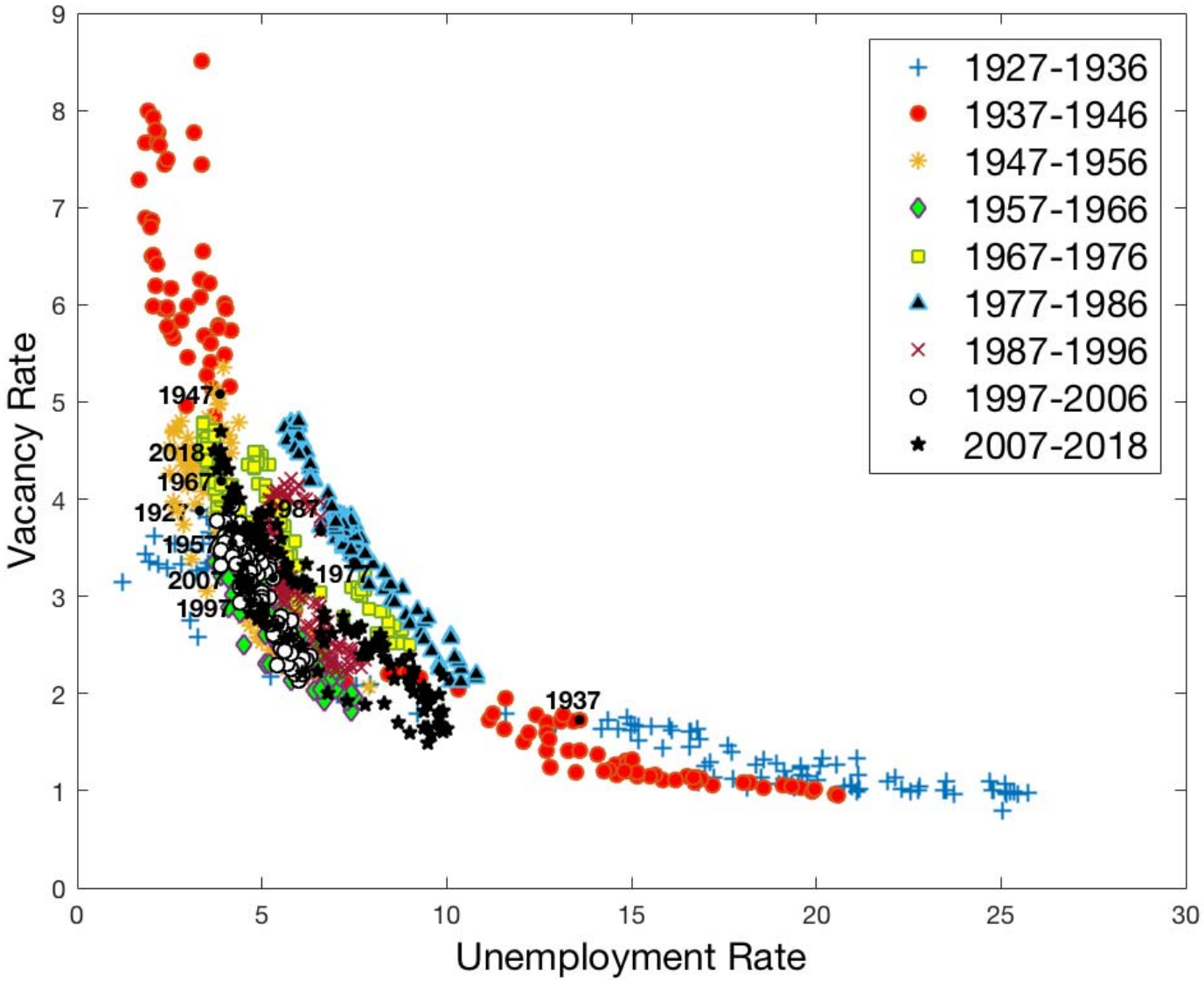

Vacancy Rate
Unemployment Rate
0
1
2
3
4
5
6
7
8
9
0
5
10
15
20
25
30
1927-1936
1937-1946
1947-1956
1957-1966
1967-1976
1977-1986
1987-1996
1997-2006
2007-2018
1947
2018
1967
1927
1957
2007
1997
1987
1977
1937

Introduction

Stylized long-run facts:

1. No secular movement in the Beveridge curve

$$u = \frac{\delta_t}{\delta_t + A_t p(v/u)}.$$

2. No secular movement along the Beveridge curve

$$u_t,\ v_t,\ v_t/u_t \text{ stationary.}$$

From the perspective of search theory, the above observations imply that the efficiency A_t of the search technology has not improved from 1926 to 2017. Telephone? Fax? Mobile phone? PC? Internet? All irrelevant!

Environment

Workers:

Firms:

Labor market:

Exogenous processes:

Environment

Workers:

- *population*: measure 1;
- *objective*: max pv of income $\{b_t, w_t\}$ discounted at rate r.

Firms:

Labor market:

Exogenous processes:

Environment

Workers:

Firms:

- *population*: positive measure;
- *technology*:
  maintain vacancies v_t at flow unit cost k_t;
  CRTS technology: 1 unit of labor –> $y_t z$ units of output;
- *objective*: max pv of income $y_t z - w_t$ discounted at rate r.

Labor market:

Exogenous processes:

Environment

Workers:

Firms:

Labor market:

- u_t and v_t come together through a matching fn $A_t M(u_t, v_t)$;
- u meets v at rate $A_t p(\theta_t)$, where $\theta_t = v_t/u_t$ and $p(\theta_t) = M(1, \theta_t)$;
- upon meeting u and v observe quality $z \sim F$, decide whether to match and bargain over terms of trade.

Exogenous processes:

Environment

Workers:

Firms:

Labor market:

Exogenous processes:

- search efficiency A_t grows at the rate g_A;
- production efficiency y_t grows at the rate g_y;
- unemp. benefit b_t grows at the rate g_b;
- vacancy cost k_t grows at the rate g_k.

Definition of a BGP

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta, h_{ue}, h_{eu}, u, g_z\}$ s.t.

1. Reservation quality R_t:

$$y_t R_t = b_t + A_t p(\theta)\gamma \int_{R_t} S_t(z) dF(z)$$

2. Surplus of a match S_t:

$$rS_t(z) = y_t(z - R_t) + \dot{S}_t(z)$$

3. Market tightness θ_t:

$$k_t = A_t \frac{p(\theta_t)}{\theta_t}(1-\gamma) \int_{R_t} S_t(z) dF(z)$$

Definition of a BGP

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta, h_{ue}, h_{eu}, u, g_z\}$ s.t.

4. Stationarity of UE, EU, u and θ_t:

$$\begin{aligned} A_t p(\theta)(1 - F(R_t)) &= h_{ue}, \\ G_t'(R_t)\dot{R}_t &= h_{eu}, \\ (1-u)h_{eu} &= u h_{ue}, \\ \theta_t &= \theta. \end{aligned}$$

5. Distribution G_t of workers across z such that every quantile $z_t(x)$ grows at some constant rate g_z:

$$(1-u)G_t'(z_t(x))z_t(x)g_z + uA_t p(\theta)[F(z_t(x)) - F(R_t)] = (1-u)G_t'(R_t)R_t g_z.$$

Necessary Conditions for a BGP

N1 A BGP may exist only if F is Pareto with some coefficient α.

Sketch: The stationarity condition for the UE rate is:

$$A_t p(\theta)(1 - F(R_t)) \; = h_{UE}, \; \forall t \geq 0.$$

Differentiating with respect to t, we obtain

$$g_A = \frac{F'(R_t)}{1 - F(R_t)} R_t g_z$$

The differential equation for F has the unique solution

$$F(z) = 1 - \left(\frac{z_\ell}{z}\right)^\alpha.$$

Necessary Conditions for a BGP

N2 A BGP may exist only if g_b and g_k are equal to $g_y + g_z$.

Sketch: Combining the equilibrium conditions for R_t and θ, we obtain:

$$y_t R_t = b_t + \frac{\gamma}{1-\gamma}\theta k_t, \ \forall t \geq 0.$$

The condition above can only be satisfied if

$$g_b, g_k = g_y + g_z$$

Existence and Uniqueness of a BGP

Impose the necessary conditions on the fundamentals.

1. Reservation quality R_t grows at the constant rate $g_z = g_A/\alpha$:

$$y_t R_t = b_t + A_t p(\theta)\gamma \underbrace{\int_{R_t} S_t(z)dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

2. Market tightness θ_t is constant

$$k_t = A_t \frac{p(\theta)}{\theta}(1-\gamma) \underbrace{\int_{R_t} S_t(z)dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

Existence and Uniqueness of a BGP

Impose the necessary conditions on the fundamentals.

3. Quality distribution $G_t(z)$ grows at constant rate g_z and starts at

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^{\alpha}.$$

4. Unemployment u is constant at

$$u = \frac{g_A}{g_A + A_0 p(\theta)[1 - F(R_0)]}.$$

5. UE and EU rates are both constant.

Existence and Uniqueness of a BGP

We have established the following.

Proposition 1: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + g_A/\alpha$.

If a BGP exists, it is unique and such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Pareto truncated at R_t growing at rate $g_z = g_A/\alpha$;

(**iii**) labor productivity grows at rate $g_y + g_A/\alpha$.

Search on the Job

Workers search on the job with relative intensity $\rho \in [0, 1]$.

Proposition 2: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + g_A/\alpha$.

Any BGP is such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Fréchet truncated at R_t and grows at rate $g_z = g_A/\alpha$;

(**iii**) labor productivity grows at rate $g_y + g_A/\alpha$.

Population Growth

Population grows at rate g_N and matching fn is $A_t N_t^{\beta} M(u_t, v_t)$.

Proposition 3: Take arbitrary growth rates $g_y > 0$, $g_A > 0$, $g_N > 0$ such that the overall search efficiency improves over time, i.e. $g_A + \beta g_N > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + (g_A + \beta g_N)/\alpha$.

Any BGP is such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Pareto truncated at R_t and grows at rate $g_z = (g_A + \beta g_N)/\alpha$;

(**iii**) labor productivity grows at rate $g_y + (g_A + \beta g_N)/\alpha$.

Back-of-the-Envelope Calculations

The theory raises some quantitative questions:

1. Can't infer growth of search efficiency from time trends of u, θ, h_{ue}, h_{eu}.
   a. Measure technological improvements in search process?
   b. Measure contribution to economic growth of improvements in search process?

2. Can't infer returns to scale in search from time trends or cross-sections of u, θ, h_{ue}, h_{eu}
   a. Measure returns to scale in search process?
   b. Measure the contribution to economic growth of returns to scale in search process?

Back-of-the-Envelope Calculations

Average number of applications per vacancy are

$$A_t N_t^{\beta} q(\theta_t).$$

- In the model, applications per vacancy grow at rate $\beta g_N + g_A$.
- In the data, applications per vacancy were 24 in 1981 (EOPP) and 45 in 2010 (Career Builder, SnagAJob).
- These observations suggest

$$\beta g_N + g_A = 2.2\%$$

Back-of-the-Envelope Calculations

Relative number of applications per vacancy in two markets of sizes N_1 and N_2 with the same search technology is

$$\frac{A_{1,t}N_{1,t}^{\beta}q(\theta_{1,t})}{A_{2,t}N_{2,t}^{\beta}q(\theta_{2,t})} = \left(\frac{N_{1,t}}{N_{2,t}}\right)^{\beta}.$$

- In the model, elasticity of applications per vacancy wrt size is β.
- In the data, elasticity of applications per vacancy wrt size is 0.52.
- These observations suggest $\beta = 0.52$ and

$$\beta g_N = 0.52 \cdot 1.1\% = 0.6\%$$

$$g_A = \underbrace{g_A + \beta g_N}_{2.2\%} - \underbrace{\beta g_N}_{0.6\%} = 1.6\%$$

Back-of-the-Envelope Calculations

Contribution of Declining Search Frictions

	Pareto coefficient		
1981-2010	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
labor productivity growth		1.9%	
cont. of search technology	0.4%	0.2%	0.1%
cont. of IRS in search	0.15%	0.07%	0.04%
cont. of declining search frictions	0.55%	0.27%	0.14%

Back-of-the-Envelope Calculations

Returns to Scale and Productivity across Cities

	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
0.5 million workers	0.91	0.95	0.98
1 million workers	1	1	1
10 million workers	1.34	1.16	1.08