A Simple Method for Bounding the Elasticity of Growing Demand with an Application to Assessing the Structural Remedy in the *DuPont* Decision

Wallace P. Mullin
George Washington University

Christopher M. Snyder
Dartmouth College
National Bureau of Economic Research

November 2018

Abstract: We propose a method for bounding the demand elasticity in a growing, homogeneous-product market. Based on the idea that growing demand curves do not cross, shifts in market equilibrium over time can be used to "funnel" the demand curve into a narrow region, thus bounding its slope. The method requires only minimal data—market price and quantity over a time span as short as two periods—and is simple to operationalize. We apply the method to assess the effectiveness of the antitrust remedy in the 1952 *DuPont* decision, ordering the incumbent manufacturers to license their patents for commercial plastics. Commentators have suggested that the incumbents may have preserved the monopoly outcome by gaming the licensing contracts. The upper bounds on demand elasticities that we compute are significantly less than 1 in many post-remedy years. Such inelastic demand is inconsistent with monopoly, suggesting the remedy may have been effective.

JEL Codes: C18 (Econometric Methods: Estimation: General), D22 (Production and Organizations: Firm Behavior: Empirical Analysis), L40 (Antitrust Issues and Policies: General), L65 (Industry Studies: Manufacturing: Plastics)

Keywords: demand estimation, bounds, elasticity, antitrust, plastics, remedy, patents

Contact Information: Mullin: Department of Economics, George Washington University, Monroe Hall 340, 2115 G Street, NW, Washington DC 20052; email: wpmullin@gwu.edu. Snyder: Department of Economics, Dartmouth College, 301 Rockefeller Hall, Hanover NH 03755; email: chris.snyder@dartmouth.edu.

Acknowledgments: The authors thank David Balan, Eric Edmonds, Liran Einav, Sharat Ganapati, Erzo Luttmer, Paul Novosad, Jose Manuel Paz y Mino, and seminar participants at Tufts University, the University of Oregon, and the 2018 International Industrial Organization Conference (Indianapolis) for valuable discussions. The authors are grateful for extensive comments by three anonymous referees, which substantially improved the paper. The authors thank John Cocklin for insights on locating historical data sources and Marvin Lieberman and the Dartmouth FIVE Project for sharing their data on the chemicals industry.

1. Introduction

Whether structural remedies for antitrust violations are effective in fostering competition has been an ongoing concern for scholars and policymakers. An early advocate of structural remedies, Adams (1951) documented a series of earlier antitrust cases that the agencies won but imposed remedies too weak to restore competition, the so-called pyrrhic victories of antitrust.

To understand why promising remedies might fail to achieve their goals, consider the *DuPont* decision of 1952, which compelled the incumbent manufacturers in the commercial plastics market to license their manufacturing patents to all applicants. As Areeda, Kaplow, and Edlin (2013) note, remedies do not typically set the licensing terms, leaving them subject to commercial negotiation. This raises the opportunity for incumbent manufacturers to preserve something close to the monopoly outcome by specifying exorbitant fees. Even if incumbents set fees that are deemed "reasonable," they can withhold the tacit knowledge necessary for entrants to compete effectively against them.

In this paper, we study the effectiveness of the antitrust remedy ordered in the aforementioned *DuPont* decision. The case involved two incumbent chemical manufacturers: the U.S. firm DuPont and the U.K. firm Imperial Chemical Industries (ICI). The two firms signed a Patents and Processes agreement in 1929, granting each company exclusive licenses for the patents and secret processes controlled by the other and dividing the global market into exclusive territories between them. The U.S. government brought suit under the Sherman Act, alleging an illegal market division. The judge ruled in favor of the government, ordering the defendants to cancel their exclusive-territory arrangements, requiring them to license the patents behind several of their products, most significantly, two types of plastic widely used for commercial purposes: low-density and high-density polyethylene.

The remedy ostensibly had the desired procompetitive effect: eleven manufacturers entered by the end of the decade; prices steadily declined and output rose (Backman 1964, p. 71). However, the same price declines and output increases may have arisen in a monopoly market experiencing substantial cost declines, plausibly true for plastics in the 1950s and 60s. The entrants may merely have produced their share of the monopoly quantity, returning most of the rents to the incumbents.

Our study will provide formal evidence for the effectiveness of the remedy that cuts through

these criticisms. Formal study is hindered by a paucity of historical data, just yearly aggregate price and quantity data for polyethylene, only available for post-remedy years. We offer a new method that allows solid conclusions to be drawn from these fairly minimal data. The method bounds the demand elasticity for a homogeneous product in a given year based on the assumption that demand is growing, sandwiching the demand curve in the given year between the demand curves in earlier and later periods.

Figure 1 provides intuition for how the method works. In this example, the researcher has price and quantity data for two years. The equilibrium point in the initial year is e_0 and in the second is, say, e_1'. The researcher wants to bound the slope the period-1 inverse demand curve through e_0. With so little data, there may be little hope to say anything more than inverse demand lies somewhere between the horizontal dotted line, corresponding to an infinitely elastic demand curve, and the dotted vertical line, corresponding to an infinitely inelastic one. In fact we can say more. Positing a functional form for demand, say linear, and assuming demand is nondecreasing over time, a curve like D can be ruled out because that would put the later equilibrium point e_1' on a lower demand curve. To preserve nondecreasing demand, the demand curve through e_0 must be at least as steep as the line labeled D' connecting e_0 and e_1'. The comparison of the two equilibrium points leads to a lower bound on the steepness of inverse demand, which translates into an upper bound on the elasticity of demand through e_0.

The tightness of the bound is data-driven. Suppose that the observed equilibrium point in the later period is e_1'' rather than e_1'. This leads to a tighter bound on the elasticity. The inverse demand through e_0 must be at least as steep as D'' to keep e_1'' from lying on a lower demand curve. The demand curve is funneled from the entire shaded region down into just the dark-shaded part, only leaving room for a relatively inelastic demand. A researcher who finds that demand is as inelastic as D'' may be able to rule out monopoly. Intuitively, the drop from e_0 to a point like e_1'' may be so steep that a monopolist would never have dropped price this much for such a small increase in quantity, even if marginal costs had dropped to zero. The more likely conclusion may be that competitive pressure drove the firms onto the inelastic region of demand.

The example in Figure 1 implicitly assumes demand is linear, at least locally in the reference period. We develop a variant of the methodology that can narrow the bounds on the demand elasticity considerably if the researcher is willing to assume that demand is consistently linear across

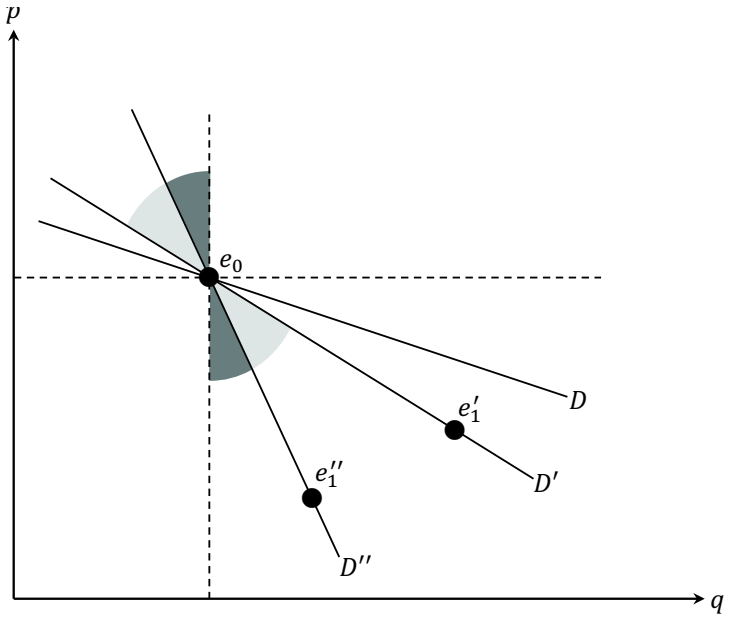


Figure 1: Intuition for the Method Bounding the Demand Elasticity

all periods and over its whole range. The variant incorporates information from the intercepts of other year's demands to iteratively narrow the bounds on the elasticity in a given year. Our methods generalize beyond the linear case. We redo the analysis for another widely assumed functional form, logit, and show in the application that the elasticity bounds are often virtually identical to those assuming linear demand. The methodology section provides general formulas allowing any functional form that the researcher chooses to impose. We also provide bounds that only require a weak concavity or convexity assumption rather than an explicit functional form.

Bounding methodologies that look promising in theory may turn out to produce such wide bounds in practice as to be uninformative. In our application to the polyethylene market, however, the upper bound on the demand elasticity is substantially and statistically significantly below 1 in many sample years. The bounds are robust to alternative products, functional forms, and method variants. Such inelastic demand is inconsistent with monopoly, suggesting the remedy may have been effective. Thus our formal evidence lends support to the effectiveness of structural antitrust remedies.

Our work is related to several literatures. One is the literature assessing the effectiveness of structural remedies for antitrust. In addition to the aforementioned, pioneering study by Adams (1951), studies by the U.S Federal Trade Commission (FTC) staff allso make important contribu-

tions to the literature. These studies include U.S. Federal Trade Commission (1999), examining the efficacy of divestiture remedies accompanying merger approvals from 1990–94; Farrell, Pautler, and Vita (2009), describing a similar FTC exercise focusing on a sample of hospital mergers; and U.S. Federal Trade Commission (2017), studying all 89 FTC merger orders from 2006–12. This last study judged remedies to be at least a qualified success in 83% of cases.

We also contribute to the industrial-organization literature following Manski (1995) that seeks to bound rather than point-estimate elasticities and other parameters. Haile and Tamer (2003) derive bounds for bids and reserve prices in their incomplete model of English auctions. Ciliberto and Tamer (2009) identify parameter sets in the presence of multiple equilibria, applying their method to an entry game among U.S. airlines. Pakes (2010) and Pakes, Porter, Ho, and Ishii (2015) develop moment inequalities in single and multi-agent settings based on the revealed-preference argument that an observed choice is better in expectation than alternative choices in the feasible set. Our paper applies to a different setting and adopts a different approach—dealing with aggregate price and quantity data in a homogeneous-product market rather than microdata from a differentiated-product market. However, we share the basic idea with the bounds literature that equilibrium observations can identify weak inequalities among structural parameters.

Our paper is also related to the literature on patent licensing. The most germane papers in this vast literature are those regarding patent pools and those using historical evidence to shed light on contemporary issues in intellectual property. Stocking and Watkins (1946) were the first to formally identify the Patents and Processes agreement between DuPont and ICI as a patent pool.[1] Lerner, Strojwas and Tirole (2007) classify distinguishing features of pro-competitive patent pools. Pools of complementary patents are more likely to be procompetitive and are more likely to allow independent licensing than pools of substitute patents. Lampe and Moser (2013) study the first U.S. patent pool, formed in 1856 involving sewing machines. Perhaps the closest paper in this literature is Watzinger, Fackler, Nagler and Schnitzer (2017), who examine induced innovation in the context of the 1956 Bell System consent decree. The United States filed a monopolization suit against Bell in 1949, alleging that it had foreclosed entry into various related markets such as

[1]The formation of a patent pool was hardly exceptional during that period, as Tirole (2014) notes in his Nobel lecture: "A little known fact is that, prior to 1945, most high-tech industries of the time were run by patent pools. But the worry about cartelization through joint marketing led to a hostile decision of the U.S. Supreme Court in 1945 and the disappearance of pools until the recent revival of interest."

telecommunications equipment. The consent decree allowed Bell to remain vertically integrated into telecommunications but forced it to license its existing patents royalty-free. The authors find that, while follow-on innovation did not expand in telecommunications equipment, it did in other sectors.

Finally, our paper is related to economic commentaries specifically on the remedy ordered the *DuPont* case. Some of the commentary is quite critical. In his classic, two-volume history, Reader (1975, p. 417) argues that the prosecution arose out of political pressure at the Department of Justice. Hounshell and Smith (1988, p. 206) criticize the court's verdict for a different reason. "Judge Ryan ...refused to accept the argument that ICI's and DuPont's agreement resulted in the genuine exchange of scientific and technical information. But the overwhelming historical evidence demonstrates this was indeed the case." By contrast, we provide supporting evidence that the licensing remedy was effective in ending the monopoly and spurring competition in plastics.

The paper is structured as follows. The first part of the paper covers methodology. Section 2 models the situation to which the methodology will be applied. Section 3 presents our methods in the special case of linear demand. We focus on the simple, linear case in this section for pedagogical purposes, but the methods are much more general. Section 4 generalizes the bounds to whatever demand curve in a broad class the researcher chooses to impose. It also provides bounds under an assumption about curvature (weak concavity or convexity) rather the specific functional form demand takes. The rest of the paper is devoted to empirical application of the methods to the *DuPont* decision. Section 5 provides institutional background, Section 6 describes the data, and Section 7 presents results. Section 8 provides discusses standard errors. Our elasticity bounds involve extreme order statistics, rendering standard bootstrapping methods invalid. We instead use the bootstrapping method for extreme order statistics proposed by Zelterman (1993), requiring some discussion provided in the section. Section 9 discusses robustness issues. Section 10 concludes. Appendix A provides proofs not included in the text. Appendix B provides the full details of the analysis of general demand curves sketched in Section 4. The last part of Appendix B applies the general results to the case of logit demand, which along with linear demand is a featured functional form in our empirical work. Appendix C, available online, provides supplementary figures documenting showing that the empirical results are not sensitive to the decisions made in constructing the dataset.

2. Model

This section lays out a general model of a growing market for a homogeneous product used in the analysis. Each period t, the interaction between producers and consumers on the market leads to an equilibrium $e_t \equiv (q_t, p_t)$, where $q_t \geq 0$ is market quantity and $p_t \geq 0$ is market price. The researcher observes the market equilibrium over a set of periods $T \equiv \{\ldots,-2,-1,0,1,2,\ldots\}$, where period $t = 0$ is the chosen reference period for which we will provide elasticity bounds. Let $E \equiv \{e_t \,|\, t \in T\}$ denote the set of time-series observations of equilibrium.

The analysis can be streamlined without much loss of generality if ties between price observations or quantity observations are ruled out, accomplished by the following definition of distinctness.

Assumption 1 (**Distinct Equilibria**). *For all $e_{t'}, e_{t''} \in E$ such that $t' \neq t''$, $q_{t'} \neq q_{t''}$ and $p_{t'} \neq p_{t''}$.*

Assuming that equilibrium points are distinct entails little loss of generality if one supposes that two observations are never exactly equal if measured to arbitrary precision. We also assume that the market is nontrivial in the sense of involving positive prices and quantities in each period.

Assumption 2 (**Nontrivial Equilibria**). *For all $e_t \in E$, $q_t, p_t > 0$.*

We will characterize each side of the market in turn starting with producers. Since the goal of our application will be to determine whether the antitrust remedy was effective in changing producer conduct, it is natural to consider producer conduct as an unknown to be determined. Thus, we will be fairly agnostic about producer behavior in the model, whether characterized by perfect competition, some form of oligopoly, or monopoly.

On the other side of the market, consumers are price takers whose behavior is captured by the demand curve $q = D_t(p)$. The index on the demand curve allows it to shift over time. Assume this function obeys the law of demand, formally that $D_t(p)$ is nonincreasing in p.

Assumption 3 (**Law of Demand**). *For all $t \in T$, $D_t(p') \geq D_t(p'')$ for all $p'' > p' \geq 0$.*

Assume that market demand is growing over time, formally that $D_t(p)$ is nondecreasing in t.

Assumption 4 (**Growing Demand**). *For all $t', t'' \in T$ such that $t' < t''$, $D_{t'}(p) \leq D_{t''}(p)$ for all $p \geq 0$.*

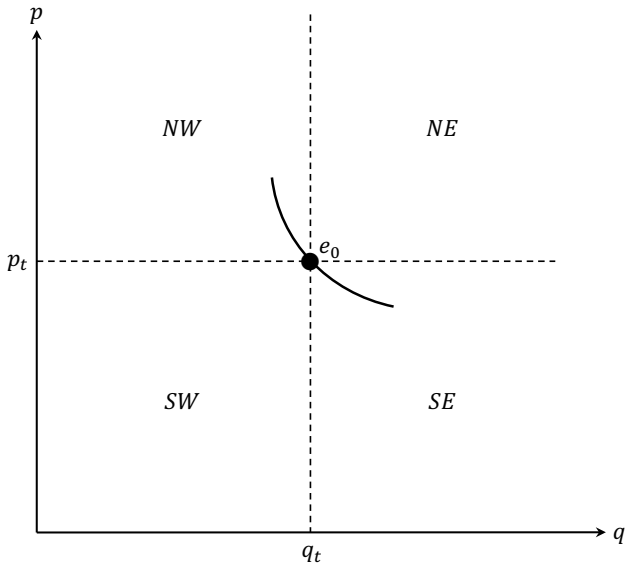


Figure 2: Sets Determined by Compass Position

The assumption requires the demand curve to shift out at least weakly from one period to the next. The shift could be parallel, or it could involve some clockwise or counterclockwise rotation as long as the rotation is not so acute that it leads the demand curves to intersect.

Not all equilibrium configurations E are consistent with growing demand. To aid the discussion of which inconsistent configurations are ruled out, we introduce notation for the subsets depicted in Figure 2 determined by compass directions relative to the reference equilibrium point, e_0. Translating the geometric into an algebraic definition, we have $NW \equiv \{(q_t, p_t) \in E \mid q_t < q_0,\ p_t > p_)\}$ and analogously for the sets *NE*, *SE*, and *SW*. The fact that the compass-set definitions involve strict inequalities leaves points on the dotted lines through e_0 in the figure unclassified, but this is without loss of generality under Assumption 1, which precludes equilibrium points from sharing coordinates.

The compass sets can be further partitioned depending on when the equilibria occur. For example, define $NW^- \equiv \{e_t \in NW \mid t < 0\}$ and $NW^+ \equiv \{e_t \in NW \mid t > 0\}$. In words, NW^- is the subset of equilibrium points in *NW* that occur before e_0 and NW^+ the subset that occur after e_0. Four compass directions and two relative times entail eight possible combinations and thus eight possible subsets. The other six subsets (NE^-, NE^+, SE^-, SE^+, SW^-, and SW^+) are defined analogously to NW^- and NW^+. All these time-augmented compass sets are implicitly defined relative to the reference equilibrium point e_0, which can be taken to be any elements of E.

Consider positioning a new equilibrium point e_t that occurs after e_0 in one of the compass sets in the figure. Regardless of where e_t is positioned in *NW*, *NE*, or *SE*, it would be possible to draw a downward-sloping demand curve through e_0 that lies below e_t, thus respecting Assumption 4 that demand is growing. However, there is no way to position e_t in SW^+ without having it lie on a lower demand curve. Thus, in a market with growing demand, $SW^+ = \emptyset$. Similar reasoning applied to the relative position of equilibrium point e_t occurring before e_0 can be used to argue that, in a market with growing demand, $NE^- = \emptyset$. We have the following lemma.

Lemma 1. *Assumption 4 that demand is growing implies that $SW^+ = NE^- = \emptyset$ taking any element of E as the reference equilibrium point e_0.*

3. Bounds Assuming Linear Demand

This section presents our methods for bounding the demand elasticity. The methods all require the researcher to impose an assumption on the functional form, or at least the curvature, of demand. Throughout this section we assume demand is linear. Section 4 extends the methods to general functional forms; but to build intuition here, we start here with the simple linear case.

Fix the reference period in which the we will bound the demand elasticity at $t = 0$. Assume demand in this period is given by

$$D_0(p) = a_0 - b_0 p. \tag{1}$$

Demand other periods can be different from (1) and need not even be linear. The demand specification in (1) involves two parameters, but further requiring the line to pass through the equilibrium point e_0 pins it down to a single-parameter family.

Focus for now on b_0 as the key parameter. Letting $\tilde{D}(p, b_0, e_0)$ be the linear demand with parameter b_0 passing through equilibrium point $e_0 = (q_0, p_0)$, we have

$$D_0(p) = \tilde{D}(p, b_0, e_0) = q_0 + b_0(p_0 - p). \tag{2}$$

Assumption 3 (law of demand) holds if and only if $b_0 \geq 0$.

Knowing e_0 and b_0, one can solve for the absolute value of the demand elasticity in period 0,

$$\epsilon_0 \equiv -\tilde{D}_p(p_0, b_0, e_0)\frac{p_0}{q_0} = \frac{b_0 p_0}{q_0}, \tag{3}$$

where $\tilde{D}_p$ denotes the partial derivative of the demand function with respect to its first argument. Note that $b_0 \geq 0$ implies $\epsilon_0 \geq 0$. For brevity, we will drop the "absolute value" modifier and simply call ϵ_0 the demand elasticity.

3.1. Method Incorporating Local Information

Begin with our simplest of our methods for bounding demand elasticities. We refer to this method as *incorporating local information* because it uses no information beyond the location of other equilibrium points e_t, observed in sample, to constrain the shape of the demand curve through the reference point e_0. This stands in contrast with a method introduced later that uses the intercepts of other demand curves to constrain the shape of the demand curve through e_0. We refer to this later method as *incorporating limiting information*.

We will derive bounds on demand steepness b_0 via pairwise comparisons of that period's equilibrium point e_0 with the location of other equilibrium points, e_t. Using equation (3), bounds on b_0 can then be translated into the desired bounds on ϵ_0.

Before proceeding with the formal analysis, we provide some intuition on how the method works using Figure 3. Each panel compares the reference equilibrium point e_0 to another equilibrium point e_t. Without further information, all we know is that the linear inverse demand through e_0 must be somewhere between the perfectly horizontal and perfectly vertical ones delineated by the dotted lines. What further information the comparison to e_t can provide depends on which of the eight time-augmented compass subsets into which e_t falls relative to e_0. By Lemma 1, two of these sets are empty when demand is growing, leaving the six non-empty sets into which e_t can fall, corresponding to the panels in the figure: SE^+, NW^+, NE^+, NW^-, SE^-, and SW^-.

In Panel A, $e_t \in SE^+$. Because of the plus superscript, we must have $t > 0$. The inverse demand through e_0 cannot be less steep than the line connecting e_0 and e_t; otherwise e_t would have to lie on a lower demand curve, which would violate Assumption 4 of growing demand since $t > 0$. The line connecting e_0 and e_t thus provides a lower bound on the steepness of the inverse demand through

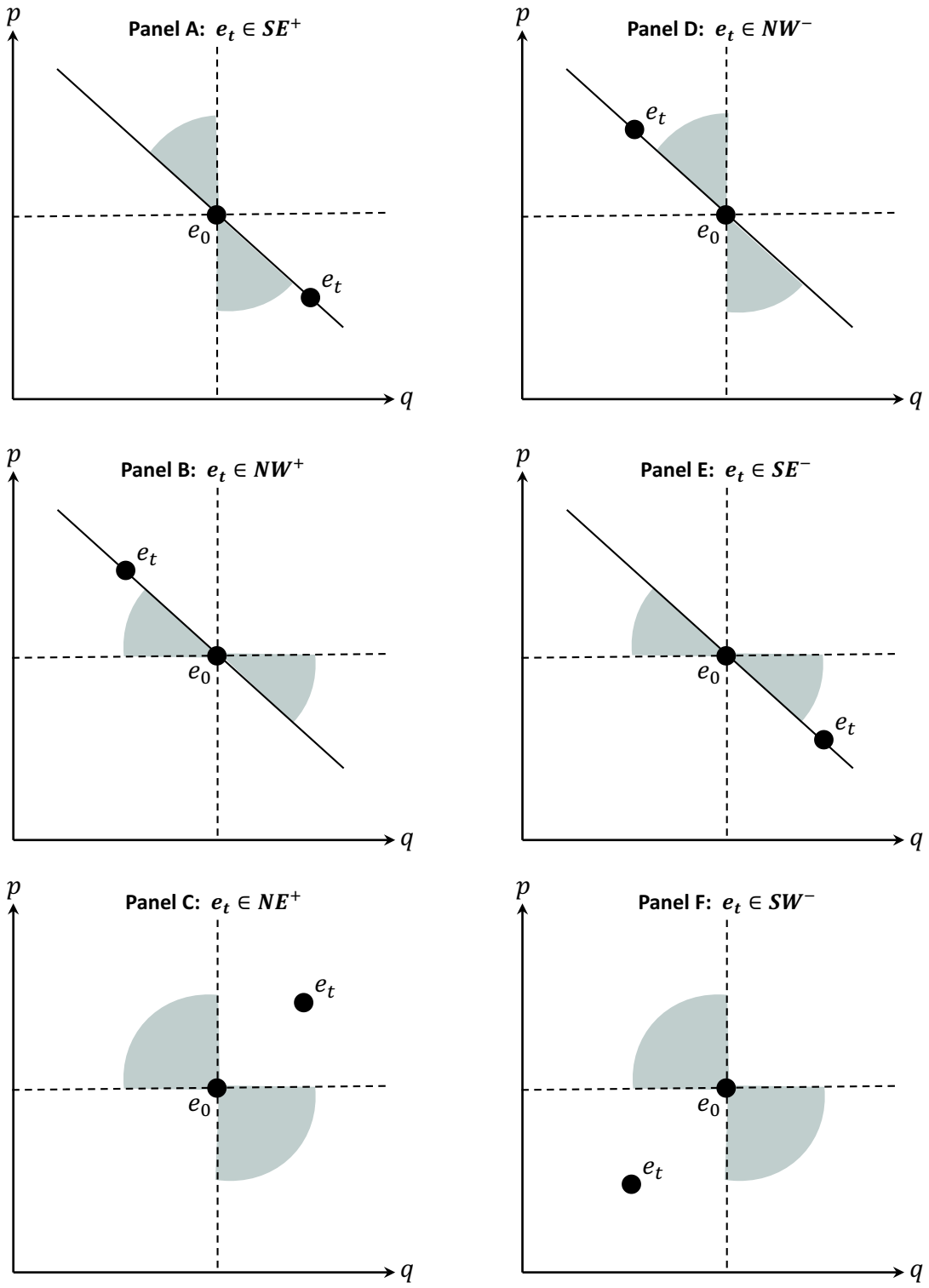


Figure 3: Incorporating Local Information from Pairwise Comparisons to Derive Upper and Lower Bounds

e_0, allowing us to to narrow the location of the inverse demand curve from somewhere between the dotted lines down into the shaded funnel. Recalling that b_0 measures the steepness of the direct (i.e., non-inverse) demand, the lower bound on the steepness of the inverse demand translates into an upper bound on b_0.

In Panel B, $e_t \in NW^+$. With this compass orientation for e_t, the line connecting e_0 and e_t provides an upper bound on the steepness of the inverse demand through e_0; any steeper and e_t would be forced to lie on a lower demand curve, violating the assumption of growing demand. The upper bound on steepness of inverse demand translates into a lower bound on b_0. In Panel C, $e_t \in NE^+$. In this case, regardless of which inverse demand between the dotted lines we draw through e_0, it is always possible to place e_t on a higher demand curve. Thus, in this case the pairwise comparison provides no information to narrow down the location of the inverse demand through e_0.

Panels D–F illustrate cases in which e_t occurs before e_0, thus falling into time-augmented compass sets with minus rather than plus superscripts. Examining each panel, the reader can verify that the bounds are the same as in Panels A–C except that they apply to the opposite compass direction. Thus, $e_t \in NW^-$ generates an upper bound on b_0, $e_t \in SE^-$ generates a lower bound, and $e_t \in SW^-$ provides no information.

We proceed with the formal analysis by comparing e_0 to a later equilibrium point, e_t, with $t > 0$. Then

$$\tilde{D}(p, b_0, e_0) = D_0(p) \leq D_t(p), \tag{4}$$

where the equality follows from (2) and the inequality from the assumption of growing demand. Since (4) holds for all $p \geq 0$, it must hold in particular for p_t. Substituting p_t into (4) yields $\tilde{D}(p_t, b_0, e_0) = q_0 + b_0(p_0 - p_t) \leq D_t(p_t) = q_t$, or rearranging,

$$b_0(p_0 - p_t) \leq q_t - q_0. \tag{5}$$

The bounds on b_0 that can be derived from equation (5) depend on the compass position of e_t relative to e_0. There are three subcases to consider depending on whether e_t is in NW^+, NE^+, or SE^+. The remaining set SW^+ is empty by Lemma 1. To streamline the subsequent analysis, we introduce the function $B(e_{t'}, e_{t''})$, which denotes the absolute value of the slope of the linear

demand through points $e_{t'}$ and $e_{t''}$:

$$B(e_{t'}, e_{t''}) \equiv \left| \frac{q_{t''} - q_{t'}}{p_{t''} - p_{t'}} \right|. \tag{6}$$

Suppose $e_t \in NW^+$. Cross multiplying (5) by $p_0 - p_t$ yields

$$b_0 \geq \frac{q_t - q_0}{p_0 - p_t} = B(e_0, e_t). \tag{7}$$

The first step follows from $e_t \in NW^+$, which implies $p_0 < p_t$ by definition of the compass set. The second step follows because the numerator and denominator of the middle fraction are both negative for $e_t \in NW^+$. Condition (7) provides a lower bound on the demand slope.

Next, suppose $e_t \in NE^+$. Cross multiplying (5) by $p_0 - p_t$ yields

$$b_0 \geq \frac{q_t - q_0}{p_0 - p_t}. \tag{8}$$

The right-hand is negative since the numerator is positive and the denominator is negative for $e_t \in NE^+$. Hence, (8) is a weaker condition than the maintained assumption $b_0 \geq 0$. Thus, this case contributes no useful information to bound b_0.

Next, suppose $e_t \in SE^+$. Cross multiplying (5) by $p_0 - p_t$ yields

$$b_0 \leq \frac{q_t - q_0}{p_0 - p_t} = B(e_0, e_t). \tag{9}$$

Cross multiplying did not change the direction of the inequality because $p_0 > p_t$ for $e_t \in SE^+$. The second step follows because both numerator and denominator of the middle fraction are positive for $e_t \in SE^+$. Condition (9) provides an upper bound on the demand slope.

The pairwise comparison of e_0 and e_t can be repeated for $t < 0$. Sparing the details, analysis similar to the preceding can be used to show $b_0 \leq B(e_0, e_t)$ when $e_t \in NW^-$, $b_0 \geq B(e_0, e_t)$ when $e_t \in SE^-$, and no useful information is contributed when $e_t \in SW^-$. We have proved the following proposition.

Proposition 1. *Suppose demands in all periods satisfy Assumption 3 (law of demand) and Assumption 4 (growing demand). If demand in the reference period is linear, i.e., $D_0(p) = \tilde{D}(p, b_0, e_0)$,*

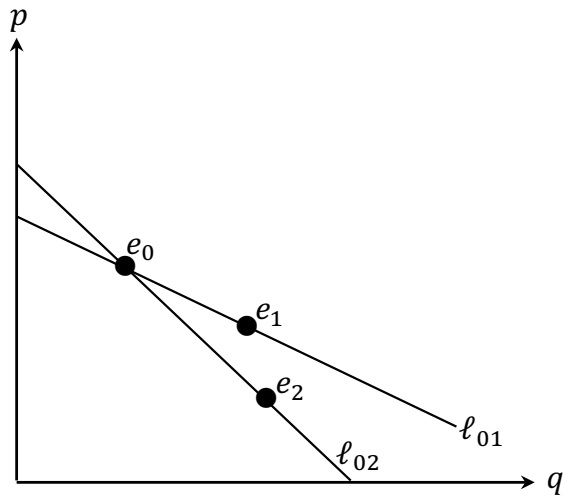


Figure 4: Illustrating Method Incorporating Local Information

then $b_0 \in [\underline{b}_0, \bar{b}_0]$*, where*

$$\underline{b}_0 \equiv 0 \vee \sup_{e_t \in SE^- \cup NW^+} B(e_0, e_t) \tag{10}$$

$$\bar{b}_0 \equiv \inf_{e_t \in NW^- \cup SE^+} B(e_0, e_t). \tag{11}$$

A few technical notes about Proposition 1 are in order. The $\vee$ operator denotes the join; i.e., $x \vee y$ denotes the maximum of x and y. The use of this operator in (10) indicates the imposition of a floor of 0 on top of the supremum. We will use this operator repeatedly later as well as $\wedge$ for the meet; i.e., $x \wedge y$ denotes the minimum of x and y. The supremum is taken in (10) rather than the maximum and the infimum in (11) rather than the minimum even though the sets involved are discrete so that the possibility that one of these sets is empty can be accommodated. An empty set does not have a maximum or minimum but does have a supremum and infimum; we use the conventional definitions $\inf \emptyset = \infty$ and $\sup \emptyset = -\infty$. If the set $SW^- \cup NW^+$ over which the supremum is taken in (10) is nonempty, then imposing a 0 floor on the supremum is superfluous because B is defined to be an absolute value. However, if the set over which the supremum is taken happens to be empty, then $\underline{b}_0 = -\infty$ without the imposition of the 0 floor. Note finally that the proposition only requires demand to be linear in the reference period in which we are deriving the elasticity bound. Functional forms for demands in other periods can be arbitrary.

Bounds on b_0 can be translated into bounds on ϵ_0 using the elasticity formula (3). In particular, $\epsilon_0 \in [\underline{\epsilon}_0, \bar{\epsilon}_0]$, where $\underline{\epsilon}_0 \equiv \underline{b}_0 p_0/q_0$ and $\bar{\epsilon}_0 \equiv \bar{b}_0 p_0/q_0$.

Proposition 1 prescribes a simple algorithm for bounding the elasticity of demand.[2] Figure 4 provides an illustration with three equilibrium points. Taking the initial equilibrium as the reference point e_0, suppose we want to bound the steepness, b_0, and ultimately the elasticity, ϵ_0, of the demand curve through e_0. We first draw lines ℓ_{01} and ℓ_{02} connecting e_0 to the other equilibrium points. The steeper of the two, ℓ_{02}, provides the a lower bound on the steepness of the inverse demand curve through e_0, which translates into the upper bound $\bar{b}_0$ on the steepness demand stated in Proposition 1. Using (3), $\bar{b}_0$ can be translated into an upper bound on the demand elasticity: i.e., $\bar{\epsilon}_0 = \bar{b}_0 p_0/q_0$. Because SE^- and NW^+ happen to be empty, pairwise comparisons do not yield a lower bound $\underline{\epsilon}_0$ on the elasticity in this illustration (apart from 0 derived from non-negativity).

3.2. Assessment in Cournot Example

Before moving to more complex methods and general specifications, in this subsection we take stock of the performance of the bounding method presented so far. We will investigate which market conditions leads it to perform well and which poorly in a simple Cournot model. Besides being a tractable representation of oligopoly, the Cournot model provides fairly robust insights because it nests both monopoly and perfect competition as special cases.

Consider a two-period model indexed by $t = 0, 1$ in which n_t firms with homogeneous marginal and average cost equal to c_t engage in Cournot competition in a market with linear demand, $q_t = a_t - b_t p_t$. For concreteness, we will take the initial period as the reference period and determine the conditions under which the bounds $[\underline{\epsilon}_0, \bar{\epsilon}_0]$ tightly bound the actual elasticity, ϵ_0, then. While the findings in this subsection do not rise to the status of general propositions—applying just to the Cournot example considered here—we still set them off as "results" so that they can be referenced while relegating the technical details behind them to Appendix A.

Our first result clarifies what growing demand entails when demand is linear.

Result 1. *Assumption 4 that demand is growing hold in this subsection's model if and only if*

$$\frac{a_1}{a_0} \geq \max\left\{1, \frac{b_1}{b_0}\right\}. \tag{12}$$

The result indicates that the linear demand curve can grow in several ways. Linear demand can grow increasing a_t holding b_t constant, resulting in a parallel shift out from the origin. Demand

[2] Stata code for this and subsequent procedures is available upon request.

can also grow holding a_t constant but reducing b_t, resulting in an rotation of the inverse demand curve outward from the origin, pivoting through the horizontal intercept. It is even possible for demand to grow when the inverse demand curve rotates the opposite way as long as the rotation does not overwhelm the parallel shift out, guaranteed by $a_1/a_0 \geq b_1/b_0$.

Solving for a firm's Nash equilibrium quantity, and using this to derive market quantity and price, we obtain equilibrium point $e_0 = (q_0, p_0)$, where

$$q_0 = \frac{n_0}{1+n_0}(a_0 - b_0 c_0), \quad p_0 = \frac{1}{1+n_0}\left(\frac{a_0}{b_0} + n_0 c_0\right) \tag{13}$$

and analogously for e_1.

Growing demand, equivalent to condition (12) in this subsection's model, is essential for our bounds not to fail, i.e., for them to contain the true parameter rather than having it fall outside. To demonstrate failure of the bounds under shrinking demand, consider a monopoly market ($n_0 = n_1 = 1$) in which $e_1 \in SE^+$ experiencing a parallel, downward shift from one period to the next, i.e., $b_0 = b_1$ and $a_0 > a_1$. Then

$$\bar{b}_0 = B(e_0, e_1) = b_0 \left[\frac{b_0(c_0 - c_1) - (a_0 - a_1)}{b_0(c_0 - c_1) + (a_0 - a_1)}\right]. \tag{14}$$

The first equation follows from (11), and the second from substituting (13) into (6). Now $e_1 \in SE^+$ implies $q_0 < q_1$, in turn implying

$$a_0 - a_1 < b_0(c_0 - c_1). \tag{15}$$

Condition (15) together with $a_0 > a_1$ implies that the last expression in (14) is strictly less than b_0, implying $\bar{\epsilon}_0 < \epsilon_0$. Thus, the true elasticity lies above our purported upper bound.

While condition (12) ensures bounds $\underline{\epsilon}_0$ and $\bar{\epsilon}_0$ do not fail, a remaining problem is that they may be trivially uninformative, including all nonnegative values. By (10), $\underline{b}_0$ is trivially equal to 0—and thus $\underline{\epsilon}_0$ is trivially equal to 0—unless $SE^- \cup NW^+$ is nonempty. Of course $SE^- = \emptyset$ because there is no equilibrium point prior to e_0. Thus $\underline{\epsilon}_0 > 0$ only if NW^+ is nonempty, requiring $e_1 \in NW^+$, in turn requiring output to fall from period 0 to 1. In view of (13), one can show that the only way for output to fall when demand is increasing is for marginal cost to rise or for the industry structure to become less competitive. One can similarly show that for $\bar{\epsilon}_0$ to be a nontrivial, finite

value, marginal cost must fall or industry structure must become more competitive. We have the following result.

Result 2. *$\underline{\epsilon}_0$ is nontrivial (i.e., $\underline{\epsilon}_0 > 0$) only if $c_1 > c_0$ or $n_1 < n_0$; $\bar{\epsilon}_0$ is nontrivial (i.e., $\bar{\epsilon}_0 < \infty$) only if $c_1 < c_0$ or $n_1 > n_0$.*

Result 2 suggests that obtaining a nontrivial bound on ϵ_0 requires an appropriate market shock. As the conditions required for the lower bound to be nontrivial are the opposite of those required for the upper bound, we see that a single shock can inform the lower bound or the upper bound but not both. With many periods and equilibrium points, nontrivial values of both lower and upper bounds can be obtained if shocks in different directions are experienced, allowing some pairwise comparisons to contribute to the lower bound and some to the upper bound. In our application to the *DuPont* case, there is never a shock that moves a later equilibrium point into NW^+, so we will only be able to identify a nontrivial upper, not lower, bound.

If luck has it that demand remains constant across the two periods, then one of our bounds will hit the true elasticity exactly, as the next result states.

Result 3. *Suppose $a_0 = a_1$ and $b_0 = b_1$. Then one of $\underline{\epsilon}_0$ or $\bar{\epsilon}_0$ equals ϵ_0. In particular, $\underline{\epsilon}_0 = \epsilon_0$ if $e_1 \in NW^+$; and $\bar{\epsilon}_0 = \epsilon_0$ if $e_1 \in SE^+$.*

Improbable luck is required for demand to remain constant while satisfying Assumption 4 because the assumption would then only be satisfied by the narrowest of margins. If, instead of remaining constant, demand increases from one period to the next, whether due to a parallel shift or rotation outward from the origin, $\bar{\epsilon}_0$ will no longer hit ϵ_0. The greater the demand increase, the wider the gap between the two. A tension thus underlies our method: a sizable increase in demand provides assurance that Assumption 4 is satisfied, without which the researcher cannot be sure $\bar{\epsilon}_0$ is a bound at all. If demand increases too much, however, the bound ceases to be very informative. Our bound works best if a market with moderately growing demand experiences a sharp drop in marginal cost or increase in competitiveness.

In the absence of identifying shocks, our bounds can perform quite poorly. Extreme cases can be constructed in which our upper elasticity bound is infinite despite the true elasticity being close to 0. For example, consider a perfectly competitive market ($n_t \to \infty$) with growing demand that does not experience a cost shock ($c_0 = c_1$). Since neither marginal cost nor competitiveness

change from one period to the next, the fact that demand is growing implies $p_1 \geq p_0$, in turn implying $\bar{b}_0 = \bar{\epsilon}_0 = \infty$. The true elasticity is $\epsilon_0 = b_0 c_0/(a_0 - b_0 c_0)$, which approaches 0 as $c_0 \to 0$. For our upper bound on the elasticity to be close to 0, it must be the case both that the true elasticity is close to 0 and that the market experiences shocks during the sample period capable of revealing the low elasticity. In the absence of such shocks, our method will be uninformative. The upper bounds on elasticities near 0 that we will find in the *DuPont* application provide proof of concept that real-world markets may experience shocks sufficient for our method to be informative.

3.3. Method Incorporating Limiting Information

The elasticity bounds can be tightened by exploiting information about the relative position of demand curves for limiting values of prices, $p \to 0$ and $p \to \infty$, rather than observed prices. The method delivers tighter bounds at the cost of requiring more demanding assumptions—that whatever functional form the researcher posits for demand extends over the whole domain of the demand curve rather than just locally and across all periods rather than just in the reference period.

Before delving into technical details, we provide some intuition on how limiting information can help tighten the elasticity bounds using Figure 5. The figure revisits the simple example introduced in Figure 4, carrying over the features from that previous figure as the solid lines. Recall the goal of that simple example was to bound the elasticity ϵ_0 of demand through the reference equilibrium point e_0. The method of incorporating local information from pairwise comparisons allowed us to conclude that the inverse demand through e_0 must be at least as steep as line ℓ_{02}, providing a lower bound of the steepness of inverse demand, translating into an upper bound $\bar{b}_0$ on the steepness of demand and an upper bound $\bar{\epsilon}_0$.

The new features of Figure 5, drawn as dotted lines and open circles, show how to tighten the bound by incorporating information on the position of the limits of demand curves through other equilibrium points. The pairwise comparison of points e_1 and e_2 leads us to conclude that the inverse demand through e_2 must be as steep as line ℓ_{12} for e_1 not to lie on a higher demand curve. But notice that ℓ_{12} intersects ℓ_{02}. Unless the inverse demand curve through e_0 is steeper than ℓ_{12}, parts of the curve will lie above the curve through e_2, violating the assumption of growing demand. For the whole inverse demand through e_0 to be lower than the whole inverse demand through e_2, the demand curves cannot cross even for prices approaching 0, implying in the case

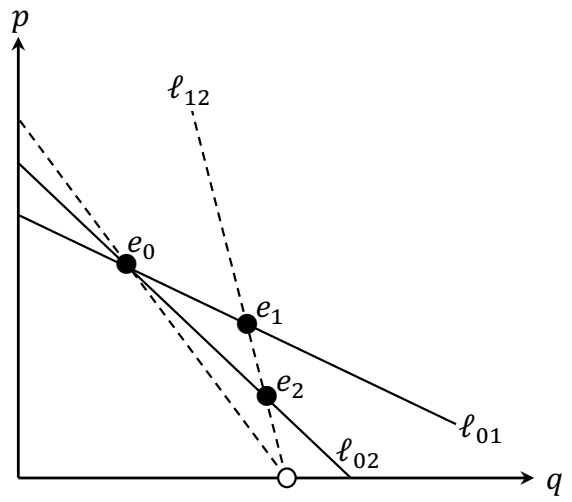


Figure 5: Illustrating Method Incorporating Limiting Information

of linear inverse demands that their horizontal intercepts cannot cross. To ensure their horizontal intercepts do not cross, the inverse demand through e_0 must be at least as steep as the dotted line that connects e_0 with the horizontal intercept of ℓ_{12}, drawn as the open circle. This new line through e_0 is even steeper than ℓ_{02}, tightening the lower bound on the steepness of inverse demand, and thus tightening the upper bound on the demand elasticity.

To facilitate a formal description of the procedure, some new notation is in order. Re-express the bounds incorporating local information presented in the previous subsection by adding a single star to them to indicate that these bounds constitute the first step of an iterative procedure, thus writing $\underline{b}_t^* \equiv \underline{b}_t$, $\bar{b}_t^* \equiv \bar{b}_t$, $\underline{\epsilon}_t^* \equiv \underline{\epsilon}_t$, and $\bar{\epsilon}_t^* \equiv \bar{\epsilon}_t$.

We will derive new bounds emerging from an examination of the limits $p \to 0$ and $p \to \infty$ in turn. Start by considering the limit $p \to 0$ and comparing e_0 to later equilibrium points e_t, $t > 0$. Assumption 4 of growing demand implies $\tilde{D}(p, b_0, e_0) \leq \tilde{D}(p, b_t, e_t)$ for all $p \geq 0$, implying in particular that $\tilde{D}(0, b_0, e_0) \leq \tilde{D}(0, b_t, e_t)$. Substituting from for $\tilde{D}$ from (2) into this inequality, and further substituting $p = 0$, yields $q_0 + b_0 p_0 \leq q_t + b_t p_t$. Notice this is a condition on the relative position of the inverse demand curves' horizontal intercepts. Rearranging,

$$b_0 \leq \frac{1}{p_0}(q_t - q_0 + b_t p_t) \leq \frac{1}{p_0}(q_t - q_0 + \bar{b}_t^* p_t), \tag{16}$$

where the second inequality follows from Proposition 1. Condition (16) provides an additional upper bound on b_0 incorporating information on a different equilibrium point's upper bound $\bar{b}_t^*$

from the prior iteration.

Analogous bounds can be obtained by examining the other limit $p \to \infty$. Additional bounds for both limits can be obtained by comparing e_0 to equilibrium points e_t occurring before rather than after. For brevity, we relegate the derivation of these bounds to the proof, provided in Appendix A, of the next proposition. Combining the bounds incorporating limiting information for all the equilibrium comparisons, if the tightest of them is tighter than the corresponding bound from the first stage incorporating local information, we update the second-stage bound, indicated with two stars in the superscript, to this tighter value. Otherwise, the second-stage bound is just set to first-stage bound incorporating local information. We have the following proposition. Appendix A fills in missing details from the proof sketched in the preceding paragraphs.

Proposition 2. *Suppose the demand curve in every period $t \in T$ is linear, i.e., $D_t(p) = \tilde{D}(p, b_t, e_t)$, and satisfies Assumptions 3 (law of demand) and 4 (growing demand). Whichever element of E the researcher chooses for e_0, the reference equilibrium point, $b_0 \in [\underline{b}_0^{**}, \bar{b}_0^{**}]$, where*

$$\underline{b}_0^{**} \equiv \underline{b}_0^{*} \vee \sup_{t<0} \left\{ \frac{1}{p_0}(q_t - q_0 + \underline{b}_t^{*} p_t) \right\} \vee \sup_{t>0} \left\{ \frac{q_0}{p_t - p_0 + q_t / \underline{b}_t^{*}} \right\} \tag{17}$$

$$\bar{b}_0^{**} \equiv \bar{b}_0^{*} \wedge \inf_{t<0} \left\{ \frac{q_0}{p_t - p_0 + q_t / \bar{b}_t^{*}} \;\middle|\; p_t + \frac{q_t}{\bar{b}_t^{*}} > p_0 \right\} \wedge \inf_{t>0} \left\{ \frac{1}{p_0}(q_t - q_0 + \bar{b}_t^{*} p_t) \right\}. \tag{18}$$

Some technical remarks about the proposition are in order. Note that, as do equations (10)–(11), (17)–(18) involve suprema and infima rather than maximums and minimums to accommodate possibly empty sets, as will be the case when computing bounds for the initial and final equilibrium points. Note also that it is conceivable that continuing the iterations could result in yet tighter bounds, for example computing $\bar{b}_0^{***}$ by replacing $\bar{b}_t^{*}$ with $\bar{b}_t^{**}$ on the right-hand side of (18), and so on. We conjecture that convergence is achieved by the stage shown in (17) and (18), so there is no gain from further iterating.[3]

The bounds $\underline{b}_0^{**}$ and $\bar{b}_0^{**}$ incorporating limiting information are at least as tight as $\underline{b}_0^{*}$ and $\bar{b}_0^{*}$. This is true by definition because the latter bounds appear on the right-hand side of (17) and (18). If the limiting information does not tighten the bounds, by default they return to the first stage that incorporates just local information. The formulas in (17) and (18) are sufficiently complicated that

[3] Our Stata code that implements the bounds in Proposition 2 allows continued iteration. In the empirical application to the *DuPont* decision and all Monte Carlos we have tried, however, iterations stop at $\underline{b}_t^{**}$ and $\bar{b}_t^{**}$. We have managed a general proof that iteration stops at $\underline{b}_t^{**}$ and $\bar{b}_t^{**}$ for many cases, but a handful of cases remain open questions.

it is hard to tell whether $\underline{b}_0^{**}$ and $\bar{b}_0^{**}$ can ever be strictly tighter than $\underline{b}_0^{*}$ and $\bar{b}_0^{*}$. Figure 5, which we now have the language to say derives $\bar{b}_0^{**}$ in the simple example with those three equilibrium points, suggests that $\underline{b}_0^{**}$ and $\bar{b}_0^{**}$ can be strictly tighter than $\underline{b}_0^{*}$ and $\bar{b}_0^{*}$ in some cases. We will indeed see that substantial tightening is possible when we turn to the empirical application to the *DuPont* decision.

Substituting the new bounds into the elasticity formula (3), we have $\epsilon_0 \in [\underline{\epsilon}_0^{**}, \bar{\epsilon}_0^{**}]$, where $\underline{\epsilon}_0^{**} \equiv \underline{b}_0^{**} p_0/q_0$ and $\bar{\epsilon}_0^{**} \equiv \bar{b}_0^{**} p_0/q_0$.

4. Generalizations

This section describes several generalizations of the bounds obtained under the assumption of linear demand. The generalization in Section 4.1 continues to require the researcher to assume a functional form for demand but allows the form to be chosen from a quite general class beyond linear. The generalization in Section 4.2 requires the researcher to make an assumption on curvature (weak concavity or convexity) rather than on the specific functional form demand takes.

4.1. General Functional Forms

This section generalizes the bounding methods from linear to a broad class of functional forms. Suppose the researcher specifies some (possibly nonlinear) form for demand, $q = D_t(p)$. The functional form may be assumed to apply just in the reference period $t = 0$, or across the sample period. While this demand curve may start out as a multiple-parameter family, assume that once it is required to pass through e_t, this pins it down to a single-parameter family indexed by θ_t:

$$D_t(p) = \tilde{D}(p, \theta_t, e_t). \tag{19}$$

As in the linear-demand case, writing the price-sensitivity parameter with a subscript, θ_t, indicates that it may vary over time. The elasticity of demand in the general case is defined as

$$\epsilon_t \equiv -\tilde{D}_p(p_t, \theta_t, e_t)\frac{p_t}{q_t}. \tag{20}$$

Assume $\tilde{D}(p, \theta_t, e_t)$ is continuously differentiable of all orders in all arguments. Assume in-

creases in $\theta_t \in [0, \infty)$ cause demand to rotate. As an accounting convention, assume that the rotation is such that demand becomes steeper (and inverse demand less steep) when θ_t increases. Thus

$$\tilde{D}_\theta(p, \theta_t, e_t) < 0 \quad \text{if } p > p_t \tag{21}$$

$$\tilde{D}_\theta(p, \theta_t, e_t) = 0 \quad \text{if } p = p_t \tag{22}$$

$$\tilde{D}_\theta(p, \theta_t, e_t) > 0 \quad \text{if } p < p_t, \tag{23}$$

where $\tilde{D}_\theta$ denotes the partial derivative of demand with respect to its second argument. Further, impose the following Inada conditions:

$$\lim_{\theta_t \to 0} \tilde{D}(p, \theta_t, e_t) = q_t \tag{24}$$

$$\lim_{\theta_t \to \infty} \tilde{D}(p, \theta_t, e_t) = \begin{cases} 0 & p > p_t \\ \infty & p < p_t. \end{cases} \tag{25}$$

Equation (24) implies that $\tilde{D}(p, \theta_t, e_t)$ becomes perfectly inelastic for arbitrarily small θ_t. Given $\tilde{D}(p, \theta_t, e_t)$ must pass through e_t, as it becomes perfectly inelastic, its inverse approaches a vertical line at quantity q_t for all $p \geq 0$. Equation (25) implies that $\tilde{D}(p, \theta_t, e_t)$ becomes perfectly elastic as θ_t becomes arbitrarily large. Together, (24) and (25) in effect say the domain of θ_t is rich enough to allow changes in θ_t to trace out all possible demand elasticities from perfectly elastic to perfectly inelastic.

For this general class of demand curves, we have worked out analogs to all the bounds derived for linear demands. We have worked out bounds on the price sensitivity parameter using both local and limiting information, and the translation of these bounds into bounds on the elasticity. Appendix B provides details behind the general analysis and precise formulas for the bounds.

An important special case of the general-demand analysis is logit demand. Next to linear, logit is perhaps the next most widely used functional form for demand among applied industrial-organization researchers since its microfounding by McFadden (1973), widely used, for example, in structural estimation of differentiated-product demand following Berry, Levinsohn, Pakes (1995). For reference, Appendix B provides a specification of logit demand in a homogeneous-

product market and derives corollaries of the general-demand propositions in this special case. We draw on these corollaries in our empirical analysis, which provides a parallel set of results for logit alongside those for linear demand.

4.2. Assumed Curvature Rather Than Functional Form

5. Institutional Background

That completes the discussion of general methods. We next turn to the empirical application, beginning with some institutional background on the plastics market and the *DuPont* decision.

The use of plastic in consumer goods—in the products themselves and their packaging—is so widespread today that it is hard to envision the world in the 1950s and 1960s when plastic was initially developed and diffused commercially. Polyethylene was among the first commercially developed plastics. We focus on two types, low- and high-density. Low-density polyethylene was used, among other things, to make Tupperware food-storage containers (Clarke 1999, p. 2) and high-density polyethylene, among other things, to make Hula Hoops (Fenichell 1996, p. 264). Polyethylene remains the highest-volume commercial plastic in terms of global sales.

In the late 1930s, the U.S. Department of Justice sought to prosecute DuPont, a U.S. manufacturer of polyethylene, and its U.K. co-conspirator Imperial Chemical Industries (ICI), for their Patents and Processes agreement. Signed in 1929, the agreement granted each company exclusive licenses for the patents and secret processes controlled by the other and divided the global market into exclusive territories between them. Views differ on whether the main motive for the agreement was the sharing of complementary technologies or the monopolization achieved by market division (Hounshell and Smith 1999, p. 190). The Department of Justice viewed the Patents and Processes agreement as an illegal market division, in violation of Section 1 of the Sherman Act. The military significance of the chemical industry led Franklin Roosevelt's administration to lobby to suspend legal action during the Second World War (Reader 1975, p. 432). After the war, the case proceeded to trial. The prosecution won a liability verdict in 1951, and Judge Sylvester Ryan ordered a remedy in 1952.

Judge Ryan's order cancelled the exclusive-territory arrangements between DuPont and ICI and required them to license patents and secret processes involved in the manufacture of several

of their products to all applicants at reasonable royalty rates. The incumbents' most significant patents covered three products: polyethylene, nylon, and neoprene. Judge Ryan did not order the compulsory licensing of neoprene. Of the remaining products, commentators have contended that only the polyethylene patents garnered substantial commercial interest (Whitney 1958, p. 217). Thus, we focus on the structural remedy's effect on the polyethylene market.

Whether compulsory licensing solves the monopolization problem is the subject of debate in the antitrust literature. In their seminal legal casebook, Areeda, Kaplow, and Edlin (2013) note that the terms of such licensing typically remain subject to commercial negotiations of the parties. This leaves open the possibility that the licensor preserves the monopoly outcome by specifying exorbitant rates.[4] At one point, ICI was asking for a fixed fee of $500,000 and royalties amounting to eight percent of sales revenue (Fortune, 1954). Since this royalty applied to revenue not profit, it would have added to markups, whether as much as the monopoly level may be hard for industry outsiders to know. Expanding the incumbents' scope to manipulate the licensing outcome in their favor, patents often do not relate all the information necessary for entrants to replicate a production process. While Judge Ryan ordered incumbents to provide accompanying manuals and training, the question remains whether these materials were sufficient to allow the entrants to operate on an even footing with incumbents.

The remedy ostensibly had a dramatic effect on the polyethylene market (Backman 1964, p. 71). Seven manufacturers entered in 1953–56 and four more in 1956–59. Prices steadily declined and output rose. On the face of these facts, one might be tempted to conclude that Judge Ryan's remedy achieved its purpose of turning a monopoly into a more competitive market. However, the same price declines and output increases may have arisen in a monopoly market experiencing substantial cost declines, plausibly realistic for plastics in the 1950s and 60s. The fact of entry seems to disprove monopoly unless it is thought that the entrants are merely producing their share of the monopoly quantity, returning most of the rents to the licensor. Thus our study will try to produce evidence for effective remedy that cuts through these criticisms.

To the extent the remedy was successful, a contributing factor may have been that the market

[4]Judge Ryan himself foresaw the potential difficulty in defining a reasonable royalty rate: "While it is true that as to [royalty rates] question might well be raised as to whether they were arrived at after arm's length commercial negotiations, they do nevertheless furnish guide posts for future determination of the amount at which such royalties should be set. But, in any event, as to these products and all others, there is also available for judicial finding the sum a prudent licensee would pay under all existing circumstances." (US vs. ICI, 1952, p. 227–228.)

was monopolized not by a single firm but by two independent firms, which used the Patents and Processes agreement to facilitate collusion. With the agreement cancelled by Judge Ryan's remedy, the independent operators may have been thrown into competition together to license their technologies. This may have disciplined exorbitant licensing fees and thwarted an attempt to preserve the monopoly outcome. That said, the question remains whether incumbents with a history of collusion needed the explicit legal agreement to maintain a cooperative relationship.

6. Data

Our dataset consists of annual price and quantity data, aggregated across all firms in the U.S. market, for two plastic products, low- and high-density polyethylene, in the period following the imposition of the remedy in the *DuPont* (1952) case. The foundation of our dataset is provided by Table 1 of Lieberman (1984), which presents the data he obtained from the annual reports, *Synthetic Organic Chemicals: United States Production and Sales*, issued by the U.S. Tariff Commission. The earliest date that price and quantity are reported for polyethylene is 1958, so Lieberman's data begin then. Lieberman ends the series in 1972 because in subsequent years the OPEC crisis created an oil shock which, together with the subsequent recession, disrupted supply and demand in the plastics market.

Rather than taking Lieberman's data directly, we returned to the original U.S. Tariff Commission source for two reasons. First, we noticed some typographical errors. Second, he recorded production for his quantity variable. Since we are interested in estimates of demand, we went back and collected sales for our quantity variable.[5] Our final dataset consists of prices (measured in nominal dollars per pound) and sales quantities (measured in billion pounds) for low- and high-density polyethylene from 1958–72. The price series is an average wholesale price, computed by dividing total annual industry revenue by industry quantity sold.

Table 1 provides descriptive statistics. Prices are roughly the same across the products. Quantity sold in the market for low-density polyethylene was about twice that in the market for high-density polyethylene.

[5] Production equals the sum of sales, change in inventories, contract production using the customer's raw materials, and production consumed directly by the manufacturer. While we believe sales best captures quantity demanded, we repeat the empirical analysis using production as our quantity measure. The results, presented in Figure C2 in online Appendix C, are qualitatively close to those using sales for quantity.

Table 1: Descriptive Statistics

		Low-density polyethylene		High-density polyethylene	
Variable	Units	Mean	Std. dev.	Mean	Std. dev.
Revenue	Billion $ per year	0.37	0.10	0.13	0.06
Price	$ per pound	0.19	0.07	0.22	0.09
Sales quantity	Billion pounds per year	2.37	1.27	0.78	0.61
Production quantity	Billion pounds per year	2.57	1.41	0.92	0.69

Source: U.S. Tariff Commission (various years).

Figure 6 displays the evolution of equilibrium over time for the two products in the two panels. The curves are of course not demand; each dot is an equilibrium point resulting from the interaction of demand and a supply relation in the given year. The figure shows how these equilibrium points shift over time. The predominant pattern is for equilibrium to shift to the southeast each year. With one exception, the equilibrium never shifts in the direction (southwest) that by Lemma 1 entails a violation of Assumption 4 that demand is growing over time.[6]

The lone exception is 1963 for low-density polyethylene. Our analysis excludes that product-year. As a robustness check, we redo the analysis preserving the requirement that $SW^+e_t = \emptyset$ by dropping 1962 rather than 1963. The results, reported in Figure C1 in online Appendix C, are quantitatively quite similar. We think the most likely cause of this violation of $SW^+e_t = \emptyset$ is that the market may have been growing but not fast enough to offset a small perturbation in price and/or quantity or a small observation error that made it look like demand shrunk that year. Section 9 discusses several approaches to robust estimation that allows for such perturbations.

7. Empirical Results

This section presents empirical results using our methods for bounding elasticity of demand ϵ_t. The nature of our dataset happens to allow only meaningful upper, not lower, bounds to be obtained. We are not able to obtain meaningful lower bounds because SE^-e_t and NW^+e_t happen to be empty for all $e_t \in E$ in our dataset. A lower bound of 0 is obtained by default—as a consequence of the law of demand (Assumption 3). Thus ϵ_t will always be bounded in an interval extending from 0 to

[6]The equilibrium set E in this application satisfies Assumptions 1 and 2 that E is distinct and nontrivial.

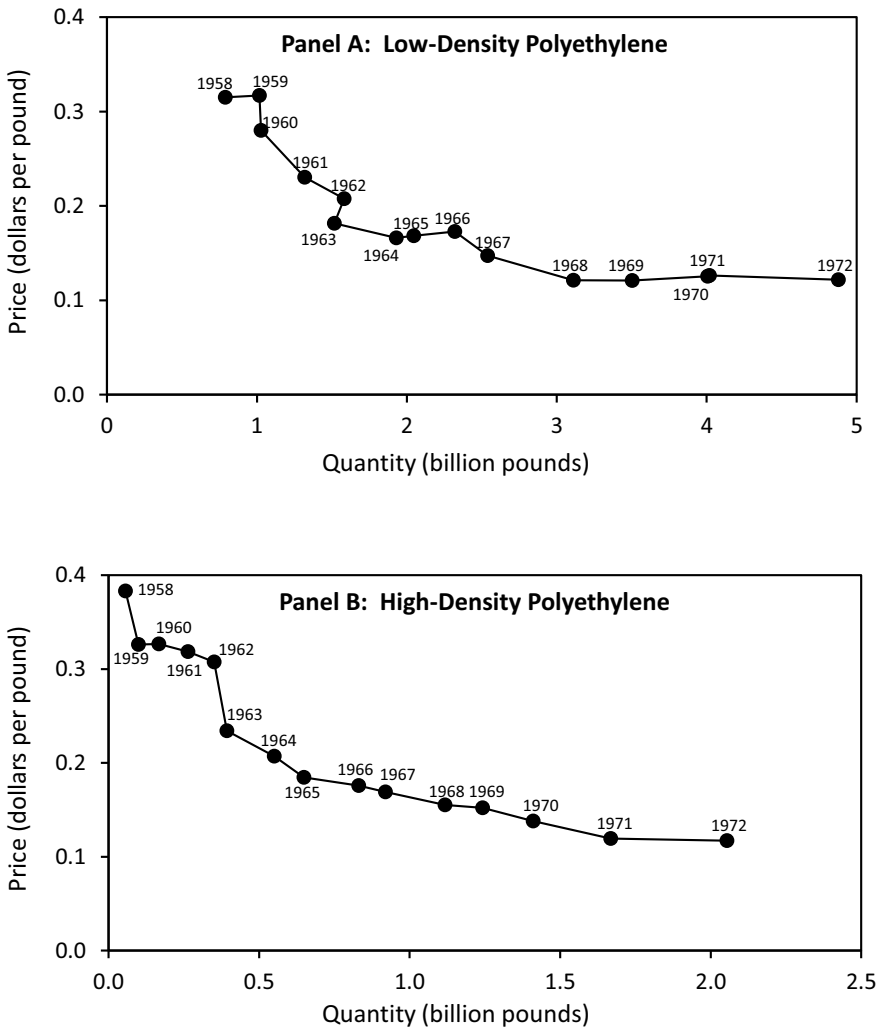


Figure 6: Evolution of Equilibrium in the Plastics Market. *Source:* U.S. Tariff Commission (various years).

the upper bound we derive. To gauge robustness to functional form, we perform all our analyses assuming both linear and logit demands based on the bounds formulas provided in Section 3 for linear demand and Appendix B for logit demand. To gauge robustness to alternative products, we estimate elasticity bounds for both low- and high-density polyethylene.

This section focuses on the bounds themselves; discussion of standard errors is deferred to the next section. Because our upper bounds involve an extreme order statistic (in particular the minimum of results from pairwise comparisons), standard bootstrapping methods are invalid. We instead use Zelterman's (1993) bootstrapping method for extreme order statistics, which requires some discussion.

The results are presented graphically in Figure 7, low-density polyethylene in Panels A–B and

Table 2: Upper Bounds on Demand Elasticity

	Low-density polyethylene				High-density polyethylene			
	Incorporating local information		Incorporating limiting information		Incorporating local information		Incorporating limiting information	
	Linear	Logit	Linear	Logit	Linear	Logit	Linear	Logit
1958	2.50	1.98	0.41	0.38	5.18	3.86	5.18	2.75
1959	0.09***	0.09**	0.09	0.09***	2.48	3.24	2.49	2.08
1960	0.08	0.08	0.08*	0.08*	3.85	3.09	2.15	1.43
1961	0.61***	0.66**	0.06*	0.06*	1.86	1.57	2.15	0.85
1962	0.68*	0.79***	0.06***	0.06***	0.50	0.49	0.50	0.49
1963					0.34***	0.35*	0.34***	0.35
1964	0.52***	0.64***	0.05***	0.05***	0.75***	0.87**	0.29***	0.30***
1965	0.57***	0.72***	0.05***	0.05***	0.69***	0.86***	0.25***	0.26***
1966	0.64***	0.62***	0.05***	0.05***	0.77***	1.06	0.23***	0.25***
1967	0.49***	0.51***	0.04***	0.04***	0.74***	1.08	0.22***	0.24***
1968	0.42***	0.60***	0.03***	0.03***	0.65***	1.07	0.20***	0.21***
1969	0.44***	0.66***	0.03***	0.03***	0.63***	1.12	0.20***	0.21***
1970	0.49***	0.78**	0.03***	0.03***	0.54***	1.01	0.18***	0.19***
1971	0.49***	0.79**	0.03***	0.03***	0.44***	0.86**	0.15***	0.16***
1972	0.49***	0.84*	0.03***	0.03***	0.43***	0.94	0.15***	0.16***

Notes: Missing entries correspond to observation dropped to preserve $SW^{+}e_t = \emptyset$ for all e_t, which by Lemma 1 is a necessary condition for Assumption 4 (growing demand over time) to hold. Significantly different from 1 in a one-tailed test at the *ten-percent level, **five-percent level, ***one-percent level based on the Zelterman (1993) bootstrap.

high-density polyethylene in Panels C–D. For each product, the top panels present results using the method incorporating local information from pairwise equilibrium comparisons, generating the upper bound $\bar{\epsilon}_t^*$. The bottom panels present results using the method incorporating limiting information, generating the upper bound $\bar{\epsilon}_t^{**}$. For reference, the precise numerical elasticities are provided in Table 2.

The figure displays bootstrapped confidence intervals around upper bounds $\bar{\epsilon}_t^*$ or $\bar{\epsilon}_t^{**}$ as whiskers at the top of the bars. The confidence intervals are based on Zelterman (1993) bootstrapped standard errors, discussed in the next subsection. We chose to display a two-tailed 90% confidence interval because this allows easy visualization of the one-tailed test of whether the elasticity bound is less than 1 at the standard (5%) significance level. The starred significance levels in Table 2 are also based on the Zelterman (1993) bootstrap.

Focus first on the results in Panel A for low-density polyethylene using the method incorpo-

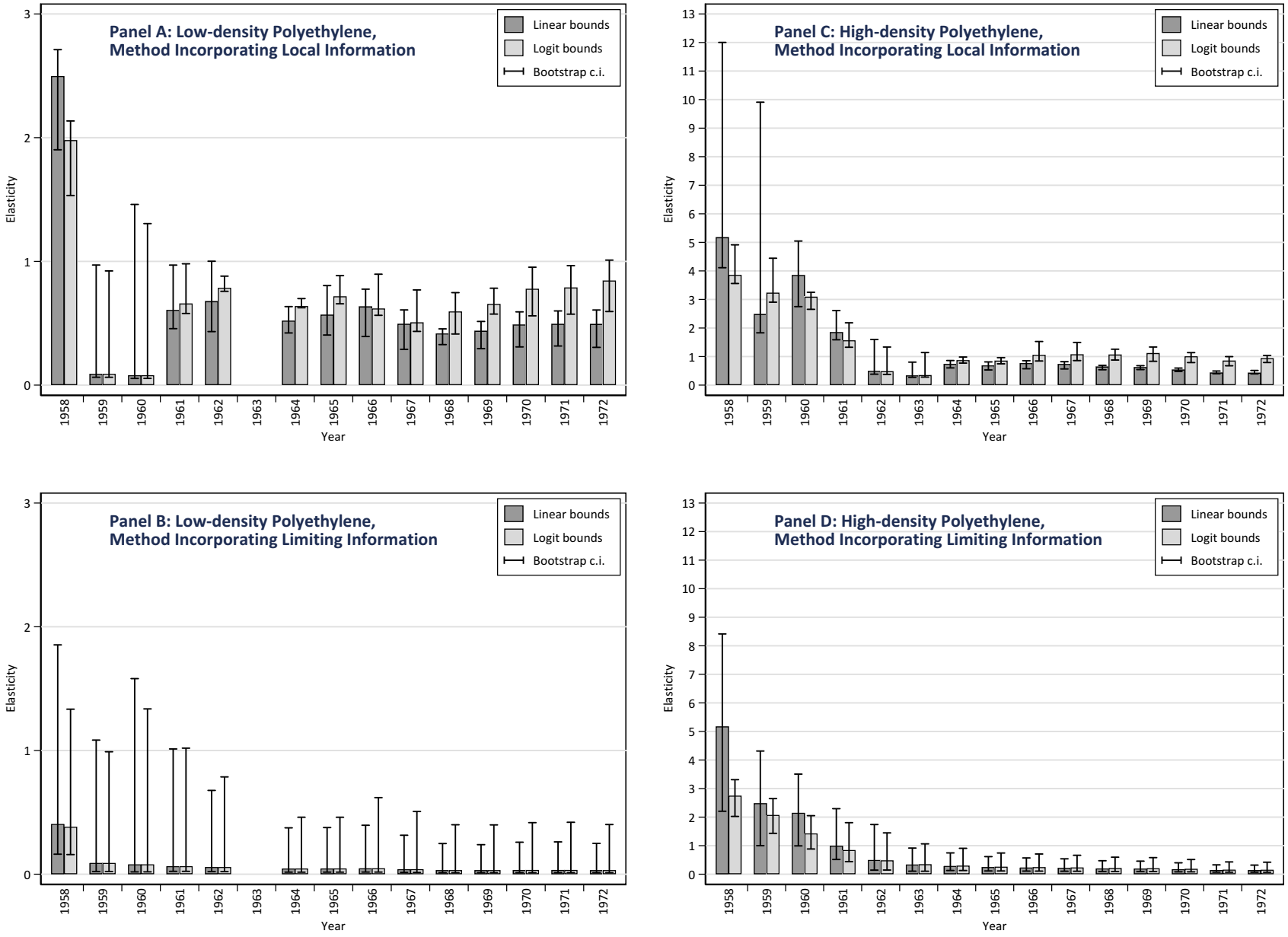


Figure 7: Elasticity Bounds. *Notes:* Shaded bars are elasticity bounds assuming linear demand (dark bars) or logit demand (light bars). Whiskers at top of bars are 90% two-sided confidence intervals on the upper elasticity bound bootstrapped using Zelterman's (1993) technique.

rating local information. Within the panel, focus first on the dark-shaded bars, representing the elasticity bounds imposing linear demand. The results are remarkable. Aside from the first year, in which the upper bound is quite high at 2.50, the upper bound on the elasticity is consistently low, never higher than 0.68—which is still quite inelastic.

The upper bound can be much lower than this, in particular, 0.09 and 0.08 in 1959 and 1960. The robustness of the extremely low bounds in those years is called into question, however, by the wide confidence intervals around them, in 1960 even straying above the threshold of 1 that might be consistent with monopoly behavior. The next section will provide a more detailed discussion of robustness. For now, we will note that a consistent finding in our results—holding across specifications, methods, and products—is that the upper bounds in the early years in the sample are either quite high, or have wide confidence intervals, or both.[7]

Setting aside the admittedly weak results for the early years and focusing on the remaining 1961–72 sample, we see that the upper bound is below 0.5 a majority of the time, significantly less than 1 at the 1% level in a one-tailed test for all of these years except 1962, in which the test is significant at the 10% level. Such low elasticities are inconsistent with a monopoly outcome, which in theory leads to an equilibrium in the elastic region of demand.

The light-shaded bars represent elasticity bounds assuming logit rather than linear demand. The results are consistent across the two functional forms in Panel A. The logit bound can be tighter or looser than that under linear: in the early years, logit demand generates tighter bounds (or at least tighter bootstrap whiskers); in later years, linear demand tends to generate tighter bounds. Overall, the two specifications track each other closely, rising and falling together over the sample, suggesting the results are robust to functional-form assumptions. Setting aside the weak results in the early years (1958–60), in the later years (1961–72), the elasticity bound under logit is as low as 0.51. The bound is significantly less than 1 at the 1% level for a majority of these years at least the 5% level for all these later years except 1972, when it is significant at just the 10% level.

Panel B presents results for low-density polyethylene using the method incorporating limiting

[7] We suspect the weak results for early years are not a symptom of an idiosyncratically high elasticity in that year; we have no evidence to suggest that polyethylene demand was markedly different in early compared to later years. Two other causes are more likely. First, early years lack prior-year information to bound the elasticities. Second, our methods work by first bounding the demand slope, multiplying by price and dividing by quantity to derive the elasticity. As Figure 6 shows, polyethylene prices were relatively high and quantities low in the early years. For a given slope bound, adjusting by price and quantity will naturally lead to high elasticity bounds.

information. The estimates for linear and logit demand are almost identical; both are are sharply tighter than in Panel A. When limiting information is incorporated, the upper bounds on the elasticity shrink down to around 0.4 in the first year and extremely close to 0 in all later years. The confidence intervals suggest that the extremeness of these estimates may not be particularly robust: the whisker representing the upper confidence-interval threshold is an order of magnitude higher than the estimated elasticity bound, which shrinks as low as 0.03 in the last five years. The low elasticity bounds obtained in the first stage from the pairwise comparison of the years 1959 and 1960 incorporating local information generates extreme limits that propagate across all the other years in the second stage incorporating that limiting information.

Bootstrapping helps correct for this propagation problem because a single, extreme pairwise comparison from the first stage will not be drawn for some second-stage pseudosamples, resulting in larger bootstrapped bounds and wider confidence intervals. Even after bootstrapping widens the confidence interval in this way, the upper confidence-interval thresholds in Panel B are only half those in many of the later years in Panel A. Our results thus document the possibility that the method incorporating limiting information can substantially tighten the results compared to the method incorporating even after correcting the propagation problem via bootstrapping. This is not to say that the method incorporating limiting information is better. First, the method incorporating limiting information does not always result tighten the thresholds: for example, the upper confidence-interval thresholds are looser in Panel B than A in the years 1959–61. Second, if the method incorporating limiting information does deliver tighter bounds, they come at the cost of stronger maintained assumptions (the assumed functional for demand must hold globally, not just locally) and less robustness.

Panels C–D present elasticity bounds for the other product, high-density polyethylene. In this market, the bounds are much wider in the early years compared to low-density polyethylene, necessitating a different vertical-axis scale to display. The bounds narrow considerably by 1963, the upper bound reaching as low as 0.34 for linear demand and 0.35 for logit demand in that year. The bounds remain well within the inelastic region under linear demand. Under logit demand the upper bound rises up above 1 in 1966 and remains around that level for the rest of the sample. The bound is significantly less than 1 at at least the 10% level in only four of the years. It should be emphasized that the logit-demand bounds do not imply that the elasticity was this high but that this

method at least cannot rule such high elasticities out. Overall, the upper bound on the elasticity is higher in the high-density market than low-density market, but even in the high-density market, elastic demand and monopoly is precluded in some years in the post-remedy period.

Panel D applies the method incorporating limiting information to the high-density polyethylene market. As in Panel C, the bounds (or the whiskers for the upper confidence-interval threshold) are not particularly tight in the first five years; but by 1964 the upper bound has shrunk below 0.3, and continues to shrink steadily, down to around 0.16 by 1972. After 1964, the bounds are all significantly less than 1 at the 1% level. Regardless of the functional form assumed, incorporating limiting information provides extremely tight bounds on the elasticity of demand in the high-density market. For nearly a decade, demand in each market was well below the level consistent with monopoly according to this method.

8. Zelterman Bootstrap

A natural way to generate error bounds around our estimates is to apply a resampling technique such as boostrapping to the pairwise comparisons between equilibrium points. Standard bootstrap methods are invalid in our context because our upper elasticity bound involves an extreme order statistic, the minimum over pairwise comparisons between a given equilibrium point and others. As Bickel and Freedman (1981) note, it is impossible to draw a pseudosample generating an order statistic more extreme than that generated with the original data because any pseudosample is a subset of the original data. Hence the bootstrapped distribution will be bounded by—rather than centered on—the extreme order statistic estimated from the original data.

Zelterman (1993) provides a technique for circumventing this problem. Instead of sampling the data directly, to bootstrap the maximum order statistic, he proposes sampling the spacings between the highest k observations. Suitably normalized, these spacings asymptotically are i.i.d. exponentials. The crucial fact for Zelterman's purposes is not that the asymptotic distribution is exponential but that it is i.i.d., implying that a bootstrap pseudosample drawn with replacement from the normalized spacings has the same distribution. The maximum order statistic can be bootstrapped by adding the spacings from the pseudosample to the order statistic in the sample anchored k positions away from the maximum. Zelterman classifies the technique as semiparametric since the

result that the normalized spacings are asymptotically i.i.d. exponential holds for a wide range of distributions of the underlying data.[8] The sole parameter is k, anchoring the order statistic to which the sampled spacings are appended when simulating the maximum order statistic.

Formally, consider applying Zelterman's technique to bootstrap a confidence interval around $\bar{\epsilon}_t^*$, the upper bound on the demand elasticity in period t using the method incorporating local information. Although it is an upper bound, $\bar{\epsilon}_t^*$ is a minimum order statistic: the minimum of the upper bounds derived from pairwise comparisons between e_t and the other equilibrium points. Let $\bar{\epsilon}_t^{[1]} \leq \bar{\epsilon}_t^{[2]} \leq \cdots \leq \bar{\epsilon}_t^{[T-1]}$ be the elasticity bounds from these pairwise comparisons ordered from smallest to largest; superscripts on these terms thus denote the order of these order statistics. We have $\bar{\epsilon}_t^* = \bar{\epsilon}_t^{[1]}$.

As Zelterman's (1993) analysis is predicated on a maximum rather than a minimum order statistic but $\bar{\epsilon}_t^*$ is a minimum order statistic, we will bootstrap the reciprocal of $\bar{\epsilon}_t^*$ rather than $\bar{\epsilon}_t^*$ itself, reversing the reciprocal in the final step to recover the elasticity bound. Defining the reciprocal $\bar{r}_t^{[i]} = 1/\bar{\epsilon}_t^{[i]}$, the inequalities between the order statistics on the reciprocals are the reverse of those between the elasticity bounds: $\bar{r}_t^{[1]} \geq \bar{r}_t^{[2]} \geq \cdots \geq \bar{r}_t^{[T-1]}$. Let $d_t^{[i]} \equiv i(\bar{r}_t^{[i]} - \bar{r}_t^{[i+1]})$ denote the normalized spacing between two of these reciprocals. The normalization needed to generate the exponential distribution asymptotically is simply multiplication by i, the degree of the order statistics involved in the spacing.

Let $\mathbf{d}_t \equiv \{d_t^{[i]} \,|\, i = 1, \ldots, k\}$ denote the set of the normalized spacings observed in the data between the k highest reciprocals. Pseudosamples $\tilde{\mathbf{d}}_t = \{\tilde{d}_{ti}\}$ of size k are drawn with replacement from $\mathbf{d}_t$. (In practice, we take 10,000 draws of bootstrap pseudosamples to provide the precision demanded by 99% confidence intervals.) For each pseudosample, the bootstrap maximum order statistic is computed as

$$\tilde{r}_t^* \equiv \bar{r}_t^{[k+1]} + \sum_{i=1}^{k} \tilde{d}_{ti}/i. \tag{26}$$

[8] Weissman (1978) provides the following sufficient condition for the asymptotic result: there exists real numbers $a_t > 0$, $b_t, t = 1, 2, \ldots$, such that for all $x \in (-\infty, \infty)$, the distribution function $F(x)$ for the underlying data satisfies

$$\lim_{t \to \infty} F^t \left(\frac{x - b_t}{a_t} \right) = \exp(-\exp(-x)).$$

In words, the highest order statistic—properly normalized by constants a_t and b_t—converges in distribution to a Gumbel extreme value as the sample size grows large. The sufficient condition is satisfied for many common distributions including the normal, lognormal, logistic, gamma, Pareto, Gompertz, Weibull, and Gumbel.

Intuitively, k draws of normalized spacings are subtracted (after reversing the normalization by dividing by i) from the observed $(k+1)$st order statistic $\bar{r}_t^{[k+1]}$, which anchors the procedure, to arrive at the simulated maximum $\tilde{r}_t^*$. Following Zelterman's recommendation, we take $k = \lfloor (T-1)/3 \rfloor$.[9,10] Taking the reciprocal of this bootstrapped maximum order statistic recovers the bootstrapped elasticity bound: $\tilde{\epsilon}_t^* = 1/\tilde{r}_t^*$.

A numerical example serves to illustrate the Zelterman bootstrap. According to Table 2, the upper bound on the demand elasticity for low-density polyethylene derived under the assumption of linear demand using the method incorporating local information is $\bar{\epsilon}_t^* = 0.52$ for the year $t = 1964$. Suppose we want to bootstrap this estimate. This is the minimum of the elasticity bounds from pairwise comparisons, the five lowest of which are shown in the second column of Table 3. Setting the key parameter for the technique to $k = 4$, the five lowest pairwise bounds are all that are relevant for the computation. Taking reciprocals in the next column, the entry in the first row now becomes the maximum order statistic among them. The last column computes the normalized spacings. This is the set from which pseudosamples are drawn with replacement to compute bootstrapped estimates. Suppose, for example, that the pseudosample $\{0.40, 0.40, 0.64, 0.24\}$ was drawn. This would generated the bootstrapped reciprocal

$$\tilde{r}_t^* = 1.22 + \frac{0.40}{1} + \frac{0.40}{2} + \frac{0.64}{3} + \frac{0.24}{4} = 2.09, \tag{27}$$

equal to the anchor reciprocal in the last row plus the four draws of normalized spacings with normalizations reversed. Reversing the reciprocal generates the bootstrapped value of the elasticity bound: $\tilde{\epsilon}_t^* = 1/2.09 = 0.48$. This procedure can be repeated (we do this 10,000 times) to generate a bootstrapped sample of elasticity bounds from which standard errors and confidence intervals can be derived.

Bootstrapping the bound $\bar{\epsilon}^{**}$ incorporating limiting information is more involved because it is a

[9]Zelterman notes that the asymptotic argument requires $k \to \infty$ as well as $T \to \infty$. Our sample of pairwise comparisons is small since it comes from a relatively short time series. We still proceed to apply the technique and the recommendation for k but note that we may be straining the asymptotic arguments behind the bootstrap. On a positive note, despite the small size of T and k in our application, the bootstrapped confidence intervals appear quite sensible and well behaved.

[10]The bootstrap procedure is written in Stata code, available on request from the authors. Even for the logit specification, which involves numerical solution of a nonlinear equation for each draw, running the procedure with 10,000 draws only takes a few minutes on a desktop computer.

Table 3: Numerical Example of Zelterman Bootstrapping

$[i]$	$\bar{\epsilon}_t^{[i]}$	$\bar{r}_t^{[i]}$	$d_t^{[i]}$
1	0.52	1.92	
			$(1.92-1.52)\times 1 = 0.40$
2	0.66	1.52	
			$(1.52-1.46)\times 2 = 0.12$
3	0.68	1.46	
			$(1.46-1.38)\times 3 = 0.24$
4	0.72	1.38	
			$(1.38-1.22)\times 4 = 0.64$
5	0.82	1.22	

Notes: Entries in $\bar{\epsilon}_t^{[i]}$ column are the five lowest of the actual elasticity bounds from pairwise comparison of the 1964 equilibrium point to other year's.

two-stage estimator, and the outcome of the second stage depends in a complicated way on the first stage. We adopt what in our view is the most natural alternative: applying Zelterman's technique to simulate a bootstrapped estimate from the normalized spacings observed in each separate stage, and then taking the minimum over the result from each stages to generate the final bootstrapped bound $\tilde{\epsilon}_t^{**}$.[11,12]

9. Robustness

Our method implicitly assumes price and quantity are perfectly observed. Observation errors can create potential problems for our method. These problems are likely to be relatively inconsequential if the market is growing rapidly over the sample but may be acute if the market experiences periods of only slow growth.

[11] An alternative would be to bootstrap the demand parameter rather than the elasticity directly. In the case of linear demand, b_t could be bootstrapped. Then the first stage bound ($\bar{b}^*$) could be treated equivalently to the pairwise comparisons between e_t and the other equilibrium points incorporating limiting information in the second stage (the expressions in braces in equation (18)). We tried this alternative, and the results were nearly identical to the alternative we adopted.

[12] While the bootstrapped confidence intervals around $\bar{\epsilon}^{**}$ should typically be tighter than around $\tilde{\epsilon}_t^*$, randomness in the draws can lead the reverse to happen in isolated cases. We see this in for example in Table 2, in the 1961 entries for low-density polyethylene: the one-tailed test is more significant for the first-stage method incorporating local information than the second-stage method incorporating limiting information. In virtually all of the rest of the entries, the second-stage confidence intervals are tighter than the first-stage.

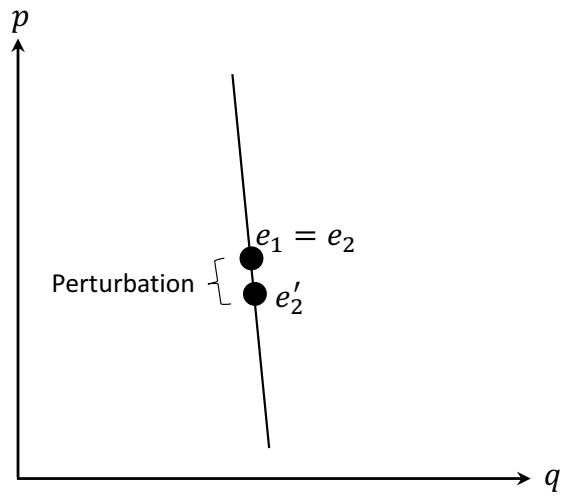


Figure 8: Vulnerability of Method to Observation Errors in Slow-Growing Market

Figure 8 illustrates the potential vulnerability. To take an extreme example, suppose a monopoly serves a market that is unchanging over time. Suppose that the true equilibrium point e_1 continues into period 2 as e_2, but due to an observation error or other small perturbation, e_2' is instead recorded in the data. Using our method, say assuming linear demand, pairwise comparison of e_1 and e_2' implies that the inverse demand curve in both periods would have to be at least as steep as the line drawn. If the perturbation happened to nudge e_2' almost directly south of e_1, we would conclude that demand was almost perfectly inelastic, erroneously rejecting monopoly control of the market. Rapid market growth—driving e_1 and e_2 apart instead of lying on top of each other—would attenuate the effect of a small perturbation from e_2 to e_2' on the estimated elasticity bound.

The concern is not just theoretical. In our application, one might be concerned that the extremely low elasticity bounds of 0.08 and 0.09 in Panel A of Figure 7 were generated by a single perturbation from 1959 to 1960 in the low-density polyethylene market rather than being a robust consequence of many pairwise comparisons. The configuration of the 1959 and 1960 equilibrium points in Panel A of Figure 6 resembles that in Figure 8, with the points lying close enough together that a small southward perturbation could generate an almost vertical line connecting them and a elasticity bound close to zero.

We take several approaches to addressing the robustness concern. One approach has already been discussed: bootstrapping confidence intervals. If the estimates are unduly influenced by the position of one or two equilibria, we will see higher estimates in bootstrapped subsamples from

which they are omitted, widening the confidence intervals.

This section takes a different approach. Rather than starting with an estimator that is potentially vulnerable to small perturbations and drawing a confidence interval around it to gauge its vulnerability, we propose a series of estimators that are robust to begin with. Section 9.1 introduces estimators that leave out the most influential equilibrium points. We will see that the leave-out estimator appears to stabilize after leaving out the single most influential equilibrium point, the result corresponding closely to the bootstrapped 95% upper confidence-interval threshold. Section 9.2 proposes biennial estimators that only require demand to grow over a two-year horizon rather than year to year.

9.1. Leave-Tightest-Out Estimators

Let $\bar{LT}_t^*(1)$ denote the estimator that leaves the most influential observation out of the calculation of the upper bound on the demand elasticity, labeled the leave-tightest-one-out estimator for short. Formally, $\bar{LT}_t^*(1) \equiv \max_{t' \neq t} \bar{\epsilon}^* \langle e_{t'} \rangle$, where $\bar{\epsilon}^* \langle e_{t'} \rangle$ is the upper bound on the elasticity restricting the sample to $E \setminus \{e_{t'}\}$. Analogously, define the leave-tightest-two-out estimator $\bar{LT}_t^*(2) \equiv \max_{t',t'' \neq t} \bar{\epsilon}^* \langle e_{t'}, e_{t''} \rangle$. The leave-out estimators associated with the method incorporating limiting information can also be defined: $\bar{LT}_t^{**}(1) \equiv \max_{t' \neq t} \bar{\epsilon}^{**} \langle e_{t'} \rangle$ and $\bar{LT}_t^{**}(2) \equiv \max_{t',t'' \neq t} \bar{\epsilon}^{**} \langle e_{t'}, e_{t''} \rangle$. Leave-out estimators with orders higher than two can also be defined.[13]

Figure 9 presents the results from the leave-tightest-out estimators. The shaded bars repeat the elasticity bounds from Figure 7. The difference are the whiskers, which before represented bootstrapped confidence intervals but now represent the extension to the upper bound when the one or two most influential comparisons are left out. The thick whisker is the extension due to the leave-one-out estimator and the thin whisker to the leave-two-out estimator.

Across the four panels A–D, we see that the leave-tightest-out estimators lead to a jump in the upper bound for the early years and across all years for the method incorporating limiting information. These results confirm the claim that the tight bounds for 1959 and 1960 were due to

[13] Although the jackknife is also a leave-out estimator, its resemblance to $\bar{LT}_t^*(1)$ and our other leave-out estimators ends there. The jackknife averages over estimates from leave-out samples whereas $\bar{LT}_t^*(1)$ takes the worst case—in the case of upper bounds their maximum—over leave-out samples. Further, the jackknife averages over pseudovalues (a suitably weighted difference between the original and leave-out estimates), whereas $\bar{LT}_t^*(1)$ is computed directly from the leave-out estimates, and the computation does not involve the original estimate. The standard jackknife is invalid in our setting for the same reason given by Bickel and Freedman (1981) that the standard bootstrap is invalid.

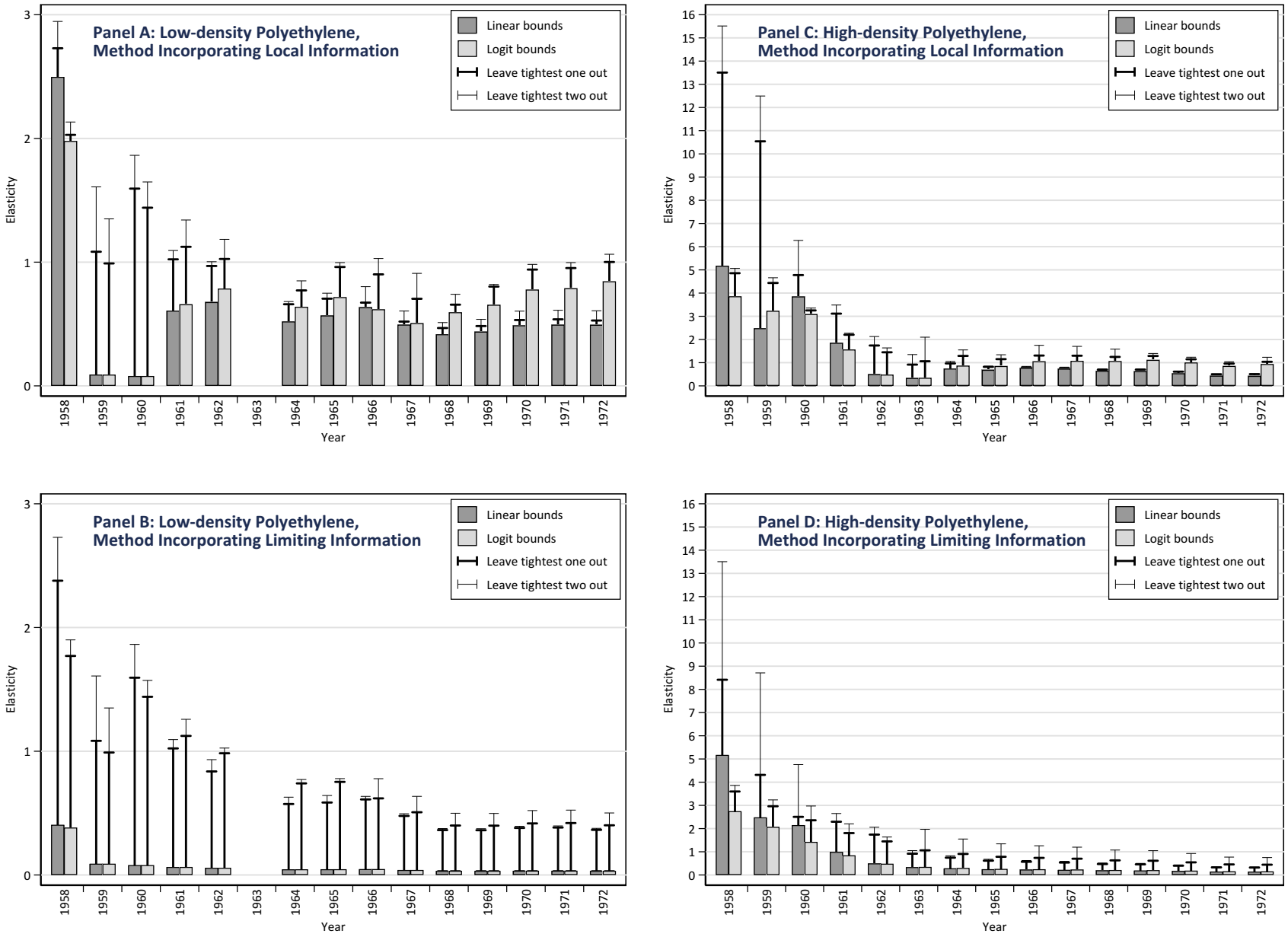


Figure 9: Leave-Tightest-Out Estimators. *Notes:* Shaded bars are the same elasticity bounds as in Figure 7 for linear demand (dark bars) and logit demand (light bars). Whiskers above bars represent leave-out estimators: the thick whisker represents the leave-tightest-one-out estimator, and the thin whisker the leave-tightest-two-out estimator.

the positioning of 1960 almost vertically below 1959. When one or the other is left out, this causes the upper bound in 1959 and 1960 in Panel A to jump up to a level more consistent with the other years. This extremely tight bound in the first stage led to extreme limits that propagated across the other years in the second stage incorporating limiting information. When one or the other year is left out of the first stage, we see in Panel B that the bounds incorporating limiting information jump from near zero to something more moderate.

While the leave-tightest-out estimators lead to higher elasticity bounds than the original in Figure 9, we can still take some reassurance from the figure. First, while the bounds jump up in some early years, in later years, the leave-tightest-out estimators remain below the threshold of 1 ruling out monopoly. In particular, for linear demand in the market for low-density polyethylene in Panel A, $\bar{LT}_t^*(1)$ is below 0.5 in a number of later years. For both linear and logit demand, $\bar{LT}_t^{**}(1)$ is below 0.5 for in all of the last six years in Panel B. The leave-out bounds are nearly as tight in the later years for high-density polyethylene in Panels B and D.

Also reassuring is the stability of the leave-out estimators: in years after the first three, the thin whisker does not extend much beyond the thick one, implying that the leave-one-out and leave-two-out estimators are quite close. Leaving one pairwise comparison out appears to be sufficient to protect against the vulnerability to small perturbations illustrated in Figure 8.

Panel A of Figure 10 compares the leave-tightest-one-out estimator against the 95% bootstrapped upper confidence threshold from Figure 7. The figure just shows the comparison for one product (low-density polyethylene) and one method (incorporating local information). Comparisons for other products and methods are qualitatively similar for other products and methods. For space considerations, they are relegated to online Appendix C (Figures C3–C5). The leave-tightest-one-out estimator in Panel A is almost identical to the bootstrap threshold, lying almost precisely on the 45-degree line. Hence, both approaches to robustness generate almost equivalent results. Most dots lie inside the dotted box with dimensions less than 1, the threshold below which monopoly behavior is ruled out.

Panel B plots the leave-tightest-two-out estimator versus the 95% bootstrapped upper confidence threshold. Given that the bootstrap threshold is nearly equivalent to the leave-tightest-one-out estimator, and the leave-tightest-one is by construction more conservative than the leave-tightest-two-out estimator, we expect the latter to be more conservative than the bootstrap thresh-

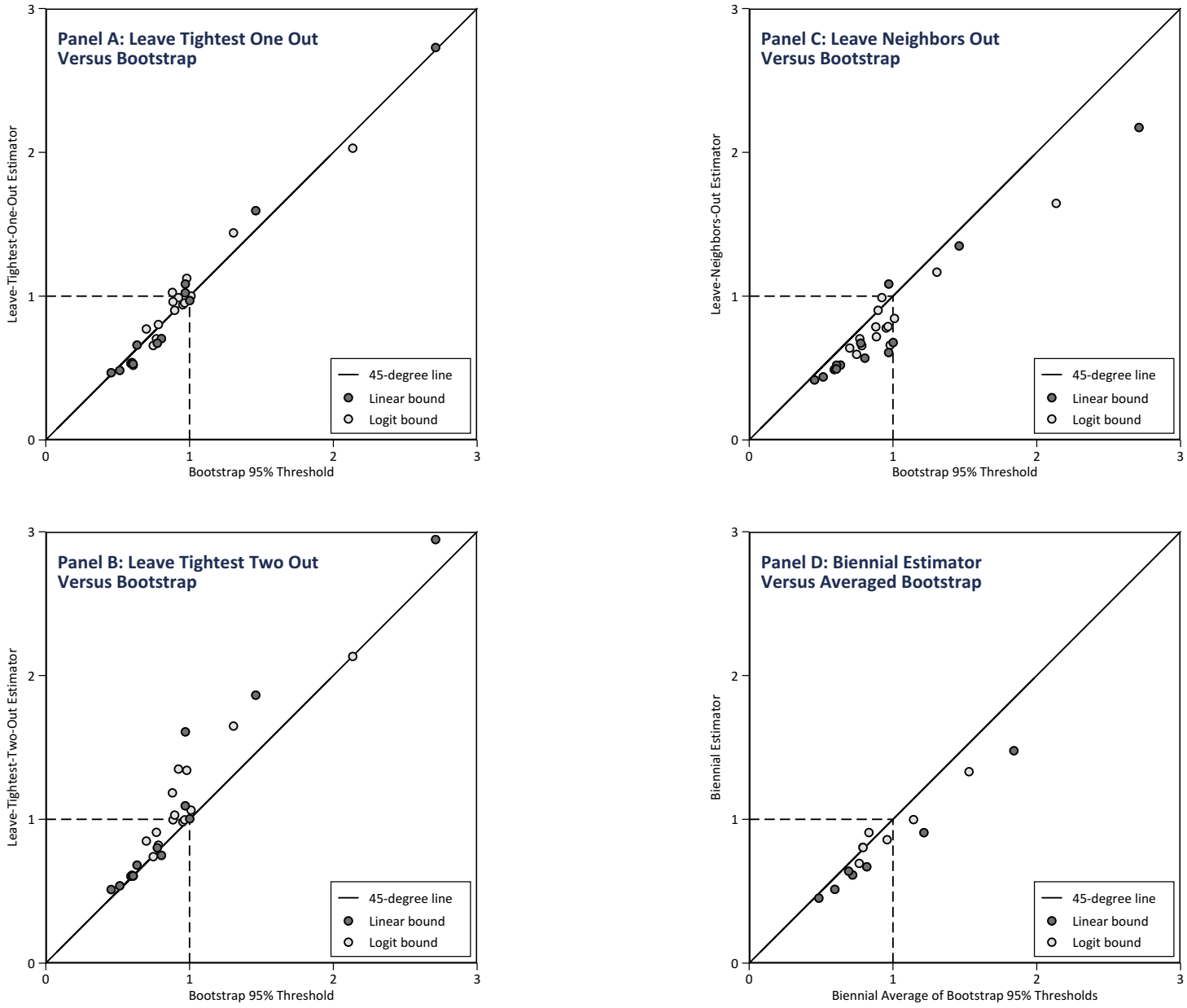


Figure 10: Comparing Various Robust Estimators. *Notes:* All panels cover the same product—low-density polyethylene—and the same method—that incorporating local information. Other products and methods covered in Figures C3–C5 in online Appendix C. The horizontal axis is the bootrapped 95% confidence upper threshold from a one-tailed test, equivalent to the top of the whisker for the 90% bootstrap confidence interval from a two-tailed test in Figure 7, bootstrapped using Zelterman's (1993) method.

old. Indeed we see this in the panel, as the dots lie above the 45-degree line. While the dots are above the 45-degree line, they are quite close, reinforcing our earlier conclusion that the bootstrap threshold (and by extension the leave-tightest-one-out) reflects much of the robustness of more conservative estimators.

9.2. Biennial Estimators

Assumption 4 that demand is growing is essential for the validity of our bounds. As we saw in Section 3.2, the true elasticity need not be contained within our bounds if the assumption is violated. Few markets experienced the explosive growth evidenced by plastics during out sample period. Yet even with plastics, it may be hard to rule out a pause in growth over the decade and a half, perhaps due to a slowdown in the economy (the NBER identifies two brief recessions during our sample, from April 1960 to February 1961, and from December 1969 to November 1970), perhaps due to variation in when sales are booked relative to year end. A more conservative assumption is that demand is certain to grow biennially if not yearly. This subsection proposes two estimators leveraging this more conservative assumption.

The first estimator based on biennial growth leaves the year before and after t out of pairwise comparisons with e_t. Formally, this leave-neighbors-out estimator is defined as $\bar{LN}^*_t(1) \equiv \bar{\epsilon}^*\langle e_{t-1}, e_{t+1}\rangle$ for the method incorporating local information and $\bar{LN}^{**}_t(1) \equiv \bar{\epsilon}^{**}\langle e_{t-1}, e_{t+1}\rangle$ for the method incorporating limiting information. Estimators leaving out neighbors in wider windows around t can also be defined. It is not ex ante obvious how the leave-neighbors-out estimator, say $\bar{LN}^*_t(1)_t$, compares to the leave-tightest-one-out estimator, $\bar{LT}^*_t(1)$. On the one hand, for years other than the endpoints of the sample, $\bar{LN}^*_t(1)$ leaves out two comparison years, while $\bar{LT}^*_t(1)$ leaves out just one. On the other hand, the one comparison left out by $\bar{LT}^*_t(1)$ is by definition the tightest while those left out by $\bar{LN}^*_t(1)$ may not be.

Panel C of Figure 10 shows that the bootstrapped 95% confidence threshold is more conservative than the leave-neighbors-out estimator as almost all the dots lie below the 45-degree line. Since the leave-tightest-one-out estimator is nearly equivalent to the 95% confidence threshold, this implies that leaving the tightest one out is as or more robust than leaving neighboring years out.

The second estimator based on biennial growth merges pairs of years together (1958–59, 1960–

61, and so forth), treating each biennial pair as the unit of observation. Letting uppercase distinguishing variables related to the merged pair of years starting in t, quantity is simply the sum $Q_t = q_t + q_{t+1}$ and price is average revenue per unit: $P_t = (p_t q_t + p_{t+1} q_{t+1})/(q_t + q_{t+1})$. Obviously, this approach effectively cuts the dataset in half. Panel D of Figure 10 compares the bounds computed for this merged dataset to the bootstrapped 95% thresholds computed using the original data averaged over the years making up the pair. The dots are all quite close to the 45-degree line, suggesting that the two approaches provide similar levels of robustness. If anything, the average bootstrap threshold is slightly more conservative, as virtually all dots lie slightly below the 45-degree line. Most are inside the box, indicating that the bounds are below the elasticity value of 1 used to reject monopoly; some are well inside the box. The practical suggestion from this comparison is that if one is concerned about violations of the assumption of growing demand in isolated years, one should look at the broader pattern of the bootstrap thresholds across multiple years rather than taking a low value in a single year as disproof of monopoly.

10. Conclusion

This paper provided a methodology for bounding the elasticity of demand that works in growing markets for homogeneous products. The underlying idea is that the demand curve through a given equilibrium point cannot be either so steep or so flat that it passes below earlier equilibria or above later equilibria without violating the assumption that demand is nondecreasing over time. These inequality conditions place bounds on the elasticity of demand in any given year. The method requires minimal information, working with as few as two time-series observations on aggregate prices and quantities.

A potential drawback of any methodology delivering bounds rather than point estimates is that the resulting bounds may be so wide as to be uninformative. In our empirical application to the polyethylene market in the 1958–72 period after a licensing remedy was ordered, the methodology turned out to quite informative. A good snapshot of the results is provided by Table 2. There, with few exceptions, we see a cascade of three-starred entries in every year in 1964–72, indicating an upper bound on the demand elasticity that is significantly less than 1 at the 1% level. This is true across products (low- versus high-density polyethylene), demand specifications (linear versus

logit), and methodologies (incorporating local versus limiting information). The main exception is the column for high-density polyethylene assuming logit demand using the method incorporating only local information. While we do not see the same cascade of three-starred results in that column, we still see elasticity bounds significant at the 10% level or better in four of the years.

Based on our finding that the elasticity of demand was bounded in the elastic region, in many years bounded well away at strong significance levels, we reject the contention that monopoly behavior effected by the Patents and Processes agreement between DuPont and ICI continued after Judge Ryan's remedy in the *DuPont* case.

The configuration of equilibrium points in 1959 and 1960 in the low-density polyethylene market, with the 1960 lying close to but almost vertically below 1959, raised a question of robustness. Assuming price and quantity are perfectly observed, this configuration provides strong evidence against monopoly. A monopolist would never have dropped its price even by the modest amount experienced unless it would have generated a more substantial gain in quantity, leaving competitive pressure as a plausible explanation of what drove the price drop. These considerations were reflected in the extremely low upper bounds on the demand elasticity for those years, as low as 0.08. This first-stage local comparison propagated into extremely low second-stage bounds incorporating limiting information for all years. This happened because the vertical intercepts for the inverse demands through 1959 and 1960 (or the inverse-demand limits for logit demand) needed to be extremely high to reflect the extremely inelastic demands, forcing later years to also have extremely high vertical intercepts and low elasticities. If the configuration of equilibrium points in 1959–60 were not an actual vertical drop but a small perturbation to fairly stable equilibrium, the extremely tight bounds may have been spurious.

We dealt with the robustness issue in two ways. First, we provided bootstrapped confidence intervals around the estimates using a procedure due to Zelterman (1993) for extreme order statistics. Second, we proposed estimators leaving out the most influential equilibria for pairwise comparison. The leave-one-out estimator generated almost exactly the same result as the 95% bootstrapped upper confidence threshold across products, functional-forms, and methods. The tight bounds for early years such as 1959 and 1960 using the method incorporating local information were shown not to be robust, nor were the extremely tight bounds across all years using the method incorporating limiting information. However, the elasticity bounds remained significantly less than 1 in

many years, a robust finding across products, functional forms, and methods. After one the most influential observation was left out, leaving more out did not appear to have much effect on the more robust findings in the last decade of our sample.

The *DuPont* application serves as a proof of concept for our bounding methodology, which could be applied to any growing market involving homogeneous products. Based on our investigation into robustness, we strongly suggest using a leave-one-out version of our estimator or displaying bootstrapped confidence intervals. We hope other researchers will find value in our methodology and find it easy to apply using the Stata code available on request.

In theory, one should be able to obtain analogous elasticity bounds in the opposite case of a declining market simply by reversing all the signs and inequalities. This is theoretically true but requires several practical caveats. First, even for markets experiencing secular declines, it may be hard to contend that the demand curve is definitively shifting back each year. Consumers may be losing their taste for a product, but natural population growth may offset this taste change. Second, even if suppliers are not investing much, the spillover of technology from other industries may lower costs in the market, shifting supply out. Equilibrium points may end up shifting to the southwest over time, offering little useful information to bound elasticities.

Appendix A: Proofs

This appendix provides technical details omitted from the sketches of the proofs provided in the text.

Proof of Result 1: To prove necessity, note that Assumption 4 requires $D_1(0) \geq D_0(0)$, implying

$$a_1 \geq a_0. \tag{A1}$$

Assumption 4 also requires $D_1(a_0/b_0) \geq D_0(a_0/b_0) = 0$, implying $a_1 - b_1 a_0/b_0 \geq 0$, in turn implying

$$\frac{a_1}{a_0} \geq \frac{b_1}{b_0}. \tag{A2}$$

Conditions (A1) and (A2) together are equivalent to (12).

To prove sufficiency, suppose (12) holds. Then

$$D_1(p) = a_1 - b_1 p = \frac{a_1}{a_0}\left(a_0 - \frac{b_1 a_0 p}{a_1}\right) \geq a_0 - b_0 p = D_0(p), \tag{A3}$$

where the inequality uses (A1) and (A2), just shown to be equivalent to (12). Hence, Assumption 4 holds. *Q.E.D.*

Proof of Result 2: Suppose $c_1 \leq c_0$, $n_0 \geq n_1$, and the maintained assumptions hold (in particular, Assumption 4). Then

$$q_1 = \frac{n_1}{1+n_1}(a_1 - b_1 c_1) \tag{A4}$$

$$\geq \frac{n_0}{1+n_0}(a_1 - b_1 c_1) \tag{A5}$$

$$\geq \frac{n_0}{1+n_0}\left(a_1 - \frac{a_1 b_0 c_0}{a_0}\right) \tag{A6}$$

$$= \frac{a_1 q_0}{q_0}. \tag{A7}$$

Equations (A4) and (A7) follow from (13). Condition (A5) follows from $n_0 \geq n_1$ and $c_1 \leq c_0$. Condition (A6) follows from (A2), which the previous proof showed is entailed by (12). Result 1 showed that (12) is equivalent to Assumption 4.

Conditions (A4)–(A7) imply $q_1 \geq a_1 q_0/a_0$, implying $q_1 \geq q_0$ by (A1), entailed by Assumption 4. But $q_1 \geq q_0$ implies $e_1 \notin NW^+$, implying $NW^+ = \emptyset$. As argued in the text, this implies $\underline{\epsilon}_0 = 0$.

Similarly, one can show that $c_1 \geq c_0$ and $n_0 \leq n_1$ imply $p_1 \geq p_0$ under maintained assumptions. Thus $SE^+ = \emptyset$, implying $\bar{\epsilon}_0 = \infty$ by (11). *Q.E.D.*

Proof of Result 3: Suppose $a_0 = a_1$ and $b_0 = b_1$. Substituting these values into (13), and then substituting the resulting equilibrium points into (6), tedious calculations can be used to show $B(e_0, e_1) = b_0$. By Assumption 1 that equilibrium points are distinct, $p_0 \neq p_1$, implying $p_0 < p_1$ or $p_0 > p_1$. First suppose $p_0 < p_1$. Then $e_1 \in NW^+$, implying $\underline{b}_0 = b_0$ by (10), implying $\underline{\epsilon}_0 = \epsilon_0$. Similar reasoning implies $\bar{\epsilon}_0 = \epsilon_0$ if $p_0 > p_1$. *Q.E.D.*

START HERE

Proof of Proposition 2: The text worked out the bound in (16) emerging from an examination of the limit $p \to 0$ comparing e_t to equilibrium points $e_{t'}$ occurring after it. Here, we complete the proof by deriving bounds emerging from the same limit ($p \to 0$) but now comparing e_t to equilibrium points $e_{t'}$ occurring after it. We also derive bounds from the other limit, $p \to \infty$, comparing e_t to equilibrium points $e_{t'}$ occurring both before and after it.

Consider the limit $p \to 0$ and compare e_t to $e_{t'}$ for $t' < t$. One can apply the analysis in the text preceding the proposition, just reversing the inequalities, to obtain

$$b_t \geq \frac{1}{p_t}(q_{t'} - q_t + b_{t'} p_{t'}) \geq \frac{1}{p_t}(q_{t'} - q_t + \underline{b}^*_{t'} p_{t'}). \tag{A8}$$

This additional lower bound incorporates information on a different equilibrium point's lower bound $\underline{b}^*_{t'}$ from a prior iteration.

Next, consider the limit $p \to \infty$. Start by supposing e_t to $e_{t'}$ for $t' > t$. The fact that $\tilde{D}(p, b_t, e_t) \leq \tilde{D}(p, b_{t'}, e_{t'})$ for all $p \geq 0$ implies $\lim_{p\to\infty} \tilde{D}(p, b_t, e_t) \leq \lim_{p\to\infty} \tilde{D}(p, b_{t'}, e_{t'})$. These limits are just the vertical intercepts of the respective inverse demands. Hence, the preceding inequality implies $p_t + q_t/b_t \leq p_{t'} + q_{t'}/b_{t'}$, or after rearranging,

$$b_t \left(p_{t'} - p_t + \frac{q_{t'}}{b_{t'}} \right) \geq q_t. \tag{A9}$$

We will show that the factor in parentheses is positive. It is immediate that it is positive when $p_{t'} > p_t$ since $q_{t'} > 0$ and $b_{t'} \geq 0$. Suppose instead that $p_{t'} < p_t$. Then $e_{t'} \in SW^+ e_t \cup SE^+ e_t$. Lemma 1 rules out $e_{t'} \in SW^+ e_t$ under Assumption 4, leaving $e_{t'} \in SE^+ e_t$. Then

$$b_{t'} \leq \bar{b}_{t'} \leq B(e_t, e_{t'}) = \frac{q_{t'} - q_t}{p_t - p_{t'}} < \frac{q_{t'}}{p_t - p_{t'}}. \tag{A10}$$

The first and second steps follow from Proposition 1. The next step follows from the definition of $B(e_t, e_{t'})$ from (6) and from $q_{t'} > q_t$ and $p_t > p_{t'}$ for $e_{t'} \in SE^+ e_t$. The last step follows from $q_t > 0$ for nontrivial equilibrium set E. Cross multiplying by $p_t - p_{t'}$, which is positive, and rearranging proves that the term in parentheses is positive.

Cross multiplying (A9) by the positive factor in parentheses yields

$$b_t \geq \frac{q_t}{p_{t'} - p_t + q_{t'}/b_{t'}} \geq \frac{q_t}{p_{t'} - p_t + q_{t'}/\underline{b}^*_{t'}}. \tag{A11}$$

Again, we have an additional lower bound incorporating information on a different equilibrium point's lower bound $\underline{b}^*_{t'}$ from the prior iteration.

Continue to consider the limit $p \to \infty$, but now suppose instead that $t' < t$. Applying the analysis behind (A9), just reversing inequalities, yields

$$b_t \left(p_{t'} - p_t + \frac{q_{t'}}{b_{t'}} \right) \leq q_t. \tag{A12}$$

An important difference with (A9) is that the factor in parentheses in (A12) cannot be unambiguously signed when $t' < t$. When the factor in parentheses is non-positive, (A12) holds for all b_t. In

that case, the condition provides no useful bounding information. When the factor in parentheses is positive, cross multiplying by it preserves the direction of the inequality, yielding

$$b_t \leq \frac{q_t}{p_{t'} - p_t + q_{t'}/b_{t'}} \leq \frac{q_t}{p_{t'} - p_t + q_{t'}/\bar{b}^*_{t'}}. \tag{A13}$$

To emphasize, condition (A13) only provides a potentially useful upper bound if the denominator of the last fraction is positive, i.e.,

$$p_{t'} + \frac{p_{t'}}{\bar{b}^*_{t'}} > p_t. \tag{A14}$$

If (A14) does not hold, we must ignore (A13) lest we conclude that b_t is bounded above by the negative number on the right-hand side of (A13), violating the law of demand (Assumption 3), which implies $b_t \geq 0$.

The second-stage lower bound $\underline{b}^{**}_t$ in (17) is set to the lower of the first-stage bound $\underline{b}^*_t$ and the supremum of the bounds (A8) and (A13) incorporating limiting information. Similarly, the second-stage upper bound $\bar{b}^{**}_t$ in (18) is set to the lower of the first-stage bound $\bar{b}^*_t$ and the infimum of the bounds incorporating limiting information (16) and (A12). *Q.E.D.*

Appendix B: Elasticity Bounds for General Demand Curves

B1. Method Incorporating Local Information

As in the linear-demand case, here in the general-demand case we will derive bounds on the price-sensitivity parameter (here denoted θ_t) from the pairwise comparison of reference equilibrium point e_t to other equilibrium points $e_{t'}$. Bounds on θ_t can then be translated into bounds on ϵ_t via equation (20).

Start by supposing $t < t'$. To respect Assumption 4 of growing demand, for all $p \geq 0$ we must have $\tilde{D}(p, \theta_t, e_t) = D_t(p) \leq D_{t'}(p) = \tilde{D}(p, \theta_{t'}, e_{t'})$. The preceding inequality must hold in particular for $p = p_{t'}$, implying

$$\tilde{D}(p_{t'}, \theta_t, e_t) \leq \tilde{D}(p_{t'}, \theta_{t'}, e_{t'}) = q_{t'}. \tag{B1}$$

Let $\Theta(e_t, e_{t'})$ be the solution to the equation formed by treating the preceding condition as an equality, i.e., the value of $\theta_t \geq 0$ solving

$$\tilde{D}(p_{t'}, \theta_t, e_t) = q_{t'}. \tag{B2}$$

One can show—as done in the proof of the next proposition—that if $e_{t'} \in SE^+ e_t$, $\Theta(e_t, e_{t'})$ exists, is unique, and provides an upper bound on the θ_t satisfying (B1). One can further show that if $e_{t'} \in NW^+ e_t$, then $\Theta(e_t, e_{t'})$ exists, is unique, and provides a lower bound on the θ_t satisfying (B1). If $e_{t'} \in NE^+ e_t$, then (B2) has no solution over $\theta_t \geq 0$ since (B1) is satisfied as a strict inequality for all $\theta_t \geq 0$. In this case, pairwise comparison of e_t and $e_{t'}$ yields no bounding information. The case in which $e_{t'} \in SW^+ e_t$ can be ignored because the set is empty under Assumption 4 of growing demand.

Pairwise comparison of e_t and $e_{t'}$ can be repeated for $t' < t$. Growing demand then implies the same inequality as (B1) with the direction reversed, leading to the same bound but for the opposite compass direction. For details, see the proof of the following proposition, provided in Appendix A.

Proposition 3. *Suppose the sequence of general demands $\tilde{D}(p,\theta_t,e_t)$ defined in* (19) *satisfies Assumptions 3 and 4. Then $\theta_t \in [\underline{\theta}_t, \bar{\theta}_t]$ for all $t \in \{1,\ldots,T\}$, where*

$$\underline{\theta}_t \equiv 0 \vee \sup_{e_{t'} \in SE^- e_t \cup NW^+ e_t} \Theta(e_t, e_{t'}) \tag{B3}$$

$$\bar{\theta}_t \equiv \inf_{e_{t'} \in NW^- e_t \cup SE^+ e_t} \Theta(e_t, e_{t'}). \tag{B4}$$

Proof. The basic logic behind the proof is sketched out in the text preceding the proposition. Here we fill in the detail that if (21)–(25) hold and $e_{t'} \in SE^+ e_t$, then the solution $\Theta(e_t, e_{t'})$ to (B2) exists, is unique, and provides an upper bound on θ_t satisfying (B1).

Suppose $e_{t'} \in SE^+ e_t$. Condition (24) implies $\lim_{\theta_t \to 0} \tilde{D}(p_{t'}, \theta_t, e_t) = q_t < q_{t'}$, where the inequality follows from $e_{t'} \in SEe_t$. Condition (25) implies $\lim_{\theta_t \to \infty} \tilde{D}(p_{t'}, \theta_t, e_t) = \infty$ since $p_{t'} < p_t$ for $e_{t'} \in SEe_t$. Condition (23) implies that $\tilde{D}(p_{t'}, \theta_t, e_t)$ is increasing in θ_t since $p_{t'} < p_t$. Together, these results imply that $\tilde{D}(p_{t'}, \theta_t, e_t)$ is below $q_{t'}$ for low θ_t and monotonically increases in θ_t until it rises above $q_{t'}$ for high θ_t. Thus the solution $\Theta(e_t, e_{t'})$ of (B2) exists and is unique. We see that (B1) is satisfied for $\theta_t \leq \Theta(e_t, e_{t'})$ and violated for $\theta_t > \Theta(e_t, e_{t'})$, implying $\Theta(e_t, e_{t'})$ provides an upper bound on θ_t satisfying (B1). *Q.E.D.*

Proposition 3 prescribes the following algorithm for bounding θ_t. To obtain the lower bound $\underline{\theta}_t$, pair e_t with each of the equilibrium points in $SW^- e_t$ and $NW^+ e_t$, compute $\Theta(e_t, e_{t'})$ for each pair by solving the nonlinear equation (B2). Equation (B2) is well-behaved. In particular—as shown in the proof of Proposition 4—the left-hand side of (B2) is monotonic in θ for $e_{t'} \in NWe_t \cup SEe_t$. Thus standard methods (including Newton-Raphson or even a straightforward grid search) can be used to rapidly solve (B2) and derive $\Theta(e_t, e_{t'})$. The largest $\Theta(e_t, e_{t'})$ across $e_{t'}$ is taken as $\underline{\theta}_t$. If both $SW^- e_t$ and $NW^+ e_t$ are empty, we set $\underline{\theta}_t$ to respect the non-negativity constraint on θ_t; i.e., $\underline{\theta}_t = 0$. To obtain the upper bound $\bar{\theta}_t$, pair e_t with each of the equilibrium points in $NW^- e_t$ and $SE^+ e_t$, compute $\Theta(e_t, e_{t'})$ for each pair by solving the nonlinear equation (B2). The smallest of these solutions $\Theta(e_t, e_{t'})$ over all $e_{t'}$ becomes $\bar{\theta}_t$. If both $NW^- e_t$ and $SE^+ e_t$ are empty, the formula (B4) yields $\bar{\theta}_t = \inf \emptyset = \infty$, correctly implying we obtain no upper bound in this case.

The bounds on θ_t can be translated into bounds on the elasticity, as the next proposition states.

Proposition 4. *Suppose conditions* (21)–(25) *hold. Then, defining*

$$\underline{\epsilon}_t \equiv -\tilde{D}_p(p_t, \underline{\theta}_t, e_t)\frac{p_t}{q_t}, \quad \bar{\epsilon}_t \equiv -\tilde{D}_p(p_t, \bar{\theta}_t, e_t)\frac{p_t}{q_t}, \tag{B5}$$

we have $\epsilon_t \in [\underline{\epsilon}_t, \bar{\epsilon}_t]$.

Proof. The proof is almost immediate. The only detail that needs to be filled in is that the elasticity as defined in (20) is nondecreasing in θ_t. Conditions (21)–(23) together with the continuous differentiability of $\tilde{D}$ of all orders in all arguments obviously imply $\tilde{D}_{p\theta}(p_t, \theta, e_t) \leq 0$. This inequality in turn implies $\frac{\partial}{\partial \theta}[-\tilde{D}_p(p_t, \theta, e_t)] \geq 0$, in turn implying that ϵ_t as defined in (20) is nondecreasing in θ_t, implying $\epsilon_t \in [\underline{\epsilon}_t, \bar{\epsilon}_t]$. *Q.E.D.*

B2. Method Incorporating Limiting Information

Next we turn to an extension of the method for incorporating limiting information from linear to general demand curves. Start by considering the limit $p \to 0$ and comparing equilibrium points e_t and $e_{t'}$ for $t' > t$. We have

$$1 \geq \lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta_t, e_t)}{\tilde{D}(p, \theta_{t'}, e_{t'})} \right] \geq \lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta_t, e_t)}{\tilde{D}(p, \bar{\theta}^*_{t'}, e_{t'})} \right], \tag{B6}$$

where a single star has been added as a superscript to the bounds incorporating local information from the previous subsection to indicate that they constitute the first step of an iterative procedure: i.e., $\underline{\theta}^*_t \equiv \underline{\theta}_t$ and $\bar{\theta}^* \equiv \bar{\theta}_t$. To see the first inequality, Assumption 4 that demand is nondecreasing over time implies $\tilde{D}(p, \theta_{t'}, e_{t'}) \geq \tilde{D}(p, \theta_t, e_t)$. Dividing by $\tilde{D}(p, \theta_{t'}, e_{t'})$ and taking limits gives the first inequality. To see the second inequality in (B6), note that in the limit as $p \to 0$, eventually $p \leq p_{t'}$. But (23) then implies $\tilde{D}_\theta(p, \theta, e_{t'}) \geq 0$ in the limit as $p \to 0$, in turn implying $\tilde{D}(p, \bar{\theta}^*_{t'}, e_{t'}) \geq \tilde{D}(p, \theta_{t'}, e_{t'}) \geq$ in the limit as $p \to 0$ since $\bar{\theta}^*_{t'} \geq \theta_{t'}$ by Proposition 3.

It can be further argued that (23) implies that the numerator on the right-hand side of (B6)—and thus the entire right-hand side—is increasing in θ_t. Treating (B6) as an equality,

$$\lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta_t, e_t)}{\tilde{D}(p, \bar{\theta}^*_{t'}, e_{t'})} \right] = 1, \tag{B7}$$

thus provides an equation in θ_t that can be solved to provide a upper bound on θ_t.

Repeating the analysis for each $t' > t$, the lowest of the solutions to (B7) across $t' > t$ is an upper bound that may be tighter than $\bar{\theta}^*_t$ from the method incorporating local information. The upper bound may be further tightened by examining the opposite limit, $p \to \infty$, comparing equilibrium points e_t and $e_{t'}$ for $t' < t$. The analysis is similar to that above and omitted for brevity.

Also similar and omitted for brevity are the analysis of the limit $p \to 0$ comparing equilibrium points e_t and $e_{t'}$ for $t' > t$ and the analysis of the limit $p \to \infty$ for $t' < t$. These analysis turn out to contribute to lower rather than upper bounds. The following proposition organizes all of the bounding information provided by these analyses.

Proposition 5. *Suppose the sequence of general demands $\tilde{D}(p, \theta_t, e_t)$ defined in* (19) *satisfies Assumptions 3 and 4 as well as conditions* (21)–(25)*. Then $\theta_t \in [\underline{\theta}^{**}_t, \bar{\theta}^{**}_t]$ for all $t \in \{1, \ldots, T\}$, where*

$$\underline{\theta}^{**}_t \equiv \underline{\theta}^*_t \vee \sup_{t' < t} \left\{ \theta \;\middle|\; \lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta, e_t)}{\tilde{D}(p, \underline{\theta}^*_{t'}, e_{t'})} \right] = 1 \right\} \vee \sup_{t' > t} \left\{ \theta \;\middle|\; \lim_{p \to \infty} \left[\frac{\tilde{D}(p, \theta, e_t)}{\tilde{D}(p, \underline{\theta}^*_{t'}, e_{t'})} \right] = 1 \right\} \tag{B8}$$

$$\bar{\theta}^{**}_t \equiv \bar{\theta}^*_t \wedge \inf_{t' < t} \left\{ \theta \;\middle|\; \lim_{p \to \infty} \left[\frac{\tilde{D}(p, \theta, e_t)}{\tilde{D}(p, \bar{\theta}^*_{t'}, e_{t'})} \right] = 1 \right\} \wedge \inf_{t' > t} \left\{ \theta \;\middle|\; \lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta, e_t)}{\tilde{D}(p, \bar{\theta}^*_{t'}, e_{t'})} \right] = 1 \right\}. \tag{B9}$$

The proposition implicitly prescribes an algorithm for computing bounds incorporating limiting information that is an iterative procedure. For example, the upper bounds $\bar{\theta}^*_{t'}$ incorporating local information are computed in a first iteration; the $\bar{\theta}^*_{t'}$ for all equilibrium points $e_{t'} \in E$ are then used to compute the new upper bound $\bar{\theta}^{**}_t$ for the single equilibrium point e_t in the second iteration. Upper bounds from the first iteration are used in the computation of upper bounds in the second iteration; lower bounds from the first iteration are used to compute lower bounds in the second iteration.

More specifically, to compute the upper bound $\bar{\theta}_t^{**}$, one solves the equation for θ_t,

$$\lim_{p\to\infty}\left[\frac{\tilde{D}(p,\theta_t,e_t)}{\tilde{D}(p,\underline{\theta}_{t'}^{*},e_{t'})}\right]=1 \tag{B10}$$

for each $t'<t$ and takes the smallest solution. One then solves a similar equation, just involving a different limit, for θ_t,

$$\lim_{p\to 0}\left[\frac{\tilde{D}(p,\theta_t,e_t)}{\tilde{D}(p,\underline{\theta}_{t'}^{*},e_{t'})}\right]=1 \tag{B11}$$

for each $t'>t$ and takes the smallest solution. The new bound $\bar{\theta}_t^{**}$ is set to whichever is smallest of (a) the upper bound from the method incorporating local information, $\bar{\theta}_t^{*}$, (b) the smallest solution over $t'<t$ to (B10), and (c) the smallest solution over $t'>t$ to (B11). The lower bound $\underline{\theta}_t^{**}$ is computed analogously.

To translate the bounds on θ_t into bounds on the elasticity ϵ_t, the arguments behind Proposition 4 continue to apply. Defining

$$\underline{\epsilon}_t^{**}\equiv-\tilde{D}_p(p_t,\underline{\theta}_t^{**},e_t)\frac{p_t}{q_t},\quad \bar{\epsilon}_t^{**}\equiv-\tilde{D}_p(p_t,\bar{\theta}_t^{**},e_t)\frac{p_t}{q_t}, \tag{B12}$$

we have $\epsilon_t\in[\underline{\epsilon}_t^{**},\bar{\epsilon}_t^{**}]$.

B3. Application to Logit Demand

This subsection provides a specification of logit demand in a homogeneous-product market and discusses the application of the general propositions in this appendix to this special case.

In the context of a homogeneous product market under study, logit demand can be specified as

$$D_t(p)=\frac{n_t\exp(-\theta_t p)}{1+\exp(-\theta_t p)}=\frac{n_t}{1+\exp(\theta_t p)}, \tag{B13}$$

where n_t is interpreted as a market-size parameter and θ_t as a price-sensitivity parameter. For demand to be nonnegative, $n_t\geq 0$; for the law of demand (Assumption 3) to hold, $\theta_t\geq 0$.

This specification of demand involves two independent parameters, but further requiring the curve to pass through the equilibrium point e_t pins it down to a single-parameter family. Focus for now on price sensitivity, θ_t, as this key parameter. Given θ_t and equilibrium point (q_t,p_t), equation (B13) can be solved for the market-size parameter:

$$n_t=q_t[1+\exp(\theta_t p_t)]. \tag{B14}$$

Substituting for n_t from equation (B14) into (B13) yields an expression for logit demand in terms of the single unknown parameter θ_t and known equilibrium point $e_t=(q_t,p_t)$:

$$\tilde{D}(p,\theta_t,e_t)=\frac{q_t[1+\exp(\theta_t p_t)]}{1+\exp(\theta_t p)}. \tag{B15}$$

The elasticity of logit demand is

$$\epsilon_t=-\tilde{D}_p(p_t,\theta_t,e_t)\frac{p_t}{q_t}=\frac{\theta_t p_t}{1+\exp(-\theta_t p_t)}. \tag{B16}$$

To apply Propositions 3–5, covering general demand curves, to the logit special case, we need to verify that conditions (21)–(25) are satisfied by logit demand. Condition (21)–(23) can be verified by differentiating (B15):

$$\tilde{D}_\theta(p,\theta,e_t) = \frac{q_t[(p_t-p)\exp(\theta_t p_t)\exp(\theta_t p)+p_t\exp(\theta_t p_t)-p\exp(\theta_t p)]}{[1+\exp(\theta_t p)]^2}. \tag{B17}$$

It is easy to see that (B17) is negative if $p > p_t$, verifying (21); that (B17) equals 0 if $p = p_t$, verifying (22); and that (B17) is positive if $p < p_t$, verifying (23).

To verify (24), substitute $\theta_t = 0$ into (B15), yielding $\tilde{D}(p,0,e_t) = q_t$, . Finally, to verify (25),

$$\lim_{\theta_t\to\infty}\tilde{D}(p,\theta,e_t) = q_t \lim_{\theta_t\to\infty}\frac{\exp(\theta_t(p_t-p))+\exp(-\theta_t p)}{1+\exp(-\theta_t p)} = q_t \lim_{\theta_t\to\infty}\exp(\theta_t(p_t-p)). \tag{B18}$$

It is easy to see that (B18) equals 0 if $p > p_t$ and equals ∞ if $p < p_t$, verifying (25).

Application of Propositions 3–5, covering general demand curves, to the special case of logit demand is fairly straightforward; but a few computational details deserve mention. In the general case, the solution of equation (B2) for θ_t is the $\Theta(e_t,e_{t'})$ having a prominent role in the general propositions. In the special case of logit demand, (B2) can be written after rearranging as

$$q_t[1+\exp(\theta_t p_t)] = q_{t'}[1+\exp(\theta_t p_{t'})]. \tag{B19}$$

Proposition 5 bounds the price-sensitivity parameter for general demand curves using the method incorporating limiting information. To apply this proposition to a given demand curve requires the relevant limits to be computed and substituted into the the bounds formulas. For reference, we report the computations for logit demand in the following corollary.

Proposition 6. *Suppose the logit demand* $\tilde{D}(p,\theta_t,e_t)$ *defined in* (B15) *satisfies Assumptions 3 and 4. Then* $\theta_t \in [\underline{\theta}_t^{**},\bar{\theta}_t^{**}]$ *for all* $t \in \{1,\ldots,T\}$*, where*

$$\underline{\theta}_t^{**} \equiv \underline{\theta}_t^{*} \vee \sup_{t'<t}\left\{\frac{1}{p_t}\ln\left(\frac{q_{t'}}{q_t}[1+\exp(\underline{\theta}_{t'}^{*}p_{t'})]-1\right)\right\} \vee \sup_{t'>t}\underline{\theta}_{t'}^{*}. \tag{B20}$$

$$\bar{\theta}_t^{**} \equiv \bar{\theta}_t^{*} \wedge \inf_{t'<t}\bar{\theta}_{t'}^{*} \wedge \inf_{t'>t}\left\{\frac{1}{p_t}\ln\left(\frac{q_{t'}}{q_t}[1+\exp(\bar{\theta}_{t'}^{*}p_{t'})]-1\right)\right\}. \tag{B21}$$

The complex formulas in Proposition 6 for logit demand look very different from those in Proposition 2 for linear demand. We will see in the empirical application to the *DuPont* decision, however, that they generate nearly identical bounds on the elasticities. Reassuringly, the bounds incorporating limiting information are more robust to functional-form assumptions than the formulas might suggest.

References

Adams, Walter. (1951) "Dissolution, Divorcement, Divestiture: The Pyrrhic Victories in Antitrust," *Indiana Law Journal* 27: 1–37.

Areeda, Phillip, Louis Kaplow, and Aaron Edlin. (2013) *Antitrust Analysis: Problems, Text, and Cases* (seventh edition). New York: Aspen Publishers.

Backman, Jules. (1964) *Competition in the Chemical Industry*. Washington: Manufacturing Chemists Association.

Berry, Steven, James Levinsohn, and Ariel Pakes. (1995) "Automobile Prices in Market Equilibrium," *Econometrica* 63: 841–890.

Bickel, Peter J. and David A. Freedman. (1981) "Some Asymptotic Theory for the Bootstrap," *Annals of Statistics* 9: 1196–1217.

Ciliberto, Federico and Elie Tamer. (2009) "Market Structure and Multiple Equilibria in Airline Markets," *Econometrica* 77: 1791–1828.

Clarke, Alison J. (1999) *Tupperware: The Promise of Plastic in 1950s America.* Washington: Smithsonian Institution Press.

Farrell, Joseph, Paul Pautler, and Michael Vita. (2009) "Economics at the FTC: Retrospective Merger Analysis with a Focus on Hospitals," *Review of Industrial Organization* 35: 369–385.

Fenichell, Stephen. (1966) *Plastic: The Making of a Synthetic Century.* New York: Harper Collins.

Fortune Magazine. (1954) "The Polyethylene Gamble," Vol. 49 (February): 134–136, 166, 170, 172, 174.

Haile, Philip A. and Elie Tamer. (2003) "Inference with an Incomplete Model of English Auctions," *Journal of Political Economy* 111: 1–51.

Hounshell David A. and John K. Smith, Jr. (1988) *Science and Corporate Strategy: Du Pont R & D, 1902–1980*. Cambridge: Cambridge University Press.

Lampe, Ryan and Petra Moser. (2013) "Patent Pools and Innovation in Substitute Technologies—Evidence from the 19th Century Sewing Machine Industry," *Rand Journal of Economics* 44: 757–778.

Lerner, Josh, Marcin Strojwas, and Jean Tirole. (2007) "The Design of Patent Pools: The Determinants of Licensing Rules," *Rand Journal of Economics* 38: 610–625.

Lieberman, Marvin B. (1984) "The Learning Curve and Pricing in the Chemical Processing Industries" *Rand Journal of Economics* 15: 213–228.

Manski, Charles F. (1995) *Identification Problems in the Social Sciences.* Cambridge: Harvard University Press.

McFadden, Daniel. (1973) "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka, ed., *Frontiers in Econometrics*. New York: Academic Press.

Motta, Massimo. (2004) *Competition Policy Theory and Practice:*. Cambridge: Cambridge University Press.

Pakes, Ariel. (2010) "Alternative Models for Moment Inequalities," *Econometrica* 78: 1783–1822.

Pakes, Ariel, J. Porter, Kate Ho, and Joy Ishii. (2015) "Moment Inequalities and their Application," *Econometrica* 83: 315–334.

Reader, W. J. (1975) *Imperial Chemical Industries: A History: Volume II: The First Quarter Century, 1926-1952* London: Oxford University Press.

Stocking, George W. and Myron W. Watkins. (1946) *Cartels in Action*. New York: The Twentieth Century Fund.

Tirole, Jean. (2014) "Market Failures and Public Policy," Nobel Prize Lecture, December 8.

"United States v. Imperial Chemical Industries," 100 *F. Supp.* 504 (Southern District of New York, 1951).

"United States v. Imperial Chemical Industries," 105 *F. Supp.* 215 (Southern District of New York, 1952).

"United States v. Imperial Chemical Industries," 254 *F. Supp.* 685 (Southern District of New York, 1966).

U. S. Federal Trade Commission. (1999) "A Study of the Commission's Divestiture Process," white paper prepared by the Staff of the Bureau of Competition.

U. S. Federal Trade Commission. (2017) "The FTC's Merger Remedies 2006-2012," white paper prepared by the Staff of the Bureau of Competition.

U. S. Tariff Commission. (various years) *Synthetic Organic Chemicals: United States Production and Sales*. Washington, D.C.: Government Printing Office.

Watzinger, Martin, Thomas Fackler, Markus Nagler, and Monika Schnitzer. (2017) "How Antitrust Enforcement Can Spur Innovation: Bell Labs and the 1956 Consent Decree," CESifo Working Paper Series no. 6351.

Weissman, Ishay. (1978) "Estimation of Parameters and Large Quantiles Based on the k Largest Observations," *Journal of the American Statistical Association* 73: 812–815.

Whitney, Simon. (1958) *Antitrust Policies: American Experience In Twenty Industries* (Volume I). New York: The Twentieth Century Fund,

Yale Law Journal. (1953–1954) "Patent Dedication as an Antitrust Remedy: New Light on Hartford-Empire." 63: 717–728.

Zelterman, Daniel. (1993) "A Semiparametric Bootstrap Technique for Simulating Extreme Order Statistics," *Journal of the American Statistical Association* 88: 477–485.

Online Appendix C: Supplementary Figures

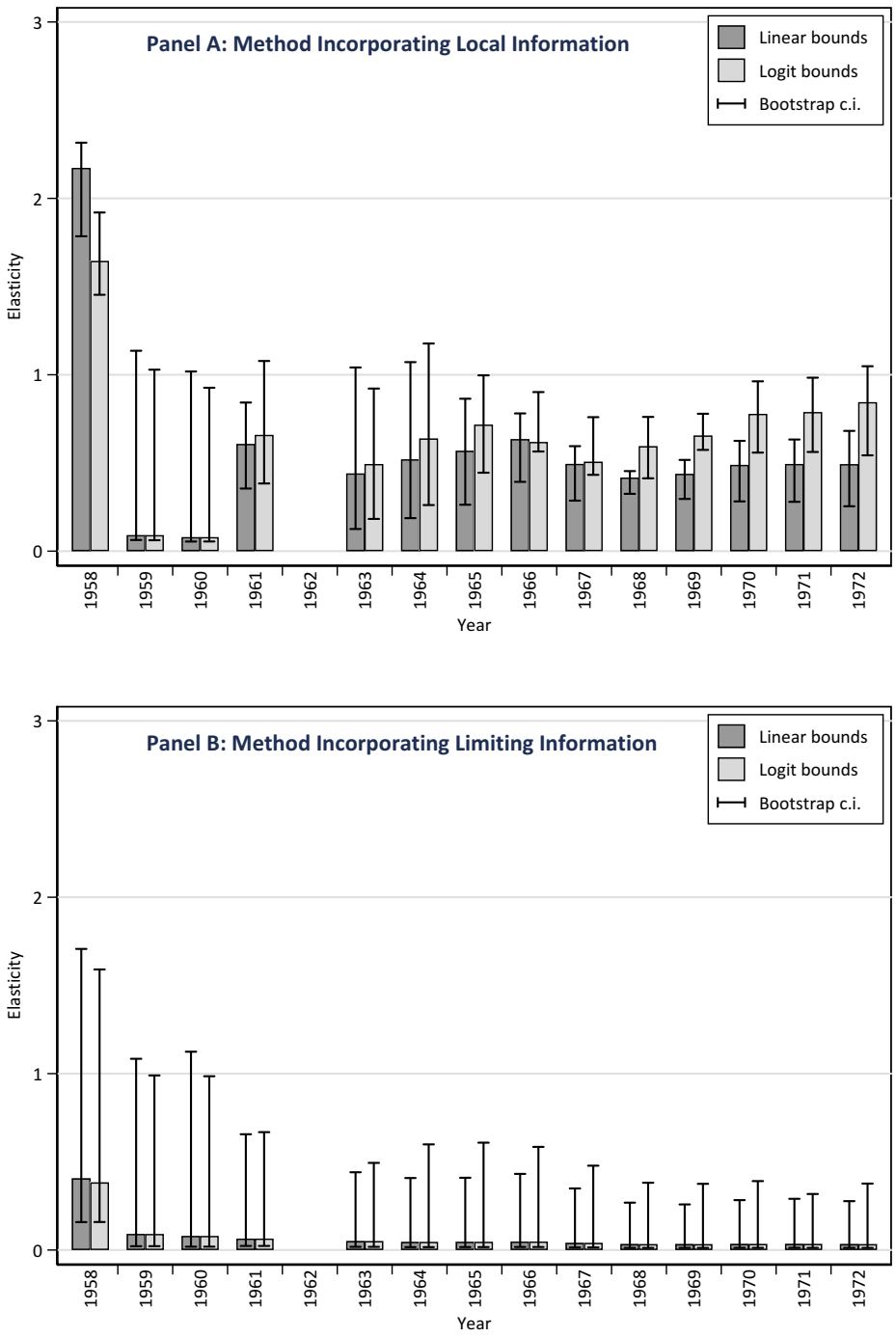


Figure C1: Elasticity Bounds for Low-Density Polyethylene Dropping Alternate Year to Preserve Growing Demand. *Notes:* Comparable to Panels A and B of Figure 7. Those panels drop 1963 to preserve the condition that $SW^{+}e_t = \emptyset$ for all e_t, which by Lemma 1 is necessary for Assumption 4 (growing demand over time) to hold. The same goal is is achieved here by dropping 1962. See Figure 7 for additional notes.

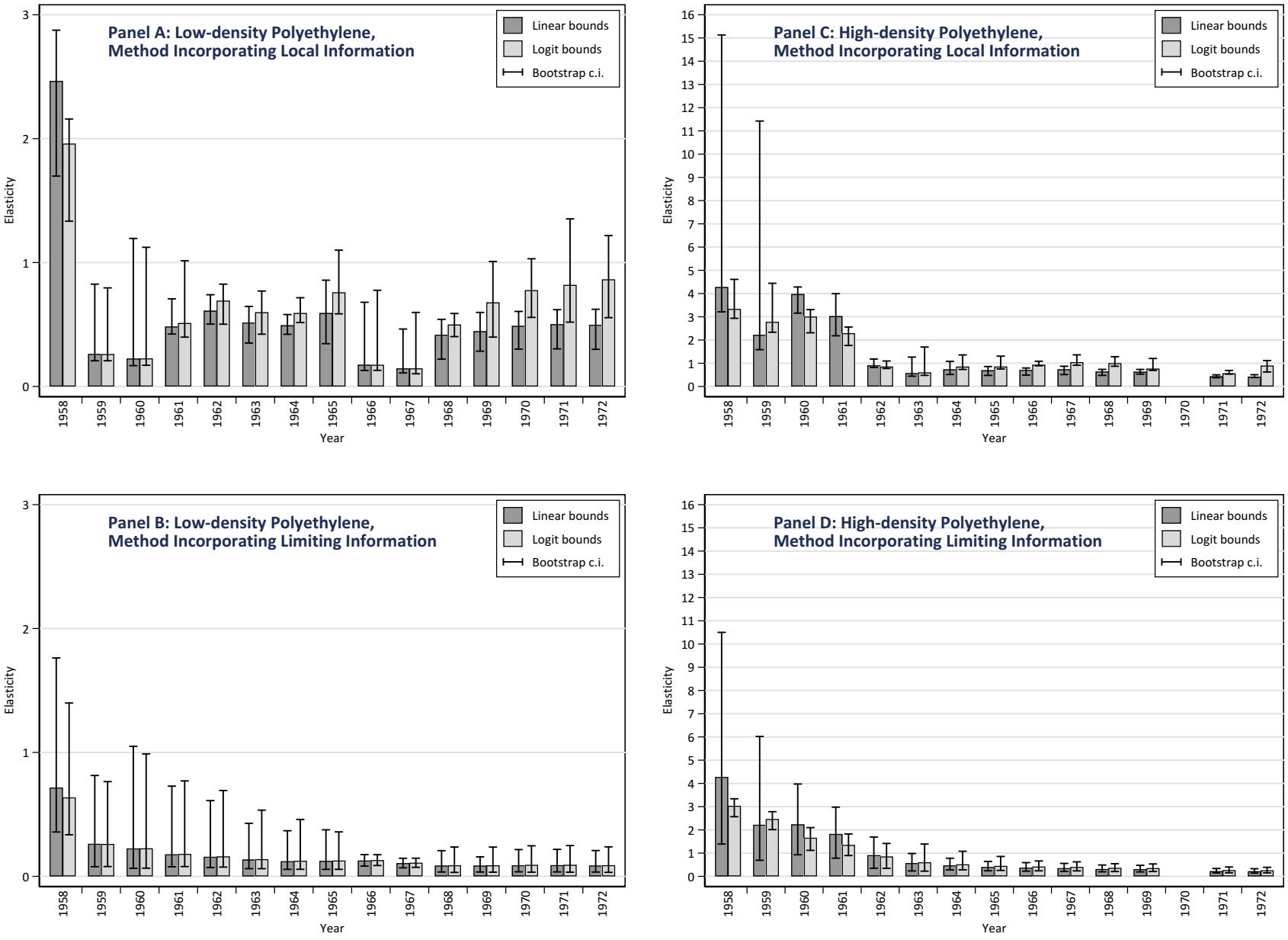


Figure C2: Elasticity Bounds Using Production Data for Quantity. When production rather than sales data is used for the quantity, 1963 no longer needs to be dropped for low-density polyethylene because the condition that $SW^{+}e_t = \emptyset$—which by Lemma 1 is necessary for Assumption 4 (growing demand over time) to hold—is no longer violated. For high-density polyethylene, 1970 now has to be dropped to preserve the condition $SW^{+}e_t = \emptyset$. See Figure 7 for additional notes.

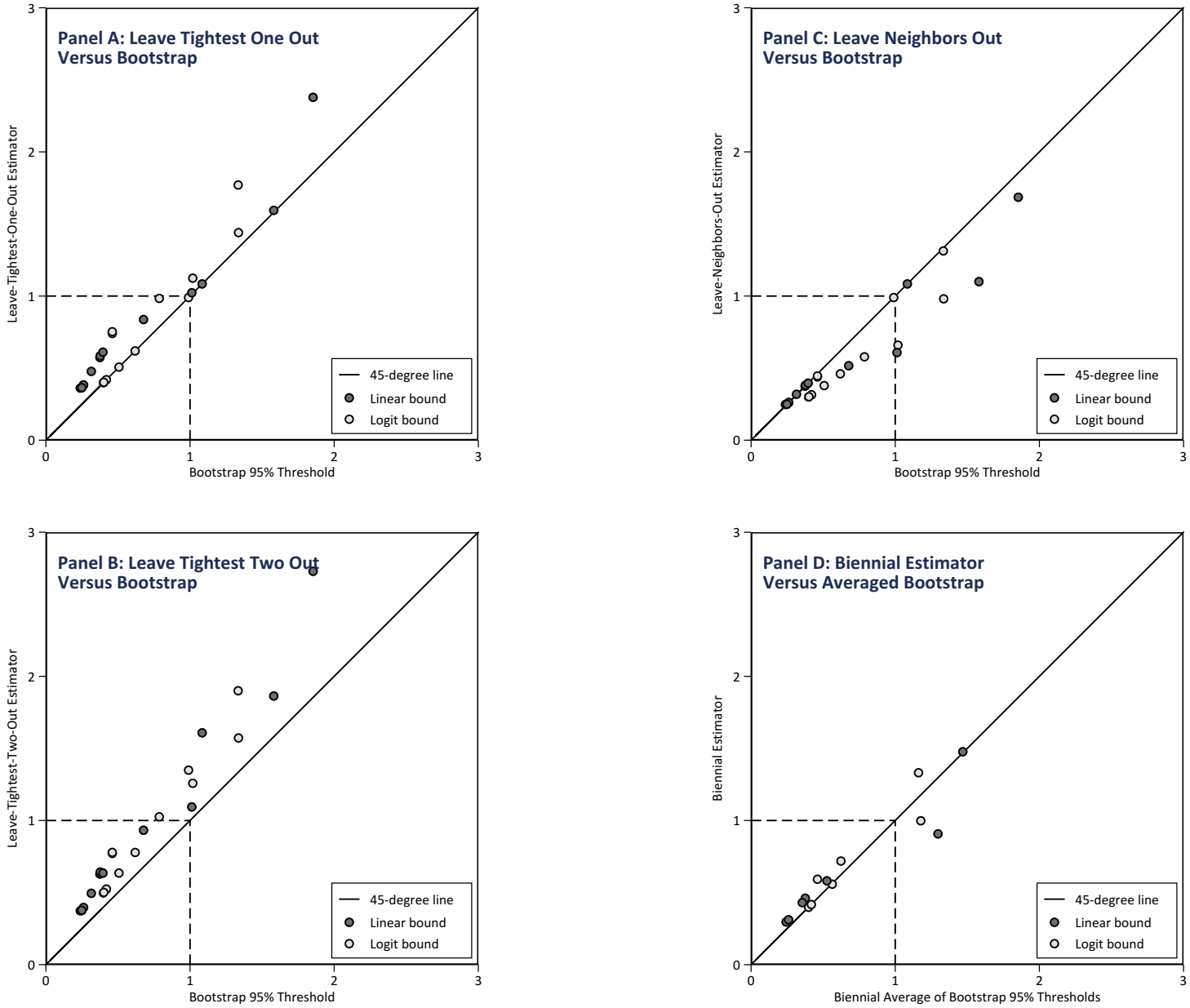


Figure C3: Comparing Various Robust Estimators for Low-Density Polyethylene Using Method Incorporating Limiting Information. *Notes:* The same robust estimators are compared in each panel as in Figure 10, and the same product (low-density polyethylene) is analyzed. Here we use a different method, that incorporating limiting information. See Figure 10 for additional notes.

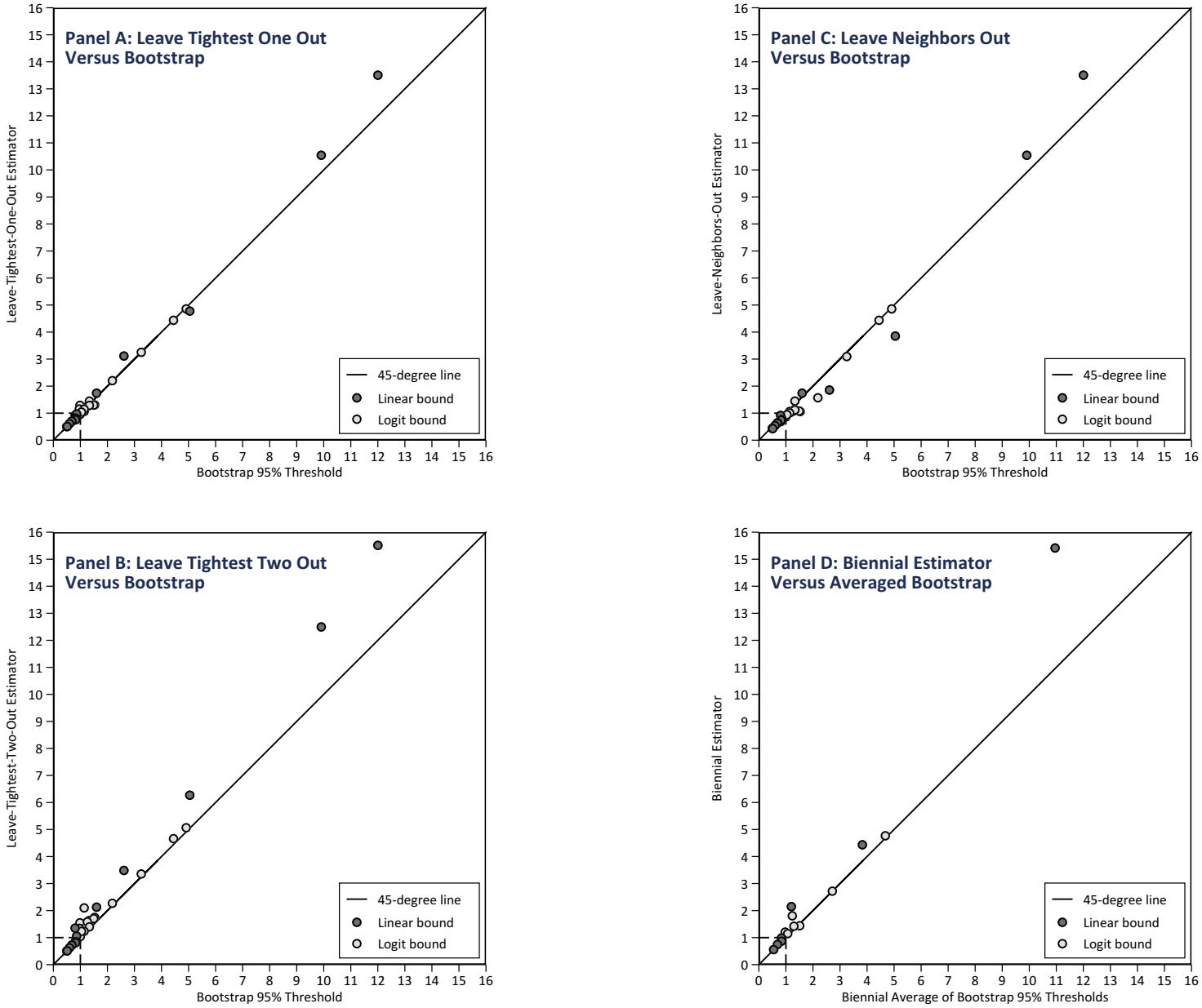


Figure C4: Comparing Various Robust Estimators for High-Density Polyethylene Using Method Incorporating Local Information. *Notes:* The same robust estimators are compared in each panel as in Figure 10, and the same method is used (incorporating local information). Here we analyze a different product, high-density polyethylene. See Figure 10 for additional notes.

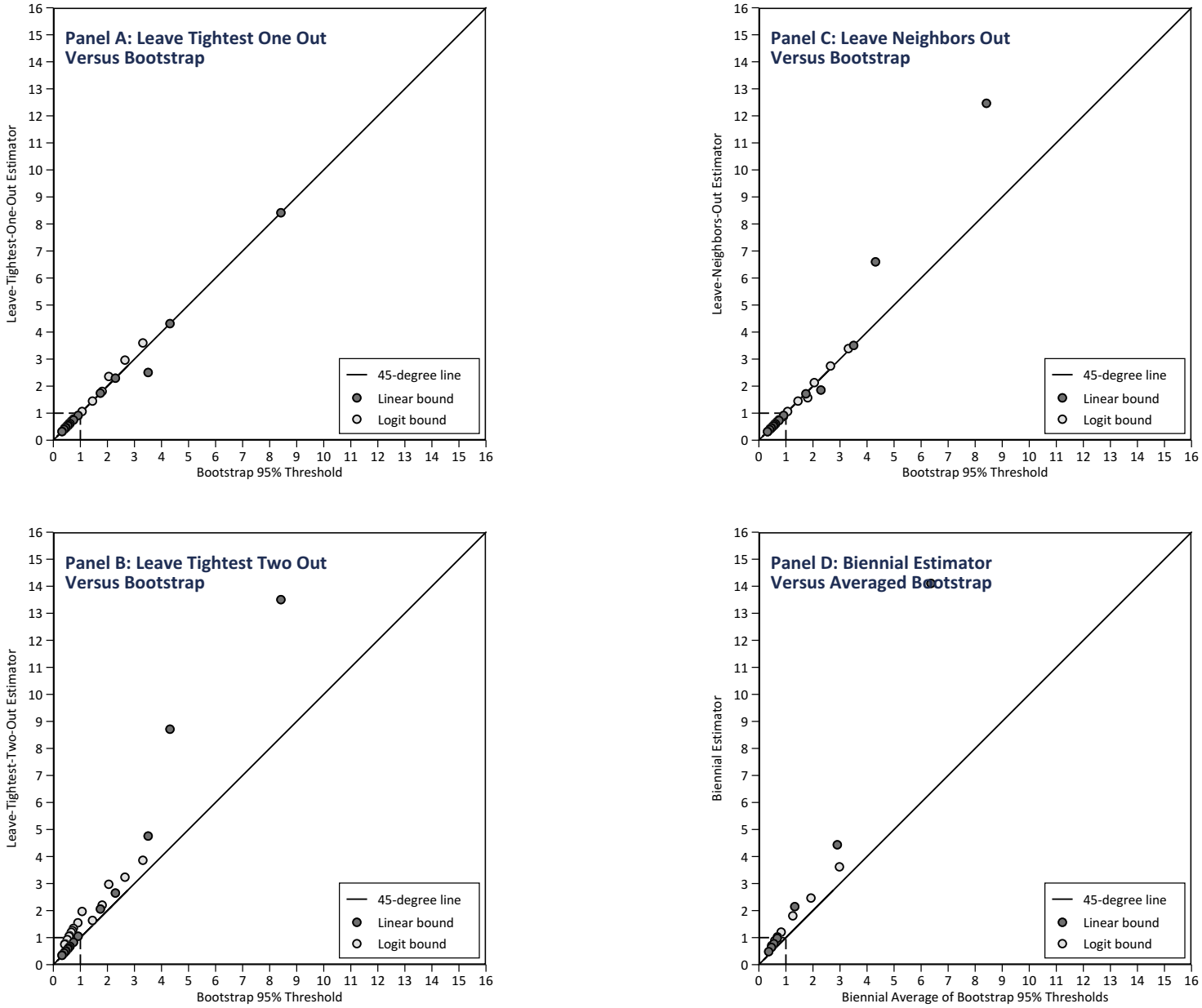


Figure C5: Comparing Various Robust Estimators for High-Density Polyethylene Using Method Incorporating Limiting Information. *Notes:* The same robust estimators are compared in each panel as in Figure 10, and the same method is used (incorporating local information). Here we analyze a different product, high-density polyethylene, using a different method, that incorporating limiting information. See Figure 10 for additional notes.