Networks, Barriers, and Trade

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March 21, 2019

Abstract

We study a class of non-parametric trade models with global production networks. We characterize their properties in terms of interpretable and measurable sufficient statistics. We provide both reduced-form results useful for measuring the sources of growth and structural results useful for making predictions or conducting counterfactuals. Our main objective is to show theoretically and quantitatively that simple stylized models can give highly misleading results. As an example, accounting for nonlinear (non-Cobb-Douglas) input-output networks, with realistic complementarities in production, significantly raises the gains from trade relative to estimates in the literature. As another example, value-added production functions, no matter how well-calibrated, are incapable of correctly predicting the costs of tariff and non-tariff barriers to trade simultaneously. Better quantitative accuracy demands the use of more complicated computational models. This paper helps bridge the gap between computation and theory.

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1 Introduction

Trade economists increasingly recognize the importance of using large-scale computational general equilibrium models for studying trade policy questions. One of the major downsides of relying on purely computational methods is their opacity; computational models can be a black box, and it can be hard to know whether or not the forces inside the model are the right ones. On the other hand, it is by now well-recognized that relying on simple stylized models, while transparent and parsimonious, can result in unreliable quantitative predictions when compared to the large-scale models.

This paper seeks to provide a theoretical map of territory usually explored only by machines. It studies output and welfare in open economies with disaggregated and interconnected production structures. We address two types of questions: (i) how to measure and decompose the *sources* of output and welfare change, and (ii) how to predict the response of output and welfare to changes in trade costs or taxes. Our analysis is non-parametric and quite general, which helps us to isolate the common forces and sufficient statistics necessary to answer these questions without committing to a specific parametric set up.

Our main message is that for a broad range of questions in open-economy settings, accounting for the details of the production structure (like, the input-output table and the elasticities of substitution) is theoretically and quantitatively very important. Simple stylized models, no matter how deftly calibrated, can get both the magnitude and the direction of effects wrong.

In analyzing the structure of open-economy general equilibrium models, we emphasize their similarities and differences to the closed-economy models used to study growth and fluctuations. To fix ideas, consider the following fundamental theorem of closed economies. For a perfectly-competitive economy with a representative household and inelastically supplied factors

$$\frac{\mathrm{d}\log W}{\mathrm{d}\log A_i} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log A_i} = \frac{sales_i}{GDP}, \tag{1}$$

where W is real income or welfare, Y is real output or GDP, and A_i is a Hicks-neutral shock to some producer i. Equation (1), also known as Hulten's Theorem, shows that the sales share of producer i is a sufficient statistic for understanding, to a first order, the impact of a shock on aggregate welfare, aggregate income, and aggregate output.[1] Specifically, Hulten's theorem implies that, to a first order, any information beyond the sales share (for example, about the details of the production network, the returns to scale, or the elasticities of substitution) is macroeconomically irrelevant.

[1] We call this a fundamental theorem since it is a consequence of the first welfare theorem. Although versions of this result existed for a long time, at least since Domar (1961), the modern treatment is due to Hulten (1978).

In this paper, we examine the extent to which the logic of (1) can be transported into international economics. We provide the open-economy analogues of equation (1), and show that although versions of Hulten's theorem continue to hold in open-economies, the sales-shares are no longer such universal sufficient statistics. Ultimately, there are two main barriers to blindly applying Hulten's theorem in an open-economy: first, in an open-economy, output and welfare are no longer the same since welfare depends on terms-of-trade but output does not; second, much of trade policy concerns the effects of tariffs, which knocks out the foundation of marginal cost pricing and Pareto efficiency that Hulten's Theorem is built on. Our generalizations make clear precisely the conditions under which a naive-application of (1) to an open-economy is valid. Even when not directly applicable, it is be helpful to think in terms of (1), and deviations from it.

Notwithstanding the differences between open and closed economies, we also prove that, under some conditions, there exists a useful isomorphism between the two types of models. In particular, for any open-economy with nested-CES import demand there exists a companion (dual) closed economy, and the welfare effects of iceberg shocks in the open-economy are equal to the output effects of productivity shocks in the closed economy. This means that we can use results from the closed-economy literature, principally Hulten (1978) and Baqaee and Farhi (2017a), to characterize the effects of iceberg shocks on welfare up to the second-order. Our formulas provide a generalization of the influential insights of Arkolakis et al. (2012) to environments with disaggregated, and non-linear, input-output connections. Compared to the Cobb-Douglas loglinear production networks common in the literature (for example Costinot and Rodriguez-Clare, 2014 and Caliendo and Parro, 2015), we find that accounting for nonlinear production networks significantly raises the gains from trade. Accounting for nonlinear input-output networks is as, or more important, as accounting for intermediates in the first place. For example, for the US, the gains from trade increase from 4.5% to 9% once we account for intermediates with a Cobb-Douglas network, but they increase further to 13% once we account for realistic complementarities in production. The numbers are even more dramatic for more open economies, for example, the gains from trade for Mexico go from 11% to 16% to 44.5%.

For most of the paper, we restrict ourselves to efficient economies, but extending the results to allow for arbitrary distorting wedges (e.g tariffs or markups) is straightforward.To our knowledge, this is the first paper in the literature that derives comparative statics with respect to tariffs (in terms of model primitives) in a general production environment with intermediate goods. We show that, in general, the losses to society as a whole, and to the output of each country, from the imposition of tariffs or other distortions can be computed by an appropriate summing up of Harberger triangles. We provide explicit formulas for what these Harberger triangles are equal to in terms of microeconomic primitives. We show

that the existence of global value chains dramatically increases the costs of protectionism by inflating both the area of each triangle *and* the weight used to aggregate the triangles. We show that simple (non-input-output) models, regardless of how they are calibrated, get either the area of the triangles or their weight wrong, resulting in incorrect predictions.

We include some empirical and quantitative applications of our results. We use our growth-accounting formulas to measure the size and direction of reallocations of resources across countries over time, and relate these to movements in the terms-of-trade for each country. Finally, as a proof-of-concept, we use a calibrated model to illustrate the quantitative importance of the forces we emphasize. We consider three exercises, each of which highlights a conceptual point: (1) reversing globalization, (2) Brexit, and (3) US tariffs on China. First, we consider the welfare changes associated with a reversal of globalization, achieved either via import tariffs or non-tariff trade barriers. We show that the latter is far more costly than the former, and link this to the fact that deadweight loss triangles are much smaller than the loss trapezoids generated by non-trade barriers. We also show that for both tariffs and non-tariff barriers, accounting for the existence of global value chains is crucial. Quantitatively, the losses from protectionism are tripled or quadrupled once we account for the existence of input-output connections.

Second, we consider a simple Brexit scenario where the remaining members of the European Union place tariffs or non-tariff trade barriers on British imports. For this scenario, we focus on the distributional consequences of the shock across different primary factors. We find that the magnitude and even the sign of the distributional consequences depend on whether or not we account for input-output linkages. In particular, whereas low and medium skilled British workers gain from Brexit when we use value-added production functions, they lose once we account for the production network.

Finally, we compute the effects of US tariffs on Chinese imports. We find that taking the network connections into account, magnifies the gains to Mexico by a factor of three, the gains to the US by a factor of five, and the losses to China by a factor of six. This last exercise emphasizes the interconnected nature of global trade, and the importance of considering the trading system as a whole when evaluating policy counterfactuals.

The outline of the paper is as follows. In Section 2, we set up the model and define the objects of interest. In Section 3, we derive some growth-accounting results useful for measurement. In Section 4, we derive comparative statics in terms of microeconomic primitives, useful for prediction. In Section 5, we establish the dual relationship between closed and open economies which can be used to generalize some of the results in Arkolakis et al. (2012) and Costinot and Rodriguez-Clare (2014). In Section 6, we extend our analysis to allow for distortions like tariffs, and we show that analytically, the costs of tariffs are very different to those of iceberg shocks. Our empirical and quantitative applications are in Section 7.

Related Literature

At a high-level, this paper is related to the classic papers of Hulten (1978), Harberger (1964), and Jones (1965). We extend Hulten (1978) and prove growth-accounting formulas for open-economies; we extend Harberger (1964) and show that deadweight-loss triangles can measure the losses from tariffs in general equilibrium; we extend the hat-algebra of Jones (1965) beyond the $2 \times 2 \times 2$ cases he considered, and use it to answer counterfactual questions.

More broadly, our paper is related to three literatures: the literature on production networks, the literature on growth accounting, and the literature on the gains from trade. We discuss each literature in turn starting with the one on the gains (or losses) from trade. As far as we are aware, this is the first paper to characterize the comparative static response of income and output to changes in iceberg costs and tariffs non-parametrically and in a model with a rich input-output structure. In particular, our results generalize some of the results in Arkolakis et al. (2012) and Costinot and Rodriguez-Clare (2014) to environments with non-linear input-output connections. Our framework generalizes the input-output models emphasized in Caliendo and Parro (2015), Caliendo et al. (2017), Morrow and Trefler (2017), Fally and Sayre (2018),, and Bernard et al. (2019). Our results about the effects of trade in distorted economies also relates to Epifani and Gancia (2011), Arkolakis et al. (2015), Berthou et al. (2018), Bai et al. (2018). Our results also relate to work with non-parametric or semi-parametric models of trade like Adao et al. (2017) and Lind and Ramondo (2018), although, our analysis does not rely on the invertibility, or stability, of factor demand systems. Finally, our characterization of how factor shares and prices respond to shocks is related to an incredibly deep literature, for example, Trefler and Zhu (2010), Elsby et al. (2013), Davis and Weinstein (2008), Feenstra and Sasahara (2017), Burstein and Vogel (2017), Artuç et al. (2010), Dix-Carneiro (2014), Galle et al. (2017), among others.

The literature on production networks has primarily been concerned with the propagation of shocks in closed economies, typically assuming a representative agent. For instance, Long and Plosser (1983), Acemoglu et al. (2012), Atalay (2017), Baqaee (2018), Carvalho et al. (2016), among many others. For an overview of this literature, see Carvalho and Tahbaz-Salehi (2018) and the references therein. A recent focus of the literature, particularly in the context of open economies, has been to model the formation of links, for example Chaney (2014), Lim (2017), Tintelnot et al. (2018), and Kikkawa et al. (2018). Our approach is different: rather than modelling the formation of links as a binary decision, we use a Walrasian environment where the presence and strength of links are determined by cost minimization subject to some exogenous production technology. We build on the results in Baqaee and Farhi (2018) which show how shocks propagate in input-output networks with heterogenous consumers.

Finally, our growth accounting results are related to closed-economy results like Solow (1957), Hulten (1978), as well as the literature extending growth-accounting to open economies, including Kehoe and Ruhl (2008), and Burstein and Cravino (2015). Perhaps closest to us are Diewert and Morrison (1985) and Kohli (2004) who introduce output indices which account for terms-of-trade changes. Our real income and welfare-accounting measures share their goal, though our decomposition into pure productivity changes and reallocation effects is different. In explicitly accounting for the existence of intermediate inputs, our approach also speaks to how one can circumvent the double-counting problem and spill-overs arising from differences in gross and value-added trade, issues studied by Johnson and Noguera (2012) and Koopman et al. (2014). Relative to these other papers, our approach has the added bonus of easily being able to handle inefficiencies and wedges.

2 Framework

In this section, we do the spadework of setting up the model and defining the key statistics of interest. We assume that there are no distortions. In Section 6, we extend our results to environments with distortions (e.g. markups, tariffs, taxes).[2]

2.1 Model

There is a set of households H, a set of producers N and corresponding goods, a set of factors F. Each household, producer, and factor is assigned to a country in a set C. The sets of households, producers, and factors belonging to country c are H_c, N_c, and F_c.

Factors

In each country, the representative household c is endowed with some share Φ_{cf} of the supply L_f of each factor f. We take the quantities and ownership structure of factors as exogenously given.[3]

[2] Distortions can be represented as wedges (implicit or explicit taxes). Tariffs, markups, and financial frictions are wedges, but iceberg trade costs are not.

[3] In Appendix H, we discuss how to endogeneize factor supply by using a model à la Roy (1951) and discuss the connection of our results with those in Galle et al. (2017).

Households

Each household h in country c maximizes a homogenous-of-degree-one demand aggregator[4]

$$C_h = \mathcal{W}_h(\{c_{hi}\}_{i \in N}),$$

subject to the budget constraint

$$\sum_{i \in N} p_i c_{hi} = \sum_{f \in F} \Phi_{hf} w_f L_f + T_h,$$

where c_{hi} is the quantity of the good produced by producer i and consumed by household h, p_i is the price of good i, w_f is the wage of factor f, and T_h is an exogenous lump-sum transfer. The lump-sum transfer can be used to allow for trade imbalances link in Dekle et al. (2008).

Producers

Each producer i in country c produces a different good using a constant-returns-to-scale production function with the associated production function

$$y_i = A_i F_i \left(\{x_{ik}\}_{k \in N}, \{l_{if}\}_{f \in F_c} \right),$$

where y_i is the total quantity of good i produced, x_{ik} is intermediate inputs from k, l_{if} is factor inputs from f, and A_i is an exogenous Hicks-neutral productivity shifter. For convenience, we also sometimes define $l_{if} = 0$ for $f \notin F_c$.

Generality

The assumption that production features constant returns to scale is without loss of generality. Indeed, as pointed out by McKenzie (1959), neoclassical production functions are constant-returns-to-scale without loss of generality, since any decreasing returns production function can always be written as a constant returns production function by adding quasi-fixed factors.

The assumption that each producer produces only one output good is without loss of generality. Indeed, one can always represent another output as a negative input in its production function. Joint production is therefore allowed by the model.

[4]In mapping our model to data, we interpret domestic "households" as any agent which consumes resources without producing resources to be used by other agents. Specifically, this means that we include domestic investment and government expenditures in our definition of "households" when we map this model to the data.

The assumption that productivity shifters are Hicks-neutral is without loss of generality. Indeed, input-augmenting technical change for producer i's use an of input k can be captured *via relabeling* as Hicks-neutral productivity changes for a new fictitious specialized producer introduced to intermediate purchases of good k by producer i with a linear technology.[5]

Finally, the assumption that there are no shocks to the composition of final demand is without loss of generality, since such shocks can be represented via relabeling as combinations of positive and negative productivity shocks.

Iceberg Trade Costs

This last observation also applies to changes in iceberg trade costs which can be captured via relabeling as negative Hicks-neutral productivity changes for new fictitious specialized producers introduced to intermediate transactions between producers. Since we can represent final demand by consumer h as the purchase of an aggregate good produced by a new fictitious specialized producer with a production function given by the corresponding demand aggregator, we can also captures changes in iceberg trade costs for consumers. These changes in iceberg trade costs could be more prevalent across countries than within countries, but the model allows for both.

Where to locate these new specialized fictitious producers is ambiguous, especially when trade is occurring across countries. We do not need to take a precise stand at this stage, but we note that this will matter for our conclusions regarding real country GDP changes. It is possible to place them in the exporting country, in the importing country, or in a new fictitious separate specialized country which contains no factors.

Equilibrium

Given productivities A_i and a vector of transfers T_h satisfying $\sum_{h\in H} T_h = 0$, a general equilibrium is a set of prices p_i, intermediate input choices x_{ij}, factor input choices l_{if}, outputs y_i, and consumption choices c_{ci}, such that: (i) each producer chooses inputs to minimize costs taking prices as given; (ii) each household maximizes utility subject to its budget constraint taking prices as given; and, (iii) the markets for all goods and factors clear so that $y_i = \sum_{h\in H} c_{hi} + \sum_{j\in N} x_{ji}$ for all $i \in N$ and $L_f = \sum_{j\in N} l_{jf}$ for all $f \in F$.

[5]The assumption that factors cannot be used in production across borders is also without loss of generality, since we can always introduce a new fictitious producer transforming a non-traded factor f in a country $c' \neq c$ into a traded intermediate input for producer i.

2.2 Definitions

In this subsection, we define the statistics of interest. Although these definitions are standard to national income accountants, and the distinctions we stress may seem tedious, it turns out that they make all the difference for the economics of the model.

Nominal Output and Nominal Expenditure

Nominal output or Gross Domestic Product (GDP) for country c is the total final value of the goods produced in the country. It coincides with the total income earned by the factors located in the country or nominal Gross Domestic Income (GDI):

$$GDP_c = \sum_{i \in N} p_i q_{ci} = \sum_{f \in F_c} w_f L_f,$$

where $q_{ci} = y_i 1_{\{i \in N_c\}} - \sum_{j \in N_c} x_{ji}$ is the net quantity of good $i \in N$ in the GDP of country c (which means that it can be positive or negative).

Nominal Gross National Expenditure (GNE) for country c is the total final expenditures of the residents of the country. In our model, it coincides with nominal Gross National Product (GNI) which is the total income earned by the factors owned by its residents and adjusted for international transfers:[6]

$$GNE_c = \sum_{i \in N} p_i c_{ci} = \sum_{f \in F} \Phi_{cf} w_f L_f + T_c,$$

where $c_{ci} = \sum_{h \in H_c} c_{hi}$, $\Phi_{cf} = \sum_{h \in H_c} \Phi_{hf}$, and $T_c = \sum_{h \in H_c} T_h$.

Nominal output or GDP and nominal GNE are *not* the same at the country level in general because they account for the value created by different sets of factors (and the different expenditures that they finance): the factors in the country vs. the factors owned by the residents of the country.

These differences are still present at the level of a household h in general:

$$GDP_h = \sum_{f \in F} \Phi_{hf} w_f L_f, \quad GNE_h = \sum_{i \in N} p_i c_{hi} = \sum_{f \in F} \Phi_{hf} w_f L_f + T_h,$$

where for household GDP, we think of the household as a set producers intermediating the uses by the different producers of the different factor endowments of the household.

[6] In our model, GNE and GNI are identical, but they are not in the data. Similarly, in the model, the expenditure and income of a household are identical, but they are not in the data. This is because our model is static. Inter-temporal trade at the country level (trade imbalances) or household level must be captured via household transfers T_h and international transfers T_c.

Of course, these differences vanish at the world level:

$$GDP = GNE = \sum_{f \in F} w_f L_f = \sum_{i \in N} p_i q_i = \sum_{i \in N} p_i c_i,$$

where $c_i = q_i = y_i$ with $c_i = \sum_{c \in C} c_{ci}$, $q_i = \sum_{c \in C} q_{ci}$, $T = 0$ with $T = \sum_{c \in C} T_c$.

Although this is not necessary, it will prove convenient to pick a specific numeraire. From now, on we use nominal world output (GDP) as the numeraire which implies the convenient normalization $GDP = GNE = 1$. All prices and transfers are expressed in units of this numeraire.

Real Output and Real Expenditure

We now define changes in real output and real expenditures at different levels of aggregation. We use Divisia indices throughout to separate quantity and price changes. We rely on their nice aggregation properties.

The change in real output (real GDP) of country c of the corresponding deflator are defined to be

$$\mathrm{d} \log Y_c = \sum_{i \in N} \chi_i^{Y_c} \mathrm{d} \log q_{ci}, \quad \mathrm{d} \log P_{Y_c} = \sum_{i \in N} \chi_i^{Y_c} \mathrm{d} \log p_i,$$

where $\chi_i^{Y_c} = p_i q_{ci} / GDP_c$.[7]

The change in real expenditure (real GNE) of country c and of the corresponding deflator are defined by aggregating changes in real expenditures across households in that country:

$$\mathrm{d} \log W_c = \sum_{i \in N} \chi_i^{W_c} \mathrm{d} \log c_{ci}, \quad \mathrm{d} \log P_{W_c} = \sum_{i \in N} \chi_i^{W_c} \mathrm{d} \log p_i,$$

where $\chi_i^{W_c} = p_i c_{ci} / GNE_c$.

Changes in real output and in real expenditure are *not* the same at the country level in general. The difference comes from two sets of reasons. First, with border cross-border factor holdings and international transfers, changes in nominal output and in nominal expenditure are not the same in general. Second, changes in the price deflators for nominal output and

[7]Note that country real output is only defined in changes, and these changes are no the differential of a level function. This means that the any discrete change in real output depends on the path of integration. The precise way to proceed is to index the economy by a continuous index (say time t), which indexes all the relevant shifters and all the equilibrium variables. We can then compute changes in real output between the initial period ($t = 0$) and some final period ($t = \tau$) as the integral of the infinitesimal real output changes along the resulting path. The differential change stated in the theorem is the real output change which obtains in the limit of small time intervals $\tau \to 0$: it is independent of the particular path of integration taken to derive it. Similar observations apply to country real expenditure when there are agents with heterogeneous preferences within the country. The same goes at the world level for real output and real expenditure, and at the household level for real output when the household own more than one factor.

nominal expenditure are not the same in general.

These differences are still present at the household level in general, where

$$\mathrm{d}\log Y_h = \sum_{f\in F} \chi_f^{Y_h}\,\mathrm{d}\log L_f, \quad \mathrm{d}\log P_{Y_h} = \sum_{f\in F} \chi_f^{Y_h}\,\mathrm{d}\log w_f,$$

$$\mathrm{d}\log W_h = \sum_{i\in N} \chi_i^{W_h}\,\mathrm{d}\log c_{hi}, \quad \mathrm{d}\log P_{W_h} = \sum_{i\in N} \chi_i^{W_h}\,\mathrm{d}\log p_i,$$

with $\chi_f^{Y_h} = \Phi_{hf} w_f l_f GDP_h$ and $\chi_i^{W_h} = p_i c_{hi}/GNE_h$.

Of course, these differences vanish at the world level so that $\mathrm{d}\log Y = \mathrm{d}\log W$ and $\mathrm{d}\log P_Y = \mathrm{d}\log P_W = \mathrm{d}\log P$, where

$$\mathrm{d}\log Y = \sum_{i\in N} \chi_i^{Y}\,\mathrm{d}\log q_i, \quad \mathrm{d}\log P_Y = \sum_{i\in N} \chi_i^{Y}\,\mathrm{d}\log p_i,$$

$$\mathrm{d}\log W = \sum_{i\in N} \chi_i^{W}\,\mathrm{d}\log c_i, \quad \mathrm{d}\log P_W = \sum_{i\in N} \chi_i^{W}\,\mathrm{d}\log p_i,$$

with $\chi_i^{Y} = p_i q_i/GDP$ and $\chi_i^{W} = p_i c_i/GNE$.

It is important to note the following aggregation properties. Changes in country real output and real expenditures aggregate up to their world counterparts. Similarly, changes in household real expenditure also aggregate up to their country and world counterparts. However, changes in household real output do not aggregate up to their country or world counterparts.[8] The reasons are easy to understand and do not reveal any inconsistency in our definitions.[9]

Finally, we comment on the relation between the change in real expenditure and the change in welfare. It follows directly from Shephard's lemma that the change in real expenditure of household h coincides with the change in the *welfare* of the household, thereby justifying the notation $\mathrm{d}\log W^h$. The notations $\mathrm{d}\log W_c$ is a slight abuse of notation since the change in real expenditure of country c cannot in general be unambiguously interpreted

[8]Namely, we have the following properties. First, $\mathrm{d}\log Y = \sum_{c\in C} \chi_c^{Y}\,\mathrm{d}\log Y_c$ and $\mathrm{d}\log W = \sum_{c\in C} \chi_c^{W}\,\mathrm{d}\log W_c$. Second, $\mathrm{d}\log W_c = \sum_{h\in H_c} \chi_h^{W_c}\,\mathrm{d}\log W_h$ and $\mathrm{d}\log W = \sum_{h\in H} \chi_h^{W}\,\mathrm{d}\log W_h$. Third, in general, $\mathrm{d}\log Y_c \neq \sum_{h\in H_c} \chi_h^{Y_c}\,\mathrm{d}\log Y_h$ and $\mathrm{d}\log Y \neq \sum_{h\in H} \chi_h^{Y}\,\mathrm{d}\log Y_h$. We have made use of the following definitions $\chi_c^{Y} = GDP_c/GDP$, $\chi_c^{W} = GNE_c/GNE$, $\chi_h^{W_c} = GNE_h/GNE_c$, $\chi_h^{W} = GNE_h/GNE$, $\chi_h^{Y_c} = GDP_h/GDP_c$, and $\chi_h^{Y} = GDP_h/GDP$.

[9]That changes in the real output changes of the households in a country do not aggregate up to the changes in the real output of the country is partly due to the fact that there are cross-border factor holdings. There is also another reason, which will become clear in Section 3, and which also explains why changes in real output of the households in the world do not aggregate up to changes in world real output: this is because we have defined changes in household real GDP by considering the household as a producer of each of its factors, and so aggregating up changes household in real output only accounts for changes in factor supplies but not for changes in the productivities accruing to the producers of the non-factor goods.

as a change in the welfare of the households of the country, unless there is a representative household in the country. This is because it requires taking a stand on distributive issues. However, there always exists a welfare criterion for which the change in the real expenditure of the country coincide with the change in the welfare of the households of the country up to the first order.[10] Similar observations apply to changes in world real expenditure $\mathrm{d}\log W$.

Input-Output Concepts

We define the Heterogenous-Agent Input-Output (HAIO) matrix to be the $(H+N+F)\times(H+N+F)$ matrix Ω whose ijth element is equal to i's expenditures on inputs from j as a share of its total revenues/income

$$\Omega_{ij}=\frac{p_j x_{ij}}{p_i y_i}.$$

The HAIO matrix Ω includes the factors of production and the households, where factors consume no resources (zero rows), while households produce no resources (zero columns). The Leontief inverse matrix is

$$\Psi=(I-\Omega)^{-1}=I+\Omega+\Omega^2+\ldots.$$

While the input-output matrix Ω records the *direct* exposures of one agent or producer to another, the Leontief inverse matrix Ψ records instead the *direct and indirect* exposures through the production network.

It will sometimes be convenient to treat good and factors together and index them by $k\in N+F$ where we use the plus symbol to denote the union of these two sets. To this effect, we must slightly extend our definitions. We also write interchangeably y_k and p_k for L_k and w_k when $k\in F$. To capture the fact that the household endowment of the goods are zero, we define $\Phi_{hk}=0$ for $k\in N$.

We define the following direct and indirect *exposures*. The exposures to a good or factor k of the real expenditure and real output of household h are given by

$$\lambda_k^{W_h}=\sum_{i\in N}\chi_i^{W_h}\Psi_{ik},\quad \lambda_k^{Y_h}=\sum_{f\in F}\chi_f^{Y_h}\Psi_{fk},$$

[10] Assume that there is a homothetic Bergson-Samuelson welfare function. We can measure changes in country welfare following a shock by the proportional reduction in the post-shock consumptions of all goods by all consumers in the country which would keep the level of country welfare unchanged. If there is no desire to redistribute at the initial point, then changes in country welfare coincide with changes in country real expenditure up to the first order, because changes in the distribution of income across consumers do not lead to first-order changes in country welfare. This is no longer true if there is a desire to redistribute at the initial point. These effects can easily be characterized using our results, but we refrain from doing so because they are not the focus of this paper.

where recall that $\chi_i^{W_h} = p_i c_{hi}/GNE_h$ and $\chi_f^{Y_h} = \Phi_{hf} w_f L_f/GDP_h$. Similarly, the exposures to a good or factor k of the of the real expenditure and real output of country c are given by

$$\lambda_k^{W_c} = \sum_{i \in N} \chi_i^{W_c} \Psi_{ik}, \quad \lambda_k^{Y_c} = \sum_{i \in N} \chi_i^{Y_c} \Psi_{ik},$$

where recall that $\chi_i^{W_c} = p_i c_{ci}/GNE_c$ and $\chi_i^{Y_c} = p_i q_{ci}/GDP_c$. Finally, the exposures to a good or factor k of the of the real expenditure and real output of the world are given by

$$\lambda_k^{W} = \sum_{i \in N} \chi_i^{W} \Psi_{ik}, \quad \lambda_k^{Y} = \sum_{i \in N} \chi_i^{Y} \Psi_{ik},$$

where recall that $\chi_i^{W} = p_i c_i/GNE$ and $\chi_i^{Y} = p_i q_i/GDP$.

Exposures in real output to good or factor k at the household, country, and world levels have a direct connection to the sales of the producer:

$$\lambda_k^{Y_h} = 1_{\{k \in F\}} \frac{\Phi_{hk} p_k y_k}{GDP_h}, \quad \lambda_k^{Y_c} = 1_{\{k \in N_c + F_c\}} \frac{p_k y_k}{GDP_c}, \quad \lambda_k^{Y} = \frac{p_k y_k}{GDP}.$$

Hence, for example, λ_k^Y is just the sales share $\lambda_k = p_k y_k/GDP$ of k in world output or *Domar weight*. Similarly $\lambda_k^{Y_c} = 1_{\{k \in N_c + F_c\}}(GDP/GDP_c)\lambda_k$ is the *local Domar weight* of k in country c, and $\lambda_k^{Y_h} = 1_{\{k \in F\}} \Phi_{hk}(GDP/GDP_h)\lambda_k$ the local Domar weight of k in household h.

We also define the following factor *income shares* as the shares in income or expenditure. The share of a factor f in the income or expenditure of household h, of country c, and of the world, are given by

$$\Lambda_f^h = \frac{\Phi_{hf} w_f L_f}{GNE_h}, \quad \Lambda_f^c = \frac{\Phi_{cf} w_f L_f}{GNE_c}, \quad \Lambda_f = \frac{w_f L_f}{GNE},$$

where, from now on, we sometimes denote factors shares and exposures to factors or factor shares with a capital Λ to distinguish them from sales shares and exposures to non-factor producers λ.

In general, the exposures to a good or factor k of real expenditure and real output are *not* the same at the level of a household or country. Similarly, when applied to a factor f, these exposures are *not* the same as the income share of that factor at the level of a household or country. These differences disappear at the world level so that $\lambda_i^Y = \lambda_i^W = \lambda_i = p_i y_i/GDP$ for a good $i \in N$ and $\Lambda_f^Y = \Lambda_f^W = \Lambda_f = w_f L_f/GDP$ for a factor $f \in F$.

2.3 A Simple Case

To build intuition, the reader is encouraged to focus on the following simple case, by assuming that there is a representative household in each country c so that H_c is a singleton. In that case, it is unambiguous that the change in country real expenditure coincides with the change in the welfare of this representative agent $h \in H_c$ so that $\mathrm{d}\log W_h = \mathrm{d}\log W_c$ and $\mathrm{d}\log P_{W_h} = \mathrm{d}\log P_{W_c}$. This simple case still contains rich economics. In particular, we still have $\mathrm{d}\log Y_c \neq \mathrm{d}\log W_c$ and $\mathrm{d}\log P_{Y_c} \neq \mathrm{d}\log P_{W_c}$ in general.

However, it is important to note that even in this simple case, $\mathrm{d}\log Y_c \neq \mathrm{d}\log Y_h$ and $\mathrm{d}\log P_{Y_c} \neq \mathrm{d}\log P_{Y_h}$ in general, even in the absence of cross-border factor holdings, because of the aforementioned observation that household real output does not aggregate up to country real output.

3 Comparative Statics: Ex-Post Sufficient Statistics

In this section, we characterize the responses to productivity shocks, factor supply shocks, and transfer shocks, of real output, and real expenditure or welfare, at the household, country, and world levels. Since iceberg trade costs can be represented as productivity shocks, these characterizations extend to shocks in iceberg trade costs.

Our results for changes in real output in Section 3.1 are expressed in terms of *initial* levels of income and expenditure shares. They essentially extend Hulten's theorem to open economies. Our results for changes in real expenditure or welfare in Section 3.2 are expressed in terms of *initial* levels and *changes* in income and expenditure shares. They highlight departures from different versions of Hulten's theorem in open economies.

The results in this section are ideal for growth and welfare accounting where information on income and expenditure shares before and after the shocks is available. They can also be used to perform counterfactuals for changes in real output. However, to perform counterfactuals for changes in real expenditure and welfare, we need to predict how sales and expenditure shares will change in response to the shocks. In Section 4, we provide a full characterizations of these changes in terms of microeconomic primitives. These results can then be combined with those of this section to form counterfactual predictions.

3.1 Growth-Accounting

We start with our growth-accounting result on the response to shocks of real output (real GDP). It can be used for growth-accounting and to perform counterfactuals. We state the result at the level of a country c and we explain how to translate it at the level of a household

h and at the world level.

Theorem 1 (Growth-Accounting)**.** *The change in real output (GDP) of country c to productivity shocks, factor supply shocks, and transfer shocks, is given by*

$$\mathrm{d}\log Y_c = \sum_{i\in N} \lambda_i^{Y_c}\, \mathrm{d}\log A_i + \sum_{f\in F} \Lambda_f^{Y_c}\, \mathrm{d}\log L_f.$$

where recall that the exposures of real output to producers and factors are given by the local Domar weights $\lambda_i^{Y_c} = 1_{\{k\in N_c\}}(p_i y_i/GDP_c)$ *and* $\Lambda_f^{Y_c} = 1_{\{f\in F_c\}}(w_f L_f/GDP_c)$*. The change* $\mathrm{d}\log Y$ *of world real output (GDP) can be obtained by simply suppressing the country index c. The change* $\mathrm{d}\log Y_h$ *of the real output (GDP) of household h can be obtained by simply replacing the country index c by the household index h.*

Theorem 1 is an adaptation of Hulten's theorem to open economies. It implies that to a first order, a unit productivity shock to i moves real output in a country c by an amount equal to producer k's local Domar weight $\lambda_i^{Y_c}$. A counterintuitive implication of this equation is that to a first order, productivity shocks to foreign producers have *no* effect on domestic real output.[11] Since discrete changes in real output are obtained by chained integration of infinitesimal changes, the same counterintuitive implication actually holds globally. [12]

As emphasized by Burstein and Cravino (2015), productivity-accounting à la Hulten (1978) is the same in an open economy as it is in a closed economy. Country c's productivity can be measured by its Solow residual and is equal to the local Domar-weighted sum of productivity shocks of domestic producers:

$$\mathrm{d}\log Y_c - \sum_{f\in F} \Lambda_f^{Y_c}\, \mathrm{d}\log L_f = \sum_{i\in N} \lambda_i^{Y_c}\, \mathrm{d}\log A_i.$$

Since iceberg trade costs can be represented as productivity shocks, an implication is that iceberg trade shocks outside of country c, for example iceberg trade costs increasing the domestic price of some of its imports or the foreign price of its exports, have no effect on its real output or on its real productivity.

In a small-open economy, since trade can be modeled as a linear technology transforming exports into imports, and shocks to the terms of trade as shocks to the productivity of that technology. Folk wisdom and naive intuition then suggest that shocks to the terms of trade

[11] There are two important caveats to this statement: (i) shocks to foreign producers may change domestic local Domar weights, and thereby change the way local shocks affect the domestic economy (a nonlinear interaction), (ii) it is ceases to be true when the economy is no longer efficient. We shall discuss both of these issues at length in Sections 5 and 6.

[12] The same reasoning applies to factor supply shocks. Transfer shocks, no matter where they occur, have no effect on domestic real output to the first order and globally.

should have the same effect as negative domestic productivity shocks. Our result reinforces and expands the results of Kehoe and Ruhl (2008) and shows that this intuition is partly invalid in the sense that shocks to productivities and iceberg trade costs outside of a country's border, even though they do affect its terms of trade, do not affect the country's real output or its aggregate productivity.

3.2 Welfare-Accounting

Theorem 1 shows that an adaptation of Hulten's theorem holds in open economies for changes in real output. This is no longer true for changes in real expenditure or welfare, for which we prove two new theorems. These theorems provide two economically-interpretable decompositions, which respectively emphasize the roles of terms-of-trade effects and of reallocation effects.

Unlike Theorem 1, these theorems are expressed in terms of changes in factor income shares $\mathrm{d}\log\Lambda_f$. They can be used for welfare accounting to *measure* the reason why real expenditure or welfare changed in equilibrium. They can also be used for counterfactual predictions by combining them with the characterizations of the changes in factor income shares provided in Section 4.

We state the results at the level of a household h, where it can be interpreted as identifying a change in welfare. We explain how to translate it at the level of a country c and at the world level.

Terms of Trade

We start with the more straightforward terms-of-trade decomposition.

Theorem 2 (Welfare-Accounting, Terms of Trade)**.** *The change in real expenditure or welfare of household h in country c in response to productivity shocks, factor supply shocks, and transfer shocks can be decomposed into the effect of the change in real output, a change in transfers, and a change in terms of trade:*

$$\mathrm{d}\log W_h = \underbrace{\chi_{Y_h}^{W_h}\,\mathrm{d}\log Y_h}_{\Delta Output} + \underbrace{(1/\chi_h^W)\,\mathrm{d}\,T_h}_{\Delta Transfers} + \underbrace{(\chi_h^Y/\chi_h^W)\,\mathrm{d}\log P_{Y_h} - \mathrm{d}\log P_{W_h}}_{\Delta Terms\ of\ Trade},$$

where the changes in terms of trade $(\chi_h^Y/\chi_h^W)\,\mathrm{d}\log P_{Y_h} - \mathrm{d}\log P_{W_h}$ *are given by*

$$\sum_{i\in N}(\lambda_i^{W_h} - (\chi_h^Y/\chi_h^W)\,\lambda_i^{Y_h})\,\mathrm{d}\log A_i + \sum_{f\in F}(\Lambda_f^{W_h} - (\chi_h^Y/\chi_h^W)\Lambda_f^{Y_h})\,\mathrm{d}\log L_f + \sum_{f\in F}(\Lambda_f^h - \Lambda_f^{W_h})\,\mathrm{d}\log\Lambda_f,$$

with $\chi_h^Y/\chi_h^W = GDP_h/GNE_h$. *The change* $\mathrm{d}\log W_c$ *of the real expenditure of country c can be obtained by simply replacing the household index h by the country index c. The change* $\mathrm{d}\log W$ *of world real expenditure can be obtained by simply suppressing the household index h.*

Consider for example a unit change in the productivity of producer i. Assume for the sake of simplicity that factor supplies are fixed. Intuitively, for given factor wages, the productivity shock affects the terms of trade of household h according to the difference between the household's exposures to producer i in real expenditure and in real output $\lambda_i^{W_h} - (\chi_h^Y/\chi_h^W)\lambda_i^{Y_h}$. The productivity also leads to endogenous changes in the wages of the different factors $\mathrm{d}\log w_f$, which given that factor supplies are fixed, coincide with the changes in their factor income shares $\mathrm{d}\log\Lambda_f$. These changes in factor wages in turn affect the household's terms of trade according to the difference $\Lambda_f^h - \Lambda_f^{W_h}$ between the share of a given factor in the household's income and of the household's exposure of real expenditure to that factor.

At the country level, and unlike real output, real expenditure *does* respond to productivity shocks outside the country in general because these shocks affect the terms of trade. In particular, while shocks to iceberg trade costs outside a country do not affect its real output or its productivity, they do affect its real expenditure. At the world level, there are no terms-of-trade effects (and no transfers).

Furthermore, changes in real output and real expenditure and of their corresponding deflators at the household and country levels aggregate up to their world counterparts. This implies that changes in household and country terms of trade sum up to zero:

$$\sum_{h\in H}\chi_h^W[(\chi_h^Y/\chi_h^W)\,\mathrm{d}\log P_{Y_h} - \mathrm{d}\log P_{W_h}] = \sum_{c\in C}\chi_c^W[(\chi_c^Y/\chi_c^W)\,\mathrm{d}\log P_{Y_c} - \mathrm{d}\log P_{W_c}] = \mathrm{d}\log P_Y - \mathrm{d}\log P_W = 0,$$

where $\chi_h^W = GNE_h/GNE$, $\chi_c^W = GNE_c/GNE$, $\chi_h^Y = GDP_h/GDP$, and $\chi_c^Y = GDP_c/GDP$. Terms-of-trade effects can therefore be interpreted as zero-sum distributive effects. The same goes for transfers.

Reallocation

We now present the reallocation decomposition. We follow Baqaee and Farhi (2017b) and define the $(H+N+F)\times(H+N+F)$ *allocation matrix* $\mathcal{X}$ as follows: $\mathcal{X}_{ij} = x_{ij}/y_j$ is the share of the quantity y_j of good j used by some agent i, where the indices i and j the households, factors, and producers. Feasible allocations are entirely described by a feasible allocation matrix $\mathcal{X}$, a vector of productivities A, and a vector of factor supplies L. In particular, the

equilibrium allocation gives rise to an allocation matrix $\mathcal{X}(A, L, T)$ which, together with A, and L, completely describes the equilibirum.[13]

Equilibrium welfare of household h can therefore be written as a function $\mathcal{W}_h(A, L, \mathcal{X})$ with $\mathcal{X} = \mathcal{X}(A, L, T)$. Differentiating yields a decomposition into two components: the direct or "pure" effect of changes in technology $\mathrm{d}\log A$ and $\mathrm{d}\log L$, holding the distribution of resources $\mathcal{X}$ constant; and the indirect effects arising from the equilibrium changes in the allocation of resources $\mathrm{d}\,\mathcal{X}$. In other words, this decomposition breaks down changes in welfare into: "pure" technology effects capturing changes in welfare from increased production of each good, holding fixed every agent's share of each good; and changes in welfare from changes in the shares of each good consumed by each agent. The following theorem characterizes this decomposition.

Theorem 3 (Welfare-Accounting, Reallocation)**.** *The change in real expenditure or welfare of household h in country c in response to productivity shocks, factor supply shocks, and transfer shocks can be decomposed into the "pure" effects of changes in technology and the effects of changes in the allocation of resources:*

$$\mathrm{d}\log W_h = \underbrace{\frac{\partial \log \mathcal{W}_h}{\partial \log L}\,\mathrm{d}\log L + \frac{\partial \log \mathcal{W}_h}{\partial \log A}\,\mathrm{d}\log A}_{\Delta Technology} + \underbrace{\frac{\partial \log \mathcal{W}_h}{\partial \mathcal{X}}\,\mathrm{d}\,\mathcal{X}}_{\Delta Reallocation},$$

where the "pure" effects of changes in technology are given by

$$\frac{\partial \log \mathcal{W}_h}{\partial \log L}\,\mathrm{d}\log L + \frac{\partial \log \mathcal{W}_h}{\partial \log A}\,\mathrm{d}\log A = \sum_{f\in F} \Lambda_f^{W_h}\,\mathrm{d}\log L_f + \sum_{i\in N} \lambda_i^{W_h}\,\mathrm{d}\log A_i,$$

and the effects of changes in the allocation of resources are given by

$$\frac{\partial \log \mathcal{W}_h}{\partial \mathcal{X}}\,\mathrm{d}\,\mathcal{X} = \sum_{f\in F} (\Lambda_f^h - \Lambda_f^{W_h})\,\mathrm{d}\log \Lambda_f + (1/\chi_h^W)\,\mathrm{d}\,T_h.$$

The change $\mathrm{d}\log W_c$ *of the real expenditure of country c can be obtained by simply replacing the household index h by the country index c. The change* $\mathrm{d}\log W$ *of world real expenditure can be*

[13]Since there may be multiplicity of equilibria, technically, the competitive equilibrium gives a correspondence from A to $\mathcal{X}$. In this case, we restrict attention to perturbations of isolated equilibria.

obtained by simply suppressing the household index h.[14,15]

Consider for example a unit change in the productivity of producer i. Assume for the sake of simplicity that factor supplies are fixed. Intuitively, for given factor wages, the pure technology effect the productivity shock is given by the exposure $\lambda_i^{W_h}$ of the real expenditure of household h to this producer. The productivity shock also leads to endogenous changes in the wages of the different factors $\mathrm{d}\log w_f$, which, given that factor supplies are fixed, coincide with the changes in their factor income shares $\mathrm{d}\log \Lambda_f$. The reallocation effect depends, for each factors f, on the change $\mathrm{d}\log \Lambda_f$ in the wage of that factor, and on the difference $\Lambda_f^h - \Lambda_f^{W_h}$ between the share of a that factor in the household's income and of the household's exposure of real expenditure to that factor.[16]

As mentioned above, these results can also be applied at the country level and at the world level. At the country level, and unlike real output, real expenditure *does* respond to productivity shocks outside the country in general because these shocks trigger reallocations of resources. In particular, while shocks to iceberg trade costs outside a country do not affect its real output or its productivity, they do affect its real expenditure. At the world level, there are no reallocation effects.[17]

Furthermore, the "pure" technology effect and the reallocation effect at the household and country levels aggregate up to their world counterparts. This implies that the effects of

[14]When applied at the country level, the theorem and its interpretation goes through unchanged when agents in the country have homogeneous preferences. When there are agents with heterogeneous preferences within the country, the overall expression for $\mathrm{d}\log W_c$ as a function of $\mathrm{d}\log A_i$ and $\mathrm{d}\log L_f$ is still correct, but the decomposition is harder to interpret. This is because in general, we can no longer define the level of country c real expenditure via a function $\mathcal{W}_c(A, L, \mathcal{X})$. Then term $\sum_{f\in F}\Lambda_f^{W_c}\,\mathrm{d}\log L_f + \sum_{i\in N}\lambda_i^{W_c}\,\mathrm{d}\log A_i$ then coincides with the changes in real expenditure at initial shares (not chain-weighted), holding the allocation of resources constant, which is equivalent, up to the first order, with the change in real expenditure valued at the initial prices, holding the allocation of resources constant. The term $\sum_{f\in F}(\Lambda_f^h - \Lambda_f^{W_h})\,\mathrm{d}\log\Lambda_f + (1/\chi_h^W)\,\mathrm{d}\,T_h$ is the residual of the chain-weighted change in real expenditure. With some abuse of notation, we still write $(\mathrm{d}\log\mathcal{W}_c/\,\mathrm{d}\log L)\,\mathrm{d}\log L + (\mathrm{d}\log\mathcal{W}_c/\,\mathrm{d}\log A)\,\mathrm{d}\log A$ for the former and $(\mathrm{d}\log\mathcal{W}_c/\,\mathrm{d}\,\mathcal{X})\,\mathrm{d}\,\mathcal{X}$ for the latter. These subtleties can be ignored when country welfare is computed via a Bergson-Samuelson welfare function with no desire to redistribute at the initial point. Similar observations apply at the world level.

[15]We have applied our decomposition to changes in real expenditure or welfare, but we could also apply it to changes in real output. The "pure" technology effects would then be the changes in real output that take place if the allocation of resources were held constant, and the reallocation effects would be the changes in real output that would be due to the reallocation of resources. The point is that the reallocation effect would be zero, and the "pure" technology effects would be given as a function of shocks to productivities and factor supplies by Theorem 1.

[16]The same reasoning applies to shocks to factor supplies. Shocks to transfers contribute only through the reallocation effect directly by increasing income via the term $(1/\chi_h^W)\,\mathrm{d}\,T_h$ and also indirectly via the endogenous changes in the factor income shares $\mathrm{d}\log\Lambda_f$ that they bring about.

[17]This follows immediately from the results since since $\Lambda_f^Y = \Lambda_f^W = \Lambda_f$ for all factors f and since $\mathrm{d}\,T = 0$.

household and country reallocations sum up to zero:

$$\sum_{h\in H} \chi_h^W (\mathrm{d}\log \mathcal{W}_h / \mathrm{d}\,\mathcal{X})\,\mathrm{d}\,\mathcal{X} = \sum_{c\in C} \chi_c^W (\mathrm{d}\log \mathcal{W}_c / \mathrm{d}\,\mathcal{X})\,\mathrm{d}\,\mathcal{X} = (\mathrm{d}\log \mathcal{W} / \mathrm{d}\,\mathcal{X})\,\mathrm{d}\,\mathcal{X} = 0.$$

Reallocation effects can therefore be interpreted as zero-sum distributive changes.

It is easy to imagine that if all production functions and all demand aggregators are Cobb-Douglas, then the allocation matrix is constant and so are factor income shares, implying that $\mathrm{d}\log\Lambda_f = 0$ for all factors f. Indeed, this will follow immediately from the results in Section 4. In this case, if in addition there are no shocks to transfers, then there are no reallocation effects and only "pure" technology effects. A Cobb-Douglas economy therefore provides a useful benchmark where there are only "pure" technology effects and no reallocation effects in changes in real expenditure or welfare.[18]

3.3 Terms of Trade vs. Reallocation

Theorems 2 and 3 provide two decompositions of changes in welfare or real expenditure with different economic interpretations: the terms-of-trade and reallocation decompositions. The goal of this section is to we compare them.

Two Hulten-Like Results

To frame our discussion, it is useful to start by stating two different versions of Hulten's theorem for real expenditure or welfare in open economies. We present them at the level of a country c, and we explain how to adapt them to the level of a household h and at the world level.

Corollary 1 (Welfare, Two Hulten-like Results)**.** *In the following two special cases, Hulten-like results give changes in the real expenditure or welfare of a country c as exposure-weighted sums of productivity and factor supply shocks (and do not feature changes in factor shares).*

i Assume that the country receives no transfers from the rest of the world (balanced trade) and faces exogenous international prices (small-open economy). Then there are only real output effects but

[18]Cole and Obstfeld (1991) also who showed that a Cobb-Douglas economy is a useful benchmark in another sense: in a Cobb-Douglas economy, in the absence of initial transfers, and with log utilities, there are no gains from allowing agents to trade financial assets to insure each other against country-specific productivity shocks. These observations are related. Indeed, with log utilities, assuming that there are no initial transfers but not that the economy is Cobb Douglas, perfect risk sharing requires transfers $\mathrm{d}\,T_h = \chi_h^W = -\sum_{f\in F}\Lambda_f^h\,\mathrm{d}\log\Lambda_f$ ensuring that $\mathrm{d}\log W_f = -\,\mathrm{d}\log P_{W_f} = \sum_{i\in N}\lambda_i^{W_h}\,\mathrm{d}\log A_i + \sum_{f\in F}\Lambda_f^{W_h}\,\mathrm{d}\log w_f$ or equivalently $\mathrm{d}\log W_f = \sum_{i\in N}\lambda_i^{W_h}\,\mathrm{d}\log A_i + \sum_{f\in F}\Lambda_f^{W_h}\,\mathrm{d}\log L_f - \sum_{f\in F}\Lambda_f^{W_h}\,\mathrm{d}\log\Lambda_f$. These transfers become zero so that $\mathrm{d}\,T_h = 0$ when the economy is Cobb Douglas, and we then get $\mathrm{d}\log W_f = \sum_{i\in N}\lambda_i^{W_h}\,\mathrm{d}\log A_i + \sum_{f\in F}\Lambda_f^{W_h}\,\mathrm{d}\log L_f$.

no terms-of-trade effects and no transfer effects, so that the change in real expenditure or welfare is given by:

$$\mathrm{d}\log W_c = \mathrm{d}\log Y_c = \sum_{f\in F} \Lambda_f^{Y_c}\,\mathrm{d}\log L_f + \sum_{i\in N} \lambda_i^{Y_c}\,\mathrm{d}\log A_i.$$

ii *Assume either that the world economy is Cobb-Douglas or, if it is not, that we keep the allocation of resources (the allocation matrix) constant. Then there are only "pure" technology effects but no reallocation effects, so that the change in real expenditure or welfare is given by:*

$$\mathrm{d}\log W_c = \sum_{f\in F} \Lambda_f^{W_h}\,\mathrm{d}\log L_f + \sum_{i\in N} \lambda_i^{W_h}\,\mathrm{d}\log A_i.$$

The change $\mathrm{d}\log W_c$ *of the real expenditure of country c can be obtained by simply replacing the household index h by the country index c. The change* $\mathrm{d}\log W$ *of world real expenditure can be obtained by simply suppressing the household index h.*

Corollary 1 shows that in some special cases, we can continue to use exposures to predict the effects of productivity and factor supply shocks on real expenditure or welfare in open economies.

The two Hulten-like results are very different. Indeed, and focusing on productivity shocks, the elasticities $\mathrm{d}\log W_c / \mathrm{d}\log A_i$ of real expenditure to productivity shocks are given by exposures in real output $\lambda_i^{Y_c}$ in case (i) and by exposures in real expenditure $\lambda_i^{W_h}$ in case (ii).

The intuitions underlying the two Hulten-like results changes in real expenditure or welfare in open economies (e.g. a household or country) are also very different. The original Hulten theorem applies in a closed economy (e.g. the world) where there are neither terms-of-trade effects nor reallocation effects. In case (i), there are no terms-of-trade effects but there are reallocation effects. In case (ii), there are no reallocation effects, but there are terms-of-trade effects.[19]

Moving away from these special cases, this shows that in the terms-of-trade and reallocation decompositions, we can interpret real output effects and "pure" technology effects as Hulten-like terms, and terms-of-trade effects (together with transfers) and reallocation effects as adjustment terms. As we saw earlier, these adjustment terms are of a zero-sum distributive nature in that they sum up to zero at the world level.

[19]The reason why we get a general Hulten-like result for changes in real output in open economies in Theorem 1 is also due to the aforementioned fact that, in contrast to the fact that there are in general reallocation effects in changes in real expenditure (except in special cases like (ii)), there are no reallocation effects in changes in real output. Theorem 1 is in turn a crucial part of case (i) and so the absence of reallocation effects also plays a role in the obtention of this Hulten-like result. In one way or another, Hulten-like results originate from the absence of reallocation effects.

Comparing the Terms-of-Trade and Reallocation Decompositions

With Corollary 1 in hand, we can discuss the similarities and differences between our two decompositions.[20] We start with the similarities. Both decompositions can be applied at the level of a household, a country, and for the world. Both decompositions isolate a distributive zero-sum term, which aggregates up to zero at the level of the world economy. These different distributive terms are responsible for departures from two different versions of Hulten's theorem.

We now turn to the differences. The decompositions have different robustness and aggregation properties, and data requirements. In these regards, the reallocation decomposition has several advantages. First it is based on general equilibrium counterfactual: "pure" changes in technology coincide with the change in real expenditure that would arise under the feasible counterfactual allocation which keeps the allocation of resources constant. This is not the case for the terms-of-trade decomposition: changes in real output are *not* the changes in real expenditure that would arise under a feasible counterfactual allocation.

Second, we will see in Section 4 that this particular general equilibrium counterfactual is extremely useful conceptually and intuitively in order to unpack our counterfactual results. This is because reallocation effects (but not "pure" technology effects) depend *only* on substitution by the different producers and households in the economy. By contrast, terms-of-trade effects also include technology effects.

Third, the reallocation decomposition is not sensitive to irrelevant changes in the environment, because it does not use changes in real output. This is not the case for the terms-of-trade decomposition: for example, assuming that changes in iceberg trade costs apply to the importers of a good or to its exporter simply produces different representations of the same underlying changes in the economy and is immaterial for changes in real expenditure, but it does modify the changes in terms of trade of the importers and of the exporter.

Fourth, the reallocation decomposition can be aggregated from the household level to the country level. This is not the case for the terms-of-trade decomposition which uses household and country real output, since changes in household real output do not aggregate up to their country counterpart.

In Section 7, we provide an empirical implementation of our decomposition in a growth- and welfare- accounting exercise at a yearly frequency assuming a representative household in every country. In this context, we find that terms-of-trade effects are typically small and that reallocation effects are typically large. Roughly speaking, for a typical country and a typical year, real expenditure and real output change roughly by the same amount, terms-of-trade effects are small, and reallocation effects are large. Case (i) of Corollary 1 therefore

[20] We refer the reader to Appendix **??**, which provides some illustrative examples.

does a good job of capturing the typical year-to-year times series for a typical country. Importantly, this is not because most countries are approximately closed since this would imply that reallocations effects would be approximately zero. Furthermore, this does not mean that that we should expect terms-of-trade effects to be small for specific counterfactual questions. For example, terms-of-trade effects in response to changes in iceberg costs underly the entirety of gains-from-trade calculations.

4 Comparative Statics: Ex-Ante Sufficient Statistics

In Section 3, we related responses to shocks of real output and real expenditure or welfare to exposures and changes in factor income shares (ex-post sufficient statistics). In this section we characterize the responses to shocks of factor income shares as a function of sufficient-statistic microeconomic primitives: the HAIO matrix and elasticities of substitution in production and in consumption (ex-ante sufficient statistics). The results of the two sections can be combined. We also characterize the responses to shocks of all prices and quantities. We focus on productivity shocks because shocks to factor supplies and to icerberg trade costs can be captured as such. Transfer shocks are covered in Appendix I.

4.1 Setup

We start by introducing our setup and defining some new input-output concepts.

Nested-CES Economies

Throughout this section, we restrict our attention to the class of models that belong to the nested-CES class, where each production function and each demand aggregator is a nested-CES function, with an arbitrary number of nests and arbitrary elasticities. The reason for this is pedagogical clarity and ease of intuition. We refer the reader to Appendix D for a discussion of how to generalize all of our results and intuitions to arbitrary economies with non-nested-CES production functions and demand aggreators.

Since we restrict our attention to nested-CES models, we can always relabel the network and rewrite the HAIO matrix in such a way that each household consumes a single good and each producer corresponds to a single CES nest, with a single elasticity of substitution. We adopt this representation, which we call the *standard form* of the economy.

Input-Output Concepts

We use the following matrix notation throughout. For a matrix X, we define $X^{(i)}$ to be its ith row and $X_{(j)}$ to be its jth column.

We define the *input-output covariance operator* to be

$$Cov_{\Omega^{(k)}}(\Psi_{(i)}, \Psi_{(j)}) = \sum_{l \in N+F} \Omega_{kl} \Psi_{li} \Psi_{lj} - \left(\sum_{l \in N+F} \Omega_{kl} \Psi_{li} \right) \left(\sum_{l \in N+F} \Omega_{kl} \Psi_{lj} \right).$$

It is the covariance between the ith and jth columns of the Leontief inverse using the kth row of the input-output matrix as the probability distribution.

We define two propagation matrices, the $(N+F) \times (N+F)$ propagation-via-substitution matrix Γ to be

$$\Gamma_{ij} = \sum_{k \in N+F} (\theta_k - 1) \frac{\lambda_k}{\lambda_i} Cov_{\Omega^{(k)}} \left(\Psi_{(i)}, \Psi_{(j)} \right),$$

and the $(N+F) \times F$ propagation-via-redistribution matrix Ξ to be

$$\Xi_{if} = \frac{1}{\lambda_i} \sum_{h \in H} (\lambda_i^{W_h} - \lambda_i) \Phi_{hf} \Lambda_f,$$

where we write λ_i and Λ_i interchangeably when $i \in F$ is a factor. The reason for these labels will become apparent shortly.

4.2 Comparative Statics

Armed with these definitions, we now state our ex-ante comparative-static results.

Factor Shares and Sales Shares (Domar Weights)

We start with a characterization of the responses of factor income shares.

Theorem 4 (Factor Shares and Sales Shares (Domar Weights))**.** *The changes in the factor income shares (factor Domar weights) in response to a productivity shock to producer i are the solution of the following system of linear equations:*

$$\frac{\mathrm{d} \log \Lambda_f}{\mathrm{d} \log A_i} = \Gamma_{fi} - \sum_{g \in F} \Gamma_{fg} \frac{\mathrm{d} \log \Lambda_g}{\mathrm{d} \log A_i} + \sum_{g \in F} \Xi_{fg} \frac{\mathrm{d} \log \Lambda_g}{\mathrm{d} \log A_i}.$$

Given these changes in factor income shares, the changes in producer sales shares (producer Domar

weights) in response to a productivity shock to producer i are:

$$\frac{\mathrm{d}\log\lambda_j}{\mathrm{d}\log A_i} = \Gamma_{ji} - \sum_{g\in F}\Gamma_{jg}\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_i} + \sum_{g\in F}\Xi_{jg}\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_i}.$$

More generally, we can use the characterization of the responses of factor shares to characterize the responses of the input-output matrix, of the Leontief inverse matrix, and of all the income shares and all the exposures in real output and real expenditure or welfare at the household, country, and world levels. [21]

Consider for example the response $\mathrm{d}\log\Lambda_f/\mathrm{d}\log A_i$ of the income share of factor f to a positive unit shock to the productivity of producer i. For fixed factor prices, every producer k will substitute across its inputs in response to this shock. Suppose that $\theta_k > 1$, so that producer k substitutes (in expenditure shares) *towards* those inputs j that are more reliant on producer i, captured by Ψ_{ji}, the more so, the higher is $\theta_k - 1$. Now, if those inputs are also more reliant on factor f, captured by a high $Cov_{\Omega^{(k)}}\left(\Psi_{(f)}, \Psi_{(i)}\right)$, then substitution by k will increase demand for factor f and hence the income share of factor f. These substitutions, which happen at the level of each producer k, must be summed across producers leading to the first propagation-via-substitution term Γ_{fi}.

This shock also also triggers changes in factor prices which then sets off additional rounds of substitution in the economy that we must account for. The change in the price of each factor g is given by $\mathrm{d}\log w_g/\mathrm{d}\log A_i = \mathrm{d}\log\Lambda_g/\mathrm{d}\log A_i$. The effect on the share of factor f is the same as that of a set of equivalent negative productivity shock to the different factors, leading to the second propagation-via-substitution term $-\sum_{g\in F}\Gamma_{fg}\,\mathrm{d}\log\Lambda_g/\mathrm{d}\log A_i$.

These changes in factor prices also change the distribution of income across consumers. This in turn affects the demand for the factor f since the different households are differently exposed, directly and indirectly, to factor f, leading to the propagation-via-redistribution term $\sum_{g\in F}\Xi_{fg}\,\mathrm{d}\log\Lambda_g/\mathrm{d}\log A_i$.

These formulas show that Cobb-Douglas assumptions, prevalent in the literature which incorporates production networks in trade models for their analytical convenience, are also special (see e.g. Costinot and Rodriguez-Clare, 2014; Caliendo et al., 2017). Whenever $\theta_k =$

[21] For example, we have:

$$\frac{\mathrm{d}\log\Omega_{lm}}{\mathrm{d}\log A_i} = (\theta_l - 1)\frac{1}{\Omega_{lm}}Cov_{\Omega^{(l)}}(I_{(m)}, \Psi_{(i)}) + \sum_{g\in F}\frac{1}{\Omega_{lm}}(\theta_l - 1)Cov_{\Omega^{(l)}}(I_{(m)}, \Psi_{(g)})\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_i},$$

$$\frac{\mathrm{d}\log\Psi_{lm}}{\mathrm{d}\log A_i} = \sum_{k\in N+F}(\theta_k - 1)\frac{\Psi_{lk}}{\Psi_{lm}}Cov_{\Omega^{(k)}}(\Psi_{(m)}, \Psi_{(i)}) + \sum_{g\in F}\sum_{k\in N+F}(\theta_k - 1)\frac{\Psi_{lk}}{\Psi_{lm}}Cov_{\Omega^{(k)}}(\Psi_{(m)}, \Psi_{(f)})\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_i},$$

where I is the identity matrix.

1, the term accounting for expenditure substitution by producer k in the propagation-via-substitution matrix Γ is equal to zero. We will return to this issue in Section 5 and show that taking into account the fact that many empirical estimates of elasticities of substitution across industries in production and in final demand are below one significantly increases estimates of gains from trade.

Real Output and Real Expenditure or Welfare

Theorem 4 gives the endogenous responses of factor shares to shocks as a function of microeconomic primitives. These were left implicit in Theorems 1, 2, and 3. Theorem 4 can therefore be used in conjunction with Theorems 1, 2, and 3, to characterize the responses to shocks of real output, real expenditure or welfare, as a function of microeconomic primitives, up to the first order, at the household, country, and world levels.

Theorem 4 also gives the endogenous responses to shocks of producer sales shares or Domar weights. Since the response of real output to productivity shocks is given by the corresponding local Domar weight, Theorem 4 can also be used to give the response of real output to shocks, up to the second order:

$$\frac{\mathrm{d}\log Y_c}{\mathrm{d}\log A_j} = \lambda_j^{Y_c}, \quad \frac{\mathrm{d}^2\log Y_c}{\mathrm{d}\log A_j\,\mathrm{d}\log A_i} = \frac{\mathrm{d}\,\lambda_j^{Y_c}}{\mathrm{d}\log A_i} = \lambda_j^{Y_c}\left(\frac{\mathrm{d}\log\lambda_j}{\mathrm{d}\log A_i} - \sum_{f\in N_c}\Lambda_f^{Y_c}\frac{\mathrm{d}\log\Lambda_f}{\mathrm{d}\log A_i}\right),$$

where $\mathrm{d}\,\lambda_j/\,\mathrm{d}\log A_i$ and $\mathrm{d}\log\Lambda_f/\,\mathrm{d}\log A_i$ are given by Theorem 4.[22]

Prices and Quantities

Armed with Theorem 4, it is straightforward to characterize the responses to shocks of prices and quantities.[23]

Corollary 2. *(Prices and Quantities) The changes in the wages of factors and in the prices and quantities of goods in response to a productivity shock to producer i are given by:*

$$\frac{\mathrm{d}\log w_f}{\mathrm{d}\log A_i} = \frac{\mathrm{d}\log\Lambda_f}{\mathrm{d}\log A_i},$$

[22]Note that country real output is only defined in changes, and these changes are no the differential of a level function when there are heterogeneous. This means that the any discrete change in real output depends on the path of integration. The precise way to proceed is to index the economy by a continuous index (say time t), which indexes all the relevant shifters and all the equilibrium variables. We can then compute changes in real output between the initial period ($t = 0$) and some final period ($t = \tau$) as the integral of the infinitesimal real output changes along the resulting path. We can then approximate the change in real output for small τ up to the first and second order. The corresponding limits are independent of the path.

[23]Recall that prices are expressed in the numeraire where $GDP = GNE = 1$ at the world level.

$$\frac{\mathrm{d}\log p_j}{\mathrm{d}\log A_i} = -\Psi_{ji} + \sum_g \Psi_{jg}\frac{\mathrm{d}\log w_g}{\mathrm{d}\log A_i},$$
$$\frac{\mathrm{d}\log y_j}{\mathrm{d}\log A_i} = \frac{\mathrm{d}\log \lambda_j}{\mathrm{d}\log A_i} - \frac{\mathrm{d}\log p_j}{\mathrm{d}\log A_k},$$

where $\mathrm{d}\log\Lambda_f/\mathrm{d}\log A_i$ *is given in Theorem 4.*

These results on the responses of prices and quantities to productivity shocks, and hence by implication to shocks to factor supplies and to iceberg trade costs, generalize the classic results of Stolper and Samuelson (1941) and Rybczynski (1955).[24]

4.3 Aggregating Elasticities

In this section, we show how to use Theorem 4 for various questions involving the aggregation of elasticities: trade elasticities, the factor bias of trade, and the elasticities of factor demand systems.

Trade Elasticities

We start by defining a class of aggregate elasticities. Consider two sets of producers I and J. Let $\lambda_I = \sum_{i\in I}\lambda_i$ and $\lambda_J = \sum_{j\in J}$ be the aggregate sales shares of producers in I and J, and let $\chi_i^I = \lambda_i/\Lambda_I$ and $\chi_j^J = \lambda_j/\Lambda_J$. Let k be another producer. We then define the following aggregate elasticities capturing the bias towards I vs. J of a productivity shock to m as:

$$\varepsilon_{IJ,m} = \frac{\partial(\lambda_I/\lambda_J)}{\partial\log A_m},$$

where the partial derivative indicates that we allow for this elasticity to be computed holding some things constant.

To shed light on trade elasticities, we proceed as follows. Consider a set of producers $S \subseteq N_c$ in a country c. Let J be denote a set of domestic producers that sell to to producers in S, and I denote a set of foreign producers than sell to producers in S. Without loss of generality, using the flexibility of network relabeling, we assume that producers in I and J are specialized in selling to producers in S so that they do not sell to producers outside of S.

Consider an iceberg trade cost modeled as a negative productivity shock $\mathrm{d}\log(1/A_m)$ to some producer m. We then define the trade elasticity as $\varepsilon_{IJ,k} = \partial(\lambda_J/\lambda_I)/\partial\log(1/A_m) = \partial(\lambda_I/\lambda_J)/\partial\log A_m$. As already mentioned, the partial derivative indicates that we allow for this elasticity to be computed holding some things constant. There are therefore different

[24]See Appendix I for a discussion of how our, by taking a limit, our results can be applied to economies where traded goods are perfect substitutes as assumed by these theorems.

trade elasticities, depending on exactly what is held constant. Different versions of trade elasticities would be picked up by different versions of gravity equations regressions with different sorts of fixed effects and at different levels of aggregation.

There are several possibilities for what to hold constant, ranging from the most partial equilibrium to the most general equilibrium. At one an extreme, we can hold constant the prices of all inputs for all the producers in I and J and the relative sales shares of all the producers in S:

$$\varepsilon_{IJ,m} = \sum_{s\in S}\sum_{i\in I}\chi_i^I(\theta_s-1)\frac{\lambda_s}{\lambda_i}Cov_{\Omega^{(s)}}(I_{(i)},\Omega_{(m)}) - \sum_{s\in S}\sum_{j\in J}\chi_j^J(\theta_s-1)\frac{\lambda_s}{\lambda_j}Cov_{\Omega^{(s)}}(I_{(j)},\Omega_{(m)}),$$

where $I_{(i)}$ and $I_{(j)}$ are the ith and jth columns of the identity matrix. An intermediate possibility is to hold constant the wages of all the factors in all countries:

$$\varepsilon_{IJ,k} = \sum_{i\in I}\chi_i^I\Gamma_{ik} - \sum_{j\in J}\chi_j^J\Gamma_{jk}.$$

And at the other extreme, we can compute the full general equilibrium:

$$\varepsilon_{IJ,m} = \sum_{i\in I}\chi_i^I\left(\Gamma_{im} - \sum_{g\in F}\Gamma_{ig}\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_m} + \sum_{g\in F}\Xi_{ig}\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_m}\right) - \sum_{j\in J}\chi_j^J\left(\Gamma_{jm} - \sum_{g\in F}\Gamma_{jg}\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_m} + \sum_{g\in F}\Xi_{jg}\frac{\mathrm{d}\log\Lambda_g}{\mathrm{d}\log A_m}\right),$$

$\mathrm{d}\log\Lambda_f/\mathrm{d}\log A_m$ is given in Theorem 4.

The trade elasticity is a linear combination of microeconomic elasticities of substitution, where the weights depend on the input-output structure. Except at the most microeconomic level where there is a single producer s in S and in the most partial-equilibrium setting where we recover $\epsilon_s - 1$, this means that the aggregate trade elasticity is typically an endogenous object, since the input-output structure is itself endogenous.[25] Furthermore, in the presence of input-output linkages, it is typically nonzero even for trade shocks that are not directly affecting the sales of I to J, except in the most partial-equilibrium setting.

Factor Bias of Trade

To shed light on the factor bias of trade, we specialize the the two sets of producers I and J to be sets of factors, and we consider as above an iceberg trade cost modeled as a negative

[25] In Appendix J, we provide necessary and sufficient conditions for the trade elasticity to be constant in the way.

productivity shock $\mathrm{d}\log(1/A_m)$ to some producer m. Then $\varepsilon_{IJ,m}$ measures the bias of the trade shock towards I vs. J.

For example, consider a country c, two factors f and g in F_c, and let $I = \{f\}$ and $J = \{g\}$.The factor bias towards factor f vs. factor g of the trade shock is then simply $\varepsilon_{fg,m} = \partial \log(\Lambda_f/\Lambda_g)/\partial \log A_m$. Holding constant the wages of all the factors in all countries, we find $\varepsilon_{fg,m} = \Gamma_{fm} - \Gamma_{gm}$. In full general equilibrium, we find via Theorem 4 that $\varepsilon_{fg,k} = \mathrm{d}\log\Lambda_f/\,\mathrm{d}\log A_m - \mathrm{d}\log\Lambda_g/\,\mathrm{d}\log A_m$.[26]

Elasticities of Factor Demand Systems

Working with a model with a representative household in every country, Adao et al. (2017) show that economies of the sort that we consider can be represented as economies in which only factors are traded within and across borders, and households have preferences over factors. Their construction can be adapted to economies with heterogeneous agents within a country.

Theorem 4 can be used to flesh out this representation by locally characterizing its associated reduced-form Marshallian demand for factors in terms of sufficient-statistic microeconomic primitives: the expenditure share of household h on factor f is given by $\Psi_h f$; the elasticities $\partial \log \Psi_h f/\partial \log A_i$ holding factor prices constant then characterize its Marshallian price elasticities as well as its Marshallian elasticities with respect to iceberg trade shocks, since shocks to factor prices and to iceberg trade costs can be represented as productivity shocks via relabeling:[27]

$$\frac{\partial \log \Psi_{hf}}{\partial \log A_i} = \sum_{k \in N+F} \frac{\Psi_{hk}}{\Psi_{hf}} (\theta_k - 1) Cov_{\Omega^{(k)}}(\Psi_{(f)}, \Psi_{(i)}).$$

This representation is the right way of representing an economy with intermediate inputs as a value-added economy. As we shall emphasize multiple times below, all other attempts are basically flawed.[28]

[26]We could also use this notion to shed light on the country bias of trade, by letting $I = F_c$ and $J = F_{c'}$ for two countries c and c', in which case $\varepsilon_{F_c F_{c'},m}$ would measure the elasticity of the relative nominal output or GDP of c vs. c' with respect to the trade shock.

[27]This formula also clarifies the conditions under which the reduced-form factor demand system is locally stable with respect to a single shock $\mathrm{d}\log A_i$ shifting technologies or preferences if and only if $\partial \log \Psi_h f \partial \log A_i = 0$, with a similar conditions for a combination of such shocks.

[28]For some welfare counterfactuals, this representation can be put to use by directly specifying and estimating a parsimoniously-parametrized factor demand system, thereby eschewing altogether the specification and estimation of the full model. This parsimony advantage must of course be traded of against the cost of misspecification, and, as our formulas make clear, the "true" functional form of the factor demand system is likely to be extremely complex for realistic economies. Furthermore, implementing such an approach in theory and in practice is more difficult in the presence of intermediate goods since shocks to iceberg costs of trading goods between two countries must be first be translated into shocks to costs of trading the factors used directly

4.4 Simple Illustrative Examples

We provide a two simple illustrative examples illustrating the endogeneity of the trade elasticity and the nonlinearities of the effects of trade and productivity shocks. We refer the reader to Appendix J for another example illustrating the factor bias of trade, and showing adverse trade shocks can reduce the capital share in all countries.

Example: Trade Elasticity in a Round-About Economy

In many trade models, the trade elasticity, defined holding factor wages constant, is an invariant structural parameter. A notable exception is provided by Yi (2003). Yi observes that the response of trade volumes to reductions in trade costs has varied greatly over time — large reductions in trade barriers resulted in relatively small movements in the volume of trade during the middle of the 20th century, whereas small reductions caused large changes in the 80s and 90s. Yi's explanation for these facts is the existence of vertical specialization, or in our language, the presence of intermediate inputs, which can cause the trade elasticity to increase as trade costs fall even when all the structural microeconomic elasticities are constant and uniform. The example, which uses the analysis in Section 34, gives a flavor of Yi's basic insight.

Consider the two-country, two-good economy depicted in Figure 1. The representative household in each country only consumes the domestic good, which is produced using domestic labor and imports with a CES production function with elasticity of substitution θ. We consider the imposition of a trade cost hitting imports by country 1 from country 2. For the sake of illustration, not assume that the trade cost does not hit the exports of country 1 to country 2.

The trade elasticity holding factor wages constant is given by

$$\frac{\theta - 1}{1 - \Omega_{21}\Omega_{12}},$$

where Ω_{ij} is the expenditure share of i on j, e.g. its intermediate input import share. We see that as the intermediate input shares increase, the trade elasticity becomes larger. Simple trade models without intermediate goods are incapable of generating these kinds of patterns.

Of course, since the intermediate input shares Ω_{ij} are themselves endogenous (depending on the iceberg shock), this means that the trade elasticity varies with the iceberg shocks.

and indirectly to produce them, which must be handled by expanding the set of factors to reflect the fact that the same factor may be crossing boundaries a different number of times along different global supply chains. These difficulties are compounded in the presence of tariffs.

In particular, if $\theta > 1$, then the trade elasticity increases as iceberg costs on imports fall in all countries since intermediate input shares rise. This increase is nonlinear since the trade elasticity has finite limits when the trade cost goes to zero and to infinity. [29]

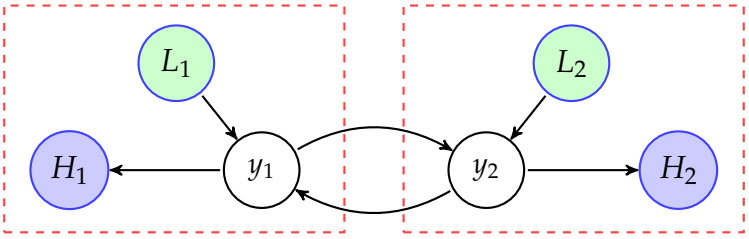


Figure 1: The solid lines show the flow of goods. Green nodes are factors, purple nodes are households, and white nodes are goods. The boundaries of each country are denoted by dashed box.

Example: Baumol's Cost-Disease and Export-Led Growth

We illustrate the nonlinear effects of trade and productivity shocks on real output discussed in Section 4.2 via a simple example showing how opening up to trade and relying on export-led growth can overcome Baumol's cost disease. Baumol's cost disease is a phenomenon whereby in the presence of complementarities across sectors, the relative sales of sectors with relatively faster productivity growth shrink over time as a result of their higher productivity growth. It reduces down the growth rate of aggregate productivity over time. As discussed by Baqaee and Farhi (2017a), Baumol's cost disease is a manifestation of nonlinearities.

Consider the economy depicted in Figure 2. Countries 1 and 2 produce varieties of wine and cloth. The representative household in each country consumes a composite of foreign and domestic varieties of wine and cloth. We assume that the elasticity of substitution across foreign and domestic varieties of wine or cloth is $\theta > 1$, but that the elasticity of substitution between wine and cloth is $\sigma < 1$. To simplify the algebra, assume that there is no home-bias, so that both households consume the same basket of wine and cloth. Finally, we assume that wine and cloth have the same size at the initial point so that $\lambda^{Y}_{cloth} = \lambda^{Y}_{wine} = 1/2$ and $\lambda^{Y_1}_{cloth_1} = \lambda^{Y_1}_{wine_1} = 1/2$. The relative size of country 1 at the initial point is χ^{Y}_1. It is also the share of country 1's varieties in the overall baskets of wine and cloth. It therefore also indexes the degree of openness of country 1: when $\chi^{Y}_1 = 1$, it is a closed world economy; when $\chi^{Y}_1 = 0$, country it is a small open economy.

[29] In Appendix J we show that there it is possible to generate "trade re-switching" examples where the trade elasticity is non-monotonic with the trade cost (or even has the "wrong" sign) in otherwise perfectly respectable economies. These examples are analogous to the "capital re-switching" examples at the center the Cambridge Cambridge Capital controversy.

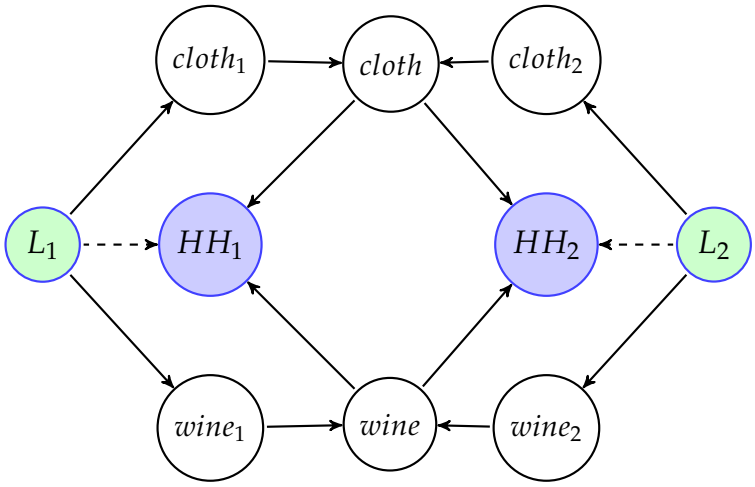


Figure 2: The solid lines show the flow of goods while the dashed lines show the flow of wage payments.

The effect on the real output or GDP of country 1 from an increase $\Delta log A_{cloth_1}$ in the productivity of its cloth is given up to the second order by[30]

$$\Delta \log Y_1 \approx \log \frac{d \log Y_1}{\Delta \log A_{cloth_1}} \Delta \log A_{cloth_1} + \frac{1}{2} \frac{\mathrm{d}^2 \log Y_1}{\mathrm{d} \log A_{cloth_1}^2} (\Delta \log A_{cloth_1})^2,$$

with

$$\frac{d \log Y_1}{d \log A_{cloth_1}} = \lambda_{cloth_1}^{Y_1} = \frac{1}{2}, \quad \frac{\mathrm{d}^2 \log Y_1}{\mathrm{d} \log A_{cloth_1}^2} = \frac{\mathrm{d} \lambda_{cloth_1}^{Y_1}}{\mathrm{d} \log A_{cloth_1}} = \frac{(\chi_1^Y)^2(\sigma - 1)}{2} + \frac{(1 - \chi_1^Y)(\theta - 1)}{4}.$$

The second-order term capture the extent to which large shocks have larger or smaller proportional effects than small shocks, conditional on the size of the sector. Cumulating rates of productivity growth over time is equivalent to increasing the size of the shock. The second-order term therefore captures the strength of Baumol's cost disease. When it is negative, Baumol's cost disease obtains. When it is positive, we have a form of reverse Baumol's cost disease.

As we increase the relative size and openness χ_1^Y of country 1, the effect on its real output of the productivity of its cloth becomes smaller because Baumol's cost disease becomes stronger. In the small-open economy limit $\chi_1^Y \to 0$, we have $\mathrm{d}^2 \log Y_1 / \mathrm{d} \log A_{cloth_1}^2 = (\theta - 1)/4 > 0$, and so there is reverse Baumol's cost disease. In the large-closed-economy limit $\chi_1^Y \to 1$, we have $\mathrm{d}^2 \log Y_1 / \mathrm{d} \log A_{cloth_1}^2 = (\sigma - 1)/2 < 0$ and so there is Baumol's cost disease.

Of course, if country 1 is small and closed, then it is as if it were a large-closed economy.

[30] We slightly abuse notation since Y_1 is not a function.

Opening up to international trade turns it into a small-open economy. Trade can therefore overcome (and indeed overturn) Baumol's cost-disease by allowing export-led growth.

5 Duality Between Open and Closed Economies

In Section 3.1, we showed that shocks to a country's terms of trade do not act like domestic productivity shocks in the sense that they do not affect its real output or productivity. In this section, we show that such shocks do act like productivity shocks on the country's real expenditure or welfare. We do so by establish a useful duality between the effects of foreign shocks to iceberg trade shocks in an open economy and the effects of domestic productivity shocks in a closed one. By bridging the open and closed economy literatures, this result allows us bring results from the (simpler) closed models to understand the behavior of open ones.

We prove that, under some conditions, the effects of iceberg trade shocks on real expenditure or welfare in an open economy can be analyzed by studying the effects of productivity shocks on real output in a dual closed economy. We can therefore shed light on the gains from trade in an open economy by leveraging the characterizations of the linear and nonlinear effects of productivity shocks on real output in closed economies provided respectively in Hulten (1978) and Baqaee and Farhi (2017a). These duality results build on a formula in Feenstra (1994) and can be seen as a generalization of the ACR results (Arkolakis et al., 2012). Unlike the other results in the paper, they do rely on the nested-CES parametric restriction.

5.1 Duality Mapping

Consider an open economy c of the nested-CES form written in standard form. Each node of the network is then a producer $i \in N_c$ with a simple CES production function with a single elasticity of substitution θ_i with associated unit-cost function[31]

$$p_i = \frac{1}{A_i}\left(\sum_{j\in N}\Omega_{ij}p_j^{1-\theta_i}\right)^{\frac{1}{1-\theta_i}}.$$

We construct a dual closed economy with the same set of producers $i \in N_c$ with CES production functions with the same set of elasticities θ_i and a HAIO matrix $\check{\Omega}$ given by $\check{\Omega}_{ij} = \Omega_{ij}/\Omega_{ic}$, where $\Omega_{ic} = \sum_{j\in N_c}\Omega_{ij}$ is the *domestic input share* of i. The unit-cost function

[31] Our results go through even when producers which do not directly use foreign inputs do not have CES production functions, but we assume for simplicity that they do.

of producer i in the dual closed economy is given by

$$p_i = \frac{1}{\check{A}_i} \left(\sum_{j \in N_c} \check{\Omega}_{ij} p_j^{1-\theta_i} \right)^{\frac{1}{1-\theta_i}}.$$

We denote by $M_c \subseteq N_c$ the set of importing producers: the domestic producers which directly use foreign inputs in non-zero amounts. For such an importing producer $i \in M_c$, we sometimes use the notation $\epsilon_i = \theta_i - 1$ since this also corresponds to the trade elasticity of this producer. We denote by $D_c = N_c - M_c$ the complement set of domestic producers.

5.2 Duality Results

To facilitate the exposition, we restrict ourselves to the case where the country c of interest has only one primary factor, which we call labor. We extend all the results to the case of multiple domestic factors in Appendix E. We also assume that there is a representative household in country c, who owns the domestic factor, and receives no transfers, but the results can be extended to the heterogeneous household case.

Let W_c denote the welfare of the representative agent. Let $\check{W}_c$ denote the welfare of the dual closed economy. Since the "inverted-hat" economy is closed, welfare is equal to real output $\check{W}_c = \check{Y}_c$.

Theorem 5 (Exact Duality)**.** *The discrete change in welfare $\Delta \log W_c$ of the original open economy in response to discrete shocks to iceberg trade costs or productivities outside of country c is equal to the discrete change in real output $\Delta \log \check{Y}_c$ of the dual closed economy in response to discrete shocks $\Delta \log \check{A}_i = -(1/\varepsilon_i) \Delta \log \Omega_{ic}$. This duality result is global in that it holds exactly for arbitrarily large shocks.*

This duality allows us to leverage results from the literature on the real output effects of productivity shocks in closed-economy models to characterize the welfare effects of trade shocks in open economy models. We use Hulten's theorem to characterize these effects to the first order, and Baqaee and Farhi (2017a) to characterize them to the second order.

Corollary 3 (First-Order Duality)**.** *A first-order approximation to the change in welfare of the original open economy is:*

$$\Delta \log W_c = \Delta \log \check{Y}_c \approx \sum_{i \in M_c} \check{\lambda}_i \Delta \log \check{A}_i,$$

where applying Hulten's theorem, $\check{\lambda}_i$ is the sales share or Domar weight of producer i in the dual closed economy.

Conditional on the size of the associated productivity shocks, the presence of intermediate inputs amplifies the effects of trade shocks much in the same way that the effect of intermediate inputs amplifies the effects of productivity shocks in closed economies. This is because (gross) sales shares are greater than (net) value-added shares, reflecting and intermediate-input multiplier discussed by, among others, Jones (2011). This observation is behind the findings of Costinot and Rodriguez-Clare (2014) that allowing for intermediate inputs significantly increases gains from trade.

Corollary 4 (Second-Order Duality)**.** *A second-order approximation to the change in welfare of the original open economy is:*

$$\Delta \log W_c = \Delta \log \check{Y}_c \approx \sum_{i \in M_c} \check{\lambda}_i \Delta \log \check{A}_i + \frac{1}{2} \sum_{i,j \in M_c} \frac{d^2 \log \check{Y}_c}{\mathrm{d} \log \check{A}_j \, \mathrm{d} \log A_i} \Delta \log \check{A}_j \Delta \log \check{A}_i,$$

where applying Baqaee and Farhi (2017a),

$$\frac{d^2 \log \check{Y}_c}{\mathrm{d} \log \check{A}_j \, \mathrm{d} \log \check{A}_i} = \frac{\mathrm{d} \check{\lambda}_i}{\mathrm{d} \log \check{A}_j} = \sum_{k \in N_c} (\theta_k - 1) \check{\lambda}_j Cov_{\check{\Omega}^{(k)}} \left(\check{\Psi}_{(i)}, \check{\Psi}_{(j)} \right).$$

To discuss these results, it is useful to assume that there is an industry structure: domestic producers are grouped into industries; the goods produced by the different producers in any given industry are aggregated into an industry good by a producer with a CES production function; and all other agents only use the goods of the producers in a given industry via the industry good. In this case, all domestic producers in a given industry are uniformly exposed to any other given domestic producer. This implies that in Corollary 4, only the elasticities of substitution across industries receive non-zero weights. The elasticities of substitution across producers within industries receive a zero weight, and they only matter via their influence on the productivity shocks $\Delta \log \check{A}_i$ through the trade elasticities. In fact, the HAIO matrix $\check{\Omega}$ of the dual closed economy can be specified entirely at the industry level where the different producers are the different industries. Given the productivity shocks $\Delta \log \check{A}_i$, Theorem 5 and Corollaries 3 and 4 can then be applied at the industry level, with this industry level HAIO matrix, and with only elasticities of substitution across industries.

Many cases considered in the literature have such an industry structure, and impose the additional assumption that all the elasticities of substitution across industries in production and in consumption are unitary. This makes the dual closed economy Cobb-Douglas. Such assumptions are made for example by ACR, Costinot and Rodriguez-Clare (2014), and Caliendo and Parro (2015). In this Cobb-Douglas case, the dual closed economy is exactly log-linear in the dual productivity shocks $\Delta \log \check{A}_i$. The effects of shocks to iceberg trade costs or to productivities outside of the country then coincide with the first-order effects of

the dual shocks given by Corollary 3. Their second-order effects given by Corollary 4 are zero (the same goes for their higher-order effects).

For example, we can recover the basic result of ACR by assuming that there is a single industry i producing only from labor so that $\check{\lambda}_i = 1$. In this case, we get $\Delta \log W_h = \Delta \log \check{A}_i$ as an exact expression. We can also recover the extension of the ACR result by Costinot and Rodriguez-Clare (2014) to allow for multiple industries and input-output linkages, but restricting elasticities of substitution across industries to be unitary. In this case, we get $\Delta \log W_h = \sum_{i \in M_c} \check{\lambda}_i \Delta \log \check{A}_i$ as an exact expression.

Our results therefore generalize the insights of ACR and Costinot and Rodriguez-Clare (2014) to models with input-output linkages and where elasticities of substitution across industries are not unitary. In such models, the dual closed economy is no longer Cobb-Douglas. Deviations from Cobb Douglas generate nonlinearities, which can either mitigate or amplify the effects of the shocks, depending on whether the second-order effects are positive or negative. This in turn depends on the elasticities of substitution and on the input-output covariances: intuitively, nonlinearities tend to amplify negative shocks if domestic producers with elasticities below unity (complementarities) are larger and more heterogeneously exposed to the trade shock than producers with elasticities above unity (substituabiities). We now solidify this intuition by way of a simple example.

Example: Gains from Trade with a Universal Input

We provide a simple example of a country c depicted in Figure 3. The only traded good is energy E.[32] The representative household in the country consumes domestic goods 1 through to N with some elasticity of substitution σ, with equal sales shares $1/N$ at the initial point. Some fraction of goods, goods 1 through to M, are made via labor L and a composite energy good E with an elasticity of substitution θ, with an initial energy share $(N/M)\check{\lambda}_E$. The composite energy good is a CES aggregate of domestic and foreign energy with elasticity of substitution $\theta_E > 1$. Domestic energy E, as well as the rest of the consumption goods, goods $M+1$ through to N, are made using only domestic labor. We assume that $\theta < 1$ and $\theta < \sigma$.

Consider an increase in iceberg trade costs which increases the cost of import of foreign energy. The welfare effect of this trade shock is the same as that of a negative productivity shock

$$\Delta \log \check{A}_E = -\frac{1}{\varepsilon_E} \Delta \log \check{\Omega}_{Ec} < 0,$$

to the energy producer of the dual closed economy, where $\varepsilon_E = \theta_E - 1$ is the trade elasticity of the energy composite good E and $\Delta \log \Omega_{Ec}$ is the change of its domestic expenditure

[32] This example is an open-economy version of an example in Baqaee and Farhi (2017a).

share.

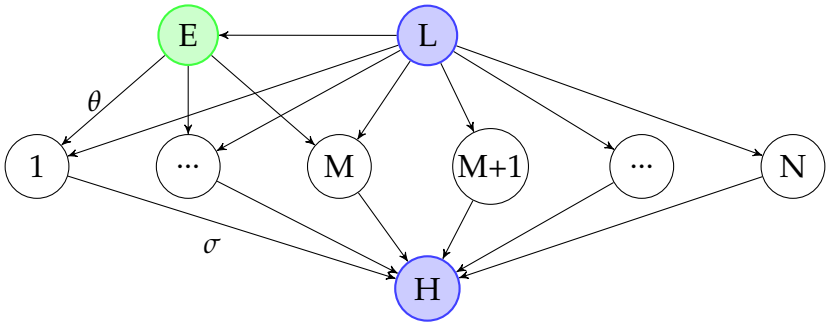


Figure 3: An illustration of the economy with a key input E. Each industry has different shares of labor and energy and substitutes across labor and energy with elasticity $\theta < 1$. The household can substitute across goods with elasticity of substitution $\sigma > \theta$. Energy is a traded good, which can either be produced domestically or sourced from the rest of the world, with an elasticity of substitution $\theta_E > 1$ between the two.

By Corollary 4 the impact of the trade shock on welfare up to the second order is

$$\Delta \log W_c \approx \check{\lambda}_E \Delta \log \check{A}_E + \frac{1}{2}\check{\lambda}_E \left[\check{\lambda}_E \left(1 - \frac{M}{N}\right)(\sigma - 1) + \left(1 - \frac{N}{M}\check{\lambda}_E\right)(\theta - 1)\right] (\Delta \log \check{A}_E)^2.$$

When $M = N$, so E becomes a universal input, and the elasticity of substitution in consumption σ drops out of the expression. This is because the different goods 1 through N are uniformly exposed to the trade shock, and so substitution by the household is irrelevant. Since $\theta < 1$, nonlinearities captured by the second-order term amplify the negative welfare effects of the trade shock. This is because complementarities between energy and labor imply that the sales share of energy $\check{\lambda}_E$ increases with the shock, thereby amplifying its negative effect.

When $M < N$, the elasticity of substitution in consumption σ matters. Since $\sigma > \theta$, the nonlinear adverse effect of the trade shock is reduced compared to the case $M = N$ when we keep the initial sales share of energy $\check{\lambda}_E$ constant. This is because the household can now substitute away from energy-intensive goods, which mitigates the increase of the sales share of energy $\check{\lambda}_E$, and hence the negative welfare effects of the shock. These effects are stronger, the lower is M, i.e. the more heterogeneous are the exposures of the different goods to energy.

These effects are absent in the cases analyzed by ACR and Costinot and Rodriguez-Clare (2014) who make the Cobb-Douglas assumption $\sigma = \theta = 1$, which renders the model log-linear and eliminates all nonlinearities.

Quantitative Gains from Trade: Intermediate Inputs and Nonlinearities

We apply our duality results and the World Input-Output Database (WIOD) (see Timmer et al., 2015) to study the gains from trade by comparing the welfare losses from moving different countries to autarky. We use data from 2008, which is the final year in the 2013 release of the data. The dataset has 41 countries, one of which is an aggregate Rest-of-World country, and each country has 30 industries. For more details on the industry classification scheme and the treatment of the data, see Appendix B.

We assume that production takes a nested CES form, where σ is the elasticity of substitution across industries in consumption, ζ is the elasticity of substitution between value-added and intermediate inputs, θ is the elasticity of substitution across industries in intermediate input use, ε_i is the elasticity of substitution between domestic and foreign varieties in industry i. The dual productivity shocks to the importing producers corresponding to a move to autarky of the original open economy are given by $\Delta \log \check{A}_i = -(1/\varepsilon_i) \log \Omega_{ic}$.

To calibrate the elasticities of substitution, we use the estimates from Caliendo and Parro (2015) for the trade elasticities ε_i. Our benchmark calibration is the far right column,where we set the elasticity of substitution across industries $\theta = 0.2$, the one between value-added and intermediates $\zeta = 0.5$, and the one in consumption $\sigma = 0.9$. These elasticities are broadly consistent with the estimates of Atalay (2017), Boehm et al. (2015), Herrendorf et al. (2013), and Oberfield and Raval (2014). Overall, the evidence suggests that these elasticities are all less than one (sometimes significantly so).

(σ, ζ, θ)	VA $(1,1,1)$	$(1,1,1)$	$(1, 0.5, 0.6)$	$(0.9, 0.5, 0.2)$
FRA	9.8%	18.5%	24.7%	30.2%
JPN	2.4%	5.2%	5.5%	5.7%
MEX	11.5%	16.2%	21.3%	44.5%
USA	4.5%	9.1%	10.3%	13.0%

Table 1: Gains from trade for a selection of countries. The first column is a multi-sector value-added economy with no intermediate inputs and with the Cobb-Douglas assumption. The second column allows for intermediate inputs but maintains the Cobb-Douglas assumption in the direction of complementarities. The other columns allow for intermediate inputs and relax the Cobb-Douglas assumption. The microeconomic trade elasticities are the same across all columns and taken from Caliendo and Parro (2015), so the size of the trade shock to each industry is the same across all columns.

The results of this exercise are in Table 1 for different values of the elasticities of substitution (σ, ζ, θ). The first column replicates the results of a multi-sector value-added model without intermediate inputs and with the Cobb-Douglas assumption $(\sigma, \zeta, \theta) = (1, 1, 1)$, reported in Costinot and Rodriguez-Clare (2014). The second column replicates the results

of an a model which allows for intermediate inputs but maintains the Cobb-Douglas assumption, also reported in Costinot and Rodriguez-Clare (2014). As expected, allowing for intermediate inputs increases gains from trade. This is because of the first-order or log-linear effect captured by Corollary 3: it reflects the fact that abstracting away from intermediate inputs reduces the volume of imports relative to GDP. The other columns continue to allow for intermediate inputs, but deviates from the Cobb-Douglas assumption, giving rise to nonlinearities. Moving across columns towards more complementarities increases the gains from trade. This is because of the nonlinear effect captured by Corollary 4: more complementarities magnify gains from trade by increasing nonlinearities.

The magnitudes of these different effects are different across countries. The importance of accounting for intermediate inputs is largely independent of the degree of openness of the country. By contrast, the importance of accounting for nonlinearities does depend on the degree of openness: the more open the country, the larger are the dual productivity shocks, and the more nonlinearities matter. Overall, it seems that nonlinearities are as important as intermediate goods to the study of gains from trade.

Japan is an interesting example where the losses from going to autarky are relatively moderate and do not increase much with complementarities. This is partly because Japan is relatively closed, but it may also be related to Japan's various self-sufficiency regulations, borne out of the widespread shortages during and after World War II, which seek to limit the import shares in certain critical sectors.

6 Tariffs and Other Distortions

So far, we have maintained the assumption of frictionless competitive markets. In this section, we extend our results to allow for tariffs, or more generally for arbitrary distortions modeled as explicit or implicit taxes or wedges. We start by summarizing how to generalize the ex-post and ex-ante comparative statistic results of Sections 3 and 4. However, to avoid repetitions, we relegate the details results to Appendix F. We instead focus on a different insightful perspective by showing that the losses from tariffs and other distortions are given, up to a second order, by a Domar-weighted sum of deadweight loss Harberger triangles (Harberger, 1954) in every market where they are introduced.

6.1 Allowing for Distortions

Let μ represent the $N \times 1$ matrix of tax wedges, where the ith element is an ad valorem tax on the output of producer i. To state our results, we assume that the revenue generated by the wedge μ_i are *included* in the revenue of producer i (the producer collects the tax revenue as

part of its revenue, and then pays the government). This is merely an accounting convention, and it is straightforward to convert our results for situations where the revenue generated by the wedge are not included in revenues. This allows us to capture tariffs on exports as taxes to specialized exporters who buy domestic goods and sell them across borders. This also allows us to capture tariffs on imports as taxes on specialized importers who buy foreign goods and sell them in the domestic market. These wedges can also capture other distortions such as markups, taxes, and financial constraints.

6.2 Generalizing the Comparative Statics in Sections 3 and 4

The growth- and welfare accounting results of Section 3 and the structural counterfactual results of Section 4 can be generalized to allow for arbitrary distortions. This allows to characterize the ex-ante and ex-post comparative static effects of shocks to productivities, factor supplies, and iceberg trade costs, in the presence of pre-existing tariffs and other distortions. This also allows to characterize the ex-ante and ex-post comparative static effects of tariffs and other distortions.

Distortions introduce important and interesting differences. For example, in the presence of distortions such as tariffs or markups, shocks to productivities, factor supplies, and iceberg trade costs outside of a country now typically have first-order effects on its real output and on its aggregate productivity. As a result, the Hulten-like result for real output fails. This is because there are now non-trivial first-order reallocation effects in real output. It follows that the small-open-economy Hulten-like result for real expenditure also fails. The Cobb-Douglas Hulten-like result for real expenditure or welfare continues to hold for shocks to productivities, factor supplies, and iceberg trade costs, because the absence of reallocation effects is preserved. All these Hulten-like results fail for shocks to distorting wedges. Even when these results fail, our growth- and welfare-accounting results can be generalized by characterizing the new corresponding reallocation effects as a function of changes in distorting wedges and changes in factor income shares. These changes in factor income shares are in turn characterized as functions of microeconomic primitives by generalizations of our propagation equations.

As far as we are aware, this is the first time such comparative statics have been derived in the literature, and so we consider them to be an important contribution of this paper. However, to avoid repetitions, we relegate these results and the corresponding analysis to Appendix F.

instead, in the body of the paper, we focus on a different perspective based on Harberger triangles combined with a generalization of our propagation equations to allow for distortions. As usual, we present our result in two ways, using ex-post and ex-ante sufficient

statistics.

6.3 Costs of Tariffs and Other Distortions: Ex-Post Sufficient Statistics

We write $\mu_i = \exp(\Delta \log \mu_i)$ and provide second-order approximations for small tariffs $\Delta \log \mu_i$ around an efficient (undistorted) equilibrium with no tariffs or other distortions.

Real Output

We start by characterizing changes in real output at the country and world level. Since changes in real output are equal to changes in real expenditure at the world level, we also characterize changes in real expenditure at the world level.

Theorem 6. *Starting at an efficient equilibrium, we have the following expressions, valid up to the second order, for the changes in real output at the country and world levels, for changes in real expenditure at the world level, in response to the introduction of small tariffs or other distortions:*

i changes in world real output and real expenditure are given by

$$\Delta \log Y = \Delta \log W \approx \frac{1}{2} \sum_{i \in N} \lambda_i \Delta \log y_i \Delta \log \mu_i;$$

ii changes in country c real output are given by:

$$\Delta \log Y_c \approx \frac{1}{2} \sum_{i \in N_c} \lambda_i^{Y_c} \Delta \log y_i \Delta \log \mu_i.$$

Hence, for both the world and for each country, the reduction in real output from tariffs and other distortions is given by the sum of all the deadweight-loss triangles $1/2 \Delta \log y_i \Delta \log \mu_i$ weighted by their corresponding local Domar weights. Harberger (1954) had shown that the classic welfare deadweight loss triangle intuition from partial equilibrium models could be applied to general equilibrium models to measure changes in welfare in the presence of income effects and with compensating transfers. Theorem 6 shows that it can be used in general equilibrium, in the presence of income effects, and in the absence of compensating transfers, to measure changes real output. To the best of our knowledge this is a new result in growth-accounting.

Conditional on matching both the Domar weights λ_i and $\lambda_i^{Y_c}$ and the changes in the quantities $\Delta \log y_i$ of all goods, the details of the production structure are irrelevant. In particular, conditional on these sufficient statistics, we do not need to know anything about whether or not there are international (or domestic) production networks. However, as we

shall show, input-output linkages do affect Domar weights and changes in the quantities of all goods, and so accounting for input-output linkages does matter. As we shall see, accounting for global value chains matters a great deal for the quantitative effects of tariffs: the triangles $1/2\Delta \log y_i \Delta \log \mu_i$ are larger, and they are also aggregated with larger weights given by sales shares λ_i and $\lambda_i^{Y_c}$rather than value-added shares.[33]

To give some intuition for Theorem 6, we focus on the country level result for simplicity. Starting at an efficient equilibrium, the introduction of tariffs or other distortions leads to changes $\Delta \log y_i$ in the quantities of goods $i \in N_c$ in country c and to changes in the wedges $\Delta \log \mu_i$ between prices and marginal costs. The price-cost margin $p_i \Delta \log \mu_i$ measure the wedge between the marginal contribution to country real output and the marginal cost to real output of increasing the quantity of good i by one unit. Hence, $\lambda_i^{Y_c} \Delta \log \mu_i$ is the marginal proportional increase in real output from a proportional increase in the output of good i. Integrating from the initial efficient point to the final distorted point, we find that $(1/2)\lambda_i^{Y_c} \Delta \log y_i \Delta \log \mu_i$ is the contribution of good i to the change in world real output.

Note that changes in the quantities of changes $\Delta \log y_i$ in the quantities of goods $i \in N_c$ in country c can be driven by changes in tariffs or other distortions in the country or in other countries. This shows that changes in tariffs or other distortions outside of the country affect the aggregate productivity of the country. This is in a sharp contrast to the results that we derived earlier for efficient economies.

Tariffs vs. Iceberg Trade Costs

It is instructive to compare the costs of tariffs to the costs of an increase in iceberg costs. In response to a change in iceberg costs outside of the country , the change in country real output is zero at any order of approximation. At the world level, in response to a change $\Delta \log \tau_i = \Delta \log(1/A_i)$ in iceberg trade costs, the change in real output or real expenditure is given up to a second-order by the sum of trapezoids rather than triangles:

$$\Delta \log Y = \Delta \log W \approx -\sum_{i \in N} \lambda_i \left(1 + \frac{1}{2} \Delta \log \lambda_i\right) \Delta \log \tau_i.$$

In contrast to equivalent shocks to tariffs, the first-order effect of shocks iceberg trade costs have nonzero first-order effects. This explains why the costs of non-tariff trade barriers are typically so much higher than tariff trade barriers in trade models.

[33]This partly affirms Paul Krugman's view that trade in intermediate inputs does not alter the basic qualitative logic of why trade barriers are harmful. Even in models with global value chains, tariffs are harmful because they cause deadweight losses captured by Harberger trianles. See *Does Trade in Intermediate Goods Alter the Logic of Costs From Protectionism?* by Paul Krugman (2018). However, it does turn out to alter the quantitative message though: in models with global value chains, the triangles are larger, and for a given set of triangles, they are more important.

Welfare

Theorem 6 shows how country and world real output respond to changes in tariffs or other distortions. These results cannot be used to speak to changes in welfare. Indeed, at the country level, changes in tariffs and other distortions typically lead to first-order changes terms of trade. These effects are characterized in Appendix F. But even at the world level where these effects washout, changes in real expenditure no longer coincide with changes in welfare even when we assume a Bergson-Samulelson welfare function with no desire to redistribute across agents at the initial point. This is because distributive effects across households manifest themselves at the second-order. We did not need to concern ourselves with this second-order effect in our first-order characterizations for efficient economies in Sections 3 and 4.

We introduce a homothetic Bergson-Samuelson social welfare function

$$W^{BS} = \sum_h \overline{\chi}_h^W \log W_h,$$

where recall that $\overline{\chi}_h^W$ is the initial income share of household h at the efficient equilibrium. These welfare weights are chosen so that there is no incentive to redistribute across agents at the initial equilibrium.

We compute the change in welfare from the introduction of tariffs or other distortions in terms of the proportional increase in the consumption of all goods by all households at the final distorted equilibrium which would keep the welfare function constant at the level of the initial efficient steady state. Formally, we measure changes in welfare by $\Delta \log \delta$, where δ solves the equation

$$W^{BS}(\overline{W}_1, \ldots, \overline{W}_H) = W^{BS}(W_1/\delta, \ldots, W_H/\delta),$$

where $\overline{W}_h$ and W_h are the values at the initial efficient equilibrium.

Corollary 5. *Starting at an efficient equilibrium, the change in world welfare in response to the introduction of small tariffs or other distortions is given up to the second order by*

$$\Delta \log \delta \approx \Delta \log Y + Cov_{\chi_h^W} \left(\Delta \log \chi_h^W, \Delta \log P_{W_h} \right).$$

The change in world welfare is the sum of the change in world real output and of a redistributive effect. The redistributive effect is positive whenever the covariance between the changes in household income shares and the changes real expenditure deflators is positive. It captures a familiar deviation from perfect risk sharing. It would be zero if households could engage in perfect risk sharing before the introduction of the tariffs or other distor-

tions. In our applications, this redistributive effect is quantitatively small and so changes in world welfare are approximately equal to changes in world real output.

6.4 Costs of Tariffs and Other Distortions: Ex-Ante Sufficient Statistics

Theorem 6 and Corollary 5 express the effects of tariffs and other distortions in terms of endogenous individual output changes. In this subsection, we provide formulas for these individual output changes, and hence for the effects of tariffs and other distortions, in terms of primitives: microeconomic elasticities of substitution and the HAIO matrix. We state these results only for world real output and world welfare, but in Appendix G, we also provide results for country real output.

Theorem 7. *Starting at an efficient equilibrium, the change in world real output in response to the introduction of small tariffs or other distortions is given up to the second order by*

$$\begin{aligned}\Delta \log Y \approx -\frac{1}{2}\sum_{l\in N}\sum_{k\in N}\Delta\log\mu_k\Delta\log\mu_l\sum_{j\in H+N}\lambda_j\theta_j Cov_{\Omega^{(j)}}(\Psi_{(k)},\Psi_{(l)})\\ +\frac{1}{2}\sum_{l\in N}\sum_{g\in F}\Delta\log\Lambda_g\Delta\log\mu_l\sum_{j\in H+N}\lambda_j\theta_j Cov_{\Omega^{(j)}}(\Psi_{(g)},\Psi_{(l)})\\ +\frac{1}{2}\sum_{l\in N}\sum_{h\in H}\chi_h^W\Delta\log\chi_h^W\Delta\log\mu_l(\lambda_l^{W_h}-\lambda_l),\end{aligned}$$

where changes in factor shares solve the system of linear equations given by

$$\Delta\log\Lambda_f=-\sum_{i\in N}\Gamma_{fi}\Delta\log\mu_i-\sum_{g\in F}\Gamma_{fg}\Delta\log\Lambda_g+\sum_{g\in F}\Xi_{fg}\Delta\log\Lambda_g+\sum_{i\in N}\Xi_{fi}\Delta\log\mu_i,$$

changes in household income shares are given by

$$\chi_h^W\Delta\log\chi_h^W=\sum_{g\in F}\Phi_{hf}\Lambda_g\Delta\log\Lambda_g+\sum_{i\in N}\Phi_{hi}\lambda_i\Delta\log\mu_i,$$

the definition of Ξ is extended for $f\in F$ and $i\in N$ by $\Xi_{fi}=\frac{1}{\Lambda_f}\sum_{h\in H}(\Lambda_f^{W_h}-\Lambda_f)\Phi_{hi}\lambda_i$, and Φ_{hi} is the share of the revenue raised by the tariff or other distortion on good i which accrues to household h. Changes in world welfare $\Delta\log\delta$ are given by Corollary 5 using changes in real output $\Delta\log Y$, in household income shares $\Delta\log\chi_h^W$, and in household real expenditure deflators $\Delta\log P_{W_h}=\sum_{i\in N}\lambda_i^{W_h}\Delta\log\mu_i+\sum_{g\in F}\Lambda_g^{W_h}\Delta\log\Lambda_g$.

Some general lessons are immediately available. First, all the terms scale with the square of the tariffs or other distortions $\Delta\log\mu$. There is therefore a sense in which misallocation

increases with the tariffs and other distortions. Second, all the terms scale with the elasticities of substitution θ of the different producers. There is therefore a sense in which elasticities of substitution magnify the costs of these tariffs and other distortions. Third, all the terms also scale with the sales shares λ of the different producers and with the square of the Leontief inverse matrix Ψ. There is therefore also a sense in which accounting for intermediate inputs magnifies the costs of tariffs and other distortions. Fourth, all the terms mix the tariffs and other distortions, the elasticities of substitution, and of properties of the network. Hence, in general, the costs of tariffs and other distortions depends on how they are distributed over the network.

We now provide some intuition for the results. Consider the formula for the change in real output $\Delta \log Y$. For a given producer $l \in N$, there are terms in $\Delta \log \mu_l$ on the three lines. Taken together, these terms sum up to $(1/2)\lambda_l \Delta \log \mu_l \Delta \log y_l$. They give the Harberger triangle $(1/2)\lambda_l \Delta \log \mu_l \Delta \log y_l$ corresponding to good l in terms of microeconomic primitives. The three lines break it down into three components, corresponding to three different effects responsible for the change in the quantity $\Delta \log y_l$ of good l.

The term $(1/2) \sum_{k \in N} \Delta \log \mu_k \Delta \log \mu_l \sum_{j \in H+N} \lambda_j \theta_j Cov_{\Omega^{(j)}}(\Psi_{(k)}, \Psi_{(l)})$ on the first line corresponds to the change $\Delta \log y_l$ in the quantity of good l coming from *substitutions* by all producers j in response to changes in all tariffs and other distortions $\Delta \log \mu_k$, holding factor wages constant.

The term $(1/2) \sum_{g \in F} \Delta \log \Lambda_g \Delta \log \mu_l \sum_{j \in H+N} \lambda_j \theta_j Cov_{\Omega^{(j)}}(\Psi_{(g)}, \Psi_{(l)})$ on the second line corresponds to the change $\Delta \log y_l$ in the quantity of good l coming from *substitutions* by all producers j in response to the endogenous changes in factor wages brought about by all the changes in tariffs and other distortions.

The term $\sum_{h \in H} \chi_h^W \Delta \log \chi_h^W \Delta \log \mu_l (\lambda_l^{W_h} - \lambda_l)$ on the third line corresponds to the change $\Delta \log y_l$ in the quantity of good l coming from *redistribution* across agents with different spending patterns, in response to the endogenous changes in factor wages brought about by all the changes in tariffs and other distortions.

Note that in contrast to the expressions for changes in sales shares, the two substitution terms feature, for each producer j, the elasticity of substitution θ_j and not $\theta_j - 1$. This is because they characterize the change in the *quantity* of good l, and not the change in its *sales*. The neutral case leading to no changes in no longer Cobb Douglas but Leontief: when all the elasticities of substitution are zero, there are no changes in the quantity of good l, and no effect of tariffs or other distortions on real output.

The system of linear equations for the changes in factor income shares $\Delta \log \Lambda_f$ is similar to the one that we encountered in Section 4, with shocks to tariffs or other distortions acting like negative productivity shocks, and with a new term $\sum_{i \in N} \Xi_{fi} \Delta \log \mu_i$ representing distribution effects arising from differential ownerships across households with different

expenditure patterns of the revenues raised by the tariffs or other distortions of good j.

The formula for the changes in the household income shares $\chi_h^W \Delta \log \chi_h^W$ is also similar to the one that we encountered in Section 4. The second term in this equation $\sum_{i \in N} \Phi_{hi} \lambda_i \Delta \log \mu_i$ is new and represents the part of the revenue raised by the tariff or other distortion on good j which accrues to household h.

Round-about Example

Consider again the example in Figure 1. Suppose that the two countries are symmetric, and that each country introduce a symmetric tax $\Delta \log \mu$ on its imports from the other country. Because of symmetry, changes in country real output, country real expenditure or welfare, changes in world real output changes in world real expenditure or welfare, are all the same. Hence, using Theorem 7, the effects of the tariffs on any of these variables is given by

$$\frac{1}{2}(\lambda_{12} \Delta \log y_{12} \Delta \log \mu + \lambda_{21} \Delta \log y_{21} \Delta \log \mu) = -\theta \frac{\Omega}{2(1-\Omega)^2} (\Delta \log \mu)^2,$$

where y_{12} is the quantity of imports from country 2 by country 1, λ_{12} is the corresponding sales share, and y_{21} and λ_{21} are defined similarly. Because of symmetry $y_{12} = y_{21}$ and $\lambda_{12} = \lambda_{21}$.

The losses increase with the elasticity of substitution θ and with intermediate input share Ω. This is both because the relevant sales shares $\lambda_{12} = \lambda_{21} = \Omega/[2(1-\Omega)]$ become larger and because the reductions in the quantities of imports $-\Delta \log y_{12} = -\Delta \log y_{21} = [\theta/(1-\Omega)]\Delta \log \mu$ become larger. The latter effect occurs because when the intermediate input share increases, goods effectively cross borders more times, and hence get hit by the tariffs more times, which increases the relative price of imports more and leads to a larger reduction in their quantity.

7 Applications

In this section, we conduct some empirical applications of our results. We lay down a structural multi-factor production network model, calibrated to fit world input-output data. We quantify the way trade costs (tariffs or iceberg) affect output, welfare, and factor rewards.

We consider three scenarios which relate to recent events: (1) a universal increase in international (tariff or non-tariff) barriers; (2) an increase of trade barriers between Great Britain and the European Union; and (3) an increase in trade barriers between the United States and China. The numbers are interesting by themselves, but more importantly for us, they demonstrate and quantify some of the mechanisms that we have emphasized analytically.

These applications also help us illustrate the importance of global input-output linkages. To bring this point out, we compare the results of the benchmark model to alternative calibrations which trivialize the input-output connections in different ways commonly used in the trade literature.

We end the section by applying our welfare-accounting formulas to perform our terms-of-trade and reallocation decompositions of changes in real expenditure in different countries.

7.1 Structural Model

The benchmark model consists of 40 countries as well as a "rest-of-the-world" composite country, each with four factors of production: high-skilled, medium-skilled, low-skilled labor, and capital. Each country has 30 industries each of which produces a single industry good. Due to lack of data, we assume that there is a representative household in each country, and that this household only derives income from domestic factors. Unbalanced trade is captured via international transfers.

The model has a nested-CES structure. Each industry produces output by combining its value-added (consisting of the four domestic factors) with intermediate goods (consisting of the 30 industries). The elasticity of substitution across primary factors is γ, and across intermediates is ζ, the elasticity of substitution between factors and intermediate inputs is θ, the elasticity of substitution in consumption goods across industries is σ. When a household or producer i in country c purchases inputs of some industry j, it consumes a CES aggregate of goods from this industry sourced from various countries with some trade elasticity ε_j.

We use data from the world input-output database to calibrate the model's expenditure shares. The world input-output database (WIOD) contains the expenditures of each industry in each country on intermediate input purchases from every other industry in every other country, as well as on the four primary factors. It also contains data on final consumption demand.

We use the data from the year 2008 to calibrate the CES share parameters in the model. To calibrate the elasticities of substitution, we use the same elasticities as in Section 5. That is, we set the elasticity of substitution across industries $\zeta = 0.2$, the one between value-added and intermediates $\theta = 0.5$, the elasticity of substitution in consumption $\sigma = 0.9$, and the trade elastiities ε_j following the estimates from Caliendo and Parro (2015). Finally, we set the elasticity of substitution among primary factors $\gamma = 0.5$. See Appendix B for more details.

Value-Added Calibrations

The benchmark model features input-output linkages.

To emphasize the importance of taking input-output lnkages into account and to connect with our analytical results, we compare the benchmark model with input-output linkages to two alternative value-added calibrations which are common in the trade literature. These alternative calibrations both assume that all production takes place with value-added production functions (no intermediates) but trivialize the input-output connections in two different ways. We call these two calibrations the low-trade value-added (LVA) and the high-trade value-added (HVA) economies. As we shall see, these two value-added calibrations are problematic, because they are not exact representations of the benchmark economy.[34]

Low-trade value-added (LVA): This is the implementation used by Costinot and Rodriguez-Clare (2014) in their handbook chapter for their value-added calibration, and it is also how Arkolakis et al. (2012) mapped their model to the data. The value-added produced by each producer matches the one in the data. It is then assumed that the fraction of the value added of i which is sold to each country is equal to the corresponding fraction of the sales of i in the data. We call this the low-trade value-added economy because it reduces the overall value of trade as a share of GDP.

High-trade value-added (HVA): The value-added produced by each producer matches the one in the data. However, it is then assumed that the valued of i which is sold to each foreign country is equal to the corresponding sales of i in the data. The residual value added is sold in the domestic country of i. We call this the high-trade value-added economy because it preserves the volume of trade relative to GDP.

Scenario I: Reversing Globalization

This scenario illustrates the importance of taking into account input-output connections and the importance of modeling the difference between iceberg shocks and tariffs. In Table 2, we report the impact on the welfare of a few countries, as well as the effect on world welfare, of a 10% increase in either the iceberg costs of trade or a 10% increase in import tariffs. We compare the response of our benchmark economy with intermediate goods to those of the LVA and HVA economies with no intermediate goods.

Across the board, and as suggested by the discussion in Section 6.3, an increase in iceberg trade costs (or other non-tariff barriers to trade) is significantly more costly than an equivalent increase in tariffs. For example, US welfare actually increases by 0.1% in response to

[34]In Section , we explained that the only correct way of representing this economy with intermediate inputs as a value-added economy is to Adao et al. (2017) by assuming that only factors are traded within and across borders, and that households have preferences over factors. We also explained that this working with this representation is challenging in the presence of intermediate inputs, trade costs, and tariffs.

	AUS	BEL	BGR	CAN	CHN	DEU	IRL	LUX	USA	World
Benchmark Iceberg	-2.0%	-6.7%	-5.9%	-3.3%	-1.3%	-3.0%	-8.3%	-16.9%	-1.1%	-2.26%
HVA Iceberg	-2.1%	-5.2%	-5.5%	-3.9%	-1.5%	-3.4%	-7.1%	-10.8%	-1.0%	-2.34%
LVA Iceberg	-0.8%	-2.3%	-1.9%	-1.4%	-0.5%	-1.2%	-3.6%	-5.0%	-0.3%	-0.86%
Benchmark Tariff	-0.7%	-2.3%	-0.7%	-1.2%	-0.2%	-0.9%	-3.7%	-5.4%	0.1%	-0.43%
HVA Tariff	-0.5%	-0.6%	-0.1%	-1.5%	-0.1%	-0.7%	-2.1%	-5.3%	0.3%	-0.23%
LVA Tariff	-0.4%	-0.3%	0.3%	-0.7%	0.1%	-0.5%	-1.3%	-0.7%	0.1%	-0.17%

Table 2: Percentage change in real income for a subset of the countries in response to a universal 10% change in iceberg trade costs or import tariffs. We compare the results from the benchmark economy with intermediate goods and input-output linkages with economies that assume only value-added production functions (HVA and LVA).

increases in tariffs, but decreases by 1.1% in response to increases in trade costs. World welfare decreases by 0.4% in response to increases in tariffs, but decreases by the much bigger amount of 2.3% in response to increases in trade costs. Hence, drawing inferences about increases in tariffs by studying increases in iceberg trade costs, as sometimes happens in the literature, can be highly misleading.

Next, we compare the benchmark model to the value-added economies. The reduction in world welfare from increases in iceberg trade costs is 2.3% for the benchmark economy, it is also 2.3% for the HVA economy, but it is only 0.9% for the LVA economy. The HVA economy does a better job than the LVA economy because it preserves the volume of trade, and hence, by Theorems 1, 2, and 3, the response of world welfare in that model is, at the first order, identical to that of the benchmark model.[35] The response of country welfare is different at the first order, but for the shock that we consider, these differences seem to be relatively small for most (but not all) countries. The LVA economy, which is the much more common calibration in the literature, is hopeless. Since LVA reduces the volume of trade to GDP, it greatly understates ,at the first order, the welfare effects of shocks to iceberg trade costs.

The reduction in world welfare from an increase in tariffs is 0.43% for the benchmark economy, but it is only 0.23% for the HVA economy, and it is even less at 0.17% for LVA economy. In this case, neither the LVA nor the HVA economy does a good job of replicating the benchmark model. Theorem 6 helps shed light on why this happens. The losses in GDP are approximately the sum over specialized exporters i affected by the tariffs of $(1/2)\lambda_i \Delta \log y_i \times \log(1.1)$, where $\log(1.1)$ is the log change in the tariffs.

Since the HVA economy preserves the volume of trade, λ_i are the same for the benchmark and the HVA economy. Nevertheless, the response of the HVA economy is half that

[35]That means, as long as the shocks are sufficiently small (ruling out nonlinearities), we should expect the benchmark and HVA economies to deliver similar world welfare results.

of the benchmark. This is because in the HVA economy, the reduction in exports $\Delta \log y_i$ in response to tariffs is significantly lower. The LVA economy is still hopeless, since it gets both the output elasticity $\Delta \log y_i$ wrong and the trade volumes λ_i wrong.

There are two reasons why the HVA economy underestimates the reduction in exports $\Delta \log y_i$ in response to tariffs. First, the impact of the tariffs on the price of traded goods is smaller. In the true economy, the universal tariff affects global value chains, and the tariffs are compounded each time unfinished goods cross borders, as in the round-about example of Section 6.4. This effect is necessarily absent in HVA where unfinished goods only cross borders once.

Second, in the HVA economy, imported goods are a larger share of each agent's basket. This mechanically lowers the elasticity of that agent's demand for imported goods since increases in the price of imported goods increase the price index of the basket by more. To see this mechanism, consider a small open economy with a simple CES demand system with elasticity $\theta_i = 1 + \varepsilon_i$. Let i be the only imported good. The demand for i is $y_i = (p_i/P)^{-\theta_i} Y$ where P is the price index and Y is total output. Assume that the economy is initially efficient. In response to the introduction of a tariff $\Delta \log \tau_i$ on good i, the change in the price of good i is $\Delta \log p_i = \Delta \log \tau_i$. Because the economy is efficient at the initial point, we have $\Delta \log Y = 0$ and $\Delta \log P = -\lambda_i \Delta \log \tau_i$, where λ_i is the share of expenditures on i. The change in the quantity of imports of good i is $\Delta \log y_i = -\theta_i(1 - \lambda_i)\Delta \log \tau_i$. Hence, as i becomes a larger part of the basket, λ_i increases and the demand for i becomes more inelastic. This intuition is not unique to assuming CES demand.

Scenario II: Brexit

We consider the effect of a 10% increase in icerberg tade costs or tariffs of British goods into the EU. For this scenario, we do not delve into the details, and instead focus on some of the distributional consequences for the different factors. The detailed results can be found in in Appendix C: in Table 5 for iceberg trade costs and in Tables 6 and 7 for tariffs.

As was to be expected, increases in iceberg trade costs and tariffs have approximately the same effects for Britain, but they have very different effects for other EU countries. Focusing on Britain, in the benchmark economy, the overall change in welfare is approximately -0.6%. This welfare reduction is distributed unequally across the different factors of production. The changes in real incomes of capital, low-, medium-, and high-skilled labor are $(-0.9\%, -0.1\%, -0.3\%, -0.7\%)$. All factors of production lose, but the losses are largest for capital and high-skilled workers.

In the HVA economy, the overall change is approximately the same at -0.6%. This was to be expected given that the increases in tariffs act on Britain approximately like increases

in iceberg trade costs, and given that the HVA economy matches the volume of trade of the benchmark economy and than the data. However, the distribution of this welfare reduction across factors displays some significant differences. The changes in real incomes of capital, low-, medium-, and high-skilled labor are $(-1.3\%, -0.1\%, -0.2\%, -0.6\%)$.

In the LVA model, the overall change in welfare is approximately half at -0.3%. This was to be expected given that the LVA economy features a lower volume of trade than the benchmark economy and than the data. The changes in the real incomes of capital, low-, medium-, and high-skilled labor are $(-0.9\%, 0.4\%, 0.1\%, -0.1\%)$. The real income of low and medium-skilled labor actually increases. Basically, the price of capital and the wage of high-skilled labor falls by enough to offset the reduction in the wage of the low and medium skilled workers.

Overall, we see significant differences between the benchmark economy and the HVA economy, and even bigger ones between the benchmark economy and the LVA economy. Accounting for intermediate goods and input-output linkages significantly changes the picture of Brexit.

Scenario III: Trump Tariffs

The last scenario illustrates how tariffs between two countries can affect a third country via supply chains. The Trump administration has recently imposed a 10% tariff on a broad range of Chinese imports, with the possibility that these tariffs will be increased further to 25%. Meanwhile, the administration renegotiated a free-trade deal with Mexico. In this subsection, we analyze an impressionistic version of this policy, computing the effects of the imposition of a tariff on China while maintaining free trade with Mexico.

We stress at the outset that our results should be interpreted as long-run effects, once prices have adjusted and factors and inputs have been reallocated. This long-run focus explains the difference between our findings and those of recent studies (Fajgelbaum et al., 2019; Amiti et al., 2019), who investigate the effects of tariffs in the short run over a horizon of less than a year.

	AUS	BRA	CAN	CHN	KOR	MEX	TWN	USA	World
Benchmark	-0.01%	0.00%	0.02%	-0.24%	-0.02%	0.03%	-0.06%	0.01%	-0.01%
LVA	-0.01%	0.00%	0.01%	-0.04%	0.00%	0.01%	-0.01%	0.00%	0.00%

Table 3: The change in welfare for a subset of countries when the USA imposes a 10% import tariff on all Chinese trade. We compare the results from the benchmark economy with intermediate goods and input-output linkages with an economy that assumes only value-added production functions (LVA).

In Table 3, we show the effects on the welfare of a few countries of a 10% tax on Chinese imports in the benchmark economy and in the LVA economy. We start with the benchmark economy. As one might expect, China is affected most negatively by this policy with a change in welfare of -0.24%, followed by some of China's close trading partners, like Taiwan, Korea, and Australia. The United States gains very slightly by 0.01% from the imposition of tariffs, illustrating almost fully offsetting effects on tariff revenues 0.12% and real factor income -0.11%. Interestingly, the country that gains the most from the tariffs is Mexico at 0.03% due to the resulting increase in the demand for Mexican goods.

Table 8 in Appendix C breaks down how the different factor prices change in response to the tariffs. This table shows that the overall trend, across most countries including the USA and Mexico but excluding China, is that capital loses and labor gains. For most countries except China and Mexico, most of labor's gains accrue to high-skilled workers. For example, in the USA, capital loses by -0.36%, low-skilled labor gains by 0.08%, medium skilled labor gains by 0.03%, and high-skilled labor gains by 0.13%. For Mexico, most of the gains go to low- and medium-skilled labor instead, by 0.41% and 0.63% respectively. In China, all factors lose but low-skilled labor and high-skilled labor actually lose the most, by -0.17% and -0.44% respectively.

In the LVA economy, the welfare effects are much smaller overall. The most impressive difference is for China, which now loses by only -0.04% compared to -0.24% in the benchmark economy. Once again, taking into account input-output linkages matters for assessing Trump tariffs.

7.2 Application to Welfare Accounting

We end our applications by decomposing the change in real expenditure in different countries over time. We implement our two decompositions: the reallocation decomposition and the terms-of-trade decomposition. We abstract away from distortions. Unlike our previous applications, these decompositions are non-parametric in the sense that they do not require taking a stand on the various elasticities of substitution.

The left column of Figure 4 displays the cumulative change in each component over time of the reallocation decomposition, for a few countries (Canada, China, and Japan). We choose these three countries because they depict a systematic pattern: industrializing countries, like China, and commodities- or services-dependent industrialized countries, like Canada, are experiencing positive reallocation, whereas manufacturing-dependent industrialized countries, like Japan, are experiencing negative reallocation.

The right column of Figure 4 displays the terms-of-trade decomposition. We see that real expenditure or welfare and real output move roughly in line with-one-another, and

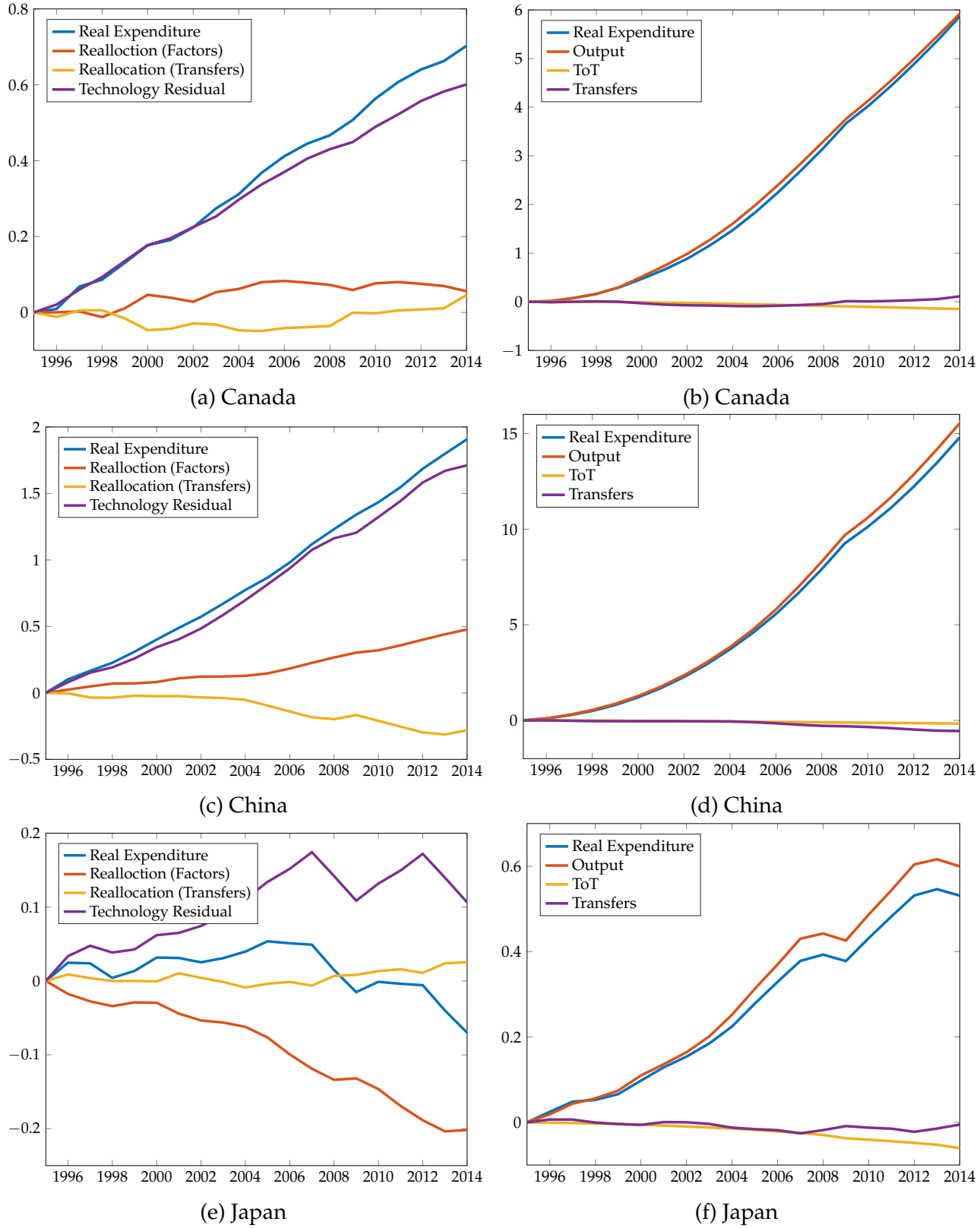


Figure 4: Welfare accounting according to the reallocation decomposition (left column) and according to the terms-of-trade decomposition (right column), for a sample of countries, using the WIOD data.

that terms-of-trade effects are small. This happens despite the presence of large reallocation effects. This indicates that none of these countries can be taken to be approximately closed, which would imply no reallocation effects.

Finally, it is interesting to note that the difference between the reallocation effect on the one hand, and the terms of trade effect and the transfer effect on the other hand identifies the following technological residual:[36]

$$\sum_{i\in N}((\chi_c^Y/\chi_c^W)\,\lambda_i^{Y_c}-\lambda_i^{W_c})\,\mathrm{d}\log A_i+\sum_{f\in F}((\chi_h^Y/\chi_c^W)\Lambda_f^{Y_c}-\Lambda_f^{W_c})\,\mathrm{d}\log L_c.$$

This residual is a measure of the difference between country c's technological change and its exposure to world technical change, including the effects of changes in productivities and in factor supplies. For a closed economy, it is always zero. By comparing the two columns of Figure 4, we can see that (and by how much) China and Canada are experiencing faster growth in productivities and factor supplies in their domestic real output than in their consumption baskets, while the pattern is reversed for Japan.

8 Conclusion

In this paper, we establish a unified framework for studying output and income in efficient and distorted open-economies. We provide ex-post sufficient statistics for measurement and ex-ante sufficient statistics for conducting counterfactuals. Our formulas provide new characterizations of the gains from trade, the losses from trade protectionism, the aggregation of trade elasticities, and the distributional consequences of trade policy.

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[36] That is, we compute $(\partial\log\mathcal{W}_h)(\partial\mathcal{X})\,\mathrm{d}\,\mathcal{X}-(1/\chi_c^W)\,\mathrm{d}\,T_c-(\chi_c^Y/\chi_c^W)\,\mathrm{d}\log P_{Y_c}+\mathrm{d}\log P_{W_c}$.

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