Low Interest Rates, Market Power, and Productivity Growth*

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Abstract

We show theoretically that low interest rate gives industry leaders a strategic advantage over followers, and this advantage becomes more dominant as interest rate approaches zero. Consequently, as interest rate declines, market structure becomes more monopolistic and for sufficiently low level of interest rate, productivity growth slows down as well. We test the key prediction of our model by analyzing excess return for industry leaders over followers in response to interest rate decline. A decline in ten year yield generates positive excess return for leaders, and the magnitude of excess return rises as interest rate approaches zero. Our model helps explain why the fall in long-term rate has been associated with rising market concentration, reduced dynamism, widening productivity-gap between leaders and followers and eventually slower productivity growth in a single coherent framework.

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1 Introduction

Long term interest rates have fallen globally since the eighties reaching their lowest level in recent years. The very low rates have sparked an extensive debate on causes and consequences. In this paper, we analyze the *supply side* consequences of falling long-term rate. In particular, suppose interest rate falls due to demand side weakness as some have suggested. How does the supply side of the economy respond?

Traditionally, lower interest rate is viewed as expansionary for the supply side. Consider a typical firm making investment decision. A fall in interest rate, all else equal, increases the net present value of future cash flows leading the firm to increase investment. Thus the supply-side relationship of growth against interest rate is typically downward sloping in endogenous growth models[1]. However, the traditional way of modelling the supply-side does not take market structure into account. Is it reasonable to assume that significant reduction in long-term rate would not impact the nature of competition within an industry?

We analyze this question by introducing competition on the supply-side. We build a model rooted in the dynamic competition literature (e.g., Aghion, Harris, Howitt and Vickers (2001)) where two firms compete in an industry for market share by investing in productivity enhancing technology. Firms decide whether to invest at each point in time. The investment decision is only a function of the current productivity gap between the leader and the follower, which is the key state variable of the model. A larger productivity gap gives the leader a larger share of industry profits. Productivity gap is stochastic with the gap reducing by one step at an exogenous poisson rate.

We show that the model solution splits the state space in two regions. If productivity gap between leader and follower is low enough, the industry is in a "competitive region" with both firms investing in an effort to escape competition. In the "monopolistic region", follower does not invest due to a "discouragement effect": the probability of overtaking the leader is too small relative to the cost of investment. At the end of the monopolistic region even the leader stops

[1]See e.g. Aghion and Howitt (1992), Klette and Kortum (2004), Grossman and Helpman (1991) and Romer (1990). The supply-side relationship between growth and interest rate is flat in exogenous growth models such as Solow and Ramsey.

investing as the perceived threat of getting overtaken becomes too small to worry about.

What happens in this supply side model with competition when interest rate falls? We have the traditional effect. A decline in interest rate causes both the leader and the follower to increase investment. However, the increase in investment is stronger for leader than follower as the leader experiences a larger absolute increase in value from the fall in rates.

The unequal response from falling rates induces a second strategic effect that works in the opposite direction. In particular, the equilibrium expected productivity gap between leader and follower *increases* due to the unequal investment responses. The increase in expected productivity gap in turn discourages the follower from investing. Thus as interest rate falls, the expected time that an industry spends in the monopolistic region increases.

In one of the key theoretical results of the paper, we show that as interest rate goes to zero, the strategic effect always dominates the traditional effect and the industry spends almost all the time in the monopolistic region. This induces an inverted-U shaped supply side relationship between growth and interest rates. Starting from a high level of interest rate, growth increases as interest rate falls - as in standard endogenous growth models - because the traditional effect dominates the strategic effect. However, as interest rate keeps falling, the endogenous rise in market structure causes the strategic effect to dominate at some point, and growth starts to fall. Our key theoretical result shows that this backward bend in the supply-curve *must* happen before interest rate hits zero.

How plausible is the supply side mechanism highlighted in our model that links a fall in interest rate with an increase in industry concentration? In general this is a difficult question to answer since industry concentration moves slowly in practice. However, we take advantage of the fact that stock prices are forward-looking and incorporate expectations regarding changes in industry concentration in response to interest rate shocks. Using CRSP-Compustat merged data from 1980 onwards, we analyze the performance of portfolio that goes long industry leaders and short industry followers in response to changes in the ten year treasury rate.

Our model has the unique prediction that the long-short portfolio would generate excess returns that keep getting larger as the level of interest rate approaches zero. We find that this is

exactly the case in the data. The estimated effect is large in magnitude and robust to a number of tests we highlight in the paper.

The model outlined in this paper is also useful in explaining a number of important macro trends in a single coherent framework. For example, the decline in ten year interest rate globally has been associated with a rise in industry concentration, markups and profit share, and a decline in business dynamism. These facts are well-documented in the literature[2]. The model is also consistent with the fact that market concentration increased first as interest rate started declining, and productivity growth slowed down later when long-term interest rates approached zero[3].

Our model also helps explain the cross-sectional pattern in productivity slowdown. In the model, the slowdown in productivity growth is associated with *larger* average productivity gap between industry leader and follower. Using firm-level data from the OECD, Berlingieri and Criscuolo (2017) and Andrews et al. (2016) show that productivity gap between 90th versus 10th percentile firms within industries has been increasing since 2000. Moreover, productivity gap between leaders and followers has risen most in industries where productivity growth has slowed down the most [4].

While we emphasize the consistency of our model with important macro trends, we would not want to give the impression that the mechanism highlighted in our model is the only relevant force behind the broad macro trends. Factors like technological change and changes in regulatory regime that others have mentioned are also plausible explanations. Our main point is to highlight that lower interest rates can directly influence market structure and through it productivity growth.

Our paper provides a new and different explanation for "secular stagnation", the observation that growth has slowed down as long-term rates trudged towards zero. Popular explanations of

[2]See De Loecker and Eeckhout (2017), Barkai (2018), Autor, Dorn, Katz, Patterson and Van Reenen (2017), Gutiérrez and Philippon (2016, 2017), Grullon, Larkin and Michaely (2016), Davis and Haltiwanger (2014), Decker, Haltiwanger, Jarmin and Miranda (2016), Haltiwanger (2015), Hathaway and Litan (2015), and Andrews, Criscuolo and Gal (2016)

[3]Productivity growth slowdown started in 2005, well before the 2007-08 recession, which suggests that - consistent with our model - structural as opposed to cyclical factors are behind the decline.

[4]Relatedly Gutiérrez and Philippon (2016, 2017), Lee, Shin and Stulz (2016) show sharp decline of investment relative to operating surplus and that investment gap is especially pronounced in concentrated industries. Moreover, Cette, Fernald and Mojon (2016) show in two-variable VAR that a negative shock to long-term interest rate leads to a fall in productivity growth, consistent with our framework.

secular stagnation,e.g., Summers (2014), focus almost exclusively on the demand side, and use frictions such as zero lower bound or nominal rigidity to generate long and persistent slowdown in growth[5]. In our framework, the initial decline in interest rate may come from weakness on the demand side. However, the key point of departure is the insight that if interest rate falls to a low enough level, then the rise in market competition may itself result in a constraining force on growth. Thus one does not need to rely on liquidity traps, zero lower bound or nominal rigidity to explain the persistent growth slowdown.

There has been work in the past that links interest rates to the *level* of productivity (e.g. Caballero, Hoshi and Kashyap (2008); and Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2017)). Our paper differs in its explicit modelling of the supply-side to investigate the relationship between interest rates and the *growth rate* of productivity.

Our paper contributes to the endogenous growth literature. Our framework on competition within an industry differs from the seminal work of Aghion et al. (2001) in that catch-up innovation by the follower happens step-by-step. The follower can thus not leap-frog the leader instantly. While leap-frogging does happen at times, especially during episodes of large technological disruptions, we feel that the more typical process of innovation is gradual.

In deriving our central theoretical result—that as interest rate converges to zero, aggregate innovation has to decline because the average distance between leaders and follower diverges—we also provide a technical contribution to the literature on dynamic patent races. Models of dynamic competition as stochastic games are notoriously difficult to analyze, and even seminal contributions in the literature either rely on numerical methods or impose significant restrictions on the state space to keep the analysis tractable (). By deriving first-order approximations of the recursive value functions when the discount rate is small, we are able to provide sharp, analytic characterizations of the asymptoptic equilibrium in the limiting case with discounting tends to zero, even as the ergodic subset of the state space becomes infinitely large. The techniques we developed here should be applicable to other stochastic games of strategic interactions with large state space.

[5]See e.g., Krugman (1998), Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), Benigno and Fornaro (n.d.) and Eggertsson, Mehrotra and Robbins (2017).

2 Supply-side model with investment and strategic competition

We start with a supply side model of innovation and strategic competition where firms take interest rate r as exogenously given. Our model generates productivity growth as a function of the interest rate and can be seen as the supply-side block of an endogenous growth model. We later close the model in general equilibrium through a demand-side Euler equation.

Setup Time is continuous. The economy has a continuum of markets indexed by $\nu \in [0,1]$, each with two forward-looking and risk-neutral firms engaging in dynamic competition. For expositional simplicity, we drop the market subscript ν and discuss the game in a given market.

Let $z_1(t), z_2(t) \in \mathbb{Z}_{>0}$ denote the productivity levels of the two market participants, and let $s(t) = |z_1(t) - z_2(t)| \in \mathbb{Z}_{\geq 0}$ be the state variable that captures their productivity gap. When $s = 0$, the two participants are said to be neck-to-neck; when $s > 0$, one of the firm is a temporary leader (L) while the other is a follower (F). The leader earns flow profit π_s and the follower earns π_{-s} given productivity gap s.

Assumption 1. *The flow profits π_s, $-\pi_{-s}$, and $(\pi_s + \pi_{-s})$ are non-negative, bounded, weakly concave and weakly increasing in s.*

Under the assumption, a higher productivity gap s is associated with higher joint profits and more unequal profits between the leader and the follower. We interpret state s to be more *competitive* than state s' if $s < s'$ and more *concentrated* if $s > s'$.

Our results hold under as long as the collection of profit functions satisfy Assumption 1, which can nest several microfounded models of duopoly competition. For concreteness, a particular collection that satisfies the assumption is

$$\pi_s = \begin{cases} 0 & s \leq 0, \\ 1 - e^{-\lambda s} & s > 0. \end{cases}$$

This profit function can be microfounded by a Bertrand pricing game in which the two participates produce perfect substitutes (for instance, see Peters (2016)).

Each firm can invest in order to improve its productivity, which evolves in step-increments. In the baseline model, we model investment decisions in each state s as a binary decision $\eta_s \in \{0, \eta\}$. Should the firm chooses to invest ($\eta_s = \eta$), it pays a flow cost c in exchange for a Poisson rate η with which the firm's productivity improves by one. Specifically, given investment decisions $\{\eta_s, \eta_{-s}\}$ over interval Δ at time t, state s transitions according to

$$s(t+\Delta) = \begin{cases} s(t)+1 & \text{with probability } \Delta \cdot \eta_s \\ s(t)-1 & \text{with probability } \Delta \cdot (\kappa + \eta_{-s}) \\ s(t) & \text{otherwise,} \end{cases}$$

where κ is the exogenous Poisson rate of technology diffusion, enabling the follower to close productivity gap by one step. The key assumption embodied in the investment technology is that catching up is a gradual process: the productivity gap has to be closed step-by-step, and the follower cannot "leapfrog" the leader by overtaking leadership with one successful innovation. This assumption plays an important role in our results and is the key difference between our model and the setup in Aghion et al. (2001). On the other hand, that κ is a state-independent constant is not a crucial assumption for us. We discuss state-dependent κ as an extension in section 3.

Interest rate is modeled as the rate at which firms discount future payoffs. Each firm's value $v_s(t)$ in state s at time t can be expressed as the expected present-discount-value of future profits net of investment costs:

$$v_s(t) = \mathbb{E}\left[\int_0^\infty e^{-r\tau}\left\{\pi(t+\tau) - c(t+\tau)\right\} \middle| s\right].$$

We look for a stationary symmetric Markov-perfect equilibrium such that the value functions and investment decisions depend on the state but not the time index. The stationary HJB equation for the two firms are respectively

$$rv_s = \pi_s + (\kappa + \eta_{-s})(v_{s-1} - v_s) + \max\{0, \eta(v_{s+1} - v_s) - c\} \tag{1}$$

$$rv_{-s} = \pi_{-s} + \eta_s \left(v_{-(s+1)} - v_{-s} \right) + \kappa \left(v_{-(s-1)} - v_{-s} \right) + \max \left\{ 0, \eta \left(v_{-(s-1)} - v_{-s} \right) - c \right\}. \quad (2)$$

Definition 1. A stationary symmetric Markov-perfect equilibrium is a collection of value functions and investment decisions $\{\eta_s, \eta_{-s}, v_s, v_{-s}\}_{s=0}^{\infty}$ that satisfy the infinite system of equations in (1) and (2).

Given the investment decisions $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$, an equilibrium induces an ergodic Markov process over state transitions and generates a unique stationary distribution over the state space.

Definition 2. A steady-state of the economy $\{\mu_s\}_{s=0}^{\infty}$ induced by equilibrium investments $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$ is a distribution over the state space such that

$$2\mu_0\eta_0 = (\mu_{-1} + \kappa)\,\mu_1$$

$$\mu_s\eta_s = \left(\eta_{-(s+1)} + \kappa\right)\mu_{s+1} \quad \text{for all } s > 0,$$

and

$$\sum_{s=0}^{\infty} \mu_s = 1.$$

In a steady-state, the aggregate investment is $I \equiv c \cdot \sum_{s=0}^{\infty} \mu_s \left(\eta_s + \eta_{-s}\right)$ and the productivity growth rate of the economy is[6]

$$g \equiv \lim_{\Delta \to 0} \left(\mathbb{E}\left[\frac{z_1(t+\Delta) + z_2(t+\Delta)}{2} \right] - \mathbb{E}\left[\frac{z_1(t) + z_2(t)}{2} \right] \right).$$

Because joint profits form an increasing and bounded sequence, it must converge; let $\bar{\pi} \equiv \lim_{s\to\infty} (\pi_s + \pi_{-s})$ denote the limit. We impose the following assumption on the investment technology η.

Assumption 2. *1)* $2c > \bar{\pi}$*; 2)* $\eta > \kappa$.

Because firms are risk-neutral, we can alternatively interpret the binary investment decision as a continuous one, i.e. firms can choose bounded investment $\eta_s \in [0, \eta]$ with marginal cost c. The first part of the assumption ensures that the upperbound on investment is "not too tight", in the

[6] In the definition, the aggregate productivity growth rate is measured as the average productivity growth across individual firms. This is without loss of generality because in a steady-state, leaders and followers in all markets have the same expected growth rate of productivity: $\lim_{\Delta\to 0} \left(\mathbb{E}\left[z_1(t+\Delta)\right] - \mathbb{E}\left[z_1(t)\right]\right) = \lim_{\Delta\to 0} \left(\mathbb{E}\left[z_2(t+\Delta)\right] - \mathbb{E}\left[z_2(t)\right]\right)$.

sense that it is unprofitable for both firms to undertake the maximum investment simultaneously and permanently given the maximum joint flow payoffs. The second part of the assumption states that when the follower does not invest, the leader's endogenous investment can be large enough to overcome the productivity diffusion, such that his market power is expected to increase over time.

Analysis of the Steady-state Equilibrium In the model, firms not only invest for gains in the flow profits in higher states; more importantly, they invest in order to enhance the respective market positions, the advancement of which enables firms to sustain or reach for even higher profits in the future. For the follower, closing the productivity gap by one step enables him further closes the gaps in the future and eventually catches up with the leader. For the leader, widening the productivity gap brings higher profits, the option value to further increase the lead in the future, as well as the increased expected duration of market leadership, because it would now take the follower additional steps to catch up.

To understand the equilibrium, we first describe the structure of value functions given any sequence of (potentially non-equilibrium) investment decisions $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$. The fact that firms are forward-looking implies that value function in each state can be written as a weighted average of flow payoffs in all ergodic states induced by the investments, i.e.,

$$v_s = \sum_{s'=-\infty}^{\infty} \lambda_{s'|s} \left(\frac{\pi_{s'} - c \cdot \mathbf{1}(\eta_{s'} = \eta)}{r} \right), \quad \text{where} \sum_{s'=-\infty}^{\infty} \lambda_{s'|s} = 1 \ \text{ for all } s. \tag{3}$$

The term $\left(\frac{\pi_{s'} - c \cdot \mathbf{1}(\eta_{s'} = \eta)}{r}\right)$ represents permanent-value in state s', or in other words, the present-discount value of the flow payoffs in state s' if a firm stays in state s' permanently. In equilibrium, the value in state s can be written as a weighted average of the permanent-value across all ergodic states. The weight $\lambda_{s'|s}$ can be interpreted as the present-discounted value of time that the firm is going to be in state s', given that the current state is s. The weights $\left\{\lambda_{s'|s}\right\}_{s'=-\infty}^{\infty}$ is a measure conditional on the current state. When the current state s is high, the firm is expected to spend more time in higher states, and the conditional distribution $\left\{\lambda_{s'|s+1}\right\}_{s'=-\infty}^{\infty}$ first-order-stochastically dominates $\left\{\lambda_{s'|s}\right\}_{s'=-\infty}^{\infty}$ for all s.

Lastly, the conditional distribution $\{\lambda_{s'|s}\}_{s'=-\infty}^{\infty}$ depends on the equilibrium investment decisions as well as the interest rate. Holding investment decisions fixed, $|\lambda_{s'|s+1} - \lambda_{s'|s}|$ is lower when the interest rate r is low. This is because under low r, the value functions place higher weights on future payoffs, and the temporary difference in value due to the different starting states becomes less important. Specifically, holding investment decisions constant,

$$rv_s \to \bar{v} \quad \text{for all } s \in \mathbb{Z}.$$

We now analyze the equilibrium structure.

Lemma 1. *In an equilibrium, value functions are monotone in the state: $v_{s+1} \geq v_s$ and $v_{-(s+1)} \leq v_s$ for all $s \geq 0$.*

Lemma 2. *Consider a generic equilibrium. If $\eta_0 = \eta$, then investment decisions follow "cut-off" rules: there exists integers $(n, k) \in \mathbb{Z}^2_{>0}$ such that*

$$\eta_s = \begin{cases} \eta & \text{for } s \leq n \\ 0 & \text{for } s > n, \end{cases} \qquad \eta_{-s} = \begin{cases} \eta & \text{for } s < k \\ 0 & \text{for } s > k, \end{cases}$$

and $\eta_{-k} \in [0, \eta]$. That is, leader strictly invests in states 1 through n, and strictly does not invest after state n; the follower strictly invests in states 1 through $(k-1)$, weakly invests in state k, and does not invest after state k.

For certain parameter range, there exists a trivial equilibrium in which neither firm invests in state 0 (i.e., $\eta_0 = 0$). State 0 is therefore an absorbing state, and aggregate investment is zero in the steady-state. This equilibrium arises when the flow profit is state 0 is high relative to π_1, and firms in the neck-to-neck state do not have sufficient incentive to invest, despite that this state has the least joint profits. We do not focus on this uninteresting equilibrium in our analysis.

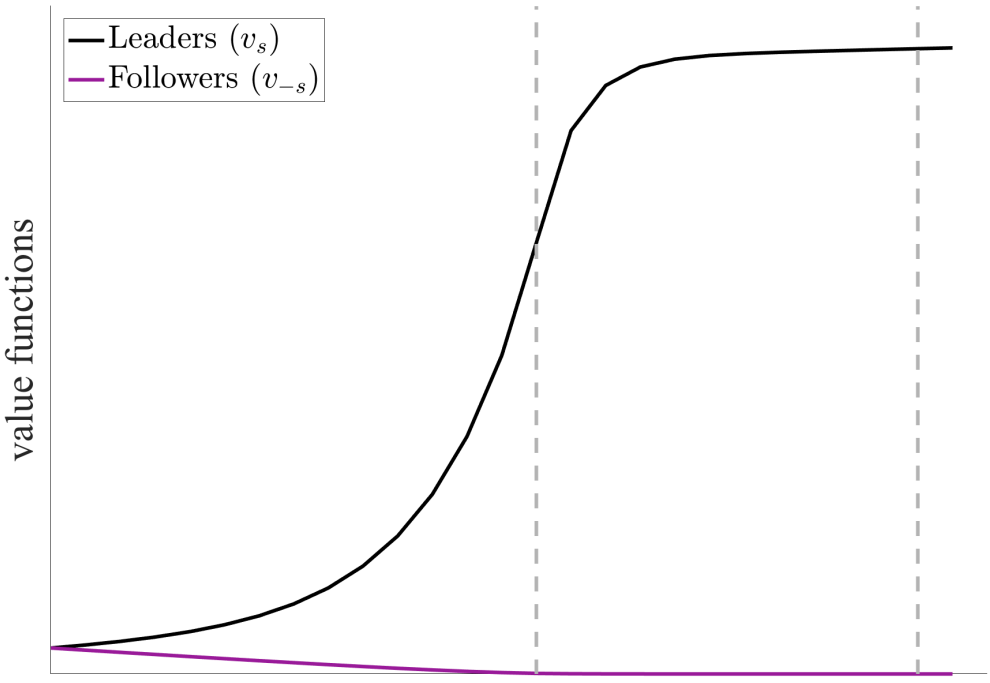
The proposition establishes that in a non-trivial equilibrium, investments are weakly decreasing in the productivity gap s. This is a manifestation of two effects. First, when the follower is too far behind, the firm value is low and the marginal value of catching up by one step is not worth the investment cost. This is also known as the "discouragement effect" in the dynamic contest

literature (Konrad (2012)). Second, the leader's strategy is monotone due to a "lazy monopolist" effect: when the leader is far ahead of the follower, he ceases investment because the marginal gain in value brought by advancing market position is no longer worth the investment costs. The proposition also illustrates in a generic equilibrium, the leader is never indifferent between the two actions while the follower is indifferent in at most one state, k, which is the last state in which he weakly invests. For expositional purposes, in what follows we assume the follower strictly invests in this state, with $\eta_{-k} = \eta$.

Proposition 1. *The leader weakly invests more than the follower in any state: $n \geq k$.*

The proposition shows that the leader invests in more state than the follower does, a phenomenon we refer to as *leader dominance*. To aid the understanding of this result, we depict the value functions in figure 1. The black curve represents the value function of the leader, while the purple curve represents that of the follower. The two verticle lines respectively represent k and n, the last states in which the follower and the leader weakly invests.

Figure 1: Value functions

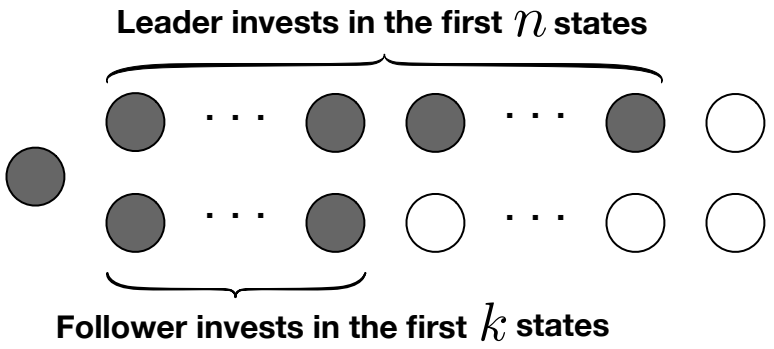


The intuition for *leader dominance is* as follows. In states that the follower does not invest, the productivity gap closes at a slow rate κ. Conversely, in states that the follower does invest, the gap closes at a fast rate $(\eta + \kappa)$. Firms are motivated to invest because of the high future flow payoffs in high states after consecutive successful investments. The leader is motivate to invest in all states $s \leq n$ in order to reach state $n + 1$, so that he can enjoy the flow payoff π_n without having to pay the investment costs in the state. The state $(n + 1)$ is especially attractive if the follower does not invest in that state, because the leader can enjoy the payoff for a longer expected duration before the state stochastically transitions down to n, after which he has to incur investment cost again. The follower, on the other hand, is also motivated by future payoffs. He incurs investment costs in exchange for the possibility of closing the gap and catching up with the leader, and for the possibility of eventually becoming the leader himself in the future so that he can enjoy the high flow payoffs. In other words, investment decisions for both forward-looking firms are motivated by the high flow profits in the high states, and the incentive to reach these states is stronger for the leader because the leader is closer to those high-payoff states.

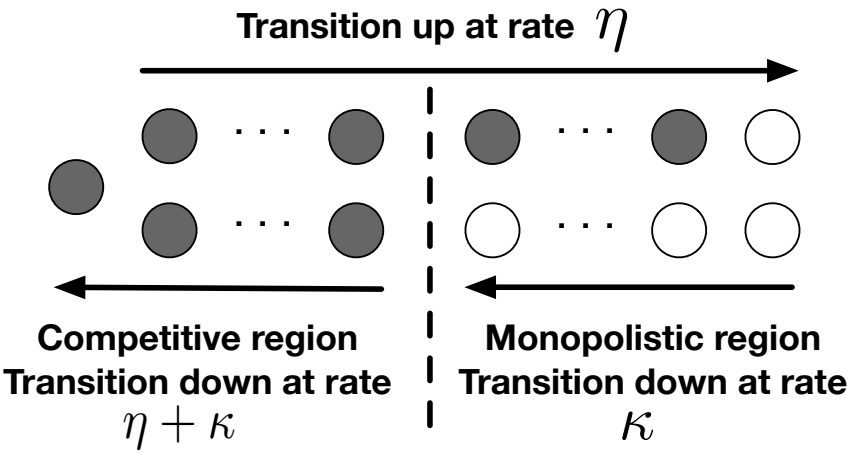
Another way to describe the intuition is to consider the contradiction brought by $n < k$. Suppose the leader stops investing before the follower does. In this case, the high flow payoff π_{n+1} is transient for the leader as market leadership is very much fleeting due to the high rate of downward state transition, implying the payoffs for being a leader is low. Because firms are forward-looking and their value functions depend on future payoffs, the low value of being a leader "trickles down" to affect the value function in all states. Therefore, low payoffs of being a leader in turn diminishes the incentive for the follower to invest, generating a contradiction to the presumption that follower invests more than the leader does.

The structure of an equilibrium can therefore be represented by the following diagram. States are represented by circles, going from state 0 on the very left and state $(n + 1)$ on the very right. The coloring of a circle represent investment decisions: states in which the firm weakly invests are represented by dark circles, while white ones represent those in which the firm does not invest. The top row represents leader's investment decisions while the bottom row represent the follower's investment decisions.

The fact that the leader invests in more states than the follower does enables us to partition



the ergodic state space into two regions: one in which both firms invest and another in which the follower does not. In the first region, the state transitions up with Poisson rate η and transitions down with rate $(\eta + \kappa)$. In expectation, the state s decreases over time in this region, and the market structure tends to move towards being more competitive. For this reason, we refer to this as the *competitive* region. Note that this label is not a reflection of the static profits, which can be very much concentrated on the leader. Rather, the label reflects the fact that joint profits tend to decrease dynamically. In the second region, the state transitions up with Poisson rate η and transitions down with rate κ. Relative to the first region, the downward transition rate is much lower while the upward transition rate is the same; thus the market structure tends to move towards being more concentrated. For this region, we refer to this as the *monopolistic* region.



The aggregate productivity growth rate in the economy is a weighted average of the productivity growth rate in each market and it depends on both the state-dependent investment decisions as well as the distribution of market structure. In a steady-state, the distribution of market structure itself is a function of the investment decisions $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$. Let $\mu^C \equiv \sum_{s=1}^{k} \mu_s$ be the fraction

of markets in the competitive region in a steady-state and let $\mu^M \equiv \sum_{s=k+1}^{n+1} \mu_s$ be the fraction of markets in the monopolistic region.

Proposition 2. *In a stationary distribution $\{\mu_s\}_{s=0}^{\infty}$ induced by cut-off strategies (n,k), the aggregate productivity growth rate is*

$$g = \mu^C \cdot (\eta + \kappa) + \mu^M \cdot \kappa.$$

Steady-state growth rate is increasing in k and decreasing in n.

The proposition shows that steady-state growth is increasing in the fraction of markets in the competitive region and decreasing in the fraction of markets in the monopolistic region. In the competitive region, both firms are investing and, as a result, productivity improvements are rapid, state transition rate is high, dynamic competition is fierce, and market power tends to decrease over time. On the other hand, only the leader is investing in the monopolisitic region, and the rate of state transition and productivity growth is low. The fraction of markets in the competitive region, hence the steady-state growth rate, is increasing in k and decreasing in n.

Proposition 3. *Let $\{e\}$ be the set of equilibria. Let v_s^e be the value function of state s in equilibrium e, and let (n^e, k^e) denote represent the investment strategies in the equilibrium. Generically, all equilibria can be Pareto-ranked. That is, there exists an equilibrium $(\star)$ such that $v_s^\star \geq v_s^e$ for all s and al equilibria e. In this Pareto-optimal equilibrium, $n^\star \leq n^e$ and $k^\star \geq k^e$ for all e, and steady-state growth rate is maximized.*

This proposition shows the existence, for any interest rate r, of a Pareto-optimal equilibrium in which firm value is maximized across all states and features the highest steady-state growth rate among all equilibria. In what follows, we refer to it as equilibrium $(\star)$ and its investment cutoffs as $(n^\star, k^\star)$, while we keep using (n,k) to represent any generic equilibrium.

Now consider the effects of a decline in the interest rate. Conventional intuitions suggest that ceteris paribus, when firms discount future profits at a lower rate, the incentive to invest should increase because the cost of investment is lower relative to future benefits. This intuition does hold in our model, as manifested in the following proposition.

Proposition 4. *The cutoff states $n^\star$ and $k^\star$, are both decreasing in r. Moreover, in any non-trivial equilibrium,*

$$\lim_{r \to 0} k = \lim_{r \to 0} (n - k) = \infty.$$

The proposition suggests that as interest rate declines, both the leader and the follower weakly raise investment in all states in equilibrium ($\star$). Moroever, as interest rate converges to zero, firms become infinitely patient and they sustain investment despite being arbitrarily far behind or ahead.

Let M_s denote the expected time for a firm currently in state s to reach state zero. It is a measure of dynamic monopoly power, as it captures the expected duration in which the firm stays as a leader. Let $M \equiv \sum_{s=1}^{\infty} \mu_s M_s$ denote the average duration of market leadership. Our main theoretical contribution concerns the behavior of aggregate variables as $r \to 0$.

Theorem. *As $r \to 0$,*

1. *The fraction of markets in the competitive region vanishes:*

$$\lim_{r \to 0} \mu^C = 0;$$

   the monopoly region becomes absorbing:

$$\lim_{r \to 0} \mu^M = 1.$$

2. *Market leadership becomes permanently persistent:*

$$\lim_{r \to 0} M = \infty.$$

3. *Aggregate investment declines and productivity growth slows down:*

$$\lim_{r \to 0} I = \lim_{r \to 0} g = \kappa.$$

4. *Joint flow profits converges to the limit:*

$$\lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s \pi_s = \bar{\pi}.$$

5. *Productivity gap between leaders and followers diverges:*

$$\lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s s = \infty.$$

6. *Relative market valuation of leaders and followers diverges:*

$$\lim_{r \to 0} \frac{\sum_{s=0}^{\infty} \mu_s v_s}{\sum_{s=0}^{\infty} \mu_s v_{-s}} = \infty.$$

The theorem states that as $r \to 0$, all markets are in the monopolistic region and leaders almost surely stay permanently as leaders. In this region, leaders invest only to counteract the exogenous diffusion of productivity, and as a result, aggregate investment and productivity growth rate decline and are both fully determined by the exogenous rate κ, there lower bound c.f. Proposition 2. The rest of the statements then follows.

To understand this result, note that

$$\mu^C \to \mu_0 \frac{2\eta + \kappa}{\kappa}, \quad \mu^M \to \mu_{n+1} \frac{\eta}{\eta - \kappa}, \quad \text{and} \quad \mu_0 = \left(\frac{\eta + \kappa}{\eta}\right)^k \left(\frac{\kappa}{\eta}\right)^{n-k} \mu_n.$$

The fact that $\mu^C \to 0$ implies that $\left(\frac{\kappa}{\eta}\right)^{n-k}$ converges to zero at a faster rate than $\left(\frac{\eta}{\eta+\kappa}\right)^k$, or equivalently,

$$\lim_{r \to 0} \left(\frac{n-k}{k}\right) > \frac{\log\left(\frac{\eta}{\kappa}\right)}{\log\left(\frac{\eta+\kappa}{\eta}\right)}.$$

Proposition 4 shows that in any equilibrium, both k and $(n-k)$ diverges as $r \to 0$. Why is $\left(\frac{n-k}{k}\right)$ asymptotically bounded below? The intuition is threefolds.

First, as the interest rate converges to zero, the value of being in state 0 diverges at a rate slower than r^{-1} (i.e., $rv_0 \to 0$). This is because holding $(n-k)$ constant, v_0 is decreasing in the k, the size of the competitive region. In this region, the state tends to transition downwards: for every step of productivity improvement made by the leader, the follower is expected to advance by more than one step on average. As a result, starting from state 0, it is exponentially more difficult to get out of the region and as k increases. Because both firms invest at the maximum rate in this region, the joint flow payoffs are negative, and v_0 is therefore low.

Second, the leader stops investing in state $n+1$ despite low r implies that $rv_{n+1} \to \pi - c\kappa/\eta$ converges to a positive constant, and therefore v_0 vanishes relative to v_n. This observation in turn implies that the decline in value as the leader falls from the monopolistic region into the competitive region is steep, as manifested by the "cliff" around $s = k$ in the leader's value function in Figure 1. In fact, one can show that the slope of the value function for the leader at the cliff,

$(v_{k+1} - v_k)$, diverges at the same rate as v_{n+1}, which is on the same order as $\left(\frac{\eta+\kappa}{\eta}\right)^k$:

$$v_{k+1} - v_k = \Theta\left(v_{n+1}\right) = \Theta\left(\left(\frac{\eta+\kappa}{\eta}\right)^k\right).$$

The third step is to realize that the slope of leader's value function around state n converges to a constant, $v_{n+1} - v_n = \Theta\left(c\right)$. Since the slope of value functions at state n is increasing in the slope at state k, which diverges and is on the same order as $\left(\frac{\eta+\kappa}{\eta}\right)^k$, implies that $(n-k)$ must be sufficiently large to neutralize the impact of $\left(\frac{\eta+\kappa}{\eta}\right)^k$ on $(v_{n+1} - v_n)$.

3 Model Extensions

We extend the basic model presented earlier in four ways. First, we show that our results survive even if the catch-up rate κ increases with the state. For example, it is plausible to assume that it is easier to catch up for the follower the further away from technological frontier the follower is. Second, we extend our model to allow for variable investment intensities, demonstrating numerically that our results are not artifacts of the binary action assumption. The numerical example is also useful in illustrating in greater detail the mechanism behind our key result. Third, we discuss transitional dynamics. Fourth, we have so far exclusively focused on the supply-side of the economy. We demonstrate how to close the model in general equilibrium by augmenting our model with an Euler equation from the demand-side.

3.1 State-dependent catch up

The no-leapfrogging assumption—that the follower has to catch up one step at a time no matter how far behind—is crucial for our results and is a key distinction between our model and the seminal work of Aghion et al. (2001). Intuitively, leader invests in the monopolistic region for two reasons: 1) to capture rising flow profits and 2) to escape dynamic competition. By getting one step further ahead of the follower, the leader is able to secure his dynamic rents for longer duration. When the discount rate is low, it is this second, escaping competition motive that dominates;

indeed, our results survive even if flow profits stay constant as a function of the state ($\pi_s = \bar{\pi}$ for all $s > 0$, $\pi_s = 0$ otherwise). Conversely, if the follower can close an arbitrarily large number of gaps with one successful innovation, such prospect of leapfrogging kills leader's dynamic incentive to get further ahead, leaving higher flow profits as the only reason for the leader to invest. If flow profits do not increase with the state, the leader does not invest at all.

That being said, our results do not crucially depend on the constancy of exogenous catch up rate κ and investment cost c. To see this, consider an extension in which the leader's investment cost forms an increasing sequence c_s with upperbound $\bar{c}$, and the exogenous catch up rate $\kappa_s \nearrow \bar{\kappa}$ (for simplicity, we maintain the follower's investment cost constant at c). These state-dependent parameters do introduce the possibility that n is bounded even as $r \to 0$; for instance, a leader could be discouraged from investing in state s—even if he is patient—if the cost c_s or the exogenous catch up rate κ_{s+1} is sufficient high. On the other hand, Theorem 1 is an asymptotic result and is derived under fairly general conditions. Intuitively, as long as the state-to-state changes in κ_s and c_s are insufficient to deter a patient leader from continuing to invest in every state, then $n \to \infty$, and for low enough interest rate r, parameters κ_s and c_s relevant to the steady-state distribution of market structures will approach the respective upperbounds $\bar{\kappa}$ and $\bar{c}$, and Theorem 1 prevails. A sufficient condition for n to stay unbounded is for the variations in c_s and κ_s to be small relative to flow profits, i.e. $\pi > 2\frac{\bar{c}\bar{\kappa} - c\kappa_1}{\eta}$.

Proposition 5. *Suppose the investment cost of the leader form an increasing sequence $c_s \nearrow \bar{c}$ for $s > 0$, and so does the exogenous catch up rate $\kappa_s \nearrow \bar{\kappa}$. Suppose also that the follower's investment cost is constant, with $c_{-s} = c_0 = c$. Then $\lim_{r\to 0} g = \kappa \ln \lambda$ as long as $\pi > 2\frac{\bar{c}\bar{\kappa} - c\kappa_1}{\eta}$.*

3.2 Variable investment intensity

Our model can be extended to allow for variable investment intensities. Investment decision becomes a continuous variable $\eta \geq 0$ with a convex cost function $c(\eta)$. HJB equations can be modified as

$$rv_s = \max_{\eta \geq 0} \pi_s - c(\eta) + (\kappa + \eta_{-s})(v_{s-1} - v_s) + \eta(v_{s+1} - v_s)$$

$$rv_{-s} = \max_{\eta \geq 0} \pi_{-s} - c(\eta) + (\kappa + \eta)\left(v_{-(s-1)} - v_{-s}\right) + \eta_s\left(v_{-(s+1)} - v_{-s}\right)$$

Under this extension, changes in investment intensity is "smoothed out" over many states, resulting in a single-peaked steady-state distribution of productivity gaps. Below, we provide numerical illustrations of the investment function, the steady-state distribution, and firms' value functions, as well as how these functions change in response to lower interest rate. We also provide numerical illustrations of how steady-state levels of productivity growth and market power vary with interest rates. In generating these numerical plots, we assume the investment cost function is quadratic, and that profit functions follow

$$\pi_s = \begin{cases} 0 & s \leq 0 \\ 1 - e^{-\lambda s} & s > 0 \end{cases}.$$

The top panel in figure 2 shows 1) both leader's and follower's investment intensity decreases in states with higher monopoly power; 2) investment intensities are higher under low interest rates. The lower panel in figure 2 shows that 1) both players raise investment following a decline in r; 2) leader's response is stronger than follower's.

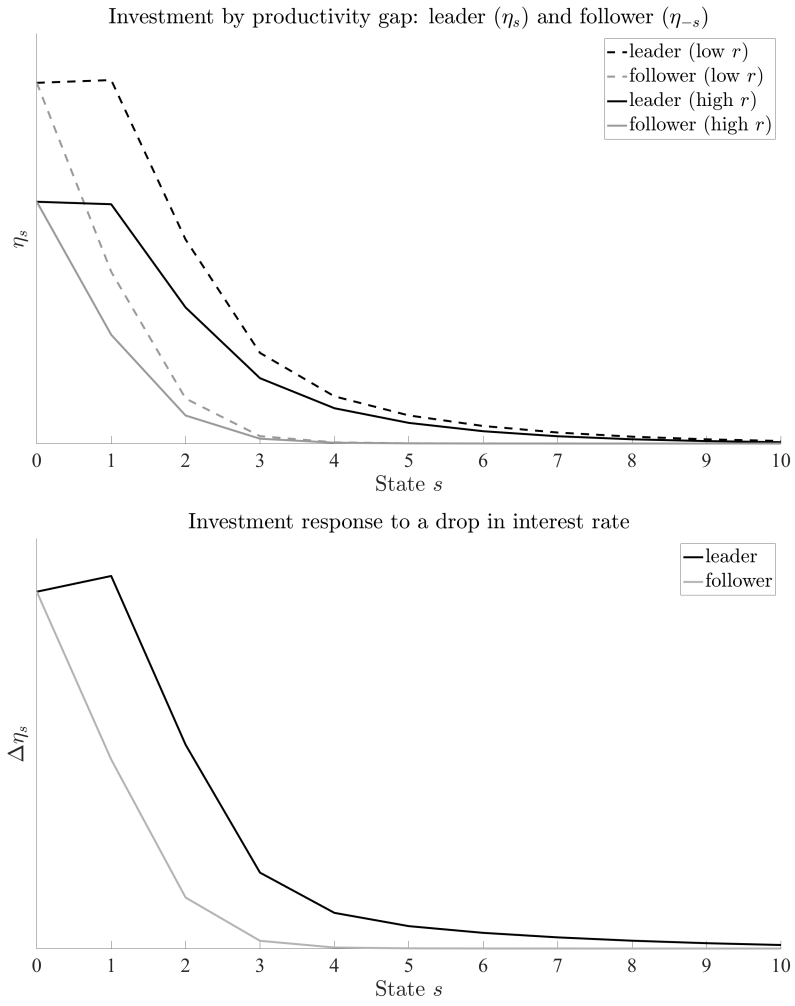
The top panel of figure 3 shows that, following a decline in r, the steady-state distribution of market structure shifts to the right, and aggregate market power increases. When r declines, increase in leader's value is larger than changes in follower's value. This is the key driver behind leader's stronger investment response following a drop in r. The bottom panel of figure 3 numerically verifies that our central result $g \to \kappa \cdot \ln \lambda$ holds under the extension with variable investment intensities.

3.3 Transitional dynamics: productivity and market power

Our earlier analysis illustrates that, starting from a high level of interest rate, declining r is at first expansionary—measured by *steady-state* growth—and only becomes contractionary when r falls sufficiently low; yet, *steady-state* market power tend to rise when r declines at any level.

Something parallel is true for transitional dynamics after unanticipated, permanent changes in interest rates, though for different reasons. Starting from a steady-state, a decline in interest

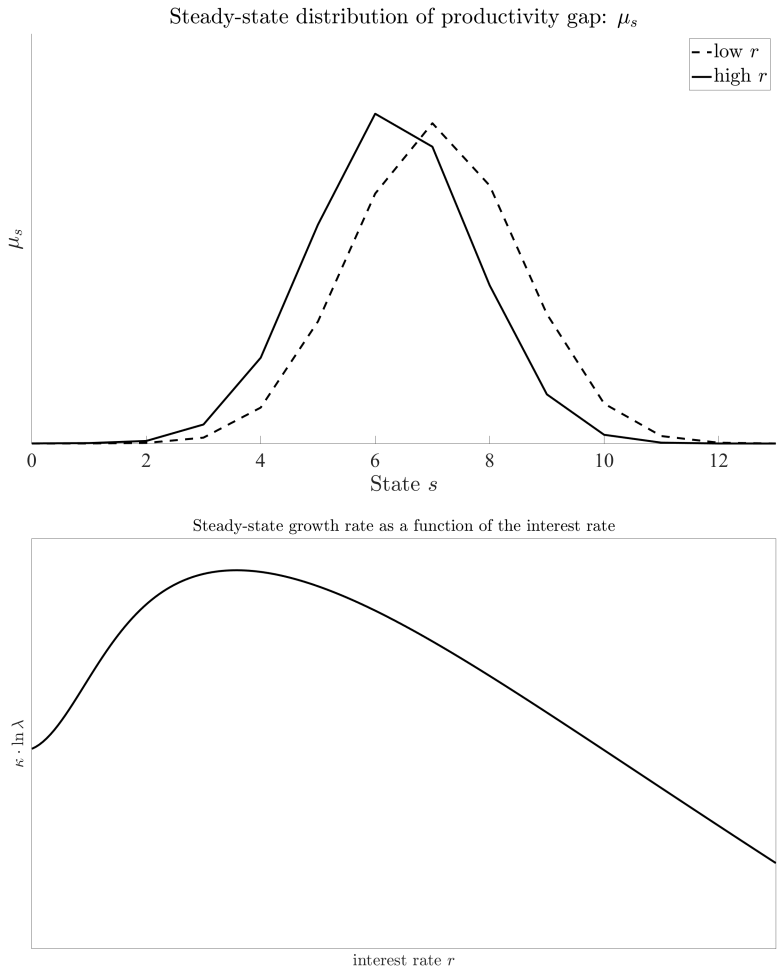
Figure 2: Investment functions



rate—a change that firms expect to be permanent—immediately moves market participants onto a new equilibrium, featuring higher investments given any market structure. The equilibrium distribution of market power starts to rise, although it moves slowly, as depicted in top panel of figure 4.

Over time, as the distribution of market structures converges to the new steady-state and as average market power increases, equilibrium growth rate and investment eventually declines to the new steady-state level. Whether productivity growth is higher or lower in the new steady-state, relative to the inital one, depends on the level of starting interest rate before the shock hits. If the starting rate is low (i.e. to the left of the peak in Figure **??**), then growth rate would be lower in the new steady-state. Productivity growth along the transitional path in such a case is depicted

Figure 3: Steady-state distribution and growth



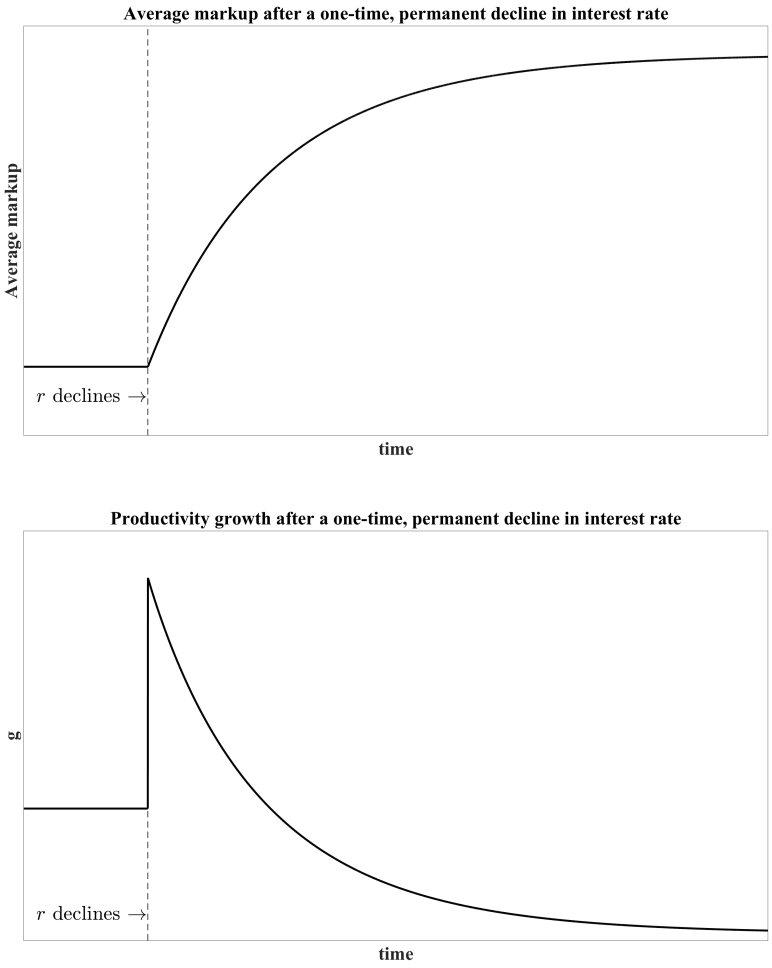
in bottom panel of figures 4.

3.4 Closing the model in general equilibrium

The focus of this paper is on how competition and productivity growth is impacted by changes in the interest rate. As such we have so far treated the interest rate as exogenous, and traced out the implied supply-side relationship between competition and interest rate, or growth and interest rate as shown in figure xx.

It is relatively straightforward to close our model in general equilibrium by adding an Euler equation from the demand-side, following Aghion et al. (2001) and Benigno and Fornaro (n.d.).

Figure 4: Time-path of average markup and productivity growth following a shock to r



Doing so explicitly identifies where movement in interest rate might come from, i.e. shifts in the demand curve, and solves for equilibrium interest rate and growth rate as well.

As before, suppose the economy has a continuum of markets indexed by $i \in [0,1]$ and measure one of identical consumers with preferences

$$\int_0^\infty \left(Y(t) - L(t)\right)^{\frac{\theta-1}{\theta}} e^{-\rho t} dt, \quad \text{where } Y(t) \equiv \int_0^1 \ln y_i(t)\, di$$

where $y_i(t)$ is the quantity consumed of the composite good produced by market i at time t, and $L(t)$ indicates the labor supplied at time t. Each market i is a duopoly market, with

$$y_i = \left[y_{1i}^{\frac{\sigma-1}{\sigma}} + y_{2i}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where y_{1i} and y_{2i} indicates the output respectively of firm 1 and 2 in market i. Each firm produce using labor, with $y_{1i} = z_{1i}\ell_{1i}$ (analogously for firm 2). Productivity z's follow proportional increments $\lambda > 1$, with $z_{1i} = \lambda^m$ for some m. Innovation at each time instance t is a binary decision that requires c units of labor, in exchange for a Poisson rate η of improving productivity proportionally by λ. Within each market, firms engage in either Bertrand or Cournot competition.

Several comments are in order. First, the assumption that labor supply is perfectly elastic ensures that the wage rate is the same as the price of consumption good at each time instance. Since both production and innovation activities use labor as inputs, this assumption eliminates the need to keep track of relative prices (between wage rate and consumption good). This assumption is also taken by Aghion et al. (2001); it simplifies notations but is not crucial for our results. For instance, in a model with exogenous labor supply, then along any balanced growth path, the relative price of labor is constant, hence endogeneity of labor supply—or its elasticity—only affects the *level* of output but should play no role in affecting aggregate productivity growth.

Second, the assumption that consumer demand is Cobb-Douglas across varieties and is CES within varieties plays an important role. The first part ensures that total revenue in each market is independent of productivities in other markets—hence profits of firm 1 in market i is a function of only z_{1i} and z_{2i} but not z_{1j} and z_{2j} for $j \neq i$. The second assumption implies that demand is homothetic within each market, hence, with either Bertrand or Cournot competition, the microfoundation generates firm profits that are functions of only the log-productivity gap within the market itself.

Formally, let $s_i \equiv \frac{|\ln z_{1i} - \ln z_{2i}|}{\ln \lambda}$ be the log-productivity gap in market i, which we refer to as the "state variable" of market i at time t. It can be shown that, under either Bertrand or Cournot competition, profits for both firms, as a share of aggregate output $Y(t)$, can be written as a function of only the state variable s.

It is easy to see that the supply-side of this general equilibrium model—and the associated production and investment decisions—are nested under the model we provide in the main text, generating an inverted-U relationship between steady-state productivity growth rate g and inter-

est rate r. The Euler equation of the consumer follows

$$g(t) = \dot{Y}(t) = \frac{1}{\theta}(r(t) - \rho),$$

which is an upward-sloping relationship in the g–r space. On a balanced growth path, both g and r are constant, along with a stable distribution of productivity gaps across markets in the economy.

Interest rate shocks that we refer to in the main text can be simply seen as shocks to consumer discount rate ρ (as in Krugman (1998)). An issue with this interpretation is that interest rate is bounded below by the discount rate ρ, and any positive growth rate is incompatible with interest rate being close to zero even as $\rho \to 0$. This issue is an artifact of the demand-side being frictionless; any incomplete markets and idiosyncratic risk would generate additional terms in the Euler equation that push down the real interest rate for any level of growth rate. For instance, Benigno and Fornaro (n.d.) use idiosyncratic, uninsurable unemployment risk to microfound the following Euler equation

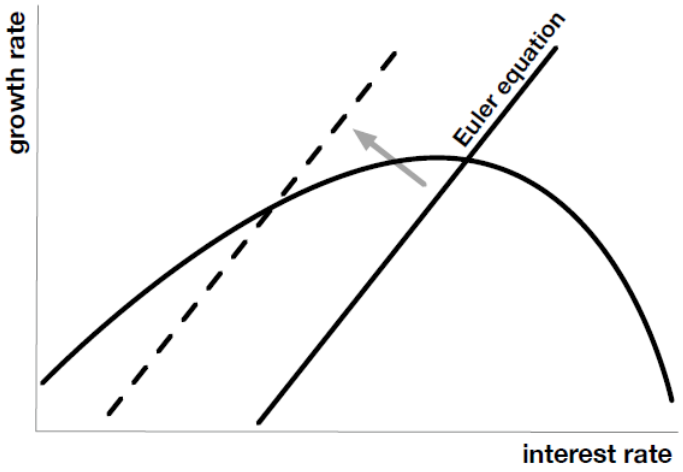
$$g(t) = \frac{1}{\theta}(r(t) - \rho + b).$$

The term b measures the severity of the unemployment risk under their specific microfoundation model, but it can be more broadly seen as a catch-all term for any shock on the demand side that pushes consumers towards savings more and consuming less, including changes in preferences, tightened borrowing constraints (e.g. Eggertsson and Krugman (2012)), or structural shifts such as an aging population and rising inequality (e.g. Summers (2014)).

Figure 5 shows the demand curve with the upward sloping vertical line. An inward shift in the demand curve lowers interest rate and increases concentration. If prevailing interest rates are low, i.e. economy is in the upward sloping region of the supply curve, then a fall in interest rate is also contractionary as productivity growth slows down.

Our model presents an alternative interpretation of "secular stagnation". As in traditional secular stagnation explanations, an initial inward shift in demand curve can lower equilibrium interest rate to very low levels. However, "stagnation" does not have to come from monetary constraints such as zero lower bound or nominal rigidities. Instead, a large fall in interest rates can make the economy more monopolistic for reasons laid out in our model, thus lowering innovation

Figure 5: Supply and demand relationships in growth models



and productivity growth as well.

4 Testing Model's Empirical Predictions

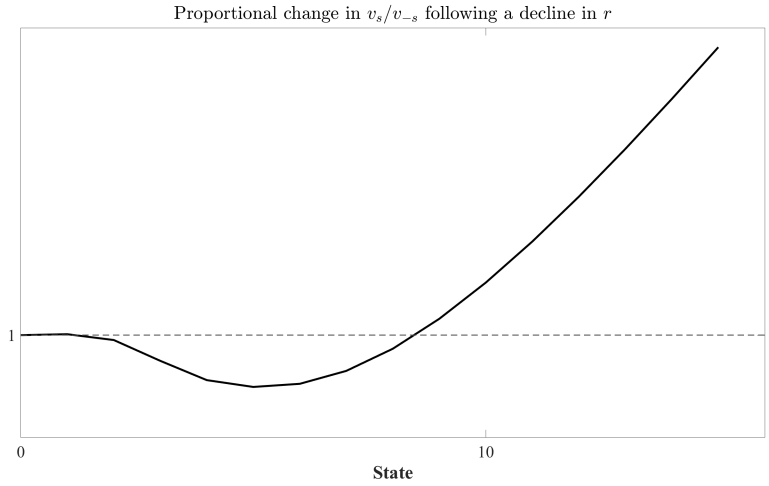
How plausible is the supply-side mechanism that links lower interest rates with increased concentration and market power? We first show that our model generates unique predictions on how the relative market values of leaders versus followers in an industry respond to a reduction in interest rates. We then test these predictions using data from publicly listed firms in the U.S.

4.1 Interest rate shocks: asymmetric effects on firm value

How does a fall in interest rate affect valuation of industry leader versus follower in our model? A decline in r lowers the discount rate of future cash flows, raising market values of all firms. Intuitively, the effect is especially pronounced for leaders near the right tail of the monopolistic region, in which the leader is expected to enjoy long periods of high rents while facing little threats of the follower catching up. Conversely, the valuation gain for the leader is relatively muted towards the left end of the monopolistic region and could even be lower, relative to the gain experience by the follower, in the competitive region. Over these states, the follower's investment phases out in the pre-shock equilibrium but is especially responsive to declines in interest rate,

thereby posing significant threats for the leader to lose dynamic rents in the new equilibrium. Figure 6 illustrates these intuitions and plots, as a function of the state variable, the on-impact changes in relative valuations between leaders and followers ($\frac{v'_s}{v_s} \Big/ \frac{v'_{-s}}{v_{-s}}$, where v'_s indicates the value function in state s after a decline in r).

Figure 6: Relative changes in leader's and follower's value function following a decline in r

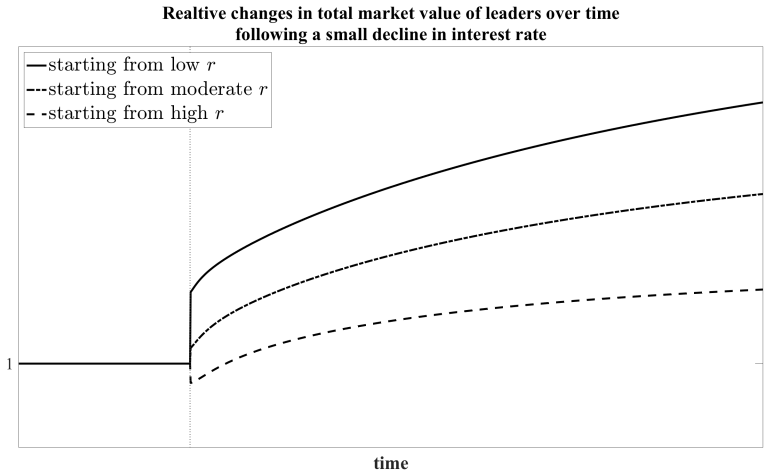


Now consider aggregating the asymmetric response to the entire economy. Suppose a decline in interest rate happens to a steady-state economy at time $t_0 > 0$, and let $\chi(t) \equiv \frac{\sum_{s=0}^{\infty} \mu_s(t) v_s(t)}{\sum_{s=0}^{\infty} \mu_s(t) v_{-s}(t)} \Big/ \frac{\sum_{s=0}^{\infty} \mu_s(0) v_s(0)}{\sum_{s=0}^{\infty} \mu_s(0) v_{-s}(0)}$ track the over-time proportional changes in the *total* market valuation of all leaders relative to all followers, as compared to the relative valuation in the pre-shock steady-state.

$\chi(t)$ is equal to one before t_0. Following a decline in interest rate at t_0, market valuation changes immediately, resulting in a discrete jump in $\chi(t_0)$, the magntiude of which is an average of the on-impact valuation response in each individual state, weighted by the distribution over market structure in the initial steady-state. If the initial level of interest rate is high, the steady-state features significant mass of markets in the competitive region and small fractions of markets towards the right-end of the monopolistic region. Consequently, the average leader in the economy could experience losses in valuations relative to the average follower. In this case, χ jumps to below one at time t_0, as it places little weights over the states in which the leader experiences large valuation gains. Conversely, under low levels of initial r, a decline in r can significantly raise $\chi(t_0)$ above one, for there are large fractions of markets with high monopoly power, and leaders in these states experience relatively large gains. When r is close to zero, the jump in χ has to be above one. To

demonstrate these intuitions, figure 7 plots the time path of $\chi(\cdot)$ following a sudden and permanent decline in r, starting from three different levels of initial interest rates.

Figure 7: market value of leaders respond more to decline in r, especially when initial r is low



Note that, over any time interval $[t, t+\Delta]$, market structure is ergodic and the identity of market leaders does not necessarily stay constant as time passes. This is why at very high interest rate, relative valuation of leader actually declines with a fall in interest rate. Nevertheless, when the shock initially hits, leaders at time $t_0 - \epsilon$ tend to remain leaders at time $t_0 + \epsilon$ for small enough $\epsilon > 0$. Hence, the asymmetric response in market valuations, as illustrated in figure 7, enables us to perfom a tight empirical test of our model mechanism through sudden changes in stock market returns. That is, starting from a moderate level of interest rate and following an unexpected, further decline in long-run r, market leaders should experience significant excess returns relative to market followers. More importantly, the excess return for leader verus follower should be *higher* when the level of interest rates is lower. This last prediction is the strongest test of our model's dynamics. We implement a triple difference-in-differences specification below to test this key prediction.

4.2 Empirical Results

We use CRSP-Compustat merged data set from 1980 onwards to compute excess returns for industry leaders versus followers in response to an interest rate shock. We use the 10-year tresury

yield as our default measure of long-run interest rate, and show robustness to alternative duration of interest rate yields[7]. The 10-year yield has the advantage that it has the longest available historical data at the long-end of the yield curve. We use Fama French definition of industries as our default, but show robustness to SIC industry definition as well.

We define "industry leaders" using a few different approaches. Our default leader definition is based on size. We classify stocks as industry leaders if they are in the top 5 percent of firms in the industry based on market value at the beginning of when excess returns are computed. We show robustness to a number of different leader-classifications as well. Since number of firms in an industry can vary significantly, we also define top 5 firms in an industry as leaders instead of top 5 percent. We also use EBITDA and sales to sort firms instead of market value. Our results are robust to these alternative definitions.

The key empirical test implied by the model at the firm level can be written as,

$$\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t}\Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t}\Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t} \tag{4}$$

where $\Delta \ln\left(P_{i,j,t}\right)$ is defined as the log change in the stock price for firm i in industry j from date $t-91$ days to t (i.e. one quarter growth). $D_{i,j,t}$ is an indicator variable equal to 1 if firm i is in the top 5% of market capitalization in its industry j at date $t-91$. Firms with $D_{i,j,t}$=1 are called leaders while the rest are called followers. i_t is the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. X are potential control variables such as firm leverage interacted with both the level of interest rate and the change in interest rate. All regressions are value-weighted and standard errors are dually clustered by industry and date.

$\alpha_{j,t}$ are industry-date fixed effects. There are two important coefficients of interest. β_3 estimates whether leaders' excess return is stronger when the level of interest rate is lower, which is the key prediction of our model. β_1 estimates the implied excess return for leaders at the zero lower bound.

[7]We use the nominal rate given the difficulty in measuring real rate. Since inflation expectations have been well-anchored during the time period we analyze, we feel nominal yield is the appropriate measure to use given the measurement error tradeoff.

Table 1 shows the results of estimating (4) on our merged CRSP-Compustat data set from 1980 onwards. We only display the relevant coefficients in the tables, but the actual regression includes all variables specified in the equation. Column (1) estimates equation (4) without interactions with the level of interest rate. The coefficient β_1 is negative and significant, i.e. leaders stock value rises by more on average in response to a decline in interest rate. Thus leaders earn positive excess return in general when interest rate falls.

Column (2) includes interactions with the level of interest rate as well and hence estimates the full specification (4). The coefficient β_3 is positive and significant as predicted by our model, i.e. leaders excess return is *higher* in response to a fall in interest rate when the level of interest rate is lower. This is succinctly captured by β_1 that captures the leaders excess return at the zero lower bound. The excess return at zero lower bound in column (2) (3.88) is three times the average excess return of 1.19 in column (1).

There may be a concern that our measure of industry leaders is spuriously correlated with balance sheet factors that are more sensitive to interest rate movements. For example, perhaps leaders are more levered and a fall in interest rate helps lower the interest burden for them. To test for this, and other related concerns, we include a number of firm level characteristics as controls by including all the interaction of the firm level characteristic with the change in interest rate as well as the level of interest rate. We include the following firm-level characteristics, firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and percent of pre-tax income that goes to taxes. The number of observations decreases becasue we have to limit sample to Compustat firms with the available data on firm financials. Column (3) shows that the inclusion of this extensive list of firm-level controls does not change our coefficients of interest materially.

Column (4) provides another example of controlling for a potentially spurious firm-level attribute. What if industry leaders are spuriously more cyclical? If a fall in interest rate represents changing economic expectations, industry leaders might generally be more responsive to changing market conditions, irrespective of the level of interest rate. To test for this possibility, we take market beta of each firm, estimated using historical data as of $t-1$, and interact a firm's market beta with both the change in interest rate and the level of interest rate in column (4). As before, our main coefficients of interest are not materially affected. Table A1 in the appendix also shows

robustness of our main findings to alternative definitions of industry leader: top 5 instead of top 5 percent, SIC instead of Fama French industries, and sorting on EBITDA and sales instead of market value.

Table 2 performs an equivalent time-series version of the excess return test implemented in table 1. In particular, the results are based on the following specification,

$$R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t \tag{5}$$

where R_t is the market-capitalization weighted average of returns for a stock portfolio that goes long in industry-leader stocks and goes short in industry-follower stocks from date $t-91$ to t. Since observations have overlapping differences, we compute standard errors using Newey-West procedure with a maximum lag length of 60 days to account for built-in correlation. β_2 estimates whether excess return for leaders is stronger at lower interest rates, while β_1 estimates the implied excess return for leaders in response to a fall in interest rates at the zero lower bound.

Columns (1) and (2) confirm our earlier results. A fall in interest rate is associated with positive excess return for industry leaders, and the excess return is larger in magnitude the lower in the level of interest rate. Column (3) shows that our results are not driven by excess market return of HML factors. We obviously cannot include small minus big size factor since that is very highly correlated with our portfolio measure. Column (4) shows that our results are driven by both positive and negative change in interest rate. In particular, the excess return results are materially unchanged whether we focus on only positive changes in interest rate or only negative changes in interest rate.

There may be a concern that industry leaders tend to have high price-to-earnings ratio and such growth firms benefit disproportionately from a decline in long-term interest rates since more of their earnings are in the future. We test for this concern by constructing a "PE portfolio" that is long the top 5% of firms by PE in an industry and short the rest. However, including the PE portfolio return does not change our coefficients of interest, and our leader-minus-follower portfolio is itself negatively corrected with the PE portfolio.

Our default specification constructs returns and interest rate changes at quarterly frequency. We think this is the appropriate frequency for us since we want to capture interest rate movements that are deemed more permanent and also want to allow sufficient time for the markets to incorporate any perceived advantages due to market competition. Figure 8 plots the histograms of interest rate changes in our sample, from daily to annually frequency. On average interest rates went down during this time period. However, there is substantial variation with the change in interest rate being positive on a high fraction of days. As we have already shown, out key findings are symmetric to whether the change in interest rate is positive or negative.

As we move from daily to annual frequency, the range of interest rate changes naturally increases. This is another reason to focus on longer term differences since we want market to incorporate sufficiently large change in interest rate when forming expectations. In any event, in the interest of full transparency, table A2 in the appendix repeats our core specification for interest rate changes at frequencies ranging from daily to annual. For reasons discussed above, the effect tends to be stronger when interest rate change is computed over longer horizons.

So far we have used the 10-year treasury rate as our default measure of interest rate shocks. Do the excess return results depend on whether interest rate shock is at the short versus the long end of the yield curve? Statistically this is a somewhat hard test to perform since interest rate movements along the yield curve tend to be stronger correlated. Table A3 in the appedix shows the correlation matrix of quarterly changes in forward rates of varying non-overlapping durations. The correlations are generally quite high, leading to problems of collinearity in joint testing. The lowest correlation is in the range of 0.7 to 0.75 between change in 0-2 forward rate and longer term forward rates (e.g. 10-30).

Table 3 estimates equation (5) using forward rate of varying duration. The main takaway is that our result holds for interest rate shocks throughout the yield curve (columns (1) through (6)). When we put the interest rate shock from both 0-2 and 10-30 forward rates together in columns (7) and (8), the result shows that both ends of the yield curve are independently important, with some evidence that the longer end of the yield curve is more important.

Figure 9 shows are key result in a more non-parametric fashion. The top panel plots the follow-

ing two variables against each other: (i) rolling 2-year correlation of interest rate change ($t-91$ days to t) and industry leader minus follower excess returns over the same period, and (ii) 10-year treasury yield at $t-91$. There is a strong positive correlation between the two variables. As the level of interest rate declines, the correlation measured on the y-axis becomes more negative, i.e. excess return is higher in response to an interest rate decline.

The bottom panel plot the coefficients $\{\beta_{2,j}\}$ of the specification $R_t = \sum_{j=1}^{4} \beta_{1,j} D_t\{4(j-1) <= i_{t-1} <= 4j\} + \sum_{j=1}^{4} \beta_{2,j} D_t\{4(j-1) <= i_{t-1} <= 4j\}\Delta i_t + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. $D_t\{4(j-1) <= i_{t-1} <= 4j\}$ is defined here as an indicator variable equal to 1 when the interest rate 91 days prior is between $4(j-1)$ and $4j$ for$j = 1, 2, 3, 4$. Consistent with the top panel, the bottom panel shows that response of excess return to interest rate decline monotonically strengthens as the level of interest rate declines.

Figure 10 plots the implied impulse response function of changes in interest rate when interest rate is at zero. In particular, the figue plots the coefficients $\{\beta_{0,j}\}$ of the specification $R_{t+j} = \alpha_j + \beta_{0,j}\Delta i_t + \beta_{1,j}\Delta i_{t-1} + \beta_{2,j}\Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_{t+j} is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date t to $t+j$. Δi_t is defined here as the change in the interest rate from date t to $t+91$. The impulse response function shows that excess return at zero interest rate continues to build up through the 91 days over which the change in interest rate is computed. Once the shock is over, the excess return is persistent.

5 Macro empirical evidence

The previous section focused on firm-level predictions of our model in response to interest rate shock. In this section we discuss how our model unifies a number of important macro trends in a single coherent framework.

A central prediction of the model is that lower interest rate increases the share of industries that are in the monopolistic region of the state space. As a result, lower interest rates should be

associated with higher profit share, higher markups, and higher concentration. The left panel of figure 11 plots the profit share of GDP against the 10-year nominal U.S. Treasury rate and shows that there is indeed a sharp rise in profit share as long-term interest rate moved towards zero. The right panel plots the share of market value that goes to the top-5 firms within an industry against the 10-year treasury rate. There is a clear trend towards increase in market concentration as the long-term rate declines.

Our model also shows that as interest rate declines, "business dynamism", defined as the likelihood of follower overtaking the leader declines. We proxy for firm dynamism using establishment entry and exit information for the United States from 1985 to 2014[8]. Figure 12 shows that lower interest rates are associated with both a decline in the entry and exit rates of establishments in the United States. Our interpretation of this decline in business dynamism is that it reflects higher market power of industry leaders and the reduced incentives to enter new markets by followers.

A more novel prediction of our model is that as interest rate declines and moves towards zero, the steady state average gap in productivity growth between leaders and followers widens. Using firm level productivity data from OECD countries, Berlingieri and Criscuolo (2017) provide evidence in support of this prediction. The two panels of Figure A1 summarize their findings. The left panel uses data from the OECD multi-prod data set and shows that the gap between the labor productivity of market leaders and followers increases as interest rates decline over the 2000s. Leaders are defined to be firms in the 90th percentile of the labor productivity distribution for a given 2-digit industry, and followers are defined to be firms in the 10th percentile. The gap between leaders and followers has been increasing steadily from 2000 to 2014 as long term interest rates have fallen.

The right panel extracts the year fixed effects from a country by industry level panel containing this same gap. As it shows, there is a steady increase in the productivity gap between market leaders and followers across countries and industries. A similar result is in Andrews et al. (2016), who show a widening gap in labor productivity of frontier versus laggard firms in both manufacturing and services for OECD firms. Andrews et al. (2016) also show cross-sectional evidence supporting

[8]The establishment entry and exit rates time series for the United States comes from the US Census' Business Dynamics Statistics database.

the predictions of the model. They show that industries in which the gap between leader and follower multi-factor productivity is rising the most are the same industries where sector-aggregate multi-factor productivity is falling the most. This empirical finding is constistent with the model. Low interest rates lead to a large gap between the leader and follower in productivity investment; as a result, total investment in productivity of the industry falls.

Finally, our model showed that as interest rate gets close to zero, productivity growth also slows down due to the accumulated rise in market concentration. This is also consistent with the growth slowdown globally. The left panel of figure 13 plots log total factor productivity for OECD countries and the 10-year nominal U.S. Treasury rate against time. As is well-known in the literature, and apparent from the figure, there is a marked slowdown in productivity growth around 2005 - well before the Great Recession. This suggests that there is a common global factor, not related to the Great Recession, that may be responsible for the growth slowdown. Our model suggests the very low level of long-term rates as one possible factor. The right panel plots the OECD average productivity growth rate against the 10-year rate and shows the slowdown in productivity growth as interest rate gets close to zero[9].

In summary, lower interest rates are associated with higher market concentration, a decline in business dynamism, a widening of the gap between the productivity of leaders and followers, and a slowdown in productivity growth. This is true both in the model and in the data.

6 Conclusion

The focus of this paper is on understanding how the supply-side of the economy responds to a reduction in long-term interest rates driven by demand-side forces. The existing literature in growth either assumes no supply-side response to declining interest rates, or a positive response driven by increased incentive to invest in the face of higher discounted present value of future profits. Our point of departure from this literature lies in explicitly modeling competition within

[9] As in the model, investment as a share of GDP has also declined with lower interest rates. In related papers, Jones and Philippon (2016) show that increase in industry concentration is associated with lower firm investment and Gutiérrez and Philippon (2016) show that investment is not correlated with market valuation and profitability after 2000s.

an industry and analyzing how lower interest rates effect the nature of competition. We build on the dynamic contests literature to show that in a fairly general set up and without relying on any financial or other forms of frictions, the effect of lower interest rates on growth in a low interest rate regime can be negative.

A reduction in long-term interest rates tends to make market structure less competitive within an industry. The reason is that while both leader and follower within an industry increase their investment in response to a reduction in interest rates, the increase in investment is always stronger for the leader. Thus in equilibrium the gap between the leader and follower increases as interest rate declines, making an industry less competitive and more concentrated. This negative effect of lower interest rates on industry competition tends to lower growth and overwhelms the traditional positive effect of lower interest rates on growth when interest rates are sufficiently low. There is thus a hump-shaped inverted-U supply-side relationship between growth and interest rate in our framework.

Our model delivers a distinctive *upward* sloping supply-curve in a low interest rate regime. We think that this insight is useful in understanding the slowdown in productivity growth in recent decades and the broader discussion regarding "secular stagnation". The slowdown in productivity growth is global as it shows up in almost all advanced economies. The slowdown started well before the Great Recession, suggesting that cyclical forces related to the crisis are unlikely to be the trigger. And the slowdown in productivity is highly persistent, lasting well-over a decade. Thus explanations that rely on traditional Keynesian forces such as price stickiness or zero lower bound are less likely to be sufficient.

This paper introduces the possibility of low interest rates as the common global "factor" that drives the slowdown in productivity growth. The mechanism that our theory postulates delivers a number of important predictions that are supported by empirical evidence. A reduction in long term intrest rate increases market concentration and market power in our model. A fall in interest rate also makes industry leadership and monopoly power more persistent. There is strong support for these predictions in the data, both in aggregate time series as well as in firm-level panel data sets.

For example, productivity gap has indeed expanded between leaders and followers, and industries where this gap has widened the most tend to have slower aggregate growth. Similarly, market value of leaders within an industry has risen relative to followers as long-term interest rate declined. All of these are quite unique predictions of our model that are confirmed in data.

A natural implication of our analysis is that we need to understand more carefully the forces behind the reduction in long-term interest rate in the first place. This is a fruitful area of research going forward.

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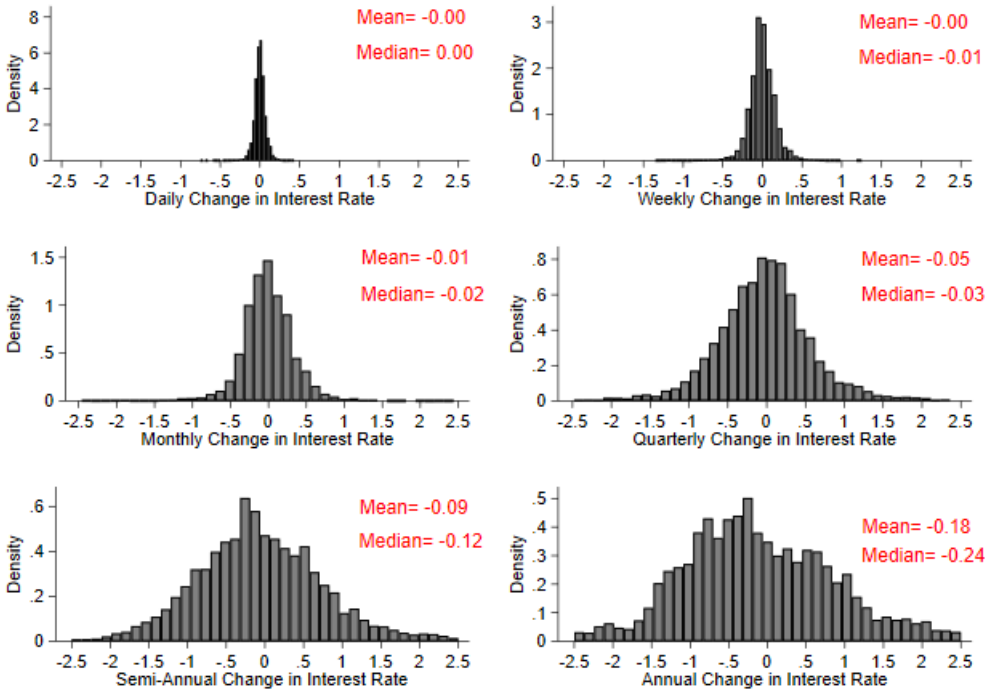
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Figure 8:

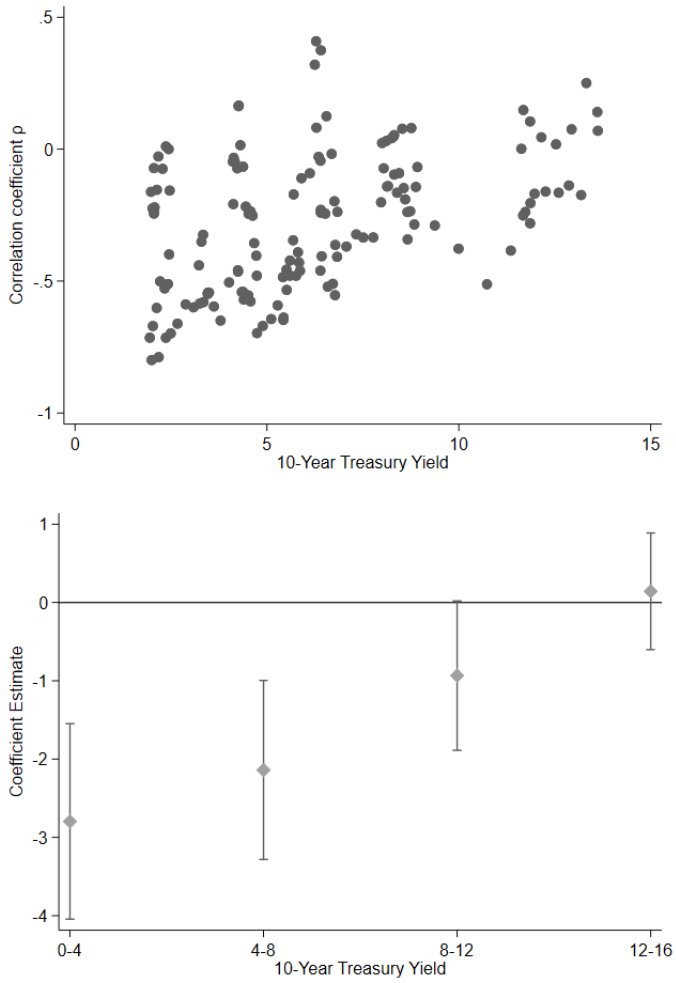
Distribution of Interest Rate Changes at Varying Frequencies



The panels plots the histograms of interest rate changes in our sample, from daily to annually.

Figure 9:

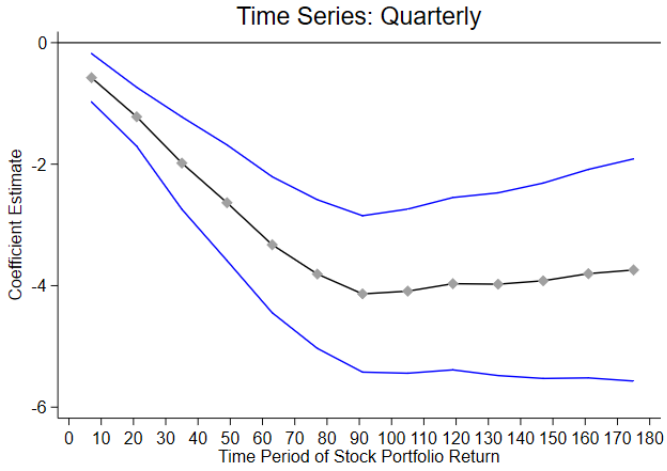
Leaders See Higher Returns from a Drop in Interest Rates as Interest Rate Goes to Zero



The top panel plots the rolling 2-year correlation of interest rate changes and top-5 portfolio returns against the 10-year Treasury yield 91 days prior. The bottom panel plot the coefficients $\{\beta_{2,j}\}$ of the specification $R_t = \sum_{j=1}^{4} \beta_{1,j} D_t\{4(j-1) <= i_{t-1} <= 4j\} + \sum_{j=1}^{4} \beta_{2,j} D_t\{4(j-1) <= i_{t-1} <= 4j\}\Delta i_t + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. $D_t\{4(j-1) <= i_{t-1} <= 4j\}$ is defined here as an indicator variable equal to 1 when the interest rate 91 days prior is between $4(j-1)$ and $4j$ for$j = 1, 2, 3, 4$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-91$. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Figure 10:

Impulse Response of Changes in Interest Rate when Rate is Zero



The figue plots the coefficients $\{\beta_{0,j}\}$ of the specification $R_{t+j} = \alpha_j + \beta_{0,j}\Delta i_t + \beta_{1,j}\Delta i_{t-1} + \beta_{2,j}\Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_{t+j} is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date t to $t+j$. Δi_t is defined here as the change in the interest rate from date t to $t+91$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date t. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Figure 11: Aggregate profit share, market concentration and interest rate

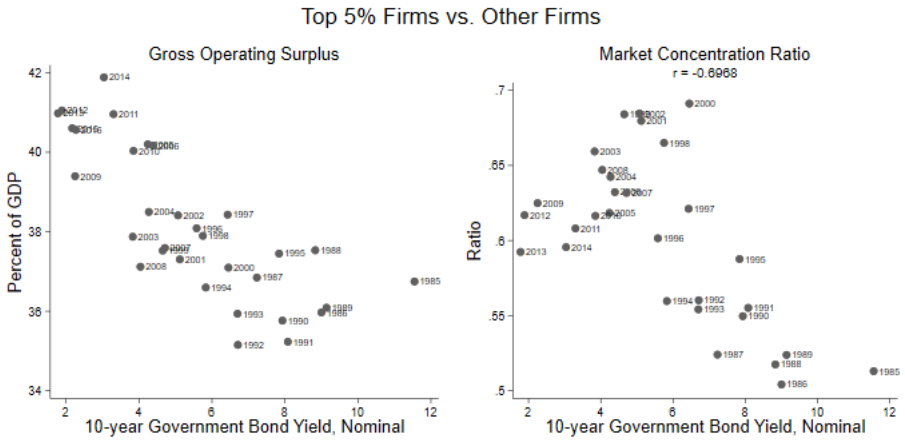


Figure 12: Business Dynamism

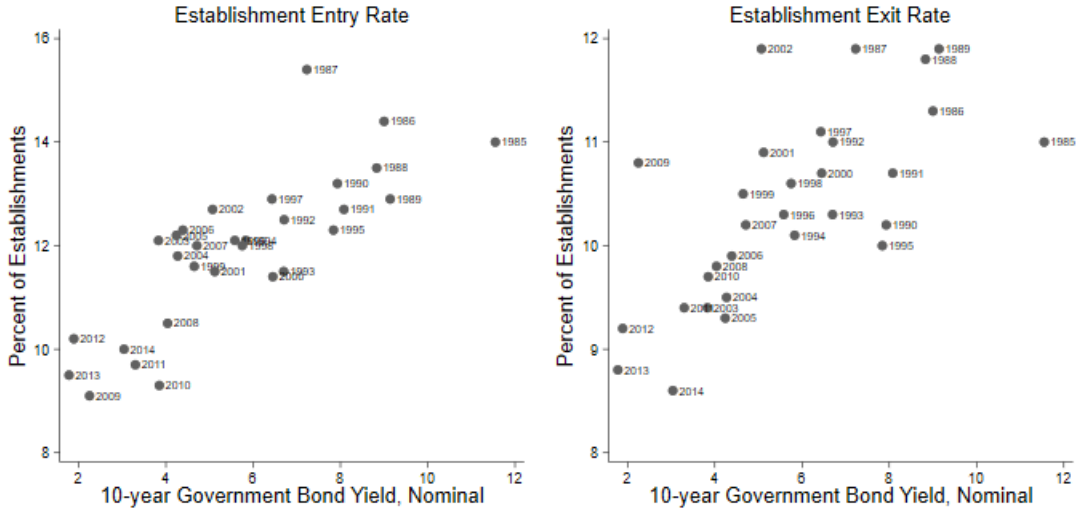


Figure 13: Productivity growth and interest rates

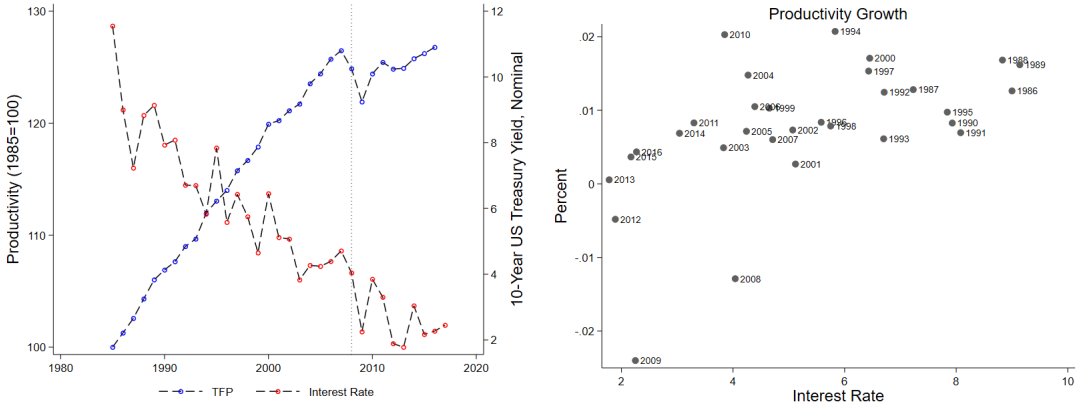


Table 1: Differential Interest Rate Responses of Leaders vs. Followers: Top 5 Percent

	Stock Price Growth			
	(1)	(2)	(3)	(4)
Top 5 Percent=1 x Δi	-1.187***	-3.879**	-4.407***	-4.181***
	(0.260)	(1.113)	(0.842)	(0.529)
Top 5 Percent=1 x Δi x Lagged i		0.293**	0.346***	0.301***
		(0.095)	(0.075)	(0.045)
Firm β x Δi				14.10***
				(0.794)
Firm β x Δi x Lagged i				-1.260***
				(0.082)
Sample	All	All	All	All
Controls	N	N	Y	
Industry-Date FE	Y	Y	Y	Y
N	61,313,604	61,313,604	44,568,088	61,299,546
R-sq	0.404	0.404	0.417	0.410

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t} \Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t} \Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t}$ for firm i in industry j at date t. $\Delta \ln\left(P_{i,j,t}\right)$ is defined here as the log change in the stock price for firm i in industry j from date $t-91$ to t (one quarter growth). $D_{i,j,t}$ is defined here as an indicator equal to 1 at date t when a firm i is in the top 5% of market capitalization in its industry j on date $t-91$. Firms with $D_{i,j,t}$=1 are called leaders while the rest are called followers. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. Controls X include a firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and percent of pre-tax income that goes to taxes. Industry classifications are the Fama-French industry classifications (FF). Standard errors are dually clustered by industry and date.

Table 2: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent

	Portfolio Return				
	(1)	(2)	(3)	(4)	(5)
Δi_t	-1.152***	-3.815***	-2.263***	-3.654***	-3.212***
	(0.309)	(0.641)	(0.601)	(0.948)	(0.775)
i_{t-1}		0.0829	0.0323	0.159*	0.146*
		(0.050)	(0.045)	(0.071)	(0.071)
$\Delta i_t \times i_{t-1}$		0.293***	0.116*	0.327***	0.262**
		(0.059)	(0.056)	(0.081)	(0.101)
Excess Market Return			-0.168***		
			(0.023)		
High Minus Low			0.0368		
			(0.045)		
$(\Delta i_t > 0)$=1 $\times$ Δi_t				0.346	
				(1.715)	
$(\Delta i_t > 0)$=1 $\times$ $\Delta i_t \times i_{t-1}$				-0.102	
				(0.170)	
PE Portfolio Return					-0.278***
					(0.075)
N	9,016	9,016	9,016	9,016	7,402
R-sq	0.044	0.089	0.229	0.092	0.195

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-91$. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. Standard errors are Newey-West with a maximum lag length of 60 days prior. In column 4, the terms $(\Delta i_t > 0) = 1$ and $(\Delta i_t > 0) = 1$ x i_{t-1} were suppressed from the table. Their coefficients are 0.0222 (0.602) and -0.0616 (0.086), respectively.

Table 3: Portfolio Returns Response to Interest Rate Changes: Along the Yield Curve

	30-Year		2-Year		10-30 Forward		2-Year & 10-30 Fwd.	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δi_t	-1.130**	-4.531***						
	(0.349)	(0.825)						
$\Delta i_t \times i_{t-1}$		0.361***						
		(0.077)						
$\Delta i_{t,0,2}$			-0.587*	-3.532***			-0.131	-2.063*
			(0.244)	(0.834)			(0.349)	(0.971)
$\Delta i_{t,0,2} \times i_{t-1}$				0.280***				0.145
				(0.069)				(0.081)
$\Delta i_{t,10,30}$					-1.085**	-4.158***	-0.932	-3.138**
					(0.355)	(0.835)	(0.524)	(1.044)
$\Delta i_{t,10,30} \times i_{t-1}$						0.333***		0.290**
						(0.079)		(0.108)
N	8,006	8,006	8,065	8,065	8,006	8,006	8,006	8,006
R-sq	0.036	0.079	0.022	0.063	0.031	0.066	0.031	0.084

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date t in columns 1-2 and $R_t = \alpha + \beta_0 i_{t-1} + \beta_{1,1} \Delta i_{t,0,2} + \beta_{1,2} \Delta i_{t,10,30} + \beta_{1,3} \Delta i_{t,0,10} + \beta_{2,1} \Delta i_{t,0,2} i_{t-1} + \beta_{2,2} \Delta i_{t,10,30} i_{t-1} + \beta_{2,3} \Delta i_{t,0,10} i_{t-1} + \varepsilon_t$ at date t in columns 3-8. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t - 91$ to t. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t - J$. i_t is defined as the nominal 30-year Treasury yield, with i_{t-1} being the interest rate J days prior and Δi_t being the change in the interest rate from date $t - 91$ to t. $i_{t,0,2}$, $i_{t,0,10}$ and $i_{t,10,30}$ are the 2-year and 10-year Treasury yield and 10 to 30 forward Treasury yield, respectively. Standard errors are Newey-West with a maximum lag length of 60 days prior. We cannot reject that the main and interaction coefficients in columns 7 and 8 are not equal.

Appendix A

Figure A1: Widening productivity gap between leaders and followers

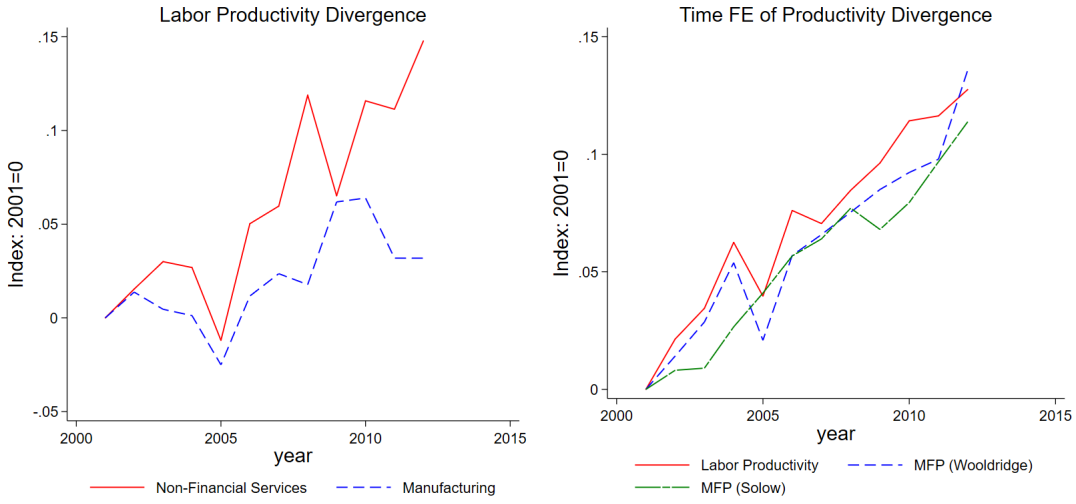


Figure A2: Top 5% average wage-employment ratio and interest rates

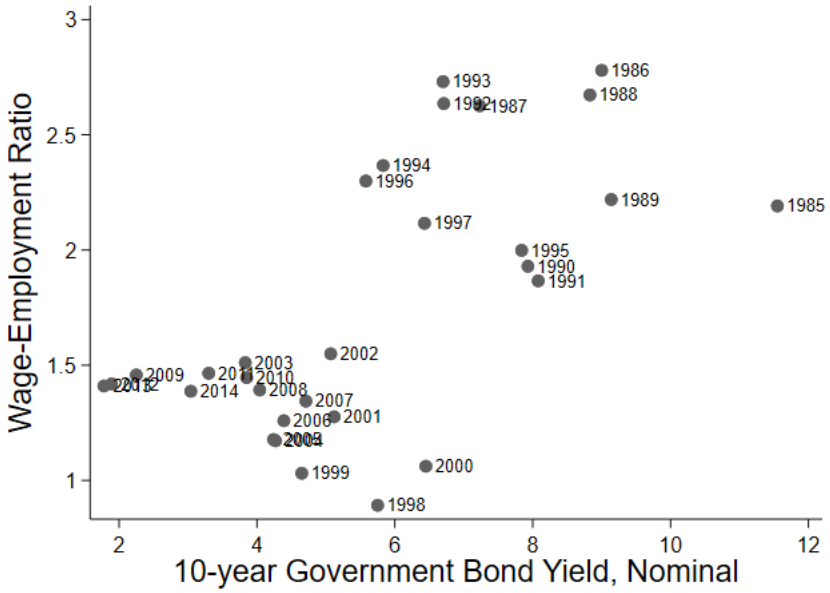
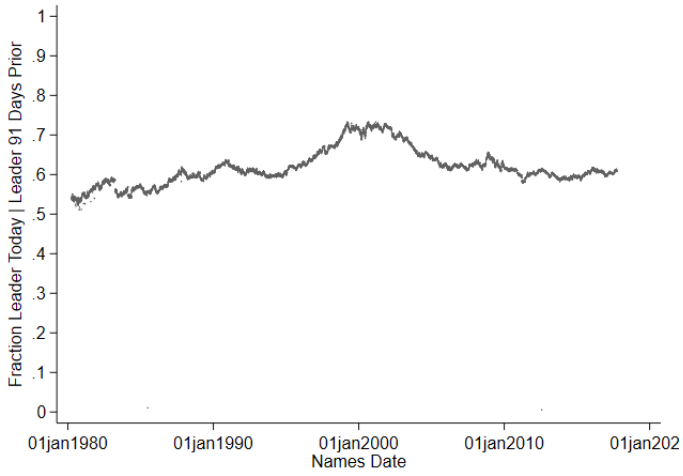


Figure A3: Fraction of Leaders who were Leaders a Quarter Prior



The panels plots the time series of the proportion of firms who are leaders today, given they were leaders 91 days prior.

Table A1: Differential Interest Rate Responses of Leaders vs. Followers: Robustness Checks

	Top 5		SIC		EBITDA		SALES	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Top 5 Percent=1 x Δi	-1.107***	-3.846**	-1.203***	-3.900***	-1.502***	-4.766***	-1.232**	-3.755**
	(0.273)	(1.220)	(0.222)	(0.936)	(0.290)	(1.080)	(0.353)	(1.314)
Top 5 Percent=1 x Δi x Lagged i		0.303**		0.293***		0.369***		0.283*
		(0.105)		(0.081)		(0.092)		(0.111)
Sample	All	All	All	All	All	All	All	All
Industry-Date FE	Y	Y	Y	Y	Y	Y	Y	Y
N	61,313,604	61,313,604	61,277,070	61,277,070	39,419,672	39,419,672	48,776,553	48,776,553
R-sq	0.404	0.404	0.404	0.405	0.428	0.429	0.412	0.413

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t} \Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t} \Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t}$ for firm i in industry j at date t. The definitions are the same as in Table 2 except for $D_{i,j,t}$. In columns 1 and 2, leaders are chosen by the top 5 number of firms by market capitalization within an industry and date. In columns 3 and 4, leaders are chosen by the top 5% of firms by market capitalization within an industry and date, where we change the definition of industry to be the 2-digit Standard Industry Classification (SIC) codes. In columns 5 and 6, leaders are chosen by the top 5% of firms by earnings before interest, taxes, depreciation, and amortization (EBITDA) within an industry and date. In columns 7 and 8, leaders are chosen by the top 5% of firms by sales within an industry and date. Standard errors are dually clustered by industry and date.

Table A2: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent, Different Frequencies

	Yearly		Semi-Yearly		Monthly		Weekly		Daily	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Δi_t	-1.060**	-5.569***	-1.187***	-4.593***	-1.005***	-2.383***	-0.991***	-1.823***	-0.887***	-1.263***
	(0.403)	(1.134)	(0.345)	(0.764)	(0.197)	(0.463)	(0.169)	(0.309)	(0.162)	(0.202)
i_{t-1}		0.380**		0.148		0.0257		0.0127		0.00361
		(0.134)		(0.080)		(0.019)		(0.008)		(0.003)
$\Delta i_t \times i_{t-1}$		0.493***		0.385***		0.151***		0.0929**		0.0437
		(0.106)		(0.073)		(0.040)		(0.034)		(0.027)
Sample	All	All	All	All	All	All	All	All	All	All
N	9,037	9,037	8,962	8,962	9,081	9,081	9,099	9,099	9,080	9,080
R-sq	0.024	0.095	0.040	0.101	0.036	0.051	0.019	0.023	0.008	0.009

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t - 91$ to t. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t - J$. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate J days prior and Δi_t being the change in the interest rate from date $t - 91$ to t. For columns 1 and 2, $J = 364$; columns 3 and 4, $J = 28$; columns 5 and 6, $J = 7$; columns 7 and 8, $J = 1$, where 1 is one trading day. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Table A3: Correlation Table of Forward Rates

Variables	0-2	2-3	3-5	5-7	7-10	10-30
0-2	1.00					
2-3	0.88	1.00				
3-5	0.82	0.93	1.00			
5-7	0.71	0.84	0.88	1.00		
7-10	0.70	0.82	0.83	0.86	1.00	
10-30	0.75	0.84	0.89	0.92	0.93	1.00

Correlation table of forward rates. P-values in parentheses.

Appendix B: Proofs

Let $\lim_{s\to\infty}\pi_s = \pi$, $\lim_{s\to\infty}\pi_{-s} = 0$. Throughout, we assume investment upperbound is high enough, so that $(2+\alpha)\,c > \pi$. Throughout, let $\sim$ denote asymptotic equivalence, i.e. $x \sim y$ iff $\lim_{r\to 0}(x-y) = 0$.

Proposition 1. *Let n and k respectively be the last state in which the leader and the follower investments. Then $n \geq k$.*

Proof. Note that when discount rate r is sufficiently high, the only equilibirum that survives would be the trivial one, in which no firm invests at all. Using the bruteforce approach, we can find upperbounds on r such that firms start to invest in state 0, 1,..., and verify by enumeration that the result holds for these high discount rate. We now prove the generic (and more relevant) case $r < \kappa + \eta$.

First, suppose $n < k-1$. Then

$$\begin{aligned} v_{-(k-1)} - v_{-k} &> c/\eta > v_k - v_{k-1} \\ v_{k-1} + v_{-(k-1)} &> v_k + v_{-k} \end{aligned}$$

a contradiction to monotonicty in investment decisions. Next suppose $n = k-1$. Holding value functions in other states constant, the value in state v_{-k} would have been $\frac{\kappa}{r+\kappa}v_{-(k-1)}$ if follower does not invest. Hence

$$\frac{rv_{-(k-1)}}{r+\kappa} > v_{-(k-1)} - v_{-k} > c/\eta$$

or

$$rv_{-(k-1)} > c\frac{r+\kappa}{\eta}.$$

Thus

$$v_{-k} = \frac{-c+\kappa v_{-(k-1)}}{r+\kappa} > \frac{-c}{r+\kappa} + c\frac{\kappa}{r\eta}$$

The fact that leader does not invest in state k shows

$$\underbrace{\frac{\pi+\kappa v_k}{r+\kappa}}_{\leq v_{k+1}} - v_k < c/\eta$$

or $rv_k \geq \pi - c\frac{r+\kappa}{\eta}$. Since $v_k = \frac{\pi+(\kappa+\eta)v_{k-1}}{r+\kappa+\eta}$, we know

$$rv_{k-1} \geq \pi - c\frac{(r+\kappa+\eta)}{(\kappa+\eta)}\frac{r+\kappa}{\eta}$$

Thus

$$r\left(v_{-(k-1)} + v_{k-1}\right) > \pi - \frac{rc}{(\kappa+\eta)}$$

which is a contradiction since $r < \kappa + \eta$ and there's at least one firm investing in every state between 0 and k. □

Proposition 2. *In the stationary distribution, growth rate follows*

$$g/\ln\lambda = \mu^C \cdot (\eta+\kappa) + \mu^M \cdot \kappa,$$

where $\mu^C \equiv \sum_{s=1}^{k} \mu_s$ *and* $\mu^M \equiv \sum_{s=k+1}^{n+1} \mu_s$.

Proof. Let $\tilde{g} \equiv g/\ln\lambda$ and $\alpha \equiv \kappa/\eta$. Steady-state $\{\mu_s\}_{s=0}^{\infty}$ is characterized by

$$\begin{aligned}
2\mu_0 &= (1+\alpha)\,\mu_1 \\
\mu_1 &= (1+\alpha)\,\mu_2 \\
&\vdots \\
\mu_{k-1} &= (1+\alpha)\,\mu_k \\
\mu_k &= \alpha\mu_{k+1} \\
&\vdots \\
\mu_n &= \alpha\mu_{n+1}
\end{aligned}$$

Thus

$$\left(2\mu_0 + \sum_{s=1}^{k-1} \mu_s\right) \eta = \left(\sum_{s=1}^{k} \mu_s\right) (\eta + \kappa) = \mu^C (\eta + \kappa)$$

$$\sum_{s=k}^{n} \mu_s \eta = \sum_{s=k+1}^{n+1} \mu_s \kappa = \mu^M \kappa.$$

To derive the aggregate growth rate of the economy, note that in a steady-state, the growth rate of the distribution of productivities should be equal to the growth rate of the frontier, i.e.

$$\tilde{g} = 2\mu_0 \eta + \sum_{s=1}^{n} \mu_s \eta.$$

To see this formally, note that the growth rate of each product line is a weighted average of the growth rate of two competing firms:

$$g(\nu) = a(\nu) g_1(\nu) + (1 - a(\nu)) g_2(\nu),$$

The weights $\{a(\nu)\}$ should be functions of the state variable. If both leader and follower in a state makes a successful innovation, the total effect on the product line should add up to one. We can therefore rewrite the weights as $a_s^L + a_{s+1}^F = 1$, i.e. if leader in state s makes an innovation (so that state goes into $s+1$) and that follower in state $s+1$ makes an innovation, the total weights should be equal to one. The productivity growth of the economy can be written as

$$\tilde{g} = \sum_{s=0}^{n+1} \mu_s \left(p_s a_s^L + p_{-s} a_s^F\right),$$

where p_s and p_{-s} are respectively the flow rate of leader and follower making a successful innovation in state s; a_s^L and a_s^F are the associated weights on aggregate growth. We also have

$$\mu_s p_s = \mu_{s+1} p_{-s+1} \quad \text{for all } s > 0$$

hence

$$\begin{aligned}
\tilde{g} &= \mu_0 p_0 + \sum_{s=1}^{n+1} \mu_s \left(p_s a_s^L + p_{-s} a_s^F \right) \\
&= \mu_0 p_0 + \sum_{s=1}^{n} \mu_s p_s a_s^L + \sum_{s=1}^{n+1} \mu_s p_{-s} a_s^F \\
&= \mu_0 p_0 + \sum_{s=1}^{n} \mu_s p_s a_s^L + \sum_{s=1}^{n} \mu_s p_s \left(1 - a_s^L \right) + 2\mu_0 p_0 \left(1 - a_0^L \right) \\
(\text{substitute } p_s = \eta) &= 2\mu_0 \eta + \sum_{s=1}^{n} \mu_s \eta.
\end{aligned}$$

Given that $\sum_{s=0}^{n+1} \mu_n = 1$, we can rewrite as

$$\begin{aligned}
\tilde{g} &= 2\mu_0 \eta + \mu_1 \eta + \cdots + \mu_n \eta \\
&= \left(2\mu_0 + \sum_{s=1}^{k-1} \mu_s \right) \eta + \sum_{s=k}^{n} \mu_s \eta \\
&= \mu^C (\eta + \kappa) + \mu^M \kappa
\end{aligned}$$

as desired. □

Theorem 1. $g \to \kappa \ln \lambda$.

Proof. We can rewrite $\tilde{g}$ as

$$\tilde{g} = \mu_0 \eta + (1 - \mu_{n+1}) \eta$$

thus finding μ_0 and μ_{n+1} is sufficient to pin down $\tilde{g}$. We know

$$\mu_0 = \frac{1}{2} \alpha^{n-k} (1 + \alpha)^k \mu_{n+1}.$$

Since distribution sums to 1, we can write

$$\begin{aligned}
1 &= \sum_{s=0}^{n+1} \mu_s \\
&= \mu_{n+1}\left(1+\alpha+\cdots+\alpha^{n-k-1}\right)+2\mu_0\left(\frac{1}{2}+(1+\alpha)^{-1}+\cdots+(1+\alpha)^{-k}\right) \\
&= \mu_{n+1}\frac{1-\alpha^{n-k}}{1-\alpha}+2\mu_0\left(-\frac{1}{2}+\frac{1-(1+\alpha)^{-(k+1)}}{1-(1+\alpha)^{-1}}\right) \\
&= \mu_{n+1}\left(\frac{1-\alpha^{n-k}}{1-\alpha}+\alpha^{n-k}(1+\alpha)^{k}\left(-\frac{1}{2}+\frac{1-(1+\alpha)^{-(k+1)}}{1-(1+\alpha)^{-1}}\right)\right) \\
(\text{as } r\to 0) &\to \mu_{n+1}\left(\frac{1}{1-\alpha}+\alpha^{n-k}(1+\alpha)^{k}\left(-\frac{1}{2}+\frac{1}{1-(1+\alpha)^{-1}}\right)\right)
\end{aligned}$$

If we can show $\alpha^{n-k}(1+\alpha)^{k} \to 0$, then $\mu_n \to 1-\alpha$, $\mu_0 \to 0$, and $g = \kappa$. This implies size of the monopolistic region: $\left(\sum_{s=k+1}^{n+1}\mu_s\right) \to 1$. Note also that aggregate investment is

$$\begin{aligned}
I &= c\left(\sum_{s=0}^{k} 2\mu_s + \sum_{s=k+1}^{\mu_n}\mu_s\right) \\
&= c\left[\mu_{n+1}\left(\alpha+\cdots+\alpha^{n-k-1}\right)+2\mu_0\left(1+(1+\alpha)^{-1}+\cdots+(1+\alpha)^{-k}\right)\right]
\end{aligned}$$

Thus $\mu_0 \to 0$ implies

$$I \to c\mu_{n+1}\frac{\alpha}{1-\alpha} = c\kappa/\eta.$$

Next we show $\alpha^{n-k}(1+\alpha)^{k} \to 0$. The proof takes several steps.

We prove the simple case $\pi_0 = 0$, $\pi_s = \pi$ for all $s > 0$. Since the proof relies on asymptotics, the mathematics for the general case (with variable profits) is very similar.

Let $w_s \equiv v_s + v_{-s}$ be the sum of leader and follower's value, and let $\Delta w_s \equiv w_{s+1} - w_s$. First, we derive a few properties of these value functions.

Lemma 3. *The following statements are true:*

1. $\Delta w_0 \sim \frac{rw_0+2c}{2\eta}$.
2. $rv_{n+1} \sim \pi - c\alpha$.

3. $\Delta v_n \sim c/\eta$.

Proof

1. follows from $rw_0 = -2c + 2\eta\,(w_1 - w_0)$.

2. leader stops investing in v_{n+1} iff

$$rv_{n+1} \geq \pi - c\frac{r+\kappa}{\eta}.$$

   Likewise, we can show $\pi - c\frac{r+\kappa}{\eta} \geq rv_n$. We further know that

$$rv_{n+1} = \frac{r\pi}{r+\kappa} + \frac{\kappa}{r+\kappa} rv_n$$

   Thus $rv_n \sim rv_{n+1} \sim \pi - c\alpha$.

3. follows from 2): $v_{n+1} - v_n = \frac{\pi}{r+\kappa} - \frac{rv_n}{r+\kappa} \sim c/\eta$.

Math Preliminaries

Consider the following recursive formulation of value functions:

$$ru_{s+1} = v + p\,(u_s - u_{s+1}) + q\,(u_{s+2} - u_{s+1})$$

The HJB equation states that, starting from state s, there's a Poisson rate p of moving up one state, and rate q of moving down; the flow payoff is v and discount rate is r.

Given u_s and $\Delta u_s \equiv u_{s+1} - u_s$, we can solve for all u_{s+t} with $t > 0$ as recursive functions of u_s and Δu_s. The recursive formulation does not have a nice closed-form representation, as the number of terms quickly explodes as we expand it out. However, asymptotic analysis as $r \to 0$ is feasible, as we only need to keep track of the terms that asymptotically dominate in the expansions.

Proposition A. *Let* $\delta \equiv \frac{ru_s - v}{q}$, $a \equiv p/q$, $b \equiv r/q$, *then*

$$\begin{aligned} u_{s+t} - u_s &\sim (\Delta u_s)\frac{1-a^t}{1-a} + \delta\frac{t-1-\frac{a-a^t}{1-a}}{1-a} \\ &+\Delta u_s \cdot b\frac{(t-1)\left(1+a^t\right)(1-a)-(2-a)\left(a^t-a\right)}{(1-a)^3} \\ &+\delta b\frac{1}{(1-a)^3}\left(\frac{(t-2)(t-1)}{2}(1-a)-(t-3)a^t-a(2-a)(t-1)+2a(1-a)\right) \end{aligned}$$

$$\begin{aligned} u_{s+t} - u_{s+t-1} &\sim \Delta u_s\frac{a^{t-1}-a^t}{1-a} + \delta\frac{1+\frac{a^t-a^{t-1}}{1-a}}{1-a} \\ &+\Delta u_s b\frac{\left((t-1)\left(1+a^t\right)-(t-2)\left(1+a^{t-1}\right)\right)(1-a)-(2-a)\left(a^t-a^{t-1}\right)}{(1-a)^3} \\ &+\delta b\frac{1}{(1-a)^3}\left(\frac{(t-2)(t-1)-(t-2)(t-3)}{2}(1-a)-(t-3)a^t+(t-4)a^{t-1}-a(2-a)\right) \end{aligned}$$

Proof. The proof involves some tedious algebra; the general idea is to keep expanding the recursion, dropping terms that are asymptotically dominated, and iteratively apply formulas for summation of power series.

The recursive formulation can be re-written as

$$\frac{ru_{s+1}-v}{q} = \frac{p}{q}(u_s - u_{s+1}) + u_{s+2} - u_{s+1}$$

$$\begin{aligned} u_{s+2} - u_{s+1} &= a(u_{s+1}-u_s) + \frac{r(u_{s+1}-u_s)+ru_s-v}{q} \\ &= a\Delta u_s + b\Delta u_s + \delta \end{aligned}$$

$$u_{s+2} - u_s = (1+a)\Delta u_s + b\Delta u_s + \delta$$

Likewise,

$$u_{s+3} - u_{s+2} = a^2\Delta u_s + (1+2a)b\Delta u_s + (1+a)\delta + o\left(r^{-2}\right)$$

$$u_{s+3} - u_s = \left(1+a+a^2\right)\Delta u_s + (1+1+2a)b\Delta u_s + (1+1+a)\delta + b\delta + o\left(r^{-2}\right)$$

Applying the formula iteratively, one can show that

$$u_{s+t} - u_{s+t-1} = a^t \Delta u_s + \delta \sum_{z=1}^{t-1} a^z + b\Delta u_s \sum_{z=1}^{t} z a^{z-1} + b\delta \sum_{z=1}^{t-1} \sum_{m=1}^{z} m a^{m-1} + o\left(r^{-2}\right)$$

$$u_{s+t} - u_s = \Delta u_s \sum_{z=0}^{t} a^z + \delta \sum_{z=1}^{t} \sum_{m=0}^{z-1} a^m + b\Delta u_s \sum_{z=1}^{t} \sum_{m=1}^{z} m a^{m-1} + b\delta \sum_{x=1}^{t-1} \sum_{z=1}^{x} \sum_{m=1}^{z} m a^{m-1} + o\left(r^{-2}\right)$$

One arrives at the proposition by subsituting the following formulas for power series summation:

$$\sum_{z=0}^{t} a^z = \frac{1 - a^{t+1}}{1 - a}$$

$$\sum_{z=1}^{t} \sum_{m=0}^{z-1} a^m = \frac{t - \frac{a - a^{t+1}}{1-a}}{1 - a}$$

$$\sum_{z=1}^{t} \sum_{m=1}^{z} m a^{m-1} = \frac{t\left(1 + a^{t+1}\right)(1-a) - (2-a)\left(a^{t+1} - a\right)}{(1-a)^3}$$

$$\sum_{x=1}^{t-1} \sum_{z=1}^{x} \sum_{m=1}^{z} m a^{m-1} = \frac{1}{(1-a)^3} \left(\frac{t(t-1)}{2}(1-a) - (t-2)a^{t+1} - a(2-a)t + 2a(1-a) \right).$$

□

In the economic problem we are interested in, the formulas can be simplified into the following cases:

1. when $a < 1$:

$$u_{s+t} - u_{s+t-1} \sim \Delta u_s a^{t-1} + \frac{\delta}{1-a} + \frac{b\Delta u_s}{(1-a)^2}$$

   (a) if $r\Delta u_s \to 0$,

$$u_{s+t} - u_s \sim \Delta u_s \frac{1}{1-a} + \delta \frac{s - \frac{1}{1-a}}{1-a}$$

(b) if $r\Delta u_s \not\to 0$,

$$r\left(u_{s+t} - u_s\right) \sim \frac{r\Delta u_s}{1-a}$$

2. when $a > 1$, $r\Delta u_s \to 0$, and $\Delta u_s + \frac{\delta}{a-1} \not\sim 0$,

$$r\left(u_{s+t} - u_s\right) \sim \left(\Delta u_s + \frac{\delta}{a-1}\right)\frac{ra^s}{a-1}$$

$$r\left(u_s - u_{s-1}\right) \sim \left(\Delta u_s + \frac{\delta}{a-1}\right) ra^{s-1}$$

When $\Delta u_s + \frac{\delta}{a-1} \sim 0$, it can be shown that

$$u_{s+t} - u_s \sim -\frac{b\delta}{(1-a)^4} \cdot a^{t+1}$$

If $r\Delta u_s$ doesn't vanish, $r\left(u_{s+t} - u_s\right)$ explodes; thus the case is irrelevant for our economic problem.

Comments. In case 1, $b \to 0$, thus $b\delta$ vanishes relative to δ, and $b\Delta u_s a^t$ vanishes relative to $b\Delta u_s a^t$. The terms that potentially dominate those multiplied by $\Delta u_s a^t$, $b\Delta u_s$, and δ. In case 2, terms that dominate are $\Delta u_s a^s$ and δa^s.

Proof for $\alpha^{n-k}(1+\alpha)^k \to 0$.

We now apply Proposition A to show $\alpha^{n-k}(1+\alpha)^k \to 0$. The lemma earlier establishes Δw_0 as a function of w_0. Based on the proposition, we can write w_k and Δw_k as functions of w_0 and Δw_0; we then find w_n and Δw_n as function of w_k and Δw_k. Since the lemma also establishes $rv_{n+1} \sim \pi - c\alpha$ and $\Delta v_n \sim c/\eta$, we will have enough asymptotic restrictions to establish the claim.

First, we find rw_k and Δw_k. Applying case 2 ($a > 1, r\Delta u_s \to 0$) of the Proposition, we have

$$\begin{aligned} r\left(w_k - w_0\right) &\sim \frac{rw_0 + 2c}{2\eta}\frac{r(1+\alpha)^k}{\alpha} + \left(\frac{rw_0 - (\pi - 2c)}{\alpha\eta}\right)\frac{r(1+\alpha)^k}{\alpha} \\ &= \left(\frac{rw_0 + 2c}{2} + \frac{rw_0 + 2c - \pi}{\alpha}\right)\frac{r(1+\alpha)^k}{\alpha\eta} \\ &\equiv C \end{aligned}$$

$$r\Delta w_k \sim C\alpha$$

Next, we apply case 1b ($a < 1, r\Delta u_s \not\to 0$):

$$\begin{aligned} rw_{n+1} &\sim \frac{r\Delta w_k}{1-\alpha} + rw_k \\ &\sim \frac{C}{1-\alpha} \end{aligned}$$

which implies $\frac{C}{1-\alpha} + rw_0 \sim \pi - c\alpha$. We also have (apply case 1 again)

$$\begin{aligned} w_{n+1} - w_n &= \left(\alpha^{n-k} + \frac{r}{\eta}\frac{1}{(1-\alpha)^2}\right)\Delta w_k + \frac{rw_k - (\pi - c)}{1-\alpha} \\ &\sim \left(\alpha^{n-k} + \frac{r}{\eta}\frac{1}{(1-\alpha)^2}\right)\frac{C\alpha}{r} + \frac{C + rw_0 - (\pi - c)}{\eta(1-\alpha)} \\ &= \alpha^{n-k}\frac{C\alpha}{r} + \frac{C}{\eta(1-\alpha)^2} + \frac{rw_0 - (\pi - c)}{\eta(1-\alpha)} \\ \left(\text{substitute } \frac{C}{1-\alpha} + rw_0 \sim \pi - c\alpha\right) &\sim \alpha^{n-k}\frac{C\alpha}{r} + \frac{c}{\eta} \\ &\sim c/\eta \end{aligned}$$

Thus

$$\begin{aligned} 0 &\sim \alpha^{n-k}\frac{C\alpha}{r} + \frac{c}{\eta} + \frac{rw_0}{\eta(1-\alpha)} \\ \iff 0 &\sim \alpha^{n-k}(1+\alpha)^k\left(\frac{rw_0 + 2c}{2} + \frac{rw_0 + 2c - \pi}{\alpha}\right) + \frac{rw_0}{(1-\alpha)} \end{aligned}$$

We know $rw_0 \geq 0$. Under the assumption $(2+\alpha)c > \pi$, we must have $rw_0 \sim 0$ and $\alpha^{n-k}(1+\alpha)^k \to 0$.

We can indeed find the relative rate of divergence between $n-k$ and k. Starting from w_n and Δw_n, note that

$$\frac{rw_{n+1} - (\pi - c)}{\eta - \kappa} = \frac{c(1-\alpha)}{\eta - \kappa} = c/\eta = \Delta w_n$$

Thus we can apply case 2 ($\Delta u_s + \frac{\delta}{a-1} \sim 0$) to find rw_k as a function of $r^2\alpha^{-n+k}$, from which we can

then show

$$\frac{n+k}{2} \sim k\frac{\ln(1+\alpha)}{-\ln\alpha}.$$

The other claims in the theorem follow naturally.

Proposition prop. $\lim_{r\to 0} k = \lim_{r\to 0}(n-k) = \infty$.

Proof. The proof of Theorem 1 above has already established this claim. We now sketch an alternative proof, through which one can better understand the structure of the model. We first prove the second part. Note that leader stops investment in state $n+1$ iff

$$\eta\left(\frac{\pi}{r+\kappa} + \frac{\kappa}{r+\kappa}v_{n+1} - v_{n+1}\right) \leq c$$

LHS is the flow payoff from investmenting, RHS is the cost. Re-arranging yields

$$rv_{n+1} \geq \pi - c\frac{r+\kappa}{\eta}.$$

Now consider value functions x, y, and z, for alternating between a state with flow payoff $\pi - c$ and π as follows:

$$\begin{aligned} rx &= \pi + \kappa(y-x) \\ ry &= \pi - c + \eta(x-y) + \kappa(z-y) \\ rz &= \pi - c + \eta(y-z) \end{aligned}$$

It is easy to construct $\epsilon > 0$ such that $rx < \pi - c\frac{\kappa}{\eta} - \epsilon$ for all r. Further, it can be shown that $rx > rv_s$ for all s for all k. Hence,

$$rv_s < rx \leq \pi - c\kappa/\eta - \epsilon \quad \text{for all } s.$$

Which establishes that $\lim_{r\to 0}(n-k) \to \infty$.

Suppose follower stops investing in state s, which means

$$
\begin{aligned}
c/\eta &> v_{-(s-1)} - v_{-s} \\
&= v_{-(s-1)} - \frac{\kappa v_{-(s-1)} + \eta v_{-(s+1)}}{r+\kappa+\eta} \\
&\geq \frac{r}{r+\kappa} v_{-(s-1)}
\end{aligned}
$$

For any finite s, we can find a low r to reach a contradiction to the inequality $rv_{-(s-1)} < c\frac{r+\kappa}{\eta}$. $\square$