Five Facts About Beliefs and Portfolios

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Abstract

We administer a newly-designed survey to a large panel of retail investors who have substantial wealth invested in financial markets. The survey elicits beliefs that are crucial for macroeconomics and finance, and matches respondents with administrative data on their portfolio composition and their trading activity. We establish five facts in this data. (1) Beliefs are reflected in portfolio allocations. The sensitivity of portfolios to beliefs is small on average, but varies significantly with investor wealth, attention, trading frequency, and confidence; (2) It is hard to predict when investors trade, but conditional on trading, beliefs changes affect both the direction and the magnitude of trades. (3) Beliefs are mostly characterized by large and persistent individual heterogeneity; demographic characteristics struggle to explain why some individuals are optimistic and some are pessimistic. (4) Investors disagree about both expected cash flows and discount rates. At the investor level, higher expected cashflows are associated with higher discount rates. (5) Expected returns and the probability of rare disasters are negatively related. Our results challenge the rational expectation framework for macro-finance, and provide important guidance for the design of behavioral models.

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I Introduction

Researchers are increasingly turning to survey data to calibrate and test macro-finance models. The unique benefit of survey data is that it can provide direct evidence on the beliefs of different agents about future economic outcomes such as returns and cash flows, which play a fundamental role in both rational expectation models and behavioral models. Despite the potential for survey data to shed light on previously unobservable elements of these theories, its use has been criticized on many fronts. Critics have variously argued that survey data is often based on small and unrepresentative samples, that it is ridden with measurement error, that it asks qualitative questions that are not informative for models, and that it may not reveal those beliefs on which agents actually base their actions.

In this paper, we provide new evidence on the link between beliefs elicited through surveys and real actions taken by survey respondents. To do this, we administer a newly-designed online belief survey to a large panel of retail investors with substantial wealth invested in financial markets. The survey elicits investors' beliefs about future stock returns, GDP growth, and bond returns, and was explicitly designed to address prevailing criticisms of existing survey data. The survey design trades off asking quantitative questions about moments that are crucial for economic theory with keeping the questions sufficiently simple that they can be answered by non-specialists. The survey is short to avoid discouraging investors from taking it again over time, thus allowing us to obtain a large number of repeat responses. We elicit the most important beliefs in different ways, to make use of recent advances in instrumental variable techniques to tackle concerns related to measurement error. For example, for subjective expectations of 1-year stock returns, we elicit both point estimates as well as distributions about different possible realizations.

The survey was administered to U.S.-based clients of Vanguard, one of the world's largest asset management firms. The respondents are individuals relevant for macro-finance models: they participate in stock and bond markets and have substantial wealth, with the average respondent holding nearly half a million dollars of assets at Vanguard. The survey has been conducted every two months since February 2017. In this paper, we study the first ten survey waves, which generated more than 20,000 responses. Many individuals respond to multiple survey waves, providing a substantial panel component to our data. We link respondents' survey answers to anonymized administrative data on their investment holdings and transactions at Vanguard. This allows us to explore the relationship between the elicited beliefs and real-world high-stakes investment behavior. While Vanguard clients are likely to disproportionately include passive buy-and-hold investors, whenever possible, we verify that the patterns in our survey data line up with the corresponding patterns in other surveys covering different investor populations.

Our most general finding is that survey data is informative about individuals' portfolio decisions. We find a robust and statistically significant relationship between beliefs and portfolio composition, both across individuals and within individuals over time. In this sense, we conclude that survey-based evidence is "here to stay," and that theoretical work has to continue to confront survey-based evidence. We organize our empirical results around five facts. These facts highlight

empirical patterns about beliefs as well as their relationships with portfolios. For each fact, we discuss the implications for macro-finance theory.

Fact 1 summarizes our main findings on the relationship between beliefs and portfolios. We first establish that there is a statistically strong relationship between beliefs and portfolio composition, but that the average sensitivity of an investor's equity share to that investor's perceived risk and expected return of the stock market is substantially lower than predicted by benchmark frictionless macro-finance models. Controlling for various measures of risk perception, a 1 percentage point increase in expected returns over the next year is associated with a 0.7 percentage points higher equity share. This relationship is not primarily driven by measurement error in beliefs and the associated attenuation bias. We find that the perceived variance of stock returns has both an economically and statistically weak relationship with portfolios, and that a better measure of risk is the probability of a large stock market drop (a rare disaster). However, even this tail probability has only has a small effect on portfolios, with an increase of 1% in the probability of a stock market drop of more than 30% decreasing the portfolio equity share by 0.1%.

This relatively small response of equity shares to beliefs about stock returns is qualitatively and quantitatively consistent with evidence documented across a number of other studies that link equity market participation and equity shares to expected stock market returns (e.g., Vissing-Jorgensen, 2003; Dominitz and Manski, 2007; Kézdi and Willis, 2011; Amromin and Sharpe, 2013; Ameriks et al., 2016; Drerup, Enke and Von Gaudecker, 2017), though many of these studies relied on self-reported equity shares or a binary indicator for holding stocks rather than administrative data. Our contribution to this literature is twofold. First, we establish this fact for a large sample of wealthy investors, while accounting for key dimensions of measurement error. Second, and importantly for the interpretation of this result, we also explore the empirical relevance of a number of possible explanations for the low observed average sensitivity. The second part of Fact 1 summarizes our findings: the sensitivity of portfolios to beliefs is increasing in wealth; it is also higher in tax-advantaged accounts, and increasing in investors' trading frequency, investors' attention to their portfolios, and investors' confidence in their own beliefs. An idealized investor who is wealthy, pays attention to her portfolio, trades often, and is confident in her beliefs has a sensitivity that is about five times larger than the average sensitivity, though it still falls short of the sensitivities generated by frictionless benchmark models.

We next investigate the role that belief changes play in explaining trading activity. Fact 2 establishes that belief changes have minimal explanatory power for predicting when trading occurs (the extensive margin), but explain both the direction and magnitude of the trading conditional on a trade occurring (the intensive margin). At a broad level, these results confirm that beliefs as elicited by surveys play an important role in explaining actual investment decisions such as trading. This is complementary to the analysis of portfolio shares across individuals, since trading focuses on active portfolio changes, while cross-sectional portfolio shares are also affected by market movements. We find it natural that predicting whether trading occurs over short horizons is difficult, since it can often be caused by factors other than beliefs about risk and returns, such as

liquidity needs induced by life events. Our findings are consistent with models of infrequent trading that endogenously or exogenously generate a flat hazard function of trading based on belief changes. We find it interesting and encouraging, however, that conditional on a trade occurring, beliefs play an important role in explaining both the direction and the magnitude of the trade.

At a fundamental level, these first two facts challenge the notion, most forcefully articulated by Cochrane (2011, 2017), that survey data and the associated critiques of rational-expectations models of macro-finance can be discarded because respondents do not understand the questions or because their answers reveal beliefs that do not correspond to their investments.[1] While all belief surveys are imperfect and subject to valid concerns about multiple dimensions of measurement error, we document that survey answers are nevertheless relevant to understanding individual behavior. As a result, our findings support the view, expressed most prominently by Greenwood and Shleifer (2014) and Barberis et al. (2015), that the challenges that survey data pose for macro-finance models should be taken seriously.

While it is beyond the scope of this paper to provide a fully specified theoretical model to explain our findings, we find it useful to discuss how our facts are likely to shape theory. A natural starting point are the most recent behavioral finance models, such as Barberis et al. (2015), which were built explicitly to match newly uncovered survey evidence. Our analysis sheds direct light on the transmission mechanism of variation in beliefs into quantities and equilibrium prices, and can guide the future evolution of these models by showing what works, what does not, and what modeling options might resolve the conflicts. Existing behavioral finance models typically imply a portfolio demand with a similar sensitivity to beliefs as in rational-expectations models, while imposing expectations that are more consistent with survey evidence. Our evidence shows that, for the average investor, the sensitivity of portfolio demand is much weaker than implied by common calibrations of these models. As a result, current frictionless behavioral models are likely to overstate the power of expectation changes in explaining asset price movements.

Our first two facts suggest ways to improve these models to jointly match expectations, portfolio dynamics, and asset prices. We find that trading occurs infrequently, and that the timing of trading does not depend meaningfully on beliefs; however, conditional on trading, beliefs matter for the size and direction of the trade. A possible model adjustment to capture this feature would be to introduce infrequent random trading, á la Calvo, and portfolio adjustment costs. It remains an open question whether a model in which behavioral agents are less reactive to their beliefs can quantitatively match asset prices. One possible avenue for the behavioral agents' expectations to have a larger impact on equilibrium prices is to have the behavioral group be substantially wealthier (in aggregate) than rational arbitrageurs. This would amplify the importance of changes in demand by the behavioral agents. Alternatively, one could make the demand of other classes of agents present in this class of models, like rational arbitrageurs, even more inelastic than the demand of behavioral agents. Our results on the heterogeneity of sensitivities along investor

[1] Greenwood and Shleifer (2014) and Adam, Matveev and Nagel (2018) refute the idea that surveys simply reflect risk-neutral expectations.

characteristics (like confidence or attention) can be useful in guiding such modeling.

After analyzing the relationship between investors' beliefs and their portfolios and trading behavior, we next decompose the variation in beliefs across individuals and over time. Fact 3 establishes that individual beliefs are mostly characterized by heterogenous and persistent individual fixed effects. Some individuals are optimistic and some are pessimistic, and their beliefs are persistent and far apart. While there is some co-movement in beliefs across individuals over time, the time variation in average beliefs only accounts for about 1% of the total variation in beliefs in the panel. Instead, between 50% and 60% of all belief variation in our panel is due to individual fixed effects, while the rest is due to idiosyncratic individual variation or measurement error. We also find that the heterogeneity in beliefs is not explained by observable characteristics such as gender, age, wealth, attention, confidence, and geographic location. These characteristics sometimes have strong statistical relationships with beliefs, but their explanatory power is limited.[2] We provide evidence that this is not the result of measurement error in eliciting beliefs. A likely explanation is that individual fixed effects reflect combinations of individual characteristics and personal experiences that researchers are yet to discover.

Fact 3 provides a powerful characterization of beliefs across agents, which can be compared to the assumptions of macro-finance models. Consider two stylized descriptions of existing models. At one extreme are models in which all individuals hold the same beliefs and the only variation is in the time series. This first class of models generates variation in asset prices with swings in individual beliefs over time, which directly translate into changes in the representative agent's beliefs. These models thus derive their aggregate implications from only a small component of the total variation in beliefs, and are silent on a major feature of the data, namely the heterogeneity across individuals. Our empirical results suggest that it might be important for these models to explicitly consider the aggregation of the heterogeneity in individual beliefs.

At the other extreme are models in which agents have permanent and constant differences in beliefs. This second class of models generates variation in asset prices by movements in wealth-weighted aggregate beliefs. Intuitively, this requires several steps: individual heterogeneity in beliefs induces heterogeneity in portfolios; given these differences in portfolios, shocks over time redistribute wealth across individuals; this changes wealth-weighted beliefs and thus asset prices. This class of models captures the individual static heterogeneity in the data, but relies on having this belief heterogeneity strongly reflected in portfolios. Fact 1 highlights that the average quantitative relation between beliefs and portfolios is limited. If individuals with different beliefs do not have very different portfolios, then shocks do not substantially redistribute wealth between optimists and pessimists, thus minimizing the variation in aggregate wealth-weighted belief. Above, we discussed possible paths to align these models more closely with the data.

[2]This finding is closely related to the literature linking expectations to demographic characteristics and personal experiences. It is common in this literature to find strong statistical relationships but low explanatory power between expectations and variables such as wealth, gender, IQ, place of birth, current location, own past experience, or friends' past experiences (see, for example, Malmendier and Nagel, 2011; Kuchler and Zafar, 2015; Das, Kuhnen and Nagel, 2017; Bailey et al., 2017, 2018; Coibion, Gorodnichenko and Kamdar, 2018; D'Acunto et al., 2019).

We next explore which beliefs individuals disagree most about. Fact 4 shows that investors disagree about both expected returns and expected cash-flow growth, and that their beliefs are positively correlated across these objects: at each point in time, individuals who expect higher cash flow growth also tend to expect higher returns in both the short run and the long run. We interpret these patterns through the lense of the Campbell and Shiller (1988) identity, which links expected returns, cash flows, and expected future price-dividend ratios: if investors disagree about one element of the decomposition, they must also disagree about some of the other elements. Quantitatively, the effect of higher expected cash-flows is partially offset by higher discount rates, but to make the identity balance with current (observable) prices, investors must also implicitly disagree about the expected long-run price-dividend ratio. While existing theories usually model disagreement in either cash flows or expected returns (e.g., Cutler, Poterba and Summers, 1990; De Long et al., 1990*a*,*b*; Barberis et al., 2015), our results thus show that disagreement about both objects is important and that, at least for quantitative evaluations, their dynamics should be modeled jointly.

Our final fact, Fact 5, establishes that when individuals expect large stock market declines ("rare disasters") to occur with higher probability, they also expect stock market returns to be lower. This holds both across individuals and within individuals over time. This relates to an important strand of the macro-finance literature, which has emphasized that expectations of rare but potentially catastrophic events, sometimes referred to as rare disasters, can help explain portfolio holdings and asset prices (e.g., Rietz, 1988; Barro, 2006; Gabaix, 2012). This literature stresses that the expected probability and size of rare disasters is one of the most important moments to understand risks and returns. In standard equilibrium models with rare disasters, expected returns and the probability of disaster are positively related. The intuition is that a higher probability of disasters induces individuals to demand higher compensation for holding stocks, and thereby increases equilibrium expected returns. The relationship in the data appears with the opposite sign, thus posing a challenge to rational-expectations models of rare disasters.

Our results are not inconsistent with the importance of rare disasters for macro-finance theory more generally, and we find evidence that disaster expectations are reflected in portfolio choices. In fact, it is likely that modifications to existing models that incorporate at least some behavioral agents can more closely align model predictions and data. When these behavioral agents believe that rare disasters have become more likely, they do not take into account that other agents might think the same, and therefore do not incorporate the fact that stock prices might already be lower to account for the increased risk. Rather, they think that prices are still high, and expect returns to be lower going forward. They therefore sell stocks to reduce their exposure, by more than they would if they anticipated that the compensation for holding on to the risk is increasing in equilibrium. In turn, this force may induce more selling pressure (and therefore a bigger price effect) in response to an increase in the disaster probability. It is beyond the scope of this paper to provide a full model of such a behavioral approach to rare disasters, but we expect that it will prove an interesting area of future research.

We conclude this introduction by summarizing the desired characteristics of a behavioral

model that would be consistent with our five facts. This largely reflects the way we would write such a model, based on our evidence, rather than the only possible model. The model would have three key ingredients: (i) large and persistent heterogeneity in beliefs, (ii) infrequent trading and portfolio adjustment costs, (iii) overconfidence and partial equilibrium thinking. These would be the salient features of the majority of investors; we would then add a small, in terms of wealth share, competitive fringe of rational arbitrageurs. It is an open question how well such a model would perform in quantitatively matching aggregate asset prices in addition to the main features of beliefs and portfolios documented in this paper.

Further Related Literature. A large and growing literature focuses on exploring the role of beliefs in explaining a large number of economic outcomes (see Gennaioli and Shleifer, 2018, for a review of some of this literature). In this literature, Manski (2004) was among the first and most prominent to argue for using survey data about expected equity returns and risks to better understanding individuals' investment behaviors. Over time, a series of papers has connected survey expectations to the behavior of respondents. For example, the Vanguard Research Initiative has provided substantial advances in linking survey evidence to retirement choices (Ameriks et al., 2016, 2015*a*,*b*, 2017, 2018).[3] As part of this agenda, Ameriks et al. (2016) find a low sensitivity of equity investment to stock market expectations, a fact also emphasized by Vissing-Jorgensen (2003), Dominitz and Manski (2007), Kézdi and Willis (2011), Amromin and Sharpe (2013), Merkle and Weber (2014) and Drerup, Enke and Von Gaudecker (2017). Our work builds on this literature by exploring a quantitative survey of a large panel of wealthy investors, specifically designed to inform theoretical models, which is matched to administrative data on these investors' portfolios and trading behaviors. This allows us to discover new facts, and deepen our understanding of existing patterns both quantitatively and in terms of their variation across people and time.[4]

II Survey Description

To explore the structure of investors' beliefs and the relationship between those beliefs and investors' portfolio allocations, we designed a new online survey of U.S.-based individual investors who hold accounts at Vanguard, one of the world's largest asset management firms with about $5.1 trillion in assets under management. We first provide a high-level overview of the survey questions. The exact phrasing and the survey interface are presented in the Appendix. We then explore the survey sample, the response rates, and the demographics of respondents and non-respondents, which allow us to analyze the dimensions of selection into responding.

[3]The GMS-Vanguard project that this paper is based on is a distinct and complementary activity to the Vanguard Research Initiative.

[4]Survey-based evidence has also started to inform the debate in macroeconomics about inflation expectations and firms' real investments (e.g., Cummins, Hassett and Oliner, 2006; Bacchetta, Mertens and Van Wincoop, 2009; Coibion and Gorodnichenko, 2012; Gennaioli, Ma and Shleifer, 2016; Bordalo et al., 2018; Fuhrer, 2018). In part based on these surveys, there is growing interest in macroeconomic theory with a strong behavioral content (e.g., Gabaix, 2016; Farhi and Werning, 2017; García-Schmidt and Woodford, 2019).

II.A Survey Design

The survey includes questions on three broad topics: expected stock-market returns, expected GDP growth rates, and expected bond returns. In the implementation of the survey, we randomized whether individuals were first asked about their expectations on stock returns or GDP growth rates. The questions on bond returns were always asked last.

Expected Stock Market Returns. We asked respondents about their expectations for the return of the U.S. stock market. We elicited point estimates for the expected annualized returns over the coming year and the coming ten years, as well as subjective probabilities that the return over the next year would fall into one of five buckets: less than -30%, between -30% and -10%, between -10% and 30%, between 30% and 40%, and more than 40%.[5] This set-up allows us to elicit investors' beliefs about 1-year expected stock returns through both point predictions, the direct question, and probabilistic assessments, the implied expectation from the elicited distribution of returns (see Manski, 2018). For all distribution questions, we randomized the ordering of the buckets (i.e., lowest to highest vs. highest to lowest) across survey respondents, and the survey enforced that the assigned probabilities add up to 100%. As shown in the Appendix, the survey interface also showed real-time histograms of the survey responses as they were entered in order to help individuals visualize the probability distribution implied by their numerical answers.

Expected Real GDP Growth Rates. We asked respondents about their expectations for the annualized growth rate of real GDP. We elicited point estimates for the expected growth rates over the coming three years and the coming ten years. In addition, we asked about the probabilities that individuals assigned to the annualized GDP growth rate over the coming three years falling into one of five buckets: less than -3%, between -3% and 0%, between 0% and 3%, between 3% and 9% and more than 9%.[6] We again randomized the ordering of buckets across respondents.

Expected Bond Returns. The final set of questions elicited respondents' expectations about U.S. government bond returns and interest rate changes. We first asked respondents for their expectations of the 1-year return of a 10-year U.S. government zero coupon bond. In addition, we elicited individuals' expectations about the yield curve in one year: in particular, we asked what respondents expected the yields-to-maturity of 1-year, 5-year and 10-year zero coupon bonds to be one year in the future. Prior to answering the question, individuals were shown the current yield curve to familiarize them with the concept.

Difficulty and Confidence. At the end of every block of questions (stock, GDP, and bonds), the survey asked individuals how confident they were about their answers (on a five-point scale from

[5] These buckets were chosen such that the tails correspond to extreme events that still have substantial probability mass based on historical frequency. Between 1927 and 2014, annualized stock market returns fell into the lowest bucket 3.7% of the time, the second bucket 11.6% of the time, the middle bucket 65.9% of the time, the fourth bucket 13.8% of the time, and the highest bucket 5% of the time.

[6] As with the stock return questions, these thresholds were chosen to allow individuals to report their expectations over relatively extreme positive and negative events. Between 1929 and 2015, annualized U.S. real three-year GDP growth fell into the lowest bucket 4.2% of the time, the second bucket 5.4% of the time, the third bucket 33.3% of the time, the fourth bucket 50.8% of the time, and the highest bucket 6.3% of the time.

"not at all confident" to "extremely confident"), and how difficult they found the questions (on a five-point scale from "not at all difficult" to "extremely difficult").

II.B Survey Sample and Response Rate

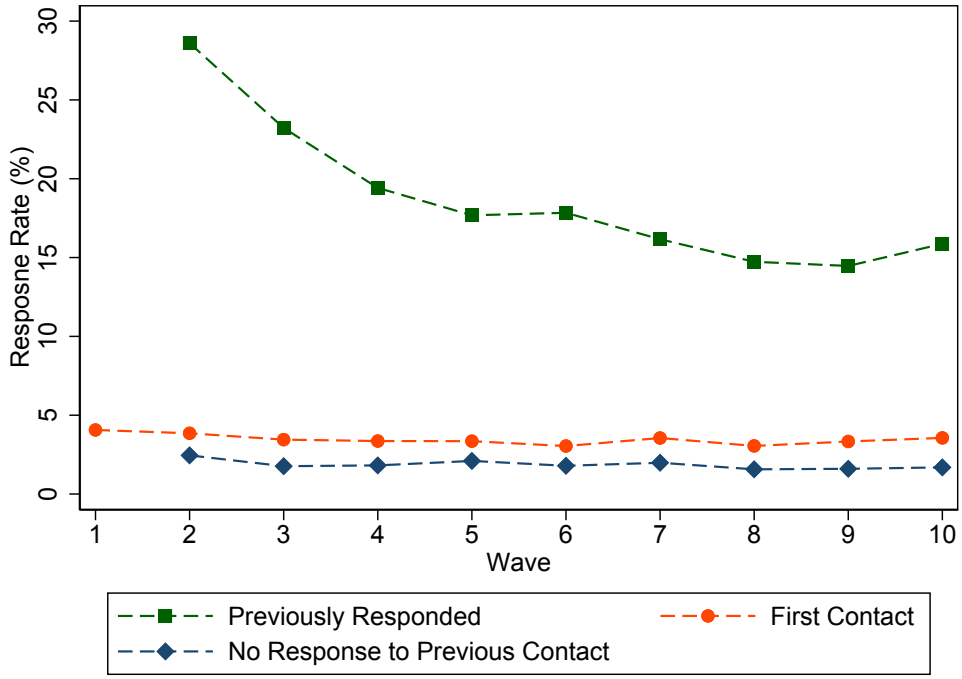
The online survey is conducted every two months among U.S.-based Vanguard investors. In this paper, we explore the first ten waves of the survey, starting in February 2017 and going through August 2018. In the first wave, 40,000 Vanguard clients were invited by email to participate in the survey. These clients were randomly selected such that 80% of the sample was drawn from clients in Vanguard's RIG division, which mostly includes retail brokerage accounts and individually-managed tax-advantaged retirement accounts such as IRAs. The remaining 20% were clients from Vanguard's IIG division, which manages employer-sponsored retirement accounts such as defined contribution plans. Additional requirements to be potentially invited to participate in the survey are that clients: (i) had to have opted into receiving Vanguard statements via email, (ii) had to be between the ages of 21 and 74, and (iii) had to have total Vanguard assets of at least $10,000. If individuals respond to the survey in any wave, they are recontacted in every subsequent wave. Those individuals who do not respond to the first wave in which they are contacted are then recontacted in two subsequent waves. If they respond in neither of these waves, they are dropped from the sample. Individuals can at any point opt out of the study and are, in this case, never contacted again. In the second wave, an additional 25,000 clients were invited to participate (in addition to those carried over from wave 1). Waves 3 to 5 invited 13,000 new clients each; from wave 6 onwards, the number of new clients contacted in each wave was increased to 14,500.

Response Rates. Figure I shows the response rates for the first ten waves, where we count only fully completed surveys as "responses."[7] The orange-circle line shows that the response rate among individuals contacted for the first time was relatively stable at 3%-4% across waves. The response rates among individuals who were previously contacted but had not yet responded, given by the blue-diamond line, were between 1.5% and 2% across waves. The green-square line shows the response rate among individuals who had responded to at least one previous survey. The steady-state re-response rate of these individuals is between 15% and 20%. It declines somewhat over time, though much of this decline is driven by compositional effects: in later survey waves, the average time since the last response of individuals who have previously responded is higher.

These response rates translate into about 2,000 survey responses per wave; across the ten waves, we received 20,207 total responses. The top panel of Figure II reports the number of responses in each wave, with different shades of gray tracking the first wave in which an individual responded. The figure shows that, starting in wave 5, we receive more responses from individuals who are re-respondents than from individuals who are responding for the first time. The bottom panel of Figure II shows that about 35% of responses come from individuals who have responded to one survey only (though some of these may end up responding to future surveys). Over 25%

[7] In the analysis below, we drop a number of partially completed surveys. Most of these answer at most one question before exiting the survey. These are treated as non-responses in the statistics in this section.

Figure I: Survey Response Rates



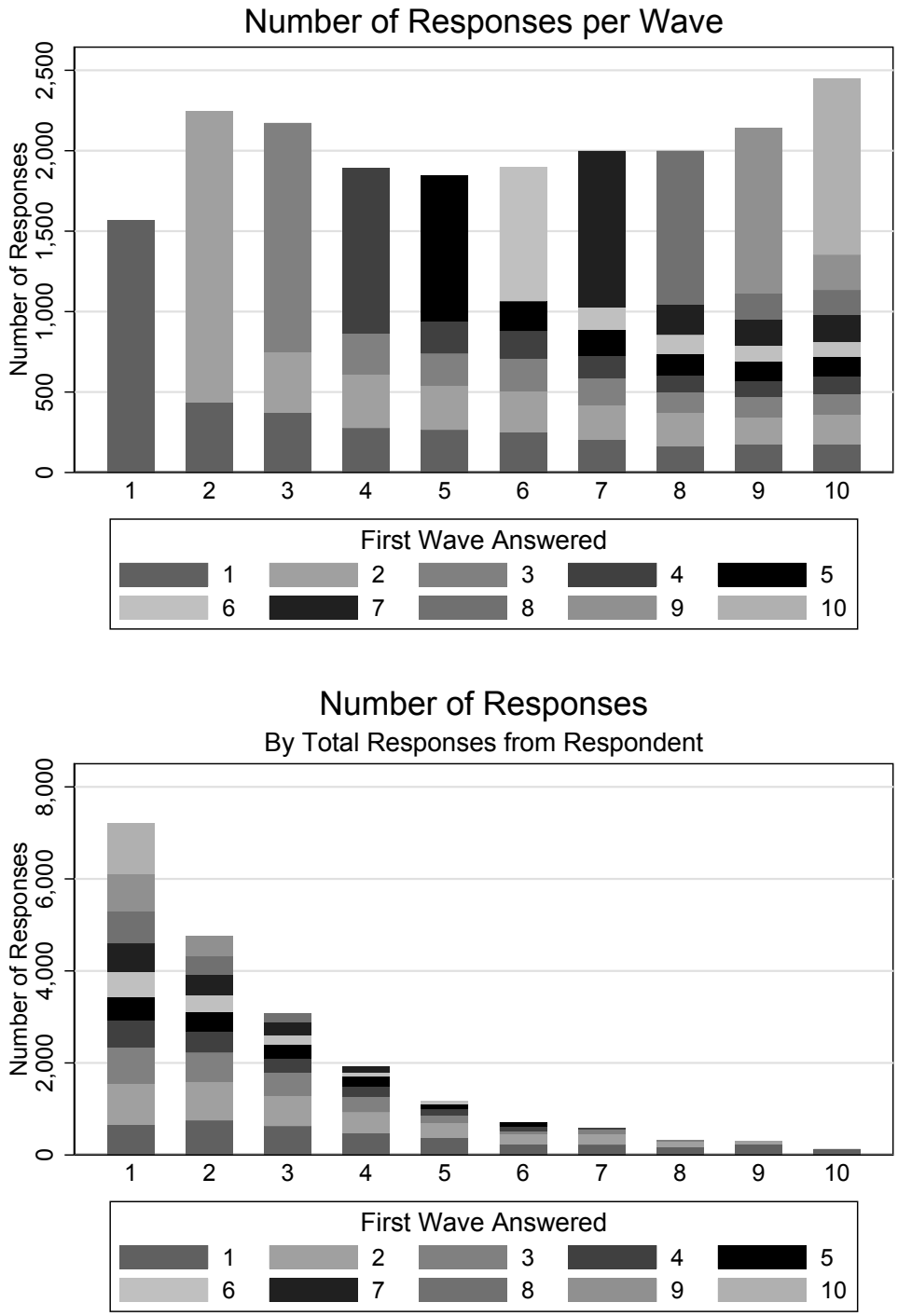
Note: Figure shows the response rates to the GMS-Vanguard Survey in each of the ten waves between February 2017 and August 2018. The orange line (circles) shows the response rates for individuals contacted for the first time. The blue line (diamonds) shows the response rates for individuals that were contacted in previous waves, but who had not yet responded. The green line (squares) shows the response rate for individuals who had previously responded.

of responses come from individuals who have responded to at least four survey waves, and 10% come from individuals who have responded to at least six survey waves.

Demographics of Respondents. In addition to the information provided by survey respondents directly, we observe data on demographic characteristics such as age, gender, and location for all individuals that were invited to take the survey. We also observe administrative data on their portfolios, including details on total portfolio values, portfolio allocations, portfolio returns, and trading behavior. We combine the CUSIP-level information on individual security holdings with information from Morningstar for mutual funds to calculate the portfolio share held in equities, fixed income instruments, cash, and other investments.[8] Table I shows summary statistics on these demographics and portfolio characteristics for the survey respondents and the non-respondents. Like in all surveys, there is selection into who responds to the survey. We distinguish two different types of selections: (i) selection into who is a Vanguard client, and (ii) selection into which Vanguard clients answers the survey. From its inception, Vanguard's business model as a *mutual* investment management company that is owned by the investors in its funds has concentrated

[8]Investments in mutual funds are apportioned depending on the portfolio composition of each fund (e.g., 60% equity and 40% fixed income). Cash includes cash-equivalent investments such as money-market funds. The category "other investments" includes alternative investments such as commodities, real estate, and derivatives.

Figure II: Number of Responses by Wave



Note: Figure shows number of responses to the GMS-Vanguard Survey. The top panel shows the number of responses per wave. The bottom panel shows the total number of responses separately by how many survey waves a person has responded to. In both panels, the colors correspond to the waves in which these individuals first answered.

Table I: Demographics - Survey Respondents and Non-Respondents

	Survey Respondents							Non-respondents	Difference
	Mean	SD	P10	P25	P50	P75	P90	Mean	
Age (years)	58.7	13.4	37	50	61	69	74	51.4	7.39***
Male	0.68	0.5	0	0	1	1	1	0.54	0.13***
Region									
Northeast	0.23	0.42	0	0	0	0	1	0.24	-0.01***
Midwest	0.21	0.41	0	0	0	0	1	0.19	0.02***
South	0.31	0.46	0	0	0	1	1	0.31	0.00
West	0.25	0.44	0	0	0	1	1	0.25	0.00
Total Vanguard Wealth ($)	467,139	778,740	26,294	67,618	201,577	545,689	1,144,906	249,010	218,130***
Length of Vanguard Relationship (Years)	15.07	8.82	3	7	15	21	27	12.83	2.24***
Active Trades / Month	1.44	3.86	0	0.10	0.44	1.51	3.46	0.88	0.57***
Monthly Portfolio Turnover (%)	2.25	6.47	0.00	0.10	0.62	2.15	5.00	1.95	0.30***
Days with Log-Ins / Month	4.40	6.08	0.05	0.38	1.43	5.86	14.76	0.88	3.52***
Total Time Spent / Month (Minutes)	32.3	77.3	0.50	3.46	11.17	31.36	76.66	4.9	27.34***
Portfolio Shares (%)									
Equity	68.5	26.4	31.3	53.6	72.3	90.0	100.0	71.3	-2.85***
Fixed Income	21.3	21.9	0.0	1.5	15.7	33.8	51.0	17.8	3.54***
Cash	8.7	18.9	0.0	0.0	1.1	6.7	26.2	9.0	-0.29
Other/Unknown	1.5	7.3	0.0	0.0	0.0	0.0	2.1	1.9	-0.40***

Note: Table shows summary statistics on both the survey respondents and non-respondents. Age, gender, location, total wealth at Vanguard and length of Vanguard relationship are measured as of August 2018. Other variables are presented as monthly averages between January 2017 and August 2018, covering the period of the ten survey waves.

on low-fee and relatively passive investment strategies. It is likely that individuals who become Vanguard clients value this investment approach. This is important to keep in mind when considering which of our results are likely to generalize to a broader set of investors. For example, clients at brokerage firms that target day-traders or professional hedge funds are likely to have higher portfolio turnover and more leverage. This does not mean that our sample is not interesting or quantitatively relevant. As of 2018, Vanguard manages more than $5.1trn in investments, so that the investors targeted in our study hold a non-trivial fraction of global assets. With the rising popularity of low-fee passive investment strategies both at Vanguard and more generally, we find that our sample is one that is likely to become even more relevant to understand investments and asset prices. In addition, as we show below, the patterns in our survey responses line up closely with those from other surveys that target very different types of investors (for example, CFOs).

To better understand the selection of Vanguard clients on observable characteristics, we next review research by Cogent Wealth Reports (2018), who compare the average Vanguard retail client to a nationally representative sample of investors with $100k or more in investable assets. In 2018, Vanguard clients were more likely to be older and more likely to be retired than the comparison sample. In addition, average total investable assets of Vanguard's clients (both inside and outside of Vanguard) were about $1.5m, which far exceeds the $824k national average in the comparison sample. The average Vanguard client held 73% of her portfolio in risky assets compared to 62% for the comparison sample; in addition, 85% of an average client's portfolio at Vanguard was in passive-like instruments, compared to 66% in the comparison sample.

Among Vanguard clients, our survey respondents are a selected sample from the representa-

tive pool of individuals invited to take the survey. Since we observe administrative data on both respondents and non-respondents, we can quantify the selection on a number of relevant dimensions. Our average respondent is about 59 years old, which is about 7.4 years older than the average non-respondent. Respondents are substantially wealthier than non-respondents, with average wealth held at Vanguard of $467k for respondents relative to $249k for non-respondents. 68% of respondents are male, while this number is 54% among non-respondents. The average respondent has been a Vanguard client for over 15 years, relative to about 13 years for non-respondents. Respondents are also likely to transact more frequently (an average of 1.4 trades vs. 0.9 trades a month during our sample), and their monthly portfolio turnover is larger: respondents turn over about 2.25% of their portfolio every month, while non-respondents have an average portfolio turnover of 1.95% per month. Respondents are also more likely to log into their Vanguard accounts (an average of 4.4 days with log-ins per month for respondents vs. 0.9 days with log-ins per month for non-respondents).[9] Portfolio allocations of respondents and non-respondents are relatively similar, though respondents are somewhat more likely to hold fixed income assets in their portfolios (21.3% vs. 17.8% fixed income share) and somewhat less likely to hold equity (68.5% vs. 71.3% equity share). Overall, while the sample of respondents is not fully representative of the Vanguard population, the direction of the selection is potentially helpful: since asset prices are likely to be disproportionately affected by the beliefs of wealthier people, the sample-skew towards wealthy Vanguard clients over-represents this particularly interesting group of individuals.

While there are differences between non-respondents and respondents, there is also large heterogeneity within the sample of respondents, who come from all over the United States. While the average age is 59 years, this ranges from 37 years at the 10th percentile of the distribution to 74 years at the 90th percentile. The 10-90 percentile range of assets held at Vanguard is $26k to $1.14m. Activity on the Vanguard site, both in terms of log-ins and in terms of trading activity, also differs substantially across our survey respondents. At the 10th percentile of the distribution, respondents spend about 0.5 minutes per month on the Vanguard site, while this number is 76.7 minutes at the 90th percentile of the distribution. There are also substantial cross-sectional differences in portfolio allocations among our respondents. At the 10th percentile of the distribution, the equity share is 31.3%, while at the 90th percentile, it is 100%. Similarly, at the 10th percentile of the distribution, respondents have none of their portfolios invested in fixed income securities, while at the 90th percentile of the distribution, respondents have more than 50% invested in fixed income assets. While the average cash allocation in respondents' portfolios is 8.7%, this ranges from 0% at the 10th percentile of the distribution to 26.2% at the 90th percentile of the distribution.

[9] Responding to the survey does not involve respondents logging into their Vanguard accounts; the process of answering the survey does therefore not lead to a mechanical increase in log-ins.

II.C Survey Responses: Summary Statistics

Table II shows summary statistics across the 20,208 survey responses.[10] The average respondent takes about 8.6 minutes (517 seconds) to answer all survey questions. The 10-90 percentile range for the total time to respond is 7.2 minutes to 13.9 minutes. Therefore, all respondents spend a significant amount of time answering the questions, rather than carelessly clicking through the survey; this is consistent with the non-compensated nature of the survey requiring a certain intrinsic interest from participants.

The average expected 1-year stock market return is 5.23%, while the average annualized expected 10-year stock market return is 6.32%. These numbers are lower than the historical average annual stock market return of 11.9% between 1927 and 2014. The fact that individuals' expected stock returns are often reported to be below the historical average is a common pattern across surveys (e.g., Dominitz and Manski, 2007; Hurd and Rohwedder, 2012). There is substantial heterogeneity in the expected 1-year stock market return across responses. At the 10th percentile of the distribution, individuals reported a 1-year expected stock return of 0.5%, while at the 90th percentile they expected a return of 10%. The across-responses standard deviation of expectations of 1-year returns is 5.32%, nearly twice as large as the standard deviation of expectations of annualized 10-year returns, which is 2.96%. This suggests that individuals expect some medium-run mean reversion in stock returns. Panels A and B of Figure III show the distributions of responses to these two questions. When we ask individuals about their expectations of annualized GDP growth, the means for the next three years and the next ten years are quite similar at 2.83% and 2.95%, respectively. This is also close to the historical U.S. real average growth rate of 3.3% between 1929 and 2015. The distributions of answers over the two horizons also have similar standard deviations at about 1.9%. Panels C and D of Figure III show the full distributions of expectations for annualized GDP growth over the next three and ten years, respectively.

Table II also shows that respondents put substantial probabilities on relatively large short-run stock market declines and GDP declines. The average individual assigns a 5.1% chance to the 1-year return of the stock market being less than -30%; the median respondent assigns a 3% chance to such an event. These numbers are quite close to the fraction of years between 1927-2014 that had such a drop, which was 3.7%.[11] As with the the other answers, there is substantial across-answer

[10] Even among the respondents who completed the entire survey flow, some respondents skipped a few questions. We verified that restricting our analysis to the sample of respondents who provided answers to every question does not affect our conclusions. We also explored the presence of extreme outlier responses, such as individuals reporting that the expected return on the U.S. stock market over the coming year was 400% or -100%. Since such outliers have extreme effects on the analysis, in our baseline analysis we set extreme outlier answers (below the bottom percentile, and above the top percentile) for each unbounded expectation question equal to missing. It is often the case that the same individuals report multiple answers outside the accepted ranges. Naturally, there are some critical judgment calls involved in selecting these cutoffs, which involve trading-off retaining true extreme beliefs with excluding answers from individuals who probably misunderstood the question or the units. We have done extensive sensitivity analysis to confirm that our results are robust to a wide range of choices for the cutoff values. We also confirmed that the results are robust to winsorizing extreme answers rather than setting them equal to missing, and to to dropping all answers of individuals who report extreme answers to at least one question.

[11] Recall that we randomize the ordering whether the buckets are ordered, smallest-to-largest or largest-to-smallest. We find that this ordering does have a small effect on the probabilities assigned to the smallest bucket. Among those

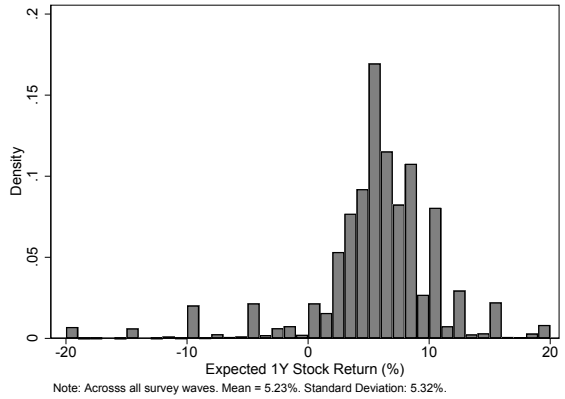
Table II: Summary Statistics - Survey Responses

	Mean	SD	P10	P25	P50	P75	P90
Expected Stock Returns							
Expected 1Y Stock Return (%)	5.23	5.32	0.5	4	5	8	10
Expected 10Y Stock Return (% p.a.)	6.32	2.96	3	5	6	8	10
Probability 1Y Stock Return in Bucket (%)							
Less than -30%	5.1	8.2	0	0	3	5	10
-30% to -10%	13.6	13.1	0	5	10	20	30
-10% to 30%	70.8	22.5	40	60	75	90	100
30% to 40%	7.6	10.8	0	0	5	10	20
More than 40%	2.9	6.5	0	0	0	5	10
Expected GDP Growth							
Expected 3Y GDP Growth (% p.a.)	2.83	1.92	1.5	2	2.5	3	4
Expected 10Y GDP Growth (% p.a.)	2.95	1.94	1.5	2	2.5	3	5
Probability p.a. 3Y GDP Growth in Bucket (%)							
Less than -3%	4.5	7.7	0	0	2	5	10
-3% to 0%	12.0	12.0	0	3	10	20	25
0% to 3%	57.8	26.1	20	40	60	80	90
3% to 9%	22.4	23.3	0	5	15	30	60
More than 9%	3.3	8.7	0	0	0	5	10
Expected Bond Returns							
Expected 1Y Return of 10Y zero coupon bond (%)	1.38	3.13	-2	1	2	3	4
Expected yield-to-maturity of zero coupon in 1Y (%)							
1Y zero coupon bond	1.83	0.96	1	1	2	2.2	3
5Y zero coupon bond	2.59	1.08	1.5	2	2.5	3	4
10Y zero coupon bond	3.30	1.42	2	2.5	3	4	5
Difficulty ("Not at all difficult", …, "Very difficult")							
Expected Stock Returns	2.31	0.97	1	2	2	3	3
Expected GDP Growth	2.42	0.97	1	2	2	3	4
Expected Bond Returns	2.85	0.99	2	2	3	3	4
Confidence ("Not at all confident", …, "Very confident")							
Expected Stock Returns	3.06	0.85	2	3	3	4	4
Expected GDP Growth	3.00	0.84	2	3	3	3	4
Expected Bond Returns	2.61	0.84	2	2	3	3	4
Time of responding to survey (seconds)	517	433	238	307	414	579	834

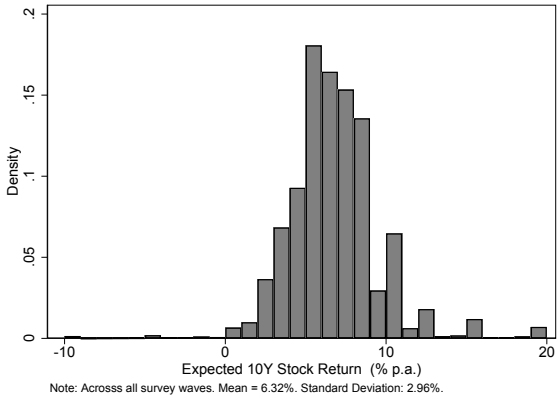
Note: Table shows summary statistics of the answers across all 20,208 responses to the first ten waves of the Vanguard-GMS survey. The possible answers for "difficulty" are 1 = "Not at all difficult", 2 = Not very difficult", 3 - "Somewhat difficult", 4 - "Very Difficult, 5 - "Extremely Difficult." The possible answers for confidence are 1 = "Not at all confident", 2 = "Not very confident", 3 = "Somewhat confident", 4 = "Very confident", and 5 = "Extremely Confident."

Figure III: Distribution of Responses to Expectation Survey

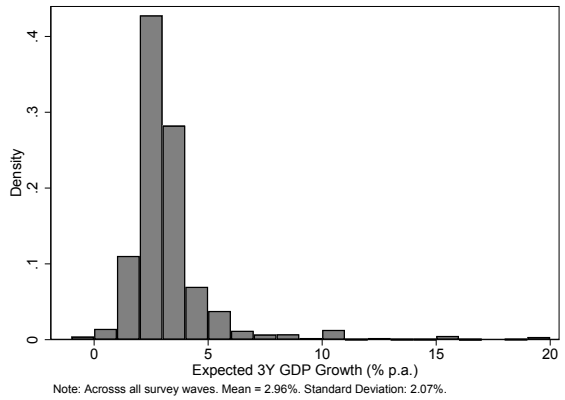
(A) 1-Year Expected Stock Returns



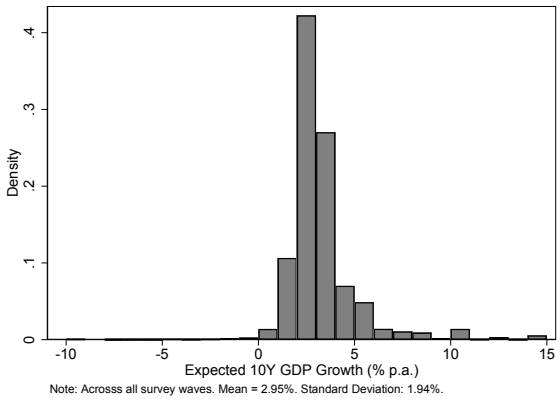
(B) 10-Year Expected Stock Returns (annualized)



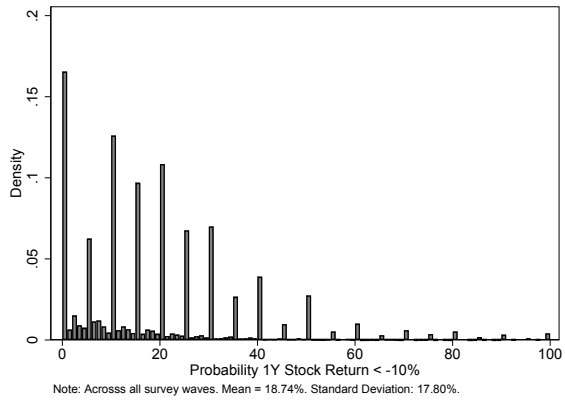
(C) 3-Year Expected GDP Growth (annualized)



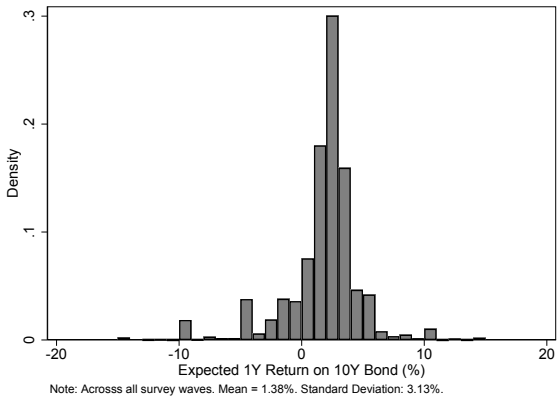
(D) 10-Year Expected GDP Growth (annualized)



(E) Probability of Stock Market Disaster



(F) 1-Year Expected Bond Returns



Note: Figure shows histograms of the answers across the 20,208 responses to the first ten waves of the Vanguard-GMS survey.

heterogeneity. Answers at the 25th percentile of the distribution correspond to a 0% chance of returns lower than -30%, while those at the 90th percentile of the distribution correspond to a 10% probability of such events. Panel E of Figure III shows the full distribution of answers; the bunching of reported probabilities at multiples of five is consistent with reporting behavior across many surveys. Similarly, in the case of GDP growth, individuals assign an average probability of an annualized decline in GDP of more than 3% over the coming three years of 4.5%. This is very close to the historical frequency of such events at 4.2%.

Individuals were also asked about the expected 1-year return of 10-year U.S. Treasury zero-coupon bonds. The average response was 1.38%, with a 10-90 percentile range of -2% to 4% (Panel F of Figure III). Lastly, individuals were asked about their expectations for the yield curve in one year. Table II provides summary statistics on these answers, though they are not the subject of this paper. On average, individuals expected the yield curve to remain upward-sloping.

Most individuals report finding the survey questions relatively easy to understand, though the questions on bond returns were perceived to be more difficult than the questions on expected stock market returns and expected GDP growth. There also appears to be a relatively wide range of confidence that individuals have in their answers. For each of the three survey blocks, individuals at the 10th percentile of the distribution report being "not very confident" in their answers, while individuals at the 90th percentile reported being "very confident."

II.D Time-Series Dynamics: A Comparison with Other Surveys

We conclude our initial analysis by comparing the time-series variation of average beliefs in our survey with similar measures obtained from existing surveys. Since these existing surveys often cover longer time spans, this analysis allows us to determine whether the time-series variation over our sample period was similar in magnitude to the variation in other periods. In addition, by comparing patterns of average beliefs across different surveys over the same time horizon, we can explore the extent to which the various surveys capture similar belief movements, despite differences in their samples and survey designs.

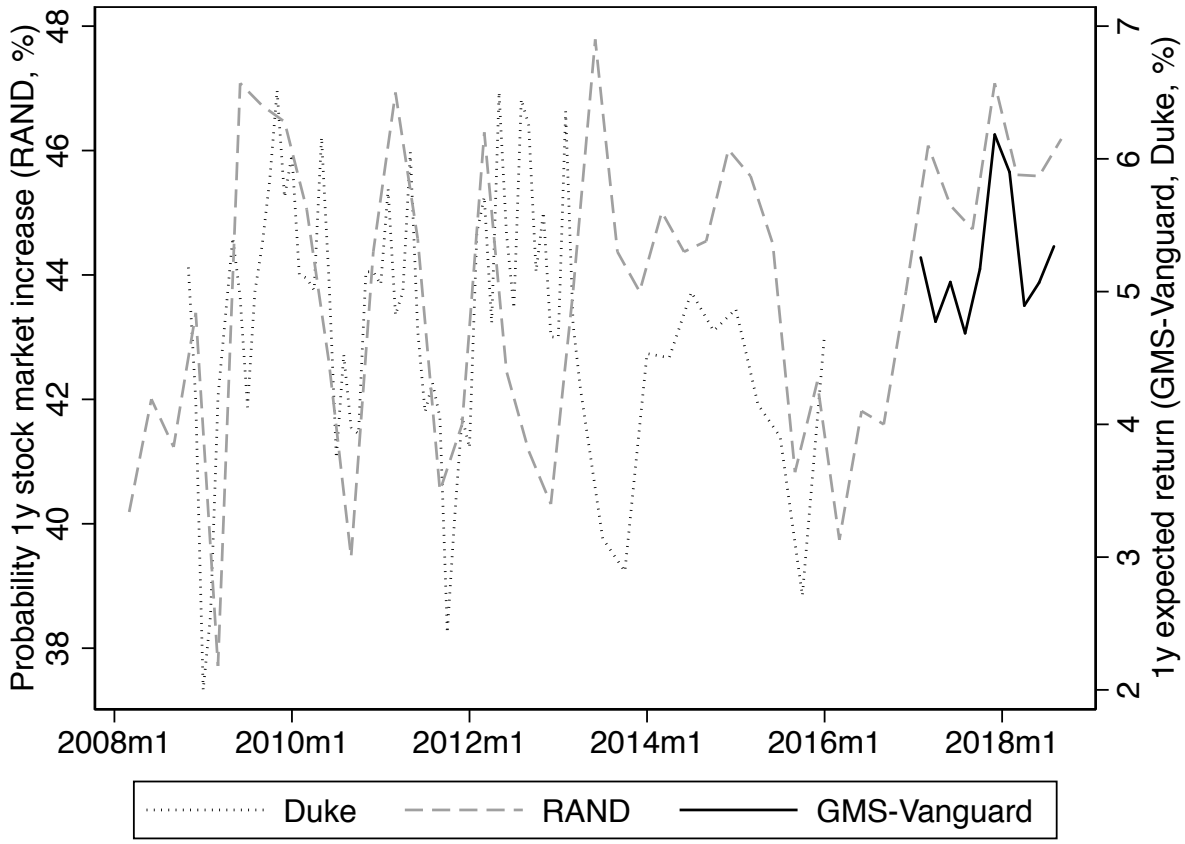
Specifically, for each wave we aggregate our survey responses by averaging responses across individuals, focusing on the question about expected 1-year stock returns. We then explore the time-series dynamics of this average expected return. We compare this time-series to similar ones from four existing surveys: Robert Shiller's investor survey, the Duke (Graham-Harvey) CFO survey, the American Association of Individual Investors survey (AAII), and the RAND American Life Panel Survey (Financial Crisis).[12] The Duke CFO survey asks explicitly about expected 1-year stock returns and is therefore directly comparable with our survey. All other surveys ask questions that are related to expected returns, but cannot be directly mapped to them. In those

individuals to whom the most negative bucket was presented first, the average probability assigned to a return lower than -30% is 5.8%, while it is 4.4% for those individuals to whom the most positive bucket is presented first. We verified that the conclusions in this paper are independent of the order in which the buckets are presented.

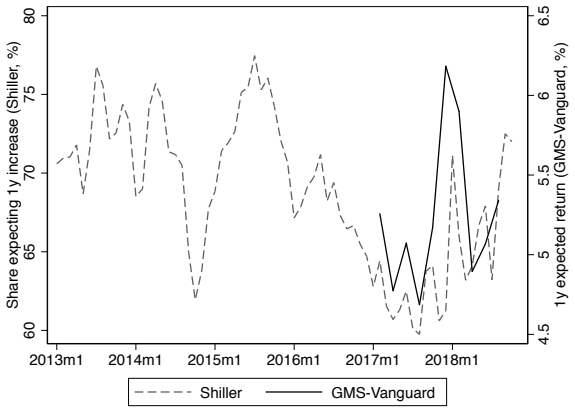
[12]See Greenwood and Shleifer (2014) for a detailed description of these surveys, except the RAND survey, which was not part of their study. Section V.A.1 provides details about the RAND survey.

Figure IV: Comparison with other surveys

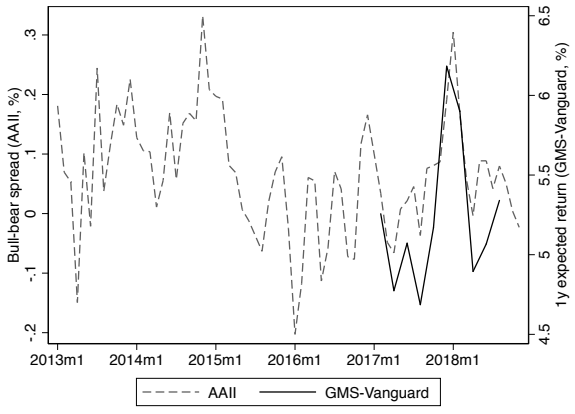
(A) GMS-Vanguard vs. Duke CFO and RAND



(B) GMS-Vanguard vs. Shiller



(C) GMS-Vanguard vs. AAII



Note: The figure compares average beliefs about the 1-year stock market return in the GMS-Vanguard survey with questions from other surveys. Panel A reports the average 1-year expected stock returns from the Duke CFO survey, and the average probability of a 1-year stock market increase from the Rand survey. Panel B reports the share of investors expecting an increase in market values in one year, from Shiller's investor survey. Panel C shows the bull-bear spread from the AAII survey.

cases, we follow the approach of Greenwood and Shleifer (2014) and use the survey questions that are most closely related to 1-year expected stock returns. For the Shiller survey, we use the share of the respondents that report expecting an increase in stock market valuations over the next year; for the RAND survey, we calculate the average (across respondents) probability of a stock market increase over the next year; and for the AAII survey, we compute the difference between the percentage of bullish and bearish investors. Figure IV plots the time series of our survey together with the other surveys. For readability, we separate the plots into different panels that focus on comparisons between our GMS-Vanguard survey and at most two other surveys (Panel A: Duke CFO and RAND; Panel B: Shiller; Panel C: AAII).

Since the beginning of our survey in February 2017, the average belief about 1-year expected stock returns has experienced significant variation. It started at just above 5% in the first survey wave, and reached above 6% by the end of 2017, only to fall back to around 5% towards the end of our sample. These patterns are shared across the other surveys. First, quantitatively, Panel A of Figure IV shows that both the level and the variation of average beliefs during our sample period is quite similar to the one displayed by the Duke CFO survey. Second, from a qualitative perspective, the peak in optimism our survey displays at the end of 2017 is shared by all other surveys, despite differences in elicitation methods and target samples. Figure IV highlights that the variability across other surveys since 2017 is roughly similar to the variation in beliefs in these surveys since 2013, though it is naturally lower than during the Great Recession.

Taken together, these results highlight that despite differences in the investors being surveyed and in the survey design and methodology, the main time-series features of our survey align both quantitatively and qualitatively with those of existing surveys. In addition, the average variability of beliefs during our sample period is comparable to the one observed over the prior years.

III Beliefs and Portfolios

In this section, we explore the relationship between respondents' beliefs and their portfolio allocations. Our focus is on understanding the asset allocation, that is, the share of each respondent's portfolio that is invested in stocks, fixed income instruments, and cash. We first explore the role of expectations about the 1-year stock return in determining portfolio equity shares. We then analyze the role of other moments (like tail events) of the belief distribution, of stock market expectations over different horizons, and of beliefs about bond returns and GDP growth. We also explore the margins of substitution that drive our results, analyzing whether more optimistic individuals substitute towards equity from cash or from fixed income investments.

III.A Expected 1-Year Stock Returns and Portfolios

To estimate the sensitivity of portfolio shares to beliefs, we run the following regression:

$$EquityShare_{i,t} = \alpha + \beta\, E_{i,t}[R_{1y}] + \gamma X_{i,t} + \psi_t + \epsilon_{i,t} \tag{1}$$

The unit of observation is a survey response by individual i in wave t. The dependent variable is the equity share in the individual's Vanguard portfolio at time t. The variable of interest is β, which captures the increase in an individual's equity share for each percentage point increase in the expected 1-year stock return.

Table III: Expected Returns and Portfolios

	Equity Share (%)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Expected 1Y Stock Return (%)	0.749***	0.785***	1.085***		0.966***	1.142***		0.884***
	(0.052)	(0.051)	(0.083)		(0.103)	(0.084)		(0.075)
Expected 1Y Stock Return (%) x (Assets < $100k)				0.576***				
				(0.105)				
Expected 1Y Stock Return (%) x (Assets >= $100k)				0.874***				
				(0.057)				
Expected 1Y Stock Return (%) x Above Median Time							1.120***	
							(0.138)	
Expected 1Y Stock Return (%) x Below Median Time							1.178***	
							(0.112)	
Controls + Fixed Effects	N	Y	Y	Y	Y	Y	Y	Y
Specification	OLS	OLS	OLS	OLS	IV	ORIV	ORIV	ORIV
Sample			E(Return) 0%-15%					Interior equity share
R-Squared	0.022	0.107	0.101	0.105				
N	19,486	19,478	17,671	19,301	19,301	19,301	19,301	16,343

Note: Table shows results from regression 1. The unit of observation is a survey response, the dependent variable is the equity share. Columns 2-8 also control for the respondents' age, gender, region of residence, wealth, and the survey wave. Standard errors are clustered at the respondent level. Significance levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

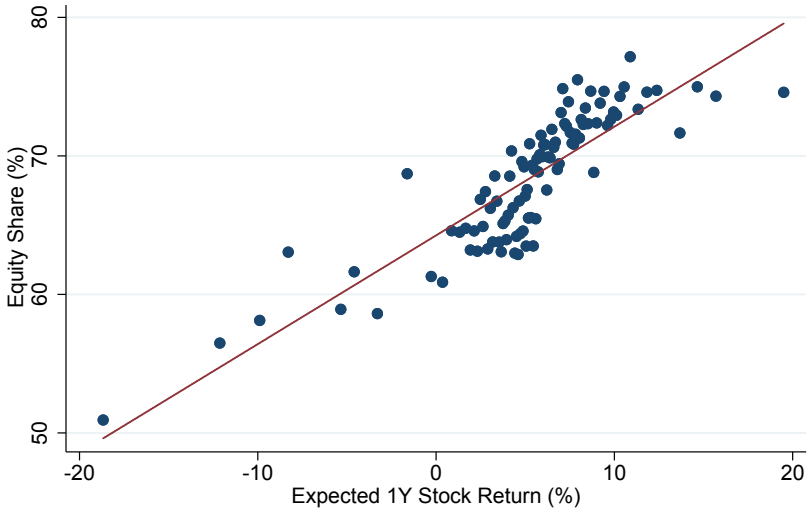
Column 1 of Table III shows estimates from this OLS regression without controlling for any additional covariates. An extra percentage point of expected 1-year stock return is associated with a 0.75 percentage point increase in respondents' equity shares. In Column 2 of Table III, we control for demographic characteristics such as age, gender, wealth, and region of residence, as well as survey-wave fixed effects. Figure V shows a binscatter plot of the resulting relationship. The estimated sensitivity of portfolio shares to beliefs is similar to the one in Column 1: an extra percentage point of expected return is associated with a 0.79 percentage point increase in the equity share.[13] Consistent with this, in the Appendix we document that there is a weak relation between beliefs and demographic characteristics. Despite the small magnitude of the average relationship

[13]While the estimates of β are the primary object of interest, the coefficients on some of the control variables are also interesting. Males and females do not have significantly different equity shares. Equity shares are strongly declining in age, with individuals above 70 years of age having about 20 percentage points lower equity share than individuals below the age of 40. Equity shares also do not differ significantly across regions or across wealth quintiles. Despite the statistically significant relationship between some of these demographics and portfolio shares, the increase in the R^2 between columns 1 and 2 is modest, and mostly driven by the inclusion of age controls.

between beliefs and equity shares, the wide heterogeneity in beliefs across individuals still implies substantial variation in their equity shares. A one-standard-deviation increase in expected 1-year stock returns is associated with a $\frac{0.785 \times 5.32}{26.2} = 0.16$ standard deviation increase in equity shares.

Figure V shows that the estimated relationship might be sensitive to beliefs at the two extremes. Therefore, we next run regression 1 on a sample of respondents that report expected returns between 0% and 15%. This drops about 10% of our responses.[14] Column 3 of Table III shows the restricted specification. Indeed, the estimated elasticity within that range is higher, but not dramatically so, suggesting that the relationship is primarily not driven by the outliers.

Figure V: Expected 1-Year Stock Returns and Equity Share



Note: Figure shows a conditional binscatter plot of survey respondents' expected 1-year stock returns and the equity share in their portfolios, conditional on the respondents' age, gender, region, wealth, and the survey wave.

While survey responses have a statistically strong relationship with individuals' portfolios, the estimated average sensitivity of equity shares to beliefs is low compared to predictions from benchmark theory models such as Merton (1969), who provides that for a power-utility investor:

$$EquityShare_{i,t} = \frac{1}{\gamma} \frac{E_i[R] - R_f}{Var_i[R]}. \tag{2}$$

Here, γ is the individual's coefficient of relative risk aversion, $E_i[R]$ is the individual's expected stock return, R_f is the risk-free rate, and $Var_i[R]$ is the individual's subjective variance of equity returns. In the classic work of Merton (1969), individuals have rational expectations, while here we allow individuals to have arbitrarily formed expectations as long as the variance is non-zero and finite. We measure $EquityShare_{i,t}$ in the Vanguard data, and $E_i[R]$ with the survey answer to 1-year expected stock returns. For a back of the envelope calculation, we assume that individuals

[14] One possibility is that extreme responses are more likely to be affected by measurement error, and that looking at moderate responses may help filter out at least some of the measurement error. See below for a more formal treatment of measurement error.

have a common measure of the variance $Var_i[R] = Var[R]$.[15] Similarly, we assume a common and constant coefficient of relative risk aversion. We later relax both of these assumptions. In this simplified setting, the β estimated in Table III corresponds to $\beta = \frac{1}{\gamma Var[R]}$. The historical standard deviation of stock market returns is around 16% a year. The simple model thus implies that a β of 0.785 requires a coefficient of relative risk aversion of $\gamma = 50$. This is considerably higher than most estimates in the experimental literature, which usually find values of γ between 3 and 10. To obtain a realistic coefficient of relative risk aversion, let us say around 4, we would need an estimate of β of around 10. In other words, the estimated sensitivity is an order of magnitude too small to align with the simplest frictionless model.[16] This relatively small response of equity shares to beliefs about stock returns is with evidence documented across a number of other studies that link equity market participation and equity shares to expected stock market returns (e.g., Vissing-Jorgensen, 2003; Dominitz and Manski, 2007; Kézdi and Willis, 2011; Amromin and Sharpe, 2013; Ameriks et al., 2016; Drerup, Enke and Von Gaudecker, 2017), though many of these studies had to rely on self-reported equity shares or a binary indicator for holding stocks rather than administrative data.

As we discuss below, the relatively low sensitivity of portfolios to beliefs provides a challenge for a number of asset pricing models. However, in many of these models it is the wealth-weighted beliefs that drive asset prices, rather than beliefs that are equally weighted across all investors. We thus explore whether the sensitivity of portfolios to beliefs is different for wealthier individuals. Column 4 of Table III shows that respondents with more than \$100k in assets have a sensitivity that is about 50% larger than that of individuals with lower wealth; in unreported results, we find that the sensitivity does not increase substantially for even higher levels of wealth. This means that the wealth-weighted sensitivity is higher than the equally-weighted sensitivity, but not by nearly enough to generate the quantity movements required by frictionless asset pricing models.

There are a number of possible explanations for this relatively low estimated average sensitivity, and our contributions in this section focuses on reconciling the estimates we obtain with the theoretical predictions. The first set of explanations involves measurement error in the key measure of beliefs, $E_{i,t}[R_{1y}]$, and the associated attenuation bias that such measurement error would entail. The second set of explanations centers around possible frictions in the transmission of beliefs to portfolios. Indeed, the Merton (1969) model is based on a number of strong assumptions, including that investors continuously pay attention to their portfolios, that they continuously rebalance their portfolios, that they are supremely confident in their beliefs, and that there are no

[15] We later relax this assumption and measure the variance of 1-year expected stock returns that is implied by the distribution question. We find the simple calculation with a common variance across individuals parametrized to the historical variance to be appealing for several reasons: (i) in many models, it is easy to learn the variance of returns but hard to learn the mean; (ii) equation 2 is particularly sensitive to measurement errors in the denominator; and (iii) model misspecification is likely and other moments (e.g., tail event probabilities) may also be important.

[16] Note that, in this model, γ drives both the sensitivity of the portfolio share to changes in expected returns, and the unconditional level of the equity share. In particular, for $\gamma = 4$, an average risk premium of 6%, and a standard deviation of 16%, we obtain an average equity share of 58%, close to the average share we see in the data. When $\gamma = 50$, the equity share drops to 4.7%. This means that high risk aversion can explain the low sensitivity of portfolios to beliefs, but at the cost of missing the average level of the portfolio share.

other frictions to trading, such as the tax implications from realizing capital gains. As a result, the high sensitivity implied by the Merton (1969) model, while it is a useful benchmark, is likely to be an upper bound for real world applications.

III.A.1 Possible Explanation I: Measurement Error

We next explore the extent to which measurement error of various types might account for the relatively low estimated sensitivity of portfolio allocations to beliefs. A first possible explanation is that classical measurement error in beliefs may induce attenuation bias in our estimates of β.[17] To explore the extent to which this can account for the low sensitivity, we exploit the fact that, for each survey response, we obtain two separate estimates of the same explanatory variable, $E_{i,t}[R_{1y}]$. The first is the expected return as reported directly by respondents in the survey. The second is the implied mean of the distribution over possible returns reported by each respondent.[18] Both of these methods elicit the same belief using very different styles of question, and are therefore likely to have independent measurement errors. This setting allows us to minimize attenuation bias by instrumenting for the reported expected return with the implied expected return from the distribution (the correlation across the two measures is 0.43). Consistent with this procedure reducing attenuation bias, the results in Column 5 of Table III show that the estimated sensitivity increases by about 20% when using this instrumental variables (IV) regression.

In principle, we could run two IV regressions, one in which we instrument for the stated expectation with the implied mean as in Column 5, and one in which we instrument for the implied mean with the stated expectation. Under standard assumptions, both IV strategies should provide consistent estimates of β, though the point estimates will only be identical with infinite data. To maximize the efficiency of our estimators in finite data, we next use the *Obviously Related Instrumental Variables (ORIV)* strategy proposed by Gillen, Snowberg and Yariv (2015), which consolidates the information from these different formulations to provide an estimator that is more efficient than either of the IV strategies alone.[19] Column 6 of Table III shows that this approach further increases the estimated sensitivity to $\beta = 1.142$, with lower standard errors than the IV estimate in Column 5. Classical measurement error therefore accounts for a non-trivial component of the low sensitivity, though the effect is not sufficiently strong to bridge the gap between the measured sensitivity and the one implied by the benchmark Merton (1969) model. Given this finding, it certainly seems like a good idea for future surveys to include various ways of eliciting the same beliefs, thereby allowing researchers to use IV and ORIV techniques to reduce the

[17] We refer to classical measurement error as the concern that the reported belief is a noisy measure of individuals' true beliefs, $E_{i,t}[R_{1y}] = E_{i,t}[R_{1y}]^{True} + \epsilon_{i,t}$, where $\epsilon_{i,t}$ represents i.i.d. and mean zero measurement error.

[18] To construct the implied mean from the distribution, we first compute, for each bucket, the average historical return conditional on the return being in that bucket, and then we weight these estimates by the subjective probabilities of each bucket reported by the respondent. Our results are unchanged if we take the mid-points of the buckets, and assign a value of -40% for the lowest open-ended bucket ($\leq -30\%$) and a value of 50% for the highest open-ended bucket ($\geq 40\%$). Our findings are also robust to picking other values for the open-ended buckets.

[19] Intuitively this estimator averages the point estimates from the two orderings of the IV, with the standard error further shrinking by observing two not-perfectly correlated draws. See original reference for a formal treatment.

attenuation bias associated with measurement error.

A different type of measurement error, which we term Frame-of-Mind Measurement Error (FRAME) can further complicate the estimation. This type of measurement error corresponds to a common critique of quantitative surveys that suggests that "individuals do not truly understand what they are responding." Specifically, FRAME suggests that individuals have a true pessimistic or optimistic "frame of mind," and that their portfolios respond to that true belief, but that they struggle to express this belief quantitatively in answering the survey. Two individuals who are equally pessimistic might therefore provide different numerical answers to express this frame of mind (e.g., one person maps "really negative" into a -5% expected return, and a second person maps it into a -20% expected return). The challenge with FRAME is that it is likely correlated across different elicitation methods, as well as across waves; it therefore cannot be solved through ORIV techniques or by averaging responses across waves. In fact, FRAME is observationally equivalent to true belief heterogeneity. While we acknowledge this possibility, we find it unlikely to be the main explanation for the low estimated sensitivity. Indeed, the next section shows substantial heterogeneity in the sensitivity that has a meaningful economic interpretation.

One last approach to dealing with measurement error is to explore the hypothesis that the time spent by individuals to answer the questions may allow us to identify individuals who are more or less subject to measurement error. Column 7 of Table III shows that we obtain similar estimates for the sensitivity among people in the top and bottom half of the time-spent distribution. Similarly, in unreported results we find that cutting out the top and bottom 10% of the time-spent distribution has little effect on the estimated sensitivity.

III.A.2 Possible Explanation II: Frictions

We next explore the extent to which deviations from the frictionless model can help us further account for the differences between our estimates and the predictions from the benchmark model.

Corner Solutions. The preceding analysis uses all responses, including those from individuals with portfolios that include either 0% or 100% in equity. Since Vanguard customers generally cannot short stocks or take substantial leverage within the account, the boundaries of 0% and 100% represent natural constraints on the investment set. One concern, therefore, could be that we measure a low sensitivity because individuals who are extremely optimistic (pessimistic) cannot express their view by taking positions in excess of 100% equity (taking negative equity positions). Column 8 of Table III shows that this is not the case; to the contrary, excluding the extreme portfolios lowers the measured sensitivity from 1.142 in the unrestricted Column 6 to 0.884 in Column 8. This occurs because, on average, very optimistic and very pessimistic agents do in fact hold 100% and 0% of their portfolios in equity, respectively. Contrary to the conjecture, and despite the constraints, these individuals have portfolios that are more in line with their beliefs (i.e., more sensitive to the beliefs) than those investors with interior portfolio allocations. For the remainder of the paper, we include the extreme portfolios in the analysis, but treat them separately whenever it is informative to do so (e.g., in the analysis of trading behavior).

Default Options and Automatic Enrollment in Defined Contribution Plans. As described in Section II, our survey sample includes investors holding three different types of accounts: retail accounts, individually managed tax-advantaged retirement accounts, and employer-sponsored retirement accounts such as defined contribution plans. Investments in the first two types of accounts usually represent an active decision of the investor. Within the defined contribution plans, it has become increasingly common to automatically enroll eligible employees into retirement accounts at prespecified contribution rates and into prespecified assets.[20] Among retirement investors, a robust empirical finding is that the default investment choices are very sticky (e.g., Madrian and Shea, 2001; Beshears et al., 2009). Among Vanguard investors, Clark, Utkus and Young (2015) found that 89% of participants under automatic enrollment remain 100% invested in the default option after 12 months; this number falls only slightly to 80% after 36 months. These numbers suggest that many investors in defined contribution plans may never have made an active portfolio allocation decision; we would thus not expect their portfolio allocations to vary with their beliefs. Consistent with this, Column 1 of Table IV shows that the average sensitivity of portfolios to beliefs is only about half the sample average among those investors in institutionally-managed defined contribution plans than it is in individually-managed plans. This highlights a first important deviation from the assumption of the benchmark model, namely that a substantial amount of wealth is invested through sticky default options rather than through active allocations.

Tax Implications. A second friction that reduces the passthrough from changes in beliefs to portfolios is the presence of capital gains taxes that would regularly arise in the rebalancing process. To test for the importance of this friction, we exploit the fact that we observe both standard and tax-advantaged individually-managed accounts. Columns 2 and 3 of Table IV highlight that we find a higher elasticity of portfolios to beliefs in tax-advantaged accounts. This is even the case when we limit the sample to clients who hold both types of accounts, thus controlling for potential differences in individual's aversion to realize losses or gains. This evidence suggests that the presence of capital gains taxes provides another important friction that inhibits the transmission of beliefs to portfolios relative to the predictions from frictionless models.

Infrequent Trading. Another plausible contributor to the low estimated sensitivity of portfolios to beliefs is that even those investors that actively choose their portfolios only adjust them infrequently (e.g., Duffie, Sun et al., 1990; Gabaix and Laibson, 2001; Agnew, Balduzzi and Sunden, 2003; Peng and Xiong, 2006; Abel, Eberly and Panageas, 2007; Alvarez, Guiso and Lippi, 2012). To the extent that investors change their beliefs over time and report their current belief in the survey, the contemporaneous portfolios may thus not be fully reflective of current beliefs. Researchers have focused on several complementary explanations for infrequent portfolio adjustments. The first is the cost of monitoring portfolio allocations, which can cause investors to only infrequently pay attention to their portfolios. The second explanation is that even after paying attention to

[20] Indeed, by the end of 2017, 46% of Vanguard plans had adopted automatic enrollment (about half of those enrolled all eligible employees, and the other half enrolled newly eligible employees only). Across all Vanguard plans (including those with and without automatic enrollment), Vanguard (2017) shows that 81% had selected a target-date fund as the default investment option (7% had selected a balanced fund, the rest a money market or stable value fund).

Table IV: Expected Returns and Portfolios – Heterogeneity

	Equity Share (%)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Expected 1Y Stock Return (%)	0.736***	1.140***	1.356***				
	(0.223)	(0.144)	(0.100)				
Expected 1Y Stock Return (%) x Monthly Turnover < 0.5%				0.731***			
				(0.159)			
Expected 1Y Stock Return (%) x Monthly Turnover ∈ [0.5%,2.5%]				1.338***			
				(0.158)			
Expected 1Y Stock Return (%) x Monthly Turnover > 2.5%				2.073***			
				(0.206)			
Expected 1Y Stock Return (%) x Monthly Vanguard Visits ∈ (0,1)					0.750***		
					(0.167)		
Expected 1Y Stock Return (%) x Monthly Vanguard Visits ∈ (1,6)					1.144***		
					(0.146)		
Expected 1Y Stock Return (%) x Monthly Vanguard Visits ∈ (6,31)					1.781***		
					(0.185)		
Expected 1Y Stock Return (%) x Low Confidence						0.469*	
						(0.202)	
Expected 1Y Stock Return (%) x Medium Confidence						1.242***	
						(0.098)	
Expected 1Y Stock Return (%) x High Confidence						1.783***	
						(0.413)	
Expected 1Y Stock Return (%) x Not Idealized							1.133***
							(0.084)
Expected 1Y Stock Return (%) x Idealized							3.455***
							(0.950)
Controls + Fixed Effects	Y	Y	Y	Y	Y	Y	Y
Specification	ORIV	ORIV	ORIV	ORIV	ORIV	ORIV	ORIV
Sample	DC Plans	Retail Account	Retail Account Tax Adv.				
N	1,921	11,343	14,898	17,904	19,301	19,301	19,301

Note: Table shows results from regression 1. The dependent variable is the equity share. In column 1 it is the equity share in institutionally-managed retirement plans (defined contribution plans); in column 2 it is the equity share in individually-managed retail accounts; and in column 3 it is the equity share in individually-managed tax-advantaged retail accounts. In columns 4-6, it is pooled across the three types of accounts. In column 6, "Low confidence" corresponds to individuals who report ed being "not at all confident" or "not very confident" in their answers about expected stock returns; "medium confidence" corresponds to individuals who report being "somewhat confident" or "very confident" about their answers; and "high confidence" corresponds to individuals who report being "extremely confident." "Idealized" respondents in column 7 are those whose behavior most closely corresponds to that of the assumptions in the frictionless model: they have average monthly portfolio turnover of at least 2.5%, they have at least six log-ins a month, and they report to be extremely confident in their beliefs. Standard errors are clustered at the respondent level. Significance levels: * ($p<0.10$), ** ($p<0.05$), *** ($p<0.01$).

their portfolios, a number of additional costs may prevent investors from trading; these can include physical transaction costs coming from brokerage commissions and capital gains taxes (see above), and mental costs coming from the need to determine optimal behavior based on beliefs and current portfolios (see Gabaix, 2016).

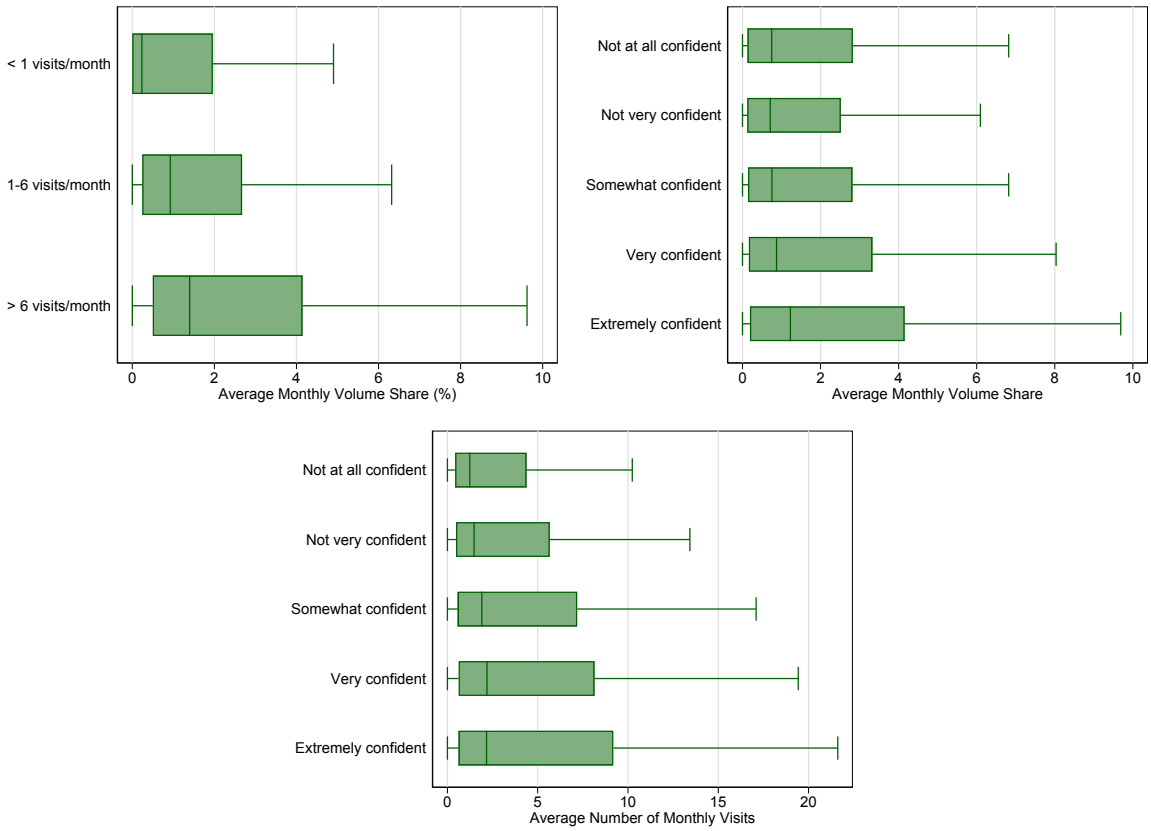
We next explore whether infrequent trading can help us understand the low sensitivity of portfolio allocations to beliefs. We first split individual respondents into three groups depending on their trading behavior during the sample period. Specifically, we classify individuals by the average monthly turnover in their portfolios, but our results are robust to other definitions of "infrequent trading", such as the average monthly number of trades. Column 4 of Table IV shows that individuals with a monthly portfolio turnover of at least 2.5% have a sensitivity of equity shares to beliefs that is about three times as large as that of individuals with a monthly portfolio turnover of less than 0.5%. However, even for those investors who have a sizable portfolio turnover, the sensitivity remains at 2.07, and thus below the benchmark value of 10 from the frictionless model discussed above. Nevertheless, these findings suggest that trading intensity is an important determinant of how strongly (and quickly) beliefs are reflected in portfolio holdings.

We next explore the role that a specific motivation behind infrequent trading, namely investor attention, plays in explaining this relationship. We measure investor attention by the frequency with which investors log into their Vanguard accounts. Logging into one's Vanguard account is correlated with, but different from actual trading activity, as shown in the top-left panel of Figure VI. Column 5 of Table IV shows that individuals who log into their Vanguard accounts more frequently also have a higher sensitivity of equity shares to beliefs. Indeed, individuals who log in more than six times per month have an sensitivity that is about 2.4 times as large as that of individuals who log in less than once per month. These results suggest that investor attention to their portfolios is an important driver of infrequent trading, and therefore of the attenuated relationship between beliefs and portfolio allocations.[21]

Confidence. A further mechanism that is potentially important in understanding how differences in beliefs translate into portfolio holdings is the confidence that individuals have in their own beliefs. Indeed, a large literature suggests that individuals who are more confident in their own beliefs are more likely to trade on them (e.g., De Long et al., 1990*a*; Kyle and Wang, 1997; Daniel, Hirshleifer and Subrahmanyam, 1998; Gervais and Odean, 2001; Barber and Odean, 2001; Statman, Thorley and Vorkink, 2006; Glaser and Weber, 2007; Grinblatt and Keloharju, 2009; Hoffmann and Post, 2016; Drerup, Enke and Von Gaudecker, 2017). To explore the role that investor confidence has on the extent to which beliefs are reflected in portfolios, we exploit the fact that the survey directly elicits how confident individuals are about their answers. Figure VI shows that while individuals that are more confident log in or trade slightly more often, most of the variation in trading and attention is within individuals with the same reported confidence. This means that

[21] Note that while the frequency of trading and log-ins strongly affects the relationship between beliefs and portfolios, they are not correlated with the level of beliefs themselves. Indeed, the correlation between the average monthly volume share and the expected 1-year stock market return is 0.02, and the correlation between the average monthly number of Vanguard log-ins and the expected 1-year stock market return is 0.05.

Figure VI: Trading Volume, Attention, and Confidence



Note: Figure shows the distributions of the average monthly volume share by log ins (top left panel), the average monthly volume share by confidence (top right panel) and the average number of monthly Vanguard visits by confidence (bottom panel). The box plots show the 5th, 25th, 50th, 75th and 95th percentiles.

any variation in sensitivity by confidence is picking up a conceptually different object than variation in sensitivity by trading frequency or attention. Column 6 of Table IV shows that individuals who report being "extremely confident" in their stock market beliefs have an almost four times higher sensitivity of portfolio shares to beliefs than individuals who report being "not at all confident" or "not very confident." However, even the sensitivity for the most confident group is substantially below the benchmark of 10 described above.[22]

Unobservable Heterogeneity: Labor Income, Other Investments, Risk Aversion. There are a number of other possible explanations for the relatively low sensitivity of equity shares to stock market beliefs. One possible set of explanations falls under the category of "optimists take risks

[22]Mapping confidence as reported in the survey to economic theory is not straightforward. One possibility is that confidence captures the degree of certainty that individuals have about the entire distribution of outcomes. Individuals who are less confident think that there is a higher probability that the outcomes will be drawn from a distribution different from the one that they reported in the survey. One possibility that we ruled out is that confidence simply reflects individuals' uncertainty about the expected outcome. While confidence is inversely related to the standard deviation of outcomes implied by the distribution questions, the relation is far from perfect, and the effect of confidence on actions such as portfolio risk-taking goes well beyond the effect induced by the standard deviation.

outside of Vanguard portfolios." For example, more optimistic respondents might have other accounts, at a different firm, and predominantly take risks in those accounts. Note that to explain our results it is not enough that all respondents choose a safer portfolios at Vanguard than in their other accounts; it would have to be that pessimists are disproportionately likely to do so. Another example would be that more optimistic respondents have occupations that are riskier; since their labor income is riskier, then they tilt their portfolio investment toward safer assets.

Another possible explanation is that more optimistic respondents are substantially more risk averse. This seems unlikely, since the implied variation in risk aversion across individuals would have to be extraordinarily large. To address this issue, we modified the specification in equation 1, and instead regressed the logarithm of portfolio shares on the logarithm of expected excess returns and the logarithm of the subjective variance (see Appendix **??**). The disadvantage of this log-log specification is that we have to exclude observations in which either the portfolio share or the expected excess returns are zero or negative.[23] The advantage is that, according to the Merton (1969) model, the coefficient of relative risk aversion enters this specification in a linearly additive manner, and therefore individual heterogeneity in risk aversion can be controlled for by individual fixed effects. We find no evidence that heterogenous risk-aversion overturns the low sensitivity of portfolios to beliefs that we have established thus far. In addition, Ameriks et al. (2016) observe one cross-section of both expected returns and risk-aversion (elicited via lottery-type questions) and conclude that risk-aversion heterogeneity is not sufficient to explain the low estimated average sensitivity of portfolios to beliefs.[24]

Summary. The evidence above has documented that investors' portfolios systematically vary with their beliefs. However, quantitatively, the average relationship is smaller than what would be predicted by simple frictionless models. Some of the low estimated sensitivity can be attributed to various forms of measurement error, but this cannot account for the entire gap. Instead, we identified a number of deviations from the frictionless benchmark model that can help explain the overall low sensitivity. For example, we find that the sensitivity is highest in actively managed tax-advantaged accounts that do not offer default options for investments and do not incur capital gains taxes upon transacting. We also find that individuals who trade more extensively, individuals who pay more attention to their portfolios, and individuals who are more confident in their beliefs all have a higher sensitivity of equity shares to stock market expectations. However, the individuals who are most similar to the frictionless benchmark on each of these three dimensions still do not have a sensitivity near that of the frictionless model. In column 7 of Table IV,

[23] Negative expected excess returns are not necessarily inconsistent with positive equity holdings and rationality. As Campbell and Viceira (1999) point out, it is possible for a rational investor to hold equities even at zero or negative expected excess return, if the investor has a long horizon, risk aversion greater than one, and believes in mean reverting stock returns.

[24] We also explored heterogeneity of the sensitivity of portfolio shares with respect to stock market beliefs along a number of demographic characteristics of the respondents. We find that the equity shares of the portfolios of older individuals are particularly sensitive to their 1-year expected returns. Similarly, the portfolios of male respondents are substantially more sensitive to beliefs than the portfolios of female respondents. We did not find significant heterogeneity in the responsiveness of portfolios to beliefs based on the region where individuals are located.

we explore the sensitivity of those respondents whose behavior comes closest to that described by the frictionless model on all three dimensions: individuals who are extremely confident in their beliefs, who pay substantial attention, and whose trading volume is significant. For respondents in that group, we estimate a β of 3.5, though the standard error around this estimate is quite large. This estimate is five times larger than the estimate obtained on the full sample, but still far from the benchmark of 10 implied by the simple frictionless model described above. Importantly, even these selected individuals still have frictions in their behaviors; for example, even in this group, individuals do not log in more than 14 times a month on average. Overall, our findings suggest that investor attention, adjustment costs, tax implications, and confidence are important features that mediate the transmission from beliefs to portfolio allocations, and should therefore play a more prominent role in the formulation of macro-finance models going forward. We sum up these findings in the following fact on the relationship between investor beliefs and portfolios.

Fact 1. *Portfolio shares vary systematically with individuals' beliefs. However, the average estimated sensitivity of an investor's portfolio share in equity to that investor's expected stock market returns is lower than predicted by frictionless asset pricing models. This sensitivity is higher in tax-advantaged accounts, and is increasing in wealth, investor trading frequency, investor attention, and investor confidence.*

III.B Other Beliefs and Portfolios

In the previous analyses, we explored the role that an individual's expectations of 1-year stock returns have on her portfolio allocation. We next explore how beliefs about other moments of stock returns as well as beliefs about bond returns and GDP growth affect these portfolio allocations. Since we are not able to instrument for most of these other beliefs, we return to OLS specifications.

In Column 1 of Table V, we control for the subjective standard deviation of 1-year stock returns.[25] This completes our analysis of the Merton (1969) model by allowing individual-level variation in both the level and the standard deviation of expected returns. A higher standard deviation of returns has a statistically insignificant negative effect on the equity share. The estimated sensitivity of portfolio shares to 1-year expected stock returns is unchanged when controlling for the standard deviation. The low response of portfolios to the subjective standard deviation of returns could be either due to measurement error in the standard deviation of returns, or due to model misspecification, whereby it is actually the negative tail probability that matters, rather than the standard deviation of returns.

Once we move away from the first and second moment of returns, or when we bring in beliefs about long-run stock returns, we lose a simple clear asset pricing model that can be used to benchmark the quantitative relationship between beliefs and portfolios. We therefore view the es-

[25] To construct the implied standard devation from the distribution question, we first split each bucket into ranges of 5 percentage points. For each of these ranges, we compute the historical probability of being in that range. We then weight these probabilities by the subjective probability of each bucket reported by the respondent. We finally calculate the standard deviation based on the mid-points of the narrower ranges, and their associated subjective probabilities. This allows us to also calculate subjective standard deviations also for the (small minority) of individuals who put all the probability mass in the middle bucket.

Table V: Beliefs and Portfolios: Long-Run Returns, Variance, Tail Risk, and Bond Returns

	Equity Share (%)						Fixed Income Share (%)	Cash Share (%)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Expected 1Y Stock Return (%)	0.779***	0.748***		0.649***	0.669***	0.675***	-0.321***	-0.367***
	(0.051)	(0.052)		(0.055)	(0.056)	(0.058)	(0.046)	(0.051)
Standard Deviation 1Y Stock Return (%)	-0.077							
	(0.051)							
Probability 1Y Stock Return < -30%		-0.108***	-0.171***	-0.095**	-0.080**	-0.083**	0.028	0.038
		(0.030)	(0.030)	(0.030)	(0.030)	(0.032)	(0.026)	(0.026)
Probability 1Y Stock Return∈ [-30%,-10%)			-0.141***					
			(0.020)					
Probability 1Y Stock Return∈ [30%,40%)			0.002					
			(0.023)					
Probability 1Y Stock Return > 40%			0.030					
			(0.037)					
Expected 10Y Stock Return (%)				0.366***	0.375***	0.414***	-0.322***	-0.109
				(0.073)	(0.075)	(0.082)	(0.067)	(0.063)
Expected 1Y Return of 10Y bond (%)					-0.193*	-0.190*	0.521***	-0.331***
					(0.087)	(0.089)	(0.073)	(0.068)
Expected 3Y GDP Growth (% p.a.)						-0.016	-0.210	0.133
						(0.151)	(0.126)	(0.100)
Expected 10Y GDP Growth (% p.a.)						0.070	0.078	-0.189*
						(0.130)	(0.110)	(0.081)
Controls + Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y
Specification	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
R-Squared	0.107	0.107	0.091	0.108	0.107	0.108	0.083	0.029
N	19,301	19,301	19,673	18,907	18,478	17,928	17,928	17,928

Note: Table shows summary results from regression 1, where we also include other beliefs elicited by the survey. The dependent variable in columns 1 to 6 is the equity share; in column 7 it is the fixed income share, and in column 8 it is the cash share. Standard errors are clustered at the respondent level. Significance levels: * ($p<0.10$), ** ($p<0.05$), *** ($p<0.01$).

timates presented in the rest of this section as providing guidance for future asset pricing theories wanting to focus on the relevant moments of the belief distributions.

In Column 2 of Table V, we control for the probability that individuals assign to a stock market decline of more than 30%. The probability of these rare disasters playes a prominent role in many macro-finance theories of portfolio formation and, in general equilibrium, of asset returns (Rietz, 1988; Barro, 2006; Gabaix, 2012). It is therefore ex-ante likely to be a major source of model misspecification for equation 2 and the associated predictions for the sensitivity of portfolios to stock market beliefs. Indeed, we find that a higher probability of a rare disaster is associated with declines in the equity portfolio share. A one-standard deviation increase in the perceived stock market disaster probability is associated with about a one percentage point lower equity share.

In Column 3, we separately include the probabilities that individuals assign to each of the five buckets of possible realizations of equity returns. Since these probabilities add up to 100%, we drop the middle bucket. Shifting subjective probability mass from the middle bucket to the low-outcome buckets is associated with substantial declines in the equity share, while shifts to high-

outcome buckets lead to only small and statistically insignificant increases in the equity share. This is consistent with concavity in the utility function, so that moving mass to negative states in which marginal utility is high has disproportionately large effects on portfolio choice.

In Column 4, we include an individual's beliefs about the average annualized stock return over the coming ten years in addition to the beliefs about the expected stock returns over the coming year. Short horizon and long horizon stock-market return expectations are positively correlated (see Section VI below), but long-run expectations matter for portfolio allocation even after controlling for short-run expectations. Interestingly, the magnitude of the effects are similar for long-run and short-run expectations. These results suggest that individuals choose their portfolio for the long-run, particularly since they do not adjust it frequently, and do not behave myopically by only focusing on their short-run expectations. In Column 5, we also include controls for a respondent's beliefs about the 1-year return of a 10-year risk-free bond. Holding fixed beliefs about equity returns, increased optimism about bond returns is associated with a lower equity share. Finally, column 6 also includes controls for GDP growth expectations, but these do not have an effect on portfolio shares over and above the stock market and bond market expectations. This is consistent with the vast majority of models in which expectations of cash flows contribute to the level of asset prices, but only expectations about returns influence portfolio choice.

Overall, the findings in this section suggest that the relationship between beliefs and portfolio allocations is more complex than suggested by the simple Merton (1969) model. First, the subjective risk of large stock market declines has larger effects on portfolio allocations than the subjective variance. Second, long-run stock market beliefs matter in addition to short-run beliefs. Third, beliefs about other investments, including fixed income investments, also influence the optimal equity share. We hope that these findings can help guide the development of future macro-finance models that explore the relationship between beliefs, portfolios, and ultimately asset prices.

III.C Substitution Patterns: Equity, Fixed Income, and Cash

The previous discussion explored the relationship between various beliefs (about stock returns, bond returns, and GDP growth) and the equity share in investors' portfolios. In Columns 7 and 8 of Table V, we instead use the fixed income share and the cash share as the dependent variables, allowing us to explore the substitution between stocks, bonds, and cash. The majority of increase in equity shares of individuals who expect higher stock market returns comes from individuals substituting away from cash rather than individuals substituting away from fixed income securities. This is despite the fact that the average fixed income share is substantially larger than the average cash share. Similarly, we find that increases in expected bond returns are associated with increases in the fixed income share, with much of the adjustment coming from reductions in the cash share instead of the equity share. The sensitivity of the bond portfolio shares to bond expected returns is even lower than the corresponding one for equities.[26] We conclude that fixed

[26] A back-of-the-envelope calculation is again very illustrative. We can apply the Merton (1969) formula in equation 2 by replacing equities with bonds and using the historical standard deviation of long-term Treasury-bond returns

income offers a similar, even more extreme, picture as equities: portfolios co-move with beliefs, but less so than implied by frictionless benchmark models.

IV Trading and the Pass-Through of Beliefs To Portfolios

The previous section showed that infrequent trading contributes to the low measured sensitivity of portfolios to beliefs in a cross-section of survey respondents. In this section, we inspect this mechanism further, by looking at investors' trading behavior over time. First, we explore the *extensive* margin of trading: can we predict the incidence of trading with changes in beliefs or other observables? Second, we inspect the *intensive* margin of trading: do changes in beliefs affect the direction and magnitude of trading, conditional on a trade taking place? Third, we inspect whether the allocation of new funds that come into an account in the form of cash reflects investors' beliefs.

Before proceeding with the analysis, we provide a brief overview of how we measure trading; the Appendix provides additional details. We observe administrative data on all transactions for each client with a retail account.[27] These include, among other things, money moving in and out of the Vanguard accounts, purchases and sales of securities, and purchases, sales, and exchanges of shares in mutual funds. Since we observe beliefs only when an investor answers the survey, we aggregate all trades that occur between two survey responses. We can then relate trading behavior to the beliefs at the time of each survey as well as changes in beliefs between surveys. We also aggregate trades within each asset class: equity, fixed income, cash and cash-equivalents, and other investments. For each time window between two surveys, we compute the changes in portfolio shares across these asset classes that are induced by active trading during the window.[28] This is different from the actual change in portfolio shares during each window, because changes in market values also affect the final portfolio shares. The advantage of using trading data is precisely that we can separately identify the component of the change in portfolio shares that can be attributed to active trading decisions.[29]

We first explore how trading behavior varies with investors' beliefs. For every investor, we identify the time windows between each pair of consecutive survey responses. These windows differ across investors who may respond to differenst survey waves. For each window, we compute the total trading activity during that window, the beliefs at the beginning of the window, and the change of beliefs during the window. We also track the investors' beginning-of-window port-

around 5%. The estimate of $\beta = 0.521$ from column 7 of Table V implies a coefficient of relative risk aversion of 860.

[27] We are in the process of analyzing similar data for trading in institutionally managed retirement accounts.

[28] For example, if a respondent with 100 dollars in her account holds 60% of her portfolio in equity at the beginning of the window, say January 15th 2018, and on January 20th she buys 10 more dollars of equity using cash already available in the account, then we predict *based on trading* the end-of-window equity share to be 70%. Trades that occur within an asset class, for example selling one stock to buy another, do not contribute to changes in asset class shares.

[29] In most periods, when trading has not taken place, the portfolio changes purely reflect market movements and they are less likely to be aligned with belief changes. The decision of the agent not to trade to offset market movement is only partially informative. For example, in the presence of fixed costs of trading, whether mental or physical, there is an inaction band with respect to market movements in which lack of trading does not imply that beliefs and portfolio shares are fully aligned.

folio allocations.[30] We then regress the change in equity portfolio share *due to trading* for individual i in window w, given as $\Delta EquityShare_{i,w}$, on the expected 1-year stock return as of the beginning of the window, $E_{i,w-}[R_{1y}]$, the change in this expectation during the window, $\Delta E_{i,w}[R_{1y}]$, and the equity share at the beginning of the window, $EquityShare_{i,w-}$:

$$\Delta EquityShare_{i,w} = \alpha + \beta E_{i,w-}[R_{1y}] + \gamma \Delta E_{i,w}[R_{1y}] + \delta EquityShare_{i,w-} + \phi X_{i,w} + \epsilon_{i,w}, \tag{3}$$

where $X_{i,w}$ includes a set of window-length fixed effects, as well as controls for age, gender, region of residence, wealth, wave fixed effects, and dummies for equity shares of 0% and 100%. Column 1 of Table VI reports the main coefficients; in the Appendix, we also report the coefficients on the control variables. A one-percentage-point increase in expected returns at the beginning of the window predicts a 0.1 percentage point increase in the equity share due to trading over the following window; a one-percentage-point change in beliefs over the window predicts a 0.13 percentage point change in the equity share. While these sensitivities are statistically significant, they are quantitatively small, and significantly smaller than what we obtained from the cross-sectional analysis in Section III.[31] Finally, Column 1 also shows that investors with high equity shares at the beginning of the window tend to subsequently trade to reduce their equity exposures, potentially a sign of rebalancing of their positions.

The low sensitivities in Column 1 could reflect two different mechanisms. First, they could simply reflect the fact that individuals trade infrequently, so that the average sensitivity to beliefs appears low (extensive margin). Alternatively, they could reflect a low sensitivity of trading to beliefs even conditional on investors trading (intensive margin). Our trading data allows us to disentangle the two explanations. We analyze them below.

The Extensive Margin of Trading. A large academic literature has aimed to explain trading volume in financial markets via a mix of changes in beliefs and overconfidence (e.g., Harrison and Kreps, 1978; Hong and Stein, 1999; Scheinkman and Xiong, 2003; Hong and Stein, 2007). A related literature has studied models of the optimal frequency and size of trading in the presence of limited information and transaction costs (e.g., Duffie, Sun et al., 1990; Gabaix and Laibson, 2001; Abel, Eberly and Panageas, 2007; Alvarez, Guiso and Lippi, 2012). A natural question in our setup, therefore, is whether changes in beliefs are associated with trading.

Column 3 of Table VI reports the results of a regression similar to equation 3, except that the dependent variable is an indicator of whether the investor has traded at all during the window, and the change in expected returns over the window is replaced with the corresponding absolute value. The predictive ability of belief changes for trading activity is statistically and economically small. Beginning-of-window expected returns that are one percentage point higher only increase

[30] For example, if an investor has answered waves 1, 2 and 5 of the survey, we would identify two windows: the 2-months period between wave 1 and wave 2, and the 6-months period between wave 2 and wave 5. Each window would appear as a separate observation in the analysis below.

[31] Since these regressions are analyzing changes over time in portfolio choice, rather than levels as in Section III, they make use of a different source of variation; while one would expect these two approaches to produce similar results in a frictionless world, this is not necessarily the case if trading frictions are present.

Table VI: Trading Analysis

	Δ Equity Share (%)	Probability Trade	Probability Trade	Probability Buy	Δ Equity Share (%)
	(1)	(2)	(3)	(4)	(5)
Δ Expected 1Y Stock Return (%)	0.131***			1.095***	0.408***
	(0.020)			(0.243)	(0.070)
Expected 1Y Stock Return (%)	0.105***		0.205*	1.253***	0.401***
	(0.020)		(0.096)	(0.253)	(0.073)
Lagged Equity Share (%)	-0.045***		-0.065**	-0.369***	-0.156***
	(0.005)		(0.024)	(0.054)	(0.015)
|Δ Expected 1Y Stock Return (%) |			0.198		
			(0.126)		
Extreme Equity Share Dummies	Y	Y	Y	Y	Y
Time Betweeen Wave Dummies	Y	Y	Y	Y	Y
Other Fixed Effects and Controls	Y	N	Y	Y	Y
Specification				Conditional on Trading	Conditional on Trading
R-Squared	0.029	0.317	0.331	0.501	0.187
N	7,738	7,994	7,738	1,998	1,998

Note: Table shows results from regression 3. The unit of observation is a window between two consecutively survey responses by the same individual. The dependent variable in columns 1 and 5 is the change in the equity share due to trading between the two survey waves. The dependent variable in columns 2 and 3 is an indicator for whether there was any active trading between the two survey waves. The dependent variable in column 4 is an indicator of whether the individual increased her portfolio share in equity during the window as a result of trading between the two survey waves. All columns control for the length of time between two consecutive answers, and for dummies capturing extreme start-of-period equity shares of 0% or 100%. All other columns, except column 2, also control for the respondents' age, gender, region of residence, wealth, and the survey wave. Columns 4 and 5 conditional the sample on windows with active trades.

the probability of trading during the subsequent window by 0.2 percentage points. Belief changes over the window have an effect of similar magnitude, but the coefficient estimates are not statistically significant. While the R^2 of the regression is high at 33%, Column 2 of Table VI, which excludes the beliefs and portfolio shares entirely from the regression, displays a similarly high R^2 of 32%. This shows that the reason for the high R^2 in Columns 2 and 3 is the inclusion of window-length fixed effects: trading is mechanically more likely to occur the longer the length of the window. The incremental explanatory power of beliefs and portfolios in predicting the extensive margin of trading is just above 1%.

On the one hand, this difficulty with predicting who trades over a given time horizon might not be surprising, since trading happens infrequently and is often driven by liquidity needs or life events such as job transitions that we do not observe. On the other hand, these results support models with a probability of trading that does not vary much with beliefs. Such models range from imposing a flat hazard function exogenously (i.e., agents are selected at random to trade with a constant probability per period), to deriving it as the optimal equilibrium choice.[32]

[32]We also explored the role of attention, as proxied by the frequency of log-ins to the Vanguard website, in predicting trading. We find that people who pay more attention are more likely to trade based on beliefs and belief changes, but the effect is small and concentrated among those investors who pay extremely close attention. This further reinforces

The Intensive Margin of Trading. In our next analysis, we condition on time windows during which individuals trade actively, and ask whether the direction and the magnitude of the trade can be explained by belief changes. We focus on trades that change the shares allocated to different asset classes by at least one percentage point. We start by predicting the direction of trading. Specifically, Column 4 of Table VI reports the results of a regression similar to equation 3, except that the dependent variable is now an indicator of whether the investor has actively increased her equity share by at least one percentage point during the window. Beliefs predict the direction of trading conditional on a trade actually occurring: an investor who expects future returns to be one percentage point higher is around 1.2 percentage points more likely to buy equities in a given window. This coefficient is very similar for the beginning-of-period beliefs and for the change in beliefs over the window. Since we do not know exactly at which point in the window the change in beliefs occurred, we do not interpret these results strongly in light of a particular model of trading behavior. Rather, it is likely that both the beginning-of-window beliefs and the change during the window help capture the individuals' true beliefs at the time of trade. Column 5 of Table VI explores the magnitude of trading conditional on a trade occurring. The dependent variable is the change in the equity share due to trading: that is, the regression is the same as for Column 1, but the results are conditional on trading taking place (again, measured by the equity share changing by at least 1% in any direction due to active trading). Conditional on trading, the sensitivity of trading to beliefs increases by a factor of four compared to the unconditional results in Column 1: a one-percentage-point increase in investors' beliefs about future returns corresponds to a 0.4 percentage point increase in the equity share due to trading. When we condition on larger trades (at least a 5% change in equity share) the magnitudes double, with estimates around 0.8%.

These results show that, consistent with the analysis in previous sections, trading occurs infrequently, but when it does occur, it lines up with beliefs with economically and statistically significant magnitudes.[33] We summarize the results of this section in the following fact:

Fact 2. (Intensive and Extensive Margin of Trading). *While belief changes have little to no explanatory power for predicting when trading occurs (extensive margin of trading), they explain both the direction and magnitude of trading conditional on a trade occurring (intensive margin of trading).*

The Allocation of New Funds. A final question we investigate is how respondents allocate "new funds" across the different asset classes depending on their beliefs. We measure these new funds as cash entering a Vanguard account from outside sources. Specifically, we look at all cases in which, during a window, we see a net inflow of outside money into the account that is at least

the conclusion that a constant hazard function of trading is a good approximation of the data.

[33] We have explored a variety of additional trading results, which we briefly summarize here. First, the results in Table VI are robust to including subjective risk measures, like the probability of extremely negative returns, though this probability itself does not appear to significantly affect trading behavior. Second, we have repeated the analysis by looking at the 10-year expected return instead of (or in addition to) the 1-year expected return. When both the short-term and long-term expectations are included in the regression, the 10-year expected return adds new information and behaves similarly to short-term expected returns. Third, we have explored the allocation to other asset classes, where the results are statistically weaker. For example, trading in bonds seems to have a statistically weak relation with beliefs about bond returns, even after conditioning on trading.

20% of the existing Vanguard assets. Of course, we do not observe whether these are new funds, such as labor income, or proceeds from other asset sales outside of Vanguard. Since investors devote some time to deciding an allocation for funds when they first transfer them to their trading account, this represents a particularly informative window to observe how beliefs affect portfolio formation. We repeat the regression of Column 5 of Table VI, but condition on a large inflow occurring during the window. We find that when investors actually trade during that window (that is, they actively allocate the new money), the sensitivity of equity shares to beliefs increases significantly, to 0.75, for both levels and changes. On the contrary, when little or no trading occurs during the window in which new funds are brought in, we find that the sensitivity drops to around 0.002.

IV.A Beliefs and Portfolios: Implication for Current Behavioral Models

The macro-finance literature is increasingly building on survey evidence to guide theoretical models. Indeed some recent models in this literature are explicitly designed to jointly match survey expectations data and asset prices (e.g., Barberis et al., 2015). More specifically, the existing models often build on a modification of Mertonian portfolio demand in CARA-normal set ups, but rather than imposing rational expectations, they impose expectations that are more consistent with survey evidence. These models' predictions for asset prices rest on two modeling blocks: (i) that beliefs are volatile and change over time, and (ii) that individual portfolios react strongly to changes in these beliefs.[34] Indeed, most behavioral models imply the same sensitivity of portfolios to beliefs as the benchmark rational-expectations models. Our results show that the sensitivity assumed in these models may only describe the behavior of a select group of investors who pay attention to their portfolios, who trade regularly, and who are confident in their beliefs. For the majority of investors, infrequent trading and lack of confidence in beliefs reduces the passthrough from beliefs to portfolios. As a result, the frictionless behavioral models, in their current form, are likely to overstate the power of expectation changes in explaining asset price movements. Our research opens a new challenge to improve the models: a successful model has to match both expectations and portfolio dynamics jointly with asset prices.

It is beyond the scope of this paper to fully specify such a model, but we find it useful to describe some possible avenues arising from our work. First, one of the central facts we uncover is that infrequent trading meaningfully reduces the pass-through of beliefs into asset demand. We find that the timing of trading does not depend meaningfully on beliefs, but that, conditional on trading, beliefs matter for the size and direction of the trade. A parsimonious way to model such behavior is to introduce infrequent random trading, whereby an agent is selected at random

[34] The first block is familiar from a long literature in finance going back to Shiller (1981) that has attempted to explain the excess volatility of asset prices compared to fundamentals via the excess volatility of agents' expectations (of either fundamentals or returns) compared to rational expectations. The second block is more nuanced: for movements in expectations to move asset prices in general equilibrium, these same movements of expectations have to move asset demand in partial equilibrium. The intuition is that if expectations move asset demand, fixed asset supply implies that prices have to adjust for markets to clear. Indeed, the smaller the response of quantities to beliefs, the larger the movement in prices that is required to clear markets.

based on a memory-less distribution to have the possibility of trading in a given period This is reminiscent of the Calvo (1983) adjustment model in macroeconomics for firm pricing decisions.

It is an open question whether a model in which behavioral agents have a lower sensitivity of portfolio demand to beliefs can match asset prices. In this direction, one further possibility for adjustments to the basic models is to explore the relative wealth and demand sensitivities of different classes of agents. For example, behavioral models such as those reviewed by Barberis (2018), often feature two types of investors (e.g., rational arbitrageurs and behavioral investors), each modeled with its own demand for stocks similar to equation 2. Asset prices are then determined by the dynamics of expectations of the behavioral agents, modulated by the relative wealth shares of the two agents and their relative demand sensitivities. One possible avenue therefore is to have the behavioral group be substantially wealthier (in the aggregate) than the arbitrageurs. This would induce the changes in demand by the behavioral agents, even with a relatively insensitive demand, to be larger than the arbitrageurs' capacity to absorb them, therefore generating bigger asset price movements. Alternatively, one could make the demand of the behavioral agents more sensitive than that of the arbitrageurs; belief surveys like ours could be used to explore the correlation between deviations from rationality and the sensitivity of beliefs and portfolio allocations.

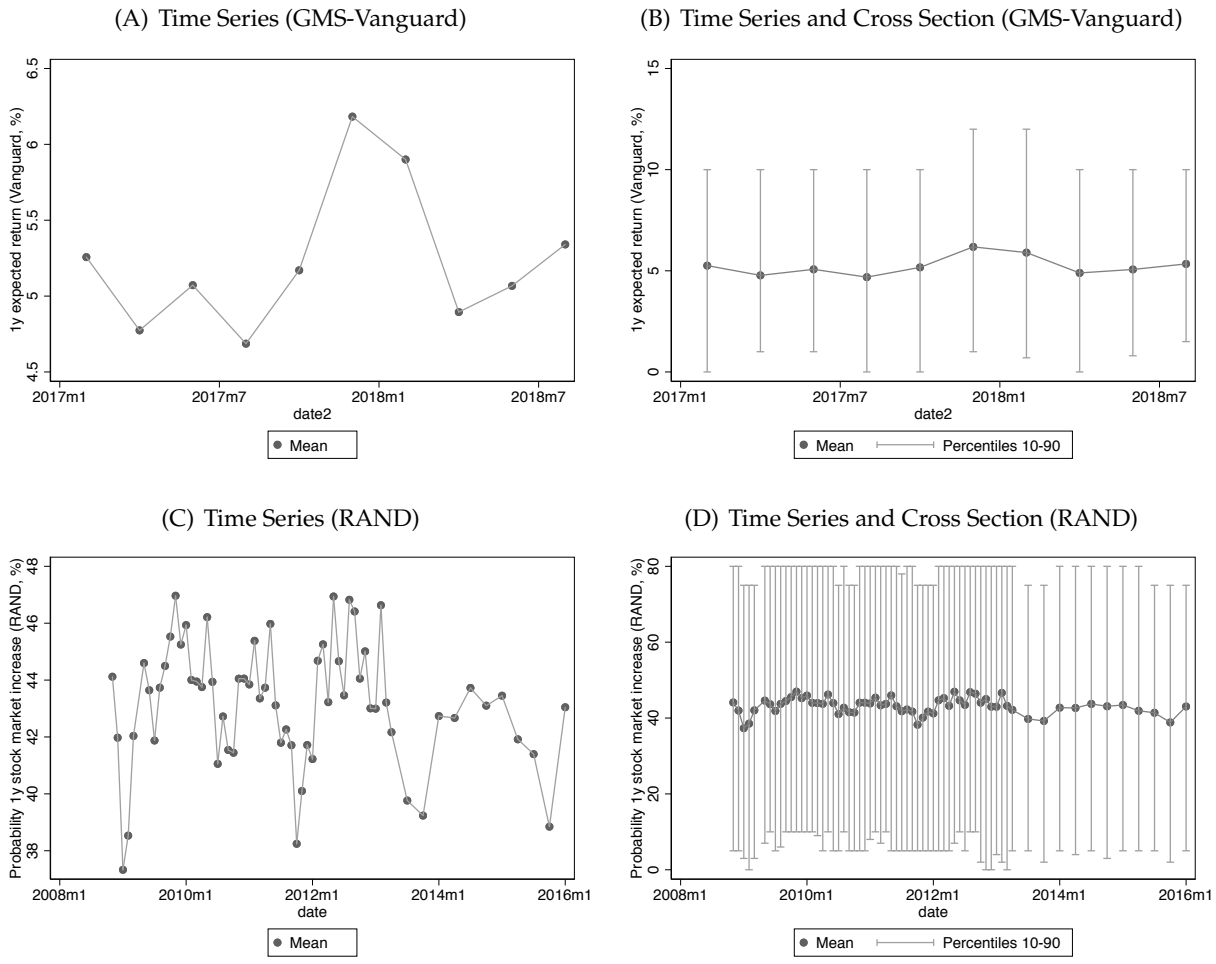
V Variance Decomposition of Beliefs

Section II documented substantial heterogeneity in investors' beliefs. In this section, we further explore this heterogeneity by decomposing the variance of beliefs into three components: fixed individual characteristics, common variation in individual beliefs over time, and a residual component that captures both idiosyncratic individual time variation and measurement error.

To motivate this variance decomposition, we first explore a graphical representation of the time-series and cross-sectional variation in our data. Panel A of Figure VII shows the time-series of average 1-year expected return in the GMS-Vanguard survey. As discussed in Section II.D, the average expected return in our sample displays meaningful time-series variation, with a maximum increase of almost 2 percentage points and a subsequent decrease of similar magnitude. Panel B of Figure VII shows the same time series of average expected returns as in Panel A, but also includes the 10th and 90th percentiles of the cross-sectional distribution of answers in each wave. The cross-sectional variation in expected returns swamps the time-series variation. The bottom row of Figure VII highlights that this pattern is not unique to our survey. Indeed, the RAND survey discussed in Section II.D also displays substantially larger cross-sectional variation than time-series variation in the probabilities that investors assign to a stock market increase.[35]

[35]Similar results also hold in the Duke CFO survey. Whereas we do not have access to the disaggregated data to compute the percentiles month by month, we observe the cross-sectional standard deviation of the responses, which is about 5 times larger than the time-series standard deviation of the average belief. Since the RAND survey was not included in Greenwood and Shleifer (2014), which documents significant co-movement in the time series behavior of a number of different surveys, it is useful to recall that Figure IV shows the time series of RAND to be consistent with the dynamics of other surveys. For the period in which they overlap, the correlation between the RAND time series and the Shiller index is 0.29, and the correlation between the RAND time series and the Duke CFO survey is 0.49. These

Figure VII: Time-Series and Cross-Sectional Variation: GMS-Vanguard and RAND



Note: The figure reports the time series of the average beliefs from the GMS-Vanguard survey (top row, 1-year expected return question) and from the RAND survey (bottom row, probability of a 1-year stock market increase). The right panels in each row also plots the 10th and 90th percentiles of the survey answers in each wave.

There are two interpretations consistent with this finding. First, individual responses may display substantial idiosyncratic variation both across individuals and over time: the same individual might at different points in time report very different beliefs. This would generate, in each survey wave, a large amount of cross-sectional variation in responses. Second, the observed cross-sectional variation could be due to *persistent* heterogeneity in beliefs; that is, the *same* investors are responsible for the dispersion in beliefs observed in different waves. Our survey contains a substantial panel element that allows us to disentangle the two alternative explanations. We find that belief variation is mostly summarized by individual fixed effects, that is, individuals tend to have very large and persistent differences in their views, which strongly dominate the time variation in beliefs, at least during our sample period. We also find that this difference in beliefs is difficult to

correlations are in the same range of those reported by Greenwood and Shleifer (2014) for other survey measures.

explain with observable characteristics of the individuals.

V.A The Dominance of Individual Fixed Effects

We denote the belief expressed by individual i at time t as $B_{i,t}$. For example, for the question about the 1-year expected stock market returns, we have $B_{i,t} = E_{i,t}[R_{1y}]$. For the (unbalanced) panel of these beliefs, we begin by running the following regressions:

$$B_{i,t} = \chi_t + \epsilon_{1,i,t}, \tag{4}$$

$$B_{i,t} = \phi_i + \epsilon_{2,i,t}, \tag{5}$$

$$B_{i,t} = \phi_{3,i} + \chi_{3,t} + \epsilon_{3,i,t}. \tag{6}$$

Equation 4 estimates a set of time (i.e., survey wave) fixed effects, χ_t, that absorb the common time-series variation of respondents' beliefs. Equation 5 estimates a set of individual fixed effects, ϕ_i, that absorb the average belief over time of each respondent. Equation 6 jointly estimates both individual and time fixed effects. We estimate these regressions both unweighted and weighted by respondent wealth, and only include individuals who responded to at least 3 waves.

Table VII: Decomposing the Variation in Beliefs: Individual and Time Fixed Effects

	R^2 (%) - Unweighted			R^2 (%) - Wealth-Weighted			
	Reg (4)	Reg (5)	Reg (6)	Reg (4)	Reg (5)	Reg (6)	N
Expected 1Y Stock Return (%)	1.0	62.3	63.0	1.4	62.0	63.0	1,983
Expected 10Y Stock Return (% p.a.)	0.3	52.7	53.0	1.1	55.2	55.6	1,955
Probability 1Y Stock Return < -10%	0.2	56.7	56.8	0.4	56.9	57.1	2,023
St.d. Expected 1Y Stock Return (%)	0.3	62.3	62.6	0.8	64.6	65.0	2,023
Confidence (Stock Qs)	0.2	66.1	66.2	0.4	68.5	68.6	2,004
Expected 3Y GDP Growth (% p.a.)	0.6	53.8	54.2	1.5	53.9	54.9	1,985
Expected 10Y GDP Growth (% p.a.)	0.4	49.3	49.6	1.8	49.7	50.0	1,971
Probability p.a. 3Y GDP Growth < 0%	0.3	54.2	54.4	0.7	57.3	57.7	2,015
St.d. Expected p.a. 3Y GDP Growth (%)	0.2	43.7	43.9	1.7	43.5	43.8	2,012
Confidence (GDP Qs)	0.1	67.8	67.8	0.3	68.2	68.5	1,997
Expected 1Y Return of 10Y bond (%)	0.4	57.3	57.7	2.7	59.5	60.2	1,951
Confidence (Bond Qs)	0.1	66.2	66.3	0.2	67.6	67.7	1,967

Note: Table reports the R^2s corresponding to the three regressions 4, 5, and 6 (both unweighted and wealth-weighted), and the number of individual respondents N. We only inlude respondents that have responded to at least three waves. Each row corresponds to a different survey question that is used as the dependent variable.

Table VII reports the R^2 of each of the three regressions (unweighted in the left panel and wealth-weighted on the right panel), for a subset of questions from our survey. Most of the panel variation in beliefs is absorbed by individual fixed effects. Consider for example the first row, corresponding to the question on 1-year expected stock returns. Time fixed effects capture about 1% of the total panel variation, whereas individual fixed effects capture more than 62% of the total variation.[36] This large difference in explanatory power is common across all beliefs. The same is true

[36]These findings echo results in Dominitz and Manski (2011), who show that individuals' responses for the probability of a positive equity return over the coming twelve months were correlated across two waves of the Michigan Survey

when we decompose the heterogeneity in individuals' confidence in their beliefs: most of the variation is across individuals rather than over time. These conclusions change little when beliefs are weighted by the wealth of the respondent.

One possible concern with this analysis is that the relatively short time period over which we observe survey responses might make the fixed effects appear more important than they truly are, for example if individuals' beliefs followed a persistent autoregressive process. We investigated this possibility in two ways. First, we added an autoregressive term to regression 6. This allows us to distinguish whether the observed persistence of beliefs is due to a true "fixed effect" of each individual (i.e., a long-run mean belief), as opposed to a temporary but persistent fluctuation of individuals' beliefs around their long-run means. For all the specifications reported above, the autoregressive terms were small, statistically insignificant, and did not account for much of the variation. Second, we increased the minimum number of waves that an individual has to answer to be included in the analysis. This allows us to address concerns that the high explanatory power of individual fixed effects arises mainly from in-sample fitting of noise. Table VIII shows how the R^2 changes for each question as we increase the minimum number of answers per individual.[37] We find at most a modest deterioration in the importance of individual fixed effects (as measured by the R^2) as we increase the minimum number of answers. We conclude that in-sample fitting of noise does not drive the explanatory power of the individual fixed effects.

While we cannot ultimately rule out that, as the survey continues in different economic environments, the relative importance of individual fixed effects will change, their overwhelming importance compared to time fixed effects in our sample is striking. In particular, the persistent cross-sectional variation is large even relative to time-series variation that other surveys have displayed during periods like the Great Recession. We next confirm this result using the (now discontinued) RAND survey, which also features a panel structure and a longer sample than ours.

V.A.1 The Dominance of Individual Fixed Effects in Other Surveys

The variance decomposition presented above can be performed on any survey with a panel structure. Among the existing surveys discussed in Section II, the RAND survey is the only one with a substantial panel dimension. It covers the period from November 2008 to January 2016, and thus includes part of the financial crisis and the following stock market recovery. The surveys were generally run at a monthly frequency, though some months were skipped; overall, there are 61 survey waves. A total of 4,734 individuals participated in the survey, 3,166 of whom responded at least 10 times, 1,780 at least 30 times, and 1,032 at least 50 times. Unfortunately, the RAND survey does not directly elicit beliefs expected returns; therefore, we focus on the closest available substitute, namely beliefs about the probability of a stock market increase one year ahead.

As highlighted in Figure VII, the time series variation of the average belief in the Rand survey

of Consumers. The authors also found substantial heterogeneity in this probability across individuals.

[37] The table also reports the number of individuals who respond a certain number of times. The number of observations is of course greater since each individual has answered multiple times. For example, 282 individuals repond to at least six waves, but the total number of survey responses from these individuals is 1,692.

Table VIII: Decomposing the Variation in Beliefs: Robustness

	R^2 (total, %)				Number of Individuals			
	#Resp≥3	#Resp≥4	#Resp≥5	#Resp≥6	#Resp≥3	#Resp≥4	#Resp≥5	#Resp≥6
Expected 1Y Stock Return (%)	62.3	59.6	58.2	54.9	1,983	968	510	282
Expected 10Y Stock Return (% p.a.)	52.7	50.5	49.4	48.5	1,955	976	499	285
Probability 1Y Stock Return < -10%	56.7	55.9	56.1	55.5	2,023	1,001	521	287
St.d. Expected 1Y Stock Return (%)	62.3	61.4	61.8	61.3	2,023	1,001	521	287
Confidence (Stock Qs)	66.1	64.4	63.2	64.3	2,004	984	510	284
Expected 3Y GDP Growth (% p.a.)	53.8	51.5	46.6	46.5	1,985	984	512	283
Expected 10Y GDP Growth (% p.a.)	49.3	45.1	46.2	44.2	1,971	969	504	282
Probability p.a. 3Y GDP Growth < 0%	54.2	52.4	50.9	50.8	2,015	1,001	521	289
St.d. Expected p.a. 3Y GDP Growth (%)	43.7	39.7	37.5	37.1	2,012	1,000	518	287
Confidence (GDP Qs)	67.8	65.4	63.4	63.4	1,997	985	508	286
Expected 1Y Return of 10Y bond (%)	57.3	53.9	52.7	52.9	1,951	974	507	284
Confidence (Bond Qs)	66.2	64.7	64.0	63.1	1,967	974	506	282

Note: The left panel reports the R^2s corresponding to regression 5. The right panel reports the number of individuals that respond the required number of times. Across columns, we increase the minimum number of responses for an individual to be included in the sample from three to six.Each row corresponds to a different survey question that is used as the dependent variable.

is swamped by the cross-sectional variation. We next explore whether the cross-sectional variation in the RAND survey is also mostly explained by individual fixed effects. To do this, we perform the same variance decomposition for the RAND survey as the one we reported in Tables VII and VIII for the GMS-Vanguard survey. The corresponding tables for the RAND survey are reported in the Appendix, and the findings are both qualitatively and quantitatively consistent with those from the GMS-Vanguard survey: across all questions, time fixed effects explain around 1% of the panel variation, and the individual fixed effects explain about 50-60% of the variation. Interestingly, the results are robust to increasing the minimum number of waves that an individual has to have responded to in order to be included from 3 all the way to 50.

V.B Beliefs and Demographics

Having established the overwhelming importance of individual fixed effects in explaining the panel variation in beliefs, it is a natural question to ask which observable characteristics explain why some individuals are optimistic and others are pessimistic. We find that observable individual characteristics have little or no explanatory power for beliefs. This is true despite including measures such as gender, age, wealth, attention (days with visits to the Vanguard website), confidence, and geographic location. To establish this finding, we run the following regression:

$$\phi_{3,i} = \alpha + \Gamma X_i + \epsilon_i, \tag{7}$$

where $\phi_{3,i}$ are the individual fixed effects estimated in regression 6, and X_i are the following individual characteristics: dummy variables for age groups, wealth quintiles, region of residence,

gender, and confidence, and the number of days with Vanguard log-ins in an average month.[38] Table IX focuses on the R^2s from these regressions, which capture the share of variation in the fixed effects that is explained by the demographics. The observed characteristics have only minimal explanatory power, with values for the R^2 between 2% and 5%. Importantly, when we restrict the analysis to explaining fixed effects that are estimated on more observations, and which should therefore be more precise, there is only a modest increase in R^2. We conclude that classic measurement error in beliefs cannot explain the low predictive power of demographics for beliefs. Despite the low overall explanatory power of demographics for beliefs, the Appendix shows that demographic characteristics sometimes have a statistically significant relationship with beliefs.

Table IX: Beliefs Heterogeneity and Demographics

R^2	#Resp≥1	#Resp≥2	#Resp≥3	#Resp≥4	#Resp≥5
Expected 1Y Stock Return (%)	2.7	4.0	5.0	4.9	5.6
Expected 10Y Stock Return (% p.a.)	2.0	3.3	3.1	4.4	8.5
Probability 1Y Stock Return < -10%	3.1	5.0	7.1	9.2	11.9
St.d. Expected 1Y Stock Return (%)	8.2	10.7	13.3	16.3	17.4
Expected 3Y GDP Growth (% p.a.)	2.8	3.5	3.7	4.2	11.2
Expected 10Y GDP Growth (% p.a.)	3.8	4.4	4.0	6.1	10.8
Probability p.a. 3Y GDP Growth < 0%	6.1	8.7	9.4	12.3	10.8
St.d. Expected p.a. 3Y GDP Growth (%)	3.3	5.0	5.4	7.5	9.6
Expected 1Y Return of 10Y bond (%)	3.3	4.3	3.8	4.8	7.9

Note: The table reports the R^2 statistics corresponding to regression 7. In each column, going from left to right, we increase from 1 to 5 the minimum number of responses for an individual to be included in the sample. Each row corresponds to a different question in the survey.

These results relate to the literature linking expectations to demographic characteristics and personal experiences. It is common in this literature to find strong statistical relationships but low explanatory power for expectations using variables such as wealth, gender, IQ, place of birth, current location, own past experience, or friends' past experiences (see, for example, Malmendier and Nagel, 2011; Kuchler and Zafar, 2015; Das, Kuhnen and Nagel, 2017; Bailey et al., 2017, 2018; Coibion, Gorodnichenko and Kamdar, 2018; D'Acunto et al., 2019).[39] Our results above highlight that the low R^2s in these analyses is unlikely to be primarily due to classic measurement errors in beliefs. An exciting possibility is that these individual fixed effects therefore reflect combinations of individual characteristics and experiences that economic research has yet to discover. Indeed, we hope that this research effort will succeed in identifying additional characteristics that are both statistically related to beliefs and can explain a substantial part of the cross-sectional variation.

We collect the findings from the variance decomposition of beliefs in the following fact, before discussing its implications for macro-finance theory.

[38] For dynamic variables such as age and wealth, we take the average over the sample. For geographic location and gender, we take the value of the most recent observation.

[39] We have thus far not included direct measures of personal experience, like own stock market performance, in our regressions, though some elements of personal or cohort experience are captured by age and geographic location.

Fact 3. *Variation in individual beliefs as well as confidence in beliefs is mostly characterized by heterogeneous individual fixed effects: between 50% and 60% of variation across responses is due to individual fixed effects and only 1% is due to common time series variation. The remaining variation is accounted for by idiosyncratic individual variation over time and measurement error. The persistent heterogeneity in individual beliefs is not explained by observable characteristics such as gender, age, wealth, attention, confidence, and geographic location.*

V.C The Importance of Belief Heterogeneity for Macro-Finance Theory

Fact 3 highlights that the largest share of the total variation in beliefs in the GMS-Vanguard survey is due to persistent heterogeneity across individuals, as opposed to aggregate or within-individual variation in beliefs over time. These patterns have important consequences for the design of macro-finance models. For this discussion, it is useful to consider two stylized classes of models used in the literature. At one extreme are models in which all individuals hold the same beliefs and the only variation is in the time series (e.g., Barberis et al., 2015; Jin and Sui, 2018). At the other extreme are models in which agents have permanent and constant differences in beliefs (e.g., Geanakoplos, 2009). The first class of models generates variation in asset prices by swings in individuals' beliefs over time, which then translate into swings in the representative agent's beliefs. The second class of models generates variation in asset prices by movements in wealth-weighted aggregate beliefs. Intuitively, this transmission requires several steps: first, individual heterogeneity in beliefs induces different portfolio shares; second, shocks over time redistribute wealth across individuals given the differences in portfolios, which then affects equilibrium prices.[40]

Our research highlights that both of these stylized models miss central features of the data, and suggests a path to enrich these models to better align with the empirical evidence. The first class of models is silent on a major feature of the data, the cross-sectional individual heterogeneity, and gets all its action from what appears to be a small component of the total variation in beliefs. In light of our results, we believe that the literature would benefit from explicitly modeling the aggregation of the heterogeneity in individual beliefs, and exploring the implications for equilibrium prices and quantities. The second class of models captures the persistent individual heterogeneity, but gets all its action by having this heterogeneity strongly reflected in portfolios. The findings in Section III highlight the challenges with this second step. Beliefs differences are reflected in portfolios in a statistically strong manner, but their average impact on portfolios is quantitatively limited: in addition to the low estimated sensitivity, the R^2 of the regression of portfolios on beliefs is only about 2.2%, highlighting the limited sense in which optimists have riskier portfolios (see column 1 in Table III above).[41] If individuals with vastly different beliefs do not, on average, have very different portfolios, then shocks do not substantially redistribute wealth between optimists

[40] For example, optimists have portfolios that load more than pessimists' portfolios on good economic outcomes; if these outcomes are realized, then the optimists' wealth share increases compared to the pessimists' share and the more optimistic wealth-weighted beliefs are reflected in the new equilibrium prices.

[41] Among the controls, the key contributor to the explanatory power is age. We find that older people have significantly lower equity shares.

and pessimists, thus reducing the resulting variation in aggregate wealth-weighted beliefs. In Section III we discussed a number of possible adjustments to the second class of models that might help better fit the data, including taking explicit account of infrequent trading and confidence in beliefs. Whether models with these adjustments can generate large swings in asset prices is an interesting area of future research.

VI Covariation in Discount Rates and Expected Cash Flows

In this section, we explore how beliefs about different objects relate to each other. While our survey separately elicits beliefs about stock market returns and economic growth, these objects are naturally linked in equilibrium. To relate these variables, we make the imperfect but useful abstraction to consider the economy's GDP to be the "dividend" paid to the holders of the stock market, so that we can proxy for beliefs about dividend growth using beliefs about GDP growth. This allows us to apply the Campbell and Shiller (1988) decomposition in our context, which links prices, expected stock market returns, and expected cash-flow growth:

$$pd_{i,t} = \frac{1-\rho^n}{1-\rho}k + E_{i,t}\sum_{j=0}^{n-1}\rho^j(\Delta d_{t+1+j} - r_{t+1+j}) + \rho^n E_{i,t}pd_{t+n}, \tag{8}$$

where pd_t is the logarithm of the price-dividend ratio, Δd_{t+1} is the growth of dividends between times t and $t+1$, and r_{t+1} is the return of the stock market between between times t and $t+1$.[42] This decomposition has to hold individual by individual, so that we use the individual expectation operator $E_{i,t}[.]$ to stress that this reflects individual beliefs about future growth and returns.[43] The current price-dividend ratio is observable; we thus assume that investors agree on its value: $pd_{i,t} = pd_t$. We then map the survey answers to the terms in equation 8 in the following way:

$$Q^i_{3d,t} = E_{i,t}\Delta d_{t+1} = E_{i,t}\Delta d_{t+2} = E_{i,t}\Delta d_{t+3}, = E^i_t\Delta d_S \tag{9}$$

$$Q^i_{r1,t} = E_{i,t}r_{t+1} = E_{i,t}r_S, \tag{10}$$

$$Q^i_{d10,t} = \frac{3}{10}E_{i,t}\Delta d_S + \frac{7}{10}E_{i,t}\Delta d_L, \tag{11}$$

$$Q^i_{r10,t} = \frac{1}{10}E_{i,t}\Delta r_S + \frac{9}{10}E_{i,t}r_L. \tag{12}$$

Equation 9 imposes that the average annualized expected growth rate of GDP over the next three years, i.e., the answer to the question about short-term growth, maps into the expected growth rate of dividends in each of the next three years ($E_{i,t}\Delta d_S$). Equation 10 simply sets up the notation

[42] The parameters ρ and κ are log-linearization constants. For more details, see Campbell and Shiller (1988). Here it suffices to think of ρ as a number close to 1, and κ as a positive constant.

[43] As noted by Campbell (2017), the decomposition does not require that individuals have rational expectations, but only that their expectations do not violate mathematical identities. Indeed, since the formula is an identity, it holds for all possible future realizations. We require that expectations satisfy the mild property that the formula must also then hold in expectation.

for short-term (i.e., one year) expected stock market returns ($E_{i,t}r_S$). Equation 11 maps the survey question about long-run economic growth ($Q^i_{d10,t}$) into long-run dividend growth ($E_{i,t}\Delta d_L$), defined as dividend growth between years 3 and 10. Equation 12 similarly maps the survey question about ten-year stock market returns ($Q^i_{r10,t}$) into long-run stock market returns ($E_{i,t}\Delta r_L$), defined as those between years 1 and 10.[44] For simplicity, we have introduced notation in this section assuming that the time interval between t and $t+1$ is one year. In bringing this decomposition to the data, we maintain the assumption that the horizon of returns and cash-flow growth is expressed in years, even if we measure the decomposition at a two-month frequency.

From the answers to the four survey questions about long-run and short-run economic growth and stock market returns, we therefore obtain estimates of the terms: $E_{i,t}\Delta d_S$, $E_{i,t}\Delta d_L$, $E_{i,t}r_S$, $E_{i,t}\Delta r_L$. We rewrite equation 8 by grouping the terms of interest:

$$pd_t = \frac{1-\rho^{10}}{1-\rho}k + \frac{1-\rho^3}{1-\rho}E_{i,t}\Delta d_S + \frac{\rho^3-\rho^{10}}{1-\rho}E_{i,t}\Delta d_L - E_{i,t}r_S - \frac{\rho-\rho^{10}}{1-\rho}E_{i,t}r_L + \rho^{10}E_{i,t}pd_{t+10}. \tag{13}$$

Since pd_t is observable at each point in time, and having calibrated $\rho = 0.96$, we use this identity to back out the expectation of each individual about the price-dividend ratio ten years into the future, $E_{i,t}pd_{t+10}$.[45] Once we have obtained this "residual" term, we can then interpret this equation as follows. All investors agree on the (observable) price-dividend ratio pd_t. However, they disagree about its determinants. Some investors, for example, might expect higher dividend growth, but to square it with the observed price-dividend ratio they need to also expect either higher returns or a lower price-dividend ratio ten years from now. This identity, therefore, disciplines the relationship between the survey answers given by each individual.[46]

Table X reports the correlations of the variables $E_{i,t}\Delta d_S$, $E_{i,t}\Delta d_L$, $E_{i,t}r_S$, $E_{i,t}\Delta r_L$ and $E_{i,t}pd_{t+10}$. Short-run and long-run expected returns are positively correlated (correlation of 0.21). Panel A of Figure VIII further illustrates this point by presenting conditional binscatter plots showing the relationship between short-run and long-run expectations. Interestingly, even those respondents who expect negative returns over the next year expect long-term returns to be positive. Similarly, short-term and long-term dividend growth are positively correlated (correlation of 0.41), a pattern that is also evidenced by Panel B of Figure VIII. Table X and Panels C and D of Figure VIII also show that expectations of stock market returns and economic growth are positively related at all horizons. In unreported results, we confirm that these correlations also hold when we compare

[44] The Campbell-Shiller decomposition requires that dividend growth and returns are either both nominal or both real. In our survey, however, the stock returns questions are nominal and the GDP growth questions are real. Our decision was driven by a desire to ask about objects that are salient to the respondents. In the general press, stock market returns are almost always reported in nominal terms, while economic growth is reported in real terms. In mapping the answers to this decomposition, we make the assumption that expected inflation is constant, in which case equation 8 still applies with an adjustment to the constant.

[45] We calibrate ρ using the average log price-earning ratio over the period 1881-2018, as provided by Robert Shiller.

[46] Note that the use of this decomposition here is different to the standard time-series context of Campbell and Shiller (1988). In the time series, this decomposition is typically used to attribute the time-variation of the price-dividend ratio to changes in expectations about either cash-flow growth or future returns. The emphasis is on variation over time of aggregate (and usually rational) expectations. Here, we instead use the decomposition in the cross-section of individuals to explore the heterogeneity in the their *joint* beliefs about returns and cash-flow growth.

Table X: Within-Response Correlation Across Answers to Different Survey Questions

Corr/std	$E_{i,t}r_S$	$E_{i,t}r_L$	$E_{i,t}\Delta d_S$	$E_{i,t}\Delta d_L$	$E_{i,t}pd_{t+10}$
$E_{i,t}r_S$	1				
$E_{i,t}r_L$	0.21	1			
$E_{i,t}\Delta d_S$	0.25	0.17	1		
$E_{i,t}\Delta d_L$	0.07	0.23	0.41	1	
$E_{i,t}pd_{t+10}$	0.34	0.76	-0.20	-0.36	1

Note: Table shows the correlation across answers given by the same individual in the same survey wave. $E_{i,t}pd_{t+10}$ is obtained by imposing equation 13 and the observed contemporaneous price-dividend ratio.

beliefs within individuals over time: when individuals become more optimistic about short-run returns, they also become more optimistic about long-run returns. The same holds for short-term and long-term cash flow expectations. In addition, when individuals become more optimistic about returns, they also become more optimistic about cash-flows.

We then use the Campbell-Shiller decomposition to perform a variance decomposition of the cross-sectional disagreement about the various elements of equation 13. Since equation 13 is an identity, the variance of any term can be decomposed into the fractions explained by the covariation with the remaining terms. The contribution of each term to total variation is not required to be positive, but the sum of all individual terms contributions is always 100%.[47]

Table XI: A Behavioral Campbell-Shiller Variance Decomposition

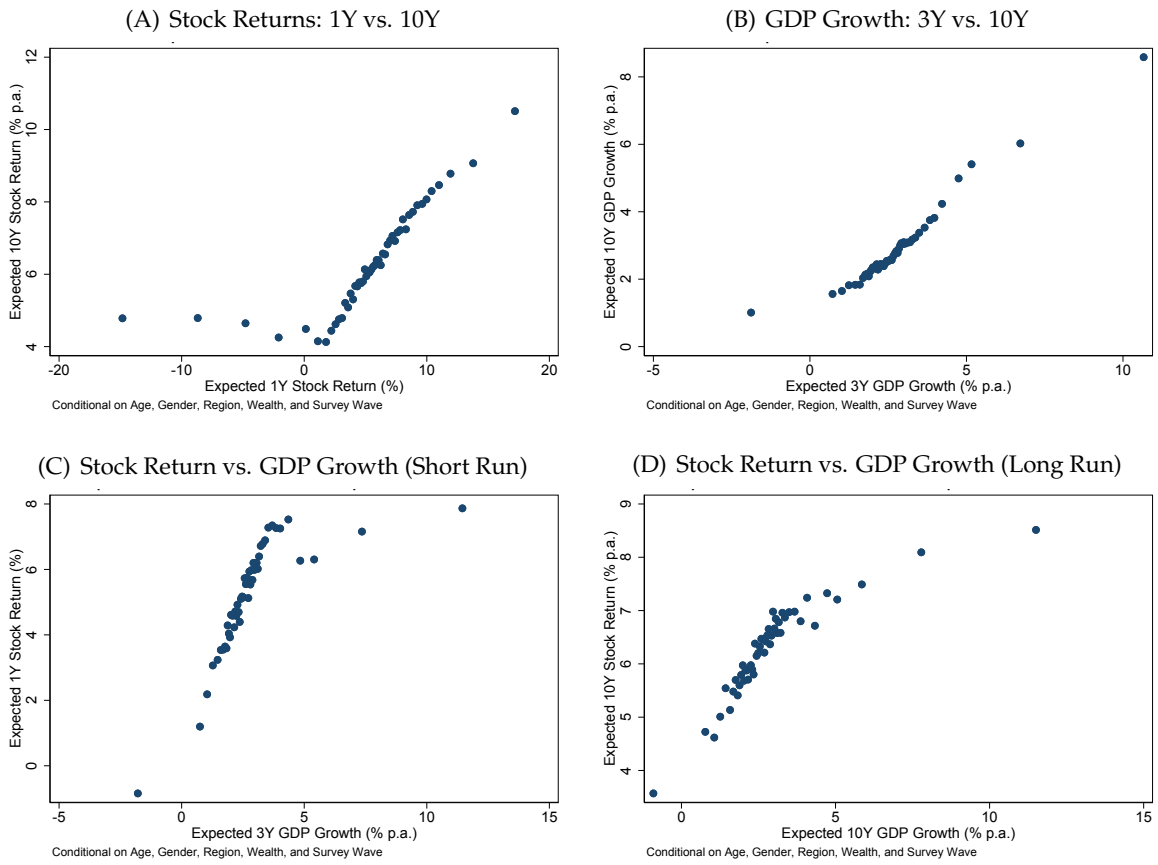
	$-E_{i,t}r_S$	$-\frac{\rho-\rho^{10}}{1-\rho}E_{i,t}r_L$	$\frac{1-\rho^3}{1-\rho}E_{i,t}\Delta d_S$	$\frac{\rho^3-\rho^{10}}{1-\rho}E_{i,t}\Delta d_L$	$\rho^{10}E_{i,t}pd_{t+10}$	$-pd_t$
$-E_{i,t}r_S$	1	-0.06	0.27	0.04	0.08	-0.06
$-\frac{\rho-\rho^{10}}{1-\rho}E_{i,t}r_L$	-0.73	1	0.68	0.43	0.66	-0.03
$\frac{1-\rho^3}{1-\rho}E_{i,t}\Delta d_S$	0.22	0.04	1	-0.19	0.04	0.02
$\frac{\rho^3-\rho^{10}}{1-\rho}E_{i,t}\Delta d_L$	0.14	0.13	-0.90	1	0.17	-0.16
$\rho^{10}E_{i,t}pd_{t+10}$	1.40	0.89	0.93	0.75	1	1.23
$-pd_t$	-0.04	0.00	0.02	-0.02	0.04	1

Note: Variance decomposition of equation 13. In the columns we report the term the variance of which is being decomposed using the variation in each of the terms in the rows.

Table XI reports the variance decomposition of each of the terms, one per column, in terms of all the remaining terms reported in the rows. The interpretation of each column is that positive numbers absorb variation while negative numbers add further variation for the remaining terms to account for. Consider, for example, the first column which reports the decomposition of the

[47] Generally, if $X+Y+Z=0$, then one can write $-X=Y+Z$, and $Var(X)=Cov(-X,-X)=Cov(-X,Y)+Cov(-X,Z)$. Scaling by the variance of X, we have: $\frac{Var(X)}{Var(X)}=\frac{Cov(-X,Y)}{Var(X)}+\frac{Cov(-X,Z)}{Var(X)}=100\%$. The total (100%) variance of X can be decomposed into the sum of two components, a fraction due to the covariation of X with Y, the term $\left(\frac{Cov(-X,Y)}{Var(X)}\right)$, and a fraction due to the covariation of X with Z, the term $\left(\frac{Cov(-X,Z)}{Var(X)}\right)$. The fractions sum up to 100% by definition.

Figure VIII: Relationships Among Different Beliefs Within the Same Survey



Note: Figure shows conditional binned scatterplots across different answers from within the same survey response.

variance of short-run returns $E_{i,t}r_S$. The covariance between short-run expected returns and short-run dividend growth, for example, helps explains 22% of the variance of short-term expected returns. Similarly, the covariance between short-term expected returns and long-term dividend growth helps explain another 14% of the total variance of short-run expected returns. Going back to equation 13, we can interpret this as follows. Investors that have higher expectations of future returns also tend to have higher expectations of dividend growth. This helps match the observed price-dividend ratio, since part of the positive effect (higher current price-dividend ratio) of higher expected future cashflows is offset by discounting these cashflows at higher rates. In contrast, expectations about long-run expected returns explain a *negative* portion of the total variance of short-run expected returns (-73%). Short and long-run expected returns are positively correlated; in the variance decomposition, this means that long-run expected returns add variance to short-run expected returns – holding the current price-dividend ratio fixed. In equation 13, when short-run returns are high, long-run returns are also high, thus creating a larger deviation from the current price-dividend ratio that needs to be accounted for by the remaining terms in the equation. Expected dividend growth in both the short-run and the long-run offsets some of this variation,

but this is not enough to balance the equation (i.e., the sum of the variance contribution of long-run returns and short-run and long-run dividend growth does not sum to 1). The slack is absorbed by the residual term: the expectation of the price-dividend ratio in ten years.

A similar pattern occurs in Column 3 of Table XI, which decomposes the variance of short-run growth expectations. Long-run growth expectations are positively correlated with short-term ones, thus adding to the total variance (-90%). Expected returns at all horizons absorb some of the variation, but not enough to completely offset it, so that the slack is again absorbed by the residual term: the expectation of the price-dividend ratio in ten years. Row 5 of Table XI shows that the residual term accounts for large amounts of the variation in all cases. In part this is due to the fact that measurement error tends to be absorbed by this residual. Despite this large residual, we find interesting economic relations among the observable terms, summarized in Fact 4.

Fact 4. *Both expected returns and expected cashflows contribute to variation in asset prices. Across individuals, higher expected cash flows are partially offset by higher expected returns (i.e., by higher discount rates) in both the short run and the long run. However, variation in expected returns and expected cashflows does not offset, given the current price dividend ratio, and the slack is absorbed by a residual term: the expectation of the price dividend ratio in ten years.*

Finally, Column 6 of Table XI decomposes the variance of the price dividend ratio, which reflects only time-series variation. With only 10 survey waves (spaced two months apart), our time series variation is limited. With that caveat in mind, the results suggest that higher price-dividend ratios are associated with higher expected returns at all horizons. This effect is consistent with the return extrapolation patterns documented by Greenwood and Shleifer (2014), and runs contrary to the implications of rational asset pricing models, which usually explain higher price-dividend ratios with lower expected returns. Similarly, higher price-dividend ratios are associated with lower expected growth (in the long run). This runs contrary to the behavioral finance literature that explains higher price-dividend ratios with higher expected cash flows.[48]

Implications for Macro-Finance Models. The behavioral finance literature usually either models expectations and disagreement about cash-flows or about returns (Cutler, Poterba and Summers, 1990; De Long et al., 1990*a*,*b*; Barberis et al., 2015). Our research shows that both dimensions are important to understand the data and that, at least for quantitative evaluations, their dynamics have to be modeled jointly. Across individuals, we find that higher expected cash flows come together with higher expected returns, thus in part mitigating the effect on current prices. However, the effect is not strong enough, and much of the variation remains unexplained and is absorbed in our setting by the expectation of the price-dividend ratio ten years into the future. Whereas the standard time-series applications of the Campbell and Shiller (1988) decomposition is informative about the sources of variation of the price dividend ratio over time, our application of this decomposition to the cross-section characterizes the heterogeneity across-individuals in their joint

[48] De la O and Myers (2017) have applied this decomposition to survey data in a time-series context and found an important role for expected short-run growth in cash-flows.

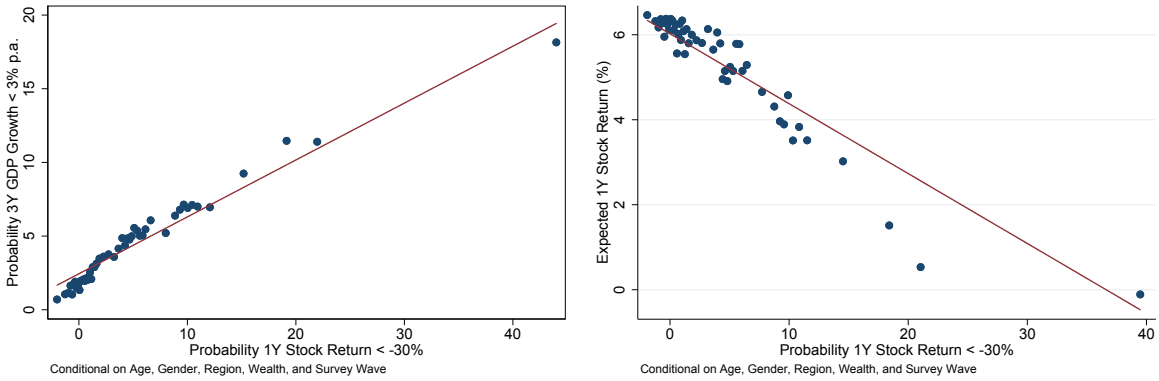
beliefs about cash flows and returns. It therefore guides macro-finance models by quantifying the relative amount of cross-sectional disagreement about these crucial processes.[49]

VII A Behavioral Approach to Rare Disasters

In the previous sections, we focused on those moments of the belief distribution that have been of most central interest to the asset pricing literature, such as the mean and variance of expected returns and growth rates. However, an important strand of this literature has emphasized that expectations of rare but potentially catastrophic negative events, sometimes called rare disasters, can help explain portfolio holdings and asset prices (Rietz, 1988; Barro, 2006; Gabaix, 2012). This literature has stressed that the expected probability and size of rare disasters are some of the most important moments for understanding risks and returns. In this section, we explore a number of features of these disaster probabilities, and discuss their implications for the theoretical literature.

Indeed, our survey was specifically designed to elicit the expectations of disaster probabilities for both stock returns (i.e., 1-year stock returns of less then -30%) and GDP growth (i.e., annualized 3-year GDP growth of less then -3%). Section III.B showed the importance of subjective disaster probabilities for portfolio formation: holding fixed the mean, respondents with a higher perceived probability of stock market disasters also had lower equity shares (see Column 2 of Table V).

Figure IX: Stock Disasters, GDP Disasters, and Expected Returns



Note: The left panel shows a conditional binned scatter plot of survey respondents' subjective probability that the 1-year stock returns are below 30% and their expectation that annualized average GDP growth over the next three years is below -3%. The right panel shows a conditional binned scatter plot of survey respondents' subjective probability that the 1-year stock returns are below 30% and their expected 1-year stock returns. Both panels condition on the respondents' age, gender, region, wealth, and the survey wave.

We first explore the relationship between individuals' expectations of the probabilities of stock market disasters and GDP disasters. The left panel of Figure IX shows that expectations of the two types of disasters are positively related at the individual level (the slope of the regression line is

[49] For example, some behavioral models like Jin and Sui (2018) imply, consistent with our results, that investors with high cash-flow growth expectations also expect high returns. Our analysis estimates the magnitude of these relationship, and therefore produces a new empirical moment that can be used inform the calibration of these models.

0.38); in unreported results, we find that this is also true within individuals and over time. These findings suggest that expectations of rare stock market disasters come with expectations of lower cash flows and are not just purely the result of expecting higher future returns (i.e., stock market disasters are not purely due to discount rate variation).

Table XII: Expected Stock Returns and Rare Disasters Beliefs

	Expected 1Y Stock Return (%)					
	(1)	(2)	(3)	(4)	(5)	(6)
Probability 1Y Stock Return < -30%	-0.165***		-0.255***		-0.076***	-0.100***
	(0.010)		(0.018)		(0.015)	(0.026)
Probability 1Y Stock Return < -10%		-0.131***				
		(0.004)				
Probability 1Y Stock Return < -30% x Low Bucket First				-0.155***		
				(0.012)		
Probability 1Y Stock Return < -30% x High Bucket First				-0.180***		
				(0.016)		
Controls + Fixed Effects	Y	Y	Y	Y	Y	Y
Individual Fixed Effects					Y	Y
Specification			Prob ∈ [0.1%,10%]			Prob ∈ [0.1%,10%]
R-Squared	0.069	0.192	0.036	0.527	0.810	0.845
N	18,049	18,049	10,396	18,049	18,057	9,606

Note: Table shows results from regression 14. The unit of observation is a survey response, the dependent variable is the expected one year stock return. All columns control for the respondents' age, gender, region of residence, wealth, and the survey wave. Columns 5-6 include individual fixed effects, wave fixed effects, and a dummy for the randomization order of the buckets in the distribution question. Columns 3 and 6 restrict the sample to individuals who report expected probabilities of a stock market disaster between 0.1% and 10%. Standard errors are clustered at the respondent level. Significance levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

We next analyze the relationship between expected returns and expected disaster probabilities. The right panel of Figure IX shows that individuals with a higher subjective probability of stock-market rare disasters have lower expected stock-market returns. To explore this relationship more formally, we run the following regression:

$$E_{i,t}[R_{1y}] = \alpha + \beta \, \text{Prob}_{i,t}[R_{1y} < -30\%] + \gamma X_{i,t} + \psi_t + \epsilon_{i,t}, \tag{14}$$

where the coefficient of interest is β. We additional control for demographic characteristics such as age, gender, wealth, and region of residence, as well as survey-wave fixed effects. The negative relationship between expected return and disaster probability is extremely robust. Column 1 of Table XII is equivalent to the regression line in the right panel of Figure IX. The estimate of β implies that a 5 percentage point increase in individuals' subjective probability of a stock market disaster reduces their expected return by 0.8 percentage points. Column 2 shows that a similar negative relationship occurs when we consider the probability of less extreme outcomes, returns below -10%. Column 3 restricts the data to those answers that report a probability of a stock market return less than -30% to be between 0.1% and 10%. Consistent with the FRAME effect discussed in earlier sections, we find that excluding very extreme responses increases the sensi-

tivity from -0.16 to -0.25. Column 4 shows that the results are not meaningfully affected by the order in which the buckets are presented to the respondent in the distribution question (high-to-low vs. low-to-high). Column 5 includes individual fixed effect, and Column 6 does the same but restricts the probabilities to be in the same range as those in Column 3. We collect the findings in this section in the following Fact.

Fact 5. *Higher expectations of stock market disasters are associted with lower expected stock market returns and higher expected probability of GDP growth disasters. Conditional on expected stock market returns, higher expectations of stock market disasters are associated with lower equity shares.*

Connection to Rare Disaster Models. In equilibrium models with rare disasters, expected returns and the probability of disasters are positively related. The intuition is that a higher probability of disaster induces individuals to demand a higher compensation for holding the stock market and this makes equilibrium expected returns higher. The closest mapping between our empirical results and theoretical models is between Column 5 of Table XII and the time-varying rare-disaster framework of Gabaix (2012). In that framework, a representative agent with rational expectations prices assets in an economy affected by time-varying rare disasters.[50] When the representative agent expects a higher probability of a disaster she also expects returns to be higher. The relationship in the data appears with an opposite sign relative to the theory. This poses a challenge to rational expectations models of rare disasters.

However, we do not think that our results are inconsistent with the importance of rare disasters for asset pricing more generally. In fact, it is likely that modifications to existing models that incorporate at least some behavioral agents can more closely align model predictions and data (see, for example, Jin, 2015). One possible approach would be to model some agents as overconfident or as thinking in partial equilibrium. When these agents believe that rare disasters have become more likely, they do not take into account that other agents might think the same, and therefore do not incorporate the fact that stock prices might already be lower to account for the increased risk. Instead, they think that prices are still high, and expect returns to be lower going forward. They therefore sell stocks to reduce their exposure by more than they would if they anticipated that the compensation for holding this risk has increased in equilibrium. In turn, this force may induce more selling pressure (and therefore a bigger price effect) in response to an increase in the disaster probability. Of course, there might also be rational agents who take the opposite side of these trades; these rational agents require higher returns to compensate for the risk and to absorb the stocks that the behavioral agents are selling. It is beyond the scope of this paper to provide a full model of the behavioral approach to rare disasters, but we expect that it will prove an interesting area of future research.

[50] The model jointly specifies the dynamics of the probability and size of disaster, which we map for simplicity into pure variation in probability. Comparing our estimates with the model of Barro (2006) is more indirect since, in that model, the disaster probability and expected (rational) returns are both constant over time. A rough comparison can be obtained by studying comparative statics of the model: again, a higher probability of disaster corresponds to a higher expected return.

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