Interest Rate and Prices in a World of Cryptocurrencies*

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Abstract

Can currency competition jeopardize the control of interest rate and prices by the central banks? Yes, it can. In a two-currency world, the growth rate of the supply of cryptocurrency imposes a lower bound on the nominal interest rate and on the attainable inflation rate. In a world of multiple competing currencies issued by profit-maximizer agents, the central bank loses complete control of the nominal interest rate and of the inflation rate, which are both determined by structural factors. Therefore, they are not subject to manipulation, a result welcomed by proponents of private money creation. Some fixes to the classical problem of indeterminacy of exchange rates are also proposed.

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In the recent year, cryptocurrencies have attracted the attention from consumers, media and policymakers.[1] Cryptocurrencies are digital currencies, not physically minted. The monetary history has shown other examples of uncoined money. For centuries, since Carlemagne, an "imaginary" money had existed which, however, did only serve the purpose of a unit of account and never that of media of exchange, unlike cryptocurrencies.[2]

The coexistence of multiple currencies within the borders of the same nation is not a recent phenomenon either. Medieval Europe was characterized by the presence of multiple media of exchange with various metallic contents.[3] More recently, some nations faced dollarization or eurization.[4]

The monetary framework in which digital currencies are nowadays emerging is quite peculiar. They have appeared within the borders of nations dominated by a single fiat government currency exactly at a time in which the central banks issuing them have succeeded in controlling their value and calming down inflation.

In this perspective, the objective of this work is to ask whether the presence of multiple currencies can jeopardize the primary objective of central banks – the control of prices and inflation– or eventually limit their operational tools – e.g. the interest rate. The short answer is: yes it can.

The analysis relies on a simple endowment perfect-foresight monetary economy along the lines of Lucas and Stokey (1987) in which currency provides liquidity services . In the benchmark single-currency model results are well understood in the literature: the central bank can control the rate of inflation by setting the nominal interest rate; the (initial) price level is instead determined by an appropriate real tax policy.[5] Through the combination of these two policies (interest-rate targeting and fiscal policy) the path of the price level is determined in all periods and the central bank can achieve any desired inflation rate by appropriately choosing the nominal interest rate.

I first embed in this benchmark a privately-issued currency which is perfectly substitutable, for liquidity services, with the government's currency .

[1] See BIS (2018).

[2] See Einaudi (1936) for an analysis of the "imaginary" money since Carlemagne to the French revolution.

[3] Cipolla (1956,1982,1990) describes several cases in the monetary history of coexistence of multiple currencies.

[4] See Calvo and Vegh (1997) for an analysis of dollarization.

[5] See among others Canzoneri, Cumby and Diba (2001), Niepelt (2004), Sims (1994, 2000, 2013) and Woodford (1995, 2001, 2003).

The private currency is "minted" each period according to a constant growth rate μ. The first important result is that the presence of another currency limits in a significant way the maneuvering space of the central bank and the achievable equilibrium allocation. The gross nominal interest rate set by the central bank cannot be lower than $\beta^{-1}(1+\mu)$, for an equilibrium to exist, with β being the intertemporal discount factor in consumers' preferences. As a consequence the equilibrium inflation rate in terms of government's currency cannot be lower than the growth rate of private currency, μ. When private currency is set to grow at a positive rate, i.e. $\mu > 0$, price stability is not even attainable.

A pervasive result in this environment, along the lines of Kareken and Wallace (1981), is the indeterminacy of the exchange rate between the two currencies which, however, does not bring about any indeterminacy of the price level (in units of government's currency) if fiscal policy is set in the same way as in the benchmark single-currency case.[6] Therefore, the path of prices can be determined in all periods even in a monetary model with a competing currency but the rate at which prices grow is bounded by the growth rate of the supply of private currency.

I then extend the framework to allow for multiple currencies under a market of profit-maximizing issuers paying a fixed entry cost. Results are even more striking in this case: the central bank completely loses the control of the nominal interest rate, which is determined by structural factors (the intertemporal discount factor, the exit rate and the fixed cost of entry). As a consequence the inflation rate is also determined by the same structural factors and therefore out of the control of the government authority. Instead the (initial) price level in units of government's currency remains determinate if fiscal policy is appropriately set as in the benchmark case. Currency competition can also pin down the real money balances in each currency which are instead not determined in the two-currency economy.

I then turn back to the issue of exchange-rate determination in the two-currency economy. This can be solved if private currency is issued in a centralized system in which the issuer has some taxation power or if the private sector manufactures enough debt securities denominated in units of the private currency. In this case, the private issuer of currency can determine

[6]It creates intedeterminacy of the equilibrium path of government's money supply, however, without any consequence for the equilibrium path of consumption, except that at time zero.

its price level, and the exchange rate, by making an appropriate real transfer. Both cases show that currencies which are not the liability of any agent, like cryptocurrencies, can hardly have their exchange rate determined which can be therefore subject to non-fundamental sunspot shocks.

This paper is related to a recent literature which, spurred by the increasing number of cryptocurrencies, has revived interest in monetary models with multiple currencies. Fernandez-Villaverde and Sanches (2018) evaluate the role of competing private currencies whose money supply is determined from the profit maximization motive of entrepreneurs.[7] They obtain substantially different results to mine. They find that an appropriately-defined price stability equilibrium can arise under certain restriction on the function characterizing the cost of producing private money. However, when the marginal cost goes to zero price stability cannot be an equilibrium. Motivated by these results, they argue in favour of Friedman's (1960) view that a purely private system of fiduciary currencies would necessarily lead to instability in the price level. Moreover, they find that competition does not allow to achieve the efficient allocation. These results might be a consequence of the assumption that the number of players in their model is fixed with no possibility of entry. In my model, instead, there is free entry upon paying a fixed cost while creation of money does not entail any other cost afterward. I do not find, in my multiple-currency competition model, any result of equilibrium multiplicity nor any hyperinflationary equilibria as they found. Moreover, when the fixed cost of entry goes to zero, private issuance of money achieves the first best by reducing to zero any liquidity premium.[8] If anything, a model of currency competition would point towards Hayek (1975)'s view that unfettered competition in currency market is beneficial for society. As in my framework, they also find that the presence of competing currency can restrict the set of possible equilibria. But, even in this case, their results are quite different to mine. They find that there cannot be an equilibrium in which the real interest rate on money equals the rate of time preference unless private money is pushed out of the market. In my model, the real interest rate is always tight in equilibrium to the rate of time preference. The restrictions that I find are on nominal variables: nominal interest rate and prices.

[7] Klein (1974) is an early example of a model of currency competition with profit-maximizing suppliers.

[8] Marimon et al. (2012) also find that currency competition can achieve efficiency. However, they also do not model entry and therefore each profit-maximizing issuer of currency would set its price level to infinite.

Schilling and Uhlig (2018) also analyze the coexistence and competition between traditional fiat money and cryptocurrencies. They are however more concerned about the determination of the price of cryptocurrencies and derive interesting bounds and asset-price relationships. Regarding government policy, they assume that the government has always full control of the inflation rate. With respect to monetary policy, they emphasize the connection between the indeterminacy of the cryptocurrency prices and the government money supply. This result is also shared by my two-currency model but disappears in the multiple-currency optimizing framework. Garratt and Wallace (2017) are also interested in the determination of the exchange rate between currencies and revisit Kareken and Wallace (1981) indeterminacy result. In my analysis, exchange-rate indeterminacy, whenever present, is not a much relevant channel for understanding how competing currencies influence the results of traditional monetary-policy analysis. I also suggest ways to overcome the indeterminacy problem in a centralized system of private money creation.

Woodford (1995) is the benchmark reference for the analysis of price determination through interest-rate targeting and fiscal rules in a single-currency economy. He also studies a "free-banking" regime in which deposits, in units of the single currency, compete with government money in providing liquidity services. In this case, he also finds that the nominal interest rate and the inflation rate are determined by structural factors. However, he does not analyze multiple-currency models as I do here.

The paper is structured as follows. Section 1 presents the model with a two-currency system and solve for the equilibrium. Section 2 discusses the implications of having a competing currency for the standard monetary-policy framework. Section 3 discusses how it is possible to solve for the indeterminacy problem of the private-currency exchange rate. Section 4 extends the model to currency competition with multiple profit-maximizing issuers and evaluates the implications. Section 5 concludes.

1 Model

I consider an economy with a two-currency system: one currency is issued by the government, the other is privately issued. The model follows the lines of Lucas and Stokey (1987) and Benigno and Robatto (2018) in which there are two goods: a "credit" and a "cash" good. The "cash" good can only be

purchased by money.

Consumers have preferences of the following form:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \ln C_t + X_t \right\}, \tag{1}$$

where β is the intertemporal discount factor with $0 < \beta < 1$; C and X are respectively the "cash" and the "credit" good. The cash good C can be purchased by money

$$C_t \leq \frac{M_{t-1}}{P_t} + \frac{M^*_{t-1}}{P^*_t} \tag{2}$$

in which M_t is the money issued by the government while M^*_t is the privately-issued money; P_t is the price of the consumption good in terms of the government's unit of account while P^*_t is the price of goods in terms of the privately-issued unit of account. Money can take the form of physical cash (coins or banknotes) or digital money. In this benchmark analysis, I am assuming that money does not carry any interest rate, later I am going to relax it. In writing the cash constraint (2), I am assuming that the two types of cash are perfect substitute in the purchase of the consumption good C. This assumption is made for simplifying the analysis without losing much generality. I am also putting the model in the worst situation of having the private money providing same liquidity services as the government money in order to challenge at most the results known from the single-currency framework.[9]

In the credit market the consumption good X_t is subject to the following budget constraint

$$\frac{B_t}{P_t(1+i_t)} + \frac{M_t}{P_t} + \frac{M^*_t}{P^*_t} + X_t \leq \frac{B_{t-1}}{P_t} + \left(\frac{M_{t-1}}{P_t} + \frac{M^*_{t-1}}{P^*_t} - C_t \right) + Y + \frac{T_t}{P_t} + \frac{T^*_t}{P^*_t}, \tag{3}$$

in which B is a risk-free interest-bearing security issued in units of the government's currency; Y is the constant endowment of the two consumption goods; T_t are the transfers in units of government's currency which are made by the government while T^*_t are the transfers made by the private issuer of currency in units of its currency.

I am making two important assumptions in writing the budget constraint (3): first, interest-bearing securities are not providing any liquidity services; second, they are only denominated in units of the government's currency. I

[9] Schilling and Uhlig (2018) makes also the assumption of perfect substitutability.

am going to discuss the relevance of this assumption later. Moreover, B_t can be positive –in which case it is an asset for the household– or negative –in which case it is debt. I am indeed allowing for the private sector to borrow issuing debt denominated in units of the government currency, but not in units of the privately-issued currency. Moreover, this debt, in the case it is issued, is paid back with certainty being subject to an appropriate borrowing constraint.

Before addressing this limit, note that absence of arbitrage opportunities has two implications on equilibrium. First, it requires that

$$\frac{P_{t+1}}{P_t} = \frac{P^*_{t+1}}{P_t} \tag{4}$$

for each $t \geq t_0$ and therefore that the exchange rate S_t –defined as the price of privately-issued currency in units of government's currency (i.e. $S_t \equiv P_t/P^*_t$)– is constant over time at a value S which has to be determined. This result is a consequence of the assumption of perfect substitutability between the two monies in the cash constraint.

The second implication of absence of arbitrage opportunities is that the risk-free nominal interest rate in the government's unit of account is non-negative, $i_t \geq 0$. Indeed, were it negative, households would make infinite profits by borrowing at negative interest rates and investing the proceedings in cash.

Considering these two results, I can write the flow budget constraint (3) as

$$\frac{W_t}{(1+i_t)P_t} + C_t + X_t + \frac{i_t}{1+i_t}\frac{M_t}{P_t} + \frac{i_t}{1+i_t}\frac{SM^*_t}{P_t} \leq \frac{W_{t-1}}{P_t} + Y + \frac{T_t}{P_t} + \frac{ST^*_t}{P_t},$$

in which I have defined nominal wealth, in units of government's currency, as

$$W_t \equiv B_t + M_t + SM^*_t.$$

I can then write the natural borrowing limit as

$$\frac{W_{t-1}}{P_t} \geq -\sum_{T=t}^{\infty} Q_{t,T}\left(Y + \frac{T_T}{P_T} + \frac{ST^*_T}{P_T}\right) > -\infty \tag{5}$$

for each $t \geq t_0$ for an appropriate discount factor $Q_{t,T}$ with $Q_{t,t} \equiv 1$ and

$$Q_{t,T} = \frac{P_T}{P_t}\prod_{j=1}^{T}\frac{1}{1+i_{t+j}},$$

for $T > t$.

The natural borrowing limit (5) is the maximum amount of net debt that the consumer can carry in a certain period of time and repay with certainty, i.e. assuming that future consumption and asset holdings will be equal to zero. The assumption that the borrowing limit is finite is a requirement for obtaining a bounded solution of consumption in the optimization problem. This requirement can be equivalently written as

$$\sum_{T=t}^{\infty} Q_{t,T} \left(\frac{T_T}{P_T} + \frac{ST_T^*}{P_T} \right) < \infty.$$

I now turn to the optimality conditions. The first-order condition with respect to the consumption of the cash good implies that

$$C_t = \frac{1}{1+\lambda_t}$$

for each $t \geq t_0$ in which $\lambda_t \geq 0$ is the Lagrange multiplier associated with the cash constraint (2). The optimality condition with respect to the holding of the interest-bearing security B_t implies that

$$1 + i_t = \frac{1}{\beta} \frac{P_{t+1}}{P_t} \tag{6}$$

for each $t \geq t_0$ while those with respect to M_t and M_t^* imply:

$$1 + \lambda_{t+1} = \frac{1}{\beta} \frac{P_{t+1}}{P_t} \tag{7}$$

for each $t \geq t_0$ from which it follows that $\lambda_{t+1} = i_t$ for each $t \geq t_0 + 1$. Finally, the intertemporal budget constraint holds with equality,

$$\frac{W_{t_0-1}}{P_{t_0}} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(Y + \frac{T_t}{P_t} + \frac{ST_t^*}{P_t} \right) = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(C_t + X_t + \frac{i_t}{1+i_t} \frac{M_t}{P_t} + \frac{i_t}{1+i_t} \frac{SM_t^*}{P_t} \right). \tag{8}$$

I now turn to specify the budget constraint of the two monetary authorities. The government's monetary authority is subject to the following budget constraint

$$M_t^g - \frac{B_t^g}{1+i_t} = T_t + M_{t-1}^g - B_{t-1}^g$$

where M_t^g is the supply of cash, while B_t^g are the assets held by the government if positive or the debt issued, if negative. In the first case, I am implicitly assuming that the private sector manufactures risk-free debt denominated in the government's unit of account and the government, through the central bank, holds it. In the second case, I am implicitly assuming that the government issues also interest-bearing liabilities which are held by the private sector and which are risk-free by definition, since the government's liabilities define the unit of account.

The private issuer of currency is instead subject to the following budget constraint

$$M_t^{*p} = T_t^* + M_{t-1}^{*p},$$

since the only liability it issues is cash. I am going to discuss later how this budget constraint can characterize creation of private money in the same way as cryptocurrencies do nowadays.

1.1 Equilibrium

Equilibrium in the goods market implies that the sum of the consumption of the two goods is equal to the constant endowment

$$C_t + X_t = Y_t.$$

Equilibrium in the market of the interest-bearing security denominated in the government's unit of account requires that

$$B_t + B_t^g = 0,$$

while equilibrium in the market of cash in the two currencies implies that supply and demand equalize

$$M_t = M_t^g,$$
$$M_t^* = M_t^{*p}.$$

I now summarize the set of equilibrium conditions in a compact way.

The first equation to consider is the Euler equation (6) derived from household's optimality condition with respect to the interest-bearing security B :

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t} \tag{9}$$

which holds for each $t \geq t_0$. The Euler equation relates the nominal interest rate to the real rate and future gross inflation rate.

The second equilibrium condition is the intertemporal budget constraint of the consumer (8) which, using equilibrium in the good and assets markets, can be written as

$$\frac{M^g_{t_0-1} - B^g_{t_0-1} + SM^*_{t_0-1}}{P_{t_0}} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{T_t}{P_t} + \frac{ST^*_t}{P_t} \right) = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{i_t}{1+i_t} \left(\frac{M^g_t}{P_t} + \frac{SM^{*p}_t}{P_t} \right) \right]. \tag{10}$$

The mirror image of the intertemporal budget constraint of the consumer is an aggregate intertemporal budget constraint for both issuers of currency. In (10), the initial value of the real liabilities of both agents plus the present-discounted value of real transfers should be equal to the seigniorage revenues that both issuers get by providing non-interest bearing liabilities which have non-pecuniary benefits for the consumer.

In equilibrium, for a bounded consumption path to exist, it should be the case that both sums should be finite, i.e.

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{T_t}{P_t} + \frac{ST^*_t}{P_t} \right) < \infty, \tag{11}$$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{i_t}{1+i_t} \left(\frac{M^g_t}{P_t} + \frac{SM^{*p}_t}{P_t} \right) \right] < \infty. \tag{12}$$

Although there is an aggregate intertemporal budget constraint pooling together both issuers of currency, each of them has a different flow budget constraint restricting their policy decisions. The following constraint applies to the government

$$M^g_t - \frac{B^g_t}{1+i_t} = T_t + M^g_{t-1} - B^g_{t-1} \tag{13}$$

given initial conditions $M^g_{t_0-1}$ and $B^g_{t_0-1}$.

The private issuer of currency instead follows the budget constraint

$$M^{*p}_t = T^*_t + M^{*p}_{t-1}, \tag{14}$$

since it issues only cash, given an initial condition $M^{*p}_{t_0-1}$.

From the optimality condition of the household, consumption of good C is inversely related to the nominal interest rate

$$C_t = \frac{1}{1 + i_{t-1}} \tag{15}$$

for each $t \geq t_0 + 1$ while

$$C_{t_0} = \frac{1}{1 + \lambda_{t_0}}$$

at the initial time t_0. The liquidity constraint implies that

$$\frac{M^g_{t-1}}{P_t} + \bar{S}\frac{M^{*p}_{t-1}}{P_t} \geq C_t \tag{16}$$

for each $t \geq t_0$ with equality whenever $i_{t-1} > 0$.

To close the set of equilibrium conditions I need to specify the monetary policy regime.[10] There are three degrees of freedom according to which I can specify monetary policy for the two authorities. I assume that the government authority sets the path $\{i_t, T_t\}^{\infty}_{t=t_0}$ eventually as a function of other variables. In the flow budget constraint (13), it means that, given initial conditions $M^g_{t_0-1}$ and $B^g_{t_0-1}$, the government is setting the size of the balance sheet $M^g_t - B^g_t/(1 + i_t)$ and let the private sector to optimally allocate its wealth among the two government securities. I assume that the private issuer of currency sets the transfer $\{T^*_t\}^{\infty}_{t=t_0}$ which, given the initial condition on $M^{*p}_{t_0-1}$, directly implies the path of its money supply according to (14). This specification is perfectly in line with how some cryptocurrencies, like Bitcoins, are issued through a proof-of-work system that determines the new units which are released at each point in time.

Definition 1 *An equilibrium is a set of sequences* $\{C_t, P_t, i_t, M^g_t, M^{*p}_t, B^g_t, T_t, T^*_t\}^{\infty}_{t=t_0}$ *with* $C_t, P_t, i_t, M^g_t, M^{*p}_t \geq 0$ *and non-negative values* $\bar{S}$ *and* λ_{t_0} *that are consistent with the monetary-policy regime and that satisfy (9), (13), (14), (16) (with equality whenever* $i_{t-1} > 0$*) for each* $t \geq t_0$ *and (15) for each* $t \geq t_0 + 1$ *together with the intertemporal constraint (10), the bounds (11), (12) and the constraint* $C_{t_0} = (1 + \mu_{t_0})^{-1}$*, given initial conditions* $M^g_{t_0-1}, B^g_{t_0-1}, M^{*p}_{t_0-1}$.

I restrict my attention to a certain class of policy rules.

[10] The analysis of all the possible monetary policy regimes is out of the scope of this work.

Definition 2 *Assume the following specification of the monetary policy regime. The government sets a constant interest rate policy $i_t = i$ at each $t \geq t_0$ and the following transfer policy*

$$\frac{T_t}{P_t} = \frac{i}{1+i}\frac{M_t^g}{P_t} - (1-\beta)\tau \tag{17}$$

for each $t \geq t_0$ and for some τ different from zero. The issuer of private currency sets $T_t^* = \mu M_{t-1}^{*p}$ at each $t \geq t_0$ with $\mu > -1$, to achieve a constant growth rate, $1+\mu$, of its money supply.

Let us focus on the implications of the monetary-policy regime defined above. First, the constant government's interest-rate policy, using the Fisher equation (9), implies that the inflation rate is constant

$$\frac{P_{t+1}}{P_t} = \beta(1+i). \tag{18}$$

Inflation will be positive whenever $(1+i) > \beta^{-1}$, negative when $(1+i) < \beta^{-1}$. A price stability policy is achieved when the nominal interest rate is set equal to the inverse of the consumer's discount factor, i.e. $(1+i) = \beta^{-1}$.

The government is also setting a transfer policy in which it rebates all the seigniorage revenues to the consumers – the first element on the right-hand side of (17)– on top of a constant which is proportional to a value τ different from zero. According to the terminology of the literature on price-level determination, the transfer rule (17) is in the class of active transfer policies.

Finally, I have assumed that the issuer of private currency sets a constant growth rate of its money, which can be also negative if it is able to destroy money using some electronic algorithm.

The assumptions on the monetary policy regime have important implications for the conditions under which an equilibrium may exist. Assume $i > 0$, then the cash constraint (16) holds with equality. After plugging in the money supply rule of the issuer of private currency and $C_t = 1/(1+i)$, I can obtain

$$\frac{M_t^g}{P_{t+1}} + S\frac{(1+\mu)^{t+1-t_0} M_{t_0-1}^{*p}}{P_{t+1}} = \frac{1}{1+i}$$

in which I can substitute (18) to obtain

$$\frac{M_t^g}{P_{t+1}} + S\left(\frac{1+\mu}{\beta(1+i)}\right)^{t+1-t_0}\frac{M_{t_0-1}^{*p}}{P_{t_0}} = \frac{1}{1+i}.$$

From the above equation it follows that for an equilibrium to exist it should necessarily be the case that $1+\mu \leq \beta(1+i)$ otherwise in a finite period of time the money supply of the government is going to be negative, which is not feasible.

Note that when $i=0$, then the cash constraint (16) holds with inequality and therefore it is possible for real money balances to grow without bound.

Moreover, under the monetary-policy regime defined above I can write explicitly the following summations

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{ST_t^*}{P_t} \right) = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{S\mu M_{t-1}^{*p}}{P_t} \right) = \mu \frac{SM_{t_0-1}^{*p}}{P_{t_0-1}} \sum_{t=t_0}^{\infty} \left(\frac{1+\mu}{1+i} \right)^{t-t_0} \tag{19}$$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{i_t}{1+i_t} \left(\frac{SM_t^*}{P_t} \right) = \frac{i(1+\mu)}{1+i} \frac{SM_{t_0-1}^{*p}}{P_{t_0-1}} \sum_{t=t_0}^{\infty} \left(\frac{1+\mu}{1+i} \right)^{t-t_0} \tag{20}$$

where in the first equality of (19) I have substituted in the transfer rule of the private issuer of currency and in the second equality the equilibrium condition (18). Similarly in (20). The first sum is finite whenever $\mu < i$ for any $i \geq 0$, while (20) requires that $\mu < i$ only when $i > 0$.

I can collect the results of the above discussion in the following Lemma.

Lemma 3 *Given the monetary policy regime specified in Definition 2, a necessary condition for an equilibrium to exist is: i)* $1+\mu \leq \beta(1+i)$ *when* $i > 0$ *and ii)* $\mu < 0$ *when* $i = 0$.

I now turn to analyze whether given the above specification of the monetary policy regime the price level and exchange rate can be determined.

2 Equilibrium implications of competing currencies

The monetary regime I have specified in Definition 2 captures in a stylized way the recent experience of digital currencies which set some algorithm to determine the time increase of the supply of their money.

I now investigate whether the presence of another currency influences the monetary policy set by the central bank. Note that in the benchmark case of a single currency, given the monetary regime of Definition 2, the central

bank has full control of the price level P and its path. Indeed by setting the nominal interest rate at a certain target i it can also set the inflation rate Π at any desired constant number given by $\Pi = \beta(1+i)$ which is a function of the interest rate chosen. Moreover, the initial price level and therefore all the path of prices is fully determined by the fiscal policy rule (17). These conclusions change substantially with the addition of a competing currency.

Given Lemma 3, the first result is that, for an equilibrium to exist, the government should set the nominal interest rate to satisfy the following inequality $1+i \geq \beta^{-1}(1+\mu)$ if it wants to fix a positive nominal interest, otherwise it could set the interest to zero provided $\mu < 0$. Therefore the rate of growth of the private currency constraints the freedom of the government in choosing its interest rate.

There are as well implications for which objectives in terms of the inflation rate the government can reach. Consider first the case in which the government wants to set a positive interest rate. Using the requirement $(1+i) \geq \beta^{-1}(1+\mu)$ into the Fisher equation (9), I obtain that in equilibrium the inflation rate Π in terms of the government's unit of account should satisfy the following inequality $\Pi \geq (1+\mu)$. Therefore given the exogenous money-growth rate of the private issuer of currency, the inflation rate in government's currency is in equilibrium bounded below by that number. In particular, if μ is positive, price stability cannot be an equilibrium. Consider now the case in which $\mu < 0$. For an equilibrium to exist, the government can also set $i = 0$ according to Lemma 3 and instead in this case prices are falling at the rate β which is not necessarily bounded above by $1+\mu$.

I now investigate whether the presence of a competing currency can create problems to the determination of the price level. The following Proposition also collects the results already derived.

Proposition 4 *Given the monetary policy regime of Definition (2) and an active monetary policy for the issuer of private currency who sets the growth rate of its money supply at* $1+\mu$*, an equilibrium exists if and only if i) the government sets a positive interest rate and* $(1+i) \geqslant \beta^{-1}(1+\mu)$ *or ii) the government sets* $i = 0$ *given* $\mu < 0$*. In both cases, the path of the price level in government's unit of account is determined, but the exchange rate* S *and consumption at time* t_0 *are not determined.Moreover the inflation rate of government currency is bounded by the growth rate in the supply of private currency, i.e.* $P_{t+1}/P_t \geq (1+\mu)$ *when the nominal interest rate is positive.*

Proof. Consider first the case in which $i > 0$ and $(1+i) \geqslant \beta^{-1}(1+\mu)$. Since

$0 < \beta < 1$ then $i > \mu$. When instead $i = 0$, in equilibrium it should be also the case that $i > \mu$. Therefore in any equilibrium $i > \mu$. Consider the flow budget constraint (14) and iterate it forward to obtain

$$\frac{SM_{t_0-1}^{*p}}{P_{t_0}} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{ST_t^*}{P_t} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{i_t}{1+i_t} \frac{SM_t^{*p}}{P_t} + \lim_{T\to\infty} \beta^{T-t_0} \frac{SM_T^{*p}}{P_{T+1}}. \quad (21)$$

Note that I can write

$$\lim_{T\to\infty} \beta^{T-t_0} \frac{SM_T^{*p}}{P_{T+1}} = \frac{SM_{t_0-1}^{*p}}{\beta P_{t_0}} \left(\frac{1+\mu}{1+i}\right)^{T+1-t_0} = 0$$

using in the first equality $P_{T+1} = [(1+i)\beta]^{T+1-t_0} P_{t_0}$ and $M_T^{*p} = (1+\mu)^{T+1-t_0} M_{t_0-1}^{*p}$. The second equality follows since $i > \mu$ in equilibrium. Moreover under the latter inequality also the two sums in (21) are finite. I can therefore write

$$\frac{SM_{t_0-1}^{*p}}{P_{t_0}} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{ST_t^*}{P_t} = \frac{i}{1+i} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{SM_t^{*p}}{P_t}.$$

Using the above equilibrium condition into (10) I obtain:

$$\frac{M_{t_0-1}^g - B_{t_0-1}^g}{P_{t_0}} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{T_t}{P_t} = \frac{i}{1+i} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{M_t^g}{P_t}. \quad (22)$$

After plugging the active monetary policy (17), (22) simplifies to

$$\frac{M_{t_0-1}^g - B_{t_0-1}^g}{P_{t_0}} = \tau, \quad (23)$$

which is going to determine the price level P_{t_0}. Note that in the case $M_{t_0-1}^g - B_{t_0-1}^g > 0$, i.e. when the government is a net debtor, then it should be $\tau > 0$. On the opposite, when $M_{t_0-1}^g - B_{t_0-1}^g < 0$ it should be $\tau > 0$. Given a constant interest rate policy i, it then follows from (9) that all the path of prices $\{P_t\}_{t=t_0}^{\infty}$ is determined, moreover using (15) the path of consumption starting from period $t_0 + 1$ is also determined.

Consider first the case $i > 0$, then

$$\frac{M_t^g}{P_t} + S\frac{M_t^{*p}}{P_t} = \beta$$

for each $t \geq t_0$ and in particular at time $t_0 + 1$ in which case

$$\frac{M^g_{t_0}}{P_{t_0}} + S\frac{M^{*p}_{t_0-1}(1+\mu)}{P_{t_0}} = \beta \tag{24}$$

Moreover using (17) into the budget constraint of the government I obtain

$$M^g_{t_0-1} - B^g_{t_0-1} = (1-\beta)P_{t_0}\tau + \frac{M^g_{t_0}}{1+i} - \frac{B^g_{t_0}}{1+i}. \tag{25}$$

Since P_{t_0} is determined in (23), then (24) and (25) are only two restrictions to determine S, $B^g_{t_0}$ and $M^g_{t_0}$. Therefore they are not sufficient to determine the path of the three variables. Note also that the liquidity constraint at time t_0 is not useful to determine the variables of interest. Indeed,

$$\frac{M^g_{t_0-1}}{P_{t_0}} + S\frac{M^{*p}_{t_0-1}}{P_{t_0}} \geq C_{t_0} \tag{26}$$

cannot disentangle S and C_{t_0}.

Consider now the case in which the government sets $i = 0$ provided $\mu < 0$. Equation (25) also applies when $i = 0$, but now (24) is replaced by

$$\frac{M^g_{t_0}}{P_{t_0}} + S\frac{M^{*p}_{t_0-1}(1+\mu)}{P_{t_0}} > 1$$

which cannot determine S and $M^g_{t_0}$. Note instead that (26) still applies and therefore C_{t_0} is also not determined. ■

There are two interesting implications of Proposition 4. First, the presence of another currency does not bring about any problem of determination of the price level – the one expressed in units of the government's currency. The price level is uniquely determined as in the benchmark single-currency framework but the rate of change in prices is affected the presence of another currency as previously discussed.

The second result is the indeterminacy of the exchange rate as in Kareken and Wallace (1981) and recently restated by Garratt and Wallace (2017) and Schilling and Uhlig (2018). In my model, this indeterminacy of the exchange rate translates into indeterminacy of consumption at time t_0 and of the equilibrium path of government's money starting from period $t_0 + 1$. Instead, the path of consumption starting from period $t_0 + 1$ is fully determined by the path of nominal interest rates set by the government. To see the result of

indeterminacy from a different angle, suppose that S is instead determined, therefore (24) determine $M^g_{t_0}$ and (25) determines $B^g_{t_0}$ and so forth using the above two equations in the subsequent periods. Moreover, given initial conditions $M^g_{t_0-1}$ and $M^{*p}_{t_0-1}$, the value of S and P_{t_0} determines the amount of initial real money balances. If the left-hand side of (26) is larger than the unitary value, then $C_{t_0} = 1$ and $\lambda_{t_0} = 0$. If it is smaller, then C_{t_0} is constrained by this value and $\lambda_{t_0} = C^{-1}_{t_0} - 1$. The interesting result is that even if the exchange rate is not determined and therefore can take any positive value, this indeterminacy does not affect the determination of the price level P which can be under full control, with the caveats underlined above, of the government given the monetary-policy regime specified above.

3 Determinacy of the private currency's exchange rate

I now discuss whether it is possible to solve the indeterminacy problem of the cryptocurrency's exchange rate. To this end I need to explore alternative monetary regimes for the issuer of private money. A characteristic of cryptocurrencies is that they are not the liability of any agent in the economy (a decentralized system), while government's currency is the liability of the central bank (a centralized system). The constant money-supply grow rule assumed for the private-currency issuer is suitable to describe the behavior of a decentralized issuer as well as of a centralized issuer. On the other side, the transfer policy (17) is specific to a centralized system in which an agent can back appropriately the value of its liabilities. To get determinacy, I need to move out of the decentralized system and assume that there is a centralized agent issuing private currency which can follow at some point time an "active" transfer policy of the same kind as (17).

Definition 5 *Assume the following specification of the monetary policy regime. The government sets a constant interest rate policy $i_t = i$ at each $t \geq t_0$ and the following transfer policy*

$$\frac{T_t}{P_t} = \frac{i}{1+i}\frac{M^g_t}{P_t} - (1-\beta)\tau \tag{27}$$

for each $t \geq t_0$ and for some τ different from zero. The issuer of private currency sets $T^*_t = \mu M^{*p}_{t-1}$ for each $t_0 \leq t < \tilde{t}$ with $\mu > -1$, to achieve a

constant growth rate, $1+\mu$*, of its money supply and sets a real transfer policy*

$$\frac{T_t^*}{P_t^*} = \frac{i}{1+i}\frac{M_t^{*p}}{P_t^*} - (1-\beta)\tau^* \tag{28}$$

for each $t \geq \tilde{t}$ *and some* $\tau^* > 0$*.*

The difference with respect to the monetary-policy regime of Definition 2 is that I have now assumed a switch to a real transfer policy for the issuer of private currency after and including time $\tilde{t}$ which can eventually coincide with the initial period t_0 or can be postponed far in the future. This modification of the monetary regime is sufficient for an equilibrium to have determinacy of all the variables and in particular of the price level, expressed in units of government's currency, and of the nominal exchange rate. A requirement in the following Proposition is a positive value of the nominal interest rate set by the government.

Proposition 6 *Given the monetary-policy regime of Definition (5), in an equilibrium with a positive nominal interest rate* $i_t = i > 0$ *for each* $t \geq t_0$*, the equilibrium is determinate.*

Proof. When the nominal interest rate is positive, (16) holds with equality and

$$\frac{M_{t-1}^g}{P_t} + \frac{M_{t-1}^{*p}}{P_t^*} = \frac{1}{1+i}$$

holds in equilibrium which implies that the real money balances are non-negative and bounded above at each point in time. Therefore in equilibrium $\lim_{t\to\infty}\beta^t M_{t-1}^{*p}/P_t^* = 0$ and $\lim_{t\to\infty}\beta^t(M_{t-1}^g - B_{t-1}^g)/P_t$ considering the transversality condition of the consumer's problem.[11] Using $\lim_{t\to\infty}\beta^t(M_{t-1}^g - B_{t-1}^g)/P_t$, the flow budget constraint of the government (13) and $P_{t+1} = \beta P_t(1+i)$ I obtain

$$\frac{M_{t_0-1}^g - B_{t_0-1}^g}{P_{t_0}} + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{T_t}{P_t} = \frac{i}{1+i}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{M_t^g}{P_t},$$

in which I can use (27) to obtain

$$\frac{M_{t_0-1}^g - B_{t_0-1}^g}{P_{t_0}} = \tau,$$

[11] When $i = 0$, it is not necessarily the case that $\lim_{t\to\infty}\beta^t M_{t-1}^{*p}/P_t^* = 0$.

which can determine the price level P_{t_0} at time t_0. Using $P_{t+1} = \beta P_t(1+i)$ starting from $t = t_0$ I can determine the sequence $\{P_t\}_{t=t_0}^{\infty}$. Using $\lim_{t\to\infty} \beta^t M_{t-1}^{*p}/P_t^* = 0$, $P_{t+1} = \beta P_t(1+i)$ and (14) starting from period $t = \tilde{t}$, I obtain

$$\frac{M_{\tilde{t}-1}^{*p}}{P_{\tilde{t}}^*} + \sum_{t=\tilde{t}}^{\infty} \beta^{t-\tilde{t}} \frac{T_t^*}{P_t^*} = \frac{i}{1+i} \sum_{t=\tilde{t}}^{\infty} \beta^{t-\tilde{t}} \frac{M_t^{*p}}{P_t^*}$$

in which I can substitute (28) and $M_{\tilde{t}-1}^{*p} = (1+\mu)^{\tilde{t}-t_0} M_{t_0-1}^{*p}$ to obtain

$$\frac{(1+\mu)^{\tilde{t}-t_0} M_{t_0-1}^{*p}}{P_{\tilde{t}}^*} = \tau^* \tag{29}$$

which determines $P_{\tilde{t}}^*$. Using (4) I can determine all the sequence $\{P_t^*\}_{t=t_0}^{\infty}$ and therefore the nominal exchange rate S. Using $P_{t+1} = \beta P_t(1+i)$, (14) for time $t \geq \tilde{t}$ and (28) I obtain

$$\frac{M_{t-1}^{*p}}{P_t^*} = (1-\beta)\tau^* + \beta \frac{M_t^{*p}}{P_{t+1}^*}, \tag{30}$$

which can determine the path of private money supply after and including period $\tilde{t}$ whereas in previous periods the path of private money supply is determined by the transfer rule $T_t^* = \mu M_{t-1}^{*p}$ for each $t_0 \leq t < \tilde{t}$ given initial condition $M_{t_0-1}^{*p}$. Consumption is determined by (15) for each $t \geq t_0 + 1$ while at time t_0 is determined by

$$C_{t_0} = \min\left\{ \frac{M_{t_0-1}^g}{P_{t_0}} + S\frac{M_{t_0-1}^{*p}}{P_{t_0}}, 1 \right\}.$$

The sequence of government's money supply is determined by

$$\frac{M_t^g}{P_{t+1}} + \frac{M_t^{*p}}{P_{t+1}^*} = \frac{1}{1+i}$$

for each $t \geq t_0$ given the path of all the other variables involved in the equation. Using $P_{t+1} = \beta P_t(1+i)$, (13) for time $t \geq t_0$ and (27) I obtain

$$\frac{M_{t-1}^g - B_{t-1}^g}{P_t} = (1-\beta)\tau + \beta\left(\frac{M_t^g - B_t^g}{P_{t+1}}\right),$$

which determines the sequence $\{B_t^g\}_{t=t_0}^{\infty}$ given that all the other variables are already determined. Note that the existence of an equilibrium imposes some

restriction on the parameter μ and the time $\tilde{t}$, indeed it should be the case that

$$\left(\frac{1+\mu}{\beta(1+i)}\right)^{\tilde{t}+1-t_0} \frac{M^{*p}_{t_0-1}}{P_{t_0}} \leq \frac{1}{1+i}$$

given P_{t_0} determined in equilibrium. ■

It is worth noting that the monetary-policy regime of Definition (5), under a positive nominal interest, does not necessarily imply the existence of an equilibrium. But, if an equilibrium exists under that regime, it features determination of prices and exchange rate.

The monetary-regime specified above could be questioned of being not much of practical relevance since it requires the issuer(s) of private currency not only to operate in a centralized system but also to have some taxation power on consumers. This taxation power could be postponed in time but eventually it should be there at some point in time to back the value of currency.

The proposal can be more appealing if I could relax the taxation requirement. This is possible if the private sector does also manufacture securities denominated in units of the private currency. In which case, (29) changes to

$$\frac{M^{*p}_{\tilde{t}-1} - B^{*p}_{\tilde{t}-1}}{P^{*}_{\tilde{t}}} = \tau^{*}$$

where $B^{*p}_{\tilde{t}-1}$ are the time $\tilde{t}-1$ holdings of the bonds issued by the private sector in the hands of the private currency's issuer. In the case in which the net asset position of the issuer of private currency is positive, i.e. $B^{*p}_{\tilde{t}-1} > M^{*p}_{\tilde{t}-1}$, price and exchange rate determination can be also achieved by providing a real transfer to the private sector rather than a real tax. This would represent a "central bank theory of price determination", as opposed to the fiscal theory of the price level along the lines of Benigno (2017).

4 Multiple-currency competition

I consider now an environment in which there can be multiple currencies. Each currency provider now operates in a centralized system since I am assuming that the supply of money is chosen to maximize the present discounted value of the rents derived from it.

At time $t_0 - 1$, there is only government money, but then starting from time t_0 there can be entry of other currency providers subject to a fixed cost. I generalize the liquidity constraint to

$$C_t \leq \frac{M_{t-1}}{P_t} + \sum_{j=1}^{J_t} \frac{M_{t-1}^j}{P_t^j}$$

for each $t \geq t_0 + 1$ in which I have included a generic number J_t of other currencies (to be determined in equilibrium) on top of government currency: M_{t-1}^j are money holdings of currency j and P_t^j is the price of goods in term of currency j. At time t_0, the cash constraint is instead given by

$$C_{t_0} \leq \frac{M_{t_0-1}}{P_{t_0}}.$$

At each time $\bar{t}$ with $\bar{t} \geq t_0$ a mass $N_{\bar{t}}$ of currency providers enters the market paying a fixed cost $\Phi > 0$. A generic currency providers j entering the market at time $\bar{t}$ chooses the sequence $\{M_t^j\}_{t=\bar{t}}^{\infty}$ in order to maximize its liquidity rents, which I am going to be specify later, subject to an exogenous exit which occurs with probability $\delta > 0$. I assume that if a currency provider exits at time $t + 1$, the money supply M_t^j chosen in the previous period is going to still provide full liquidity services in period $t + 1$ but then it will completely lose any value at the end of period. Therefore, the first-order condition at time t with respect to money holdings M_t^j of a generic security j issued by a currency provider entering the market at time $\bar{t}$ is

$$\frac{1}{P_t^j} = \beta(1 - \delta + \lambda_{t+1}) \frac{1}{P_{t+1}^j}, \tag{31}$$

since the payoff of the security is going to be one only with probability $1 - \delta$ while the security is going to provide full liquidity services in any case. The above condition holds for each $t \geq \bar{t}$.

Given that the first-order condition (7) still applies, comparing it with (31) I obtain that

$$\frac{P_{t+1}^j}{P_t^j} = \frac{P_{t+1}}{P_t} - \beta\delta \tag{32}$$

saying that the return on private money (the inverse of its inflation rate) should be higher than that of government money in order to compensate for its exogenous disappearance rate.

Given the exogenous exit rate, the number J_t of private currency providers is going to follow the law of motion $J_t = N_t + (1-\delta)J_{t-1}$ given $J_{t_0-1} = 0$.

Each issuer is subject to the flow budget constraint

$$M^j_{t-1} + T^j_t = M^j_t,$$

starting from its entry time $\bar{t}$ with $M^j_{\bar{t}-1} = 0$. Per-period real profits derived from issuing currency at time t are given by the liquidity premium in period $t+1$, λ_{t+1}, multiplied by the real money balances, i.e. $\lambda_{t+1}M^j_t/P^j_{t+1}$. I assume that each suppliers maximizes the present-discounted value of the real profits by choosing the sequence $\{M^j_t\}^{\infty}_{t=\bar{t}}$ under the circumstances that he is going to be still on the market, therefore it maximizes:

$$\beta \sum_{t=\bar{t}}^{\infty} [(1-\delta)\beta]^{t-\bar{t}} \lambda_{t+1} \frac{M^j_t}{P^j_{t+1}} \tag{33}$$

starting from its entry period $\bar{t}$.

The following results arise in an equilibrium with competition.

Proposition 7 *Given a fixed cost Φ of entry into the market and an exogenous probability δ of exit, in an equilibrium with currency competition with profit-maximizing suppliers: i) consumption is constant and equal to*

$$C_t = 1 - z^{\frac{1}{2}} > 0, \tag{34}$$

for each $t \geq t_0 + 1$ with $z \equiv (1-(1-\delta)\beta)\Phi/\beta$ and assuming $z < 1$; ii) the supply of real money balances of each private supplier of currency is constant and equal to

$$\frac{M^*_t}{P^*_{t+1}} = z^{\frac{1}{2}} - z > 0, \tag{35}$$

iii) the number of private suppliers of currency is given by

$$J_t = z^{-\frac{1}{2}} \left(1 - \frac{M^g_t}{P_{t+1}} \frac{1}{1 - z^{-\frac{1}{2}}} \right). \tag{36}$$

Proof. I consider the optimization problem of a generic supplier of private currency. Each supplier internalizes that he/she has some power on the rents given by λ_t. If liquidity rents are zero, i.e. $\lambda_t = 0$, the supply of currency

will be infinitely elastic. Instead when $\lambda_t > 0$, rents are inversely related to the overall supply of liquidity as shown by the following equation

$$C_t = \frac{1}{1+\lambda_t} = \frac{M_{t-1}}{P_t} + \sum_{j=1}^{J_t} \frac{M_{t-1}^j}{P_t^j}. \tag{37}$$

Considering the above constraint into the objective function (33) and deriving it with respect to the sequence $\{M_t^j\}_{t=\bar{t}}^{\infty}$, I get

$$C_{t+1}\left(1 - C_{t+1}\right) = \frac{M_t^j}{P_{t+1}^j}, \tag{38}$$

for each $t \geq \bar{t}$ in which $\bar{t}$ is the entering time of a generic supplier j of private currency. Therefore, the above equality holds for each $\bar{t} \geq t_0$. Since the left-hand side is independent of j, the right-hand side is also independent of j and therefore the equilibrium is symmetric. By defining $M_t^j/P_{t+1}^j = M_t^*/P_{t+1}^*$ for each j I can write (38) as

$$C_{t+1} = \left(\frac{M_t}{P_{t+1}} + J_{t+1}\frac{M_t^*}{P_{t+1}^*}\right) = 1 - \frac{\frac{M_t^*}{P_{t+1}^*}}{\left(\frac{M_t}{P_{t+1}} + J_{t+1}\frac{M_t^*}{P_{t+1}^*}\right)}, \tag{39}$$

which holds for each $t \geq t_0$. Free entry occurs until there are no rents left in the market, i.e. at each point in time the present-discounted value of profits of each supplier of currency entering the market is equal to the fixed cost

$$\beta \sum_{t=\bar{t}}^{\infty} [(1-\delta)\beta]^{t-\bar{t}} \lambda_{t+1} \frac{M_t^j}{P_{t+1}^j} = \Phi, \tag{40}$$

for each $\bar{t} \geq t_0$. I can further write (40) using (37) and (38) as

$$\sum_{t=\bar{t}}^{\infty} [(1-\delta)\beta]^{t-\bar{t}} (1 - C_{t+1})^2 = \frac{\Phi}{\beta}. \tag{41}$$

Since (41) applies at each $\bar{t} \geq t_0$, it implies that

$$C_t = 1 - z^{\frac{1}{2}}, \tag{42}$$

for each $t \geq t_0 + 1$. Moreover, using (38) and (42), I obtain that

$$\frac{M_t^*}{P_{t+1}^*} = z^{\frac{1}{2}} - z,$$

for each $t \geq t_0$. Combining the above equation with (39) and (42), I obtain that the number of supplier in the market, at each point in time, is given by

$$J_t = z^{-\frac{1}{2}} \left(1 - \frac{M_t}{P_{t+1}} \frac{1}{1 - z^{-\frac{1}{2}}} \right).$$

■

There are some interesting implications that can be derived from the results of Proposition 7: *i*) the nominal interest rate and *ii*) the inflation rate in each currency are constant and function of structural parameters.[12] To see the first result, combine the constant consumption level obtained above, equation (42), with the equilibrium condition $C_t = 1/(1 + i_{t-1})$ for each $t \geq t_0 + 1$. It follows that the nominal interest rate is constant and pin down by the entry cost Φ, the exogenous exit rate δ and the intertemporal discount factor β through the parameter z

$$1 + i_t = \frac{1}{1 - z^{\frac{1}{2}}}, \tag{43}$$

for each $t \geq t_0$. An implication of currency competition is therefore that the government loses completely control on the nominal interest rate.[13] Moreover, to see that also the inflation rate in each currency is determined by structural factors, combine (43) with the Euler equation (9) to obtain

$$\frac{P_{t+1}}{P_t} = \frac{\beta}{1 - z^{\frac{1}{2}}} \qquad \frac{P^*_{t+1}}{P^*_t} = \frac{\beta}{1 - z^{\frac{1}{2}}} - \delta\beta, \tag{44}$$

where the second equality follows from (32).

We collect this

Corollary 8 *Given a fixed cost Φ of entry into the market and an exogenous probability δ of exit, in an equilibrium with currency competition with profit-maximizing suppliers: i) the nominal interest rate and ii) the inflation rates are determined by structural factors*

$$1 + i_t = \frac{1}{1 - z^{\frac{1}{2}}},$$

[12] Woodford (1995) finds a similar result but in a context of competition of securities denominated in the same currency.

[13] The real interest rate is instead always tied to the rate of time preference. This is in contrast to the results of Fernandez-Villaverde and Sanchez (2017) who finds that there cannot be an equilibrium in which the real interest rate is equal to the rate of time preference.

$$\frac{P_{t+1}}{P_t} = \frac{\beta}{1 - z^{\frac{1}{2}}} \qquad \frac{P^*_{t+1}}{P^*_t} = \frac{\beta}{1 - z^{\frac{1}{2}}} - \delta\beta.$$

Note that in the special case in which the entry cost is zero, there are infinite numbers of currency suppliers in the market which abate to zero all liquidity premia. The efficient allocation is reached.[14] Therefore the nominal interest rate is at the zero-lower bound, consistently with complete satiation of liquidity, while prices in units of government currency are falling at the rate β and all other prices in units of the private currencies are falling at the rate $\beta(1-\delta)$. This results is perfectly in line with Hayek (1974) who advocated for private money creation as a way to achieve efficiency in the provision of liquidity. The fact that the government loses control of the nominal interest rate and of the inflation rate is also in line with his view that the government should not be let to exercise price manipulation in the system. Indeed, it is unfettered competition that controls the inflation rate.

A further important result of Proposition (7) is that real money balances for each of the private suppliers of currency are constant and well determined.[15] They are not affected by any indeterminacy of the exchange rate which was instead pervasive in the two-currency model of Section 1. Therefore this does not create any instability in the demand of real money balances of government currency.

The key question that remains to ask is whether any possible indeterminacy of price level, in units of government currency, can arise due to the fact that the interest rate and inflation are determined by exogenous factor? The answer is no and follows in a similar way previous arguments with some caveats. To this end, I need to redefine appropriately the monetary regime.

Definition 9 *Assume the following specification of the monetary policy regime. The government sets the sequence of money supply* $\{M^g_t\}^{\infty}_{t=t_0}$ *and the following transfer policy*

$$\frac{T_t}{P_t} = \frac{i_t}{1+i_t}\frac{M^g_t}{P_t} - (1-\beta)\tau \tag{45}$$

for each $t \geq t_0$ *and for some* τ *different from zero.*

[14] This result is in contrast with Fernandez-Villaverde and Sanchez (2017) who instead find that private money creation can be socially wasteful.

[15] This is again in contrast with Fernandez-Villaverde and Sanches (2017) who instead find multiplicity of equilibria among which hyperinflationary equilibria in which real money balances converge to zero.

As a difference with respect to the monetary policy regime assumed in the benchmark analysis, see Definition (2), I am substituting the interest rate policy with a money-supply policy.

Proposition 10 *Given the monetary policy regime of Definition (9) and the results of Proposition (7), in an equilibrium under currency competition the path of the price level in government currency is determined and also consumption at time* t_0.

Proof. To determine the price level in government's currency, consider first the flow budget constraint of the consumer. In this more general case it can be written as

$$\begin{aligned}\frac{B_t}{P_t(1+i_t)}+\frac{M_t}{P_t}+J_t\frac{M_t^*}{P_t^*}+X_t \;\leq\; & \frac{B_{t-1}}{P_t}+\left(\frac{M_{t-1}}{P_t}+(1-\delta)J_{t-1}\frac{M_{t-1}^*}{P_t^*}-C_t\right)+\\ & +Y+\frac{T_t}{P_t}+J_t\frac{T_t^*}{P_t^*}.\end{aligned}$$

In this case, the intertemporal budget constraint of the consumer together with goods market equilibrium implies the following aggregate resource constraint:

$$\frac{M_{t_0-1}^g-B_{t_0-1}^g}{P_{t_0}}+\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\frac{T_t}{P_t}+J_t\frac{T_t^*}{P_t^*}\right)=\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{i_t}{1+i_t}\frac{M_t^g}{P_t}+J_t\beta\frac{M_t^*}{P_{t+1}^*}\lambda_{t+1}\right],$$

which can be written, using the law of motion $J_t=(1-\delta)J_{t-1}+N_t$, as

$$\begin{aligned}&\frac{M_{t_0-1}^g-B_{t_0-1}^g}{P_{t_0}}+\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{T_t}{P_t}+\sum_{\bar{t}=t_0}^{\infty}\beta^{\bar{t}-t_0}N_{\bar{t}}\sum_{t=\bar{t}}^{\infty}[(1-\delta)\beta]^{t-\bar{t}}\frac{T_t^*}{P_t^*}\\ =\;&\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{i_t}{1+i_t}\frac{M_t^g}{P_t}+\sum_{\bar{t}=t_0}^{\infty}\beta^{\bar{t}-t_0}N_{\bar{t}}\sum_{t=\bar{t}}^{\infty}[(1-\delta)\beta]^{t-\bar{t}}\beta\lambda_{t+1}\frac{M_t^*}{P_{t+1}^*}. \quad (46)\end{aligned}$$

Consider now the flow budget constraint of a generic issuer of private currency, in real terms,

$$\frac{M_{t-1}^*}{P_t^*}+\frac{T_t^*}{P_t^*}=\frac{M_t^*}{P_t^*}=\beta(1-\delta+\lambda_{t+1})\frac{M_{t-1}^*}{P_t^*}$$

where to obtain the second equality I have used (31). I can iterate the above budget constraint forward starting from the entry period $\bar{t}$ to obtain

$$\sum_{t=\bar{t}}^{\infty}[(1-\delta)\beta]^{t-\bar{t}}\frac{T_t^*}{P_t^*}=\beta\sum_{t=\bar{t}}^{\infty}[(1-\delta)\beta]^{t-\bar{t}}\frac{M_t^*}{P_{t+1}^*}\lambda_{t+1} \tag{47}$$

in which I have used the fact that M_t^*/P_{t+1}^* is bounded in all periods given (35) and therefore $\lim_{t\to\infty}(1-\delta)^t\beta^t M_t^*/P_{t+1}^*=0$. Note moreover that the right-hand side is also finite and equal to Φ given the zero-rent condition in the market. Profits maximization in a market of currency competition implies therefore that an intertemporal budget constraint holds for each private currency supplier. I can then use (47) into (46) to obtain

$$\frac{M_{t_0-1}^g-B_{t_0-1}^g}{P_{t_0}}+\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{T_t}{P_t}=\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{i_t}{1+i_t}\frac{M_t^g}{P_t}\right],$$

which boils down to

$$\frac{M_{t_0-1}^g-B_{t_0-1}^g}{P_{t_0}}=\tau,$$

having used (45). Therefore the price level at time t_0 is determined by the above equation. Given the cash constraint $C_{t_0}\leq M_{t_0-1}^g/P_{t_0}$ consumption at time t_0 is determined by $C_{t_0}=\min(1,M_{t_0-1}^g/P_{t_0})$. Given that the interest rate is constant at (43), the path of the price level is determined by $P_{t+1}=\beta(1+i)P_t$ for each $t\geq t_0$. Note from Proposition (7) that given that the government monetary authority sets $\{M_t^g\}_{t=t_0}^{\infty}$ and $\{P_t\}_{t=t_0}^{\infty}$ is determined, it follows that J_t is also determined. Moreover, using the flow budget constraint of the government and the real transfer policy (45), I obtain that

$$\frac{M_{t-1}^g-B_{t-1}^g}{P_t}=(1-\beta)\tau+\beta\left(\frac{M_t^g-B_t^g}{P_{t+1}}\right),$$

which determines the sequence $\{B_t^g\}_{t=t_0}^{\infty}$ given that all other variables are determined. ■

The result of this Section contrasts with the view of Friedman (1960) who was arguing that a pure private system of fiduciary currencies would lead necessarily to instability of the price level. Fernandez-Villaverde and Sanchez (2017) reaches a similar conclusion arguing that "in a monetary system with competitive issuers the supply of each brand becomes unbounded when the

marginal cost goes to zero...Private entrepreneurs always have an incentive to mint just a little bit more of the currency." These incentives are closed in my model by the assumption of free entry in the market of currency that abates to zero all rents. Moreover, using standard devices of the literature on the determination of the initial price level using an appropriate transfer policy the full path of prices is determined.

5 Conclusion

I have studied models of coexistence in the same economy between government currency and privately-issued currencies. The presence of competing currencies can limit the use of the interest rate by the central banks and also the attainable equilibrium inflation rate. In a market with free entry embedded by multiple currencies with profit-maximizer issuers, the central bank completely loses control of the interest rate and inflation rate which are both determined by the structural factors (rate of time preference, entry cost, exit rate).

I have kept the analysis as simple as possible to shed light on an important topic that has recently received much attention. In particular, I have assumed that private and government currencies are perfect substitute delivering the same liquidity services. The analysis can be extended to imperfect substitutability without major implications for the results. Most interesting is to model endogenously the acceptability or not of currencies as medium of exchange. A topic I leave to future research. Another future avenue of research is the extension to a multi-country world to study the competition among different reserve currencies and national currencies.

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