Bargaining on Appellate Courts

Giri Parameswaran[a], Charles M. Cameron[b] , and Lewis A. Kornhauser[c]

[a]Haverford College, gparames@haverford.edu
[b]Princeton University, ccameron@princeton.edu
[c]New York University School of Law, lewis.kornhauser@nyu.edu

December 1, 2018

Abstract

We present a sequential bargaining model of a multi-member appellate court that extends similar models developed to study legislatures. The judges decide a case by majority rule, and announce a legal rule compatible with the case decision. The members of the dispositional majority bargain over which rule to announce. Unique to U.S. Courts, the announced policy must have the support of a majority of the bench, which often requires a super-majority of those in the dispositional majority coalition. We study the properties of equilibria, including the extent of strategic voting, as a function of case location and salience. We show that the median judge is pivotal over case dispositions, but not the announced policy. As delay becomes costless, the dispositional majority endogenously separates into two cohesive factions. The announced policy is the either the ideal policy of some pivotal judge (not necessarily the median), or the result of asymmetric Nash Bargaining between representative leaders of the factions, with bargaining weights proportional to factional size.

Key Words: Bargaining, Judicial Politics,
JEL Codes: C78, H8, K40

1 Introduction

American appellate courts perform a dual function when they decide cases. First, they decide which party to the dispute prevails. Second, they give reasons for their judgment in an opinion that typically announces a rule or policy which will be applied in future cases.[1] Much of the existing literature focuses exclusively on one of these functions, conceiving of the court either as a 'jury' (see Fischman (2008), Fischman (2011) or Iaryczower and Shum (2012)) or as a 'legislature' (see Hammond, Bonneau and Sheehan (2005), Bonneau et al. (2007)). However, as we outline below, these two functions interact with, and constrain, one another. For example, the court is constrained to choose policies that rationalize the disposition of case. The composition of the dispositional majority also determines which judges participate in crafting policy.

In this paper, we build a theoretical model of adjudication that explores the implications for decision-making that are a consequence of this dual role. We present several results that are novel in the literature: amongst them that: (i) the median judge on the bench is pivotal in the court's choice of disposition, but not policy; (ii) the median judge may act strategically, so that the disposition of the court is at odds with her sincere belief about the ideal disposition; (iii) the court's chosen policy depends on the composition of the dispositional majority, and typically depends on the identity of the opinion writer; and (iv) even when the author exerts no agenda-control, the chosen policy will not coincide with the ideal policy of either the median judge on the bench, or the median judge within the dispositional majority. Indeed, we show that the equilibrium policy depends on ideal policies of all judges in the dispositional majority, and that all but the most extreme judges may potentially be pivotal. Furthermore, we show that the case disposition, the composition of the dispositional majority, and the policy rules that result, are all sensitive to the facts of the case being decided. An implication of our model is that the judicial nominations are not simply about 'moving the median'.

The model adapts the sequential bargaining approach developed by Baron and Ferejohn and others to study legislatures (Baron and Ferejohn (1989), Banks and Duggan (2006), Jackson and Moselle (2002), McCarty (2000), Kalandrakis (2010), inter alia) to the study of appellate courts. We know of no prior application of sequential bargaining models to institutions that use the peculiar voting rule that U.S. appellate courts commonly use: pure majority rule

[1] Some independent regulatory agencies employ procedures modeled on those of the U.S. Supreme Court. More generally, appellate procedures differ dramatically across legal systems. English courts, for example, follow a *seriatim* practice in which each judge announces her own dispositional vote and reasons for it. The French Cour de Cassation, by contrast, uses a *per curiam* practice in which the court announces a single, unsigned, ostensibly unanimous opinion. For a more general discussion, see Kornhauser (2008).

over the disposition followed by k-majority rule over policy with an electorate restricted to the members of the dispositional majority. We follow Baron and Ferejohn (1989) and subsequent papers and focus on stationary sub-game perfect equilibria. These equilibria are unique and, arguably, have a focal quality. We are able to completely characterize stationary sub-game perfect policy offers and policy voting strategies for all ideological make-ups of all Courts and all discount rates, conditional on the majority dispositional coalition.

As in Rubinstein (1982), Binmore (1987), and Baron and Ferejohn (1989), an important parameter is $\delta \in [0, 1]$, typically interpreted as the bargainers' impatience during bargaining. (Alternatively, it can be thought of as the cost of making counter-proposals.) In the judicial context, we think δ is better understood as a *toughness* parameter that measures the parties' willingness to compromise. High levels of δ lead to "tough" bargaining in which non-writers demand compromises from the opinion author if they are to join the majority opinion. Low levels of δ lead to softer bargaining in which non-writers are willing to join opinions with few compromises. Plausibly, δ rises with a case's importance, and with greater resources per case. Thus, changes in δ are readily interpretable as changes in substantively important attributes of cases such as their importance, or attributes of different courts such as case load and resources. In this paper, we pay particular attention to the behavior of judges as $\delta \to 1$.

The model makes makes several substantive contributions to our understanding of appellate adjudication, as well as offering a novel application of the sequential bargaining approach to political institutions.

First, the model analyzes the impact of *strategic dispositional voting* on opinion content and case dispositions. Because strategic dispositional voting appears to be common in some appellate courts such as the U.S. Courts of Appeals, this analysis is arguably important.[2] In the model, strategic dispositional voting can be quite consequential for opinion content. However, when the location of the case before the court lies within the interval spanned by the judges' ideal points, strategic dispositional voting does not alter the winning disposition. On the other hand, if the case is extreme (but not too extreme) and δ is sufficiently small, strategic dispositional voting may actually change the disposition of the case. Under strategic dispositional voting, unanimous dispositions are the predominant outcome (as they are on the U.S. Courts of Appeals).

[2]Using a data set of immigration decisions concerning asylum and a theoretical model in which judges on a three-judge panel decide only the disposition, Fischman (2008) estimates that at least one judge votes strategically in 45% of the cases and two judges vote strategically in 8% of the cases.

Second, the *toughness or case importance* δ strongly affects the character of the bargaining. Although agreement is immediate, many values of δ lead ex ante to a *distribution of opinion locations* rather than a single location. The distribution reflects probabilistic opinion assignment during the bargaining game over policy and the fact that values of δ even modestly lower than 1 lead to a pronounced first-mover advantage for the realized assignee. Indeed, for sufficiently low values of δ, the assignee is able to impose her most-preferred policy on the majority coalition. However, as δ goes to 1, the distribution of equilibrium opinion locations converges to a single point, so the first-mover advantage degrades.

We provide an explicit characterization of the limit equilibria, extending insights presented in Parameswaran and Murray (2018) and Predtetchinski (2011). We verify the well known result, that under simple majority rule (which in our modelcvtranslates to the case of a unanimous dispositional vote), the limit cpolicy coincides with the median judge's ideal (see Cho and Duggan (2009)) . However, when the decision is non-unanimous, the limit policy neither privileges the median judge on the bench (as predicted by Hammond, Bonneau and Sheehan (2005)) nor the median judge within the majority coalition (as predicted by Cardona and Ponsatí (2007)). Nor need it coincide with some 'average' policy. Instead, the limit policy coincides with the asymmetric bilateral Nash Bargaining solution between two 'decisive' judges in the dispositional majority. The bargaining weights are determined endogenously in equilibrium, as a function of the ideal policies of all agents in the dispositional majority. We show that the limit policy is generically a strict subset of the 'core'. Moreover, depending on the arrangement of judges' ideal policies, the limit policies may coincide with the ideal policies of a subset of judges in the dispositional majority (not necessarily the median).

Third, the *location of the selected case* affects both the disposition of the case and the content of the majority opinion under both sincere and strategic voting. Hence, the model suggests that case selection matters in appellate jurisprudence because some cases conduce to particular policy outcomes rather than others. Again, a full discussion is beyond the scope of this paper, but a "bargaining vehicle" theory of case selection is quite different from current theories of certiorari highlighting control or learning in the judicial hierarchy (Cameron, Segal and Songer (2000), Carrubba and Clark (2012), Beim (2017)).

The paper is organized in the following way. Section II presents the model. Section III examines policy bargaining within the dispositional majority. For clarity of exposition, we focus on bargaining majorities created by sincere dispositional voting. Section IV analyzes dispositional voting. Section V concludes. All proofs are presented in the Appendix.

2 The Model

2.1 Cases, Dispositions and Rules

There is a court consisting of n judges, (assumed odd), which must decide a case. A *case* z encodes the details of an event, that has occurred, for example, the level of care exercised by a manufacturer or the intrusiveness of a search by the police. Let $Z = [0, 1]$ be the case space. A judicial *disposition* $d \in D = \{0, 1\}$ of the case determines which party prevails in the dispute between the litigants. Judges dispose of cases by applying a legal rule.

A legal rule $r : Z \to D$ maps the set of possible cases into dispositions. Let $X = 2^Z$ be the space of possible rules. We focus on an important class of legal rules, *cutpoint*-based doctrines, which take the form:

$$r(z; y) = \begin{cases} 1 & \text{if } z \geq y \\ 0 & \text{otherwise} \end{cases}$$

where y denotes the cutpoint. For example, in the context of negligence, the defendant is not liable if she exercised at least as much care as the cutpoint y.[3] Let X^C be the space of cut-point rules. We have $X^C = \{\{[0, y), [y, 1]\} \mid y \in [0, 1]\}$. It should be clear that rules live in an entirely different space to cases. The special structure of cut-point rules allows us to summarize them in terms of a threshold in case-space.

2.2 Decision Making by the Court

Decision-making by the justices occurs in two distinct stages. In the first stage, each judges cast a dispositional vote ($d^j \in \{0, 1\}$), and the disposition of the case is determined by simple majority rule. The dispositional votes of each judge separate the judges into dispositional majority ($M \subset \{1, .., n\}$) and minority coalitions. By construction, $|M| \geq \frac{n+1}{2}$.

In the second stage, the justices in the dispositional majority must agree upon a legal rule y that rationalizes the chosen disposition. Consistency requires that $y < z$ if $d = 1$ and

[3] Other examples include allowable state restrictions on the provision of abortion services by medical set providers; state due process requirements for death sentences in capital crimes; the degree of procedural irregularities allowable during elections; the required degree of compactness in state electoral districts; and the allowable degree of intrusiveness of police searches. Many other examples of cutpoint rules may suggest themselves to the reader.

$y > z$ otherwise. The judges in the dispositional majority bargain over the legal rule to be implemented.

We formalize this by studying a bargaining model á la Baron and Ferejohn (1989) and Banks and Duggan (2000). Initially, a judge from the dispositional majority is recognized to propose a policy y that is consistent with the majority's disposition. Upon seeing the proposal, each justice in the dispositional majority either votes to endorse the proposed opinion by 'joining' or declines to endorse the opinion by 'concurring'. To become the policy of the court, the proposal requires the assent of a majority *of the entire court*, not just the dispositional majority. Thus, in many cases, the dispositional majority will bargain under an effective super-majority rule. If the proposal is accepted, it is implemented and the bargaining game ends. Else, the justices retire, and the process repeats itself in the following period, and this continues until a policy of the court emerges. Delay within the bargaining game is costly, and the judges share a common discount factor $\delta \in (0, 1)$.

In the first period of bargaining, we allow the identity of the proposing judge to be non-random, reflecting current practice where the most senior judge in the dispositional majority determines who will write the opinion. However, in subsequent bargaining periods, we assume judges are recognized probabilistically. Let p_j^M denote the probability that judge j is recognized when the majority coalition is M, and let $p^M = (p_1^M, ..., p_n^M)$. In principle, for each M, our model could entertain any generic probability vector p^M satisfying $p_i^M = 0$ for all $i \notin M$ (although see footnote XXX). However, we simplify the analysis by assuming that:

$$p_j^M = \begin{cases} \frac{p_j^N}{\sum_{i \in M} p_i^N} & \text{if } j \in M \\ 0 & \text{otherwise} \end{cases}$$

where $N = \{1, .., n\}$ is the unanimous majority coalition. Hence, we allow for some judges to be more likely to be recognized than others, perhaps reflecting their particular interest or expertise in an area of the law, but require that the composition of coalitions not change the relative likelihood of being recognized.

2.3 Judicial Preferences

Following Carrubba et al. (2012) and Cameron and Kornhauser (2008) , we assume that judges' preferences exhibit both *expressive* and *policy* components. Policy utility depends on the actual policy implemented by the dispositional majority, and stems from the judge's

concern for how future cases will be decided. Expressive utility depends neither on the policy chosen, nor on the actual disposition of the case, but rather, on the judge's individual vote in the instant case.[4] Whereas policy preferences are consequentialist – they depend on actual outcomes – expressive preferences simply reflect the judge's desire to be seen to decide cases correctly, regardless of if or how their vote changes actual outcomes. As will become clear, absent an expressive component of utility, judges would never have an incentive to dissent. Rather than taking an *ad hoc* approach to specifying these preferences, we present a framework that makes sense of both components in a cohesive way. We begin by specifying the dispositional preferences of a given judge, and build both expressive and policy preferences from this.

Suppose judge j has ideal threshold x^j, and that $0 \leq x^1 \leq .... \leq x^n \leq 1$, so that the judges are ordered by ideal threshold. Judge j's *dispositional utility* is:

$$u_D(d^j; z, x^j) = \begin{cases} 0 & \text{if } d^j = r(z; x^j) \\ l(z - x^j) & \text{if } d^j \neq r(z; x^j) \end{cases}$$

where $l(\cdot)$ is a quasi-concave 'loss' function that satisfies $l(0) = 0$ and $l(\cdot) < 0$ otherwise. (I.e. l has a single peak at 0.) The strict quasi-concavity of l implies that dispositional preferences satisfy the *increasing differences in dispositional values (IDID)* property (see Cameron, Kornhauser and Parameswaran (2019)), which implies that voting insincerely is costly, and this cost becomes larger the further is the case from the threshold x^j. Intuitively, judges feel more strongly about 'incorrectly' deciding 'clear-cut' cases (those far from the threshold), than 'contestable' ones (those close to the boundary that separates acceptable and unacceptable conduct). The expressive component of a judge's utility is simply the dispositional utility associated with the outcome she votes for.

Consider a generic cut-point rule y. The choice of y implicates the disposition of future cases that will be decided by courts.[5] For simplicity, suppose one case arises in each future period, and that cases are independent draws from a continuous distribution $F(z)$ that admits a density fz). The judge's *policy* utility is her expected per-period dispositional utility from having the rule implemented, given the distribution over cases that are likley to arise. We

[4]Cameron and Kornhauser (2008) treats the utility of casting join vs concur votes as expressive; in contrast, here the value of such votes comes from the policy resulting from votes.

[5]To be clear, the court faces a single live decision problem: to choose a policy rule. Future courts are bound by this rule, and will mechanically dispose of future cases accordingly. Hence, our model is 'static', although we motivate judicial preferences by implicitly appealing to a dynamic model.

have:

$$u_P(y; x^j) = \int u_D(r(z, y); z, x^j) dF(z)$$

The *IDID* property implies that policy utility $u_P(y; x)$ is strictly quasi-concave in y for every x, although it is not necessarily concave. Moreover, the *IDID* property implies that, whenever $x^i > x^j$, $\frac{\partial u_P(y;x^i)}{\partial y} > \frac{\partial u_P(y;x^j)}{\partial y}$, or equivalently, $\frac{\partial^2 u_P(y,x)}{\partial x \partial y} > 0$.[6] Hence, preferences satisfy the Spence-Mirrlees condition. The benefit from marginally increasing the policy y is monotone in the judges' ideal policies.

Example 1. Two special cases are worth noting: Suppose cases are uniformly distributed on $[0, 1]$. Then:

- If $l(z - x) = -1$, then $u_P(y; x) = -|y - x|$.
- If $l(z - x) = -2(z - x)^2$, then $u_P(y; x) = -(y - x)^2$.

Overall utility is:

$$u_P(y; x^j) + D u_D(d^j; z, x^j)$$

where $D \geq 0$ denotes the relative importance of the expressive component of utility.

To motivate judicial preferences further, consider a dynamic model in which the court decides a single case in each future period, and that the court discounts the future at rate $\rho \in (0, 1)$.[7] Then, judge j's lifetime expected utility given the current case z, the chosen policy y and the distribution over future cases F is:

$$u_D(r(z; y); z, x^j) + \frac{\rho}{1 - \rho} u_P(y; x^j)$$

Setting $D = \frac{1-\rho}{\rho}$, gives D the natural interpretation as the importance to utility of the current case relative to the future stream. As $D \to 0$, the court becomes perfectly future oriented, whereas as $D \to \infty$, the court ignores the future entirely. Notice that this formulation almost coincides with our formulation of the judge's overall utility.[8]

[6] To see this, note that, for any policy y and any ideal policy x, $\frac{\partial u_P(y;x)}{\partial y} = (-1)^{mathbb1[y<x]} l(y - x)$. Then, $\frac{\partial^2 u_P(y,x)}{\partial x \partial y} = (-1)^{mathbb1[y>x]} l'(y - x) > 0$, since $l'(z) > 0$ if $z < 0$ and $l'(z) < 0$ otherwise.

[7] To clarify, the discount factor δ captures the cost of delay in the bargaining phase of the court's deliberations in a single case. The discount factor ρ is reflects the judges' present bias, and the passage of time *between* cases.

[8] The difference is that the latter depends on the actual disposition of the instant case, whereas the former depends only on judge j's dispositional vote in the instant case.

We should note the role of the "legal status quo" within the bargaining game, as this point has engendered some controversy among judicial scholars. Although there is a prior legal policy, this policy effectively reverts to a null policy when the Court takes the case – policy is in limbo until the Court resolves the case. In fact, the only way for policy to revert to the status quo *ante* is for the Court to re-enact it anew in the majority opinion. So, the players do not receive a 'status quo' payoff during rounds of bargaining, but rather zero each round there is no agreement on a new policy. We finesse the issue of whether the Court would ever take another case in a policy area, that is, whether the announced policy is actually time-consistent and renegotiation-proof. In effect, we assume the Court commits to its announced policy, but we do not study the commitment mechanism. None of the current models of collegial courts address the time-consistency of the bargaining outcome (but note Rasmusen (1994) and Cameron, Kornhauser and Parameswaran (2019)). Recent legislative models of sequential policy making with evolving status quos determined by earlier rounds of policy-making are suggestive (Baron and Ferejohn (1989), Kalandrakis (2010)) but we do not pursue this point any farther in this paper.

2.4 Strategies and Continuation Values

Given the repeated game structure of bargaining, strategies can be quite complex, as they may be history dependent. We restrict attention to stationary strategies, which require the players to choose equivalent strategies in every structurally equivalent sub-game.

A strategy for judge j is a triple (d^j, y^j, A^j), where:

- $d^j(z;\delta)$ denotes the judge's dispositional vote, given a case z, the vector of ideal policies $\mathbf{x} = (x^1, .., x^n)$, and the discount factor δ.
- $y^j(z, M, \delta)$ denotes the policy proposed by the judge, whenever she is in the dispositional majority and recognized to make a proposal, where $M \subseteq \{1, .., n\}$ is the composition of the dispositional majority.
- $A^j(z, M, \delta)$ denotes the set of proposals that the judge will accept, whenever she is in the dispositional majority.

We solve for stationary sub-game perfect equilibria.

3 Bargaining over Policies within the Dispositional Majority

In this section we characterize the outcome of policy bargaining for a generic dispositional majority. In section 4, we find the optimal dispositional coalition, given the policy bargaining that is anticipated to follow.

3.1 Equilibrium Characterization

Let z be the case, and suppose the dispositional majority coalition $M \subseteq \{1, ..., n\}$ contains $m \in \{k, ..., n\}$ members, where $k = \frac{n+1}{2}$. Without confusion, we re-label the judges in the coalition, preserving the ordering of ideal policies, so that $M = \{1, ..., m\}$ with $x^1 \leq .... \leq x^m$. (Once the majority coalition has been determined, the preferences of the non-majority judges become inconsequential, so we are free to disregard them, and focus on the m remaining judges.) Similarly, let p_i denote the recognition probability of (relabled) judge j (where we omit the superscript denoting the coalition for convenience).

We note that we are now free to focus solely on policy utility, since dispositional utility was determined at the time of the dispositional vote. If the majority disposition was 1, the majority must choose a policy in the interval $[0, z]$, whilst if the disposition was 0, it must choose a policy in the interval $[z, 1]$. Generically, the court's policy must be contained in $[\underline{x}, \overline{x}]$, where $\underline{x} \in \{0, z\}$ and $\overline{x} \in \{z, 1\}$.

Banks and Duggan (2000) show, in a framework that embeds ours, that equilibria must be in no delay. Let $y^i \in [\underline{x}, \overline{x}]$ be the equilibrium proposal of judge i in the majority. The payoff to judge j in the continuation play is: $v_P(x^j) = \sum_{i=1}^{m} p_i u_P(y^i; x^j)$. Then, judge j will accept any proposal that has weakly higher utility than her continuation payoff. We have:

$$A^j = \{y \in [\underline{y}, \overline{y}] \,|\, u_P(y; x^j) \geq \delta v_P(x^j)\}$$

It is easily verified that $u_P(y; x)$ is single-peaked in y for each x (i.e. it is strictly quasi-concave). This implies that A^j is an interval for each judge. We have $A^j = [\underline{y^j}, \overline{y^j}]$ where $\underline{x} \leq \underline{y^j} < \overline{y^j} \leq \overline{y}$ are the left and right left and right certainty equivalents induced by the continuation game.

For any winning policy coalition (i.e. any $C \subset M$ containing at least $k = \frac{n+1}{2}$ judges), let $A_C = \cap_{j \in C} A^j$ be the set of policies that is jointly acceptable to all judges in coalition C. Since each A^j is a convex set, so will be A_C. The social acceptance set $A = \cup_{\{C \subset M \,:\, |C| \geq k\}} A_C$ is the set of policies that are acceptable to *some* winning policy coalition. We establish in the proof of Proposition 1 that A is a non-empty interval, $A = [\underline{y}, \overline{y}]$.

That fact that preferences satisfy the Spence-Mirrlees condition has the following implication: For any two policies, y, y' with $y' > y$, if player i prefers y' to y, then every player j with $x^j > x^i$ will also prefer y' to y. Similarly, if judge i prefers y to y', then so will all judges j with $x^j < x^i$. It follows that, in equilibrium, the sub-coalitions who support and reject a given proposal will both be connected.[9] Since the proposer only needs the support of $k = \frac{n+1}{2}$ judges, it suffices to either earn the support of the left-most k judges $\{1, ..., k\}$ in the dispositional majority, or the right-most k judges $\{m - k + 1, ..., m\}$, where judge $m - k + 1$ is the k^{th} judge from the right. It follows that judges $\{m - k + 1, ..., k\}$ must be in every equilibrium coalition. Moreover, judges $m - k + 1$ and k are *decisive* in the sense that, a proposal that loses their support cannot be winning. Following Parameswaran and Murray (2018), we refer to these as the *left* and *right decisive judges*, respectively. It is evident that the boundaries of the social acceptance set will depend on the preferences of the left and right decisive judges.

Note that, if $m = n$, so that the policy coalition need only be a simple majority of the dispositional coalition, then the left and right decisive judges will both coincide with the median judge. By contrast, for any $m < n$, $m - k + 1 < k$, and so, generically, the decisive judges will be non-median players, with distinct preferences.

Given a social acceptance set $A = [\underline{y}, \overline{y}]$, it is straight-forward to find the equilibrium offer of each judge. Clearly, any judge whose ideal policy lies in the social acceptance set will simply propose their ideal policy, and this will be accepted. By contrast, judges whose ideal policies lie outside the social acceptance set will propose the closest policy in A (i.e. one of the boundary points) to their ideal policy.

For notational convenience, we index the left and right decisive judges by l and r, so that $l = m - k + 1$ and $r = k$. Given the above discussion, we have the following result, which has been previously noted by Cho and Duggan (2003),Cardona and Ponsati (2011),Parameswaran and Murray (2018), amongst others:

[9] To be clear – the decision rule does not require connected coalitions. Any policy coalition containing at least $k = \frac{n+1}{2}$ judges will be decisive, and in principle, such coalitions may be disconnected. However, disconnected coalitions will never optimally arise, in equilibrium.

Proposition 1. *For $\delta < 1$, the bargaining game admits a unique equilibrium, characterized by a pair $\left(\underline{y}, \overline{y}\right)$, with $\underline{x}\underline{y} \leq overline y \leq \overline{x}$, such that:*

1. *When judge j is recognized, she will propose:* $y^j = \begin{cases} \underline{y} & x^j < \underline{y} \\ x^j & x^j \in \left[\underline{y}, \overline{y}\right] \\ \overline{y} & x^j > \overline{y} \end{cases}$

2. *The pair $\left(\underline{y}, \overline{y}\right)$ satisfies:*

   - $\underline{y} = \min\{y \geq \underline{x} | u_P(y; x^r) \geq \delta \sum_j p_j u_P(y^j, x^r)\}$
   - $\overline{y} = \max\{y \geq \overline{x} | u_P(y; x^l) \geq \delta \sum_j p_j u_P(y^j, x^l)\}$

Roughly speaking, Proposition 1 states that judges with 'moderate' preferences will be able to successfully implement their ideal rule in equilibrium, whilst judges with (relatively) 'extreme' preferences must offer a compromise rule. All 'extreme left' judges will pool on the same proposal $\underline{y}$, whilst all 'extreme right' judges will pool on the same proposal $\overline{y}$. What constitutes 'moderate' and 'extreme' is itself determined in equilibrium, and depends on the preferences of the left and right decisive judges. $\overline{y}$ is the highest policy that the left decisive judge is willing to accept, given her continuation payoff. Similarly, $\underline{y}$ is the lowest policy that the right decisive judge is willing to accept, given her continuation utility. Any proposal in the region $\left[\underline{y}, \overline{y}\right]$ is equilibrium consistent.

To make sense of Proposition 1, let $E[y] = \sum_j p_j y^j$ be the expected policy that will be proposed (and accepted) in the continuation game. As we show in the appendix, if $E[y]$ is proposed, it will receive unanimous support.[10] Take a judge j in the dispositional majority, and suppose her ideal policy lies below the expected policy chosen in the continuation game (i.e. $x^j < E[y]$). Since $\delta < 1$, judge j can offer a policy slightly below $E[y]$ and still retain unanimous support. Decreasing the offer further, she will eventually lose the support of the right-most judge in the dispositional majority, then the second-most-right judge, and so on. Since it suffices to have the support of the right decisive judge, judge j will continue decreasing the offer until either she reaches her ideal policy, or the support of the right decisive judge would be lost. Hence, the lowest acceptable policy is pinned down by the preferences of the right decisive judge. A similar argument shows that the highest acceptable policy is determined by the left decisive judge.

[10] In typical bargaining games, this result follows from the concavity of players' preferences. In our model, preferences are not concave. However, the *IDID* property implies that policy preferences inherit some concave-like features.

Proposition 1 does not provide closed form expressions for $\underline{y}$ and $\overline{y}$. Instead, as is common in bargaining models, these are defined recursively. Given a conjecture $(\underline{y}, \overline{y})$, part 1 of the proposition determines each judge's optimal proposal. The implied values $\left(\hat{\underline{y}}, \hat{\overline{y}}\right)$can then be imputed by part 2 of the proposition. If the conjectured and implied thresholds coincide, they constitute an equilibrium. In fact, as the proof demonstrates, the mapping from conjectured to implied thresholds is a contraction mapping. This implies that the equilibrium is unique.

3.2 Comparative Statics on δ

The discount rate δ parameterizes the cost of delay in bargaining. Given the bargaining equilibrium, this entails that it is also a measure of the degree of agenda control that the proposer exerts. When $\delta = 0$, delay is so costly that the non-proposing judges will accept any feasible policy. The proposer thus has complete control over the agenda and will propose the feasible policy closest to her ideal.

As the following lemma shows, as δ increases, the range of proposals that are equilibrium consistent narrows. Moreover, in the limit as $\delta \to 1$, the proposer loses control of the agenda entirely, and in equilibrium, all judges will makes the same proposal.

Lemma 1. *In any equilibrium,* $\overline{y}(\delta) > \underline{y}(\delta)$ *whenever* $\delta < 1$. *Moreover,* $\overline{y}(\delta) - \underline{y}(\delta)$ *is decreasing in* δ, *and* $\lim_{\delta \to 1} \left(\overline{y}(\delta) - \underline{y}(\delta)\right) = 0$.

Lemma 1 states the familiar result that when delay is costly ($\delta < 1$), the proposer can pull the chosen policy somewhat towards her ideal. This results in equilibrium proposals that are dispersed around a mean. The lemma additionally shows that as $\delta \to 1$, the proposer's advantage disappears, and the best the proposer can do is to announce the expected policy. (See Predtetchinski (2011) and Parameswaran and Murray (2018).) The intuition is that, as $\delta \to 1$, delay becomes smaller, and so the decisive judges can both be more insistent that the equilibrium policy not be too far from the expected continuation policy. This causes the acceptance set to narrow. In the limit as delay becomes costless, the acceptance set collapses to a singleton, and so all judges make identical proposals. Thus, δ parameterizes the proposer's degree of agenda-control, and captures the extent to which policy outcomes depend on the particular whims of the judge chosen to author an opinion.

3.3 Limit Equilibria

In this subsection, we characterize the policy μ that is uniquely proposed in the limit as $\delta \to 1$. We focus on this limit for two reasons. First, as we will argue, this policy is a focal policy within the core, and is thus the policy that median-voter-type logic would select. (More on this, below.) Second, μ provides an approximate measure of the 'average' policy that would be proposed when $\delta < 1$. (Note – Proposition 1 implicitly characterizes the proposal that each judge will make in equilibrium. The true average policy is found by taking the expectation over these. However, because the proposition does not provide an explicit characterization, we cannot directly compute the average. In general, though, we might expect the average policy to be in a neighborhood of the limit equilibrium.)

A virtue of our bargaining approach (with $\delta < 1$) is that take seriously the influence that opinion authors exert on policy, constrained by the requirement to build a policy coalition. By contrast, much of the existing literature applies Median-Voter-like logic to predict the equilibrium policy of the court (see Hammond, Bonneau and Sheehan (2005), Carrubba et al. (2012)). Naturally, policy under this approach is not author dependent.

The Median Voter Theorem is the special case (under simple-majority rule) of the equilibrium concept of the *core*. The *core* is the set of policies for which there does not exist some other policy that is preferred to it by a winning coalition. Under simple majority rule, the core is uniquely the median judge's ideal policy (see Black (1948) and Downs (1957)). Under super-majority rule, the core is the set of policies bounded by the ideal policies of the left and right decisive voters. This is generically an interval and it contains the median voter's ideal policy. Hence, under super-majority rule, the notion of the core is insufficient to generate a unique equilibrium prediction analogous to the median voter theorem. We need a refinement criterion.

When committees deliberate under simple majority rule, Cho and Duggan (2009) show that the limit equilibrium of the associated bargaining game selects the median voter's ideal policy. Generalizing to decision procedures other than simple majority rule, Banks and Duggan (2000) establish an equivalence between the core and the equilibria of the bargaining game when $\delta = 1$. (Formally, the bargaining game admits a continuum of equilibria, each associated with a particular core policy that is proposed and accepted by all players.) Hence, there is an intimate connection between the core and the equilibria of the bargaining game when delay is costless. Moreover, when $\delta < 1$, the bargaining game admits a unique equilibrium, whilst for $\delta = 1$, under super-majority rule, there are generically a continuum of equilibria

that coincide with the core. Taking the limit as $\delta \to 1$, then, selects the core policy that is robust to introducing small costs to making counter-proposals. We think of this limit as a refinement of the core, whenever it contains multiple policies.

Parameswaran and Murray (2018) provide an explicit characterization of this robust policy. For each $i \in 1, .., m$, let $P_i = \sum_{j \leq i} p_j$ be the probability that the proposer is weakly to the left of judge i. Let $b_{i,i+1}$ be defined as follows:

$$b_{i,i+1} = \arg\max_y u_P(y, x^l)^{P_i} \cdot u_P(y, x^r)^{1-P_i}$$

To make sense of this policy, consider the following story: Suppose the judges in the dispositional majority separate into two connected factions $\{1, ..., i\}$ and $\{i+1, ..., m\}$ – which we dub the left and right factions. P_i is the probability that the recognized proposer in the continuation game belongs to the left faction. Suppose further that the factions behave as cohesive units and delegate decision making to the left and right decisive judges, respectively. (We can thus think of the left and right decisive judges as factional leaders.) The factional leaders then engage in asymmetric Nash Bargaining, with bargaining weights proportional to their faction's recognition probability, to determine the equilibrium policy. $b_{i,i+1}$ is the outcome that results from this procedure.

Let μ denote the limit equilibrium policy. Parameswaran and Murray (2018) show that the limit equilibrium of the bargaining game is closely related to this asymmetric Nash Bargaining outcome. They show the following result:

Proposition 2. *Let* $i^* = \min\{i \,|x_i > b_{i,i+1}\}$. *Then:* $\mu = \min\{x_i^*, b_{i^*-1,i^*}\}$

Proposition 2 characterizes the common equilibrium policy that all judges will propose when the cost of making counter-proposals becomes arbitrarily small. Depending on the arrangement of the judges' ideal policies, the limit proposal may either: (i) coincide with the ideal policy of one of the judges having ideal policies within the core, or (ii) be the solution to the asymmetric Nash Bargaining problem between the left and right decisive judges, with bargaining weights proportional to the (recognition probability weighted) support for each faction.

In strong contrast to existing results, our analysis shows that the equilibrium policy will generically not coincide with either the median judge on the bench, or the median judge in the dispositional majority. This should not be surprising. The logic of the median voter theorem is particular to decision making under simple majority rule. But, as we have argued,

policy-making by the court often proceeds under an effective super-majority rule, and in such cases, the median judge need not be pivotal.

Parameswaran and Murray (2018) describe the logic of this result in significant detail and we refer the interested reader to that paper. Here, we briefly provide intuition for the result. Conjecture that $\mu \in (x^{i-1}, x^i)$. Since $\underline{y} \to \mu$ and $\overline{y} \to \mu$, then for δ large enough, $x^{i-1} < \underline{y}(\delta) < \mu < \overline{y}(\delta) < x^i$.Hence, when δ is large, judges 1,...,$i-1$ will propose $\underline{y}(\delta)$ and judges $i, ..., m$ will propose $\overline{y}(\delta)$. It is as if the judges separate into cohesive factions, with all members of the same faction making the same equilibrium proposal. Furthermore, $\underline{y}(\delta)$ and $\overline{y}(\delta)$ were determined by the preferences of the right and left decisive judges, respectively. Hence, we can think of the left decisive judge as bargaining on behalf of the left faction, and the right decisive agent as bargaining on behalf of the right faction, with the bargaining weights being proportional to the (recognition probability weighted) size of their respective factions. Parameswaran and Murray (2018) show that, as $\delta \to 1$, this coincides with the asymmetric Nash bargaining solution between the left and right decisive agents.[11]

Notice that the separation into factions was endogenous to the policy chosen in equilibrium. Hence, for the asymmetric Nash bargaining solution to indeed be equilibrium consistent, it must be that this solution lies in the interval (x_{i-1}, x_i) — so that players separate into the factions as conjectured. As the proof of Proposition **??** shows, there is a unique player i^* that determines the composition of factions, in equilibrium. There are two possibilities. For a range of recognition probabilities and ideal policy arrangements, the factions $\{1, ..., i^* - 1\}$ and $\{i^*, ..., n\}$ induce a faction-size weighted Nash Bargaining solution that is equilibrium consistent — i.e. which falls in the required interval (x_{i^*-1}, x_{i^*}). Outside this range of parameters, the following problem arises: If i^* is conjectured to be in the left faction, then the location of the induced Nash Bargaining solution will cause i^* to defect to the right faction, and vice versa. Player i^* is *pivotal*. The only possibility is that the limit equilibrium coincides with i^*'s ideal policy, x_{i^*}.

The following example may help in building further intuition for the results in Proposition 2:

Example 2. Suppose $m = 6$ and $k = 5$, so that $l = 2$ and $r = 5$ (i.e. the court splits 6-3 in the dispositional vote), and assume each judge is recognized to counter-propose with equal probability. For simplicity, suppose $u_P(y, x) = 1 - |y - x|$ (which, we have previously shown is the policy utility that obtains when the dispositional loss function is constant), and

[11]Binmore, Rubinstein and Wolinsky (1986) establish a similar result in the Rubinstein two-person alternating offers bargaining setting.

normalize : $x_1 \leq 0 = x_2 \leq ... \leq x_5 = 1 \leq x_6$. Then the Nash Bargaining solutions are: $b_{1,2} = \frac{5}{6}$, $b_{2,3} = \frac{2}{3}$, $b_{3,4} = \frac{1}{2}$, $b_{4,5} = \frac{1}{3}$ and $b_{5,6} = \frac{1}{6}$. Notice that $b_{i,i+1}$ is decreasing in i. Then:

$$\mu = \begin{cases} b_{2,3} = \frac{2}{3} & x^3 > \frac{2}{3} \\ x^3 & \frac{1}{2} \leq x^3 \frac{2}{3} \\ b_{3,4} = \frac{1}{2} & x^3 < \frac{1}{2} < x^4 \\ x^4 & \frac{1}{3} \leq x^4 \leq \frac{1}{2} \\ b_{4,5} = \frac{1}{3} & x^4 < \frac{1}{3} \end{cases}$$

Let us check the logic of the Proposition through the example. Suppose $x^4 = 0.25 < \frac{1}{3}$. Conjecture that $\mu \in (x^4, x^5)$. Then, per the logic in the previous paragraph, for δ large enough, it must be that judges 1 through 4 choose $\underline{y}(\delta)$ and judges 5 and 6 choose $\overline{y}(\delta)$. It follows that the equilibrium policy will be the asymmetric Nash Bargaining solution where the the left decisive voter has weight $\frac{2}{3}$ and the right decisive voter has weight $\frac{1}{3}$. This solution is $b_{4,5} = \frac{1}{3}$, which lies between $x^4 = 0.25$ and $x^5 = 1$, as conjectured.

By contrast, suppose that $x^4 = 0.4$, and conjecture again that $\mu \in (x^4, x^5)$. The same logic would lead us to conclude that $\mu = b_{4,5} = \frac{1}{3}$. But this no longer satisfies the requirement that $x^4 < \mu < x^5$, since $x^4 = 0.4 > \frac{1}{3}$. Our conjecture is not equilibrium consistent. We can similarly show that conjecturing $\mu \in (x^3, x^4)$ is not consistent. Under this arrangement of ideal policies, judge 4 is pivotal. Conjecturing that her ideal policy is below the limit expected policy causes us conclude that it is, in fact, above. Instead conjecturing that his ideal policy is above the limit expected policy causes us to conclude that it is below. The only consistent alternative is that the limit policy coincides with x^4.

Let us note some features of the equilibrium mapping. First, for each judge whose ideal policy lies strictly within the core, there is some arrangement of ideal policies for which they are pivotal. For example, with 5-member majority coalition, the core is $[x^1, x^5]$. Then, it is possible that equilibrium policies reflect the ideal policies of any of judges 2, 3 and 4, as the cost of delay vanishes. In particular, the median judge in the majority is not generically privileged. Additionally, there are arrangements of ideal policies under which no judge is pivotal, and the equilibrium policy is simply the solution to the asymmetric Nash Bargaining problem between judges l and r. Note that, although the ideal policies of the remaining judges does not affect the equilibrium policy, these will matter in so far as they determine the weights attributed to each faction.

Second, for different arrangements of ideal policies, the limit equilibrium selects *some* policy

in the core. However, not every policy in the core can be sustained in equilibrium. In particular, policies too close to x^l and x^r are not sustainable. These intuitions hold even if several judges had identical preferences and voted as a block. For example, suppose $m = 5$ and $x^2 = x^3 = x^4$, so that the middle 3 judges constitute a homogeneous middle bloc. Then this middle bloc will be pivotal provided that $x^2 \in [b_{4,5}, b_{1,2}]$. For x^2 outside this interval, the 'extreme' judges will continue to exert some influence over the location of the equilibrium policy. By contrast, suppose $x^3 = x^4 = x^5$, so that there is a right-wing bloc. Notwithstanding that this bloc contains the median justice in the majority, it will never be pivotal, and the equilibrium policy will *never* reflect the bloc's ideal. Moreover, if $x^2 \in (b_{2,3}, b_{1,2})$, then judge 2 will be pivotal.

In this paper, we do not take up the issue of nominations to the bench. However, in concluding this section, we briefly note the stark implications of Proposition **??** for the president's optimal nomination's choice. An important implication of the proposition is that equilibrium outcomes depend not only on the relative ordering of the ideal policies of the judges, but their absolute location in state space. The president's nomination problem is, thus, not simply a 'move-the-median' game. The president could nominate two different judges, both occupying the same relative position in the ordering, but with different implications for the equilibrium policies chosen.

4 Dispositional Voting

We now consider dispositional voting, which occurs before the bargaining within the dispositional majority. We begin with some terminology. By "utility maximizing voting," we meant that the judge votes on the disposition to maximize its utility, correctly anticipating the effect of its dispositional vote on the choice of rule. By "sincere voting" we mean that the judge votes strictly in accord with its most preferred rule, without considering the effects of this dispositional vote on the composition of the dispositional majority and hence the content of the opinion. Using the notation defined earlier, sincere voting by judge j corresponds to $d^j = r(z, x^j)$. Sincere dispositional voting may sometimes be compatible with utility maximizing voting. "Sophisticated" dispositional voting means voting "insincerely" at the dispositional stage ($d^j \neq r(z, x^j)$), in order to affect subsequent bargaining over the opinion. So, sophisticated voting means voting "incorrectly" at the dispositional stage in order to join the dispositional majority and participate in bargaining over the opinion's content.

4.1 First Round Assignment

In the first stage, each judge's dispositional vote depends on their belief about the equilibrium policies that will result, given differently composed majority coalitions. In the previous section, we determined the policy that each judge would propose, in equilibrium. The policy that will likely be proposed depends on the judge who is selected by the chief justice (or the most senior judge in the majority) to draft the proposal. For each majority coalition $M \subset \{1, ..., n\}$, let $s(M, d, z) \in \{1, .., n\}$ denote the judge who is selected to make the first proposal. We must have $s \in M$ – the selected judge must be in the majority coalition. Moreover, let $\gamma(M, d, z) = y_s$ be the policy that the selected judge will propose in equilibrium.

The function s depends on the particular incentives faced by the chief (or most senior) justice. In a naive model, we might suppose that the chief is purely motivated to maximize her utility from the case. But this would imply that the chief justice always assigns the opinion to herself – an assumption at odds with the actual practice of recent chiefs. Indeed, the court has maintained a practice of trying to share the workload of opinion writing amongst its members. Such a policy might be rationalized by noting that opinion writing is costly, and so the chief makes her assignment choice taking into account the associated direct and opportunity costs. Other factors may also be at play. Given the many additional incentives that would need to be incorporated, it is clear that providing micro-foundations for the chief justice's selection is outside the scope of this paper.

Instead, we take a reduced form approach, taking the selection function s as given. We make the following assumption:

Assumption 1. *Let $M \subset \{1, ..., n\}$ be a majority coalition, and suppose $j \notin M$. Then $|\gamma(M \cup \{j\}) - x^j| \leq |\gamma(M) - x^j|$.*

Assumption 1 is a rationality condition, analogous to the weak axiom of revealed preference. It says that introducing a new member to the coalition shouldn't cause the chief to select a judge who would implement a policy that is worse from j's perspective. To build intuition, suppose $j > \max\{i \in M\}$, so that introducing judge j pulls the center of mass of the coalition to the right. Doing so might cause some previously-feasible left-wing policies to no longer be feasible, which would require the chosen policy to move in j's direction. Moreover, any left-wing policy that can now be implemented, could have been implemented even when j was not in the coalition. So, if she were to switch, one of her choices must have been sub-optimal.

Our reduced form approach is intended to capture a structurally sound decision making problem for the chief. Generically, the chief will not be indifferent between her choices. Moreover, we know that in an extensive form game with perfect information, pure strategy equilibria always exist. Hence, our focus on pure strategies for the chief justice (by assuming her selection is a singleton set) is not overly restrictive. (Additionally, we know that all judges make the same proposal in equilibrium as $\delta \to 1$, and so in this limit, the chief's selection is inconsequential. Any judge will do.)[12]

4.2 Optimal Dispositional Coalitions

Let $M^0(z)$ and $M^1(z)$ be the sets of agents who, if voting sincerely, would choose dispositions '0' and '1', respectively. Let d^* be the disposition of the court, and let M^* be the equilibrium majority coalition.

Lemma 2. *All judges who agree with the disposition of the court will vote sincerely. Formally, for each $i \in \{0,1\}$, if $d^* = i$, then $M^i \subseteq M^*$.*

Lemma 2 states that, if a judge is not dispositionally pivotal, and a sincere vote would put her in the majority, then she will vote sincerely. The intuition is straight-forward: The judge cannot affect the dispositional outcome. If she joins the majority, she can (weakly) draw the equilibrium closer to her ideal policy, whilst avoiding the hedonic cost of being insincere. By contrast, voting strategically, incurs both the explicit cost of being insincere and the opportunity cost of forgoing the ability to affect the policy in the second stage. For these agents, it is a strictly dominant strategy to vote sincerely.

Judges who disagree with the disposition of the court face a more interesting trade-off. Voting strategically enables them to influence the equilibrium proposal, but incurs the cost of insincerity. As we will see through the following results, there will often exist multiple equilibria in the voting game. To understand why, suppose the disposition of the court is $d^* = 1$. Let M be a coalition consisting of at least k judges, with $M^1 \subset M$, and γ_M be the associated equilibrium policy proposal. Take two judges, i and j, who, if they voted sincerely, would find themselves in the minority. It follows that $\gamma_M \leq z < x^i \leq x^j$. We consider two scenarios that illustrate the sources of multiplicity.

[12]Note – if we allowed mixed strategies, the results that follow would all continue to hold, provided that the variance in the first round proposal is not too large.

First, suppose $\gamma(M) = \gamma(M \cup \{i\}) = \gamma(M \cup \{j\}) < \gamma(M \cup \{i,j\})$. In this scenario, adding one judge to coalition M has no effect on the equilibrium policy, whereas adding both judges does. (To see why this might be the case, note that adding judges to the coalition changes the identity of the judges whose preferences determine $\underline{y}$ and $\overline{y}$. We showed in Lemma ## that adding a judge to coalition whose ideal policy is above the mean, will cause both $\underline{y}$ and $\overline{y}$ to increase. It might be that adding one judge to the coalition does not shift $\underline{y}$ sufficiently to change the equilibrium choice,but adding two judges will.) If $D > 0$, but is not too large, then the choices for judges i and j are strategic complements. Judges i and j face a coordination game; they either both want to vote sincerely, or both strategically. As long as the judges are coordinated, their choices are equilibrium consistent.

Second, suppose $\gamma(M) < \gamma(M \cup \{i\}) = \gamma(M \cup \{j\}) = \gamma(M \cup \{i,j\})$. In this scenario, adding either judge to coalition M favorably affects the equilibrium policy offered during the bargaining stage. However, having added one judge, the marginal effect of the adding the second judge is zero. The judges choices are now strategic substitutes (for $D > 0$ not too large). They are playing a game of chicken; each wants to vote strategically if and only if the other votes sincerely.

The first type of multiplicity is perverse, in the following sense. Both judges i and j would prefer to coordinate on the equilibrium where they vote strategically. Nevertheless, the equilibrium where they both vote sincerely can be sustained by beliefs that that the other judge will vote sincerely. In the most extreme case, when D is sufficiently small, we can have an equilibrium in which all judges sincerely believe that the disposition should be 0, but all choose 1 believing that all others will do the same. On collegial courts, it is not unreasonable to assume that such beliefs can be dispelled by communication between the judges, and that a coalition of judges can conspire to jointly affect a favorable deviation.

To rule out perverse equilibria of this sort, we focus on equilibria that are coalition-proof (see Bernheim, Peleg and Whinston (1987)).[13] An equilibrium fails to be coalition-proof, if there is a mutually beneficial deviation for some subset of agents, such that no sub-subset of those agents then have a further incentive to deviate from the deviation. Whereas Nash equilibria need only survive unilateral deviations, Coalition-proof Nash equilibria (CPNE) must also survive joint deviations by stable coalitions. The notion of coalition-proofness thus refines

[13]In standard bargaining models, the usual refinement is to require that strategies are *weakly dominant*. This, in effect, requires each player to vote as though they were pivotal. Such a refinement is reasonable, in so far as the players' payoffs are constant under any scenario in which their vote is not pivotal. However, in our model, the agent's will generically have a strict preference between their choices, even if their vote does not change the ultimate dispositional outcome. Hence, the weak-dominance criterion has no bite if applied to overall utilities, and is too strong if applied to dispositional outcomes alone.

the set of Nash Equilibria, by ruling out equilibria in which a subset of agents are trapped in a situation that is inferior, but which from which they could jointly and stably escape. When strategic complementarity creates multiple equilibria, coalition-proofness selects the equilibrium that is 'most plausible', in the sense of ensuring that those complementarities are exploited as far as possible.

Selecting a focal equilibrium from amongst the multiplicity that might arise when judges' choices are strategic substitutes is less straight-forward. Our approach is motivated by the following result:

Lemma 3. *Let M be a Nash equilibrium coalition. There exists a connected coalition M' with $\#M' = \#M$ and such that M' is also a Nash equilibrium coalition. Moreover, the M' equilibrium can be sustained over a larger range of value of D than the M coalition.*

To make sense of Lemma 3, first note that the net benefit of voting strategically decreases as the ideal policy moves further away from the equilibrium policy. (This becomes evident by examining equations (A3) and (A4) in the appendix.) Hence, more extreme judges should be less inclined to vote strategically than moderate judges. If a majority coalition is disconnected, then a relatively extreme judge finds it optimal to vote strategically, whilst a relatively moderate judge finds it optimal to vote sincerely. Lemma 3 formalizes the intuition, that if this true, the moderate judge must be willing to vote strategically if the extreme one does not. Moreover, the moderate judge should be more willing to do so, in the sense that he would continue to vote strategically even if the cost of insincerity somewhat increased.

Disconnected coalitions arise when judges' choices are strategic substitutes. For the remainder of this paper, we will focus attention on connected coalitions. Since for every equilibrium coalition that is disconnected, there is a corresponding connected coalition that induces the same disposition and (roughly) the same equilibrium policy, limiting attention to connected coalitions is not substantively restrictive. Additionally, this equilibrium selection mechanism has two desirable features. First, as the second part of Lemma 3 tells us, connected coalitions are equilibrium consistent over a larger range of the cost parameter D than a corresponding disconnected coalition. It follows that connected coalitions are 'robust', in the sense that they are least brittle to perturbations in the cost of insincerity. Second, as an empirical matter, coalitions appear for the most part to be connected.[14]

[14]We again stress, that our focus on connected equilibria is limited to the equilibrium selection problem when multiple equilibria exist. If it were the case that there existed a unique equilibrium that was disconnected, then that necessarily be the equilibrium predicted by our model. We discard disconnected equilibria, only in so far as there is a connected coalition that produces substantially similar equilibrium policies.

We are now ready to characterize the main results in this section.

Proposition 3. *There exist functions $j_0^*(D,z) \in \{1,...,k\}$ and $j_1^*(D,z) \in \{k,...,n\}$, which uniquely define the (connected) majority coalition in a CPNE, as follows:*

- *Suppose $d^* = 1$. Then $M^* = \{1,...,j_1^*(D,z)\}$.*
- *Suppose $d^* = 0$. Then $M^* = \{j_0^*(D,z),...,n\}$*

Moreover, $j_0^*(D,z)$ is (weakly) increasing in D and $j_1^*(D,z)$ is (weakly) decreasing in D. As such, $\#M^*$ is weakly decreasing in D.

Proposition 3 states one of the main claims of this paper. This is a unique coalition proof Nash equilibrium in which the optimal coalition is connected. The functions j_0^* and j_1^* pin down the composition of this coalition, by identifying the most extreme judge to join the coalition. If $j_0^* = 1$ or $j_1^* = n$, then the equilibrium coalition consists of all members of the court, and the disposition is unanimous. Moreover, Proposition 3 highlights the relationship between the cost of insincerity and the size of the majority coalition. As this cost increases, the majority coalition shrinks. The intuition is straight-forward. When $D = 0$, insincerity is costless, and so strategic voting is weakly dominant for every judge. As D increases, the hedonic cost of ruling incorrectly increases faster for extreme agents relative to moderate ones. (This follows since this cost is increasing in the distance of the case from each judge's ideal policy.) Hence, the most extreme judge will be the first to find strategic voting to be prohibitively costly. As D increases further, the next most extreme judge leaves the coalition, and so on, until the only judges left in the coalition are those voting sincerely. This motivates the following result:

Corollary 1. *If the cost of insincerity is sufficiently large, then all judges will vote sincerely. The median justice is dispositionally pivotal. (Formally, there exists $\bar{D} > 0$ s.t. whenever $D > \bar{D}$, $d^j = r(z;x^j)$ for each j, and $d^* = d^k$.)*

The intuition behind Lemma 1 is straight-forward. If D is sufficiently large, then the penalty for voting strategically overwhelms the policy benefit from doing so. All judges will vote sincerely. By construction, the sincere coalitions are connected, and the larger coalition must contain the median judge. Hence, the disposition of the court must coincide with the median judge's ideal disposition.

In the analysis thus far, we have characterized the optimal dispositional choice of each judge, given their belief about the eventual disposition of the court. We showed, importantly, that judges have an incentive to vote strategically, in order to take part in the bargaining process, and affect the opinion of the court. equilibrium. We complete this section by turning our attention to determining the disposition of the court.

Proposition 4. *In the unique connected coalition-proof Nash equilibrium, the median justice is dispositionally pivotal.*

In the previous section, we showed that the policy of the court need not reflect the ideal policy of the median justice in the majority, or even the median judge on the bench. However, Proposition 4 shows that the median judge is pivotal in determining the disposition of court. The intuition follows immediately from the preceding results. Since they are connected and contain at least a bare majority of judges, every equilibrium coalition must contain the median judge. Hence, the dispositional vote of the median judge must coincide with the disposition of the court.

We stress that, whilst the median justice is pivotal, it need not follow that the disposition of the court coincides with the median judge's sincere assessment of the case; she may vote strategically. To see when this will happen, let $M_0^*(z, D)$ and $M_1^*(z, D)$ denote the majority coalitions that result if the disposition of the court is 0 and 1, respectively. For concreteness, suppose $x^k < z$, so that the median judge would choose disposition 1 if she voted sincerely. Then, sincerity is optimal provided:

$$u_P(\gamma(M_1^*), x^r) \geq u_P(\gamma(M_0^*), x^r) - Du_D(z, x^r)$$
$$D \geq \frac{2(\gamma(M_0^*) - \gamma(M_1^*))}{z - x^k}\left[x^k - \frac{\gamma(M_0^*) + \gamma()}{2}\right]$$

where $\gamma(M_0^*) \geq z \geq \gamma(M_1^*)$. Notice that if $\left|\gamma(M_0^*) - x^k\right| \geq \left|\gamma(M_1^*) - x^k\right|$, then the right hand term is non-positive, and so sincere voting is optimal for every $D \geq 0$. By contrast, if $\left|\gamma(M_0^*) - x^k\right| < \left|\gamma(M_1^*) - x^k\right|$, then strategic voting will be optimal provided that the cost of insincerity is not too large. To make sense of this, note that the condition implies that the policy induced by voting strategically is closer to the median judge's ideal policy than is the policy induced by voting sincerely. For example, this might be the case, if the median judge is slightly left of center, all other leftist judges are on the extreme left, and rightist judges are moderate.

We note that the median judge's motivation for voting strategically differs from that of the other judges. For non-pivotal judges (i.e. other than the median), the benefit of voting

strategically is that the judge is included in the majority coalition, and may thus direct the court's equilibrium policy towards their ideal. By contrast, since she is pivotal, the median judge will always be in the majority coalition. Her motivation is to vote strategically is choose amongst equilibrium policies, rather than to purchase the right to participate in the second-stage bargaining.

<<< Comparative statics on z to be written >>>

5 Conclusion

In this paper, we presented a sequential bargaining model of a multi-member appellate court. When deciding cases, the court must decide both the case disposition, and a legal rule that rationalizes the disposition and which provides guidance to lower courts about how to decide future cases. In our model, these decisions are made sequentially, by majority rule. In the first stage, the judges cast dispositional votes, with a majority deciding the disposition of the court. The dispositional vote also determines the subset of judges who participate in the determination of the court's legal rule. The policy of the court is determined by bargaining between the members of the dispositional majority, and requires a majority of the full bench (rather than merely the dispositional majority). Judges are assumed to have preferences over both case dispositions and policy rules.

Our framework highlights several important features of judicial decision making. First, since the dispositional vote acts as a gateway into the policy making stage, each judge may have an incentive to cast a strategic dispositional vote in the first stage, in order to influence the subsequent second stage policy. Moreover, the costs and benefits of voting strategically depend upon the nature of the case to be decided. This is true for two reasons. First, since the court's announced policy must be consistent with its disposition, there are limits to the amount by which a strategically voting judge may 'moderate' the court's policy. Second, dispositional preferences are assumed to satisfy the increasing differences in dispositional values property, which causes the immediate cost of voting strategically to depend on the case being decided. Judges may more profitably vote strategically if the case appears 'contestable' from their perspective, than if it appears clear cut.

We show that, in equilibrium, the median judge is pivotal over case dispositions. Furthermore, we show that equilibrium coalitions are connected — meaning that the most extreme judges are the least likely to vote strategically. By contrast, moderate judges may frequently

find themselves voting contrary to their preferred outcome, in order to affect the court's policy outcome.

Second, the sequential structure of our game highlights that, although policy making requires a simple majority of the entire bench, when dispositional majorities are non-unanimous, the dispositional majority faces an effective super-majority requirement. We characterize the equilibria of unidimensional spatial bargaining games under conditions under for any (super)-majority condition. These equilibria will generically depend upon the ideal policies of the agents in the dispositional majority as well as the location of the case. A important novelty of this paper is in characterizing the limit equilibria of the bargaining game as $\delta \to 1$, which we interpret as the limit as the cost of proposing counter-proposals becomes arbitrarily small — an assumption which we think is reasonable given the institutional setting of the court. We show that, in the limit, it as if the dispositional majority endogenously separates into two factions. The announced policy is the either the ideal policy of some pivotal judge (not necessarily the median of the dispositional majority), or the result of asymmetric Nash Bargaining between representative leaders of the factions, with bargaining weights proportional to factional size. Importantly, in the limit, the chosen policy will never coincide with the ideal policy of the median judge – and so whilst the median judge decides the disposition of the court, she does not also determine the policy of the court. Our result thus stands in contrast to both median voter results and median-of-the-majority results that have been proposed in the existing literature.

Relatedly, our analysis provides microfoundations for the emergence of policy coalitions within the court — which gives a theoretical basis to commonly discussed notions of court separating into left- and right-wing blocs.

Finally, our model highlights the importance of the case-space approach to modelling courts, by show-casing the importance of case location in both the composition of dispositional majorities (and the likelihood of strategic voting) and the equilibrium policies that result.

6 Appendix

Proof of Proposition 1. The proof structure is identical to that in Parameswaran and Murray (2018), and we refer the interested reader to that paper. The only difference between our model and that paper is that policy preferences in this model need not be concave. However, we show that, after a suitable change of variables, preferences are indeed concave.

Recall, $u_P(y;x) = -|\int_x^y l(z-x)f(z)dz|$. For any z, let $\zeta = F(z)$ and let $\chi^j = F(x^j)$. Define $\lambda(\zeta,\chi) = l(F^{-1}(\zeta) - F^{-1}(\chi)$. Then $l(z-x) = \lambda(\zeta-\chi)$. It follows that:

$$\begin{aligned} u_P(y;x) =& -|\int_x^y l(z-x)f(z)dz| \\ =& -|\int_\chi^\gamma \lambda(\zeta,\chi)d\zeta \\ =& v_P(\gamma;\chi) \end{aligned}$$

Now,

$$\frac{\partial^2}{\partial\gamma^2} = \begin{cases} -\frac{\partial}{\partial\gamma}\lambda(\gamma,\chi) < 0 & \text{if } \gamma > \chi \\ \frac{\partial}{\partial\gamma}\lambda(\gamma,\chi) < 0 & \text{if } \gamma < \chi \end{cases}$$

Hence, v_P is concave. □

Proof of Proposition 2 . Take any $i \in \{1,...,m\}$, and suppose $\mu \in (x^{i-1}, x^i)$. Then, by Proposition **??**, there exists $\bar{\delta} < 1$ s.t. for $\delta > \bar{\delta}$, $x^{i-1} < \underline{y}^r(\delta) < \overline{y}^l(\delta) < x^i$. (For clarity, we make explicit the dependence of $\underline{y}^r$ and $\overline{y}^l$ on δ.) Then, by Proposition 1, all judges $j \in \{1,..,i-1\}$ will propose $\underline{y}^r$ and all judges $j \in \{i,..,n\}$ will propose $\overline{y}^l$. Again by Proposition 1, this implies that:

$$u_P^r\left(\underline{y}^r\right) = \delta\left[(1-P_i)\,u_P^r\left(\underline{y}^r\right) + P_i u_P^r\left(\overline{y}^l\right)\right] \tag{1}$$

$$u_P^l\left(\overline{y}^l\right) = \delta\left[(1-P_i)\,u_P^l\left(\underline{y}^r\right) + P_i u_P^l\left(\overline{y}^l\right)\right] \tag{2}$$

where $P_i = \sum_{j\geq i} p_j$. By the implicit function theorem, this system of equations pins down $\underline{y}^r$ and $\overline{y}^l$ in terms of the model parameters.

Now, let $E[y] = (1-P_i)\,\underline{y}^r + P_i\overline{y}^l$. Note, by construction, that $\underline{y}^r < E[y] < \overline{y}^l$. Then $\overline{y}^l - E[y] = \frac{1-P_i}{P_i}\left(E[y] - \underline{y}^r\right)$. We affect the following change of variables: Let $\varepsilon = E[y] - \underline{y}^r$. Then, we have: $\underline{y}^r = E[y] - \varepsilon$ and $\overline{y}^l = E[y] + \frac{1-P_i}{P_i}\varepsilon$. Equations (1) and (2) become:

$$\left(1-\delta(1-P_i)\right)u_P^r\left(E[y]-\varepsilon\right) = \delta P_i u_P^r\left(E[y] + \frac{1-P_i}{P_i}\varepsilon\right) \tag{3}$$

$$\left(1-\delta P_i\right)u_P^l\left(E[y] + \frac{1-P_i}{P_i}\varepsilon\right) = \delta(1-P_i)\,u_P^l\left(E[y]-\varepsilon\right) \tag{4}$$

By the implicit function theorem, and since u is continuously differentiable, we have:

$$\left[\begin{array}{cc} (1-\delta(1-P_i))\,u^r_{P,y}\left(\underline{y}_r\right)-\delta P_i u^r_{P,y}\left(\overline{y}_l\right) & -(1-\delta(1-P_i))\,u^r_{P,y}\left(\underline{y}_r\right)-\delta(1-P_i)\,u^r_{P,y}\left(\overline{y}_l\right) \\ (1-\delta P_i)\,u^l_{P,y}\left(\overline{y}^l_P\right)-\delta(1-P_i)\,u^l_{P,y}\left(\underline{y}^r\right) & \left(\frac{1-P_i}{P_i}-\delta(1-P_i)\right)u^l_{P,y}\left(\overline{y}_l\right)+\delta(1-P_i)\,u^l_{P,y}\left(\underline{y}_r\right) \end{array}\right]\left(\begin{array}{c}\frac{\partial E[y]}{\partial\delta}\\ \frac{\partial\varepsilon}{\partial\delta}\end{array}\right)=$$

$$\left(\begin{array}{c}(1-P_i)\,u^r_P\left(\underline{y}_r\right)+P_i u^r_P\left(\overline{y}_l\right)\\ P_i u^l_P\left(\overline{y}_l\right)+(1-P_i)\,u^l_P\left(\underline{y}_r\right)\end{array}\right)$$

Taking limits as $\delta\to 1$, we have:

$$\left[\begin{array}{cc} 0 & -u^r_{P,y}(\mu) \\ 0 & \frac{1-P_i}{P_i}u^l_{P,y}(\mu)\end{array}\right]\left(\begin{array}{c}\lim_{\delta\to1}\frac{\partial E[y]}{\partial\delta}\\ \lim_{\delta\to1}\frac{\partial\varepsilon}{\partial\delta}\end{array}\right)=\left(\begin{array}{c}u_r(\mu)\\ u_l(\mu)\end{array}\right)$$

These imply that:

$$\lim_{\delta\to1}\frac{\partial\varepsilon}{\partial\delta}=-\frac{u^r_P(\mu)}{u^r_{P,y}(\mu)}=\frac{P_i}{1-P_i}\frac{u^l_P(\mu)}{u^l_{P,y}(\mu)}$$

The second equality provides an equation that uniquely defines the limit equilibrium.

Next, we note that equation defining μ_i coincides with the first order condition of the i^{th} Nash Bargaining problem. Recall, that problem was: $\max_{y\in X}u^l_P(y)^{1-P_i}u^r_P(y)^{P_i}$. Since utilities are concave, the maximizer must be the solution to the first order condition: $(1-P_i)\frac{u^l_{P,y}(b_{i-1,i})}{u^l_P(b_{i-1,i})}+P_i\frac{u^r_{P,y}(b_{i-1,i})}{u^r_P(b_{i-1,i})}=0$. Re-arranging gives the desired result.

Notice that $b_{i-1,i}$ is increasing in P_i. (To see this, re-arrange the first order condition to give: $\frac{u^l_{P,y}(b_{i-1,i})}{u^r_{P,y}(b_{i-1,i})}\cdot\frac{u^r_P(b_{i-1,i})}{u^l_P(b_{i-1,i})}=-\frac{P_i}{1-P_i}$. We know that $b\in\left[x^l,x^r\right]$. By single-peakedness, over this region we know that $u^l_P(b)$ is strictly decreasing in b and $u^r_P(b)$ is strictly increasing in b, and so $\frac{u^r_P(b)}{u^l_P(b)}$ is strictly decreasing in b. Similarly, by concavity (after transformation) of u, $u^l_{P,y}(b)$ is decreasing in b and $u^r_{P,y}(b)$ is increasing in b, and so $\frac{u^r_{P,y}(b)}{u^l_{P,y}(b)}$ is weakly decreasing in b. Hence, the left hand term is strictly decreasing in b. The right hand term is also strictly decreasing in P. Hence, as P increases, so must b.) Then, since P_i is decreasing in i, it follows that $b_{i-1,i}$ is decreasing is as well.

Since we conjectured $\mu\in(x^{i-1},x^i)$, then the limit equilibrium policy coincides with i^{th} Nash Bargaining solution provided that $x^{i-1}<b_{i-1,i}<x^i$. Now, since x^i is increasing and $b_{i-1,i}$ is decreasing in i, then by the definition of i^*, $x^i<b_{i,i+1}$ for all $i<i^*$ and $x^i\geq b_{i,i+1}$ for all $i\geq i^*$. Moreover, for $i<i^*$, $x^{i-1}\leq x^i<b_{i,i+1}\leq b_{i-1,i}$, which is inconsistent. Similarly, for $i>i^*$, $b_{i-1,i}\leq x^{i-1}\leq x^i$, which is inconsistent. Hence, if $b_{i-1,i}\in(x^{i-1},x^i)$, then $i=i^*$. Note however, that the converse need not be true. Setting $i=i^*$ gives two possibilities: (i) $x^{i^*-1}<b_{i^*-1,i^*}<x^{i^*}$, or (ii) $x^{i^*-1}\leq x^{i^*}\leq b_{i^*-1,i^*}$ (with at least one inequality strict).

The former case is equilibrium consistent, and since the equilibrium is unique, we have $\mu = b_{i^*-1,i^*}$.

Suppose the latter case prevails. It follows that the limit equilibrium is not contained in any of the open intervals (x^{i-1}, x^i), and so $\mu \in \{x^1, ..., x^m\}$. (In fact, since $\underline{y}^r < x^r$ and $\overline{y}^l > x^l$ for all δ, and since $\lim_{\delta\to 1} \underline{x}^r = \mu = \lim_{\delta\to 1} \overline{x}^l$, then $x^l \leq \mu \leq x^r$, and so $\mu \in \left\{x^l, ..., x^r\right\}$.) Suppose $\mu = x^i$ for some $i \in \{l, ..., r\}$. Let $I = \{j | x^j = x^i\}$ and denote $I = \{i^-, ..., i^+\}$, where $i^- \leq j \leq i^+$ for all $j \in I$. (Obviously, I may be a singleton, in which case $i^- = i = i^+$.) Let $\Pi_i^- = \sum_{j<i^-} p_j$ and $\Pi_i^+ = \sum_{j>i^+} p_j$ and $\Pi_i = \sum_{j\in I} p_j$. Then, for δ sufficiently large, (1) becomes:

$$u_P^r(\underline{y}_r) = \delta \left[\Pi_i^- u_P^r(\underline{y}_r) + \Pi_i u_P^r(x^i) + \Pi_i^+ u_P^r(\overline{y}^l)\right]$$

Since $\underline{y}_r < x^i < \overline{y}^l$, there exists $\gamma \in (0, 1)$ s.t. $x^i = \gamma \underline{y}^r + (1-\gamma)\overline{y}^l$. We can write (1) as:

$$\begin{aligned} u_P^r(\underline{y}^r) = \delta &\left[(\Pi_i^- + \Pi_i\gamma)u_P^r(\underline{y}^r) + (\Pi_i^+ + \Pi_i(1-\gamma))u_P^r(\overline{y}_l)\right] \\ &+ \delta \left[\Pi_i\gamma(u_P^r(\underline{y}_r) - u_P^r(x^i)) + \Pi_i(1-\gamma)(u_P^r(\overline{y}^l) - u_P^r(x^i))\right] \end{aligned} \tag{5}$$

Notice (5) is the sum of two terms, with the first term being analogous to the expression in (1), and the second term being a 'correction' term.

We repeat the procedure for equation (2), and then apply the change of basis method above, and take limits as $\delta \to 1$. Since $\underline{y}^r \to x^i$ and $\overline{y}^l \to x^i$, the 'correction' term in (5) goes to zero. It follows that $\mu = b(\rho^*)$, where $\rho^* = \Pi_i^+ + \Pi_i(1 - \lim_{\delta\to 1} \gamma(\delta))$. Now, there must be some k s.t. $b_{k,k+1} < b(\rho^*) = x^i < b_{k-1,k}$. Moreover, it must be that $k \in I$, since $b_{i^+,i^+ +1} < b(\rho^*) < b_{i^- -2,i^- -1}$, by construction. But then, we can choose i appropriately s.t. $b_{i,i+1} < x^i < b_{i-1,i}$. But this requires $i = i^*$. □

Proof of Lemma 2. Let z be an arbitrary case. Suppose $d^* = 0$. (The other case is analogous.) Then $M^0 = \{j |\, x^j > z\}$. Moreover, all feasible second stage policies must satisfy $y \geq z$. Suppose there is a j, such that $j \in M^0$ and $j \notin M^*$. Then

$$\begin{aligned} E\left[u^j\left(d^j = 1; d^{-j}\right)\right] &> E\left[u^j\left(d^j = 0; d^{-j}\right)\right] \\ u_P(\gamma_M, x^j) + Dl(z - x^j) &> u_P\left(\gamma_{M\cup\{j\}}, x^j\right) \\ \left|x^j - z\right| &> -l^{\left(-1/D\left(\left(u_P(\gamma_{M\cup\{j\}}, x^j) - u_P(\gamma_{M\cup\{j\}}, x^j)\right)\right)\right.} \end{aligned}$$

By assumption 1, $\left|\gamma_{M^*\cup\{j\}} - x^j\right| \leq |\gamma_{M^*} - x^j|$, which implies that the RHS term is non-negative. By contrast, the LHS term is strictly negative whenever $D > 0$, which is a contradiction. Hence $j \in M^*$. □

Proof of Lemma 3. Let M be an equilibrium coalition, and suppose M is not connected. WLOG, suppose $d^* = 1$, so that, by Lemma 2, $M^1(z) \subset M$. Since M^1 is a connected coalition and M is disconnected, M must contain members of $M^0(z)$. Then there exists $i < j$ with $i, j \in M^0(z)$, $i \notin M$ and $j \in M$. Then $z < x^i \leq x^j$. Let M' be identical to M except that judge j is replaced by judge i. Using arguments made previously, suppose $\gamma_M = \gamma_{M'}$. (Intuitively, judge i and j cannot bring their ideal policy to bear. If recognized, they will each select y^H, and so the chief should be indifferent between selecting either to propose. Moreover, each judge affects the equilibrium policies offered by other judges only in so far as their presence affects the relative ordering of coalition members, and they both affect this ordering in the same way.) Since M is an equilibrium, it must be that:

$$u_P(\gamma_M, x^j) - Dl(z - x^j)_P(\gamma_{M-\{j\}}, x^j) \tag{6}$$

and:

$$u_P(\gamma_M, x^i) \geq u_P(\gamma_{M\cup\{i\}}, x^i) - Dl(z - x^i) \tag{7}$$

Clearly, (6) requires $\gamma_{M-\{j\}} < \gamma_M$. Then, since $x^i \leq x^j$, it follows that:

$$u_P(\gamma_{M'}, x^i) - Dl(z - x^i) \geq u_P(\gamma_{M'-\{i\}}, x^i)$$

noting that $\gamma_{M'} = \gamma_M$ and that $M' - \{i\} = M - \{j\}$ by construction. Moreover, if $x^i < x^j$, then the inequality is strict, and continues to be so for some $D' > D$ and even for some $\gamma_{M'} < \gamma_M$. Moreover, since $x^j > x^i$, it follows that $u_P(\gamma_{M'}, x^j) \geq u_P(\gamma_{M'\cup\{j\}}, x^j) - Dl(z - x^j)$, and again this inequality is strict if $x^i < x^j$. This confirms that M' is also an equilibrium coalition. We can iterate in this way until we arrive at a connected coalition. □

Proof of Proposition 3. Let z be an arbitrary case. Consider a CPNE with $d^* = 1$. (The other scenario is analogous.) We proceed iteratively. Begin with smallest connected coalition that is a candidate to be an equilibrium, M_{j^*}. This is an equilibrium if, for every $j \in M_{j^*}^c$:

$$\begin{aligned} u_P(\gamma_{M_{j^*}}, x^j) &\geq u_P(\gamma_{M_{j^*+1}}, x^j) - Dl(z - x^j) \\ x^j &\geq \frac{\gamma_{M_{j^*}} + \gamma_{M_{j^*+1}}}{2} + \frac{D}{D - 2\left(\gamma_{M_{j^*}} - \gamma_{M_{j^*+1}}\right)}\left(z - \frac{\gamma_{M_{j^*}} + \gamma_{M_{j^*+1}}}{2}\right) \end{aligned}$$

There are two possibilities. If this condition holds, then M_{j^*} is a Nash equilibrium coalition of the game (although not necessarily CPNE). Stop. If this condition does not hold, then

M_{j^*} is not equilibrium consistent. Then, consider the coalition M_{j^*+1}. We have just shown that no $j \in M_{j^*+1}$ has a favorable deviation to $d^j = 0$. Again take $j \in M^c_{j^*+1}$. Iterate this procedure, until the condition is satisfied for every j in the candidate set. If the condition is not satisfied for every $i \in \{j^*, ..., n-1\}$, then we have showed that no $j \in M_n$ will want to deviate from choosing $d^j = 1$, and so M_n is equilibrium consistent.

So far, we have found the smallest equilibrium consistent connected coalition. Let M_{i^*} be this coalition. There may be larger equilibrium consistent coalitions as well. Starting from coalition M_{i^*}, consider joint deviations to coalitions M_{i^*+2}, M_{i^*+3},... etc. until the first mutually beneficial deviation is found. If no such deviation exists, then M_{i^*} is a CPNE. If a beneficial deviation is found, then repeat the procedure, considering joint deviations from this new coalition. The CPNE is the largest coalition for which no additional joint deviations exist.

The comparative statics on D follow immediately by noting that the RHS of the above equation is decreasing in D. □

References

Banks, Jeffrey S. and John Duggan. 2000. "A Bargaining Model of Collective Choice." *The American Political Science Review* 94(1):pp. 73–88.

Banks, Jeffrey S and John Duggan. 2006. "A General Bargaining Model of Legislative Policy-making." *Quarterly Journal of Political Science* 1(1):49–85.

Baron, David P and John A Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83(04):1181–1206.

Beim, Deborah. 2017. "Learning in the judicial hierarchy." *The Journal of Politics* 79(2):591–604.

Bernheim, B Douglas, Bezalel Peleg and Michael D Whinston. 1987. "Coalition-proof nash equilibria i. concepts." *Journal of Economic Theory* 42(1):1–12.

Binmore, Ken. 1987. "Perfect equilibria in bargaining models." *The Economics of Bargaining, Basil Blackwell, Oxford* .

Binmore, Ken, Ariel Rubinstein and Asher Wolinsky. 1986. "The Nash Bargaining Solution in Economic Modelling." *The RAND Journal of Economics* pp. 176–188.

Black, D. 1948. "On the rationale of group decision-making." *The Journal of Political Economy* 56(1):23–34.

Bonneau, Chris W, Thomas H Hammond, Forrest Maltzman and Paul J Wahlbeck. 2007. "Agenda control, the median justice, and the majority opinion on the US Supreme Court." *American Journal of Political Science* 51(4):890–905.

Cameron, Charles M, Jeffrey A Segal and Donald Songer. 2000. "Strategic Auditing in a Political Hierarchy: An Informational Model of the Supreme Court's Certiorari Decisions." *American Political Science Review* 94(01):101–116.

Cameron, Charles M and Lewis A Kornhauser. 2008. "Modeling Collegial Courts (3): Judicial Objectives, Opinion Content, Voting and Adjudication Equilibria.".

Cameron, Charles M, Lewis A Kornhauser and G Parameswaran. 2019. "Stare Decisis and Judicial Log-Rolls: A Gains-from-Trade Model." *RAND Journal of Economics (forthcoming)* .

Cardona, Daniel and Clara Ponsatí. 2007. "Bargaining one-dimensional social choices." *Journal of Economic Theory* 137(1):627–651.

Cardona, Daniel and Clara Ponsati. 2011. "Uniqueness of stationary equilibria in bargaining one-dimensional policies under (super) majority rules." *Games and Economic Behavior* 73(1):65–75.

Carrubba, Cliff, Barry Friedman, Andrew D Martin and Georg Vanberg. 2012. "Who controls the content of Supreme Court opinions?" *American Journal of Political Science* 56(2):400–412.

Carrubba, Clifford J and Tom S Clark. 2012. "Rule creation in a political hierarchy." *American Political Science Review* 106(3):622–643.

Cho, Seok-ju and John Duggan. 2003. "Uniqueness of stationary equilibria in a one-dimensional model of bargaining." *Journal of Economic Theory* 113(1):118–130.

Cho, Seok-ju and John Duggan. 2009. "Bargaining foundations of the median voter theorem." *Journal of Economic Theory* 144(2):851–868.

Downs, A. 1957. "An economic theory of political action in a democracy." *The Journal of Political Economy* 65(2):135–150.

Fischman, Joshua B. 2008. "Decision-making under a norm of consensus: A structural analysis of three-judge panels.".

Fischman, Joshua B. 2011. "Estimating Preferences of Circuit Judges: A Model of Consensus Voting." *The Journal of Law and Economics* 54(4):781–809.

Hammond, Thomas H, Chris W Bonneau and Reginald S Sheehan. 2005. *Strategic behavior and policy choice on the US Supreme Court*. Stanford University Press.

Iaryczower, Matias and Matthew Shum. 2012. "The value of information in the court: Get it right, keep it tight." *The American Economic Review* 102(1):202–237.

Jackson, Matthew O and Boaz Moselle. 2002. "Coalition and party formation in a legislative voting game." *Journal of Economic Theory* 103(1):49–87.

Kalandrakis, Tasos. 2010. "Minimum winning coalitions and endogenous status quo." *International Journal of Game Theory* 39(4):617–643.

Kornhauser, Lewis A. 2008. "Modelling Courts." *Theoretical Foundations of Law and Economics* p. 1.

McCarty, Nolan M. 2000. "Presidential pork: Executive veto power and distributive politics." *American Political Science Review* 94(1):117–129.

Parameswaran, Giri and Jacob Murray. 2018. "Limit Equilibrium of Unidimensional Bargaining Games: A Core Refinement.".

Predtetchinski, Arkadi. 2011. "One-dimensional bargaining." *Games and Economic Behavior* 72(2):526–543.

Rasmusen, Eric. 1994. "Judicial Legitimacy as a Repeated Game." *Journal of Law, Economics, and Organization* pp. 63–83.

Rubinstein, Ariel. 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica: Journal of the Econometric Society* pp. 97–109.