Economics of Proof-of-Stake Payment Systems

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First draft: November 2018

Latest draft: January 2019

Abstract

In this paper we develop a valuation framework for a Proof-of-Stake (PoS) payment system. Active network participants (proposers and validators) are required to stake tokens, and receive payments in return for their services. This connects cash flows to token holdings, and allow for valuation using conventional methods. As an application of our framework, we analyse security properties of the PoS system. We show that while high token valuation relative to the flow of transactions is central to network security, valuation bubbles have adverse security implications.

1 Introduction

Recent years have sparked significant interest in blockchain-powered payment systems, or cryptocurrencies. In theory, cryptocurrencies offer a number of advantages over traditional payment systems, including reduced fees, improved security, and a more robust trust model. However, despite their great promise, cryptocurrencies have gained limited traction as an alternative to traditional payment systems. There are many reasons for this, including one fundamental, technical drawback. Namely, the consensus mechanism used in Bitcoin and related cryptocurrencies is poorly-suited to high-rate applications like payments processing.

To understand this claim, notice that a blockchain is simply an immutable, sequential data structure that is maintained by a distributed set of nodes. At a high level, the core consensus problem in blockchains is the following: given a blockchain data structure, what should be the next block in the blockchain? In the context of cryptocurrencies, a "block"

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is simply a list of transactions; hence, the blockchain is simply a ledger of transactions. *Nakamoto consensus*, first proposed in the original Bitcoin paper Nakamoto (2008), solves this problem using a technique called proof-of-work (PoW). In PoW, so-called *miners* race to solve a computationally-intensive puzzle. The first miner to solve the puzzle (and provide proof of that solution) is allowed to choose the next block in the blockchain. The act of producing a new block and appending it to the chain is called *block proposal*, and comes with associated rewards for the miner—typically a combination of transaction fees and a freshly-minted tokens. A key property of PoW computational puzzles is their difficulty, which must be high to protect the security of the blockchain.[1] Hence, PoW exhibits an unfavorable tradeoff between security and computational complexity Bagaria, Kannan, Tse, Fanti, and Viswanath (2018); indeed, Bitcoin alone is estimated to consume more energy than some developed nations Lee (2017). Moreover, common PoW designs like Nakamoto consensus are fundamentally limited in terms of the throughput and latency figures they can support Bagaria et al. (2018). Because of these drawbacks, PoW cryptocurrencies have struggled to support the volume of traffic flowing in industrial-grade payment systems.

The drawbacks of PoW are well-documented. To address them, an alternative called *proof-of-stake* (PoS) has been proposed. In PoS systems, the next block proposer is selected randomly from the set of all users, with probability proportional to the fraction of total stake (tokens) held by each user. In principle, this selection process can be significantly more efficient than PoW mining. However, PoS is just an access control mechanism—on its own, it does not specify how to achieve consensus. For example, it is unclear how to securely implement PoS proposer election in the first place, as well as how to use the results of that procedure. Many open questions remain surrounding the development of PoS cryptocurrencies, particularly regarding their security and economic properties.

In this paper, we analyze the economic properties of PoS payment systems. While analogous studies have been conducted on PoW cryptocurrencies, we find that PoS systems in particular exhibit traits that lend themselves naturally to analysis using traditional valuation tools. In particular, because individual token holdings in the PoS stake systems determine the allocation of rewards within the system, as we discuss in our PoS primer below, to-

[1]For example, in Bitcoin, the entire network solves one puzzle every 10 minutes on average.

kens effectively confer cash flow rights on their holders – they can therefore be valued using standard valuation methods.

We propose a dynamic equilibrium model for quantifying the *value* of a PoS payment system as a function of system parameters like transaction volume, token velocity, and token supply schedule. In PoW cryptocurrencies, token valuation is important because it gives investors and regulators a way to reason about the worth of the system. While this is also true of PoS cryptocurrencies, token valuation plays an additional role in PoS: it gives a quantitative measure of the *security* of the system. Because of the way PoS systems are architected, higher token values imply better security. Our model therefore depends critically on PoS-specific implementation choices, and allows us to approximate the effects of changing various parameters in the PoS consensus protocol.

We analyze the effect of valuation bubbles on network security within our valuation framework. This analysis highlights some important connections between network design features and properties of the network as a financial system, suggesting directions for further investigation.

The ideas proposed in this paper differ from comparable analyses for PoW systems in a few key respects. First, the primary cost in PoW systems is the physical cost of running a mining rig: electricity, hardware, and cooling, for instance. Rewards in PoW systems are designed to compensate for these costs. In contrast, the physical costs of PoS systems are relatively low. The primary expense for PoS participants is the fact that they must deposit tokens as collateral in order to reap rewards; we discuss this point in detail in Section 3. This observation allows us to tie PoS cryptocurrency valuations to fiat quantities through the opportunity cost of holding tokens. Such a connection has not been possible in prior PoW valuations.

Another key difference is the fact that in PoS systems, it is (relatively) easy for users to shift resource allocations. A user can choose to use her tokens to participate in the consensus mechanism one day, and then use those same tokens to purchase goods the next. This is not the case in PoW: a miner that invests its resources in mining rigs cannot easily convert those mining rigs into spendable Bitcoins, and vice versa. Because of such low frictions, PoS payment systems are amenable to traditional arbitrage-based valuation techniques. At

the same time, the same easy of reallocation may have undesirable implications for network security, as our analysis of valuation bubbles indicates.

Because of the differences between the PoS and PoW systems, the valuation framework presented here is quite different from prior work on valuing PoW cryptocurrencies. We expect our framework to provide a useful starting point for developing more sophisticated models based on specific consensus mechanisms or protocols.

1.1 Outline

We begin this paper by describing prior work analyzing the economics of cryptocurrencies, and highlight why this analysis does not apply to PoS systems. In Section 3, we explain how PoS cryptocurrencies work, in order to motivate our modeling of the system. In Section 4 we introduce our equilibrium valuation model, followed by a stochastic version in Section 4.2. Finally, we discuss security implications in Section 5.1.

2 Related Work

Our paper is most closely related to the small but growing literature on crypto-currency valuation. Existing papers are primarily concerned with valuation in the Proof-of-Work setting, motivated by the Bitcoin design.

Cong, Li, and Wang (2018) develop a dynamic model of cryptocurrency adoption and valuation. Their paper postulates that cryptocurrency yields a flow of benefits to the users, which increases with the degree of currency adoption. This setting is more suitable for describing the dynamics of token adoption, with token valuation described in a relatively reduced form. Related to this, are the studies by Pagnotta and Buraschi (2018) and Athey, Parashkevov, Sarukkai, and Xia (2016). The latter emphasize the effect of learning about eventual success of the cryptocurrency on its price volatility, while the former focuses on the link between network security and valuation, and the negative feedback from loss of security on token valuation, which may in turn lead to multiple equilibria. Sockin and Xiong (2018) also consider the possibility of multiple equilibria in a static model of utility tokens, which allow their holders to participate and benefit from membership in a digital platform. This paper is one of a few that consider the Proof-of-Stake consensus protocol, in addition to the

more common analysis under the Proof-of-Work setting.

In our analysis we do not explicitly incorporate the feedback from network security to token values, and focus on the reverse direction: implications of valuation dynamics for network security. What distinguishes our paper from the prior work is that we establish and explicit connection between transactions on the network, and associated transaction fees, and token values – our valuation model is conventional and based on absence of arbitrage. Our results do not hinge on assumed "convenience yield" (benefits) of holding tokens, or on assumptions about token velocity.

Liu and Tsyvinski (2018) is an empirical study of the risk-return properties of cryptocurrencies. Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2018) is another related study, estimating risk-return restrictions on historical Bitcoin prices. Our analysis highlights that the opportunity cost of holding tokens is central to token valuation.

3 Proof-of-Stake Primer

This section gives a brief primer on common design choices in proof-of-stake (PoS) blockchains. These design choices will inform our models in Section 4. Recall that the central question in blockchains is how to determine (and agree upon) the next block in the blockchain. As such, most existing PoS architectures have three key architectural components:

1. **Proposer election**: One or more nodes are selected to coordinate the protocol for block production.

2. **Block proposal**: The proposers (nodes elected in the previous phase) form new blocks, add them to the blockchain data structure, and broadcast them to the rest of the network.

3. **Block finalization**: This step is not yet standard in many cryptocurrencies, but it consists of a mechanism for periodically "finalizing" the blockchain up to a certain point. Once a block is finalized, it and all prior blocks cannot be reverted without the nodes responsible for the change incurring significant penalties. The finalization procedure is run by a (possibly separate) set of nodes called *validators*.

It is useful to understand common design choices for each of these architectural components in order to create meaningful economic models.

3.1 Proposer election

Most blockchain protocols work by specifying a designated node or set of nodes that are responsible for block generation. The process of choosing those nodes is sometimes called proposer election, and is generally executed via a distributed protocol. For example, in Bitcoin, that distributed protocol is proof-of-work.

In PoS systems, a common approach to proposer election relies on verifiable random functions (VRFs). This approach is used in most PoS cryptocurrencies, including Qtum, Peercoin, BlackCoin, Particl, Cardano, and Algorand (Chen and Micali (2016)).[2] The idea is that each node will locally execute a pre-defined function that takes only public inputs, such as the blockchain contents, the node's own public key, and the current time. If this function evaluates to 1, the node is a proposer; otherwise, it is not. For example, consider the PoSv3 protocol, which is used in Particl, Peercoin, BlackCoin, and Qtum. Every time interval (e.g., 16 seconds), each node computes the following:[3]

$$\text{Hash (BlockchainContents || Coin || CurrentTime)} \overset{?}{\leq} \text{DifficultyParameter} \qquad (1)$$

where Hash$(\cdot)$ denotes a hash function, 'BlockchainContents' is a condensed representation of the blockchain contents, 'Coin' denotes a user's token ID, 'CurrentTime' the current quantized timestamp, and 'DifficultyParameter' a system-wide parameter that ensures the number of proposers is not too high. We are omitting many details from this expression, but notice that the user can compute this function for each coin it possesses, with an equal probability of succeeding each time. Hence its probability of being elected proposer on any of these coins is proportional to the number of coins the user owns.

This VRF-based approach has the advantage of being fast, distributed, and random. Note that each node can check its own status as a proposer without consulting any other nodes. In practice, more than one node may be elected proposer due to randomness in the

[2]See Qtum (https://qtum.org/), Peercoin (https://peercoin.net/), BlackCoin (https://blackcoin.org/), Particl (https://particl.io/), and Cardano (https://www.cardano.org/en/home/) for technical background.

[3]This expression has been simplified for clarity.

protocol, even if the expected number of proposers per time slot is less than one. A common, critical property of many VRF proposer election mechanisms is that they require input coins to be "old enough". For example, (1) could be modified such that only coins that were last spent at least m blocks ago can be used as valid inputs. From the perspective of our economic models, this requires users to hold tokens for m blocks before they can participate in proposal. In this sense, users must decide if they want to hold their tokens (to be eligible to be a proposer) or spend them. This decision will be made explicit in our model in Section 4.

3.2 Block proposal

The purpose of block proposal is to provide a suggested ordering of transactions that can be accepted or rejected by the rest of the network. For efficiency reasons, this is typically done by the elected proposers (see Section 3.1). There are many variants of block proposal, but most commonly, for a given block, each proposer at a given time slot assembles the most recent transactions into a block, and then shares its block with the rest of the network for approval. Once a proposer has been elected, the main uncertainty in the system is regarding where to append the block. In some blockchains like Algorand, there is no forking; in other words, the likelihood of the system producing two or more concurrently-proposed blocks (for instance) is chosen to be arbitrarily low Chen and Micali (2016); Gilad, Hemo, Micali, Vlachos, and Zeldovich (2017). Hence the block proposer always perceives the blockchain as a single chain, and appends its block to the end of that global blockchain. In chain-based protocols, such as those using PoSv3 (e.g, Qtum, Particl), forking is much more common; this happens both because of network delays (a proposer may not know about the most recent block) and the fact that multiple proposers may be elected at the same time. In chain-based protocols, proposers typically append their block to the longest chain known to the proposer at the time.

For our purposes, the details of block proposal matter primarily because of how they affect the incentives in the system. Proposers are rewarded for creating blocks that end up in the final longest chain. These rewards manifest in two ways: block rewards and transaction fees. **Block rewards** refer to a predetermined reward given to the proposer of each block

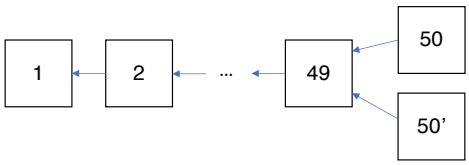


Figure 1: Example blockchain evolution. In a finalization protocol that finalizes every 50 blocks, the validators must choose between block 50 and 50'.

in the longest chain. In most cryptocurrencies, this block reward starts out large and decays (often to zero) over time.

The second class of rewards is **transaction fees**. Transaction fees are (typically) paid by the sender of a transaction in addition to the desired transfer amount. The transaction fee is generally paid to the proposer of the block in the main (longest) chain that includes the transaction. However, these fees can also be split between the proposer of a block and a set of *validators* that finalize transactions, as described in Section 3.3. Regardless, the higher the transaction fee of a particular transaction, the higher the reward reaped by the proposer of the block including that transaction (and possibly block validators who finalize it). In practice, many cryptocurrencies allow users to choose their own transaction fees, above a certain minimum. Another option is to set transaction fees algorithmically, e.g., by correlating them with the amount of the underlying transaction. In this work, we will make assumptions only on the *aggregate* behavior of transaction fees as a function of the total value of transactions processed in the system. These assumptions are discussed in Section 4.

3.3 Finalization

Some cryptocurrencies include (or are starting to include) a procedure called *finalization*. Although finalization is not yet widespread, it is being considered for adoption in a few major cryptocurrencies. Ethereum, for instance, has proposed a finalization mechanism in its roadmap Buterin and Griffith (2017), and major cryptocurrencies like Ripple and Stellar provide similar finality guarantees Mazieres (2015). Since Ethereum's finalization procedure is concrete, we describe it here. Finalization relies on a set of nodes that deposit stake in order to be recognized as *validators*. As long as their stake is locked up, validators can participate in the finalization procedure. The finalization procedure consists of periodically

(e.g., every 50 blocks) running a voting protocol among the validators. The purpose is to choose one chain from the set of candidate chains that have appeared in the system. For instance, suppose the system forms a perfect blockchain until block 49, at which point two proposers independently append blocks to the same parent block. Then we have a blocktree that looks like Figure 1. The job of the validators is to choose one of the forks, and seal that choice for posterity. If the validators choose block 50', for example, then subsequent blocks will build on 50', not 50. The mechanics of the voting procedure are not important for this work; the key point is that validators receive a portion of transaction fees as a reward for participating in the finalization procedure. Hence, they are being rewarded for locking up (depositing) their stake. The longer they remain validators, the longer they reap these rewards.

3.4 Slashing

Finalization procedures require validators to deposit stake for security reasons; the stake is used as collateral in case a validator misbehaves. At a high level, 'misbehavior' is behavior that can result in one of two outcomes: (1) a finalized block getting un-finalized, or (2) conflicting blocks getting finalized. If a validator misbehaves by, say, trying to finalize two blocks at the same height, the validator's deposit will be *slashed*, or confiscated. In Ethereum's finalization protocol, Casper the Friendly Finality Gadget (FFG), 4% of the slashed funds are given to the node that reports the misbehavior, and the remaining 96% is burned so that nobody can spend it Ethereum. Casper FFG imposes a lower limit on the deposit to become a validator; at the time of writing, this minimum was 1500 ETH, close to $190,000. Hence, any misbehavior would result in the validator losing a substantial sum of money. Our security analysis in Section 5.1 extends the baseline model to include the effects of slashing. Notice that although slashing is described in the context of finalization, the concept can also be used to protect against other kinds of misbehavior. For example, one could require proposers to deposit stake, and slash proposers who propose multiple blocks at once.

A natural question is how to enforce slashing in blockchains, since they are inherently distributed. A key challenge is that nodes must first come to consensus on the fact that a

particular validator misbehaved. Casper FFG manages enforcement by having all actions recorded in transactions: a node signals its intent to join the validator pool by sending a special deposit transaction to the network, and all of its votes for blocks are also recorded in transactions (votes can be viewed as endorsements). All transactions are signed with the validator's private key, so if an observer notices two transactions from a validator voting for conflicting blocks, the observer can prove to the rest of the network that the validator was misbehaving. The observer can then submit its own transaction with a link to the two offending transactions to claim its 4% finder's fee. Since all valid transactions are included in the blockchain, they eventually get confirmed through a combination of the block proposal mechanism and/or the finalization procedure. This highlights a very important point: because we require transaction confirmation to enforce good behavior in finalization, the blockchain formation mechanism must itself have strong security properties. In other words, it is not enough to rely on finalization alone for transaction confirmation: block proposal and finalization each improve and strengthen the security guarantees of the other. A principled economic analysis of the interplay of incentives between these systems has not been studied (to the best of our knowledge), and is an interesting direction for future work.

4 Valuation Model

In this section, we present an economic model of PoS cryptocurrencies. The main economic implication of the PoS designs discussed in Section 3 is that network participants who serve as block proposers or validators can earn rewards by committing their tokens to such activities. For example, proposers must keep their tokens unspent for some period of time (Section 3.2), and validators must deposit their stake to participate in finalization (Section 3.3). In our model, for the sake of simplicity, we do not distinguish between the two functionalities; we refer to both simply as 'validation'. Thus, as long as tokens are used for validation, each token generates a stream of cash flows for its holder in the form of additional tokens. This simplified model does not necessarily represent more complex consensus structures; however, we use it as a starting point for more nuanced models of PoS consensus.

A key property of PoS cryptocurrencies is that tokens can be re-allocated across various uses: the same token can be used for validation, for consumer and merchant transactions,

and for off-chain routing of payments. Our analysis assumes, as an approximation, that tokens can be re-allocated in a frictionless manner. If a specific design deviates from this idealized frictionless model, e.g., due to constraints imposed on movements of tokens within the system, one would need to enrich the model with an explicit description of such frictions.

In a frictionless system, individual optimization by the network participants implies that expected rates of return on alternative feasible uses of the tokens must be equalized. In particular, any holder of a token may exchange it for fiat currency and invest the proceeds in a portfolio of financial assets with a return risk profile similar to that of the tokens. Thus, in a system where tokens can be easily exchanged without frictions for fiat currency, the rate of return on holding tokens must equal the opportunity cost: the expected rate of return on the risk-matched investment strategy in financial markets.

4.1 A stationary model with fee-based rewards

We first outline a parsimonious model in which rewards for validation activity are paid entirely in transaction fees generated by retail transactions rather than in newly minted tokens. This model abstracts away from many empirically-relevant details to focus on the main mechanism of token valuation.

Consider a payment network. Suppose that in each period, the network processes retail transactions in the amount of Y_t, measured in the fiat currency, "dollars." Assume that the aggregate float of tokens is fixed, and without loss of generality equal to one. Let p_t denote the equilibrium price of the tokens in terms of the fiat currency. Assume that the network is in a stationary growth regime: the volume of transactions is expected to grow at a constant rate g_Y, so that $\mathbb{E}_t[Y_{t+s}] = Y_t \exp(g_Y s)$.

In this model we do not distinguish between block proposal and block validation functions, and refer to block proposers and block validators simply as "validators." We assume that the protocol for transactions is such that, whenever transactions take place, some tokens must be transferred to the validators as rewards for their activities, and, importantly, individual validators collect their pro-rata share of the total rewards, in proportion to their token stake. The latter assumption rules out individual strategies like selfish mining (e.g., Eyal and Sirer (2018)), which may distort the allocation of rewards among validators in relation

to their individual token balances. We assume that the aggregate amount of fees generated by the network grows at the same long-run rate as the aggregate transaction volume (the ratio of the two series is a stationary process). To simplify the derivations in our model, we further restrict aggregate fees to be a constant multiple of the transaction volume – thus, validators receive fees in the collective amount of cY_t dollars per period (actual rewards are in tokens, which validators sell to consumers in exchange for dollars). We should note that this assumption applies to the total volume of fees, rather than the fee structure for individual transactions. Our assumption of aggregate fee dynamics is consistent with multiple alternative fee schedules, and, in essence, states that the total volume of fees is co-integrated with the total volume of transactions. Importantly, we do not assume that fees for individual transactions are set as a fixed fraction of the transaction value. Equilibrium fee structure depends closely on the design details of the payment systems (e.g., on the protocol for block formation and fee distribution among various actors).[4] How fees should be determined in a payment systems, and related implications for economic efficiency of the system are important questions for future research and beyond the scope of this paper. The valuation framework we develop here, however, should be useful in addressing such questions.

We assume that validators have unrestricted access to financial markets and behave competitively: they take market prices, and, importantly, *the design of the payment network*, as exogenous and not affected by their individual actions. Also, we assume that the risk premium associated with a financial claim paying Y_t dollars per period is constant and equal to λ_Y (in equilibrium, this determines the opportunity cost of capital associated with validation activity). Finally, we also assume that there are no physical costs associated with block validation activities.

We look for a stationary equilibrium, in which $\phi \in [0, 1]$ tokens are held by the validators, and $1 - \phi$ are held by consumers for transaction purposes. We assume that validators have no use for tokens outside of their validation activity, and therefore stake their entire token balance. The equilibrium distribution of token holdings, ϕ, is endogenous and determined as a part of the solution.

[4]See Lewenberg, Sompolinsky, and Zohar (2015), Pass and Shi (2017), Kiayias, Russell, David, and Oliynykov (2017), and Huberman, Leshno, and Moallemi (2017) for analysis of fee mechanisms and alternative design suggestions.

In equilibrium, the total market value of all the tokens held by the validators is ϕp_t, which is the value of a financial claim on the perpetual stream of cash flows in the amount of cY_t per period. Assuming no valuation bubbles, the market value of this cash flow stream is given by the valuation formula for a perpetuity with constant growth:

$$p_t\phi = \lim_{T\to\infty} \int_0^T c\mathbb{E}_t[\exp(-\lambda_Y s)Y_{t+s}]\, ds = \frac{cY_t}{\lambda_Y - g_Y}. \tag{2}$$

To pin down the value of the tokens, we need to make an assumption about consumer's demand for tokens. In a market with infinite token velocity, consumers would hold no balances, which would imply that in equilibrium $\phi = 1$ and $p_t = (cY_t)/(\lambda_Y - g_Y)$. More generally, consumers hold balances of tokens for liquidity reasons. We assume that consumer balances are proportional (with a constant coefficient k) to the rate of transactions. Then,

$$p_t(1-\phi) = kY_t \tag{3}$$

Combining (2) and (3), we find the equilibrium token value:

$$p_t = \left(k + \frac{c}{\lambda_Y - g_Y}\right) Y_t. \tag{4}$$

Note that, because consumer demand for token balances is non-negative, $(cY_t)/(\lambda_Y - g_Y)$ serves as a lower bound on the token value.

The equilibrium value of the tokens in (4) consists of two terms. The first term, kY_t, reflects consumer demand for holding token balances. The value of this term depends on k, which is inversely related to the equilibrium token velocity. It is important to acknowledge that token velocity is an equilibrium outcome, related to a number of properties of the payment network and the broader market. Wider adoption of the tokens could raise demand for token balances, while efficient channels for transactions between tokens and fiat currency would enable consumers to support the desired volume of transactions with lower token balances and result in higher token velocity.

The relation between token value and token velocity is commonly invoked when discussing the valuation of cryptocurrencies in PoW systems. While token velocity and equilibrium token value are certainly related in equilibrium, the relation between the two, like its analog

in traditional monetary economics, is not a true structural relation and it does not provide a reliable anchor for token valuation.

The second term in the valuation equation (4), $cY_t/(\lambda_Y - g_Y)$, reflects the demand for tokens from validators. This term is proportional to the overall volume of retail transactions, Y_t, and to the rate at which fees are charged for transactions. All else equal, broader adoption and utilization of the payment system (higher Y_t) result in higher value of the tokens. Importantly, the above equation does not suggest that token value is increasing in the level of fees. Our analysis here focuses on a single stationary equilibrium and does not explicitly describe how the system responds to changes in parameters: higher fee levels would eventually lead to lower transaction volume.[5]

Our analysis in this section also relies critically on the assumption of competitive behavior by the network participants. To what extent this assumption offers a good approximation of agent behavior in this environment depends on individual opportunities and incentives to engage in strategic behavior. Ultimately, individual incentives and token valuation are closely linked in PoS systems, and must be analyzed jointly. This represents an important direction for future research.

4.2 An extension with increasing token supply

We now extend our valuation model to allow for increasing token supply and transitory dynamics. Specifically, we allow validators to be rewarded in newly minted tokens in addition to the fees collected from consumer transactions.

We now allow the transaction volume growth parameter to vary in time, and denote it by $g_Y(t)$. We also explicitly describe randomness in the evolution of transaction volume Y_t:

$$\frac{dY_t}{Y_t} = g_Y(t)\,dt + \sigma(t)\,dZ_t, \tag{5}$$

where $g_Y(t)$ and $\sigma(t)$ are bounded continuous functions of time, and Z_t is a Brownian motion.[6] Here Z_t is a simple model for the randomness in the system, and $\sigma(t)$ represents the

[5]The effect of transaction fees on token value is analogous to the concept of the Laffer Curve in the theory of taxation.

[6]We implicitly assume that all elements of the model, e.g., the growth rate of Y_t, are properly restricted so that tokens have a finite value in equilibrium.

instantaneous volatility of growth in transaction volume. Investors require compensation for being exposed to dZ_t shocks based on the comparable investment opportunities in financial markets – we denote the market price of risk of dZ_t shocks by η. For simplicity, we assume that η is constant. Then, under the risk-neutral valuation measure Q, the transaction volume follows

$$\frac{dY_t}{Y_t} = g_Y^Q(t)\,dt + \sigma(t)\,dZ_t^Q, \tag{6}$$

where

$$g_Y^Q(t) = g_Y(t) - \eta\sigma(t), \tag{7}$$

and Z_t^Q is a Brownian motion under measure Q.

As above, p_t denotes the total value of tokens at time t and validators hold fraction ϕ_t of all tokens. The market clearing condition requires that validators and consumers collectively hold all tokens, and thus

$$\phi_t p_t + kY_t = p_t, \tag{8}$$

where k is a consumer demand parameter, which we again assume to be constant. Thus,

$$\phi_t = 1 - k\frac{Y_t}{p_t}. \tag{9}$$

We assume that token supply grows deterministically over time. Specifically, we assume that new tokens are issued at a bounded rate $g_S(t)$. In expectation under the risk-neutral valuation measure, validators earn the risk-free rate of return r_f, which we assume to be constant. Thus, we obtain the valuation equation:

$$\phi_t p_t = cY_t\,dt + e^{-r_f dt}E_t^Q\big[(\phi_t + (1-\phi_t)g_S(t)\,dt)p_{t+dt}\big]. \tag{10}$$

On the left-hand side, $\phi_t p_t$ is the total market value of the tokens staked by the validators at time t. On the right-hand side, we have two terms. The first term, $cY_t\,dt$, is the flow of transaction fees that accrue to the validators over the infinitesimal period $[t, t+dt)$. The second term, $e^{-r_f dt}E_t^Q\big[(\phi_t + (1-\phi_t)g_S(t)\,dt)p_{t+dt}\big]$, is the discounted expected value (under

measure Q)) of the validators' token holdings at the end of the period.[7] Then,

$$E_t^Q\big[dp_t\big] = r_f p_t\, dt - \frac{p_t}{p_t - kY_t}\left(cY_t + kY_t g_S(t)\right)\, dt. \tag{12}$$

We look for an equilibrium token price process p_t of the form $p_t = p(t, Y_t)$, where $p(t, Y)$ is a sufficiently smooth function of its arguments. Applying Ito's lemma, we obtain a PDE on the token price function:

$$\frac{\partial p}{\partial t} + \frac{\partial p}{\partial Y} g_Y^Q(t) Y + \frac{1}{2}\frac{\partial^2 p}{\partial Y^2}\sigma(t)^2 Y^2 - r_f p + \frac{p}{p - kY}(c + kg_S(t))Y = 0. \tag{13}$$

Equation (13) is the valuation PDE. This equation has multiple solutions, and we look for a non-negative solution without valuation bubbles. Specifically, we look for a solution with suitably bounded growth in Y, and subject to a boundary condition

$$p(t, 0) = 0, \tag{14}$$

which requires that token value vanishes at zero transaction volume (recall that zero is an absorbing boundary for the transaction volume process). The above equation has a linear solution,

$$p(t, Y) = A(t)Y, \tag{15}$$

where the unknown function $A(t)$ is a bounded, positive solution of the ODE:

$$\frac{dA(t)}{dt} + g_Y^Q(t)A(t) - r_f A(t) + \frac{A(t)}{A(t) - k}(c + kg_S(t)) = 0. \tag{16}$$

In the stationary case of $g_S(t) = g_S$, $\sigma(t) = \sigma$, and $g_Y(t) = g_Y$, the total value of tokens p_t is a constant multiple of the transaction volume Y_t:

$$\frac{p_t}{Y_t} = k + \frac{c + kg_S}{r_f + \eta\sigma - g_Y}. \tag{17}$$

[7] To see how this term is determined, note that all new tokens accrue to the validators. Therefore, if validators start the period $[t, t + dt)$ with a fraction ϕ_t of the tokens, they end the period with

$$\frac{\phi_t + g_S(t)\, dt}{1 + g_S(t)\, dt} = (\phi_t + g_S(t)\, dt)(1 - g_S(t)\, dt) + o(dt) = \phi_t + (1 - \phi_t)g_S(t)\, dt + o(dt) \tag{11}$$

tokens.

This solution describes the total value of tokens under constant growth rate in the number of outstanding tokens. Note that, $r_f + \eta\sigma = \lambda_Y$, which is the expected return on the financial claim paying a cash flow stream equal to the aggregate flow of transaction fees, cY_t. We thus recover the valuation formula (4) by setting the token rewards to zero, $g_S = 0$.

The above solution highlights the valuation effect of rewarding validators in newly minted tokens. In addition to collecting transaction fees, validators also collect proceeds from seignorage. Comparing with (4), we see that this effectively raises the flow of proceeds to validators from c to $c + kg_S$. The second term, kg_S is intuitive: transfers to validators due to seignorage are proportional to the level of token balances held by consumers. If consumers hold no token balances between transactions, any benefit validators derive from collecting rewards in newly minted tokens is completely offset by the decline in the market value of tokens in their stake.

5 Network Security and Valuation Bubbles

In this section we are concerned with a particularly important type of malicious behavior: the majority, or "51%" attack. Feasibility of such attacks depends on the tradeoff between the benefit and cost of attacks. The direct economic benefit is derived from the attackers' ability to double-spend coins on multiple transactions (the attackers may also be motivated by the objective of destroying credibility of a particular blockchain system, in which case the benefit of the attack is harder to quantify). The cost depends on the requirements for a successful attack, which in turn depend on the defensive mechanisms implemented in the network.

The list of empirical examples of successful majority attacks is growing. For example, on May 16th of 2018 attackers stole $17.5M by executing the 51% attack on Bitcoin Gold (BTG). As recently as January 5th of 2019, Ethereum Classic (ETC) suffered a successful 51% attack, which allowed the attackers to double-spend approximately $1,1M worth of tokens. The overall value of Ethereum Classic tokens declined by roughly 20%, or $100M, over the week following the attack.

In a recent paper, Budish (2018) analyses the incentives behind the majority attacks in the PoW systems, like Bitcoin. In such systems, majority attacks require attackers' to gain

majority of hashing power over a period of time, and hence the cost of the attack depends on the cost and liquidity of the mining equipment. In the PoS system, attackers face a very different tradeoff: to execute the majority attack, one must gain a sufficient degree of majority in the validation pool, which requires holding a significant number of tokens. [8] The cost of the attack is that ones' token holdings would be slashed (confiscated) based on the network security protocol. Majority attacks are only feasible if their economic benefits exceed the cost of executing the attack – hence, network security hinges critically on token valuation.

5.1 A security condition

Suppose there are N regularly spaced finalization points per year. We call the span of time between two consecutive finalization points an *epoch*. If Y_t is the current expected annualized rate of arrival of dollar transaction volume on the network, a successful double-spend attack would yield on average Y_t/N dollars worth of transactions. At this point, we need to recognize a simplification introduced within our model: we model the flow of transactions as smooth. This assumption is not accurate at very short time scales – it is more realistic to think of Y_t in the model as an expected rate of transaction arrival, with the actual transaction volume per epoch being a lot more random than implied by the process (5). Thus, Y_t/N is an expected value of transactions in the next epoch, and attacks are likely to be launched selectively to take advantage of epochs with relatively high transaction values. Our analysis thus yields an estimate of the level of revenue, $\gamma Y_t/N$, at which a double-spend attack becomes profitable, for a given set of network and market parameters. We use γ to express our answer, which effectively formulates it in units of time: a particular epoch is attractive for an attacker if it contains contains value in excess of a factor of γ of the expected epoch value.

To quantify the cost of executing an attack, suppose the network protocol requires the 2/3 majority to validate a group of blocks between finalization points. This is the case in Ethereum's Casper FFG, for instance Buterin and Griffith (2017). In that case, an attacker could execute a double-spend attack while controlling 2/3 of the total pool of tokens staked

[8] Here, we are assuming that users only consider a transaction confirmed once it has been finalized. In reality, there may be confirmation rules that do not rely only on finalization, but finalization is at least a sufficient condition for confirmation, by definition.

by validators. Such an attack would result in an attacker losing 1/3 of the validation pool in tokens (half of the attacker's stake) to slashing. To see this, consider a situation where the same tokens are used for two separate transactions, and both get validated. First time around, all the "honest" validators vote on the transaction, which accounts for 1/3 of the validation pool, and attacker contributes another 1/3 to the vote. Second time around, the attacker validates transactions unilaterally, using its 2/3 control of the validation pool. As a result, 1/3 of all the tokens staked by the validators ends up slashed as a penalty for validating invalid blocks. We denote the fraction of the validation pool lost due to slashing by $\mathcal{Z}$: under alternative rules it would differ from 1/3.

Note that validators earn a fair rate of return on their capital. Thus, the only other cost the attacker faces in addition to slashing is the subsequent decline in the value of its remaining stake (because of the network losing credibility following a successful attack) We do not model this value, which yields a somewhat conservative estimate of the cost of the attack (our argument for $\mathcal{Z} = 1/3$ in the previous paragraph may understate the cost of the attack by a factor of two, at the most).

Suppose the system is following a steady-state growth path, and there is no token supply. Using the valuation equation (2), the economic loss to slashing equals the benefit of double-spending if

$$\mathcal{Z}\frac{cY_t}{\lambda_Y - g_Y} = \gamma\frac{Y_t}{N}, \tag{18}$$

and therefore

$$\gamma = \frac{N\mathcal{Z}c}{\lambda_Y - g_Y}. \tag{19}$$

To get a sense of the economic magnitudes involved, suppose that $N = 52,000$ (this places finalization points roughly ten minutes apart, $\lambda_Y = 7\%$, $g_Y = 2\%$, and $c = 0.10\%$. These numbers are real (inflation-adjusted). They imply that transaction volume on the network grows roughly at the same rate as GDP; the expected return on a claim on the flow of fees is roughly in line with expected real returns on the stock market (in the absence of empirical data, one could plausibly argue for a higher or a lower value); and average transaction fees are at a somewhat arbitrary level of 0.1%, which is much lower than the transaction fees charged on smaller transactions by credit cards, but higher than the cost of wiring funds through the traditional systems. With the assumed parameters, in steady state, the security

threshold $\gamma \approx 350$. Thus, a double-spending attack may be worth executing if the value stolen exceeds two and a half days worth (approximately 350 ten-minute periods) of average transaction volume on the network. As long as the value of individual epochs is lower than this thresholds, double-spending attacks would not be launched by a profit-maximizing actor (this says nothing about an attacker motivated to undermine the network itself).

5.1.1 Valuation bubbles and network security

In this section we show that valuation bubbles may adversely affect network security. Consider a network with a constant growth rate and volatility of the transaction volume, zero supply of new tokens, and stochastic consumer demand for token holdings. Specifically, we assume that consumer demand follows a continuous-time Markov process, taking on one of two values: $k_H > k_L$. The transition rate from state k_i to k_j is ν_{ij}. We assume that this process is independent of the transaction volume process Y_t, and shocks to consumer demand carry no risk premium – hence, the same transition rates apply under the risk-neutral valuation measure Q.

The time-varying consumer demand for token balances captures, in reduced form, speculative trading by investors in tokens. The state with high token balances, $k_t = k_H$, can be interpreted as a state in which investors are relatively optimistic about future token price appreciation. Their belief is in fact irrational: starting in the high state, returns from holding tokens are relatively low. In contrast, we assume throughout that validators hold rational expectations about future token prices.[9] Note that while we allow for transient speculative consumer demand, we continue to rule out rational valuation bubbles, which are associated with unbounded expected asymptotic token price appreciation.

Under the above assumptions, token values in a stationary equilibrium are given by

$$p(t, Y) = A(k_t)Y, \tag{20}$$

[9] We model speculative bubbles directly through time-varying consumer demand for convenience. One can, alternatively, model biased believes of investors directly. This would not change the qualitative conclusions in this section: we will make this analysis available in an Internet Appendix.

where $A(k_t)$ satisfies the following system of equations:

$$(g_Y^Q - r_f)A(k_H) + \frac{A(k_H)}{A(k_H) - k_H}c + \nu_{HL}(A(k_L) - A(k_H)) = 0, \tag{21}$$

$$(g_Y^Q - r_f)A(k_L) + \frac{A(k_L)}{A(k_L) - k_L}c + \nu_{LH}(A(k_H) - A(k_L)) = 0. \tag{22}$$

The security thresholds γ_H and γ_L are then given by

$$\gamma_i = \mathcal{Z}N(A(k_i) - k_i), \quad i \in \{L, H\}. \tag{23}$$

The above equation reflects that $(A(k_t) - k_t)Y_t$ is the market value of the tokens stakes by the validators, at time t.

Intuitively, network security may be compromised in a high-valuation state, k_H. This happens because, starting in this state, consumer demand for balances is expected to decline, and hence expected capital gains on holding tokens are low. To compensate for that, and allow validators to earn a fair expected rate of return on their token stakes, the overall size of the validator pool must be low in equilibrium, so that the yield from transaction fees offsets the low expected changes in the token price. This implies that the overall value of the validation pool is relatively low, and hence the network is volnurable to a majority attack.

A numerical illustration helps confirm this intuition. Assume that interest rate r_f is 1%; growth rate of transaction volume, g_Y, is 2%; the market price of risk shocks to Y_t, η, is 0.5; volatility of transaction volume growth σ is 0.12; and the transaction fee parameter is $c = 0.1\%$. These numbers are consistent with the numerical example is Section 5.1.

To set the baseline, assume first that $k_L = k_H = 15/365$, i.e., consumer demand for balances is constant, and equal to approximately 15 days worth of spending. In this case, $A(k_H) = A(k_L) = 0.061$, and $\gamma(k_H) = \gamma(k_L) = 347$ – we obtain the same security threshold as in the previous section. Note that the security condition is determined by the value of the validators' holdings, rather than by the total value of all outstanding tokens. Hence, we obtain the same security threshold as in the previous section, independently of the assumed value of consumer demand.

Next, we let consumer demand differ across states: we assume that $k_H = 1/4$, $k_L = 15/365$, $\nu_{LH} = 0.1$, and $\nu_{HL} = 1$. Thus, consumer balances in the high (optimistic) state

are equal to approximately three months worth of spending, compared to 15 days in the low state. Transition rates across the two states are highly asymmetric, so that the network spends only 9% of the time in the high state, and transitions out of that state, on average, in a year. In contrast, it takes on average ten years to enter the high state, starting from a low state.

Tokens are more valuable in a high state, due to higher consumer demand for balances:

$$A(k_H) = 0.253, \quad A(k_L) = 0.177. \tag{24}$$

The market value of the validators' holdings is highly asymmetric across the two states: in the low state, validators hold a fraction $\phi_L = 0.768$ of the entire supply of tokens. In contrast, in the high state, they hold only $\phi_H = 0.011$ of all the tokens. As we discuss above, low holdings by validators are caused by the anticipated token price decline associated with the impending "burst of the bubble." Security thresholds are, accordingly, very different across the two states: $\gamma_H = 50$, and $\gamma_L = 2,359$. Note that the possibility of a valuation bubble forming in the future makes the network more secure in the low state, by raising the value of the validators' holdings. The opposite happens in the high state. Under the assumed parameters, the security threshold declines from 347 in the baseline case (without the valuation bubble), to 50. This means that a profitable double-spend attack requires capturing roughly eight hours worth of average transaction volume in a single epoch, compared to two and a half days in the baseline case.

Certain design feature may be introduced to help reduce the negative effect of valuation bubbles on validators' willingness to stake tokens. For instance, one may lock in validators' holdings over extended time periods, making it impossible for them to respond to highly transient valuation changes. In return, validators would then require a higher expected rate of return in equilibrium, to compensate them for the loss of liquidity. Understanding the net effect of such arrangements on network security, and network dynamics in general, is an important question for future research.

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