Private Equity Indices Based on Secondary Market Transactions*

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Abstract

Measuring the performance of private equity investments (buyout and venture) has historically only been possible over long horizons because the IRR on a fund is only observable following the fund's final distribution. We propose a new approach to evaluating performance using actual prices paid for limited partner shares of funds in secondary markets. We construct indices of buyout and venture capital returns using a proprietary database of secondary market prices between 2006 and 2017. These transaction-based indices exhibit significantly higher betas and volatilities, and lower alphas and Sharpe ratios than NAV-based indices built from *Preqin* and obtained from *Burgiss*. There are a number of potential uses for these indices. In particular, they provide a way to track the returns of the buyout and venture capital sectors on a quarter-to-quarter basis and to value illiquid stakes in funds.

JEL classification: G11, G23, G24

Key words: Private Equity, Secondary Market for Private Equity Funds, Market Index

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1. Introduction

In recent decades, private equity has become an important asset class for institutional investors. A 2017 survey of institutional investors finds that 88% are invested in private equity, with nearly a third having an allocation greater than 10% (Whyte (2017)). A typical private equity investment begins with capital commitments at the fund's creation and ends with the final distribution, which is often 12 to 15 years after the initial capital commitment. The return on the fund is determined by the returns on the individual portfolio companies in which the fund invests. These returns, however, are only fully observable following the fund's final distribution.[1] Therefore, it is difficult for investors to know the value of an investment in a private equity fund while the fund still owns portfolio companies. The underlying value of these portfolio companies fluctuates with firm-specific and economy-wide news in a manner similar to that of public equities, but these fluctuations are usually not fully reflected in the valuations that funds report to their investors. Moreover, since returns are measured at such irregular, infrequent intervals, it can be quite challenging to estimate standard performance parameters such as factor alphas and betas (Axelson, Sorenson, and Strömberg (2014)).

While active markets for trading investments in private equity funds did not exist prior to 2000, in the early 2000s, a secondary market developed on which limited partners (LPs) could transact their stakes in private equity funds. In this paper, we use data obtained from a large intermediary in this market to evaluate the risk and return of private equity funds in a similar manner to the way in which investors regularly use public equity markets to understand the risk and return of publicly traded companies. Using these data we construct transaction-based indices for both buyout and venture capital funds, and use these indices to address a number of questions about the private equity market. These indices provide new insights into the performance of private equity as an asset class. In contrast to the existing literature, we find that

[1] Funds do report "Net Asset Values (NAVs)" to their investors, which are accounting-based valuations of the fund. These NAVs are adjusted to reflect the fund's actual value, but at any point in time, the gap between the NAV and the value of an investor's stake in a fund can be substantial.

neither buyout nor venture funds outperform public markets on a risk-adjusted basis.[2] We also find that NAV-based indices, such as the *Burgiss* index, tend to significantly understate the volatility of private equity as well as its covariance with other asset classes.

The primary difficulty in constructing an index from secondary market data for private equity is accounting for the fact that not every fund trades in every period, and many funds in our sample do not trade at all. In the subsample of funds that could be matched with cash flow data from the *Preqin* database, there are 1,119 fund transactions for 630 funds (372 buyout and 258 venture) from 2006 through 2017, implying that the average fund in our data trades 1.8 times in our sample. Moreover, there are many other funds that never trade through the intermediary that provides our sample. We employ two approaches to construct our indices in light of these challenges.

First, we show that if funds transact with uniform i.i.d. probability, we can construct an index that tracks the price of a broad portfolio of funds, even if funds within the portfolio do not transact in our sample every period. Second, we account for the possibility that fund transactions are not random, and that the decision to transact in the secondary market could be related to fund market values or other characteristics. To account for such possible sample selection, we create a hedonic index using the approach of Heckman (1979). We estimate the parameters of an econometric model using observed transaction prices and each period create an inferred price for every fund, including those that do not transact, using a broad universe of funds. We then use these inferred prices to construct our index. Other indices based on secondary markets also use some type of interpolation to infer the prices of non-traded assets. Bond indices, for instance, often employ "matrix pricing" to infer the prices of non-traded bonds. In addition, we account for measurement error when estimating performance parameters by applying bias adjustments (e.g. Scholes and Williams (1977) and Blume and Stambaugh (1983)).

[2] For example, current evidence suggests both buyout and venture funds outperform on a risk-adjusted basis (Cochrane (2005), Korteweg and Sorensen (2010), Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2016)). It should be emphasized, however, that our study is on a more recent sample period than these other studies, so the results are not comparable and not necessarily inconsistent with the prior studies.

A striking observation about the transactions-based indices is that they are much more cyclical and exposed to market-wide risk than other indices based on reported NAVs. We estimate the beta of the transactions-based buyout index to be greater than two. As emphasized by Axelson, Sorensen and Strömberg (2014), buyout funds are essentially portfolios of highly levered firms. Even if the portfolio firms prior to the buyout have unlevered betas slightly less than 1.0, tripling their leverage (as typically done in a buyout) should lead to a portfolio beta larger than two, consistent with our findings. For comparison, we estimate the beta of a buyout index that we construct based on the NAVs reported by *Preqin* using the same set of firms that make up the transaction-based indices, and the beta of a buyout index produced by *Burgiss*. In contrast to the estimates for the transactions-based indices, these estimates of beta are both less than 0.5. Indices based on NAVs have much lower betas than the transactions-based indices because NAVs are smoothed over time and do not reflect the information contemporaneously incorporated into market prices.

An implication of the high estimated betas is that, even though the hedonic buyout index averages a 20% return over the sample period, its CAPM alpha is estimated to be insignificant from zero. The relatively high average return, therefore, is just enough to compensate investors for their exposure to market-wide risk. This finding is in contrast to our estimates of alpha for the two NAV based indices, as well as those in much of the literature based on fund cash flows, which are significantly positive.

The betas of transaction-based venture capital indices are also higher than that of the corresponding NAV-based indices. We estimate betas for the transactions-based indices to be about 1.0, and for the NAV-based indices to be about 0.3. Since venture funds usually do not leverage their portfolio companies, there is not a Modigliani-Miller ratcheting of betas as in buyouts, which is why venture betas tend to be lower than buyout betas. Nonetheless, the estimated betas of the transactions-based indices are large enough to affect inferences about performance. NAV-based indices of venture funds exhibit positive alphas that are statistically significant while the transactions-based indices' alphas are *negative* and marginally significant in some specifications. We do note, however, that the period for which we have transactions data was a

poor one for the venture capital industry and the negative estimate of alpha should not be used as an estimate of venture capital performance over other periods.

Besides altering perceptions of performance, the lack of information from secondary markets can also distort investors' portfolio decisions. For example, following the Financial Crisis of 2008 a number of investors incorrectly believed that their portfolio weight in private equity had substantially increased, since NAVs of their private equity positions dropped much less in value than the market values of their stock holdings. Our analysis suggests that at the time of the Crisis, private equity funds' values had fallen substantially below their NAVs by at least as much as public equities, so, properly measured, the fraction of institutional portfolios made up by private equity did not increase in the financial crisis. Our analysis also suggests, however, that current market values of buyout funds are substantially above their NAVs, which is likely to be relevant to any investor allocating capital across asset classes.

The indices we develop based on secondary market transactions allow us to evaluate the risk-adjusted, net of fee performance of broad private equity portfolios. While there are a number of papers that have estimated private equity performance, none rely on secondary market data, which is ideal for measuring the risk and returns of securities. After all, asset price theory is built on the concept of return, and secondary markets are the only means by which we can observe achievable returns measured at frequent regular intervals. For this reason, our results differ in some regards to what has been reported in the literature.

Finally, our indices also allow us to value individual funds at any given point in time and to estimate the extent to which general partners over- or understate net asset values relative to market value over the course of the business cycle. While the indices we construct are similar to stock and bond indices that reflect prices achievable to the marginal investor who transacts in secondary markets, we note that the performance of such indices can differ from that of the typical buy-and-hold investor for two reasons. First, the duration of expected cash flows can be held relatively constant in secondary market transactions while the duration

of cash flows for a buy-and-hold investor may dramatically shrink over the life of the investment.[3] Second, achievable returns of all asset classes in secondary markets are influenced to some degree by transaction costs and illiquidity. In secondary markets for private equity, most transaction costs appear to be laid on the seller and buyers may earn a small premium as market makers if positions are held to the end of a fund's life (Nadauld, Sensoy, Vorkink, and Weisbach, (2018), NSVW hereafter). In contrast, our index returns reflect the return from buying *and* selling at secondary market prices. To the extent that buyers earn a premium for market making, this premium is largely canceled out when the position is sold. In addition, any attempt to adjust secondary market returns for transaction costs or illiquidity would render them unachievable, and to that extent, somewhat meaningless.

Our primary objective in this paper is to develop methods to measure achievable private equity returns in secondary markets. Transactions costs and illiquidity are a feature of many secondary markets. Further, without the information such markets provide, valuation of private equity in real time is challenging at best, and achievable returns for private equity positions can only be measured over extremely long horizons.

The paper proceeds as follows. In Section 2 we describe the secondary market data we use to construct our indices. In Section 3 we describe our empirical methodology. Section 4 describes our contribution relative to the work of others. In Section 5 we describe our data, and present the indices. In Section 6 we discuss some implications and applications of transactions-based private equity indices. Section 7 concludes.

2. Data

A large intermediary in the private equity secondary market provides us with data on all the secondary market transactions intermediated by their firm that is similar to data used in NSVW (2018).[4] These data identify the fund name, the vintage, the total capital committed by the seller, the amount

[3] For example, the holding period return from buying and holding a corporate bond to maturity can differ from the return earned by engaging in secondary bond markets.
[4] The data used below extend the data from NSVW (2018) through the end of 2017.

unfunded by the seller, the purchase price, and the transaction date for funds that transacted from June of 2000 through December of 2017. Since the database contains only five transactions before 2006, we eliminate these and conduct our analysis using transactions that take place between 2006 and 2017.

We clean the data as detailed in Appendix A and pull the most recent transaction for each fund each calendar quarter. After cleaning the data, we are left with 3,404 fund transactions for 2,424 funds, which we refer to as the *transaction sample*. This transaction sample is made up of 2,066 buyout transactions and 1,338 venture transactions for 1,427 buyout funds and 997 venture funds.

We obtain data on other fund characteristics, such as calls, distributions, NAV, fund LP type, and size, from *Preqin*, and clean these data as detailed in Appendix A. Within each calendar quarter we sum all contributions and distributions (separately) for a given fund. A reporting lag causes our data to be missing information for the last quarter of 2017, and hence, our data from *Preqin* extend from the first quarter of 2006 through the third quarter of 2017 with quarterly information on 2,287 unique funds, of which 1,251 are buyout and 1,036 are venture. We refer to these data as the *Preqin sample.*

We then merge the transactions sample with the *Preqin* sample, some of which is done by hand (see Appendix A for details). After this process, we end up with a sample on 630 funds (372 buyout and 258 venture) for which 1,246 transactions occurred through the intermediary providing our data between 2006 and 2017. We refer to these data as the *merged sample.*

LPs transact in the secondary market for many reasons, primarily, according to practitioners, for portfolio rebalancing purposes.[5] However, transactions on funds that are 10 years or older, referred to as tail-end transactions by practitioners, are essentially transactions in which the buyer is speculating about the value of the one or two particular portfolio firms left in the fund. For this reason, the prices of tail-end transactions do not reflect the fundamental economics of a fund, given that most of the fund is liquidated before the time of the transaction. Similarly, transactions on young funds, those 3 years or younger, primarily reflect liquidity demands of investors and so prices of young transactions do not contain much

[5] NSVW (2018) provide discussion and analysis of participants and motivations for secondary market trading of private equity.

economic content other than demand for immediate liquidity. Appendix B describes other institutional features of the secondary market that impact secondary market prices.

Transactions of funds in the 4-9 year range are most likely to reflect the value of the fund's portfolio firms. To the extent that common factors affect all private equity funds, these transactions are likely to be informative about other funds' values as well. Consequently our index is constructed with transactions on funds that are between 4 and 9 years old. We merge all transactions on 4-9 year-old funds and refer to this sample as the *baseline merged sample*.

Table 1 reports summary statistics for the various samples, with Panel A containing statistics for buyout funds and Panel B for venture funds. This table reports statistics for both the merged transaction/*Preqin* sample and its complement, i.e., all quarter-fund observations in *Preqin* for which no transactions occurred. Table 1 also reports statistics separately for the *baseline merged sample* (the intersection of the set of funds in the *Preqin* universe that are four to nine years old with the transactions sample) and its complement.

The first three rows of each panel report the mean, first quartile (Q1) and third quartile (Q3) for transaction prices as a fraction of NAV, denoted throughout the paper as $\pi_{i,t}$. Consistent with prior findings, funds on average transact at a discount relative to NAV (see NSVW (2018)). Discounts are smaller for baseline transactions than for other, less typical deals. The overall average $\pi_{i,t}$ for buyout funds is between 0.82 to 0.83 but for baseline transactions is 0.89. Similarly, the overall average $\pi_{i,t}$ for venture funds is between 0.80 to 0.83 but for baseline transactions is 0.92. Among venture baseline transactions, the third quartile for $\pi_{i,t}$ is 1.21, suggesting that many venture funds transact at a premium to NAV.

The deviation between a fund's NAV and its market price will depend on the performance of the investments that the fund has made but not exited. A fund that has made poorly performing investments

will have a market price less than NAV, so a trade for less than NAV is not necessarily reflective of a liquidity discount.[6]

Funds that transact in our sample tend to be larger than the average fund in *Preqin*. The average fund size in the buyout merged sample is about $4.7 billion, compared to an average fund size in the buyout compliment sample of about $1.8 billion. Similar patterns are found for venture, though venture funds in our data are about 80% to 90% smaller than buyout funds. The average fund age of transacting funds tend to be around eight to ten years in our data. In addition to being larger, buyout funds that transact display higher average $PMEs$, in the range of 1.16 to 1.18 compared to 1.12 for non-transacting funds. Transacting venture funds display average PMEs between 0.95 and 1.04 compared to average non-transacting fund PMEs of 1.01.[7]

Figure 1 reports the number of transactions per quarter for the merged sample. Buyout and venture transactions are highly correlated in the time series, though the buyout market has been more active between 2009 and 2016 relative to the venture market. Venture transactions picked up substantially in 2014 and 2015 before tapering off slightly in 2016 and 2017.[8] Table 1 indicates the average number of transactions in the merged sample per quarter is about 16 for buyout funds and about 10 per quarter for venture funds.

3. Constructing an Index Measuring Private Equity Returns

Private equity returns are a function of transaction prices, fund contributions and fund distributions. While we observe quarterly distributions and contributions for a large sample of funds, we observe market prices only when there is a transaction intermediated by the firm that provides us with our data. In this

[6] NSVW (2018) develop this issue further. They argue that a second measure of a trade's discount relative to fundamental value is the difference in returns between buyers and sellers. If transactions always occurred at fundamental values and expected returns do not change over time, then buyer and seller returns should be approximately the same. Instead, these authors find that buyers of LP stakes outperform sellers, suggesting that, on average, transaction prices tend to be lower than fundamental values.

[7] We calculate the *PME* for each fund using all cash flows up to the most recent date for which we have cash flow data in *Preqin*, using NAV as the terminal value for funds that have not liquidated.

[8] Deal volume in our sample is also influenced by fluctuations in the market share of our data provider.

sample of transactions, no fund transacts in every quarter and some funds never transact through the intermediary providing our data. In addition, the transactions that do occur are highly nonsynchronous. Our observed index returns therefore contain measurement error coming from two sources: non-trading and nonsynchronous trading.[9] Non-trading, after accounting for any sample selection in the funds that trade, can be modeled as i.i.d. measurement error in our index returns similar to the setting described in Blume and Stambaugh (1983). Nonsynchronous trading induces spurious autocorrelation and cross-autocorrelation with the market (see Scholes and Williams (1977) and Lo and MacKinlay (1990)). In addition, both sources of measurement error induce biases in estimated variances and covariances using observed index returns. We next show how we construct our indices and correct for biases in estimated moments that arise from measurement error.

3.1 Calculating Returns and Accounting for Measurement Error

Let $P_{i,t}$ denote the secondary market price of a \$10 million commitment to private equity fund i at the end of quarter t. Suppose at the end of quarter $t-1$ a purchasing LP acquires from a selling LP a \$10 million commitment to each of N different private equity funds. If the purchasing LP holds for one period and then sells all of their positions at the end of quarter t, the log quarterly buy-and-hold return for the portfolio is

$$r_t = \log\left(\sum_i P_{i,t} + D_{i,t} - C_{i,t}\right) - \log\left(\sum_i P_{i,t-1}\right), \quad (1)$$

where $D_{i,t}$, $C_{i,t}$ represent the total distributions and contributions associated with the position in fund i during quarter t. For convenience equation (1) can be rewritten using simple averages,

$$r_t = \log(\bar{P}_t + \bar{D}_t - \bar{C}_t) - \log(\bar{P}_{t-1}), \quad (2)$$

[9] Nontrading in a given period can be modeled as extensive nonsynchronous trading as in Lo and MacKinlay (1990) if all securities trade at some point in time. In our data, however, some funds never transact.

where an overline represents a simple average. Equations (1) and (2) define the log-return on a price-weighted portfolio.[10] The moments and estimated performance parameters of our indices are similar for log returns and for arithmetic returns. To facilitate the exposition, we choose to use log returns to explain our methodology for estimating $\bar{P}_t$ and the moments of r_t. We make the following three assumptions regarding fund returns and transactions:

1. True (unobserved) log returns are independent and identically distributed across time.
2. Each fund is assigned a random transaction time relative to the end of the quarter, $t - s_i(t)$, where t defines the end of the quarter, $s_i(t)$ is independent and identically distributed across time, and $0 < s_i(t) < 1$ (following Scholes and Williams (1979)).
3. In quarter *t*, a set of $k_t < N$ funds actually transact in quarter t.

Let $\bar{P}_{k,t-s(t)}$ denote the observed average price using the k_t funds that transact, let $\tilde{r}_t = \log(\bar{P}_t) - \log(\bar{P}_{k,t-s(t)})$, and let $\bar{\delta}_{k,t}$ denote an i.i.d random variable. Under the three assumptions above, we show that an observed portfolio return, r_t^O, can be constructed such that

$$r_t^O = r_t + \Delta\bar{\delta}_{k,t} - \Delta\tilde{r}_{i,t} \tag{3}$$

where r_t represents the true portfolio return. The random variable $\tilde{r}_t$, which arises from nonsynchronous trading, is similar to the portfolio return from buying each of the k_t funds at their assigned transaction times, and selling them all simultaneously at their end-of-quarter market values. The random variable $\bar{\delta}_{k,t}$ arises from i.i.d measurement error in the average fund price. We show that equation (3) holds in two cases: first, for the case that all funds transact with independent uniform probability, and second, for the case of sample selection in which certain types of funds are more likely to transact than others.

3.1.1 Case 1: Transactions with Independent Uniform Probability

[10] The analysis of this section can be extended to portfolios with other weighting methods by defining $P_{i,t}$ to be the price of the appropriately sized position in fund i.

The observed transaction price for any fund can always be written as the product of the population average across all N funds, $\bar{P}_{t-s(t)}$, and a fund-specific scaling constant, $(1+\delta_{i,t})$,

$$P_{i,t-s(t)} = \bar{P}_{t-s(t)}(1+\delta_{i,t}), \tag{4}$$

where $\delta_{i,t} > -1$ and the population average of $\delta_{i,t}$ across all N funds is identically equal to zero, $\bar{\delta}_t = 0$. If we let $\bar{\delta}_{k,t}$ denote the average value of $\delta_{i,t}$ across the k_t funds that transact, then the observed estimate of the average price using these k_t funds, $\bar{P}_{k,t-s(t)}$, is given by

$$\bar{P}_{k,t-s(t)} = \bar{P}_{t-s(t)}(1+\bar{\delta}_{k,t}), \tag{5}$$

where $\bar{\delta}_{k,t}$ is the average value of $\delta_{i,t}$ across the k_t funds that transact. If funds transact with independent uniform probability, then $\bar{\delta}_{k,t}$ is independent across time and mean zero.

Using equation (5) we can write the observed log average price across all funds as

$$\log(\bar{P}_{k,t-s(t)}) = \log(\bar{P}_t) + \log(1+\bar{\delta}_{k,t}) - \tilde{r}_t, \tag{6}$$

where

$$\tilde{r}_t = \log(\bar{P}_t) - \log(\bar{P}_{k,t-s(t)}). \tag{7}$$

Equation (6) implies we can write the observed log portfolio return[11], r_t^O, as

$$\begin{aligned} r_t^O &= \log(\bar{P}_{k,t-s(t)}) - \log(\bar{P}_{k,t-1-s(t-1)}) \\ &= r_t + \Delta\bar{\delta}_{k,t} - \Delta\tilde{r}_{i,t} \end{aligned} \tag{8}$$

where we use the first-order approximation,

$$\Delta\bar{\delta}_{k,t} \approx \log(1+\bar{\delta}_{k,t}) - \log(1+\bar{\delta}_{k,t-1}). \tag{9}$$

The bottom line of (8) is identical to equation (3).

3.1.2 Case 2: Sample Selection

[11] Equation (8) holds if $\overline{D}_t$ and $\overline{C}_t$ are zero, or more generally if $\overline{D}_t = \overline{C}_t$. Otherwise the observed return is $r_t^O = \log(\bar{P}_{k,t-s(t)} + \overline{D}_t - \bar{C}_t) - \log(\bar{P}_{k,t-1-s(t-1)})$ which can be written as $r_t + \Delta\bar{\delta}_{k,t} - \Delta\tilde{r}_{i,t}$ plus a small additive term.

Because some types of funds are more likely to transact than others, estimating $\bar{P}_t$ as the average of observed prices could potentially lead to sample selection bias. We therefore develop a hedonic model as a way to obtain an estimate of $\bar{P}_t$ at a point in time conditional on the fund's characteristics.[12] Our objective is to understand the price changes of a portfolio of differentiated goods over time when some transaction prices are not observed.

Let the N funds in the index over quarter t classify a full "population" of funds. Our objective is to estimate the population average price using a sample of funds that transact from this population. Let $\boldsymbol{X_{N,t}}$ denote an $N \times p$ matrix of p characteristics observable by the end of quarter t for the full population of N funds, regardless of whether the fund transacts or not. Let $\boldsymbol{\pi_{N,t-s(t)}}$ denote the $N \times 1$ vector of scaled market prices for transactions in quarter t with element i defined as $\pi_{i,t-s(t)} = P_{i,t-s(t)}/NAV_{i,t}$. To begin, suppose we observe the full population of scaled prices. Finally, let $\boldsymbol{\theta}_{N,t}$ denote a $p \times 1$ vector of population parameters. For clarity in the exposition below, we will drop time subscripts. The vector of parameters, $\boldsymbol{\theta_N}$, comes from the regression,

$$\boldsymbol{\pi}_N = \boldsymbol{X}_N\boldsymbol{\theta}_N + \boldsymbol{\epsilon}_N, \tag{10}$$

where $\boldsymbol{\epsilon}_N$ represents a vector of i.i.d. residuals. The average "fitted" price, $\bar{P}_N^f$, can be obtained as

$$\bar{P}_N^f = \frac{1}{N}[(\boldsymbol{X_N\theta_N})'\boldsymbol{NAV_N}]. \tag{11}$$

where $\boldsymbol{NAV_N}$ denotes the $N \times 1$ vector of NAVs. Using the full population of N funds, the average fitted price is identical to the average price across all N funds since residuals are mean zero,

$$\bar{P}_N^f = \bar{P} \tag{12}$$

where $\bar{P}$ is the average price across all N funds.[13]

[12] For discussion of hedonic models and examples of their uses in other contexts, see Gatzlaff and Haurin (1998), Pakes (2003) and Hwang, Quigley, and Woodward (2005).

[13] To see this note that $(\boldsymbol{X_N\theta_N})'\boldsymbol{NAV_N} = (\boldsymbol{\pi_N - z_N})'\boldsymbol{NAV_N}$. The average fitted price is equal to the population average price across all N funds only if NAV_i is independent of the regression residual. We can in fact ensure this is true by including NAV as one of the explanatory variables in $\boldsymbol{X_N}$.

Now consider that we only observe a sample of $k_t < N$ funds that transact in period t. Let $\boldsymbol{\pi_k}$ denote the vector of observed scaled market prices, and let $\boldsymbol{X_k}$ denote the $k \times p$ matrix of fund characteristics for funds that transact. Suppose we use the observed sample of data to run the regression

$$\boldsymbol{\pi}_k = \boldsymbol{X_k}\boldsymbol{\theta_k} + \boldsymbol{\epsilon_k}, \tag{13}$$

and use the estimated vector of coefficients, $\boldsymbol{\theta}_k$, to estimate fitted prices for the full population of N funds,

$$\bar{P}_N^{f,k} = \frac{1}{N}[(\boldsymbol{X_N}\boldsymbol{\theta_k})'\boldsymbol{NAV_N}]. \tag{14}$$

The value $\bar{P}_N^{f,k}$ represents the estimated average fitted price for all N funds based on parameters, $\boldsymbol{\theta_k}$, estimated using information in only k funds. Because we observe fund characteristics and NAVs for the full population of N funds, the expected value of $\bar{P}_N^{f,k}$ is given by

$$E\left[\bar{P}_N^{f,k}\right] = \tfrac{1}{N}[(\boldsymbol{X_N}\boldsymbol{E}[\boldsymbol{\theta_k}])'\boldsymbol{NAV_N}]. \tag{15}$$

Equation (15) implies the average estimated fitted price is an unbiased estimator of the average price for the full population of N funds, provided that $\boldsymbol{\theta_k}$ is an unbiased estimator of $\boldsymbol{\theta_N}$,

$$E[\boldsymbol{\theta_k}] = \boldsymbol{\theta_N}. \tag{16}$$

Let y_i^* denote a latent random variable that determines whether a fund is selected for transaction, which may depend on fund characteristics. If $y_i^* \geq 0$ then fund i is selected for transaction in period t, and otherwise, it is not. We only observe prices for funds such that $y_i^* \geq 0$. Stack the values of y_i^* for the k funds with observed prices into the $k \times 1$ vector $\boldsymbol{y_k^*}$. Also, let the $k \times 1$ vector $\boldsymbol{\epsilon_{N(k)}}$ denote the k-element subset of $\boldsymbol{\epsilon_N}$ corresponding to the funds that transact in the quarter. It follows that the expected OLS estimate of $\boldsymbol{\theta_k}$ can be written

$$E\left[\boldsymbol{\theta_k^{OLS}}\right] = \boldsymbol{\theta_N} + E\left[(\boldsymbol{X_k'X_k})^{-1}\boldsymbol{X_k'}\,\boldsymbol{\epsilon_{N(k)}} \;\middle|\; \boldsymbol{y_k^*} > \boldsymbol{0}\right]. \tag{17}$$

Equation (17) highlights the fact that $\boldsymbol{\theta_k^{OLS}}$ is an unbiased estimator of $\boldsymbol{\theta_N}$ if population residuals, the elements of $\boldsymbol{\epsilon_N}$, are independent of the selection variable y_i^*.

Panel A of Figure 1 illustrates an example when population residuals are not mean zero, conditional on $y_i^* \geq 0$. Suppose we expect larger funds to have higher $\pi_{i,t-s(t)}$, and we also expect larger funds to

transact more often. We therefore need to estimate the relation between size and $\pi_{i,t-s(t)}$ to infer the prices of smaller funds that do not transact. The dark line in this panel shows the relation between size and price using the population of all funds. Points crossed out represent observations we do not observe. The dashed line represents the relation using only observed observations. Larger funds (to the right) do indeed transact more often and have higher prices, but the population residuals of transacting funds are mean positive since they are almost all above the solid line. This happens because there is some other omitted variable positively correlated with price that is also correlated with selection, y_i^*. Holding size fixed, a higher realization of this variable results in a higher likelihood of being observed (higher y_i^*), and a higher price, causing observed observations in the chart to be shifted up relative to observations that are crossed out. Using observed transactions, this results in a biased intercept. Panel B illustrates an example when the omitted variable is also positively correlated with size. In this case, the covariance between population residuals and size is not zero, conditional on $y_i^* \geq 0$.

Based on the above examples, a natural solution to make $\boldsymbol{\theta_k^{OLS}}$ an unbiased estimate of $\boldsymbol{\theta_N}$ is to include variables positively associated with selection, y_i^*, as elements in $\boldsymbol{X_k}$ and $\boldsymbol{X_N}$. Doing so should reduce any dependence between the population residuals and selection y_i^*. Despite the number of variables we include, however, the likelihood exists that some residual dependence remains.

A more comprehensive solution is to abandon OLS and estimate the parameters of the sample selection model of Heckman (1979), in which we use a system of equations to simultaneously model both price and selection. In specifying the complete joint distribution for the residuals of these equations we can estimate and account for any residual dependence between price and selection as we estimate the model parameters. The approach provides unbiased estimates, $\boldsymbol{\theta_k}$, such that

$$\boldsymbol{\theta_k} = \boldsymbol{\theta_N} + \boldsymbol{e_k} \tag{18}$$

where $\boldsymbol{e_k}$ is a mean zero $p \times 1$ vector that is reasonably independent of other variables used in the estimation of average price. Equation (18) implies that we can write the estimated average price as

$$\bar{P}_N^{f,k} = \bar{P}\left(1 + \bar{\delta}_k\right), \tag{19}$$

where in this case,

$$\bar{\delta}_k = \frac{1}{N}\left[\frac{(\boldsymbol{X_N e_k})' \boldsymbol{NAV_N}}{\bar{P}}\right] \tag{20}$$

is an i.i.d random variable, as it is in equation (5). Following equations (6) and (7), we can again show that observed returns are given by equation (3). Observed returns, then, can be described by (3) in the case that funds transact with uniform i.i.d. probability, and in the case that there is sample selection.

3.1.3 Sample Moments Adjusted for Measurement Error

Using equation (3) and the three assumptions specified at the beginning of section 3.1, we can derive some moments of the observed log returns:

$$\begin{aligned}
E[r_t^O] &= E[r_t] \\
Var[r_t^O] &= Var[r_t] + 2Var\left[\bar{\delta}_{k,t}\right] + 2Var\left[\tilde{r}_{i,t}\right] - 2Cov(r_t, \tilde{r}_t) \\
Cov\left[r_t^O, r_{m,t}\right] &= Cov\left[r_t, r_{m,t}\right] - Cov\left[\tilde{r}_t, r_{m,t}\right] \\
Cov\left[r_t^O, r_{t-1}^O\right] &= -Var\left[\bar{\delta}_{k,t}\right] - Var\left[\tilde{r}_{i,t}\right] + Cov\left(r_t, \tilde{r}_{i,t}\right) \\
Cov\left[r_t^O, r_{m,t-1}\right] &= Cov\left[\tilde{r}_t, r_{m,t}\right].
\end{aligned} \tag{21}$$

where $r_{m,t}$ is the observed public equity market return with no measurement error. The first line of (21) indicates that measured portfolio returns are unbiased. The second line of (21) indicates that the observed portfolio variance may be either over- or understated (see discussion in Scholes and Williams (1979) and Lo and MacKinlay (1990)). The third line of (21) indicates the covariance of the observed portfolio return with the market is understated due to the reduced contemporaneous overlap in these measured returns from non-synchronous trading. The fourth and fifth lines of (21) provide results for the observed autocovariance and cross-autocovariance with the market.

The moments defined in equation (21) enable us to derive unbiased estimates of the following moments for the portfolio return of all funds with synchronous trading:

$$
\begin{aligned}
E[r_t] &= E[r_t^O] \\
Var[r_t] &= Var[r_t^O] + 2Cov\left[r_t^O, r_{t-1}^O\right] \\
Cov\left[r_t, r_{m,t}\right] &= Cov\left[r_t^O, r_{m,t}\right] + Cov\left[r_t^O, r_{m,t-1}\right].
\end{aligned}
\tag{22}
$$

The moments defined in (22) can be used to define unbiased estimates of other parameters of interest, such as the CAPM alpha and beta, as well as the contemporaneous correlation between the fund portfolio return and the market.

3.2 Details of Index Construction

Observed returns for the indices we construct are all calculated as

$$
r_t^O = \frac{\bar{P}_t^E + \bar{D}_t - \bar{C}_t}{\bar{P}_{t-1}^E} - 1\,. \tag{23}
$$

where $\bar{P}_t^E$ denotes an estimate of $\bar{P}_t$ across the population of all funds at the end of quarter t in the *Preqin* universe. Equation (23) is the arithmetic analog to equation (8). We use three different approaches to obtain $\bar{P}_t^E$. First, we estimate $\bar{P}_t^E$ as $\bar{P}_{k,t-s(t),}$ the simple average across all funds that happen to transact. Since this approach ignores the possibility of sample selection, we refer to indices created in this manner as "naïve" indices. Second, we estimate $\bar{P}_t^E$ using a hedonic model that accounts for sample selection. Third we use $\overline{NAV_t}$ as an estimate for $\bar{P}_t$, where $\overline{NAV_t}$ is the average NAV across the full population of funds in the *Preqin* universe.

3.2.1 Naïve Indices

As a baseline for comparisons, we first create simple indices of private equity returns that do not adjust for selection: "naïve" indices. Naïve indices are price weighted. They represent an estimate of the return on a portfolio strategy that uses all capital at the end of each quarter to buy an equal sized commitment

in each fund and then liquidates at the end of the quarter after collecting distributions and paying out any capital calls.

To create naïve indices, we estimate $\bar{P}_t^E$ as $\bar{P}_{k,t-s(t),}$ the simple average across all funds that happen to transact. One way to express the average observed price, $\bar{P}_{k,t-s(t)}$, is

$$\bar{P}_{k,t-s(t)} = \bar{\pi}_{k,t-s(t)}\overline{NAV}_t + Cov(\pi_{k,t-s(t)}, NAV_t). \quad (24)$$

An advantage of (24) is that it enables us to use the information in the full population of $NAVs$ in the *Preqin* universe to estimate average price, regardless of whether funds transact in that quarter or not. While NAV is not the market price it is likely to contain some pricing information. To create the naïve indices we use only funds that are 4-9 years old. In particular, we use the baseline transactions sample to estimate $\bar{\pi}_t$, we use funds in the baseline merged sample to estimate $Cov(\pi_t, NAV_t)$, and we use the *Preqin* Universe for funds that are four to nine years old to estimate $\overline{NAV}_t$, $\bar{D}_t$, and $\bar{C}_t$. We then use (24) to compute average price for the naïve index, and (23) to calculate returns on this index.

We estimate $\bar{\pi}_t$, $\overline{NAV}_t$, and $Cov(\pi_t, NAV_t)$ for each quarter. For buyout funds, we have sufficient observations to estimate $\bar{\pi}_{k,t-s(t)}$ in every quarter using the baseline transactions sample. As seen in Table 1, the average number of transactions per quarter for this sample is 19.1 with a first quartile of 8 and a third quartile of 27. In the baseline merged sample for buyout funds, the average number of observations per quarter is only 7.7, and there are four quarters for which the number of buyout transactions is less than or equal to one. When computing the buyout naïve index, we use the covariance estimated from the previous period for these quarters.

For venture funds, we do not have sufficient observations to estimate $\bar{\pi}_{k,t-s(t)}$ in every quarter using the baseline transactions sample. As seen in Table 1, the average number of transactions per quarter for this sample is 9.8, but there are seven quarters for which there are zero venture transactions. To create the venture index, we therefore first create a naïve index using all funds (both buyout and venture). For this total naïve index, we compute the fraction of capital invested in buyout funds each quarter, and take an average across quarters, $\bar{w}_b$. We then compute the venture index return each quarter as

$$r_{vn,t} = \frac{r_{Tn,t} - \bar{w}_b r_{Bn,t}}{1 - \bar{w}_b} \tag{25}$$

where $r_{Tn,t}$ represents the return on the total naïve index and $r_{Bn,t}$ is the return on the buyout naïve index.

3.2.2 Hedonic Indices

To create indices that account for sample selection we choose a set of observable characteristics and estimate a vector of coefficients, $\boldsymbol{\theta}$, that captures the linear relation between prices and characteristics as in (10). We then use the vector of coefficients to estimate the average price across all funds in a given quarter as in (14). We create both price-weighted and size-weighted Hedonic indices.

We use the merged transaction-*Preqin* sample, for buyout funds and venture funds separately, to estimate $\boldsymbol{\theta}$ by OLS. We then use the *Preqin* universe, for buyout funds and venture funds separately, to obtain an estimate of $\boldsymbol{\theta}$ by estimating the parameters of a sample selection model. Following Heckman (1979) we model the transaction process as

$$\begin{aligned} \pi_{i,t-s(t)} &= \boldsymbol{x}'_{i,t}\boldsymbol{\theta} + \epsilon_{i,t} \\ y^*_{i,t} &= \boldsymbol{z}'_{i,t}\boldsymbol{\gamma} + v_{i,t} \\ y_{i,t} &= \begin{cases} 1 \text{ if } y_{i,t} \geq 0 \\ 0 \text{ otherwise} \end{cases} \end{aligned} \tag{26}$$

where $\boldsymbol{x}_{i,t}$ and $\boldsymbol{z}_{i,t}$ are vectors of characteristics for fund i in quarter t observable across all funds in the *Preqin* universe. As in section 3.1.2, the variable $y^*_{i,t}$ is a latent variable that characterizes when a transaction occurs, such that fund i transacts in quarter t if and only if $y^*_{i,t} \geq 0$. We refer to the first equation of (26) as the *pricing equation*, and the second equation as the *selection equation*.

Appendix C outlines the econometrics of Heckman selection models. Valid selection models require a variable in the selection equation that is omitted from the pricing equation that meets the exclusion restriction. In a standard instrument sense, the variable meeting the exclusion restriction in the selection

equation must be independent of the error term in the pricing equation.[14] We propose the fraction of LPs invested in a fund that are pension funds as a variable that plausibly satisfies the exclusion restriction. Pension funds are typically buy-and-hold investors with the main investment objective of matching the duration of liabilities. Empirically, NSVW (2018) document that pension funds are less likely than other investors to sell their stakes in a private equity fund than other investors. In addition, the characteristics of the limited partners are unlikely to be correlated with innovations in transaction prices. While certain seller types may sell for different reasons, buyers face obstacles to find out who sellers are. The intermediary who provides our data keeps the names of clients confidential and all documents and other information shared with potential buyers is redacted to remove any reference to the seller or other LPs. Competition among bidders should cause the same fund to sell for the same price, regardless of pension fund ownership.

To implement the sample selection model given in (26) we need to select the explanatory variables $\boldsymbol{x}_{i,t}$ and $\boldsymbol{z}_{i,t}$. Table 2 lists the explanatory variables we consider. The first 6 rows of Table 2 list the "state variables" that are constant across funds and vary only over time. The final 6 rows of Table 2 list the fund-specific variables that vary both across funds and over time.

After estimating $\boldsymbol{\theta}$ using the full merged sample, we infer prices of funds that do not transact in a given quarter as $\boldsymbol{x}_{i,t}'\boldsymbol{\theta}$, using all funds in the baseline merged sample. Hence our indices are based only on the returns of funds that are four to nine years old. Since we can estimate returns for the full population of funds in the baseline merged sample, we create index returns by simply taking a price-weighted or size-weighted sum of returns. This gives an index return identical to (23) where prices are appropriately scaled to reflect investment positions consistent with portfolio weights.

[14] See Heckman (1990), Leung and Yu (1996), Andrews and Schafgans (1998), Korteweg and Sorensen (2011), and Wooldridge (2010) pp. 803-808. Although parameter estimates are still consistent with no exclusion restriction under the parametric assumptions of the model (specifically, the assumption of normality), they are less efficient and more sensitive to model assumptions.

We compute hedonic indices for both buyout and venture using the sample selection model. For comparison we also compute the hedonic indices using OLS. We compute size-weighted and price-weighted versions of the hedonic indices.

3.2.3 Construction of Other Indices

For comparison we also compute NAV-based indices, using $\overline{NAV_t}$ as an estimate for $\bar{P}_t$ in (23), where $\overline{NAV_t}$ is the average NAV across the full population of funds in the Prequin universe for funds that are four to nine years old. We also compare our indices to the buyout and venture indices created by *Burgiss*.

4. Alternatives to Secondary Market Based Indices

4.1. Prior Work Measuring Private Equity Risk and Return

The absence of an observable time series of market values at regular intervals for private equity has limited the ability of researchers to evaluate investment performance and value LP stakes in funds using standard empirical tools motivated by asset pricing theory. Basic parameters such as factor betas and alphas, volatility, correlations and average returns, have been estimated using non-traditional methods.

Prior studies about the investment performance of private equity can be broadly classified into one of four groups, depending on the type of data used. First, many studies use fund-level data on cash flows paid to and received by limited partners. Second, other studies use data on private equity funds' investments in their portfolio companies and the distributions they receive from these companies. Third, some studies use venture financing rounds and exit events (IPO, acquisition, and failure), which provide intermittent estimates of market value at these times. Finally, some studies use other proxies for market value, such as NAV or the prices of similar publically listed securities.

Papers that use fund-level cash flows have relied on the *PME* approach, which measures the performance of a fund relative to the public equity market at the same time.[15] Recent studies that use relatively high-quality fund-level cash flow data find the PME for buyout funds to be in the range of 1.19-1.23 and for venture funds to be in the range of 1.06-1.36, suggesting that these two types of private equity funds beat the S&P 500 even after the fees that LPs pay (see Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), Korteweg and Nagel (2016), and Robinson and Sensoy (2016)). Other studies use fund-level cash flows to estimate CAPM betas by estimating cross-sectional regressions of fund $IRRs$ on the $IRRs$ of the S&P 500 measured over the life of each fund (see Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), and Driessen, Lin, and Phalippou (2012)). These papers generally find betas for both private equity types to be in the range of 1.08 to 1.23. Exceptions are Kaplan and Schoar (2005) who find a buyout beta of 0.41, and Driessen Lin, and Phalippou (2012) who find a venture beta of 2.73.[16]

Papers relying on cash flows between private equity funds and their portfolio firms generally estimate cross-sectional regressions of excess $IRRs$ on the excess $IRRs$ of factor portfolios in the cross section.[17] Among other things, these papers find CAPM alphas for buyout funds to be in the range of 9.3% to 16.3% with betas in the range of 0.95 to 2.3 (see Frazoni, Nowark, and Philippou (2012) and Axelson, Sorensen, and Strömberg (2014)). It is important to note that, in contrast to the estimates presented below, these studies estimate risk and return *gross of fees*. Axelson, Sorensen, and Strömberg (2014) argue that management fees, because they are a function of performance, covary positively with market returns, so will have a positive beta, leading gross of fee betas to be larger than net of fee betas.

[15] The *PME* approach was originally developed by Kaplan and Schoar (2005). $PMEs$ are calculated by discounting all cash flows of the fund at a rate equal to the total return on the S&P 500 index, and then dividing the future value of cash inflows by the future value of cash outflows. A fund with a PME above 1.0 therefore has outperformed the passive index over the evaluation period.

[16] Other papers that investigate fund-level cash flows include Chen, Baierl and Kaplan (2002), Phalippou and Gottschalg (2009), and Phalippou (2012).

[17] The use of IRR is necessary since deal-level cash flows sometimes include intermediate cash flows occurring because of interim recapitalizations or equity injections.

A number of papers use valuations in venture financing rounds to measure the risk-adjusted performance of venture funds. These estimates are also gross of fees, so estimates of both risk and return will tend to be somewhat inflated relative to that received by investors. Since portfolio firms that receive more rounds of financing tend to be the better performing investments, these papers have to adjust for the sample selection implicit in their reliance of financing rounds. Papers that estimate the parameters of sample selection models find CAPM alphas for venture firms in the range of 32% to 38% with betas of 1.9 to 2.7 (see Cochrane (2005) and Korteweg and Sorensen (2010)). Other papers use venture financing events to create hedonic and repeated sales indices that account for sample selection and find alphas in the range of 4% and betas in the range of 0.6 to 1.3 (see Peng (2001) and Hwang, Quigley, and Woodward (2005)). An important caveat to these papers is that in venture capital deals, securities issued in different rounds are not the same as one another. Since securities issued in later rounds tend to have more rights than those in earlier rounds, post-money valuations tend to overstate firms' actual valuations (see Gornall and Strebulaev (2018)).

Overall, evidence in the literature suggests that both buyout and venture funds tend to perform well as investments. The average PME for both types of funds is usually estimated to be greater than 1, which implies that these funds outperform public markets. This outperformance could reflect positive alpha, the greater risk of private equity funds relative to the market, or both. Estimates of fund betas, however, are somewhat mixed, with some studies finding betas in the range of 1.0 and other studies finding betas well above 2.0.

4.2. Advantages of Secondary Market Indices as Measures of Private Equity Performance

For most asset classes, investors rely on secondary-market transaction-based indices to measure the asset class's risk and return. Because such secondary markets did not exist in private equity, risk and return have been measured using a variety of alternative approaches. In this subsection, we discuss each of these approaches.

Two of the most common methods to measure private equity performance are a fund's *PME*, and its *IRR*. While PME does help us understand the performance of private equity relative to a given

benchmark portfolio, its ability to shed light on risk-adjusted performance is limited.[18] For example, PME does not guide the researcher in choosing the correct benchmark portfolio, cannot account for multiple factor exposures, and cannot be manipulated to estimate the alphas and betas of factor models.[19] Moreover, standard asset pricing theory is built on the concept of returns. The issues with *IRR* are a staple of most introductory finance classes and are discussed in most textbooks (see for example, Brealey, Myers and Allen (2016)). Most importantly, however, for the purposes of measuring private equity performance, both *PME* and *IRR* can only be measured after a fund has completed all of its distributions, so can only be used to measure performance over the fund's 10-15 year life.

Using the cross section of returns to estimate standard performance parameters for private equity can also be problematic for two reasons. First, it is impossible to estimate parameters such as the volatility of an entire asset class using cross sectional data. Studies that estimate volatility using the cross-sectional dispersion of returns usually estimate the expected volatility of a single fund or deal. For example, Cochrane (2005) uses cross-sectional variation in returns from financing events to estimate the average annualized volatility of the return from investing in a venture startup to be 107%. An investor with a broad portfolio, however, would be affected by the volatility of a portfolio of startup firms (through their impact on the funds' returns) rather than the expected volatility of a single position. Portfolio volatility primarily depends on the covariance structure across positions in addition the volatility of individual investments. Consequently, the volatility of a venture index is likely to be a better representation of the risk exposure faced by investors than the volatility of individual portfolio firms. In addition, the correlation of a portfolio's return with that of other asset classes cannot be estimated using cross-sectional data.

Second, as noted by Axelson, Sorensen, and Strömberg (2014), the irregular intervals over which returns or $IRRs$ are measured can affect the inferences about the private equity return distribution one can

[18] Korteweg and Nagel (2016) provide an extension of the PME that does allow for risk adjustments.
[19] Kaplan and Schoar (2005) estimate alpha as $PME - 1$. Phalippou and Gottschalg (2009) estimate alpha as the constant that would need to be added to the chosen benchmark discount rate to drive the PME to 1.0. These are appropriate methods only if beta relative to the chosen benchmark portfolio is 1.0.

draw from the cross-section of returns. The IRR of a fund is only observable when the fund is complete, and hence, the IRR interval will vary widely across funds, and to the extent that some funds return capital at different times than other funds, across funds as well. The irregular intervals at which venture financing events occur will also cause variation in measured return horizons. Using $IRRs$ or returns compounded over irregular intervals can result in surprisingly large biases when estimating CAPM parameters. Axelson, Sorensen, and Strömberg (2014) simulate deal-level cash flows and estimate the CAPM using cross-sectional variation in $IRRs$. In some reasonable specifications, the beta is underestimated relative to the true beta, on average, by 116%. In other specifications, beta is overestimated by 123%.

Given the difficulties of measuring performance using the cross section of $IRRs$ or returns, other authors create indices of venture capital returns using post-money valuations from each financing round as estimates of market value. Peng (2001) and Hwang, Quigley, and Woodward (2005) develop hedonic and repeat sales indices using venture financing events and methods that are similar to those we use to create our indices. Such indices can also be problematic, however, for three reasons. First, financing events represent prices at which some investors acquire shares in startups, but not all investors can purchase at that price and generally investors are not allowed to exit at that time. Second, these financing rounds occur non-randomly in time, and are more likely to occur in bull markets than in bear markets. Third, in venture-backed firms, shares from different financing rounds have different rights and therefore different values. Newly created shares in financing events often have rights and therefore are worth more than old shares. Consequently, implied valuations based on post-money valuations are likely to be upward biased (see Gornall and Strebulaev 2018). Finally, returns from venture financing events are gross of fees, making it difficult to understand the return earned by limited partners that invest directly in private equity funds.

Jegadeesh, Kraussl, and Pollet (2015) take another approach to estimating private equity performance based on the returns to publicly traded private equity securities. In general, these securities are publicly traded funds of funds and private equity firms. These returns are likely to be correlated with those received by limited partners in private equity funds but are different for several reasons. Most importantly, the publicly traded private equity firms tend to be the general partners, whose claim is the

present value of future fees and carried interest earned by the fund, which are not equal to the returns earned by the limited partners who invest in the funds. In addition, large buyout firms such as Blackrock and KKR hold a variety of investments other than private equity, including hedge funds, real estate, advisory services, etc. Finally, some of the publicly traded private equity firms are funds of funds, which are a function of the claims of limited partners. However, these funds of funds charge an extra layer of fees that sometimes vary with fund performance, making it difficult to infer the returns earned by limited partners in the underlying funds held by the publicly traded funds of funds.

The indices we develop based on secondary market transactions enable us to evaluate the risk-adjusted performance of broad private equity portfolios using standard empirical methods that avoid some of the pitfalls of these alternative methods that have been used in prior studies. We use our indices to create time-series of arithmetic returns quoted at regular intervals. In contrast to a single fund or deal, the index returns have relatively little idiosyncratic risk implying that parameter estimates should be less sensitive to using either log or arithmetic returns.[20] The indices we create include bankrupt funds. Moreover, we build the indices from data on the actual secondary market transactions, so they represent the net of fee returns investors actually receive.

5. Estimates of Secondary Market Based Private Equity Indices

5.1 Pricing Parameters

Table 3 presents our estimates of the parameters for the sample selection model for both buyout and venture funds, using the merged sample described in Section 2. The selection model involves two stages, a selection equation and a pricing equation. Panel A reports estimates of the selection equation, while Panel B reports estimates of the pricing equation. Panel B contains the parameters we use to infer the prices of funds that don't transact, using both OLS and the Heckman sample selection model.

[20] Our estimates of alpha are nearly identical whether we use simple returns, log returns, or the continuous time adjustment discussed above.

The estimates presented in Panel A suggest that a number of variables are associated with fund selection. Among the economy-level variables, the estimates indicate that funds are more likely to transact when public equity markets are valued highly (the aggregate market-to-book ratio for equities (MTB_t)), and when private equity has been performing well, (the average fund's PME). The coefficient on MTB_t for buyout funds is 0.20 (t-statistic = 1.7) and the coefficient on the average PME_t is 1.05 (t-statistic = 1.8). Both coefficients are higher and also statistically significant for venture funds. Both fund types are also more likely to transact when the valuation confidence index is high, and when the crash confidence index is low. These represent times when institutional investors believe the public market is overvalued, but the likelihood of a crash is low.

Among the fund-specific variables, fund size strongly affects the likelihood that a stake in the fund is traded on the secondary market for buyouts and venture. Not surprisingly, larger funds are much more likely to transact than smaller funds. In addition, fund age also affects the likelihood of a transaction. The coefficients on both $AGE < 4$ (dummy equal to one if fund is less than 4 years old) and $4 \leq AGE \leq 9$ (dummy equal to one if fund is 4 to 9 years old) are negative and statistically significant. Funds older than 9 years (the group excluded in the age-dummy-variable classification) are the most likely type of fund to be traded on the secondary market (although there are fewer such funds, so the most common transaction is for funds between 4 and 9 years old, which we refer to as "baseline transactions").

Finally, and importantly for our econometric design, stakes in private equity funds held by pension funds are less likely to transact than stakes held by other types of investors for both buyout and venture. The coefficient on $Frac.Pension_i$ (fraction of LPs that are pension funds) is -0.44 for buyout and -0.23 for venture, both of which are significantly different from zero. Consistent with the views of practitioners communicated to us through private conversations, pension funds organizationally appear to have less pressure to change portfolio strategies than other investors, so are less likely to sell stakes in the secondary market. This negative significant relationship between $Frac.Pension_i$ and the likelihood of transacting suggests that this variable satisfies the exclusion restriction, since the identity of the investors affects the

likelihood of a trade but there is no reason why it should affect the pricing of deals. This argument is bolstered by the fact that the identity of sellers is withheld from buyers in the bidding process.

The bottom rows in Panel A test the null hypothesis that there are omitted variables correlated with both fund selection and fund premiums, $H_0: \rho = 0$. The Heckman (1979) sample selection model helps ensure that our parameter estimates are unbiased even if $\rho \neq 0$, while if $\rho = 0$ then OLS estimates of the pricing equation are also unbiased. The low t-statistic on ρ for buyout funds (-0.3) as well as the Wald, likelihood ratio, and Lagrange multiplier test fail to reject the hypothesis that ρ equals zero. In contrast, the high t-statistic on ρ for venture funds (-7.0) and the low p-values on the Wald, and likelihood ratio test statistics indicate that we can reject the null hypothesis that $\rho = 0$ for venture funds. These findings are consistent with the OLS estimates of the pricing equation in Panel B. For buyout funds, the Heckman and OLS test statistics are virtually identical, suggesting that both approaches provide consistent estimates. In contrast, the estimated parameters are somewhat different for the Heckman and OLS models using venture funds, which, combined with the nonzero estimated ρ, suggests that the OLS estimates for venture funds potentially suffer from selection bias.

The results in Table 3 Panel B for the pricing equation indicate that the scaled market price ($\pi = Price/NAV$), the dependent variable in our pricing model, tends to be higher for both buyout and venture funds when the crash confidence index is high. These are times when a crash in the public market appears likely from the view of institutional investors. The other estimated parameters on state variables differ across buyout and venture funds.

Of the fund-specific variables, π tends to be higher for funds with higher valuations as measured by $NAV_{i,t}$, and for transactions aged between 4 and 9 years (the omitted category from our age dummy variables). We observe higher valuations for funds that are 4 to 9 years old, in part, because such funds are likely to have more capital invested than younger or older funds. Since, on average, portfolio firms increase in value and NAVs reflect historical cost, NAVs of funds with more invested capital tend to have relatively high deviations from fundamentals.

The bottom row of Panel B reports the R-square of the pricing regression. Our hedonic pricing model is able to explain about 34% of the variation in scaled prices for buyout funds, and about 30% of the variation in scaled prices for venture funds.

5.2. Private Equity Indices over Time

Figure 3 graphs the size weighted hedonic indices for both buyout and venture over the 2006-2017 sample period. For comparison we also present the performance of the public equity market, as reported on Ken French's website, the index based on *Preqin* reported NAVs we describe above in equation (26), and the *Burgiss* Index that is used by practitioners.

Figure 3 illustrates that the transactions-based indices are much more volatile than the two NAV based indices (*Preqin* and *Burgiss*). NAVs adjust slowly to changes in the value of the private equity funds' assets. They do not immediately adjust to information likely to affect future cash flows in the manner of public equity markets. Consequently, the indices based on NAVs tend to be "smoother" than the transactions based private equity indices or public equity markets.

For this reason, NAV based indices do not reflect changes in information to the degree that our transactions-based index does. For example, the only sharp decline in equity markets during our sample period occurred during the 2008 Financial Crisis. During this period, the NAV indices declined somewhat because assets were written down, but not as much as overall public equity indices. In contrast, the transactions-based indices decline at least as much as public equity markets; during 2008 the public equity market index declined 37% while the transactions-based buyout index declined 51% and the transactions-based venture index declined 33%. Even though private equity positions were not written down completely during the 2008 crisis, their value appears to have declined by as much or more than the public equity markets at that time.

5.3. Risk and Return of Private Equity Indices

5.3.1. Buyout Indices

Table 4 reports estimates of risk and return for the buyout transaction-based index we create as well as for alternative buyout indices.[21] Panel A reports estimates for hedonic indices that are size-weighted and price-weighted, as well as the naïve index that is price-weighted. Panel B reports moment estimates for size-weighted *Preqin* NAV-based indices as well as differences relative to the Hedonic indices.

Consistent with Figure 3, the estimates in Panel A of Table 4 indicate that buyout funds have performed very well over our sample period (2006 to 2017). The two hedonic indices each have an expected annual return for a baseline fund between 18-20% and the naïve index has an expected return of 29%. These values are relatively high because our sample period was a good one for the buyout industry.

In addition to the expected returns, the estimated betas of the transactions-based indices are also large. The estimated betas of the two hedonic buyout indices are over 2 (2.43 and 2.25) and for the naïve buyout index is 1.34. These estimates are substantially larger than those reported in the prior literature (an exception is Axelson, Sorensen, and Strömberg (2014)). However, betas of this magnitude *are* consistent with theory. Private equity funds are portfolios of equity positions in leveraged buyouts. Since buyouts tend to be much more highly levered than public firms, Modigliani-Miller Proposition 2 implies that buyouts should have substantially higher betas than public firms. For example, Axelson, Jenkinson, Strömberg, and Weisbach (2013) report a mean debt to total capital ratio of 70% in their sample of 1,157 LBOs. In contrast, typical large publicly-traded firms have approximately a 20-25% debt to total capital ratio. Even if the firms experiencing buyouts are less risky than average so have asset betas somewhat smaller than 1, the equity portion of the LBO should nonetheless have a beta greater than 2 given these leverage ratios (see Axelson, Sorensen, and Strömberg (2014) for more discussion).

A consequence of these relatively high estimates of beta is that even though the expected return from the transactions-based buyout indices are high (at least 20%), the estimated alphas are not statistically

[21] In this version of the paper, we ignore measurement error in index values when estimating standard errors for index moments, and compute standard errors by GMM and the delta method using the approach of Newey and West (1987) to account for any time series dependence. Future versions of the paper will develop inference following the subsampling methodology of Politis and Romano (1992, 1994) which will enable us to account for measurement error in index values.

significantly different from zero, equal to exactly zero for the hedonic indices. The high returns earned by buyout funds appear to reflect the "beta risk" coming from the leverage in the buyouts.

Panel B of Table 4 compares the estimates from the size-weighted hedonic transactions-based index to the more commonly used NAV based indices, both the one we constructed from the *Preqin* database and the one distributed by *Burgiss*. Both these indices have lower expected returns than the transactions-based indices, with expected returns ranging from 10 to 13 percent. In addition, there is a large difference in volatility between the transactions-based indices and the NAV-based ones, with the standard deviation of the NAV-based indices less than that of the public equity market (9% for both *Preqin* and *Burgiss* compared to 16% for the public equity market), and the standard deviation for the transactions-based index equal to 91%. Some volatility of the transactions-based index may be the result of noise in the transactions prices, but the low volatility of the NAV-based indices undoubtedly comes from the way NAVs are computed, where firms only update NAV occasionally if there is not a new round of financing.

Probably because of this updating process, the estimated betas for these indices are less than .5, equal to .45 for both *Preqin* and *Burgiss.* It is hard to reconcile betas this low for buyout funds with theory; buyout funds take companies whose assets presumably have betas of close to 1, and add substantial leverage to them. The most likely explanation for low NAV-based indices is that NAVs do not tend to adjust at the time of market movements because the information in these movements is not incorporated into the NAVs at the time it becomes public.

Regardless of the reason for the low estimated beta for the NAV indices, they have an important implication. Given these betas, the estimates of alpha become positive and statistically significant (5% for *Burgiss* and 8% for *Preqin*). These positive alphas differ from the zero alphas estimated for the transactions-based indices. It is interesting that even though the transactions-based indices have higher average returns than the NAV based ones, because the estimated betas are so much higher, the transactions-based buyout indices are associated with an insignificant positive alpha while the NAV-based buyout indices are associated with a positive one.

5.3.2. Venture Capital Indices

Table 5 reports estimates of risk and return for venture funds in the same format as Table 4. As has been documented elsewhere and is evident from Figure 1, venture did not perform well as an asset class during our sample period, 2006-2017. The expected return on the size-weighted hedonic venture index is 4% and is 6% on the naïve index, both of which trail the 9.43% average return on the public equity market. The estimated beta is around 1 for the hedonic indices (1.04 for the size weighted and 1.99 for the price weighted), and is 1.50 for the naïve index. Given the low expected return and these betas, the estimated alphas are negative and marginally significant for the two hedonic indices.

These estimates markedly contrast with estimates using the more traditional NAV-based indices. For both the index we constructed from *Preqin* NAVs and the one created by *Burgiss*, the expected returns are higher (8% and 11%), the estimated betas are lower (0.31), and the estimated alphas are positive (5% and 7%) and statistically significantly different from zero. The smoothing coming from the way that NAVs are constructed tends to lower the estimated beta because the effect of market-wide factors are incorporated with only a long lag. Similar to buyout funds, the standard approach to measuring private equity performance tends to understate the systematic risk and therefore to overstate the alpha of venture funds.

5.4. The Impact of the 2008 Financial Crisis

An important issue in interpreting the results is the role of the 2008 Financial Crisis. It is often thought (hoped) that the Financial Crisis was a one-time event that will not be repeated, so many studies exclude this period in the hope of having a sample that will be more representative of the future. On the other hand, a fund's beta is relevant to an investor because it measures the sensitivity of fund values to market declines, and the Financial Crisis was the only major market-wide decline in equity markets during our sample period. In addition, because the cash flows produced by private equity funds for investors come through the highly pro-cyclical exits of portfolio companies, investors in private equity funds such as endowments who rely on cash flows to fund operations should be particularly concerned about the way their value changes during market downturns.

For these reasons, we focus on the estimates from Tables 4 and 5 based on the entire sample, but for completeness, present the estimates of buyout risk and return without the Financial Crisis in Table 6. To calculate the hedonic indices without the Financial Crisis, we use the same parameter estimates for the pricing equation as in Tables 4 and 5, and then omit years 2008 and 2009 when estimating index returns.

The estimated betas are much lower when the crisis years are excluded, the betas for both buyout and venture indices are both less than 1.0. Excluding the financial crisis, the alphas for buyout are higher, and significantly different from zero. The differences in the transactions-based index when the Financial Crisis is excluded emphasizes the fact that private equity returns during this period were poor, and by excluding them, we potentially overstate the performance of private equity funds.

6. Implications and Applications

6.1. "Matrix Pricing" of Private Equity Funds.

Unlike some securities, private equity funds rarely trade, so for most funds, there is not a recent price from which one can value the fund. For this reason, most funds are valued by limited partners at the NAV, which can deviate substantially from the best available estimate of the fund's fundamental value. These valuations are used for a number of purposes by investors in funds, including portfolio allocation decisions across asset classes, and spending decisions, which are often made by universities and foundations and are based on a fixed percentage of a portfolio's assessed value. More accurate pricing of limited partner stakes in private equity funds could have substantial consequences for investors since it could lead to the portfolio allocation decisions and spending rules corresponding to better estimates of the underlying values of an institution's private equity investments.

One possible approach to valuing stakes in private equity funds more accurately would be to follow the practice of "matrix pricing" commonly used to price bonds. This practice consists of pricing bonds that do not trade based on the prices of bonds that do trade. The idea is that the same fundamentals affect similar bonds in the same manner so prices of bonds that do not trade likely move approximately the same amount as prices of bonds that do trade. Since private equity funds that invest in one type of asset are likely affected

by a number of the same shocks to their fundamentals, they can be priced by a comparable approach using the transactions-based index we construct.

For any fund, the fund's history of quarterly cash inflows and outflows could be combined with the quarterly returns of the hedonic indexes to calculate market values in the following way. Beginning with the first contribution in the amount *C* which occurs in quarter *q0,* represented as *C(q0), s*ubtract any distributions *D* that occur in quarter *q0,* represented as *D(q0).* At the end of the initial quarter *q0,* the book value, and absent extreme circumstances, the market value of the investment in the fund, *V,* should be *V(q0) = C(q0) - D(q0).* Incorporating subsequent changes in market values via the index, the end of the next quarter market value could be calculated as $V1 = V0(1+r) + C(q1) - D(q1)$, where *r* is the hedonic-predicted return for a given fund type (either buyout or venture). At the end of quarter 2, the market value could be calculated as $V2 = V1(1+r) + C(q2) - D(q2)$. This process could be repeated for each quarter to obtain a market value for any quarter between the origination date and the fund's liquidation date. In circumstances where the full history of calls and distributions is not available, the beginning value could be the first available NAV at time *t*. In this circumstance, value at time 1 could be calculated as $V1 = NAV(q0)(1+r) + C(q1) - D(q1)$. The resulting market values at time *t,* represented as *V(T),* can be compared to NAVs at time t, denoted *NAV(T),* by taking the ratio of the two, *V(T)/NAV(T).* The resultant market-to-book ratio measures the differences between market and book values in each quarter.

We perform this calculation for each fund in our sample for every quarter in our sample period (2006 to 2017), and report the estimated market to book ratios for the 4th quarter of each year in Table 7. Because our hedonic indexes are estimated with transactions from funds that are between 4 and 9 years old, we only calculate market values for 4-9 year-old funds. Following the procedure described above, beginning in Q1 of the fifth year for each fund, we use the fund-specific predicted quarterly return and fund-specific calls and distributions to calculate a market value for each fund. For the purposes of reporting quarterly average market-to-book ratios, we sum market values for each fund in a quarter and divide by the sum of fund NAVs in the quarter. Because each fund in our sample is a different age in calendar time, we report results for vintage year between 2006 and 2017.

Panel A of Table 7 reports the Q4 average market-to-book ratio for buyout funds. Averages for each of the vintages are reported across the bottom of the table. Average market-to-book ratios range from a low of 0.91 for the 2003 vintage of funds to high of 1.39 for 2005 vintage buyout funds. Market-to-book ratios are considerably lower during the crisis, ranging between 0.65-0.77 in 2008, for the 3 vintages that were old enough for our hedonic estimation. Funds that invested during the Financial Crisis, 2007 and 2008 vintages, presumably did so at lower valuations, and report high average market-to-book ratios in subsequent years. For example, these estimates indicate that by the end of 2017, a 2008 buyout fund has a NAV that is understated by 44% relative to its market value.

Panel B reports market-to-book ratios for venture funds. These ratios are substantially lower than the market-to-book ratios reported for buyout funds. This difference reflects the fact that returns were lower for venture than buyout during our sample period (see Figure 1 and Tables 4 and 5). The market-to-book ratios for venture funds were less impacted by the Financial Crisis; ratios reach 0.8, 0.8, and 0.92 for the three age-eligible vintages during 2008. The 2004 vintage reports the highest average market-to-book ratios, with 2009 vintage funds reporting the lowest.

Individual funds could mark their values to market using the hedonic approach in one of two ways. The most accurate approach would be to generate fund-specific market values using the hedonic-estimated coefficients applied to the fund's attributes. A simpler approach that would nonetheless be a substantial improvement over using NAVs would involve multiplying a fund's NAV by the average market-to-book ratio of the industry. For example, an LP would multiply the NAV of each 4-9 year-old fund in his or her portfolio by the appropriate ratio from Table 8. For younger funds, the deviation between NAVs and market value is likely to be smaller but could be estimated using the coefficients from the hedonic regressions. Tail-end funds will have only a few portfolio companies left and their values will vary depending on the fortunes of these particular investments, so matrix pricing is likely to be less useful for valuing these funds.

7. Conclusion

Measuring the performance of private equity investments (buyout and venture) has historically only been possible over long horizons because a fund's return is only observable following the fund's final distribution. However, in recent years, a secondary market has developed in which investors in private equity funds can trade their stakes. We construct indices of this market using a proprietary database of secondary market prices of private equity stakes between 2006 and 2017. This index provides a new way to measure the risk and return of private equity. In addition, it can be used by investors for a number of purposes, including using the estimated risk and return to make better portfolio decisions, and valuing an LP's private equity investments by a method similar to the "matrix pricing" approach commonly applied to thinly traded bonds.

Our estimates indicate that the transactions-based indices of buyout and venture capital track the public equity market much more closely than the NAV-based indices we consider, one of which we construct from *Preqin* data and one of which is the commonly used one produced by *Burgiss*. The hedonic buyout indices have betas greater than 2, consistent with the notion that buyouts that increase leverage also increase beta. The hedonic venture indices have betas slightly higher than 1. Both of these findings are in marked contrast to the NAV-based indices, whose betas are estimated to be less than .5. Since NAVs adjust with a long lag, estimated betas from NAV-based indices likely understate the extent to which private equity returns depend on market-wide factors.

The high beta on the transactions-based indices has important implications about the inferences we make about private equity performance. Despite having an average return of over 20% during our sample period, the buyout indices have an estimated alpha of close to zero. The high beta estimates raise the required return to a level where even this high return level is not abnormally large. This finding is counter to our estimates of alpha for NAV-based indices as well as estimates of alpha in the literature, which are consistently positive and statistically significantly greater than zero.

For the transactions-based venture capital indices, we estimate a *negative* alpha, again in contrast to the positive estimated alpha for the NAV-based indices. This negative alpha is likely reflective of our

sample period being a poor one for the venture capital industry rather than a statement about the underlying economics of venture capital; undoubtedly if there had been a secondary market during the booming 1990s and one had constructed a transactions-based index, it would have had a positive alpha. The point, however, is that when one uses an index for which prices adjust to market-wide information in real time, one estimates much higher betas and consequently lower alphas.

These indices also have a number of potential uses for investors. Better estimates of private equity risk and return should affect the optimal portfolio decisions of investors when deciding on the allocation to private equity in their portfolios. In addition, the index can be used to provide more accurate values for stakes in private equity funds that investors hold in a manner similar to the "matrix pricing" approach commonly used to price illiquid bonds. Our estimates suggest that the use of NAV for valuation as is done by most LPs can be misleading, and NAVs often substantially misstate the value of an investor's private equity holdings.

Undoubtedly, there are uses for the index we have not discussed in this paper. For example, one could design derivative contracts based on an index of private equity returns. These derivatives could potentially be useful to investors and General Partners for hedging. Better indices of private equity performance such as the ones presented here clearly have much to offer the private equity community.

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Appendix A: Data Details

We first classify funds in the transactions data as either buyout or venture funds based on the specified fund type. Specifically, we classify funds as buyout funds that are labeled as "Buyout," "Small Buyout," "Mid-Cap Buyouts," "Mid-Cap Buyout," "Buyout/Growth," "Mega Buyouts," "Large-Cap Buyouts," and "All Private Equity". We classify funds as venture funds that are labeled "Venture," "Venture (General)," "Early Stage," "Early Stage VC," "Early Stage: Seed," "Early Stage: Start-up," "Expansion/Late Stage," and "Growth, and Mezzanine". After classifying funds as either buyout or venture and eliminating all other transactions we have 5214 fund transactions from 2006 through 2017 of which 3277 are for buyout funds, and 1937 are for venture funds.

We hand check similar fund names to develop a list of unique fund names. We then clean the transactions data as follows:

1) Eliminate transactions with a price less than zero.
2) Eliminate transactions with a NAV less than zero.
3) Eliminate transactions that have the same price for every fund in the portfolio transaction.
4) Eliminate two transactions that appear to be obvious data errors.
5) Eliminate transactions for which the total amount committed by the seller minus the unfunded commitment is less than zero.
6) Eliminate transactions for which the total capital committed is less than or equal to zero.
7) Eliminate transactions for which the fund name is missing.

9) If multiple transactions occur on the most recent transaction date for a given fund/quarter, use only the transaction based on the highest total commitment.
10) If multiple transaction records exist with the same fund name, vintage, and commitment on the most recent transaction date for a given fund/quarter, take the mean transaction price (as a percent of NAV) as the price for this fund/quarter.

12) Of the remaining transactions eliminate any for which the price, as a percent of NAV, is greater than 3 standard deviations away from the mean price across funds for a given quarter.

After cleaning the data as described above, we are left with 3,404 fund transactions of which 2066 are for buyout funds and 1,338 are for venture funds. These transactions are for 2,424 unique funds, 1,427 of which are buyout and 997 of which are venture. These 3,404 transactions are the "Transactions Sample" highlighted in the first column of Table 1.

We also classify funds in the Preqin data as buyout or venture based on the fund type as specified by Preqin . Specifically, we classify funds as buyout funds that are labeled "Buyout." We classify funds as venture funds that are labeled "Venture (General)," "Venture Debt," "Growth," "Early Stage," "Early Stage: Seed," "Early Stage: Start-up," and "Expansion / Late Stage."

Using the Preqin data, we compute the realized NAV-based return for each fund, i, in every quarter, t, as $(NAV_{i,t} - C_{i,t} + D_{i,t})/NAV_{i,t-1} - 1$, where $C_{i,t}$, $D_{i,t}$, denote the calls and distributions made by fund i during quarter t, and $NAV_{i,t}$ is the end-of quarter NAV. For each fund-quarter we also compute the internal rate of return, $IRR_{i,t}$ from fund initiation through the end of the quarter, treating $NAV_{i,t}$ as a terminal cash flow in the final quarter, t. We then clean the Preqin data as follows:

1) Eliminate any fund-quarters for which $NAV_{i,t-1} = 0$, or the NAV-based return is otherwise missing. Note that we retain records for which $NAV_{i,t} = 0$. Once NAV hits zero, however, we no longer include the fund in the sample.
2) Eliminate stale NAVs, those with a report date prior to the end of each quarter.
3) Identify fund-quarters for which the NAV-based return is in the 1% tails across all fund-quarters. For these observations, eliminate the fund-quarter record only if IRR_{t-1} or IRR_t is more than 5000 basis points away from the IRR self-reported by the fund after and nearest to t in calendar time. We obtain year-end self-reported IRRs from Preqin.

We then merge our Preqin data with the explanatory variables described in Table 3. We find that a few fund-quarters are missing data on either fund size or the fraction of LPs that are pension funds. After eliminating these observations, our filtered Preqin dataset represents a full panel that includes data on fund cash flows and explanatory variables. This panel contains 41,464 fund-quarter observations from 2006 through 2017. The observations are for 2,287 unique funds, 1,251 of which are buyout, and 1,036 of which are venture. These 41,464 observations are the Preqin Universe highlighted in the second and third columns of Table 1.

To merge the transaction and cash-flow data, we first identify funds with identical fund names in the two databases and designate these as a match. We then identify fund names in the transaction and Preqin data that are "similar" and that also have the same vintage. Fund names A and B are considered similar if fund name A contains the first 10 characters of fund name B anywhere in the fund name string. We then hand check this list to determine which funds match. After merging we have transactions and Preqin cash-flow data in the merged sample for 630 funds, 372 of which are buyout and 258 of which are venture. These 630 funds account for 1,119 transactions, 716 of which are buyout, and 403 of which are venture. These 1,119 transactions are the merged sample highlighted in the second column of Table 1.

Appendix B. Portfolio Transactions in the Secondary Market

A unique feature of the PE secondary market is the fact that individual funds are frequently sold as part of a larger portfolio transaction. For example, a seller might offer to sell their ownership stake in five unique funds, hoping to sell a portfolio of holdings in one large transaction. In a portfolio transaction, the buyer submits an offer price for the entire portfolio of funds, and the buyer and seller enter into a contract to eventually transfer ownership based on the portfolio offer price. Given that the construction of our price index relies on the market prices paid for individual funds, rather than one price for a portfolio of funds, it is important to consider the economics that govern how prices get assigned to individual funds in a portfolio transaction to determine whether and how portfolio transactions could influence the index.

Once a buyer and seller are in contract to transfer ownership of the portfolio of funds, the process moves to a second phase. During the diligence process, buyers assign prices to each of the individual funds in the portfolio, subject to the constraint that the size-weighted average of the individual prices equals the winning offer price. The price allocation process can be nuanced because although the buyer bids on the full portfolio, they may in reality only have strong demand for certain funds in the offered portfolio. The buyer's assignment of prices to individual funds will reflect their demand for those funds. High prices are assigned to funds the buyer most demands and lower prices are assigned to funds they demand less, again, subject to the constraint that size weighted-average prices equal the full portfolio bid. Conversations with industry experts indicate that there are times when prices allocated to individual funds result in certain of the funds being excluded from the final transaction. Thus, the prices assigned to individual funds are a reflection of demand for the funds, albeit filtered through the portfolio purchasing process.

While the assignment of prices to individual funds is ultimately a reflection of demand, the concern is whether prices determined through a portfolio process are somehow systematically different from single-fund sale prices. Any bias in our index stemming from portfolio transactions would have to display time series properties given that our object of interest from the index is quarter-over-quarter returns. The fraction of portfolio transactions each quarter is volatile, but shows no consistent trend quarter-over-quarter.

Appendix C. The Econometrics of Heckman Selection Models

The OLS estimate of $\boldsymbol{\theta}$ is unbiased if $\epsilon_{i,t}$ is uncorrelated with $y^*_{i,t}$ as discussed in section 3.1.2. By including all variables of $\boldsymbol{z_{i,t}}$ associated with $\pi_{i,t-s(t)}$ into $\boldsymbol{x_{i,t}}$ we ensure that $\epsilon_{i,t}$ is uncorrelated with $\boldsymbol{z}'_{i,t}\boldsymbol{\gamma}$. If any residual dependence remains between $\epsilon_{i,t}$ and $v_{i,t}$, then the OLS estimate of $\boldsymbol{\theta}$ is biased.

To account for any residual dependence between $\epsilon_{i,t}$ and $v_{i,t}$ we simultaneously estimate all parameters of (26) by MLE.[22] We assume $\epsilon_{i,t}$ and $v_{i,t}$ are jointly distributed normal,

$$\begin{pmatrix}\epsilon_{i,t}\\ v_{i,t}\end{pmatrix} \sim N\left[\begin{pmatrix}0\\0\end{pmatrix}, \begin{matrix}\sigma_\epsilon & \rho\sigma_\epsilon\\ \rho\sigma_\epsilon & 1\end{matrix}\right]. \tag{27}$$

Under this assumption it is well known that the log-likelihood function of the model given in (26) is:

$$\mathcal{L}(\theta,\gamma,\rho,\sigma_\epsilon;\boldsymbol{x},\boldsymbol{z},\boldsymbol{\pi}) = \sum_{N_0}\log\left[1-\Phi\left(\boldsymbol{z}'_{i,t}\boldsymbol{\gamma}\right)\right] + \tag{28}$$

$$\sum_{N_1}\left[-\log\sigma_\epsilon + \log\phi\left(\frac{\pi_{i,t}-\boldsymbol{x}'_{i,t}\boldsymbol{\theta}}{\sigma_\epsilon}\right) + \log\Phi\left(z'_{i,t}\gamma + \frac{\frac{\rho}{\sigma_\epsilon}\left(\pi_{i,t}-\boldsymbol{x}'_{i,t}\boldsymbol{\theta}\right)}{\sqrt{1-\rho^2}}\right)\right]$$

where N_0 represents the set of observations for which no transaction prices are observed, and N_1 is the set of observations for which we do observe transaction prices.[23] We use the full *Preqin* universe panel to maximize (**Error! Reference source not found.**). We estimate the standard errors of the parameters using the quasi-maximum-likelihood approach of White (1982), which accounts for heteroscedasticity and any cross-sectional or time-series dependence, and can be valid even if the true density of $\epsilon_{i,t}$ and $v_{i,t}$ is not normal.

[22] Monte Carlo experiments indicate that MLE is often more efficient than the two-step approach originally proposed by Heckman (1979) (see Puhani (2000)). In addition, MLE allows for straightforward computation of robust asymptotic standard errors, is convenient for conducting standard model diagnostics, and imposes the natural restriction that $|\rho| \leq 1$ where ρ represents the correlation between $\epsilon_{i,t}$ and $v_{i,t}$. Regardless, our results are virtually unchanged using either approach to estimate the parameters of the model given in (26).

[23] See, for example, Hall (2002).

Figure 1. Transactions per Period, Baseline Transactions Sample, Funds 4-9 Years Old

This figure illustrates the number of transactions we observe per period in the baseline transactions sample for funds that are four to nine years old, as documented in Table 1. This sample contains 900 buyout transactions and 461 venture transactions.

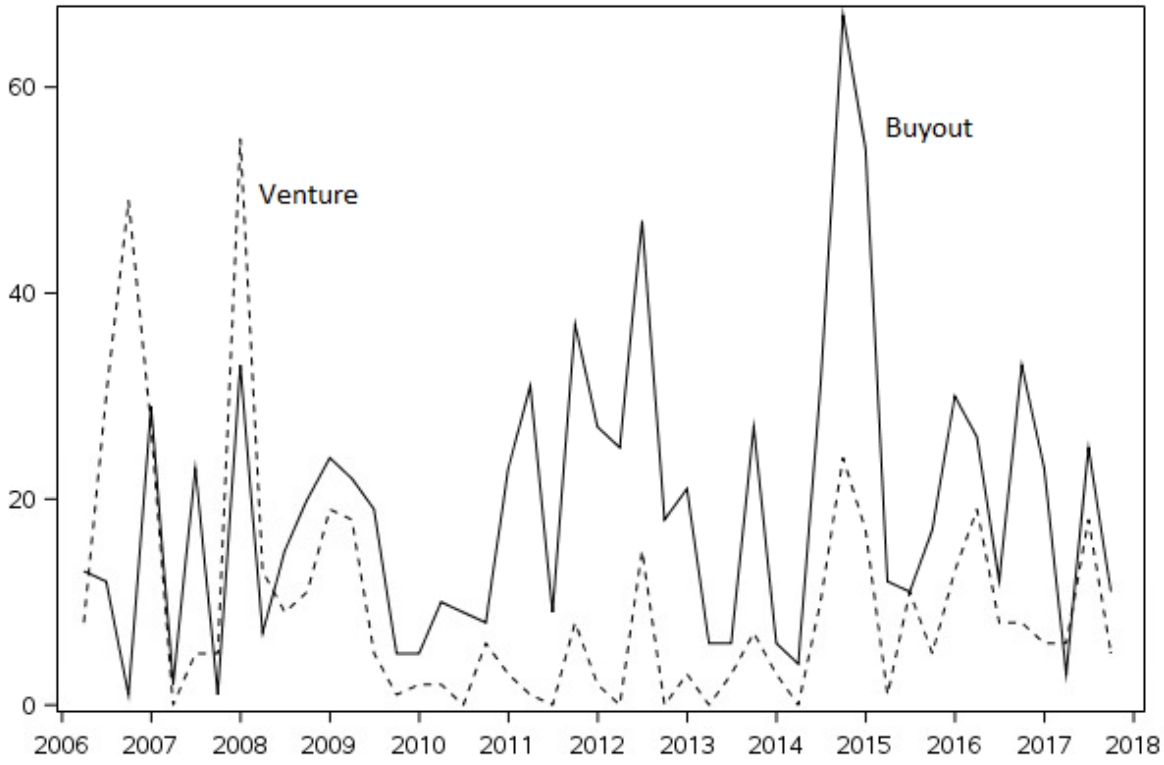
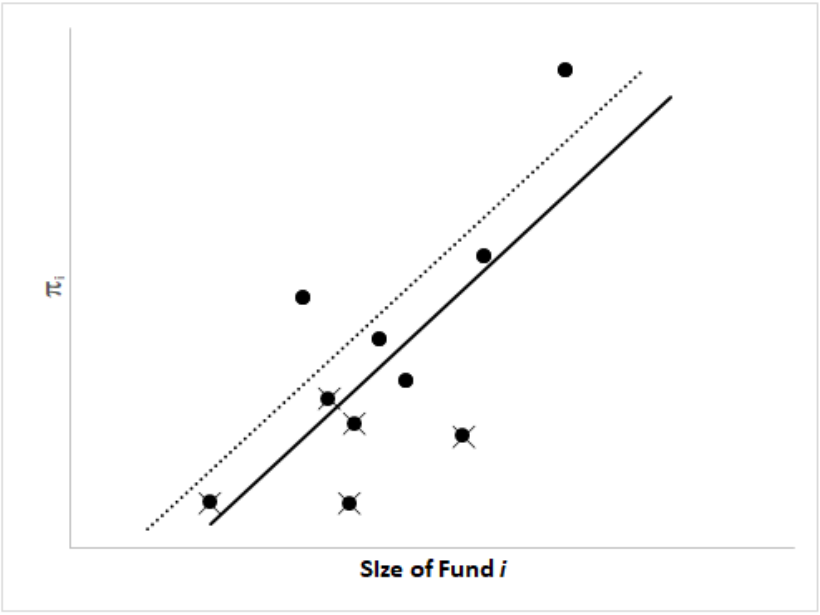


Figure 2. Sample Selection Examples

This figure illustrates examples of when the OLS estimator of $\boldsymbol{\theta_N}$ is biased, when $k<N$ observations are used. The OLS estimator is biased when the population error term of the pricing equation is not independent of selection, $y^*_{i,t}$, as illustrated by equation (17). In Panel A, the population residual is not mean zero, conditional on selection. In Panel B, the covariance between the population residual and size is not zero, conditional on selection.

Panel A.



Panel B.

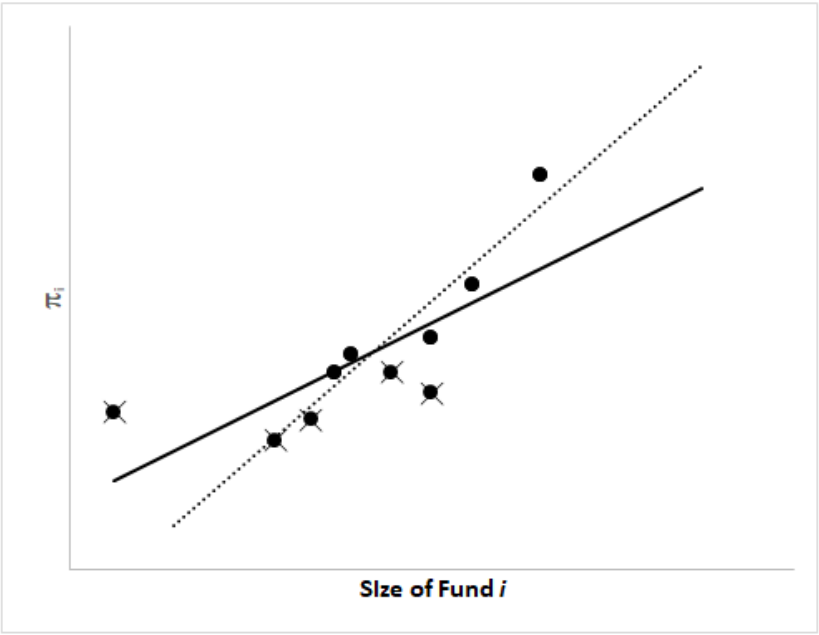


Figure 3. Buyout and Venture Indices Over Time

This figure illustrates the value of investing $1 in an index at the beginning of 2006 in each index as labeled. Panel A. is for buyout funds and Panel B. is for venture funds. The market index is based on the public market return as posted on Ken French's website. We build the Preqin indices based on NAVs reported in Preqin using funds in the Preqin Universe that are 4-9 years old. The transactions indices are the size-weighted hedonic indices.

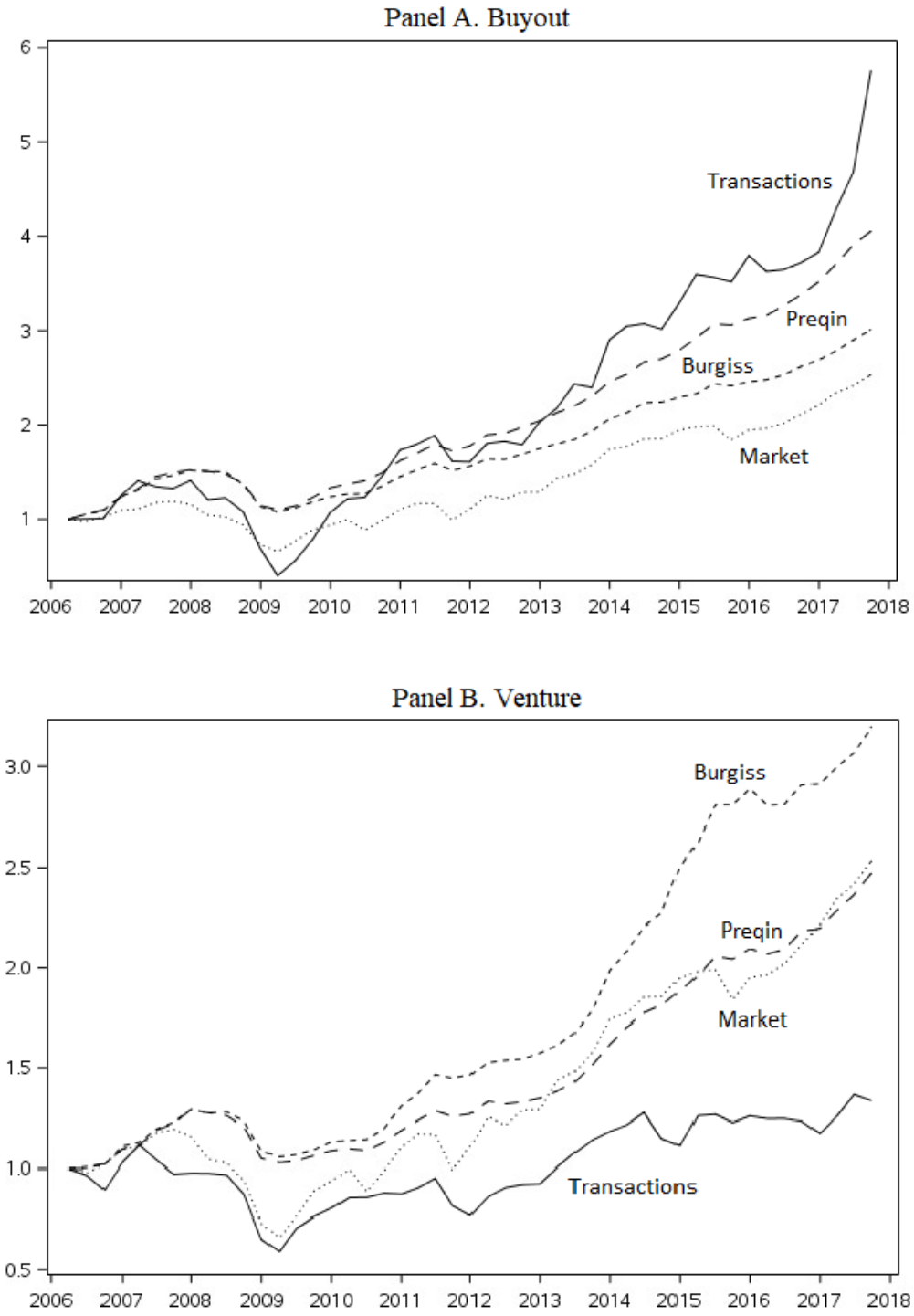


Table 1. Summary Statistics

This table reports summary statistics for the data samples we use. Panel A is for buyout funds while Panel B is for Venture funds. The "transactions full sample" is the cleaned sample of all transactions as described in the text. The "transactions baseline sample" is the subset of transactions in the transactions sample that is for funds 4-9 years old. The "*Preqin* Universe" is the cleaned sample of cash flow data and other characteristics from *Preqin* as described in the text. We create the "full merged sample" by merging the transactions full sample with the *Preqin* universe. The "compliment" sample in the *Preqin* universe represent those fund-quarter records in *Preqin* for funds that do not transact. We also report summary statistics for the *Preqin* universe for funds that are four to nine years old. We create the "baseline merged" sample by merging funds in the *Preqin* universe with the baseline sample. The "compliment" sample for the *Preqin* universe for funds that are four to nine years old represent the fund-quarter records in *Preqin* for funds that are four to nine years old that do not transact. $\pi_{i,t}$ is the fund price as a fraction of NAV. $Size(\$MM)$ represents total commitments in $US millions. Age is fund age in years. $Trans\ per\ Qtr$ is the number of transactions per quarter. PME is the Kaplan Schoar (2005) PME using NAV as terminal value for funds that have not yet liquidated. N is the total number of observations in the sample. "Mean" is the average across funds and across time, "Q1" is the 25th percentile, and "Q3" is the 75th percentile.

Panel A. Buyout

		Transactions		*Preqin Universe*		*Preqin Universe*	
			4-9 Yrs Old	*Full Sample*		*4-9 Yrs Old*	
		Full Sample	*Baseline*	*Full Merged*	*Compliment*	*Baseline Merged*	*Compliment*
$\pi_{i,t}$	Mean	0.83	0.89	0.82		0.89	
	Q1	0.70	0.80	0.69		0.80	
	Q3	1.00	1.03	1.00		1.02	
Size ($MM)	Mean	4699.2	5403.4	4715.8	1844.5	5416.4	1921.4
	Q1	1396.0	1800.0	1398.0	400.0	1800.0	410.0
	Q3	6000.0	7626.8	6000.0	2100.0	7626.8	2175.5
Age (years)	Mean	9.4	6.8	8.0	6.4	6.50	6.37
	Q1	6.0	5.0	5.0	3.0	5.00	5.00
	Q3	12.0	8.0	11.0	9.0	8.00	8.00
Trans per Qtr	Mean	44.0	19.1	15.2		7.7	
	Q1	14.0	8.0	6.0		4.0	
	Q3	56.0	27.0	25.0		12.0	
PME	Mean	1.19	1.16	1.18	1.12	1.16	1.18
	Q1	0.94	0.96	0.93	0.89	0.94	0.94
	Q3	1.40	1.31	1.38	1.30	1.32	1.34
N		2,066	900	716	20,346	364	9,138

Table 1. *Continued*

Panel B. Venture							
		Transactions		*Preqin Universe*		*Preqin Universe*	
			4-9 Yrs Old	*Full Sample*		*4-9 Yrs Old*	
		Full Sample	*Baseline*	*Full Merged*	*Compliment*	*Baseline Merged*	*Compliment*
$\pi_{i,t}$	Mean	0.80	0.88	0.84		0.92	
	Q1	0.60	0.70	0.63		0.75	
	Q3	0.99	1.08	1.01		1.21	
Size ($MM)	Mean	650.1	711.8	645.7	415.3	711.8	407.4
	Q1	286.5	376.4	285.0	165.0	376.4	171.0
	Q3	830.0	865.4	830.0	500.0	865.4	500.0
Age (years)	Mean	10.8	7.0	10.0	7.3	6.70	6.54
	Q1	7.0	6.0	6.0	4.0	6.00	5.00
	Q3	14.0	8.0	14.0	11.0	8.00	8.00
Trans per Qtr	Mean	28.5	9.8	8.6		3.2	
	Q1	6.0	2.0	2.0		0.0	
	Q3	40.0	13.0	14.0		4.0	
PME	Mean	1.03	0.95	1.04	1.01	0.95	0.98
	Q1	0.56	0.60	0.64	0.64	0.67	0.61
	Q3	1.23	1.14	1.21	1.16	1.16	1.17
N		1,338	461	403	19,999	152	8,938

Table 2. Explanatory Variable Descriptions

This table describes the explanatory variables used in our hedonic models. The first six variables are "state variables" that are the same across all funds and vary only across time. The last six variables are "fund specific variables" that vary across funds and also (potentially) across time.

State Variables	MTB	The average end-of-month market-to-book ratio over the transaction quarter, calculated using all stocks with share code 10 or 11 in CRSP.
	Volatility	The annualized standard deviation of the value-weighted portfolio of all stocks in CRSP with share code 10 or 11 using daily data over the transaction quarter.
	Value Confidence Ind.	The average end-of-month value of the Valuation Confidence Index over the transaction quarter from the International Center for Finance at Yale. Institutional Investors are asked to report their assement of stock market value relative to fundamental value. The Valuation Confindence Index is one minus the percentage of respondants who think that the market is overvalued. We scale this variable by 100.
	Crash Confidence Ind.	The average end-of-month value of the Crash Confidence Index over the transaction quarter from the International Center for Finance at Yale. Institutional Investors are asked to report the probability of a catostrophic market crash in the next six months. The Crash Confidence Index is the percentage of respondants who think that the probability is less than 10%. We scale this variable by 100.
	Avg. Market NAV	The average log NAV reported at the end of the transaction quarter across funds in our Preqin Universe.
	Average Market PME	The average Kaplan Schoar (2005) PME at the end of the transaction quarter across funds.
Fund SpecificVriables	Log Size	The log size of the transacting fund (total commitments of all limited partners).
	NAV	The log NAV for the transacting fund reported as of the end of the transaction quarter.
	PME	The Kaplan Schoar (2005) PME of the transacting fund as of the end of the transaction quarter, calculated using data from Preqin.
	AGE< 4	Age dummy that equals 1 if the year of transaction quarter minus the vintage year for the transacting fund is less than 4.
	4<=AGE<=9	Age dummy that equals 1 if the year of the transaction quarter minus the vintage year for the transacting fund is greater than or equal to 4 and less than or equal to 9.
	Frac. Pension	The fraction of the transacting fund's limited partners that are pension funds.

Table 3. Sample Selection Model Parameters

This table reports the estimates of the Heckman (1979) sample selection model of equation (26). Panel A reports estimates of $\boldsymbol{\gamma}$ for the selection equation while Panel B reports estimates of $\boldsymbol{\theta}$ for the "pricing equation", while. "Heckman" refers to the sample selection model, while "OLS" refers to the model estimated by simple OLS with no selection equation. Variables are described in Table 2. Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Selection Equation					
		Buyout		Venture	
		estimate	(t-stat)	estimate	(t-stat)
	Intercept	-5.18	-(1.2)	-31.83	-(6.4)***
State Variables	MTB	0.20	(1.7)*	1.61	(9.5)***
	Volatility	0.24	(0.8)	-1.08	-(2.3)**
	Value Confidence Ind.	0.46	(1.8)*	2.89	(8.3)***
	Crash Confidence Ind.	-1.36	-(3.0)***	-3.85	-(6.4)***
	Avg. Market NAV	-0.07	-(0.3)	1.25	(4.3)***
	Average Market PME	1.05	(1.8)*	5.65	(7.0)***
Fund Specific	Log Size	0.42	(24.9)***	0.39	(15.2)***
	NAV	0.03	(1.9)*	-0.01	-(0.5)
	PME	-0.11	-(2.2)**	0.01	(0.5)
	AGE < 4	-0.86	-(14.0)***	-0.94	-(11.0)***
	4<=AGE<=9	-0.37	-(7.0)***	-0.50	-(8.7)***
	Frac. Pension	-0.44	-(5.1)***	-0.23	-(2.6)**
	ρ	-0.02	-0.15		
		$\chi^2(1)$	p-value	$\chi^2(1)$	p-value
	Wald	0.02	(0.88)	26.21	(0.00)
	Likelihood Ratio	0.01	(0.94)	4.17	(0.04)
	Lagrange Multiplier	0.00	(0.97)	0.15	(0.70)

Table 3. *Continued*

Panel B: Pricing Equation								
		Buyout				Venture			
		OLS		Heckman		OLS		Heckman	
		estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
	Intercept	-14.33	-(7.7)***	-14.29	-(7.6)***	-2.80	-(1.0)	7.36	(1.5)
State Variables	MTB	0.18	(2.3)**	0.18	(2.2)**	0.31	(3.1)***	-0.17	-(0.8)
	Volatility	0.41	(3.3)***	0.41	(3.2)***	-0.25	-(0.8)	0.17	(0.5)
	Value Confidence Ind.	-0.33	-(2.2)**	-0.33	-(2.1)**	0.21	(0.9)	-0.61	-(1.6)
	Crash Confidence Ind.	0.71	(3.0)***	0.72	(2.9)***	1.06	(2.5)**	2.24	(3.9)***
	Avg. Market NAV	0.97	(8.0)***	0.97	(8.0)***	0.09	(0.5)	-0.30	-(1.3)
	Average Market PME	-0.47	-(1.7)*	-0.47	-(1.7)*	0.75	(1.6)	-1.11	-(1.3)
Fund Specific	Log Size	0.01	(1.1)	0.01	(0.5)	0.00	(0.0)	-0.12	-(2.6)***
	NAV	0.03	(3.6)***	0.03	(3.5)***	0.03	(2.5)**	0.04	(2.4)**
	PME	0.10	(3.3)***	0.10	(3.3)***	-0.02	-(0.7)	-0.02	-(0.6)
	AGE< 4	0.04	(0.9)	0.04	(0.8)	0.00	(0.0)	0.29	(2.2)**
	4<= AGE<=9	0.10	(3.4)***	0.10	(3.2)***	0.02	(0.5)	0.17	(2.3)**
	R-square	34%		34%		22%		30%	

Table 4. Buyout Indices 2006-2017

This table reports moment estimates for buyout indices using data from 2006-2017. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices

	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.20	(1.9)*	0.18	(1.8)*	0.29	(2.7)***
β	2.43	(4.2)***	2.25	(4.0)***	1.34	(2.0)**
α	-0.02	-(0.3)	-0.02	-(0.3)	0.16	(1.6)
σ	0.91	(5.9)***	0.88	(5.7)***	0.40	(6.4)***
Sharpe	0.22	(1.8)*	0.20	(1.7)*	0.71	(2.5)**
Corr Mkt	0.42	(3.2)***	0.40	(3.1)***	0.54	(1.9)*
Autocorr	0.37	(2.7)***	0.35	(2.6)***	-0.13	-(0.7)

Panel B: Size-Weighted NAV Indices

	Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.13	(3.4)***	0.10	(2.8)***
β	0.45	(4.4)***	0.45	(4.3)***
α	0.08	(2.8)***	0.05	(1.8)*
σ	0.09	(4.0)***	0.09	(4.0)***
Sharpe	1.26	(1.9)*	0.97	(1.6)
Corr Mkt	0.78	(10.9)***	0.77	(10.4)***
Autocorr	0.55	(4.9)***	0.50	(3.7)***

Differences
Relative to Hedonic Size-Weighted Index

	Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.08	(0.9)	0.10	(1.2)
β	1.97	(3.8)***	1.98	(3.8)***
α	-0.10	-(1.4)	-0.07	-(1.0)
σ	0.82	(5.4)***	0.82	(5.4)***
Sharpe	-1.05	-(1.8)*	-0.75	-(1.5)
Corr Mkt	-0.36	-(6.9)***	-0.35	-(6.8)***
Autocorr	-0.18	-(1.1)	-0.14	-(0.7)

Table 5. Venture Indices 2006-2017

This table reports moment estimates for buyout indices using data from 2006-2017. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices

	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.04	(0.8)	0.03	(0.6)	0.06	(0.4)
β	1.04	(4.5)***	0.99	(4.6)***	1.50	(3.0)***
α	-0.06	-(1.7)*	-0.07	-(2.0)**	-0.02	-(0.1)
σ	0.69	(4.0)***	0.70	(4.2)***	0.68	(6.4)***
Sharpe	0.05	(0.6)	0.03	(0.5)	0.16	(0.7)
Corr Mkt	0.23	(3.8)***	0.22	(3.8)***	0.34	(2.2)**
Autocorr	0.23	(1.9)*	0.23	(2.0)**	-0.46	-(5.2)***

Panel B: Size-Weighted NAV Indices

	Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.08	(3.1)***	0.11	(3.4)***
β	0.31	(3.7)***	0.31	(3.5)***
α	0.05	(2.2)**	0.07	(2.8)***
σ	0.07	(4.5)***	0.08	(6.0)***
Sharpe	1.08	(1.8)*	1.24	(2.4)**
Corr Mkt	0.70	(6.7)***	0.61	(4.7)***
Autocorr	0.43	(3.5)***	0.40	(3.2)***

Differences
Relative to Hedonic Size-Weighted Index

	Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)
E[r]	-0.04	-(1.2)	-0.07	-(1.7)*
β	0.73	(4.2)***	0.73	(4.0)***
α	-0.11	-(3.1)***	-0.13	-(3.6)***
σ	0.63	(3.8)***	0.62	(3.7)***
Sharpe	-1.04	-(1.9)*	-1.19	-(2.5)**
Corr Mkt	-0.47	-(6.8)***	-0.38	-(4.7)***
Autocorr	-0.21	-(1.8)*	-0.17	-(1.6)

Table 6. Buyout Indices 2006-2017 Excluding the Financial Crisis

This table reports moment estimates for buyout indices using data from 2006-2017 excluding the years 2008 and 2009. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices, and Panel C is for the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. For the naïve index, β and α are also bias adjusted. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices

	Buyout				Venture			
	Hedonic Size Weighted		Naïve Price Weighted		Hedonic Size Weighted		Naïve Price Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.23	(4.3)***	0.29	(4.6)***	0.06	(1.7)*	0.09	(0.6)
β	0.92	(4.0)***	0.49	(1.0)	0.72	(2.9)***	0.72	(0.9)
α	0.11	(1.9)*	0.22	(2.2)**	-0.04	-(0.8)	0.06	(0.3)
σ	0.41	(1.7)*	0.30	(6.2)***	0.31	(0.9)	0.61	(5.9)***

Panel B: Size-Weighted NAV Based Indices

	Buyout				Venture			
	Prequin		Burgiss		Prequin		Burgiss	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.17	(8.1)***	0.14	(6.5)***	0.12	(6.9)***	0.14	(5.8)***
β	0.29	(6.8)***	0.30	(7.2)***	0.22	(6.3)***	0.23	(3.6)***
α	0.12	(7.9)***	0.09	(6.4)***	0.08	(6.0)***	0.11	(5.1)***
σ	0.06	(6.1)***	0.06	(5.6)***	0.05	(12.4)***	0.06	(11.3)***

Differences Relative to Hedonic Size-weighted Indices

	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.07	(1.5)	0.09	(2.0)**	-0.06	-(1.7)*	-0.08	-(2.2)**
β	0.64	(3.2)***	0.62	(3.0)***	0.50	(2.0)**	0.49	(1.8)*
α	-0.01	-(0.1)	0.02	(0.4)	-0.12	-(2.6)**	-0.14	-(2.9)***
σ	0.35	(1.5)	0.35	(1.5)	0.26	(0.8)	0.24	(0.7)

Table 7. Market-to-Book Ratios

This table reports year-end average market-to-book ratios. Market values for each fund are calculated using the following procedure. We begin by assuming that the market value of the fund is equal to NAV in years one through four of the fund's life. We then calculate the market value each quarter from years 5-9 for fund *i* using the following formula:

$$market\ value_{i,t} = market\ value_{i,t-1} * (1 + r_t) + Calls_t - Distributions_t\ .$$

For the first quarter in year five, we use NAV as the preceding quarter's market value. The aggregate market-to-book ratio reported in this table is calculated as the sum of the individual fund's market value within each quarter divided by the sum of the individual fund's NAV in each quarter. We report the resultant market-to-book ratio for Q4 of each year, with the exception of 2017, where we report values as of Q2 due to data limitations. Panel A reports results for buyout funds. Panel B reports results for venture funds.

Panel A. Buyout

	Vintage Year											
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
2006	1.14	--	--	--	--	--	--	--	--	--	--	--
2007	1.04	1.02	--	--	--	--	--	--	--	--	--	--
2008	0.77	0.65	0.76	--	--	--	--	--	--	--	--	--
2009	0.97	0.86	1.00	1.04	--	--	--	--	--	--	--	--
2010	1.24	1.03	1.23	1.28	1.05	--	--	--	--	--	--	--
2011	1.20	0.92	1.12	1.16	0.91	0.87	--	--	--	--	--	--
2012	--	0.98	1.25	1.28	0.97	1.01	1.05	--	--	--	--	--
2013	--	--	1.80	1.71	1.19	1.23	1.25	1.13	--	--	--	--
2014	--	--	--	1.86	1.14	1.17	1.17	0.99	1.02	--	--	--
2015	--	--	--	--	1.28	1.28	1.31	1.05	1.02	1.03	--	--
2016	--	--	--	--	--	1.22	1.28	0.91	0.91	0.93	1.00	--
2017	--	--	--	--	--	--	1.44	1.01	1.07	1.10	1.17	1.13
Average	1.06	0.91	1.19	1.39	1.09	1.13	1.25	1.02	1.00	1.02	1.08	1.13

Table 7. *Continued*

Panel B. Venture												
	Vintage Year											
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
2006	1.03	--	--	--	--	--	--	--	--	--	--	--
2007	0.88	0.87	--	--	--	--	--	--	--	--	--	--
2008	0.80	0.80	0.92	--	--	--	--	--	--	--	--	--
2009	0.98	0.99	1.14	1.01	--	--	--	--	--	--	--	--
2010	0.89	1.02	1.09	0.99	0.92	--	--	--	--	--	--	--
2011	0.72	0.83	0.98	0.80	0.83	0.90	--	--	--	--	--	--
2012	--	0.86	0.99	0.86	0.90	0.99	1.01	--	--	--	--	--
2013	--	--	0.97	0.90	0.94	1.07	1.09	0.97	--	--	--	--
2014	--	--	--	0.72	0.70	0.85	0.91	0.79	0.90	--	--	--
2015	--	--	--	--	0.63	0.78	0.87	0.82	0.94	0.97	--	--
2016	--	--	--	--	--	0.72	0.86	0.68	0.88	0.92	0.96	--
2017	--	--	--	--	--	--	0.95	0.74	0.96	1.02	1.06	0.90
Average	0.88	0.90	1.02	0.88	0.82	0.89	0.95	0.80	0.92	0.97	1.01	0.90