The Origins and Effects of Macroeconomic Uncertainty

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March 2018 §

Abstract

We construct and estimate a dynamic stochastic general equilibrium model that features a sizable and endogenous term and equity premia. We incorporate stochastic volatility in a wide array of structural shocks to study the macroeconomic origins and effects of uncertainty. Uncertainty fluctuations arise from changes in stochastic volatility and the endogenous response to uncertainty itself. Our benchmark estimation implies sizable effects of uncertainty on asset prices and the macroeconomy. Importantly, uncertainty shocks have heterogeneous effects depending on their origin. To inspect the mechanisms, we introduce a novel analytical decomposition of the impact of macroeconomic uncertainty on the real economy through four distinct endogenous risk propagation channels. The relative importance of these four channels is determined by the sign, magnitude, and dynamics of the term and equity premia. Using asset price data provides vital information for disciplining the effects of uncertainty in our estimation.

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§We thank Nick Bloom and conference participants at SITE.

1 Introduction

It is well-established that broad measures of macroeconomic and financial market uncertainty vary significantly over time.[1] There is also an emerging literature showing that uncertainty shocks are important for explaining business cycle fluctuations in Dynamic Stochastic General Equilibrium (DSGE) models. These papers typically consider only one source of uncertainty changes.[2] However, there is empirical and anecdotal evidence suggesting that increases in measures of uncertainty are related to heterogeneous sources (e.g., Bloom (2014)). In this paper, we quantitatively explore how uncertainty shocks, deriving from demand, supply, and policy sources, jointly affect business cycle and asset pricing dynamics in a general equilbrium model. We show how the effect of uncertainty (i) varies significantly depending on the origin and (ii) decompose its effects into four distinct endogenous risk propagation channels, two of which the effects are determined by the sign, magnitude, and dynamics of the term and equity premia.

In our quantitative analysis, we build and estimate a dynamic stochastic general equilibrium (DSGE) model along the lines of Christiano, Eichenbaum, and Evans (2005), but with the following departures. First, we allow for stochastic volatility changes in each of the seven structural shocks modeled as a Markov chain. Second, we assume that the representative household has Epstein and Zin (1989) recursive preferences to capture sensitivity towards low-frequency consumption growth and discount rate risks. Third, we use an iterative solution method to endogenously capture sizable and time-varying risk premia. By modeling stochastic volatility as regime changes, we obtain conditionally log-linear solution that facilitates estimation using a modification of the standard Kalman filter. Our solution method captures both the first- and second-order effects of uncertainty on agents' decision policies, and also allows us to analytically decompose four distinct risk propagation channels. Lastly, we use asset pricing data in our estimation.

Figure 1 illustrates the strong relation between real activity, measured as detrended GDP, the slope of the nominal yield curve, and macroeconomic volatility.[3] As the economy enters a

[1] See, for example, Baker, Bloom, and Davis (2016) and Jurado, Ludvigson, and Ng (2015)

[2] Some examples include Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Basu and Bundick (2017).

[3] Detrended GDP is obtained by applying a bandpass filter. Similar results hold if GDP is detrended using an HP filter. The slope of the term structure is computed as the difference between the five-year yields and the one-year yield, Macroeconomic volatility is measured as a five-year moving average of the standard deviation of GDP growth.

recession, the slope of the yield curve tends to go up and so does macroeconomy volatility. In our model, movements between a low and a high volatility regime generate changes from low to high uncertainty that trigger a decline in real activity and a stepening of the term structure, consistently with the data. Importantly, part of this uncertainty is due to the endogenous response of the macroeconomy to shift in volatility. We find these effects of uncertainty on business cycle and risk premia dynamics to be sizable, both on the macroeconomy and the term structure of interest rates. Uncertainty shocks together explain over 10% of the variation in investment growth, around 6% for consumption growth, and over 18% for the slope of the nominal yield curve. These shocks also lead significant countercyclical variation in the nominal term premium and equity premium. The effects of uncertainty are even more sizeable when focusing on fluctuations at business cycle frequencies. An economy that were exclusively affected by uncertainty shocks would present business cycle fluctuations as large as 17% with respect to an economy that presents *both* Uncertainty and traditional level shocks.

The origins of the uncertainty shocks also play an important role. Volatility shocks to preference, total factor productivity (TFP), markup and government spending shocks generate positive co-movement between consumption, investment, and output while volatility shocks to monetary policy, relative price of investment and liquidity shocks generate negative co-movement between consumption and investment. Quantitatively, the most important volatility shocks for business cycles are the ones associated with preferences and total factor productivity. The changes in uncertainty associated with preferences and TFP shocks produce positive co-movement between consumption, investment, and output, which is often a challenge for standard macroeconomic models. We also find that uncertainty changes in TFP are the main driver of the equity premium while uncertainty changes in preferences are the main driver of term premia.

Using our approximate model solution method, we identify and quantify four distinct propagation channels for uncertainty shocks that are labeled as a precautionary savings motive, an investment risk premium, inflation risk premium, and a nominal pricing bias. The *precautionary savings* term reflects the prudence of the representative household towards uncertainty about future income.[4] This prudence term arises in our solution through the households' consumption-savings

[4]Formally, the precautionary savings term relates to the convexity of marginal utility (e.g., Kimball (1990)).

Euler equation. The *investment risk premium* term emerges through the firms' investment Euler equation, which is tightly-linked to the equity premium. The *inflation risk premium* term shows up through the Fisherian equation, and is closely-related to the nominal term premium. Finally, the *nominal pricing bias* arises in the Phillips curve due to the presence of nominal rigidities that makes firms more prudent when setting nominal goods prices.

Our decomposition of the different risk propagation channels show that different forces contribute to generating empirically realistic macroeconomic and asset pricing dynamics. The precautionary savings, investment risk premium, and the nominal pricing bias terms are the most important risk propagation channels for business cycles. The parameters governing price stickiness, capital adjustment costs, and elasticity of labor supply are critical for determining the effects of these risk propagation channels. Price stickiness and labor supply elasticity are critical for determining the sign and magnitude of the precautionary savings channel, while capital adjustment costs are similarly important for determining the sign and magnitude of the investment risk premium channel. The degree of price stickiness and labor supply elasticity determines the sensitivity of labor demand shifts to uncertainty changes, as highlighted in Basu and Bundick (2017). The degree of capital adjustment costs effects the covariance of the return on investment and the stochastic discount factor, which determines the effect of the investment risk premium channel.

Our paper connects to the macroeconomics literature studying the impact of uncertainty shocks on the macroeconomy (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Bachmann and Bayer (2014), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), Bianchi, Ilut, and Schneider (2014), Basu and Bundick (2017), Schaal (2017), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017)). We complement these papers but differ along the following dimensions. First, these papers only consider one source of time-varying uncertainty while we allow for volatility changes for a rich set of demand, supply, and policy shocks. Second, we provide a novel decomposition of the effects of uncertainty into four endogenous risk propagation channels. This decomposition elucidates the economic channels in which macroeconomic uncertainty changes transmits to business cycle and asset pricing fluctuations. Third, we estimate the model using Bayesian likelihood-based methods to quantify the effects of uncertainty. Fourth, we use asset price data, namely the nominal term premium and the equity premium to discipline the effects of uncertainty, as they are directly related to two of the four risk propagation

channels.

The pricing of volatility risks relates to the endowment economy model of Bansal and Yaron (2004). We build on this work by considering a general equilibrium framework with production, where the dynamics of stochastic consumption volatility risks are linked to the time-varying second moments of an array of structural macroeconomic shocks and to the endogenous response of the macroeconomy to changes in the volatility of these shocks. Furthermore, our production-based setting allows us to consider the endogenous feedback between risk premia and business cycle fluctuations via uncertainty shocks. The role of preference shocks for generating a positive real term premia relates to the endowment economy model of Albuquerque, Eichenbaum, Luo, and Rebelo (2016). We show that time discount factor shocks endogenously generate sizable positive inflation risk premia in a New Keynesian framework. More broadly, our paper relates to an emerging literature studying asset prices in New Keynesian models (e.g., Bekaert, Cho, and Moreno (2010), Hsu, Li, and Palomino (2014), Rudebusch and Swanson (2012), Dew-Becker (2014), Weber (2015), Kung (2015), and Campbell, Pflueger, and Viceira (2014)).

2 Simplified Model

To illustrate the key model mechanisms and our solution method we consider a simplified version of the New Keynesian model used for the structural estimation. In the next section, we embed this framework into a richer medium-sized DSGE model (e.g., Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007))

Household A representative household has a recursive utility over streams of consumption C_t and hours worked L_t:

$$V_t = \left((1-\beta_t)\, u(C_t, L_t)^{1-1/\psi} + \beta_t \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right)^{\frac{1}{1-1/\psi}} \quad (1)$$

$$u(C_t, L_t) = C_t e^{-\tau_0 \frac{L_t^{1+\tau}}{1+\tau}} \quad (2)$$

where γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution. In the limit, when $\psi \to 1$, preferences specified above become

$$V_t = u(C_t, L_t)^{(1-\beta_t)} \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{\beta_t}{1-\gamma}}$$

This is the case that we consider below.[5]

Also define a preference shock $\tilde{b}_t$, such that: $\beta_t = \frac{1}{1+\hat{\beta}e^{\tilde{b}_t}}$. Preference shock $\tilde{b}_t$ follows AR(1) process, with volatility of innovations depending on aggregate volatility regime ξ_t

$$\tilde{b}_{t+1} = \rho_\beta \tilde{b}_t + \sigma_{\beta,\xi_{t+1}} \varepsilon_{\beta,t+1}$$

Volatility regime ξ_t follows a Markov-switching process with a transition probability matrix H.

The representative household owns capital $\overline{K}_{t-1}$, pre-determined at time $t-1$, which it supplies to competitive capital markets at a real rental rate r_t^k. The household accumulates capital according to the following law of motion:

$$\overline{K}_t = \overline{K}_{t-1}(1-\delta_0) + \Phi_K \left(\frac{I_t}{\overline{K}_{t-1}} \right) \overline{K}_{t-1} \tag{3}$$

$$\Phi_K \left(\frac{I_t}{\overline{K}_{t-1}} \right) = \frac{\phi_{K,1}}{1-1/\varphi_K} \left(\frac{I_t}{\overline{K}_{t-1}} \right)^{1-1/\varphi_K} + \phi_{K,2} \tag{4}$$

where $\Phi_K \left(\frac{I_t}{\overline{K}_{t-1}} \right)$ function reflects capital adjustment cost. I_t is time t investment. Household also supplies labour service L_t to a competitive labour market at a real wage W_t.

The time t budget constraint of the household is

$$P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} = P_t D_t + P_t W_t L_t + B_t + P_t \overline{K}_{t-1} r_t^k$$

where P_t is a nominal price of the consumption good, B_{t+1} is an amount of nominal one-period bonds held by household at time t with maturity at time $t+1$, R_t is the gross nominal interest rate set at time t by the monetary authority, D_t is a real dividend income received from the intermediate firms

The optimization problem of the household, results in the following intertemporal first order

[5]We also considered specifications in which the parameter is freely estimated, but we found it to be poorly identified, with the prior essentialy dictating its value. Our results do not depend on this parameter being restricted to 1.

condition:

$$1 = E_t\left[M_{t+1}\frac{P_t}{P_{t+1}}\right]R_t \tag{5}$$

where

$$M_{t+1} = \frac{1-\beta_{t+1}}{1-\beta_t}\beta_t\left(\frac{V_{t+1}^{1-\gamma}}{E_t\left[V_{t+1}^{1-\gamma}\right]}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-1} \tag{6}$$

is the Stochastic Discount Factor (SDF). The intratemporal condition is

$$W_t = \tau_0 L_t^{\tau} C_t \tag{7}$$

And first order condition with respect to investment decision gives us:

$$q_t = E_t\Big[M_{t+1}\Big(r_{t+1}^k + q_{t+1}\left(1-\delta_0+\Phi_K\left(\frac{I_{t+1}}{\overline{K}_t}\right)-\phi_{K,1}\left(\frac{I_{t+1}}{\overline{K}_t}\right)^{1-1/\varphi_K}\right)\Big)\Big] \tag{8}$$

where q_t denotes shadow value of capital:

$$q_t = \frac{1}{\phi_{K,1}}\left(\frac{I_t}{\overline{K}_{t-1}}\right)^{1/\varphi_K} \tag{9}$$

It is useful to define return on investment:

$$R_{t+1}^i = \frac{r_{t+1}^k + q_{t+1}\left(1-\delta_0+\Phi_K\left(\frac{I_{t+1}}{\overline{K}_t}\right)-\phi_{K,1}\left(\frac{I_{t+1}}{\overline{K}_t}\right)^{1-1/\varphi_K}\right)}{q_t} \tag{10}$$

Using this definition we can rewrite (8) in an asset pricing form:

$$1 = E_t\Big[M_{t+1}R_{t+1}^i\Big] \tag{11}$$

Final Goods A representative firm produces the final (consumption) good in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods $X_{i,t}$ as input in

the CES production technology

$$Y_t = \left(\int_0^1 X_{i,t}^{\frac{\nu-1}{\nu}} \, di \right)^{\frac{\nu}{\nu-1}}$$

where ν is elasticity of substitution between intermediate goods. The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity ν

$$X_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\nu}$$

where P_t is the nominal price of the final good and $P_{i,t}$ is the nominal price of intermediate good i.

Intermediate Goods The intermediate goods sector is characterized by a continuum of monopolistic competitive firms. Each intermediate goods firm hires labour L_{it} and rents capital $K_{i,t}$ on competitive markets and produces output X_{it} using constant returns to scale technology:

$$X_{it} = K_{it}^{\alpha} (e^{n_t} L_{it})^{1-\alpha} \tag{12}$$

where n_t is a stochastic productivity trend:

$$\begin{aligned} \Delta n_t &= \mu + x_t \\ x_t &= \rho_x x_{t-1} + \sigma_{x,\xi_t} \varepsilon_{x,t} \end{aligned}$$

μ is the unconditional mean of productivity growth and x_t is a TFP growth shock. Note that standard deviation σ_{x,ξ_t} depends on aggregate volatility regime ξ_t

The intermediate firms face a cost of adjusting the nominal price a lá Rotemberg (1982), measured in terms of the final good as

$$G(P_{i,t}, P_{i,t-1}, Y_t) = \frac{\phi_R}{2} \left(\frac{P_{i,t}}{\Pi_{ss} P_{i,t-1}} - 1 \right)^2 Y_t$$

where $\Pi_{ss} \geqslant 1$ is the steady-state inflation rate and ϕ_R is the magnitude of the price adjustment costs. The source of funds constraint is

$$P_t D_{i,t} = P_{i,t} X_{i,t} - P_t W_t L_{i,t} - P_t r_t^k K_{i,t} - P_t G(P_{i,t}, P_{i,t-1}, Y_t)$$

where $D_{i,t}$ is the real dividend paid by the firm. The objective of the firm is to maximize shareholder's value $V_t^{(i)} = V^{(i)}(\cdot)$ taking the pricing kernel M_t, the competitive real wage W_t, real rental rate of capital r_t^k and the vector of aggregate state variables $\Psi_t = (P_t, Z_t, Y_t)$ as given:

$$V^{(i)}\left(P_{i,t-1};\Psi_t\right) = \max_{P_{i,t},L_{i,t},K_{i,t}} \left\{ D_{i,t} + E_t\left[M_{t+1}\, V^{(i)}\left(P_{i,t};\Psi_{t+1}\right)\right]\right\},$$

subject to

$$\begin{aligned} D_{i,t} &= \frac{P_{i,t}}{P_t} X_{i,t} - W_t L_{it} - r_t^k K_{i,t} - G(P_{i,t}, P_{i,t-1}, Y_t) \\ X_{i,t} &= Y_t \left(\frac{P_{i,t}}{P_t}\right)^{-\nu} \\ X_{it} &= K_{it}^{\alpha}(Z_t L_{it})^{1-\alpha} \end{aligned}$$

The optimization problem results in first order condition corresponding to price setting decision:

$$\begin{aligned} &(1-\nu)\left(\tfrac{P_{i,t}}{P_t}\right)^{-\nu}\tfrac{Y_t}{P_t} + \nu W_t \tfrac{L_{i,t}}{1-\alpha}\left(\tfrac{P_{i,t}}{P_t}\right)^{-1}\tfrac{1}{P_t} \\ -\phi_R\left(\tfrac{P_{i,t}}{\Pi_{ss}P_{i,t-1}} - 1\right)\tfrac{Y_t}{\Pi_{ss}P_{i,t-1}} &+ E_t\left[M_{t+1}\phi_R\left(\tfrac{P_{i,t+1}}{\Pi_{ss}P_{i,t}} - 1\right)\tfrac{Y_{t+1}P_{i,t+1}}{\Pi_{ss}P_{i,t}^2}\right] = 0 \end{aligned} \tag{13}$$

and first order condition for amount of capital:

$$r_t^k = \frac{\alpha}{1-\alpha} W_t \frac{L_{i,t}}{K_{i,t}} \tag{14}$$

Central Bank The central bank follows a modified Taylor rule that depends on output and inflation deviations:

$$\ln\left(\tfrac{R_t}{R_{ss}}\right) = \rho_r \ln\left(\tfrac{\mathcal{R}_{t-1}}{\mathcal{R}_{ss}}\right) + (1-\rho_r)\Big(\rho_\pi \ln\left(\tfrac{\Pi_t}{\Pi_{ss}}\right) + \rho_y \ln\left(\tfrac{\widehat{Y}_t}{\widehat{Y}_{ss}}\right)\Big) + \sigma_{R,\xi_t}\varepsilon_{R,t} \tag{15}$$

where R_t is the gross nominal short rate, $\widehat{Y}_t \equiv \frac{Y_t}{Z_t}$ is detrended output, and $\varepsilon_{R,t}$ is a monetary policy shock. Variables with an ss subscript denote steady-state values.

Symmetric Equilibrium We choose a symmetric equilibrium, where all intermediate firms make identical decisions: $P_{i,t} = P_t$, $X_{i,t} = X_t = Y_t$, $K_{i,t} = K_t$, $L_{i,t} = L_t$, $D_{i,t} = D_t$. Also,

we assume nominal bonds are in zero net supply $B_t = 0$. Clearing of capital market implies $K_{i,t} = \overline{K}_{t-1}$. The aggregate resource constraint is

$$Y_t = C_t + I_t + \frac{\phi_R}{2}\left(\frac{\Pi_t}{\Pi_{ss}} - 1\right)^2 Y_t \tag{16}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate

Log-linearization and risk-adjustment Our goal is to study the effects of uncertainty on both asset prices and the macroeconomy. To achieve that, we log-linearize the model, capturing the effects of uncertainty through time-varying risk-adjustment terms (e.g., Jermann (1998), Lettau (2003), Backus, Routledge, and Zin (2010), Uhlig (2010), Dew-Becker (2012), Malkhozov (2014), and Bianchi, Ilut, and Schneider (2014)). The idea behind this approximation method is that all expectational equations are approximated assuming that the variables are conditionally log-normal. The approximate solution indeed satisfies this condition, because it is linear in the state variables. Then we solve a resulting system of linear expectational difference equations augmented with an iterative procedure meant to capture a risk-adjustment component. This procedure allows us to solve rational expectation models in which uncertainty is controlled by a Markov-switching process by using solution methods that have been developed for log-linear approximations. It is important to emphasize that we allow risk to affect not only asset prices, but also the policy functions controlling the macroeconomic variables. This is crucial to study the effects of uncertainty on the macroeconomy.

Let's define a risk-free rate $R_{f,t}$ as a return on a theoretical risk-free asset, which pays one unit of consumption good in every state of the world next period. Risk-free rate satisfies the following asset pricing equation:

$$1 = E_t\Big[M_{t+1}R_{f,t}\Big]$$

or

$$\frac{1}{R_{f,t}} = E_t\Big[M_{t+1}\Big] \tag{17}$$

As described above, the log-linearization approach, that we are using, approximates all expectational equations assuming that the variables are conditionally log-normal. So, log-linearizing (17),

we get

$$-\widetilde{r}_{f,t} = E_t\Big[\widetilde{m}_{t+1}\Big] + \frac{1}{2}Var_t\Big[\widetilde{m}_{t+1}\Big] \tag{18}$$

Here and below, variables with tilde denote log-deviations from deterministic steady state of the corresponding variables[6]. We can log-linearize expression for Stochastic Discount Factor (6), using our risk-adjustment approach[7]:

$$\widetilde{m}_{t+1} = \overline{\beta}\widetilde{b}_{t+1} - \widetilde{b}_t + (1-\gamma)(\widetilde{v}_{t+1} - E_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}]) - (\widetilde{c}_{t+1} - \widetilde{c}_t) - \gamma\widetilde{x}_{t+1}$$
$$-\frac{1}{2}(1-\gamma)^2 Var_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}]$$

and then substitute it out in (18) to get:

$$\tilde{c}_t = E_t\Big[\tilde{c}_{t+1}\Big] - \widetilde{r}_{f,t} + (1-\overline{\beta}\rho_\beta)\tilde{b}_t + \rho_x\widetilde{x}_t \underbrace{-\frac{1}{2}Var_t\Big[\widetilde{m}_{t+1}\Big] + \frac{1}{2}(1-\gamma)^2 Var_t\Big[\tilde{v}_{t+1} + \widetilde{x}_{t+1}\Big]}_{\text{Precautionary savings motive}} \tag{19}$$

which is an Euler equation with respect to risk-free rate of return. $\tilde{b}_t$ is the preference shock. And two risk-adjustment terms $-\frac{1}{2}Var_t\Big[\widetilde{m}_{t+1}\Big] + \frac{1}{2}(1-\gamma)^2 Var_t\Big[\tilde{v}_{t+1} + \widetilde{x}_{t+1}\Big]$ represent a precautionary savings motive.

Log-linearizing and risk-adjusting intertemporal first order condition of the household (5) and combining it with expression for log risk-free rate (18), we get:

$$\widetilde{r}_t = \widetilde{r}_{f,t} + E_t\Big[\widetilde{\pi}_{t+1}\Big] + \underbrace{Cov_t\Big[\widetilde{m}_{t+1}; \widetilde{\pi}_{t+1}\Big] - \frac{1}{2}Var_t\Big[\widetilde{\pi}_{t+1}\Big]}_{\text{Inflation Risk Premium}} \tag{20}$$

where $\widetilde{r}_t$ is a nominal short term interest rate. The risk adjustment term $Cov_t\Big[\widetilde{m}_{t+1}; \widetilde{\pi}_{t+1}\Big] - \frac{1}{2}Var_t\Big[\widetilde{\pi}_{t+1}\Big]$ is an inflation risk premium and it reflects the fact that payoff of a nominal short term bond in real terms is uncertain. Indeed rate of return on this bond in consumption units depends on realization of inflation next period. Therefore covariance of inflation with SDF determines inflation

[6] For capital $\widetilde{k}_t = \log K_t - \log K_{ss}$

[7] $E_t\Big[e^{(\widetilde{v}_{t+1} + \widetilde{x}_{t+1})(1-\gamma)}\Big]$ is approximated as $\exp\Big((1-\gamma)E_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}] + \frac{(1-\gamma)^2}{2}Var_t[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}]\Big)$

risk-premium on short term nominal bond. If inflation tends to be high when marginal utility of wealth is high (high SDF state), then nominal short term bonds are risky and investors demand a risk premium for holding them.

Now let's turn to an investment decision of the household. We can log-linearize and risk adjust equation (11) and use (18) to get:

$$E_t[\widetilde{r}_{i,t+1} - \widetilde{r}_{f,t}] = -Cov_t[\widetilde{m}_{t+1}; \widetilde{r}_{i,t+1}] - \frac{1}{2}Var_t[\widetilde{r}_{i,t+1}] \tag{21}$$

Risk adjustment terms on the right represent an investment risk premium. If return on investment is low when marginal utility of wealth is high (high SDF state), investment return is risky and will command a risk premium. So, in equilibrium households will choose a level of investment such that expected investment return will be higher than risk-free rate to compensate them for risk, they are exposed to.

Log-linearizing expression for gross investment return (10), we get

$$\widetilde{r}_{i,t+1} = \overline{\beta}e^{-\mu}(1-\delta_0)q_{t+1} - \widetilde{q}_t + (1 - \overline{\beta}e^{-\mu}(1-\delta_0))\widetilde{r}^k_{t+1} + \overline{\beta}e^{-\mu}\tfrac{1}{\varphi_K}\widehat{\left(\tfrac{I}{K}\right)}(\widetilde{i}_{t+1} - \widetilde{k}_{t+1}) \tag{22}$$

Substituting $\widetilde{r}_{i,t+1}$ in (21), we get an investment decision equation:

$$\begin{aligned}\widetilde{q}_t = \overline{\beta}e^{-\mu}(1-\delta_0)E_t[q_{t+1}] + \left(1 - \overline{\beta}e^{-\mu}(1-\delta_0)\right)E_t\left[\widetilde{r}^k_{t+1}\right] \\ +\overline{\beta}e^{-\mu}\tfrac{1}{\varphi_K}\widehat{\left(\tfrac{I}{K}\right)}E_t[\widetilde{i}_{t+1} - \widetilde{k}_{t+1}] - \big(\widetilde{r}_{f,t} \underbrace{-Cov_t\left[\widetilde{m}_{t+1}; r^i_{t+1}\right] - \frac{1}{2}Var_t\left[r^i_{t+1}\right]}_{\text{Investment Risk Premium}}\big)\end{aligned} \tag{23}$$

Note that risk-adjustment terms in this equation represent investment risk premium. Expression for $\widetilde{q}_t$ is obtained by log-linearizing (9):

$$\widetilde{q}_t = \tfrac{1}{\varphi_K}\left(\widetilde{i}_t - \widetilde{k}_t\right)$$

Next, let's consider price setting decision of intermediate firms. We apply the same risk-adjustment technique to log-linearize (13) and get Phillips Curve equation:

$$\tilde{\pi}_t = \overline{\beta} E_t\left[\tilde{\pi}_{t+1}\right] + \kappa_R(\tilde{w}_t + \tilde{l}_t - \tilde{y}_t) + \underbrace{\frac{1}{2}\overline{\beta}^*\Big(2Cov_t\Big[\tilde{m}_{t+1} + \tilde{y}_{t+1} + \widetilde{x}_{t+1}; \tilde{\pi}_{t+1}\Big] + 3Var_t\Big[\tilde{\pi}_{t+1}\Big]\Big)}_{\text{Nominal Pricing Bias}} \quad (24)$$

where risk-adjustment terms represent nominal pricing bias and $\kappa_R = \frac{\nu-1}{\phi_R}$.

The rest of the equations, which we need to close the system, don't have terms which depend on expectations of endogenous variables. As a result, a simple log-linearization would suffice and no risk-adjustment terms are needed:

Capital accumulation equation (3):

$$\widetilde{k}_{t+1} + \widetilde{x}_{t+1} = (1-\delta_0)e^{-\mu}\widetilde{k}_t + (1-(1-\delta_0)e^{-\mu})\widetilde{i}_t$$

Production function (12) (in symmetric equilibrium $Y_t = X_{i,t}$):

$$\tilde{y}_t = \alpha\tilde{k}_t + (1-\alpha)\tilde{l}_t$$

Resource constraint (16):

$$\tilde{y}_t = \frac{C_{ss}}{Y_{ss}}\tilde{c}_t + \frac{I_{ss}}{Y_{ss}}\tilde{i}_t$$

Taylor rule (15):

$$\tilde{r}_t = \rho_r\tilde{r}_{t-1} + (1-\rho_r)\Big(\rho_\pi\tilde{\pi}_t + \rho_y\tilde{y}_t\Big) + \sigma_{R,\xi_t}\varepsilon_{R,t}$$

Rental rate of capital (14):

$$\widetilde{r}^k_t = \widetilde{w}_t + \widetilde{l}_t - \widetilde{k}_t$$

Intratemporal first order condition of household (7):

$$\tilde{w}_t = \tilde{c}_t + \tau \tilde{l}_t$$

Finally, extra equation describing dynamics of v_t is obtained by log-linearization and risk-adjustment of (1). Note that this equation is only required to compute the risk-adjustment terms. The dynamics of the model, conditional on the volatility regime, don't depend on it.

$$\widetilde{v}_t = (1-\overline{\beta})(\widetilde{c}_t - \tau_0 \widetilde{l}_t) + \overline{\beta}\left(-\mu \widetilde{b}_t + E_t\Big[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}\Big] + \frac{1}{2}(1-\gamma) Var_t\Big[\widetilde{v}_{t+1} + \widetilde{x}_{t+1}\Big]\right)$$

Based on log-linearization of the model above, we can identify 4 channels, through which uncertainty affects the economy. These are: a *precautionary savings motive* represented by risk adjustment terms in Euler equation (19); *inflation risk premium*, represented by risk-adjustment terms in equation for short term nominal interest rate (20); *investment risk premium*, represented by risk adjustment terms in investment decision equation (23); *nominal pricing bias*, represented by risk-adjustment terms in Phillips Curve (24).

2.1 Solution

The key step to implement our solution method consists of realizing that in a model in which stochastic volatility is modelled as a Markov-switching process, uncertainty at time t only depends on the regime in place at time t, ξ_t. Thus, the system of equations presented above can be written by using matrix notation (Sims (2002)) as:

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Gamma_\sigma Q_{\xi_t} \varepsilon_t + \Gamma_\eta \eta_t + \Gamma_{c,\xi_t} \tag{25}$$

where the DSGE state vector S_t contains all variables of the model known at time t, Q_{ξ_t} is a regime-dependent diagonal matrix with all standard deviation of the shocks on the main diagonal, ε_t is a vector with all structural shocks, η_t is a vector containing the expectation errors, and the

Markov-switching constant Γ_{ξ_t} captures the effects of uncertainty:

$$\Gamma_{c,\xi_t} = \begin{pmatrix} a_1 Cov_t[c_1' S_{t+1}; d_1' S_{t+1}] \\ a_2 Cov_t\left[c_2' S_{t+1}; d_2' S_{t+1}\right] \\ \vdots \end{pmatrix}$$

where we have used the fact that uncertainty at time t only depends on the regime in place at time t, ξ_t.

However, we cannot compute the volatility terms in Γ_{c,ξ_t} without knowing the solution for S_t. This is because we need to know how the economy reacts to the exogenous shocks ε_t. Therefore, we employ the following iterative procedure. First, note that given some $\Gamma_{c,\xi_t} = \widetilde{\Gamma}_{c,\xi_t}$, the solution to (25) can be characterized as a MS-VAR, of the kind studied by Hamilton (1989), Chib (1996), and Sims and Zha (2006):

$$S_t = TS_{t-1} + RQ_{\xi_t}\varepsilon_t + C_{\xi_t} \tag{26}$$

Taking (26) as given, we can now compute the implied level of uncertainty, i.e. the implied $\widetilde{\Gamma}_{c,\xi_t}$. In particular:

$$Cov_t\left[c_1' S_{t+1}; d_1' S_{t+1}\right] = E_t\left\{Cov_t\left[c_1' S_{t+1}; d_1' S_{t+1}|\xi_{t+1}\right]\right\} + Cov_t\left\{E_t\left[c_1' S_{t+1}|\xi_{t+1}\right]; E_t\left[d_1' S_{t+1}|\xi_{t+1}\right]\right\} =$$

$$= c_1' E_t\left[RQ_{\xi_{t+1}}(RQ_{\xi_{t+1}})'\right] d_1 + c_1' Var_t\left[C_{\xi_{t+1}}\right] d_1 \tag{27}$$

where we used the law of total covariance: $Cov(X,Y) = E(Cov(X,Y|Z)) + Cov(E(X|Z), E(Y|Z))$. Note that the changes in constant induced by the risk adjustment are themselves a source of uncertainty. Given the new value for $\widetilde{\Gamma}_{c,\xi_t}$, we repeat the iteration: (1) compute a new solution to (25); (2) update $\widetilde{\Gamma}_{c,\xi_t}$. This iterative procedure continues until the desired level of accuracy is reached. It is worth emphasizing that only C_{ξ_t} depends on Γ_{c,ξ_t}, while the matrices T and R do not depend on it. So, we only need to iterate on C_{ξ_t}. Furthermore, standard conditions for existence and uniqueness of a stationary solution apply. Thus, we know that a finite level of uncertainty exists as long as a solution exists and the shocks are stationary.

In the solution (26), the matrices T and R are equivalent to a standard log-linear solution, so conditional on the volatility regime, the dynamics of the model are the same as in a standard log-linear solution. Volatility matters in two ways. First, like in loglinearized models, it affects

the size of the innovations. This term is captured by Q_{ξ_t}. Second, it affects uncertainty and, as a result, it also affects the risk adjustment term C_{ξ_t}. This term is not present in a standard log-linear approximation. It adjusts the levels of the variables, determines the dynamics in response to a volatility regime change, and it contributes to generate uncertainty.

Let us rewrite (26) in the following form to emphasize dependence on of the solution on structural parameters θ^p, the stochastic volatilities θ^v and transition probability matrix H

$$S_t = T\left(\theta^p\right) S_{t-1} + R\left(\theta^p\right) Q\left(\xi_t, \theta^v\right) \varepsilon_t + C\left(\xi_t, \theta^v, \theta^p, H\right) \tag{28}$$

It is important to emphasize that the constant $C_{\xi_t} = C\left(\xi_t, \theta^v, \theta^p, H\right)$ in (26) depends on the structural parameters θ^p because for a given volatility of the exogenous disturbances, different structural parameters determine different levels of uncertainty. In a standard log-linearization this term would always be zero. As shown below, this hybrid approach allows us to capture salient asset pricing features despite having approximated a model with a conditionally linear solution. Furthermore, it is worth emphasizing that given that agents are aware of the possibility of regime changes, uncertainty also depends on the possibility of moving across regimes. This is why the constant term depends on the transition matrix H. Finally, given that in the model regime changes enter additively, the conditions for existence and uniqueness of a solution are not affected by the presence of regime changes. The model can then be solved by using solution algorithms developed for fixed coefficient general equilibrium models such as Blanchard and Kahn (1980) and Sims (2002). Of course, the model can also be solved by using the solution algorithms explicitly developed for MS-DSGE models such as Farmer, Waggoner, and Zha (2009), Farmer, Waggoner, and Zha (2011), Cho (2016), and Foerster, Rubio-Ramírez, Waggoner, and Zha (2016).

2.2 Nominal Bond Prices and Yield Curve

Define $P_t^{(n)}$ as a n-period nominal bond price at time t. This bond price satisfies the following asset pricing equation:

$$P_t^{(n)} = E_t\Big[M_{t+1} \frac{P_{t+1}^{(n-1)}}{\Pi_{t+1}}\Big]$$

Applying the same log-linearization and risk-adjustment technique, that we used above, we get

$$\widetilde{p}_t^{(n)} = E_t\Big[\widetilde{m}_{t+1} - \widetilde{\pi}_{t+1} + \widetilde{p}_{t+1}^{(n-1)}\Big] + \frac{1}{2}Var_t\Big[\widetilde{m}_{t+1} - \widetilde{\pi}_{t+1} + \widetilde{p}_{t+1}^{(n-1)}\Big] \tag{29}$$

where variables with tilde, as before, denote log deviations of corresponding variables from deterministic steady state. Using this equation we solve for nominal bond prices iteratively, starting from $n = 2$ (Note that gross short term nominal interest rate is an inverse of a price of one-period nominal bond: $R_t = 1/P_t^{(1)}$ and therefore $\widetilde{p}_t^{(1)} = -\widetilde{r}_t$). Given (26) solution to (29) is given by:

$$\widetilde{p}_t^{(n)} = T_p S_{t-1} + R_p Q_{\xi_t}\varepsilon_t + C_{p,\xi_t}$$

Having solved for $\widetilde{p}_t^{(n-1)}$ and knowing solution of the model (26), we can compute $Var_t\Big[\widetilde{m}_{t+1} - \widetilde{\pi}_{t+1} + \widetilde{p}_{t+1}^{(n-1)}\Big]$ in a way similar to (27) to get solution for $\widetilde{p}_t^{(n)}$.

Given a price of a n-period nominal bond $P_t^{(n)} = P_{ss}^{(n)} e^{\widetilde{p}_t^{(n)}}$, yield on this bond is given by

$$y_t^{(n)} = -\tfrac{1}{n}\log P_t^{(n)}$$

where $P_{ss}^{(n)}$ is a price of a n-period nominal bond in a deterministic steady state

2.3 Risk Propagation Channels

In order to understand the role played by the different channels presented above in the propagation of uncertainty shocks, we consider a calibration of the simplified model that is qualitatevely in line with the results for the estimated model. Thus, the results presented here are qualitatevely in line with the ones obtained based on a richer model. The simplified model allows to emphasyze the key forces at work and to focus on two shocks that empirically play an important role: Preference shocks and TFP shocks.

Figure 2 displays impulse response functions to an increase in the volatility of the preference shock (panel a) and of the TFP growth shock (panel b). The overall effect of increasing uncertainty about preferences and TFP growth (solid red line) generates positive co-movement between consumption, investment, and output. The other lines illustrates the contribution of each of the four

risk propagation channels, precautionary savings (dashed line), investment risk premium (line with circles, inflation risk premium (line with crosses), and the nominal pricing bias (dotted line) to the overall effect of heightened uncertainty. These individual propagation channels have different effects on each variable conditional on the source of the uncertainty shock and different effects across the shocks conditional on a particular variable. Importantly, the degree of price adjustment and capital adjustment costs are important for determining the sign and magnitude of these channels.

With higher supply or demand uncertainty, the precautionary savings channel increases the desire for saving, reflected in the drop in consumption, but a rise in labor hours, investment, and output. This effect is reflected in the variance of marginal utility growth, given by Eq. (19). In this calibration, we only have a moderate degree of price adjustment costs, however with a sufficiently high degree of price stickiness, higher uncertainty will generate a large enough downward shift in labor demand that translates to a fall in investment, labor hours, and output, which is the mechanism that Basu and Bundick (2017) uses to produce positive comovement between macroeconomic aggregates. Importantly, our decomposition in this qualitative example illustrates how other risk propagation channels can also help to flip the positive investment response from the precautionary savings channel, namely through the investment risk premium and nominal pricing bias channels.

The sign of the investment risk premium channel depends mainly on the covariance between the return on investment and the real stochastic discount factor (see equation 23). This covariance is determined by the response to the level shocks, and the impulse responses are depicted in Figure 3. Note that both the preference and TFP shocks give rise to negative comovement between the return on investment and the real stochastic discount factor, which translates into a positive investment premium, on average. Therefore, when uncertainty increases (both in demand and supply), the investment premium rises (marginal q falls), and in equilibrium, investment falls significantly. The fall in investment is also associated with a drop in labor hours and output. Through the resource constraint, the fall in investment leads to an increase in consumption. Notice that the investment risk premium channel is by far the most important (by an order of magnitude) for investment dynamics relative to other macroeconomic quantities. The investment premium is closely tied to the equity premium in the model, and consequently, we require this term to be positive and to increase with risk in our structural estimation.

The nominal pricing bias channel depends on (i) the variance of inflation and (ii) the covariance between the inflation and the real stochastic discount factor, output, and TFP growth (see Eq. 24). The variance term relates to a precautionary price-setting effect highlighted in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) that creates a desire for firms to increase prices more when uncertainty is higher in the presence of sticky prices. However, the sign of the overall effect of the nominal pricing bias also depends on the comovement between inflation and the stochastic discount factor, output, and TFP, which are determined by the responses to the level shocks depicted in Figure 3. A demand shock generates positive comovement between inflation and these three variables, while a TFP shock generates negative comovement. Consequently, the response of the macroeconomy to an increase in TFP growth volatility has the opposite sign with respect to the response to an increase in the volatility of preference shocks. Therefore, the source of the uncertainty shock plays an important role in determining its effects through this channel.

The inflation risk premium channel depends on the covariance between the stochastic discount factor and inflation (see Eq. 20). In Figure 3, we can see that a preference shock generates positive comovement between inflation and the real stochastic discount factor (therefore, positive nominal term premium), while the technology shock generates negative comovement (therefore, negative nominal term premium). Given the different signs in the covariances, the second moment demand and supply shocks have the opposite responses (Figure 2). As with the investment risk premium channel, we can discipline this inflation risk premium channel using asset pricing data, namely, nominal bond yields across different maturities.

3 Quantitative Model and Estimation

In this section, we augment the model with additional features standard in DSGE literature in order to take the model to the data using Bayesian likelihood-based methods. We introduce habit formation and preference for liquidity, by specifying the utility kernel in the following form

$$u(C_t, L_t, B_{t+1}) = \zeta_{C,t}(C_t - h\overline{C}_{t-1})e^{-\tau_0 \frac{L_t^{1+\tau}}{1+\tau}} e^{\zeta_{B,t} \frac{B_{t+1}}{R_t P_t Z_t^*}}$$

where $\zeta_{B,t}$ is a liquidity shock with standard deviation of innovations depending on aggregate volatility regime ξ_t. This shock captures time-variation in liquidity premium on short-term gov-

ernment bonds. We allow for variable capital utilization and specify depreciation of capital in (3) as a function of capital utilization. We introduce a markup shock, by modeling time-variation in elasticity of substitution between intermediate goods. We also allow for partial indexation to past inflation in a price adjustment cost function. We modify the Taylor rule to have an inflation target adjustment parameter in the stochastic steady state. We also introduce exogenous government spending, financed by lump-sum taxes on households. Finally, we model a declining trend in the relative price of investment good and introduce a stationary shock to it as a temporary deviation from this trend. Overall the estimated model has seven exogenous shocks: preference shock, TFP growth shock, monetary policy shock, markup shock, shock to relative price of investment, government spending shock and liquidity shock. The aggregate volatility regime ξ_t follows a Markov-Switching process with two regimes. We allow volatility of all seven exogenous shocks to differ across these regimes. Appendix contains more detailed description of the estimated model.

4 Empirical Analysis

We estimate the model by using Bayesian methods. We consider the sample 1984:Q2-2015:Q4. The priors for the model parameters are combined with the likelihood to obtain the posterior distribution. Details about the priors can be found in the Appendix. We use eleven observables: GDP per-capita growth, Inflation, FFR, consumption growth, investment growth, price of investment growth, one-year yield, two-year yield, three-year yield, four-year yield, and five-year yield. Given that we have more observables than shocks, we allow for observation errors on all yields. Finally, we also repeated our estimation exluding the zero-lower-bound period, with no significant change in the results.

4.1 Parameter estimates

Table 1 reports the posterior mode values for the structural parameters. A few comments are in order. First, we fix the elasticity of intertemporal substitution to 1. Second, the parameters controlling the magnitude of the price adjustment cost (ϕ_R) and the average markup (ν) cannot be

separately identified. Thus, we define and estimate parameter $\kappa_R = \frac{\nu-1}{\phi_R}$.[8] The resulting estimated value for κ_R implies a level of price stickiness in line with the existing new-Keynesian literature. Third, in line with previous results in the literature, we find a more than one-to-one response of the FFR to inflation. The fact that the response is well above 1 guarantees that the Taylor principle is satisfied.

Table 2 reports the estimates for the volatilities of the shocks and the persistence for the two regimes. Figure 4 reports the probability of the High volatility regime (Regime 2). The high volatility regime is less persistent than the low volatility one. This in itself contributes to generate additional uncertainty, as regime changes are more likely when the economy is under this regime.

4.2 The Effects of Uncertainty

We are interested in characterizing the level of uncertainty across the two regimes. Uncertainty is computed taking into account the possibility of regime changes, following the methods developed in Bianchi (2016). For each variable z_t, we measure uncertainty by computing the conditional standard deviation $sd_t(z_{t+s}) = \sqrt{\mathbb{V}_t(z_{t+s})} = \sqrt{\mathbb{E}_t\left[z_{t+s} - \mathbb{E}_t(z_{t+s})\right]^2}$, where $\mathbb{E}_t(\cdot) \equiv \mathbb{E}(\cdot|\mathbb{I}_t)$ and $\mathbb{I}_t$ denotes the information available at time t. We assume that $\mathbb{I}_t$ includes knowledge of the regime in place at time t, the data up to time t, and the model parameters for each regime. These assumptions are consistent with the information set available to agents in our model and so our measure of uncertainty reflects uncertainty supposedly faced by the agent in the model across the two regimes. Uncertainty is computed taking into account that future regimes are unknown.

Figure 5 reports the results. The horizontal axis reports the time horizon s used to compute uncertainty. The figure clearly shows that Regime 2 is characterized by higher uncertainty. For all variables and for all horizons uncertainty is higher under this regime. This is for two reasons. First, the larger volatility of the exogenous shocks implies substantial fluctuations both in the term structure and the real economy. Second, Regime 2 is less persistent than Regime 1. As a result, when agents are under this regime they expect the possibility of large swings in real activity and the term structure. Both these channels will be studied below.

[8] The average markup (ν) affects the steady state of the model. For the purpose of computing the steady state we fix this parameter to 6, a value that implies an average net markup of 20% and that is considered in the ballpark (see Gali (1999)).

It is also interesting to notice that uncertainty for consumption, investment and GDP is hump-shaped. In other words, uncertainty is not monotonically increasing with the time horizon. Agents are more uncertain about the short run than the long run. This is because two competing forces are at play. On the one hand, events that are further into the future are naturally harder to predict, as the possibility of shocks and regime changes increase. On the other hand, in the long run the probability of still being in the high uncertainty regime declines.

Figure 6 presents a simulation to understand the effects of changes in uncertainty on the macroeconomy and the term structure of interest rates. We take the most likely regime sequence, as presented in Figure 4 and we simulate the economy based on the parameters at the posterior mode. The top left panel reports the behavior of GDP and the slope of the yield curve. An increase in uncertainty determines both a drop in real activity and an increase in the slope. This generates a negative co-movement between the slope of the yield curve and real activity, as in the data. The two panels on the right of the figure compare the movements in the slope and GDP induced by the increase in uncertainty with the actual series. We can see that the estimated sequence of volatility regimes produces business cycle fluctuations of GDP and slope of yield curve in the model similar to fluctuations observed in the data.

The fluctuations in uncertainty also lead to significant breaks in the term premium. Term premium is defined as the difference between the yield on an 20-period bond and the expected average short-term yield (1 quarter) over the same 20 periods (following Rudebusch, Sack, and Swanson (2006)). The expected value is computed taking into account the possibility of regime changes using the methods developed in Bianchi (2016). The bottom left panel of Figure 6 shows that periods of high uncertainty are associated with an increase in term premia. In the low uncertainty regime, we obtain an annualized theoretical term premium of 0.82%, while in the high uncertainty regime the annualized term premium is 1.26%.

In order to explore the importance of uncertainty shocks with respect to other disturbances, we compute a variance decomposition. We consider the unconditional variance as implied by the model when only one shock is active and compare it to the overall variance. The results are reported in left panel of Table 3. Uncertainty shocks play an important role in explaining fluctuations in the slope of the term structure. In fact, the contribution of changes in the volatility regime is larger than any other shock and explains 25% of unconditional variance and 62% of business cycle

fluctuations in the slope. Liquidity shock also play an important role and explains another 24% of unconditional variance of the slope. Uncertainty shocks also explain a small, but still sizeable fraction of the unconditional variance of GDP, consumption, and invesment growth (5% to 6%).

However, a better measure of the importance of uncertainty shocks is represented by their ability to generate sizable business cycle fluctuations. Thus, we explore how much variation in endogenous variables can be generated by uncertainty shocks at a business cycle frequency. We do it by computing the volatility of business cycle fluctuations in an economy where only uncertainty shocks are present and comparing it to the volatility of business cycle fluctuations in an economy in which both uncertainty and level shocks are active (right panel of Table 3). Uncertainty shocks can explain a substantial part of variation in consumption, investment, and output over the business cycle. In particular 19% of variation in consumption and 17% of variation in output and investment over a business cycle is attributed to uncertainty shocks. Uncertainty shocks also explain 27% of business cycle variation in Fed funds rate.

4.3 Yield Curve

In what follows, we provide more details about the fit of the model and the mechanisms at play. We start by inspecting the ability of the model of matching movements in the term structure. As a first basic check on the fit of the model, Figure 7 presents the yields as implied by our model and as they are in the data. Recall that we have observation error on all yields: The model implied yields are obtained by removing such observation errors. Clearly, observation errors do not play a key role in matching the observed path for yields. The last penel of the figure also shows that the model tracks very well the behavior of the slope of the term structure. As we will see below, variations of premia over the business cycle play a key role in generating such close fit.

Table 4 reports the nominal and real yield curve as implied by our estimated model. Our model generates an upward-sloping real and nominal yield curve with sizable premia. It is worth emphasyzing that in our estimated model risk premia are substantially more important than liquidity premia in generating nominal premia and the only determinant of *real* premia. The right side of Table 4 shows that the overall nominal term premium is 1.12%, generating an unnconditional slope of the term structure very much in line with the data (1.05%). The risk premium accounts for the bulk of it: 0.93% Vs. 0.19%. The real term premium is *.24%* and it is all due to the

risk premium. Notice that this does not mean that liquidity *shocks* are irrelevant, because they contribute to generate risk.

Risk premia are primarily the result of the persistent shocks to time discount rates coupled with recursive preferences. To understand the role of preference shocks, Figure 8 presents the impulse responses to such a shock for some key variables. A negative time preference shock (less patience) induces the agent to consume more and save less, which decreases the wealth-to-consumption ratio. A drop in the wealth-to-consumption ratio implies a decline in the return on a claim to aggregate consumption. When the agent prefers an early resolution of uncertainty (i.e., $\psi > 1/\gamma$), a decrease in the return on the consumption claim increases marginal utility. When the shock is persistent, this leads to a sharp increase in marginal utility.

A persistent negative time preference shock also increases the real rate persistently, which erodes the payoffs of long real bonds more than short ones. Given that a negative time preference shock is associated with high marginal utility, long real bonds provide less insurance against bad states of the world relative to short real bonds. In equilibrium, this is reflected in an upward-sloping real yield curve and positive real term premia.

The time preference shock endogenously generates a negative relation between marginal utility and inflation, which translates into positive inflation risk premia increasing with maturity. A persistent negative time preference shock increases aggregate demand, which raises inflation persistently. The negative time preference shock is also associated with high marginal utility as discussed above. Persistently higher inflation erodes the value of long nominal bonds more than short nominal bonds during high marginal utility states. Consequently, the nominal yield curve is upward-sloping and nominal term premia are positive.

4.4 Responses to Uncertainty Shocks

Figure 11 shows impulse response functions with the average effect of an increase in uncertainty from all seven shocks from our estimated model. The model generates positive comovement between consumption, investment, output, as there is an economic contraction following heightened macroeconomic uncertainty. Also, higher uncertainty increases the nominal slope and term premia, consistent with the observed dynamics in the data. Next, we consider the individual effects of each of second moment shocks, and within each second moment shock, we decompose the effects into

the four risk propagation channels, precautionary savings, investment risk premium, inflation risk premium, and nominal pricing bias, that were discussed in the context of the simplified model from the section above. The results show that the sorigins of uncertainty are important to understand its effects. An increase in the volatility of the shocks to preference, TFP, markups, and government spending produce positive co-movement between consumption, investment, and output, while an increase in the volatility of the shocks to monetary policy, relative price of investment, and liquidity shocks generate negative comovement.

Figure 12 decomposes the effect of uncertainty increases for preferences (Panel a) and in TFP (Panel b), which are quantitatively the two most important sources of uncertainty fluctuations for determining the overall effects documented in Figure 11. Note that in contrast to the responses from the calibrated example in our simplified model (Figure 2), the precautionary savings channel generates positive comovement between consumption, investment, and output. In our estimated model, the magnitude of price adjustment costs is sufficiently high that it flips the sign of investment (due to a larger downward shift in labor demand). The other three channels, are similar qualitatively as the simplified model. The investment and inflation risk premium channels are disciplined by the equity and term premium, respectively.[9] The precautionary savings channel is the most important for driving the effects of changes in uncertainty in demand. For changes in TFP uncertainty, precautionary savings is most important for the consumption, GDP, inflation and FFR responses, but the investment risk premium channel is the most important for investment response. Thus, our results highlight that positive commovement in response to uncertainty can arise also in response to supply shocks, as long as the effects of the risk channel are large enough.

Figure 13 shows impulse responses from a positive first moment markup shock (panel a) and a second moment markup shock (panel b). The markup uncertainty shock also generates positive comovement between consumption, investment, and output, but in contrast to the uncertainty shock to preferences and TFP, the second moment markup shock works almost exclusively through the nominal pricing bias channel. Finally, Figure 14 plots the responses to an increase in uncertainty in monetary policy and liquidity. These are two examples in which uncertainty shocks generate negative comovement between consumption, investment, and output.

[9]Recall that we require the equity premium to be positive and positively related to uncertainty.

5 Conclusion

This paper quantitatively explores the effect of macroeconomic uncertainty on business cycle and asset pricing fluctuations. We build and estimate a DSGE model that features realistic bond and equity risk premia. We estimate the model using macroeconomic data, the term structure of interest rates, and imposing restrictions on the behavior of the equity premium. Our model allows for stochastic changes in the volatility of a rich set of structural shocks. Uncertainty shocks arise as a result of changes in stochastic volatility, but also because of the endogenous response to the macroeconomy to uncertainty itself. We decompose the effect of uncertainty through the lens of four endogenous risk propagation mechanisms: precautionary savings, investment risk premium, inflation risk premium, and nominal pricing bias. The effects arising from the investment and inflation risk premia channels are disciplined by the equity and nominal term premia, respectively.

We find sizable effects of changes in uncertainty. While the model generates a positive comovement in consumption, investment, and output to an overall increase in macroeconomic uncertainty, we find that uncertainty has significantly different effects depending on the origins of the shock. The most important sources of uncertainty for business cycle fluctuations are changes in the volatility of preference and TFP shocks. Changes in TFP uncertainty are most important for explaining variation in the equity premium while changes in preference uncertainty are most important for explaining variation in the nominal term premium. Overall, this paper highlights the importance of accounting for the origins of macroeconomic uncertainty and for using asset prices to discipline the propagation channels for uncertainty.

6 Appendix

Household Assume that the household has recursive utility over streams of consumption C_t and labor L_t:

$$\begin{aligned} V_t &= u(C_t, L_t, B_{t+1})^{(1-\beta_t)} \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{\beta_t}{1-\gamma}} \\ u(C_t, L_t, B_{t+1}) &= (C_t - h\overline{C}_{t-1}) e^{-\tau_0 \frac{L_t^{1+\tau}}{1+\tau}} e^{\zeta_{B,t} \frac{B_{t+1}}{R_t P_t Z_t^*}} \end{aligned}$$

where γ is the coefficient of risk aversion, elasticity of intertemporal substitution is equal to 1 for this preferences. $Z_t^* = e^{n_t} \Upsilon^{\frac{\alpha}{1-\alpha}t}$ is a stochastic trend of the economy. B_{t+1} is an amount of nominal one-period bonds held by household at time t with maturity at time $t+1$. P_t - nominal price of consumption good. Also define $\beta_t = \frac{1}{1+\hat{\beta} e^{\tilde{b}_t}}$. where $\tilde{b}_t$ is a preference shock

$$\tilde{b}_{t+1} = \rho_\beta \tilde{b}_t + \sigma_{\beta,\xi_{t+1}} \varepsilon_{\beta,t+1}$$

ξ_t - Markov-switching process with transition matrix H, which determines aggregate volatility regime. Liquidity shock $\widetilde{\zeta}_{B,t}$ follows AR(1) process:

$$\widetilde{\zeta}_{B,t+1} = \rho_{\zeta_B} \widetilde{\zeta}_{B,t} + \sigma_{\zeta_B,\xi_{t+1}} \varepsilon_{\zeta_B,t+1}$$

Households own capital stock $\overline{K}_{t-1}$, predetermined at time $t-1$, and rent out capital services $K_t = U_t \overline{K}_{t-1}$ to a competitive capital markets at the real rate r_t^k, where U_t is a capital utilization. Household accumulates capital according to the following law of motion:

$$\begin{aligned} \overline{K}_t &= \overline{K}_{t-1} \left(1 - \delta(U_t)\right) + \Phi_K \left(\frac{I_t}{\overline{K}_{t-1}} \right) \overline{K}_{t-1} \\ \Phi_K \left(\frac{I_t}{\overline{K}_{t-1}} \right) &= \frac{\phi_{K,1}}{1 - 1/\varphi_K} \left(\frac{I_t}{\overline{K}_{t-1}} \right)^{1-1/\varphi_K} + \phi_{K,2} \\ \delta(U_t) &= \delta_0 + \delta_1 (U_t - U_{ss}) + \frac{\delta_2}{2} (U_t - U_{ss})^2 \end{aligned}$$

Note, that depreciation rate depends on utilization rate of capital U_t. Household also supplies labour service L_t to a competitive labour market at a real wage W_t. The time t budget constraint

of the household is

$$P_tC_t + P_t(e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1}I_t + \frac{B_{t+1}}{R_t} = D_t + P_tW_tL_t + B_t + P_t\overline{K}_{t-1}r_t^kU_t - P_tT_t$$

where P_t is a nominal price of the consumption good, B_{t+1} is an amount of nominal one-period bonds held by household at time t with maturity at time $t+1$, R_t is the gross nominal interest rate set at time t by the monetary authority, D_t is a real dividend income received from the intermediate firms, T_t - lump-sum taxes. Υ is an average rate of decline in price of investment good relative to consumption good, $\zeta_{\Upsilon,t}$ is a shock to this relative price:

$$\zeta_{\Upsilon,t+1} \quad = \quad \rho_\Upsilon\zeta_{\Upsilon,t} + \sigma_{\zeta_\Upsilon,\xi_{t+1}}\varepsilon_{\zeta_\Upsilon,t+1}$$

The household's intertemporal first order condition is

$$1 = E_t\left[M_{t+1}\frac{\mathcal{P}_t}{\mathcal{P}_{t+1}}\right]\mathcal{R}_t + \frac{1}{Z_t^*}\overline{\zeta}_B e^{\tilde{\zeta}_{B,t}}(C_t - h\overline{C}_{t-1})$$

where

$$M_{t+1} = \frac{1-\beta_{t+1}}{1-\beta_t}\beta_t\left(\frac{V_{t+1}}{\left(E_tV_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{u(C_{t+1},L_{t+1})}{u(C_t,L_t)}\right)^{-1}\left(\frac{u_1'(C_{t+1},L_{t+1})}{u_1'(C_t,L_t)}\right)$$

or

$$M_{t+1} = \frac{1-\beta_{t+1}}{1-\beta_t}\beta_t\left(\frac{V_{t+1}}{\left(E_tV_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{C_{t+1}-h\overline{C}_t}{C_t-h\overline{C}_{t-1}}\right)^{-1}$$

is the stochastic discount factor. The intratemporal condition is

$$W_t = \tau_0 L_t^\tau(C_t - h\overline{C}_{t-1})$$

FOC for investment decision:

$$\frac{(e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1}}{\phi_{K,1}}\left(\frac{I_t}{\overline{K}_{t-1}}\right)^{1/\varphi_K} = E_t\Big[M_{t+1}\Big(r_{t+1}^kU_{t+1} +$$
$$+\frac{(e^{\zeta_{\Upsilon,t+1}}\Upsilon^{t+1})^{-1}}{\phi_{K,1}}\left(\frac{I_{t+1}}{\overline{K}_t}\right)^{1/\varphi_K}\left(1-\delta(U_{t+1})+\Phi_K\left(\frac{I_{t+1}}{\overline{K}_t}\right)-\phi_{K,1}\left(\frac{I_{t+1}}{\overline{K}_t}\right)^{1-1/\varphi_K}\right)\Big)\Big]$$

FOC for capital utilization:

$$r_t^k = \frac{(e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1}}{\phi_{K,1}} \left(\frac{I_t}{\overline{K}_{t-1}}\right)^{1/\varphi_K} \delta'(U_t)$$

Final Goods A representative firm produces the final (consumption) good in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods $X_{i,t}$ as input in the CES production technology

$$Y_t = \left(\int_0^1 X_{i,t}^{\frac{1}{1+\lambda_{p,t}}} \, di\right)^{1+\lambda_{p,t}}$$

where $\lambda_{p,t}$ determines elasticity of substitution between intermediate goods. $\lambda_{p,t}$ follows AR(1) process in logs.

$$\log \lambda_{p,t} - \log \overline{\lambda_p} = \rho_\chi(\log \lambda_{p,t-1} - \log \overline{\lambda_p}) + \sigma_{\chi,\xi_t}\varepsilon_{\chi,t}$$

The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity $\nu_t = \frac{1+\lambda_{p,t}}{\lambda_{p,t}}$

$$X_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$$

where P_t is the nominal price of the final good and $P_{i,t}$ is the nominal price of intermediate good i.

Intermediate Goods The intermediate goods sector is characterized by a continuum of monopolistic competitive firms. Each intermediate goods firm produces X_{it} using labor L_{it} and capital $K_{i,t}$ with constant returns to scale technology:

$$X_{it} = K_{it}^{\alpha}(Z_t L_{it})^{1-\alpha}$$

where Z_t is an aggregate technology shock :

$$\begin{aligned} Z_t &= e^{n_t} \\ \Delta n_t &= \mu + x_t \\ x_t &= \rho_x x_{t-1} + \sigma_{x,\xi_t}\varepsilon_{x,t} \end{aligned}$$

μ is the unconditional mean of productivity growth, and ρ_x is the persistence parameter of the autoregressive process x_t.

The intermediate firms face a cost of adjusting the nominal price a lá Rotemberg (1982), measured in terms of the final good as

$$G(P_{i,t}, P_{i,t-1}, Y_t) = \frac{\phi_R}{2}\left(\frac{P_{i,t}}{\Pi_{ss}^{\kappa_2}\Pi_{t-1}^{(1-\kappa_2)}P_{i,t-1}} - 1\right)^2 Y_t$$

where $\Pi_{ss} \geqslant 1$ is the steady-state inflation rate, ϕ_R is the magnitude of the price adjustment costs. The source of funds constraint is

$$P_t D_{i,t} = P_{i,t}X_{i,t} - P_t W_t L_{i,t} - P_t r_t^k K_{i,t} - P_t G(P_{i,t}, P_{i,t-1}, Y_t)$$

where $D_{i,t}$ is the real dividend paid by the firm. The objective of the firm is to maximize shareholder's value $V_t^{(i)} = V^{(i)}(\cdot)$ taking the pricing kernel M_t, the competitive real wage W_t, real rental rate of capital r_t^k and the vector of aggregate state variables $\Psi_t = (P_t, Z_t, Y_t)$ as given:

$$V^{(i)}\left(P_{i,t-1};\Psi_t\right) = \max_{P_{i,t},L_{i,t},K_{i,t}}\left\{D_{i,t} + E_t\left[M_{t+1}\ V^{(i)}\left(P_{i,t};\Psi_{t+1}\right)\right]\right\},$$

FOC corresponding to price setting decision:

$$\begin{aligned} &\left(1 - \tfrac{1+\lambda_{p,t}}{\lambda_{p,t}}\right)\left(\tfrac{P_{i,t}}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}\ \tfrac{Y_t}{P_t} + W_t\tfrac{L_{i,t}}{1-\alpha}\left(\tfrac{1+\lambda_{p,t}}{\lambda_{p,t}}\right)\left(\tfrac{P_{i,t}}{P_t}\right)^{-1}\ \tfrac{1}{P_t} \\ -\phi_R\left(\tfrac{P_{i,t}}{\Pi_{ss}^{\kappa_2}\Pi_{t-1}^{(1-\kappa_2)}P_{i,t-1}} - 1\right)\tfrac{Y_t}{\Pi_{ss}^{\kappa_2}\Pi_{t-1}^{(1-\kappa_2)}P_{i,t-1}} &+ E_t\left[M_{t+1}\phi_R\left(\tfrac{P_{i,t+1}}{\Pi_{ss}^{\kappa_2}\Pi_t^{(1-\kappa_2)}P_{i,t}} - 1\right)\tfrac{Y_{t+1}P_{i,t+1}}{\Pi_{ss}^{\kappa_2}\Pi_t^{(1-\kappa_2)}P_{i,t}^2}\right] = 0 \end{aligned}$$

FOC for capital:

$$r_t^k = \frac{\alpha}{1-\alpha} W_t \frac{L_{i,t}}{K_{i,t}}$$

Central Bank The central bank follows a modified Taylor rule that depends on output and inflation deviations:

$$\ln\left(\frac{\mathcal{R}_t}{\mathcal{R}_t^*}\right) = \rho_r \ln\left(\frac{\mathcal{R}_{t-1}}{\mathcal{R}_t^*}\right) + (1-\rho_r)\left(\rho_\pi \ln\left(\frac{\Pi_t}{\Pi_{ss} e^{\overline{\pi}^*}}\right) + \rho_y \ln\left(\frac{\widehat{Y}_t}{\widehat{Y}_{ss}}\right)\right) + \sigma_{R,\xi_t}\varepsilon_{R,t}$$

where $\mathcal{R}_t$ is the gross nominal short rate, $\widehat{Y}_t \equiv \frac{Y_t}{Z_t^*}$ is detrended output. Variables with an ss subscript denote steady-state values.

$\overline{\pi}^*$ is a stochastic steady state correction for inflation target, this correction doesn't affect deterministic steady state of the model and doesn't enter directly price adjustment cost function.

Symmetric Equilibrium We choose a symmetric equilibrium, where all intermediate firms make identical decisions $P_{i,t} = P_t$, $X_{i,t} = X_t = Y_t$, $K_{i,t} = K_t$, $L_{i,t} = L_t$, $D_{i,t} = D_t$. Also, we assume nominal bonds are in zero net supply $B_t = 0$. The aggregate resource constraint is

$$Y_t = C_t + (e^{\zeta_{\Upsilon,t}}\Upsilon^t)^{-1} I_t + \frac{\phi_R}{2}\left(\frac{\Pi_t}{(\Pi_{ss})^{\kappa_2}(\Pi_{t-1})^{(1-\kappa_2)}} - 1\right)^2 Y_t + G_t$$

where $\Pi_t \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}$ is the gross inflation rate and G_t are government spending, which follows exogenously specified AR(1) process in logs:

$$\log G_{t+1} - \log G_{ss} = \rho_g(\log G_t - \log G_{ss}) + \sigma_{g,\xi_{t+1}}\varepsilon_{g,t+1}$$

Note that volatility of innovations depends on aggregate volatility regime ξ_t. Government spending is financed by lump-sum taxes on households: $G_t = T_t$

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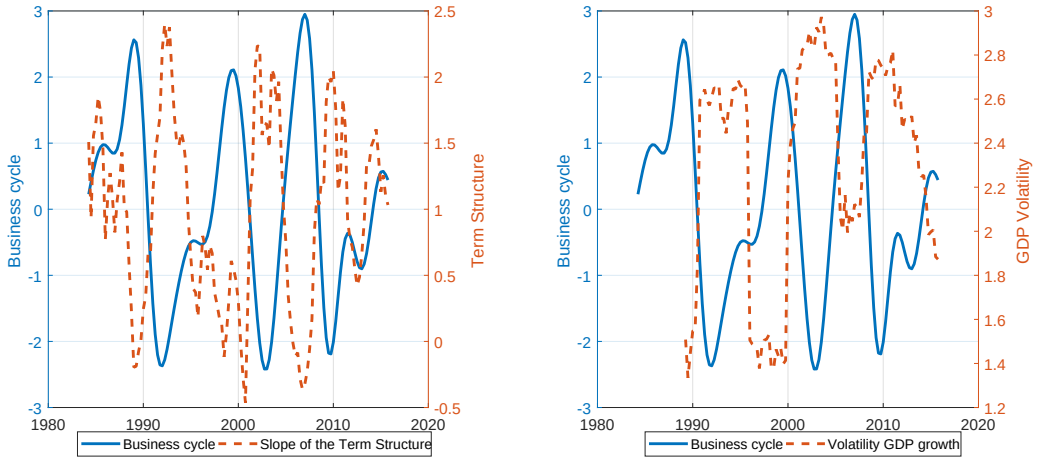
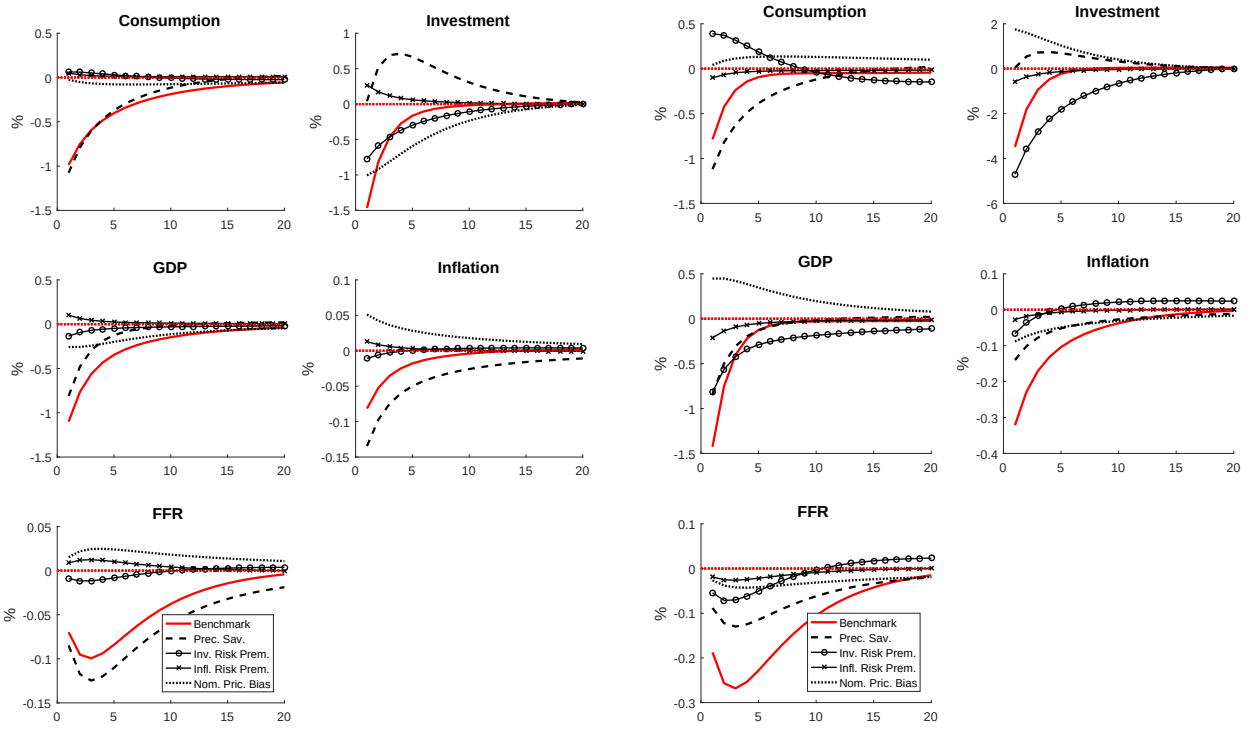


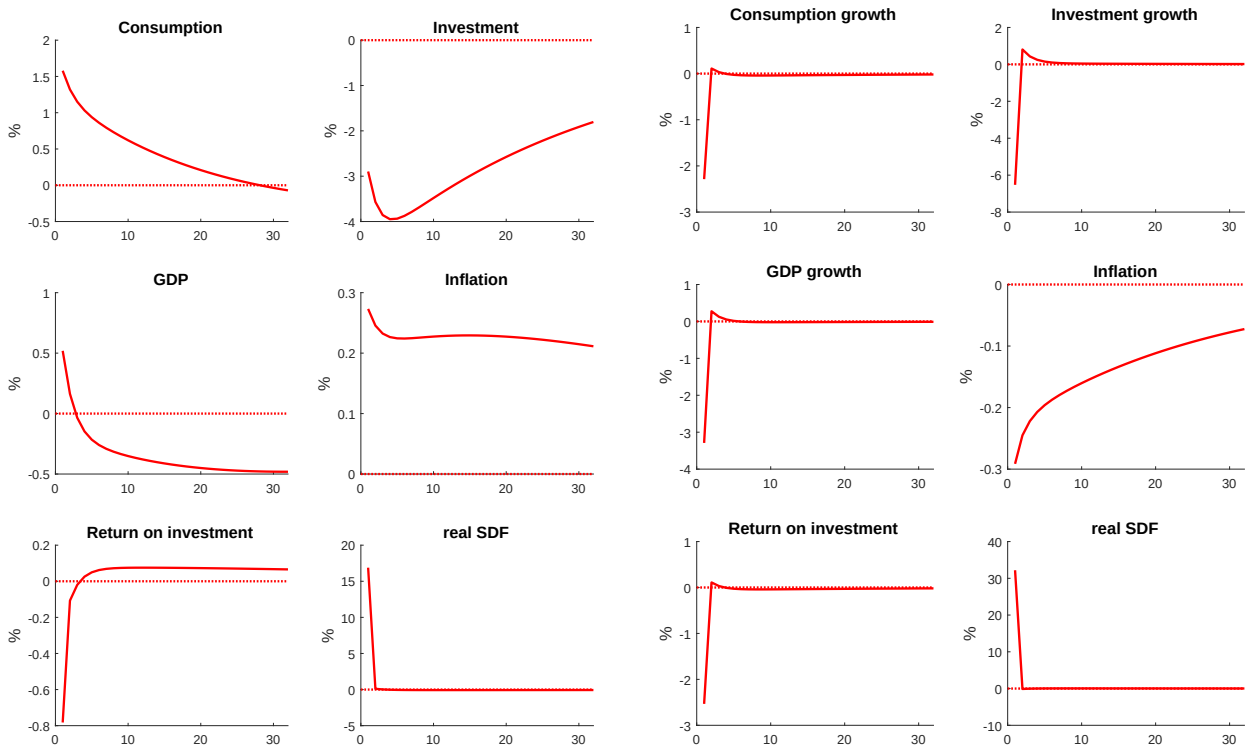
Figure 1: Panel A plots the comovement between the slope of the yield curve (dashed line) and the cyclical component of GDP (solid line) and Panel B plots the comovement between the volatility of GDP growth (dashed line) and the cyclical component of GDP growth (solid line) from the data.



(a) Increase in volatility of a preference shock

(b) Increase in volatility of TFP growth shock

Figure 2: Calibrated model. IRFs represent a change in the expected path of corresponding variables when volatility regime changes. The units of the y-axis are annualized percentage deviations from a steady state. Units on the x axis are quarters. Red solid line depicts an IRF to volatility regime change in a benchmark model. Black dashed line shows contribution of a precautionary savings motive and represents an IRF to a volatility regime change in a model, where inflation risk premia and risk premia on investment return along with varince term in Phillips curve do not change across volatility regimes. Black line with circles shows contribution of a channel operating through change in risk premium on investment return. This line represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where risk premia on investment return does not vary across volatility regimes. Black line with crosses shows contribution of inflation risk premium channel and represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where inflation risk premia does not vary across volatility regimes. Black dotted line shows contribution of a channel operating through a change in variance term in Phillips curve (nominal pricing bias). This line represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where variance term in Phillips curve does not vary across volatility regimes. We use the following parameter values: $\overline{\beta} = 0.996$, $\rho_\beta = 0.98$, $\alpha = 0.33$, $\psi = 1$, $\gamma = 10$, $\mu = 0.005$, $\rho_x = 0.1$, $\kappa_R = 0.005$, $\pi_{ss} = 0.007$, $\rho_r = 0.7$, $\rho_\pi = 1.5$, $\rho_y = 0.1$, $\delta_0 = 0.02$, $\varphi_K = 3$, $\tau = 5$, $\sigma_{b,1} = 0.0125$, $\sigma_{b,2} = 0.0250$, $\sigma_{x,1} = 0.0063$, $\sigma_{x,2} = 0.0125$.



(a) IRF to a preference shock

(b) IRF to a negative TFP growth shock

Figure 3: The units of the y-axis are annualized percentage deviations from a steady state (0 on these axis is normalized to be stochastic steady state). Units on the x axis are quarters. We use the following parameter values: $\overline{\beta} = 0.996$, $\rho_\beta = 0.98$, $\alpha = 0.33$, $\psi = 1$, $\gamma = 10$, $\mu = 0.005$, $\rho_x = 0.1$, $\kappa_R = 0.005$, $\pi_{ss} = 0.007$, $\rho_r = 0.7$, $\rho_\pi = 1.5$, $\rho_y = 0.1$, $\delta_0 = 0.02$, $\varphi_K = 3$, $\tau = 5$, $\sigma_{b,1} = 0.0125$, $\sigma_{b,2} = 0.0250$, $\sigma_{x,1} = 0.0063$, $\sigma_{x,2} = 0.0125$.

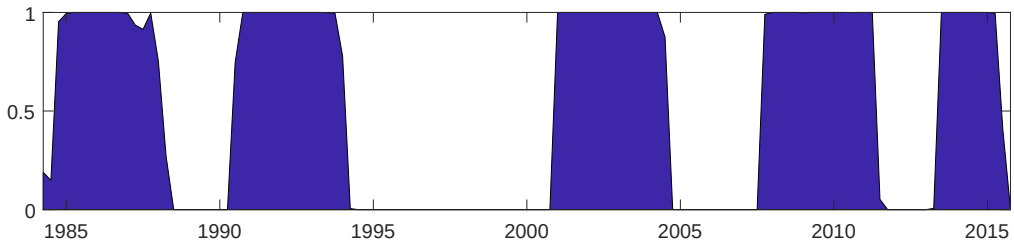


Figure 4: The figure reports the probability of Regime 2, the high uncertainty regime.

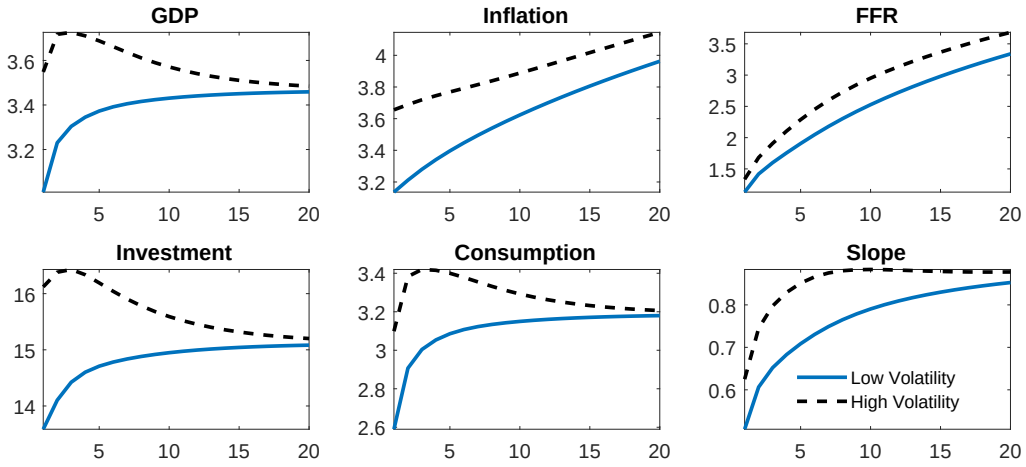


Figure 5: Uncertainty. The figure reports the level of uncertainty at differnt horizons. Uncertainty is computed taking into account the possibility of regime changes.

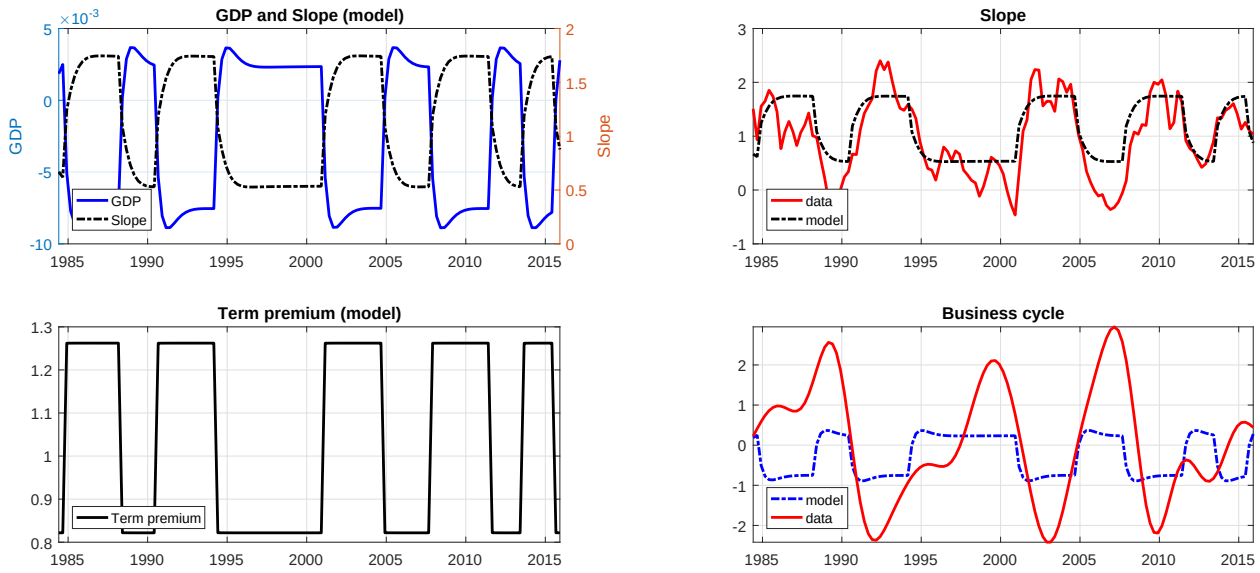


Figure 6: This figure plots selected variables from simulation of the model with estimated volatility regime sequence (all Gaussian shocks are set to zero in this simulation). Top left panel plots simulated path of GDP, expressed in log-deviations from steady state, and slope of the yield curve, expressed as a difference between 5-year yield and 1-year yield. Top right panel plots simulated slope of the yield curve and slope of the yield curve observed in the data. Bottom left panel plots simualted dynamic of nominal term premium in the model, expressed as a difference between 5-year nominal yield and an expected average yield on 1-quarter nominal bond over the next 20 quarters. Bottom right panel plots simulated path of GDP in the model and cyclical component of GDP in the data (obtained using bandpass filter). Units on the y-axis for GDP fluctuations are percentage points (model and data). Units on the y-axis for Term premium and Slope are annualized percent (data and model).

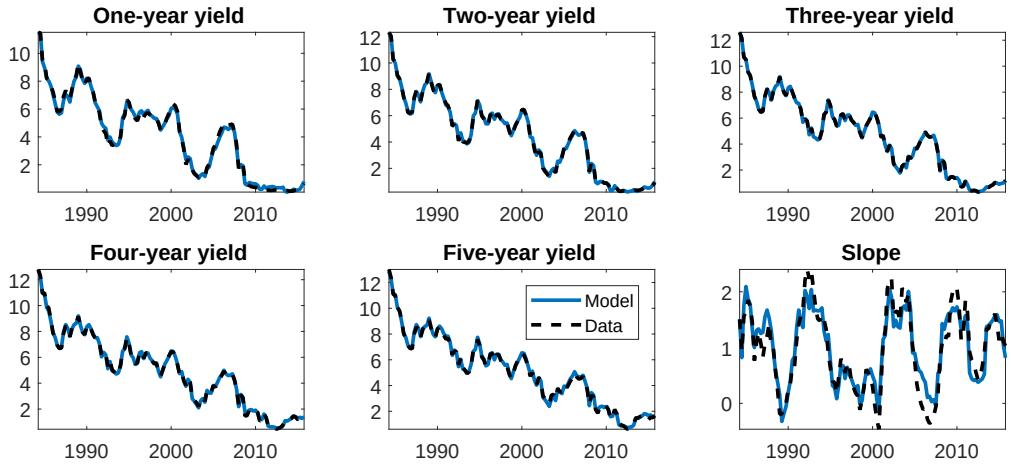


Figure 7: The figure compares the yields as implied by our model (blue solid line) with the yields observed in the data (black dashed line). The model implied yields are obtained by removing the estimated observation errors.

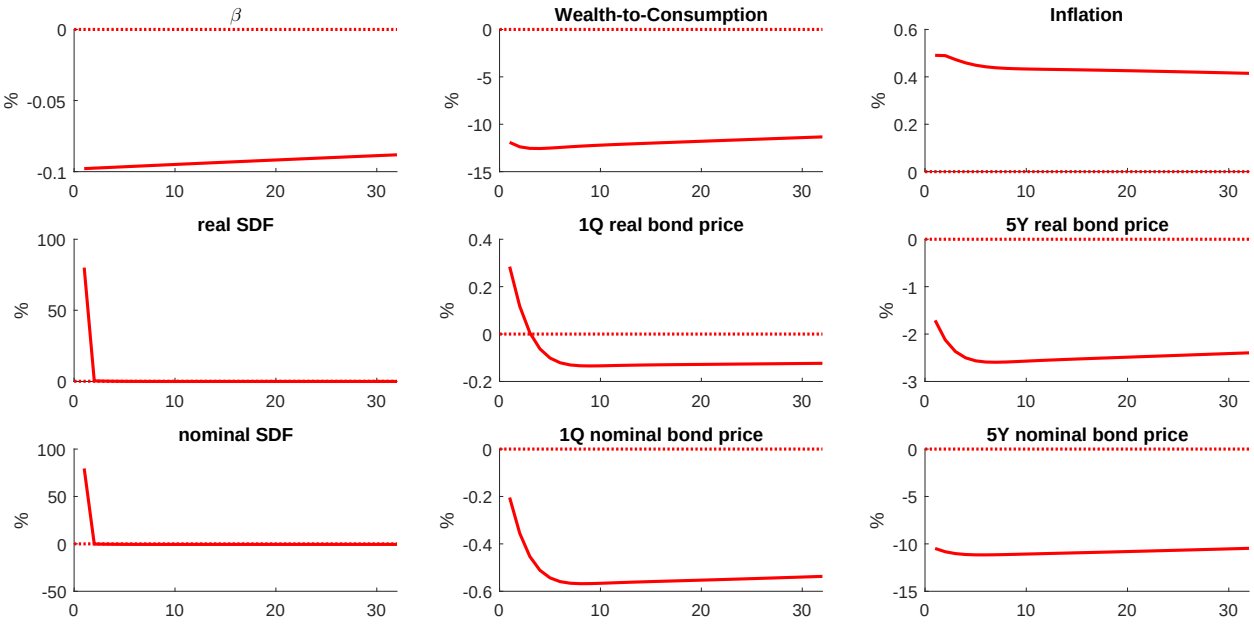
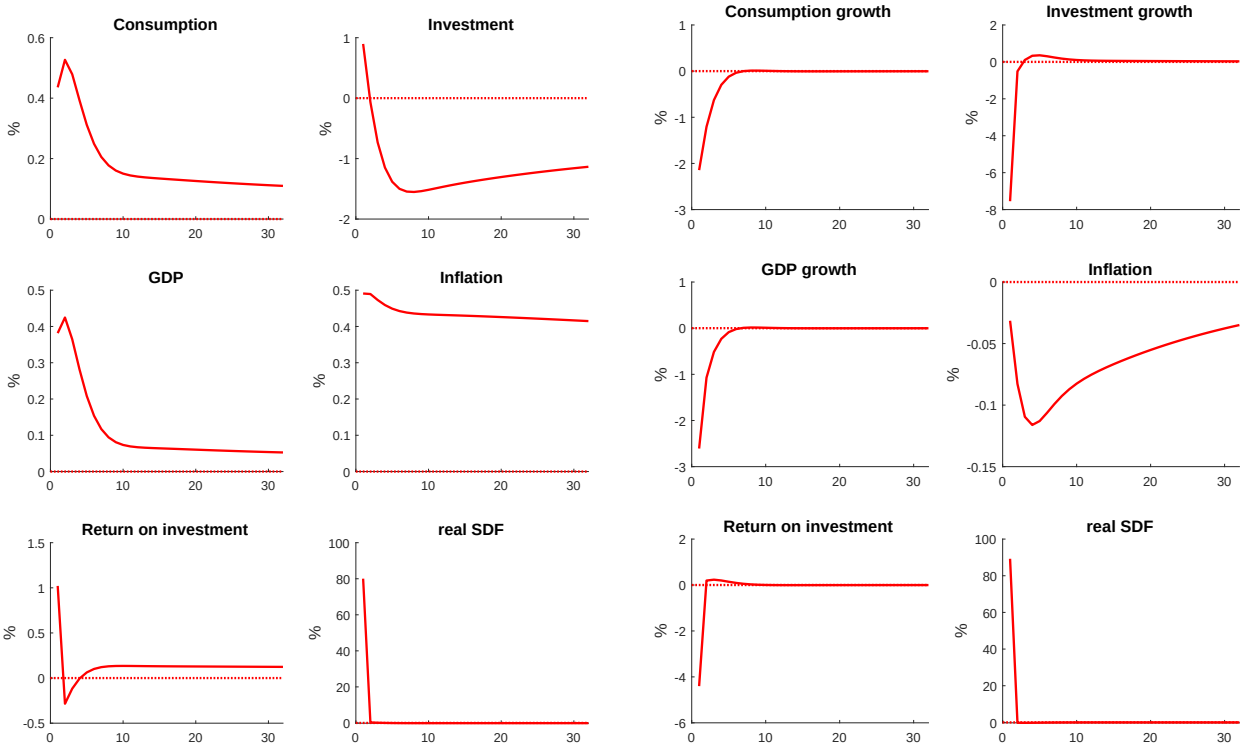


Figure 8: Estimated model. IRF to a preference shock. The units of the y-axis are annualized percentage deviations from a steady state (0 on these axis is normalized to be stochastic steady state). Units on the x axis are quarters. The top left panel plots β_t - loading on continuation utility in Epstein - Zin value function.



(a) IRF to a preference shock

(b) IRF to a negative TFP growth shock

Figure 9: Estimated model. The units of the y-axis are annualized percentage deviations from a steady state (0 on these axis is normalized to be stochastic steady state). Units on the x axis are quarters.

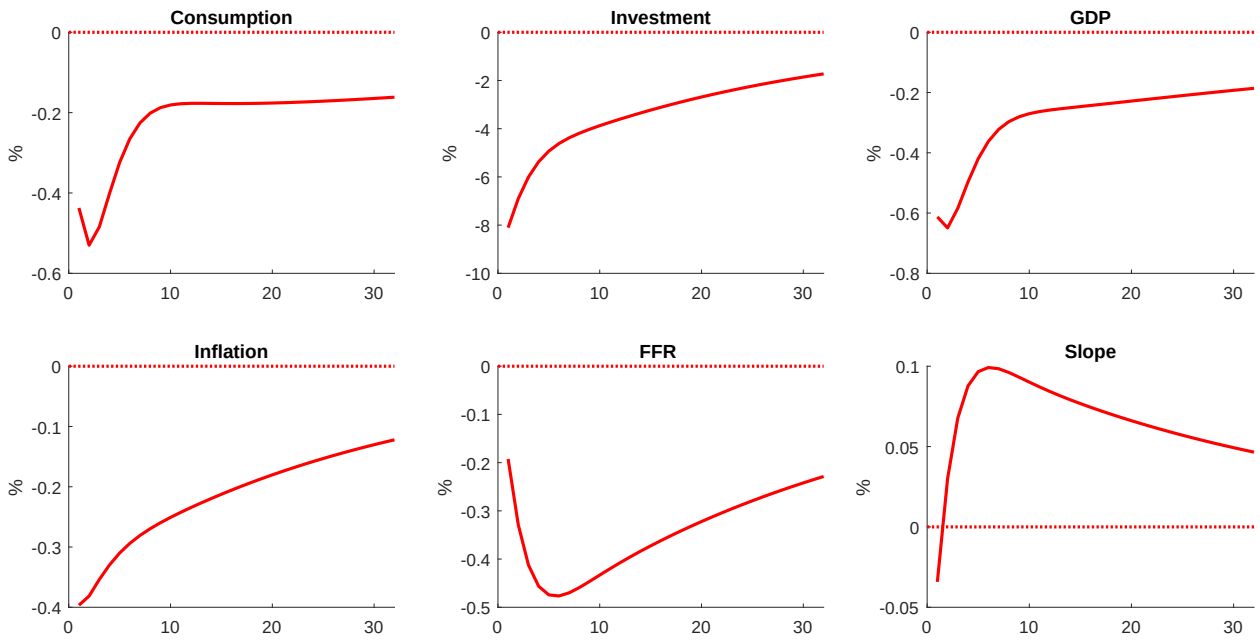


Figure 10: Estimated model. IRF to a liquidity shock. The units of the y-axis are annualized percentage deviations from a steady state (0 on these axis is normalized to be stochastic steady state). Units on the x axis are quarters. Middle bottom panel plots Fed Funds Rate (1-quarter nominal interest rate). Bottom right panel plots slope of the yield curve expressed as a difference between 5-year and 1 year nominal yields.

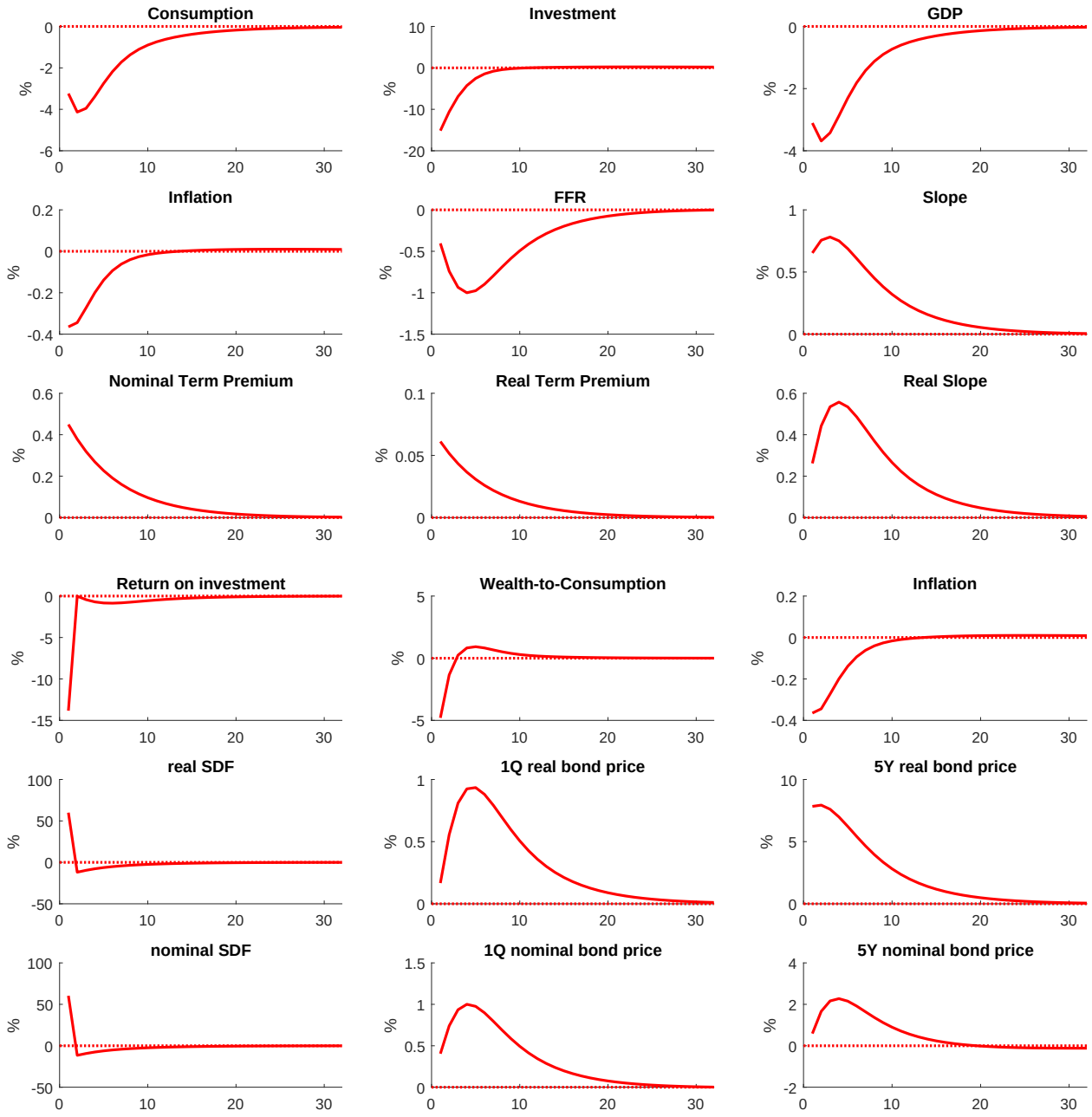
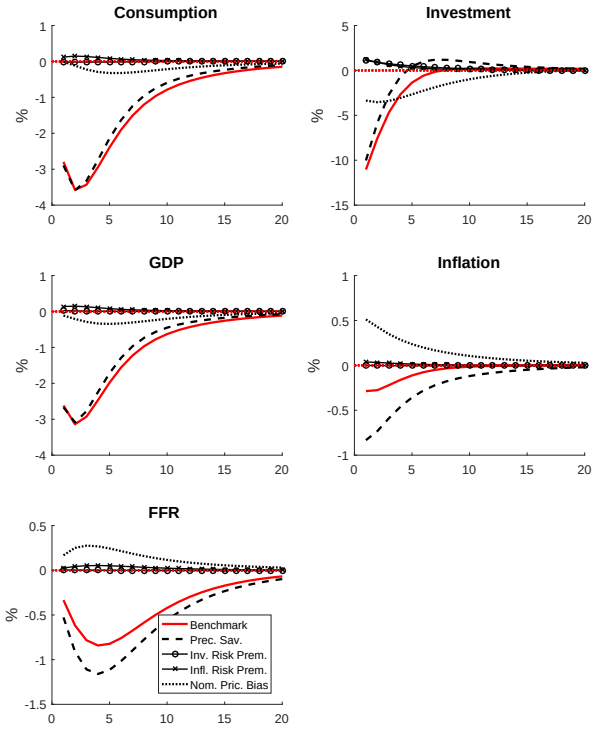
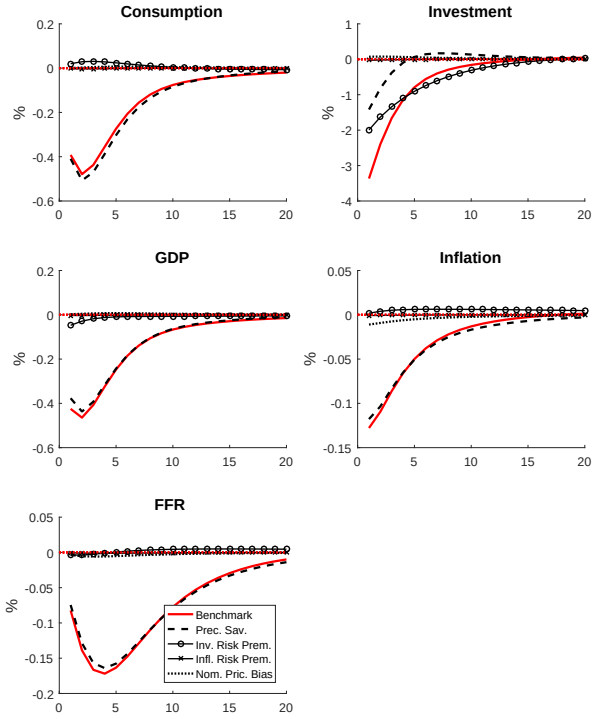


Figure 11: This figure plots IRF to a change in aggregate volatility regime from volatility regime 1 (low volatility regime) to a volatility regime 2 (high volatility regime). IRFs represent a change in the expected path of corresponding variables when volatility regime changes. The figure plots IRFs of consumption, investment, GDP, inflation, Fed Funds Rate (1-quarter nominal interest rate), slope of nominal yield curve expressed as a difference between 5-year and 1-year nominal yields, nominal term premium defined as a difference between 5-year nominal yield and an expected average yield on 1-quarter nominal bond over the next 20 quarters, real term premium defined as a difference between 5-year real yield and an expected average yield on 1-quarter real bond over the next 20 quarters, real slope expressed as a difference between 5-year and 1-year real yields. The units of the y-axis are annualized percentage deviations from a steady state. Units on the x axis are quarters.

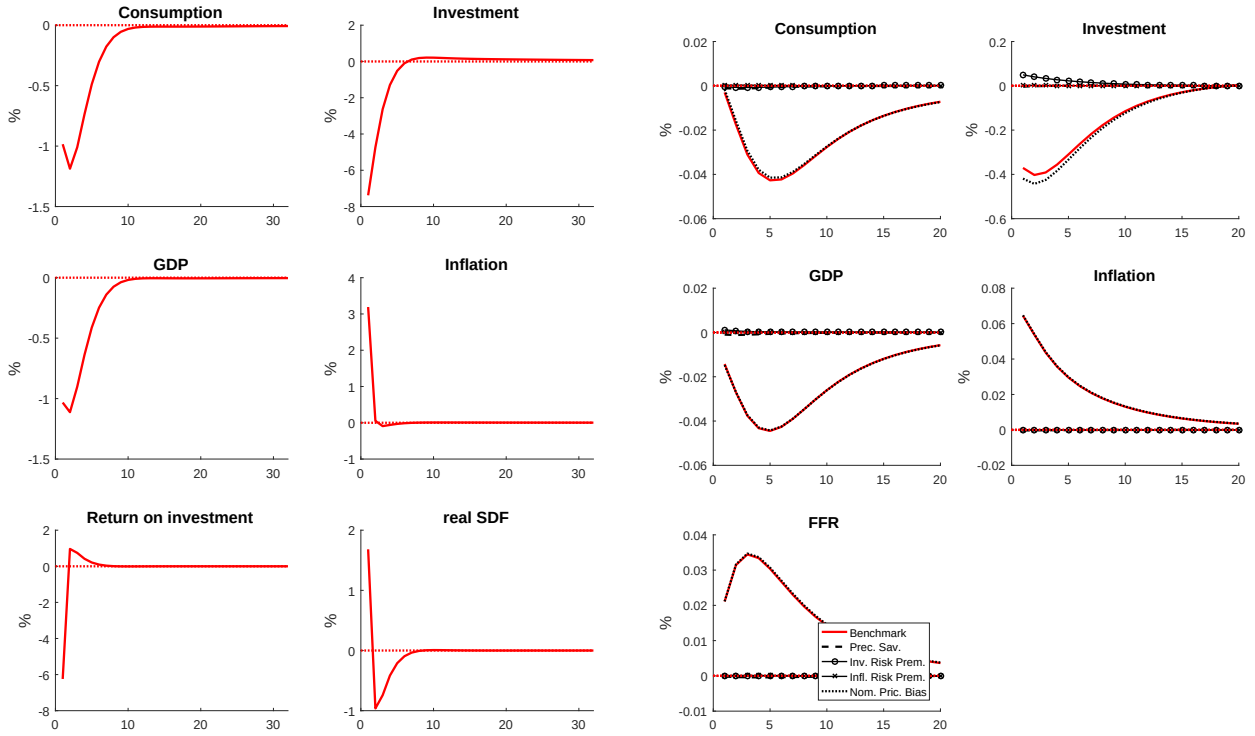


(a) Increase in volatility of a preference shock



(b) Increase in volatility of a TFP growth shock

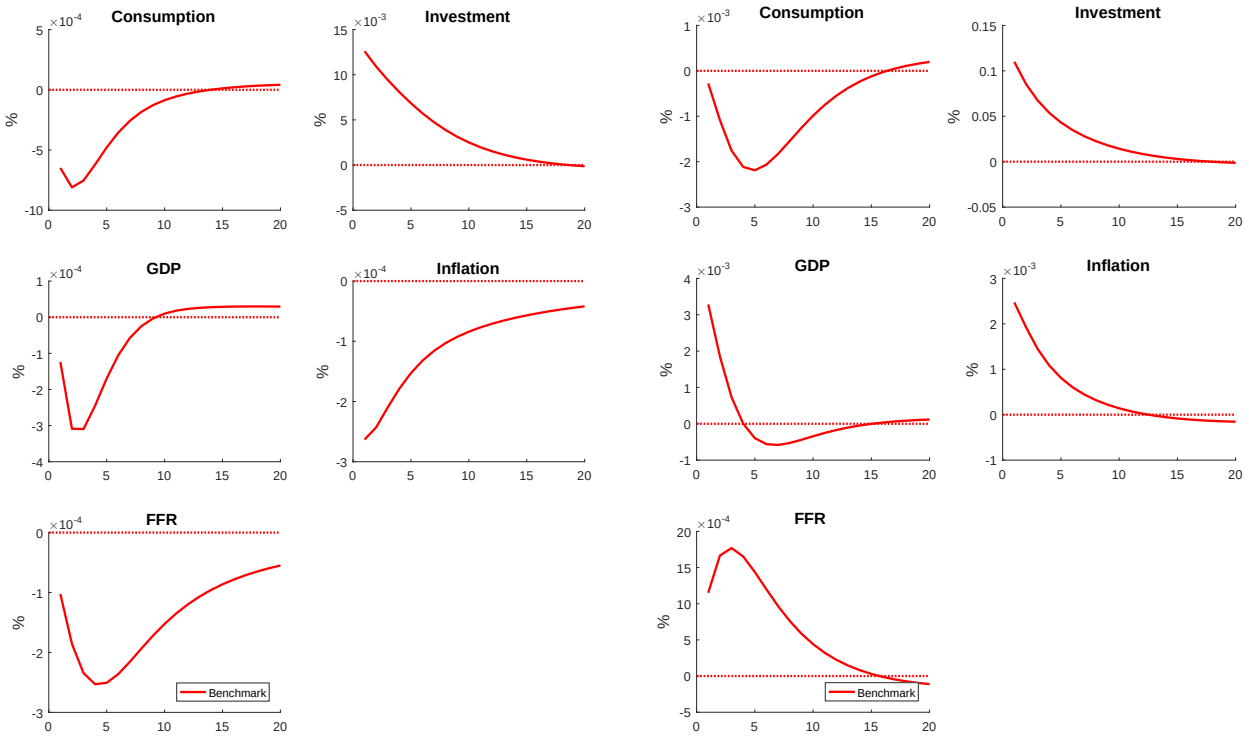
Figure 12: Estimated model. IRFs represent a change in the expected path of corresponding variables when volatility regime changes. The units of the y-axis are annualized percentage deviations from a steady state. Units on the x axis are quarters. Red solid line depicts an IRF to volatility regime change in a benchmark model. Black dashed line shows contribution of a precautionary savings motive and represents an IRF to a volatility regime change in a model, where inflation risk premia and risk premia on investment return along with varince term in Phillips curve do not change across volatility regimes. Black line with circles shows contribution of a channel operating through change in risk premium on investment return. This line represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where risk premia on investment return does not vary across volatility regimes. Black line with crosses shows contribution of inflation risk premium channel and represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where inflation risk premia does not vary across volatility regimes. Black dotted line shows contribution of a channel operating through a change in variance term in Phillips curve (nominal pricing bias). This line represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where variance term in Phillips curve does not vary across volatility regimes.



(a) IRF to a markup shock

(b) Increase in volatility of markup shock

Figure 13: Estimated model. Left panel plots IRF to a markup shock. The units of the y-axis are annualized percentage deviations from a steady state (0 on these axis is normalized to be stochastic steady state). Units on the x axis are quarters. Right panel plots IRF to increase in volatility of markup shock. IRFs represent a change in the expected path of corresponding variables when volatility regime changes. The units of the y-axis are annualized percentage deviations from a steady state. Units on the x axis are quarters. Red solid line depicts an IRF to volatility regime change in a benchmark model. Black dashed line shows contribution of a precautionary savings motive and represents an IRF to a volatility regime change in a model, where inflation risk premia and risk premia on investment return along with varince term in Phillips curve do not change across volatility regimes. Black line with circles shows contribution of a channel operating through change in risk premium on investment return. This line represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where risk premia on investment return does not vary across volatility regimes. Black line witch crosses shows contribution of inflation risk premium channel and represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where inflation risk premia does not vary across volatility regimes. Black dotted line shows contribution of a channel operating through a change in variance term in Phillips curve (nominal pricing bias). This line represents a difference between IRF to a volatility regime change in a benchmark model (red solid line) and an IRF to a volatility regime change in a model, where variance term in Phillips curve does not vary across volatility regimes.



(a) IRF to increase in volatility of a monetary policy shock

(b) IRF to increase in volatility of a liquidity shock

Figure 14: Estimated model. IRFs represent a change in the expected path of corresponding variables when volatility regime changes. The units of the y-axis are annualized percentage deviations from a steady state. Units on the x axis are quarters.

Parameter	Description	Posterior Mode
A. Preferences		
$\overline{\beta}$	Mean of subjective discount factor	0.9924
ρ_β	Persistence of preference shock	0.9967
h	Degree of habit formation	0.6002
ψ	Elasticity of intertemporal substitution	1.0000
γ	Risk aversion	19.5626
τ	Elasticity of labor supply	27.2957
$\overline{\zeta}_B$	Liquidity preference parameter	$0.1503 * 10^{-2}$
ρ_{ζ_B}	Persistence of liquidity shock	0.9768
B. Productivity		
μ^*	Average economic growth	0.0031
ρ_x	Persistence of productivity growth shocks	0.1240
C. Technology		
α	Capital share in production	0.0505
δ_0	Average capital depreciation	0.0161
δ_2	Coeff in capital depreciation func.	7.5023
φ_K	Capital adj. cost parameter	1.4067
$\log(\Upsilon)$	Average decline in price of investment	0.0050
ρ_Υ	Persistence of shocks to price of investment	0.9377
κ_R	Slope of Phillips Curve	0.0016
ρ_χ	Persistence of markup shocks	0.0329
κ_2	Degree of indexation to past inflation	0.9722
D. Monetary and Fiscal Policy		
ρ_r	Degree of monetary policy inertia	0.7213
$\rho_{\pi,1}$	Sens. of interest rate to inflation	1.2456
ρ_y	Sensitivity of interest rate to output	0.3206
ρ_ξ	Persistence of monetary policy shocks	0.0010
π_{ss}	Inflation in deterministic steady state	0.0085
π^*	Risk adjustment of inflation target	0.0172
η_G	St.st. share of gov.spending in GDP	0.1580
ρ_g	Persistence of gov. spending shocks	0.9800

Table 1: This table reports estimated parameter values (quarterly).

	Regime 1	Regime 2
Regime persistence	0.96	0.88
Prob. of regime	0.76	0.24
Preference, σ_b	0.0273	0.0453
TFP growth, σ_x	0.0096	0.0112
Monetary, σ_R	0.0013	0.0017
Markup, σ_χ	0.0078	0.0093
Price of inv., σ_{ζ_Υ}	0.0030	0.0052
Gov. spending, σ_g	0.0225	0.0259
Liquidity, σ_{ζ_B}	0.0006	0.0007

Table 2: Standard deviation of exogenous shocks. This table reports estimated standard deviations of shocks (quarterly). The model has 7 exogenous mean-zero normally distributed shocks: preference shock, TFP growth shock, monetary policy shock, markup shock, shock to relative price of investment, government spending shock and liquidity shock. The model has 2 volatility regimes, which follow Markov-switching process. We refer to Regime 1 as low volatility regime and Regime 2 as high volatility regime. Top panel reports persistence of corresponding regimes (probability to stay in the same regime next quarter) and unconditional probability of a given regime.

	Unconditional variance decomposition				Uncertainty and business cycle	
	Preference	TFP growth	Liquidity	Uncertainty	Median	Conf. Inter.
GDP	1.49	69.24	3.36	4.94	17.33	(13.38; 21.74)
Inflation	73.69	0.37	4.11	0.04	6.9	(5.41; 8.71)
FFR	88.19	0.23	7.7	0.53	27.24	(21.31; 34.64)
Investment	1.13	25.39	30.39	6.02	17.62	(13.41; 21.86)
Consumption	2.31	64.77	2.19	6.71	19	(14.4; 24.67)
Slope	7.39	4.24	23.91	25.15	61.57	(50.62; 73.79)

Table 3: Panel on the left presents the contribution of the different shocks to the unconditional variance of the macroeconomic variables and the slope of the yield curve. Panel on the right analyzes the importance of uncertainty shocks in generating business cycle fluctuations with respect to the traditional level shocks. Specifically, we use the posterior mode parameter values to simulate two economies 1,000 times. In the first economy, only uncertainty shocks occur. In the second economy, we had level shocks on top of the same uncertainty shocks. For each simulation and for each variable we extract business cycle fluctuations using a bandpass filter. Finally, for each simulation we compute the ratio between the volatilities of the business cycle fluctuations for the two economies.

	Yields						Slope		
	1Q	1Y	2Y	3Y	4Y	5Y	Total	Risk	Liquidity
Nominal	2.12	2.41	2.64	2.85	3.05	3.24	1.12	0.93	0.19
Real	0.79	0.80	0.87	0.92	0.98	1.03	0.24	0.24	-

Table 4: Panel on the left reports unconditional means of nominal and real yields in the estimated model for the following maturities: 1-quarter and 1,2,3,4,5 years. Panel on the right reports slopes of the corresponding term structures, defined as a difference between yields on 5-year and 1-quarter bonds. First column in the right panel reporst total value, while the next two column decompose difference between 5-year and 1-quarter yield into risk premium and liqidity premium. Values are annualized percent. 1-quarter real yield corresponds to a risk free rate $r_{f,t}$ in the model. Real bond prices are computed as $P_{r,t}^{(n)} = E_t[M_{t+1}P_{r,t+1}^{(n-1)}]$, where M_{t+1} is a real SDF.