The economics of banning gasoline vehicles

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Abstract

We analyze the dynamics of recent policy proposals to ban gasoline vehicles. Our model captures declining electric vehicle damages; declining electric vehicle production costs due to exogenous changes or to learning by doing; stock effects; and the introduction of complementary infrastructure such as charging stations. We derive conditions under which it is socially optimal to ban gasoline vehicles in the long run. Under these conditions, we derive two classes of solutions. In one solution, it is optimal to ban gasoline cars before beginning production of electric cars. This solution obtains if preference heterogeneity is limited. In the other solution, it is only optimal to ban gasoline cars after beginning the production of electric cars.

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1 Introduction

At least four countries are considering implementing a ban on the sale of new gasoline vehicles within the next few decades: Britain (proposed date of ban is 2040), France (2040), India (2030), and Norway (2025).[1] Presumably, such a policy would an inefficient way to lower carbon emissions especially relative to taxes or cap and trade. However, the fact that these policies are being discussed justifies an economic analysis of the relevant issues.

Electric vehicles are likely substitutes for banned gasoline vehicles. The market for electric vehicles is expanding rapidly, with market shares around two percent of sales in many countries and as high as 50 percent in Norway.[2] Electric vehicles are currently more expensive than gasoline vehicles, but prices are declining and are expected to decline further over time. Other potential substitutes are fuel cell vehicles or natural gas vehicles, but current market shares of these vehicles are much lower than market shares of electric vehicles.

In this paper we consider a theoretical model of the transition from gasoline to electric vehicles. We construct an optimal control problem for a planner who determines the production levels for gasoline and electric vehicles over time to maximize welfare. Welfare includes the utility from using the vehicles, the costs of producing them, and the pollution damages associated with their use. We assume that gasoline vehicles and electric vehicles are perfect substitutes, but there is heterogeneity of preferences among consumers with respect to the price that makes one indifferent between purchasing a gasoline vehicle and an electric vehicle. This heterogeneity is the distinguishing characteristic of our work relative to other papers that have examined the transition from dirty to clean vehicles or from dirty to clean fuels more generally (Chakravorty et al 2012, Amigues et al 2016, Bahel and Chakravorty 2016, Creti et al 2018). The distribution of indifference prices in the population of consumers plays a critical role in the solution to the planner's problem.

Our model accounts for a number of important dynamic aspects to the problem. First, the model allows for declining damages from electric vehicles. Holland et al (2018) document an extraordinary decline in air pollution from electricity generation in recent years and a decline

[1] http://money.cnn.com/2017/07/26/autos/countries-that-are-banning-gas-cars-for-electric/index.html. Accessed 9/1/2017.

[2] See Holland et al 2016 and Yuksel et al 2016 for analyses of the economic implications of electric vehicles.

in damages from electric vehicles. Our model assumes that damages decline exogenously over time. Second, we allow for declining production costs of electric vehicles. The resurgence of electric vehicles was led by high-cost but high performance Teslas. Prices of other electric vehicles have typically been \$20-\$30,000 higher than comparable gasoline vehicles primarily due to the high initial costs of batteries. However, production costs of electric vehicles have declined dramatically due to improvements e.g., in battery technology or electric motors. We allow for production costs to decline either exogenously, or due to learning by doing in the manufacturing of electric vehicles. Third, both gasoline and electric vehicles are durable goods. Stocks of durable goods decay naturally over time but can be increased through the production of new goods. We model the stock effects of production and depreciation for both electric and gasoline vehicles. The final dynamic consideration is the production of complementary infrastructure for electric vehicles such as charging stations. Complementary infrastructure increases the utility of electric vehicles, and thus changes the distribution of the indifference prices among the consumers. In the most complete version of the model, the planner simultaneously selects the production of vehicles and the roll out of infrastructure.

We first consider the basic question of whether or not a ban is justified by the economic fundamentals in the model. To do this we analyze the long run steady state. We give a simple condition under which the planner does indeed want to stop production of gasoline vehicles in the long run. The condition is a function of the indifference price of the consumer with the strongest preference for gasoline vehicles, the long production costs of the vehicles, and the long run pollution damages. A basic numerical calibration based on plausible values for the parameters of the model for the United States shows that the ban is much more strongly influenced by percentage changes in production costs than by equal percentage changes in pollution damages.

Next, under the assumption that the ban is justified in the long run, we turn to the question of when electric vehicle production should be started and when gasoline vehicle production should be halted (i.e. the point in time in which the ban takes effect.) We show that there are two types of solutions to the planner's problem, depending on the distribution of indifference prices. In one solution, which we refer to as the *gap solution*, the planner optimally bans gasoline vehicles before production of electric vehicles starts. In the gap

during which no vehicles are produced, the stock of gasoline vehicles decays naturally until the point at which it is optimal to begin production of electric vehicles. For this solution, we characterize the stop time for gasoline vehicles and the start time for electric vehicles as the simultaneous solution to a system of two equations. The gap solution arises, for example, if there is no heterogeneity in consumer preferences, so that the indifference price is the same for all consumers. In the gap solution, it is optimal to ban gasoline vehicles strictly before production of electric vehicles begins.

A second solution occurs when there is significant heterogeneity of consumer preferences. We call this solution the *no gap solution* because the planner begins the production of electric vehicles before the ban on gasoline vehicles takes effect. This solution is more intuitively appealing than the gap solution, as the production of at least one type of vehicle occurs at all times. In this case, we can characterize the stop time for gasoline vehicles as the solution to a single equation and the start time for electric vehicles as a solution to a single equation as well. Moreover, the start time for electric vehicles depends directly on the indifference price of the consumer with the strongest preference for electric vehicles. In the no-gap solution, it is only optimal to ban gasoline vehicles after starting production of electric vehicles.

Numerical analysis of the calibrated model shows the relationship between heterogeneity in the distribution of indifference prices and the resulting solution (gap or not gap). The optimal time to implement a ban on gasoline vehicles in the Unites States is quite sensitive to the parameters that characterize the heterogeneity of preferences.

2 Model

Consider a continuous time model in which $G(t)$ denotes the stock of gasoline vehicles and $g(t)$ the production of gasoline vehicles at time t. The state equation for the stock of gasoline vehicles is

$$\dot{G} = -aG + g,$$

where $a > 0$ captures the retirement of vehicles due events such as accidents and mechanical failure. Likewise, $X(t)$ and $x(t)$ denote the stock and production of electric vehicles and the

state equation for the stock of electric vehicles is

$$\dot{X} = -aX + x.$$

Each vehicle has production costs and costs from operating the vehicle, which include externalities from emissions of air pollution. Let c_g denote the production costs of a gasoline vehicle, and let δ_g denote the social costs of driving a gasoline vehicle per unit of time. These social costs are equal to the sum of the operating costs and of damages from air pollution from tailpipes. We assume that both c_g and δ_g are constant across consumers and over time.[3]

Electric vehicles are characterized initially by both greater production costs *and* greater social costs than the traditional technology that they will be replacing.[4] Let production costs of an electric vehicle at time t be $c_x(t)$ with $\dot{c}_x \leq 0$ and $\ddot{c}_x \geq 0$. Decreases in c_x over time are due mostly to improvements in battery technology. Let $\delta_x(t)$ denote the social costs of driving an electric car per unit of time at time t. The social costs are equal to the sum of operating costs and damages from air pollution from electric power plants. We assume $\dot{\delta}_x \leq 0$. Decreases in δ_x over time are due to decreases in damages as the electricity grid gets cleaner.[5]

Consumers derive utility from driving the vehicles, and from the perspective of an individual consumer, electric vehicles and gasoline vehicles are perfect substitutes. In other words, in a simple static choice model, there is a fixed price ratio $\eta(i)$ at which consumer i would switch from purchasing one type of vehicle to the other. For this reason, we refer to $\eta(i)$ as the *indifference ratio* for consumer i. We assume there is a unit mass of consumers on $[0, 1]$ and the distribution of η is non-decreasing over this range (so that consumer 1 has the strongest preferences for electric vehicles.) Figure 1 shows a sample linear distribution for η. The linear distribution is characterized by the slope s and the value for $\eta(1)$. These variables describe different features of the consumers' preferences. The slope s indicates the degree of heterogeneity of consumer preferences. If the slope is zero, then we have homogeneous preferences. As the slope increases, we have greater heterogeneity. The value for

[3]This assumption is consistent with assuming that all vehicles in the stock are driven the same number of miles (normalized to one) in a unit of time.

[4]Holland et al 2016.

[5]Holland et al 2018.

$\eta(1)$ indicates the intensity of preferences for electric vehicles. If $\eta(1) < 1$, then consumers have strong preferences for gasoline vehicles. As $\eta(1)$ increases, the intensity of preferences for electric vehicles increases. Also shown in Figure 1 is the percentage of consumers that would buy the gasoline vehicle at parity prices, which is denoted by ρ and is defined by the intersection of the distribution for η and the horizontal line at 1.

Figure 1: Distribution for η

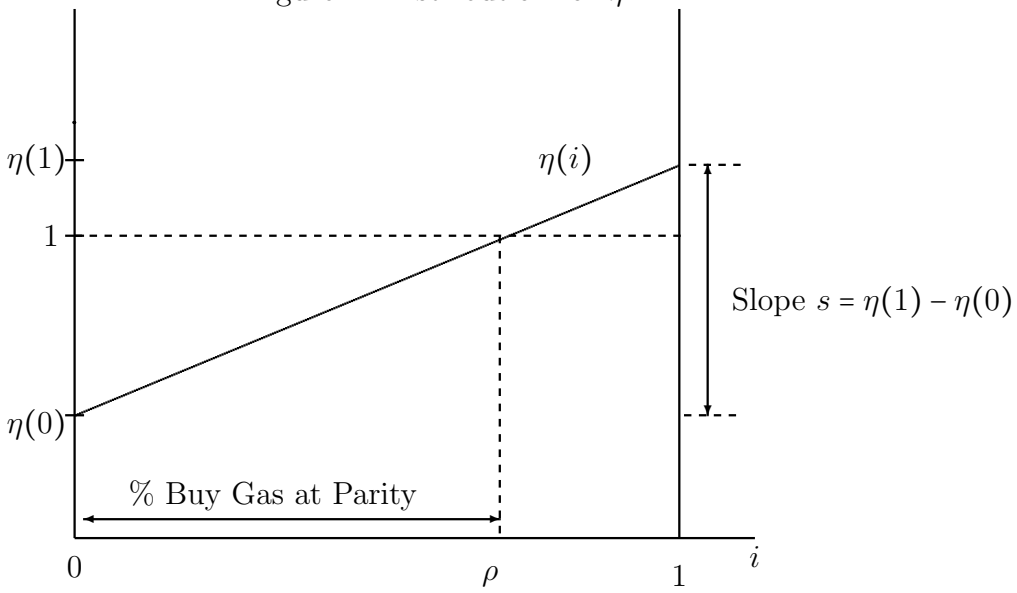


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The distribution of consumer preferences η and the stock of vehicles G and X determine the aggregate utility function $U(G, X)$ as follows. At each instant of time, we assume a planner, or alternatively the market for vehicles, allocates the stock of vehicles to the consumers such that the sum of utility across all consumers is maximized. The value of this sum yields the aggregate utility function. See Online Appendix A for more discussion and a derivation of the aggregate utility function. In general, the aggregate utility function is a complicated function of X and G. But in the special case in which preferences are homogenous (η is constant) we have a *representative consumer model*, and aggregate utility is given by

$$U(G, X) = U(G + \eta X).$$

We use the notation U_G and U_X to denote the partial derivative of aggregate utility with respect to G and X respectively.

The planner determines the production of electric and gasoline vehicles over a time horizon T. In this optimal control problem, the state variables are vehicle stocks X and G and the control variables are the production levels x and g. We assume that the state variables are continuous.[6] Let r be the interest rate. At time 0 there are no electric vehicles and the stock of gasoline vehicles is in steady state. The salvage values for vehicle stocks at terminal time are c_g and c_x (in current value.) The planner's problem is

$$\begin{aligned}
\max_{x,g} \quad & c_x e^{-rT} X(T) + c_g e^{-rT} G(T) + \int_0^T e^{-rt} \left(U(G,X) - c_g g - c_x x - \delta_g G - \delta_x X \right) dt \\
\text{s.t.} \quad \dot{X} &= -aX + x \; ; \; X(0) = 0 \\
\dot{G} &= -aG + g \; ; \; G(0) = G^{ss} \\
\bar{x} &\geq x \geq 0 \\
\bar{g} &\geq g \geq 0,
\end{aligned}$$

where $\bar{x}$ and $\bar{g}$ are maximum production constraints and G^{ss} is the initial steady state stock of gasoline vehicles.

The treatment of social costs in the model deserve additional explanation. On the gasoline side, because damages from gasoline vehicles are constant over time, we can simply multiply the stock of gasoline vehicles G by social costs δ_g. On the electric side, the calculation is the same, but the interpretation is more complicated because social costs are decreasing over time. At first blush, this suggests that we should keep track of each vintage of electric vehicles, and associate the various vintages with the social costs at the time they were produced. Our formulation is appropriate, however, under the assumption that most of the decrease in damages from electric vehicles over time is due to improvements in the electricity grid, rather than efficiency improvements to the electric drivetrains. An improvement in the grid leads to a contemporaneous decrease in damages from the entire fleet of electric vehicles because all vehicles are charged using the current grid.

[6] This rules out using impulse functions for the controls x and g.

The derivation of necessary conditions for the planner's problem are given in Online Appendix B. These conditions include the state equations for X and G, the corresponding adjoint equations

$$\dot{\beta} = (a + r)\beta + \delta_x - U_X$$

$$\dot{\alpha} = (a + r)\alpha + \delta_g - U_G,$$

and the following conditions for interior values for g and x. If x in interior, then

$$\beta = c_x.$$

If g is interior, then

$$\alpha = c_g.$$

2.1 Long Run Steady State

First consider the long run steady state. In this steady state, all improvements to electric vehicles and the electricity grid have have been completed, so social costs have converged to some constant $\hat{\delta}_x$ and production costs have converged to some constant $\hat{c}_x$. Our first proposition describes parameter combinations required to justify a ban on gasoline vehicles in the long run steady state. All proofs are in Online Appendix C.

lrprop **Proposition 1.** *In the long run steady state, if*

$$\eta(0) \geq \frac{(a + r)\hat{c}_x + \hat{\delta}_x}{(a + r)c_g + \delta_g}$$

then gasoline vehicles are banned.

Proposition 1 show that the preferences of the consumer who likes gasoline vehicles the most combines with production costs (discounted for time and decay) and social costs from use to determine whether or not gasoline cars are banned in the long run steady state. We can use this result, for example, to place requisite bounds on the social cost of carbon. We will carry out such calculations in the simulation section.

2.2 Transition From Gasoline to Electric Vehicles

In this section we focus on the transition from gasoline to electric vehicles. Toward that end, we assume that the parameters of the problem are such that gasoline vehicles will not be produced in the long run steady state. The initial steady state has gasoline production with g interior and this production continues for some time period $[0, t^g]$. Electric vehicles are produced with x interior for some time period $[t^e, T]$. We refer to t^g and t^e as *transition times*. With this set-up, there are two solutions to the planner's problem. In the *gap solution* we have $t^g < t^e$, implying there is a time period in which neither type of vehicle is produced. In the *no gap solution* we have $t^g > t^e$, implying there is a time period in which both vehicles are produced.

2.2.1 Gap Solution

First we describe the gap solution. During $[0, t^g]$, gasoline vehicles are produced in steady state with interior g. From the interior condition for g we have $\alpha = c_g$ and hence $\dot{\alpha} = 0$. Using the adjoint equation for α we can write an equation the defines G^{ss}. We have

$$U_G(G^{ss}, 0) = (a + r)c_g + \delta_g. \tag{1}$$ defgss

At time t^g, production of gasoline vehicles stops. From then on, the stock of gasoline vehicles decays exponentially according to

$$G^{gap}(t) = G^{ss} e^{-a(t-t^g)} \quad \text{for } t \in [t^g, T]. \tag{2}$$ eqggap

At time t^e, production of electric vehicles starts with interior x. From the interior condition for x we have $\beta = c_x$. Let $X^{gap}(t)$ denote the stock of electric vehicles during the period $[t^e, T]$. From the adjoint equation for β it follows that

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G^{gap}, X^{gap}).$$

Given G^{gap}, this equation implicitly determines X^{gap}.

We can now characterize the transition times t^e and t^g in the gap solution.

gapprop **Proposition 2.** *In the gap solution, the transition times t^e and t^g are the solutions to the system of equations described by*

$$U_X(G^{gap}(t^e), 0) = (a+r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e) \tag{3}$$ gapone

and

$$\int_{t^g}^{T} e^{-(a+r)\tau} U_G(G^{ss}, 0)d\tau = \int_{t^g}^{t^e} e^{-(a+r)\tau} U_G(G^{ss}e^{-a(\tau - tg)}, 0)d\tau + \int_{t^e}^{T} e^{-(a+r)\tau} U_G(G^{gap}(\tau), X^{gap}(\tau))d\tau. \tag{4}$$ gaptwo

Equation (3) equates marginal utility equal to marginal costs at t^e. Notice that (3) is a function of both t^e and t^g (because $G^{gap}(t^e)$ is a function of t^g). The marginal utility depends on the stock of gasoline vehicles, which in turn depends on how long it has been since production of these vehicles was stopped. The expression on left hand side of (4) is the discounted (for both time and stock decay) marginal utility in a counterfactual in which we continue to produce gas cars in steady-state during $[t^g, T]$. The expression on right hand side is sum of discounted marginal utility of actual production plan (no cars during $[t^g, t^e]$ and electric cars during $[t^e, T]$.) The transition time t^g is such that the regulator is just indifferent between these two options. Notice (4) also depends on t^e because this effects the actual production plan.

In the representative consumer case, we obtain the gap solution.

nobothg **Proposition 3.** *Suppose that aggregate utility is given by $U(G + \eta X)$. Then the solution to the planner's problem has $t^g < t^e$.*

The representative consumer case thus provides a useful benchmark for the analysis of the transition from gasoline to electric vehicles. If there is no heterogeneity of preferences, then the planner simply accounts for the decreasing damages and production costs of electric vehicles when determining the optimal time to introduce them. The planner bans production of gasoline vehicles before beginning production of electric vehicles because gasoline vehicles produced today will remain in the fleet for some time, and they will cause more damages than the increasingly clean electric vehicles.

2.2.2 No Gap Solution

Similar to the gap solution, the no gap solution starts at time 0 with gasoline vehicles produced in steady state with g interior. But in the no gap solution, electric production starts before gasoline production stops. At time t^e, electric production starts. During $[t^e, t^g]$, we have interior production of both types of vehicles. Using the interior conditions we have $\alpha = c_g$ which implies $\dot{\alpha} = 0$. We also have $\beta = c_x$ which implies $\dot{\beta} = \dot{c}_x$. Let the stock of gasoline and electric vehicles be denoted during this period by $G^{int}(t)$ and $X^{int}(t)$. From the adjoint equations, these stock levels satisfy

$$U_G(G^{int}, X^{int}) = (a + r)c_g + \delta_g \tag{5}$$ counterint

and

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G^{int}, X^{int}).$$

This solution to this system of equations describes the evolution of G^{int} and X^{int} during $[t^e, t^g]$.

During $[t^g, T]$ we have only interior production of electric vehicles. The stock of gasoline vehicles is given by $G^{no}(t) = G^{int}(t^g)e^{-a(t-t^g)}$. Given this, the stock of electric vehicles $X^{no}(t)$ satisfies

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G^{no}, X^{no}).$$

We now characterize the transition times in the no gap solution.

nogapprop **Proposition 4.** *In the no gap solution, the transition time t^e is the solution to*

$$\eta(1)(\delta_g + (a + r)c_g) = (a + r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e), \tag{6}$$ nogapone

and the transition time t^g is the solution to

$$\int_{t_g}^{T} e^{-(a+r)\tau} \left(\delta_g + (a + r)c_g\right) d\tau = \int_{t_g}^{T} e^{-(a+r)\tau} U_G(G^{no}(\tau), X^{no}(\tau))d\tau = 0. \tag{7}$$ nogaptwo

The interpretation of these equations is similar to that in the gap solution, but they are

conceptually simpler because they each contain only one transition time. Equation (6) is not a function of t^g because gasoline production is at steady state when the production of electric vehicles begins. This equation depends only on the value of η for the consumer who likes electric vehicles the most. The left hand side of (7) is counterfactual marginal utility of continuing production according to G^{int} described by (5). The right hand side is actual marginal utility from shutting off production of gasoline vehicles at t^g. It is not a function of t^e because electric production has already started at t^g.

We have seen that the representative consumer model leads to the gap solution. With more general preferences, either the gap or no gap solution may occur. Loosely speaking, if the heterogeneity of preferences is small, then the gap solution occurs. If, however, there is greater diversity of preferences among consumers, then the planner accounts for this diversity by extending the production lifetime of gasoline vehicles past the point at which electric vehicles are introduced.

2.3 Extensions

In this section we present several extensions that capture facets of electric vehicle adoption that will be featured in the simulation analysis.

The first extension is to introduce learning by doing in the production of electric vehicles. The marginal cost of producing electric vehicles is now a function of the cumulative electric vehicle production up to that point, denoted by an additional state variable $Z(t)$. As cumulative electric production grows, the marginal cost decreases. Thus the marginal cost of producing electric vehicles is given by a function $c_x = f(Z)$. The state equation for Z is given by

$$\dot{Z} = x.$$

The second extension is to include charging infrastructure. Let the additional state variable $W(t)$ be the stock of charging stations and let the additional control variable $w(t)$ be the number of stations produced at time t. Assuming charging stations do not depreciate,

the state equation for the stock of charging stations is

$$\dot{W} = w.$$

The utility of driving an electric vehicle depends on the availability of charging stations. As the stock of charging stations increases, the marginal utility of electric vehicles increases because it is easier to use them for long trips. Thus we can write the aggregate utility function as $U(G, X, W)$.

Including these extensions allows for a more general analysis of policy in the simulation section. A detailed discussion of the solution to the planner's problem with the extensions is given in Online Appendix D. The qualitative features of the solutions described above are generally robust to the extensions. For example, with learning by doing, Proposition 3 still holds and the equations for the transition times t_e and t_g have a similar structure as in Proposition 2.

2.4 Second Best Subsidies

In a market economy, of course, a planner does not make the decisions to produce gasoline and electric vehicles. Rather these decisions are decentralized to firms and consumers through the prices for these vehicles. Government policy frequently nudges the consumer toward a lower polluting technology by subsidizing its purchase. Consider the government's choice of subsidy on the purchase of electric vehicles. Because the welfare-maximizing subsidy will generally not lead to the same outcomes as in the planner's problem, we have a second best problem. Langer and Lemoine (2017) consider a problem that is similar in spirit but different in details in their analysis of subsidies for adopting residential solar units in California.

In theory, one determines the optimal second best subsidy in the following manner. Given the market price for electric vehicles and an arbitrary price for electric vehicles, consumers determine the purchases of vehicles in each instant of time to maximize utility. They ignore the externality from driving when making this decision. The solution to the consumer's problem describes the choices of g and x, and thus X and G, as a function of the price for electric vehicles. Using this solution, the planner selects the price of electric vehicles (i.e.

Table 1: Baseline parameter values

Parameter	Value	Description
T	30	Period length
a	0.067	Stock decay
δ_g	(295.9) + 2726	Social costs of gasoline vehicles: (externality) + operating
δ_x	$156e^{(1.4-0.08*t)} + 1535$	Social costs of electric vehicles: (externality) + operating
c_g	36113	Production costs of gasoline vehicles
c_x	$c_g + 21961e^{-0.06t}$	Production costs of electric vehicles: 60 kWh battery
$\eta(1)$	1.21	Maximum indifference ratio

uses a tax or subsidy to manipulate the market price as desired) to maximize the objective used in the analysis of the first-best problem (utility minus production costs minus social costs).

In practice, it difficult to determine an analytical solution to the general second best problem. See Holland et al (2019) for a discussion and solution in a simpler framework.

3 Simulation Components

In this section we describe the components of the simulation exercise and the calibration of parameters for these components. We consider a 30 year time frame starting in 2005. Baseline values are shown in Table 1. We now describe how we determine these values.

3.1 Social costs of gasoline vehicles: δ_g

In the theoretical model, we assume that δ_g is constant over time. To put this assumption in context, we determine how air pollution damages from gasoline vehicles has evolved over time. In particular, we calculate a historical time series of tail pipe emission regulations, fleet MPG, and sulphur content in gasoline. Combining this data with information from Argonne National Laboratory's GREET model,[7] enables us to calculate emissions in grams per mile for 5 pollutants (VOC, CO_2, SO_2, $PM_{2.5}$, and NO_X) during the years 1975-2015 (see Online Appendix E for details.) To determine damages from these emissions, we follow the methodology of Holland et al (2016). We combine the emissions in grams per mile with

[7] https://greet.es.anl.gov/

the estimates of marginal damages per gram by county using the AP2 integrate assessment model, the EPA social cost of carbon, and VMT by county.

To illustrate this data, we conduct an experiment in which we assume the 2015 fleet of vehicles has the polluting characteristics of, say, 1975 vintage passenger vehicles. Repeating this for each year in our historical time series gives the results shown in Figure 2. The major improvements in emission reductions occur in the late 1970's and the 1990's. There has been little improvement in the last decade. Looking to the future, it is not clear how much further reductions will occur. It seems possible to continue to decrease CO_2 emissions with future improvements in MPG. For other pollutants such as $PM_{2.5}$, emissions from tailpipes are now at levels below what is emitted from tire and brake wear, suggesting that prospects for further improvements are limited (Timmers and Achten 2016).

To calibrate the model, we us 2015 vintage vehicles as baseline. This gives the damages component of social costs δ_g to be 1.97 cents per mile and hence \$295.9 dollars per year per vehicle (assuming 15,000 miles per year).[8] The operating costs of \$2726 are determined by data from the American Automobile Association (see Online Appendix G).

3.2 Social costs from electric vehicles: δ_x

We follow a similar procedure to determine historical emissions and damages from electricity generation, although changes to the composition of the electricity grid and data reporting requirements make it impractical to look back past 2010. Holland et al (2018) determine the marginal damages from a unit of electricity for the three electricity interconnections in the United States from 2010-2017. Marginal damages are decreasing at approximately 6 percent per year in the Eastern interconnection and we use this result in our baseline specification. The operating costs for electric vehicles are determined by data from the American Automobile Association (see Online Appendix G) and are assumed to stay constant at a value of \$1535 over time.

[8] All dollar amounts are delineated in year 2017 dollars.

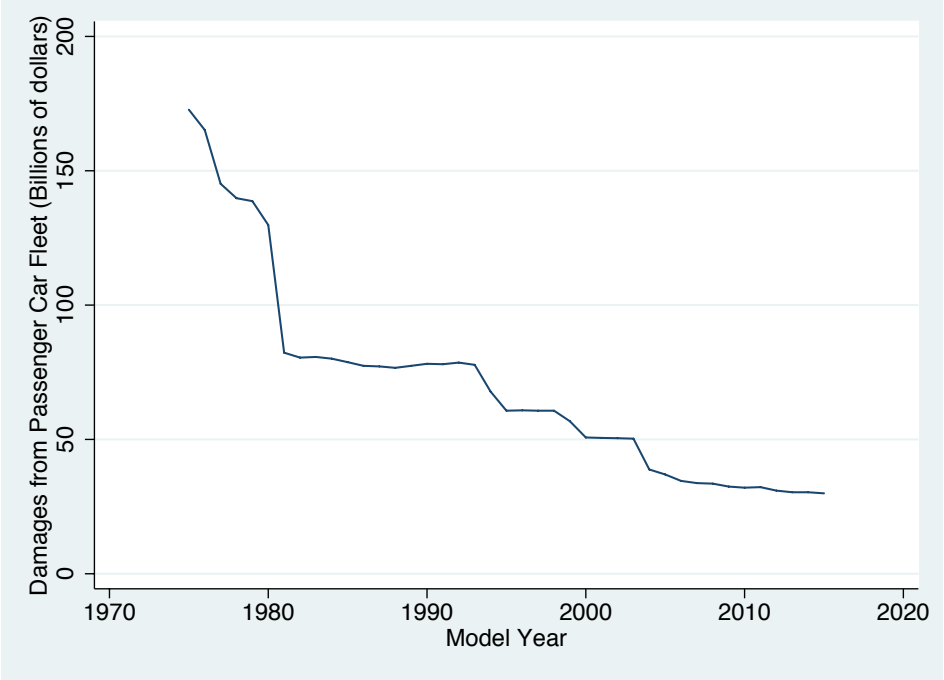


Figure 2: Damages from Gasoline Emissions of Passenger Cars over Time gascarhist

3.3 Preferences and stock decay: η and a

The distribution for η is calibrated with data from Hidrue et al (2011). Details on this calibration and the determination of $U(X, G)$ are given in Online Appendix A. The baseline value for $\eta(1)$ is 1.21. Data from the Department of Transportation and the Bureau of Economic Analysis enables us to determine $a = 0.067$ (see Online Appendix H).

3.4 Production costs: c_g, and c_x

For the production cost of gasoline vehicles, c_g, we use average sales price. This is consistent with perfect competition in market, so that vehicles are priced at marginal cost.

For the production cost of electric vehicles, we assume that most of the changes in cost will be due to decreased cost for the batteries. We assume an electric car has a 60 kWh battery. Online Appendix F contains time series data on costs of lithium ion batteries. Using this data, we determine an exponential model to predict the decrease in electric vehicle production costs over time.

4 Simulation Results

4.1 Long Run Steady State

We first analyze the results implied by Proposition 1. Assume that the long run production cost of electric vehicles is equal to the production cost of gasoline vehicles (so that $\hat{c}_x = c_g$.) Also assume that in the long run, the emissions from electric power generation are equal to zero, so that $\hat{\delta}_x$ consists of operating costs only. Under these assumptions, the critical value for $\eta(0)$ that determines whether or not gasoline vehicles are banned in the long run steady state depends on four parameters: gasoline global damages (due to CO_2, evaluated at the social cost of carbon), gasoline local damages (due to VOC, SO_2, $PM_{2.5}$, NO_X), gasoline operating costs, and electric operating costs.

Figure 3 shows the sensitivity of the critical value for $\eta(0)$ to changes in these four parameters. Consider the red line. Holding the other three parameters at their baseline value, the red line shows the relationship between the critical value of $\eta(0)$ and different

values of the electric operating costs, expressed as a percentage of its baseline value. As electric operating costs increase, the critical value of $\eta(0)$ that leads to a ban on gasoline vehicles in the long run increases. An increase in the costs of operating electric vehicles makes electric vehicles less desirable, hence the preferences of the consumer that likes electric vehicles the least must increase if the ban on gasoline vehicles is to be justified. The other three variables have a negative relationship with $\eta(0)$. For example, the blue line shows that as the social cost of carbon increases, gasoline vehicles impose greater external costs, and hence the preferences required to justify a ban decrease.

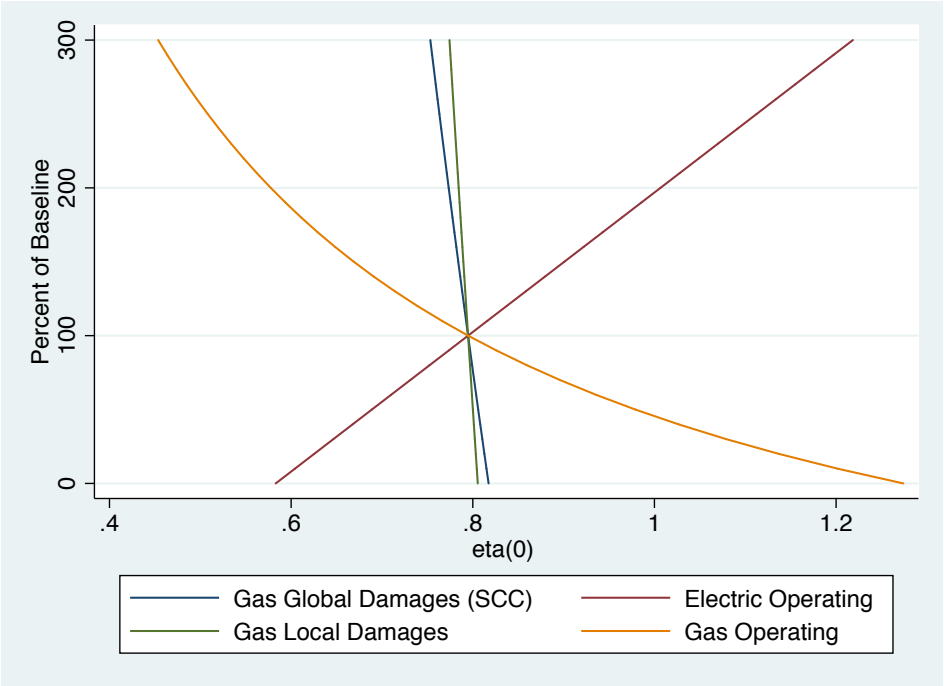


Figure 3: Parameter Combinations That Imply $G = 0$ in Long Run Steady State figprop1

The general message from Figure 3 is that the critical value of $\eta(0)$ is much more sensitive to operating expenses than it is to externalities.

4.2 Transition from gasoline to electric vehicles

Next we turn to analyzing the transition from gasoline to electric vehicles. For these results, we assume that parameter values are such that $G = 0$ in the long run steady state. Thus we are interested in the transition times, given that parameters are such that gasoline vehicles will be banned in the long run.

The results are shown in Figure 4. The red and blue curves show the values for t_g and t_e corresponding to the baseline value of $\eta(1) = 1.21$. There are three different regions represented as a function of the slope s. For small values of s we have the gap solution. Here $t_g < t_e$. As s increases, there is greater heterogeneity of preferences. This leads at first to slightly later transition times within a gap solution. But eventually we cross a threshold in which we move from the gap solution to the no gap solution. There is only a small range of parameters that gives the no gap solution. Here increases in s lead to much later transition times for gasoline vehicles, and in fact only a small increase in s is needed for the the transition time for gasoline vehicles to hit the terminal time. Finally, we reach the final region in which heterogeneity is so strong that gasoline vehicles are never banned. The threshold of this region is indicated by the point at which the red and blue curves stop. This critical value for s is equal to the value for s at the intersection of the black and blue curves in Figure 3.

Also shown in Figure 4 is an alternative set of results two other values of $\eta(1)$. The curves on the upper left show the case in which the intensity of preferences for gasoline vehicles are the strongest. The curves on the lower right show the case in which the intensity of preferences for gasoline vehicles is the weakest. Comparing across all three sets of curves shows the complicated comparative statics of intensity of preferences and heterogeneity of preferences. As we move from the upper left to the lower right, the intensity of preferences for gasoline vehicles increases, and the critical value for the heterogeneity of preferences such that gasoline vehicles are never banned increases as well.

5 Conclusion

The small ranges for which the no-gap solution exists illustrates a fundamental tension in the decision to ban gasoline vehicles. If such a ban is ever justified in the long run, then it must be the case that electric vehicles are fairly good substitutes for gasoline vehicles. But, if they are indeed good substitutes, then this suggests that the gap solution is likely to occur. Hence gasoline vehicles are banned quite early, before production of electric vehicles commences! There is only a narrow range for parameters that yields the much more intuitively appealing

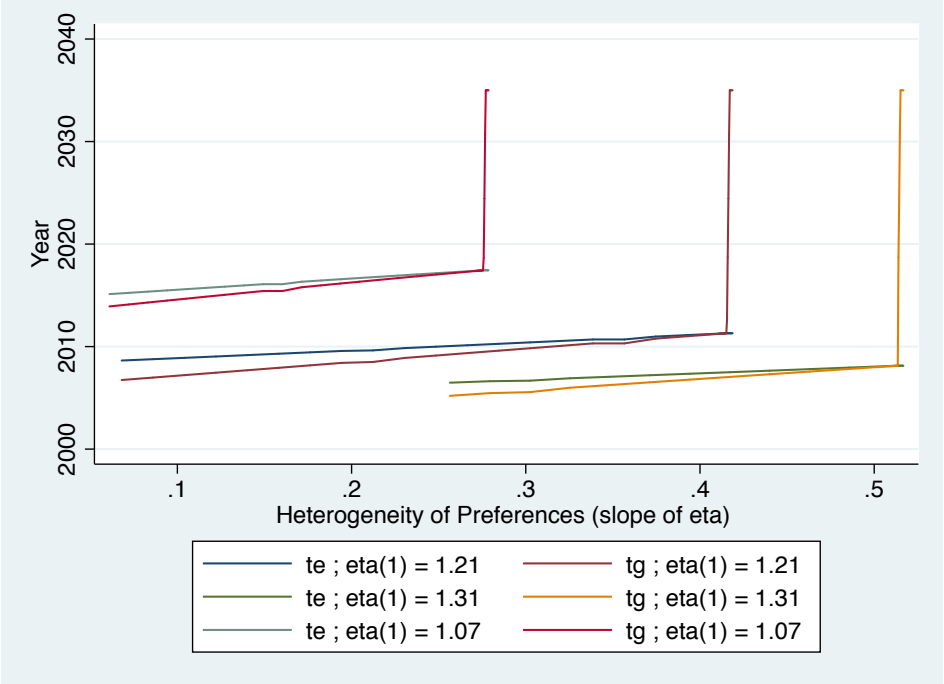


Figure 4: Transition times as a function of consumer heterogeneity exp-exp

no-gap solution. In this solution, gasoline and electric vehicles are both produced for a period of time before gasoline vehicles are banned.

The simulation results show that the optimal time to ban gasoline cars is quite sensitive to the heterogeneity of consumer preferences. When preferences are homogeneous, then we have the gap solution and the ban occurs quite early in our simulated time period. As heterogeneity increases, the time at which gasoline vehicles are banned increases, and eventually we pass the critical threshold at which we move to the no gap solution. Here a small increase in heterogeneity leads to a large increase in the time at which gasoline vehicles are banned. This suggests that optimal policy will depend critically on accurate assessment of consumer preferences for electric vehicles.

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Online Appendices

A Derivation of Aggregate Utility

agg-utility

In this section we describe how to determine the aggregate utility function given a distribution for the indifference ratio η. We also describe how we calibrate the aggregate utility function.

There is a unit mass of consumers on $[0,1]$. The total stock of electric vehicles and gasoline vehicles at time t is given by X and G. For each consumer, electric vehicles and gasoline vehicles are perfect substitutes, but the indifference ratio is not the same across consumers. Correspondingly, each consumer i has utility function $\mathcal{U}(\varrho(i) + \eta(i)\chi(i))$, where $\chi(i)$ and $\varrho(i)$ are the shares of the total stocks that are allocated the consumer and $\eta(i)$ is consumer i's indifference ratio. The distribution for $\eta(i)$ determines the degree of heterogeneity of preferences. When $\eta(i)$ is constant over i, all consumers have the same preferences, and hence the aggregate utility function has the representative consumer form.

In each instant of time, the planner, or the market for new and used vehicles, allocates the stock of vehicles to the consumers to maximize the sum of utility. Either way, the relevant optimization problem is

$$\begin{aligned} U(X,G) = \quad \max_{\varrho(i),\chi(i)} \quad & \int_0^1 \mathcal{U}(\varrho(i) + \eta(i)\chi(i))di \\ \text{s.t.} \quad & X = \int_0^1 \chi(i)di \\ & G = \int_0^1 \varrho(i)di \end{aligned}$$

We can transform this problem into a standard optimal control problem by introducing the state variables

$$c(i) = \int_0^i \varrho(s)ds \text{ and } \ell(i) = \int_0^t \chi(s)ds.$$

These variables represent the cumulative amount of cars allocated up to and including consumer i. The optimization problem becomes

$$
\begin{aligned}
U(X,G) = \max_{x,g} \quad & \int_0^1 \mathcal{U}(\varrho(i) + \eta(i)\chi(i))di \\
\text{s.t.} \quad \frac{dc}{di} &= \varrho \ ; \ c(0) = 0, c(1) = G \\
\frac{d\ell}{di} &= \chi \ ; \ \ell(0) = 0, \ell(1) = X \\
\chi &\geq 0 \\
\varrho &\geq 0
\end{aligned}
$$

The intuition for the solution to this problem can be gleaned from a simple exchange economy with linear preferences. With two consumers, the efficient solutions are on the edges of the Edgeworth box. All of one good is allocated to one consumer and the other good is split between the consumers. As we add more consumers, the partition extends accordingly. One group of consumers gets a share of one good and another group of consumers gets a share of the other good, with possibly one individual consumer getting a share of both goods. With a continuum of consumers, there is an critical consumer that defines the boundary between those getting one good and those getting the other.

The Hamiltonian is

$$H = \mathcal{A}\varrho + \mathcal{B}\chi + \mathcal{U}(\varrho + \eta\chi).$$

The adjoint equations are

$$\frac{d\mathcal{A}}{di} = 0$$

$$\frac{d\mathcal{B}}{di} = 0.$$

It follows immediately that both $\mathcal{A}$ and $\mathcal{B}$ are constant with respect to i. Maximizing H with respect to ϱ and χ gives the conditions that $\varrho = 0$ if $\mathcal{A} + \mathcal{U}' < 0$ and ϱ interior if $\mathcal{A} + \mathcal{U}' = 0$. Likewise $\chi = 0$ if $\mathcal{B} + \eta\mathcal{U}' < 0$ and χ interior if $\mathcal{B} + \eta\mathcal{U}' = 0$.

Both χ and ϱ cannot be interior during a time period. If this were the case, then we would have $\mathcal{B} = \eta\mathcal{A}$. But this can't occur more more than an instant in time because η varies with i and $\mathcal{B}$ and $\mathcal{A}$ do not.[9]

[9]There is the trivial solution $\mathcal{B} = \mathcal{A} = 0$. But this implies $\mathcal{U}' = 0$, i.e. all consumers are satiated.

We start by allocating ϱ to consumers with small values for i. The value for ϱ is given by

$$\mathcal{A} = -\mathcal{U}'(\varrho). \tag{A-1}$$ eqone

Because $\mathcal{A}$ is constant with respect to i, $\mathcal{U}'(\varrho)$ is constant with respect to i as well. Thus the solution to this equation will be a single value, call it ϱ^*. At some point i^*, we switch from allocating ϱ to allocating χ. We must have allocated the total amount of G, so

$$\int_0^{i^*} \varrho^* di = G. \tag{A-2}$$ eqtwo

From i^* to 1 we allocate x according to

$$\mathcal{B} + \eta\mathcal{U}'(\eta\chi) = 0. \tag{A-3}$$ eqthree

Let the solution to this equation be given by $\chi^*(i)$. When $i = 1$, we must have allocated all the X. Thus

$$\int_{i^*}^1 \chi^*(i) di = X. \tag{A-4}$$ eqfour

Finally, at i^*, we have both χ and ϱ interior. Thus we have $\mathcal{A} + \mathcal{U}' = 0$ and $\mathcal{B} + \eta\mathcal{U}' = 0$. It follows that

$$\eta(i^*)\mathcal{A} = \mathcal{B}. \tag{A-5}$$ eqfive

Solving these five equations gives us values for $\mathcal{A}$, $\mathcal{B}$, ϱ^*, $\chi^*(i)$, and i^* as a function of X and G. This in turn gives us $U(X, G)$. For numerical analysis, we often only require values for U_G and U_X. By the envelope theorem, we have

$$U_G = -\mathcal{A} \tag{A-6}$$ envg

and

$$U_X = -\mathcal{B} \tag{A-7}$$ envx

.

It use useful to combine several of these results into a lemma.

lem-mus **Lemma 1.** *The aggregate utility function satisfies*

$$U_X(G, X) = \eta(i^*)U_G(G, X).$$

Proof. We have

$$U_X(G, X) = -\mathcal{B} = -\eta(i^*)\mathcal{A} = \eta(i^*)U_G(G, X),$$

where the first equality follows from (A-7), the second from (A-5), and the third from (A-6). □

We can obtain an explicit solution for the aggregate utility function when the individual consumers' utility functions are logarithmic and the distribution for η is linear. Consider the individual utility function

$$\mathcal{U}(\varrho + \eta\chi) = k \ln(\varrho + \eta\chi),$$

where k is a parameter to be calibrated. With this utility function Equations (A-1) and (A-2) become

$$\mathcal{A} = -\frac{k}{\varrho} \tag{A-8}$$ eqsixp

and

$$\varrho^* i^* = G. \tag{A-9}$$ eqsixpp

Eliminating ϱ^* gives

$$\mathcal{A} = \frac{ki^*}{G}. \tag{A-10}$$ eqsix

Equation (A-3) implies

$$\chi^* = \frac{-k}{\mathcal{B}}. \tag{A-11}$$ eqsixa

Substituting this into Equation (A-4) gives

$$\int_{i^*}^{1} \frac{-k}{\mathcal{B}} di = X.$$

Since $\mathcal{B}$ is constant with respect to i, this becomes

$$\mathcal{B} = -\frac{(1 - i^*)k}{X}. \tag{A-12}$$ eqsixb

Now using (A-5) and (A-10) gives

$$\eta(i^*)\left(-\frac{ki^*}{G}\right) = -\frac{k(1-i^*)}{X}. \tag{A-13}$$ eqseven

Re-writing gives

$$\eta(i^*)i^* = (1-i^*)\frac{G}{X}. \tag{A-14}$$ eqeight

Now suppose we have a linear distribution for η, as defined in the main text:

$$\eta(i) = s(i-1) + u,$$

where $s = \eta(1) - \eta(0)$ and $u = \eta(1)$. In this case (A-14) will be a quadratic function of i^*. It follows that

$$i^* = -\frac{-\sqrt{\left(\frac{G}{X} - s + u\right)^2 + \frac{4Gs}{X}} + \frac{G}{X} - s + u}{2s}.$$

Using this, we can determine the optimal ϱ^* and χ^* from the equations (A-8) to (A-12). Then integrating the objective function using this optimal solution gives the aggregate utility function:

$$U(X,G) =$$

$$-\frac{k\left(X\left(-2s\log\left(X\left(\sqrt{\frac{G^2}{X^2}+\frac{2G(s+u)}{X}+(s-u)^2}+s+u\right)+G\right)+2(u-s)\log\left(\frac{X\sqrt{\left(\frac{G}{X}-s+u\right)^2+\frac{4Gs}{X}}-G-sX+uX}{2X}\right)+2s\log(2u)+s+u-2u\log(u)\right)-X\sqrt{\left(\frac{G}{X}-s+u\right)^2+\frac{4Gs}{X}}+G\right)}{2sX}.$$

We now describe how we calibrate the aggregate utility function. First, we get parameters for the distribution of η from data on WTP for electric vehicles in Table 9 of Hidrue et al (2011). Consider row D. Fitting a linear model to the five data points gives us an estimate of 1.21 for $\eta(1)$ and 0.87 for $\eta(0)$.

Next we determine the parameter k. Using the equation for the initial steady state (1) we have

$$U_G(G^{ss}, 0) = (a+r)c_g + \delta_g.$$

Using the values $G^{ss} = 110$ million, $a = 0.067$ (From Online Appendix H), $r = 0.05$, $c_g = 36113$, and $\delta_g = 3021.91$, gives $k = 797,184$.

The equations for the transition times in the proofs of Propositions 2 and 4 involve $U_X(G, 0)$ evaluated at various values for G. So it is useful to determine explicit formulas for these expressions. In the no gap solution, we seek to characterize $U_X(G^{ss}, 0)$. If $X = 0$, then $i^* = 1$. We have

$$U_X(G^{ss}, 0) = \eta(1)U_G(G^{ss}, 0) = \eta(1)((a + r)c_g + \delta_g), \tag{A-15}$$ uxprimativ

where the first equality follows from Lemma 1 and the second from (1).

In the no gap solution, we seek to characterize $U_X(G^{gap}(t^e), 0)$. From Lemma 1 we have

$$U_X(G^{gap}(t^e), 0) = \eta(1)U_G(G^{gap}(t^e), 0).$$

Unlike above, we don't have a direct equation for $U_G(G^{gap}(t^e), 0)$ so we can't write U_X in terms of the primitives of the model. However, we can do this if $\mathcal{U}$ is logarithmic. Because $i^* = 1$, equation (A-10) along with (A-6) implies that for any G, we have

$$U_G(G, 0) = \frac{k}{G} \tag{A-16}$$ generalg

Using (A-16) with $G = G^{ss}$ gives

$$U_G(G^{ss}, 0) = \frac{k}{G^{ss}} = (a + r)c_g + \delta_g,$$

where the second equality comes from (1). Solving for G^{ss} gives

$$G^{ss} = \frac{k}{(a + r)c_g + \delta_g}.$$

Now from (2), we have

$$G^{gap}(t^e) = \frac{k}{(a + r)c_g + \delta_g} e^{-a(t_e - tg)}.$$

Using (A-16) with $G = G^{gap}$ gives

$$U_G(G^{gap}(t_e), 0) = \frac{k}{G^{gap}(t^e)} = \frac{1}{(a+r)c_g + \delta_g} e^{a(t_e - tg)}.$$

It follows that

$$U_X(G^{gap}(t_e), 0) = -\mathcal{B} = -\eta(1)\mathcal{A} = U_G(G^{gap}(t_e), 0) = \eta(1)\frac{k}{G^{gap}(t^e)} =$$

$$\eta(1)\frac{1}{(a+r)c_g + \delta_g} e^{a(t_e - tg)}.$$

B Planners Problem

planprob

Let $\tilde{\beta}$ and $\tilde{\alpha}$ be the adjoint variables corresponding to the system equations for X and G. The Hamiltonian is

$$H = \tilde{\beta}(-aX + x) + \tilde{\alpha}(-aG + g) + e^{-rt}\left(U(G, X) - c_g g - c_x x - \delta_g G - \delta_x X\right).$$

From the Maximum Principle[10] the necessary conditions for the optimal control are

$$\begin{aligned}
\dot{X} + aX - x &= 0 && \text{(state equations)} \\
\dot{G} + aG - g &= 0 && \\
X(0) &= 0 && \text{(initial state conditions)} \\
G(0) &= G_0 && \\
-\dot{\tilde{\beta}} + \tilde{\beta}a - e^{-rt}\left(U_X - \delta_x\right) &= 0 && \text{(adjoint equations)} \\
-\dot{\tilde{\alpha}} + \tilde{\alpha}a - e^{-rt}\left(U_G - \delta_g\right) &= 0 && \\
\tilde{\beta}(T) &= c_x e^{-rT} && \text{(adjoint final conditions)} \\
\tilde{\alpha}(T) &= c_g e^{-rT}, &&
\end{aligned}$$

and, in addition, the controls g and $x$maximize the Hamiltonian subject to the feasibility constraints. Notice that the Hamiltonian is linear in the controls. Thus we have additional necessary conditions for interior controls

[10] See for example Kamien and Schwartz (2012).

$$\begin{aligned}\tilde{\alpha} - e^{-rt}c_g &= 0 \quad \text{(necessary condition for interior } g\text{)}\\ \tilde{\beta} - e^{-rt}c_x &= 0 \quad \text{(necessary condition for interior } x\text{)}\end{aligned}$$

In other words, if the control g satisfies $\bar{g} > g > 0$, then it must be the case that $\tilde{\alpha} - e^{-rt}(c_g) = 0$. We also have the boundary conditions. For example, if $\tilde{\alpha} - e^{-rt}c_g < 0$ then $g = 0$ and if $\tilde{\alpha} - e^{-rt}c_g > 0$ then $g = \bar{g}$. Finally, we note that if $\tilde{\alpha} - e^{-rt}c_g = 0$ then any g such that $\bar{g} \geq g \geq 0$ maximizes the Hamiltonian.

Using the change of variables $\beta = e^{rt}\tilde{\beta}$ and $\alpha = e^{rt}\tilde{\alpha}$ (i.e. current values instead of present values) the adjoint equations become

$$\dot{\beta} = (a + r)\beta - U_X + \delta_x \; ; \beta(T) = c_x$$

$$\dot{\alpha} = (a + r)\alpha - U_G + \delta_g \; ; \alpha(T) = c_g.$$

The necessary conditions for interior x and g become

$$\beta = c_x$$

and

$$\alpha = c_g.$$

C Proofs

proofss

Proof of Proposition 1

Proof. Consider a long run steady state in which there is interior production of both vehicles. We have $\alpha = c_g$. Because c_g is constant, we have $\dot{\alpha} = 0$. It follows from the adjoint equation for α that

$$U_G(G, X) = (a + r)c_g + \delta_g.$$

In the long run steady state, $\hat{c}_x$ is constant. So $\dot{\beta} = 0$ and the adjoint equation for α gives

$$U_X(G, X) = (a + r)\hat{c}_x + \hat{\delta}_x. \tag{A-17}$$ lrux

If the parameters are such that the solution to these equations has $G \leq 0$, then there cannot be interior production of gasoline vehicles in the long run steady state, which implies that gasoline vehicles are banned. So we find the critical value of $\eta(0)$ that makes $G = 0$. When $G = 0$, there are two possible solutions for i^*, depending on the value of X. If X is small, then $i^* > 0$. In this case, some consumers do not get assigned any vehicles. If X is large, then $i^* = 0$. In this case all consumers do get assigned a positive amount of vehicles. For the moment, suppose that X is sufficiently large so that $i^* = 0$. Then using Lemma 1, we have

$$U_X(0, X) = \eta(0) U_G(0, X).$$

Combining this with the two equations above gives the critical value for $\eta(0)$:

$$\eta(0) = \frac{(a+r)\hat{c}_x + \hat{\delta}_x}{(a+r)c_g + \delta_g}. \tag{A-18}$$ eq-etacrit

For bigger values of $\eta(0)$, consumer i has stronger preferences for electric vehicles, which makes $G < 0$.

It remains to show that X will be sufficiently large so that all consumers are assigned a positive amount of vehicles. In particular, we must have $\chi^*(0) \geq 0$. From (A-3) we have

$$\mathcal{B} + \eta(0)\mathcal{U}'(\eta(0)\chi^*(0)) = 0.$$

Let F be the inverse of $\mathcal{U}'$. Solving for $\chi^*(0)$ gives

$$\chi^*(0) = \frac{1}{\eta(0)} F\left(\frac{-\mathcal{B}}{\eta(0)}\right).$$

Because $\eta(0) > 0$, proving that $\chi^*(0) \geq 0$ is equivalent to proving that

$$F\left(\frac{-\mathcal{B}}{\eta(0)}\right) \geq 0.$$

Using (A-18), (A-7), and (A-17) gives

$$F\left((a+r)c_g + \delta_g\right) \geq 0.$$

We show this equation holds by analyzing the assumption that the initial steady state value for G is positive. We have

$$U_G(G^{ss}, 0) = (a + r)c_g + \delta_g.$$

In the initial steady state, $i^* = 1$ and $\varrho = G^{ss}$. From (A-1) we have

$$\mathcal{A} = -\mathcal{U}'(G^{ss}).$$

So we have

$$(a + r)c_g + \delta_g = U_G(G^{ss}, 0) = -\mathcal{A} = \mathcal{U}'(G^{ss}),$$

where the second equality follows from (A-6). Solving this equation for G^{ss} gives

$$G^{ss} = F\left((a + r)c_g + \delta_g\right).$$

Since $G^{ss} > 0$ we have shown that

$$F\left((a + r)c_g + \delta_g\right) \geq 0,$$

which completes the proof.

□

Proof of Proposition 2

Proof. By the continuity of the state variables, at t^e we have $X^{gap} = 0$. Thus t^e is the solution to

$$\dot{c}_x(t^e) = (a + r)c_x(t^e) + \delta_x(t^e) - U_X(G^{gap}(t^e), 0). \tag{A-19}$$

gaponeproo

This equation is a function of both t^e and t^g (through $G^{gap}(t^e)$), so we need another equation to pin down the transition times.

During $[t^g, t^e]$ the adjoint equation for α evolves according to the forward equation

$$\dot{\alpha} = (a + r)\alpha + \delta_g - U_G(G^{gap}, 0) \;\; ; \;\; \alpha(t^g) = c_g.$$

During $[t^e, T]$ the adjoint equation for α evolves according to the backward equation

$$\dot{\alpha} = (a+r)\alpha + \delta_g - U_G(G^{gap}, X^{gap}) \;\; ; \;\; \alpha(T) = c_g.$$

By the continuity of the adjoint variables, these two solutions must coincide at t^e. We have to solve two linear, first-order differential equations of the form

$$\dot{\alpha} - (a+r)\alpha = f(t).$$

Using the integrating factor method the solution is

$$\alpha = e^{(a+r)t} \int e^{-(a+r)\tau} f(\tau) d\tau + K e^{(a+r)t},$$

where K is an arbitrary constant.

For the forward equation we have

$$\alpha = e^{(a+r)t} \int_{t_g}^{t} e^{-(a+r)\tau} (\delta_g - U_G(G^{ss} e^{-a(\tau - tg)}, 0) d\tau + K e^{(a+r)t},$$

with the condition $\alpha(t^g) = c_g$. Evaluating at $t = t^g$ gives

$$c_g = K e^{(a+r)t_g}$$

so that

$$K = c_g e^{-(a+r)t_g}.$$

Thus the full solution to the forward equation is

$$\alpha = e^{(a+r)t} \int_{t_g}^{t} e^{-(a+r)\tau} (\delta_g - U_G(G^{ss} e^{-a(\tau - tg)}, 0) d\tau + c_g e^{(a+r)(t-t^g)}$$

Evaluating at $t = t^e$ gives

$$\alpha(t^e) = e^{(a+r)t^e} \int_{t_g}^{t^e} e^{-(a+r)\tau} (\delta_g - U_G(G^{ss} e^{-a(\tau - tg)}, 0) d\tau + c_g e^{(a+r)(t^e - t^g)} \qquad \text{(A-20)}$$ forata

For the backward equation we have

$$\alpha = e^{(a+r)t}\int_{t_e}^{t} e^{-(a+r)\tau}\left(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))\right)d\tau + Ke^{(a+r)t},$$

with the condition $\alpha(T) = c_g$. Evaluating at $t = T$ gives

$$c_g = e^{(a+r)T}\int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))\right)d\tau + Ke^{(a+r)T}$$

so that

$$K = c_g e^{-(a+r)T} - \int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))\right)d\tau.$$

Thus the full solution to the backward equation is

$$\alpha = c_g e^{(a+r)(t-T)} + e^{(a+r)t}\int_{t_e}^{t} e^{-(a+r)\tau}(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau)))d\tau -$$

$$e^{(a+r)t}\int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))\right)d\tau.$$

Evaluating at $t = t^e$ gives

$$\alpha(t^e) = c_g e^{(a+r)(t^e-T)} - e^{(a+r)t^e}\int_{t_e}^{T} e^{-(a+r)\tau}(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau)))d\tau. \qquad \text{(A-21)}$$

bacata

Because α is continuous at t^e, we set the right-hand side of (A-34) and (A-35) equal to each other. This gives

$$e^{(a+r)t^e}\int_{t_g}^{t^e} e^{-(a+r)\tau}(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, 0)d\tau + c_g e^{(a+r)(t^e-t^g)} =$$

$$c_g e^{(a+r)(t^e-T)} - e^{(a+r)t^e}\int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))\right)d\tau.$$

Simplifying gives

$$e^{(a+r)t^e}\left(\delta_g\int_{t_g}^{T} e^{-(a+r)\tau}d\tau - \int_{t_g}^{t^e} e^{-(a+r)\tau}U_G(G^{ss}e^{-a(\tau-tg)}, 0)d\tau - \int_{t_e}^{T} e^{-(a+r)\tau}U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))d\tau\right) =$$

$$c_g e^{(a+r)(t^e-T)} - c_g e^{(a+r)(t^e-t^g)}.$$

Dividing both sides by $e^{(a+r)t^e}$ and carrying out the first integral gives

$$\frac{\delta_g}{a+r}(e^{-(a+r)t_g} - e^{-(a+r)T}) - \int_{t_g}^{t^e} - \int_{t_e}^{T} = c_g e^{-(a+r)T} - c_g e^{-(a+r)t^g}.$$

Simplifying gives

$$\left(c_g + \frac{\delta_g}{a+r}\right)(e^{-(a+r)t_g} - e^{-(a+r)T}) = \int_{t_g}^{t^e} e^{-(a+r)\tau}(U_G(G^{ss}e^{-a(\tau-tg)}, 0)d\tau + \int_{t_e}^{T} e^{-(a+r)\tau}U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))d$$

We can write this in a slightly different way as

$$\int_{t^g}^{T} e^{-(a+r)\tau}((a+r)c_g + \delta_g)d\tau = \int_{t^g}^{t^e} e^{-(a+r)\tau}(U_G(G^{ss}e^{-a(\tau-tg)}, 0)d\tau + \int_{t^e}^{T} e^{-(a+r)\tau}U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))d\tau.$$

Finally, using the (1), we have

$$\int_{t^g}^{T} e^{-(a+r)\tau}U_G(G^{ss}, 0)d\tau = \int_{t^g}^{t^e} e^{-(a+r)\tau}(U_G(G^{ss}e^{-a(\tau-tg)}, 0)d\tau + \int_{t^e}^{T} e^{-(a+r)\tau}U_G(G^{ss}e^{-a(\tau-tg)}, X^{gap}(\tau))d\tau. \quad \text{(A-22)}$$

gaptwoproo

Thus we find t^g and t^e as the solution to the simultaneous system of equations (A-19) and (A-22). □

Proof of Proposition 3.

Proof. First we show that both g and x cannot be interior during the same time interval. Suppose both g and x are interior during some time interval. Because g is interior we have $\alpha = c_g$. Thus $\dot{\alpha} = 0$ and from the adjoint equation for α we have

$$\delta_g + (a+r)c_g = U'.$$

Because x is interior we have $\beta = c_x$. From the adjoint equation for β we have

$$-\dot{c}_x + (a+r)c_x + \delta_x = U'\eta. \quad \text{(A-23)}$$

adjbt

Eliminating U' gives

$$\delta_g + (a+r)c_g = \frac{-\dot{c}_x + (a+r)c_x + \delta_x}{\eta}.$$

Taking the time derivative gives

$$\ddot{c}_x = \frac{(a+r)\dot{c}_x + \dot{\delta}_x}{\eta}.$$

By the assumptions of the model, the right hand side is negative, and the left hand side is non-negative, so we have a contradiction.

So far we have shown that $t^g \leq t^e$. So we must rule out the case that $t^g = t^e$. Suppose that t^g does indeed equal t^e. In steady state with g interior, we have $\alpha = c_g$, thus $\dot{\alpha} = 0$, and hence $U' - (a+r)c_g - \delta_g = 0$. These equations hold at $t = t^g$. Because $t^g = t^e$, we are also producing electric vehicles with interior x. Taking the time derivative of (A-23) gives

$$-\ddot{c}_x + (a+r)\dot{c}_x + \dot{\delta}_x = \dot{U}'\eta$$

The left hand side is negative, hence $\dot{U}'$ is negative. Thus marginal utility is decreasing over time.

Now consider some point in time $\tilde{t} = t^g + \varepsilon$. Because $\dot{\alpha} = 0$ at t^g, we have $\alpha = c_g$ at $\tilde{t}$. So at $\tilde{t}$, the adjoint equation for α is

$$\dot{\alpha} = -(U' - (a+r)c_g - \delta_g).$$

Because marginal utility is decreasing over time, U' is less at $\tilde{t}$ than it is at t^g. At t^g we have $U' - (a+r)c_g - \delta_g = 0$. So at $\tilde{t}$ we have $U' - (a+r)c_g - \delta_g < 0$. Hence at $\tilde{t}$ we have $\dot{\alpha} > 0$. So this implies α will become greater than c_g in the next time instant. But, from the necessary conditions, if $\alpha > c_g$ then it is optimal for g to be positive (equal to the maximum production level.) This contradicts the definition of t^g. □

Proof of Proposition 4

Proof. At t^e we have $G = G^{ss}$ and $X = 0$. From the interior condition for x we have $\beta = c_x$ and from the adjoint condition for β we have

$$\dot{c}_x(t^e) = (a+r)c_x(t^e) + \delta_x(t^e) - U_X(G^{ss}, 0). \tag{A-24}$$ nogaponepr

Notice that this equation is only a function of t^e, not t^g as in the gap solution. Thus we can solve this single equation for t^e.

Next we determine t^g. Again we focus on the adjoint variable α. During $[t^e, t^g]$, we still have g interior, so that $\alpha = c_g$. During $[t^g, T]$ the adjoint equation for α evolves according to the backward equation

$$\dot{\alpha} = (a+r)\alpha + \delta_g - U_G(G^{no}, X^{no}) \quad ; \quad \alpha(T) = c_g.$$

By the continuity of the adjoint variables, we must also have $\alpha(t^g) = c_g$. Now follow steps similar to that for the gap case to derive an equation for t_g.

For the backward equation we have

$$\alpha = e^{(a+r)t} \int_{t_g}^{t} e^{-(a+r)\tau} \left(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau))\right) d\tau + Ke^{(a+r)t},$$

with the condition $\alpha(T) = c_g$. Evaluating at $t = T$ gives

$$c_g = e^{(a+r)T} \int_{t_g}^{T} e^{-(a+r)\tau} \left(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau))\right) d\tau + Ke^{(a+r)T}$$

so that

$$K = c_g e^{-(a+r)T} - \int_{t_g}^{T} e^{-(a+r)\tau} \left(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau))\right) d\tau.$$

Thus the full solution to the backward equation is

$$\alpha = c_g e^{(a+r)(t-T)} + e^{(a+r)t} \int_{t_g}^{t} e^{-(a+r)\tau} (\delta_g - U_G(G^{no}(\tau), X^{no}(\tau)))d\tau -$$

$$e^{(a+r)t} \int_{t_g}^{T} e^{-(a+r)\tau} \left(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau))\right) d\tau.$$

Evaluating at $t = t^g$ gives

$$\alpha(t^g) = c_g e^{(a+r)(t^g - T)} - e^{(a+r)t^g} \int_{t_g}^{T} e^{-(a+r)\tau} (\delta_g - U_G(G^{no}(\tau), X^{no}(\tau)))d\tau.$$

Using $\alpha(t^g) = c_g$ gives

$$c_g = c_g e^{(a+r)(t^g - T)} - e^{(a+r)t^g} \int_{t_g}^{T} e^{-(a+r)\tau}(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau)))d\tau.$$

This can be written as

$$c_g e^{-(a+r)t^g} = c_g e^{-(a+r)T} - \int_{t_g}^{T} e^{-(a+r)\tau}(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau)))d\tau.$$

and

$$c_g e^{-(a+r)T} - c_g e^{-(a+r)t^g} = \int_{t_g}^{T} e^{-(a+r)\tau}(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau)))d\tau.$$

and

$$\int_{t^g}^{T} -e^{-(a+r)\tau}(a+r)c_g = \int_{t_g}^{T} e^{-(a+r)\tau}(\delta_g - U_G(G^{no}(\tau), X^{no}(\tau)))d\tau.$$

and finally

$$\int_{t_g}^{T} e^{-(a+r)\tau}\left(\delta_g + (a+r)c_g\right)d\tau = \int_{t_g}^{T} e^{-(a+r)\tau} U_G(G^{no}(\tau), X^{no}(\tau))d\tau = 0. \tag{A-25}$$

nogaptwopr

This equation can be solved for t^g. Thus the transition times for the no gap solution are given by (A-24) and (A-25). Using (A-15), we can write (A-24) as

$$\eta(1)(\delta_g + (a+r)c_g) = (a+r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e) \tag{A-26}$$

nogaponepr

□

D Extensions

extend

Here we introduce learning by doing and charging infrastructure. To account for charging infrastructure, the aggregate utility function becomes $U(G, X, W)$ where W is the stock of charging stations and $U_W > 0$. To account for learning by doing, the marginal cost function for producing electric vehicles becomes

$$c_x = f(Z)$$

where $f'(Z) \leq 0$ and $f(\infty) = c_g$.

The planner's problem is

$$\begin{aligned}
\max_{x,g} \quad & f(Z(T))e^{-rT}X(T) + c_g e^{-rT}G(T) + \int_0^T e^{-rt}\left(U(G,X,W) - c_g g - f(Z)x - c_w w - \delta_g G - \delta_x X\right) dt \\
\text{s.t.} \quad \dot{X} &= -aX + x \ ; \ X(0) = 0 \\
\dot{G} &= -aG + g \ ; \ G(0) = G^{ss} \\
\dot{W} &= w \ ; \ W(0) = 0 \\
\dot{Z} &= x \ ; \ Z(0) = 0 \\
\bar{x} &\geq x \geq 0 \\
\bar{g} &\geq g \geq 0 \\
\bar{w} &\geq w \geq 0.
\end{aligned}$$

Let $\tilde{\lambda}$, $\tilde{\beta}$, $\tilde{\alpha}$, and $\tilde{\phi}$ be the adjoint variables corresponding to the system equations for Z, X, G, and W. The Hamiltonian is

$$H = \tilde{\lambda}x + \tilde{\beta}(-aX + x) + \tilde{\alpha}(-aG + g) + \tilde{\phi}w + e^{-rt}\left(U(G,X,W) - c_g g - c_w w - f(Z)x - \delta_g G - \delta_x X\right).$$

From the Maximum Principle the necessary conditions for the optimal control are

$$
\begin{aligned}
\dot{Z} - x &= 0 && \text{(state equations)}\\
\dot{X} + aX - x &= 0\\
\dot{G} + aG - g &= 0\\
\dot{W} - w &= 0\\
X(0) &= 0 && \text{(initial state conditions)}\\
Z(0) &= 0\\
G(0) &= G_0\\
W(0) &= 0\\
-\dot{\tilde{\lambda}} - e^{-rt}\left(-f'(Z)x\right) &= 0 && \text{(adjoint equations)}\\
-\dot{\tilde{\beta}} + \tilde{\beta}a - e^{-rt}\left(U_X - \delta_x\right) &= 0\\
-\dot{\tilde{\alpha}} + \tilde{\alpha}a - e^{-rt}\left(U_G - \delta_g\right) &= 0\\
-\dot{\tilde{\phi}} - e^{-rt}U_W &= 0\\
\tilde{\lambda}(T) &= f'(Z(T))X(T)e^{-rT} && \text{(adjoint final conditions)}\\
\tilde{\beta}(T) &= f(Z(T))e^{-rT}\\
\tilde{\alpha}(T) &= c_g e^{-rT}\\
\tilde{\phi}(T) &= 0,
\end{aligned}
$$

and, in addition, the controls g, x, and w maximize the Hamiltonian subject to the feasibility constraints. Notice that the Hamiltonian is linear in the controls. Thus we have additional necessary conditions for interior controls

$$
\begin{aligned}
\tilde{\alpha} - e^{-rt}c_g &= 0 \quad \text{(necessary condition for interior } g\text{)}\\
\tilde{\lambda} + \tilde{\beta} - e^{-rt}f(Z) &= 0 \quad \text{(necessary condition for interior } x\text{)}\\
\tilde{\phi} - e^{-rt}c_w &= 0 \quad \text{(necessary condition for interior } w\text{).}
\end{aligned}
$$

In other words, if the control g satisfies $\bar{g} > g > 0$, then it must be the case that $\tilde{\alpha} - e^{-rt}\left(c_g\right) = 0$. We also have the boundary conditions. For example, if $\tilde{\alpha} - e^{-rt}c_g < 0$ then $g = 0$ and if $\tilde{\alpha} - e^{-rt}c_g > 0$ then $g = \bar{g}$. Finally, we note that if $\tilde{\alpha} - e^{-rt}c_g = 0$ then any g such that $\bar{g} \geq g \geq 0$ maximizes the Hamiltonian.

Assume in what follows that T is long enough (and/or $Z(T)$ is big enough) so that

$f(Z(T)) = c_g$ and $f'(Z(T)) = 0$.

Using the change of variables $\beta = e^{rt}\tilde{\beta}, \alpha = e^{rt}\tilde{\alpha}, \lambda = e^{rt}\tilde{\lambda}$, and $\phi = e^{rt}\tilde{\phi}$ (i.e. current values instead of present values) the adjoint equations become

$$\dot{\lambda} = r\lambda + f'(Z)x \ ; \lambda(T) = 0$$

$$\dot{\beta} = (a+r)\beta - U_X + \delta_x \ ; \beta(T) = c_g$$

$$\dot{\alpha} = (a+r)\alpha - U_G + \delta_g \ ; \alpha(T) = c_g.$$

$$\dot{\phi} = r\phi - U_W \ ; \ \phi(T) = 0.$$

The necessary conditions for interior x, g, and w become

$$\lambda + \beta = f(Z), \tag{A-27}$$ sumadjointl

$$\alpha = c_g.$$

and

$$\phi = c_w.$$

Next we show a few facts about the adjoint variables.

lem-pos **Lemma 2.** *1. For all $t \in [0,T]$ we have $\lambda \geq 0$ and $\phi \geq 0$.*

2. If x is interior, then $\dot{\beta} = -r\lambda$.

3. If x is zero for $t \in [0, t^s)$ and interior for $t \in [t^s, T]$ for some t^s, then $\beta(t^s) < f(0)$ and $\beta \geq c_g$ for $t \in [t^s, T]$.

Proof. Suppose that at some point in time $\lambda < 0$. Because $f'(Z)x \leq 0$, it follows from the adjoint equation for λ that $\dot{\lambda} < 0$. Thus λ must continue to fall for the rest of the time period. But this is a contradiction with $\lambda(T) = 0$. The proof for ϕ is similar.

Next suppose that x is interior. Take the time derivative of (A-27). This gives

$$\dot{\lambda} + \dot{\beta} = f'(Z)x.$$

Using the adjoint equation for λ this implies

$$r\lambda + f'(Z)x + \dot{\beta} = f'(Z)x.$$

Simplifying gives the desired result that

$$\dot{\beta} = -r\lambda.$$

The fact that $\beta(t^s) < f(0)$ follows directly from (A-27) and the fact that $\lambda \geq 0$. To prove that $\beta \geq c_g$, we combine

$$\dot{\beta} = -r\lambda \leq 0.$$

with $\beta(T) = c_g$. □

Learning By Doing

In this section, we focus learning by doing and ignore charging infrastructure. Furthermore, we assume that we have the representative consumer model so that aggregate utility is given by $U(G + \eta X)$. In the main text, we showed that such a utility function leads to the gap solution. The next proposition shows this result is robust to the having the cost of electric vehicles be determined by learning by doing.

Proposition 5. *Consider the model with learning by doing. Suppose that aggregate utility is given by $U(G + \eta X)$. Then the solution to the planner's problem has $t^g < t^e$.*

Proof. First we show that both g and x cannot be interior during the same time interval. Suppose both g and x are interior during some time interval. Because g is interior we have $\alpha = c_g$. Thus $\dot{\alpha} = 0$ and from the adjoint equation for α we have

$$\delta_g + (a + r)c_g = U'.$$

Because x is interior we have $\lambda + \beta = f(Z)$. Taking the time derivative gives

$$\dot{\lambda} + \dot{\beta} = f'(Z)x.$$

Substituting in from the adjoint equations for λ and β gives

$$r\lambda + f'(Z)x + (a+r)\beta - U'\eta + \delta_x = f'(Z)x.$$

Simplifying gives

$$r(\lambda+\beta) + a\beta + \delta_x = U'\eta. \tag{A-28}$$ keyforx

Substituting the value for U' from above and using (A-27) gives

$$rf(Z) + a\beta + \delta_x = (\delta_g + (a+r)c_g)\eta.$$

Taking the time derivative gives

$$rf'(Z)x + a\dot{\beta} + \dot{\delta}_x = 0 \tag{A-29}$$ fstep

From Lemma 2 we know that

$$\dot{\beta} = -r\lambda.$$

So we have

$$rf'(Z)x + \dot{\delta}_x = ar\lambda.$$

The first expression on the left-hand-side is non-positive and the second expression is negative. This implies that λ is negative, which contradicts Lemma 2.

So far we have shown that $t^g \leq t^e$. So we must rule out the case that $t^g = t^e$. Suppose that t^g does indeed equal t^e. In steady state with g interior, we have $\alpha = c_g$, thus $\dot{\alpha} = 0$, and hence $U' - (a+r)c_g - \delta_g = 0$. These equations hold at $t = t^g$. Because $t^g = t^e$, we are also producing electric vehicles with interior x. From (A-28) and (A-27) we have

$$r(f(Z)) + a\beta + \delta_x = U'\eta. \tag{A-30}$$ interxu

Taking the time derivative gives

$$\eta\dot{U}' = rf'(Z)x + a\dot{\beta} + \dot{\delta}_x.$$

Every term on the right hand side is negative, hence $\dot{U}'$ is negative. Thus marginal utility is decreasing over time.

Now consider some point in time $\tilde{t} = t^g + \varepsilon$. Because $\dot{\alpha} = 0$ at t^g, we have $\alpha = c_g$ at $\tilde{t}$. So at $\tilde{t}$, the adjoint equation for α is

$$\dot{\alpha} = -(U' - (a+r)c_g - \delta_g).$$

Because marginal utility is decreasing over time, U' is less at $\tilde{t}$ than it is at t^g. At t^g we have $U' - (a+r)c_g - \delta_g = 0$. So at $\tilde{t}$ we have $U' - (a+r)c_g - \delta_g < 0$. Hence at $\tilde{t}$ we have $\dot{\alpha} > 0$. So this implies α will become greater than c_g in the next time instant. But, from the necessary conditions, if $\alpha > c_g$ then it is optimal for g to be positive (equal to the maximum production level.) This contradicts the definition of t^g. □

Equations for transition times

Next we determine expressions for the transition times in the gap solution for the model with learning by doing. During $[0, t^g]$, we have interior steady state production of gasoline vehicles, so that $\alpha = c_g$. From the adjoint equation for α,

$$(a+r)c_g + \delta_g = U'. \tag{A-31}$$ eqforgasa

During (t^g, t^e), there is no production of gasoline or electric vehicles. The stock of gasoline vehicles decays exponentially,

$$G(t) = G^{ss} e^{-a(t-t^g)},$$

and so marginal utility is

$$U'(G(t)) = U'(G^{ss} e^{-a(t-t^g)}).$$

Next consider $[t^e, T]$. Taking the time derivative of (A-27) and using the state equation for Z gives

$$\dot{\lambda} + \dot{\beta} = f'(Z)x.$$

Substituting in the adjoint equations for λ and β gives

$$r\lambda + f'(Z)x + (a+r)\beta - U'\eta + \delta_x = f'(Z)x.$$

Simplifying and using (A-27) gives

$$a\beta + r(f(Z)) + \delta_x = U'\eta \tag{A-32}$$ quasi-ba

Evaluating this equation at t^e gives

$$a\beta(t^e) + rf(0) + \delta_x(t^e) = U'(t^e)\eta. \tag{A-33}$$ eqfortsa

From the adjoint equation for α we have

$$\dot{\alpha} = \begin{cases} (a+r)\alpha + \delta_g - U'(G^{ss}e^{-a(t-t^g)}) & \alpha(t^g) = c_g \quad t^g \le t \le t^e \\ (a+r)\alpha + \delta_g - \frac{(a\beta + r(f(Z)c) + \delta_x)}{\eta} & \alpha(T) = c_g \quad t^e \le t \le T \end{cases},$$

We have to solve two linear, first-order differential equations of the form

$$\dot{\alpha} - (a+r)\alpha = f(t).$$

Using the integrating factor method the solution is

$$\alpha = e^{(a+r)t}\int e^{-(a+r)\tau} f(\tau)d\tau + Ke^{(a+r)t},$$

where K is an arbitrary constant.

For the forward equation we have

$$\alpha = e^{(a+r)t}\int_{t_g}^{t} e^{-(a+r)\tau}(\delta_g - U'(G^{ss}e^{-a(\tau - tg)})d\tau + Ke^{(a+r)t},$$

with the condition $\alpha(t^g) = c_g$. Evaluating at $t = t^g$ gives

$$c_g = Ke^{(a+r)t_g}$$

so that

$$K = c_g e^{-(a+r)t_g}.$$

Thus the full solution to the forward equation is

$$\alpha = e^{(a+r)t}\int_{t_g}^{t} e^{-(a+r)\tau}(\delta_g - U'(G^{ss}e^{-a(\tau-tg)})d\tau + c_g e^{(a+r)(t-t^g)}$$

Evaluating at $t = t^e$ gives

$$\alpha(t^e) = e^{(a+r)t^e}\int_{t_g}^{t^e} e^{-(a+r)\tau}(\delta_g - U'(G^{ss}e^{-a(\tau-tg)})d\tau + c_g e^{(a+r)(t^e-t^g)} \qquad \text{(A-34)}$$ forata

For the backward equation we have

$$\alpha = e^{(a+r)t}\int_{t_e}^{t} e^{-(a+r)\tau}\left(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau + Ke^{(a+r)t},$$

with the condition $\alpha(T) = c_g$. Evaluating at $t = T$ gives

$$c_g = e^{(a+r)T}\int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau + Ke^{(a+r)T}$$

so that

$$K = c_g e^{-(a+r)T} - \int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x))}{\eta}\right)d\tau.$$

Thus the full solution to the backward equation is

$$\alpha = c_g e^{(a+r)(t-T)} + e^{(a+r)t}\int_{t_e}^{t} e^{-(a+r)\tau}(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x)}{\eta})d\tau -$$

$$e^{(a+r)t}\int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau.$$

Evaluating at $t = t^e$ gives

$$\alpha(t^e) = c_g e^{(a+r)(t^e-T)} - e^{(a+r)t^e}\int_{t_e}^{T} e^{-(a+r)\tau}(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x)}{\eta})d\tau. \qquad \text{(A-35)}$$ bacata

Because α is continuous at t^e, we set the right-hand side of (A-34) and (A-35) equal to

each other. This gives

$$e^{(a+r)t^e}\int_{t_g}^{t^e} e^{-(a+r)\tau}(\delta_g - U'(G^{ss}e^{-a(\tau-tg)})d\tau + c_g e^{(a+r)(t^e-t^tg)} =$$

$$c_g e^{(a+r)(t^e-T)} - e^{(a+r)t^e}\int_{t_e}^{T} e^{-(a+r)\tau}\left(\delta_g - \frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau.$$

Simplifying gives

$$e^{(a+r)t^e}\left(\delta_g\int_{t_g}^{T} e^{-(a+r)\tau}d\tau - \int_{t_g}^{t^e} e^{-(a+r)\tau}U'(G^{ss}e^{-a(\tau-tg)})d\tau - \int_{t_e}^{T} e^{-(a+r)\tau}\left(\frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau\right) =$$

$$c_g e^{(a+r)(t^e-T)} - c_g e^{(a+r)(t^e-t^g)}.$$

Dividing both sides by $e^{(a+r)t^e}$ and carrying out the first integral gives

$$\frac{\delta_g}{a+r}(e^{-(a+r)t_g} - e^{-(a+r)T}) - \int_{t_g}^{t^e} - \int_{t_e}^{T} = c_g e^{-(a+r)T} - c_g e^{-(a+r)t^g}.$$

Simplifying gives

$$\left(c_g + \frac{\delta_g}{a+r}\right)(e^{-(a+r)t_g} - e^{-(a+r)T}) = \int_{t_g}^{t^e} e^{-(a+r)\tau}(U'(G^{ss}e^{-a(\tau-tg)})d\tau + \int_{t_e}^{T} e^{-(a+r)\tau}\left(\frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau.$$

We can write this in a slightly different way as

$$\int_{t^g}^{T} e^{-(a+r)\tau}((a+r)c_g + \delta_g)d\tau = \int_{t^g}^{t^e} e^{-(a+r)\tau}(U'(G^{ss}e^{-a(\tau-tg)})d\tau + \int_{t^e}^{T} e^{-(a+r)\tau}\left(\frac{(a\beta + r(f(Z)) + \delta_x)}{\eta}\right)d\tau.$$

The expression on left hand side is sum of discounted (for both time and stock decay) marginal utility in a counterfactual in which we continue to produce gas cars in steady-state during $[t^g, T]$. The expression on right is sum of discounted marginal utility of actual production plan (no cars during $[t^g, t^e]$ and electric cars during $[t^e, T]$.) Finally, we take advantage of the fact that in steady state gas production from $[0, t^g]$ we have $U'(G^{ss}) = (a+r)c_g + \delta_g$. Thus we can write

$$\int_{t^g}^{t^e} e^{-(a+r)\tau}\left(U'(G^{ss}e^{-a(\tau-tg)}) - U'(G^{ss})\right)d\tau + \int_{t^e}^{T} e^{-(a+r)\tau}\left(\frac{(a\beta + r(f(Z)) + \delta_x)}{\eta} - ((a+r)c_g + \delta_g\right)d\tau = 0.$$

The other equation for t^g and t^e comes the continuity of X. At t_e, we have $X = 0$. From (A-33) we have

$$U'(G^{ss}e^{-a(t^e-t^g)} + \eta 0) = \frac{(a\beta(t^e) + rf(0) + \delta_x(t^e))}{\eta}.$$

These expressions for the transition times have a similar structure as those in the main text. But unlike in the main text, we cannot solve them directly for the transition times, because they are a function of the endogenous variables β and Z. To determine the transition times, we must solve the system of coupled algebraic and differential equations that comprise the state equations, adjoint equations, and interior necessary conditions.

We solve the system iteratively. Given an initial guess for the transition times t_g and t_e, we determine a candidate solution to the coupled algebraic and differential equations. Using this solution, we evaluate the equations for the transition times described above. If they are satisfied, then we have found the solution to the optimal control problem. If not, then we use numerical techniques to iterate on guesses for t_g and t_e until the transition time equations are indeed satisfied.

Given t_g and t_e, the procedure to determine the candidate solution is a follows. There are three time periods defined by the transition times t_g and t_e. Period one occurs during $[0, t_g]$. Here we are in the initial steady state with only gasoline vehicle production. We have $G = G^{ss}$, $g = g^{ss}$, $x = 0$, and $X = 0$. Period two occurs during $[t_g, t_e]$. Here there is no production of either vehicle. We have $G = G^{ss}e^{-(t-tg)}$, $g = 0$, $x = 0$ and $X = 0$.

Period three occurs during $[t_e, T]$. We have interior production of electric vehicles and no production of gasoline vehicles. We have $G = G^{ss}e^{-(t-tg)}$ and $g = 0$. The solution for X and x is more complicated. The general idea is to reduce the system down to a pair of coupled differential equations for Z and λ. We solve this system, and then back substitute to determine x and X.

Consider the special case in which utility is quadratic and $\eta = 1$ so that $U' = m - k(X + G)$. When x is interior, we have from (A-30),

$$rf(Z)c_x + a\beta + \delta_x = U' = m - k(X + G). \tag{A-36}$$ stepo

Taking the time derivative, using the state equations, and using $g = 0$, gives

$$rf'(Z)xc_x + a\dot{\beta} + \dot{\delta}_x = -k(-aX + x - aG).$$

Using Lemma 2 we have

$$rf'(Z)x - ra\lambda + \dot{\delta}_x = -k(-aX + x - aG).$$

Solving for x gives

$$x = \frac{ra\lambda - \dot{\delta}_x + ak(X + G)}{rf'(Z) + k}. \tag{A-37}$$

isxpos

Using (A-36) gives

$$x = \frac{ra\lambda - \dot{\delta}_x - a(m - rf(Z) + a\beta + \delta_x)}{rf'(Z) + k}.$$

Using (A-27) gives

$$x = \frac{ra\lambda - \dot{\delta}_x - a(m - rf(Z)c_x + a(f(Z) - \lambda) + \delta_x)}{rf'(Z) + k}.$$

Notice that only the endogenous variables Z and λ appear in this equation. So we can use this expression for x to solve the coupled differential equations

$$\dot{\lambda} = r\lambda + f'(Z)x \quad ; \ \lambda(T) = 0$$

$$\dot{Z} = x \quad ; Z(t_e) = 0.$$

With this in hand, we determine the rest of the solution by substituting Z and λ into the appropriate equations for x, X, and β (which is needed to analyze the equations for the transition times.)

Finally, we have to account for corner conditions. If the initial value of costs $f(0)$ is large, and/ or the function $f(Z)$ decreases slowly in Z, then it may be optimal to never start producing electric vehicles within $[0, T]$. Another corner condition occurs when the function $f(Z)$ has a large (negative) derivative at $Z = 0$. Inspection of (A-37) shows that the numerator is positive. If the denominator is negative (which will occur if $rf'(0) + k < 0$),

then we have a contradiction to the assumption that $x \geq 0$. This means that the gap solution is invalid. So for the gap solution to occur, we must have

$$rf'(0) + k > 0.$$

Finally, we briefly discuss the case in which $f(Z(T))$ is not equal c_g and $f'(Z(T))$ is not equal to zero. In this case, the final condition for λ is not equal to zero, and so the conditions of Lemma 2 do not hold. Hence λ can be negative. And there is a third transition time t_f, because the interior production of electric vehicles does not last until T (due to end of period effects). The solution requires a third equation for the third unknown t_f, which is determined by analyzing the continuity of β at t_f.

Charging Infrastructure

In this section, we focus on charging infrastructure and ignore learning by doing. Assume aggregate utility is quadratic

$$U(X, G, W) = m_g(W)G + m_x(W)X - \frac{k_1(W)}{2}G^2 - k_2(W)XG - \frac{k_3(W)}{2}X^2,$$

The parameters $m_g(W)$, $m_x(W)$ and so on are determined as follows. First, we determine their value for a given value of s (the slope of the distribution for η), following the procedure described in Online Appendix A. We then repeat for values of s in $[0, \eta(1)]$. We then fit a quadratic function to this data that describes, for example, the value of m_g as a function of s. Then we use the equation

$$s = s_0 e^{-vW}$$

to determine m_g as a function of W.

We these equations in hand, we consider a solution to the planner's problem with four transition times. In addition to the usual t_g and t_e, we have two transition times for the infrastructure control variable w. Time t_{w1} designates the time at which charging infrastructure production starts. The time t_{w2} designates the time at which charging infrastructure production stops. There are five time periods defined by the four transition times. Either

the gap or no gap solution may occur, depending on the cost of charging stations.

First consider the no gap solution. Here we consider solutions with $t_e < t_g < t_{w1} < t_{w2}$. The transition time t_e is given by (6). To determine the other three transition times, we analyze the five time periods.

Period one occurs during $[0, t_e]$. Here we are in the initial steady state with only gasoline vehicle production. We have $G = G^{ss}$, $g = g^{ss}$, $x = 0$, $w = 0$, $W = 0$, and $X = 0$.

Period two occurs during $[t_e, t_g]$. We have interior production of gas and electric vehicles. The stock of vehicles X and G are equations analogous to (5). We also have $w = 0$ and $W = 0$

Period three occurs during $[t_g, t_{w1}]$. We have interior production of electric vehicles. The stock of gas vehicles is $G3 = G^{ss}e^{-a(t-tg)}$. From the adjoint equation for β and the fact that $\beta = c_x$, the stock of electric vehicles $X3$ satisfies

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G3, X3, 0).$$

We also have $w = 0$ and $W = 0$

Period four occurs during $[t_{w1}, t_{w2}]$. We have interior production of electric vehicles and charging infrastructure. We have $\beta = c_x$ and $\phi = c_w$. The stock of electric vehicles $X4$ and the stock of charging infrastructure $W4$ are found from the adjoint equations for β and ϕ. We have

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G3, X4, W4)$$

and

$$rc_w - U_W(G3, X4, W4) = 0.$$

Period five occurs during $[t_{w2}, T]$. We have interior production of electric vehicles. The stock of electric vehicles $X5$ satisfies

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G3, X5, W4(t_{tw2})).$$

We also have $w = 0$ and $W = 0$

As with learning by doing, we employ an iterative solution to find the transition times. Given an initial guess for t_g, t_{w1}, and t_{w2}, we use the above results to to specify a candidate

solution for all the state variables, the control variables, and the adjoint variables in each period. If the candidate solution is valid, then it must satisfy the following three equations:

- 1. Continuity of α at t_{w2}.
- 2. Continuity of ϕ at t_{w2}.
- 3. Continuity of W at t_{w1}.

If the three equations are satisfied, then we have found the solution. If not, we iterate until the equations are satisfied.

Next consider the gap solution. Here we consider solutions with $t_g < t_e < t_{w1} < t_{w2}$.

Period one occurs during $[0, t_g]$. Here we are in the initial steady state with only gasoline vehicle production. We have $G = G^{ss}$, $g = g^{ss}$, $x = 0$, $w = 0$, $W = 0$, and $X = 0$.

Period two occurs during $[t_g, t_e]$. There is no production. The stock of gas vehicles is $G2 = G^{ss}e^{-a(t-tg)}$. We also have $x = 0$, $X = 0$, $w = 0$, and $W = 0$

Period three occurs during $[t_e, t_{w1}]$. We have interior production of electric vehicles. From the adjoint equation for β and the fact that $\beta = c_x$, the stock of electric vehicles $X3$ satisfies

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G2, X3, 0).$$

We also have $w = 0$ and $W = 0$

Period four occurs during $[t_{w1}, t_{w2}]$. We have interior production of electric vehicles and charging infrastructure. We have $\beta = c_x$ and $\phi = c_w$. The stock of electric vehicles $X4$ and the stock of charging infrastructure $W4$ are found from the adjoint equations for β and ϕ. We have

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G2, X4, W4)$$

and

$$rc_w - U_W(G2, X4, W4) = 0.$$

Period five occurs during $[t_{w2}, T]$. We have interior production of electric vehicles. The stock of electric vehicles $X5$ satisfies

$$\dot{c}_x = (a + r)c_x + \delta_x - U_X(G2, X5, W4(t_{tw2})).$$

We also have $w = 0$ and $W = 0$.

As usual, we employ an iterative solution to determine the four transition times. The four relevant equations are

- 1. Continuity of α at t_{w2}.
- 2. Continuity of X at $t = t_e$.
- 3. Continuity of ϕ at t_{w2}.
- 4. Continuity of W at t_{w1}.

E Emissions from gasoline vehicles

hisgas

The historical emissions from gasoline vehicles are given in Table B. Many of these values simply reflect the emissions regulations in place at various points in time. For 1975-2003, the NO_X standard comes from data in Mondt (2000). After 2003 the NO_X standard is average for Tier 2 bins phased in from 2004 to 2009. VOC emissions include tailpipe and evaporation. For 1975-2003, the tailpipe VOC emissions come from Lee et al (2010) and after 2003 tailpipe VOC emissions come from Tier 2 Bin 5. Evaporation VOC is fixed at the value specified in GREET 2013. $PM_{2.5}$ includes tailpipe emissions (Tier 2 bin 5 standard) and tire and break wear from GREET 2013. SO_2 emissions calculated from GREET 2013 and EPA (1999). Emissions of CO_2 are derived from fleet average MPG figures (EPA 2015, Table 9.1).

F Lithium ion battery prices over time

lithium

Data on prices and production of Lithium ion batteries comes from Kittner, Lill, and Kammen (2017) and is shown in Table C. All numbers in this section are expressed in 2015 dollars. (We convert to 2017 dollars to get the numbers used in the main text.)We assume that the cost premium of electric vehicles relative to gasoline vehicles is largely driven by battery prices. The general procedure for specifying c_x is to determine an initial cost premium in

Table B: Historical Emissions from Gasoline Vehicles (g/mile) hisgastab

Model Year	CO_2	SO_2	$PM_{2.5}$	VOC	NO_X
1975	658	0.20881	0.0173	1.557	3.1
1976	596	0.18919	0.0173	1.557	3.1
1977	570	0.18071	0.0173	1.557	2
1978	526	0.16680	0.0173	1.557	2
1979	517	0.16390	0.0173	1.557	2
1980	444	0.14095	0.0173	1.557	2
1981	415	0.13173	0.0173	0.467	1
1982	400	0.12698	0.0173	0.467	1
1983	402	0.12756	0.0173	0.467	1
1984	397	0.12585	0.0173	0.467	1
1985	386	0.12257	0.0173	0.467	1
1986	375	0.11895	0.0173	0.467	1
1987	373	0.11845	0.0173	0.467	1
1988	369	0.11697	0.0173	0.467	1
1989	375	0.11895	0.0173	0.467	1
1990	381	0.12099	0.0173	0.467	1
1991	380	0.12047	0.0173	0.467	1
1992	385	0.12203	0.0173	0.467	1
1993	378	0.11996	0.0173	0.467	1
1994	381	0.12099	0.0173	0.307	0.6
1995	380	0.08192	0.0173	0.307	0.6
1996	381	0.08227	0.0173	0.307	0.6
1997	380	0.08192	0.0173	0.307	0.6
1998	380	0.08192	0.0173	0.307	0.6
1999	386	0.08334	0.0173	0.307	0.3
2000	388	0.04924	0.0173	0.307	0.3
2001	386	0.04903	0.0173	0.307	0.3
2002	385	0.04881	0.0173	0.307	0.3
2003	383	0.04860	0.0173	0.307	0.3
2004	385	0.02929	0.0173	0.132	0.07
2005	378	0.02159	0.0173	0.132	0.07
2006	381	0.00726	0.0173	0.132	0.07
2007	369	0.00702	0.0173	0.132	0.07
2008	366	0.00696	0.0173	0.132	0.07
2009	350	0.00666	0.0173	0.132	0.07
2010	344	0.00656	0.0173	0.132	0.07
2011	347	0.00661	0.0173	0.132	0.07
2012	328	0.00624	0.0173	0.132	0.07
2013	319	0.00606	0.0173	0.132	0.07
2014	319	0.00606	0.0173	0.132	0.07
2015	313	0.00596	0.0173	0.132	0.07

conjunction with a decay function that is either a function of time or cumulative production. We use 2005 as the initial year and 2035 as the terminal year. Let π_0 denote the initial cost premium. We assume that electric vehicles have a 60 khw battery.

To inform the exponential and learning by doing models, we estimate the following model for battery prices

$$\ln(\text{Price}) = constant + \alpha \text{Year} + \beta \ln(\text{Cumulative Production}) + \varepsilon.$$

Using OLS, we obtain $\alpha = -0.0641$ and $\beta = -0.160$.

For the exponential cost function we use the estimate of α to define the decay and the same initial cost premium as in the linear cost function. This gives

$$c_x = cg + 60 * 351.95e^{-0.0641t}.$$

The learning by doing model tends to hit corner conditions more than the other models, so we allow a bit more flexibility in the parameters. As is typical in the learning by doing literature, we consider a power law formulation

$$c_x = f(Z) = cg + \frac{a_1}{(a_2 + Z)^b}.$$

We interpret the parameter a_2 as the baseline level of technological production of batteries and electric cars as the base year. In this formulation, the initial cost premium is given by

$$\pi_0 = c_x(Z = 0) - c_g = \frac{a_1}{a_2^b}. \tag{A-38}$$ eqaone

As discussed in Online Appendix D, the gap solution will only occur if

$$rf'(0) + k > 0.$$

This implies

$$-r\frac{b}{a_2}\frac{a_1}{a_2^b} + k > 0.$$

Table C: Price and Cumulative Production of Lithium Ion Batteries lithprices

Year	Price (2015 Dollars per KWh)	Cumulative Production (MWh)
1991	5394.66	0.13
1992	4392.33	1.68
1993	3444.54	14.55
1994	2718.82	50.08
1995	2566.13	121.13
1996	1888.28	547.42
1997	1329.07	1257.9
1998	872.46	2288.09
1999	711.34	3815.63
2000	619.38	5982.59
2001	508.85	8504.79
2002	487.47	12092.71
2003	437.97	17350.26
2004	401.51	24526.11
2005	351.95	33371.58
2006	317.43	44916.87
2007	320.07	58806.74
2008	319.25	75616.08
2009	298.29	94954.08
2010	260.87	119308.1
2011	231.81	149029.1
2012	185.82	183816.1
2013	183.14	226555.1
2014	170.2	276384.1
2015	150	337871.1

Using (A-38) and solving for a_2 gives

$$a_2 > \frac{rb\pi_0}{k}.$$

So for a given value of b and π_0, we select a_2 such that this inequality holds, and then the value of a_1 is determined from (A-38). We then determine the optimal solution for a variety of values of b and π_0. As shown in the main text, if we select $\pi_0 = 60 * 351.95$ to be consistent with the other models and $b = 0.160$, the optimal solution is to never start building electric vehicles.

G Operating costs for gasoline and electric vehicles

opcosts

The American Automobile Association (AAA) issues an annual report on the cost of driving. For 2017, the operating costs for the average gasoline vehicle are 18.18 cents per mile, or $2726 dollars per year (assuming 15000 miles driven per year).

For the first time in 2017, AAA determined operating costs for electric vehicles. The

values are 10.23 cents per mile or $1535 dollars per year (assuming 15000 mlles driven per year.)

H Stock decay

stockdecay

Basic statistics for auto fleet: 2016 Light duty vehicle sales: 17.5 million units. Of this, 6.9 million passenger cars. (See "light vehicle production.xlsx", produced by Bureau of Economics Analysis, 2017). Perhaps better figure is average over last 5 years. 2012-2016 average production per year is 16.3 million units. Of this, 7.4 million passenger cars.

The 2015 stock of light duty vehicles: 190 million.

https://www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/publications/national_transpo

Not clear what fraction of this is passenger cars. Data from 2006 entry in "passenger cars" row suggests 135 million. But also says not comparable with later data. But light vehicle production spreadsheet gives production back to 1976. So we can get proportion of production of light duty vehicles that was passenger cars. Assuming that passenger cars and light trucks retired at same rate gives us proportion of stock that is passenger cars. Approximately 58 percent of all light duty production from 1976 to 2016 is passenger cars. So 58 percent of 190 is 110 million.

Assume that the current production of gasoline passenger vehicles is in steady state. The stock of vehicles is 110 million and the average annual production over the last five years is 7.4 million units per year. In steady state the decay rate is the annual production divided by the stock. So we have

$$a = 7.4/110 = 0.067.$$

11

[11] We have for light duty vehicles (using average values over the last 5 years for production)

$$a = 16.3/190 = 0.086$$

I Charging Infrastructure

charge

Li et al (2017) study the effect of charging stations on the demand for electric vehicles. From the last column of Table 3 in their results, we see that a one percent increase in the number of charging stations gives a 0.844 percent increase in electric vehicle sales. We want to calibrate the equation

$$s = s_0 e^{-vW}.$$

From the linear distribution of η we have

$$\hat{\rho} = 1 - \rho = \frac{\eta(1) - 1}{s}.$$

This is the fraction of consumers that buy electric cars at parity prices. We have

$$\frac{d\hat{\rho}}{dW} = \frac{e^{vw}(s-1)v}{s_0}.$$

So the elasticity is

$$\frac{d\hat{\rho}}{dW}\frac{W}{\hat{\rho}} = vW.$$

In Li et al (2017), the stock of charging stations is 6.4 thousand. Setting the elasticity equal to 0.844 and solving for v gives

$$v = 0.131875.$$

References for Online Appendices

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