Sovereigns at Risk

A dynamic model of sovereign debt and banking leverage

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June 17, 2019

Abstract

This paper develops a dynamic model with heterogeneous investors and sovereign default to analyze the dynamic link between banking sector capitalization and sovereign bond yields. The banking sector is modelled as operating under a Value-at-Risk (VaR) constraint, which can bind occasionally. As default risk rises, the constraint may bind, generating a fall in demand for sovereign bonds that can be accompanied by a rise in the risk premium if other agents are more risk averse. In turn, the rise in risk premium leads to a feedback effect through debt accumulation dynamics and the probability of government default.

Keywords: Banking, Asset Pricing, Sovereign default, Fiscal Limits

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1 Introduction

Recent events in the Eurozone suggest that understanding the determinants of bond price movements can be quite important when drafting policy responses to deal with debt sustainability. When bonds are risky, their pricing is dependent on the risk attitudes of the different types of investors in the markets. This paper sets out to establish a framework to understand the role of banking sector demand for risky sovereign bonds, in the presence of capital constraints that vary with portfolio risk.

In the framework presented here, yield movements can be amplified by the presence of such constraints. As the risk of sovereign defaults rises, yields go up for two main reasons. The first reason is standard. As the expected payouts are lower there is a first-order effect that leads to lower bond prices. The second reason why yields go up is due to a change in the marginal investor. If default risk rises and the banking sector becomes constrained, the price of bonds will need to fall even more to attract other investors and clear the market. The contribution of this paper is to provide a framework to study these interactions in the sovereign bond market. As the model will detail, this channel can have a strong effect on yields not only on impact, but also dynamically via larger debt accumulation.

Data on banking sector holdings of bonds show patterns of an important correlation with risk premium dynamics. As seen in Figure 8, the behaviour of Eurozone bond prices has changed after the onset of the sovereign debt crisis. Differences in default risk triggered a divergence across member countries. Up until 2008, Eurozone sovereign bond spreads were negligible and bond prices experienced strong comovement, indicating that investors saw the different bonds as almost perfect substitutes. After 2008 we see that bond price movements started to diverge, reaching their peaks during the European sovereign debt crisis.

Particularly interesting was the aftermath to the ECB's Long-Term Refinancing Operations (LTRO). These operations consisted in the injection of EUR 1 trillion of funding into the banking system at rates as low as 0.75% and

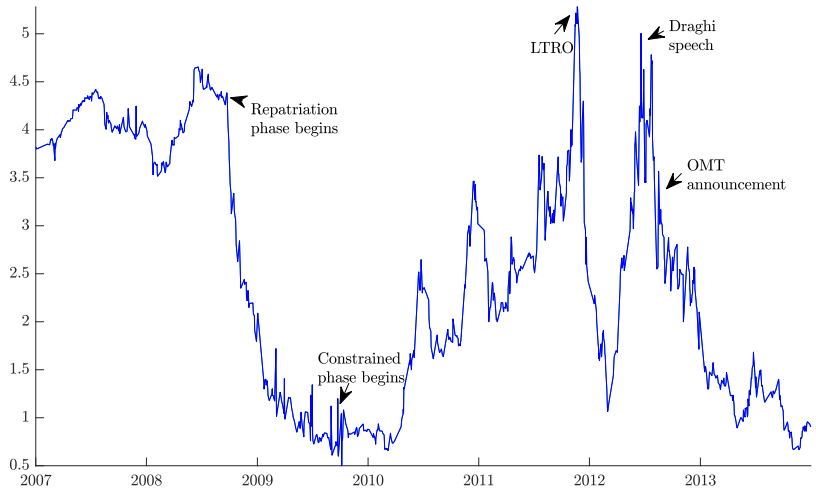


Figure 1: Spanish 1-year bond yields during the sovereign crisis. Source: Bank of Spain

a maturity of 3 years. This liquidity was distributed across two tranches, in December 2011 and February 2012. Figure 11 plots the change in sovereign bond holdings by Monetary Financial Institutions (MFI) in core and peripheral Eurozone countries. It shows how the LTRO resulted in substantial bond buying by banks of distressed countries such as Italy and Spain. On the other hand, this was not emulated by banks in core countries, which actually divested away from Eurozone bonds during the same period. Although officially not intended to deal with the rising sovereign yields, this operation became informally known as the "Sarko trade", after French President Nicolas Sarkozy told reporters that the LTRO would again enable governments to finance themselves using domestic banks. As seen in Figure 1, the impact of this policy on Spanish yields was quite striking. From December 2011 to March 2012 at the end of the LTRO period, yields fell by 345 basis points, a fall of about 75%.[1]

[1]The case of Spain is particularly striking and so it will be used as the main example in the paper. Similar patterns can be found for Italy with slightly different timings, but still very highly correlated. Figures for yields and sectoral bond holdings for both Italy and Spain can be found in Figures 9 and 10. The correlation between plotted Spanish and Italian bond holdings is 0.92 for domestic MFIs and 0.93 for the foreign holdings. For yields, the

It's particularly important to notice that there were a number of distinct stages before the LTRO and these shed some light on the importance of the marginal buyer of sovereign debt. As can be seen in Figure 2, the *pre-crisis stage* before 2009 was characterized by an increase in the holdings of sovereign bonds by foreign investors. These were around 48.79% of total holdings in 2008Q3, while domestic MFIs held only around 22.63% at this time. Once the problems in the sovereign debt market started to show, the *repatriation stage* of sovereign debt began, with domestic banks absorbing most of the net sales of foreign investors. In the quarter that followed, foreign investors sold roughly 6% of their total holdings of Spanish debt, predominantly absorbed by Spanish domestic MFI. This trend continued for a year and at the end of 2009Q3, domestic MFI held roughly 30% of total Spanish debt. Note that sovereign debt yields continued to fall in this stage, reaching their trough in September 2009 with 190 basis points (bp).

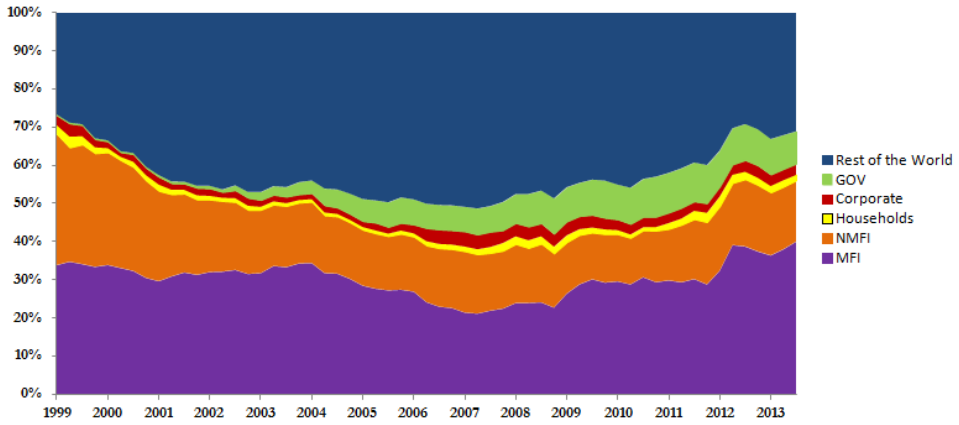


Figure 2: Spanish bond holdings by sector as percentage of total. Sector breakdown according to ESA95 classification. MFI: Monetary and Financial Institutions; NMFI: Non-monetary financial institutions; Households: Household and non-profit institutions serving households; Corporate: Non-financial Corporations; GOV: General Government; ROW; Rest of the world. Source: Bank of Spain.

At this point, the *constrained stage* began with domestic banks no longer increasing their share of sovereign debt and domestic Non-Monetary Financial

cross-country correlation is 0.88.

Institutions (NMFI) and Households (HH) picking up the slack as the rest of the world continues selling Spanish debt to domestic investors. This is also when sovereign debt yields really took off, with yields increasing to their peak of 510bp in November 2011, just before the LTRO intervention. Figure 3 shows the year-on-year percentage change in the proportion of bond holdings held by domestic MFI contrasted with the change for Households and domestic Non-Monetary and Financial Institutions (NMFI) during the constrained stage. Although the data is only quarterly, we can see that as banking sector demand falters before 2011Q4, households and NMFI start increasing their asset holdings significantly. From 2010Q3 to 2011Q3, households increased their total bond holdings by 181% and NMFI by 33%. During this period the proportion of bonds held by domestic MFI was basically flat, falling slightly by 2%.

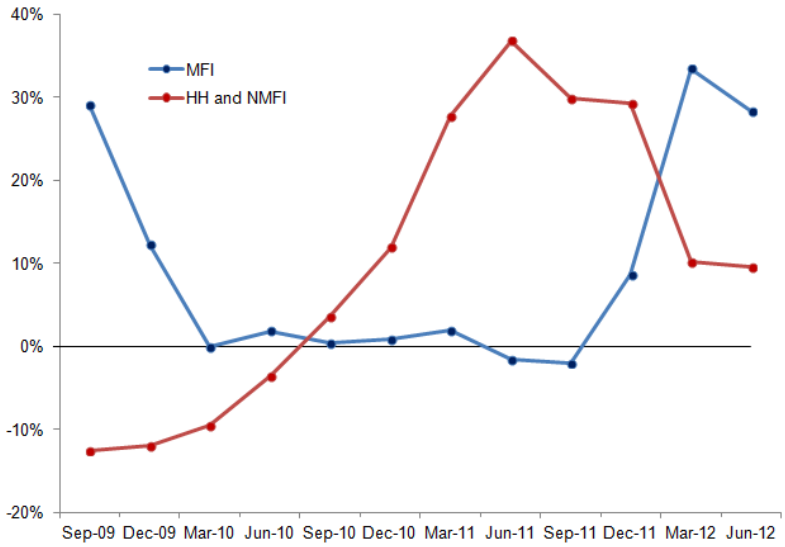


Figure 3: Year-on-year percentage change in share of sovereign bonds held by each sector. Monetary and Financial Institutions (MFI) and Households and Non-Monetary and Financial Institutions. Quarterly frequency from September 2009 to June 2012. Source: Bank of Spain

In December 2011, the first LTRO tranche was introduced, followed by a second one in February 2012. The impact on Spanish bond purchases by domestic banks was immediate, increasing their share of domestic sovereign

debt significantly in these two quarters, as can be seen in Figure 3. The impact on yields was also immediate as they fell significantly at this point. A summary of these stages can be found in Table 1.

Stage	Period	Share of assets			Yields
		MFI	NMFI + HH	ROW	
Pre-crisis	Up to 08Q3	22.63%	16.03%	48.79%	4.2 %
Repatriation	08Q4 to 09Q3	30.07%	13.62%	43.85%	1.9%
Constrained	09Q4 to 11Q3	28.72%	18.84%	40.01%	5.1%†
LTRO	11Q4 to 12Q1	39.06%	18.50%	30.32 %	2.8%

Table 1: End-of-period yields and share of sovereign bond holdings by investor type. Value at the end date of each stage.

† Although bond holding data is quarterly, yields continued to increase in 2011Q4 before the LTRO in December. The yield shown is just before the intervention in November 2011, as end of Q3 yields of 3.8% do not fully reflect the extent of the rise.

These dynamics are the main motivation for the mechanism described in the paper, which considers the importance of the identity of the marginal buyer of sovereign debt and also differences in willingness to pay for sovereign debt once it becomes risky. The paper will argue that in the constrained stage, domestic MFI were no longer the marginal buyers of Spanish debt, which lead to a jump in yields which is in turn amplified by the effect higher yields have on debt sustainability and resulting feedback. The LTRO can be seen as a temporary way to relieve domestic banks so they resume their status as the marginal buyer and provide some relief to sovereign yields.[2].

That prices of domestic sovereign bonds are important for European banks is well understood. Acharya and Steffen (2015) document that these banks were heavily invested in significant carry trade behaviour using Eurozone bonds. This was true both before and during the crisis, but particularly so for periphery country banks which did not diversify away from their exposure to peripheral sovereign debt. But the connection between banking sector capitalization and sovereign yields has been further clarified in the recent crisis. Acharya et al. (2014) analyze the case when sovereign debt problems arise due to the additional expenditures due to government bailouts or their

[2]More on the temporary nature of this effect later when discussing the results.

expectation. This is the traditional interpretation of the Irish and Spanish case, or a banking-to-sovereign channel. On the other hand, highlighting the sovereign-to-banking channel, Gennaioli et al. (2014) look at how public default hurts banks by decreasing their net worth and thus limiting their ability to finance real investment. Bocola (2016) studies how news about a possible future default affect investment through both the ability of banks to fund projects and the higher riskiness of these projects themselves. Like the present paper, it studies the effect of leverage constraints, but focusing on the lending channel, while the present paper focuses on the role of banks as important players in the sovereign debt market and the feedback effects between yields and default probabilities. Corsetti et al. (2013) show how the lending channel can generate that type of feedback whenever there is a correlation between sovereign and private yields. Although the lending channel is not pursued in this paper, the mechanism present here also provides a reason for this correlation as higher default risk can constrain the banking sector and lead to a rise in risk premia.

The model presented in this paper highlights an important feedback mechanism at work that comes from bank leverage constraints and the role of banks in sovereign debt markets. As sovereign risk increases, so does bank balance sheet risk, which can make leverage constraints bind. If banks cannot absorb the supply of bonds, then the remaining supply must be picked up by other investors in the economy who might require a higher premium to hold them. To understand bond price movements, it is then important to determine who is the marginal investor of bonds, as the properties of her stochastic discount factor will be crucial in determining the market price.

Moreover, the model includes negative dynamic effects on debt sustainability due to higher yields. First, a higher yield today will imply that the debt being rolled over in the future will be larger, increasing the probability of future default. Second, higher yields will also lead to lower output as the government raises (distortionary) taxes in order to repay the higher debt burden in the future. Higher debt today will also imply higher expected debt in the future (conditional on no default), so short term adverse debt dynamics can become a burden that persists over time.

2 Related Literature

There has been a recent call for macroeconomic models to embed endogenous leverage and default. Woodford (2010) urges to change our models of financial intermediation and Geanakoplos (2011) highlights that models that do not feature endogenous leverage and default cannot replicate the leverage cycle. The leverage cycle comes from the fact that negative shocks will be associated not only with a fall in the underlying fundamentals of the asset (i.e. the quality of the asset *supply*), but also with a drop in leverage.

In Geanakoplos (2010), the author sets out a theory of the leverage cycle[3] where both leverage and the rate of interest are endogenously determined by supply and demand, leading to a positive correlation between asset prices and leverage. In the leverage cycle, the marginal buyer plays a crucial role in determining asset prices. When the ability to leverage is high, then agents more willing to buy the asset will be able to purchase larger quantities, leading to higher asset prices. In the model described in the present paper, this mechanism will also feature prominently. When less risk averse banks are able to leverage sufficiently, bond prices will be higher. But when they are constrained and cannot lever more, then there is a change in the marginal investor (who charges a premium) and bond prices fall.

The literature on constrained financial intermediation is also very related to this paper. Bernanke and Gertler (1989) paved the way by showing how agency costs and changes in borrower net worth can amplify shocks in the economy. In another seminal paper, Kiyotaki and Moore (1997) show that this is magnified by a positive feedback between asset prices and firms net worth. An important distinction with respect to the present paper is that banks described here can default in equilibrium and face risk-based constraints on their balance sheet, which gives rise to procyclical leverage. On the other hand, credit frictions like in Kiyotaki and Moore (1997) lead to countercyclical leverage and do not feature default at equilibrium.

[3]See also Geanakoplos (1997), Geanakoplos (2003), Fostel and Geanakoplos (2012) among others.

Gertler et al. (2012) show this mechanism can be applied to financial intermediaries and consider a model where they can issue outside equity as well as short-term debt. Expanding this setup, Gertler and Kiyotaki (2015) allow for the model to consider household liquidity shocks as in Diamond and Dybvig (1983), while Gertler and Karadi (2011) study the effects of unconventional monetary policy under this type of constraints. Brunnermeier and Sannikov (2014) highlight the importance of non-linearities and off steady-state behaviour. Under continuous time, they show that debt constraints exhibit larger amplification and persistence of shocks away from the stochastic steady-state than near it[4]. He and Krishnamurthy (2013) present a model where financial intermediaries face occasionally binding capital constraints and also highlight how risk premia can be significantly amplified when they bind. In their model, occasionally binding equity issuance constraints limit the amount of funds that can be intermediated. Although leverage is again countercyclical and there is no default in equilibrium, the model is able to generate persistence and amplification in risk premia.

Another approach can be found in papers such as Adrian and Shin (2010a) and Adrian and Shin (2014), wherein the financial sector faces a Value-at-Risk constraint. Adrian and Boyarchenko (2012) embed this approach in a dynamic model with financial intermediaries facing balance sheet risk constraints. In this paper, the constraint is always binding and the probability of intermediary default is positive. They show that this generates procyclical leverage and that regulation faces a trade-off between likelihood of bank default and the price of risk. Nuño and Thomas (2017) also show that endogenous leverage constraints and moral hazard create a strong link between asset prices and bank leverage, generating banking cycles. The model described in the current paper will embed a Value-at-Risk constraint in a model with both bank and sovereign default. This approach has the advantage of being able to treat bank default explicitly and it also generates procyclical leverage. Moreover, it also fits the regulatory environment set up by the Basel Agreements and European

[4]Maggiori (2017) also exploits these non-linearities in an open economy setting to explain cross country portfolios and current account imbalances.

stress tests. The constraint tightens when balance sheet risk rises and loosens when it becomes lower. This will be central to the amplification mechanism described in the current paper.

Diamond and Rajan (2000) propose a theory of banking where banks require capital because deposits are prone to runs. In that theory, capital requirements make the bank safer, but also affect the cost of capital and the ability of banks to liquidate projects. Angeloni and Faia (2013) highlight how this generates also procyclical leverage and how even countercyclical capital requirements may be sub-optimal. Since the present paper is concerned with the interaction of risk based capital requirements with sovereign yields, rather than the optimality of such requirements, it will for now take their existence as given. A next step would be to consider how optimal capital requirements might be designed once one also accounts for the existence of such interaction.

There are several papers that show that sovereign bonds are also very important in bank behaviour and their balance sheets. As mentioned earlier, Acharya and Steffen (2015) highlight that the behaviour of banks post-2008 was very close to a carry trade strategy, with some banks holding risky sovereign portfolios that were about a third of the balance sheet size. They also highlight how the correlation between bond yields of the core and the periphery turned negative around 2010 as the risk premium increased significantly at that time. Gros (2013) also documents that domestic sovereign exposure often exceeds 100% of bank capital, leading to concerns about the ability of banks to absorb haircuts. The author also shows that banks have decreased their exposure to domestic sovereign debt. So even though regulatory risk weights for sovereign bonds had not changed, banks still scaled back their holdings of risky Eurozone sovereign bonds. This could have been in anticipation of the upcoming EU stress tests which from 2010 started to consider a "sovereign risk shock" in its adverse scenarios. These stress tests can be seen as the ability of the bank to survive a specific event, which occurs probabilistically.

Gennaioli et al. (2014) also highlight the strong link between bank balance sheets and government default. They present a two-period model of opportunistic default, wherein higher leverage leads to larger bond holdings by banks and

lower yields due to the government's lower incentive to default. In the model, leverage is determined exogenously by a measure of financial development. As the model described in the next section will show, allowing leverage to depend on the riskiness of the bank's portfolio introduces a feedback effect that can amplify changes in default risk. Acharya et al. (2014) document the strong link between banking sector health and sovereign debt sustainability. Although their main focus is the first channel, where bailout expectations deteriorate sovereign risk, empirically they also provide evidence of strong feedback effects and how increased sovereign risk is an important factor in determining the health of the financial sector. Importantly, they show that higher sovereign credit risk affects the credit risk of even *foreign* banks. This means that it is not only via bailout expectations that sovereign credit risk and bank credit risk are linked, but also there is a direct effect on bank balance sheets (as should be expected given that the proportion of sovereign debt on bank balance sheets is considerable). Related to the current work, Chari et al. (2019) study how financial repression can sometimes be optimal by providing discipline to governments who otherwise cannot commit to its policies, but do not want to also trigger a financial crisis. In the current paper, policies that increase the holdings of sovereign bonds by domestic banks can also be desirable but through their impact on yields (and associated debt sustainability) rather than a potentially complementary effect of decreasing incentives to sovereign default.

Kollmann et al. (2013) study how the support for banks had a stabilizing effect on Eurozone real variables. In their model, banks can deviate from an exogenous leverage constraint at a cost. In that case, supporting constrained banks leads to higher investment and output. The present paper can then be seen as complementary. Although it abstracts from the lending channel, it highlights the feedback effect between sovereign risk and bank balance sheet risk. Also looking at this feedback effect, Farhi and Tirole (2018) highlight how bailouts generate risk-shifting from banks, making them the *natural buyers* of domestic bonds (as in the current paper). In order to focus on the role of regulation in an international setting, the authors assume that foreign investors are always the marginal buyers of sovereign debt. In that setting, risk-shifting

will have no effect on the equilibrium price of bonds.

3 The Model

In this section, I will describe a simple theoretical framework with a mechanism that can generate large swings in bond prices in response to banking sector conditions. This framework will then be extended in sections 5 and 6 to consider the presence of other assets in the bank's balance sheet and also moral hazard that might arise from the existence of government guarantees on deposits.

To have a change in the marginal investor affecting government bond yields, we require an economy populated with a minimum of three types of agents: a government who may potentially default, a banking sector which faces occasionally binding constraints (that depend on sovereign risk) and a residual investor. To keep the framework as simple as possible, households will serve as the residual investors. It is not essential for the mechanism that households invest directly in sovereign bonds. The model would be isomorphic to one where household savings are channelled through an investment vehicle (such as a mutual or investment fund), as long it priced assets using the stochastic discount factor of the household.

The other important assumption is that banks are more willing to hold sovereign bonds than households. In that sense, they are what Geanakoplos (2010) calls *natural buyers*. In the model, banks are assumed to have lower risk aversion than households[5]. Although all that is necessary is making banks more willing to hold bonds, there are a number of papers[6] describing how compensation of CEOs tends to encourage excessive risk-taking in banks. Moreover, as Bolton et al. (2010) note, in the presence of moral hazard the

[5] As Geanakoplos (2010) notes, this higher willingness could be achieved through different assumptions with similar effects. For example, if the banks have access to better hedging techniques than the general public, or can use them as collateral (adding a collateral value). They could also simply be more optimistic, as is the case with the natural buyers in Geanakoplos (2010).

[6] For example, Acharya et al. (2016), Rajan (2005, 2008), Cheng et al. (2010) among others.

value of the stock for a levered bank is like the value of a call option. It is increasing in the volatility of the assets held and thus induces risk shifting. This risk-shifting would also lead to portfolio choices that seem less risk averse when compared to non-levered investors.

Finally, another key assumption is that banks face a constraint on the risk they can take on their balance sheet. Partially to address risk shifting, European banks face regulation from the Basel Agreements that limits the amount of risky assets they can have on their balance sheet relative to Tier 1 capital (which includes the book value of common equity, with some deductions, and certain classes of preferred equity). This is modelled by imposing on the banking sector a Value-at-Risk (VaR) constraint, which sets an upper limit to the probability of bank default[7]. As with current banking regulation, this constraint also limits the amount of risky assets banks can have on their balance sheet relative to the size of their equity. Crucially, as will be detailed later, maximal leverage will then depend on how risky the balance sheet is.

3.1 The households

The model is an infinite horizon closed economy model with a representative household and a single consumption good. The consumption good Y_t is produced using a combination of labour and a stochastic productivity shock A_t[8].

$$Y_t = A_t(1 - L_t) \tag{1}$$

with L_t being the households' endogenous choice of leisure time. Total endowment of labour is normalized to 1, so $1 - L_t$ represents total working hours.

[7]In Appendix D, following Adrian and Shin (2014), I describe how this constraint can be microfounded and seen as imposed by stakeholders that want to decrease the incentives for risk-shifting.

[8]For simplicity reasons, the model will abstract from capital. Obviously, capital has an important role to play as the interest rate will affect investment behaviour and capital accumulation. This is left for a future extension.

Labour productivity A_t has the following law of motion:

$$\log A_t = \rho^a \log A_{t-1} + \varepsilon_t^a \quad , \quad \text{where} \quad \varepsilon^a \sim N(0, \sigma_a^2) \tag{2}$$

with ρ^a determining the persistence and σ_a^2 the volatility of the productivity shock.

The representative household makes decisions regarding consumption C_t, leisure L_t and savings. Savings can be invested in deposits D_t, bond purchases B_t^H (no short-selling) or a portfolio of both. The household then maximizes utility subject to its budget constraint and takes bond prices q_t^B and deposit rates $1/q_t^D$ as given. The maximization program is as follows:

$$\max_{\{C_t, L_t, B_t^H, D_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \qquad \text{s.t.} \tag{3}$$

$$C_t + q_t^B B_t^H + q_t^D D_t = (1 - \Delta_t) B_{t-1}^H + D_{t-1} + Y_t(1 - \tau_t) + \tilde{Z}_t + \Pi_t^B - \Gamma_t \tag{4}$$

$$B_t^H \geq 0 \tag{5}$$

where β is the subjective discount factor of the households and Π_t^B are banking sector's dividends. Δ_t is the haircut on government bonds in case of sovereign default. In case of default, there will be a cost Γ_t which will be assumed to be proportional to output. Finally, $\tilde{Z}_t$ are net transfers from the government and τ_t is the proportional labour income tax rate. The felicity function u is assumed to be strictly increasing and concave in both consumption and leisure.

3.2 The government

The government requires resources for government expenditure G_t and gross transfers to households Z_t. It collects taxes via a distortionary labour tax τ_t and can also fund itself through debt B_t. The government can potentially default with an endogenous probability, at which point a real cost is incurred. The setup for the government is similar to the one in Bi (2012) and Bi and Leeper (2013).

The choice of modelling the government with this setup has two main

advantages. First and foremost, it can be seen as a reduced form to describe complex interactions between political economy considerations of governments in power and other real variables in the economy. Although reduced form, it incorporates different elements of government debt dynamics such as long periods of unsustainable policies that may leave the sovereign with large debt-to-GDP ratios. The debate on the sustainability of social security systems is an example of long-term problems that are often identified far earlier than they are addressed.

There is a large literature on models of strategic default building on Eaton and Gersovitz (1981)[9]. The key property of these models that is required for the present paper is that they generate a time-varying, state-dependent risk of default. The reduced form approach taken in the present paper also has those properties, so it is possible to approximate them through adequate calibration.

The second reason is more practical. The methodology provides a way to have a full description of the time-varying probability density function of default rates, which is required to pin down the leverage limit imposed by the Value-at-Risk constraint on the banking sector, as will be explained later. Government expenditures are stochastic and their process is as follows:

$$\begin{aligned} \log G_t &= (1-\rho^G)\log \bar{G} + \rho^G \log G_{t-1} + \varepsilon_t^G \\ &\text{with} \quad \varepsilon^G \sim N(0, \sigma_g^2) \end{aligned} \tag{6}$$

with ρ^G being the persistence and $\bar{G}$ the long-run level of government expenditures.

As in Bi and Leeper (2013), the government transfers Z_t follow a Markov switching process with two regimes. In the first regime transfers are cyclical, but stationary. In the second, transfers grow exponentially and put the solvency of the government at risk. This is in the spirit of political economy models such as Ruge-Murcia (1995, 1999) and Davig (2004). The economic intuition is that transfers can seemingly enter unsustainable paths that require political

[9]For example, recently Aguiar and Gopinath (2006) and Arellano (2008) use it to explain many key facts in emerging economies.

reform to bring them back into control (e.g. pension reforms)[10]. The Markov regimes can then also be seen as reduced forms that capture the existence of a lag between the need for political reform and its implementation. Specifically, government transfers follow the following process:

$$\log Z_t \equiv \begin{cases} \log \bar{Z} + \alpha^Z \log A_t & s_t^Z = 0 \\ \mu^Z + \log Z_{t-1} + \alpha^Z \log A_t & s_t^Z = 1 \end{cases} \tag{7}$$

$\mu^Z > 0$ measures the explosiveness of the regime and α^Z the cyclicality. s_t^Z indicates the regime, with $s_t^Z = 0$ being the stationary regime and $s_t^Z = 1$ being the unsustainable one. s_t^Z evolves according to the transition matrix P^Z.

The government's main source of funding is an income tax, which is proportional to wages (output) received. It can also borrow from bond markets if it requires additional funding to service its expenditures. The labour income tax rate τ_t follows the following feedback rule:

$$\tau_t - \bar{\tau} = \xi(B_{t-1} - \bar{B}) \tag{8}$$

where B_{t-1} is the government debt at the beginning of the period. The rule intends to represent the observation that authorities tend to raise taxes when government debt rises. The gradient ξ is a measure of how reactive tax rates are to debt. $\bar{B}$ is a target level of debt and $\bar{\tau}$ the tax rate prevailing at that level.

Since taxes are distortionary, ensuring that the first transfer regime is stable requires imposing a restriction on parameter ξ which cannot be too low. If it is too low, even in the stable regime the feedback rule might not be enough to stabilize debt-to-GDP ratios after they reach a certain level. In the preferred calibrations for ξ, this restriction is always satisfied. Note that with the possibility of explosive transfers the feedback rule is not enough to ensure

[10]Empirically, Balassone et al. (2010) highlight the role of cash transfers in unsustainable fiscal policies in the EU. Afonso et al. (2011) and Afonso and Toffano (2013) provide strong evidence of fiscal policy regimes switching in Portugal and Italy, while less so for Germany.

that the government never defaults in equilibrium.

The probability of default depends on the stochastic fiscal limit B_t^* which is drawn from a time-varying distribution $\mathcal{B}_t$. This state-dependent distribution is endogenous and depends on the ability of the government to honour its debt, as will be detailed later. The government will then partially default on its obligations if its debt is larger than the stochastic fiscal limit ($B_{t-1} > B_t^*$), and will honour the debt in full if it is lower or equal to the stochastic fiscal limit.

Since there is the possibility of default, bonds will be risky assets. Investors may buy a bond at time t for the price q_t^B, which pays $1 - \Delta_{t+1}$ units of the consumption good at time $t + 1$. If the government honours its debt in full, then $\Delta_{t+1} = 0$. If the government defaults at $t + 1$, investors receive only a fraction $1 - \Delta_{t+1}$ of the payoff, with $\Delta \in [0, 1]$. The expected payoff of a bond is then simply $\mathbb{E}_t(1 - \Delta_{t+1})$. The haircut Δ_t is determined by the following rule:

$$\Delta_t \equiv \begin{cases} 0 & \text{if } B_{t-1} < B_t^* \\ \delta_t & \text{if } B_{t-1} \geq B_t^* \end{cases} \tag{9}$$

where $\delta_t \sim \Omega(\delta)$. Ω is the distribution of haircuts conditional on a government default. The Ω distribution is based on the haircut database constructed by Cruces and Trebesch (2013), which covers 180 sovereign debt restructurings between 1970 and 2010. The empirical distribution is then approximated by a parametric Beta Distribution. The governments budget constraint can be written as:

$$B_{t-1} - q_t^B B_t = \tau_t A_t (1 - L_t) - G_t - Z_t + \Delta_t B_{t-1} \tag{10}$$

The change in total debt equals the primary deficit adjusted by debt service and the haircut Δ_t (if any). The government is also the guarantour of household deposits in banks. It is assumed that even that when the government defaults, it can still fund deposit guarantees by reducing transfers to the households.[11].

[11] Alternatively, it would be equivalent to assume the representative household (who owns

Net transfers $\tilde{Z}_t$ to households can then be smaller than gross transfers Z_t. Note that this creates an implicit seniority structure on the liabilities of the government. The most senior ones are the deposit guarantees, followed by the transfers to households and finally the most junior ones are the liabilities towards bond holders. Therefore, there will be an associated risk premium that risk averse bond holders will demand over deposits, whenever the probability of default is larger than zero. The risk premium is

$$\frac{\mathbb{E}_t(1-\Delta_{t+1})}{q_t^B} - \frac{1}{q_t^D} \tag{11}$$

with $\frac{1}{q_t^D}$ being the riskless deposit rate. The expected return on sovereign bonds will not coincide with the implied yield $\frac{1}{q_t^B}$, unless the probability of default is zero.

Additionally, there is a real cost Γ_t to sovereign default. There are many papers that seek to measure and quantify such costs. In particular, some authors have estimated how costs of sovereign default can differ when combined with a banking crisis. Papers like Borensztein and Panizza (2009), Sandleris (2016) and Laeven and Valencia (2012), all estimate significant costs of default in the short-run, which can be particularly serious when coinciding with a banking crisis. In the database provided by Laeven and Valencia (2012), the average output loss of twin crises (banking and sovereign) is extremely large and equal to 38.6% of GDP. For only sovereign crises, Sandleris (2016) estimates a cost of 15% for European economies. Sovereign crises combined with a banking crisis also have a cumulative output loss that is 7.4 percentage points larger than those without a banking crisis.

The general pattern seems to be that there are high output losses during sovereign crises, especially in advanced economies, and that these are aggravated when combined with banking crises. Moreover, the costs seem to be relatively contained within one year preceding and following default. To capture these important effects in our model, we will assume that if the government defaults there is a loss that is proportional to total output. The calibration chosen is a

the banking sector) is forced to recapitalize the bank.

10% loss, which is large but still on the conservative side given the numbers described above.

3.3 Banking sector

The banking sector is composed of financial institutions which fund themselves through equity and household deposits. They then use these funds to invest in the financial markets. In the basic model I assume that they purchase only sovereign bonds. For the moment, the banking sector will be kept as simple as possible to highlight the amplification mechanism. Extensions where banks also invest in other assets and have access to discount window funding will be discussed in a later section.

The balance sheet of the banks at the end of period t in the basic model is then simply:

Assets	Liabilities
$q_t^B B_t^B$	E_t
	$q_t^D D_t$

where E_t is the bank's equity and $q_t^D D_t$ the deposit amount. On the asset side, the bank holds sovereign bonds B_t^B which are valued at the current price q_t^B.

The banking sector is assumed to be risk neutral, but will be constrained by a Value-at-Risk condition. This condition imposes that the bank invests in such a way that the probability it cannot repay its obligations must be smaller than an exogenous parameter α. Bank default would then occur if the ex-post value of the assets is not enough to repay its liabilities . The VaR constraint can then be written as:

$$Prob\left(D_t > (1 - \Delta_{t+1})B_t^B\right) \leq \alpha \tag{12}$$

So the probability that equity is wiped out by the bank's losses (negative profits) must be less or equal than α. This constraint is not only in the spirit of the Basel Agreements, but also has the desirable property of generating

procyclical leverage, which can be observed in the data as described in Geanakoplos (2011) and Adrian and Shin (2014). Kalemli-Ozcan et al. (2012) using a panel of European and US commercial and investment banks also provide evidence for the presence of procyclical leverage for investment banks and large commercial banks[12]. On the other hand, models with debt or collateral constraints (such as He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014)) feature countercyclical leverage. This constraint can then be interpreted as either imposed by regulation, proxying for the risk behaviour of banks or a mix of both. For modelling purposes, however, it will be imposed as an exogenous constraint.

The VaR constraint also has a tight link with bank capitalization. Given prices and assets B_t^B, the larger is equity E_t, the smaller are bank's liabilities D_t relative to its assets B_t^B. It will then be able to absorb lower returns on assets before it defaults on its borrowers, so a constrained bank can then be seen as an insufficiently capitalized bank. Despite the fact that regulatory risk-weights for Eurozone sovereign bonds remained at zero during this period, starting in 2010 the Committee of European Banking Supervisors (CEBS) began to consider "sovereign risk shocks" in the adverse scenarios of its stress tests. One interpretation of the stress tests is that there is an adverse scenario with probability α, and the constraint is that the banks must be resilient enough to survive if it occurs. The probability of not being able to absorb such losses is then required to be smaller (or equal) to α, which is exactly what the Value-at-Risk constraint imposes. During the *repatriation stage* the CEBS stress tests did not consider a "sovereign risk shock" but did starting from 2010. Perhaps not coincidentally, the first stress tests of the period identified here as the *constrained stage* were also the first to include the "sovereign risk shock" .

The representative bank is assumed to be a risk neutral price taker operating in a competitive environment. So it maximizes the expected value of next period profits Π_{t+1}, under the VaR constraint while taking asset prices q_t^B and

[12]The authors also show that leverage seems acyclical for smaller banks. In Spain, the banking sector is quite concentrated with the 5 biggest banks owning more than 68% of the market share (2018).

q_t^D as given. The program is then:

$$\max \mathbb{E}_t \left[\Pi_{t+1}\right] \tag{13}$$

$$\text{s.t. } Prob\left(D_t > (1-\Delta_{t+1})B_t^B\right) \leq \alpha \tag{14}$$

where α is the Value-at-Risk threshold, or the maximum probability of default that banks are allowed. The constraint is simply saying that the probability of sovereign haircuts being large enough to wipe out equity cannot be larger than α. In the beginning of period $t+1$, the government announces whether it will default or not, and the haircut Δ_{t+1} is revealed. The banks profits can then be written as:

$$\Pi_{t+1} = (1-\Delta_{t+1})B_t^B - D_t \tag{15}$$

Banks distribute the entirety of profits to households $\Pi_{t+1} = \Pi_{t+1}^B$ and therefore equity is constant[13]. As mentioned earlier, in case of bank default governments might need to help recapitalize banks. In that case, $\Pi_t^B = -E_t$ and the government funds the difference by reducing net transfers $\tilde{Z}_t = Z_t - (\Pi_t - \Pi_t^B)$.

As Adrian and Shin (2010b) note, leverage generally fluctuates through changes in the total size of the balance sheet and not due to changes in equity. The relationship between balance sheet size and leverage for the main Spanish banks can be seen in Figure 12. Equation (15) implies that when binding the VaR constraint (12) can be rewritten as:

$$Prob\left(\Delta_{t+1} > 1 - \frac{D_t}{B_t^B}\right) = \alpha \tag{16}$$

Let $\Lambda_t = \frac{q_t^B B_t^B}{E_t}$ be the leverage ratio, which is defined as the size of the market

[13]One alternative would be to consider a dividend distribution rule such as $\Pi_t^B = \Pi_t + \xi^{\Pi}\left(E_t - \overline{E}\right)$. The current model is then the particular case of $\xi^{\Pi} = 0$.

value of assets relative to equity. We can then rewrite equation (16) as:

$$P\left(\Delta_{t+1} > 1 - \frac{q_t^B}{q_t^D}\frac{\Lambda_t - 1}{\Lambda_t}\right) \leq \alpha \tag{17}$$

Since the expression above is monotonic in Λ_t, we can use the cumulative distribution function $\Omega(\Delta_{t+1})$ and the market prices to calculate maximal leverage Λ_t. Due to the monotonicity there is an unique solution. Note that the higher the probability of default, the lower will be the leverage ratio at the binding limit.

3.4 Equilibrium

Let $S = \{A, B, Z, G, s^Z\}$ be the vector of state variables. Given a sequence of prices $\{q_t^B, q_t^D\}_{t=0}^{\infty}$ and the distribution of conditional fiscal limits $\mathcal{B}(S)$ and haircuts $\Omega(\Delta)$, define the optimal decisions of the representative household as $C(S), B^H(S), L(S), D^H(S)$, and that of the representative bank as $B^B(S), D^B(S)$. We can then define the equilibrium as follows.

Definition 1 An equilibrium is a sequence of prices $\{q_t^B, q_t^D\}_{t=0}^{\infty}$, and policy rules $C(S), B^H(S), L(S), D^H(S), B^B(S), D^B(S)$, such that:

- $C(S), B^H(S), L(S), D^H(S), B^B(S), D^B(S)$ are optimal given $\{q_t^B, q_t^D\}_{t=0}^{\infty}$
- Asset and consumption markets clear at every period t

$$\begin{aligned} Y_t &= C_t + G_t \\ B_t &= B_t^H + B_t^B \\ D_t^H &= D_t^B \end{aligned}$$

3.5 The fiscal limits

Note that the equilibrium definition described above works for any particular fiscal limit distribution $\mathcal{B}$. However, this distribution will play a key role in the model, so it is important that it connects the several aspects of the model.

Any government's ability to honour its debts is intrinsically linked with not only its debt-to-GDP ratio, but also to growth and the government's policy on expenditure and taxation. In the procedure described below, these will all be relevant in determining the probability of default.

On the revenue side, the presence of distortionary taxes imposes a limit on the ability of the government to collect tax revenue. By reducing the net wage, an increase in the tax rate will reduce the incentives of households to work. From equation (19) we see that an increase in tax rates will reduce working hours, so for a sufficiently high tax rate this may lead to a fall in revenues. Of course, revenues also depend on the other state variables in the economy (e.g. on productivity A_t) so in the current setup, the Laffer curve will be dynamic and its shape will vary with the state of the economy.

Since there is a Laffer curve, there is also a (time-varying) tax rate $\tau_t^{\max}$ that would maximize government revenues for a given state. This tax rate $\tau_t^{\max}$ can then be used to calculate the distribution of net present value of maximal surpluses that the government would be able to collect in the future. This is the fiscal limit distribution. For every state, we then have:

$$\mathcal{B}^*(A_t, G_t, s_t^Z) \sim \sum_{j=0}^{\infty} \beta^j \frac{u'_{C_{t+j}^{max}}}{u'_{C_t^{max}}} \left(\tau_t^{max}(A_{t+j}(1 - L_{t+j}^{max}) - G_{t+j} - Z_{t+j}\right) \qquad (18)$$

where the consumption and leisure choices (C_t^{max}, L_t^{max}) take into account that the tax rate is set at the maximum of the Laffer curve. This conditional distribution implies that the government's ability to honour its debt depends on the current state of the economy, including transfers (and the regime) and government expenditure. Appendix B describes how these limits are calculated in more detail.

Under this definition of fiscal limit distribution, the government will default with probability π^D, if a proportion π^D of future paths have a net present value of (maximal) future surpluses which is lower than the maturing debt at the beginning of the period.

Note that the fiscal limit distribution also depends crucially on future

expectations about the transfer regime. Even when the government is at the stable regime, it can still default if its debt is sufficiently high. This is because agents expect the government will enter the explosive regime at least in some of the future paths. By the same token, even during the explosive regime investors may still be willing to buy the bonds as they expect the governments to enter the stable regime with positive probability.

3.6 The role of the marginal investor

In the model presented, the identity of the marginal investor will matter significantly for bond pricing. To check that, let's start by comparing the First-Order Conditions (FOC) of the two agents. For the households we have:

$$\frac{u'_{L,t}}{u'_{C,t}} = A(1-\tau_t) \tag{19}$$

$$q_t^D = \beta \mathbb{E}_t \left[\frac{u'_{C,t+1}}{u'_{C,t}} \right] \tag{20}$$

$$q_t^B \geq \beta \mathbb{E}_t \left[(1-\Delta_{t+1}) \frac{u'_{C,t+1}}{u'_{C,t}} \right] \tag{21}$$

Condition (19) simply states that the marginal rate of substitution between consumption and leisure must be equal to the net wage, while the other two Euler equations illustrate the willingness to pay of households for each of the two assets. Note that the bond prices depend on the probability of sovereign default next period and the corresponding haircut.

There is also the possibility that the banking sector holds all government bonds. In this case, equation (21) may not hold with equality given that banks are less risk averse than households and the latter cannot short-sell government bonds. That simply means that the bond price is too expensive for the households and they prefer to put all their savings into deposits. The Lagrange multiplier of the short-selling constraint is positive, creating the wedge in equation (21). When banks are constrained, then the households may become the marginal buyer of bonds, so that the price q^B falls and equation

(21) holds with equality. We can then define $q_t^{B,c}$ as the price that would hold when banks are constrained and households are the marginal investor.

$$q_t^{B,c} \equiv \beta \mathbb{E}_t \left[(1 - \Delta_{t+1}) \frac{u'_{C,t+1}}{u'_{C,t}} \right] \tag{22}$$

For the banks, the first order condition is the following:

$$q_t^B \leq \mathbb{E}_t (1 - \Delta_{t+1}) q_t^D \tag{23}$$

which will hold with equality when the VaR constraint is not binding and with inequality otherwise. Similarly, we can then define $q_t^{B,u}$ as the price that would hold in the unconstrained case:

$$q_t^{B,u} \equiv \mathbb{E}_t (1 - \Delta_{t+1}) q_t^D \tag{24}$$

$$q_t^{B,u} = \beta \mathbb{E}_t (1 - \Delta_{t+1}) \mathbb{E}_t \left[\frac{u'_{C,t+1}}{u'_{C,t}} \right] \tag{25}$$

Note that since households are the only depositors, then the FOC of equation (20) must always hold.

Looking at the pricing equations, it becomes apparent that if the default probability is zero, then $\Delta_{t+1} = 0$ for all states, which means the bond is risk free. This would make deposits and bonds perfect substitutes and prices would equalize, according to both the household's and the bank's valuation. Conversely, if the probability of default is larger than zero, then $q_t^B < q_t^D$ and the implied yield is larger than deposit rates. This is true regardless of the identity of the marginal investor.

The key distinction between equations (25) and (22) is that households care about the correlation between consumption and bond returns. If this correlation is positive, then

$$q_t^{B,c} < q_t^{B,u} \tag{26}$$

and the difference can be seen as the partial equilibrium effect of a change in the marginal investor.

In the model, the main mechanism that generates a positive correlation is the output loss Γ_t during sovereign default episodes. Note that the main benefit of default is the future tax relief that comes with lower debt levels, but the timing of the model implies that this relief happens only the period after default, as tax rates depend on the debt level at the start of the period. This timing assumption has the convenience of guaranteeing that the correlation between future bond returns and consumption is positive in all points of the state space. Given the large (but still conservative) size of the output loss during default, only for extreme points of the state space will relaxing the timing assumption generate a conditional negative correlation. The data also suggests that consumption does not tend to increase in the year of default. For example, Mendoza and Yue (2012) show that consumption hits its trough during default years.

We can then define the implied spread Spr_t, as simply the excess return on bonds over deposits, conditional on no default. This variable can be compared with implied market yields, which are also the return conditional on no default.

$$Spr_t = 1/q_t^B - 1/q_t^D \tag{27}$$

The impact on the spread due to the change in the marginal investor is also amplified due to a feedback mechanism that goes from yields to default probabilities. From the government's budget constraint (10), we can see that a lower bond price will lead to a higher amount B_t to repay next period. This implies the probability of default at the beginning of next period is also now higher, as the government is now closer to its fiscal limit. There is then a feedback from higher yields, leading to higher probability of default, which in turn again raises yields. The introduction of a risk premium is therefore accompanied by higher default risk and the impact on the q_t^B is potentially strong.

Moreover, there is also a dynamic Laffer curve effect. The higher B_t, the higher is the expected tax rate at $t+1$. This leads to lower expected future consumption, leading to lower deposit rates/higher q_t^D. Equation (27) shows clearly that a rising q_t^D accompanied by falling bond prices q_t^B both contribute to widen the spread. Higher debt and lower output will then both play a role in the rise of debt-to-GDP ratios.

Finally, there is a dynamic effect that comes from debt accumulation. Debt accumulation has a persistent effect, since for a given set of exogenous state variables (A_t, G_t, s_t^Z, Z_t), the higher is B_t, the higher will be $\mathbb{E}_t(B_{t+1}|\Delta_{t+1}=0)$. So unless the country defaults, debt levels are persistent and accumulated debt today will have a cost in terms of higher future tax rates and also higher future default probabilities and yields.

3.7 Calibration

Since the model cannot be solved analytically, a numerical solution was used to analyze its properties. Given the nature of the occasionally binding constraint, the model is highly non-linear and so a global solution method was used. This method is described in more detail in Appendix C. The model's calibration is discussed in this section.

In order for the model to have a balanced growth path but also allow for the scaling of risk aversion, the preferences used for the felicity function are of the King-Plosser-Rebelo form.

$$u(C_t, L_t) = \frac{\left(C_t L_t^{\phi}\right)^{1-\gamma}}{1-\gamma} \tag{28}$$

Risk aversion γ was calibrated to 4, a value common in the literature, and ϕ was set so that leisure converges towards 0.6 in the stable regime.

Other parameters in the model were calibrated to the economy of Spain. The choice of Spain for calibration is due to the recent events surrounding the LTRO operation and the fall in yields associated with it described in Figure 1. By calibrating the model to Spain, the expected impact of that policy on bond

yields within the model can then be compared to the data. Table 2 shows the various parameter values used.

β was set to match Spain's average deposit rate. The productivity process was fitted to the Spanish TFP series calculated in the EU KLEMS database, while the process for government expenditures G was fitted to World Bank data on general government final consumption expenditures. $\overline{Z}$ was set to the mean social security spending as percentage of GDP since 1995. The probabilities of regime switching p_i^Z were calibrated following Bi (2012) to 2.5% in both regimes. Growth of transfers in the explosive regime was set to 2% a year, which was the average growth of Social Security transfers/GDP from 2002 to 2012 in Spain.

The target level of debt $\bar{B}$ was set such that the debt-to-GDP ratio equals to 60% when the economy remains deterministically in the stable regime. $\bar{\tau}$ is set such that the economy would remain at $\bar{B}$ in that case. Note that since the stochastic economy enters the unstable regime, the unconditional mean for B_t will in fact tend to be higher than $\bar{B}$. Bank equity $E_t = \overline{E}$ is set to match the MFI book equity over GDP in Spain since 1999. The shape parameters (ω_1, ω_2) of the Ω distribution of haircuts were estimated by fitting a Beta distribution to the haircut database in Cruces and Trebesch (2013). I use their preferred definition of haircuts, which is based on Sturzenegger and Zettelmeyer (2007).[14]

4 Heterogeneous investors and sovereign risk

We start from a situation when the government bond supply is absorbed by the banking sector and will look at the effect of a regime switching shock. As we can see from Figure 2, domestic financial institutions hold more than half of the total amount of bonds, of which 2/3 are held by Monetary and Financial Institutions (MFI). Banks thus play a key role as the main holders of sovereign bonds.

[14]In this definition, haircuts are computed by taking the difference between the present values of old and new instruments, discounting them at the post-restructure market rates.

Table 2: Calibration of selected parameters

Parameter	Value	Description
γ	4	Standard risk aversion value
ϕ	1.2183	match steady-state leisure at 0.6
β	0.973	match Spain's average deposit rate 2003 to 2012
ρ^a	0.817	Fitted from EU KLEMS data
σ_a	0.019	Fitted from EU KLEMS data
$\bar{G}$	18.45%	Government consumption spending (% of GDP): 1995-2012
ρ^G	0.952	Fitted from the data used for $\bar{G}$
σ_G	0.012	Fitted from the data used for $\bar{G}$
$\bar{Z}$	14.39%	Average social security funds (% of GDP): 1995-2012
$P^Z_{i,i}$	0.975	Bi (2012)
μ_z	1.02	Average growth in social security (% of GDP): 2002-2012
α^z	-0.45	Fitted using EU KLEMS data and social security funds
ω_1	0.991	$\Omega \sim Beta(\omega_1, \omega_2)$ fitted to Cruces and Trebesch (2013) data
ω_2	1.502	$\Omega \sim Beta(\omega_1, \omega_2)$ fitted to Cruces and Trebesch (2013) data
ξ	0.32	Change in tax burden per pp increase in debt-to-GDP
$\bar{B}/\bar{Y}$	60%	Target level of debt set to Stability and Growth Pact level
$\bar{E}/\bar{Y}$	23%	Match book equity over GDP of MFI in Spain 1999:2012

In the unstable regime, risk in sovereign credit rises and the Value-at-Risk constraint of banks becomes binding. The bank is no longer able to leverage and cannot absorb the newly emitted debt. The residual bond supply has to be held by households, and these will require a premium as the return on sovereign bonds is positively correlated with consumption. This will increase the spread both instantaneously and dynamically through larger debt accumulation.

Regime switching shock:

To highlight this mechanism, two different models are compared. In the first one, banks are subject to a Value-at-Risk constraint with $\alpha = 0.5\%$. This implies the banks must have a portfolio that has less than a 0.5% probability of wiping out their equity. The second model is one where banks are always unconstrained.

We show the effect of a regime switching shock that lasts for 10 periods. For 10 periods, the country is in the explosive regime and then reverts back to

the stable one. The exact length of the regime, however, is unknown to the agents of the economy so there is no assumption of perfect foresight. In the first model (with $\alpha = 0.5\%$), banks become constrained after the shock hits. In the second model, banks are always unconstrained. The difference is then the effect of a change in the marginal investor.[15].

The impulse response functions can be seen in Figure 4. The top 3 panels are very similar across the two models. The first panel plots the regime shock described above. In period 1 the transfer regime switches and remains unstable until period 11, when it reverts to the stable branch. In the second top panel, transfers increase by exactly the same amount and the small differences are due to the denominator. The third panel shows that the rise in the probability of default is almost the same on impact.

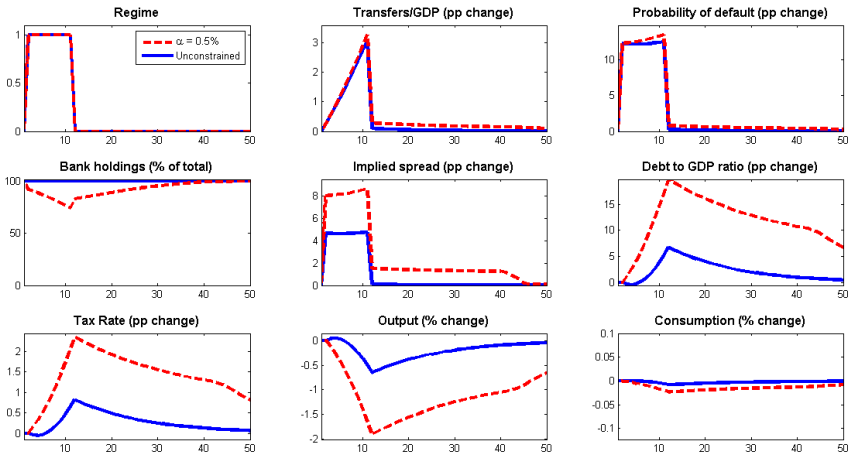


Figure 4: Impulse response of the baseline model to a 10 period regime switching shock

In the middle row, we can see in the first panel that banks become constrained when the shock hits. Their ability to leverage is reduced and they

[15] Alternatively, two models with the same α, but different levels of bank equity could have been compared. Note that it is always possible to find a level of equity that is sufficiently high, such that the regime shock described will not be enough to constrain the bank throughout the experiment

cannot absorb enough of the bond supply. Households become the new marginal investor and the impact on yields is quite significant. The implied spread increases by roughly 70% more than in the case where the banks remain unconstrained. On impact the spread increase in the case when banks do not become constrained is of 467 basis points, whilst in the case when the banking sector becomes constrained is of 801 basis points. As the effect on the probability of default is similar across models, this difference in yields comes from the risk attitudes of the new marginal investor. There can then be substantial debt repayment costs of having a constrained debt sector during a sovereign debt crisis. In the model the difference is of 334 basis points. For comparison purposes, during the LTRO period[16] the fall in yields for 1-year bonds was of 345 basis points so the model is able to match that relatively well. As will be detailed in a later section, the LTRO effect can be interpreted as policy that relaxed the constraint of banks and restored their role as marginal investors, providing relief to yields.

Naturally, this difference in yields leads to higher debt-to-GDP over time as higher interest accrues when the government rolls over its debt. This can be seen on the right panel of the middle row. The time that it takes for debt to return to its previous level is also quite long, and debt-to-GDP ratios are quite persistent. In the model with $\alpha = 0.5\%$, the half-life of the increase in debt-to-GDP due to the regime shock is around 13 years, while it is 11 years for the unconstrained model. Just before the shock reverses, faster increases in Debt-to-GDP ratios also lead to a probability of default that is about 1 percentage point higher (or 8% larger) in the constrained case.

In the bottom panel, we see that tax rates also increase during the unstable regime, as they are tightly linked to debt levels. So higher debt levels in the constrained model, lead to higher tax rates. Since taxes are distortionary, this also leads to lower output and consumption, although the latter features far more smoothing. Note that this output loss is the result of higher distortionary taxes due to the higher debt-to-GDP ratio.

[16]Specifically the difference from the 1st of December (before the December LTRO) to the 1st of March (just after the February LTRO).

5 Bank balance sheets and yield dynamics

In the baseline model, the balance sheet of banks is simplified to highlight the mechanism at work. In the current section, I extend the model to allow for other assets on the balance sheet of banks. This can be important as returns on other assets might also determine if the financial sector is constrained or not. During the crisis, many banks suffered severe losses with mortgage-backed securities and other toxic assets, leaving the banking sector in a more fragile situation which can have knock on effects on sovereign debt markets.

One additional asset F is introduced with total supply F_t. The asset's payout R_t^F is assumed to follow a lognormal distribution with mean $\overline{R}^F$ and volatility σ_R. To include these assets and keep the framework simple, sequential trading is introduced. Banks first invest in real assets, the return on those assets is realized, and then invest in sovereign bonds. Given that the bank is risk neutral when unconstrained[17], the price of these assets is $q_t^F = \frac{1}{\overline{R}^F}$. The balance sheet of the bank is:

Assets	Liabilities
$q_t^F F_t^B$	E_t
$q_t^B B_t^B$	$q_t^D D_t$
	$F_t^B R_t^F$

Where F_t^B are the asset holdings by banks, q_t^F their price and R_t^F the return on those assets. With sequential trading, during the second stage the model becomes equivalent to the baseline. Let $\tilde{E}_t$ be the available net worth after the return on the F assets

$$\tilde{E}_t \equiv E_t + F_t^B R_t^B - q_t^F F_t^B \tag{29}$$

The bank's balance sheet can be written in an analogous way as before with $\tilde{E}_t$ replacing E_t:

[17]The calibration is such that it is always the case that the bank is unconstrained in the first stage. Given $\alpha = 0.005$ the variance of the payout σ_R or asset supply F_t relative to equity E_t would need to be very large for it not to be the case.

Assets	Liabilities
$q_t^B B_t^B$	$\tilde{E}_t$
	$q_t^D D_t$

When the other assets perform poorly, then $\tilde{E}_t$ will be smaller. This will reduce the available net worth for the second stage, increasing the likelihood that the bank is constrained.

For the numerical experiments, F_t is fixed and set to 13.6 times the size of equity, which is the size of risk-weighted assets relative to Core Tier 1 Capital of Spanish banks in the 2011 EU-wide stress test data. σ_R is calibrated to the volatility of output σ_A. Bad and good returns are defined as two standard deviations differences from the unconditional mean. The impulse responses to a regime shock conditional on the asset returns can be seen in Figure 5[18].

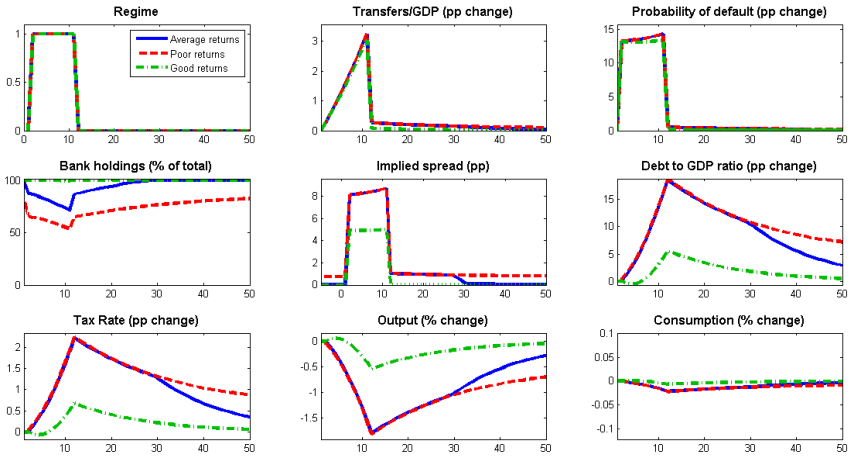


Figure 5: Impulse response to a regime switching shock conditional on asset returns

In the left panel of the top row, we see the regime switch which is common to all 3 cases. In the middle panel, bank holdings become constrained in the

[18]Since the starting yield is different across the three cases, impulse response functions are plotted as the difference to the case with no regime shock and average returns. This is done to highlight the pre-shock differences across the three models

case of low or average returns. If the returns on other assets are high, then it might be enough to increase the net worth of the bank to the point where it remains unconstrained after the regime switching shock. In that case, the implied yield plotted in the green line of the right panel[19], increases by far less than in the other two cases. Since in the case of low and average returns the household is the marginal investor once the shock hits, the implied yields are the same for the duration of the unsustainable regime. The main difference between these two cases are in what happens before and after. Before the regime shock, yields are already slightly higher when returns on other assets are low, as can be seen by the red line being above the other two. But since the probability of default is still low, the difference in spreads is not large. The intuition is that unless the probability of default is relatively high, then households will still be willing to buy debt at relatively low yields. It is then the combination of a fragile banking sector added to an uncertain sovereign debt situation that triggers the large amplification of yields due to the mechanism described.

Another important difference between these two cases, is that when returns are average the recovery can be faster relative to when they remain poor. Because banks become the marginal investor faster, at which point yields become lower. In the other graphs, we see an acceleration of the recovery in debt-to-GDP ratios at this point, with respect to when returns are low.

6 Moral Hazard

The presence of deposit insurance generates the possibility of moral hazard, distorting the willingness of banks to hold risky assets. There is a strand of the literature that builds on Kareken and Wallace (1978) and Dewatripont and Tirole (1994) which argues that the presence of government guarantees may

[19]The right panel in the top row depicts the change in implied spread relative to the case with average returns and no regime switching. This is done to highlight the difference in spreads even during the stable regime. For the same reason, the graph is extended on the left to show the periods before the shock hits.

induce excessive risk-taking, and regulation may be introduced to mitigate it[20].

Other authors also look at how these distortions and regulation interact over the cycle. Repullo and Suarez (2013) show that regulation of the Basel II form is too procyclical as capital requirements increase in bad times amplifying the need to deleverage. Malherbe (2015) extends this result to a more general setting and shows that systemic risk builds up during booms and therefore the tightness of capital requirements should be procyclical. Dewatripont and Tirole (2012) also show that incentives to *gamble for resurrection* are higher in bad times, leading to excessive risk-taking by the banking sector.

In the baseline model presented in the previous section, banks do not internalize the presence of an option value of default due to deposit insurance.[21]. Therefore, there is no significant risk-shifting and it would seem that banking regulations serve no role. In this section, I will show that the presence of the VaR constraint can serve to mitigate significantly the ability of banks to risk-shift when it is sufficiently tight.

When banks take into account the government guarantees, the distribution of returns in the maximization problem becomes truncated at the point where the bank defaults and the program can then be written as:

$$\max \mathbb{E}_t \left[\max(0, \Pi_{t+1})\right] \tag{30}$$

$$\text{s.t. } Prob\left(D_t > (1-\Delta_{t+1})B_t^B\right) \leq \alpha \tag{31}$$

Since the expectation of returns is now conditional on the bank not defaulting, it becomes trivial that the moral hazard is eliminated when $\alpha = 0$. If the bank never defaults, then the conditional expectation is the same as the unconditional.

[20]Jeanne and Korinek (2019, 2013), Hanson et al. (2011), Sandleris (2016), among others also show that pecuniary externalities are another important rationale for the presence of such regulation

[21]Note that any truncation of the profit function would lead to similar effects. When the probability of intermediary default is positive limited liability leads to similar risk-shifting motives, even without deposit insurance. One key difference is that in the case of deposit insurance, the price of deposits q_t^D is also not a function of balance sheet risk in the intermediary's maximization problem.

However, this is no longer true when $\alpha > 0$ zero and banks will be willing to pay a higher price for the sovereign bond due to the moral hazard. When unconstrained, the price banks are willing to pay is then:

$$q_t^{B,u} = q_t^D \left(1 - \mathbb{E}_t \left[\Delta_{t+1} | \Delta_{t+1} < \overline{\Delta}_t\right]\right) \tag{32}$$

$$\mathbb{E}_t \left[\Delta_{t+1} | \Delta_{t+1} < \overline{\Delta}_t\right] = \int_0^{\overline{\Delta}_t} \Delta dF(\Delta) \tag{33}$$

$$\tag{34}$$

where $\overline{\Delta}_t$ is the maximal level of haircut for which the bank doesn't default. As in equation (16), we can express it as a function of prices and Λ_t:

$$\overline{\Delta}_t = 1 - \frac{q_t^B}{q_t^D} \frac{\Lambda_t - 1}{\Lambda_t} \tag{35}$$

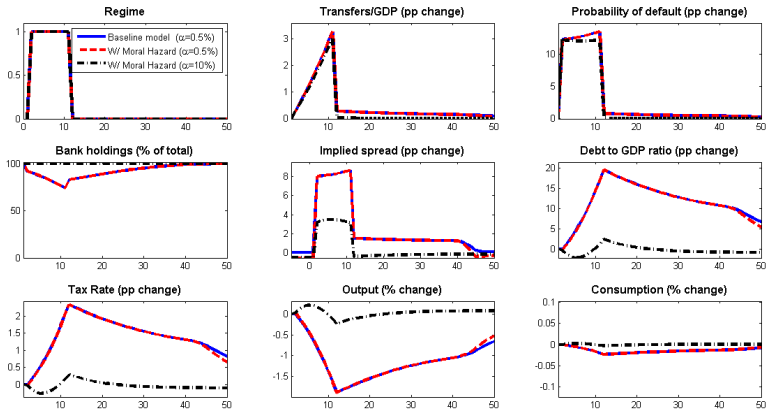


Figure 6: Impulse response to a regime switching shock in the presence of moral hazard

I compare two new models with moral hazard with the baseline one from section 4. The first new model will keep the Value-at-Risk tightness of the baseline ($\alpha = 0.5\%$), but now banks take advantage of moral hazard. The

second new model also considers moral hazard, but now in a situation where the VaR constraint is significantly looser ($\alpha = 10\%$). The remaining calibration is the same as in section 4. Figure 6 plots the impulse response functions. To highlight the differences across models, again impulse response functions are plotted as the difference to the baseline in the case without a regime shock.

As we can see from the panels, the differences between the baseline model (blue line) and the model with moral hazard and $\alpha = 0.5\%$ (red dashed line) are relatively small. Since debt-to-GDP and other state variables are set to be the same when the shock hits, the yields are the same during the period when the banking sector is constrained. Households are pricing the bonds, so moral hazard doesn't affect yields. The small differences are during the periods when the banking sector is unconstrained. Before the shock the yield is actually below the baseline as the banks are more willing to buy bonds due to the moral hazard. For the same reason, the recovery is also only marginally faster once the banks become unconstrained.

On the other hand, if the constraint is not very tight ($\alpha = 10\%$), banks are very willing to take additional risk on their balance sheet and the implied spread rises by much less. There is even a period where debt-to-GDP falls. This is because the implied yield actually falls (despite the spread rising) due to a fall in the risk free rate. The extent of risk-shifting is then much larger than the other cases and the spread rises by 319 basis points, which is less than half than it would in the absence of moral hazard (801 basis points). Note however, that the bank behaviour is now potentially extremely risky as $\alpha = 10\%$ and bank default is substantially more likely in this case.

Generally, bond prices will be increasing in α. As α increases, the maximal leverage rises. Since now the probability of the bank defaulting is higher, the haircut cutoff $\overline{\Delta}$ falls as even smaller haircuts are enough to wipe out equity. As the cutoff falls, so does the conditional expectation of haircuts $\mathbb{E}_t\left[\Delta_{t+1}|\Delta_{t+1} < \overline{\Delta}_t\right]$. The willingness to pay of banks will then be higher leading to a rise in the price q_t^B. Lower interest also triggers a fall in the probability of sovereign default, which reinforces the fall in yields. In short, the larger is the option value of default, the larger will be the willingness to pay.

A tight Value-at-Risk constraint seems then an effective way to limit risk-shifting within the banking sector. Since the probability of bank default is at most 0.5%, the expected return conditional on no bank default will only be marginally different from the unconditional return. Note that since the probability of bank default is smaller than α we have that:

$$\mathbb{E}_t \left[\Delta_{t+1} | \Delta_{t+1} < \overline{\Delta}_t \right] \geq \frac{\mathbb{E}_t(\Delta_{t+1}) - \alpha \mathbb{E}_t \left[\Delta_{t+1} | \Delta_{t+1} > \overline{\Delta}_t \right]}{1 - \alpha} \tag{36}$$

So as $\alpha \to 0$, the expression in equation (32) converges to equation (23) which is the one without moral hazard.

7 Unconventional Monetary Policy in the Euro Area

The theoretical framework sketched out in the previous sections can be used to assess the impact of unconventional monetary policy measures in the Euro Area. In this section I will look at the particular example of the recent Long-Term Refinancing Operations.

In the baseline model, the policy can be introduced by allowing the banks to access a fixed quantity of funding L_t at a lower rate q_t^{LTRO} than the current market deposit rate. Since the interest rate is lower, the same amount of funding will require a lower payment in the following period and therefore the probability of bank default becomes lower. This means that constrained banks can leverage more, allowing banks to increase asset purchases. With a combination of low LTRO interest rate $1/q_t^{LTRO}$ and amount of funding L_t, banks regain the status of marginal investor of sovereign debt, leading to a fall in yields.

The balance sheet of the bank will then be as follows:

Assets	Liabilities
$q_t^B B_t^B$	E_t
	$q_t^{LTRO} L_t$
	$q_t^D D_t$

To analyse this application of the model, I calibrate q_t^{LTRO} such that the LTRO interest rate is 198 basis points lower than the deposit rate at the time of the intervention[22]. The amount L_t is then picked to the minimum necessary to leave banks unconstrained, given the interest rate provided. Two types of interventions were analyzed and compared to the baseline (red dashed line). The first intervention (blue line) considers simple a one-off intervention the year following the shock. This is the intervention most similar in the model to the ECB's LTRO as the intervention was concentrated within a span of 3 months (December 2011 to February 2012). The second intervention (green dash-dotted line) lasts for the duration of the regime shock, also starting the period after the shock initially hits. The impulse response functions can be seen in Figure 7.

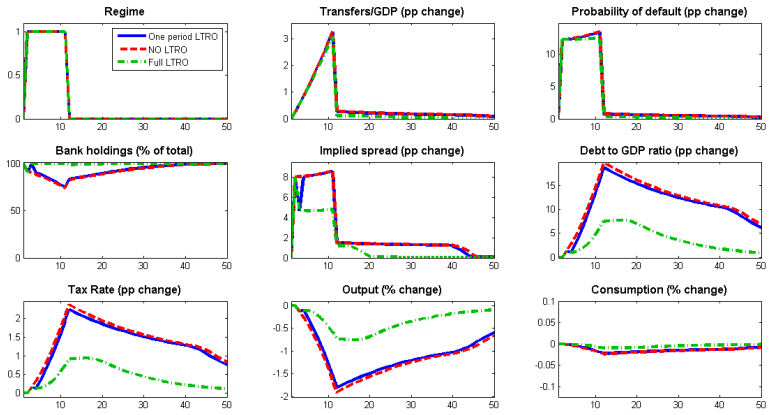


Figure 7: Impulse response to a regime switching shock in the presence of moral hazard

[22]In December 2011, the household deposit rate on deposits up to 1 year maturity was 273 basis points, while the LTRO rate was of 75 basis points.

The one-period intervention has very limited effects. In the middle panel of the second row, we see the fall in yields spreads is significant and of 335 basis points. But the effect is short lived and only has a significant impact during the period of intervention. This is very similar to the drop of 345 basis points in 1-year yields during the LTRO period[23]. The banks become the marginal investor briefly, but in the following year bonds are again priced by households and yields rise again. After intervention, yields remain lower than without it (red dashed line), but the difference is a paltry 7 basis points which is imperceptible in the graph. This difference is small because the short-lived intervention fails to stem the rise of debt-to-GDP ratios. Indeed, the effect on Spanish yields of the LTRO program was short-lived, and in August 2012, the ECB announced the possibility of direct purchases of bonds in the secondary market.

The second type of intervention is more successful. Banks remain unconstrained throughout the unstable regime and the lower yields lead to far lower debt accumulation. The debt-to-GDP ratio rises by less than a third than in the previous two cases. However, when the LTRO support is withdrawn total debt is still too large for banks to absorb and they remain constrained for another year after transfers become stable again. Importantly, the rise in debt-to-GDP ratios is only reversed once transfers are back to the stable regime. So although this intervention slows down debt accumulation significantly, it is not by itself able to solve the problem even when it is prolonged for 9 years, as is the case in the second intervention.

Although absent in the model, it is also possible that such a long intervention would generate moral hazard on the part of governments and delay the change to a sustainable fiscal policy. Since the path of debt-to-GDP only reverts when the regime switches again, it might be important to add conditionality to such interventions. Moreover, LTRO funding can also be seen as a form of subsidy to the banking sector. Although it might be justifiable in the short-run, long interventions may be less politically feasible.

The amplification mechanism described in this paper is a combination of

[23]From the 1st of December 2011 to the 1st of March 2012

the tightness of the constraint and the amount of debt in the market. In our regime switching shock, it tightens because the risk of the underlying asset goes up and the LTRO intervention relaxes it via the liability side. But the constraint could also tighten due to regulatory changes (e.g. a change in α) or because the supply of debt went up. Given that prior to the regime switching shock the banking sector is unconstrained, the same dynamics would occur if the shock was both a regime shock and a tightening of α to the calibrated level. The interpretation of this shock to α would be the introduction of the "sovereign risk shock" in the European stress tests. Moreover, the presence of foreign investors should help relieve the burden from the domestic banking sector as they also purchase some of the debt. The combination of net sales by foreign investors, introduction of stress tests and debt sustainability issues observed during the sovereign debt crisis all increase the likelihood that the constraint binds, leading to the amplification mechanism described.

8 Conclusion

This paper establishes a framework wherein banking sector capital matters for determining sovereign bond yields. Specifically, it focuses on the ability of banks to lever and how it may be impaired by constraints on balance sheet risk.

According to the baseline model, the increase in spread is much higher when the banking sector becomes constrained. The combination of unresolved fiscal problems with a constrained banking sector can lead to increases in spreads that are 70% higher than when the banking sector is well capitalized. Moreover, low returns on other assets exacerbate the problem and make the banking sector less able to deal with rising sovereign default risk.

The model is also able to rationalize the bond yield movements and changes in bond holdings that occurred around the LTRO period. As the European Central Bank injected large amounts of funds at low rates into the banking sector, this led to a burst of bond purchases by banks in troubled countries which were previously constrained. By the same token, bond purchases by

households and non-financial monetary institutions (such as pension funds) were the main net buyers of sovereign debt during the pre-LTRO period when banks were constrained and yields were rising.

Policies such as the LTRO also have short-lived effects on sovereign yields. Although they provide some borrowed time to troubled countries, to address the sovereign crisis they need to be combined with fiscal measures that bring government spending back to sustainability. The LTRO was not officially intended to solve the sovereign debt crisis, but it does seem to provide some respite, and in that sense it was a success. The model presented here shows that it works by dampening the amplification channel that comes from self-reinforcing rises in sovereign yields and balance sheet risk.

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Appendix A. Figures

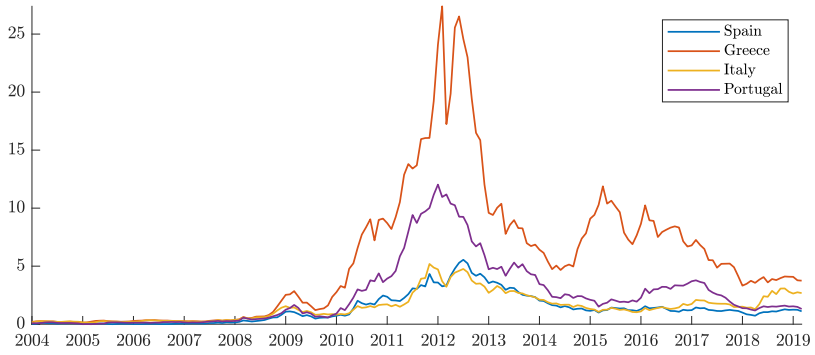


Figure 8: 10-year bond spreads over Germany for selected Eurozone countries. Source: European Central Bank

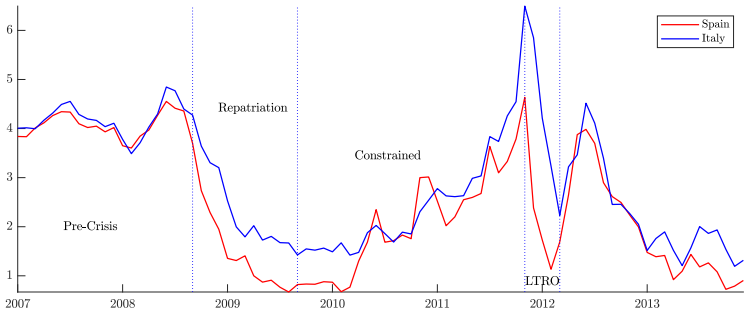


Figure 9: 1-year bond yields for Spain and Italy during the sovereign crisis. Source: European Central Bank

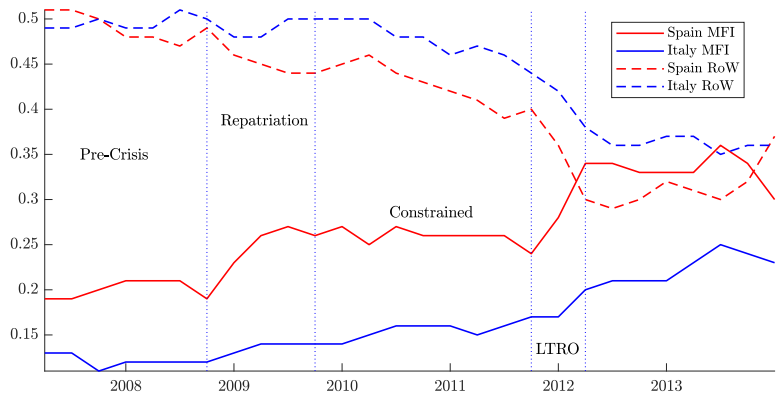


Figure 10: Spanish and Italian bond holdings by sector as percentage of total. Sector breakdown according to ESA95 classification. MFI: Monetary and Financial Institutions; ROW: Rest of the world. Source: Bruegel database of sovereign bond holdings

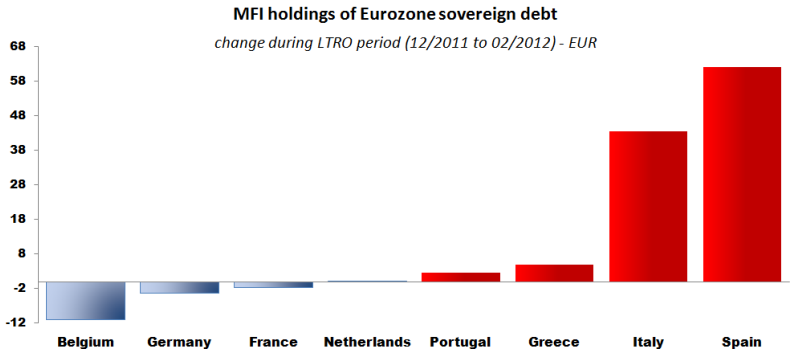


Figure 11: Change in MFI holdings of Eurozone sovereign debt over the three months from December 2011 to February 2012. Billions of Euro. Source: European Central Bank.

Appendix B. Solving for the fiscal limits

Here I provide a description of how to obtain the state-dependent fiscal limit distributions $\mathcal{B}$. The procedure is based on Bi (2012) and Bi and Leeper (2013). First note that, given the assumed preferences, the FOCs of the households only depend on the tax rate τ_t and the exogenous state variables A_t and G_t.

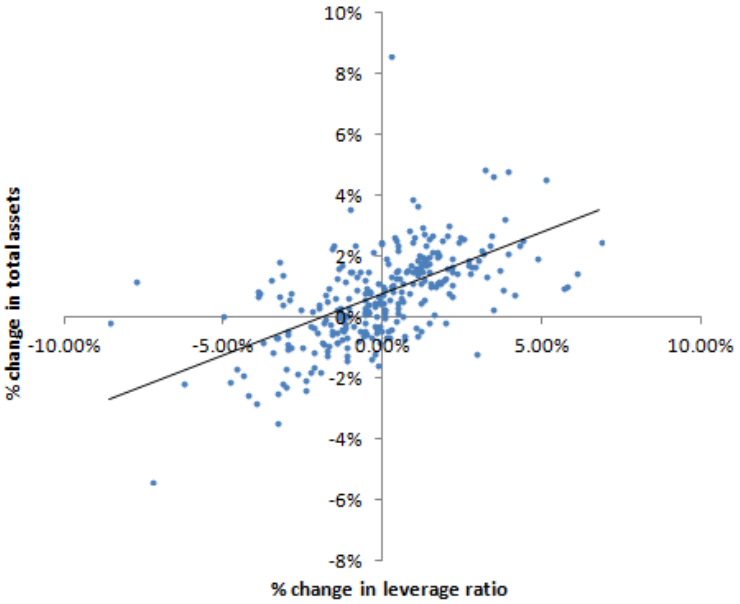


Figure 12: Leverage ratio and total asset changes for Spanish MFI. Monthly frequency 1989:12 to 2013:8. Correlation 59.8%. Source: Bank of Spain

Using King-Plosser-Rebelo preferences and the goods market clearing equation and the intratemporal FOC described in equation (19), we have:

$$L_t = \phi \frac{A_t - g_t}{A_t(1 - \tau_t + \phi)} \tag{37}$$

$$C_t = \frac{(1 - \tau_t)(A_t - g_t)}{1 - \tau_t + \phi} \tag{38}$$

Given the choice of labour and the production function, total tax is then

$$T_t = \tau_t A_t (1 - L_t) = \tau_t \frac{A_t(1 - \tau_t) + \phi g_t}{(1 - \tau_t + \phi)} \tag{39}$$

We can then find τ_t^{max} that maximizes the above expression for each pair (A_t, g_t). Equivalently, for each such pair we can find L_t^{max} and C_t^{max} by plugging in τ_t^{max} on equations (37) and (38). Since there is a unique mapping between states and τ_t^{max} we can then obtain the fiscal limit distributions by Markov Chain Monte Carlo simulation using 3 steps.

1. For each simulation i, draw the shocks (A_t,G_t,s_t^Z) for 200 periods conditional on the initial state (A_0, G_0, s_0^Z). Then compute the paths of all other variables using the corresponding $(\tau_t^{max}, L_t^{max}, C_t^{max})$ and the discounted sum of maximum fiscal surplus, defined as:

$$\mathcal{B}_i^*(A_0, G_0, s_0^Z) = \sum_{t=0}^{\infty} \beta^t \frac{u'_{C_t^{max}}}{u'_{C_0^{max}}} (\tau_t^{max}(A_t(1 - L_t^{max}) - G_t - Z_t) \qquad (40)$$

2. Repeat the simulation for 50000 times and obtain the conditional distribution of $\mathcal{B}^*(A_0, G_0, s_0^Z)$ using the simulated data.

3. Repeat the procedure for each point in the discretized state space

Note that the solution method of the model will already involve the discretization of the state space. See appendix C for more details. Finally, the resulting distributions were fitted to Generalized Extreme Value distributions. This simplifies the step for calculating Λ_t, as it requires inverting the cumulative distribution function (cdf). A brute force approach to invert the empirical cdf could have been used, but given the number of points used (50000) it would be very cumbersome. Results were checked using a non-parametric kernel approximation and there were no significant differences. Using a normal distribution, however, leads to a poor fit as the distributions are significantly skewed and feature fat tails.

Appendix C. Solution method for the non-linear model

The solution is obtained using a two-stage Euler Equation method for the non-linear model. First, note that the decision rules of the household in terms of C_t, L_t can be defined in terms of the state variables (B_{t-1}, A_t, G_t). Note also, that given the asset prices (q_t^D, q_t^B) and fiscal limit distribution $\mathcal{B}$, and future bond liabilities B_t, we can then solve the bank's problem which will determine the amount of bonds held by the bank B_t^B and deposits D_t. The main difficulty is that B_t depends on the asset prices and the asset prices will depend on B_t. So I will use an iterative procedure, where the asset prices (q_i^D, q_i^B) are taken

as given for each state i and solve for the corresponding B_t^i. With B_t^i, I tend solve for the corresponding asset prices (q_{i+1}^D, q_{i+1}^B) and iterate. The procedure entails the following steps

1. Discretize the state space for the variables (B, Z, A, G). The joint process for A and G is approximated using the procedure of Tauchen (1986) using 9 nodes. The state space for the state variable Z is discretized using 40 nodes and 200 for B.

2. Calculate the fiscal limit distributions $\mathcal{B}$ for each point in the state space.

3. Iterate on prices (q^D, q^B) and debt levels B_t.[24] Start with an initial guess $(q_i^D, q_i^B) = (q_0^D, q_0^B)$ for asset prices.

   (a) Solve for B_t^i using (q_i^D, q_i^B), the government's budget constraint (10) and plugging in tax rates and the optimal decisions of households.

   (b) Given B_t^i, calculate $(q_{i+1}^{B,c}, q_{i+1}^{B,u}, q_{i+1}^D)$ using equations (22),(24) and (20).

   (c) Given probabilities of default, set $q_{i+1}^B = q_i^{B,j}$ depending on whether the bank is constrained $(j = c)$ or not $(j = u)$.

   (d) Check for convergence. If $||(q_{i+1}^D, q_{i+1}^B) - (q_i^D, q_i^B)||$ is smaller than a threshold value stop. Else, go back to (a) and repeat.

Appendix D. Microfounding the Value-at-Risk

In this appendix, I report how the Value-at-Risk constraint can be microfounded under a contracting framework with moral hazard. This is explained in greater detail in Adrian and Shin (2014). The setup has a principal and an agent. The agent buys assets at date 0 and receives payoffs and repays creditors at date 1. The agent has some initial equity E and chooses the size of its balance sheet. Assets are funded in a collateralized borrowing arrangement. The agent

[24]Note that given the fiscal limit distributions, the default probability conditional on the exogenous states only depends on B_t. Tax rates τ_t also depend only on the debt level at the start of the period B_{t-1}, which is part of the state space.

sells the assets A for price D at date 0 and agrees to repurchase assets at data 1 for price $\overline{D}$. We then have the balance sheet relation $A = D + E$. The notional value of securities is $(1 + \overline{r})A$. The agent can invest in two assets (A_H, A_L), with densities $(f_H(), f_L())$ and expected payoffs (r_H, r_L) where $r_H < r_L$. The key assumption is that A_L has higher upside risk. Formally, there is one unique $z*$ such that $F_H(z*) = F_L(z*)$ and

$$(F_H(z) - F_L(z))(z - z*) \geq 0$$

for all z. Define $\pi_H(\overline{D}, A)$ as the price of the put option with strike price $\overline{D}$ on the portfolio of securities A_H whose current value is A. Let $\Psi \equiv \overline{D}/A$ be the ratio of promised repurchase price at date 1 to the market value of the agent's assets at date 0. Then in a competitive market we have

$$\pi_H(\overline{D}, A) = \pi_H(\Psi, 1) \equiv \pi_H(\Psi)$$

where $\Psi \equiv \overline{D}/A$. We can define $\pi_L(\Psi)$ analogously. The gross expected payoff of the creditor when the assets are good is

$$\overline{D} - A\pi_H(\Psi) = A(\Psi - \pi_H(\Psi))$$

Given the creditor's stake $\overline{D}$, its participation constraint implies that the net expected payoff must be positive

$$A(\Psi - d - \pi_H(\Psi) \geq 0$$

where $d = D/A$. On the other hand, the payoff of the borrower is the difference between the net payoffs as a whole to the creditor's net payoffs.

$$A(r_i - \Psi + d + \pi_i(\Psi))$$

with $i = \{H, L\}$. The incentive compatibility constraint is then

$$r_H - r_L \geq \pi_L(\Psi) - \pi_H(\Psi) \equiv \Delta\pi(\Psi)$$

Using the results in option pricing of Breeden and Litzenberger (1978), and given the properties of F_L and F_H, then the constraint always binds and there is an unique solution Ψ^* to the incentive compatibility constraint. This is because F_H cuts F_L precisely once from below at $z*$ and this is exactly the point that maximizes $\Delta\pi(z)$. This implies that there is a unique leverage $\lambda^* = \frac{1}{1-\Psi^*}$ that solves the incentive compatibility constraint. We can then calculate the market value of debt d^*

$$d^* = \Psi^* - \pi_H(\Psi^*)$$

The intuition is that the creditor imposes a leverage constraint to avoid risk-shifting. This leverage constraint has an implied probability of default α^*. Adrian and Shin (2014) show that when F_H and F_L are Generalized Extreme Value distributions

$$F_H(z) = \exp\left\{-\left(1+\xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}$$
$$F_L(z) = \exp\left\{-\left(1+\xi\left(\frac{z-\theta-k}{m\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}$$

then α^* is invariant to changes in the location parameter θ. So as θ varies in the business cycle, the optimal leverage constraint would be such that α^* is constant[25], which would then be equivalent to a Value-at-Risk constraint with parameter α^*.

Appendix E. Data

Data on 10-year yields for Germany, Spain, Greece and Portugal is from the European Central Bank (Table 11.15 of the Statistics Pocket Book). From the Bank of Spain, data was collected for Spanish 1-year bond yields (Table TI.1.3). Also data on bond holdings by sector (Financial Accounts: Tables 2.29 to 2.36)) and deposit rates with less than 1 year maturity (Statistical

[25]For a formal proof of this step, see Adrian and Shin (2014)

Bulletin: Table 19.9). Finally, data on MFI capital and reserves was used to calibrate book equity as percentage of GDP (Statistical Bulletin: Table 6.2). Government expenditure is taken from the World Bank's data on General Government Final Consumption Expenditure as percentage of GDP (indicator code: NE.CON.GOVT.ZS). For the calibration of transfers, Eurostat data on Social Security Funds as percentage of GDP was taken (code gov_a_main,sub-sector social security funds). Finally, data on haircuts is taken from the database constructed by Cruces and Trebesch (2013) and can be found at `https://sites.google.com/site/christophtrebesch`.