The long-run effects of monetary policy*

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Abstract

A well-worn tenet holds that monetary policy does not affect the long-run productive capacity of the economy. Merging data from two new international historical databases, we find this not to be quite right. Using the trilemma of international finance, we find that exogenous variation in monetary policy affects capital accumulation, and to a lesser extent, total factor productivity, thereby impacting output for a much longer period of time than is customarily assumed. We build a quantitative medium-scale DSGE model with endogenous TFP growth to understand the mechanisms at work. Following a monetary shock, lower output temporarily slows down TFP growth. Internal propagation of the monetary shock extends the slow down in productivity, and eventually lowers trend output. Yet the model replicates conventional textbook results in other dimensions. Monetary policy can have long-run effects.

JEL classification codes: E01, E30, E32, E44, E47, E51, F33, F42, F44

Keywords: monetary policy, interest rates, money neutrality, productivity, hysteresis, trilemma, instrumental variables.

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> When coin is in greater plenty, as a greater quantity of it is required to represent the same quantity of goods, it can have no effect either good or bad, taking a nation within itself, any more than it would make an alteration in a merchant's books; if instead of the Arabian method of notation, which requires few characters, he should make use of the Roman, which requires a great many.
>
> — *Of Money* in Hume (1758)

1. Introduction

Money has long been believed to be neutral in the long-run, as David Hume noted, and as a voluminous literature on monetary economics has argued more formally with rare exceptions.[1] However, the evidence is less clear cut than the elegant logic of the predominant theories would suggest. Barro (2013) provides evidence that high levels of inflation result in a loss in the *rate* of economic growth. Work by Caballero, Hoshi, and Kashyap (2008) and Gopinath, Kalemli-Özcan, Karkarabounis, and Villegas-Sánchez (2017) links interest rates to the *level* of productivity, whereas more recently, Benigno and Fornaro (2018) and Liu, Mian, and Sufi (2018) link low interest rates with the rate of *growth* of productivity. Our paper follows in this more recent tradition and sets out to investigate the link between monetary policy, the growth rate of productivity, and the growth rate of output in the medium to long term.

In monetary economics, most theories embrace the assumption that an exogenous shock to interest rates has only transitory effects on prices and economic activity (see, e.g., Christiano, Eichenbaum, and Evans, 1999; Ramey, 2016; Coibion, Gorodnichenko, and Ulate, 2017). However, recent research by Jordà, Schularick, and Taylor (2019) suggests that previous measures of monetary shocks might have been endogenous responses to the outlook. They introduce a new instrumental variable based on the trilemma of international finance (see, e.g., Obstfeld, Shambaugh, and Taylor, 2004, 2005; Shambaugh, 2004) to document that monetary shocks have larger and more persistent effects than previously measured, and closer in magnitude to measures obtained with narrative shocks (Romer and Romer, 2004) and market-based, high-frequency identified shocks (Gertler and Karadi, 2015). Moreover, they also document that the effects of monetary policy are state-dependent, extending similar results reported in Tenreyro and Thwaites (2016), Angrist, Jordà, and Kuersteiner (2018), and Barnichon and Matthes (2018).

Using the same instrument as Jordà, Schularick, and Taylor (2019) we show, using historical data since 1870 on 17 advanced economies, that monetary shocks have very long-lasting effects on output and productivity. In particular, we augment the data in Jordà, Schularick, and Taylor (2017) and available at `www.macrohistory.net/data`, with data on total factor productivity (TFP) and factor

[1] See, for example, the discussion of Sidrauski's work by Fischer (1983).

input from Bergeaud, Cette, and Lecat (2016) and available at http://www.longtermproductivity.com.[2]

We investigate this proposition from a variety of angles, by correcting for spillover effects, considering different samples, and examining state dependence as a function of the business cycle, the level of inflation, and importantly, credit. We also examine the soundness of our findings to alternative definitions of monetary shocks based on Romer and Romer (2004) for the postwar United States. The main result is robust to all these modifications. We then look under the hood to examine through what channels is monetary policy is exacting these changes on output growth. We find that the total hours worked response is relatively stable and small. Thus, the main effects on output growth that we uncover appear to come primarily through the responses of real capital accumulation, and to a lesser extent, of utilization adjusted TFP. These results echo the important early findings by Evans (1992) who cast doubt on the exogeneity of TFP shocks, a key tenet of mainstream DSGE models, finding that money, interest rates, and government spending Granger-caused these impulses.

Motivated by these findings, we build a quantitative medium-scale DSGE model with endogenous TFP growth to understand the mechanisms at work. TFP growth deviates from the exogenous trend whenever output is different from potential output. A contractionary monetary policy shock lowers output temporarily producing a slowdown in TFP growth. Under a standard Taylor rule, the temporary slowdown in TFP growth accumulates to yield permanently lower trend levels of output and capital. The model can generate this medium run effect on GDP while replicating the conventional textbook results, namely the utilization rates of labor and capital fall temporarily, and capital to TFP ratio exhibits a hump-shaped response. In our empirical analysis, we corroborate these conventional results (Christiano, Eichenbaum, and Evans, 2005) for the historical sample.

Our paper is related to the seminal work by Cerra and Saxena (2008) who documented that output losses in the aftermath of economic and political crises in low-income and emerging economies are highly persistent. More recently, researchers have also documented similar effects following financial crises for advanced economies (see Martin, Munyan, and Wilson, 2015; Haltmaier, 2013). Fatás and Summers (2018) document strong hysteresis effects of fiscal consolidations for advanced economies following the global financial crisis. We document a causal effect of monetary policy shocks in a historical panel data for advanced economies.

There is mixed evidence of long-run non-neutrality proposition in monetary economics. In the structural VAR literature, Bernanke and Mihov (1998) reject long-run neutrality in their identified responses to monetary policy shocks since the impulse responses are statistically indistinguishable from zero. Mankiw (2001) however interprets the non-recovery of GDP to zero in Bernanke and Mihov (1998)'s estimates as evidence of long-run non-neutrality.

Our paper is also related to the recent literature that emphasizes slow recovery following the Great Recession due to endogenous productivity growth (Anzoategui, Comin, Gertler, and Martinez,

[2]We are particularly thankful to Antonin Bergeaud for sharing some of the disaggregated series from their database that we used to construct our own series of adjusted TFP.

2018; Bianchi, Kung, and Morales, 2019), labor force participation (Erceg and Levin, 2014; Galí, 2016), or skill depletion (Kiyotaki and Zhang, 2017). This literature has its origins in the seminal work by Stadler (1990) who developed a business cycle model with endogenous technology and sticky wages to generate a persistent effect of monetary shocks on output.[3] Of these recent papers, Moran and Queraltó (2018) uses three equation VAR models to emphasize an empirical link between TFP growth and monetary policy shocks. We differ in two important ways. One, we investigate and establish that monetary policy shocks can affect level of GDP, capital stock as well as TFP in a panel of 17 countries spanning more than a hundred years. Two, we use externally identified shocks instead of ordering restrictions to identify monetary policy shocks. Our analysis further provides answer to questions raised by policymakers including Chair Yellen (2016) recently.[4] Aggregate demand shocks, particulary monetary policy shocks, do indeed have long-lasting effects on the level of output, capital stock and TFP in the economy.

2. Data and series construction

The empirical features motivating our analysis rest on two major international and historical databases. Data on macro aggregates and financial variables, including assumptions on exchange rate regimes and capital controls can be found in `www.macrohistory.net/data`. This database covers 17 advanced economies reaching back to 1870 at annual frequency. Detailed descriptions of the sources of the variables contained therein, their properties, and other ancillary information are discussed in Jordà, Schularick, and Taylor (2017) and Jordà, Schularick, and Taylor (2019), as well as references therein. Importantly, we will rely on the trilemma instrument discussed in Jordà, Schularick, and Taylor (2016), and more recently, Jordà, Schularick, and Taylor (2019) as the source of exogenous variation in interest rates. We briefly describe the instrument construction below but refer the reader to the last reference for a detailed analysis.

The second important source of data relies on the work by Bergeaud, Cette, and Lecat (2016) and available at `http://www.longtermproductivity.com`. This historical database adds to our main database observations on capital stock (machines and buildings), hours worked, and number of employees, and the Solow residuals (raw TFP). In addition, we construct time-varying capital and labor utilization corrected series using the procedure discussed in Imbs (1999) with the raw data from Bergeaud, Cette, and Lecat (2016) to construct our own series of utilization-adjusted TFP. We went back to the original sources so as to filter out cyclical variation in input utilization rates in the context of a richer production function that allows for factor hoarding. We explain the details of this correction below.

[3] See also Fatas (2000), and Barlevy (2004).

[4] "*The first question I would like to pose concerns the distinction between aggregate supply and aggregate demand: Are there circumstances in which changes in aggregate demand can have an appreciable, persistent effect on aggregate supply? ... More research is needed, however, to better understand the influence of movements in aggregate demand on aggregate supply.*"

2.1. Instrument construction

Jordà, Schularick, and Taylor (2019) developed a quasi-natural experiment utilizing the theory of trilemma in international finance: a country with fixed nominal exchange rates and open capital flows loses monetary independence. Variations in the base country interest rates can be used as an instrumental variable to identify exogenous monetary shocks to an open peg country. The sample is split into three group of countries, *bases* whose currency serve as the anchor for the second group of pegging economies, labeled as the*pegs*, and *floats* are the final group of countries that allow their currency to be freely determined in the market.

A country i is defined to be in a peg $q_{i,t} = 1$ at date t if it maintained a peg at dates $t-1$ and t in order to eliminate opportunistic pegging due to country's monetary conditions. In order to construct the instrument, changes in short term nominal interest rate in the base country are first sterilized to remove the component that can be explained by economic conditions in that country. The resulting variation is adjusted for capital openness. More formally: let $\Delta r_{i,t}$ denote the changes in short term interest rate in country i at date t, $\Delta r_{b(i,t),t}$ be the change in short term interest rate in country i's base country $b(i,t)$ at time t and $\Delta \hat{r}_{b(i,t),t}$ denote the predictable component of variations in base rate. Recognizing that countries are not perfectly open to capital flows, adjustment for capital mobility is constructed using the continuous version $k_{i,t} \in [0,1]$ of capital openness index of Quinn, Schindler, and Toyoda (2011). The resulting instrument is $z_{i,t} \equiv k_{i,t}(\Delta r_{b(i,t),t} - \Delta \hat{r}_{b(i,t),t})$.[5]

2.2. Factor utilization correction

The Imbs (1999) correction follows Burnside and Eichenbaum (1996) in endogenizing the capital utilization rate in a partial equilibrium model. We assume perfectly competitive factor markets and an aggregate production function which is constant returns to scale in effective capital and labor:[6]

$$Y_t = A_t \left(K_t u_t\right)^{\alpha} \left(L_t e_t\right)^{1-\alpha}$$

where Y_t is output, K_t is capital stock, and L_t is total hours worked. u_t and e_t denote the respective factor utilizations. A_t is the utilization adjusted TFP. We assume perfect competition in the input and the output markets. Higher capital utilization increases the depreciation of capital $\delta_t = \delta u_t^{\phi}$ where $\phi > 1$. As a result, firms choose capital utilization rate optimally. Labor hoarding is calculated assuming instantaneous adjustment of effort e_t against a payment of a higher wage $w(e_t)$, while keeping fixed employment (determined one period in advance). The firm's optimization problem is given by:

$$\max_{e_t, u_t, K_t} A_t \left(K_t u_t\right)^{\alpha} \left(L_t e_t\right)^{1-\alpha} - w(e_t) L_t - (r_t + \delta u_t^{\phi}))K_t$$

[5] See Jordà, Schularick, and Taylor (2019) for implementation details.

[6] We assume that these variables are stationary. In section 4, we construct capital-utilization in a general equilibrium model along with endogenous growth.

Households choose consumption, labor supply and effort to maximize their lifetime utility subject to their budget constraint (with complete asset markets)

$$\max_{c_t, L_t, e_t} \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \chi \frac{(e_t L_t)^{1+\nu}}{1+\nu} \right]$$

Normalizing the long-run capital-utilization and labor-utilization rates to one, the utilization rates can be derived from following :

$$u_t = \left(\frac{\frac{Y_t}{K_t}}{\frac{Y}{K}} \right)^{\frac{\delta}{r+\delta}}; \quad e_t = \left(\frac{\frac{Y_t}{C_t}}{\frac{Y}{C}} \right)^{\frac{1}{1+\nu}} \frac{L}{L_t}$$

where Y, C, L and K are the steady-state values of output, consumption, labor and capital.

The Solow residual then can be decomposed into utilization-adjusted TFP and utilization corrections:

$$TFP_t \equiv \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}} = A_t \times u_t^{\alpha} e_t^{1-\alpha}$$

To construct steady state values of Y, C, L and K, we extract a HP-filter trend from the data series. We will show later that our empirical results are robust to computing moving averages over a 10 year window, using time-varying values of α constructed from labor-income data, and reasonable parameters of the aggregate capital depreciation rate. Bergeaud, Cette, and Lecat (2016) constructed capital stock for machines and buildings separately using the perpetual inventory method with data on investment in machines and buildings and different depreciation rates. We will show robustness of our results to choosing different depreciation parameters. We wish to emphasize that our estimation of productivity assumes misallocation related-wedges are absent. We have not yet found the data to take into account markups in our productivity estimation. See Basu and Fernald (2002) and Syverson (2011) for extensive discussions on what determines productivity.

3. Monetary shocks have long lasting effects

3.1. Empirical approach

The basic empirical approach relies on local projections (Jordà, 2005) estimated with instrumental variables (LPIV). Several applications of these methods are available in the literature, though a more general discussion of the method can be found in Ramey (2016), Stock and Watson (2018) and Jordà, Schularick, and Taylor (2019). Based on the latter, we estimate in particular:

$$y_{i,t+h} = \alpha_i + \Delta r_{i,t} \beta_h + \boldsymbol{x}_{i,t} \gamma_h + v_{i,t+h}; \qquad h = 0, 1, \ldots, H; i = 1, \ldots, N; t = t_0, \ldots, T \tag{1}$$

where $y_{i,t+h}$ is the outcome variable for country i observed h periods from today; α_i are country fixed effects, $\Delta r_{i,t}$ refers to the change in the short-term government bond (3-months in duration), our

stand-in for the policy rate which we will instrument with $z_{i,t}$, the trilemma instrument borrowed from Jordà, Schularick, and Taylor (2019) as discussed earlier; and $\boldsymbol{x}_{i,t}$ collects all additional controls including lags of the outcome and interest rates, as well as lagged values of other macro aggregates. Moreover, we control for global business cycle effects through a global GDP control variable to parsimoniously soak up time effects. We estimate Equation 1 with instrumental variable methods and report cluster robust standard errors.

Table 1 reports the first-stage regression of the pegging country's short term interest rate $\Delta r_{i,t}$ on the instrument $z_{i,t}$ with controls $x_{i,t}$, country fixed effects and (robust) clustered standard errors. The t-statistic is well above 3 for full sample and post-WW2 samples illustrating that it is not a weak instrument. We refer the reader to Jordà, Schularick, and Taylor (2019) for detailed discussion on the instrument and proceed henceforth assuming the reader is on board regarding instrument relevance and strength.

Table 1: *Trilemma instrument: first stage evidence*

pegs ($q = 1$)	All years	PreWW2	PostWW2
$z_{i,t}$	0.52^{***}	0.35^{**}	0.56^{***}
t-statistic	[8.62]	[2.05]	[8.97]
Obs	672	148	524

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Full sample: $1890 - 2015$ excluding WW1: $1914 - 1919$ and WW2: $1939 - 1947$. Pre-WW2 sample: 1870-1938 (excluding 1914-1919). Post WW2 sample: $1948 - 2015$. These regressions include country fixed effects as well as up to two lags of the first difference in log real GDP, log real consumption, investment to GDP ratio, credit to GDP, short and long-term government rates, log real house prices, log real stock prices, and CPI inflation. In addition we include world GDP growth to capture global cycles. See text.

3.2. Main results

The main story in the paper can be illustrated with the response of real GDP to a shock to domestic interest rates using the trilemma instrument. Before we show the main results, we highlight the value of our instrumental variable by providing a comparison of the response of GDP per capita to a shock in the short-term domestic interest rate calculated using selection-on-observables identification versus identification with the trilemma instrument. This is shown in Table 2. The table reports the coefficient estimates of the impulse response calculated with each identification approach for the full and post-WW2 samples. Note that the label *LP-OLS* refers to identification via selection, *LP-IV* to the trilemma instrument identification. The samples are restricted to pegging economies to match the samples in both cases.[7]

Table 2 is organized as follows. We provide the coefficient estimates by row and provide a test of the null hypothesis that *LP-OLS* and *LP-IV* estimates are equivalent for the two samples considered:

[7]The plots and inference is robust to using real GDP per capita. See Table A1

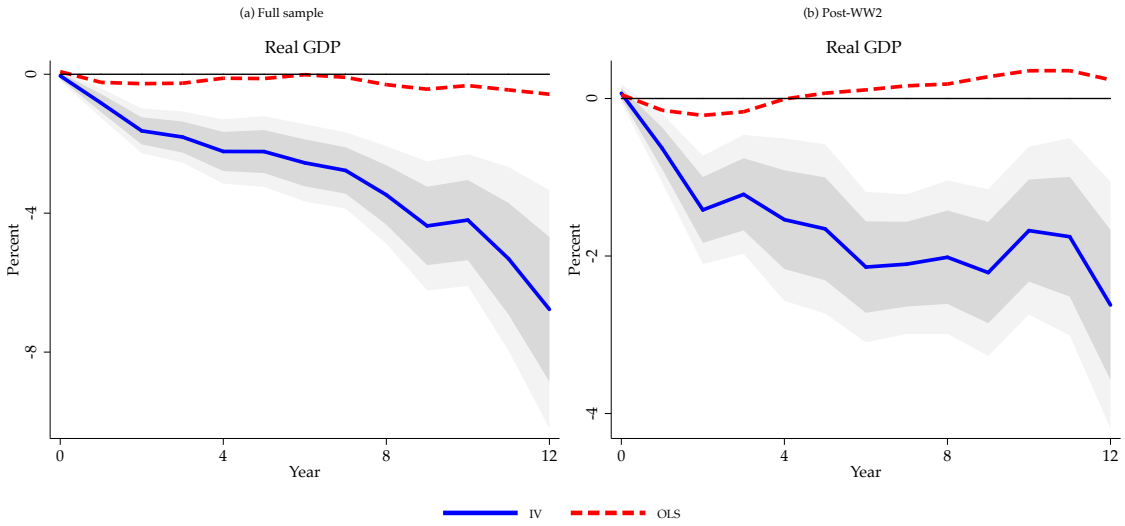
Table 2: *LP-OLS vs. LP-IV. Attenuation bias of real GDP responses to interest rates. Trilemma instrument. Matched samples*

Responses of real GDP at years 0 to 10 (100 $\times$ log change from year 0 baseline).

	(a) Full Sample		*OLS-IV*	*(b) Post-WW2*		*OLS-IV*
Year	LP-OLS	LP-IV	*p-value*	LP-OLS	LP-IV	*p-value*
	(1)	(2)	(3)	(4)	(5)	(6)
$h = 0$	0.08**	-0.04	0.18	0.05**	0.07	0.79
	(0.03)	(0.09)		(0.02)	(0.07)	
$h = 2$	-0.27	-1.63***	0.00	-0.21	-1.42***	0.00
	(0.16)	(0.39)		(0.13)	(0.42)	
$h = 4$	-0.11	-2.22***	0.00	-0.01	-1.54**	0.01
	(0.26)	(0.56)		(0.21)	(0.63)	
$h = 6$	-0.01	-2.55***	0.00	0.11	-2.14***	0.00
	(0.29)	(0.67)		(0.22)	(0.58)	
$h = 8$	-0.30	-3.47***	0.00	0.18	-2.02***	0.00
	(0.29)	(0.85)		(0.22)	(0.59)	
$h = 10$	-0.33	-4.20***	0.00	0.35	-1.68***	0.00
	(0.36)	(1.15)		(0.27)	(0.65)	
$h = 12$	-0.58	-6.77***	0.00	0.24	-2.62***	0.00
	(0.42)	(2.08)		(0.33)	(0.95)	
KP weak IV		68.34			69.18	
H_0: $LATE = 0$	0.00	0.00		0.00	0.00	
Observations	607	607		482	482	

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Cluster robust standard errors in parentheses.Full sample: $1890 - 2015$ excluding WW1: $1914 - 1919$ and WW2: $1939 - 1947$. Post WW2 sample: $1948 - 2015$. *Matched sample* indicates LP-OLS sample matches the sample used to obtain LP-IV estimates. *KP weak IV* refers to the Kleibergen-Paap test for weak instruments. $H_0 : subATE = 0$ refers to the p-value of the test of the null hypothesis that the coefficients for $h = 0, ..., 10$ are jointly zero for a given subpopulation. $OLS = IV$ shows the p-value for the Hausmann test of the null that OLS estimates equal IV estimates. See text.

Figure 1: *Baseline response to 100 bps trilemma shock: Real GDP*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. See text.

full and post-WW2. The differences between identification schemes could not be starker. Both are economically and statistically, but the *LP-IV* response is considerably larger at all horizons. We display these results graphically in Figure 1.

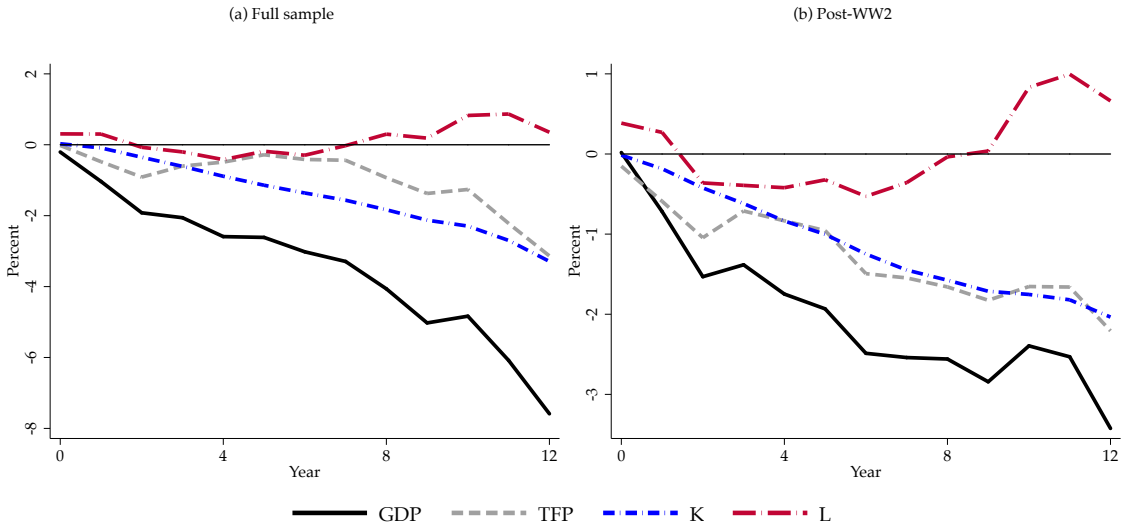
Regardless of the sample used, a 1 percentage point increase in domestic short-term interest rates have sizable and long-lasting effects on GDP. In the full sample, GDP declines by over 6 percent relative to period 0 over 12 years. The effect, while still large, is considerably more muted in the post-WW2 sample. The drop 12 years after impact is about half, at 2.7 percent. This is a far cry from traditional notions of long-run neutrality found in the literature.

What is the source of this persistent decline? We decompose the decline in GDP into its components, namely, total hours worked (that is, employees times number of hours per employee); capital stock (the sum of capital measured in machines and buildings); and utilization-adjusted total factor productivity (TFP). Using the Imbs (1999) correction, we decomposed the Solow residual (raw TFP from a Cobb-Douglas production function) into factor-utilization (capital and labor) and the subsequent residual is labeled as the utilization adjusted TFP.

Figure 2 displays the responses of each of these components to the same shock to the domestic short-term interest rate instrumented with the trilemma, both for the full and the post-WW2 samples. The figure displays the responses of total hours worked, capital and TFP adjusted without error bands to provide a clearer sense of the dynamic paths. A more detailed figure is provided in the Appendix, which includes the error bands.

Several features deserve mention. Figure 2 shows that there are similar declines in capital and utilization adjusted TFP whereas total hours worked exhibits a much flatter pattern. Because capital

Figure 2: *Baseline response to 100 bps trilemma shock: Real GDP and components*



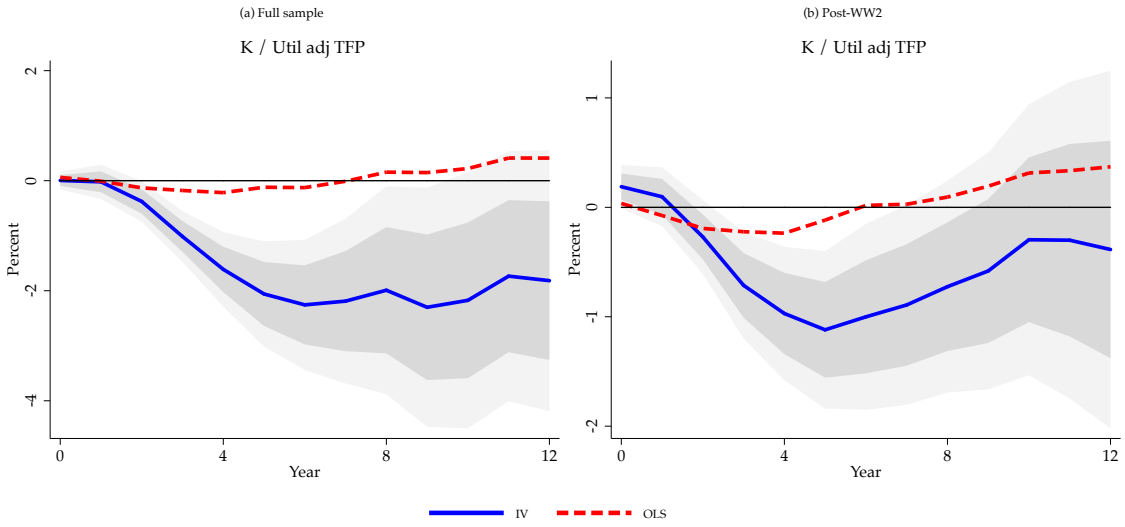
Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. See text.

enters the production function with a smaller weight, it should be clear from the figure that most of the decline in GDP is explained by utilization adjusted TFP, and then capital, with total hours worked mostly flat. To be sure, the response of hours worked conforms well with the textbook response to a monetary shock. Total hours fall in the short-run, but then recover quickly and remain mostly flat.

Capital accumulation also follows textbook dynamics in the short-run. The response is initially muted but builds up over time. But unlike the textbook model, capital does not appear to recover even 12 years after the shock. Similarly, utilization adjusted TFP appears to follow textbook dynamics: it falls gradually rather than suddenly. Over time, the decline in utilization adjusted TFP accelerates, ending at a level 3 (1) percent lower in full (post WW2) sample by year 12 relative to year 0. Figure 3 presents the evolution of capital to utilization adjusted TFP ratio in response to the trilemma shock. The hump-shaped response replicates the conventional justification for including investment adjustment costs in typical DSGE models (Christiano, Eichenbaum, and Evans, 2005).

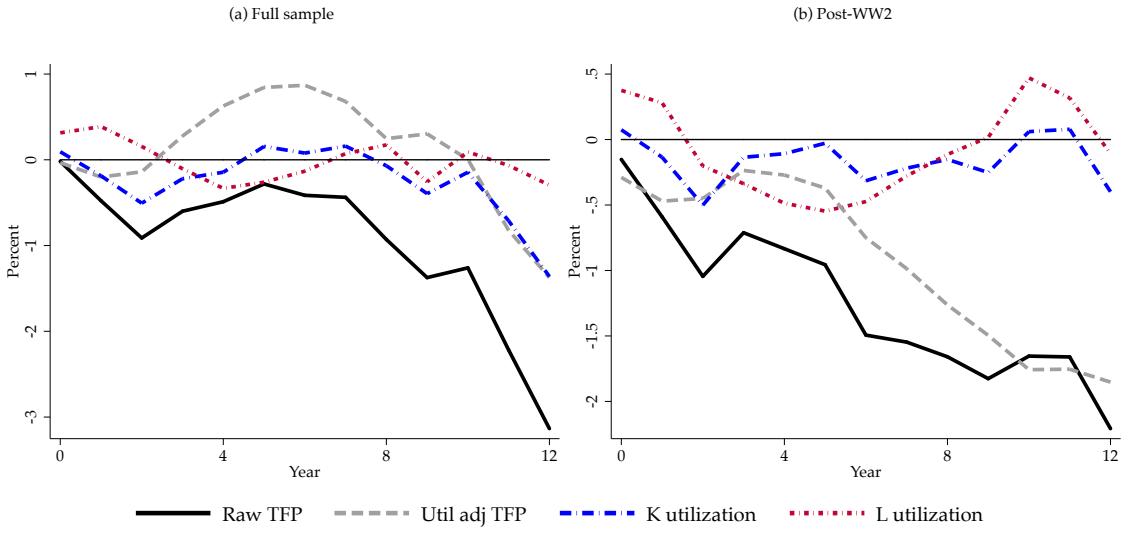
We present the responses of the various components of the Solow residual in Figure 4 for both the full and post-WW2 samples. Utilization rates of labor and capital exhibit cyclical dynamics: falling in the short-run and then recovering back to zero. In a sense, this pattern is mechanical. It reflects the restriction imposed on the calculation that utilization rates must return to zero eventually (Imbs, 1999). We later show the robustness of using a more commonly used utilization adjusted TFP series constructed by Fernald (2014) for the post-WW2 US, which requires more detailed data than is available for the historical panel of 17 countries. Reassuringly, the results for utilization adjusted TFP using the two methods are qualitatively similar.

Figure 3: *Baseline response to 100 bps trilemma shock: Capital to utilization adjusted TFP*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. See text.

Figure 4: *Baseline response to 100 bps trilemma shock: TFP and utilization*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. See text.

One explanation for the long-lasting effects of the monetary shock could be that domestic interest rates remain elevated for a long period of time as well. In other words, persistence is generated by a delayed response in interest rates. A simple check of this proposition can be done in two steps. Figure 5 shows that the short-term real interest rate does indeed take approximately 8 years to return to zero, while the nominal interest rate returns to zero after 4 years. The response of nominal interest rate is typical of what has been reported often in the literature (see, e.g., Christiano et al., 1999; Ramey, 2016). Secondly, we can calculate the responses of the main variables normalized by the response of interest rates over time to sterilize the dynamics of interest rates themselves. This is no different than calculating a multiplier (see, e.g., Ramey, 2016; Ramey and Zubairy, 2018).

Thus, in Figure 6 we show the ratio of the cumulative change in GDP to the area under the real interest rate path in Figure 5 In a given period, the difference in the level of real interest rate relative to the counterfactual path measures the tightness of monetary policy. We consider these cumulative gaps as a measure of overall monetary policy tightness (the area under the solid line in Figure 5). By year 12, the multiplier is -1 in the full sample and -0.8 in the post-WW2 sample, virtually identical and sizable.[8]

3.3. Is the US different?

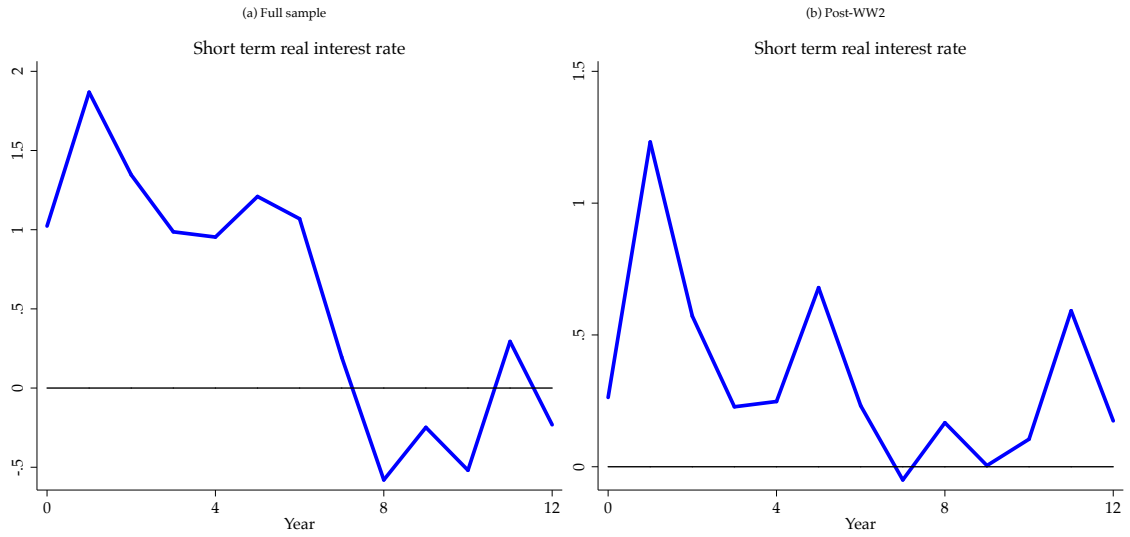
The US for most of our historical sample is not a pegging economy (with the exception of the Gold standard years before WW1). It is the quintessential base country for many economies in our sample. For this reason, the trilemma instrument mechanically sets aside any information coming from the US economy during estimation. It is natural to wonder the extent to which US data conforms with the patterns presented so far.

In this section, we examine US data post-WW2. This allows us to incorporate three useful modifications. First, we use higher frequency quarterly data. Second, we use the alternative utilization adjusted series for TFP constructed by Fernald (2014). Third, in order to achieve identification we rely on the instrumental variable constructed by Romer and Romer (2004) based on the Federal Reserve staff's implied forecast errors for the policy rate, and extended to recent years by Wieland and Yang (2016). It turns out that results based on U.S. data largely confirm the results we reported in the previous section for non-U.S. pegging economies.

Figure 7 plots the the path of real GDP along with its three components: total hours worked, capital, and utilization adjusted TFP. The responses are qualitatively similar to those in the long-run panel, although the amplitudes are more muted. Quarterly data is naturally noisier than yearly data, but smoothing over a temporary recovery in GDP 4 to 5 years after the shock, real GDP ends nearly one percent lower 8 years after impact. Utilization adjusted TFP and hours exhibit a similar u-shaped pattern, with TFP nearly back to zero by year 8. Strikingly, capital accumulation exhibits a protracted decline over the entire period, ending about 1.25 percent lower after eight years. For

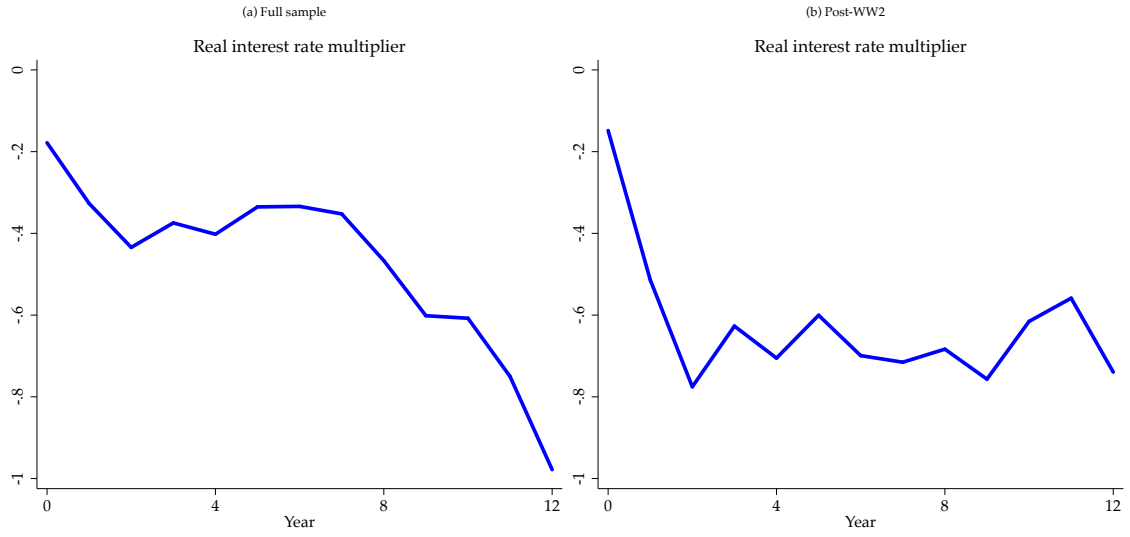
[8]For real GDP per capita, the multiplier is -1.4 in the full sample and -1.3 in the post-WW2 sample by year 12.

Figure 5: *Baseline response to 100 bps trilemma shock: Short term real interest rate*



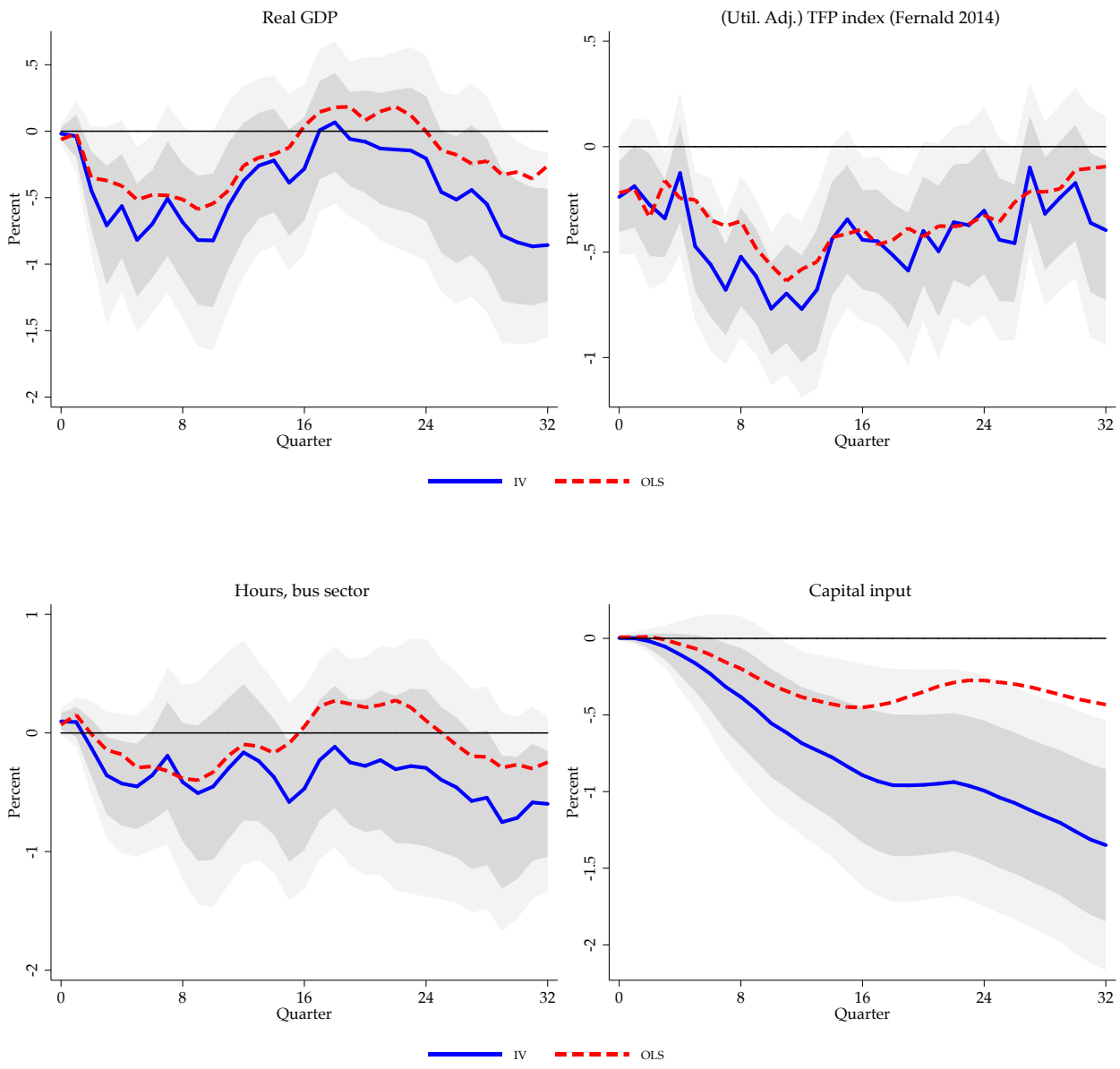
Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses of short term real interest rate for pegging economies. Inflation expectations constructed from the impulse response of consumer price level index to the same trilemma shock. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. See text.

Figure 6: *Cumulative change in real GDP / Cumulative short term expected real interest rate gap*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Plots path of cumulative change in real GDP divided by cumulative change in short term nominal interest rate for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. See text.

Figure 7: *Baseline response to 100 bps Romer and Romer (2004) shock : US postwar data*

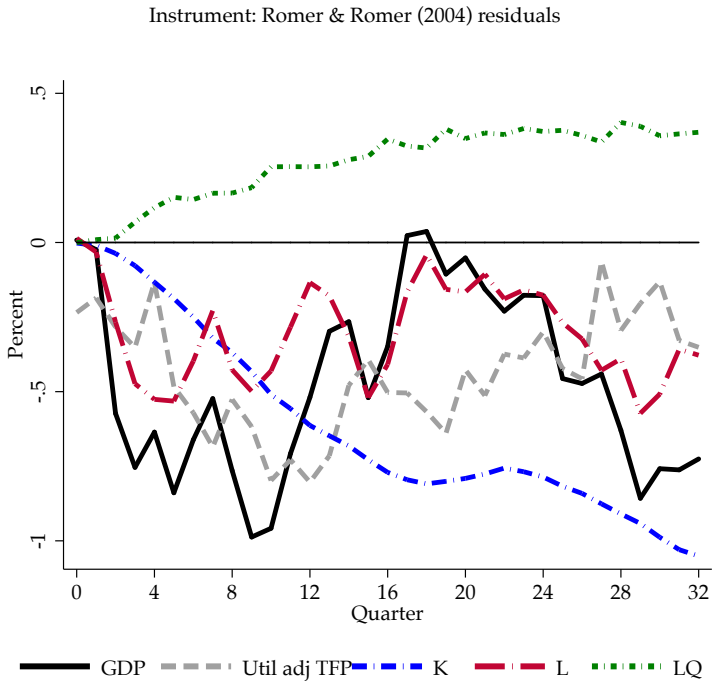


Notes: Response to a 100 bps shock in federal funds rate rate instrumented with policy forecast residuals (Romer and Romer, 2004; Wieland and Yang, 2016). Responses of real GDP, utilization-adjusted TFP (Fernald, 2014), capital stock and hours worked for US economy. Quarterly sample: 1969-Q1: 2007-Q4. Quarterly data series are taken from Fernald (2014).

comparison, we plot the various components on the same graph in Figure 8.

The data for utilization-adjusted TFP series are based on sectoral data for the US that accounts for heterogeneity across workers and types of capital. Fernald (2014) also notes that there are various

Figure 8: *Baseline response to 100 bps Romer and Romer (2004) shock: US postwar data*



Notes: Response to a 100 bps shock in overnight federal funds rate rate instrumented with policy forecast residuals (Romer and Romer, 2004; Wieland and Yang, 2016). Responses of real GDP, capital stock, total hours worked, labor composition/quality, and raw TFP for US. Quarterly sample: 1969-Q1: 2007-Q4. Quarterly data series are taken from Fernald (2014). See text.

other corrections that are not conducted in the quarterly series due to the lack of rich-industry level data. The finding that monetary policy shocks can affect utilization-adjusted TFP echo the Evans (1992)'s critique of using Solow residuals as productivity shocks in RBC models. While the construction of quarterly utilization-adjusted TFP series is detailed and thorough, our analysis suggests caution against using the quarterly-adjusted residuals as pure TFP shocks.[9]

4. A model of hysteresis

Impulse responses calculated with standard methods that internally favor reversion to the mean will tend to underestimate the value of the response at longer horizons. By relying on local projections, we allow the data to more directly speak as to its long-run properties. The evidence presented in the previous sections strongly indicate that these long-run effects are important and require further investigation. In order to think through a possible mechanism that explains our empirical findings,

[9] Ramey (2016) also documents that utilization adjusted TFP series fail Granger causality tests (See Table 11 in her paper). We are grateful to Valerie Ramey for alerting us to that analysis.

we augment a textbook New Keynesian model with endogenous growth in a stylized manner. We present the different components of the model in the next few sections, starting with the production side of the economy.

4.1. Monopolistically Competitive Producers

Assume there is a continuum of differentiated intermediated good producers that sell the intermediate good Y_{kt}. These goods can be aggregated into a Dixit-Stiglitz final composite good Y_t as follows:

$$Y_t = \left[\int_0^1 Y_{kt}^{\frac{1}{1+\lambda_p}} dk\right]^{1+\lambda_p}$$

where $\lambda_p > 0$ is the price markup. The iso-elastic demand for intermediate good k is given by:

$$Y_{kt} = \left(\frac{P_{kt}}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t$$

The zero profit condition for competitive final good producers implies that the aggregate price index is

$$P_t = \left[\int_0^1 P_{kt}^{-\frac{1}{\lambda_p}} dk\right]^{-\lambda_p}$$

Each intermediate good k is produced by a price-setting monopolist using labor L_{kt} and physical capital K_{kt}[10]:

$$Y_{kt} = (Z_t L_{kt})^{1-\alpha} \; K_{kt}^{\alpha}$$

where Z_t is the aggregate TFP. The variable Z_t denotes a non-stationary TFP series that we describe in the next subsection.[11] Firms may not be able to adjust their price in a given period, but they will always choose inputs to minimize total cost each period. The cost minimization yields the input demand functions.

$$W_t = (1-\alpha)\; mc_{kt} F_t^{1-\alpha} \left(\frac{K_{kt}}{L_{kt}}\right)^{\alpha}$$

$$R_t^k = \alpha\; mc_{kt} F_t^{1-\alpha} \left(\frac{K_{kt}}{L_{kt}}\right)^{\alpha-1}$$

The first order condition implies that the capital labor ratio at the firm level is independent on firm-specific variables:

$$\frac{K_{kt}}{L_{kt}} = \frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}$$

[10] We can append fixed cost in the production function to eliminate steady state profits. This is usually done to justify no entry and exit in the steady state in a DSGE model (Christiano et al. 2005, Justiniano, Primiceri, and Tambalotti 2013, Smets and Wouters 2007). In a micro-founded model of growth, positive rents are needed to incentivize investment in growth (Romer, 1990).

[11] Relative to the conventional literature, we will allow for an endogenous relationship between TFP growth and output gap.

Thus, (nominal) marginal cost is independent of firm specific variables:

$$P_t mc_{kt} = P_t mc_t = \frac{1}{F_t^{1-\alpha}} \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

Each firm k is assumed to set prices on a staggered basis following Calvo (1983). With probability $(1-\theta_p)$, a firm adjusts its price independent of previous history. A resetting firm chooses $\tilde{P}_{kt}$ to maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta_p^{s-t} Q_{t,s} \left[\frac{\tilde{P}_{kt}}{P_s} - mc_s\right] Y_s(k)$$

subject to demand for its product

$$Y_{kt} = \left(\frac{P_{kt}}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t$$

where the stochastic discount factor in period s relative to period t is given by:

$$Q_{t,s} = \beta^{s-t} \frac{\Lambda_s}{\Lambda_t} \frac{P_t}{P_s}$$

and Λ_t is the marginal utility of consumption defined later. The first order condition is :

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta_p^{s-t} Q_{t,s} \left[\frac{\tilde{P}_t}{\Pi_{t,s}} - (1+\lambda_p)\, mc_s\right] \left(\frac{\tilde{P}_t}{\Pi_{t,s}}\right)^{-\frac{1+\lambda_p}{\lambda_p}} Y_s = 0$$

By the law of large numbers, the law of motion of the aggregate price index P_t is given by:

$$P_t^{\frac{1}{\lambda_p}} = (1-\theta_p)(\tilde{P}_t)^{\frac{1}{\lambda_p}} + \theta_p P_{t-1}^{\frac{1}{\lambda_p}}$$

4.2. Hysteresis effects

In order to be able to capture the empirical features describe in the previous sections, we examine a richer specification of the low of motion for total factor productivity than is conventional. In particular, we assume that the law of motion for Z_t is:

$$\log Z_t = \log Z_{t-1} + \mu_t + \eta \log\left(Y_{t-1}/Y_{t-1}^{f,t-1}\right)$$

where μ_t is the exogenous component of the TFP growth rate, that may be subject to trend shocks. $Y_{t-1}^{f,t-1}$ is the flexible price level of output in period $t-1$ conditional on Z_{t-1}, and will be referred to as the *potential* output at time $t-1$. The second component denotes the endogenous component of TFP growth, where η is the elasticity of TFP growth rate with respect to fluctuations in output due to nominal rigidities. We refer to this as the *hysteresis elasticity* (to be consistent with DeLong and Summers 2012).

The above law of motion allows business cycles to affect TFP growth rate only in the presence of

nominal rigidities or inadequate stabilization. For clarity, we employed this reduced form equation.[12] A micro-founded model of innovation and productivity growth that yields this exact representation under monetary policy shocks can be found in the recent literature embedding endogenous growth into DSGE models (Bianchi et al., 2019; Garga and Singh, 2016). The effects of business cycles on TFP growth rate that are unrelated to nominal rigidities can be denoted by time varying values of μ_t, which may depend on other shocks (markup shocks, stationary TFP shocks, discount factor shocks, capital quality shocks etc.). For ease of exposition, we only focus on the hysteresis effects induced by the presence of nominal rigidities and treat μ_t to be an exogenous series. Movements in this series can be used to induce state-dependent hysteresis effects of monetary policy shocks.

This functional dependence creates a role for hysteresis stabilization by central banks in a reduced form manner (Yellen, 2016). [13] Long-run effects of monetary policy shocks depend on the value of η. To clarify, in this paper, we are only able to discuss the sign and the magnitude of η in response to temporary monetary shocks denoted as η^{mp}. We assume that $\eta^{mp} > 0$ and the exact quantitative magnitudes can extracted from the empirical IRFs estimated above.

4.3. Households

Rest of the model components are standard. We briefly summarize them here and leave the detailed derivations to the appendix.

4.3.1 Households

Each household supplies differentiated labor indexed by j. Household j chooses consumption C_t, risk-free nominal bonds B_t, investment I_t and capital utilization u_t to maximize the utility function, with external habits over consumption:

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \beta^{s-t} \left[\log(C_{j,s} - hC_{j,s-1}) - \frac{\omega}{1+\nu} L_{j,s}^{1+\nu} \right]$$

where h is the degree of habit formation, $\nu > 0$ is the inverse Frisch elasticity of labor supply, $\omega > 0$ is a parameter that pins down the steady-state level of hours, and the discount factor β satisfies $0 < \beta < 1$. We assume perfect consumption risk sharing across the households. As a result, household's budget constraint in period t is given by

$$P_t C_t + P_t I_t + B_{t+1} = B_t(1+i_t) + B_{j,t}^S + (1+\tau^w) W_t L_{j,t} + \Gamma_t + T_t + R_t^K u_t K_t^u - P_t a(u_t) K_t^u \qquad (2)$$

[12] A similar setup was used by Stadler (1990) in his seminal work.

[13] Theoretically, there is no apriori reason to expect η to be positive. While a 'cleansing' effect of recessions may induce counter-cyclicality, recessions may reduce funding access to firms to conduct R&D, skill development, and learning-by-doing. The sign on the cyclicality of TFP misallocation is also ambiguous and depends on the assumptions in a model.

where I_t is investment, $B^S_{j,t}$ is the net cash-flow from household j's portfolio of state-contingent securities. Labor income $W_t L_{j,t}$ is subsidized at a fixed rate τ_w. Households own an equal share of all firms, and thus receive Γ_t dividends from profits. Finally, each household receives a lump-sum government transfer T_t. Since households own the capital and choose the utilization rate, the amount of effective capital that the households rent to the firms at nominal rate R^K_t is :

$$K_t = u_t K^u_t$$

The (nominal) cost of capital utilization is $P_t a(u_t)$ per unit of physical capital. As in the literature (Smets and Wouters 2007) we assume $a(1) = 0$ in the steady state and $a'' > 0$. Following Christiano, Eichenbaum, and Evans (2005), we assume investment adjustment costs in the production of capital. Law of motion for capital is as follows:

$$K^u_{t+1} = \left[1 - S\left(\frac{I_t}{(1+g_{ss})I_{t-1}}\right)\right] I_t + (1-\delta_k)K^u_t$$

where $g_{ss} \equiv \bar{\mu}$ is the steady state growth rate of Z_t. Utility maximization delivers the first order condition linking the inter-temporal consumption smoothing to the marginal utility of holding the risk-free bond

$$1 = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}(1+i_t)\frac{P_t}{P_{t+1}}\right] \tag{3}$$

The stochastic discount factor in period $t+1$ is given by:

$$Q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}\frac{P_t}{P_{t+1}}$$

where Λ_t is the marginal utility of consumption given by:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \frac{h\beta}{C_{t+1} - hC_t}$$

The household does not choose hours directly. Rather each type of worker is represented by a wage union who sets wages on a staggered basis. Consequently the household supplies labor at the posted wages as demanded by firms.

We introduce capital accumulation through households. Solving household problem for investment and capital yields the Euler condition for capital:

$$q_t = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{R^K_{t+1}}{P_{t+1}}u_{t+1} - a(u_{t+1}) + q_{t+1}(1-\delta_k)\right)\right]$$

where the (relative) price of installed capital q_t is given by

$$q_t \left[1 - S\left(\frac{I_t}{(1+g_{ss})I_{t-1}}\right) - S'\left(\frac{I_t}{(1+g_{ss})I_{t-1}}\right)\frac{I_t}{(1+g_{ss})I_{t-1}}\right]$$
$$+\beta\frac{\Lambda_{t+1}}{\Lambda_t}q_{t+1}\frac{1}{(1+g_{ss})}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{(1+g_{ss})I_t}\right) = 1$$

Choice of capital utilization rate yields:

$$\frac{R_t^K}{P_t} = a'(u_t)$$

4.3.2 Wage Setting

Wage Setting follows Erceg, Henderson, and Levin (2000) and is relatively standard. Perfectly competitive labor agencies combine j type labor services into a homogeneous labor composite L_t according to a Dixit-Stiglitz aggregation:

$$L_t = \left[\int_0^1 L_{j,t}^{\frac{1}{1+\lambda_w}} dj\right]^{1+\lambda_w}$$

where $\lambda_w > 0$ is the nominal wage markup. Labor unions representing workers of type j set wages on a staggered basis following Calvo (1983), taking given the demand for their specific labor input:

$$L_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t, \qquad \text{where } W_t = \left[\int_0^1 W_{j,t}^{\frac{-1}{\lambda_w}} dj\right]^{-\lambda_w}$$

In particular, with probability $1-\theta$, the type-j union is allowed to re-optimize its wage contract and it chooses $\tilde{W}$ to minimize the disutility of working for laborer of type j, taking into account the probability that it will not get to reset wage in the future.[14] The first order condition for this problem is given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \Lambda_{t+s}\left[(1+\tau_t^W)\tilde{W}_t - (1+\lambda_w)\omega\frac{L_{j,t+s}^{\nu}}{\Lambda_{t+s}}\right] L_{j,t+s} = 0 \tag{4}$$

By the law of large numbers, the probability of changing the wage corresponds to the fraction of types who actually change their wage. Consequently, the nominal wage evolves according to:

$$W_t^{\frac{1}{\lambda_w}} = (1-\theta_w)\tilde{W}_t^{\frac{1}{\lambda_w}} + \theta_w W_{t-1}^{\frac{1}{\lambda_w}}$$

[14] We assume imperfect wage indexation in our nominal wage rigidity assumption. We ignore specifying it here for ease of exposition. See appendix for details.

where the nominal wage inflation and price inflation are related to each other :

$$\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \frac{1}{\pi_t} \frac{1}{1+g_t}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the inflation rate, $w_t \equiv \frac{W_t}{P_t Z_t}$ is the productivity adjusted real wage and g_t is the growth rate of Z_t.

4.4. Government

The central bank follows a Taylor rule in setting the nominal interest rate. It responds to deviations in inflation, output and output growth rate from time-t natural allocations.

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^{f,t}}\right)^{\phi_y}\right]^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{Y_t^{f,t}/Y_{t-1}^{f,t-1}}\right)^{\phi_{dy}} \epsilon_t^{mp} \tag{5}$$

where i_{ss} is the steady state nominal interest rate, $Y_t^{f,t}$ is the time-t natural output, ρ_R determines interest-rate smoothing and $\epsilon_t^{mp} \sim N(0, \sigma_r)$ is the monetary policy shock.

We assume government balances budget every period:

$$P_t T_t = \tau^p \int_0^1 p_{it} x_{it} di + \tau^w W_t L_t + P_t G_t$$

where G_t is the government spending, which is determined exogenously as as a fraction of GDP

$$G_t = \left(1 - \frac{1}{\lambda_t^g}\right) Y_t$$

where the government spending shock follows the process:

$$\log \lambda_t^g = (1-\rho_g)\lambda^g + \rho_g \log \lambda_{t-1}^g + \epsilon_t^g; \quad \epsilon_t^g \sim N(0, \sigma_g)$$

4.5. Market Clearing

$$Y_t = C_t + I_t + a(u_t)K_t^u + G_t$$

4.6. Simulations

The DSGE model is standard. We take the parameters estimates from common estimates in the DSGE literature Justiniano et al. (2013). We report these in Table 3. The steady state parameters imply (annualized) real interest rate of 2.40%, and an investment-GDP ratio of 17%. The new parameter in our model, relative to the business cycles literature, is η - the hysteresis elasticity.

DeLong and Summers (2012) calibrate this parameter between 0.05 and 0.1. Here we show the results for value of $\eta = 0.05$, and discuss the robustness to alternate calibrations in the following section.

Figure 9 plots the model-implied impulse responses for output, capital stock, real interest rate and capital to utilization adjusted TFP after a monetary policy shock. The exogenous shock is calibrated to generate a 100 basis point increase in nominal interest rate, and its persistence is fixed to match the persistence of real interest rate gap in the data. The IRFs for output and capital stock are plotted in deviations from an exogenous trend. For real interest rate, we plot the actual path of real interest rate, while the exogenous trend is real interest rate fixed at 2.40 pp. Capital to utilization adjusted TFP ratio is in deviation from steady state ratio. Time is in quarters.

The model can replicate the empirical estimated patterns. There is a persistent decline in capital stock, output and TFP. Capital to utilization adjusted TFP exhibits hump shaped response, with the ratio returning back to steady state.

We define the accumulated gaps in TFP growth rate as the *hysteresis*. We next show the path of output, and capital when the central bank sets interest rates following an augmented Taylor rule:

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^{f,t}}\right)^{\phi_y} \left(\frac{H_t}{H_t^{f,t}}\right)^{\phi_H}\right]^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{Y_t^{f,t}/Y_{t-1}^{f,t-1}}\right)^{\phi_{dy}} \epsilon_t^{mp}$$

where H_t is hysteresis and follows the law motion given by:

$$H_t = H_{t-1} + g_t - g_t^f$$

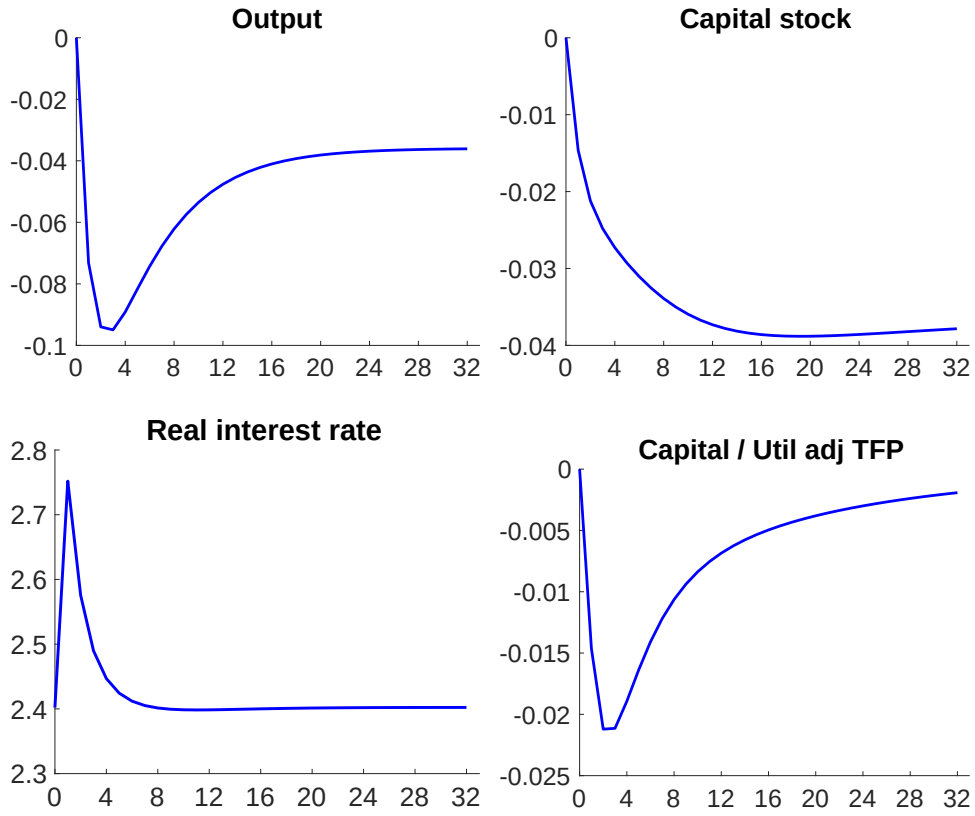
Table 3: *Parameters*

Steady State Parameters			
β	δ_k	α	μ
Discount factor	Capital depreciation rate	Capital share	Trend growth rate
0.999	0.025	0.28	2%

Parameters Characterizing the Dynamics					
ν	λ_p	λ_w	θ_p	θ_w	h
Inverse Frisch elasticity	Price s.s. markup	Wage s.s. markup	Price Calvo probability	Wage Calvo probability	(Internal) habit
1.00	0.15	0.15	0.750	0.750	0.5
$\frac{a''(1)}{a'(1)}$	$S''(1)$	ϕ_π	ϕ_y	$1 - \frac{1}{\lambda_g}$	η
Capital utilization cost	Investment adjustment cost	Taylor rule inflation response	Taylor rule (normalized) output response	Government spending share	Hysteresis elasticity
4	2	1.50	0.125	0.20	0.05

Notes: The table shows the parameter values of the model for the baseline calibration. See text.

Figure 9: *Response of Output, Capital Stock, Real interest rate and Capital to TFP ratio to 100 bps increase in nominal interest rate*



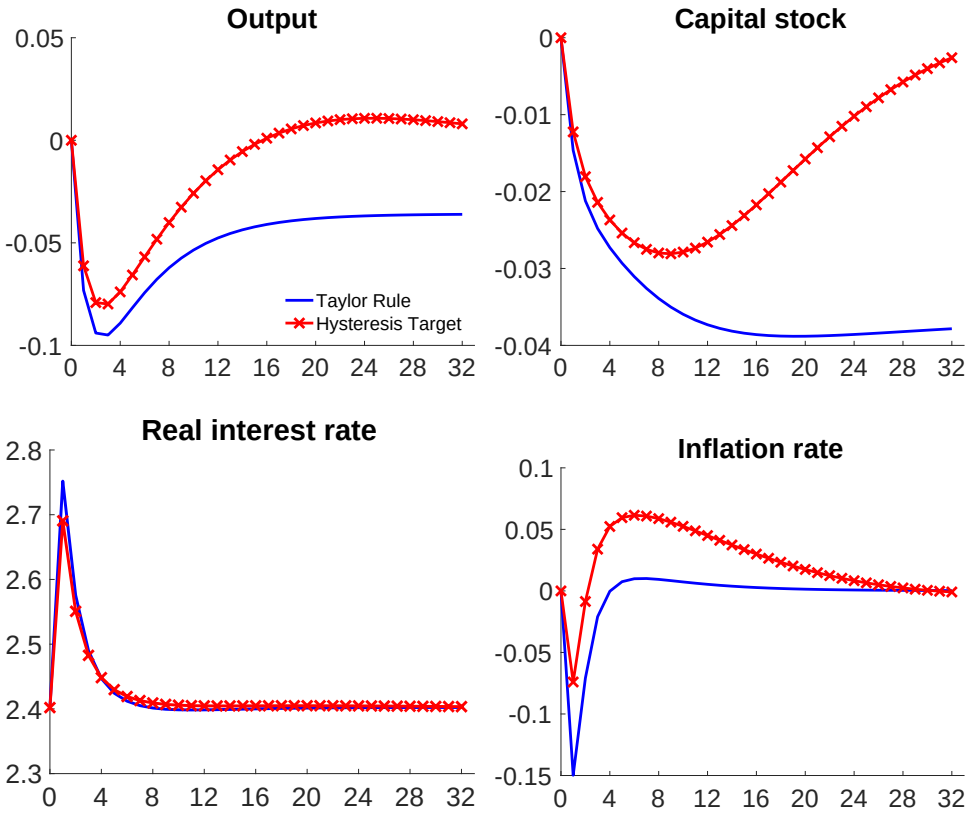
Notes: The figure plots the model-implied IRFs for output, capital stock, real interest rate and capital to utilization adjusted TFP. Time is in quarters. IRFs are traced following a one-time exogenous shock in the federal funds rate of 100 bps (annualized). The persistence of the shock is calibrated such that real interest rate returns to zero after 10 quarters. The IRFs for output and capital stock are plotted in deviations from an exogenous trend. For real interest rate, we plot the actual path of real interest rate, while the exogenous trend is real interest rate fixed at 3 pp. Capital to utilization adjusted TFP ratio is in deviation from steady state ratio.

We set $\phi_H = 0.5$ to plot the graphs. Figure 10 contrasts the path of output and capital stock after a 100 bps shock to the federal funds rate. The real interest rate increases by less than in the case of the standard Taylor rule (Equation 5). This is because, under a hysteresis targeting rule, the central bank accommodates above-target inflation at a later time in order to target zero output hysteresis. Expectations of a high inflation rate induce less contraction in the economy as well as a sharp recovery of GDP to the pre-shock trend.

5. Robustness and Discussion

In this section, we address some of the concerns relating to our empirical analysis presented above.

Figure 10: *Responses of output, capital, real interest rate and inflation rate to 100 bps increase in nominal interest rate under a Hysteresis targeting rule*



Notes: The figure plots the model-implied IRFs for output, capital, real interest rate and net inflation rate. Time is in quarters. IRFs are traced following a one-time exogenous shock in the federal funds rate of 100 bps (annualized). The persistence of the shock is calibrated such that real interest rate returns to zero after 10 quarters. The IRFs for output and capital stock are plotted in deviations from an exogenous trend.

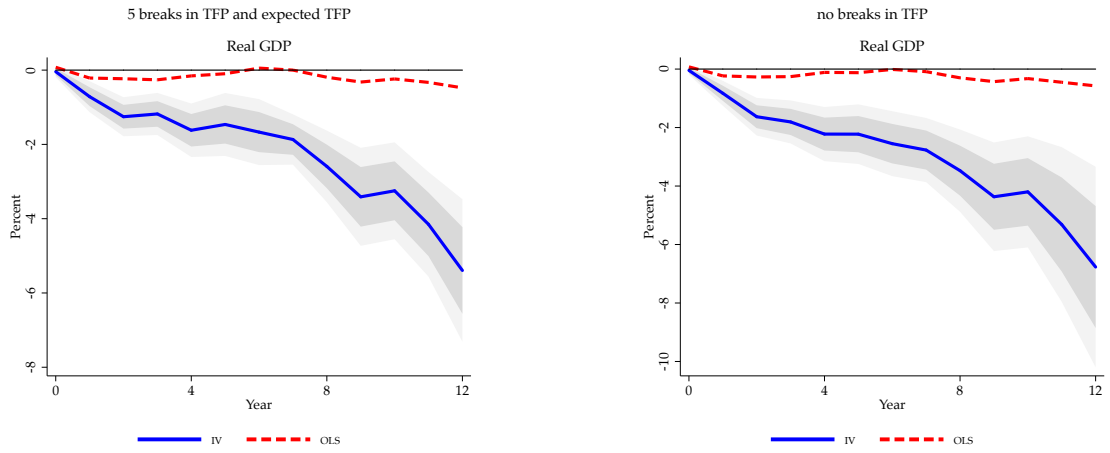
5.1. Structural breaks

Fernald (2014) and Gordon (2016) have convincingly argued that there are structural breaks in TFP growth in the US economic trajectory. It is plausible that there are structural breaks in other economies' TFP growth rates. If these structural breaks implying slowdown in TFP growth occur around the identified monetary shocks, it could bias our results leading us to attribute the persistent effects to incorrect source. To address this concern, we first estimate five structural breaks in TFP growth for each country in our sample using the UD-max statistic of Bai & Perron (1998). We report these estimated structural break dates in the appendix. Then in our baseline specification, we allow output growth to lie in either of the five regimes at horizon zero as well as horizon h: i.e.

$$y_{i,t+h} = \alpha_{i,h} + \sum_{k-1}(D_{i,k,t} + D_{i,k,t+h}) + \widehat{\Delta r}_{i,t}\beta_h + x_{i,t}\gamma_h + v_{i,t+h},$$

where $D_{i,k,t+h}$ is country-specific dummy for TFP growth regime k (Bai-Perron) at horizon $h = 0, ..., H-1$ and $k \in (1, 5)$. Notice that this specification is conservative whereby we allow horizon h

Figure 11: *Response to 100 bps trilemma shock with structural breaks: Real GDP*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands. See text and Jordà, Schularick, and Taylor (2019).

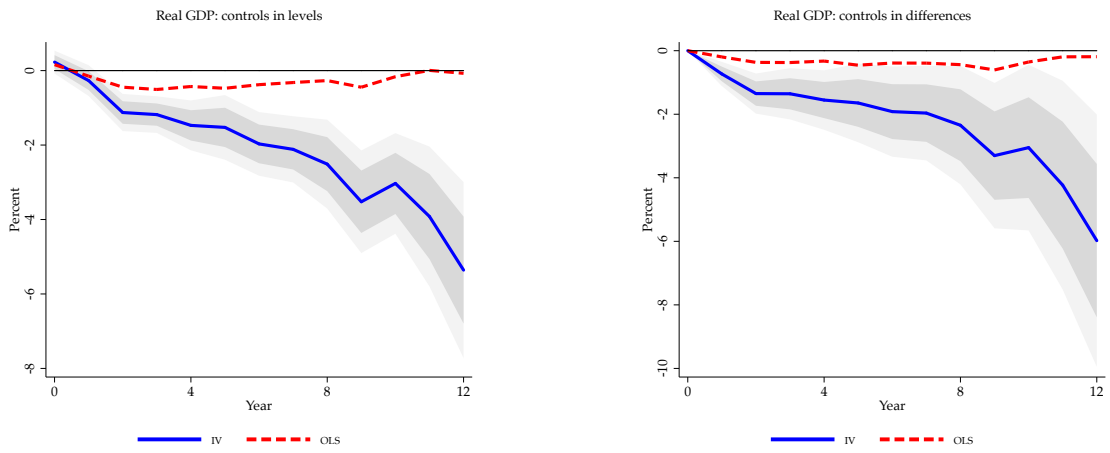
regime changes in the estimation along with horizon 0 regimes.

Figure 11 compares the estimated impulse response when including structural breaks in the left panel and the baseline specification of no structural breaks in the right panel. There is hardly a noticeable difference in the two panels.

5.2. Levels/Differences and number of lags

We report the robustness of IRFs estimated in the baseline to adding the control variables $x_{i,t}$ in levels instead of first differences in the left panel of Figure 12, as well as to including up to 5 lags of the control variables in the right panel.

Figure 12: *Response to 100 bps trilemma shock with controls in levels: Real GDP*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands. See text and Jordà, Schularick, and Taylor (2019).

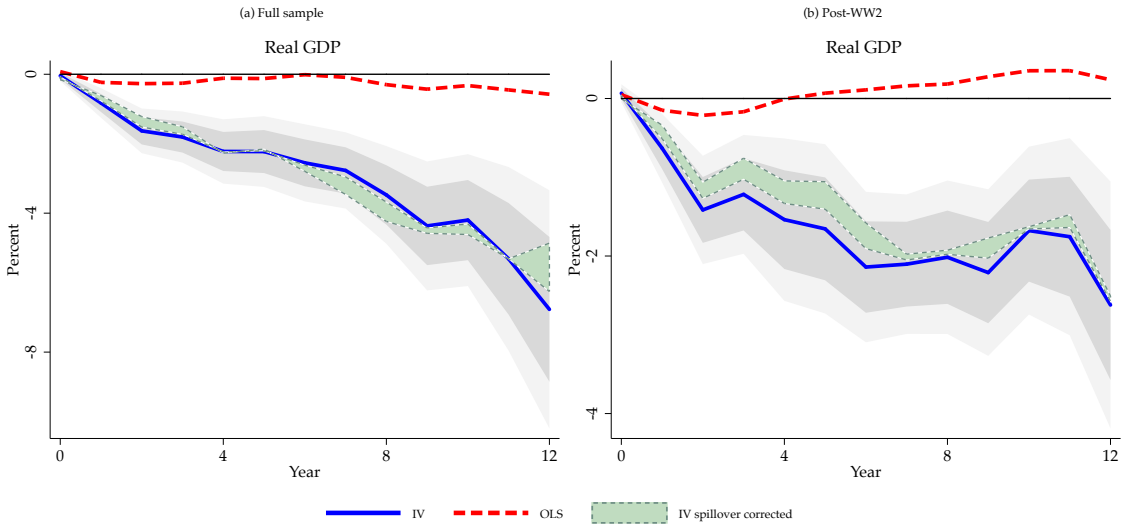
5.3. Spillover correction for the trilemma instrument

A violation of the exclusion restriction could occur if base rates affect home outcomes through channels other than movements in home rates. Additional influences via such channels are sometimes referred to as *spillover effects*. These could occur if base rates proxy for factors common to all countries. That said, these factors would have to persist despite having included global GDP to soak up such business cycle variation. We now assess such spillover effects more formally, using two separate approaches: a) a control function approach developed in Jordà, Schularick, and Taylor (2019), and b) controlling for foreign variables at each horizon to orthogonalize any spillover channels from the interest rate channel.

5.3.1 Synthetic control function approach

First test for exclusion restriction violation follows the control function approach developed in Jordà, Schularick, and Taylor (2019). It exploits the presence of subpopulation of floats in our sample which for example contain information about the global factors at the time of the trilemma shock. Figure 13 shows our spillover-adjusted estimates of response of output to a 100 bps monetary policy shock.

Figure 13: *Response to 100 bps trilemma shock with spillover corrections: Real GDP*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands, LP-IV spillover corrected estimates displayed as a light green shaded area with dashed border, using $\lambda \in [1, 18]$. See text and Jordà, Schularick, and Taylor (2019).

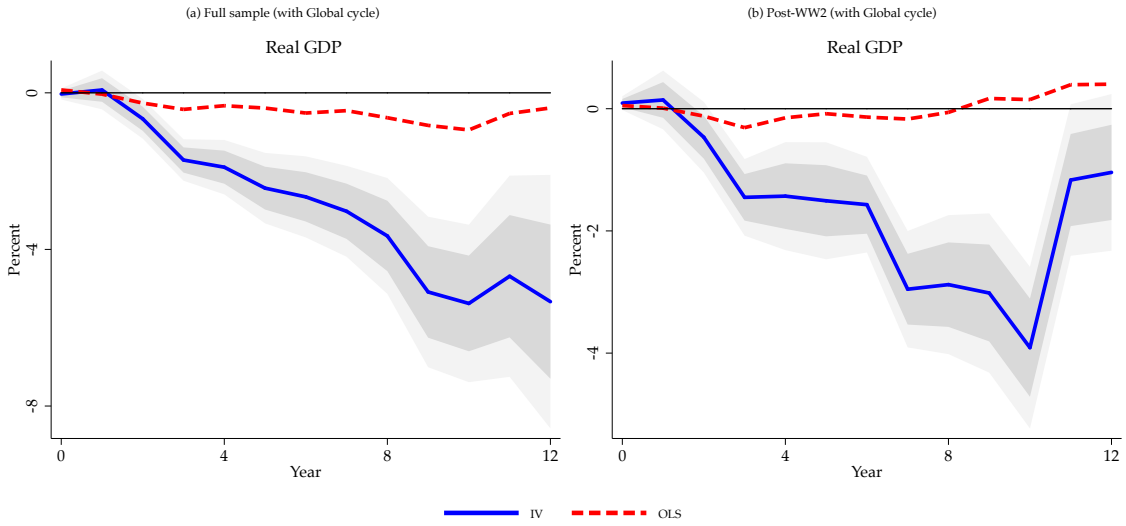
5.3.2 Orthogonalized interest rate channel

A second approach that attempts to provide validity to the exclusion restriction is directly controlling for a primary channel through which the spillover effects may originate. A monetary tightening in the base country may reduce the demand for imports from pegging economy. This reduction in demand my arise from other trading partners when the tightening affects other countries as well. Or the interest rate channel induced contraction in pegging economy by reducing demand for other countries' output may be subject to *spillbacks*. If the transmission is primarily driven by increased trade, controlling for global GDP growth can potentially absorb these effects allowing us to orthogonalize domestic demand effects with respect to international spillbacks. We augment the baseline specification as follows:

$$y_{i,t+h} = \alpha_{i,h} + \widehat{\Delta r}_{i,t}\beta_h + x_{i,t}\gamma_h + G_{t+h}\hat{\gamma}_h + \nu_{i,t+h}, \qquad \text{for } h = 0, ..., H-1$$

where G_{t+h} is global gdp growth at time $t+h$.

Figure 14: *Response to 100 bps trilemma shock with foreign variables controlled: Real GDP*



Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands. See text.

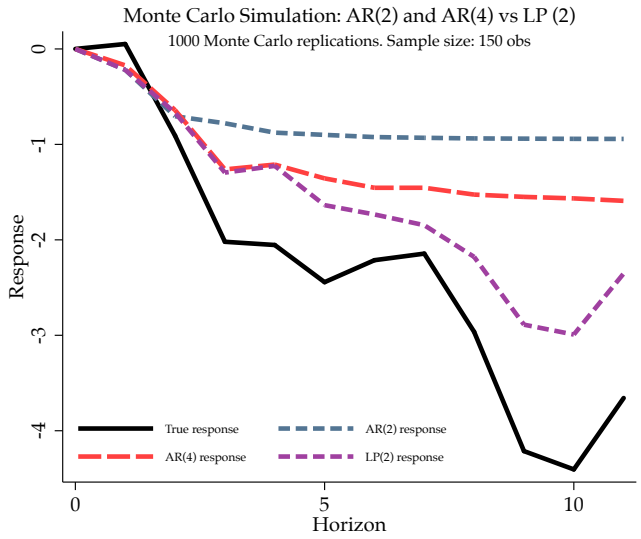
Figure 14 produces the results from the estimation with global GDP growth at each horizon. In comparison to the baseline specification, we still find very persistent and significant effects on GDP.

5.4. Why might VARs not find persistent effects?

That a misspecified VAR suffers from compounding of model mis-specification errors gives local projections approach is a well-known advantage. We provide an example here that suggests why researchers may not be able to estimate such long-run effects that we find when estimating VARs in finite samples and few lags.

Using the estimated IRFs in the baseline model for the full sample over 12 years, we recover the coefficients of an invertible MA(12) process and generate data using this data generating process. Figure 15 compares the IRFs obtained from estimating AR(2), AR(4) and a local projection with two lags. AR(4) process outperforms AR(2) in replicating the true impulse response function coefficients at horizons greater than two. The misspecification errors get compounded at higher horizons under AR(2) than under AR(4), while local projections is not prone to a similar weakness.

Figure 15: *Estimated IRFs using LP(p) and AR(p)*



Notes: Solid black line denotes the basline response of real GDP in pegging economies to a 100 bps shock in domestic interest rate instrumented with the trilemma. Sample: 1890–2015 (World Wars excluded).

6. Conclusion

In this paper, we set out to investigate the widely accepted proposition of long-run money non-neutrality. Using panel-data spanning over a century and trilemma-identified monetary policy shocks, we find that monetary policy shocks indeed have long-run effects on output, capital and TFP. Furthermore, in the quarterly data for the US since 1969, we find that capital stock is permanently lower after a temporary monetary policy shock identified using a separate instrument. At the same time, capital to TFP ratio exhibits a hump-shaped relation as commonly found in the empirical macro-economics literature using vector-autoregression estimation. We reconcile these findings using a simple extension of the Smets and Wouters (2007) medium-scale DSGE model, where TFP growth depends on deviations of output from its flexible-price counterpart. Using the model, we show that a central bank that also targets hysteresis in its policy rule can offset the long-run effects of monetary policy.

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Appendix: Data Sources and Calculations

Appendix: Trilemma Instrument Construction

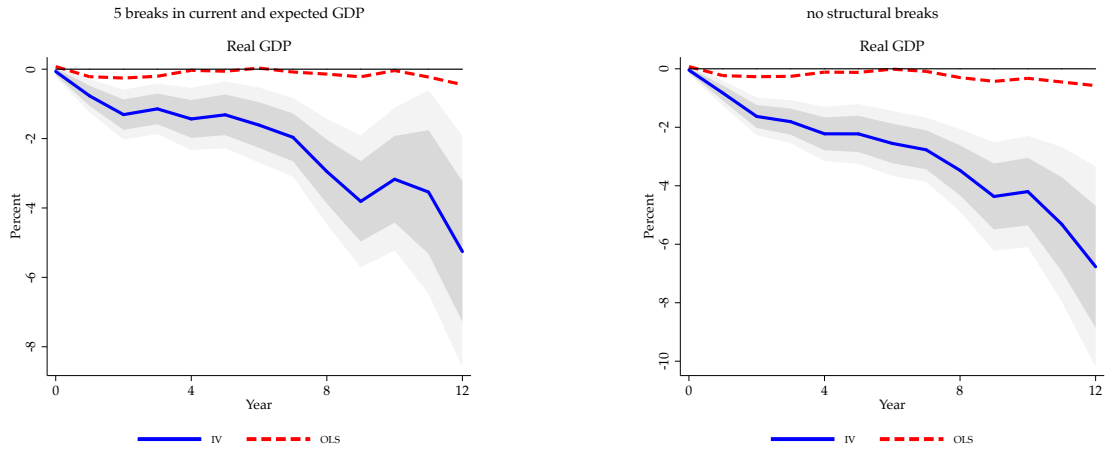
We adopt the instrument constructed in Jordà, Schularick, and Taylor (2019) in this paper. For completeness we describe the details of instrument construction and relevance in this section.

Appendix: Related Graphs

IRFs with Structural Breaks in GDP per capita

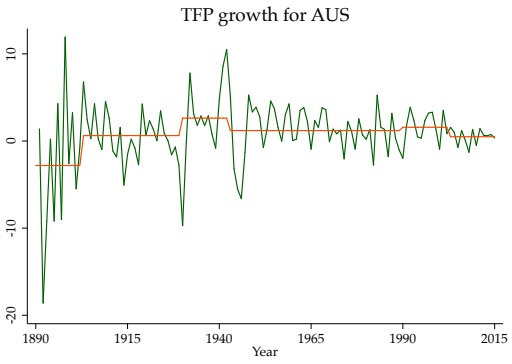
We show an alternate selection of structural breaks based on real GDP per capita.

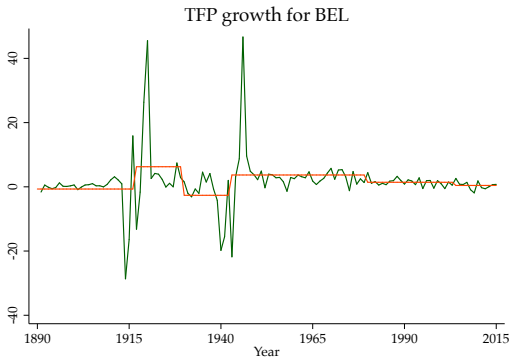
Figure A1: *Response to 100 bps trilemma shock with structural breaks: Real GDP*

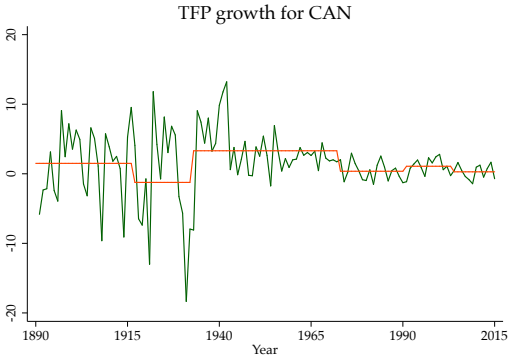


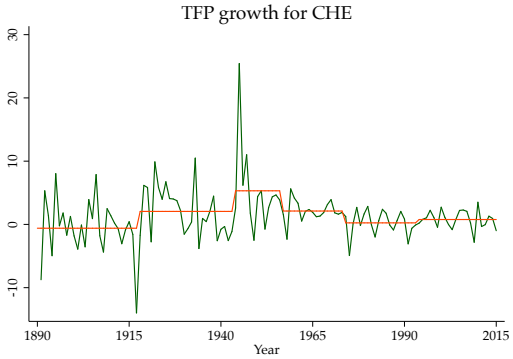
Notes: Response to a 100 bps shock in domestic interest rate instrumented with the trilemma. Responses for pegging economies. Full sample: 1890–2015 (World Wars excluded). Post-WW2 sample: 1948–2015. LP-OLS estimates displayed as a dashed red line, LP-IV estimates displayed as a solid blue line and 1 S.D. and 90% confidence bands. See text and Jordà, Schularick, and Taylor (2019).

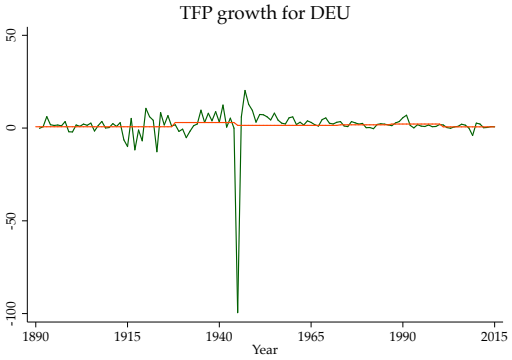
Structural Breaks in TFP growth and GDP growth

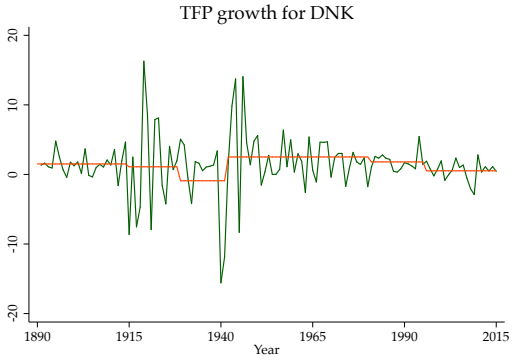


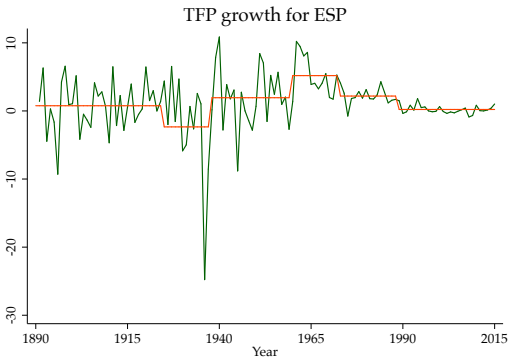


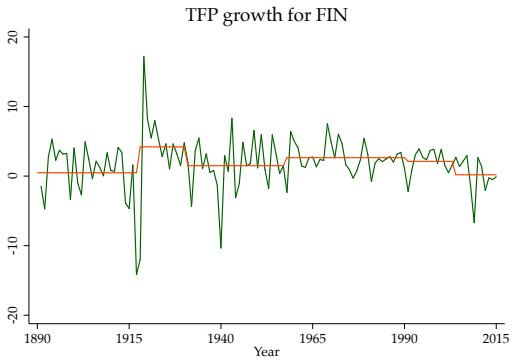


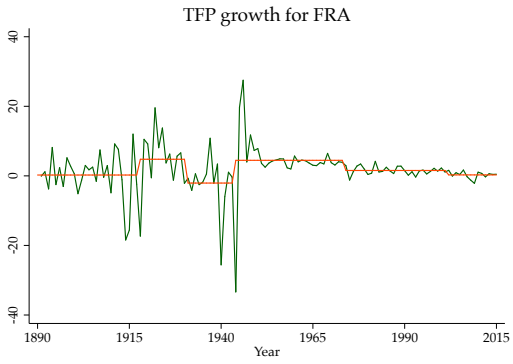


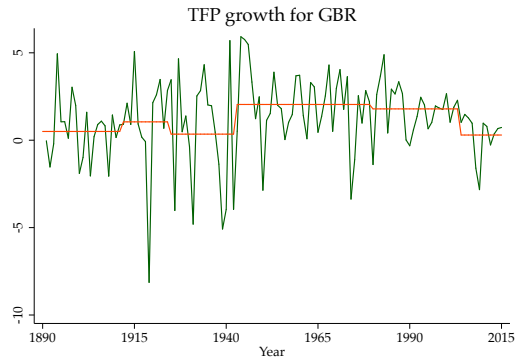


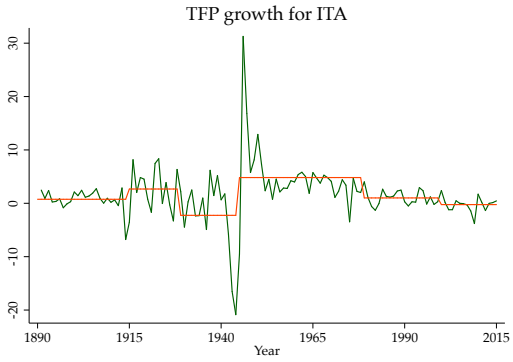


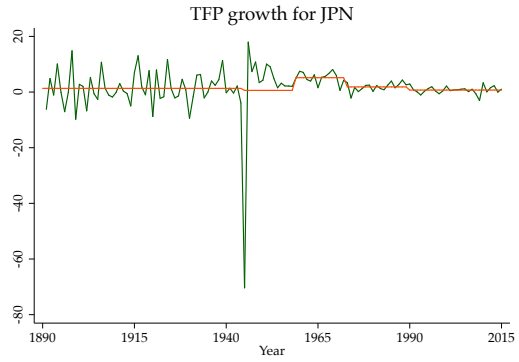


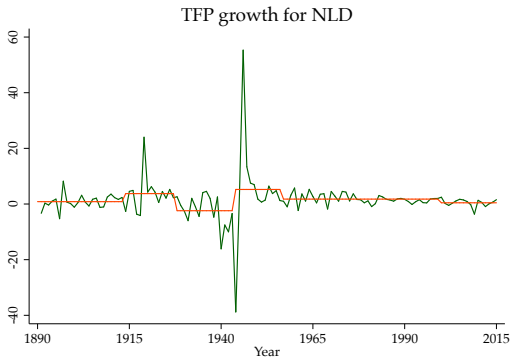


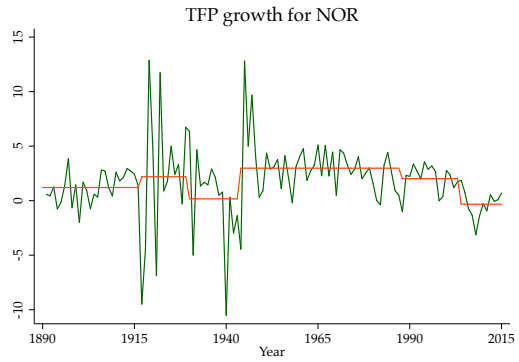

TFP growth for FRA
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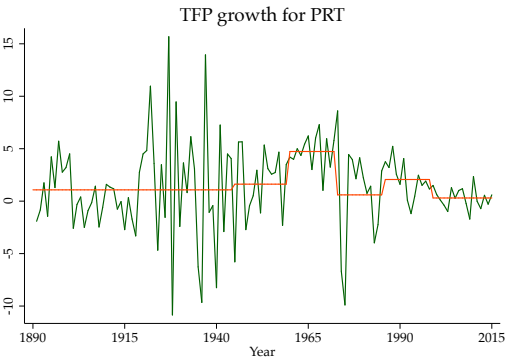

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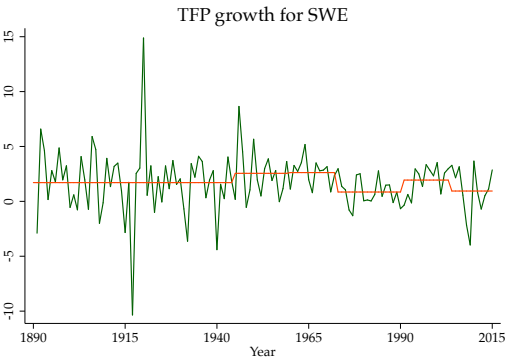

TFP growth for ITA
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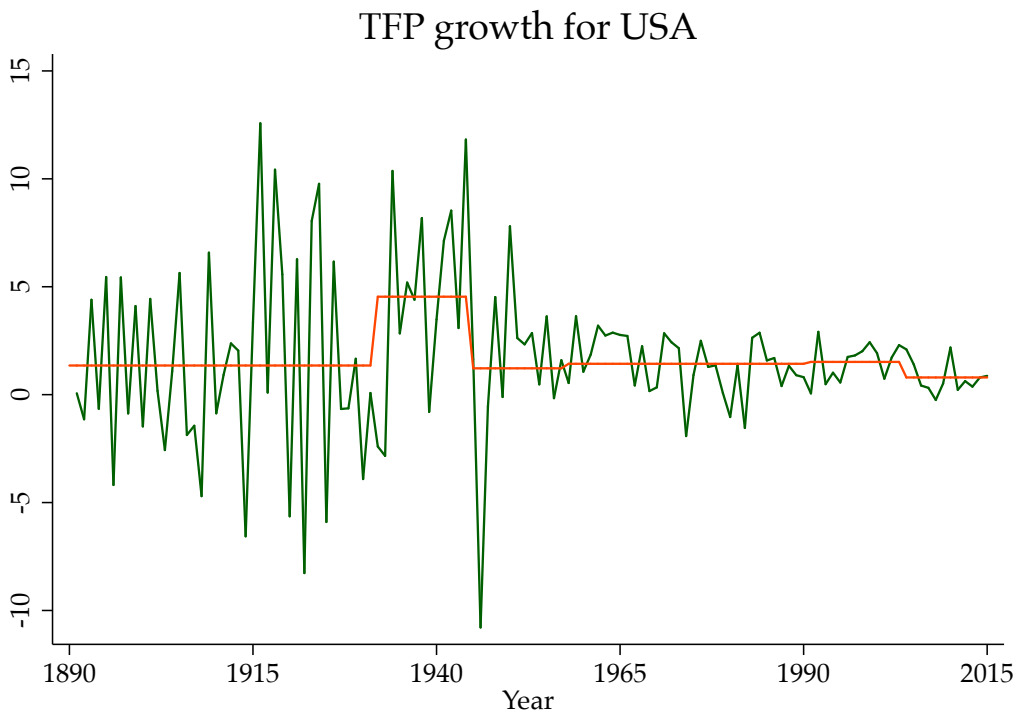

TFP growth for JPN
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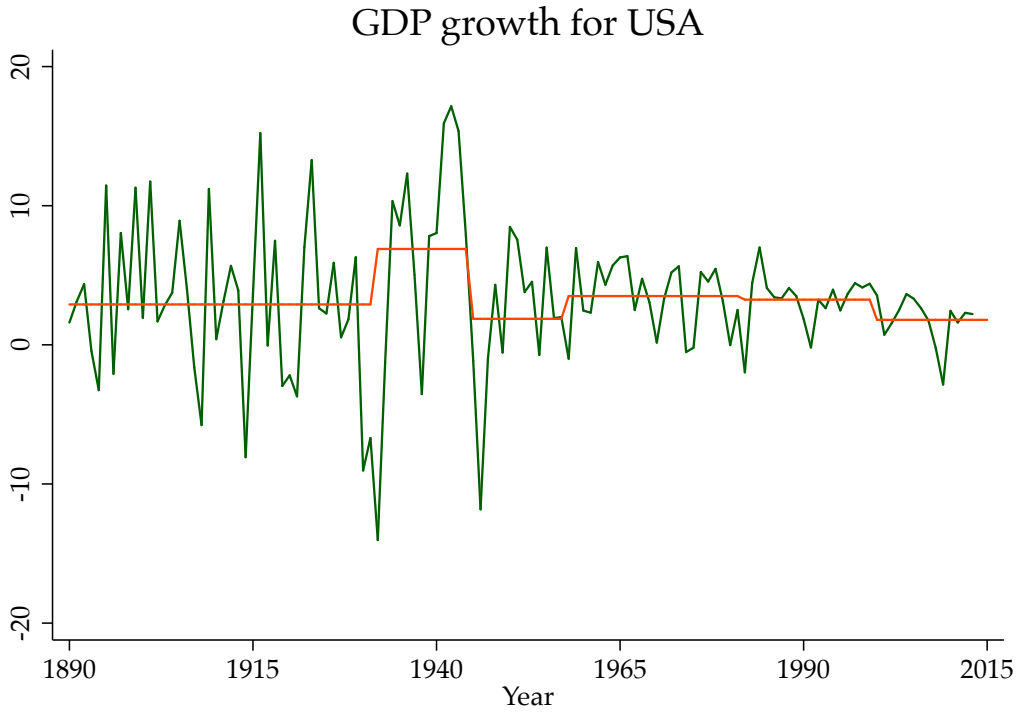

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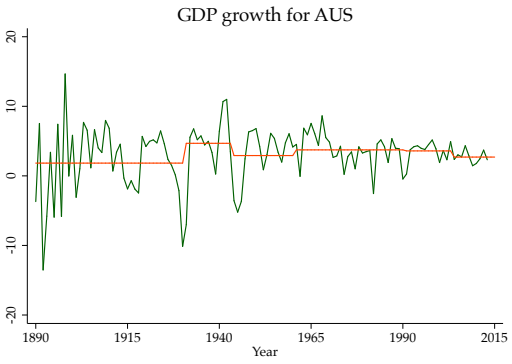

TFP growth for NOR
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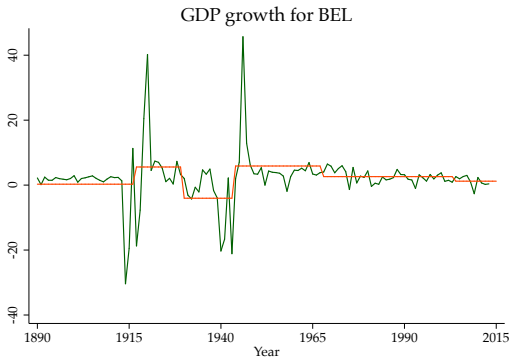

TFP growth for PRT
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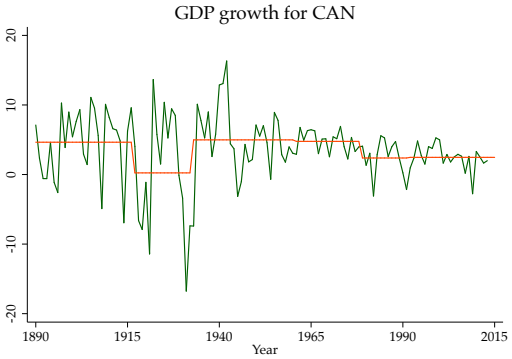

TFP growth for SWE
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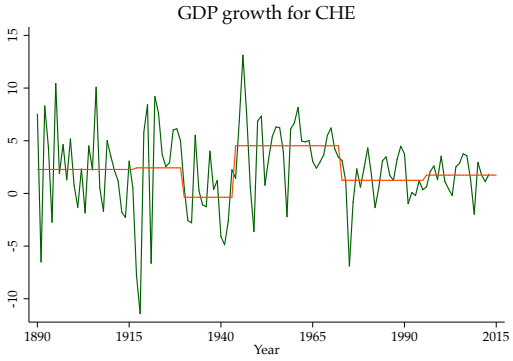

TFP growth for USA
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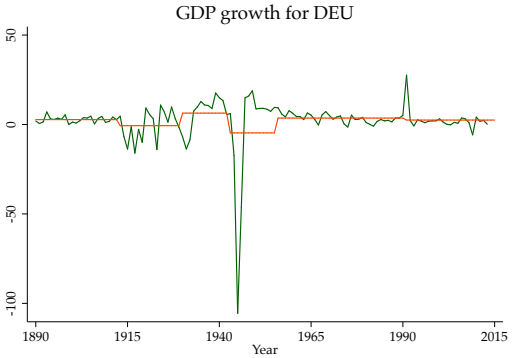

GDP growth for USA
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2015
Year

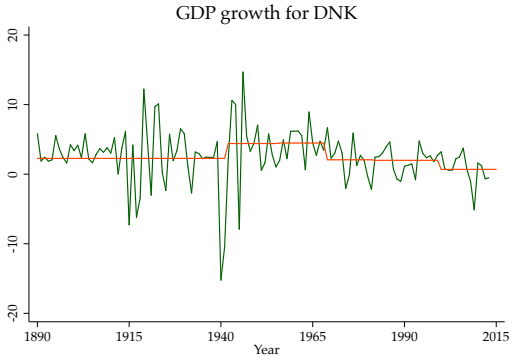

GDP growth for AUS
20
10
0
-10
-20
1890
1915
1940
1965
1990
2015
Year


GDP growth for BEL
40
20
0
-20
-40
1890
1915
1940
1965
1990
2015
Year

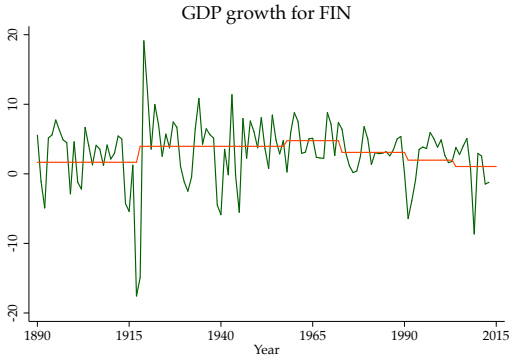

GDP growth for CAN
20
10
0
-10
-20
1890
1915
1940
1965
1990
2015
Year

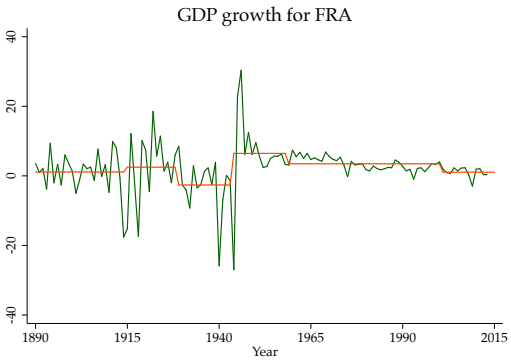

GDP growth for CHE
15
10
5
0
-5
-10
1890
1915
1940
1965
1990
2015
Year

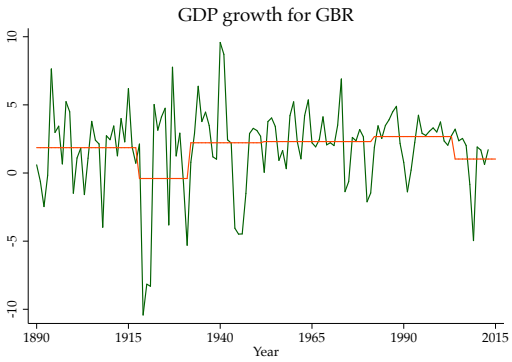

GDP growth for DEU
50
0
-50
-100
1890
1915
1940
1965
1990
2015
Year


GDP growth for DNK
20
10
0
-10
-20
1890
1915
1940
1965
1990
2015
Year

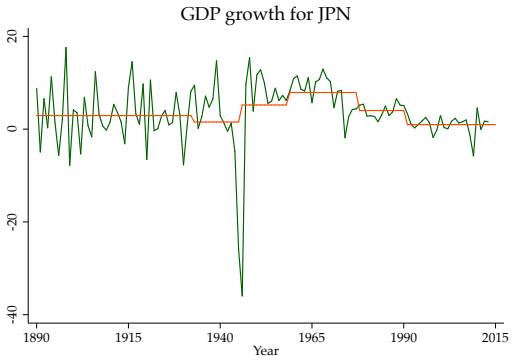

GDP growth for ESP
10
0
-10
-20
-30
1890
1915
1940
1965
1990
2015
Year

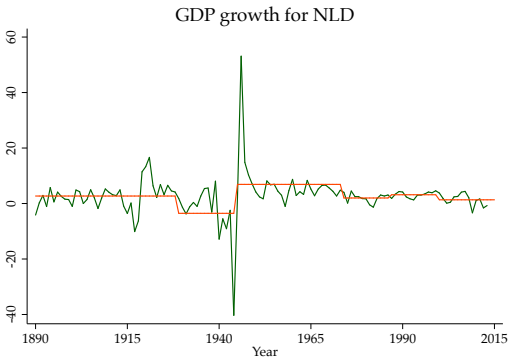

GDP growth for FIN
20
10
0
-10
-20
1890
1915
1940
1965
1990
2015
Year

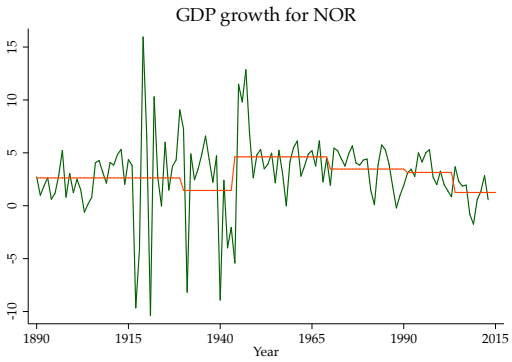

GDP growth for FRA
40
20
0
-20
-40
1890
1915
1940
1965
1990
2015
Year

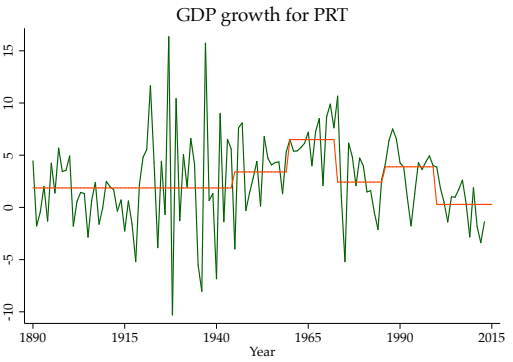

GDP growth for GBR
10
5
0
-5
-10
1890
1915
1940
1965
1990
2015
Year

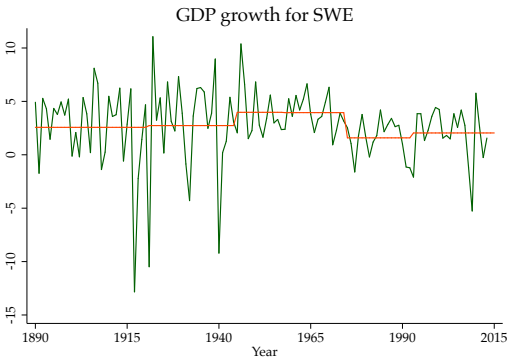

GDP growth for ITA
20
10
0
-10
-20
-30
1890
1915
1940
1965
1990
2015
Year


GDP growth for JPN
20
0
-20
-40
1890
1915
1940
1965
1990
2015
Year

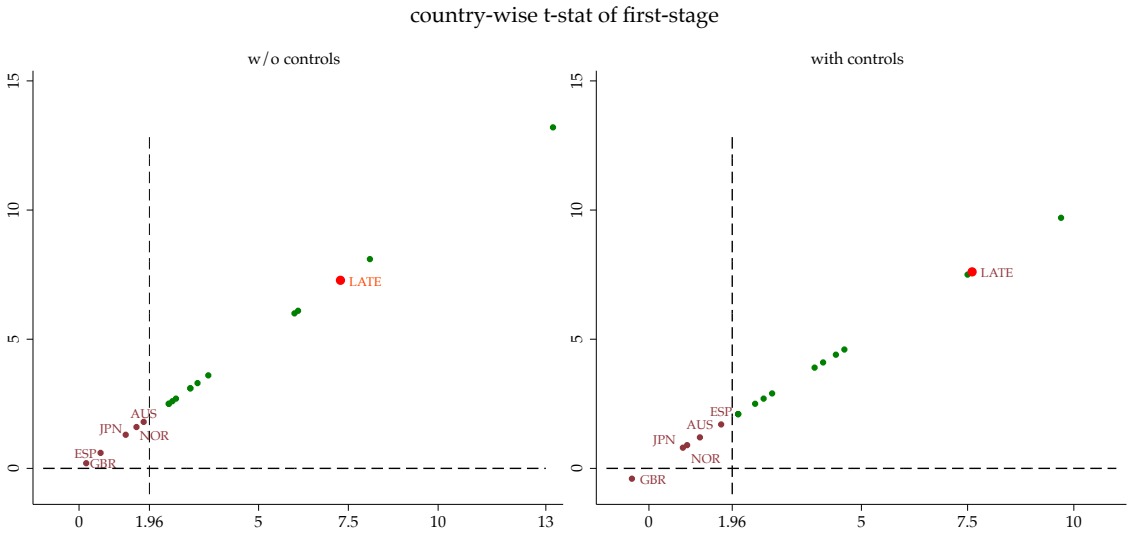

GDP growth for NLD
60
40
20
0
-20
-40
1890
1915
1940
1965
1990
2015
Year


GDP growth for NOR
15
10
5
0
-5
-10
1890
1915
1940
1965
1990
2015
Year

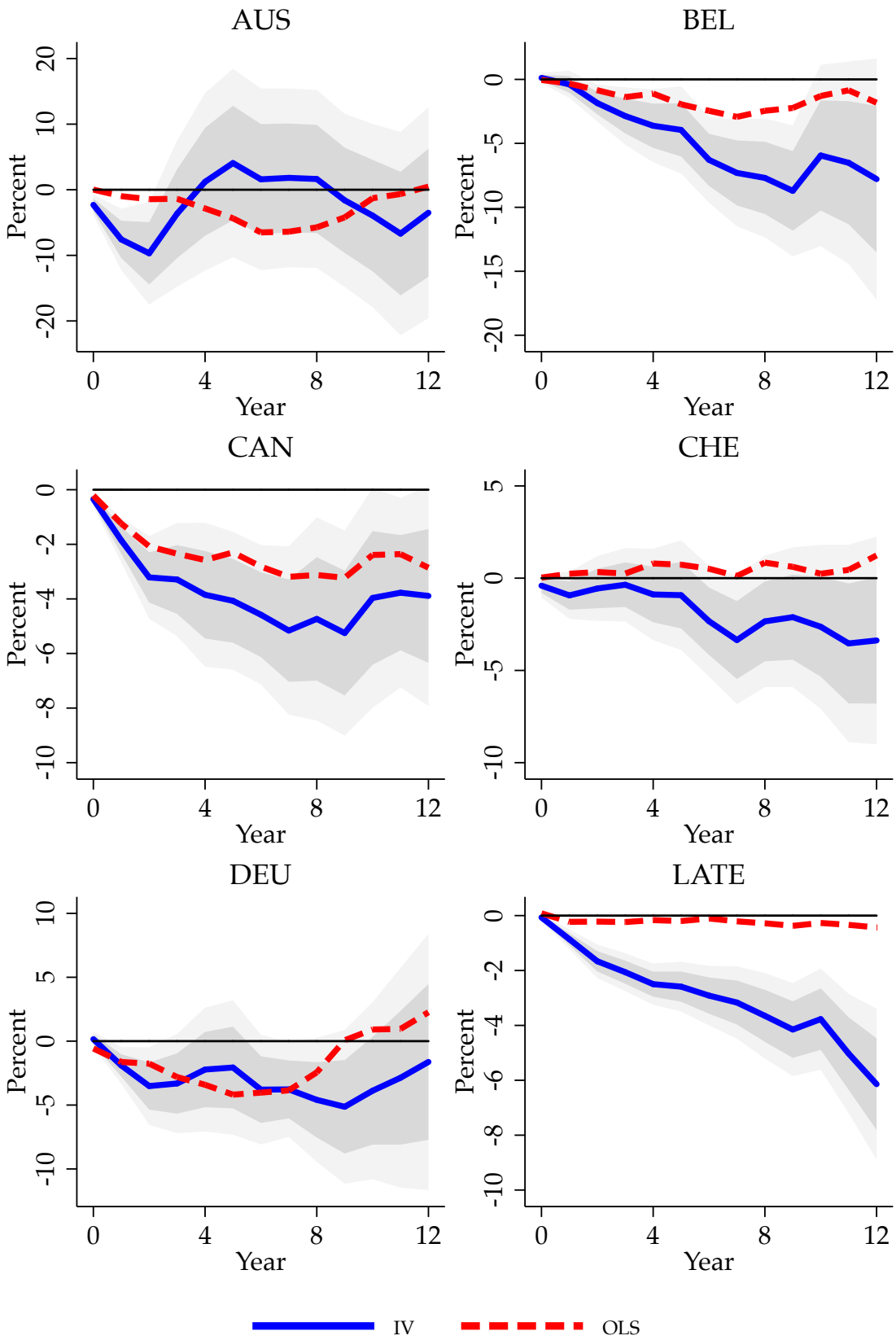

GDP growth for PRT
15
10
5
0
-5
-10
1890
1915
1940
1965
1990
2015
Year

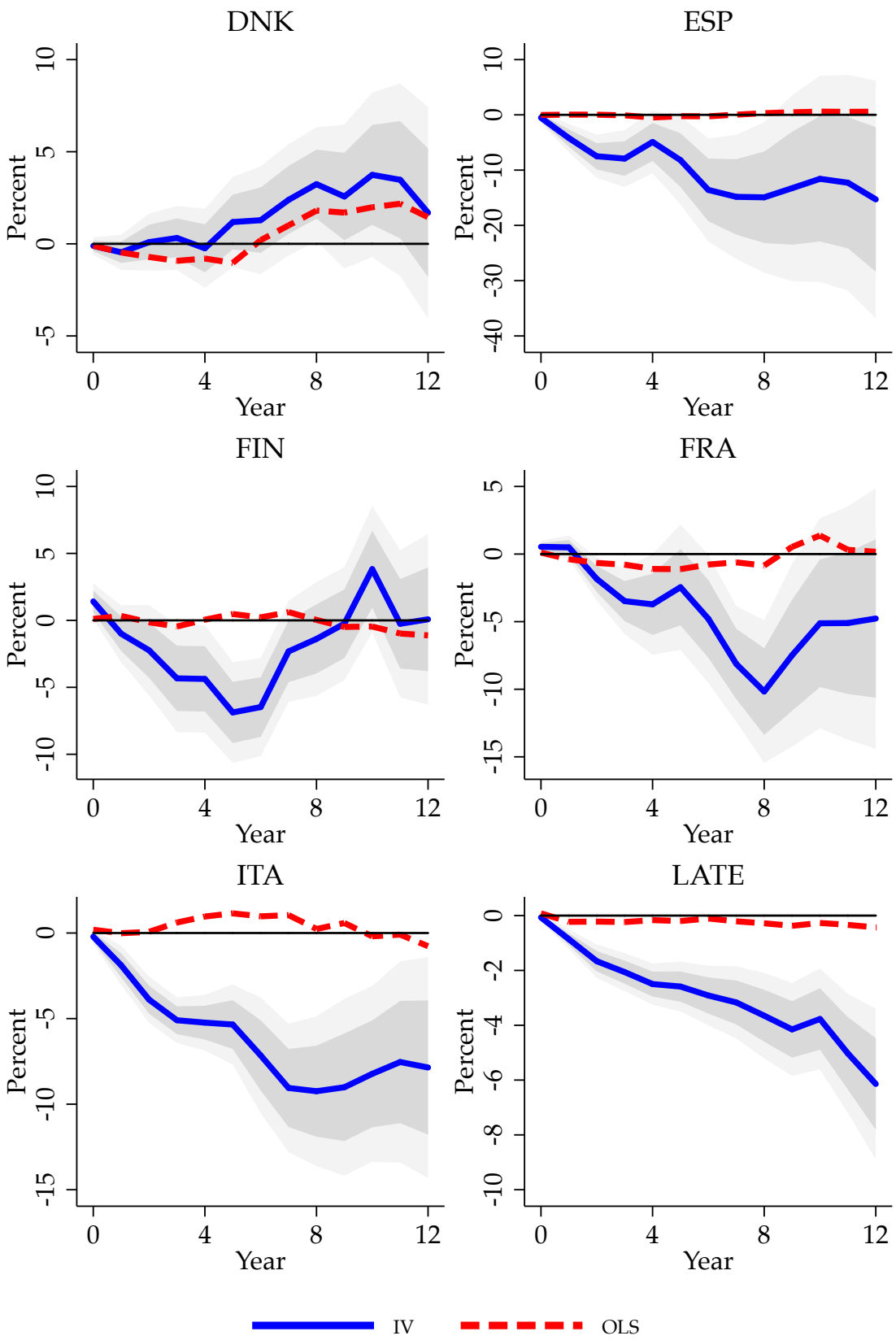

GDP growth for SWE
10
5
0
-5
-10
-15
1890
1915
1940
1965
1990
2015
Year

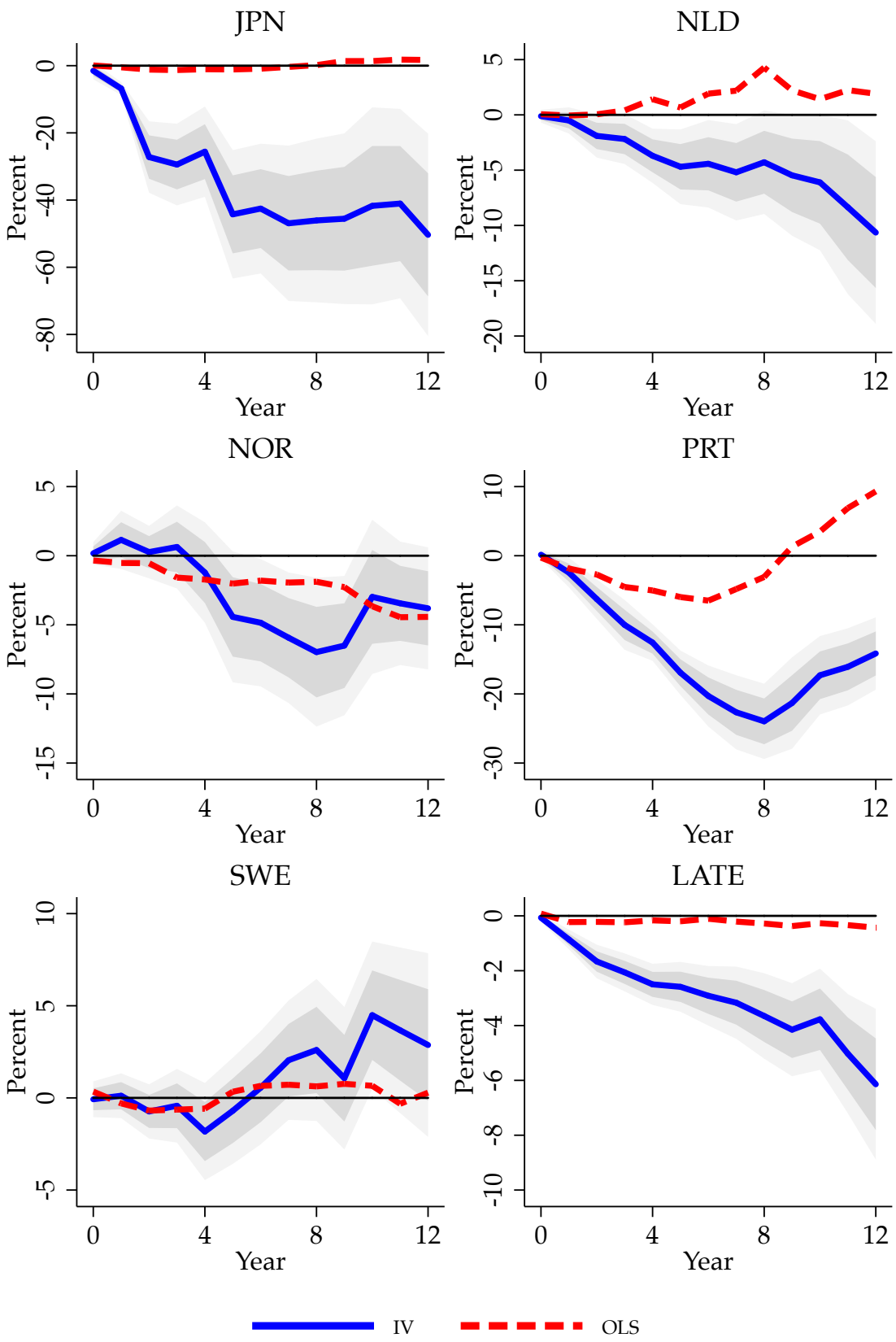
First stage t-statistic for each country



Country Specific IRFs over 1890-2015




DNK
ESP
FIN
FRA
ITA
LATE
Percent
Year
IV
OLS


JPN
NLD
NOR
PRT
SWE
LATE
Percent
Year
0
4
8
12
IV
OLS

Appendix: Related Tables

Table A1: *LP-OLS vs. LP-IV. Attenuation bias of real GDP per capita responses to interest rates. Trilemma instrument. Matched samples*

Responses of real GDP per capita at years 0 to 10 (100 × log change from year 0 baseline).

	(a) Full Sample		*OLS-IV*	*(b) Post-WW2*		*OLS-IV*
Year	LP-OLS	LP-IV	*p-value*	LP-OLS	LP-IV	*p-value*
	(1)	(2)	(3)	(4)	(5)	(6)
$h = 0$	0.07***	-0.07	0.09	0.04**	0.09	0.47
	(0.03)	(0.09)		(0.02)	(0.06)	
$h = 2$	-0.28*	-1.72***	0.00	-0.25*	-1.47***	0.00
	(0.16)	(0.34)		(0.13)	(0.38)	
$h = 4$	-0.16	-2.53***	0.00	-0.08	-1.83***	0.00
	(0.26)	(0.50)		(0.21)	(0.57)	
$h = 6$	-0.06	-2.87***	0.00	0.02	-2.37***	0.00
	(0.29)	(0.66)		(0.23)	(0.55)	
$h = 8$	-0.36	-3.55***	0.00	0.10	-2.04***	0.00
	(0.29)	(0.84)		(0.22)	(0.55)	
$h = 10$	-0.40	-4.05***	0.00	0.26	-1.63***	0.00
	(0.35)	(1.08)		(0.24)	(0.62)	
KP weak IV		79.66			84.86	
H_0: $LATE = 0$	0.00	0.00		0.00	0.00	
Observations	607	607		482	482	

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Cluster robust standard errors in parentheses.Full sample: $1890 - 2015$ excluding WW1: $1914 - 1919$ and WW2: $1939 - 1947$. Post WW2 sample: $1948 - 2015$. *Matched sample* indicates LP-OLS sample matches the sample used to obtain LP-IV estimates. *KP weak IV* refers to the Kleibergen-Paap test for weak instruments. H_0 : $subATE = 0$ refers to the p-value of the test of the null hypothesis that the coefficients for $h = 0, ..., 10$ are jointly zero for a given subpopulation. $OLS = IV$ shows the p-value for the Hausmann test of the null that OLS estimates equal IV estimates. See text.

Appendix: DSGE model

Stationary Allocation

We normalize the following variables :

$$y_t = Y_t / Z_t$$

$$c_t = C_t / Z_t$$

$$k_t = K_t / Z_t$$

$$k_t^u = K_t^u / Z_{t-1}$$

$$\mathbb{I}_t = I_t / Z_t \quad \text{capital investment}$$

$$\mathbb{G}_t = G_t / Z_t \quad \text{Govt Spending}$$

$$w_t = W_t/(Z_t P_t)$$
$$r_t^k = R_t^k/P_t$$
$$\lambda_t = \Lambda_t Z_t$$
$$\tilde{\Gamma}_t \equiv \frac{\Gamma_t}{P_t Z_t}$$

Definition 1 (normalized equilibrium). 19 endogenous variables $\{\lambda_t, g_{t+1}, i_t, \pi_t, \pi_t^w, X_{1t}^p, X_{2t}^p, X_{it}^w, X_{2t}^w, c_t, y_t, z_t, k_{t+1}^u, r_t^K, \mathbb{I}_t, q_t, u_t, k_t, w_t, L_t\}$, 3 shocks $\{\lambda_t^g, \epsilon_t^{mp}, \mu_t\}$ given the natural rate allocation variables $\{y_t^f, g_{t+1}^f\}$.

Consumption Euler Equation

$$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t (1+g_{t+1})} \frac{1+i_t}{\pi_{t+1}} \right] \tag{6}$$

$$\lambda_t = \frac{1}{c_t - \frac{h c_{t-1}}{1+g_t}} - \frac{h\beta}{c_{t+1}(1+g_{t+1}) - h c_t} \tag{7}$$

Price-Setting

$$\frac{X_{1t}^p}{X_{2t}^p} = \left(\frac{1-\theta_p \pi_t^{\frac{1}{\lambda_p}}}{1-\theta_p} \right)^{-\lambda_p} \tag{8}$$

$$X_{1t}^p = \lambda_t y_t m c_t + \theta_p \beta \pi_{t+1}^{\frac{1+\lambda_p}{\lambda_p}} X_{1t+1}^p \tag{9}$$

$$X_{2t}^p = \frac{1}{1+\lambda_p} \lambda_t y_t + \theta_p \beta \pi_{t+1}^{\frac{1}{\lambda_p}} X_{2t+1}^p \tag{10}$$

Wage-Setting

$$\frac{X_{1t}^w}{X_{2t}^w} = \left(\frac{1-\theta_w (\pi_t^w)^{\frac{1}{\lambda_w}}}{1-\theta_w} \right)^{-\lambda_w + (1+\lambda_w)\nu} \tag{11}$$

$$X_{1t}^w = \omega {L_t}^{1+\nu} + \theta_w \beta \left((\pi_{ss}^w)^{1-\iota_w} (\pi_{W,t})^{\iota_w} \right)^{-\frac{(1+\lambda_{w,t+1})(1+\nu)}{\lambda_{w,t+1}}} \pi_{W,t+1}^{\frac{(1+\lambda_w)(1+\nu)}{\lambda_w}} X_{1t+1}^w \tag{12}$$

$$X_{2t}^w = \frac{1+\tau^w}{1+\lambda_w} \lambda_t w_t L_t + \theta_w \beta \left((\pi_{ss}^w)^{1-\iota_w} (\pi_{W,t})^{\iota_w} \right)^{\frac{-1}{\lambda_{w,t+1}}} \pi_{W,t+1}^{\frac{1}{\lambda_w}} X_{2t+1}^w \tag{13}$$

$$\pi_{W,t} = \frac{w_t}{w_{t-1}} \pi_t (1+g_t) \tag{14}$$

Capital Investment

$$k_{t+1}^u = \left[1 - S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}} \frac{1+g_t}{1+g_{ss}} \right) \right] \mathbb{I}_t + (1-\delta_k) \frac{k_t^u}{1+g_t} \tag{15}$$

$$q_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t (1+g_{t+1})} \left(r_{t+1}^K u_{t+1} - a(u_{t+1}) + q_{t+1}(1-\delta_k) \right) \right] \tag{16}$$

$$q_t \left[1 - S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{1+g_t}{1+g_{ss}}\right) - S'\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{1+g_t}{1+g_{ss}}\right)\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{1+g_t}{1+g_{ss}}\right]$$
$$+\beta\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}\frac{1+g_{t+1}}{1+g_{ss}}\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t}\right)^2 S'\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t}\frac{1+g_{t+1}}{1+g_{ss}}\right) = 1 \tag{17}$$

Capital utilization rate

$$k_t = u_t\frac{k_t^u}{1+g_t} \tag{18}$$

$$r_t^K = a'(u_t) \tag{19}$$

Production Technologies

$$y_t = k_t^\alpha L_t^{1-\alpha} \tag{20}$$

$$r_t^k = \alpha\frac{y_t}{k_t} \tag{21}$$

$$w_t = (1-\alpha)\frac{y_t}{L_t} \tag{22}$$

Endogenous Growth

$$g_t = \log Z_t - \log Z_{t-1} = \mu_t + \eta\log(y_{t-1}/y_{t-1}^f) \tag{23}$$

Government

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi}\left(\frac{y_t}{y_t^f}\right)^{\phi_y}\right]^{1-\rho_R}\left(\frac{y_t/y_{t-1}}{y_t^f/y_{t-1}^f}\frac{1+g_t}{1+g_t^f}\right)^{\phi_{dy}}\epsilon_t^{mp} \tag{24}$$

Market Clearing

$$y_t = c_t + \mathbb{I}_t + a(u_t)\frac{k_t^u}{1+g_t} + \left(1 - \frac{1}{\lambda_t^g}\right)y_t \tag{25}$$

Shocks

$$\log\lambda_t^g = (1-\rho_g)\lambda^g + \rho_g\log\lambda_{t-1}^g + \epsilon_t^g; \quad \epsilon_t^g \sim N(0,\sigma_g) \tag{26}$$

$$\epsilon_t^{mp} \sim N(0,\sigma_r) \tag{27}$$

Steady State

$$1 = \beta\frac{1}{(1+g)}\frac{1+i}{\pi}$$
$$\lambda = \frac{(1+g)-h\beta}{c(1+g-h)}$$
$$mc = \frac{1}{1+\lambda_p}$$
$$\omega L^{1+\nu} = \frac{1+\tau^w}{1+\lambda_w}\lambda wL$$
$$\pi^w = \pi(1+g)$$

$$q = 1$$

$$u = 1$$

$$(1 - \frac{1-\delta_k}{1+g})k^u = \mathbb{I}$$

$$1 = \beta \left[\frac{1}{1+g} \left(r^K + (1-\delta_k) \right) \right]$$

$$k = \frac{k^u}{1+g}$$

$$r^K = a'(1)$$

$$y = k^\alpha L^{1-\alpha}$$

$$r^k = \alpha \frac{y}{k}$$

$$w = (1-\alpha)\frac{y}{L}$$

$$g = \mu$$

$$y = c + \mathbb{I} + \left(1 - \frac{1}{\lambda^g}\right) y$$

Approximate Equilibrium

We log-linearize the variables around the steady state as follows: for any variable x,

$$\hat{x}_t = \log\left(\frac{x_t}{x}\right)$$

where x is the steady state, except for the following variables

$$\hat{g}_{t+1} \equiv \log\left(\frac{1+g_{t+1}}{1+g}\right)$$

$$\hat{i}_t = \log\left(\frac{1+i_t}{1+i_{ss}}\right)$$

Definition 2 (Approximate Equilibrium)**.** An approximate competitive equilibrium in this economy with endogenous growth is defined as a sequence of 16 endogenous variables $\{\hat{\lambda}_t, \hat{g}_{t+1}, \hat{i}_t, \hat{\pi}_t, \hat{c}_t, \hat{\pi}_t^w, \hat{mc}_t, \hat{y}_t, \hat{L}_t, \hat{w}_t, \hat{k}_{t+1}^u, \hat{\mathbb{I}}_t, \hat{q}_t, \hat{r}_t^K, \hat{k}_t, \hat{u}_t\}$, which satisfy the following equations, for a given sequence of one exogenous shock $\{\hat{\epsilon}_t^{mp}\}$ and given sequence of flexible price allocations.

Consumption Euler Equation

$$(\mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{g}_{t+1}) + \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} = 0 \tag{28}$$

$$\hat{\lambda}_t = \frac{1+g}{1+g-h\beta}\left[\hat{g}_t - \left(\frac{1+g}{1+g-h}(\hat{c}_t+\hat{g}_t) - \frac{h}{(1+g)-h}\hat{c}_{t-1}\right)\right] + \mathbb{E}_t\frac{h\beta}{(1+g)-h\beta}\left[\left(\frac{1+g}{1+g-h}(\hat{c}_{t+1}+\hat{g}_{t+1}) - \frac{h}{(1+g)-h}\hat{c}_t\right)\right] \tag{29}$$

Price-Setting

$$\hat{\pi}_t = \frac{\beta}{1+\iota_p\beta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\iota_p\beta}\hat{\pi}_{t-1} + \kappa_p\hat{mc}_t \tag{30}$$

where $\kappa_p = \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p(1+\iota_p\beta)}$.
Wage-Setting

$$\hat{\pi}^w_t = \frac{\beta}{1+\iota_w\beta}\mathbb{E}_t\hat{\pi}^w_{t+1} + \frac{\iota_w}{1+\iota_w\beta}\hat{\pi}^w_{t-1} + \kappa_w[-\hat{\lambda}_t + \nu\hat{L}_t - \hat{w}_t] \tag{31}$$

where $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\iota_w\beta)(1+\nu\left(1+\frac{1}{\lambda_w}\right))} > 0$

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t + \hat{g}_t \tag{32}$$

Capital Investment

$$\hat{k}^u_{t+1} = \frac{\mathbb{I}}{k^u}\hat{\mathbb{I}}_t + \frac{1-\delta_k}{1+g}\left[\hat{k}^u_t - \hat{g}_{t+1}\right] \tag{33}$$

$$\hat{q}_t = \left[\mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{g}_{t+1}\right] + \frac{r^K}{r^K+(1-\delta_k)}\hat{r}^K_{t+1} + \frac{1-\delta_k}{r^K+(1-\delta_k)}\mathbb{E}_t\hat{q}_{t+1} \tag{34}$$

$$\hat{q}_t - S''\left(\hat{\mathbb{I}}_t - \hat{\mathbb{I}}_{t-1} + \hat{g}_t\right) + \beta S''\left(\hat{\mathbb{I}}_{t+1} - \hat{\mathbb{I}}_t + \hat{g}_{t+1}\right) = 0 \tag{35}$$

parametrize S'' from Smets and Wouters (2007), or Justiniano et al. (2013) to be between 3 and 5.
Capital utilization rate

$$\hat{k}_t = \hat{u}_t + \hat{k}^u_t - \hat{g}_t \tag{36}$$

$$\hat{r}^K_t = \frac{a''(1)}{a'(1)}\hat{u}_t \tag{37}$$

Production Technologies

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)\hat{L}_t \tag{38}$$

$$\hat{r}^k_t = \hat{y}_t - \hat{k}_t \tag{39}$$

$$\hat{w}_t = \hat{y}_t - \hat{L}_t \tag{40}$$

Endogenous Growth

$$\hat{g}_t = \hat{\mu}_t + \eta(\hat{y}_{t-1} - \hat{y}^f_{t-1}) \tag{41}$$

where η is the hysteresis elasticity

Government

$$\hat{i}_t = \rho_R \hat{i}_{t-1} + (1-\rho_R)\left[\phi_\pi \hat{\pi} + \phi_y(\hat{y}_t - \hat{y}_t^f)\right] + \phi_{dy}\left[(\hat{y}_t - \hat{y}_{t-1}) - (\hat{y}_t^f - \hat{y}_{t-1}^f) + (\hat{g}_t - \hat{g}_t^f)\right] + \hat{\epsilon}_t^{mp} \quad (42)$$

Market Clearing

$$\frac{1}{\lambda^g}\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{\mathbb{I}}{y}\hat{\mathbb{I}}_t + \frac{a'(1)k}{y}\hat{u}_t \quad (43)$$