Knowledge Diffusion, Trade and Innovation across Countries and Sectors

Jie Cai, Nan Li and Ana Maria Santacreu*

October 2, 2017

Abstract

We develop and quantify a multi-country and multi-sector endogenous growth model in which technology evolves through innovation and knowledge diffusion. We quantify the effect of a trade liberalization on innovation, comparative advantage and welfare in a framework that features intersectoral production and knowledge linkages that are consistent with the data. A reduction in trade frictions induces a reallocation of innovation and comparative advantage across sectors. Knowledge spillovers amplify the effect of a trade liberalization as countries and sectors benefit from foreign technology. In contrast to standard one sector models of trade and innovation, we find significant dynamic gains from trade. These gains are mainly driven by a reallocation of innovation towards sectors with lager comparative advantage and larger knowledge spillovers.

Keywords: Technology Diffusion; R&D; Patent Citations; International Trade

JEL Classification: F12, O33, O41, O47

*Cai, Shanghai University of Finance and Economics, cai.jie@mail.shufe.edu.cn; Li, International Monetary Fund, address: 700 19th Street NW, Washington DC 20431, email: nanli1@gmail.com; Santacreu, Federal Reserve Bank of St. Louis, email: am.santacreu@gmail.com. We are grateful to Jonathan Eaton, Dan Trefler, Lorenzo Caliendo, Fernando Parro and participants of the seminars at the Econometric Society Summer Meeting, IMF, Saint Louis Fed, Penn State University, RIDGE, Fudan University, Tsinghua University for helpful comments. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of St. Louis, or the Federal Reserve System. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

1 Introduction

The world has increasingly become a highly interconnected network of countries and sectors which not only trade goods and services between each other, but at the same time, exchange ideas with one another. Recently, a growing strand of the trade literature has examined how the benefits of trade liberalization may spread across sectors through heterogeneous production input-output linkages (Caliendo and Parro 2015). However, sectors are also linked along a different dimension—innovation and knowledge diffusion. Indeed, technological advances never happen in isolation (David 1990; Rosenberg 1982). Knowledge in one sector can be used to enhance innovation in another, and much alike the cross-sector production input-output linkages, knowledge diffusion across sectors is far from uniform. Therefore, in a world with multiple sectors, when changes in trade costs alter the knowledge composition of the economy, the latter also conditions trade patterns and aggregate growth (as shown in the empirical research by Hausmann, Hwang, and Rodrik 2007; Hidalgo, Klinger, Barabási, and Hausmann 2007). Furthermore, although trade flows often serve as a vehicle for knowledge diffusion (Alvarez, Buera and Lucas, 2014), there may be other channels of diffusion of ideas across countries and sectors. The literature so far has either treated these two as separate issues or has modeled them together as one channel (e.g. more trade necessarily implies more knowledge spillovers).

We develop a multi-sector and multi-country endogenous growth model in which technology evolves endogenously through innovation and knowledge diffusion to study the effect of changes in trade costs on innovation, comparative advantage and welfare. Knowledge diffusion occurs when a firm in an industry learns about ideas that have been developed in other countries and industries. We assume this learning to be exogenous. The production side of the model is a multi-sector version of on Eaton and Kortum 2002, as the one developed in Caliendo and Parro 2015. Different from those papers, which assume that technology is exogenous, we introduce dynamics through an endogenous process of innovation and and exogenous process of knowledge diffusion. In contrast to models that do not feature knowledge spillovers, our model delivers convergence in relative productivity at the sector level, which has been documented by several empirical studies (see Levchenko and Zhang 2016, Hausmann and Klinger 2007, Cameron, Proudman, and Redding 2005, Proudman and Redding 2000, Bernard and Jones 1996a, and Bernard and Jones 1996b.) In a Ricardian framework, this evolution of sectoral productivity changes comparative advantage and welfare. Moreover, knowledge spillovers amplify the effect of trade liberalizations on innovation and welfare, as countries and industries have access to a larger pool of ideas (Keller 2004).

Our model has implications for welfare gains from trade that differ from both static models of trade and dynamic one-sector models of trade and innovation. Relative to static models, the endogenous evolution of comparative advantage provides an additional source of welfare gains. Changes in trade costs cause changes in innovation which, through knowledge flows, spread across countries and generates changes in revealed comparative advantage, hence welfare. The effect of trade on innovation is driven by the heterogeneous multi-sector dimension of our model. Standard models of trade and innovation that do not account for heterogeneity in production, innovation,

and knowledge flows find a negligible impact of trade on innovation and welfare (Atkeson and Burstein 2010, Eaton and Kortum 1996 and Eaton and Kortum 1999). Accounting for these sources of heterogeneity across sectors is important to study the effect of openness on the reallocation of innovation and production across sectors, and ultimately on the cross-country distribution of welfare gains from trade. Our model generates dynamic gains from trade, which are larger in countries that reallocate more innovation towards sectors in which comparative advantage increases the most. Two recent papers that also emphasize the importance of this heterogeneity are Somale 2014 and Sampson 2016. Somale 2014 studies a multi-sector semi-endogenous model of trade and innovation without knowledge spillovers. He focuses on the role of innovation to generate endogenous comparative advantage. Different from his paper, we introduce knowledge spillovers in our model as an additional source of comparative advantage. Furthermore, we use data on R&D at the sector level to discipline the innovation process of the model, so that we can capture explicitly the effect of trade on innovation. With our methodology, we are able to identify what sectors in a country experience increases in R&D after a trade liberalization, and we link these results to welfare gains from trade. Sampson 2016 develops a theoretical Armington framework of innovation and learning as sources of endogenous comparative advantage. Different from his paper, our emphasis is on the quantification of the model, which allows us to do counterfactuals. Furthermore, his analysis is done assuming free trade, whereas we are interested in analyzing the effect of change sin trade costs on innovation and welfare.

Despite of its complexity, the model comes with the benefit of tractability, as we build upon the Ricardian trade model of Eaton and Kortum 2002 with Bertrand Competition (Bernard, Eaton, Jensen, and Kortum 2003). The innovation and international technology diffusion processes are modeled in a similar fashion as in Eaton and Kortum 1996 and Eaton and Kortum 1999. Technology evolves over time through two channels: (i) innovators in each sector invest final output to introduce with a new idea which, if successful, can be used to produce an intermediate good. The efficiency of innovation in a country-sector increases with the stock of knowledge of that particular sector-country; (ii) ideas diffuse both across sectors and countries according to an exogenous process of diffusion. The novel feature of this model is that sectors and countries are connected not only through trade in intermediate goods but also through knowledge diffusion.

The model is solved in two stages. Given the distribution of firm's productivity together with trade barriers, we solve for a static competitive equilibrium for the world economy. The equilibrium is static in that we take as given the technology level that determines the patterns of trade. We then allow for the technology profile to evolve endogenously due to a process of innovation and diffusion. The second stage allows us to determine the characteristics of the innovation process that drives the endogenous evolution of comparative advantage and dynamic welfare gains from trade. A similar approach has been used in Alvarez, Buera, and Lucas Jr 2008. Different form their paper, our diffusion channel produces a Frechet distribution of productivity, as in Eaton and Kortum 1999. Furthermore, Alvarez, Buera, and Lucas Jr 2008 abstract from innovation, which is a key channel in our model. Buera and Oberfield 2016 study a model with technology diffusion

and innovation that delivers the same Frechet distribution. However, different from the predictions of our model, trade has no impact on innovation in their framework, as they consider a one-sector model.

We calibrate the model to cross-country and cross-sector data on patent citations, R&D intensity and international trade. The production structure of the model delivers a gravity equation at the sector level that can be estimated to obtain both trade barriers and the level of technology for every sector-country pair, as in Levchenko and Zhang 2016 . Cross-country and cross-sector patent citations allow us to discipline the direction and speed at which knowledge in a particular sector-country is utilized in the innovation of other sectors in other countries. We assume that the diffusion lags have an exponential distribution. Finally, data on R&D intensity at the sector-country level allow us to calibrate the parameters that govern the evolution of technology through innovation.

We conduct a counterfactual exercise to study the effect of a trade liberalization on innovation, comparative advantage and welfare. In contrast to one-sector models of trade and innovation, changes in trade frictions have non-negligible effect on innovation, as there is a reallocation of R&D toward sectors in which the country has comparative advantage.[1] Knowledge diffusion is an additional source of technological progress in our paper, and it is important to explain convergence in relative productivity found in the data or to explain growth miracles as in Buera and Oberfield 2016. We find that this reallocation of R&D is more important in sectors with stronger knowledge spillovers. Our quantitative framework has implications for welfare gains from trade. A trade liberalization strengthens a country's comparative advantage, and hence the static gains from trade. In addition, there are dynamic gains from trade due to higher R&D investment in those sectors in which the country has comparative advantage. Moreover, knowledge diffusion has two opposite effects on welfare. On the one hand, it enables faster productivity convergence and makes countries more similar to each other, which dampens the gains from trade. On the other hand, it provides strong dynamic gains, because countries can innovate with access to a larger foreign knowledge pool. In our quantitative exercise, the second channel dominates and knowledge spillovers introduce additional gains from trade. Finally, we study the role of different sources of sectoral heterogeneity on innovation, comparative advantage and welfare. We find that, after a trade liberalization: (i) Larger countries experience lower gains from trade; (ii) R&D reallocates towards sectors that experience larger increases in revealed comparative advantage; this is especially the case in those countries that have larger gains from trade; (iii) an increase in innovation translates into an increase in the growth rate and income per capita on the BGP. As a result, we find significant dynamic gains from trade, which are heterogeneous across countries. Dynamic gains are larger in those countries that experience larger increases in innovation (iv) In a world in which we do not allow for knowledge spillovers, welfare gains from trade are smaller, especially for smaller countries that spend less in R&D, as they have more to gain from flows of ideas. In the extreme case of instantaneous knowledge diffusion, welfare gains from trade are larger. However, the difference in gains between our baseline

[1]These result has been found in Somale 2014 in a semi-endogenous model of growth with multiple sectors and no knowledge diffusion. Eaton and Kortum 1996 and Eaton and Kortum 1999 , Buera and Oberfield 2016, and Atkeson and Burstein 2010 find tha trade has a negligible impact on innovation.

model and that with instantaneous diffusion is much smaller than the difference between the baseline model and that without knowledge diffusion, suggesting that the world is closer to instantaneous diffusion than to zero diffusion; and (vi) A one-sector version of our model generates negligible dynamic gains.

Related Literature Our paper merges and extends several strands of existing literature. First, the literature on innovation, diffusion and international trade. Eaton and Kortum 1996 and Eaton and Kortum 1999 posit technological innovations and their international diffusion through trade as potential channels of embodied technological progress. Santacreu 2015 develops a model in which trade allows countries to adopt innovations that have been developed abroad, and thus diffusion does not take place without trade. Our main departure from these previous papers is that we allow knowledge diffusion and trade to operate separately, even though common economic forces may contribute to the development of both and diffusion and trade may benefit and reinforce each other. In addition, we extend these studies into a multi-sector environment in which sectors interact both in the product space and in the technology space.

The second strand of literature extends Eaton and Kortum 2002 to a multi-sector model of trade (Chor 2010; Costinot, Donaldson, and Komunjer 2012). A recent growing body of research in this area also explores the trade and growth implications of interdependence across different sectors through intermediate input-output relationships (Eaton, Kortum, Neiman, and Romalis 2016, Caliendo and Parro 2015). Our paper differs in several dimensions. First, our focus is on innovation and knowledge diffusion. Second, besides the factor demand linkages, this paper also simultaneously consider the intrinsic interconnections of technologies embodied in different sectors, which turns out to be significant and relevant when studying innovation and diffusion. Related to the current work, Cai and Li 2016 study knowledge spillovers across sectors within a country and how trade costs affect the distribution of endogenous knowledge accumulation across sectors. Different from our paper, however, cross-sector knowledge diffusion is not considered across countries and intermediate input demand linkages across sectors are absent. Perla, Tonetti, and Waugh 2015 study the effect of trade on growth in a symmetric country model in which firms learn from existing knowledge by other firms. Levchenko and Zhang 2016 provide evidence of relative productivity convergence across 72 countries over 5 decades: productivity grew systematically faster in initially relatively less productive sectors. These changes have had a significant impact on trade volumes and patterns, and a modest negative welfare impact.

Led by Hidalgo, Klinger, Barabási, and Hausmann 2007, several papers have shown that producing goods with strong synergy with each other can improve growth, as it is easier to adapt existing ideas and enter new sectors (e.g. Hausmann and Klinger 2007, Kali, Reyes, McGee, and Shirrell 2013, Hausmann and Klinger 2007). However, these studies mostly adopt the regression based approach which is hard to establish causality and to examine the general equilibrium implications of changing trade structure. Moreover, none of these studies consider at the same time the product complementarity along the intermediate input-output dimension.

2 The Model

We develop a general equilibrium model of trade in intermediate goods, with sector heterogeneity and input-output linkages, in which technology evolves endogenously through innovation and knowledge diffusion. The model builds upon the Ricardian trade model of Eaton and Kortum 2002 with Bertrand Competition (Bernard, Eaton, Jensen, and Kortum 2003). The innovation and diffusion processes are modeled as Eaton and Kortum 1996 and Eaton and Kortum 1999.

There are M countries and J sectors. Countries are denoted by i and n and sectors are denoted by j and k. Labor is the only factor of production and we assume it to be mobile across sectors within a country but immobile across countries. In each country, there is a representative consumer who consumes a non-traded final good and saves. A perfectly competitive final producer in the country combines the composite output of each J sectors in the domestic economy with a Cobb-Douglas production function. In each sector there is a producer of a composite good that operates under perfect competition and sells the good to the final producer and to intermediate producers from all sectors in that country. Intermediate producers use labor and composite goods of each other sector in that country to produce varieties that are traded and are used by the composite producer of that sector, either domestic or foreign. These firms operate under Bertrand competition and are heterogeneous in their productivity. Trade is Ricardian. Finally, the technology of each sector evolves endogenously through innovation and technology diffusion. The innovation process follows the quality-ladders literature in that new innovations increase the quality of the product in a given sector. Diffusion is assumed to be exogenous. Foreign firms that decide to use a domestic innovation pay royalties to the innovator. We assume that royalty payments are perfectly enforced.

2.1 Consumers

In each country there is a representative household who chooses consumption optimally to maximize her life-time utility

$$U_{nt} = \int_{t=0}^{\infty} \rho^t u\left(C_{nt}\right) dt, \tag{1}$$

where $\rho \in (0,1)$ is the discount factor, and C_{nt} represents consumption of country n at time t. The household finances R&D activities of the entrepreneurs and own all the firms.

We assume that households' preferences are represented by a CRRA utility function

$$u\left(C_{nt}\right) = \frac{C_{nt}^{1-\gamma}}{1-\gamma}$$

with an intertemporal elasticity of substitution, $\gamma > 0$.

2.2 Final production

Domestic final producers use the composite output from each domestic sector j in country n at time t, Y_{nt}^j, to produce a non-traded final output Y_{nt} according to the following Cobb-Douglas

production function

$$Y_{nt} = \prod_{j=1}^{J} \left(Y_{nt}^{j}\right)^{\alpha^j}, \tag{2}$$

with $\alpha^j \in (0,1)$ the share of sector production on total final output, and $\sum_{j=1}^{J} \alpha^j = 1$.

Final producers operate under perfect competition. Their profits are given by:

$$\Pi_{nt} = P_{nt} Y_{nt} - \sum_{j=1}^{J} P_{nt}^{j} Y_{nt}^{j},$$

where P_{nt} is the price of the final product, and P_{nt}^{j} is the price of the composite good produced in sector j from country n.

Under perfect competition, the price charged by the final producer to the consumers is equal to the marginal cost, that is

$$P_{nt} = \prod_{j=1}^{J} \left(\frac{P_{nt}^{j}}{\alpha^j}\right)^{\alpha^j}.$$

The demand by final producers for the sector composite good is given by:

$$Y_{nt}^{j} = \alpha^j \frac{P_{nt}}{P_{nt}^{j}} Y_{nt}.$$

2.3 Intermediate producers

In each sector j there is a continuum of intermediate producers indexed by $\omega \in [0,1]$ that use labor, $l_{nt}^{j}(\omega)$, and a composite intermediate good from every other sector k in the country, $m_{nt}^{jk}(\omega)$ to produce a variety ω according to the following constant returns to scale technology[2]

$$q_{nt}^{j}(\omega) = z_{n}^{j}(\omega)[l_{nt}^{j}(\omega)]^{\gamma^j} \prod_{k=1}^{J} [m_{nt}^{jk}(\omega)]^{\gamma^{jk}}, \tag{3}$$

with $\gamma^j + \sum_{k=1}^{J} \gamma^{jk} = 1$. Here γ^{jk} is the share of materials from sector k used in the production of intermediate ω is sector j, and γ^j is the share of value added. Firms are heterogeneous in their productivity $z_{n}^{j}(\omega)$.

The cost of producing each intermediate good ω is

$$c_{nt}^{j}(\omega) = \frac{c_{nt}^{j}}{z_{nt}^{j}(\omega)},$$

where c_{n}^{j} denotes the cost of input bundle. With constant returns to scale,

[2]The notation in the paper is such that every time there are two subscripts or two superscripts, the right one corresponds to the source country and the left one corresponds to the destination country.

$$c_n^j = \Upsilon^j W_{nt}^{\gamma^j} \prod_{k=1}^{J} (P_n^k)^{\gamma^{jk}},$$

with $\Upsilon^j = \prod_{k=1}^{J} (\gamma^{jk})^{-\gamma^{jk}} (\gamma^j)^{-\gamma^j}$ and W_{nt} the nominal wage rate.

2.4 Composite intermediate goods (Materials)

Each sector j produces a composite good combining domestic and foreign varieties from that sector. Composite producers operate under perfect competition and buy intermediate products ω from the minimum cost supplier.

The production for a composite good in sector j and country n is given by the Ethier 1982 CES function,

$$Q_{nt}^j = \left(\int r_{nt}^j(\omega)^{1-1/\sigma} d\omega \right)^{\sigma/(\sigma-1)}, \tag{4}$$

where $\sigma > 0$ is the elasticity of substitution across intermediate goods, and $r_{nt}^j(\omega)$ is the demand of intermediate goods from the lowest cost supplier in sector j.

The demand for each intermediate good ω is given by

$$r_{nt}^j(\omega) = \left(\frac{p_{nt}^j(\omega)}{P_{nt}^j} \right)^{-\sigma} Q_{nt}^j,$$

where

$$P_{nt}^j = \left(\int p_{nt}^j(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \tag{5}$$

Composite intermediate goods are used as final goods in the final production and as materials for the production of the intermediate goods.

$$Q_{nt}^j = Y_{nt}^j + \sum_{k=1}^{J} \int m_{nt}^{kj}(\omega) d\omega.$$

2.5 International trade

Trade in goods is costly. In particular, there are iceberg transport costs from shipping a good that is produced in sector j from country i to country n, $d_{ni}^j > 1$. We follow Bernard, Eaton, Jensen, and Kortum 2003 and assume Bertrand competition.

The p'th most efficient producer of variety ω from sector j in country i has productivity z_{pi}^j and can deliver a unit of good to country n at the cost:

$$c_{pni}^j(\omega) = d_{ni}^j \frac{c_i^j}{z_{pi}^j(\omega)},$$

With Bertrand competition, as with perfect competition, composite producers in each sector

buy from the lowest cost supplier. The lowest cost of good ω in country n is given by

$$c_{1n}^j(\omega) = min_i \left\{ c_{1ni}^j(\omega) \right\},$$

In addition, Bertrand competition implies that the price charged by the producer will be the production cost of the second lowest producer

$$c_{2n}^j(\omega) = min \left\{ c_{2ni*}^j(\omega), min_{i \neq i^*} \{ c_{1ni}^j(\omega) \} \right\},$$

where i^* satisfies $c_{1ni*}^j(\omega) = c_{1n}^j(\omega)$. The low cost supplier will not want to charge a mark-up above $\bar{m} = \sigma/(\sigma - 1)$. Hence,

$$p_n^j(\omega) = min \left\{ c_{2n}^j(\omega), \bar{m} c_{1n}^j(\omega) \right\}.$$

Ricardian motives for trade are introduced as in Eaton and Kortum 2002, since productivity is allowed to vary by sector and country. The productivity of producing intermediate good ω in country i and sector j is drawn from a Frechet distribution with parameter T_i^j and shape parameter θ. A higher T_i^j implies a higher average productivity of that sector-country, while a lower θ implies more dispersion of productivity across varieties.

$$F(z_i^j) = Pr \left[Z \leq z_i^j \right] = e^{-T_{it}^j z^{-\theta}},$$

and,

$$Pr \left[p_{ni,t}^j < p \right] = 1 - e^{-T_{it}^j \left(d_{ni}^j c_{it}^j / p \right)^{-\theta}}.$$

Because each sector j in country n buys goods from the second cheapest supplier, the cost of producing good ω in sector j and country n is $p_{nt}^j(\omega) = min \left\{ p_{nit}^j(\omega) \right\}$. Then, $c_{nt}^j(\omega)$ are realizations from

$$G_n^j(p) = 1 - \prod_{i=1}^{M} \left(Pr \left[p_{nit}^j > p \right] \right) = 1 - \prod_{i=1}^{M} e^{-T_{it}^j \left(d_{ni}^j c_{it}^j / p \right)^{-\theta}} = 1 - e^{-\Phi_{nt}^j p}$$

with $\Phi_{nt}^j = \sum_{i=1}^{M} T_{it}^j (d_{ni}^j c_{it}^j)^{-\theta}$ each country n and sector j accumulated technology. From here, we can obtain the distribution of prices of goods in sector j in country n as

$$P_{nt}^j = B \left(\Phi_{nt}^j \right)^{-1/\theta}, \tag{6}$$

with $B = \left[\frac{1+\theta-\sigma+(\sigma-1)(\bar{m})^{-\theta}}{1+\theta-\sigma} \Gamma \left(\frac{2\theta+1-\sigma}{\theta} \right) \right]^{1/(1-\sigma)}$. For prices to be well defined, we assume $\sigma < (1+\theta)$.[3]

We assume that the average productivity of each sector j in country i, T_{it}^j is driven by two components: (i) a time-varying component, A_{it}^j, which reflects the stock of knowledge of country i in sector j at time t. We refer to this component as "knowledge-related productivity", and it reflects

[3] Details of these derivations can be found in Bernard, Eaton, Jensen, and Kortum 2003.

the part of productivity that is driven by innovation and diffusion of foreign innovations through knowledge spillovers, as we describe in more detail in Section 2.7; (ii) a time-invariant component, $T^j_{i,p}$, which captures the part of productivity that is not explained by innovation or diffusion. Factor endowments, multinational production, or human capital could be factors embodied in this component.

We make the following assumption[4]

$$T^j_{it} = A^j_{it} T^j_{p,i}. \tag{7}$$

Therefore, the dynamics of the level of technology T^j_{it} are driven by the dynamics of the "knowledge-related productivity", A^j_{it}.

2.6 Expenditure shares

The probability that country i is the lowest cost supplier of a good in sector j that is to be exported to country n is

$$\pi^j_{nit} = \frac{T^j_{it}\left(c^j_{it} d^j_{ni}\right)^{-\theta}}{\Phi^j_{nt}}, \tag{8}$$

π^j_{nit} is also the fraction of goods that sector j in country i sells to any sector in country n. In particular, the share country n spends on sector j products from country i is

$$\pi^j_{nit} = \frac{X^j_{nit}}{X^j_{nt}}. \tag{9}$$

2.7 Endogenous growth: Innovation and international technology diffusion

We model the innovation process within each industry in a country as in Kortum 1997. Innovation follows the quality-ladders literature, in that a blueprint (i.e., an idea) is needed to produce an intermediate good. Ideas are developed with effort and they increase the efficiency of production of an intermediate good. In each sector j and country n, there are entrepreneurs that invest final output to come up with an idea. Within each sector, research efforts are targeted at any of the continuum of intermediate goods. In each country n and sector j, ideas are drawn at a Poissson rate: If a fraction of final output, s^j_{nt}, is invested into R&D by the entrepreneur, then ideas are created at the rate

$$\lambda^j_{nt}\left(s^j_{nt}\right)^{\beta_r} \tag{10}$$

with $\lambda^j_{nt} = \lambda^j_n A^j_{nt}$, λ^j_n a scaling parameter that captures the efficiency of innovation in sector j in country n, A^j_{nt} is the stock of knowledge of sector j in country n, and $\beta_r \in (0,1)$ a parameter of diminishing returns to investing into R&D. This process has been microfounded in Eaton and Kortum 1996 and Eaton and Kortum 1999 and it ensures that there is a balanced growth path

[4]This formulation is similar to the one introduced in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple 2013.

without scale effects. Note that λ_{nt}^{j} depends positively of the stock of knowledge, A_{nt}^{j}, that is, countries that have accumulated more knowledge have a higher efficiency of innovation.

Once an idea has arrived in sector j and country n there is no forgetting. New ideas created in each sector j and country n increase its average productivity, A_{nt}^{j}.

Ideas may also diffuse exogenously to other sectors and countries. Through diffusion, the stock of knowledge in each sector j and country n is composed of knowledge that has been developed by each sector k in country i. An idea discovered at time t in country i and sector k diffuses to country n and sector j at time $t+\tau_{ni}^{jk}$. We assume that the diffusion lag, τ_{ni}^{jk}, has an exponential distribution with parameter ε_{ni}^{jk} as the speed of diffusion, so that $Pr[\tau_{ni}^{jk} \leq x] = 1 - e^{-\varepsilon_{ni}^{jk} x}$. Therefore, the flow of ideas diffusing to country n and sector j is given by

$$\dot{A}_{nt}^{j} = \sum_{i=1}^{M} \sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \lambda_{is}^{k} \left(s_{is}^{k} \right)^{\beta_r} ds, \tag{11}$$

Therefore, the growth of the stock of knowledge in a particular sector j and country n at time t depends on the past research effort that has been done by each other sector k in country i up to time t, and that has diffused at the rate ε_{ni}^{jk}. If $\varepsilon_{ni}^{jk} \to \infty$, then there is instantaneous diffusion. If $\varepsilon_{ni}^{jk} \to 0$, then there is no diffusion.

2.7.1 The incentives to innovate

Following the quality-ladders literature, an idea is the realization of two random variables. One is the good ω to which the idea applies. An idea applies to only one good in the continuum. The good ω to which it is associated is drawn from the uniform distribution $[0, 1]$. The other is the quality of the idea, $q^{j}(\omega)$, which is drawn from the Pareto distribution $H(q) = 1 - q^{-\theta}$. In equilibrium, only the best idea for each input is actually used to produce an intermediate good in any sector and country. In that case, the idea can be used to produce an intermediate product ω in sector j and country n with efficiency $z_{n}^{j}(\omega)$. Therefore, the efficient technology $z_{n}^{j}(\omega)$ for producing good ω in country n is the best idea for producing it yet discovered. This modeling choice follows Eaton and Kortum 2006, with a few modifications.

The stock of ideas at each point in time in each sector j and country n is A_{nt}^{j}. Because there is a unit interval of intermediate goods, the number of ideas for producing a specific good is Poisson with parameter A_{nt}^{j}. This Poisson arrival implies that the probability of k ideas for producing a good by date t in sector j and country n is $\left(A_{nt}^{j}\right)^{k} e^{-A_{nt}^{j}}/k!$. If there are k ideas, the probability that the best one is below the best quality q is $[H(q)]^{k}$. Summing over all possible k, $F(q) = e^{-A_{nt}^{j} q^{-\theta}}$. Therefore, the quality distribution of successful ideas inherits the distribution of productivity of the intermediate goods produced in a country. Our probabilitic distribution assumptions for the quality of an idea imply that the probability of an idea being successful is $1/A_{nt}^{j}$. This introduces a competitive effect, by which the larger the stock of knowledge in a sector-country, the lower the probability that the new idea lowers the cost there.

Entrepreneurs finance R&D issuing equity claims to the households. These claims pay nothing if the entrepreneur is not successful in introducing a new technology in the market, and it pays the stream of future profits if the innovation succeeds. The value of a successful innovation in a particular sector is the expected flow of profits that will last until a new producer is able to produce the good at a lower cost. Because of the probabilistic distribution of productivity, entrepreneurs will be indifferent on what product ω to devote its efforts, since in expectation, all products within a sector deliver the same expected profit.

The profits of an innovator in sector j in country n have two components. First, the expected profit from selling the product in that sector an country,

$$\frac{1}{A_{nt}^{j}}\frac{\sum_{i=1}^{M}\pi_{int}^{j}X_{it}^{j}}{1+\theta}, \tag{12}$$

Second, the innovator gets royalties from those technologies that have diffused to other countries and sectors and that have been used to produce an intermediate product there. We assume that royalty payments are perfectly enforced and are proportional to the profits that successful intermediate good producers in other sectors and countries obtain from using that technology. The expected royalty payment to an entrepreneur in country n and sector j from a technology that has been diffused and adopted by a producer in sector k of country i is

$$\chi_{in,t}^{kj}\frac{\Pi_{it}^{k}}{A_{it}^{k}}=\chi_{in,t}^{kj}\frac{1}{A_{it}^{k}}\frac{\sum_{m=1}^{M}X_{mt}^{k}\pi_{mit}^{k}}{1+\theta}, \tag{13}$$

where $\chi_{in,t}^{kj}$ is the fraction of technologies developed by entrepreneurs from from sector j in country n that are used by sector k in country i. Note that $\chi_{nn,t}^{jj}=1$.

Hence, the value of an idea that has been developed in country n and sector j is

$$V_{nt}^{j}=\sum_{i=1}^{M}\sum_{k=1}^{J}\int_{t}^{\infty}\left(\frac{P_{it}^{k}}{P_{is}^{k}}\right)e^{-\rho(s-t)}\chi_{in,s}^{kj}\frac{\Pi_{is}^{k}}{A_{is}^{k}}ds, \tag{14}$$

and the first order condition for optimal R&D is

$$\beta_{r}\lambda_{nt}^{j}V_{nt}^{j}\left(s_{nt}^{j}\right)^{\beta_{r}-1}=P_{nt}Y_{nt}, \tag{15}$$

Therefore, the optimal R&D investment is a positive function of the value of an innovation, V_{n}^{j} and the efficiency of innovation λ_{n}^{j} as[5]

[5]The optimization problem of the innovator is as follows. Innovators choose the amount of final output to be allocated into R&D. In our model, s_{n}^{j} is the fraction of final output that is spent into R&D activity. Therefore, innovators choose $S_{n}^{j}=s_{n}^{j}Y_{n}$ to maximize

$$\dot{A}_{n}^{j}V_{n}^{j}-P_{n}S_{n}^{j}$$

subject to equation (11).

$$s_{nt}^{j} = \left(\beta_r \lambda_{nt}^{j} \frac{V_{nt}^{j}}{P_{nt} Y_{nt}} \right)^{\frac{1}{1-\beta_r}}. \tag{16}$$

2.8 Balance of payments

The current account balance equals the trade balance plus the net foreign income derived from net royalty payments. Total imports in country n are given by:

$$IM_{nt} = \sum_{i=1}^{M} \sum_{k=1}^{J} X_{nit}^{k} = \sum_{k=1}^{J} X_{nt}^{k} \sum_{i=1}^{M} \pi_{nit}^{k}, \tag{17}$$

Total exports in country n are given by:

$$EX_{nt} = \sum_{i=1}^{M} \sum_{k=1}^{J} X_{int}^{k} = \sum_{i=1}^{M} \sum_{k=1}^{J} \pi_{int}^{k} X_{it}^{k}$$

Net royalty payments are given by

$$RP_{nt} = \sum_{j=1}^{J} RP_{nt}^{j}$$

and

$$RP_{nt}^{j} = \sum_{i=1}^{M} \sum_{k=1}^{J} \left(\chi_{in,t}^{kj} \Pi_{i}^{kt} - \chi_{ni,t}^{jk} \Pi_{nt}^{j} \right)$$

The balance of payments implies

$$EX_{nt} = IM_{nt} - RP_{nt}$$

3 Endogenous growth along the balanced growth path (BGP)

We now derive an expression for the growth rate of the economy on the BGP that can be used to understand the main mechanisms of the model. International and sector diffusion guarantee that the "knowledge-related productivity" A_{nt}^{j} (and from equation (7) also average productivity, T_{nt}^{j}) grows at a common rate, g_A, across all countries and sectors. We normalize all the endogenous variables so that they are constant on the BGP, and denote the normalized variables with a hat.

Because we are normalizing all the variables so that they are constant on the BGP, we remove time subscripts in our derivation. From the resource constraint in equation (65), the fraction of final output that is invested into R&D, s_n^j, is constant on the BGP. This result, together with the expression for the value of an innovation, implies that

$$\hat{V}_n^j = \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\chi_{in}^{kj}}{\rho - g_y + g_A} \frac{\hat{\Pi}_i^k}{\hat{A}_i^k}$$

with $\hat{V}_n^j = \frac{V_n^j A_M^J}{W_M}$, $\hat{A}_i^k = \frac{A_i^k}{A_M^J}$, and $\hat{\Pi}_i^k = \frac{\Pi_i^k}{W_M}$; χ_{in}^{kj} is the fraction of profits that a firm in sector k, country i pays to the innovator in sector j, country n as royalties. We impose $\rho - g_y + g_A > 0$ and we derive and expression for g_y in Appendix E.

Profits are given by,

$$\hat{\Pi}_n^j = \frac{\sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{(1+\theta)}$$

We can show that, on the BGP, the fraction of profits paid by producers from sector k in country i, that use technologies from sector j in country n is

$$\chi_{in}^{kj} = \frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}}$$

Royalty payments are given by the fraction of technologies that having been diffused from (n,j) to (i,k) are used to produce intermediate goods in (i,k). The fraction of technologies from (n,j) that have diffused to (i,k) is $\varepsilon_{in}^{kj} \int e^{-\varepsilon_{in}^{kj}(t-s)} \dot{A}_{ns}^j ds / A_{nt}^j$. On the BGP, this is equal to $\frac{\varepsilon_{in}^{kj}}{\varepsilon_{in}^{kj}+g}$.

We can now use the expression for the value of an innovation, equation (14), together with the optimal investment in R&D, equation (16), to obtain

$$\hat{V}_n^j = \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}} \frac{1}{(1+\theta)} \frac{\sum_{m=1}^{M} \hat{X}_m^k \hat{\pi}_{mi}^k}{A_{it}^k} \frac{1}{\rho - g_y + g_A}$$

Therefore, the optimal R&D investment of sector j in country n is

$$s_n^j = \left(\beta_r \lambda_n^j \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}} \frac{1}{(1+\theta)} \frac{\hat{A}_n^j}{\hat{A}_i^k} \frac{\sum_{m=1}^{M} \hat{X}_m^k \hat{\pi}_{mi}^k}{\hat{Y}_n} \frac{1}{\rho - g_y + g_A} \right) \tag{18}$$

To gain some intuition on why trade has an effect on R&D, let's assume that there are no royalties, that is $\chi_{in}^{kj} = 0$. Then,

$$s_n^j = \left(\beta_r \lambda_n^j \frac{1}{(1+\theta)} \frac{1}{\rho - g_y + g_A} \frac{\sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \right)^{\frac{1}{1-\beta_r}} \tag{19}$$

with $\hat{X}_i^j = \frac{X_i^j}{W_M}$ and $\hat{Y}_n = \frac{P_n Y_n}{W_M}$. Trade affects optimal investment into R&D at the sector level to the extent that it affects the reallocation of production into particular sectors. This result differs from previous papers in the literature that find that trade has no impact on R&D intensity. In our model, R&D reallocates towards sectors in which the country has comparative advantage, through $\frac{\sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n}$. In section F, we show how in the one sector version of our model without royalties, changes in trade costs have no effect on innovation, hence on the growth rate, even when we allow

for knowledge spillovers.

Substituting equation (19) into the growth rate of stock of knowledge in

$$g_A = \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(s_i^k\right)^{\beta_r}$$

$$g_A = \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(\frac{1}{\rho - g_y + g_A} \beta_r \lambda_i^k \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}}$$

Rearranging, we obtain an expression for the growth rate of the stock of knowledge on the BGP,

$$g_A \hat{A}_n^j = \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} (\lambda_i^k)^{\frac{1}{1-\beta_r}} \hat{A}_i^k \left(\frac{1}{\rho - g_y + g_A} \beta_r \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}} \tag{20}$$

The growth rate of the stock of knowledge on the BGP depends positively on the speed of diffusion, the expected profits, and the efficiency of innovation, and it depends negatively on the dispersion parameter. Following Eaton and Kortum 1999, the Frobenius theorem guarantees that there is a unique balanced growth path in which all countries and sectors grow at the same rate g_A. The expression for the growth rate can be expressed in matrix form as:

$$g_A A = \Delta(g_A) A$$

If the matrix $\Delta(g_A)$ is definite positive, then there exists a unique positive balanced growth rate of technology $g_A > 0$ given research intensities and diffusion parameters. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country-sector pair relative level of knowledge along that balanced growth path.

In Appendix B, we report the equations of the model after normalizing the endogenous variables.

4 Welfare Gains from Trade

We compute welfare gains from trade after a trade liberalization between the baseline and the counterfactual BGP. Welfare in our model is defined in equivalent units of consumption. We can use equation (1) to obtain the lifetime utility in the initial BGP as

$$\bar{U}_i^* = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{1-\gamma} e^{g^*(1-\gamma)t} dt = \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{\rho - g^*(1-\gamma)}$$

and in the counterfactual BGP as

$$\bar{U}_i^{**} = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{1-\gamma} e^{g^{**}(1-\gamma)t} dt = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}$$

with $*$ denoting the baseline BGP, and $**$ denoting the counterfactual BGP.

Welfare gains are defined as the amount of consumption that the consumer is willing to give up in the counterfactual BGP to remain the same as in the initial BGP. We call this, λ_i, which is obtained as:

$$\bar{U}_i^*(\lambda_i) = \bar{U}_i^{**}$$

$$\frac{\left(\hat{C}_i^* \lambda_i\right)^{1-\gamma}}{\rho - g^*(1-\gamma)} = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}$$

From here,

$$\lambda_i = \frac{\hat{C}_i^{**}}{\hat{C}_i^*} \left(\frac{\rho - g^*(1-\gamma)}{\rho - g^{**}(1-\gamma)} \right)^{\frac{1}{1-\gamma}} \tag{21}$$

Welfare gains depend on changes in normalized consumption between the BGPs and the change in growth rates. From equation (65), normalized consumption in the BGP is equal to income per capita net of R&D expenditures. That is,

$$\hat{C}_i = \hat{Y}_i - \sum_{k=1}^{J} s_i^k \hat{Y}_i = \left(1 - \sum_{k=1}^{J} s_i^k \right) \hat{Y}_i \tag{22}$$

In static models or one-sector models of trade and innovation in which changes in trade costs do not have an effect on innovation, $g^* = g^{**}$ and $s_i^k = 0$. In that case, welfare gains from trade are computed as changes in the real wage. As in Caliendo and Parro 2015, we can obtain an expression for the real wage in country i as

$$\frac{W_i}{P_i} \propto \prod_{j=1}^{M} \left(\frac{W_i}{P_i^j} \right)^{\alpha^j}$$

Using the first order conditions for prices and import shares, it can be shown that

$$\frac{W_i}{P_i^j} = \left(\frac{T_i^j}{\pi_{ii}^j} \right)^{1/\theta} \frac{W_i}{c_i^j} \propto \left(\frac{T_i^j}{\pi_{ii}^j} \right)^{1/\theta} \prod_{k=1}^{J} \left(\frac{W_i}{P_i^k} \right)^{\gamma^{jk}}$$

Therefore,

$$\frac{W_i}{P_i} \propto \prod_{j=1}^{J} \left(\left(\frac{T_i^j}{\pi_{ii}^j} \right)^{\alpha^j/\theta} \prod_{k=1}^{J} \left(\frac{W_i}{P_i^k} \right)^{\alpha_i^j \gamma^{jk}} \right) \tag{23}$$

Note that this formula resembles the standard welfare formula in Arkolakis, Costinot, and Rodríguez-Clare 2012. In a one-sector version of our model, in which $j = 1$ and, γ^{jk}=0, $\alpha^j = 1$, equation (23) becomes

$$\frac{W_i}{P_i} \propto \left(\frac{T_i}{\pi_{ii}}\right)^{1/\theta} \tag{24}$$

This is the standard formula for welfare gains from trade that has been used in the literature and it depends on aggregate productivity, the home trade shares and the trade elasticity.

Our formula for real wages in equation (23) is dynamic. Dynamics are driven by the evolution of the stock of ideas captures in T_i^j. In this sense, our formula is the multi-sector version of the one derived in Buera and Oberfield 2016.

5 Quantitative Analysis

We quantify our model to evaluate the role that sector heterogeneity in production and knowledge flows has on innovation and welfare. We study the effect of a trade liberalization that consists of a uniform reduction of trade barriers of 40%. We compare the economy in the baseline and counterfactual BGPs. We consider four versions of our model: (i) our baseline model with heterogeneity in innovation, production and knowledge linkages; (ii) a model with sector heterogeneity but where diffusion is almost negligible; and (iii) a one sector model with knowledge flows across countries. In all cases, we recalibrate the parameters of the model to match the same moments of the data.

5.1 Calibration

We use data on bilateral trade flows, R&D intensity, production, and patent citations to calibrate the main parameters of the model. We assume that the world is on a BGP in 2005. Here we explain in more detail the calibration of the average productivity parameters T_i^j, the diffusion parameters ε_{in}^{jk}, and the parameters governing the innovation process—the elasticity of innovation, β_r, and the efficiency of innovation, λ_i^j. Details on the data used in the calibration are relegated to Appendix B, and the description of the calibration procedure to recover other parameters of interest is provided in Appendix C.

5.1.1 Estimation of T_i^j: Gravity equation at the sector level

To estimate the technology parameters for tradable sectors, $j \leq J-1$, we follow the procedure in Levchenko and Zhang 2016 by estimating standard gravity equations for each sector in 2005. We start from the trade shares in equation (9):

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j \left(c_i^j d_{ni}^j\right)^{-\theta}}{\Phi_n^j}. \tag{25}$$

Dividing the trade shares by their domestic counterpart as in Eaton and Kortum 2002 and, assuming $d_{nn}^j = 1$, we have

$$\frac{\pi_{ni}^j}{\pi_{nn}^j} = \frac{X_{ni}^j}{X_{nn}^j} = \frac{T_i^j \left(c_i^j d_{ni}^j\right)^{-\theta}}{T_n^j \left(c_n^j\right)^{-\theta}}. \tag{26}$$

Taking logs of both hand sides, we have

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right) - \theta \log(d_{ni}^j). \tag{27}$$

The log of the trade frictions can be expressed as

$$\log(d_{ni}^j) = D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + ex_i^j + \nu_{ni}^j \tag{28}$$

Following Eaton and Kortum 2002, $D_{ni,k}^j$ is the contribution to trade costs of the distance between country n and i falling into the k^{th} interval (in miles), defined as [0,350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, maximum). The other control variables include common border effect, B_{ni}, common currency effect CU_{ni}, and regional trade agreement RTA_{ni}, between country n and country i. We include an exporter fixed effect, ex_i^j, to fit the patterns in both country incomes and observed price levels as shown in Waugh (2010). ν_{ni}^j is the error term.

Substituting (28) back into (27) results in the following gravity equation at the sector level:

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \theta ex_i^j - \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right) - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \tag{29}$$

Define $\hat{F}_i^j = \log\left(T_i^j \left(c_i^j\right)^{-\theta^j}\right) - \theta ex_i^j$ and $F_n^j = \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right)$. We then estimate the following equation using fixed effects and observables related to trade barriers, taking θ as known.

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \hat{F}_i^j - F_n^j - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \tag{30}$$

Figure 1 plots the distance parameters that we obtain from the sectoral gravity equations, d_{in}^j, against the trade share from the data that we use to estimate the gravity equations at the sector level, using $\theta = 8.28$.

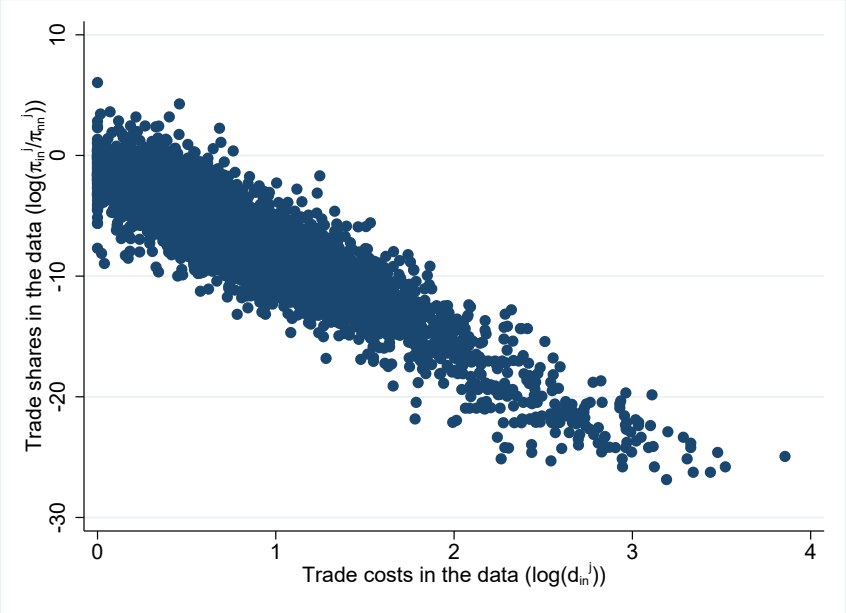


Figure 1: Trade shares and distance

The productivity of the tradable sector in country n relative to that in U.S., T_n^j/T_{US}^j, is then recovered from the estimated importer fixed effects as in

$$S_n^j = \frac{\exp(F_n^j)}{\exp(F_{US}^j)} = \frac{T_n^j}{T_{US}^j}\left(\frac{c_n^j}{c_{US}^j}\right)^{-\theta} \tag{31}$$

in which the relative cost component can be computed by expressing (58) as

$$\frac{c_n^j}{c_{us}^j} = \left(\frac{W_n}{W_{US}}\right)^{\gamma^j}\prod_{k=1}^{J-1}\left(\frac{P_n^k}{P_{US}^k}\right)^{\gamma^{jk}}\left(\frac{P_n^J}{P_{US}^J}\right)^{\gamma^{jJ}}, \tag{32}$$

where J indicates the nontradable sector. Using data on wages (in USD), estimates of price levels in the tradable sector and the nontradable sector relative to the United States, we can back up the relative cost. The nontradable relative price is obtained using the detailed consumer price data collected by the International Comparison Program (ICP). To compute the relative price of the tradable sector, we follow the approach of Shikher (2012) by combining (56), (57) and (59) and get the following expression for relative prices of tradable goods

$$\frac{P_n^j}{P_{US}^j} = \left(\frac{X_{nn}^j/X_n^j}{X_{US,US}^j/X_{US}^j}\frac{1}{S_n^j}\right)^{\frac{1}{\theta}}. \tag{33}$$

The right hand side of this expression can be estimated using the observed expenditure shares of domestic product in country n and in U.S. and the estimated importer fixed effects.

To compute the relative productivity in nontradable sectors, we combine (59), (60) and set the

trade cost in nontradable sector d_{ni}^J to infinity for all i and n. This implies $\Phi_n^J = T_n^J \left(c_n^J\right)^{-\theta}$ based on equation (60). Substituting this expression into (59), we express the nontradable good price as

$$p_n^J = \frac{c_n^J}{\left(T_n^J\right)^{1/\theta}}. \tag{34}$$

The relative technology in nontradable sector can then be constructed based on

$$\frac{T_n^J}{T_{US}^J} = \left(\frac{c_n^J}{c_{US}^J}\frac{P_{US}^J}{P_n^J}\right)^{\theta} \tag{35}$$

Again, the cost ratios are calculated following (32) and the price ratios for the non-tradable sectors are from the ICP database.

We now have estimated the relative productivity for all countries relative to U.S. in every sector. To estimate the level of productivity, we need the productivity level in the U.S. First, using OECD industry account data, we estimate the empirical sectoral productivity for each sector in the U.S. by the Solow residual (without capital in the production function)

$$\ln Z_{US}^j = \ln Y_{US}^j - \gamma^j \ln L_{US}^j - \sum_{k=1}^{J} \gamma^{jk} \ln M_{US}^{jk}, j = 1, 2, ..., J, \tag{36}$$

where Z_{US}^j denotes the measured productivity in U.S. in sector j, Y_{US}^j is the output, L_{US}^j is the labor input and M_{US}^{jk} is the intermediate input from sector k. Finicelli et al. (2013) show that trade and competition introduce selection in the productivity level, and the relationship between empirical productivity and the level of technology T_{US}^j in an open economy is given by

$$T_{US}^j = \left(Z_{US}^j\right)^{\theta} \left[1 + \sum_{i \neq US} S_i^j \left(d_{US,i}^j\right)^{-\theta}\right]^{-1}, \tag{37}$$

in which S_i^j and $d_{US,i}^j$ are estimated using (31) and (28) respectively. To obtain the exporter fixed effect in trade cost, ex_i^j, we use the importer and exporter fixed effects from the Gravity equation (30). That is, $ex_i^j = (F_i^j - \hat{F}_i^j)/\theta$. Lastly, we normalize the nontradable technology in the U.S. to one, and express all T_{US}^j relative to T_{US}^J as

$$\hat{T}_{US}^j = \left(\frac{Z_{US}^j}{Z_{US}^J}\right)^{\theta} \left[1 + \sum_{i \neq US} S_i^j \left(d_{US,i}^j\right)^{-\theta}\right]^{-1}. \tag{38}$$

Throughout our analysis we assume that θ is common across countries and set it equal to 8.28.[6]

[6]We have also run our gravity equation at the sector level using $\theta = 4$ and a sector specific θ from Caliendo and Parro 2015. We find that the technology parameters estimated under different θ are highly correlated, as it has been documented in Levchenko and Zhang 2016. In particular, the calibration of technology parameters for $\theta = 4$ and $\theta = 8.28$ is 0.98, whereas the correlation of the technology parameter when θ is common and when we use the θ from

We find that the cross-sector dispersion of the estimated relative productivity of country n relative to the United States is larger for countries that have a lower level of income per capita (see Figure 2). This result is consistent with Levchenko and Zhang 2016.

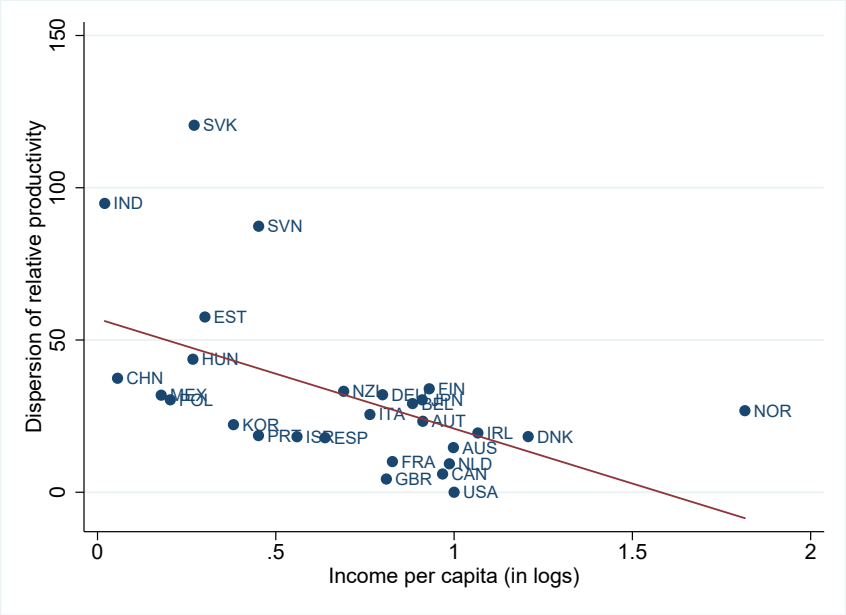


Figure 2: Dispersion of relative productivity and income per capita

5.1.2 The speed of knowledge diffusion

We discipline the speed of knowledge diffusion, ε_{ni}^{jk}, using citation data across countries and sectors obtained from the U.S. Patent and Trade Office (USPTO) for the period of 2000-2010. In the innovation literature, citation data have been used to trace the direction and intensity of knowledge flows between economic units (such as firms or countries) and across technological classes.[7] In the dataset, each patent is assigned to one of the 428 three-digit United States Patent Classification System (USPCS) technological fields (NClass) and belongs to one of the IPC (International patent clasificiation) categories. We use the probability mapping provided by USPTO to assign patents into different SIC categories, which are then mapped into one of our 19 sectors.[8]

Based on our model assumption of the exponential distribution of citation lags, we can express the share of total citations from country n sector j made in year t to patents applied in year s by

Caliendo and Parro 2015 is 0.8.

[7] Although patent statistics have been widely used in studies of firm innovations, not all innovations are patented, especially process innovations, which are often protected in other ways such as copyright, trademarks and secrecy (see Levin et al.,1987). Our measure implicitly assumes that for any sector, the unpatented and patented knowledge utilizes knowledge (patented or unpatented) from other sectors in the same manner, with the same likelihood and intensity.

[8] Details of the concordance are available at http://www.uspto.gov/web/offices/ac/ido/oeip/taf/data/sic_conc.

country i sector k as

$$\widehat{citeshare}_{ni,s}^{jk,t} \equiv \frac{citation_{ni,s}^{jk,t}}{\sum_{s=0}^{t}\sum_{ik} citation_{ni,s}^{jk,t}} = \frac{\varepsilon_{ni}^{jk} e^{-\varepsilon_{ni}^{jk}(t-s)} P_{i,s}^{k}}{S_n^{j,t}}, \tag{39}$$

where $P_{i,s}^{k}$ is the total number of patent applications in sector k of country i in year s, $S_n^{j,t} = \sum_{s=1}^{t}\sum_{i,k} \varepsilon_{ni}^{jk} e^{-\varepsilon_{ni}^{jk}(t-s)} P_{i,s}^{k}$ is the knowledge stock available to country n sector j at time t that was ever invented at $s \leq t$, and $citation_{ni,s}^{jk,t}$ is the number of citations from patents in nj,t to patents ik,s. $P_{i,s}^{k}$, similar to the term $\lambda_{is}^{k}\left(s_{is}^{k}\right)^{\beta_r}$ in Eqn.(11), measures the new knowledge generated in ik at time s, and $\varepsilon_{ni}^{jk} e^{-\varepsilon_{ni}^{jk}(t-s)}$ is the share of $P_{i,s}^{k}$ that arrives nj at time t.

In addition, we assume that $S_n^{j,t}$ is a stock variable that grows at a constant rate g_n^j, i.e. $S_n^{j,t} = S_n^{j,0} e^{t g_n^j}$. Suppose (t_0, s_0) is the base year-pair. Dividing both sides of (39) by the base year-pair observation leads to

$$\frac{\widehat{citeshare}_{ni,s}^{jk,t}}{\widehat{citeshare}_{ni,s_0}^{jk,t_0}} = \frac{\varepsilon_{ni}^{jk} e^{-\varepsilon_{ni}^{jk}(t-s)} P_{i,s}^{k}}{S_n^{j,0} e^{t g_n^j}} \Big/ \frac{\varepsilon_{ni}^{jk} e^{-\varepsilon_{ni}^{jk}(t_0-s_0)} P_{i,s_0}^{k}}{S_n^{j,0} e^{t_0 g_n^j}} = \frac{P_{i,s}^{k}}{P_{i,s_0}^{k}} e^{-\varepsilon_{ni}^{jk}[(t-s)-(t_0-s_0)]} e^{(t_0-t)g_n^j} \tag{40}$$

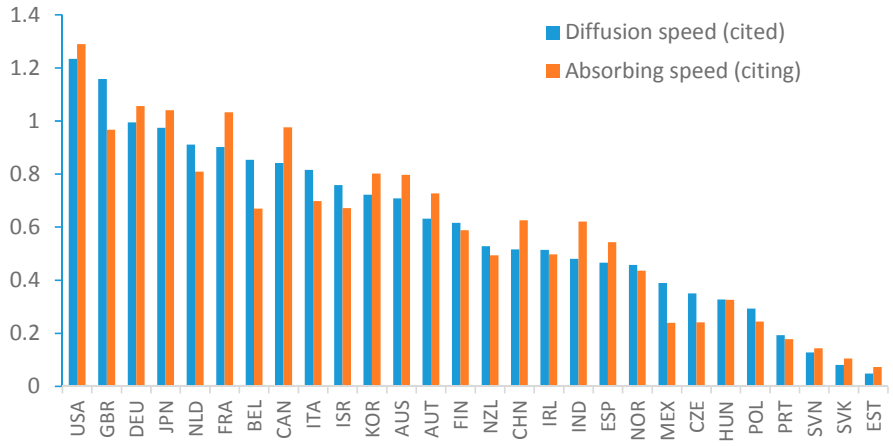
Parameters $\{\varepsilon_{ni}^{jk}, g_n^j\}$ are then estimated using General Moment of Method (GMM) by the quadratic distance between the empirical counterpart of the citation share and (40). For each (nj, ik) country-sector pair over T peroids, we have $T(T-1)/2$ observations of (t,s) year-pairs and 2 unknowns.

Figure 4 shows that the distribution of diffusion speed across countries and sectors is highly heterogeneous and skewed. In addition, Table 1 reports the average speed of diffusion by cited sector and citing sector. It shows that Chemicals, Computer, Electronic and Medical Instruments are the sectors that have the largest diffusion speed, while patents in Wood Products has the lowest diffusion speed. The citing speed (or speed of absorption) is highly correlated with the cited speed. Figure 3 shows the average speed of diffusion and absorption by country. Unsurprisingly, new knowledge in US, UK, Germany and Japan diffuse the fastest. The speed of diffusion of knowledge in US and UK on average diffuses in less than a year (captured by $1/\epsilon$). Countries which diffuse knowledge (get cited) rapidly also tend to acquire new knowledge from other countries (citing others) fast. Canada, France, and emerging innovation powerhouse like China and India are faster at acquiring new knowledge than diffusing its own knowledge.

Table 1: Average diffusion speed by sectors

ISIC	Industry	Cited	Citing
C24	Chemicals and chemical products	0.948	0.853
C30T33X	Computer, electronic and medical instruments	0.939	0.931
C01T05	Agriculture, hunting, forestry and fishing	0.932	0.912
C17T19	Textiles, textile products, leather and footwear	0.771	0.888
C29	Machinery and equipment, n.e.c.	0.752	0.850
C10T14	Mining and Quarrying	0.793	0.747
C28	Fabricated metal products, except machinery and equipment	0.667	0.736
C40T95	Nontradables	0.630	0.633
C21T22	Pulp, paper, paper products, printing and publishing	0.619	0.595
C15T16	Food products, beverages and tobacco	0.628	0.611
C25	Rubber and plastics products	0.570	0.537
C27	Basic metals	0.594	0.581
C23	Coke, refined petroleum products and nuclear fuel	0.478	0.506
C34	Motor vehicles, trailers and semi-trailers	0.406	0.377
C26	Other non-metallic mineral products	0.445	0.471
C31	Electrical machinery and apparatus, n.e.c.	0.473	0.467
C36T37	Manufacturing n.e.c. and recycling	0.336	0.319
C35	Other transport equipment	0.310	0.310
C20	Wood and products of wood and cork	0.175	0.144

Figure 3: Average speed of diffusion by country



Note: This figure presents the average diffusion speed and absorbing speed by country. Average diffusion (absorbing) speed is calculated as the average ϵ by cited (citing) country.

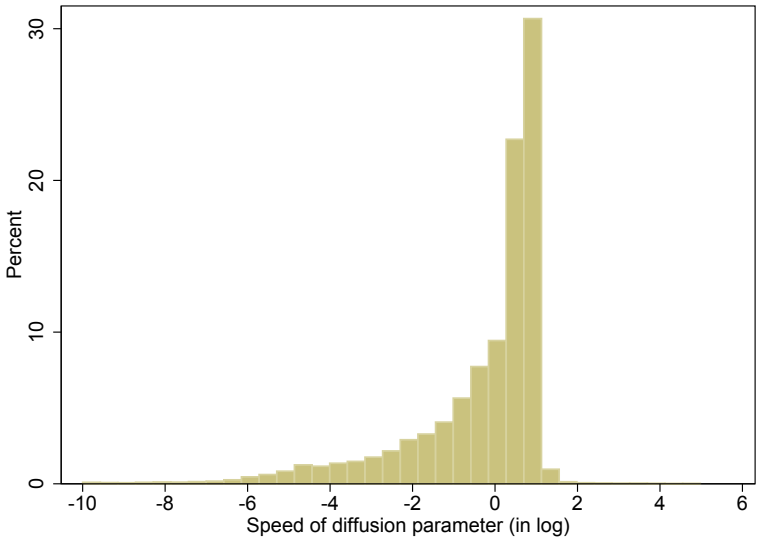
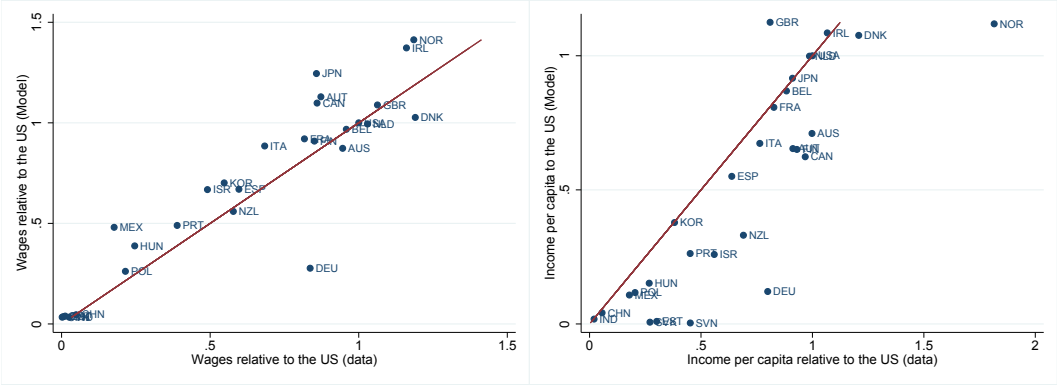


Figure 4: Distribution graph of ϵ_{ni}^{jk}

5.1.3 Parameters of innovation

We calibrate the parameters of innovation $\{\beta_r, \lambda_n^j, \hat{A}_n^j\}$ in two steps. First, we solve for the static trade equilibrium taking as given the estimated sectoral productivity T_i^j, the estimated trade barriers d_{in}^j, and production input-output linkages parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$. The static equilibrium delivers relative wages, costs, prices and trade shares on the BGP.

Figure 5 shows relative wages and income per capita in the data and in the model. Our calibration strategy delivers relative wages and relative income per capita that are broadly consistent with those observed in the data.



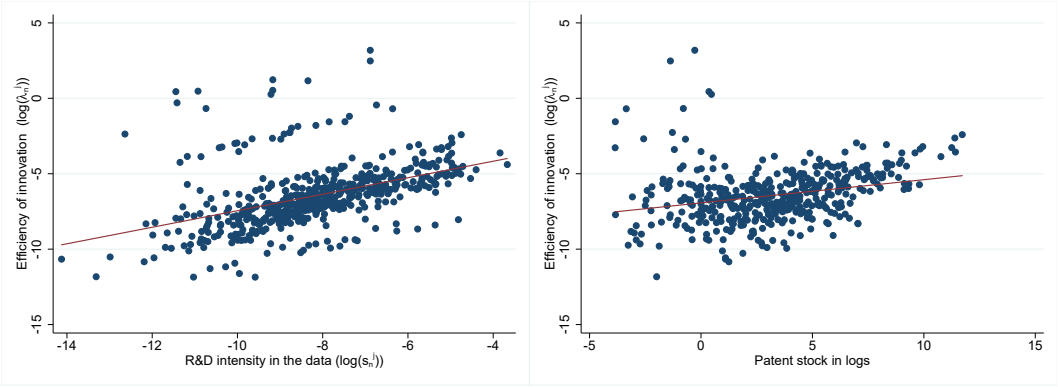
(a) Wages relative to the US

(b) Income per capita

Figure 5: Validation: Relative wages and relative income per capita

Having computed wages and trade shares, in the second step we use the estimated parameters for knowledge diffusion, ε_{in}^{kj}, data on R&D intensity at the country-sector level, s_n^j, and the expression

for the growth rate of the economy on the BGP in equation (20) to calibrate the innovation parameters $\lambda_n^j, \beta_r, A_n^j$. We proceed as follows: First, we assume a growth of income per capita (productivity) on the BGP of $g_y = 3\%$. This corresponds to a growth rate for the stock of knowledge on the BGP of $g_A = \theta \left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j\right)^{-1} g_y = 0.25$. Because all countries and sectors' stock of knowledge grows at the same rate, all countries have the same productivity growth on the BGP (see Appendix E for details on the derivation). Second, we use the Frobenius theorem and equation (20) to obtain a value for the efficiency of innovation, λ_i^k, and the elasticity of innovation, β_r. Given data for s_n^j, the estimated values for ε_{ni}^{jk}, and g_A, we can use the Frobenius theorem and iterate on equation (20) to obtain β_r and λ_n^j. We obtain that $\beta_r = 0.24$ and λ_n^j ranges from $7 * 10^{-6}$ to 24, with mean 0.10 and standard deviation 1.2. Figure 6 plots the calibrated values for λ_i^k against two measures of innovation in the data: R&D intensity (as an input of innovation and a flow variable) and the stock of patents (as an output of innovation and a stock variable). As the figure shows, there is a positive relationship between the efficiency of innovation, λ_i^k and both measures of innovation.

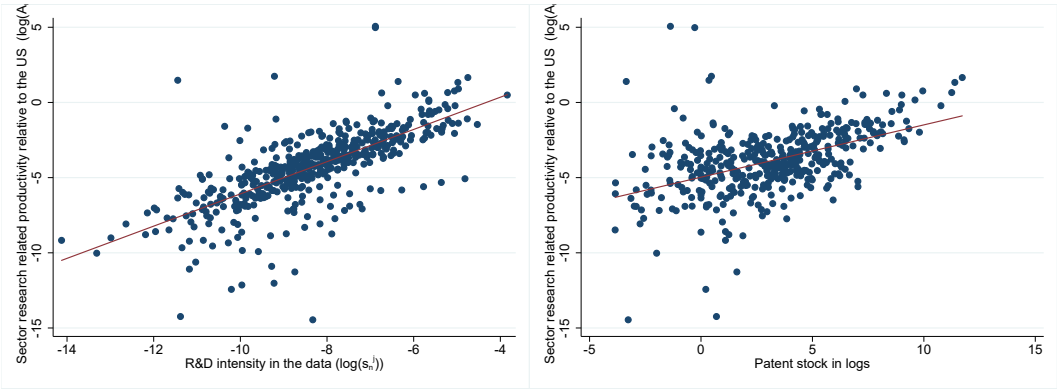


(a) R&D intensity

(b) Stock of patents

Figure 6: The productivity of innovation, R&D intensity and the stock of patents

Given these parameter values, and using again the properties of the Frobenius theorem, the associated eigenvector to the growth rate of $g_A = 0.25$ corresponds to the normalized "knowledge-related productivity" $\hat{A}_n^j$. Figure 7 shows that there is a strong positive relation between the "knowledge-related productivity", relative to the United States in sector J, $\hat{A}_i^j$, and both R&D intensity and the stock of patents.



(a) R&D intensity (b) Stock of patents

Figure 7: Research-related productivity and innovation

Figure 8 shows, by industry, the relationship between R&D intensity and the "knowledge-related productivity" across sectors. We observe a strong positive correlation between the two measures. The correlation is larger for machinery and equipment, computer, electronic and optical equipment, and electrical machinery sectors.

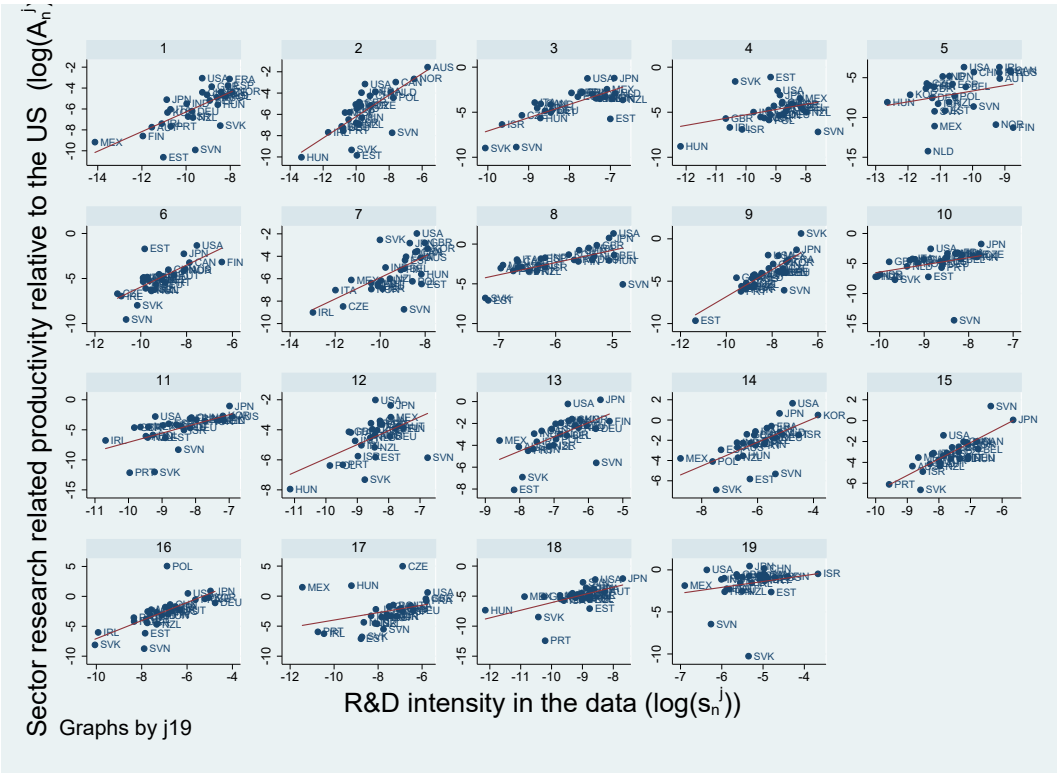


Figure 8: Research-related productivity and innovation by sector

The algorithm

The calibration of the parameters of innovation, $\{\lambda_n^j, \beta_r, \hat{A}_n^j\}$ follows a recursive algorithm. First, knowing $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, \hat{T}_n^j, d_{in}^j\}$, we use the trade structure of the model to obtain wages, prices, expenditures, trade shares, and output, from equations (56), (57), (58), (59), (60), (62), (63), (64), and (65).

Then, given $\{\varepsilon_{in}^{jk}, g_A, s_n^j\}$ we iterate over equation (20) to obtain $\{\lambda_n^j, \beta_r\}$. We do this in an iterative process in which, given a guessed value of λ_n^j, and β_r, we use R&D data, s_n^j, and keep iterating until $g_A = 0.25$. We use equation (66), (67) and the Frobenious theorem. The Frobenius theorem guarantees that there is a unique balanced growth path in which all countries and sectors grow at the same rate g_A. The expression for the growth rate can be expressed in matrix form as:

$$g_A A = \Delta(g_A) A$$

If the matrix $\Delta(g_A)$ is definite positive, then there exists a unique positive balanced growth rate of technology $g_A > 0$ given research intensities. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country and sector relative level of knowledge along that balanced growth path. We update β_r so that $g_A = 0.25$ and we update λ_n^j so that R&D intensity matches the data. Then, we obtain $\hat{A}_n^j$ from the eigenvector associated to $\Delta(g_A = 0.25)$. Knowing $\hat{T}_n^j$ from the gravity regressions, and $\hat{A}_n^j$ from the Frobenius theorem, we can obtain $T_{p,n}^j$ from equation (7).

5.2 Counterfactual Analysis

We perform a uniform reduction of trade barriers, d_{in}^j, of 40%. All other parameters are kept fixed at their calibrated values. We analyze the effect of this reform on innovation and welfare gains from trade across the baseline and counterfactual BGP. First, we describe briefly the algorithm that we develop to compute the counterfactual BGP. Then, we report our main results for our multi-country and multi-sector endogenous growth model featuring heterogeneous intelinkages in production and knowledge flows.

The algorithm

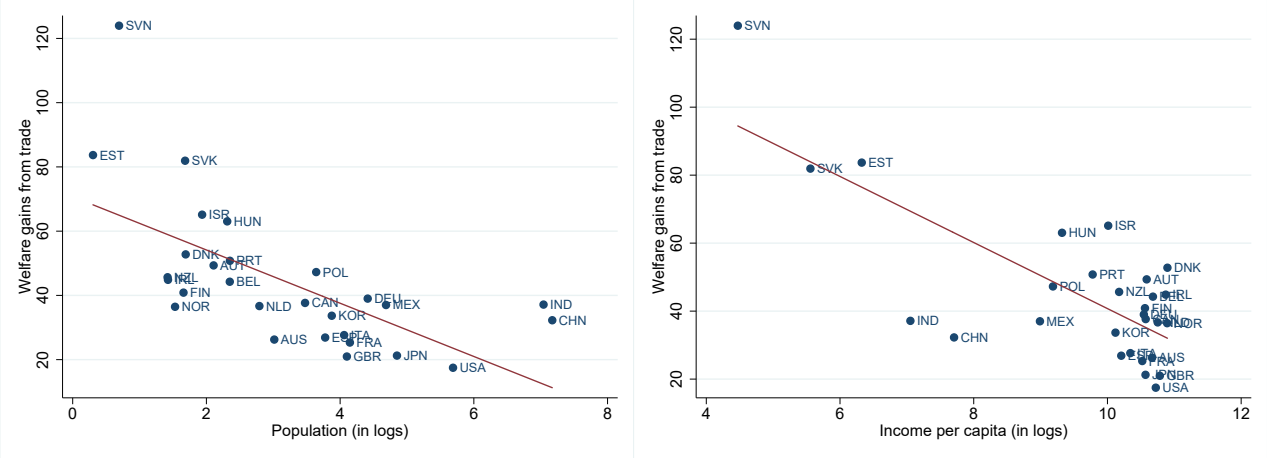
In our calibration exercise, we took the level of technology, $\hat{T}_n^j$, as given by the estimated values from the gravity regressions. However, when there are changes in trade costs, $\hat{T}_n^j$ will change across counterfactuals to the extent that $\hat{A}_n^j$ also changes. In our model, there are changes in $\hat{A}_n^j$ that are induced by changes in the innovation intensity, s_n^j and by knowledge diffusion. Our algorithm to solve for the counterfactual equilibrium uses the properties of the Frobenius theorem and allows T_n^j to evolve over time through changes in A_n^j. We proceed as follows. First, we take $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, T_{p,n}^j, \hat{T}_n^j, \beta_r, \lambda_n^j\}$ as given, and compute the static equilibrium that corresponds to the new trade barriers, d_{in}^j. With that equilibrium, we compute the new optimal R&D intensity s_n^j and use the Frobenius theorem to obtain the new g_A and associated eigenvector $\hat{A}_n^j$. We do this by

iterating over equation (20) until $g_A(t-1) = g_A(t)$. The new A_n^j delivers a new $\hat{T}_n^j$ (we keep $T_{n,p}^j$ constant across counterfactuals). We then repeat the procedure until $\hat{T}_n^j$ converges.

Welfare Gains from Trade, Innovation and RCA

We compute welfare gains from trade using equation (21). Welfare gains across two BGPs depend on two components: (1) changes in consumption, $\hat{C}_i$, and (2) changes in the growth rate, g. From equation (22), the change in consumption depends on two additional factors: (i) the change in income per capita, $\hat{Y}_i$, and (ii) the change in R&D intensity s_i^k, which has an effect both on the growth rate g and on income per capita $\hat{Y}_i$.

We find that welfare gains from trade are heterogeneous across countries, raging from 17.5% in the United States to 124% in Slovenia, with an average gain of 44.6%. The gains are larger for smaller countries and countries with lower income per capita (see Figure 9), which is consistent with the findings in Waugh 2010.



(a) Population

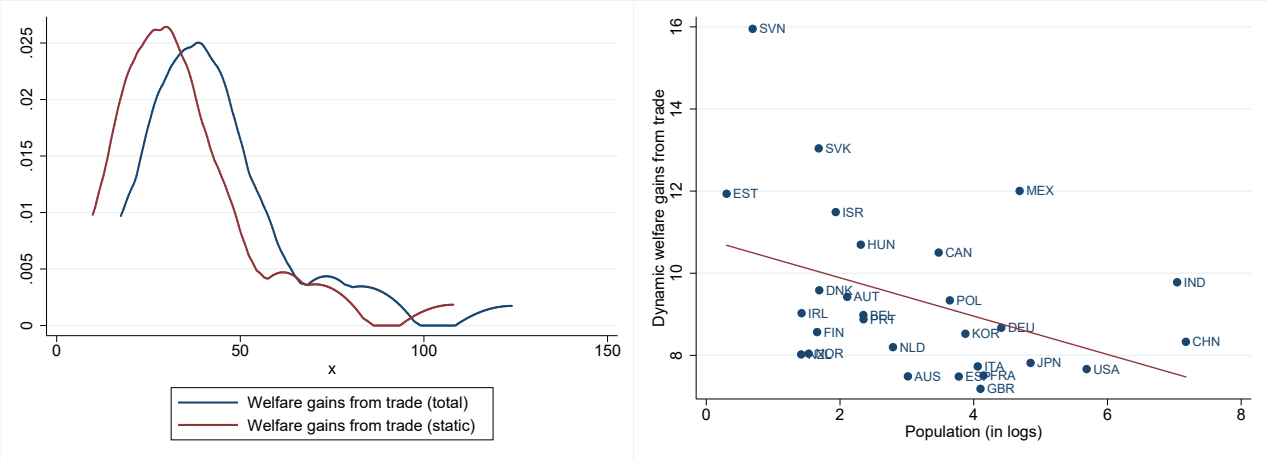
(b) Income per capita

Figure 9: Welfare gains from trade against population and income per capita

Welfare gains from trade can be divided into static and dynamic gains. Static gains correspond to those obtained in a model where the stock of knowledge is not allowed to change over time. These are the gains that are obtained in standard static models of trade and are driven by increased specialization and comparative advantage. Dynamic gains isolate the effect of R&D and knowledge spillovers in increasing income per capita, and the growth rate g. Knowledge diffusion has two opposite effects on dynamic gains from trade. On the one hand, it increases the stock of knowledge of a sector-country as it increases the pool of ideas. This has an additional effect on the efficiency of innovation, from equation (10). On the other hand, knowledge spillovers imply convergence of comparative advantage, dampening the total welfare gains from trade that are driven by differences in comparative advantage. We explore this point further in Section 5.2.

Figure 10a compares welfare gains from trade in our baseline model to those static gains in

which the stock of technology is kept constant across counterfactuals. The difference between the two is a measure of dynamic gains from trade (see Figure 10b). The cross-country distribution of static gains is shifted to the left, which implies that dynamic gains are positive. We find that, after the trade liberalization, the average country experiences dynamic gains from trade that are around 9%, with a minimum of 7.1% in the United Kingdom and a maximum of 15.95% in Slovenia. Dynamic gains are larger for smaller countries (see Figure 10b)



(a) Welfare gains from trade: Baseline against static

(b) Dynamic welfare gains from trade

Figure 10: Welfare gains from trade against population and income per capita

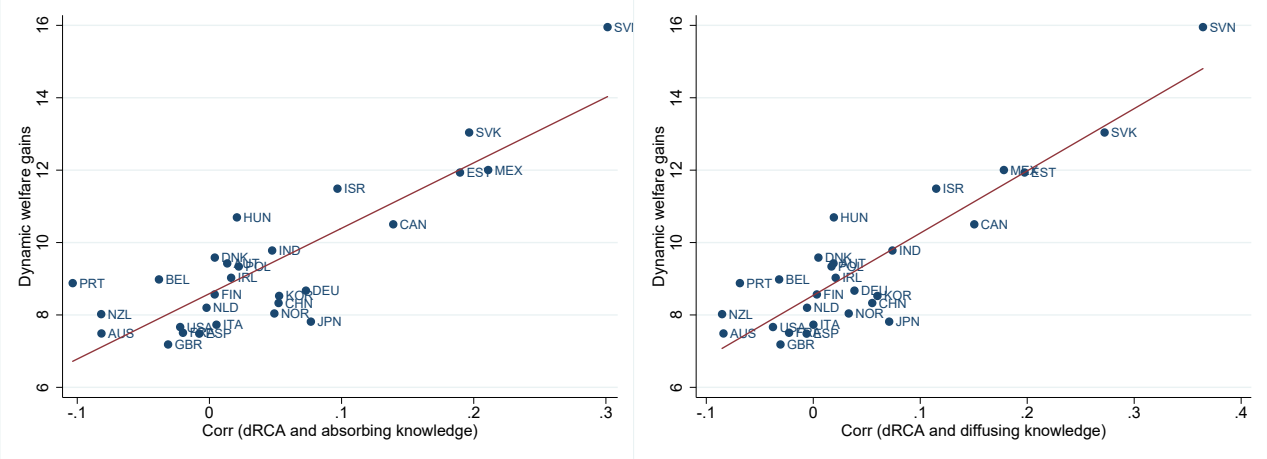
Dynamic gains from trade are driven by innovation and knowledge diffusion, which generates increases in the growth rate and in income per capita. The growth rate increases from 3% in the initial BGP to 3.1% in the counterfactual BGP. Here we analyze the direct role of innovation on dynamic gains, and we leave the analysis on the role of knowledge diffusion to Section 5.2.

After a trade liberalization, there is a reallocation of R&D and comparative advantage across. Except for Finland, New Zealand and Norway, all countries in our sample experience an increase in R&D spending after a trade liberalization. However, the increase in R&D is far from uniform across sectors within a country. In general, R&D increases are correlated with increases in revealed comparative advantage (RCA) at the sector level. We find that this correlation is stronger for those countries that experience larger gains from trade (see Figure 11).



Figure 11: Welfare gains from trade and correlation of R&D and RCA

We also find that RCA increases more in those sector that have larger knowledge spillovers, in both directions of knowledge flows: absorbing (citing) and diffusing (cited) knowledge. This correlation is larger in countries that experience larger dynamic gains from trade (see Figure 12).

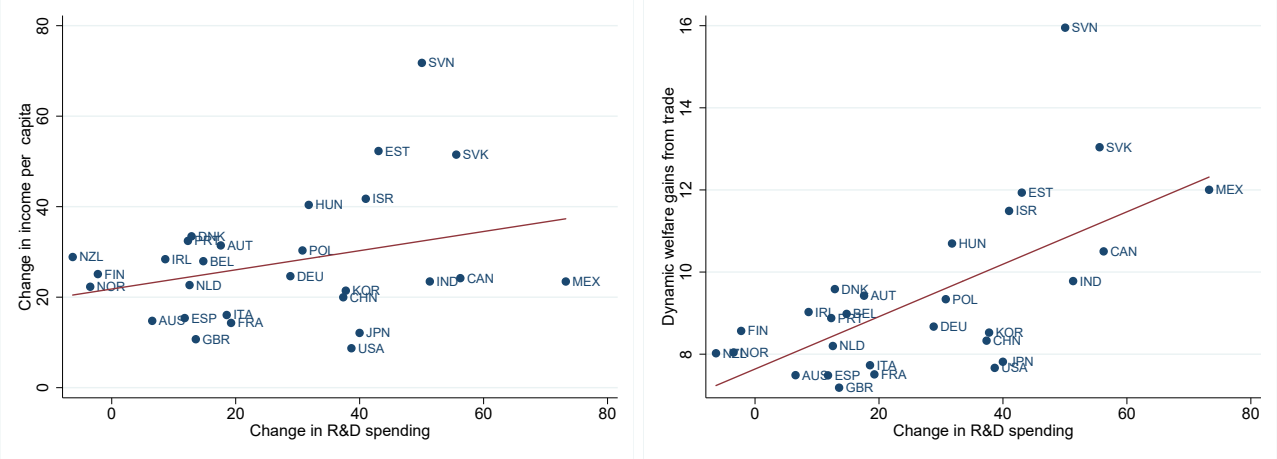


(a) Population

(b) Income per capita

Figure 12: Dynamic welfare gains and the correlation of RCA and knowledge flows

Because of the effect on innovation and knowledge diffusion, income per capita increases after a trade liberalization and, from equation (22), consumption and welfare increase. We find that countries with larger increases in R&D spending also experience larger increases in income per capita (see Figure 13a). Those countries with larger increases in R&D experience larger dynamic welfare gains from trade (see Figure 13b).



(a) Income per capita

(b) Dynamic welfare gains from trade

Figure 13: The effect of changes in innovation on dynamic welfare gains

Welfare Gains from Trade: The role of knowledge spillovers

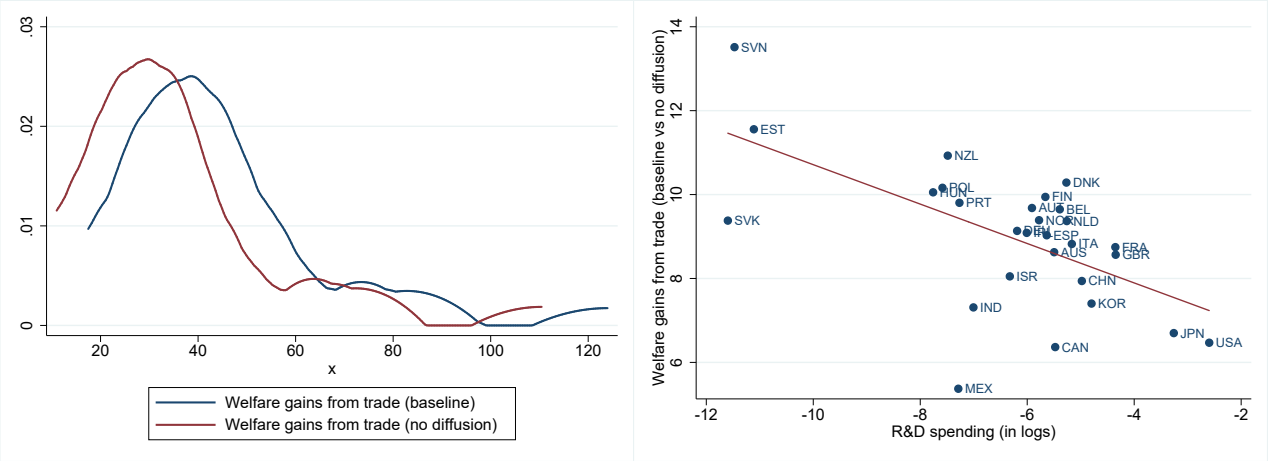
We study the role of knowledge diffusion on welfare gains from trade by recalibrating a model in which the diffusion parameters ε_{ni}^{jk} are set to a very small value of 0.0001 (we set $\varepsilon_{nn}^{jj} \to \infty$, that is, we assume instantaneous diffusion within the same country-sector pair).[9] We recalibrate the model parameters, β_r and λ_n^j, by using the same input-output linkage parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$, estimated technology, T_n^j, R&D intensity s_n^j and growth rate g_A values as those in the baseline model. We obtain $\beta_r = 0.28$ and a mean λ_n^j of 0.18 with standard deviation 2.2 .We then perform the same trade liberalization exercise as in the baseline model and evaluate the effect that a reduction of 40% in trade barriers has on innovation and welfare.

Knowledge diffusion has two opposite effects on welfare. On the one hand, it enables faster productivity convergence and makes countries more similar to each other, which dampens the gains from trade. On the other hand, it provides strong dynamic gains, because countries can innovate with access to a larger foreign knowledge pool. This has an effect on the stock of knowledge of a sector-country, and on the efficiency of innovation by equation (10).

Figure 14 shows that the welfare gains from trade in a model with very low diffusion are smaller than in a model where we allow for diffusion. The direct effect on the stock of knowledge and the effect on the efficiency of innovation that allows countries to benefit from foreign innovations through diffusion dominate the comparative advantage effect by which convergence in technology would reduce the gains from trade, as countries become more similar (see Levchenko and Zhang 2016). The differences in welfare gains between the baseline model and a model with very low diffusion are larger for less innovative countries, such as Slovenia, Estonia and Slovakia. These are

[9]The Frobenius theorem is only valid if there is at least some diffusion across all country-sector pairs. Setting ε_{ni}^{jk} to a very small number allows us to make use of the properties of the Frobenius theorem, while allowing for very slow to virtually no diffusion.

the countries that benefit more from knowledge flows. In Japan, the United States, and Korea, the difference between the gains are smaller, as these countries are already very innovative and have less to gain from the flow of ideas.

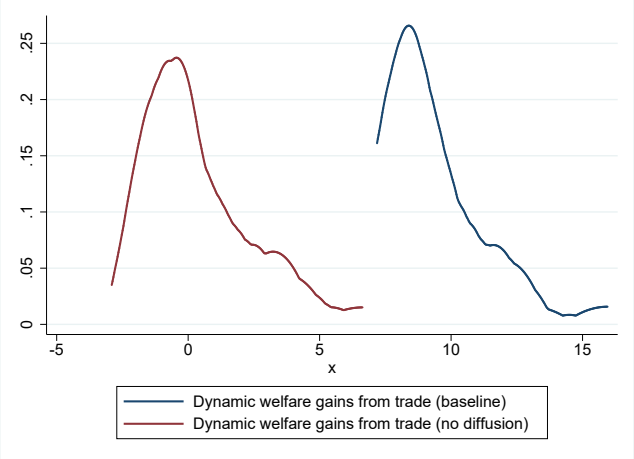


(a) Welfare gains from trade

(b) Welfare gains from trade and R&D

Figure 14: Welfare gains from trade with negligible knowledge spillovers

Static welfare gains from trade are the same in the baseline model and in the model with almost no knowledge spillovers, as they are computed keeping the level of technology constant across counterfactuals. In both cases, the initial stock of knowledge is the same, and it is estimated from the gravity regressions. Therefore all the differences in welfare gains in the two cases are driven by differences in dynamic welfare gains from trade (see Figure 15a). Dynamic gains are driven by both innovation and knowledge spillovers. Figure 15b compares, for a cross-section, the change in R&D in the baseline model with the corresponding change in R&D in a model with negligible knowledge spillovers. The figure shows that there are substantial differences in how R&D changes in the two models. All countries, except for Mexico, experience larger increases in R&D in the baseline model. This suggests that the larger dynamic gains from trade in Mexico are entirely driven by knowledge flows.



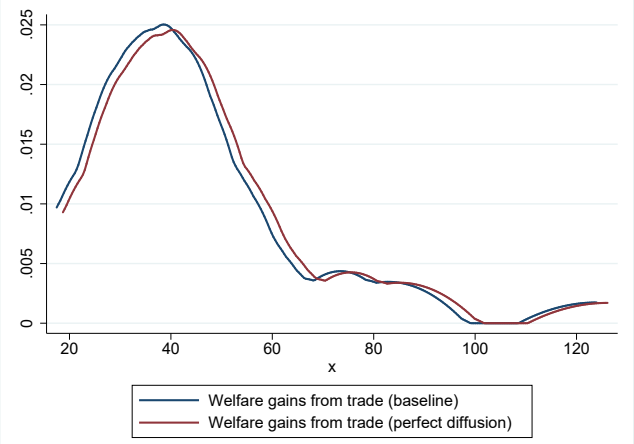
(a) Dynamic welfare gains from trade



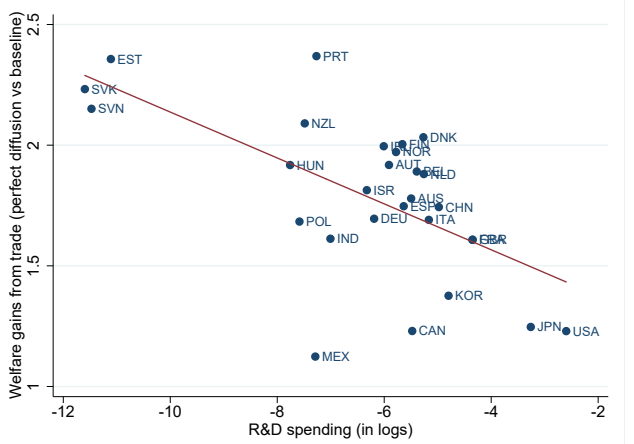
(b) Changes in R&D spending

Figure 15: Dynamic welfare gains from trade with negligible knowledge spillovers

We then look at a world where there is instantaneous (perfect) diffusion across all sectors and countries. In this exercise we set $\varepsilon_{ni}^{jk} \to \infty$. All other parameters are kept fixed as in the baseline calibration. We then obtain $\beta_r = 0.19$ and λ_n^j with mean 0.0015 and standard deviation 0.0045. We find that the welfare gains in this case are larger than in the baseline model, but the differences are small, suggesting that the world is closer to almost perfect diffusion than to no diffusion, as changes in trade costs have lower effects on welfare gains (see Figure 16). As in the case of very low diffusion, the difference between the gains are larger in countries with lower total R&D spending, that is countries in which the stock of knowledge is lower.



(a) Welfare gains from trade



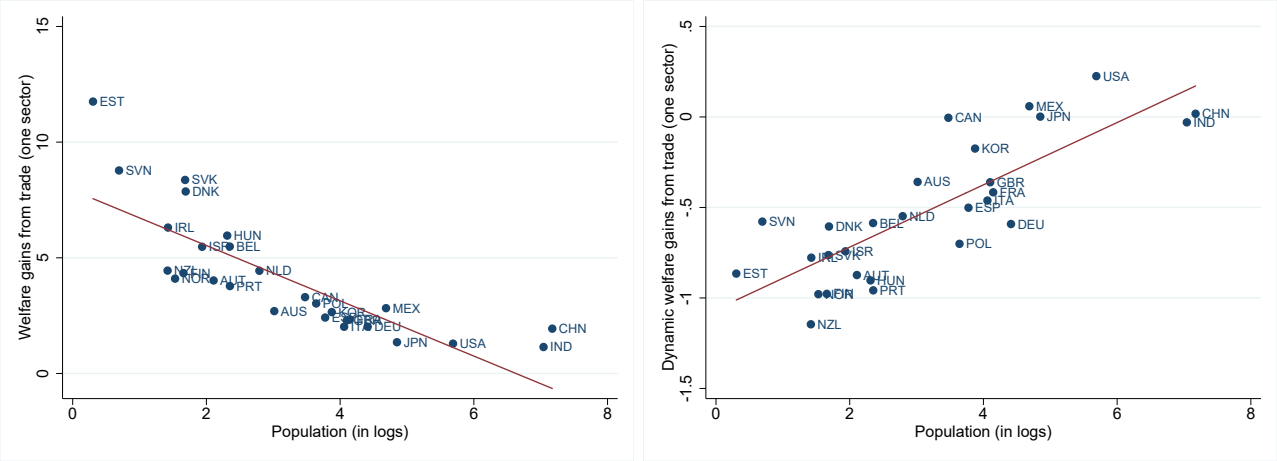
(b) Welfare gains from trade (cross-section)

Figure 16: Welfare gains from trade with negligible knowledge spillovers

Welfare Gains from Trade: The role of multiple sectors

We recalibrate our baseline model to a one-sector model. We re-estimate the technology parameters, T_n by running gravity equations at the country level. The production and knowledge linkages parameters are also recalibrated at the country level. We set $\alpha^j = 1$, $\gamma^j = 1$ and $\gamma^{jk} = 0$, for all j and k. We obtain country-level data for R&D intensity, s_n. Then, assuming the same g_A as in the baseline model, we obtain a $\beta_r = 0.28$ and λ_n with mean 0.22 and standard deviation 0.13. We find that this model delivers substantially lower gains from trade than our multi-sector growth model with production and knowledge interlinkages.

In the multi-sector version of the model R&D reallocates towards sectors that experience larger increases in comparative advantage. This resource reallocation channel is no longer present in a one-sector model and R&D increases are much smaller. As in the baseline model, welfare gains from trade are larger for smaller countries (see Figure 17a). Dynamic gains are negative for most countries, and are positive for larger countries (see Figure 17b). The reason is that in our one-sector model with royalties, dynamic gains are driven by the existence of royalty payments, which are larger countries with a larger stock of knowledge.

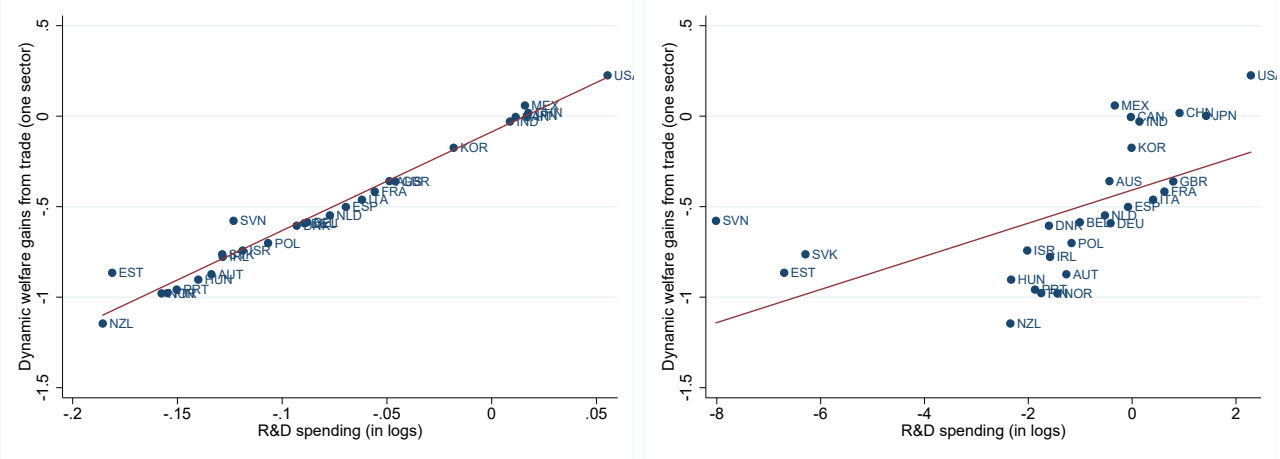


(a) Total gains

(b) Dynamic gains

Figure 17: Welfare gains from trade in a one-sector model

Figure 18a shows that more innovative countries are the ones who experience larger dynamic welfare gains from trade. These are the countries with larger increases in R&D intensity. Negative dynamic gains are driven by decreases in innovation after the trade liberalization in most countries. We find that those countries that were initially less innovative, experience decreases in their R&D intensity after a trade liberalization. The opposite is true for more innovative countries.



(a) Total gains

(b) Dynamic gains

Figure 18: Welfare gains from trade in a one-sector model

In our baseline model, changes in trade costs do generate changes in R&D intensity even when we consider a one-sector model, because of the presence of royalties. These effects are, however, much smaller than in the multi-sector model. This is independent on whether or not there are knowledge spillovers. Standard one-sector models of trade and innovation in which trade has no effect on innovation do not feature royalty payments (see Eaton and Kortum 1996 and Eaton and Kortum 1999 , Buera and Oberfield 2016, and Atkeson and Burstein 2010). In Appendix F we show that in a one sector model without royalties, changes in trade barriers have no effect on R&D and the growth rate of the economy, hence no effects on dynamic welfare gains from trade.

6 Conclusion

We develop a quantitative framework to study the effect of interlinkages between trade, knowledge flows and production on innovation and welfare. In our model, we distinguish between static gains from trade–which are driven by increased specialization and comparative advantage—and dynamic gains from trade—which are driven by innovation and knowledge diffusion. In our model, changes in trade barriers have a quantitatively important effect on innovation and welfare. After a trade liberalization, R&D reallocates toward sectors in which the country has a comparative advantage. Knowledge diffusion amplifies this effect, as revealed comparative advantage is reallocated towards sectors with larger knowledge flows. Furthermore, knowledge spillovers allow sectors in a country to benefit for a larger pool of ideas, increasing dynamic welfare gains from trade. Finally, a one-sector version of our deliver much smaller total gains in welfare and almost negligible, or even negative in some countries, dynamic welfare gains. This result reinforces the importance of modeling sectoral heterogeneity when studying the effect of trade liberalizations on innovation and welfare.

References

Alvarez, F. E., F. J. Buera, and R. E. Lucas Jr (2008). Models of idea flows. Technical report, National Bureau of Economic Research.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012). New trade models, same old gains? *The American Economic Review 102*(1), 94–130.

Arkolakis, C., N. Ramondo, A. Rodríguez-Clare, and S. Yeaple (2013). Innovation and production in the global economy. Technical report, National Bureau of Economic Research.

Atkeson, A. and A. T. Burstein (2010). Innovation, firm dynamics, and international trade. *Journal of political economy 118*(3), 433–484.

Bernard, J. Eaton, B. Jensen, and S. Kortum (2003). Plants and productivity in international trade. *The American economic review 93*(4).

Bernard, A. B. and C. I. Jones (1996a). Comparing apples to oranges: productivity convergence and measurement across industries and countries. *The American Economic Review*, 1216–1238.

Bernard, A. B. and C. I. Jones (1996b). Technology and convergence. *The Economic Journal*, 1037–1044.

Buera, F. J. and E. Oberfield (2016). The global diffusion of ideas. Technical report, National Bureau of Economic Research.

Cai, J. and N. Li (2016). Growth through intersectoral knowledge linkages.

Caliendo, L. and F. Parro (2015). Estimates of the trade and welfare effects of nafta. *The Review of Economic Studies 82*(1), 1–44.

Cameron, G., J. Proudman, and S. Redding (2005). Technological convergence, r&d, trade and productivity growth. *European Economic Review 49*(3), 775–807.

Chor, D. (2010). Unpacking sources of comparative advantage: A quantitative approach. *Journal of International Economics 82*(2), 152–167.

Costinot, A., D. Donaldson, and I. Komunjer (2012). What goods do countries trade? a quantitative exploration of ricardo's ideas. *The Review of Economic Studies*, 581–608.

David, P. A. (1990). The dynamo and the computer: an historical perspective on the modern productivity paradox. *The American Economic Review 80*(2), 355–361.

Eaton, J. and S. Kortum (1996, May). Trade in ideas: Productivity and patenting in the OECD. *Journal of International Economics 40*(3-4), 251–278.

Eaton, J. and S. Kortum (1999, August). International technology diffusion: Theory and measurement. *International Economic Review 40*(3), 537–570.

Eaton, J. and S. Kortum (2002). Technology, geography, and trade. *Econometrica 70*(5), 1741–1779.

Eaton, J. and S. Kortum (2006). Innovation, diffusion, and trade. Technical report, National Bureau of Economic Research.

Eaton, J., S. Kortum, B. Neiman, and J. Romalis (2016). Trade and the global recession. *The American Economic Review 106*(11), 3401–3438.

Ethier, W. J. (1982). Dumping. *The Journal of Political Economy*, 487–506.

Hausmann, R., J. Hwang, and D. Rodrik (2007). What you export matters. *Journal of economic growth 12*(1), 1–25.

Hausmann, R. and B. Klinger (2007). *The structure of the product space and the evolution of comparative advantage*. Center for International Development at Harvard University.

Hidalgo, C. A., B. Klinger, A.-L. Barabási, and R. Hausmann (2007). The product space conditions the development of nations. *Science 317*(5837), 482–487.

Kali, R., J. Reyes, J. McGee, and S. Shirrell (2013). Growth networks. *Journal of Development Economics 101*, 216–227.

Keller, W. (2004). International technology diffusion. *Journal of economic literature 42*(3), 752–782.

Kortum, S. S. (1997). Research, patenting, and technological change. *Econometrica*, 1389–1419.

Levchenko, A. A. and J. Zhang (2016). The evolution of comparative advantage: Measurement and welfare implications. *Journal of Monetary Economics 78*, 96–111.

Perla, J., C. Tonetti, and M. E. Waugh (2015). Equilibrium technology diffusion, trade, and growth. Technical report, National Bureau of Economic Research.

Proudman, J. and S. Redding (2000). Evolving patterns of international trade. *Review of international economics 8*(3), 373–396.

Rosenberg, N. (1982). *Inside the black box: technology and economics*. Cambridge University Press.

Sampson, T. (2016). The global productivity distribution and ricardian comparative advantage.

Santacreu, A. M. (2015). Innovation, diffusion, and trade: Theory and measurement. *Journal of Monetary Economics 75*, 1–20.

Somale, M. (2014). Comparative advantage in innovation and production. Technical report, Mimeo, Princeton University.

Waugh, M. (2010). International trade and income differences. *The American Economic Review 100*(5), 2093–2124.

Appendix

A Model Equations

There are 14 endogenous variables and we need 14 equations. The endogenous variables are

$$\{\pi_{in}^j, T_i^j, c_i^j, W_i, P_n^j, X_{ni}^j, X_n^j, P_n, Y_n, \Phi_n^j, C_n, s_n^j, V_n^j, A_n^j\}$$

The corresponding equations are:

(1) Probability of imports

$$\pi_{ni}^j = T_i^j \frac{\left(c_i^j d_{ni}^j\right)^{-\theta}}{\Phi_n^j}, \tag{41}$$

with

$$T_i^j = A_i^j T_{p,i}^j, \tag{42}$$

(2) Import shares

$$X_{ni}^j = \pi_{ni}^j X_n^j, \tag{43}$$

(3) Cost of production

$$c_n^j = \Upsilon^j W_{nt}^{\gamma^j} \prod_{k=1}^{J} (P_n^k)^{\gamma^{jk}}, \tag{44}$$

(4) Intermediate good prices in each sector

$$P_n^j = A^j \left(\Phi_n^j\right)^{-1/\theta}, \tag{45}$$

(5) Cost distribution

$$\Phi_n^j = \sum_{i=1}^{M} T_i^j \left(d_{ni}^j c_i^j\right)^{-\theta}, \tag{46}$$

(6) Price index

$$P_n = \prod_{j=1}^{J} \left(\frac{P_n^j}{\alpha^j}\right)^{\alpha^j}, \tag{47}$$

(7) Labor market clearing condition

$$W_n L_n = \sum_{j=1}^{J} \gamma^j \sum_{i=1}^{M} \pi_{in}^j X_i^j, \tag{48}$$

(8) Sector production

$$X_n^j = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} X_i^k \pi_{in}^k + \alpha^j P_n Y_n, \tag{49}$$

(9) Final production

$$P_n Y_n = W_n L_n + \frac{\sum_{j=1}^{J} \sum_{i=1}^{M} \pi_{in}^j X_i^j}{1+\theta}, \tag{50}$$

(10) Resource constraint

$$Y_n = C_n + \sum_{k=1}^{J} s_n^k Y_n, \tag{51}$$

(11) Innovation

$$\dot{A}_{nt}^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \alpha_{is}^k \left(s_{is}^k\right)^{\beta^k} ds, \tag{52}$$

(12) R&D expenditures

$$\beta^j \lambda_{nt}^j V_{nt}^j \left(s_{nt}^j\right)^{\beta^j - 1} = P_{nt} Y_{nt}, \tag{53}$$

(13) Value of an innovation

$$V_{nt}^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \int_{t}^{\infty} \left(\frac{P_{it}^k}{P_{is}^k}\right) e^{-\rho(s-t)} \chi_{in,s}^{kj} \frac{\Pi_{is}^k}{A_{is}^k} ds, \tag{54}$$

$$\Pi_{nt}^j = \frac{1}{(1+\theta)} \sum_{i=1}^{M} X_{it}^j \pi_{int}^j. \tag{55}$$

B Model Equations (normalized)

In what follows, we report the equations of the model after normalizing the endogenous variables.

(1) Probability of imports

$$\pi_{ni}^j = \hat{T}_i^j \frac{\left(\hat{c}_i^j d_{ni}^j\right)^{-\theta}}{\hat{\Phi}_n^j}, \tag{56}$$

where $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$ and $\hat{\Phi}_n^j = \frac{1}{T_M^J} \frac{\Phi_n^j}{(W_M)^{-\theta}(T_M^J)^{\Lambda_j}}$ with Λ^j defines in Appendix E.

(2) Import shares

$$\hat{X}_{ni}^{j} = \pi_{ni}^{j} \hat{X}_{n}^{j}, \tag{57}$$

(3) Cost of production

$$\hat{c}_{n}^{j} = \Upsilon^{j} \hat{W}_{n}^{\gamma^{j}} \prod_{k=1}^{J} (\hat{P}_{n}^{k})^{\gamma^{jk}}, \tag{58}$$

(4) Intermediate good prices in each sector

$$\hat{P}_{n}^{j} = B \left(\hat{\Phi}_{n}^{j} \right)^{-1/\theta}, \tag{59}$$

(5) Cost distribution

$$\hat{\Phi}_{n}^{j} = \sum_{i=1}^{M} \hat{T}_{i}^{j} \left(d_{ni}^{j} \hat{c}_{i}^{j} \right)^{-\theta}, \tag{60}$$

(6) Price index

$$\hat{P}_{n} = \prod_{j=1}^{J} \left(\frac{\hat{P}_{n}^{j}}{\alpha^{j}} \right)^{\alpha^{j}}, \tag{61}$$

(7) Labor market clearing condition

$$\hat{W}_{n} L_{n} = \sum_{j=1}^{J} \gamma^{j} \sum_{i=1}^{M} \pi_{in}^{j} \hat{X}_{i}^{j}, \tag{62}$$

(8) Sector production

$$\hat{X}_{n}^{j} = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} \pi_{in}^{k} \hat{X}_{i}^{k} + \alpha^{j} \hat{Y}_{n}, \tag{63}$$

where $\hat{Y}_{n} = \frac{P_{n} Y_{n}}{W_{M}}$.

(9) Final production

$$\hat{Y}_{n} = \hat{W}_{n} L_{n} + \frac{\sum_{j=1}^{J} \sum_{i=1}^{M} \pi_{in}^{j} \hat{X}_{i}^{j}}{1+\theta}, \tag{64}$$

(10) Resource constraint

$$\hat{Y}_{n} = \hat{C}_{n} + \sum_{k=1}^{J} s_{n}^{k} \hat{Y}_{n}, \tag{65}$$

(11) R&D expenditures

$$\beta_{r} \lambda_{n}^{j} \hat{V}_{n}^{j} \left(s_{n}^{j} \right)^{\beta_{r}-1} = \hat{Y}_{n}, \tag{66}$$

(12) Value of an innovation

$$\hat{V}_n^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}} \frac{1}{\rho - g_y + g_A} \frac{1}{\hat{A}_i^k} \frac{\sum_{m=1}^{M} \hat{X}_m^k \pi_{mi}^k}{(1+\theta)}, \tag{67}$$

C Data Description and Calculation

This appendix describes the data sources and the construction of various variables for the paper. 28 countries are included in our analysis based on data availability (mostly constrained by the availability of the R&D data): Australia, Austria, Belgium, Canada, China, Czech Republic, Estonia, Finland, France, Germany, Hungary, India, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovenia, Spain, Slovakia, Slovenia, United Kingdom, and United States. The model is calibrated for 2005. 18 tradable sectors and one aggregate nontradable sector are under consideration, and are reported in Table 2.

Bilateral trade flows at the sectoral level Bilateral trade data at sectoral level (expenditure by country n of sector j goods imported from country i, X_{ni}^j) are obtained from the OECD STAN Bilateral Trade Dataset. Values are reported in thousand U.S. dollars at current prices. Sectors are recorded at the ISIC (rev. 3) 2-3 digit level and are aggregated into the 19 sectors as listed in Table 2. We use the importer reported exports in each sector as the bilateral trade flows because it is generally considered to be more accurate than the exporter reported exports.

Value added and gross production Domestic sales in sector j, X_{nn}^j is estimated based on the *domestic* input-output table provided by OECD STAN database, which contains data at ISIC 2-digit level that can be easily mapped into our 19 sectors. OECD provides separate IO table for domestic output and imports. We sum up the values for a given row up before the Column "Direct purchases abroad by residents (imports)" to obtain X_{nn}^j. We compared this way of estimating the domestic expenditure on domestic product with an alternative calculation based on $X_{nn}^j = Y_n^j - \sum_{i \neq n}^{M} X_{in}^j$, where both gross production of country n in sector j, Y_n^j and the total exports from n to i in sector j, $\sum_{i \neq n}^{M} X_{in}^j$, are from OECD STAN Database for Structural Analysis. The first method proves to be superior as the second generates a number of negative observations for some country-sectors. However, data are missing for India, for which we use the INDSTAT (2016 version) provided by United Nations Industrial Development Organization (UNIDO).

Trade barriers and gravity equation variables Data for variables related to trade costs used in gravity equations (such as geographic distance and common border dummies) at the country-pair level are obtained from the comprehensive geography database compiled by CEPII. WTO's RTA database provides information on regional trade agreements. The currency union indicator is obtained from Rose (2004) and was updated to reflect the Euro Area membership.

Wages Average annual wage is reported by OECD Labour statistics at current price in local currency. They are transformed into U.S. dollar at the 2005 exchange rates to obtain the variable w_n in the model. However, wage data for China, India, and New Zealand are missing in this database, and are obtained from International Labor Organization (ILO).

Factor shares and final consumption shares In our analysis we used the U.S. factor shares in 2005 for all countries. Data on the share of materials from sector k used in the production in sector j, γ^{jk}, as well as the labor share of production in sector j, γ^j, come from the Input-Output Database maintained by OECD STAN. The I-O table gives the value of the intermediate input in row k required to produce one dollar of final output in column j. We then divide this value by the value of gross output of sector j to obtain γ^{jk}. Similarly, the labor share is calculated as the ratio of value added to gross output, as capital input does not exist in the model. In addition, the final consumption expenditure shares of each sector, α_n^j also come from the I-O matrix.

R&D data R&D expenditures at the country-sector level are obtained from the OECD database of Business enterprise R&D expenditure by industry (ISIC Rev 3). Since sectoral R&D data for China, India and Sweden and several sectors in other countries are missing, we obtain estimates of these missing observations using the following approach. First, we run a regression using existing country-sector specific R&D and patent data from USPTO for 2005:

$$\log(R_n^j) = \beta_0 + \beta_1 \log\left(PS_n^j\right) + \mu_n + \gamma_j + \varepsilon_n^j, \tag{68}$$

where R_n^j is the R&D dollar expenditure of country i in sector j and PS_n^j is the patent stock of country i in sector j. μ_i and γ_j are country and sector fixed effects. This relation is built on the observations that (a) at steady state, R&D expenditure should be a constant ratio of R&D stock, and (b) innovation input (R&D stock) is significantly positively related to innovation output (patent stock). In fact, the coefficient β_1 is large and significant at 99% and R^2 is close to 0.90. Assuming that the relationship captured by (68) holds for China, India and Sweden, we can obtain the fitted value of their sectoral level R&D expenditure:

$$\log(\widehat{R}_n^j) = \widehat{\beta}_0 + \widehat{\beta}_1 \log\left(PS_n^j\right) + \widehat{\mu}_n + \widehat{\gamma}_j$$

For these three countries, we have information on all the right-hand-side variables except for the country fixed effects, $\widehat{\mu}_n$. This allows us to compute the *share* of R&D in a given sector for each country,

$$\widehat{r}_n^j = \frac{\widehat{R}_n^j}{\sum_j \widehat{R}_n^j} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}.$$

Second, we then obtain the aggregate R&D expenditure as percentage of GDP, $R\&D/GDP_n^{WB}$, for each country from the World Bank World Development Indicator Database. The country-sector specific R&D can then be estimated as $s_n^j = \widehat{r}_n^j \times R\&D/GDP_n^{WB}$. For the countries with missing

Table 2: List of Industries

Sector	ISIC	Industry Description
1	C01T05	Agriculture, Hunting, Forestry and Fishing
2	C10T14	Mining and Quarrying
3	C15T16	Food products, beverages and tobacco
4	C17T19	Textiles, textile products, leather and footwear
5	C20	Wood and products of wood and cork
6	C21T22	Pulp, paper, paper products, printing and publishing
7	C23	Coke, refined petroleum products and nuclear fuel
8	C24	Chemicals and chemical products
9	C25	Rubber and plastics products
10	C26	Other non-metallic mineral products
11	C27	Basic metals
12	C28	Fabricated metal products, except machinery and equipment
13	C29	Machinery and equipment, nec
14	C30T33X	Computer, Electronic and optical equipment
15	C31	Electrical machinery and apparatus, nec
16	C34	Motor vehicles, trailers and semi-trailers
17	C35	Other transport equipment
18	C36T37	Manufacturing n.e.c. and recycling
19	C40T95	Nontradables

sectors, we estimate the fitted value using the same procedure. To maintain consistency across countries, we correct the OECD data generated total R&D with the World Bank total R&D.

$$s_n^j = R\&D/GDP_n^{WB} \times \frac{R_n^{j,OECD}}{\sum_j R_n^{j,OECD}}$$

This estimated s_n^j is the R&D intensity parameter in Equations (15) and (11) used in our quantitative analysis.

D The determinants of diffusion speed

Table 3 examines the determinants of cross-country-sector knowledge diffusion speed by estimating a gravity equation extended to include measures of linguistic and religious distance as well as common history variables that potentially affect effectiveness of interaction and communication, all obtained from CEPII. We also investigate whether trade plays any role in driving the diffusion speed once distances and historical variables are controlled for. Citing and cited country fixed effects are included to control for country-specific characteristics such as size, level of development, and geography. Since we are interested not only on cross-country but also on cross-sector knowledge diffusion, we also include patent stock in citing and cited country-sector, and directional sector-pair fixed effects to capture the innate knowledge spillover relationship between different technologies

(Cai and Li, 2016) that are independent of the source and destination countries.

Column (1) to (3) show that knowledge in sectors of country i diffuses faster to sectors of country n when the two countries are linguistically closer to each other or share a common language, share a border or in the same continent, both belong to the same regional free trade agreement (FTA) or currency union, were ever in a colonial relationship before 1945, have shared a common colonizer or were once the same country, have a different latitude. One country being landlocked or one country once being the colony of the other reduces the knowledge diffusion speed. Interestingly, geographic distance does not play a significant negative role and even an insignificant positive role on knowledge diffusion once trade linkages—that is exports between any country-sector pair combinations— are controlled for. Trade linkages are significantly and positively associated with knowledge linkages. The size of knowledge stock, as reflected in the patent stock, also matters. Higher the stock of knowledge the faster the diffusion speed, while countries with similar knowledge structure tend to diffuse slower.

Table 3: Determinants of Knowledge Diffusion Speed across Countries and Sectors

Dependent variable: $\log \varepsilon_{ni}^{jk}$			
	(1)	(2)	(3)
Georgraphic Distance$_{ni}$)	-0.071	0.038	0.014
	(-1.66)	(0.86)	(0.31)
Border$_{ni}$	0.349***	0.355***	0.348***
	(5.64)	(5.73)	(5.61)
FTA$_{ni}$	0.259***	0.214***	0.225***
	(5.66)	(4.67)	(4.88)
Currency Union$_{ni}$	0.582***	0.596***	0.585***
	(9.83)	(10.06)	(9.87)
Common language$_{ni}$	1.035***	0.997***	0.988***
	(18.77)	(18.01)	(17.82)
landlock$_{ni}$	-1.091***	-1.154***	-1.162***
	(-9.25)	(-9.80)	(-9.84)
absolute distance in lattitude$_{ni}$	0.015***	0.016***	0.016***
	(9.07)	(9.18)	(9.31)
absolute distance in longtitude$_{ni}$	-0.000	-0.000	0.000
	(-0.21)	(-0.40)	(0.01)
Common continent$_{ni}$	0.309***	0.289***	0.277***
	(5.00)	(4.68)	(4.47)
Linguistic Distance$_{ni}$	-1.732***	-1.663***	-1.663***
	(-9.80)	(-9.40)	(-9.40)
Religious Distance$_{ni}$	0.301*	0.263	0.203
	(2.12)	(1.86)	(1.41)
Colony$_{ni}$	-0.551***	-0.555***	-0.552***
	(-7.13)	(-7.17)	(-7.13)
Common Colonizer$_{ni}$	1.754***	1.730***	1.722***
	(6.23)	(6.16)	(6.14)
Colony after 1945$_{ni}$	0.998***	0.991***	1.021***
	(5.89)	(5.85)	(6.01)
Same country$_{ni}$	2.185***	2.145***	2.141***
	(20.72)	(20.36)	(20.32)
$\log X_{in}^{jk}$		0.025***	0.027***
		(5.18)	(5.48)
$\log X_{ni}^{jk}$		0.015**	0.013**
		(3.07)	(2.59)
$\log X_{in}^{kj}$		0.012*	0.010*
		(2.52)	(2.02)
$\log X_{ni}^{kj}$		0.029***	0.030***
		(5.86)	(6.17)
log Patent Stock$_{n}^{j}$			0.025***
			(3.66)
log Patent Stock$_{i}^{k}$			0.026***
			(3.87)
$Similarity_{ni}$			-0.256**
			(-2.72)
Citing country FEs	Yes	Yes	Yes
Cited Country FEs	Yes	Yes	Yes
Sector-pair FEs	Yes	Yes	Yes
R^2	0.52	0.52	0.52
Num of obs	272,916	272,916	272,916

E The Balanced Growth Path

Here, we derive an expression for the growth rate of the economy along the BGP. First, note that through technology diffusion, the level of "knowledge-related productivity", A_n^j, grows at the same rate for every country n and sector j. Therefore, we can pick country M and sector J's technology level to normalize every A_n^j and T_n^j. Normalized variables are denoted with a hat. In particular, $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$.

From equation (53), we normalize the value of an innovation as $\hat{V}_n^j = \frac{V_n^j T_M^J}{W_M}$. Then, from equation (55), profits are normalized as $\hat{\Pi}_n^j = \frac{\Pi_n^j}{W_M}$, and from equation (48), X_i^j are normalized as $\hat{X}_i^j = \frac{X_i^j}{W_M}$ for all j. Hence, expenditures grow at a constant rate for all sectors, since π_{in}^j is constant in the BGP (see equations (41) and (46)). From equations (48) and (50), $P_n Y_n$ grow at the rate of W_M. Note that $g_{w_n} = g_w$ for all n.

To derive an expression for the BGP growth rate of the real output per capita, Y_n, we start from the fact that $\frac{W_n}{P_n Y_n}$ is constant in steady-state. Hence,

$$g_{Y_n} = g_w - g_{P_n}$$

Using equation (47),

$$g_{P_n} = \sum_{j=1}^{J} \alpha^j g_{p_n^j}$$

We then derive the expression for $g_{p_n^j}$ from equations (44), (45) and (46). First, we rewrite equation (44) as

$$\frac{c_n^j}{W_n} = \prod_{k=1}^{J} \left(\frac{p_n^k}{W_n} \right)^{\gamma_n^{jk}}$$

In growth rates, it becomes

$$g_{\tilde{c}_n^j} = \sum_{k=1}^{J} \gamma_n^{jk} g_{\tilde{p}_n^k} \tag{69}$$

where $\tilde{c}_n^j = \frac{c_n^j}{W_n}$ and $\tilde{p}_n^k = \frac{p_n^k}{W_n}$. From equation (46),

$$g_{\Phi_n^j} = g_T - \theta g_{c_n^j} = g_T - \theta g_{c_i^j}$$

with $g_T = g_A$.

Hence, $g_{c_n^j} = g_{c^j}$ for all n. Normalizing by wages,

$$g_{\tilde{\Phi}_n^j} = g_T - \theta g_{\tilde{c}_n^j} \tag{70}$$

where $\tilde{\Phi}_n^j = \frac{\Phi_n^j}{W_n^{-\theta}}$

Combining equation (45) and (70) implies that

$$g_{\tilde{p}_n^k} = -\frac{1}{\theta} g_T + g_{\tilde{c}^k} \tag{71}$$

Substitution into (69) and using $\sum_{k=1}^{J} \gamma^{jk} = 1 - \gamma^j$, we get

$$g_{\tilde{c}^j} = -\frac{(1-\gamma^j)}{\theta} g_T + \sum_{k=1}^{J} \gamma^{jk} g_{\tilde{c}^k} \tag{72}$$

We can express the previous expression in matrix form so that:

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{1}{\theta} g_T \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} + \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix} \begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} \tag{73}$$

From here

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{g_T}{\theta} (I - A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} \tag{74}$$

where

$$A = \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \vdots & \ddots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix}$$

Therefore, the cost of production c_n^j can be normalized as

$$\hat{c}_n^j = \frac{c_n^j}{W_M (T_M^J)^{-\frac{1}{\theta}\Lambda_j}} \tag{75}$$

with Λ_j is the jth entry of the vector $\Lambda = (I - A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix}$.

With this, we can obtain an expression for the growth rate of real output as

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j g_{p_n^j}$$

From Equation (71), we have

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_{c^j} \right)$$

Based on Equation (75), the above equation becomes

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_w - \Lambda_j g_T \right)$$

Therefore,

$$g_{Y_n} = \frac{1}{\theta} \left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j \right) g_T = g_y, \forall n \tag{76}$$

Note that in a one-sector economy in which $\gamma^{jk} = 0, \forall n, k$ and $\gamma^j = 1, \forall j$, the growth rate is

$$g_y = -\frac{1}{\theta} g_T$$

as in Eaton and Kortum 1996 and Eaton and Kortum 1999. With multiple sectors, however, the growth rate of the economy is amplified by the input-output linkages.

F One Sector Model

We show that in a one-sector version of our model, change sin trade barriers have no effect on the optimal R&D intensity, hence on growth rates along the BGP. In the one-sector model, $\gamma^j = 1$ and $\gamma^{jk} = 0$. The one-sector version of equations (62), (63) and (64) is

$$\hat{W}_n L_n = \sum_{i=1}^{M} \pi_{in} \hat{X}_i, \tag{77}$$

$$\hat{X}_n = \hat{Y}_n, \tag{78}$$

$$\hat{Y}_n = \hat{W}_n L_n + \frac{\sum_{i=1}^{M} \pi_{in} \hat{X}_i}{1+\theta}, \tag{79}$$

Using equations (77) and (79),

$$\hat{Y}_n = \frac{1+\theta}{\theta} \hat{W}_n L_n$$

and

$$\frac{\sum_{i=1}^{M} \pi_{in} \hat{X}_i}{1+\theta} = \frac{\hat{Y}_n}{2+\theta}$$

From equations (66) and (67) in a one sector model with royalties:

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r \lambda_n \frac{\hat{V}_n}{\hat{Y}_n} = \beta_r \lambda_n \frac{1}{\rho - g_y + g_A} \frac{\sum_{i=1}^{M} \frac{\varepsilon_{in}}{g_A + \varepsilon_{in}} \frac{\sum_{m=1}^{M} \hat{X}_m \pi_{mi}}{1+\theta}}{\hat{Y}_n} \tag{80}$$

Using the previous expression

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r \lambda_n \frac{\hat{V}_n^j}{\hat{Y}_n} = \beta_r \lambda_n^j \frac{1}{\rho - g_y + g_A} \frac{1}{2+\theta} \sum_{i=1}^{M} \frac{\varepsilon_{in}}{g_A + \varepsilon_{in}} \frac{\hat{Y}_i}{\hat{Y}_n} \tag{81}$$

In this case, changes in trade costs have an effect on optimal R&D intensity to the extent that they have an effect on $\frac{\hat{Y}_i}{\hat{Y}_n}$.

If there are no royalties, the above expression becomes

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r \lambda_n^j \frac{1}{\rho - g_y + g_A} \frac{1}{2+\theta} \tag{82}$$

In this case, changes in trade costs do not have an effect on optimal R&D intensity, hence on the growth rate along the BGP.