Unequal Growth*

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VERY PRELIMINARY AND INCOMPLETE

Abstract

This paper argues that changes in household income dynamics in the Unite States over the past 50 years can account, at the same time, for the increase in income inequality and for a significant portion of the US slowdown in aggregate growth. We first apply, using US household panel data for the period 1967-2014, a simple statistical decomposition showing that aggregate growth is the sum of average growth across households plus the covariance between income growth income and levels. The data shows that, in a statistical sense, most of the growth slowdown is accounted by a fall in the covariance between income levels and income growth. It also shows that the fall in covariance is the result of income inequality increasing, coupled with a negative correlation between income growth and income levels. Second, we develop a simple structural model of household income dynamics. We introduce changes to income dynamics that are qualitatively consistent with globalization: in recent years it is harder for any household to experience sustained income growth, but, if it does so, it grows faster than in earlier years. These changes can generate patterns of inequality, aggregate growth and co-movement between growth and levels consistent with US micro and macro data.
JEL Classification Numbers: D31, O4

Key Words: Income distribution, Gibrat's law, Inequality, aggregate growth, Pareto distribution, speed of transition.

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1 Introduction

Over the past five decades, two changes in US macroeconomic landscape stand tall: one is an increase in household income inequality, the other is a slowdown in aggregate growth. This paper argues that these two changes are connected. The foundation for the connection is that for households, Gibrat's law does not hold, that is income growth is very "unequal" across the income distribution. If there are changes to fundamentals that drive a change in the income distribution (i.e. the increase in income inequality), these changes will also naturally result in a change in aggregate growth (i.e. the growth slowdown). Our main conclusion is that there is a unified explanation, namely a change in the underlying dynamics of household income, that can go a long way in accounting for the increase in inequality and the slowdown in growth. We articulate our point in several steps. First we present a simple statistical decomposition showing that aggregate growth can be written as the average of household/individual growth, plus the cross sectional covariance between growth and income levels. Higher covariance means high income households grow faster, and this results in faster aggregate growth. This decomposition suggests that a growth slowdown might arise either because the average of individual growth falls, or because the covariance between individual income growth and individual income level falls. We then use data from PSID over the period 1967-2014 to show that the slowdown in aggregate growth is mostly accounted (in a statistical sense) by the fall in covariance between individual growth and individual income level. In order to understand the fall in covariance, we further decompose the covariance as the product of the correlation between growth and level (a measure of rank mobility), the standard deviation of income levels (a measure of income inequality) and the standard deviation of income growth rates (a measure of income instability). The same PSID data reveals that the fall in covariance is mostly a result of income inequality increasing, coupled with a negative correlation between income growth and income levels. To be more precise the data shows that, at each point in time, low income households grow, on average, faster than high income households. The data also shows that, over time, high income households

have a larger share of income and low income have a lower share (i.e. there is an increase in income inequality). Putting these two facts together implies that in recent years the US economy has more income concentrated in the hands of high income slow growers and less income in the hands of the poor fast growers. Since the high income slow growers are larger, this results in a slowdown in growth. This discussion should clarify that in a world where low income households grow faster than high income households, an increase in income inequality is associated, ceteris paribus, to a slowdown in aggregate growth. However, it should also be clear that these data alone cannot help us to understand the fundamental changes that drive, at the same time, the increase in inequality and the slowdown in growth. For this reason in the second part of the paper we present a simple framework to assess whether changes to individual earning dynamics that can produce, at the same time, a fall in aggregate growth and the patterns in income inequality, rank mobility and income instability we document in the data. The set-up models individuals entering the labor markets being able to produce an initial income y_0. With a constant probability these individuals come across a successful job/project, that enables them to grow at a constant rate over time. When on a successful growth path, individual might lose their job/project and fall back to y_0. We first show that this set-up can generate an income distributions and a distribution of growth over levels that matches the data. We then show that three key changes are necessary to match the documented changes in US micro and macro data. The first is a reduction of the probability of success, the second one is a faster growth rate of individuals/household which are on the fast growth path, and the third is a fall in the initial level of income y_0. These three changes can be interpreted as a stylized version of globalization. In a global world it might more difficult to land a successful job/projet, but conditional on success, the payoffs are higher (growth faster). We conclude that

Literature Review To be Completed Gabaix et al. (2016); Benhabib and Bisin (2016); Jovanovic (2014); Atkinson, Piketty, and Saez (2011); Jones and Kim (2014); Cortes, Jaimovich, and Siu (2018); Luttmer (2011); Arkolakis (2016); Lucas (2000); Olley and Pakes (1996);

Chetty et al. (2014); Kopczuk, Saez, and Song (2010); Benabou (1996); Guvenen et al. (2015)

2 A micro decomposition of aggregate growth

In this section we present a simple statistical decomposition that connects aggregate income growth to micro-level (household orb individual) income growth, cross sectional income inequality, and the cross sectional correlation between income growth and income level. These types of decompositions have been widely used in industrial organization to connect sectoral productivity growth to productivity growth in individual firms (see, among others, Olley and Pakes 1996). We find it useful to apply this decomposition to household level data (as opposed to firms), because it connects aggregate growth with household income inequality, which has a more direct welfare content than firms income inequality.

Let y_{it} be level of income of household/individual i at time t. Let Γ_{t+T} be the economy's aggregate growth over an horizon T, which is

$$\Gamma_{t+T} = \frac{E_i(y_{it+T})}{E(y_{it})} = E\left(\frac{y_{it+T}}{y_{it}}\frac{y_{it}}{E(y_{it})}\right)$$

where $E(.)$ is the cross sectional average. Now define

$$g_{i,t+T} \equiv \frac{y_{it+T}}{y_{it}} \quad , \quad s_{i,t} \equiv \frac{y_{it}}{E(y_{it})}$$

so that $\Gamma_{t+T} = E(g_{i,t+T} \cdot s_{i,t})$ where $g_{i,t+T}$ is income growth of unit i and $s_{i,t}$ the ratio between income of unit i and average income. Then, using the definition of covariance and the fact that $E(s_{i,t}) = 1$ we get

$$\Gamma_{t+T} = cov(g_{i,t+T}, s_{i,t}) + E_i(g_{i,t+T}) \tag{1}$$

or equivalently

$$\Gamma_{t+T} = corr(g_{i,t+T}, s_{i,t})\sigma(s_{i,t})\sigma(g_{i,t+T}) + E(g_{i,t+T}) \tag{2}$$

Equation 1 suggests that what matters for aggregate growth is not only the (un-weighted) average individual growth $E(g_{i,t+T})$ but the distribution of growth opportunities, as summarized by $cov(g_{i,t+T}, s_{i,t})$. The intuition for why this is the case is straightforward: the higher the covariance, the faster higher income individuals grow; since they are high income they contribute more to aggregate growth and aggregate growth is higher. Equation 2 also suggests that $cov(g_{i,t+T}, s_{i,t})$ is linked to three cross sectional moments that have a natural economic interpretation. The first, $corr(g_{i,t+T}, s_{i,t})$, is the correlation between level and growth at the individual level. This measure captures the degree of mean reversion (or economic rank mobility) in individual income dynamics. The second, $\sigma(s_{i,t})$ is the standard deviation of $s_{i,t}$, which is essentially a measure of cross sectional income inequality. The third, $\sigma(g_{i,t+T})$, is the standard deviation of the growth rate of individual income, which is a measure of cross sectional income volatility. Equation 2 also suggests that changes in any of these three quantities will be associated, *ceteris paribus*, with changes in aggregate growth. If, for example, $corr(g_{i,t+T}, s_{i,t}) < 0$ an increase in cross sectional inequality will be associated with a reduction in $cov(g_{i,t+T}, s_{i,t})$ and thus in aggregate growth. It is useful at this point to note that this decomposition is a statistical identity, so it cannot be used to make causal inferences on growth and inequality. Nevertheless it highlights that these two statistics are connected and we find it useful to document how this connection has evolved over time. In particular the main goal of the next section is to document whether the slowdown in aggregate growth in the U.S. has been associated to a slowdown in individual growth $E(g_{i,t+T})$ or to a change in the distribution of growth opportunities.

3 A decomposition of US growth: 1967-2014

Equations 1 and 2 involve both cross sectional moments and moments related to individual income growth, so in order to bring them to the data we need panel data on household/individual income. Since our main focus is aggregate growth in the United States we also want a panel which captures well aggregate US growth. For these reasons we work with the Panel Study of Income Dynamics (PSID), which is a panel of US households, selected to be representative of the whole population, collected from 1967 to 2014. Figures 1 reports aggregate growth in per capita labor income both in the PSID and the National Income and Product Accounting (NIPA).[1] The solid lines report the actual annualized growth (computed over a 4 years horizon), while the dotted lines are polynomial trends. The figure shows that growth in PSID tracks growth in NIPA quite closely. Also, importantly for our purposes, both sources show, a marked decline in long run growth over the past 20 years, as highlighted by the trends.

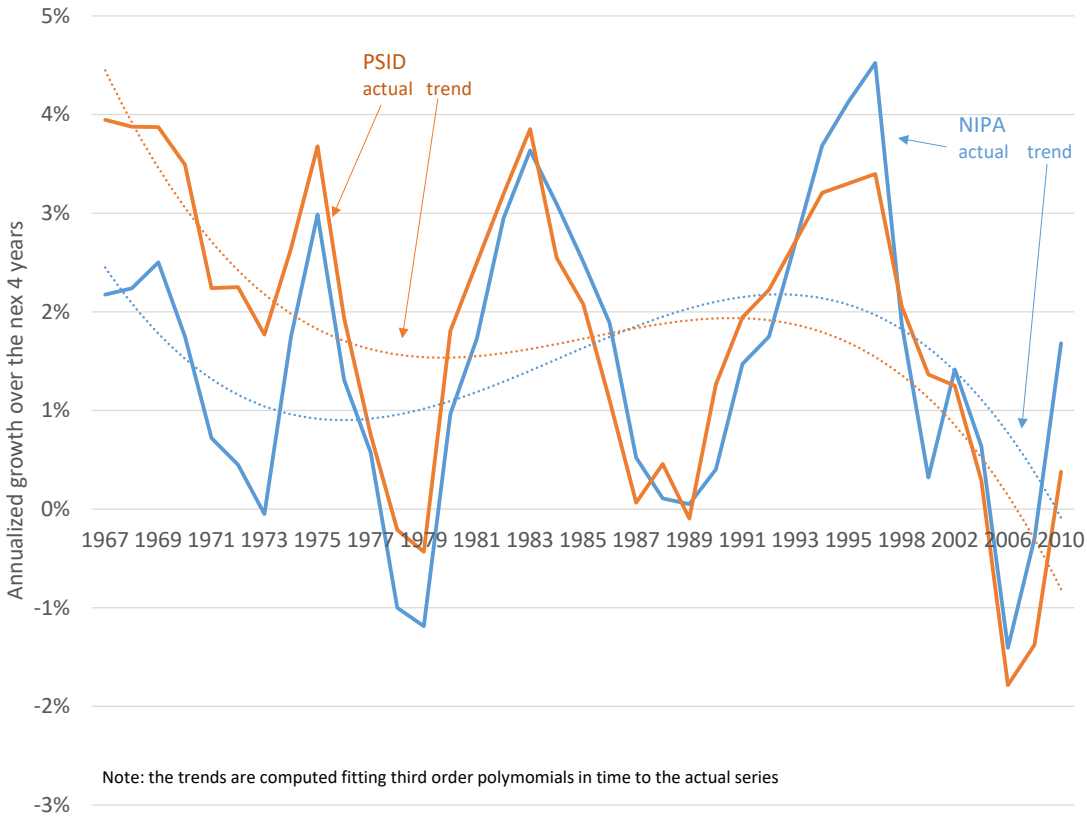
Figure 2 also shows that the PSID captures well the patterns of US household income inequality, as documented in from a much larger cross sectional survey, i.e. the March Current Population Survey. The figure plots a common measure of inequality, the 90/20 ratio in household labor income, derived from the two surveys.[2] It shows that both surveys capture the well known secular increase in income inequality in the United States. Again, relevant for our purposes, it is the fact that inequality markedly increases over the past 20 years, the same exact period in which aggregate growth slows down.

Since figures 1 and 2 show that data in PSID capture well the evolution of aggregate growth and inequality, we now proceed to compute the data equivalent in PSID of equations

[1]The income measure in PSID is total wage and salary income plus 50% of business income for each household in the sample, divided by the total number of persons in the sample. The income measure in NIPA is compensation of employees, wages and salaries disbursement plus 50% of proprietors income, per capita. All measures are deflated using PCE deflator. See the data appendix for more details on data construction and for similar figures for different (narrower and broader) income measures.

[2]The income measure in both PSID and CPS is total wage and salary income plus 50% of household business and farm income. Inequality measures are computed for households with heads between age 25 and 60. The average sample size in the PSID is around 4000 household per year, the size in CPS is 10 times larger.

Figure 1: Growth in labor income: NIPA and PSID

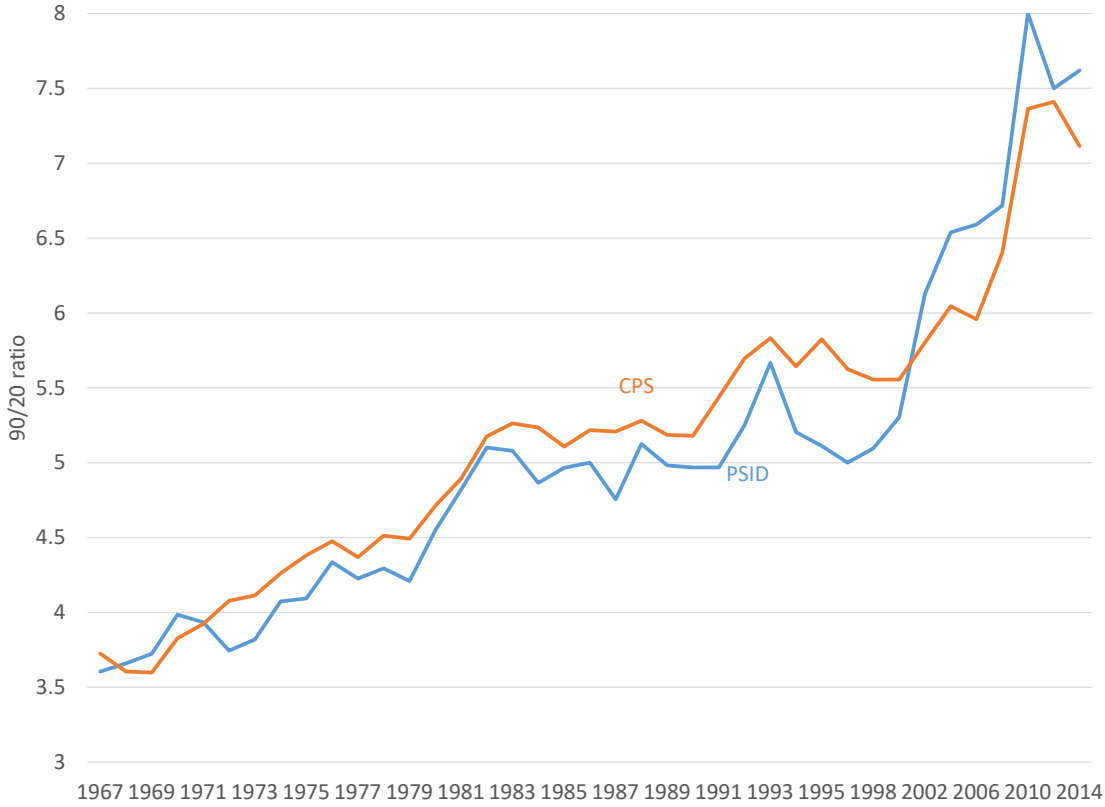


1 and 2. Figure 3 shows the growth decomposition suggested by equation **??**, where, in order to reduce noise due to measurement error in individual income, we aggregate households in 10 deciles.[3]The line labelled Γ_{t+T} reports aggregate growth rate (annualized) over 4 years for our PSID sample. The lines labelled $E(g_{i,t+T})$ report the (unweighted) average of the growth rate across deciles in our sample, and finally the line labelled $cov(g_{i,t+T}, s_{i,t})$ reports the covariance the between the growth and the normalized level.

The upshot from 3 is that the decline in aggregate growth, displayed by the declining trend in Γ_{t+T} is mostly accounted, in a statistical sense, by the decline in covariance between growth and levels, that is $cov(g_{i,t+T}, s_{i,t})$, while the un-weighted average of growth rates in each decile, $E(g_{i,t+T})$, is, over the long run, roughly unchanged. To get some better intuition

[3]Formally let I_t by the group of households who are in the i_{th} decile of the income distribution in period t. We define $g_{i,t+T} = \frac{\sum_{i \in I_t} y_{i,t+T}}{\sum_{i \in I_t} y_{i,t}}$, that is the growth rate of income in a given decile is computed using the same group of households in t and $t+T$.

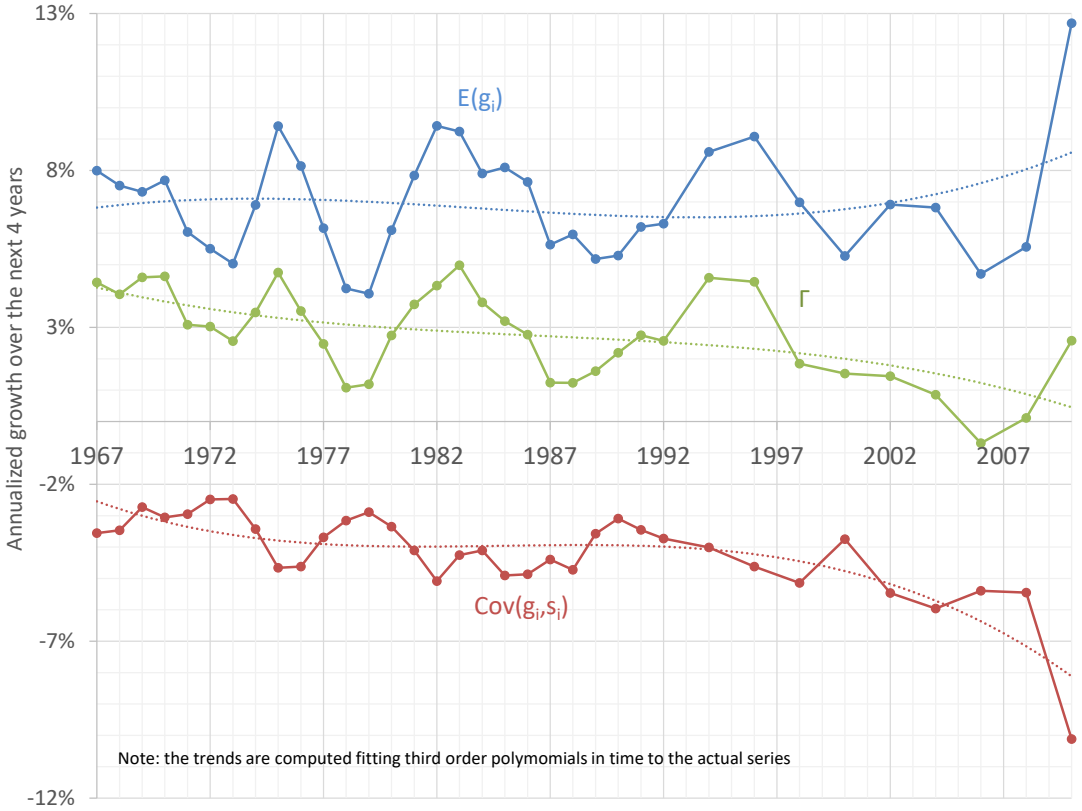
Figure 2: Inequality in labor income: PSID and CPS



for this finding, in panel (a) of figure 4 we plot the average 4 years growth in each decile for the first four years of the PSID sample v/s the 4-years growth in the last four years of the sample for which we can compute growth.[4] The panel shows that the un-weighted average of the $g_{i,t+T}$ (i.e. the area under the two curves) is roughly constant across the two subperiods (7.6% v/s 7.4%). Perhaps surprisingly, constant average results from the bottom deciles growing faster, and from the middle and top deciles growing slower. Panel (b) plots the share of average income in each decile (that is the s_i). The panel shows the increase inequality over the period, as reflected in the shares of the bottom deciles falling and the shares of the top deciles increasing. Finally panel (c) puts the information in the first two panels together and plots the decile growth rates, weighted by their respective shares, that is $g_{i,t+T}s_{i,t}$. The area under the lines offer a visual description of the decline in aggregate

[4]The first 4 years are 1967-1970, while the last 4 years for which we can compute growth are 2004,2006,2008 and 2010. We took average over four samples to smooth out cyclical components in the growth.

Figure 3: A decomposition of US aggregate growth

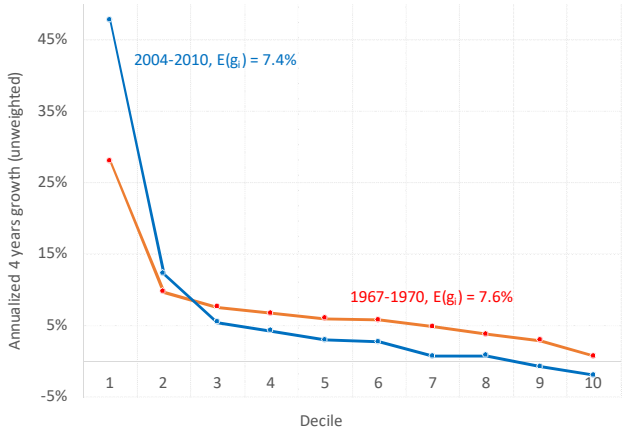


growth. Comparing growth rates (panel a) and shares (panel b) shows why growth in the later year of the sample is much weaker. The four lines in panels a and b all have the same average, yet the product of the 1967-70 lines is much higher than the product of the 2004-10 lines. This is because the covariance between the two lines has become more negative. The fast growing low deciles now have (because higher inequality) lower weight, thus dragging down growth. Similarly the slow growing high deciles now receive higher weight, similarly contributing to the reduction in growth.

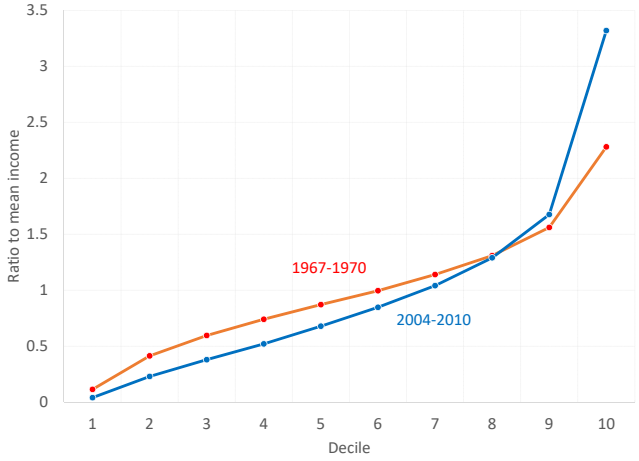
So far we have established that in a statistical sense the slowdown in growth is associated to a reduction in the covariance between growth and level. Figure 5 further decomposes the trend in the covariance, using equation 2. The figure shows that the fall in the covariance is the result of two off-setting trends. On one hand the correlation between growth and levels $(corr(g_i, s_i))$, which, in the beginning of the sample is around -0.8, becomes less negative. This would result, *ceteris paribus*, in an increase in the covariance. On the other

Figure 4: Growth and inequality by deciles of the income distribution

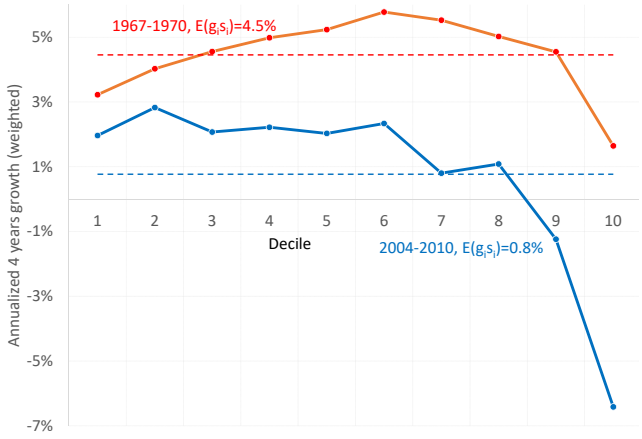
(a) Growth by decile



(b) Share of income by decile

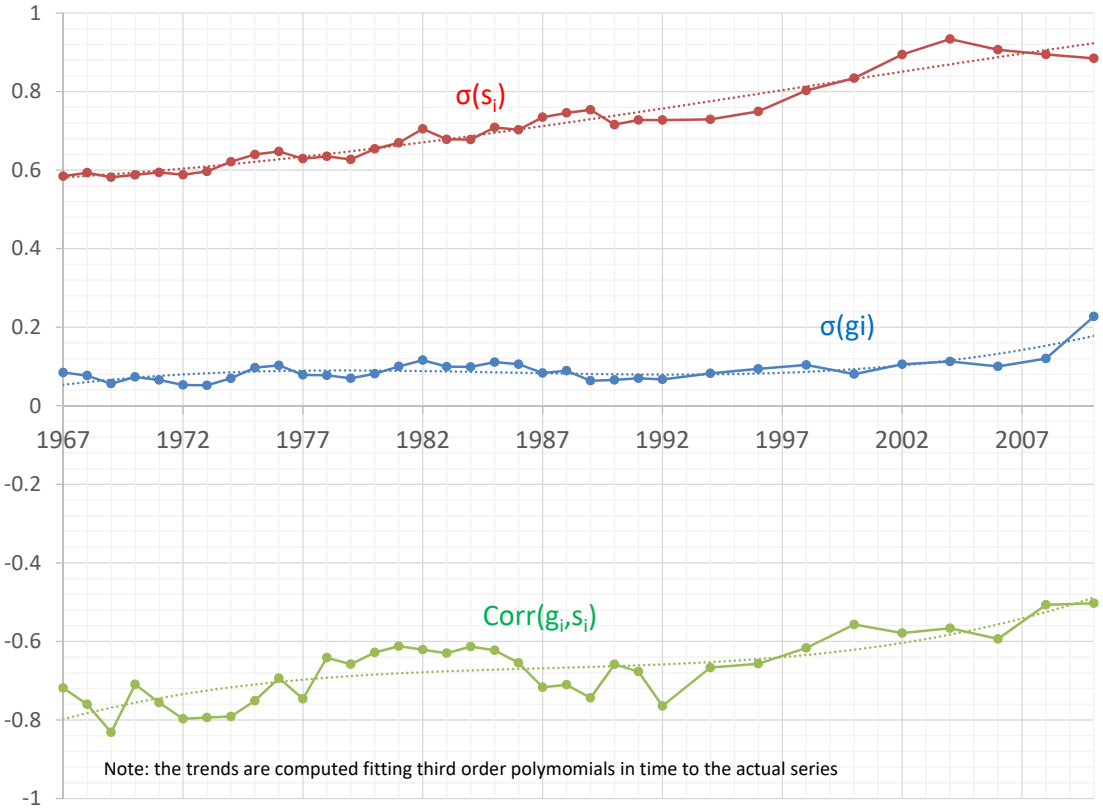


(c) Contributions to aggregate growth



hand the fact that income inequality ($\sigma(s_i)$) has increased, together with the fact that the correlation is negative, implies a decline in the covariance. Overall the increase in inequality dominates and thus a reduction of the covariance is observed. Nevertheless the increase (fall in absolute value), of the correlation is an important feature of the data. In particular it shows that, together with the increase in inequality, US households have experienced a substantial reduction in rank mobility, i.e. in recent years it is less likely for low income households to experience strong growth.

Figure 5: Decomposing the decline in covariance



3.1 Extensions of empirical analysis

TO BE COMPLETED

- Controlling for age, education and other demographics
- Documenting the fact on administrative data (SIPP gold standard, 1980-2012)
- Documenting the decomposition on CPS panel data (1980-2017)

4 A simple model

This section presents a simple dynamic model of the distribution of incomes. The model is deliberately simple and, in the steady state, produces an invariant distribution of incomes that has a Pareto tail. In the steady state the model is associated with a constant aggregate income level. We exploit the simplicity of this model and study a transition from one invariant distribution to a new one, following the change of one or more fundamental parameters. The main aim of the model is to analytically illustrate how a shock to the parameters of the income process will lead to a change of the cross sectional income inequality as well as to a change, during the transition, of the economy's aggregate growth rate. The simplicity of the model has both advantages and disadvantages. The biggest advantage is that the model can be analytically solved to analyse the transition of the distribution of incomes following a fundamental shock, highlighting the various possibile shocks that trigger more inequality, as well as their differential consequences in terms aggregate growth. On the other hand, its simplicity makes it hard to take it to the aggregate data without further adjustments to accommodate more sources of heterogeneity.

The setup considers a cross section of agents, each of which can be matched with a business "project" producing an income y of different types. Agents without a project have income y_0 and are are matched with a new project at rate φ. Each project is destroyed at a rate δ, and as long as it remains alive the income from the project grows at rate γ so that a project surviving t periods yields the income $y(t) = y_1 e^{\gamma t}$. Let $\pi = \{1, 0\}$ denote the agent's state (with or without a project, respectively).

In steady state the economy has a fraction $\omega \equiv \delta/(\varphi + \delta) \in (0, 1)$ of agents with no projects, i.e. income y_0. Noting that the distribution of project durations is exponential with parameter δ, we compute the density of y conditional on $\pi = 1$ by a change of variables which gives

$$f(y) = \frac{\alpha\, y_1^{\alpha}}{y^{(\alpha+1)}} \quad \text{where} \quad \alpha \equiv \frac{\delta}{\gamma} \tag{3}$$

which is a Pareto distribution with CDF equal to $F(y) = 1 - \left(\frac{y_1}{y}\right)^{\alpha}$. For the distribution to have a finite mean and variance we must have $\alpha > 2$ in which case the mean income conditional on $\pi = 1$ is $\mathbb{E}\left(y\middle|\pi = 1\right) = \frac{\alpha}{\alpha-1}y_1$. Thus the mean income in the population is $\mathbb{E}(y) = y_0\omega + (1-\omega)\frac{y_1\alpha}{\alpha-1}$.

4.1 Steady state moments

Let $s(y) \equiv y/\mathbb{E}(y)$, such that $\mathbb{E}s(y) = 1$. Just like the distribution of incomes, the distribution of the $s(y)$ features a mass point ω at $s_0 = y_0/\mathbb{E}(y)$ and a Pareto distribution for $s \in (s_1, \infty)$ with parameter α where $s_1 = y_1/\mathbb{E}(y)$. We use the distribution $f(y)$ to compute the income levels that correspond to the deciles of the income distribution. For each decile we compute the mean income within the decile y_i, as was done in the data, and the corresponding income share $s(y_i)$. We will use the standard deviation of income shares across deciles σ_{s_i} for $i = 1, 2, ..., 10$, as our measure of income inequality.

Next we compute the expected income growth associated to an income level y over a time horizon of length T. To do so we need to compute the expected income level T periods ahead conditional on today's state $\pi = \{0, 1\}$ and income level. Let $M(y, T) \equiv \mathbb{E}(y(T)|y(0) = y)$ denote the expected value of income in T periods for an agent with current income y and $\pi = 1$. This is

$$M(y,T) = ye^{(\gamma-\delta)T} + \int_0^T \delta e^{-\delta s} m(T-s)ds \tag{4}$$

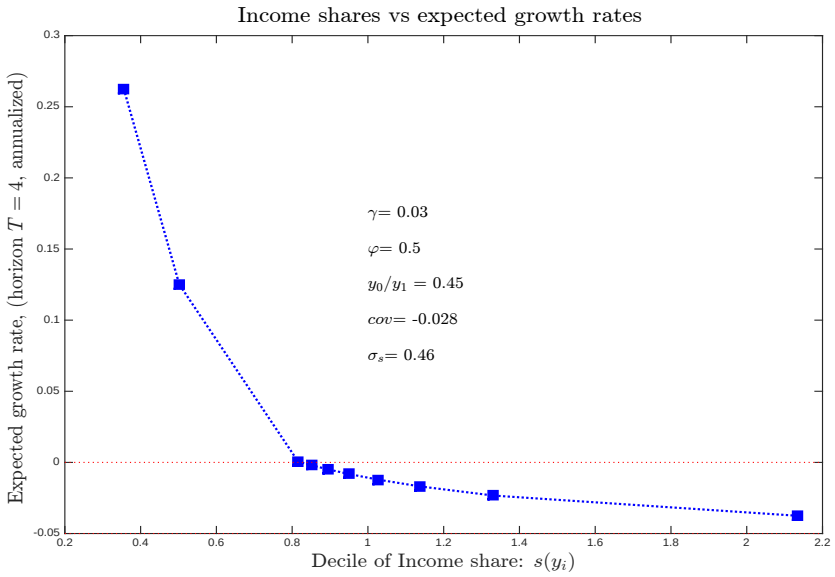
where $m(T) \equiv \mathbb{E}(y(T)|y(0) = y_0)$ is the expected value of income T periods from now for an agent whose current state is $\pi = 0$ (i.e. in a no growth state) or

$$m(T) = y_1 \int_0^T \varphi\,\theta(s) e^{(\gamma-\delta)(T-s)}ds + y_0\theta(T) \quad \text{where} \quad \theta(s) = \frac{\delta + \varphi e^{-(\varphi+\delta)s}}{\varphi+\delta} \tag{5}$$

where $\theta(s)$ is a statistic, measured at the agent level, that gives the fraction of time the agent at state $\pi = 0$ will spend in that state over a time period of length s (this takes into account the possibility of leaving the state and coming back to it). Notice that the steady

state fraction of agents at $\pi = 0$ defined above, namely $\omega = \delta/(\delta + \varphi)$, obtains in the limit as $\lim_{s\to\infty} \theta(s) = \omega$.

Figure 6: Cross sectional patterns in the model



Finally, we compute the mean expected growth rate within a decile, which we denote by $\mathbb{E}(g_i|y_i, T)$. This growth rate equals $\frac{m(T)}{y_0}$ for all agents with $\pi = 0$ and hence for deciles populated exclusively by such agents. For deciles populated by agents whose incomes are growing (i.e. agents with $\pi = 1$), the expected growth rate is obtained by weighting the individual growth rates within the decile:

$$\mathbb{E}(g_i|y_i, T) = \int_{y\in y_i} \frac{M(y, T)}{y_i} dy \quad i = 1, 2, ..., 10$$

where the notation $y \in y_i$ is a shorthand to indicate that we are averaging across all the income levels y that belong to the interval that defines the i-th decile with mean income y_i.[5]

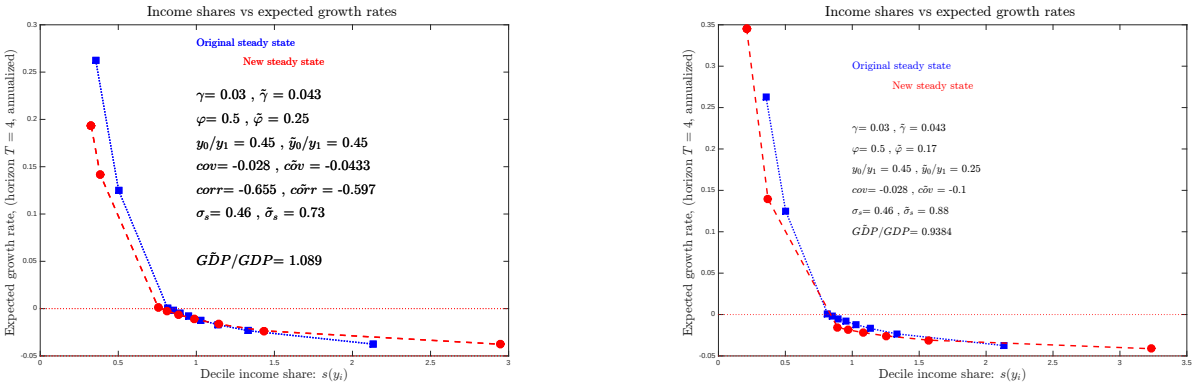
Following the same logic we can readily compute the higher moments, see Appendix A.

[5]An obvious adjustment must be done to construct the expected growth rate for the decile that mixes agents with $\pi = 0$ and agents with $\pi = 1$.

These moments can be used to compute the cross sectional moments, income shares, covariances and expected growth at each decile, that correspond to the magnitudes discussed in the empirical analysis.

Figure 7: Cross sectional patterns in the model

Constant y_0/y_1 With exogenous fall in y_0/y_1



5 Shock and transition to a new steady state

The cross sectional patterns illustrated above are mute about the length of the transition process between the initial and the final steady state. Next we provide an analytic characterization of distribution of incomes following a once and for all shock to one or more of its fundamental parameters as a function of t, the time elapsed since the shock occurred. Solving for the time evolution of a density function is generally hard because the partial differential equation involved do not admit a closed form solution. Fortunately, such difficulties can be overcome for the simple the stochastic income process considered.

Suppose at time $t = 0$ some parameters experience a once and for all change to new values $\tilde{\varphi}$ and $\tilde{\gamma}$. In particular assume that the new success rate $\tilde{\varphi}$ applies for all $t > 0$ to the pool of agents without project. The new growth rate $\tilde{\gamma}$ will only apply to successful projects

initiated after $t = 0$.

The mass of agents without a project in the cross section is equal to $\omega = \delta/(\delta + \varphi)$ before the shock. After the shock, it obeys

$$\tilde{\omega}(t) = \tilde{\omega} + (\omega - \tilde{\omega})\, e^{-(\delta+\tilde{\varphi})t} \tag{6}$$

where $\tilde{\omega} = \delta/(\delta + \tilde{\varphi})$, which is the asymptotic steady state fraction of agents who are not growing.

We also allow the income for an agent without a project to evolves exogenously, t periods after the shock, as $y_0(t)$, with $y_0(0) = y_0$. We will still assume that whenever the agent exits the $\pi = 0$ state, she will start from income y_1. Thus y_1 is the (time invariant) "initial income" agents get when they start a new project. Instead, the income of the agents without the project, y_0, may change as time elapses, an assumption that we will use to analyze the consequences of the reduction of incomes in the bottom decile recorded over the past 40 years.

5.1 Income density during transition $f(y,t)$ for $\pi = 1$

Below we solve in closed form the PDE for the Kolmogorov Forward equation to compute the density of income levels during the transition. Notice that after the shock the agents with the project (i.e. with $\pi = 1$) come in 2 types. Agents with a new project, with parameter $\tilde{\gamma}$ and agents with the old project (parameter γ).

The domain for y for the new type is $y \in (y_1, y_M(t))$ with $y_M(t) = y_1 e^{\tilde{\gamma}t}$ where t is the time elapsed since the shock. Let $\tilde{f}(y,t)$ denote the density of y at time t conditional on $\pi = 1$ and the project being a new variety. We want to characterize the density $\tilde{f}(y,t)$ during a transition towards the new invariant Pareto distribution. Note that the support of this distribution is $(y_1, y_1 e^{\tilde{\gamma}t})$ where y_1 is the injection point where mass flows in at a rate $\varphi\tilde{\omega}(t)$.

The density obeys $\tilde{f}(y,t)$ the Kolmogorov forward equation

$$\frac{\partial}{\partial t}\tilde{f}(y,t) = -\frac{\partial}{\partial y}\left(\tilde{f}(y,t)\tilde{\gamma}y\right) - \delta\tilde{f}(y,t) \tag{7}$$

We use an eigenvalue-eigenfunction decomposition to solve the above PDE by separating its variables (see Appendix B for details). Conjecture $\tilde{f}(y,t) = \sum_{j=0}^{\infty} e^{\lambda_j t} f_j(y)$ then the KFE gives

$$\lambda_j f_j(y) = -f_j'(y)\tilde{\gamma}y - (\delta + \tilde{\gamma})f_j(y) \quad \text{for} \quad j = 0, 1, 2, ...$$

So that $f_j(y) = A_j y^{-\left(1+\frac{\delta+\lambda_j}{\tilde{\gamma}}\right)}$. It is easy to show that the mass for the new type for $y \in (y_1, y_M(t))$ with $y_M(t) = y_1 e^{\tilde{\gamma}t}$ is given by $\eta(t)$ which is

$$\eta(t) = 1 - \tilde{\omega} - (\omega - \tilde{\omega})e^{-(\delta+\tilde{\varphi})t} - (1-\omega)e^{-\delta t} \tag{8}$$

which converges to $1-\tilde{\omega}$ asymptotically. We solve for λ_j, A_j by ensuring that $\int_{y_1}^{y_1 e^{\tilde{\gamma}t}} \tilde{f}(y,t)dy = \eta(t)$. This gives:

$$\tilde{f}(y,t) = (1-\tilde{\omega})\tilde{\alpha}\frac{y_1^{\tilde{\alpha}}}{y^{1+\tilde{\alpha}}} + e^{-(\delta+\tilde{\varphi})t}(\omega - \tilde{\omega})\frac{\tilde{\varphi}}{\tilde{\gamma}}\frac{y_1^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}}}{y^{1-\frac{\tilde{\varphi}}{\tilde{\gamma}}}} \tag{9}$$

Notice that the transition is completely explained by just 2 eigenvalues: the one associated to the invariant distribution $\lambda_0 = 0$, and the dominant eigenvalue $\lambda_1 = -(\delta + \tilde{\varphi})$.

We can now derive an equation for the cross-sectional distribution of incomes for the agents with $\pi = 1$ $(y > y_1)$, by taking into account the mass of survivors with the old project that fades out at rate δ, distributed over the domain $y \in (y_m(t), \infty)$ with $y_m(t) = y_1 e^{\gamma t}$.

Hence t periods after the shock the density is

$$f(y,t)\begin{cases} \tilde{f}(y,t) & \text{for} \quad y \in (y_1, y_m(t)) \\ \tilde{f}(y,t) + (1-\omega)\frac{\alpha y_1^{\alpha}}{y^{1+\alpha}} & \text{for} \quad y \in (y_m(t), y_M(t)) \\ (1-\omega)\frac{\alpha y_1^{\alpha}}{y^{1+\alpha}} & \text{for} \quad y \in (y_M(t), \infty) \end{cases} \tag{10}$$

Notice that $\int_{y_1}^{\infty} f(y,t)dy = 1 - \tilde{\omega}(t) \quad \text{for all} \quad t$. See Appendix B for more information.

The above equation illustrates the convenience of our simple model. As in the seminal paper by Gabaix et al. (2016), the transition equation for the evolution of the cross sectional distribution of incomes is based on the partial differential equation that characterizes the Kolmogorov Forward equation. Two differences stand out in comparison with their analysis: first, our simpler framework allows us to derive closed form expressions for the whole distribution function during a transition, while their more general framework restricts information to the asymptotic behavior of the income distribution, namely the set of frequencies of the distribution with the slowest dynamics, as captured by the dominant eigenvalue of the transition equation. Second, in spite of its simplicity our stochastic framework mixes two fundamental sources of distribution dynamics: the evolution of the mass of agents without a project, or those with $\pi = 0$, slowly moving from ω to $\tilde{\omega}$ according to equation (6). Moreover, another source of dynamics comes from equation (10), i.e. the dynamic evolution of the incomes for agents with $\pi = 1$. The overall dynamics of the income distribution results from the combination of these forces. One consequence is that the significance of the dominant eigenvalue can be substantially muted.

5.2 Cross sectional moments during transition with $y_0(t)$

Now we compute $\mathbb{E}(g_i|y,T,t)$ the expected income growth over a time period of length T for an agent with income y, t periods after the shock. We begin by computing the expected income levels conditional on the current y in a horizon of T periods.

If the agent is in state $\pi = 0$ then the expected value of income over a time period T is computed following the logic used for the steady state in equation (5), with the difference that the formula will now use the new parameters $\tilde{\varphi}$ and $\tilde{\gamma}$ and that the income of the poor ($\pi = 0$) at time t is $y_0(t)$.

$$\tilde{m}(t,T) = y_1 \int_0^T \tilde{\varphi}\, \theta(s) e^{(\tilde{\gamma}-\delta)(T-s)} ds + \theta(T) y_0(t+T) \quad \text{where} \quad \theta(s) = \frac{\delta + \tilde{\varphi} e^{-(\tilde{\varphi}+\delta)s}}{\tilde{\varphi}+\delta} \tag{11}$$

After t periods since the shock occurred there is a mass of agents in state $\pi = 1$ which keeps growing at rate γ. The expected value of income over an horizon T for these "old" agents with current income y is

$$M^o(y,t,T) = y e^{(\gamma-\delta)T} + \int_0^T \delta e^{-\delta s} \tilde{m}(t+s, T-s) ds \tag{12}$$

Notice that this statistic depends on t, the time elapsed since the shock, since the income of the poor $y_0(t)$ depends on calendar time.

The new projects initiated after the shock grow at rate $\tilde{\gamma}$. The expected value of income for an agent with $\pi = 1$ and current income y after t periods since the shock occurred, over an horizon T, is

$$M^n(y,t,T) = y e^{(\tilde{\gamma}-\delta)T} + \int_0^T \delta e^{-\delta s} \tilde{m}(t+s, T-s) ds \tag{13}$$

The expected cross-sectional income growth t for the incomes in each decile is readily computed at each t, as was done for the steady state. As t grows these moments change because of the time varying composition of the agents, in particular the agents with the old

projects growing at rate γ gradually disappear and as the new ones take over (as indicated by equation (8)) and because the income of the poor changes $y_0(t)$.

5.3 Transition dynamics: Three scenarios

Next we use the model to discuss three transition scenarios. All scenarios begin with a parametrization of the model with the following salient economic features. The ratio between the y_0/y_1 close to $1/2$, a cross sectional dispersion of incomes, measured on the income deciles, around to 0.45. The covariance between expected income growth and the income shares, $cov(g_i, s_i)$ equal to -0.03.

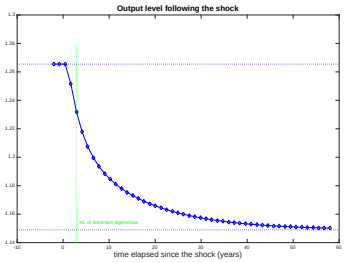
The first transition considers a reduction in the parameters φ, the possibility that a successful project arrives. This can be seen as a metafore for the fact that in a global world it becomes more difficult for your own ideas to succeed, some kind of a superstar effect: some winners will take it all, leaving many others without a chance. The second scenario supplements the reduction in φ (more difficult to succeed) with an increase in γ (better growth potential). Again, we can think of this effect through the lenses of the economics of superstar: conditional on achieving success, the payoff is now bigger. The third scenario 3 supplements scenario 2 with a gradual fall in $y_0(t)$, a phenomenon that is apparent from the CPS data.

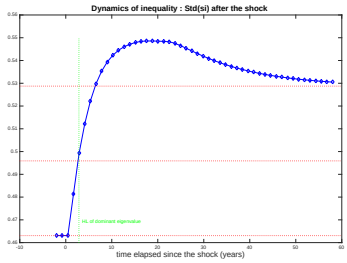
- Scenario 1: Growth and covariance fall, inequality rises but too little, fall in growth concentrated at the bottom. Mean annualized growth effect over first 10 years is -0.66%
- Scenario 2: Larger increase in ineq., fall in cov., increase in corr! Fall in growth still concentrated at the bottom. Mean annualized growth effect over first 10 years is -0.45%
- Scenario 3: Shock triggers higher γ and lower φ & falling y_0 such that y_0 halfs over a 20 year period. The mean annualized growth effect over first 10 years is -1.2%

Figure 8: Transition dynamics: three scenarios

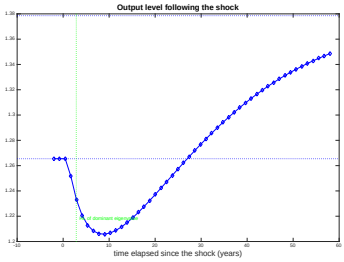
GDP Level Inequality (σ_s)

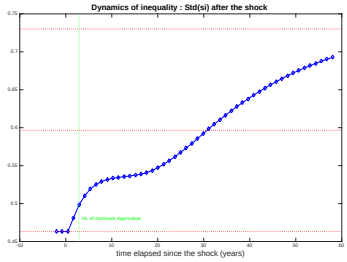
Scenario 1



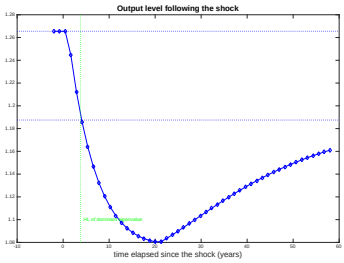


Scenario 2





Scenario 3



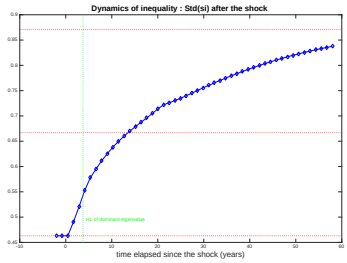
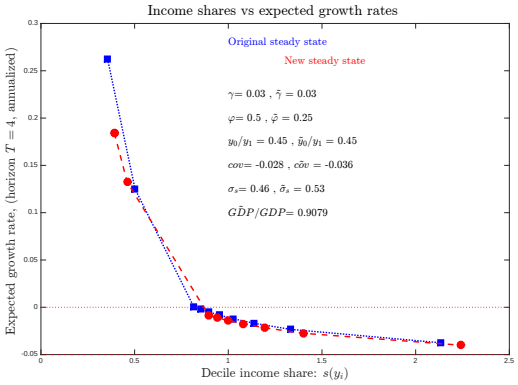
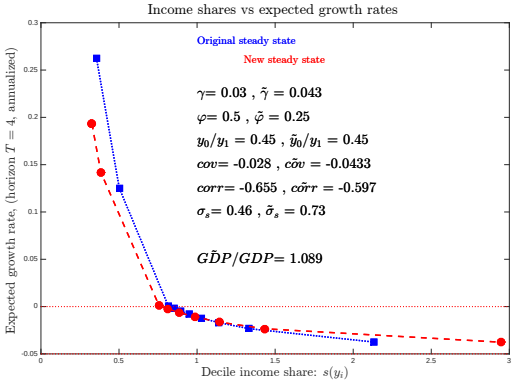


Figure 9: Cross sectional moments: three transitions

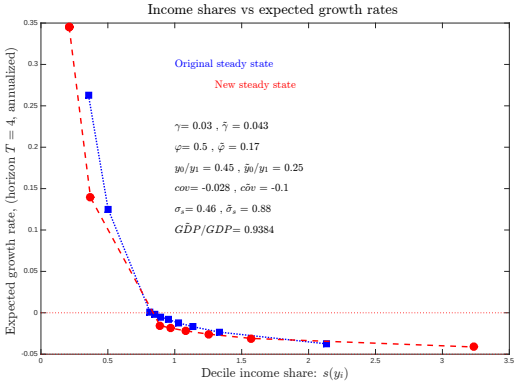
Scenario 1



Scenario 2

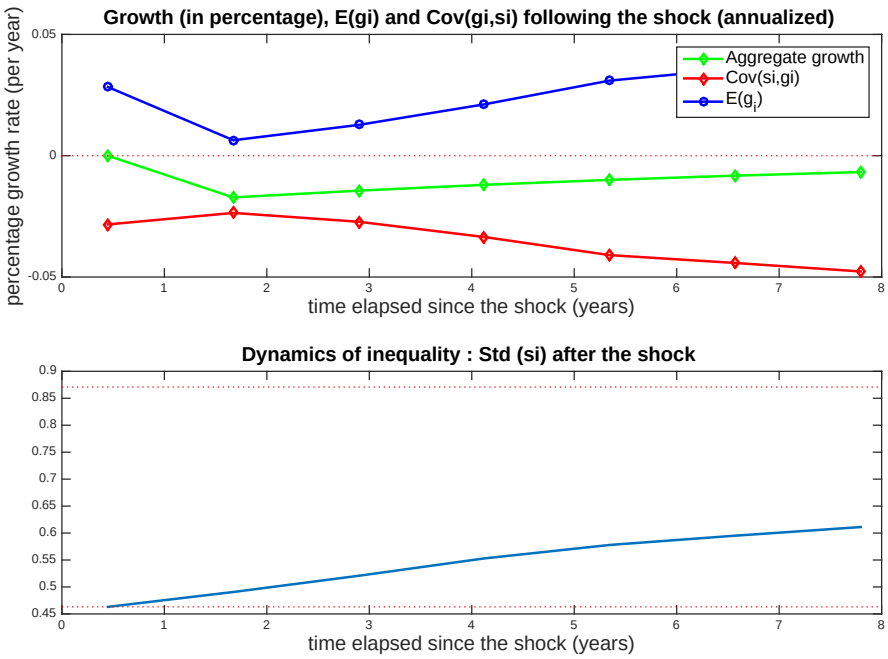


Scenario 3



OTHER MATERIAL

Figure 10: Dynamics during transition



5.4 Welfare analysis

In this section we evaluate the welfare impact of the changes described above, and we contrast our findings with the welfare impact of a slowdown in a standard macro model. TO BE COMPLETED

6 Conclusion

TO BE WRITTEN

A Steady state: Details on model computations

Integration of equation (5) gives

$$m(T) = y_1 \left(\frac{\alpha - \omega}{\alpha - 1} + \frac{\varphi\gamma}{\varphi + \gamma} \left(\frac{e^{-(\varphi+\delta)T}}{\varphi + \delta} - \frac{e^{-(\delta-\gamma)T}}{\delta - \gamma} \right) \right) + (y_0 - y_1) \frac{\delta + \varphi e^{-(\varphi+\delta)T}}{\varphi + \delta} \tag{14}$$

where we used $\alpha \equiv \delta/\gamma$. Notice that as $T \to \infty$ the expected income level converges to the average cross section income computed above.

Next we compute $M(y, T)$ by direct integration

$$\begin{aligned} M(y,T) =& y e^{(\gamma-\delta)T} + y_0 \frac{\alpha - \omega}{\alpha - 1} \left(1 - e^{-\delta T}\right) \\ &+ y_0 e^{-\delta T} \left(\frac{\delta\gamma}{(\varphi+\gamma)(\varphi+\delta)} \left(1 - e^{-\varphi T}\right) + \frac{\delta\varphi}{(\varphi+\gamma)(\delta-\gamma)} \left(1 - e^{\gamma T}\right) \right) \end{aligned} \tag{15}$$

Thus the expected income growth in the cross section over a period of length T is

$$\mathbb{E}(g_i, T) \equiv \omega \frac{m(T)}{y_0} + (1 - \omega) \int_{y_0}^{\infty} h(y) \frac{M(y,T)}{y} dy \tag{16}$$

or

$$\begin{aligned} \mathbb{E}(g_i, T) = \omega \left(\frac{\alpha - \omega}{\alpha - 1} + \frac{\varphi\gamma}{\varphi + \gamma} \left(\frac{e^{-(\varphi+\delta)T}}{\varphi + \delta} - \frac{e^{-(\delta-\gamma)T}}{\delta - \gamma} \right) \right) + (1 - \omega) \\ \int_{y_0}^{\infty} \alpha \frac{y_0^{\alpha}}{y^{\alpha+1}} \frac{M(y,T)}{y} dy \end{aligned} \tag{17}$$

or

$$\begin{aligned} \mathbb{E}(g_i, T) = \omega \left(\frac{\alpha - \omega}{\alpha - 1} + \frac{\varphi\gamma}{\varphi + \gamma} \left(\frac{e^{-(\varphi+\delta)T}}{\varphi + \delta} - \frac{e^{-(\delta-\gamma)T}}{\delta - \gamma} \right) \right) + (1 - \omega) \left(e^{(\gamma-\delta)T} \right. \\ \left. + \frac{\alpha}{\alpha + 1} \left(\frac{\alpha - \omega}{\alpha - 1} \left(1 - e^{-\delta T}\right) + e^{-\delta T} \left(\frac{\delta\gamma}{(\varphi+\gamma)(\varphi+\delta)} \left(1 - e^{-\varphi T}\right) + \frac{\delta\varphi}{(\varphi+\gamma)(\delta-\gamma)} \left(1 - e^{\gamma T}\right) \right) \right) \right) \end{aligned} \tag{18}$$

Likewise let $M_2(y, T) \equiv \mathbb{E}(y^2(T)|y(0) = y)$ denote the second moment of income in T periods

for an agent with current income y. This is

$$M_2(y,T) = y^2 e^{(2\gamma-\delta)T} + \int_{y_0}^{T} \delta e^{-\delta s} m_2(T-s)ds \tag{19}$$

where $m_2(T) \equiv \mathbb{E}(y^2(T)|y(0)=y_0)$ is the expected second moment of income T periods from now conditional on $\pi = 0$. Some analysis reveals that

$$m_2(T) = y_0^2 \left(\int_0^T \varphi\, \theta(s) e^{(2\gamma-\delta)(T-s)} ds + \theta(T) \right) \tag{20}$$

The second moment over a period of length T is

$$\mathbb{E}(g_i^2, T) \equiv \omega \frac{m_2(T)}{y_0^2} + (1-\omega) \int_{y_0}^{\infty} h(y) \frac{M_2(y,T)}{y^2} dy \tag{21}$$

We can compute it by integrating (19) and (22).

$$m_2(T) = y_0^2 \left(\frac{\frac{\alpha}{2} - \omega}{\frac{\alpha}{2} - 1} + \frac{2\varphi\gamma}{\varphi + 2\gamma} \left(\frac{e^{-(\varphi+\delta)T}}{\varphi+\delta} - \frac{e^{-(\delta-2\gamma)T}}{\delta - 2\gamma} \right) \right) \tag{22}$$

$$\begin{aligned} M_2(y,T) =& y^2 e^{(2\gamma-\delta)T} + y_0^2 \frac{\frac{\alpha}{2} - \omega}{\frac{\alpha}{2} - 1} \left(1 - e^{-\delta T}\right) \\ &+ y_0^2 e^{-\delta T} \left(\frac{2\delta\gamma}{(\varphi+2\gamma)(\varphi+\delta)} \left(1 - e^{-\varphi T}\right) + \frac{\delta\varphi}{(\varphi+2\gamma)(\delta-2\gamma)} \left(1 - e^{2\gamma T}\right) \right) \end{aligned} \tag{23}$$

therefore

$$\mathbb{E}(g_i^2, T) \equiv \omega \frac{m_2(T)}{y_0^2} + (1-\omega) \int_{y_0}^{\infty} h(y) \frac{M_2(y,T)}{y^2} dy \tag{24}$$

$$\begin{aligned} \mathbb{E}(g_i^2, T) =& \omega \left(\frac{\frac{\alpha}{2} - \omega}{\frac{\alpha}{2} - 1} + \frac{2\varphi\gamma}{\varphi + 2\gamma} \left(\frac{e^{-(\varphi+\delta)T}}{\varphi+\delta} - \frac{e^{-(\delta-2\gamma)T}}{\delta - 2\gamma} \right) \right) + (1-\omega) \left(e^{(2\gamma-\delta)T} \right. \\ &\left. + \frac{\frac{\alpha}{2}}{\frac{\alpha}{2}+1} \left(\frac{\frac{\alpha}{2} - \omega}{\frac{\alpha}{2} - 1} \left(1 - e^{-\delta T}\right) + e^{-\delta T} \left(\frac{2\delta\gamma}{(\varphi+2\gamma)(\varphi+\delta)} \left(1 - e^{-\varphi T}\right) + \frac{\delta\varphi}{(\varphi+2\gamma)(\delta-2\gamma)} \left(1 - e^{2\gamma T}\right) \right) \right) \right) \end{aligned} \tag{25}$$

B Details on the distribution $f(y,t)$ during a transition

Derivation of $\eta(t)$: notice that $\eta(t+\Delta) = (1-\delta\Delta)\eta(t) + \tilde{\varphi}\Delta\tilde{\omega}(t)$ which gives $\eta(t) + \Delta\eta'(t) = (1-\delta\Delta)\eta(t) + \tilde{\varphi}\Delta\tilde{\omega}(t)$ or, in the limit, $\eta'(t) = -\delta\eta(t) + \tilde{\varphi}\tilde{\omega}(t)$. This is a first order linear differential equation with solution $\eta(t) = Ce^{-\delta t} + 1 - \tilde{\omega}(t)$ where $\tilde{\omega}(t) = \frac{\delta}{\delta+\tilde{\varphi}} + \left(\frac{\delta}{\delta+\varphi} - \frac{\delta}{\delta+\tilde{\varphi}}\right) e^{-(\delta+\tilde{\varphi})t}$. By imposing the condition $\eta(0) = 0$ we find $C = -\varphi/(\delta+\varphi)$ and we have $\eta(t) = 1 - \tilde{\omega}(t) - (1-\omega)e^{-\delta t}$ which gives the equation in the text.

Solving the KFE and the associated boundary condition gives

$$\sum_{j=0}^{\infty} \frac{A_j\,\tilde{\gamma}}{\delta+\lambda_j} y_1^{-\frac{\delta+\lambda_j}{\tilde{\gamma}}} \left(e^{\lambda_j t} - e^{-\delta t}\right) = 1 - \tilde{\omega} - (\omega - \tilde{\omega})e^{-(\delta+\varphi)t} - (1-\omega)e^{-\delta t} \tag{26}$$

and matching coefficients gives the eigenvalue-eigenfunction pair associated to the invariant distribution $\lambda_0 = 0, A_0 = (1-\tilde{\omega})\tilde{\alpha}y_1^{\tilde{\alpha}}$, and another eigenvalue-eigenfunction pair associated to the transition $\lambda_1 = -(\delta+\tilde{\varphi}), A_1 = (\omega-\tilde{\omega})\frac{\tilde{\varphi}}{\tilde{\gamma}} y_1^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}}$ and all others $\lambda_j = A_j = 0$ for $j = 2, 3, ...$, so that finally we have equation (9).

Let us compute the cumulative distribution function at time t, denoted by $F(y,t)$. Straightforward integration gives

$$F(y,t) \begin{cases} \tilde{\omega}(t) & \text{for} \quad y \le y_0 \\ 1 - (1-\tilde{\omega})\left(\frac{y_1}{y}\right)^{\tilde{\alpha}} + (\omega-\tilde{\omega})e^{-(\delta+\tilde{\varphi})t}\left(\frac{y_1}{y}\right)^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}} & \text{for} \quad y \in (y_1, y_m(t)) \\ 1 - (1-\tilde{\omega})\left(\frac{y_1}{y}\right)^{\tilde{\alpha}} + (\omega-\tilde{\omega})e^{-(\delta+\tilde{\varphi})t}\left(\frac{y_1}{y}\right)^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}} + (1-\omega)\left(e^{-\delta t} - \left(\frac{y_1}{y}\right)^{\alpha}\right) & \text{for} \quad y \in (y_m(t), y_M(t)) \\ 1 - (1-\omega)\left(\frac{y_1}{y}\right)^{\alpha} & \text{for} \quad y \in (y_M(t), \infty) \end{cases} \tag{27}$$

Let $F_j = \{0.1, 0.2, ..., 0.9\}$ denote the thresholds for the deciles of the distribution function at time t. The associated income thresholds from the income distribution are:

$$\begin{cases} y_0 & \text{if} \quad F_j \leq \tilde{\omega}(t) \\ F_j = 1 - (1-\tilde{\omega}) \left(\frac{y_1}{y_j}\right)^{\tilde{\alpha}} + (\omega - \tilde{\omega}) e^{-(\delta+\tilde{\varphi})t} \left(\frac{y_1}{y_j}\right)^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}} & \text{if} \quad F_j \leq F(y_m(t)) \\ F_j = 1 - (1-\tilde{\omega}) \left(\frac{y_1}{y_j}\right)^{\tilde{\alpha}} + (\omega - \tilde{\omega}) e^{-(\delta+\tilde{\varphi})t} \left(\frac{y_1}{y_j}\right)^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}} + (1-\omega)\left(e^{-\delta t} - \left(\frac{y_1}{y_j}\right)^{\alpha}\right) & \text{if} \quad F_j \leq F(y_M(t), t) \\ y_j = y_1 \left(\frac{1-F_j}{1-\omega}\right)^{-\frac{1}{\alpha}} & \text{if} \quad F_j > F(y_M(t), t) \end{cases} \tag{28}$$

C Closed form expressions for cross sectional moments during the transition

We give two closed form expressions for the first and second moment of the aggregate income t periods after the shock. Equivalent magnitudes can be computed using $f(y,t)$. The aggregate income of the economy, t periods after the shock, is given by

$$\mathbb{E}(y,t) = \tilde{\omega}(t) y_0(t) + y_1 \tilde{\varphi} \int_0^t \tilde{\omega}(s) e^{(\tilde{\gamma}-\delta)(t-s)} ds + (1-\omega) \int_{y_1 e^{\gamma t}}^{\infty} h(y) y dy \tag{29}$$

or

$$\mathbb{E}(y,t) = \tilde{\omega}(t) y_0(t) + y_1 \left(\tilde{\varphi} \int_0^t \tilde{\omega}(s) e^{(\tilde{\gamma}-\delta)(t-s)} ds + (1-\omega) e^{(\gamma-\delta)t} \frac{\alpha}{\alpha - 1} \right) \tag{30}$$

Let $\bar{y}_0 = \lim_{t\to\infty} y_0(t)$, then the output in the new steady state is

$$\lim_{t\to\infty} \mathbb{E}(y,t) = \tilde{\omega}\bar{y}_0 + \frac{\tilde{\alpha} y_1 (1-\tilde{\omega})}{\tilde{\alpha} - 1}$$

The second moment for aggregate income in the economy, t periods after the shock is

$$\mathbb{E}(y^2,t) = \tilde{\omega}(t)y_0^2(t) + y_1^2\left(\tilde{\varphi}\int_0^t \tilde{\omega}(s)e^{(2\tilde{\gamma}-\delta)(t-s)}ds\right) + (1-\omega)\int_{y_1e^{\gamma t}}^{\infty} h(y)y^2dy \tag{31}$$

or

$$\mathbb{E}(y^2,t) = \tilde{\omega}(t)y_0^2(t) + y_1^2\left(\tilde{\varphi}\int_0^t \tilde{\omega}(s)e^{(2\tilde{\gamma}-\delta)(t-s)}ds + \frac{(1-\omega)\alpha}{\alpha-2}e^{(2\gamma-\delta)t}\right) \tag{32}$$

Given the second moment, we compute the variance of the income shares s_i along the transition as: $\sigma^2_{s_i,t} = \frac{\mathbb{E}(y^2,t)}{(\mathbb{E}(y,t))^2} - 1$.

Integration of equation (11) gives

$$\tilde{m}(t,T) = y_1\left(\frac{\tilde{\alpha}-\tilde{\omega}}{\tilde{\alpha}-1} + \frac{\tilde{\varphi}\tilde{\gamma}}{\tilde{\varphi}+\tilde{\gamma}}\left(\frac{e^{-(\tilde{\varphi}+\delta)T}}{\tilde{\varphi}+\delta} - \frac{e^{-(\delta-\tilde{\gamma})T}}{\delta-\tilde{\gamma}}\right)\right) + (y_0(t+T)-y_1)\frac{\delta+\tilde{\varphi}e^{-(\tilde{\varphi}+\delta)T}}{\tilde{\varphi}+\delta} \tag{33}$$

where we used $\tilde{\alpha} \equiv \delta/\tilde{\gamma}$ and $\tilde{\omega} \equiv \delta/(\delta+\tilde{\varphi})$.

After t periods the expected growth rate of income over an horizon T is

$$\mathbb{E}(g_i,t,T) \equiv \tilde{\omega}(t)\frac{\tilde{m}(t,T)}{y_0(t)} + (1-\tilde{\omega}(t))\left(\int_{y_1}^{y_1e^{\tilde{\gamma}t}} h(y,\tilde{\alpha})\frac{M^n(y,t,T)}{y}dy + \int_{y_1e^{\gamma t}}^{\infty} h(y,\alpha)\frac{M^o(y,t,T)}{y}dy\right) \tag{34}$$

where $h(y,\alpha) = \alpha y_1^{\alpha}/(y^{\alpha+1})$ from equation (3).[6] Notice that $\mathbb{E}(g_i,t,T)$ is a forward looking variable (expectation over future horizons). It jumps the moment the shock hits since agents know the new parameters will apply from that moment onwards.

In the same fashion we can compute the second moment

$$\mathbb{E}(g_i^2,t,T) \equiv \tilde{\omega}(t)\frac{\tilde{m}_2(t,T)}{y_0^2(t)} + (1-\tilde{\omega}(t))\left(\int_{y_1}^{y_1e^{\tilde{\gamma}t}} h(y,\tilde{\alpha})\frac{M_2^n(y,t,T)}{y^2}dy + \int_{y_1e^{\gamma t}}^{\infty} h(y,\alpha)\frac{M_2^o(y,t,T)}{y^2}dy\right) \tag{35}$$

[6] Verify : note that $\forall\ t$ we have (after simple algebra)

$$1 = \tilde{\omega}(t) + (1-\tilde{\omega}(t))\left(\int_{y_1}^{y_1e^{\tilde{\gamma}t}} h(y,\tilde{\alpha})dy + \int_{y_1e^{\gamma t}}^{\infty} h(y,\alpha)dy\right)$$

$$1 = \int_{y_1}^{y_1e^{\tilde{\gamma}t}} h(y,\tilde{\alpha})dy + \int_{y_1e^{\gamma t}}^{\infty} h(y,\alpha)dy = 1 - e^{-\delta t} + e^{-\delta t}$$

where

$$\tilde{m}_2(t,T) = y_1^2 \int_0^T \varphi\, \theta(s) e^{(2\gamma-\delta)(T-s)} ds + \theta(T) y_0^2(t+T) \tag{36}$$

which is

$$\tilde{m}_2(t,T) = y_1^2 \left(\frac{\frac{\tilde{\alpha}}{2} - \tilde{\omega}}{\frac{\tilde{\alpha}}{2} - 1} + \frac{2\tilde{\varphi}\tilde{\gamma}}{\tilde{\varphi} + 2\tilde{\gamma}} \left(\frac{e^{-(\tilde{\varphi}+\delta)T}}{\tilde{\varphi}+\delta} - \frac{e^{-(\delta-2\tilde{\gamma})T}}{\delta - 2\tilde{\gamma}} \right) \right) + \left(y_0^2(t+T) - y_1^2 \right) \frac{\delta + \tilde{\varphi} e^{-(\tilde{\varphi}+\delta)T}}{\tilde{\varphi}+\delta} \tag{37}$$

and

$$M_2^o(y,t,T) = y^2 e^{(2\gamma-\delta)T} + \int_0^T \delta e^{-\delta s} \tilde{m}_2(t+s, T-s) ds \tag{38}$$

$$M_2^n(y,t,T) = y^2 e^{(2\tilde{\gamma}-\delta)T} + \int_0^T \delta e^{-\delta s} \tilde{m}_2(t+s, T-s) ds \tag{39}$$

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Unequal Growth

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March 2019

Introduction

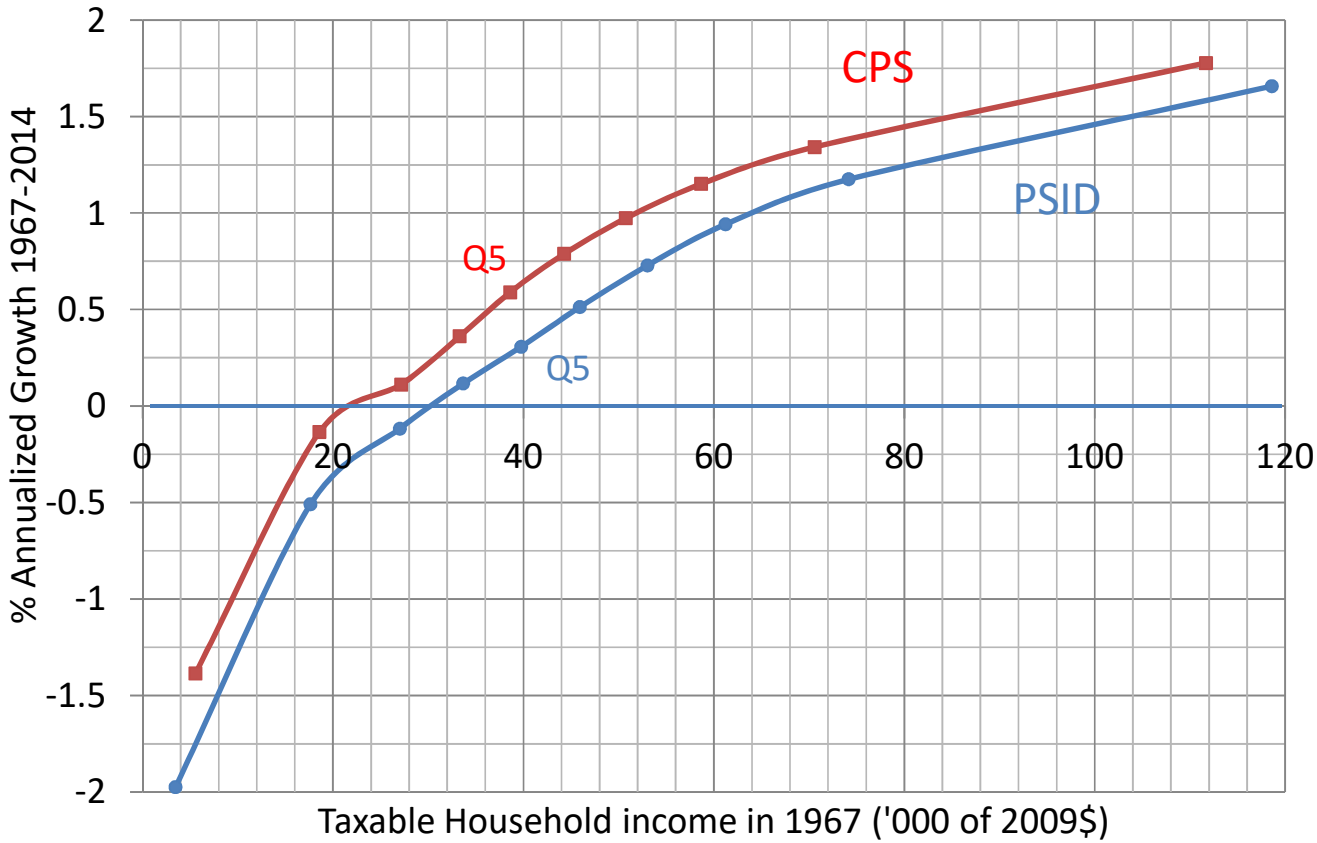
Two major changes of the U.S. economy over past 50 years

- Increase in Household Income Inequality
- Slowdown in Aggregate Growth

Often studied independently

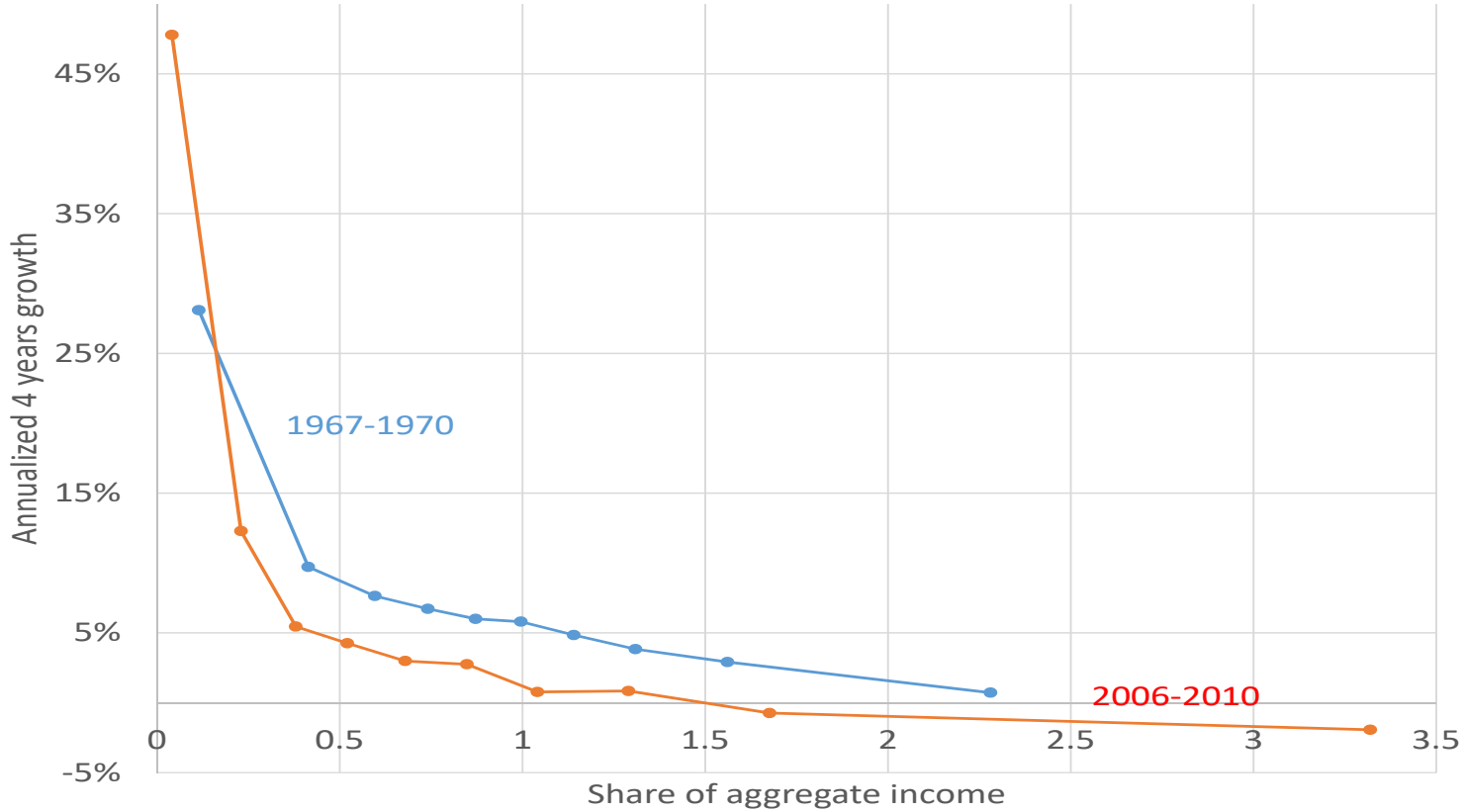
- This paper aims to link the two, empirically and theoretically
- Building block: If income growth varies systematically with income level (Gibrat Law fails), aggregate income growth depends on income distribution

Unequal growth 1967-2014: a cross sectional view



- Each point: level & growth of deciles of 1967 income distribution
- Top grew fast, bottom stagnated: not same households across time!

Unequal growth 1967-2014: a panel view (PSID)



- Poor grow faster than rich (mean reversion), early and late in sample
- Same households across time!
- How to reconcile this figure (β convergence) with previous (σ divergence)?

Setup: aggregate micro income dynamics

Change in households' income dynamics triggers

- Micro changes (previous two figures)
- Macro changes (aggregate growth)
- Inequality interacts with aggregate growth since Gibrat's law fails (income growth varies with income level)

Outline

- A micro decomposition of aggregate growth
- Empirical analysis on micro decomposition
- Simple model to interpret the data

Some Related literature

- “Inequality and Growth”, Benabou, 1996
- “Sigma Convergence versus Beta Convergence”, Young, Higgins and Levy 2007
- “Misallocation and growth”, Jovanovic 2014
- “Earnings, Inequality and Mobility in the United States”, Kopczuk, Saez and Song 2010
- “Dynamics of inequality”, Gabaix, Lasry, Lions and Moll 2016
- “Top income inequality dynamics”, Kim and Jones 2017

A micro decomposition of aggregate growth

Let y_{it} income of household i at time t

- Aggregate growth in period t over horizon T, $\Gamma_{t,T}$ can be written as

$$\Gamma_{t,T} = \frac{E_i(y_{i,t+T})}{E_i(y_{i,t})} = E_i\left(\frac{y_{i,t+T}}{y_{i,t}}\frac{y_{i,t}}{E(y_{i,t})}\right)$$

- Define $g_{i,T} = \frac{y_{i,t+T}}{y_{i,t}}$, $s_{i,t} = \frac{y_{i,t}}{E(y_{i,t})}$ so that $\Gamma_{t,T} = E_i(g_{i,T} \cdot s_{i,t})$

$$\Gamma_{t,T} = cov(g_{i,T}, s_{i,t}) + E_i(g_{i,T})$$

Aggregate growth depends on the distribution of growth opportunities

- Similar decomposition widely used in IO (Olley and Pakes, 1996)

Insights from decomposition

$$\Gamma_T = cov(g_{i,T}, s_i) + E(g_{i,T})$$

- Simple way to use micro data to evaluate the the distributional impact of a given aggregate growth performance
- Distribution of growth opportunities matters for aggregate growth
- Alternative mechanisms behind growth slowdown
  - Change of a common growth factor: Change in $E(g_i)$, not $cov(g_i, s_i)$
  - Change in $cov(g_i, s_i)$ for given $E(g_i)$
  - Warning: $Cov(g_i, s_i), E(g_i)$ not independent primitives....need a theory

Covariance decomposition

$$\Gamma = E(g_i) + corr(g_{it}, s_{it})\sigma(g_{it})\sigma(s_{it})$$

Covariance, and thus Γ, depends on patterns of inequality

- If $corr(g_i, s_i) < 0$, Cov (and Γ) decreasing in income inequality, $\sigma(s_{it})$

  Intuition

  - higher $\sigma(s_{it})$ implies more poor with (faster than avg.) growth and more rich with (slower than avg.) growth: since rich larger: $\Gamma \downarrow$

- Aggregate growth depends on:
  - Individual growth, $E(g_i)$
  - Income inequality, $\sigma(s_{it})$
  - Rank mobility, $corr(g_i, s_i)$
  - Income instability, $\sigma(g_{it})$

Plan

- (1) measure $corr(g_i, s_i)$, $\sigma(g_{it})$, $\sigma(s_{it})$ and $E(g_i)$ over 1967-2014
- (2) simple mechanism to understand driving force of changes

Panel Study of Income Dynamics (PSID)

- Long panel of about 5,000 HH, representative of U.S. population

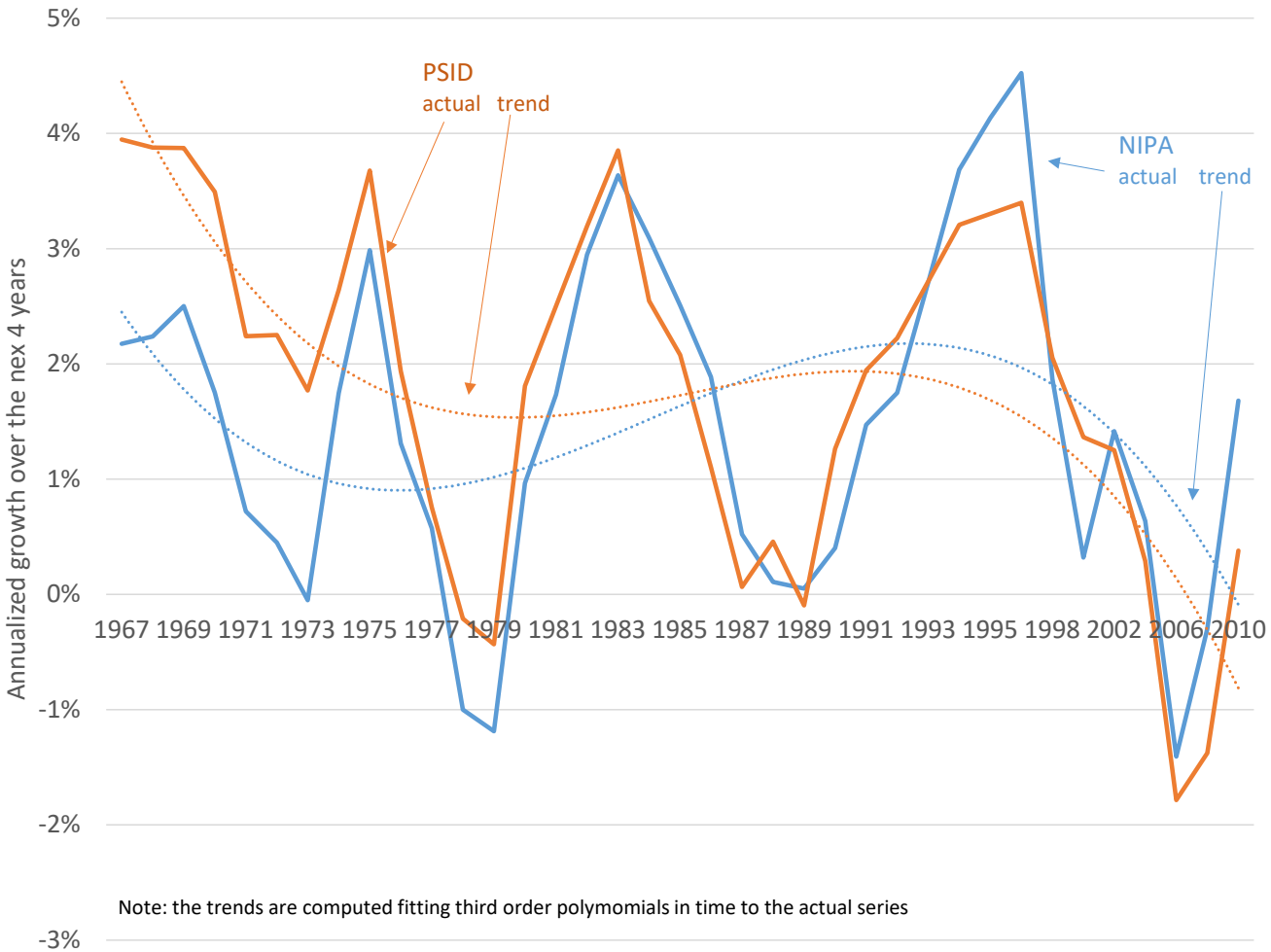
  Panel essential to identify change of individual dynamics (vs composition)

- 1967-2014 (Annual until 1996, bi-annual after)

- Publicly available

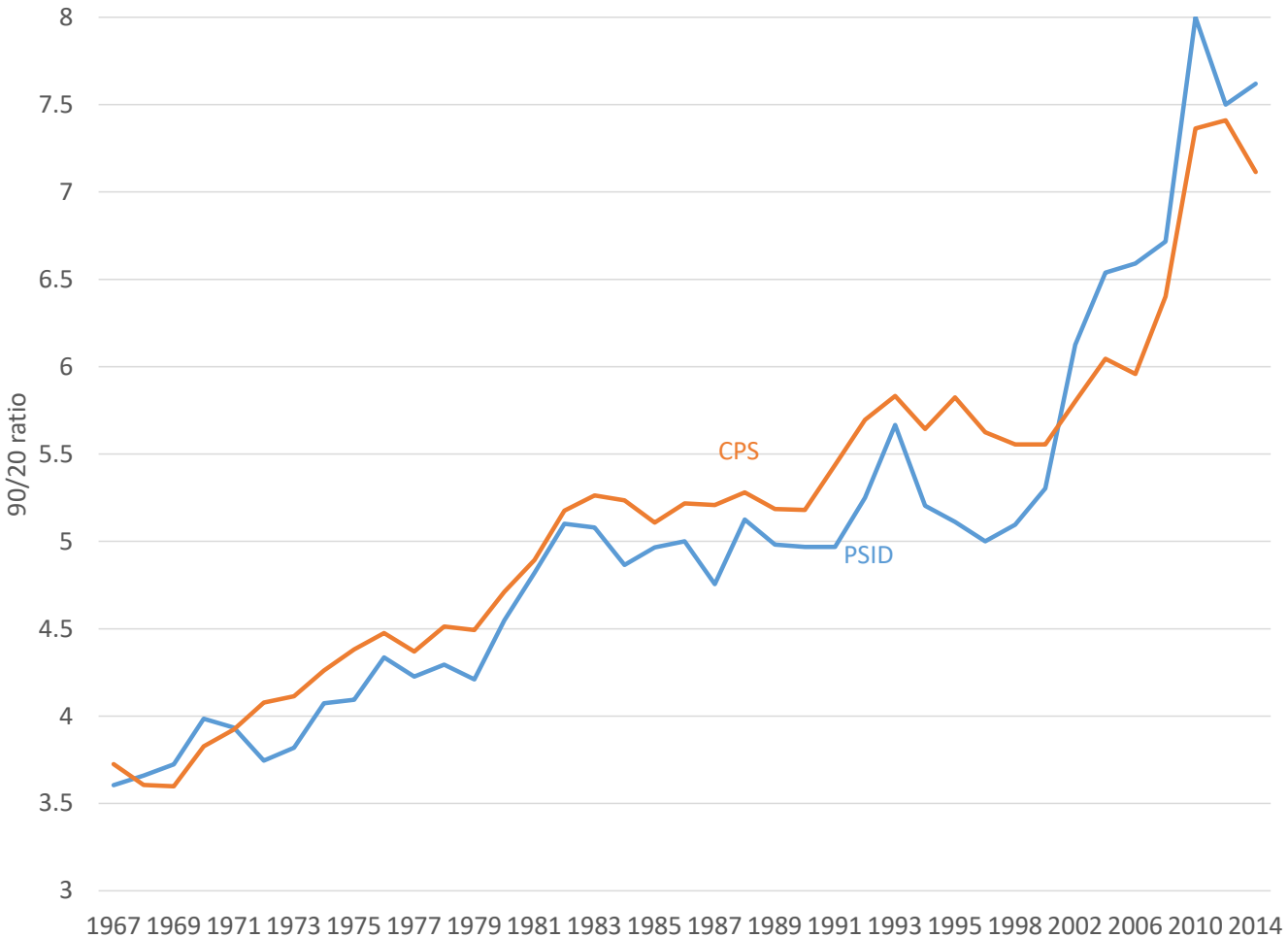
- Panel data must aggregate up to macro outcomes

PSID v/s NIPA
4 yrs growth of real labor income per capita



- Aggregate PSID matches well macro NIPA Dynamics (including recent growth slowdown)

PSID v/s CPS
Cross sectional inequality



- PSID matches well cross sectional inequality in labor income from much larger sample (CPS)

Mapping decomposition to panel data

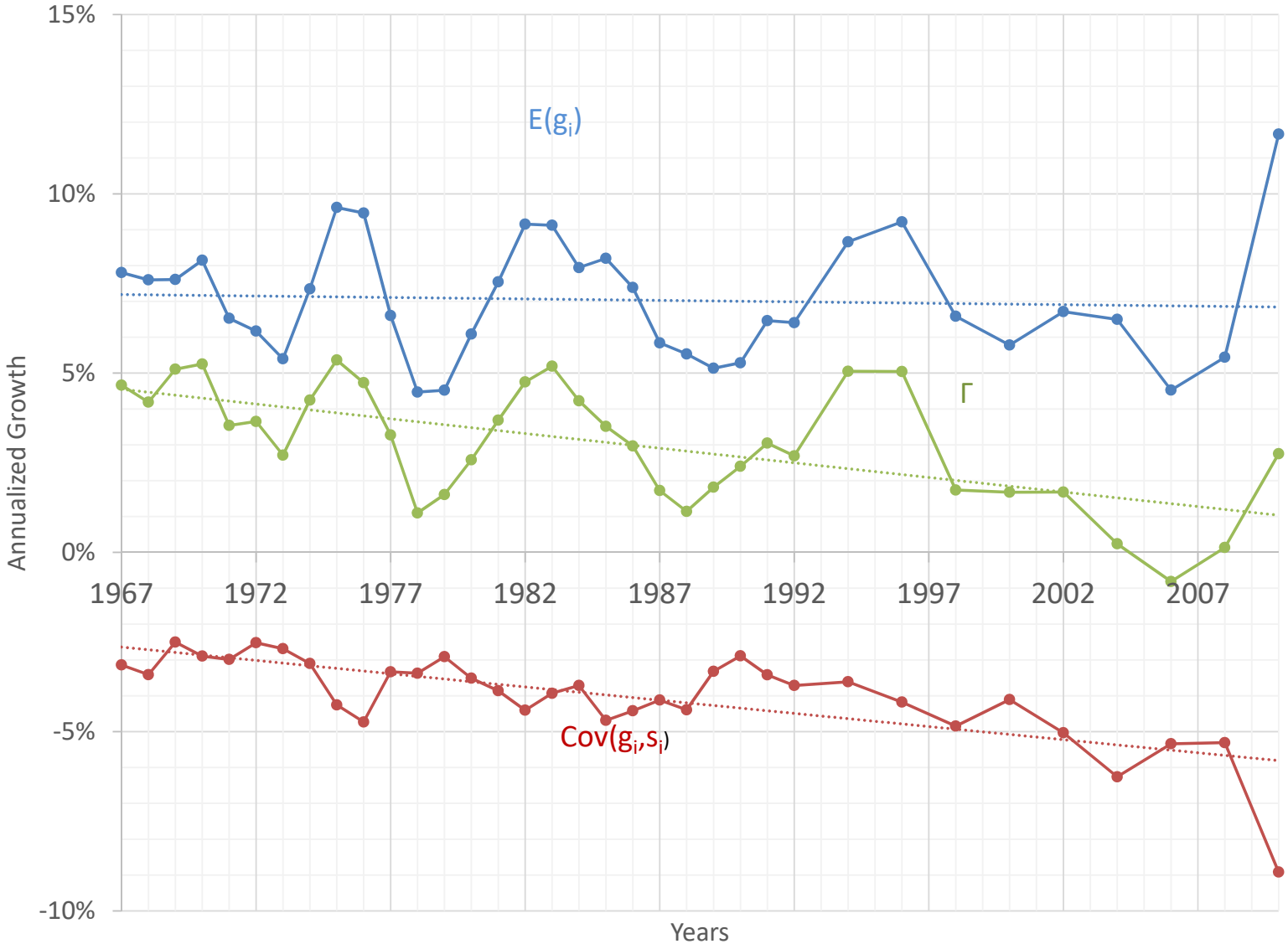
Let $T = 4$ years and $y_{j,i,t}$ be income of HH j, in decile i in year t.

$$\text{then} \quad g_{i,t+T} = \frac{\sum_j y_{j,i,t+T}}{\sum_j y_{j,i,t}} \quad \text{and} \quad s_{i,t} = \frac{\sum_j y_{j,i,t}}{\sum_i \sum_j y_{j,i,t}}$$

Aggregating by income deciles (quintiles) useful with measurement error

- Income measure: pre-tax, pre govt transfer, taxable household income (i.e. labor income + capital income)
- Sample restrictions: Households with head 25-60, with income above 20% of the pvty line, no imputed labor income, which are in sample in year t and $t + 4$ (avg. sample per year $\simeq$ 3500)
- Similar patterns from hholds with 25-40 head (age composition)

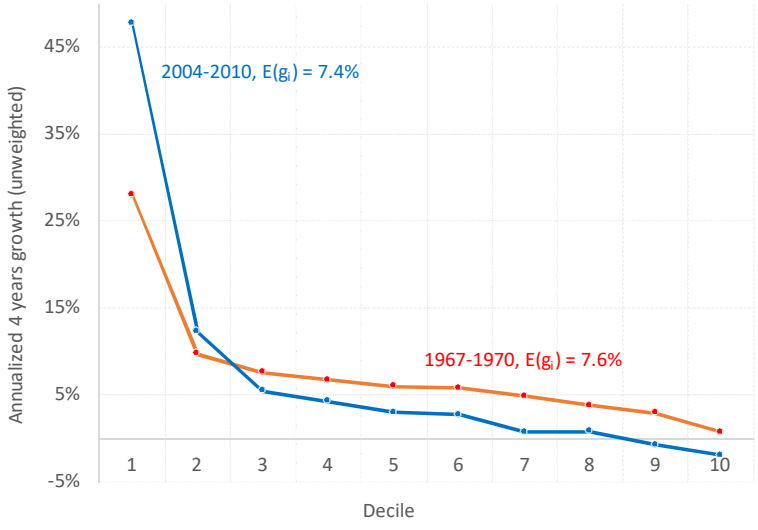
Aggregate growth decomposition (PSID)



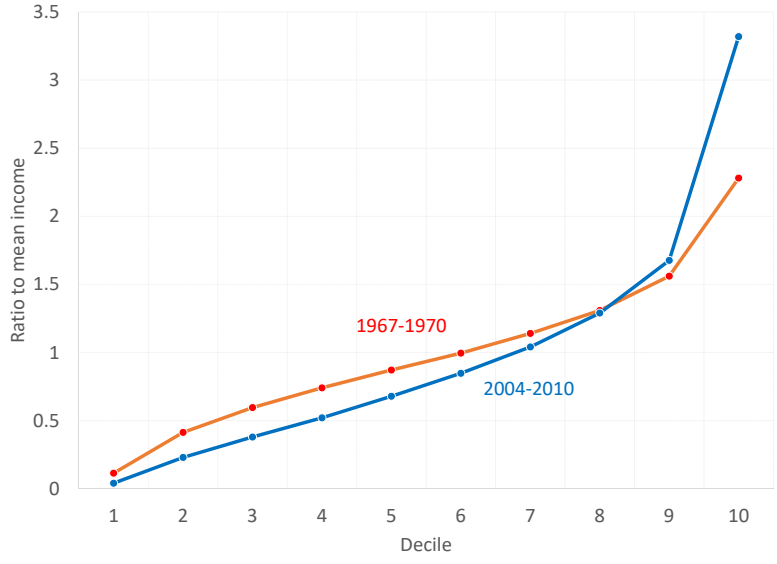
- Γ declines, $E(g_i)$ almost flat. Implies: $cov_t \downarrow$

Contribution to growth by decile

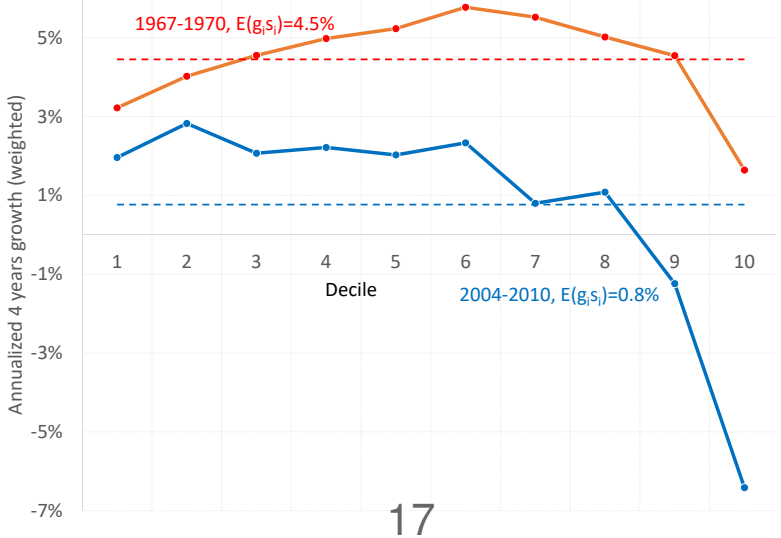
g_i



s_i



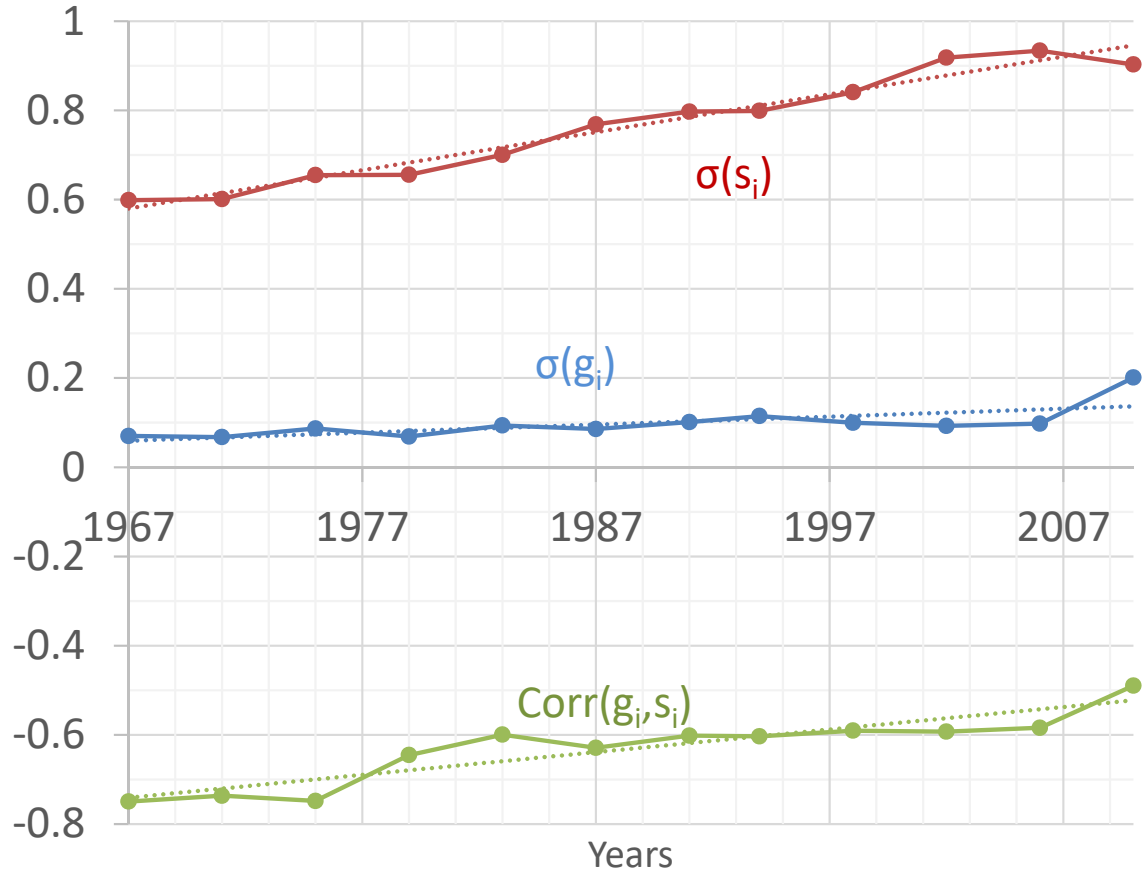
Contribution to aggregate growth $g_i \cdot s_i$



Some intuition

- Trends
  - U.S. becoming a more unequal society
  - Bottom is growing faster, top is growing slower
- Causes for growth decline
  - Slow (and slower) growing top has a larger share of income
  - Fast (and faster) growing bottom has a smaller share
- All decile contribute to decline in growth
- Most decline in aggregate growth account for (in a statistical sense), by falling covariance

Covariance decomposition (PSID)



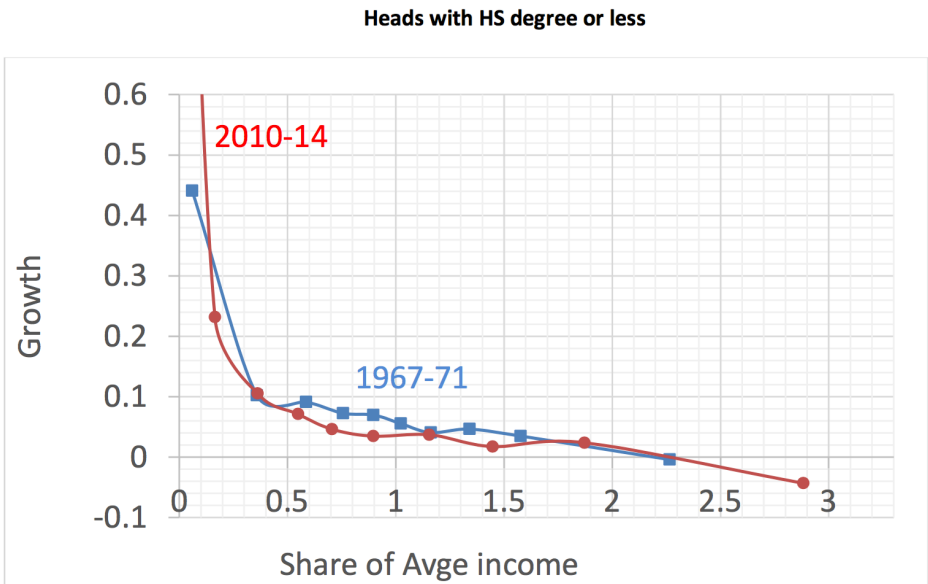
$Corr(g_i, s_i)$ increasing (toward 0) signals growth/level relation getting weaker

- less rank mobility
- less impact of increasing inequality on growth

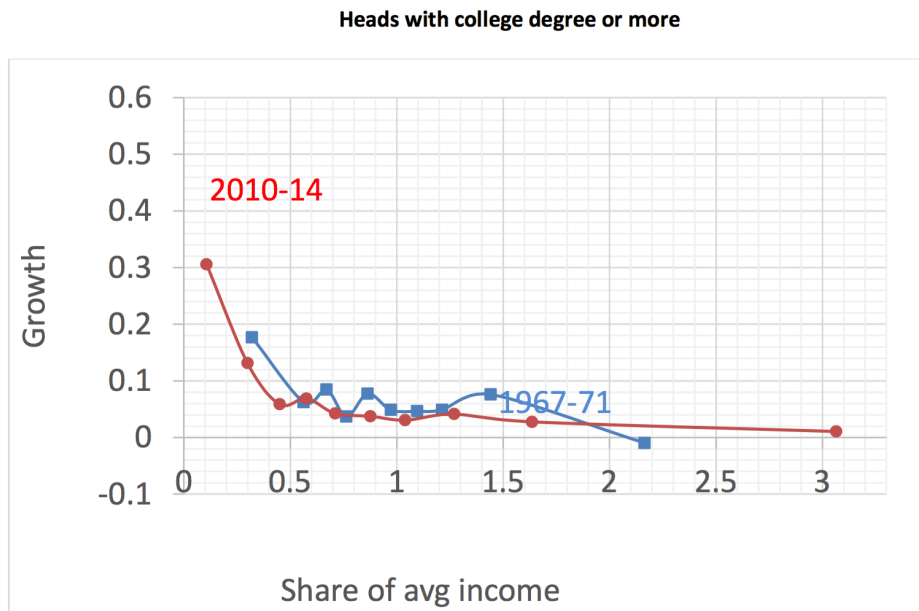
Robustness of growth decomposition

Cut sample by HH head education:

at most High-school



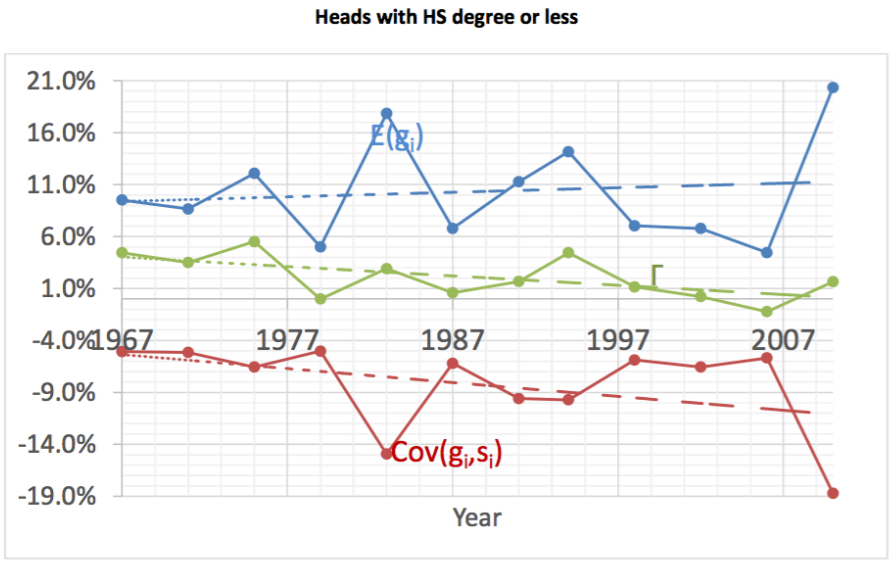
College or more



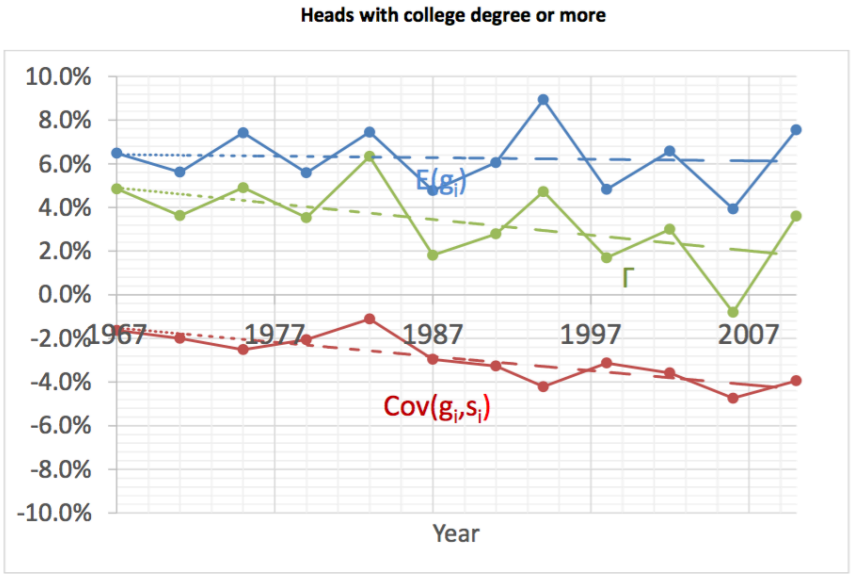
Robustness of growth decomposition (1)

Cut sample by HH head education:

at most High-school



College or more

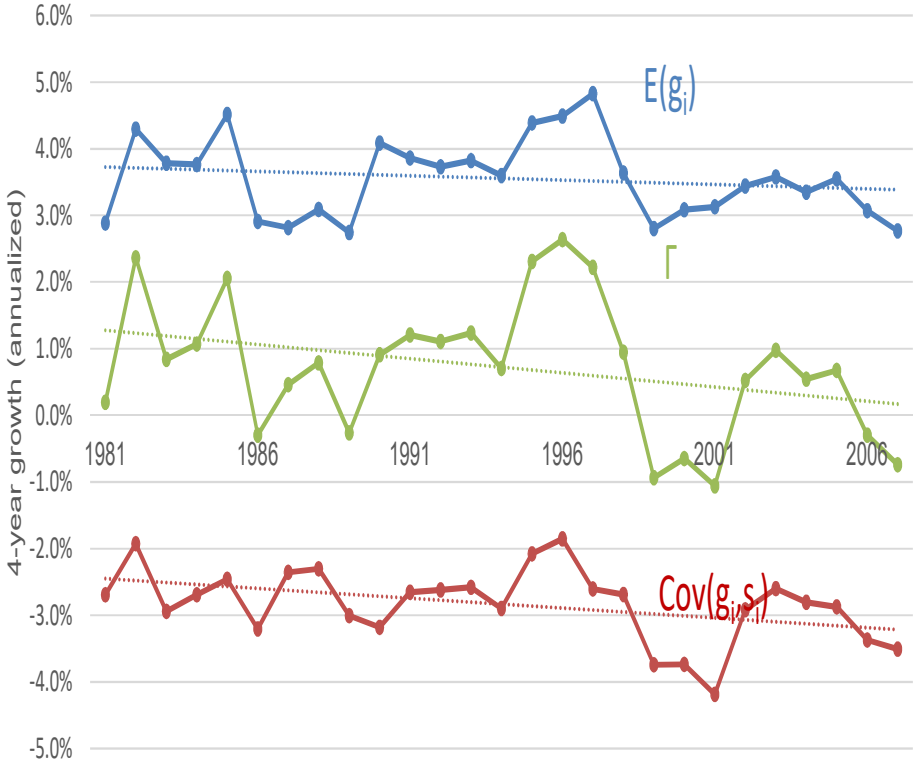


Patterns robust to several more demographics (e.g. age, race)

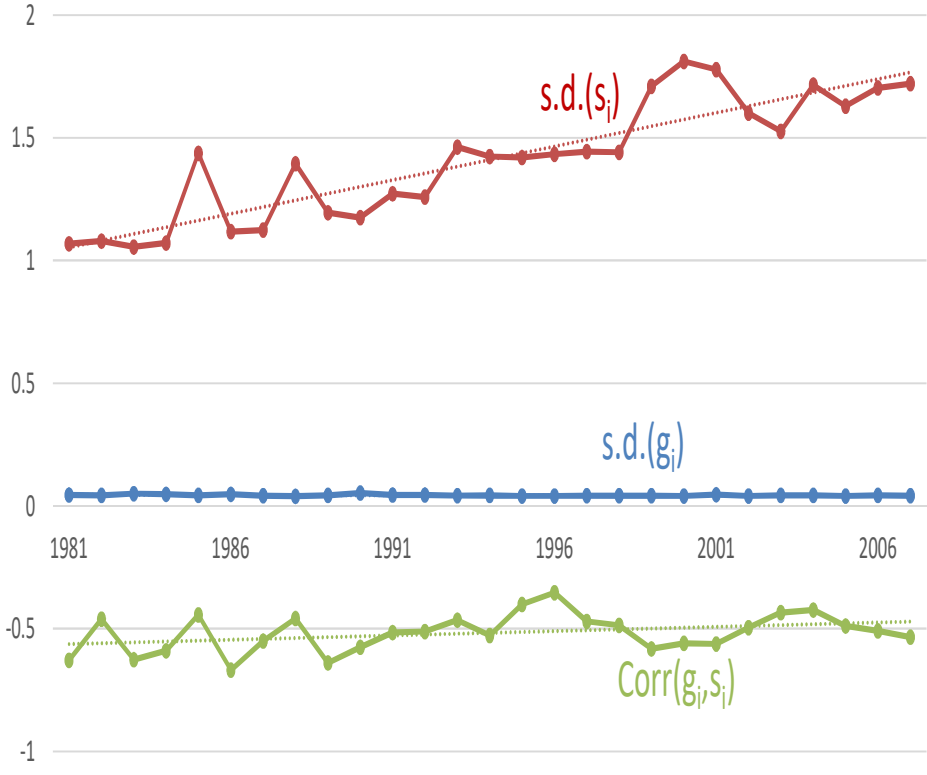
Robustness of growth decomposition (2)

Administrative data from SIPP users
Larger sample (20x), higher quality data, indiv v/s hholds, 1980-2012

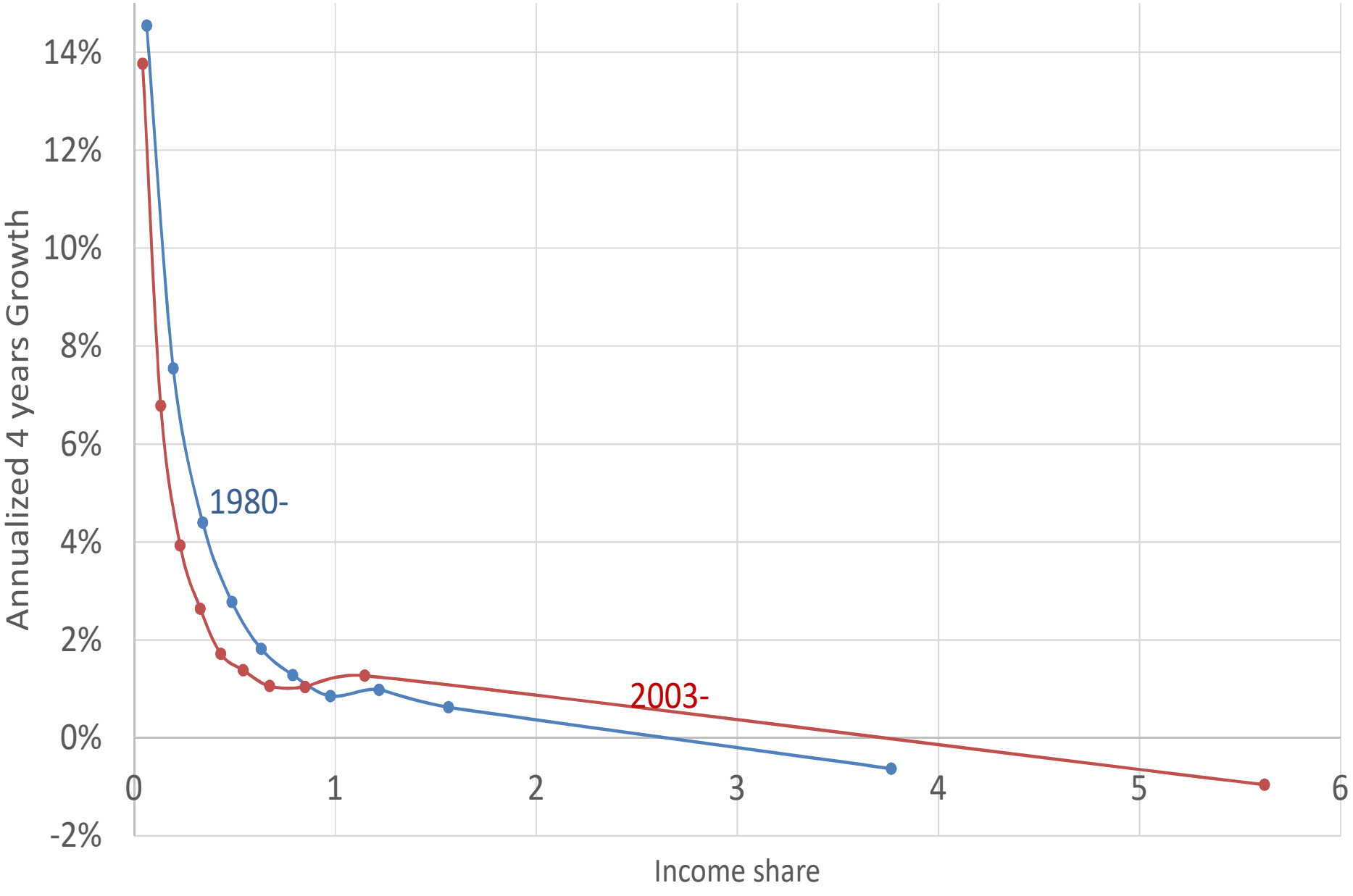
Growth decomposition



covariance decomposition



Income Growth and Income level in SIPP data



A broad brush characterization (stylized facts)

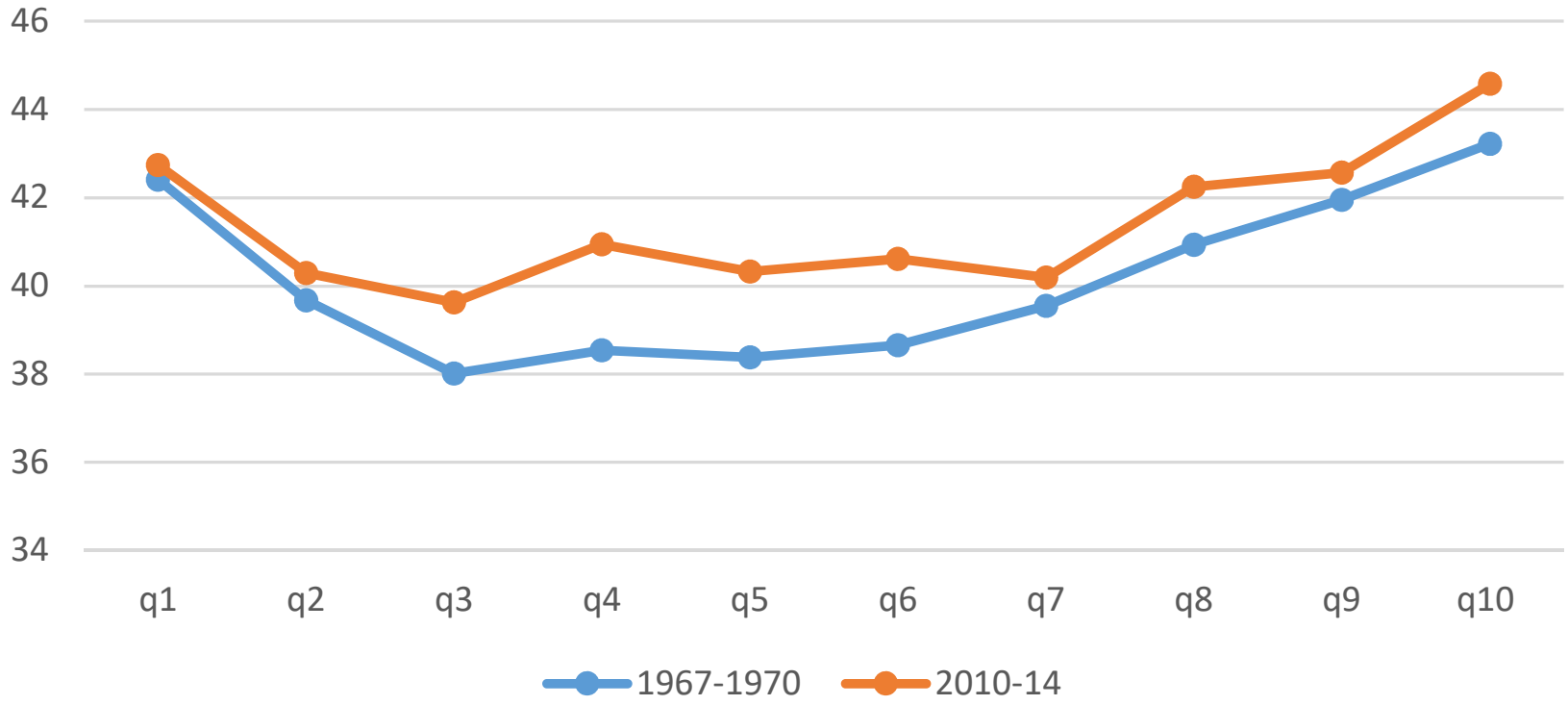
$$\Gamma_t = \underbrace{corr(g_i, s_i)\sigma(g_{it})\sigma(s_{it})}_{cov} + E(g_i)$$

- Γ_t declines over time (between 1% and 3%)
  - – significant contribution of $\Delta cov_i(g_{it}, s_{it})$
  - – small decline in "average" x-section growth $E(g_i)$
- The reduction of Γ_t and of cov_t is persistent
- Reduction of $cov(g_{it}, s_{it})$ results mostly from
  - – large increase in $\sigma(s_{it})$ (from 0.6 to 0.95) + $corr(g_{it}, s_{it}) < 0$

Who are the poor fast growers?

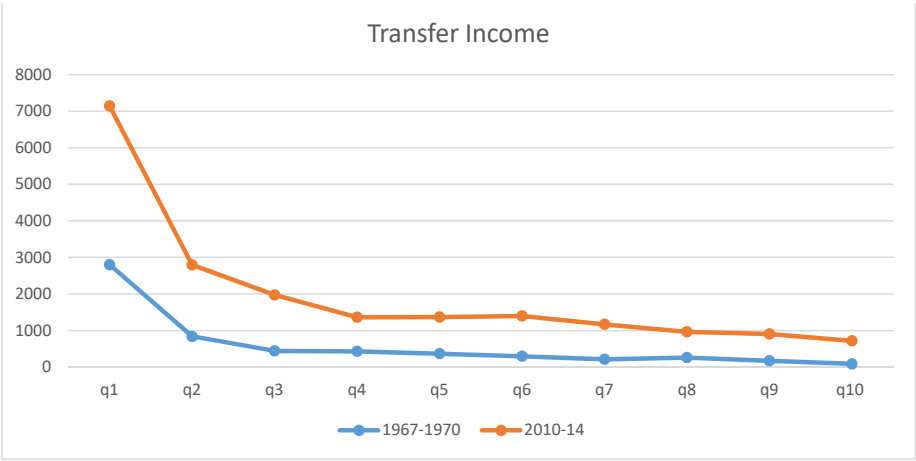
- Before writing a model for (g_i, s_i), show some characteristics of different deciles of income distribution.
- And is their type changing?

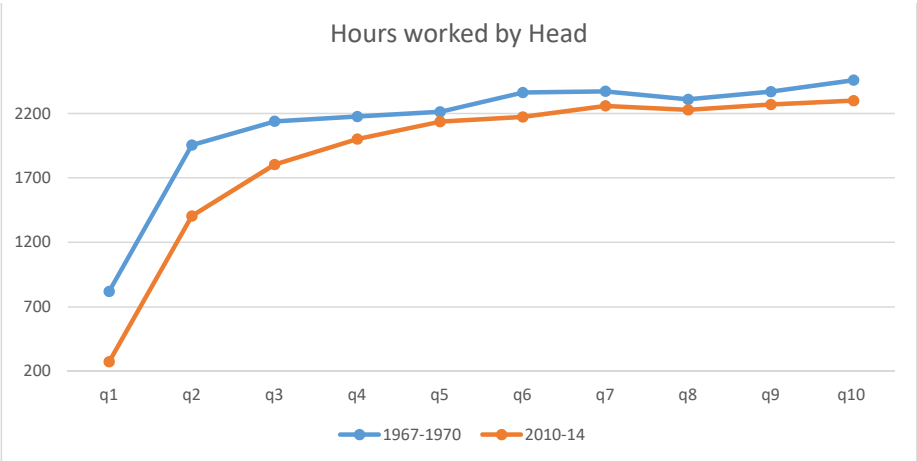
Age of different quantiles



- The poor fast growers are not all young
- Mean reversion not all explained by demographic (Some household become poor-fast growers because of shocks)

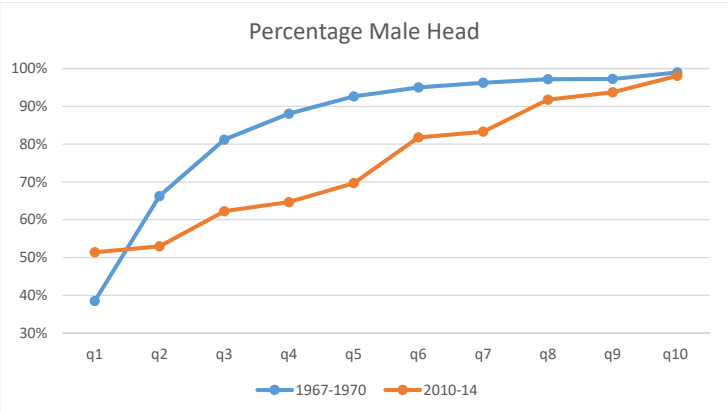
Hours and transfers

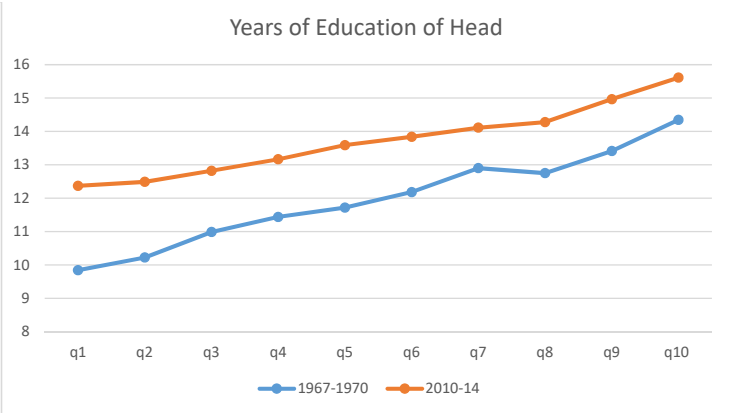


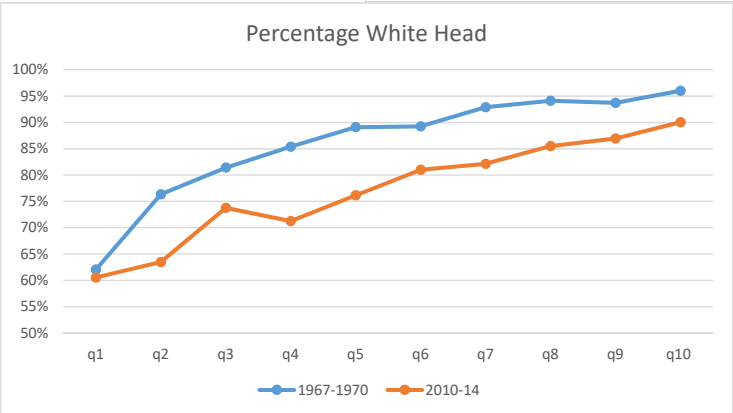


- Poor fast-growers work much less hours and receive more transfers, suggesting they are experiencing (temporary) shock to their ability to work
- For poor fast growers hours are becoming lower and transfers becoming larger!

White, Male and College Educated?







- Blue line shows in early years, poor-fast growers mostly non-white, non-male, non-educated
- Orange line same slope but flatter, showing that poor-fast growers (i.e. household experiencing shocks) more diffused across demographic strata.

Suggestions from data

- Micro data is calling for a mechanism that puts more households at risk of becoming a poor fast grower, but also of becoming very rich slow grower
- Want to study aggregate consequences of these changes

A simple model of unequal growth

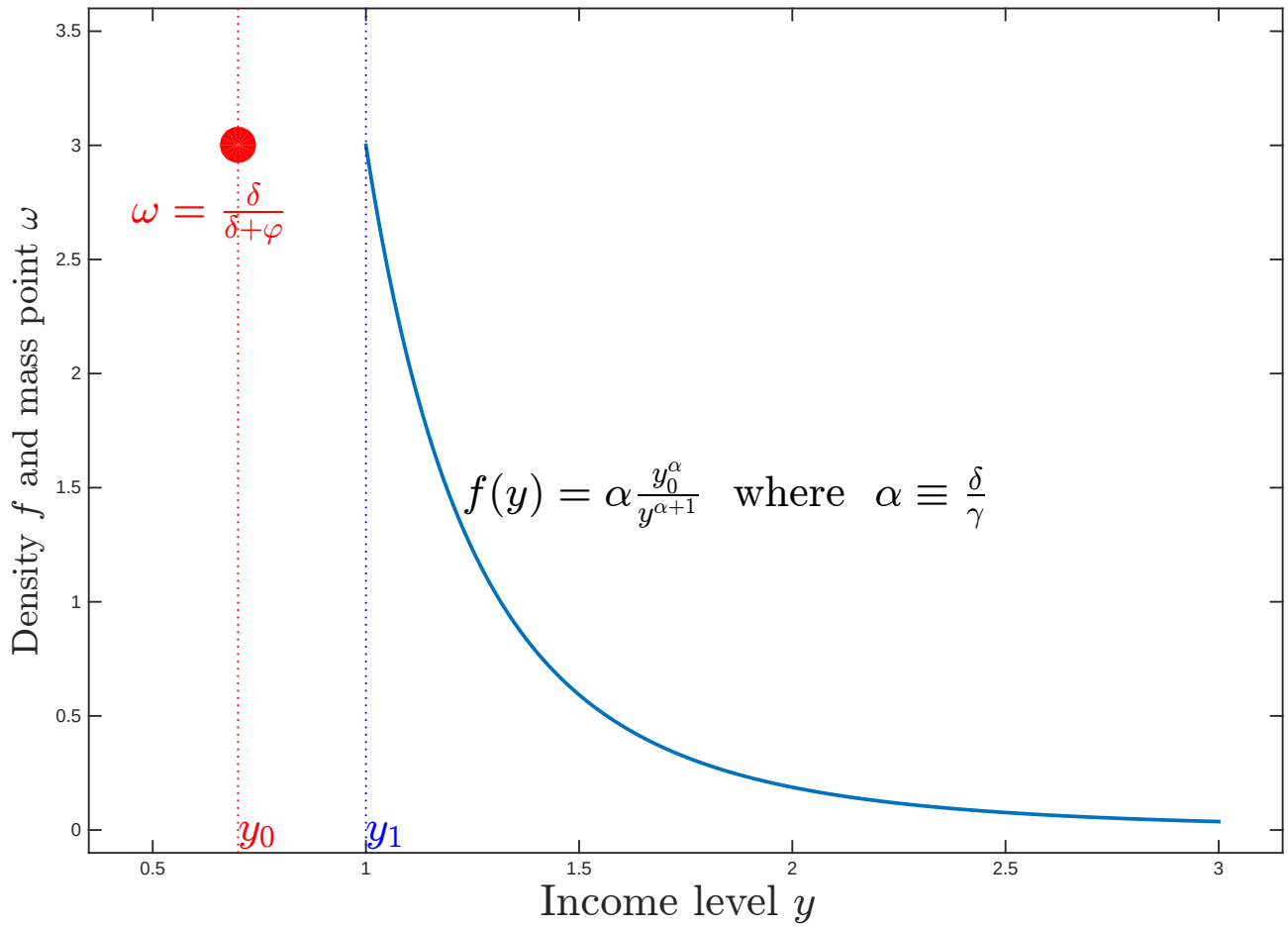
Qualitative analysis:

- Structural mechanism for $\{\Gamma_t, \sigma(s_i)_t, cov_t\}$ dynamics

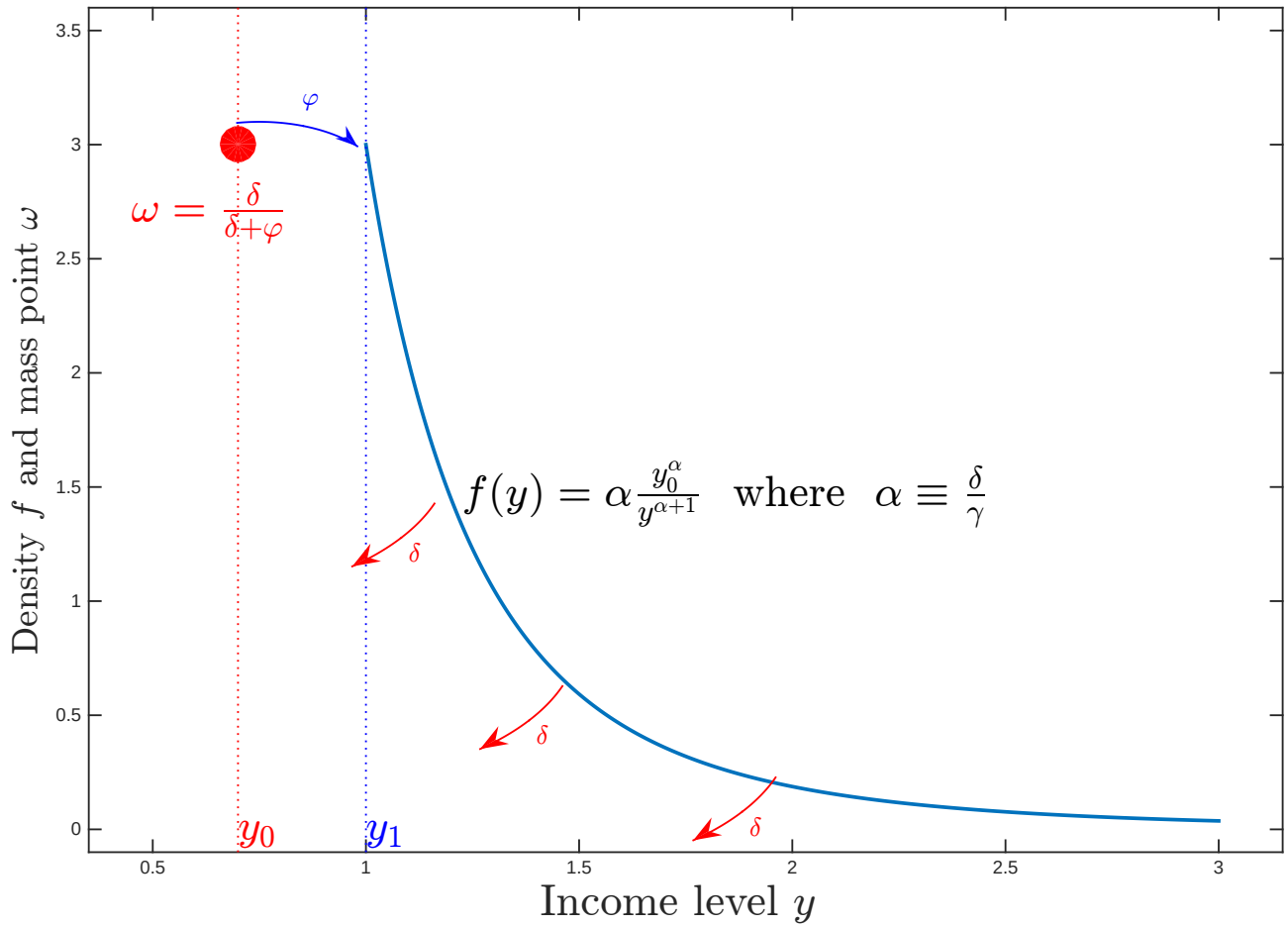
Environment: Income produced by successful projects/jobs

- New projects created at rate φ; die at rate δ
- Income from first successful project is y_1 grows at rate γ (state $\pi = 1$)
- Fraction $\omega \equiv \frac{\delta}{\delta+\varphi}$ of agents w/o project ($\pi = 0$), income y_0

Model in a nutshell (steady state)

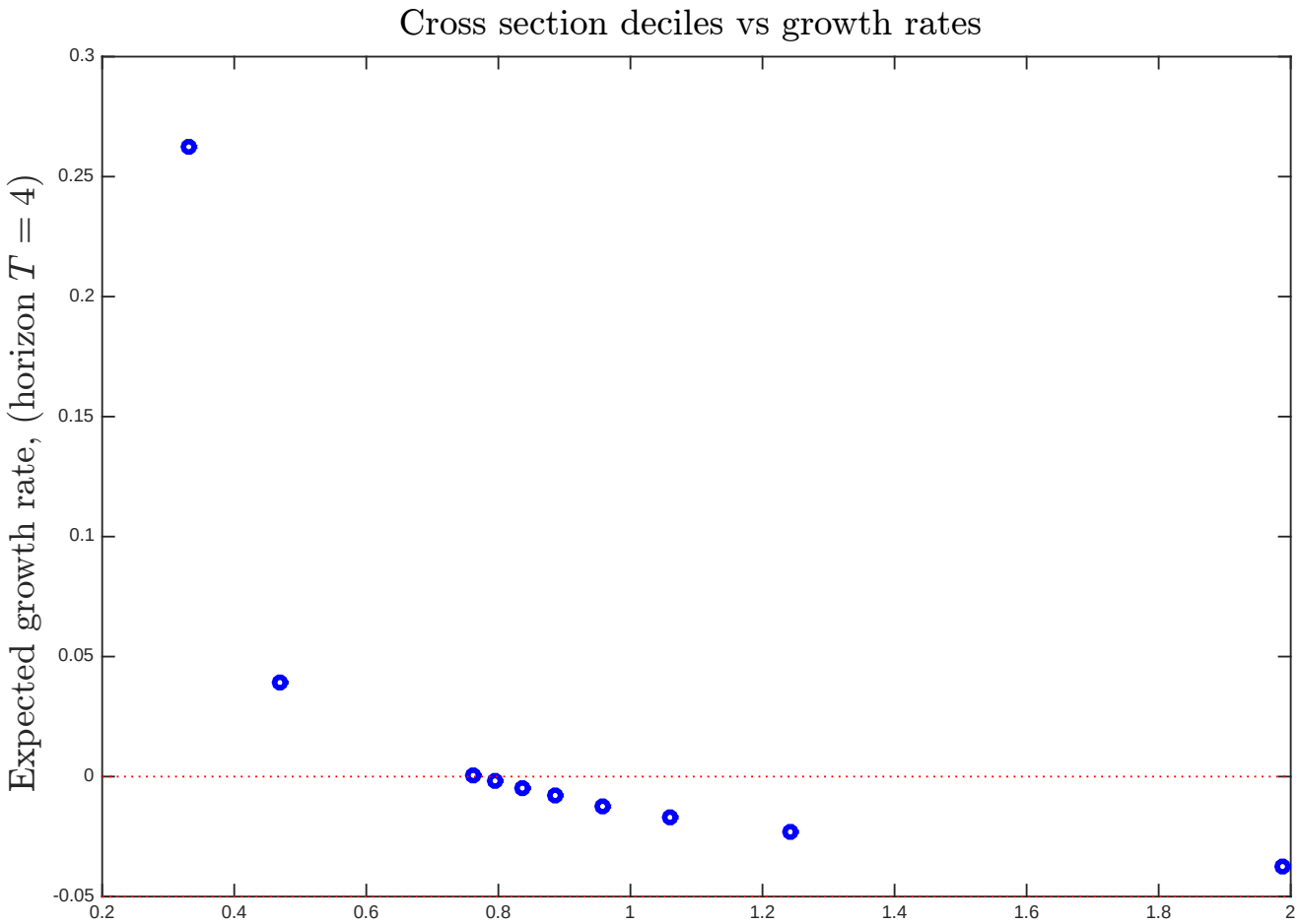


Model in a nutshell (steady state)



(1) Pareto income distribution (2) Mean Reversion

Model in a nutshell (unequal growth)



- Rich households on average grow slower

Model's essentials

- Steady state w/o growth $\Gamma = 1$ (remove common growth factor)
- New StSt yields non-trivial dynamics: $\{\Gamma_t, cov(t), \mathbb{E}(g_{i,t}), \sigma(s_i, t)\}$
- Stylized mechanism gives (analytic) dynamics behind data

Transition following change in fundamental parameters

- Suppose two parameters change: $\tilde{\varphi}, \tilde{\gamma}$
  Example: harder to succeed ($\varphi \downarrow$), new projects grow faster ($\gamma \uparrow$)

- Optional: exogenous downward trend in income of poor y_0
  data: income of poorest decile falls by 2% p.y. over 1970-2010

- Gradual convergence to new steady state

- Analytics of *transition* give: $\{\Gamma(t), \mathbb{E}(g_{i,t}), cov(t), \sigma_{s_i}(t)\}$
  – observable moments t periods after the shock occurred

Characterise transition after shocking φ and γ

$t \in (0, \infty)$ time elapsed since shock

– fraction of agents who **do not grow**: $\tilde{\omega}(t)$

$$\tilde{\omega}(t) = \tilde{\omega} + (\omega - \tilde{\omega})\, e^{-(\delta+\tilde{\varphi})t} \quad \text{where} \quad \omega_i = \frac{\delta}{\delta + \varphi_i}$$

– fraction of agents with **NEW project** (i.e. with $\tilde{\gamma}$): $\eta(t)$

$$\eta(t) = 1 - \tilde{\omega} - (\omega - \tilde{\omega})e^{-(\delta+\tilde{\varphi})t} - (1 - \omega)e^{-\delta t}$$

– density of agents with **NEW project**: $\tilde{f}(y,t)$ solves KFE

$$\frac{\partial}{\partial t}\tilde{f}(y,t) = -\frac{\partial}{\partial y}\left(\tilde{f}(y,t)\tilde{\gamma}y\right) - \delta\tilde{f}(y,t) \quad \text{s.t.} \quad \int_{y_1}^{y_1 e^{\tilde{\gamma}t}} \tilde{f}(y,t) = \eta(t)$$

Characterising transition in closed form!

Solving the PDE (using eigenvalue-eigenfunction decomposition) gives

$$\tilde{f}(y,t) = (1-\tilde{\omega})\tilde{\alpha}\frac{y_1^{\tilde{\alpha}}}{y^{1+\tilde{\alpha}}} + e^{-(\delta+\tilde{\varphi})t}(\omega-\tilde{\omega})\frac{\tilde{\varphi}}{\tilde{\gamma}}\frac{y_1^{-\frac{\tilde{\varphi}}{\tilde{\gamma}}}}{y^{1-\frac{\tilde{\varphi}}{\tilde{\gamma}}}}$$

where $\delta+\tilde{\varphi}$ is the "dominant eigenvalue" (as in Gabaix et al. 2016)

– distribution of incomes t at time t :

$$f(y,t)\begin{cases}\tilde{f}(y,t) & \text{for} \quad y \in (y_1, y_m(t)) \\ \tilde{f}(y,t) + (1-\omega)\frac{\alpha y_1^{\alpha}}{y^{1+\alpha}} & \text{for} \quad y \in (y_m(t), y_M(t)) \\ (1-\omega)\frac{\alpha y_1^{\alpha}}{y^{1+\alpha}} & \text{for} \quad y \in (y_M(t), \infty)\end{cases}$$

where $y_m(t) = y_1 e^{\gamma t} \quad , \quad y_M(t) = y_1 e^{\tilde{\gamma} t}$

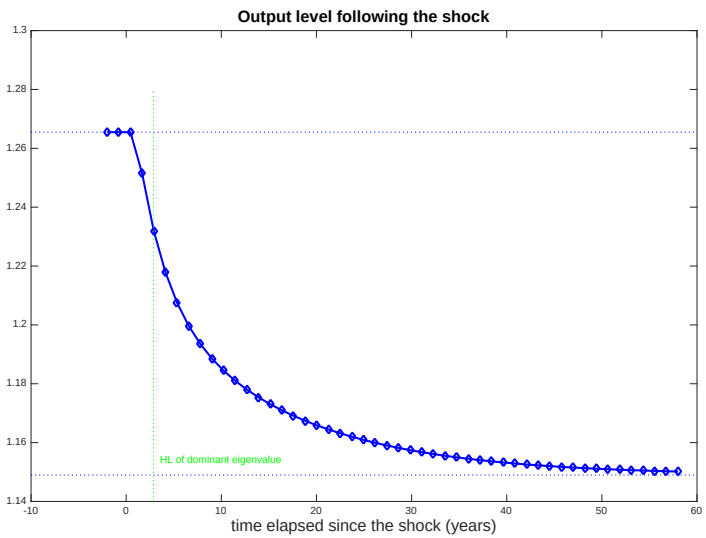
Transition dynamics: Three scenarios

- Scenario 1
  - reduce φ (more difficult to succeed)
- Scenario 2
  - increase γ (better growth potential)
  - reduce φ (more difficult to succeed)
- Scenario 3
  - Adds gradual fall in $y_0(t)$ to Scenario 2
- Three scenarios stylized features of globalization?

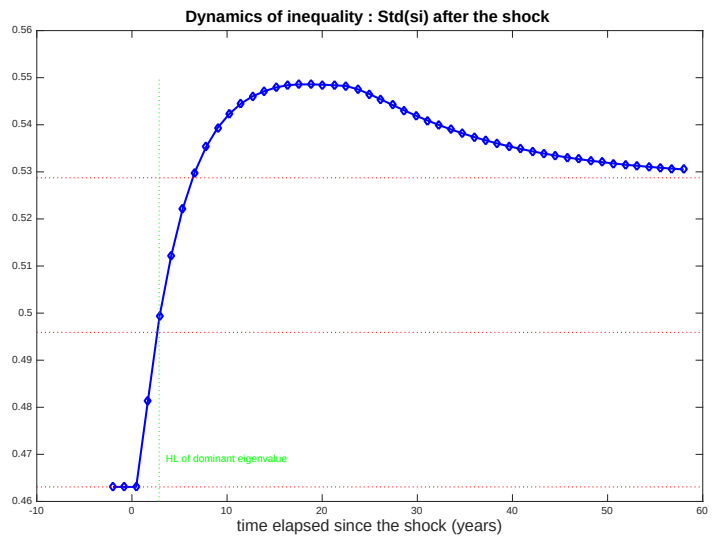
Transition dynamics (scenario 1)

Shock lowers φ: harder to have a successful project

GDP Level

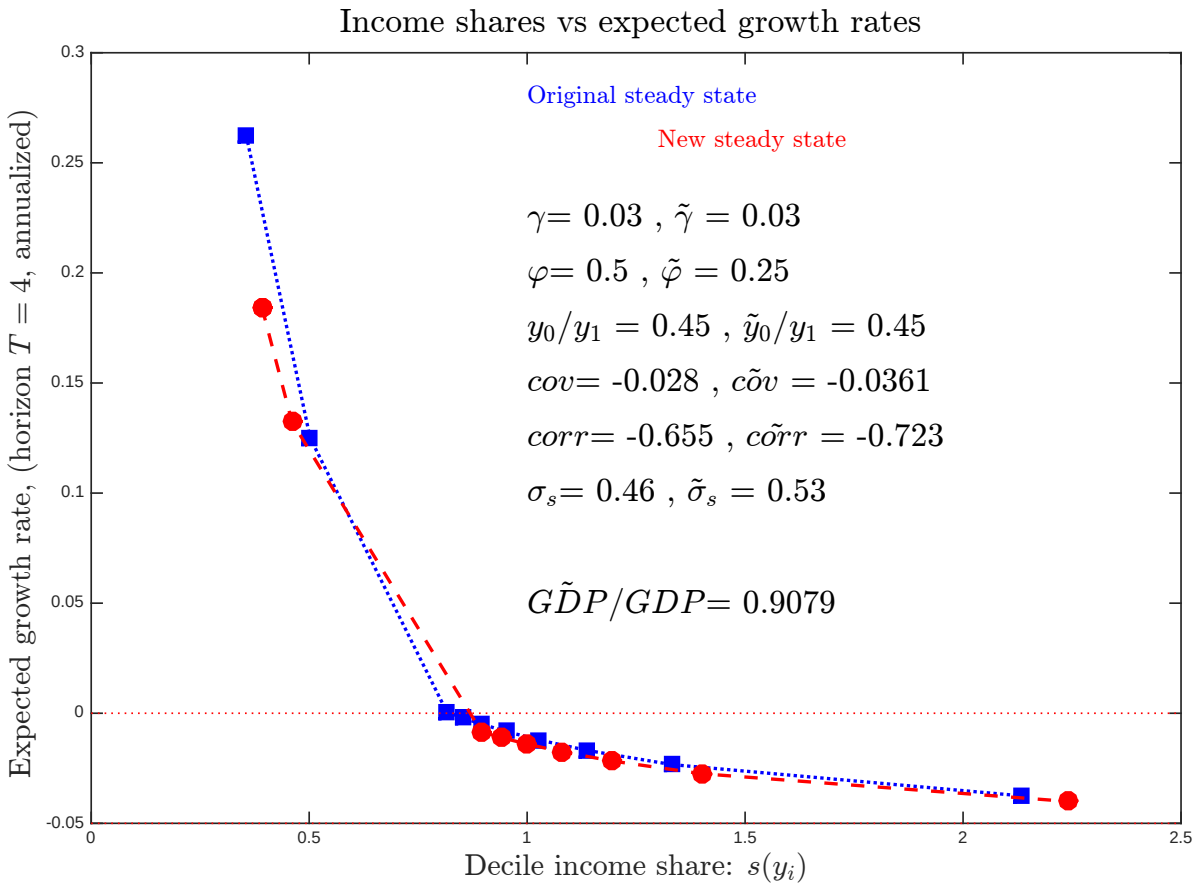


Inequality (σ_s)



Mean annualized growth effect over first 10 years is -0.66%

Cross sectional moments in Scenario 1

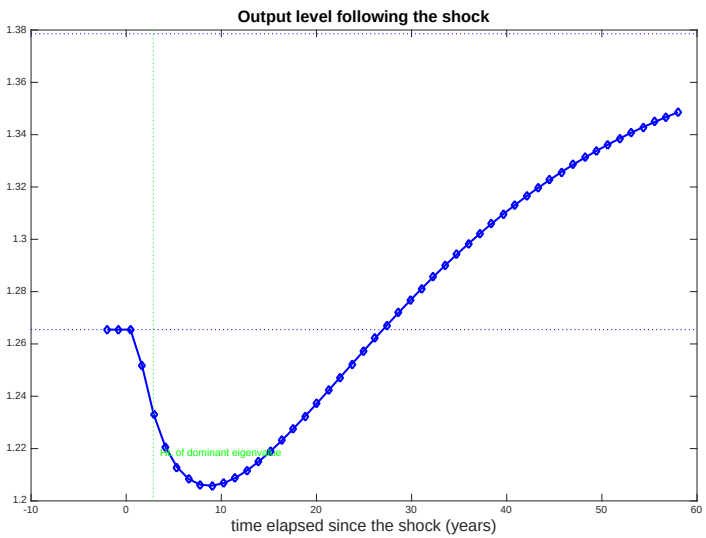


- Growth and covariance fall, inequality rises but too little, fall in growth concentrated at the bottom

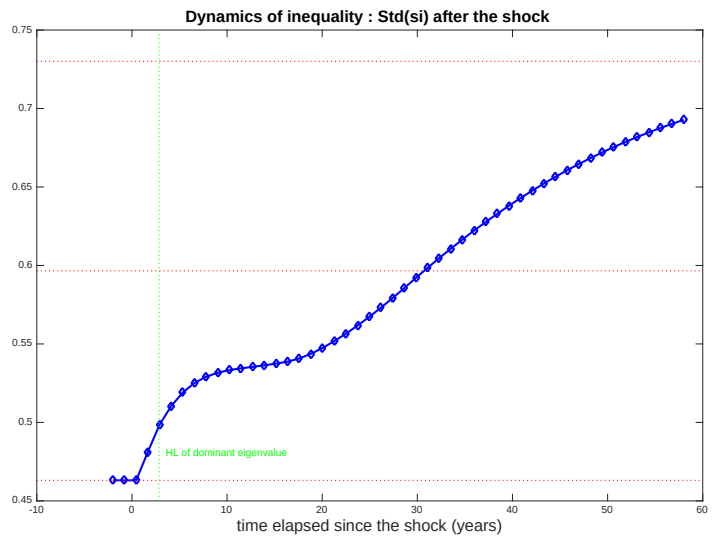
Transition dynamics (scenario 2)

Shock triggers higher γ and lower φ (constant y_0)

GDP Level

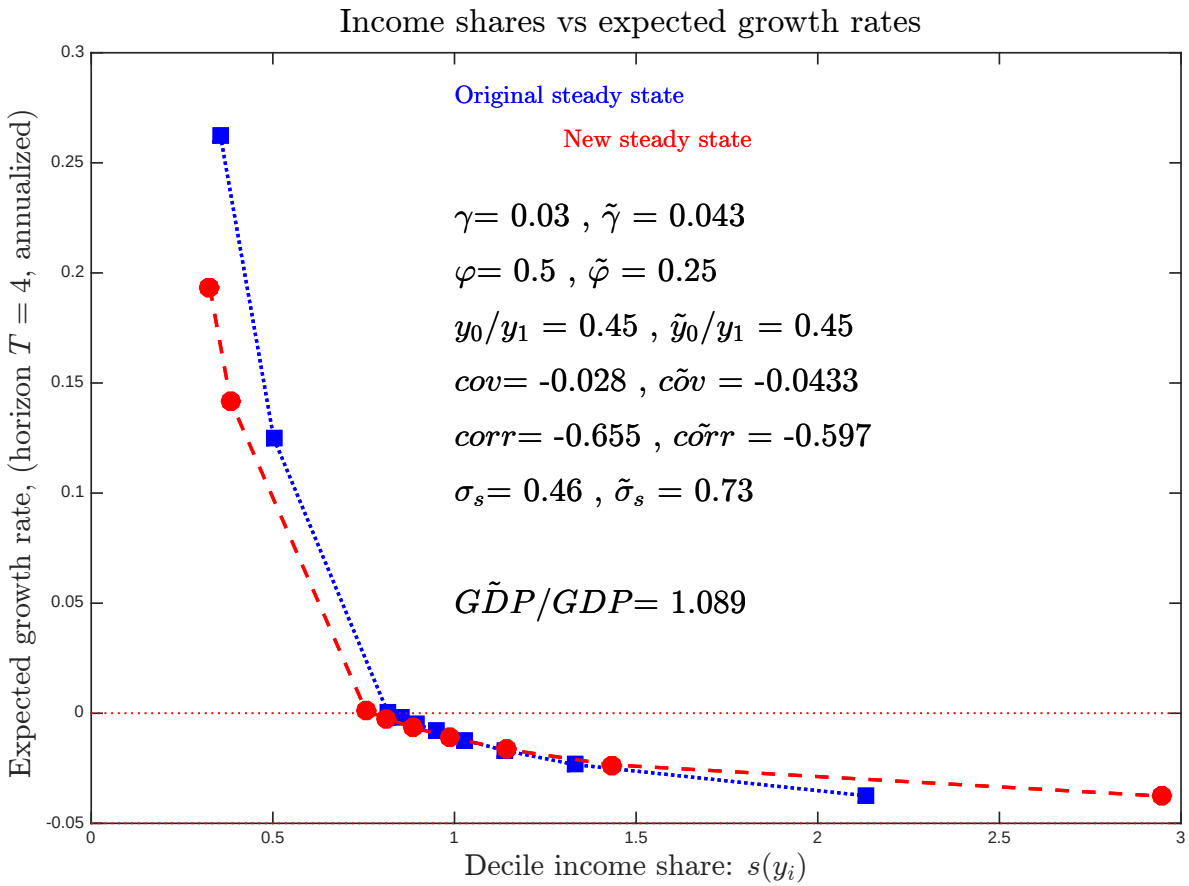


Inequality (σ_s)



Mean annualized growth effect over first 10 years is -0.45%

Cross sectional moments in Scenario 2

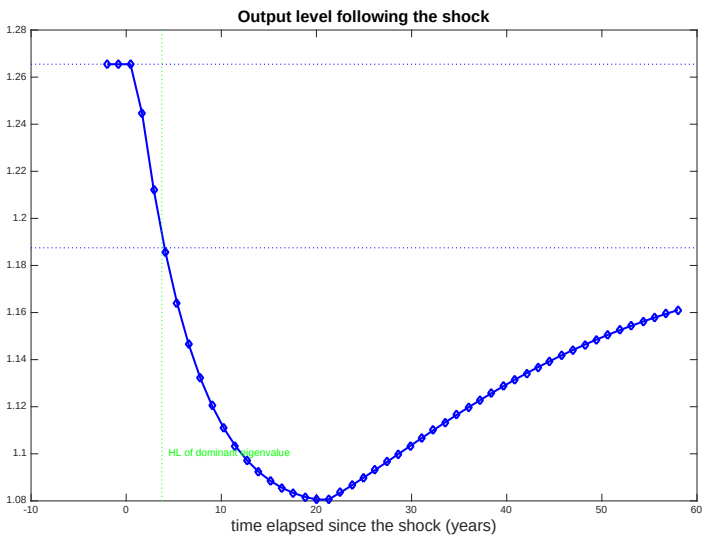


- Larger increase in ineq., fall in cov., increase in corr! Fall in growth still concentrated at the bottom

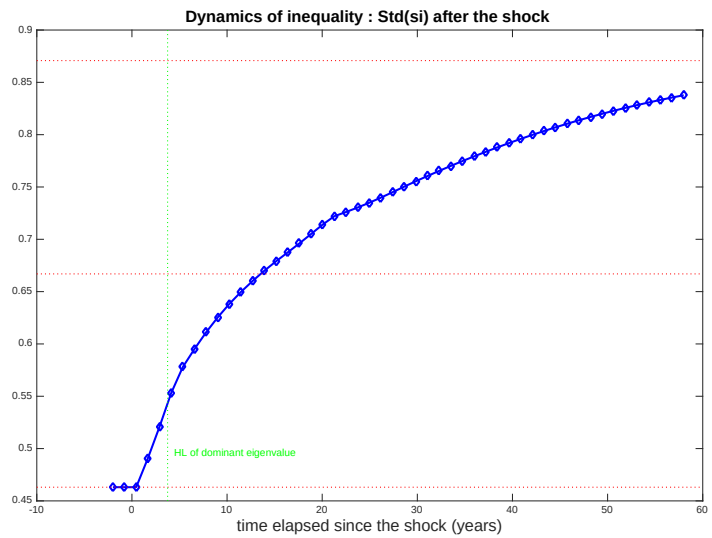
Transition dynamics (scenario 3)

Shock triggers higher γ and lower φ & falling y_0

GDP Level

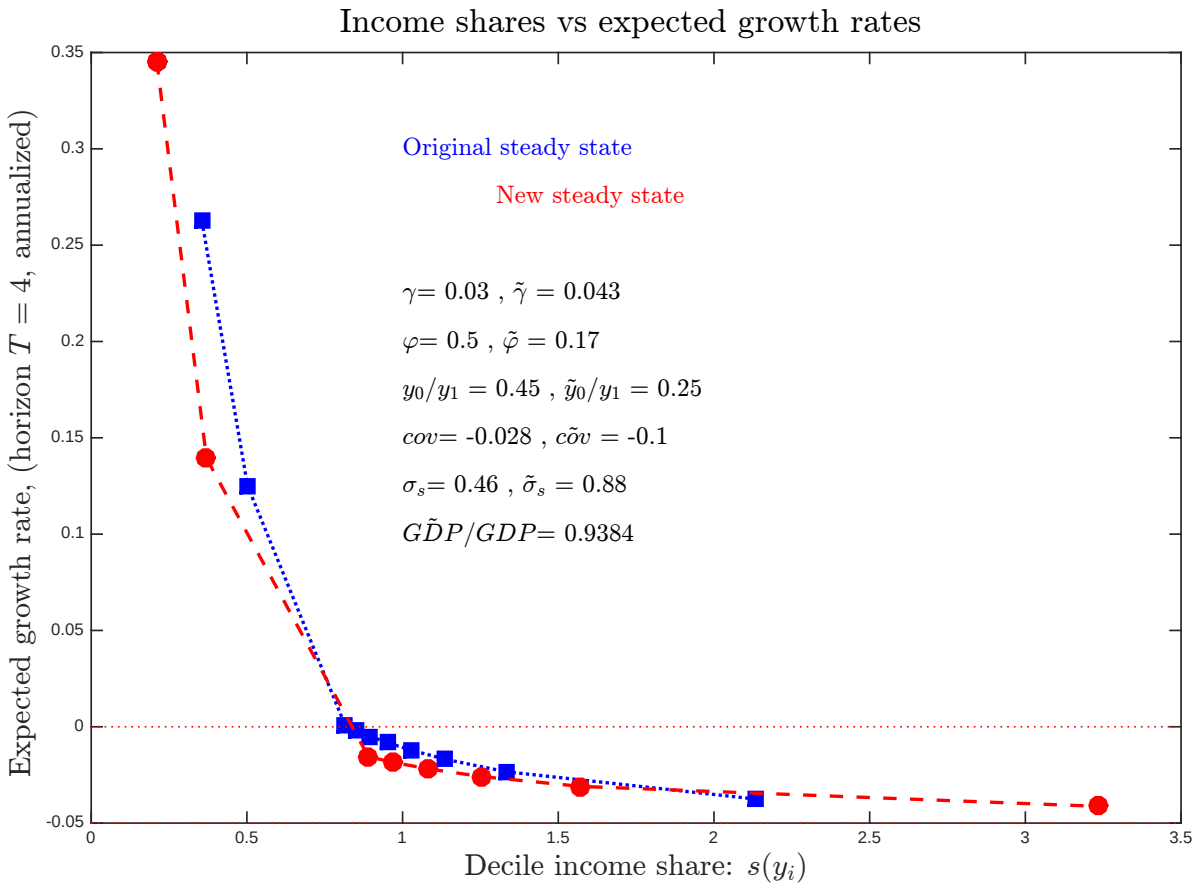


Inequality (σ_s)



Mean annualized growth effect over first 10 years is -1.2%

Cross sectional moments in Scenario 3



- Qualitative success

Summarizing model results

Positives

- Pareto income distribution
- Mean reversion (higher growth rates for poor)
- Persistent growth slowdown accounted by fall in covariance
- Provides insights on what structural change are needed to explain the facts
- Explain β convergence and σ divergence

Negatives

- Mechanical (no choices)
- Quantitative fit could be improved

Closing remarks and open issues

- $\Gamma_T = cov(g_{i,T}, s_i) + E(g_{i,T})$
- Empirically: large deviations from Gibrat's low: $Cov(g_i, s_i) < 0$
- Most US growth decline over past 40 year accounted by $Cov(g_i, s_i)$
- Empirically $Cov(g_i, s_i) \downarrow$ results mostly from $\sigma(s_i) \uparrow$

- Simple model: links inequality and growth changes
- Key shocks: bigger opportunities $\gamma \uparrow$ and risks $\varphi \downarrow y_0 \downarrow$
- model qualitatively consistent with $\Gamma \downarrow, Cov \downarrow$ and $\sigma(s_i) \uparrow$

Open issues

- Can the model generate micro and macro dynamics quantitatively consistent with data?
- What is the welfare impact of these changes (higher inequality, reduction in rank mobility)?