant portion of the parameter space.

More abundant leisure options make leisure more attractive relative to work, and households optimally choose to work less and play more. When leisure technologies are not well developed, household is at the corner with no time spent on leisure at all. The implication that there is a non-decreasing relationship between leisure technology and total leisure time is strongly supported by the empirical evidence in papers that analyze the impact of leisure

[11] Assumption that $\xi = \psi = 0$ simplifies the notation and the algebra. All of the economics of the model carries through as long as $0 < \xi < 1$, which is required for a positive relationship between H_L and M.

technologies on time allocation (see, for example, Gentzkow and Shapiro (2008), Falck et al. (2014) and Reis (2015)).

4.2 Platform technology for production of marketing activities

The second instance in the model where the bounded H_L implies more stringent conditions for a balanced growth path is the technology of the platform used to produce marketing activities. Because that technology takes H_L as an input, it requires increasing returns to scale so that marketing activity can be produced *ad infinitum* even as households' time is bounded. In particular, I assume that the marketing activity B is produced using the following technology:

$$F^B(H_L) = \begin{cases} 0 & \text{if} \quad H_L = 0 \\ \frac{1}{1-H_L} & \text{if} \quad H_L > 0 \end{cases}. \tag{14}$$

One justification for a convex production function is that, at higher levels of H_L, the marketing firm is better able to learn about consumers' tastes and preferences, and thus to provide more targeted marketing. Arguably, such information acquisition appears to play an increasingly important role in the real world. This assumption is not without controversy, however. The marginal effectiveness of marketing may decline as the cognitive and memory limits start to bite, suggesting that decreasing returns may be a more plausible alternative. Furthermore, some of the technical constraints faced by firms in this sector suggest that decreasing returns may be a better assumption. For example, the success of machine learning techniques used for targeting consumers' behavior depends on the accuracy of their predictions. But the prediction accuracy increases with a square-root of the number of observations (Varian (2017)), which is a hard, statistics-driven constraint on the growth potential of leisure platforms. All in all, the assumption of increasing returns imposed here should be interpreted as one that is necessary for the economy to admit a balanced growth path. Without convexity, there will be a limit on the size of the marketing sector, and at some point along the growth path marketing prices will rise rapidly as demand outstrips supply. The sheer size and profitability of the largest platforms suggests that the assumption of increasing returns is a useful one to describe the past experience. Investigating the validity of this assumption going forward is left for future work.

What remains to be specified is the functional form for the leisure technology production function. In the main text I assume that the production function for the number of varieties

is linear in final good used by the platform, X_M:

$$F^M(L_M, X_M) = X_M; \tag{15}$$

The linear one-to-one technology is perhaps the simplest possible formulation and alternatives are interesting to explore. In Appendix C I solve the model under the alternative assumption that leisure technologies are produced using labor (leisure scientists, $M = L_M$) and I show that the key qualitative results continue to hold under that alternative assumption.

The production function for M reflects the fact that services $m(j)$ are non-rival. Production of an extra variety of leisure – raising M – requires real effort and resources; but once a specific variety j has been produced, it provides unlimited amount of services $m(j)$ within a period. All M varieties are perishable, in that M is modeled as a flow and not a stock. Future work could usefully explore different ways of modeling M and the process of technological improvements of the leisure options.[12]

The attractiveness of these assumptions becomes clear with the following Corollary:

Corollary 1. *These assumptions imply a consolidated quasi-production function for marketing services that consolidates the platform technologies. As long as $X_M > 1$, (15) implies that $M > 1$, and by equation (13), households' supply of leisure H_L is positive. We thus have:*

$$B = \begin{cases} 0 & \text{if } X_M^* \leq 1 \\ X_M^{\frac{1}{\nu-1}} & \text{if } X_M^* > 1 \end{cases}. \tag{16}$$

That is, the platform faces a minimum scale of operation. Beyond this minimum, the consolidated production function is smooth and concave. The cost function is

$$c(B) = \begin{cases} +\infty & if\ X_M^* \leq 1 \\ B^{\nu-1} & if\ X_M^* > 1 \end{cases}. \tag{17}$$

5 Analytical characterization of the equilibrium

In what follows, I denote the asymptotic growth rate of a variable X along the sBGP with γ_x, so that $\gamma_x := \lim_{t\to\infty} \frac{\dot{x}}{x}$. The first key result is contain in the following proposition:

[12] One possibility is that the leisure time itself is an important part of the production process of leisure varieties – particularly for the digital ones, such as social media. See Arrieta Ibarra et al. (2018) for discussion along those lines.

Proposition 2. Condition for entry of the platform. *The platform is active and there is leisure-enhancing technological change if and only if*

$$(1-s)\cdot A\cdot N > \frac{1}{\kappa}, \tag{18}$$

where $s := \frac{L_A}{L_A+L_Y}$ is the share of workers employed in the R&D sector and $\kappa := \frac{\alpha}{1-\alpha}\chi^2\alpha^{\frac{2}{1-\alpha}}(\frac{1}{\nu-1})$ is a positive constant.

Proof. See Appendix B. □

Proposition (2) describes a watershed moment for an economy. On the balanced growth path, the left hand side of condition (18) is growing at a rate $\gamma_A + n$, while the right side is a constant. Therefore it is just a matter of time when (18) is satisfied. The marketing platform becomes a viable business proposition only when the economy reaches a certain size. Only then the demand for marketing services and the revenue generated through the business-to-business side is sufficient to compensate for the costs of creating the supply of leisure technology that is sufficient to move the households away from the corner solution of zero leisure.

5.1 Segmented Balanced Growth Path

Given the result in Proposition 2, the balanced growth equilibrium in this economy can be defined as follows:

Definition 2. A *segmented balanced growth path* (sBGP) is an equilibrium trajectory along which: (i) when (18) is not satisfied, per capita consumption, output and the measure of varieties A all grow at a constant rate; (ii) as $t \to \infty$, per capita consumption, output, A and M grow at possibly distinct but constant rates, and average hours H_W decrease at a constant rate.

The segmented Balanced Growth Path (sBGP) is then characterized by the following Proposition:

Proposition 3. Characterization of the sBGP. *There exists $\hat{t}$ such that (18) holds $\forall t \geq \hat{t}$.*

For $t \leq \hat{t}$, average hours worked are constant ($H_W = 1$), and per capita consumption, output, wages and "knowledge" all grow at the same constant rate given by:

$$g = \frac{\lambda n}{1-\phi}. \tag{19}$$

For $t \geq \hat{t}$, the marketing platform is active and the economy transitions to the other segment of the sBGP. As $t \to \infty$, average hours worked decline at a constant rate $\gamma_{H_W} = -\frac{1}{\nu}(\gamma_A + n)$ and the growth rate of A is:

$$\gamma_A = \frac{n\lambda\left(\frac{\nu-1}{\nu}\right)}{1-\phi+\frac{\lambda}{\nu}} < g. \tag{20}$$

Proof. See Appendix (B). □

Proposition 3 contains several key results. First, it tells us that the economy transitions endogenously from one segment of the sBGP to another, when condition (18) is first satisfied. Fundamentally, this is driven by the juxtaposition of the household's leisure supply, which is zero when the leisure technology is relatively undeveloped (low M), and the fact that the platform requires some of household's time to produce. The theory thus highlights the non-homotheticity in leisure choices and the importance of market size as the factors that determine whether leisure technologies emerge in equilibrium.[13]

Second, the balanced growth equilibrium in this economy is consistent with declining hours worked. The magnitude of the decline is governed by the elasticity of substitution across leisure varieties ν. In particular, the effect vanishes as $\nu \to \infty$, when leisure varieties are perfect substitutes and leisure technology brings no improvement to the experience of households. This is intuitive: when technology improvements have little effect on the overall attractiveness of leisure, the decline in hours worked associated with leisure technologies will be small. The declining hours along the sBGP is a unique feature of the theory developed

[13] In reality, there may be complementary factors that encourage the development of leisure-enhancing technologies. For example, the invention of household appliances (such as the washing machine) has dramatically cut the time required to complete the necessary household duties, which meant that households had more time to allocate to other activities, including leisure. This meant that the potential market for leisure services expanded, which could have directed innovations towards the leisure sector, as in the directed technical change literature (Acemoglu (2002)). More recently, other technological breakthroughs – notably the invention of the internet and the mobile phone – have opened up new avenues to reach people's attention throughout their day (Figure 15), amass large quantities of data, and target marketing with extraordinary precision, which likely boosted the productivity of platforms. Analysis of these phenomena is beyond the scope of this paper, but the present framework presents a useful laboratory to study them in the future.

here: the model qualitatively matches the key trend observed in the data without hardwiring the decline in hours into household preferences, and shows that growth can still be balanced. In a standard model with endogenous labor supply, falling hours worked on the balanced growth path can be ruled out, as this would violate household intratemporal optimality condition in finite time. Here, instead, the intratemporal optimization is the key force driving the downward trend in hours worked, with leisure technologies tilting the balance between work and leisure towards the latter. Thus the model stresses the importance of innovation that is targeted specifically at capturing people's time, attention and data, a mechanism that is arguably at the heart of the modern economy but is new to macroeconomic models.

The third key result in this proposition is that the emergence of leisure-enhancing technologies is associated with a decline in the growth rate of knowledge. The ideas production function in equation (11) puts the present model within the class of semi-endogenous growth models (e.g. Jones (1995), Kortum (1997), Segerstrom (1998)). In this framework, the long-run growth rate of knowledge is determined by the growth rate of the total pool of resources that are be used to generate ideas. Leisure-enhancing technological change acts to limit how fast this pool of resources can grow by generating decline in hours worked. This lowers the long-run growth rate of the economy.

Recent research has highlighted the importance of the growth of resources devoted to innovation in generating growth. For example, Bloom et al. (2017a) show that research productivity (defined as $\frac{\frac{\dot{A}}{A}}{L_A}$) is declining, both at the aggregate and at the industry- and firm-level. This suggests that the growth of inputs into research is the key determinant of the long-run growth rate of innovation. Diverting time towards leisure, and away from productive activities that drive the expansion of traditional technology lowers the growth rate.

The theory speaks to some of the recent discussion around how recent technologies affect behavior. For example, some studies started to explore how technologies and content available today may act to divert peoples' time and attention away from creative thinking. Some suggest that more distracted minds may lower workers' productivity (e.g. Terranova (2012), Nixon (2017)), while the nascent experimental evidence shows that leisure-technologies occupy our limited cognitive resources, significantly decreasing cognitive performance (Ward et al., 2017). The present paper can be seen as formalization of these ideas, and a way to link the micro-observations with economic performance at the macro level.

Several other implications that follow are contained in the next proposition.

Proposition 4. Consumption growth, real interest rate and the share of workers

in R&D. *The asymptotic growth rate of per capita consumption along the sBGP is*

$$\gamma_c = \frac{\lambda n \left(\frac{\nu-1}{\nu}\right)^2}{1-\phi+\left(\frac{\lambda}{\nu}\right)} - \frac{n}{\nu}, \tag{21}$$

and the asymptotic steady state real interest rate is

$$r = \rho + \gamma_c. \tag{22}$$

Both the growth rate of per-capita consumption and the real interest rate decline as the leisure technologies emerge. The consumption growth rate declines by more than the growth rate of A. Indeed, the steady state growth rate of per-capita consumption is negative if the love-of-variety effect is sufficiently strong.

The steady state share of workers employed in the R&D sector is

$$s = \left(1 + \frac{1-\alpha}{\alpha(1-\alpha-\alpha\chi)}\left(\frac{r-n}{\gamma_A}\right)\right)^{-1}. \tag{23}$$

The long-run effect of leisure technologies on s is ambiguous.

Proof. See Appendix (B). □

This proposition highlights that the growth rate of per-capita consumption declines by more than the growth rate of TFP: we have $\gamma_c < \gamma_A$ which trivially implies $g_A - \gamma_c > g_A - \gamma$. Intuitively, consumption growth is affected by both the decline in hours worked and the decline in productivity, both of which lead to a lower household income growth. Indeed, it is possible that in balanced growth equilibrium consumption per capita declines over time. This result highlights the fact that, when households choose how much leisure to enjoy, they do not take into account the impact of that decision on the pace of technological progress in equilibrium – there is a dynamic externality from leisure choices today to the future standard of living.

Lower consumption growth drives the decline in the equilibrium real interest rate – a prediction of the model that is consistent with the trend observed in the data ((Rachel and Smith, 2015), (Summers, 2015)). Intuitively, facing lower steady state growth rate of income, households must revise down their consumption growth. For that to happen, the interest rate must fall to make the current consumption more attractive relative to future consumption.

Finally, the proposition provides the formula for the share of workers employed in R&D. This share is determined by the relative profitability of R&D activities, which in turn is tightly linked to the profitability of the consumer sector (recall that the price of a patent is a discounted sum of profits that this patent accrues). The direct effect of leisure technologies is through the reduction in profits (the term $-\alpha\chi$), which lowers the incentives to innovation and hence lowers s. The overall effect is ambiguous, however, because of two indirect effects of opposite sign: lower growth rate means innovation is less productive, while lower interest rate boosts the net present discounted value of future profits from innovation activity.

Importantly, the negative effect of leisure technologies on the long-run growth rate in Proposition (3) is independent of the effect on s. This is because the share of workers in R&D only lowers the *level* of output, but not its long-run growth rate.

5.2 Measurement

The salience of leisure-enhancing technologies in today's economy raises three important questions about the way we measure economic activity. First, are the current methods of measuring economic output well-suited to capture the emergence of leisure technologies? Second, what may be a better measure of activity? And third, how does this preferred measure compare to other measures of economic activity? In particular, is the slowdown of economic growth described in Propositions (3) and (4) "real", or only a a corollary of mismeasurement?

GDP as measured by statistical agencies worldwide. It is useful start with a review of how the activity of the leisure sector of the economy is measured by statistical agencies worldwide. Bean (2016) concisely describes the current practice:

> Most of the web's popular destinations, such as Google, Facebook, and YouTube, rely on advertising to generate income. [...] Digital products and services are effectively paid for by the advertisers. As such, the 2008 UN System of National Accounts (SNA) treats them as an intermediate input in the advertising industry. Therefore, the advertisement expenditure adds to the value added of the industries supplying advertisement space, and at the same time detracts from the value added of the advertising industries. Consequently the value of digital products nuanced through selling advertising space will be accounted for in aggregate GDP only to the extent that it also translates into higher consumption of the goods and services being advertised.

In our model, advertising is a zero-sum activity: there is no increase in consumption that results from it in equilibrium. Consequently, to answer the first of the three questions posed above, it is clear that GDP, the way it is currently measured, does not capture any of the value generated by the marketing sector. The following proposition describes the behavior of GDP as currently measured in the data.

Proposition 5. GDP as currently measured. *Let $GDP_{measured}$ denote the gross value added in the traditional sector of the economy (i.e. excluding the value of leisure technologies), so that it is defined as follows:*

$$GDP_{measured} := Y - \int_0^A x(i)di - X_M.$$

Then, in equilibrium:

$$GDP_{measured} = \begin{cases} AN(1-s)\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right) & \text{if } t < \hat{t} \\ (AN)^{\frac{\nu-1}{\nu}}(1-s)^{\frac{\nu-1}{\nu}}\kappa^{-\frac{1}{\nu}}\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} - \kappa\right) & \text{if } t \geq \hat{t} \end{cases}. \tag{24}$$

The stationary growth rate of GDP as measured in the data is given by:

$$\gamma_{GDP_{measured}} = \begin{cases} g+n & \text{if } t < \hat{t} \\ \frac{\nu-1}{\nu}(\gamma_A + n) & \text{as } t \to \infty \end{cases}.$$

The asymptotic growth rate of measured GDP per capita is equal to the asymptotic growth rate of per capita consumption.

Proof. See Appendix (B). □

Proposition 5 shows how GDP as measured today by statistical agencies worldwide is affected by the emergence of the leisure economy, in terms of levels and growth rates. On impact, the level of measured GDP drops, as resources are used in the leisure sector with no corresponding increase in measured gross output (the $-\kappa$ term in the second line of (24)). Over time, the growth rate of measured GDP slows down, in line with the measure of the growth rate of per-capita consumption. From measurement point of view, this is good news: even when leisure services are omitted, GDP growth still gives a good steer on the growth of per capita consumption.

Measuring value of leisure services with the opportunity cost of time. The key challenge is to measure the value of leisure services. The intertemporal optimality condition measuring that this value with the opportunity cost of time – the wage rate – is a sensible strategy. Recall that marginal utilities of spending one extra working or enjoying leisure are equal. This is in line with some of the past literature that suggested similar valuation method (Goolsbee and Klenow (2006), Brynjolfsson and Oh (2012)). The following proposition characterizes the value of leisure services under this assumption.

Proposition 6. Valuing leisure at the current wage rate. *The value of leisure services measured as the opportunity cost of time defined by*

$$V_{leisure} := N \cdot w \cdot \int_0^M m(i)di$$

which, on the segment of sBGP when $t > \hat{t}$, is equal to:

$$V_{leisure} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} AN \left(1 - (\kappa(1-s)AN)^{-\frac{1}{\nu}}\right)$$

Proof. See Appendix B. □

Taking this methodology of valuing leisure services as given, the equilibrium GDP in this economy can be expressed a function of the primitives of the model:

$$GDP_{model} = \begin{cases} AN(1-s)\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right) & \text{if } t < \hat{t} \\ (AN)^{\frac{\nu-1}{\nu}}(1-s)^{\frac{\nu-1}{\nu}}\kappa^{-\frac{1}{\nu}}\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} - \kappa\right) + V_{leisure} & \text{if } t \geq \hat{t} \end{cases}.$$

GDP is a sum of two components: the measure of activity of the "traditional" sector, which is properly captured by statistics agencies, and the value of leisure services. The theory thus allows for a quantitative counterfactual analysis, which I pursue in the next Section.

6 Quantitative evaluation

6.1 Illustrative parametrization

To further explore the workings of the model and the quantitative properties of the balanced growth equilibrium and the transition path, it is useful to consider an illustrative

parametrization (Table 1). The parametrization corresponds to annual frequency, with the discount rate of 2% and population growth of 1% per annum. I follow Jones (1995) and set parameters of the R&D production function – λ and ϕ – to hit the long-run per-capita growth rate of the economy (with inactive marketing) of 2%.

Parameter ν is the key one in this calibration. It pins down the strength of the link between leisure enhancing technological change and the resulting shifts in time allocation. A higher ν makes this link weaker: advancements in leisure technology do not shift households' decisions by much. Conversely, the closer ν is to unity, the more sensitive households' time allocation decisions become. Calibrating this parameter is difficult, and future work may fruitfully consider the strength of the link between technology and time allocation empirically. Through the lens of the theory developed in this paper, parameter ν is the elasticity of substitution across the leisure varieties, so one way to gauge a sense of the plausible magnitudes is to consider the estimates of different elasticities from the literature. For example, Goolsbee and Klenow (2006) estimate the elasticity between internet vs. everything else of about 1.5, highlighting that such elasticies can be quite low when the composite goods are very broadly defined. Elsewhere, Broda and Weinstein (2006) study the welfare gains from increased variety as a result of the rising trade penetration in the US economy. In the process, the authors estimate thousands of elasticities of substitution between similar products imported from different countries. For example, they establish that the elasticity of substitution across cars imported from various places is around 3, while the corresponding elasticity for apparel and textiles is about 6. Within products classified as differentiated, the median elasticity is around 2 and the mean is about 5. To err on the side of caution, the parametrization that follows assumes that $\nu = 5$.

Parameter χ corresponds to the perceived effectiveness of marketing: the value of 0.05 means that each producer believes that spending twice as much on marketing as their competitors would increase their sales by 2.5%.[14] The empirical literature on the effects of marketing highlights that estimation is plagued with difficulties, not least the endogeneity of the marketing campaigns and the residual volatility that is very large compared to any potential treatment effect (DellaVigna and Gentzkow (2010), Lewis and Reiley (2014), Lewis and Rao (2015)). Some of these studies report the increases in sales that follow a large marketing campaign of about 5%, putting the value of 0.05 broadly in the right ballpark.

Finally, α is a share of consumer goods in the final good production, with $1-\alpha$ corre-

[14] To see this, note that equilibrium quantity is $x(i) = [\alpha^2 \left(\frac{b(i)}{B}\right)^{\alpha\chi} L_Y^{1-\alpha}]^{\frac{1}{1-\alpha}}$, thus the elasticity is $\frac{\alpha\chi}{1-\alpha}$, which is 0.025 for $\alpha = \frac{1}{3}$.

ρ	Household discount rate	0.02
ν	Love for variety in leisure	0.7
n	Population growth	0.01
α	Share of consumer goods in Y	0.33
χ	Perceived effectiveness of advertising	0.05
λ	Diminishing returns to labor in R&D	0.65
ϕ	Increasing returns in the ideas production function	0.65

Table 1: Illustrative parametrisation

sponding to the share of labour in the retails sector. I set α to $\frac{1}{3}$, following Jones (1995).

6.2 Effects on growth and other variables of interest

I now solve for the sBGP characterized above, including for the transition path. Figures 7 and 8 show the results. Figure 7 shows that the growth rate of knowledge declines gradually.[15] Remarkably, the decline in TFP occurs exactly at the point when the rate of technological progress that includes leisure technologies is expanding (the bottom-right panel of Figure 8 shows the growth rate of leisure technology is the highest initially). This result may help to reconcile the productivity paradox, that the weak observed productivity growth occurs at the time when technological change appears to be very rapid.

The two upper panels of Figure 8 show the time allocation choice along the sBGP. Hours worked decline at a constant rate of about -0.4% per annum, which matches the average rate of decline of weekly hours worked illustrated in Figure 2. Thus the model's key prediction is in line with the trends observed in the data.

The bottom-left panel of 8 shows that the equilibrium real interest rate falls sharply on impact, and then adjusts further to settle at a level that is about 1.3pp lower. This economically significant decline in the real rate of interest is matches the decline in real interest rates across advanced economies that we have seen over the past 20 years (Rachel and Smith, 2015), arguably the period over which the leisure enhancing technological change has been particularly rapid.

The parsimonious approach developed here may be too simplistic to give definitive answers on the question of how much of the observed slowdown in productivity and hours worked can be accounted for by leisure-technologies. It is nonetheless interesting to speculate how the model's predictions match with the historical developments. Two interpretations of

[15]The transitional dynamics are fairly slow - this is driven by the (relatively high) value of ϕ, and is a well-known result in this class of models, discussed at length in Jones (1995).

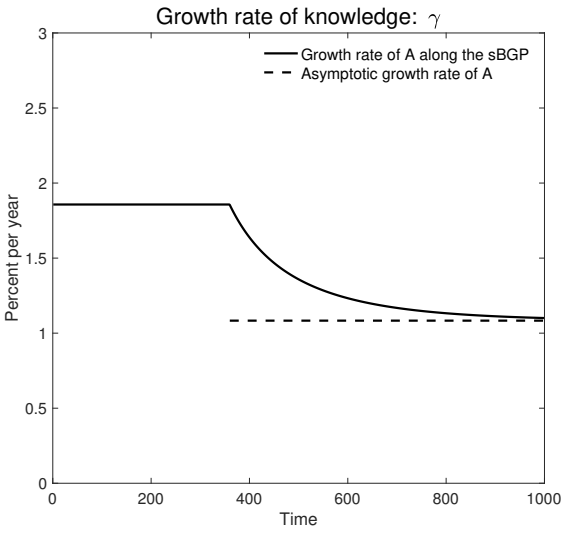


Figure 7: Growth rate of knowledge in the sBGP

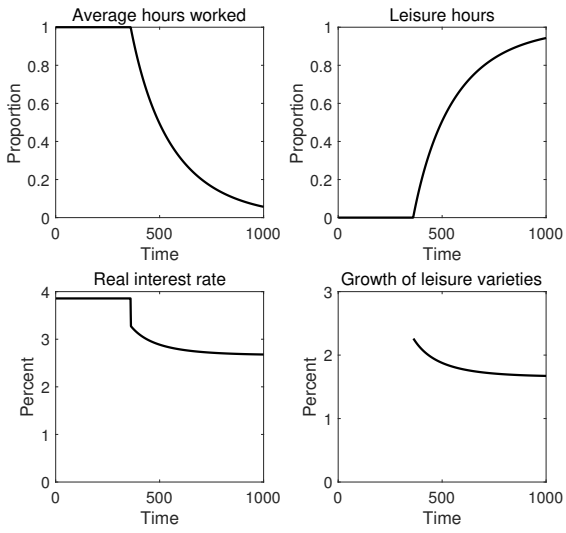


Figure 8: Some key variables on the sBGP

the findings above are possible. One is that the "watershed moment" came when the leisure sector first appeared, perhaps a century ago, when free newspapers and magazines became important and when the radio first became popular (early 1920s). The other focuses on the setup of the problem, and in particular on the feature that the *work-leisure margin is key*. In the model, improved leisure options crowd out working time, including time spent creatively on generating new ideas. According to this interpretation, the watershed moment corresponds more closely to the inventions of the internet and mobile technologies, which arguably made the competition between leisure and work much more direct and quantitatively important. In other words, improving TV services may have crowded out other leisure activities and housework, while the smartphone may be eating into work and creative hours, dragging on productivity growth as a result. Future work may usefully enrich the framework to provide more concrete quantitative answers to these issues.

6.3 Measurement

To further illustrate the measurement challenges posed by the leisure economy, Figure 9 compares the two measures of GDP along the balanced growth equilibrium defined above: $GDP_{measured}$ and GDP_{model}. The key result is that the growth of GDP_{model} declines with the emergence of the leisure sector, although that decline is not as large as slowdown in GDP as currently measured. This shows that the slowdown is driven by more than mis-measurement

alone. Why, even when valued at labor's hourly marginal product, do the leisure services not compensate for the lost output in the traditional sector of the economy? The reason is implicit in Proposition (2): leisure innovations are ultimately driven by the overall size of the traditional economy. A decline in the productivity growth in the traditional sector translates into weaker demand for marketing activities; the corresponding leisure innovations are worth less than the productivity decline.

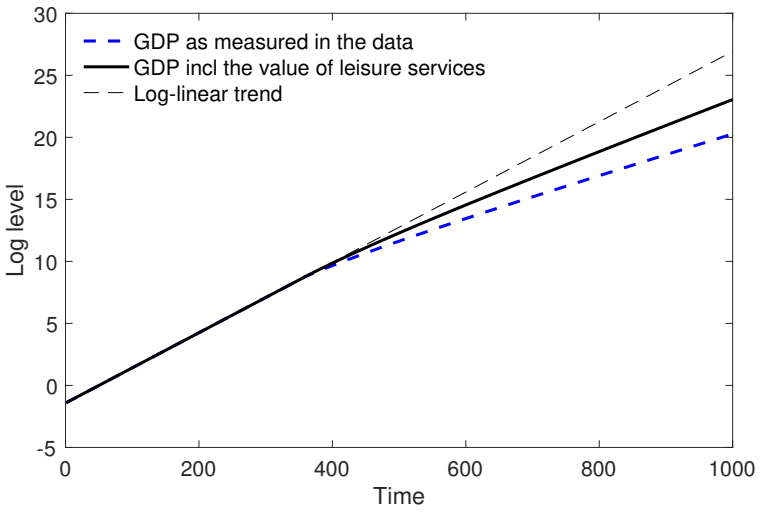


Figure 9: Level of two measures of GDP on the sBGP

Figure 10 plots the growth rates of GDP and of hourly and per-head productivity along the sBGP. Each subplot contains two metrics, analogous to those in Figure 9. Along the transition, a gap opens up between the two measures: the economy produces leisure varieties that are not captured in measured statistics. In fact, if one tries to include the value of leisure services in the statistics, and assuming that hours worked are measured correctly, the growth rate of GDP *per hour* actually shoots up. The intuition behind this result is straightforward: the measure $\frac{GDP_{measured}+V_{leisure}}{Hours\ Worked}$ rises "on impact" as the denominator declines and the numerator is little changed. In the long-run, however, the decline in the growth of TFP dominates, and we see the decline across all three growth rates in Figure 10.

Overall, then, the model provides a novel yet nuanced contribution to the productivity debate highlighted in the Introduction. The rapid leisure-enhancing technological change may be one reason why productivity has slowed down. The model also predicts that the drag on productivity growth may be a permanent feature of the modern economy.[16]

[16]Modern technologies benefit the economy through multiple channels that are completely absent from the model, which may well ultimately prevail and lead to higher productivity and prosperity.

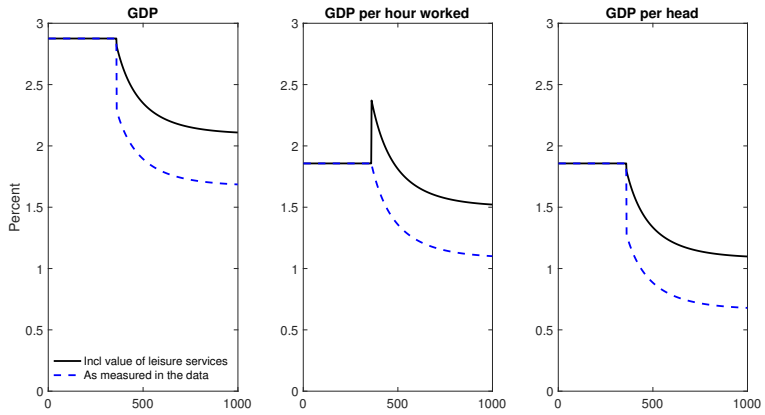


Figure 10: Growth rates of GDP and productivity

7 Welfare

Leisure-enhancing technological change introduces novel welfare considerations to the market equilibrium.[17] There are two new inefficiencies. The *static inefficiency* arises because the supply of leisure services is only driven by the profit-maximizing platform that benefits from this activity only indirectly. This means that the equilibrium level of supply may be suboptimally low. The *dynamic inefficiency* results from the adverse effects of leisure consumption today on future productivity growth, and suggests that there may be too much free leisure.

7.1 Static inefficiency

To illustrate the static inefficiency, consider a simplified static setup, where A is an exogenous constant and we normalize $N = 1$. The planner maximizes household's utility subject to the resource-, time endowment-, technological and non-negativity constraints:[18]

$$\max_{c,X,X_M,H_L} \log(c) + l \tag{25}$$

[17] In addition to the inefficiencies familiar from the literature on optimal growth: the presence of monopolistic competition and externalities to R&D.

[18] For simplicity, I assume that planner only cares about the working household.

subject to:

$$\begin{aligned} c + X_M + AX &= \int_0^A x(i)^\alpha (1 - H_L)^{1-\alpha} di \\ M &= X_M \\ x(i) &= X \\ l &= H_L M^{\frac{1}{\nu-1}} \\ 0 &\leq X_M, X, M \end{aligned}$$

Proposition 7. Socially optimal allocation in the static setting. *$B = 0$ is consistent with socially optimal allocation. In such allocation, leisure hours solve:*

$$H_L^{planner} = \arg\max_{H_L} \left(\log\left(A\hat{\alpha}(1 - \frac{\nu}{\nu - 1} H_L) \right) + H_L^{\frac{\nu}{\nu-1}} \left(\frac{A\hat{\alpha}}{\nu - 1} \right)^{\frac{1}{\nu-1}} \right)$$

where $\hat{\alpha} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$ is a constant.

Proof. See Appendix B. □

Advertising per se is socially useless in this model, so the planner provides leisure services directly, and not as a by-product of marketing activity. Analytical characterization of the planner's optimal allocation is not feasible, but the problem is easily solved numerically, using the parametrization outlined in Section 6. Figure 11 illustrates the objective of the planner as a function of leisure hours for different values of A. The black line traces out the optimal choice of leisure hours. When A is small, the optimal allocation is at the corner; for A large enough, the choice of leisure is increasing in A. Thus the supply of leisure services in the socially optimal allocation is qualitatively similar to the market allocation. The difference is quantitative: to illustrate, in decentralized equilibrium the marketing sector emerges when $A > 22,000$, whereas the planner provides positive supply of leisure services at a much lower level of A (Figure (11)). The key takeaway from the static setting is that there is under-provision of leisure services in market equilibrium.

The economics underlying the static inefficiency is not new: a similar mechanism was identified by Spence and Owen (1977), who study the television market. They highlight that the marginal cost of supplying a TV program to an additional consumer is zero, and thus efficiency calls for a zero price. Under the ad-supported business model, the pricing is indeed efficient. The authors note, however, that *"the program may not be supplied unless revenues cover the cost of producing it, a cost that is independent of the number of viewers.*

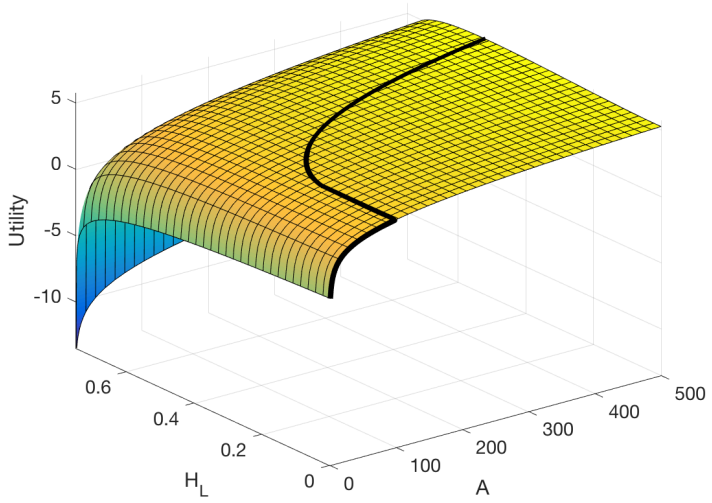


Figure 11: Planner's objective as a function of H_L for different values of A, and the optimal allocation

The revenue under advertiser support comes from advertisers who pay a price of roughly two cents per viewer per hour of prime time. The issue with respect to program selection then is whether two cents is a reasonable estimate of the average value of the program to the viewers of it. If it is not, then revenues may understate the social value of the program, and some programs with a potential positive surplus may not be profitable." This is a very similar mechanism to the one identified above: the leisure sector may not be active if the revenues from providing marketing do not compensate for the cost of supplying leisure services.

7.2 Dynamic inefficiency

The static setting offers only partial answers to the welfare implications of the leisure enhancing technological progress. A major distortion in the full dynamic model is that household's choices of leisure hours have a long-term impact on the on the growth rate of productivity. This effect is external to the household's decision – and this externality results in a inefficiency of the market equilibrium when innovation is endogenous.

To see this explicitly, consider the planner's optimal control problem. For the purposes of this problem, it is useful to make a simplifying assumption that s is given. The problem is:

$$\max_{\{c,H_W\}_{t=0}^{\infty}} \int_0^{\infty} e^{-(\rho-n)t} \left(\log(c) + l\right) dt$$

subject to

$$\begin{aligned} Y &= C + X + X_M \\ l &= H_L M^{\frac{1}{\nu-1}} \\ M &= X_M \\ 1 &= H_W + H_L \\ \dot{A} &= \zeta A^{\phi} L_A^{\lambda} \\ L_A &= s H_W N, \quad s \in [0,1] \end{aligned}$$

and C, X, X_B are all weakly positive.

There are two control variables of this problem – consumption per capita c and hours worked H_W. The stock of knowledge A is the state variable. The solution may be interior: in particular, there are two constraints that may bind in the optimal allocation: $X_B \geq 0$ and $H_W \leq 1$. Let the Lagrange multipliers on these constraints be ψ_1 and ψ_2, respectively and let μ denote the costate. We can then set up the following Hamiltonian and Lagrangian:

$$\mathcal{H} = \log(c) + (1 - H_W) X_B^{\frac{1}{\nu-1}} + \mu \left(\zeta A^{\phi} (s H_W N)^{\lambda}\right)$$

$$\mathcal{L} = \mathcal{H} + \psi_1 X_B + \psi_2 (1 - H_W).$$

The first order necessary condition with respect to hours worked is:

$$\underbrace{c^{-1} A (1-s) \hat{\alpha}}_{\text{static marginal cost of leisure}} + \underbrace{\mu \zeta \lambda A^{\phi} (s H_W N)^{\lambda - 1} s}_{\text{dynamic marginal cost of leisure}} - \psi_2 = \underbrace{X_M^{\frac{1}{\nu-1}}}_{\text{marginal benefit of leisure}} \tag{26}$$

This condition equates the marginal benefit from decreasing hours worked on the right, with the cost on the left-hand side. The cost comprises of two components: first, as usual, lowering hours worked today immediately lowers the resources available for consumption – this is the static cost of leisure; second, lowering hours worked today diminishes productivity growth going forward (this is the dynamic term). Relative to the decentralized allocation, the planner takes into account the dynamic cost of leisure, which translates into lower leisure services in the optimal allocation, relative to the decentralized equilibrium.

8 Conclusion

This paper put forward the idea that leisure-enhancing technologies, ubiquitous today and present over the past century, are key to understand the growth process. Distinguishing between the traditional productivity-enhancing innovations and the leisure-enhancing technological progress can shed new light on some of the unanswered questions and puzzles in applied theoretical work and in policy. In particular, the theory that features leisure enhancing technologies can simultaneously account for the trends in hours worked, interest rates and productivity growth. The framework also highlights the important measurement challenges and the non-trivial welfare implications of the leisure sector.

The model is relatively straightforward and the implications are transparent given the closed-form analytical solution. The framework developed here may be used in the future to extend the analysis in several interesting directions.

It may be useful to recast the mechanisms highlighted here in alternative growth modeling frameworks, for instance in the spirit of quality innovations as in Grossman and Helpman (1991) and Aghion and Howitt (1992) or in the learning and search framework of Lucas and Moll (2014). Furthermore, the model presented here was kept particularly parsimonious, especially when parametrizing production functions of the marketing platform, and future research may consider more detailed analysis of the functional forms and the key parameters. Also, the analysis rested on the assumption that leisure services are provided free of charge; future work could generalize the problem of the platform to explicitly model the provision of partly subsidized services, which could also help bring the model more directly to the data.

An open question is how the rise of the leisure sector interacts with heterogeneity, both at the household and at firm level. On the households side, it is interesting to study how time allocation decisions interact with income and wealth inequality. For example, disaggregated evidence on time allocation across the income distribution shows that poor individuals increased their leisure more than the rich (Boppart and Ngai, 2016). Allowing for household and income heterogeneity in the presence of leisure-enhancing technologies could bring out new insights and aid the debate on leisure-inequality. Considering firm heterogeneity may be important, too: the current setting is well suited to analyze equilibrium outcomes when heterogenous firms compete not only in prices but also through marketing activities. More productive firms may devote more resources to brand building, cementing their market share, with interesting implications for market power (De Loecker and Eeckhout, 2017).

A richer analysis of normative implications of leisure-enhancing technologies could be useful. The welfare analysis in the paper is based on the revealed-preference argument. In

reality, the issue is likely to be more complex. For example, recent research from psychology documents adverse effects that the recent technologies have on well-being (Verduyn et al., 2017). Over and above that, the current paper takes a neutral stance on the welfare impact of marketing activities, while in reality both positive and negative channels of influence arise. Marketing activities may provide consumers with valuable information or may be complementary with consumption, or could detract from utility if ad overload and privacy issues are a concern.

Finally, the framework built here can help answer some of the policy questions, such as optimal taxation of the providers of free leisure services. I plan to pursue these ideas in future work.

References

Acemoglu, D. (2002). Directed Technical Change. *Review of Economic Studies*, 69(4):781–809.

Acemoglu, D. and Guerrieri, V. (2008). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy*, 116(3).

Adler, G., Duval, R., Furceri, D., Kiliç Çelik, S., Koloskova, K., and Poplawski-Ribeiro, M. (2017). Gone with the Headwinds: Global Productivity.

Aghion, P., Akcigit, U., and Howitt, P. (2014). What Do We Learn From Schumpeterian Growth Theory? *Handbook of Economic Growth*, 2:515–563.

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., and Li, H. (2017a). Missing Growth from Creative Destruction. *Working Paper Series*.

Aghion, P. and Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, 60(2):323–351.

Aghion, P. and Howitt, P. (2009). *Stages of Growth BT - The Economics of Growth*. Number 10. MIT Press.

Aghion, P., Jones, B., and Jones, C. (2017b). Artificial Intelligence and Economic Growth.

Aguiar, M., Bils, M., Charles, K. K., and Hurst, E. (2017). Leisure Luxuries and the Labor Supply of Young Men. Technical report, National Bureau of Economic Research, Cambridge, MA.

Aguiar, M. and Hurst, E. (2007a). Measuring trends in leisure: the allocation of time over five decades. *The Quarterly Journal of Economics*, 122(3):969–1006.

Aguiar, M. and Hurst, E. (2007b). Measuring Trends in Leisure: The Allocation of Time Over Five Decades. *The Quarterly Journal of Economics*, 122(3):969–1006.

Aguiar, M. and Hurst, E. (2016). The Macroeconomics of Time Allocation. In *Handbook of Macroeconomics*, volume 2, pages 1–44.

Anderson, S. P. and Renault, R. (2006). Advertising Content. *American Economic Review*, 96 (1).

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics*, 00(3):668–691.

Arrieta Ibarra, I., Goff, L., Jimenez Hernandez, D., Lanier, J., and Weyl, E. G. (2018). Should We Treat Data as Labor? *American Economic Review, Papers and Proceedings*.

Bagwell, K. (2007). The economic analysis of advertising. *Handbook of Industrial Organisation*.

Bean, C. (2016). Independent Review of UK Economic Statistics. Technical report.

Becker, G. S. (1965). A Theory of the Allocation of Time. *The Economic Journal*, 75(299):493–517.

Bloom, N., Jones, C. I., Reenen, J. V., and Webb, M. (2017a). Are Ideas Getting Harder to Find?

Bloom, N., Jones, C. I., Van Reenen, J., and Webb, M. (2017b). Are Ideas Getting Harder to Find?

Boik, A., Greenstein, S., and Prince, J. (2016). The Empirical Economics of Online Attention.

Boppart, T. and Krusell, P. (2016). Labor Supply in the Past, Present, and Future: A Balance-Growth Perspective.

Boppart, T. and Ngai, L. R. (2016). Rising inequality and trends in leisure *.

Bridgman, B. (2016). Is Productivity on Vacation? The Impact of the Digital Economy on the Value of Leisure.

Broda, C. and Weinstein, D. E. (2006). Globalization and the Gains From Variety. *The Quarterly Journal of Economics*, 121(2):541–585.

Brynjolfsson, E., Eggers, F., and Gannamaneni, A. (2017a). Using Massive Online Choice Experiments to Measure Changes in Well-being.

Brynjolfsson, E. and McAfee, A. (2014). *The second machine age : work, progress, and prosperity in a time of brilliant technologies*. W. W. Norton & Company, 2014.

Brynjolfsson, E. and Oh, J. H. (2012). The attention economy: Measuring the value of free digital services on the internet. *International Conference on Information Systems, ICIS 2012*, 4:3243–3261.

Brynjolfsson, E., Rock, D., and Syverson, C. (2017b). Artificial Intelligence and the Modern Productivity Paradox : A Clash of Expectations and Statistics.

Byrne, D. M., Fernald, J. G., and Reinsdorf, M. B. (2016). Does the United States have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*.

Chamberlin, E. (1933). *The theory of monopolistic competition*. Harvard University Press.

Christensen, M. A., Bettencourt, L., Kaye, L., Moturu, S. T., Nguyen, K. T., Olgin, J. E., Pletcher, M. J., and Marcus, G. M. (2016). Direct Measurements of Smartphone Screen-Time: Relationships with Demographics and Sleep.

De Loecker, J. and Eeckhout, J. (2017). The Rise of Market Power and the Macroeconomic Implications.

DellaVigna, S. and Gentzkow, M. (2010). Persuasion: Empirical Evidence. *Annual Review of Economics*, 2(1):643–669.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, 67(3):297–308.

Falck, O., Gold, R., and Heblich, S. (2014). E-lections: Voting behavior and the internet. *American Economic Review*, 104(7):2238–2265.

Gentzkow, M. and Shapiro, J. M. (2008). Preschool Television Viewing and Adolescent Test Scores: Historical Evidence from the Coleman Study. *Quarterly Journal of Economics*, 123(1):279–323.

Goolsbee, A. and Klenow, P. J. (2006). Valuing consumer products by the time spent using them: An application to the Internet. In *American Economic Review*, volume 96, pages 108–113.

Gordon, R. J. (2016). Perspectives on the rise and fall of American growth. In *American Economic Review*, volume 106, pages 72–76.

Grossman, G. M. and Helpman, E. (1991). Quality Ladders in the Theory of Growth. *The Review of Economic Studies*, 58(1):43.

Huberman, M. and Minns, C. (2007). The times they are not changin': Days and hours of work in Old and New Worlds, 1870-2000. *Explorations in Economic History*, 44(4):538–567.

Jones, C. I. (1995). R&D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759–784.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles. *Journal of Monetary Economics*, 21(2-3):195–232.

King, R. G., Plosser, C. I., and Rebelo, S. T. (2002). Production, Growth and Business Cycles: Technical Appendix. *Computational Economics*, 20(1/2):87–116.

Kortum, S. (1997). Research, Patenting, and Technological Change. *Econometrica*, 65(6):1389–1419.

Lewis, R. A. and Rao, J. M. (2015). The unfavorable economics of measuring the returns to advertising. *Quarterly Journal of Economics*, 130(4):1941–1973.

Lewis, R. A. and Reiley, D. H. (2014). Online ads and offline sales: Measuring the effect of retail advertising via a controlled experiment on Yahoo! *Quantitative Marketing and Economics*, 12(3):235–266.

Lucas, R. E. and Moll, B. (2014). Knowledge Growth and the Allocation of Time. *Journal of Political Economy*, 122(1).

Marshall, A. (1890). *Principles of Economics*. MacMillan and Co. Ltd, London, eighth edi edition.

Nakamura, L., Samuels, J., and Soloveichik, R. (2017). Measuring the "Free" Digital Economy within the GDP and Productivity Accounts.

Ngai, L. R. and Pissarides, C. A. (2008). Trends in hours and economic growth. *Review of Economic Dynamics*, 11(2):239–256.

Nixon, D. (2017). Is the economy suffering from the crisis of attention?

Predrag Bejakovic, P. B. (2018). *Economics for the Common Good*, volume 42.

Rachel, L. and Smith, T. D. (2015). Secular drivers of the global real interest rate.

Ramey, V. A. and Francis, N. (2009). A century of work and leisure. *American Economic Journal: Macroeconomics*, 1(2):189–224.

Reis, F. (2015). Patterns of Substitution between Internet and Television in the Era of Media Streaming-Evidence from a Randomized Experiment.

Rochet, J.-C. and Tirole, J. (2003). Platform Competition in two-sided markets. *Journal of the european economic association 1 (4), 990-1029*.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy*, 98(5):S71–S102.

Segerstrom, P. S. (1998). Endogenous Growth without Scale Effects. *American Economic Review*, 88(5):1290–1310.

Spence, M. and Owen, B. (1977). Television Programming, Monopolistic Competition and Welfare. *Quarterly Journal of Economics*, 91(1):103–126.

Summers, L. H. (2015). Demand side secular stagnation. *American Economic Review*, 105(5):60–65.

Syverson, C. (2016). Challenges to mismeasurement explanations for the U.S. productivity slowdown.

Terranova, T. (2012). Attention, economy and the brain.

Varian, H. (2017). Artificial Intelligence, Economics, and Industrial Organization.

Verduyn, P., Ybarra, O., Leuven, K., Jonides, J., and Kross, E. (2017). Do Social Network Sites Enhance or Undermine Subjective Well-Being? A Critical Review. *Social Issues and Policy Review*, 11(1):274–302.

Wallsten, S. (2013). What are we not doing when were online.

Ward, A. F., Duke, K., Gneezy, A., and Bos, M. W. (2017). Brain Drain: The Mere Presence of One's Own Smartphone Reduces Available Cognitive Capacity. *Journal of the Association for Consumer Research*, 2(2):140–154.

Appendices

A Illustrative evidence

Material in this Appendix further motivates the focus of this paper and is a useful background to the analysis. I discuss the supply of leisure-enhancing innovations as well as the evidence on changing time allocation patterns.

Evidence of leisure-enhancing innovations

Figure 1 made the point that the availability of zero-price services aimed at extracting peoples' time and attention is not a new phenomenon; instead, leisure technologies have been around for at least a century. Nonetheless, the amount of free content grew significantly faster than GDP over the recent past, chiefly due to the rise of digital content. This recent trend can be observed across a wide range of indicators. For example, Figure 12 shows mobile phone apps available on the two main mobile platforms. There are now more than 3 million apps on either of the platforms, and the majority of those are available free of charge to the consumer (Figure 13).

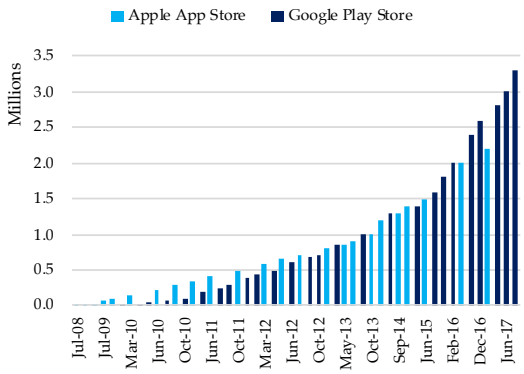


Figure 12: Number of apps worldwide
Source: Statista, Android, Google, Apple.

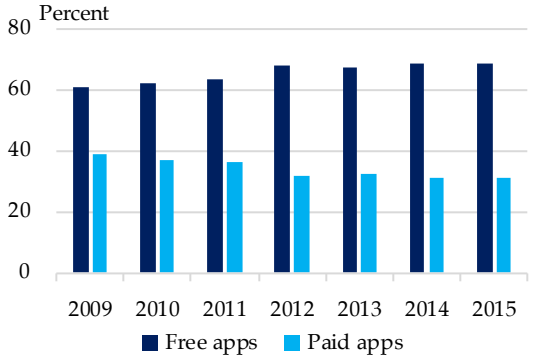


Figure 13: Free and paid apps offered in the Google Play Store
Source: Statista.

Consistent with this rapid growth of new leisure services, the leisure sector appears to be an increasingly important driver of the overall R&D spending. No explicit measure for the R&D share of the leisure economy exists; but it is possible to construct proxies by looking

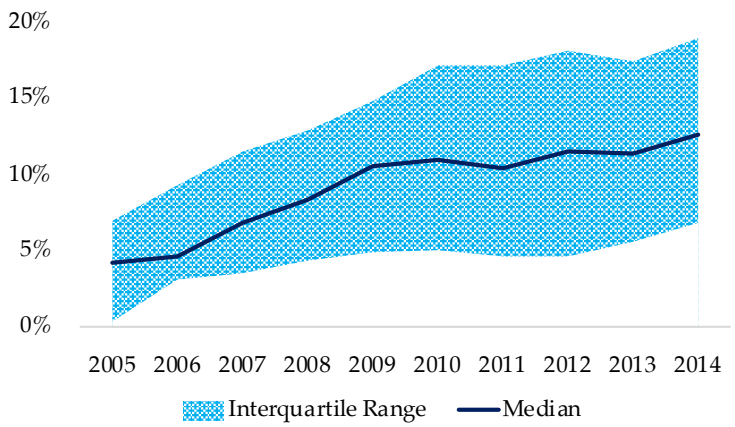


Figure 14: R&D expenditure share of the (proxy for the) leisure economy
Source: OECD. Includes data for an unbalanced panel of 39 countries. The figure shows the median and the interquartile range of the country-level share of R&D spending in the following sectors: publishing; motion picture, video and television program production; sound recording; programming and broadcasting activities; telecommunications services; computer programming, consultancy and related activities; information service activities; data processing, hosting and related activities; and web portals.

at the industries most likely engaged in the leisure-enhancing innovations. Figure 14 shows that the share of R&D spending accounted for by the sectors in the leisure economy such as video and TV program production, sound recording, broadcasting and web portals has been increasing rapidly.

Recent changes in time allocation patterns

There is some indicative evidence that the increased leisure time is, in part, substituting for time spent at work. A recent study by Christensen et al. (2016) measured smartphone screen time over the course of an average day among a sample of 653 people (Figure 15). Time spent on the phone averaged 1 hour and 29 minutes per day, corroborating the order of magnitude in the Nielsen 2016 data. Crucially, the mobile phone usage appears to be uniformly distributed throughout the day, suggesting that leisure time is, in part, substituting for time spent working. In a different study, Wallsten (2013) uses time use surveys to estimate that each minute spent on the internet is associated with loss of work-time of about 20 seconds.

Indeed, one feature of the latest technology is that it allowed leisure to "compete" with work much more directly than has been the case in the past. While it may not have been possible to watch TV at work, online entertainment is available during the work hours. The theory in this paper focuses on the work-leisure margin, which may thus be particularly relevant for the digital technologies of the past decade and going forward.

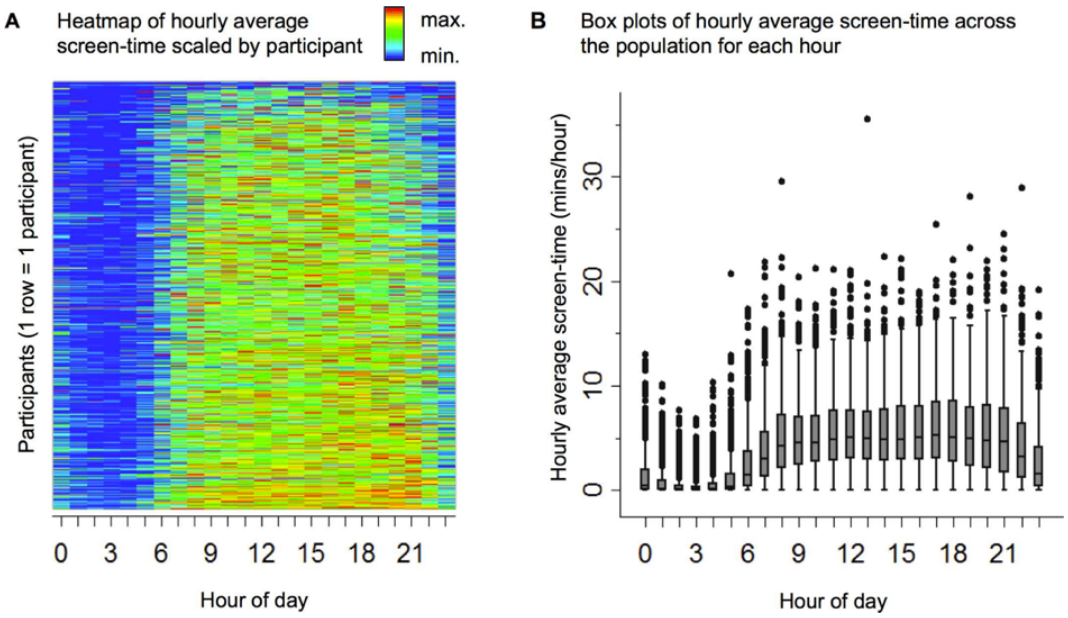


Figure 15: Mobile phone use over the course of an average day

Source: Christensen et al. (2016).

B Proofs and derivations

Proof of Lemma 1

Proof. The love of varieties implies that households like to diversify their use of leisure time. By definition, the total amount of marketable leisure time is given by $H_L \equiv \int_0^M h(j)dj$. Recall that for each activity j, household can access any amount of services $m(j)$ at no cost: these are free of charge, non-rival and non-excludable. Naturally then, availability of free services will never be constraining households leisure production, so that $m(j) \geq h(j) \forall j$. Consequently, we can simply write:

$$l = \left(\int_0^M h(j)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} dj. \tag{27}$$

Households choose how much time to devote to each individual activity to maximize production of leisure for any given amount of total leisure time. Because of the symmetry of this problem, it is immediate that the optimal choice is to spread free time evenly across the

M available leisure options: $h(j) = h \ \forall j$; and therefore

$$H_L \equiv \int_0^M h(j)dj = Mh \Rightarrow h = \frac{H_L}{M}.$$

Plugging this back into the simplified production function for leisure (27) yields the result. □

Proof of Proposition 1

Proof. Suppose $\psi \neq \xi$. Denoting the net growth rate of a variable by $\gamma_{\cdot}(t)$, equation (12) implies that

$$\gamma_{H_W}(t) = (\xi - \psi)\gamma_{H_L}(t) + \frac{1-\xi}{1-\nu}\gamma_M(t).$$

Balanced growth requires that $\gamma_{H_W}(t)$ and $\gamma_M(t)$ are constant, which implies $\gamma_{H_L}(t)$ is not constant, a contradiction. □

Proof of Proposition 2

Proof. The prove this Proposition, I first derive the optimality conditions of the different firms in the economy.

The retail sector.

The maximization problem of the retailer is:

$$\max_{x(i),L_Y} \int_0^A \left(\left(\frac{b(i)}{\bar{B}} \right)^{\chi} x(i) \right)^{\alpha} L_Y^{1-\alpha} di - \int_0^A x(i)q(i)di - wL_Y \tag{28}$$

The first order conditions yields the standard labour demand equation and the inverse demand curve for the product of firm i:

$$\alpha \left(\frac{b(i)}{\bar{B}} \right)^{\alpha\chi} x(i)^{\alpha-1} L_Y^{1-\alpha} = q(i) \tag{29}$$

where $q(i)$ is the price of variety i of the consumer good. Equation (29) shows that higher marketing expenditures of producer i, holding marketing of all other firms constant, shifts the demand curve out (for any given price $q(i)$, firm i can sell a higher quantity of its product).

Consumer sector.

Intermediate producers maximize profit, subject to the linear technology (recall that $x(i) = X$) and demand curve in (29). Each firm can also buy marketing at a given price p_B. In effect, the problem of each producer is:

$$\max_{x(i),b(i)} \alpha \left(\frac{b(i)}{\bar{B}}\right)^{\alpha\chi} x(i)^{\alpha} L_Y^{1-\alpha} - x(i) - p_B b(i). \tag{30}$$

The first order conditions to this problem yield the standard result that, for each producer, price equals mark-up over marginal cost ($q(i) = q = \frac{1}{\alpha}$). Plugging this back into equation (29), the quantity of each good produced in equilibrium is given by

$$x(i) = \alpha^{\frac{2}{1-\alpha}} L_Y. \tag{31}$$

The FOCs also yield the inverse demand curve for marketing:

$$p_B = \alpha^{\frac{2}{1-\alpha}} \chi \frac{1}{b(i)} \left(\frac{b(i)}{\bar{B}}\right)^{\frac{\alpha}{1-\alpha}\chi} L_Y. \tag{32}$$

Marketing platform.

The platform chooses the quantity of marketing supplied to maximize profits, subject to the demand curve for marketing services (32) and to the consolidated production function (16). Given the cost function 17, the profit maximization can thus be written as

$$\max_{b(i)} \int_0^A b(i) \cdot p_B di - \left[\int_0^A b(i) di\right]^{\nu-1}$$

Plugging in the inverse demand curve yields the problem (10) in the main text. The first order condition is

$$\kappa \frac{1}{\bar{B}} L_Y - \left[\int_0^A b(i) di\right]^{\nu-2} = 0$$

where $\kappa = \frac{\alpha}{1-\alpha}\chi^2 \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\nu-1})$ is a constant defined in the main text. This uses the fact that in a symmetric equilibrium $b(i) = b = \bar{B}$. Because $\bar{B} = \frac{B}{A}$ and $B = \int_0^A b(i) di$, we have:

$$\kappa \frac{A}{B} L_Y - B^{\nu-2} = 0.$$

Rearranging gives

$$B = (\kappa A L_Y)^{\frac{1}{\nu-1}} \tag{33}$$

Next, combining (33) and (16) gives the expression for the inputs used by the platform:

$$X_M = \kappa A L_Y. \tag{34}$$

Labor market clearing implies:

$$L_Y = (1-s)H_W N. \tag{35}$$

We thus have:

$$X_M = \kappa A(1-s)H_W N.$$

Combined with equations (13) and (15), this implies that

$$X_M = \kappa A(1-s)X_M^{\frac{1}{1-\nu}} N$$

when $X_M > 1$. This is the case when

$$(\kappa A(1-s)N)^{\frac{\nu-1}{\nu}} > 1$$

or equivalently

$$(1-s)\cdot A \cdot N > \frac{1}{\kappa}.$$

This is the condition for the platform to optimally choose the scale of its operation above the minimum i. provides the condition The platform will operate if its profits are positive. To verify this, note that by equation (10), when $X_M > 1$, equilibrium profits of the platform are:

$$\pi_B = p_B B - X_M = \left(\alpha^{\frac{2}{1-\alpha}}\chi - \kappa\right) A L_Y = \alpha^{\frac{2}{1-\alpha}}\chi\left(1 - \frac{\alpha}{1-\alpha}\chi\frac{1}{\nu-1}\right) A L_Y.$$

which is positive as long as $\frac{\alpha}{1-\alpha}\chi\frac{1}{\nu-1} < 1$, which I assume is satisfied (which will be the case under any reasonable set of parameter values). □

Proof of Proposition 3

Proof. The first part of the Proposition follows from the fact that the left hand side of (18) is a growing quantity (as there is positive population growth) while the right hand side is a constant.

Consider the section of the BGP where the condition (18) does not hold. Proposition 2 implies that the platform is inactive: $\mathcal{O} = 0$. It is then straightforward to see that the economy is identical to the model economy studied in Jones (1995). In that economy, the

measure of consumer goods varieties A, wages, output and consumption per capita all grow at the same constant rate given by (19). The proof is available in Jones (1995) and is omitted for brevity.

To prove the final part of the proposition, note first that the solution to the household's intratemporal optimization problem implies that average hours worked are a power function of the number of leisure varieties (recall equation (13)). Thus the growth rate of average hours must be proportional to the growth rate of varieties of leisure if the two variables grow at constant rates, so that:[19]

$$\gamma_{H_W} = \frac{1}{1-\nu}\gamma_M \tag{36}$$

Next, equation (15) implies that the M and X_M grow at the same rate in equilibrium. Equation (16) implies that this rate is given by:

$$\gamma_{X_M} = \gamma_M = \frac{\nu - 1}{\nu}(\gamma_A + n) \tag{37}$$

Combining (37) and (38) gives the expression for the growth rate of average hours worked as a function of γ_A:

$$\gamma_{H_W} = -\frac{1}{\nu}(\gamma_A + n) \tag{38}$$

We have:

$$\gamma_A = \frac{\zeta L_A^{\lambda}}{A^{1-\phi}} \tag{39}$$

On the segmented balanced growth path, the left-hand side of this equation is asymptotically constant by definition. That means that the numerator and the denominator of the fraction on the right-hand side must grow at the same rate. More formally, using the fact that $L_A = H_W \cdot N \cdot s$, differentiating (39) with respect to time gives and solving for γ yields:

$$\gamma_A = \frac{\lambda(\gamma_H + n)}{1-\phi} \tag{40}$$

Combining (38) and (40) and solving for γ yields

$$\gamma_A = \frac{n\lambda\left(\frac{\nu-1}{\nu}\right)}{1-\phi+\left(\frac{\lambda}{\nu}\right)}$$

The fact that $\gamma_A < g$ follows immediately from the comparison of the two expressions.

[19]Note that, for variables X and Y, if $Y = X^a$ then gross growth rates satisfy $\zeta_Y = \zeta_X^a$ and net growth rates satisfy $\gamma_Y = a\gamma_X$.

The asymptotic rate of decline in hours worked is:

$$\gamma_{H_W} = -\frac{1}{\nu}\left(\frac{n\lambda\left(\frac{\nu-1}{\nu}\right)}{1-\phi+\left(\frac{\lambda}{\nu}\right)}+n\right) \tag{41}$$

□

Proof of Proposition 4

Proof. On the sBGP, household budget constraint and the no-Ponzi condition imply that consumption can grow as fast as income, so that:

$$\gamma_w + \gamma_{H_W} = \gamma_c \tag{42}$$

where γ_w is the growth rate of hourly wages. Optimal labor demand in the final good sector implies that wages satisfy:

$$w = (1-\alpha)\frac{Y}{L_Y}. \tag{43}$$

Substituting (31) into the final good production function (9) and imposing the equilibrium outcome that all firms' marketing activities are equal yields the following formula for equilibrium output:

$$Y = \alpha^{\frac{2\alpha}{1-\alpha}} A L_Y \tag{44}$$

Plugging back into equation (43) yields the expression for the equilibrium wage:

$$w = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A. \tag{45}$$

Wages increase in line with A : $\gamma_w = \gamma_A$. Combining formulas (20), (41) and (42) yields:

$$\gamma_c = \frac{\lambda n\left(\frac{\nu-1}{\nu}\right)^2}{1-\phi+\left(\frac{\lambda}{\nu}\right)} - \frac{n}{\nu} < g_c.$$

which is lower than in the economy with no leisure-enhancing technological change. Direct comparison of (20) and (21) and the fact that, when $t < \hat{t}$, $g_c = g_A = \frac{\lambda n}{1-\phi}$ imply that the growth rate of consumption declines by more than the growth rate of knowledge. The condition for positive consumption growth in the presence of leisure enhancing technology is

given by:

$$\frac{\lambda n \left(\frac{\nu-1}{\nu}\right)^2}{1-\phi+\left(\frac{\lambda}{\nu}\right)} > \frac{n}{\nu}$$

Simplifying, this implies

$$\nu > 2 + \frac{1-\phi}{\lambda},$$

that is, the strength of the love-for-variety effect needs not be too strong. In the calibration in the main text, the right-side of this condition is equal to around 2.5, so that with $\nu > 2.5$, there will be positive asymptotic growth in consumption per capita along the sBGP.

To derive the Euler equation of the household formally, set up the Hamiltonian and take the optimality conditions, utilizing the Maximum Principle. The Hamiltonian is:

$$\mathcal{H}(d, c, H_L, \mu) = \log(c) + l + \mu \left(d(r-n) + w(1-H_L) - c\right)$$

And the associated optimality conditions are:

$$\begin{aligned} \mathcal{H}_c = 0 &\Rightarrow \frac{1}{c} - \mu = 0 \\ \mathcal{H}_{H_W} = 0 &\Rightarrow M^{\frac{1}{\nu-1}} - \mu w = 0 \\ \mathcal{H}_d = -\dot{\mu} + (\rho - n)\mu &\Rightarrow \mu(r-n) = -\dot{\mu} + (\rho - n)\mu \end{aligned} \tag{46}$$

The transversality condition is:

$$\lim_{t\to\infty} \left[d(t) \cdot \exp\left(-\int_0^t (r(s) - n) ds \right) \right] = 0$$

Standard manipulations imply

$$\gamma_c = r - \rho. \tag{47}$$

To find the share of workers employed in the R&D sector, consider the problem of the R&D producers, who choose how much labour to employ in order to maximize profits:

$$\max_{L_A} \dot{A} P_A - w L_A$$

where P_A is the price of a patent which gives the holder of the patent a perpetual right to produce that particular variety. The first order condition of this problem is:

$$P_A = \frac{w}{\zeta A^{\phi} {L_A}^{\lambda-1}}. \tag{48}$$

A consumer firm is willing to purchase a patent as long as the return from doing so exceeds the required rate of return on investment, r, meaning that the following no arbitrage condition holds in equilibrium:

$$\frac{\Pi}{P_A} + \frac{\dot{P}_A}{P_A} = r \tag{49}$$

The optimal choice of quantities and prices by the monopolistically competitive firms results in equilibrium profits given by:

$$\Pi(i) = \left(\frac{b(i)}{\bar{B}}\right)^{\frac{\alpha}{1-\alpha}\chi} \left[\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 - \chi\right)\right] L_Y. \tag{50}$$

In equilibrium,

$$\Pi(i) = \Pi = \left[\alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 - \chi\right)\right] L_Y \ \forall i$$

Profits are positive if $\alpha < \frac{1}{1+\chi}$, a condition that restricts the importance of marketing. Intuitively, if marketing is perceived to be very effective (χ is high), firms would purchase so much of it that it would make them unprofitable. Firms would then exit the market. This is clearly not consistent with the notion of equilibrium or indeed with the observation of the real world: while marketing expenditure does raise overall costs of running a company, firms stop short of the point where ad spending eats up their entire profits. I therefore assume that the condition $\alpha < \frac{1}{1+\chi}$ is satisfied throughout the paper; the illustrative parametrization below shows that this assumption is not restrictive.

This condition has a standard interpretation: the flow return of purchasing a patent (the dividend plus the capital gain) must equal the rate of interest.

Note that the price of a patent grows at the same rate as population: $\frac{\dot{P}_A}{P_A} = n$. To see this, note that the price of a patent will reflect the stream of profits generated by variety i, discounted by the effective discount rate, which will be the market interest rate r minus growth rate of population n:

$$P_A = \int_t^{\infty} e^{-(r-n)s} \Pi ds = \frac{\Pi}{r-n} \tag{51}$$

Combining this with the no-arbitrage condition above yields the result.

The share s can be calculated from the free-entry to R&D, which implies that profits in

the R&D sector must be zero in equilibrium. Thus:

$$wL_A = P_A \dot{A} \tag{52}$$

Substituting in for w using (43) and for P_A using the no-arbitrage condition (51), and noting that $\dot{A} = A\gamma$ gives:

$$(1-\alpha)\frac{Y}{L_Y} \cdot L_A = \frac{A\Pi}{r-n} \cdot \gamma$$

By definition, $s = L_A/(L_Y + L_A)$. By (50) we have that $A\Pi = \alpha^{\frac{2}{1-\alpha}} A L_Y \left(\frac{1}{\alpha} - 1 - \chi\right)$ when $t > \hat{t}$. Output is $Y = \alpha^{\frac{2\alpha}{1-\alpha}} A L_Y = \alpha^{\frac{2}{1-\alpha}-2} A L_Y = \frac{1}{\alpha^2}\alpha^{\frac{2}{1-\alpha}} A L_Y$ so that $A\Pi = Y\alpha^2\left(\frac{1}{\alpha} - 1 - \chi\right) = Y\alpha(1-\alpha-\alpha\chi)$. Putting those together gives:

$$\frac{s}{1-s}(1-\alpha)Y = \frac{\gamma}{r-n} \cdot \alpha(1-\alpha-\alpha\chi)Y$$

Solving for s gives:

$$s|_{t\to\infty} = \left(1 + \frac{1-\alpha}{\alpha(1-\alpha-\alpha\chi)}\left(\frac{\rho+\gamma_c-n}{\gamma}\right)\right)^{-1} \tag{53}$$

Similarly, for $t < \hat{t}$, we have:

$$s|_{t\leq\hat{t}} = \left(1 + \frac{1}{\alpha}\left(\frac{\rho+g_A-n}{g_A}\right)\right)^{-1} \tag{54}$$

Comparison of (53) and (54) implies the effect of leisure technologies on the share of workers in R&D is ambiguous. □

Proof of Proposition 5

Proof. For $t < \hat{t}$, $X_M = 0$ as the platform is inactive. Thus $GDP_{measured} = Y - \int_0^A x(i)di$. Equations (31) and (44), and the labor market clearing condition $L_Y = (1-s)N$ imply that:

$$GDP_{measured} = AN(1-s)\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right). \tag{55}$$

For $t \geq \hat{t}$, using equations (31), (44) and (34) we have:

$$GDP_{measured} = Y - \int_0^A x(i)di - X_M = AL_Y\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} - \kappa\right). \tag{56}$$

Equations (55) and (56) show that the level of measured GDP declines on impact by $\kappa A L_Y$, as resources are used in the production of leisure and marketing services.

The labor market clearing for $t \geq \hat{t}$, given by equation (35), together with equations (13) and (34) imply that

$$L_Y = (1-s)(\kappa A L_Y)^{\frac{1}{1-\nu}} N,$$

thus

$$L_Y = (\kappa A)^{-\frac{1}{\nu}} \left((1-s)N\right)^{\frac{\nu-1}{\nu}}.$$

Plugging this into equation (56) yields:

$$GDP_{measured} = (AN)^{\frac{\nu-1}{\nu}} (1-s)^{\frac{\nu-1}{\nu}} \kappa^{-\frac{1}{\nu}} \left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} - \kappa\right). \tag{57}$$

The stationary growth rate follows by differentiating expressions (55) and (57) with respect to time, noting that s is constant. The growth of measured GDP per capita is equal to $\frac{\nu-1}{\nu}\gamma_A - \frac{n}{\nu}$, which by (21) is equal to γ_c. □

Proof of Proposition 6

Proof. The first part of the Proposition can be easily proved by contradiction. Suppose that $B^{planner} > 0$ in the socially optimal allocation. This implies $X_M^{planner} > 0$. It is now possible to increase utility by lowering X_M and raising X such that the resource constraint is satisfied – thus the initial allocation cannot be socially optimal.

To prove the second part, note that, by definition:

$$V_{leisure} := N \cdot w \cdot \int_0^M m(i)di \tag{58}$$

In equilibrium, we have $m(i) = h(i)$, and so

$$V_{leisure} = N \cdot w \cdot \int_0^M h(i)di = N \cdot w \cdot H_L.$$

Equations (13), (34), (15) and (35) imply that

$$1 - H_L = (\kappa A(1-s)(1-H_L)N)^{\frac{1}{1-\nu}}.$$

Rearranging yields to the following expression for equilibrium leisure hours:

$$H_L = 1 - (\kappa A(1-s)N)^{-\frac{1}{\nu}}. \tag{59}$$

Combining equations (58) and (59) and substituting for equilibrium wages from equation (45) gives:

$$V_{leisure} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} AN\left(1 - (\kappa A(1-s)N)^{-\frac{1}{\nu}}\right)$$

□

Proof of Proposition 7

Proof. It is convenient to first separately analyze the problem of choosing the optimal quantity of each differentiated product $x(i)$. After noting that, in the static model, the total labor supply is just the hours worked of the representative household ($L_Y = H_W$), the problem amounts to the following maximization:

$$\max_{x(i)} \int_0^A x(i)^\alpha H_W^{(1-\alpha)} di - \int_0^A x(i) di.$$

Optimal $x(i)$ thus satisfies:

$$x(i) = \alpha^{\frac{1}{1-\alpha}} H_W \ \forall i, \tag{60}$$

which gives the expression for gross output in the optimal allocation:[20]

$$Y = A\alpha^{\frac{\alpha}{1-\alpha}} H_W.$$

Substituting this back into the objective function yields the maximization in two variables only:

$$\max_{X_M, H_L} \log\left((1-H_L)A\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) - X_M\right) + H_L X_M^{\frac{1}{\nu-1}}. \tag{61}$$

Let $\hat{\alpha} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$. The main difficulty of this problem is that it may well be optimal for the planner to choose a corner solution in either of the two choice variables. *If* the solution was interior, it *would* satisfy the two first order conditions:

[20] Note that in the decentralized setting, the ratio between $X \equiv Ax$ and Y is α^2 , while in the planner's problem it is α, meaning that the in the market allocation too little of each of the consumer good is produced. This is a well-known result due to market power in the the monopolistically competitive consumer goods sector.

$$H_L : \frac{1}{c}\left[A\hat{\alpha}\right] = X_B^{\frac{1}{\nu-1}}$$
$$X_B : \frac{1}{c} = \frac{1}{\nu-1} H_L X_B^{\frac{2-\nu}{\nu-1}}$$

Dividing one by the other yields the result that, in any interior optimal solution, H_L and X_B are proportional to each other:

$$X_B = \frac{1}{\nu-1} A\hat{\alpha} H_L \tag{62}$$

Equation (62) is a ray through the origin in the (H_L, X_B) space. Thus the corner solution $(H_L, X_B) = (0, 0)$ also satisfies (62), implying that equation (62) is satisfied at all times, whether at a corner or in an interior solution. It is thus possible to further reduce the dimensionality of the problem, by plugging equation (62) into the objective function. This finally yields a maximization problem in a single choice variable, H_L:

$$\max_{H_L} \; \log\left(A\hat{\alpha}(1 - \frac{\nu}{\nu-1} H_L)\right) + H_L^{\frac{\nu}{\nu-1}} \left(\frac{A\hat{\alpha}}{\nu-1}\right)^{\frac{1}{\nu-1}}. \tag{63}$$

Figure (11) in the main text characterizes the solution to this problem. □

C Leisure innovations are produced with labor

Instead of production function in equation (15), suppose leisure varieties are produced one-for-one with labor:

$$F^M = L_M.$$

We now have the following consolidated production function of the platform:

$$B = L_M^{\frac{1}{\nu-1}}$$

Thus the cost of producing B units of marketing services is given by $c(B) = B^{\nu-1} \cdot w$. The profit maximization of the platform is

$$\max_{b(i)} \int_0^A b(i) \cdot p_B di - \left[\int_0^A b(i) di\right]^{\nu-1} \cdot w$$

where $p_B = \alpha^{\frac{2}{1-\alpha}} \chi \frac{1}{b(i)} \left(\frac{b(i)}{B}\right)^{\frac{\alpha}{1-\alpha}\chi} L_Y$.

The first order condition of the platform is

$$\alpha^{\frac{2}{1-\alpha}}\frac{\alpha}{1-\alpha}\chi^2\frac{1}{\bar{B}}L_Y-(\nu-1)w\left[\int_0^A b(i)di\right]^{\nu-2}=0$$

which, using the fact that $\bar{B}=\frac{B}{A}$ and $B=\int_0^A b(i)di$, becomes

$$\kappa\frac{A}{B}L_Y-wB^{\nu-2}=0.$$

Rearranging gives

$$B=\left(\frac{\kappa AL_Y}{w}\right)^{\frac{1}{\nu-1}}$$

And thus

$$L_M=B^{\nu-1}=\frac{\kappa AL_Y}{w}$$

So scientists employed in the leisure sector are a constant proportion of the workers in the final good industry.

Wages are given by the optimality condition in the retail sector:

$$w=(1-\alpha)\frac{Y}{L_Y}=(1-\alpha)\frac{\alpha^{\frac{2\alpha}{1-\alpha}}AL_Y}{L_Y}=(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}A$$

Therefore

$$L_M=\left(\frac{\alpha}{1-\alpha}\right)^2\chi^2\frac{\alpha}{\nu-1}L_Y$$

The labor market clearing condition is

$$L_Y+L_M+L_A=H_WN.$$

The free entry condition to R&D ensures zero profit condition holds:

$$wL_A=P_A\dot{A} \tag{64}$$

Plugging in for the wage from the FOC of the retailer, and for the price of a patent from the no-arbitrage condition, and noting that $\dot{A}=A\gamma$ gives:

$$(1-\alpha)\frac{Y}{L_Y}\cdot L_A=\frac{\Pi}{r-n}\cdot A\gamma$$

Define $s = L_A/L_Y$. We have:

$$s(1-\alpha)Y = \frac{\gamma}{r-n} \cdot \alpha(1-\alpha-\alpha\chi)Y$$

thus

$$s = \frac{\frac{\gamma}{r-n} \cdot \alpha(1-\alpha-\alpha\chi)}{1-\alpha}$$

We thus have the following expressions for labor employed in the three industries:

$$\begin{aligned}
L_M &= \left(\frac{\alpha}{1-\alpha}\right)^2 \chi^2 \frac{\alpha}{\nu-1} L_Y \\
L_A &= \frac{\frac{\gamma}{r-n} \cdot \alpha(1-\alpha-\alpha\chi)}{1-\alpha} L_Y \\
L_Y &= H_W N \left(1 - \left(\frac{\alpha}{1-\alpha}\right)^2 \chi^2 \frac{\alpha}{\nu-1} - \frac{\frac{\gamma}{r-n} \cdot \alpha(1-\alpha-\alpha\chi)}{1-\alpha}\right)
\end{aligned}$$

These imply that the growth rates satisfy:

$$\gamma_M = \gamma_L = \gamma_H + n$$

and therefore

$$\gamma_H = \frac{1}{1-\nu}(\gamma_H + n)$$

which implies

$$\gamma_H = -\frac{n}{\nu}.$$

This finally yields:

$$\gamma_A = \frac{\lambda\left(n - \frac{n}{\nu}\right)}{1-\phi}.$$

This shows that with this alternative assumption the growth rate of the economy declines along the sBGP.