An Instrumental Variable Approach to Dynamic Models

work in progress, comments welcome

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Introduction

The endogeneity of market structure is a common theme in IO, but many recent approaches to firm and industry dynamics assume away serial correlation, resulting in econometrically endogenous market structure. We propose a Generalized IV approach to the identification/estimation of dynamic models in IO.

Some IO Examples

State, x_{it}	Action, a_{it}	$\mathcal{A}(x_{it})$	Transition
Capital	Investment	$\mathbb{R}^+$	$x_{it+1} = \lambda x_{it} + a_{it}$
Out/In	Entry/Exit	$\{0, 1\}$	$x_{it+1} = a_{it}$
Retail	# of Stores	$\mathcal{I}^+$	$x_{it+1} = a_{it}$
Quality	R&D	$\mathbb{R}^+$	$x_{it+1} \sim f(x_{it}, a_{it})$

Dynamics: Standard "CCP" Practice

- Much of the empirical IO literature models unobservables as
  - Private information
  - IID over time
- This simplifies estimation, as outlined in several influential papers
  - e.g. Rust (1987), Hotz and Miller (1993), Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007), Pesendorfer and Schmidt-Dengler (2008)

Standard Practice (cont'd)

- IID private shocks $\Rightarrow$ current state is econometrically exogenous.
- Policy function:

$$a_{it} = \sigma\left(x_{it}, u_{it}\right), \qquad x_{it} \perp u_{it}$$

- State does not reflect any persistent unobserved factors, only accumulated past luck

Standard Practice (cont'd)

$$a_{it} = \sigma\left(x_{it}, u_{it}\right), \qquad x_{it} \perp u_{it}$$

- This leads to two-step approaches à la Hotz-Miller
  1. obtain policy function via a (generalized) regression of a on x
  2. use this + Bellman's equation to identify structural parameters

Problems with Exogenous States

- Unobservables (e.g. market profitability) are likely persistent
- This implies that the current state will likely be endogenous

$$a_{it} = \sigma\left(x_{it}, u_{it}\right), \qquad x_{it} \not\perp u_{it}$$

- Similar endogeneity problems have been tackled in the literature on static models using IVs
- Goal: extend this to dynamic models

Existing approaches to serial correlation

Full cites and discussion in the paper.

- Discrete heterogeneity (*e.g.* two "types") in two-step methods. Helps particularly with the initial conditions problem and with computation.
- Full solution methods have to impose an initial condition model and have to pick an equilibrium in oligopoly models.
- Kalouptsidi, Scott, and Souza-Rodrigues (2018) have a similar IV logic in a special case model.
- Related large literatures include state dependence vs. persistent heterogeneity & initial conditions problems in panel data.

No existing systematic approach to identification, especially to flexible initial conditions, general models, multiple equilibria and general forms of serial correlation.

An IV approach

Essentially, we follow the classic two steps of the CCP literature:

1. Identify/Restrict policy
2. Use this + Bellman to identify/restrict structural parameters

The innovation is that in step 1, we use generalized IV methods, "GIV", to allow for endogenous states. For identification, we rely on Chesher and Rosen (2017) and, by transitivity, on a much larger literature

- e.g. Manski and Tamer (2002), Tamer (2003), Manski (2003), Chernozhukov, Hong, and Tamer (2007), Berry and Tamer (2007), Ciliberto and Tamer (2009), Beresteanu, Molinari, and Molchanov (2011), Galichon and Henry (2011), Andrews and Shi (2013)

GIV

Accommodates models that are

- incomplete
- partially identified

due to, e.g., discrete outcomes, initial conditions problem, multiple equilibria, "not strong enough" IVs.

IV Intuition

- Past exogenous variation will be correlated with current states
  - If Detroit was large and rich 50 years ago, it may have many Sears stores today
  - Past macro shocks may impact today's market structure
  - Past regulatory regimes might have persistent effects
- In general, past exogenous cost and demand shifters are natural IVs

Generalized IV in a nutshell

- For any σ, define

$$\mathcal{U}(a, x, \sigma) \equiv \{u : a = \sigma(x, u)\}$$

  i.e. the inverse image set according to σ, given (a, x)

- Then, the identified set for the true σ_0 is given by all the functions σ that, for some θ_u, satisfy

$$\underbrace{\mathbb{P}\{\mathcal{U}(a, x, \sigma) \subset \mathcal{S}|z\}}_{\textit{data+policy+IV}} \leq \underbrace{\mathbb{P}\{U \in \mathcal{S}; \theta_u\}}_{\textit{model}}$$

  for all sets $\mathcal{S}$ in an appropriate collection

- The event on the rhs is a necessary condition for that on the lhs. In a complete discrete model, the $\leq$ become equalities that define MLE.

Monopolist entry example

- We start from a simple monopolist entry example
- We characterize the sharp ID set for the policy function via GIV when $T = 1, T = 2$
- This yields the ID set for the structural parameters
- We explore through simulations the effects of:
  - IV strength
  - Number of time periods in the data
  - Variation in covariates

Example: Single Firm "Entry / Exit"

A minimal model to think about dynamics. State is In/Out in the prior period

$$a_{it} \in \{0, 1\} \qquad x_{it} \in \{0, 1\}$$

Single-period profits $\pi(a_{it}, x_{it}, w_{it}, \epsilon_{it})$.

$$\begin{aligned}
\pi(0, x_{it}, w_{it}, \epsilon_{it}) &= 0 \\
\pi(1, 1, w_{it}, \epsilon_{it}) &= \bar{\pi}(w_{it}) - \epsilon_{it} \\
\pi(1, 0, w_{it}, \epsilon_{it}) &= \bar{\pi}(w_{it}) - \epsilon_{it} - \gamma
\end{aligned}$$

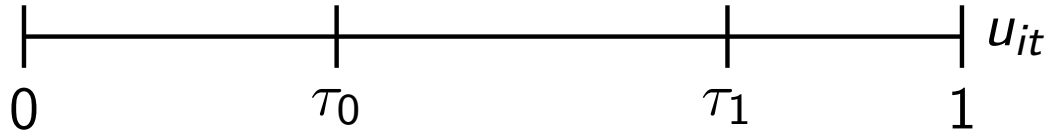
$$\epsilon_{it} = \rho \epsilon_{i,t-1} + \nu_{it}, \quad \nu_{it} \sim \mathcal{N}(0, 1).$$

Define u_{it} as the quantile of ϵ_{it}.

Without variation in w_{it}, 3 structural parameters: $\bar{\pi}, \gamma, \rho$.

Restrictions Using Only One Period

Chesher (2010), etc.

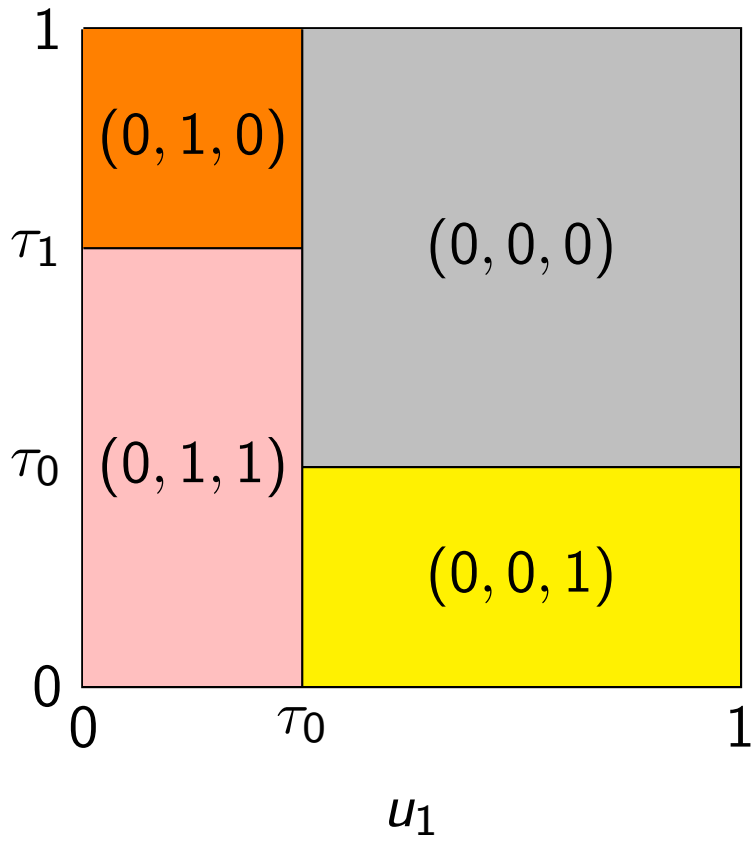


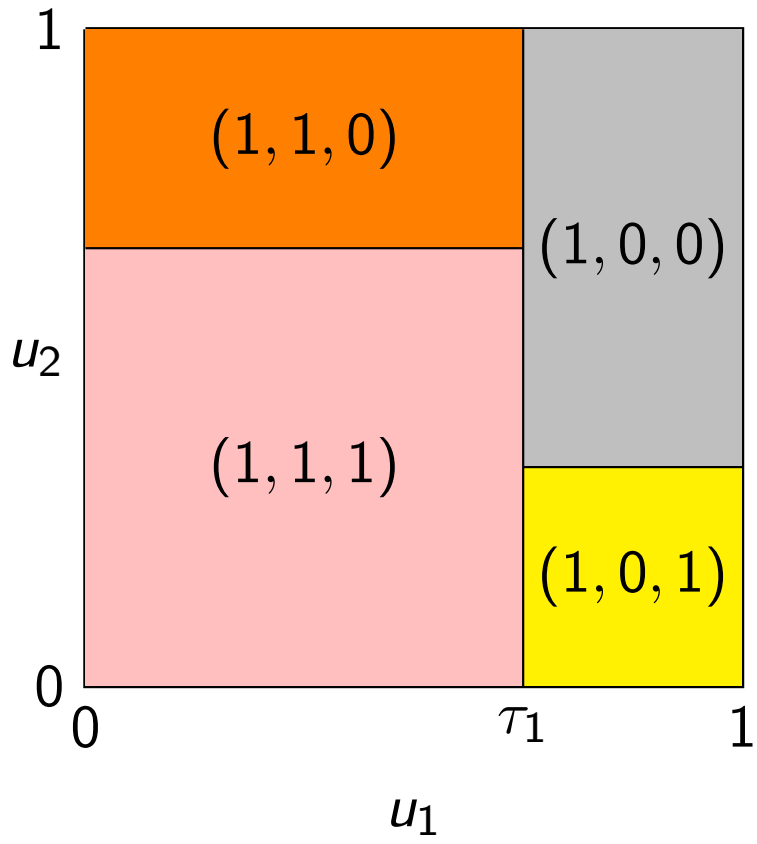
a	x	$\mathcal{U}(a,x,\sigma)$
1	1	$(0,\tau_1)$
1	0	$(0,\tau_0)$
0	1	$(\tau_1,1)$
0	0	$(\tau_0,1)$

$\mathcal{S}$	$\mathbb{P}(\mathcal{U}(a,x,\sigma)\subseteq\mathcal{S}\mid z)$	$\leq \mathbb{P}(U\in\mathcal{S};\theta_u)$
$\mathcal{U}(1,1)$	$\mathbb{P}((1,1)\|z)+\mathbb{P}((1,0)\|z)$	$\leq \tau_1$
$\mathcal{U}(1,0)$	$\mathbb{P}((1,0)\|z)$	$\leq \tau_0$
$\mathcal{U}(0,1)$	$\mathbb{P}((0,1)\|z)$	$\leq 1-\tau_1$
$\mathcal{U}(0,0)$	$\mathbb{P}((0,0)\|z)+\mathbb{P}((0,1)\|z)$	$\leq 1-\tau_0$

Elemental Inverse Image Sets, $T = 2$

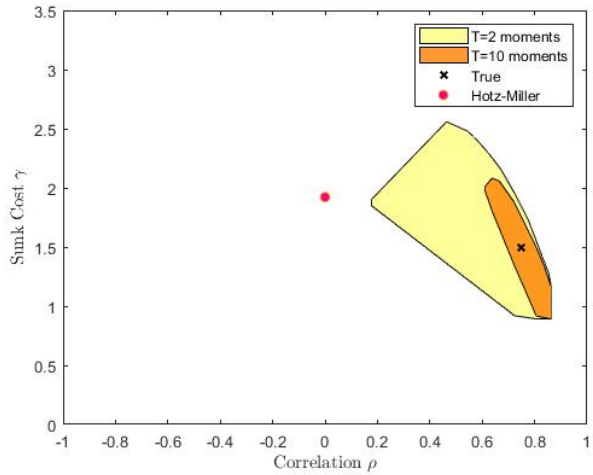
8 elemental sets $\mathcal{U}(a, x, \sigma)$, labeled (x_1, a_1, a_2)



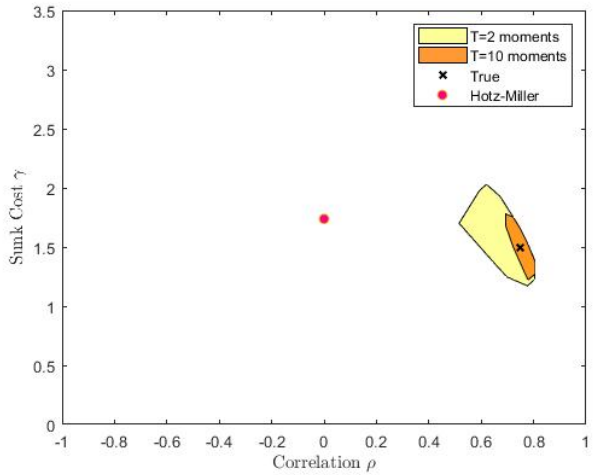


Computing the Identified Set of Structural Parameters

True $\rho = 0.75$, varying IV strength, T=2 or T=10



(a) IV strength = 0.25



(b) IV strength = 0.56

(ρ on the horizontal axis, sunk cost on the vertical)

Inference

- We apply the literature on moment inequalities for inference
- The literature on the "sharp identified set" is less useful for inference, no strict rules for what moment conditions to use & there can be very many possibilities.
- Since as T grows we have many inequalities relative to the sample size, we use Chernozhukov, Chetverikov, and Kato (2018)
- In Monte Carlos, we obtain confidence sets for
  - Structural parameters
  - Counterfactual subsidy to entry
- Inference method works well in the Monte Carlos

Dynamic Games

more preliminary

Policies now depend on the serially correlated unobservables of all players.

With serial correlation, complete information is much easier than serially correlated private info, but we can add iid private information on top of serially correlated public (as in PPHI), which makes computation (and existence) easier.

Broad Idea of the Games Approach

In common with the full solution approach, looking for [a] policy functions and [b] dynamic parameters that satisfy the best response condition.

The key advance is that we can restrict the computation to those strategies that survive the GIV conditions. In a favorable case, this would be a small set.

A computed oligopoly example is in the appendix, more research is needed.

Illustrative Empirical Application

- Small-town ready-mix concrete example of Collard-Wexler (2014)
- That paper considers exog change in N, answered by the policy function itself, with an assumption on initial conditions that closes the model.
- We consider a policy, like the environmental question in the cement example of Ryan (2012), that changes sunk and/or fixed cost. Not answered by the observed policy function.
- We allow for an incomplete model, with no assumptions on initial conditions. Does it hurt us a lot?

Empirical Example

- The number of concrete firms is shifted by "construction demand," proxied by local construction employment
- We use past income growth as IV
- We use the "Last-In-First-Out" model of Abbring and Campbell (2010)
  - Unique equilibrium (we hope)
  - Computationally equivalent to sequence of single-agent problems

Summary Statistics

Variable	Mean	St.Dev.	Min	Max
Number of plants	0.97	0.93	0.00	6.00
Construction Employment	519	819	3	17,772
Household Income Growth 1969-89	0.15	0.11	-0.16	0.69

Quasi-First State Ordered Probit: *N* of Plants

Log Construction Employment	0.14^{**}
Income Growth 1969-1989	0.22^{**}
Likelihood-Ratio Test p-value	0.00

Flow Profit Function

from an Abbring-Campbell example

$$\pi_{it} = \begin{cases} \alpha_{x_{it}} w_i - \beta + u_{it} & \text{if was in at } t-1, \text{ stays in at } t \\ \alpha_{x_{it}} w_i - \beta - \gamma + u_{it} & \text{if was out at } t-1, \text{ enters at } t \\ 0 & \text{if is out at } t \end{cases} \quad (1)$$

It would be better if static profits were estimated from data on price & qty, leaving only sunk cost (γ), fixed cost (β), and the degree of serial correlation (ρ) for the dynamic estimation.

Summary statistics. 428 markets, 1994–2005

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Quasi-First Stage Ordered probit results

Log Construction Employment	0.14**
Income Growth 1969-1989	0.22**
Likelihood-Ratio Test p-value	0.00

GIV vs MLE with $\rho \equiv 0$

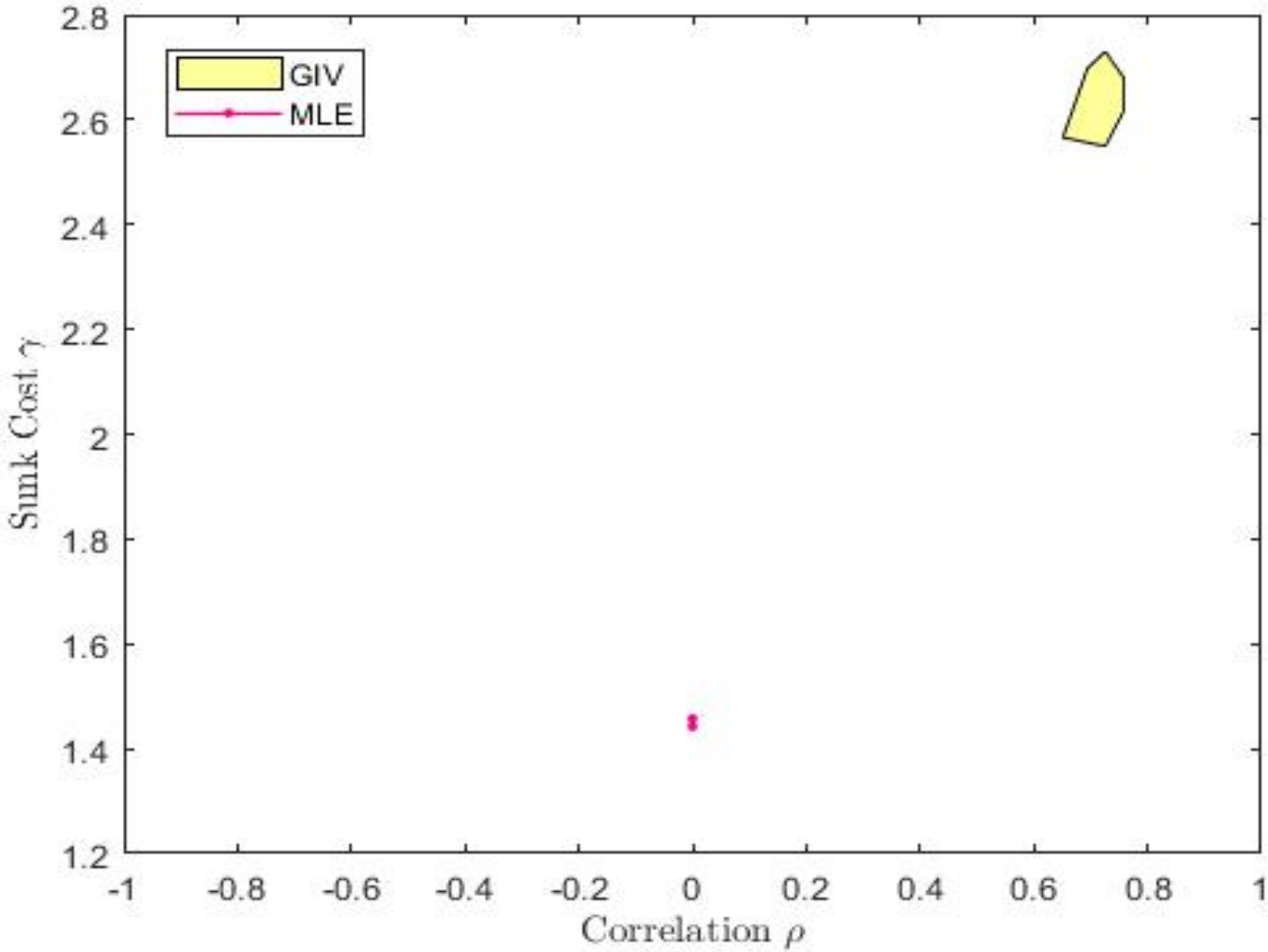


Figure: Autocorrelation (x-axis) and sunk cost (y-axis): Projection of 95% confidence sets

Counterfactual Change in Sunk Costs

after 5 years

	GIV	MLE $\rho = 0$
Change in total # of firms	(-0.09,-0.03)	(-0.18,-0.15)
Change in fraction of new firms	(-0.05,-0.01)	(-0.15,-0.14)

Table: Increase in sunk cost (95% confidence intervals)

In the results allowing serial correlation, few markets are near the margin of entry/exit and this is persistently true, so the policy has a relatively small effect on firm behavior. Incomplete model with GIV still leads to relatively precise estimates of counterfactual policy.

Conclusion

- Market structure in dynamic IO models should not be assumed to be exogenous
- GIV methods allow us to
  - deal with endogenous states
  - preserve the intuition of existing two-step methods
- IV Intuition: past exogenous demographics/regulation/shocks are correlated with today's state

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