Resource Curse or Blessing? Sovereign Risk in Resource-Rich Emerging Economies*

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Abstract

In this paper we document the stylized facts about the relationship between international oil price swings, sovereign risk and macroeconomic performance of oil-exporting economies. We show that even though being a bigger oil producer decreases sovereign risk–because it increases a country's ability to repay–having more oil reserves increases sovereign risk by making autarky more attractive. We develop a small open economy model of sovereign risk with incomplete international financial markets, in which optimal oil extraction and sovereign default interact. We use the model to understand the mechanisms behind the empirical facts.

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1 Introduction

International oil price volatility has affected the macroeconomic performance of oil exporting countries through changes in incentives. More specifically, changes in oil prices affect a country's desire to exploit their natural resource–a real asset–affecting reserves and extraction of oil. Changes in oil prices also affect a country's desire to consume and borrow or lend in financial assets, and finally they also affect incentives to repay or default, which at the same time affect sovereign risk and asset prices.

In this paper we document the stylized facts about the relationship between international oil price swings, sovereign risk and macroeconomic performance of oil-exporting economies. We show that even though being a bigger oil producer decreases sovereign risk–because it increases a country's ability to repay–having more oil reserves increases sovereign risk by making autarky more attractive. As we document in the next section using a dynamic fixed effects regression, the short-run elasticity of country risk with respect to changes in oil production is 0.04% and the long-run elasticity with respect to oil reserves is -0.12%. That is, when oil production increases by 1% country risk decreases by 0.04%, and when oil reserves increase by 1%, country risk increases by 0.12%.

We then develop a small open economy model of sovereign risk with incomplete international financial markets, in which optimal oil extraction and sovereign default interact. We use the model to understand the mechanisms behind the empirical facts.

Examining data for the 30 largest emerging market oil exporters over the period 1979-2010, we found that these countries hold an average external public debt[1] to GDP ratio of around 30% and sixteen countries in the sample have experienced between one and five default episodes. We highlight three features of the relationship between country risk and the size of the oil sector: First, as is natural to expect, a given oil exporting country is perceived by investors as less risky, the larger the share of oil in GDP and the higher the oil prices, allowing its public sector to support higher levels of public debt. Second, and perhaps less natural to expect, in the long run, country risk perception increases the higher the level of oil reserves of the country. This may reflect the fact that having a large stock of oil increases a country's outside option (the value of autarky), making default more appealing. Third, the data also shows that during default episodes, the median of oil exporting countries increases net oil exports. This evidence suggests that a country in default and excluded from international financial markets, increases its oil exports to withstand the consequences of financial autarky.

When we explore the relationship between oil price changes and macro performance, we find that increases in oil prices are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price decreases are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and a higher number of default events.

We build a small open economy model with two types of goods: a tradable and non-storable

[1]We use external public debt data from the World Bank where public and publicly guaranteed debt comprises long-term external obligations of public debtors, including the national government, political subdivisions (or an agency of either), and autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity. Data are in current U.S. dollars.

consumption good and oil. The sovereign government owns and delegates oil extraction to an oil producing company—which transfers its profits back at the end of the period—and can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets but cannot commit to repaying its debt. The relative price of oil and the consumption good are exogenously given.

We find that theory predicts that long-run reserves of the resource have two opposing effects in determining the long-term sovereign risk premium. Higher stock of reserves allow the country to have a higher extraction rate to support debt repayments, lowering default risk. However, they also allow the country to use the resource during default times, making the value of default more attractive. The tension between these two forces determines the long run default risk premium.

The paper proceeds as follows. Section 2 presents the empirical evidence, Section 3 presents the model, Section 4 presents the calibration and solution method, Section 5 shows the results and Section 6 concludes.

2 Empirical Evidence

2.1 Data

We have collected data for oil GDP, non-oil GDP, oil reserves, oil consumption, oil net exports, total public debt, total external public debt, net foreign assets, default episodes and country risk, for Saudi Arabia, Iran, Iraq, Kuwait, Venezuela, United Arab Emirates, Russian Federation, Libya, Nigeria, Kazakhstan, Qatar, China, Brazil, Algeria, Mexico, Angola, Azerbaijan, Ecuador, India, Oman, Sudan, Malaysia, Indonesia, Egypt, Yemen, Argentina, Syrian Arab Republic, Gabon, Colombia and Vietnam. The common feature among these thirty countries is that they are the group of emerging market countries with the largest proven oil reserves in the world.

The data we use was collected from different sources. First, country risk data is taken from the Institutional Investor's country credit ratings, which contains information for the 1979-2010 period of sovereign debt risk that is published biannually in the March and September issues of Institutional Investor magazine. Those ratings are based on a survey of leading international bankers, who are asked to rate each country on a scale from 0 to 100 (where 100 represents maximum creditworthiness). The answers are then weighted in accordance with the particular bank's global exposure and the level of sophistication for that country's analysis systems. A second source of information is the US Energy Information Administration dataset (EIA), which reports oil reserves, oil production, oil consumption and oil net exports information (in thousand barrels per day) for 219 countries, as well as Brent spot prices (in USD per barrel), from 1980 to 2013.

Total public debt data comes from the World Development Indicators tables (WDI) and the World Economic Outlook database (WEO). We have information, covering 1979-2010 period, for direct government fixed-term contractual obligations to others outstanding on a particular date. It includes domestic and foreign liabilities such as currency and money deposits, securities other than shares, and loans. In this case, information is not reported for Iraq and Libya. Total public external debt data

is taken from the World Bank Global Development Finance database (GDF), which has annual data for over 130 countries on total external debt by maturity and type of debtor (private non-guaranteed debt and publicly guaranteed debt). The data goes back as far as 1970 and is collected on the basis of public and publicly-guaranteed debt reported in the World Bank's Debtor Reporting System by each of the countries. This information is not available for Saudi Arabia, Canada, Iraq, Kuwait, United Arab Emirates, Libya, Qatar, and Oman. In the case of Russia, we use data reported by the Ministry of Finance.[2]

We use the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane and Milesi-Ferreti (2007) to obtain information on net foreign asset positions. It contains data for the 1970-2011 period and for 188 countries (including those in our sample), plus the euro area as a whole. Specifically, net foreign assets series are based on three alternative measures: i) the accumulated current account, adjusted to reflect the impact of capital transfers, valuation changes, capital gains and losses on equity and Foreign Direct Investment (FDI), and debt reduction and forgiveness; ii) the net external position, reported in the International Investment Positions section of the International Monetary Fund's Balance of Payments Statistics (BOPS), and net of gold holdings; iii) the sum of net equity and FDI positions (both adjusted for valuation effects), foreign exchange reserves and the difference between accumulated flows of "debt assets", and the stock of debt measured by the World Bank (or the OECD).

Default data is from Borensztein and Panizza (2006), for the 1980-2010 period. We include sovereign defaults on foreign currency bond debt and foreign currency bank debt. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue. Such rescheduling agreements covering short and long term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary.

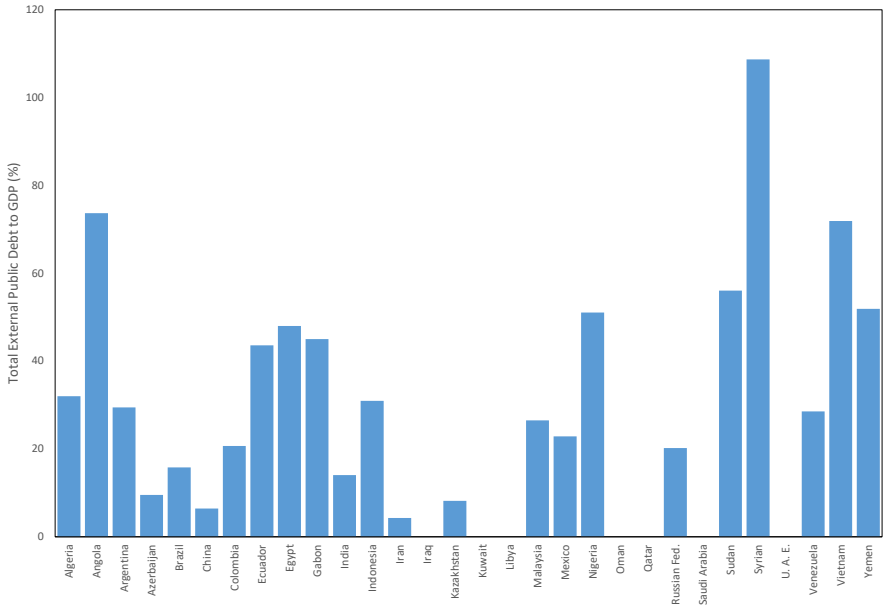
2.2 Stylized Facts

We start this section by illustrating that countries who are net oil exporters, do hold external public debt and default on it. Figure 1 shows the average external public debt as a percentage of GDP for twenty three countries in our sample. Those countries that show up as having no external public debt is because there is no data available. This subset of countries holds a minimum average external public debt to GDP ratio of around 5% (Iran) and a maximum of around 110% (Sudan). On average, these countries have a mean external public debt to GDP ratio of roughly 25% which is not negligible.

Figure 2 shows the number of default episodes for the same twenty five countries. In this case a value of zero means that it was a country that did not default on its debt during this period. We have this data for all of our sample. The number of default episodes ranges between zero and 3. The fact

[2]Russian total public external debt data is available for a monthly frequency. Since GDP information is reported only at the end of each year, we use debt information in December, in order to construct public external debt-to-GDP ratios. Russian data is only available since 1990.

Figure 1: Average External Public Debt of Net Oil Exporters (1979-2010)



that these countries do hold external public debt and default on it shows that it is relevant to better understand the connection between oil prices, macroeconomic performance and sovereign default.

We now turn our attention to the relationship between sovereign risk—measured by the Institutional Investor Index (III from now on)—and different variables. Specifically, Table 1 shows the unconditional correlation between the III and oil reserves, oil prices, external public debt and total public debt for all the countries in our sample. We can see that as expected, both external and total public debt are negatively correlated with sovereign risk and this relationship is statistically significant in nearly all of the cases. On the other hand, oil prices are positively correlated with the III, meaning that higher oil prices are associated with lower sovereign risk. This is natural, as higher oil prices imply a greater ability to repay. Finally, we can see that when we don't condition on anything, the sign of the correlation between oil reserves and sovereign risk is mixed. For some countries it is positive and for some it is negative.

Table 1 contains contemporaneous correlations reflecting short-term relationships. Now, we take a look at long-run unconditional correlations. To do so, we run a between effects regression of the III on oil production and on oil reserves to production.[3] The between effects regression is just an OLS regression on the cross-sectional equation over the average value of each variable for a given country. Figure 3 shows the in between effects regression results for the III on oil production. Again, each point in the plot represents the average values over time for a given country. The results show that in the long-run countries that have maintained high oil production have been characterized by a higher

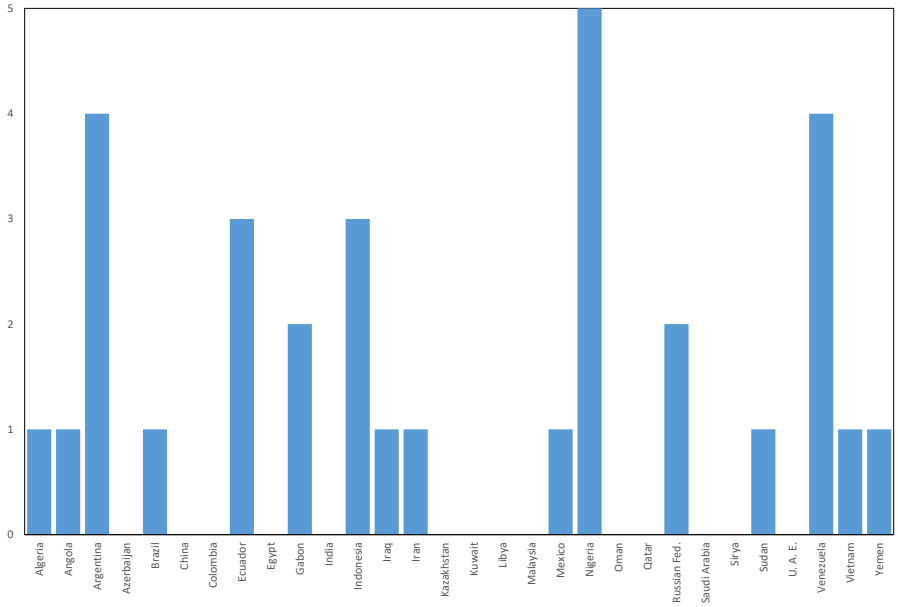
[3]Note that reserves to production (oil extraction) represents the number of years that it would take a country to deplete its reserves assuming that there are no new discoveries.

Table 1: Unconditional Correlations

Country	Oil Reserves	Real Oil Prices	Total External Public Debt to GDP	Total Public Debt to GDP
Algeria	0.4098**	0.7744***	-0.7498***	-0.7413***
	(0.1588)	(0.0767)	(0.1135)	(0.1225)
Angola	0.8044***	0.8597***	-0.7438***	-0.7002***
	(0.1034)	(0.0619)	(0.1364)	(0.1908)
Argentina	-0.0059	0.3587***	-0.6159***	-0.6353***
	(0.1741)	(0.1132)	(0.1351)	(0.1434)
Azerbaijan	0.5799**	0.9004***	-0.8409***	-0.8994***
	(0.2456)	(0.0907)	(0.1632)	(0.1652)
Brazil	0.8174***	0.8856***	-0.7383***	-0.2157
	(0.1003)	(0.0563)	(0.1157)	(0.1783)
China	-0.5770***	0.8812***	-0.8191***	0.5051***
	(0.1422)	(0.0573)	(0.1014)	(0.1726)
Colombia	-0.1126	0.8565***	-0.8025***	-0.5751***
	(0.173)	(0.0626)	(0.1023)	(0.1519)
Ecuador	0.0083	0.5082***	-0.5903***	-0.6466***
	(0.1741)	(0.1044)	(0.1384)	(0.1393)
Egypt	-0.4703***	0.3351***	-0.6809***	-0.5912***
	(0.1536)	(0.1143)	(0.1256)	(0.1498)
Gabon	-0.3720**	0.6194***	-0.7295***	-0.6806***
	(0.1616)	(0.0952)	(0.1173)	(0.1338)
India	0.0708	0.8168***	-0.8436***	0.1667
	(0.1736)	(0.07)	(0.0921)	(0.1934)
Indonesia	0.2101	0.4709***	-0.5926***	-0.9226***
	(0.1702)	(0.107)	(0.1381)	(0.0704)
Iraq	-0.5215***	0.5946***		
	(0.1508)	(0.0975)		
Iran	0.7418***	0.0991	0.1229	-0.5640***
	(0.1186)	(0.1207)	(0.1782)	(0.1533)
Kazakhstan	0.8081***	0.8809***	-0.5395***	-0.7951***
	(0.1473)	(0.073)	(0.1837)	(0.1516)
Kuwait	0.0469	0.8257***		-0.9062***
	(0.1766)	(0.0684)		(0.113)
Libya	0.4660***	0.7069***		
	(0.154)	(0.0858)		
Malaysia	0.2187	0.7522***	-0.4409***	-0.3123*
	(0.1699)	(0.0799)	(0.1539)	(0.1764)
Mexico	-0.8381***	0.7638***	-0.7459***	-0.7085***
	(0.095)	(0.0783)	(0.1142)	(0.1311)
Nigeria	0.4223**	0.8194***	-0.7410***	-0.6734***
	(0.1578)	(0.0695)	(0.1152)	(0.135)
Oman	0.7066***	0.6547***		-0.4931***
	(0.1232)	(0.0917)		(0.1588)
Qatar	0.9091***	0.8442***		-0.4797**
	(0.0725)	(0.065)		(0.2013)
Russian Federation	0.7410***	0.6909***	-0.7539***	-0.8704***
	(0.1679)	(0.0877)	(0.1434)	(0.1194)
Saudi Arabia	-0.3566**	0.8461***		-0.9083***
	(0.1652)	(0.0646)		(0.1118)
Syria	0.4064**	0.3173***	0.8676**	-0.7369***
	(0.1591)	(0.115)	(0.2224)	(0.1234)
Sudan	0.5003***	0.6235***	-0.4986***	-0.8144***
	(0.1555)	(0.0948)	(0.1487)	(0.1407)
United Arab Emirates	0.3769**	0.7213***		0.3344*
	(0.1637)	(0.084)		(0.1721)
Venezuela	-0.2995*	0.3915***	-0.4065**	-0.7707***
	(0.1687)	(0.1116)	(0.159)	(0.1301)
Vietnam	0.3896*	0.8493***	-0.8134***	-0.7283***
	(0.201)	(0.0805)	(0.1269)	(0.1713)
Yemen	0.4929	-0.4010*	0.4628	-0.5306
	(0.2751)	(0.2048)	(0.2803)	(0.346)

Standard errors in parenthesis
*** p<0.01, ** p<0.05, * p<0.1

Figure 2: Net Oil Exporters Number of Default Episodes (1979-2010)



average credit rating: the correlation is 0.28. As mentioned earlier, this is expected as higher oil production implies a greater ability to repay debt.

On the other hand Figure 4 shows the in between effects regression for the III on oil reserves to production. In this case the long-run unconditional correlation is -0.02. This result points to the fact that having oil might have two opposing effects on country risk. High oil production increases a country's ability to repay and lowers country risk, but possessing a large stock of oil increases the value of autarky triggering a "Limited Commitment" mechanism: a country can be excluded from financial markets but it can still sell their real asset—oil in this case—in international markets, making autarky more attractive.

Figure 3: In between effects regression of the Institutional Investor Index (X-Axis) on average oil production (Y-Axis): 1979-2010.

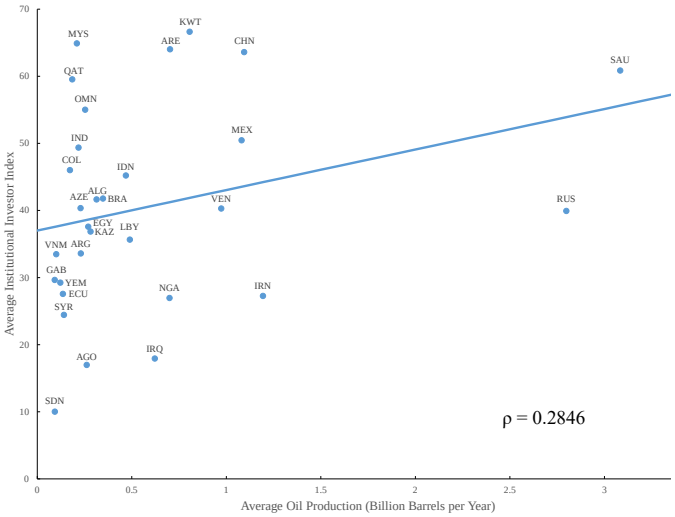
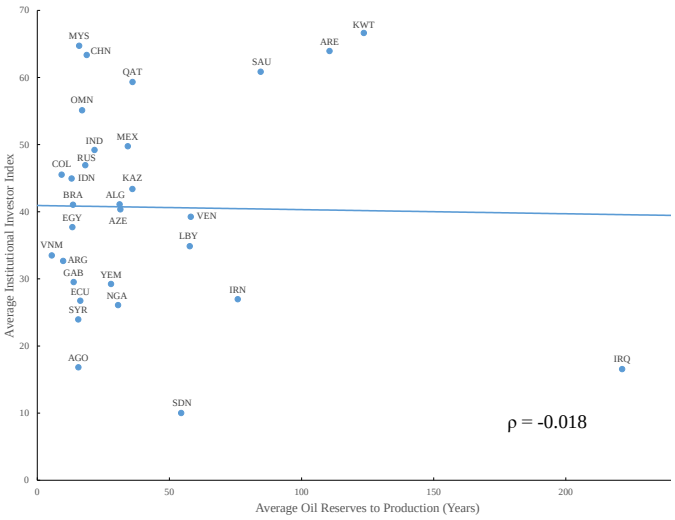


Figure 4: In between effects regression of the Institutional Investor Index (X-Axis) on average oil production (Y-Axis): 1979-2010.



Notice that our analysis has two dimensions. We want to study the short versus long-run effects of both production and reserves. To summarize, so far, these unconditional correlations point towards

two mechanisms. First, extracting more oil (production) increases a country's ability to repay its debt, decreasing country risk. Second, owning a larger stock of oil (reserves) seems to be positively correlated with country risk, and this goes in line with the idea that if a country has a larger stock of a real asset, then autarky becomes a more attractive option.

In order to study more formally the presence of these two mechanisms in the data, and establish conditional correlations we run a dynamic fixed effects estimation[4] of long-run, short-run and convergence coefficients. These allows us to put all the previous results together and be able to establish statistical significance of the relevant variables and timing. The results are shown in Table 1.[5]

Note that in each model the convergence coefficient has the expected sign and is statistically significant at the 1% level. Since the estimated coefficients take a value around -0.2, convergence in the III runs at an annual rate of about 0.2%, which means that each year the III covers about 0.2% of its distance to the "steady state". It should also be noted that convergence is slightly slower in Model (2), where the net foreign assets-to-GDP ratio is included and default is excluded.

If we focus on the short-run coefficients, we observe that an increase in oil GDP decreases country risk, and this result is statistically significant at a 5% level. The same for an increase in non-oil GDP, but this result is significant to a 1% level. In the short run, a positive change in oil reserves (which can happen if extraction is lower than discoveries of oil in a given period), decreases country risk. In the first two models this result is significant to a 10% level and to a 5% in the third model which controls for both default and net-foreign assets. As is expected, increases in external public debt increase country risk, and this result is statistically significant to a 1% level. Finally, a positive *change* in net-foreign assets increases country risk but this result is not statistically significant.

When looking at the long-run coefficients, as shown in Pesaran et al. (1999), the usual interpretation—when series are in logs—is that of an elasticity. Then, the long-run oil GDP elasticity is 0.05 in the first model, 0.08 in the second, and 0.05 in the third, which means that when oil GDP increases by 1%, the III is between 0.05% and 0.08% higher in the long-run. However, this results are not statistically significant. With respect to non-oil GDP, long-run elasticities are positive, but it is significantly different from zero at a 5% level, only in the second model where we control for net-foreign assets but not for default. In this case, other things equal, country risk rating decreases around 0.16% because of a 1% increment in non-oil GDP.

Moreover, a significant negative relationship between oil reserves and the III was found. A rise in oil reserves worsens our measure of country risk in the long term. Thus, an oil exporting economy is perceived as more risky in the future when it boosts its reserves today. This elasticity is statistically different from zero at a 10% level for models one and two and and at a 1% level for the third model where we control for net-foreign assets and default.

[4]See Appendix B to see the different methods we used for the estimation and the Hausman test that determined that dynamic fixed effects was the dominant approach.

[5] Due to data limitations, in Model (1) Yemen was excluded from the sample. In Model (2) Iraq, Libya, Yemen and Azerbaijan were excluded, and in Model (3) Iraq, Russian Federation, Yemen and Azerbaijan. Consequently, the estimation is performed taking into account 887, 785 and 828 observations, respectively.

Table 2: Dynamic Fixed Effects Regression Results for Institutional Investor Index

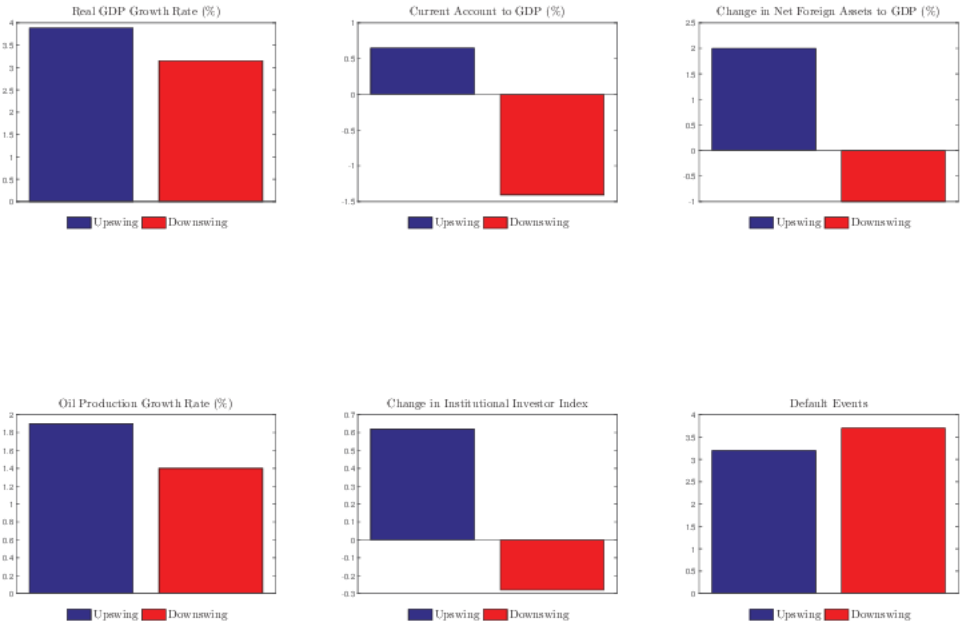
	Δ Inst. Investor Index		
	Model (1)	Model (2)	Model (3)
Convergence coefficient			
Inst. Investor Index (-1)	-0.237***	-0.200***	-0.236***
	(0.024)	(0.023)	(0.024)
Short-run coefficients			
Δ Oil GDP	0.039**	0.041**	0.039**
	(0.016)	(0.017)	(0.016)
Δ Non-oil GDP	0.395***	0.413***	0.384***
	(0.074)	(0.076)	(0.074)
Δ Oil reserves	0.045*	0.052*	0.053**
	(0.026)	(0.027)	(0.026)
Δ Ext. pub. debt	-0.066***	-0.085***	-0.069***
	(0.022)	(0.024)	(0.023)
Δ Oil Discoveries	-0.017	-0.014	-0.018
	(0.019)	(0.020)	(0.019)
Δ NFA		-0.035	-0.029
		(0.033)	(0.032)
Long-run coefficients			
Oil GDP	0.051	0.076	0.052
	(0.043)	(0.051)	(0.043)
Non-oil GDP	0.092	0.151**	0.107*
	(0.057)	(0.070)	(0.057)
Oil reserves	-0.108**	-0.109*	-0.143***
	(0.049)	(0.060)	(0.050)
Ext. pub. debt	-0.203***	-0.147***	-0.130***
	(0.037)	(0.055)	(0.045)
Oil Discoveries	0.176	0.211	0.194*
	(0.109)	(0.132)	(0.109)
NFA		0.336***	0.242***
		(0.109)	(0.089)
Default	-0.305***		-0.300***
	(0.056)		(0.056)
Constant	0.662***	0.335*	0.598***
	(0.196)	(0.194)	(0.195)
Observations	512	509	509

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

As expected, in the long-run, external public debt still has a negative effect on country risk and is again statistically significant to a 1% level for the three models. Different from the short-run, in the

Figure 5: Oil Price Swings and Macro Performance



long run the *level* of net-foreign assets does matter for country risk. Holding more net-foreign assets decreases country risk. This result is statistically significant to a 1% level. Finally, as expected, being in default increases country risk. When the probability of default increases by 1%, the III drops by 0.35%. This last result is statistically significant to a 1% level.

These results support the two mechanisms that we believe are behind the unconditional correlations presented above. Oil production decreases country risk by increasing a country's ability to repay, but greater oil reserves increase country risk by making autarky more attractive.

We now look at the relationship between oil price swings and macro performance. To construct Figure 5, we classified all the years in our sample as corresponding to an upswing (a positive change in prices) or downswing (a negative change in prices), and then constructed interval periods for the two. Table 11 in Appendix C shows how the intervals were constructed. We then averaged the different macroeconomic variables over the upswings and downswings and Figure 5 shows the results.

We can see that oil price upswings are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price downswings are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and higher number of default events.

Figure 6: Oil Prices and Default Episodes

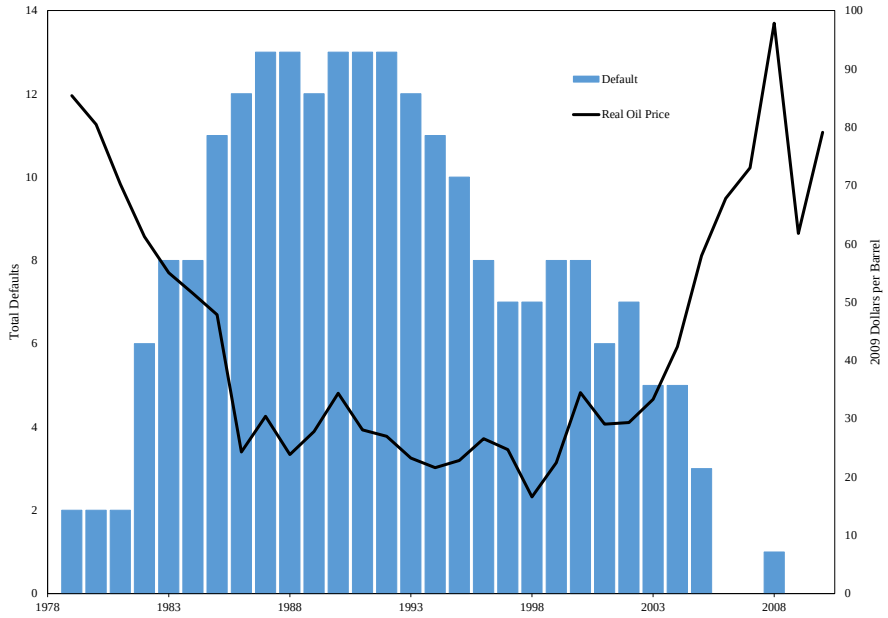
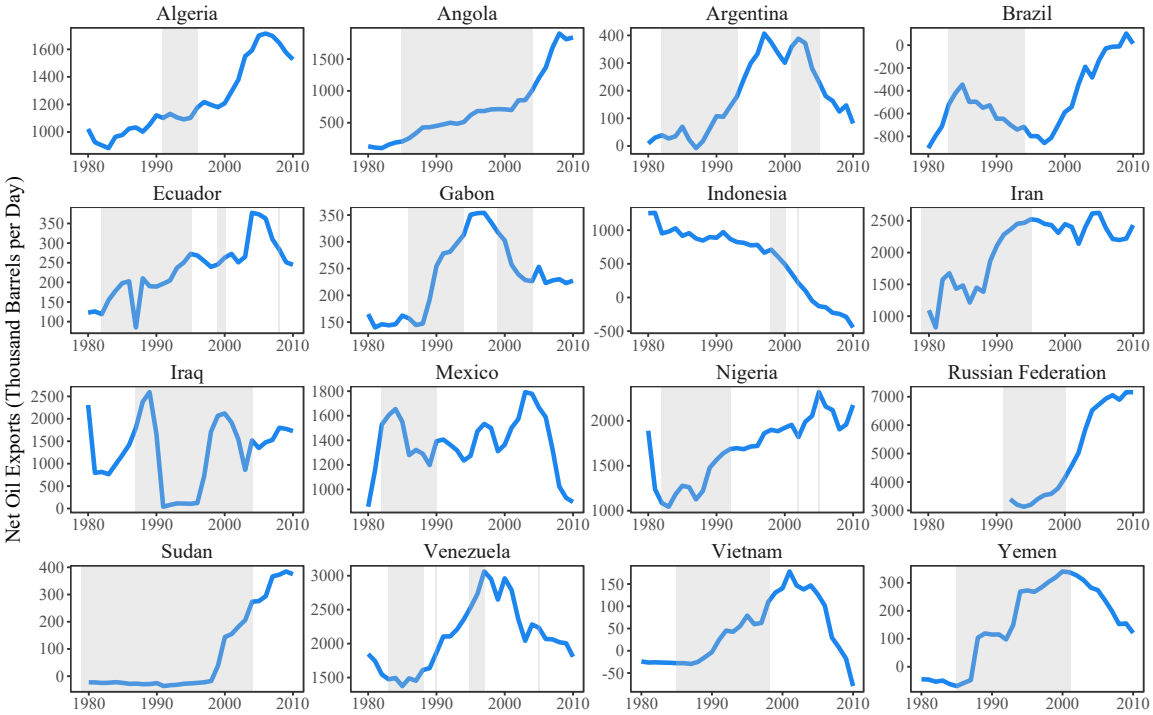


Figure 7: Oil Exports During Default Episodes



Note: Shaded area means the country is in default

3 A Model of Sovereign Default and Oil Extraction

We present an off-the-shelf general equilibrium small open economy model with sovereign default and modify it so that the sovereign receives an important part of its income from the oil industry, and the oil industry's extraction decisions are affected by country risk.

There are two agents in the model and two types of goods: a company that produces oil and the sovereign government who receives a tradable non-storable consumption good. We assume that the price of oil, p, is determined by international markets and is taken as given by the oil company, and the tradable non-storable consumption good received by the government, y, is also exogenously determined. At the beginning of the period both $p$and $y$are realized, and initial oil reserves, s, and sovereign debt standing, b, are given. At that point the sovereign government decides whether to default on its debt or repay. If the sovereign decides to default, there is no bond market that period and the oil company decides how much oil to extract, x, and sells it in the international oil market facing a trade penalty because of the default of the sovereign. Then consumption is determined.

If the sovereign government decides to repay, the bond market opens, the oil company takes as given the price of bonds, q, and decides how much oil to extract and oil reserves for tomorrow. Simultaneously the government chooses its debt level for tomorrow and consumption is determined.

In the next subsections we formalize this problem.

The problem of the oil producing company

Given today's realization of the price of oil, p, the oil company decides how much oil to extract optimally, x, from the available stock of proven oil reserves, s, by maximizing the present discounted value of their payoffs:

$$w(s,p) = \max_{\{x,s'\}} \left\{ f(x,s,p) + q\left(B',S,y,p\right) E\left[W\left(s',p'\right)\right] \right\}$$

subject to a transition function for oil reserves and oil prices respectively,

$$s' = g(x,s,p),\ p' = z_p(p), \tag{1}$$

and a feasibility constraint,

$$0 \leq x \leq s + d. \tag{2}$$

The payoff function is given by $f(x,s,p) = px - e\left(x,s\right)$, where px are the revenues generated by selling the oil at the international price and $e\left(x,s\right)$ are the extraction costs that the firm faces for extracting x units of oil. The transition function $g(x,s,p) = s - x + d$, states that tomorrow's oil reserves are equal to today's stock, minus what is extracted plus d, which stands for oil discoveries, and are assumed to be constant. Finally, the feasibility constraint (2) guarantees that extraction is less than today's reserves plus the discoveries, and that once oil has been extracted from the ground it can't be returned.

The extraction cost function ($e(x,s)$) is expressed in units of the consumption good and has the following properties: $e_s < 0$, $e_x > 0$ and $e_s(0,s) = 0$. Total extraction costs fall with oil reserves but increase with the extraction rate. Also, the marginal cost of an additional unit of reserves, conditional on not extracting oil is zero. We assume that $e(x,s)$ is given by:

$$e(x,s) = \psi \left(\frac{x}{s}\right)^{\gamma} x. \tag{3}$$

The oil company discounts the future at a rate $q(B', S, y, p)$, where B' is tomorrow's optimal debt level chosen by the sovereign, and is taken as given by the firm. As will be clear from the next Subsection, the optimal level of debt chosen by the government will depend on the optimal extraction policy of the firm, but the firm doesn't endogenize that effect, as such, we denote it by a capital letter in the problem of the firm. Likewise, note that $q$is a function of (capital) S , signaling that the firm does not endogenize the effect that their choice of reserves has on their discount factor.

The discount factor, $q(B', S, y, p)$, corresponds to the equilibrium price of the risky bond issued by the government, and as such depends on the probability of default of the sovereign such that extraction decisions are affected by the spread of the country. This assumption implies, that if the oil company were to acquire debt from foreign lenders they would face a risk premia like the one the sovereign government faces. In Appendix D, we show that this is observed in the data. Credit ratings of oil companies are highly correlated with the credit ratings of its country of origin.

The optimality conditions of the firm imply the following Euler equation

$$q(B', S, y, p) E\left[p' - e_x(x', s') - e_s(x', s')\right] = p - e_x(x, s). \tag{4}$$

Note that Equation 4 states that optimal extraction, $x^*(p, s; B')$, is such that the marginal benefit of extracting a unit of oil today (right hand side of the equation) has to be equal to the discounted expected benefit of extracting a unit of oil tomorrow (left hand side of the equation). Whenever q is different from zero, the firm's decision is inter-temporal, and the tradeoff between extracting today or tomorrow is affected by the discount factor q, which is determined in equilibrium by the probability of default of the sovereign government. More on this to follow.

Finally, at the end of each period the oil company transfers its profits $\pi^*(s, p; B')$ to the sovereign government.

The problem of the sovereign

The sovereign is a benevolent government who maximizes the utility of private agents. He receives a tradable non-storable consumption good, y, and the profits generated by the oil company, $\pi^*(s, p; b')$. He issues all possible debt abroad by issuing one-period, non-state contingent discount bonds, b', to be repaid tomorrow but he cannot commit to repaying its debt. The set of bond face values is $B = [b_{min}, b_{max}] \subset R$ where $b_{min} \leq b_{max} = 0$. As in the Eaton-Gersovitz model, when the country defaults it does not repay in the current period and the punishment is exclusion from world credit markets in the same period. Next period the country can re-enter world financial markets with a

probability λ. Consequently, the planner's payoff is given by:

$$V(b,S,y,p)=\max\left\{v^{nd}(b,S,y,p),v^{d}(S,y,p)\right\},$$

where $v^{nd}(b,S,y,p)$ is the value of no-default and $v^{d}(S,y,p)$ is the value of default.

The value of no-default solves the following constrained maximization problem:

$$v^{nd}(b,S,y,p)=\max_{\{c,b'\}}\left\{u(c)+\beta E\left[V\left(b',S'^{*}\left(s,p;b'\right),y',p'\right)\right]\right\} \tag{5}$$

subject to the budget constraint

$$c-b+A=y+px^{*}\left(s,p;b'\right)-q\left(b',S,y,p\right)b', \tag{6}$$

where $x^{*}\left(s,p;b'\right)$ and $S'^{*}\left(s,p;b'\right)$ correspond to the optimal oil extraction and tomorrow's optimal stock of oil reserves chosen by the oil company.

$q\left(b',S,y,p\right)$ is the pricing function for the risky bond. Recall that in the previous subsection, the discount factor of the firm was denoted by $q\left(B',S,y,p\right)$ to emphasize that the oil company doesn't internalize the fact that tomorrow's optimal debt level—which determines the price of the risky bond—will be affected by their optimal extraction choice, but make sure to note that $q\left(B',S,y,p\right)=q\left(b',S,y,p\right)$. Also note that the stock of oil inside the pricing function is a capital S, meaning that the sovereign takes that level of reserves as given, however in equilibrium $S=s$.

We assume that the oil company pays the extraction cost, $e\left(x^{*}\left(s,p;b'\right),s\right)$, to the government so it doesn't show up in the budget constraint. Finally, A, adjusts absorption given that the model doesn't have investment or government spending.

If the government chooses to default, then he solves the following constrained optimization problem:

$$v^{d}(S,y,p)=\max_{\{c\}}\left\{u(c)+\beta\left(1-\lambda\right)Ev^{d}\left(S'^{d},y',p'\right)+\beta\lambda EV\left(0,S'\left(p,s;0\right),y',p'\right)\right\} \tag{7}$$

subject to the budget constraint

$$c+A=y+\theta px^{d}\left(s,p\right). \tag{8}$$

We assume that if the sovereign defaults on its debt today, they will be seized a fraction $(1-\theta)$ of the oil revenues.[6] Furthermore, just as in the case of no default they receive $px^{*}\left(s,p;b'\right)$ from the oil company, we denote by $px^{d}\left(s,p\right)$ what they receive in the case of default, $x^{d}\left(s,p\right)=x^{*}\left(s,p;0\right)$. This comes from the fact that whenever the sovereign defaults, there is no bond market and $q\left(b',S,y,p\right)=0$. In this case, the oil company's optimality condition (Equation 4) reduces to $p-e_{x}\left(x,s\right)=0$, the problem of the oil company becomes intra-temporal, and the optimal extraction policy is given by

$$x^{d}\left(s,p\right)=\left[\frac{p}{\varphi\left(1+\gamma\right)}\right]^{\frac{1}{\gamma}}s.$$

[6]This is essentially a trade penalty as observed in the data and will be discussed further in the calibration section.

In other words, the extraction policy under default is constant for all pairs, $\{s, p\}$. Intuitively what is happening is that when there is a default the price of debt goes to zero and its yield goes to infinity. Then arbitrage requires that the return on oil production goes to infinity as well, and this can be accomplished by setting current extraction so that the marginal extraction cost equals the current price (see Equation 4).

Equation 7 states that the value of default is equal to todays consumption utility plus the weighted discounted expected continuation values. The weight on the continuation values is given by λ, the probability with which the sovereign can re-enter financial markets. If they do re-enter international financial markets, they do so with a debt $b' = 0$. On the other hand, with probability $(1 - \lambda)$ the sovereign will remain in the default state.

The definition of the default set and the probability of default are as in the Eaton-Gersovitz model. For a debt position $b < 0$, default is optimal for the set of realizations of $\{y, p\}$ for which $v^d(S, y, p)$ is at least as high as $v^{nd}(b, S, y, p)$:

$$D(b, S) = \left\{\{y, p\} : v^{nd}(b, S, y, p) \leq v^d(S, y, p)\right\}. \tag{9}$$

The probability of default tomorrow perceived as of today, $d(b', S'^*(b'; p, s), y, p)$, can be calculated from the default set and the transition probability functions of the endowment $z_y(y' \mid y)$ and oil prices $z_p(p' \mid p)$ as follows:

$$d(b', S'^*(b'; p, s), y, p) = \int \int_{D(b', S)} dz_y(y' \mid y)\, dz_p(p' \mid p). \tag{10}$$

Hence, the risky bond price is determined by the following non-arbitrage condition

$$q(b'; S, y, p) = \bar{q}\left(1 - d(b'; S, y, p)\right),$$

where $\bar{q}$ is the risk free rate.

It is important to note that the sovereign knows how the optimal policies of the oil company respond to changes in debt through its effect on q, and he takes into account these effects when choosing tomorrow's debt. This can be seen through Equations 5 and 6, where the sovereign evaluates how q affects x^* and s'^* and takes it into account when choosing b'. In this sense the setup is like a Stackelberg leader where the government is the leader and chooses optimal plans internalizing the optimal plans of the oil company.

3.1 Two period example

In this Subsection we study a two period version of the model described above to illustrate the implications of the model.

We assume that reserves are fixed at $\bar{s}$, there are no discoveries and the endowment of the non-tradable good that the sovereign receives is parametric and equal in both periods. In the first period there is no uncertainty—p is known—and the oil company chooses extraction today, x, and extraction

tomorrow, x', given the expected price for tomorrow

$$\max_{x\geq 0, x'\geq 0} px - e\left(x, \bar{s}\right) + q\left(b'\right) E_{p'}\left[p'x' - e\left(x'\right)\right] \tag{11}$$

subject to the resource constraint

$$x + x' = \bar{s}.$$

Recall that oil extraction costs are given by Equation 3 and in the two period case, $e\left(x, \bar{s}\right) = \frac{\varphi}{2}\left(\frac{x}{\bar{s}}\right)^{\gamma} x$ and $e\left(x'\right) = \frac{\varphi}{2}x'$ (because $x' = \bar{s} - x$). Furthermore, we assume that p' is a random variable distributed uniformly over the interval $[0, \bar{p}']$, such that its density function is

$$h\left(p'\right) = \begin{cases} \frac{1}{\bar{p}'} & \text{if } 0 < p' \leq \bar{p}' \\ 0 & \text{otherwise} \end{cases},$$

and its expected value is $E\left(p'\right) = \frac{\bar{p}'}{2}$. Under this assumption, the solution to the problem in Equation 11 implies that the optimal extraction policies (assuming an interior solution) are given by

$$x^* = \max\left\{\min\left[\left[\frac{2p - q\left(b'\right)\left(\bar{p}' - \varphi\right)}{\left(1+\gamma\right)\varphi}\right]^{\frac{1}{\gamma}} \bar{s}, \bar{s}\right], 0\right\},$$

and extraction in the second period is given by

$$x'^* = \max\left\{\min\left[\left(1 - \left[\frac{2p - q\left(b'\right)\left(\bar{p}' - \varphi\right)}{\left(1+\gamma\right)\varphi}\right]^{\frac{1}{\gamma}}\right) \bar{s}, \bar{s}\right], 0\right\}.$$

At the beginning of the second period p' is realized and x'^* is observed by the sovereign who decides whether to repay or default based on the relative utilities

$$d = \begin{cases} 1 & \text{if } u\left(y' + \theta p'x'^*\right) \geq u\left(y' + p'x'^* + b'\right) \\ 0 & \text{otherwise} \end{cases}$$

This implies that for a given level of b', the realization of the price tomorrow, p', has to be above a threshold $\tilde{p'}$ for the sovereign to repay. This threshold satisfies the condition

$$\tilde{p'}x'^* = \frac{b'}{\left(\theta - 1\right)}. \tag{12}$$

Because the default set is defined by

$$\mathcal{D}\left(b', p, \bar{s}\right) = \left\{p' \in \mathcal{P} : d\left(b'; p, p', \bar{s}\right) = 1\right\},$$

then the probability of default is determined by the price realizations between zero and $\tilde{p'}$

$$d\left(b', p, \bar{s}\right) = \int_0^{\tilde{p'}} d\left(b'; p, p', \bar{s}\right) h\left(p'\right) dp'. \tag{13}$$

This allows us to write the problem of the sovereign who chooses debt to solve the following problem

$$\begin{aligned} \max_{b'} \quad & u\left(y + px^* - q\left(b', p, \bar{s}\right) b'\right) \\ & + \beta \left\{ d\left(b', p, \bar{s}\right) \int_0^{\tilde{p'}\left(b', p, \bar{s}\right)} u\left(y' + \theta p' x'^*\right) h(p') dp' \right. \\ & \left. + \left[1 - d\left(b', p, \bar{s}\right)\right] \int_{\tilde{p'}\left(b', p, \bar{s}\right)}^{\infty} u\left(y' + p' x'^* + b'\right) h(p') dp' \right\} \quad . \end{aligned} \tag{14}$$

Note that for $x \in (0, \bar{s})$, today's optimal extraction, x^*, is increasing in today's price, p, and the stock of available oil reserves, $\bar{s}$, but decreasing in future prices, p'. Also, today's extraction, x^*, is decreasing in the sovereign bond price. This is intuitive because a higher q implies that their yield drops and the opportunity cost of extracting oil falls, giving the oil producer the incentive to adjust extraction plans so as to reduce the return on "investing" in oil production (i.e. investing in oil reserves) to match the lower return on bonds. This implies extracting less today, because extracting less today reduces the marginal cost of extraction (for a given p), and hence increases the net marginal revenue on oil extraction today (that is $p - e_x\left(x, s\right)$ rises), hence reducing the rate of return on oil reserves. This is all working through Equation 4.

As mentioned earlier, the planner is aware of the relationship mentioned above, so in choosing debt issuance he takes into account how an increase in b' will reduce q resulting in more extraction today. Finally, also notice, that there is a level of today's price $\hat{p}$ above which it is optimal to extract all the available oil reserves today

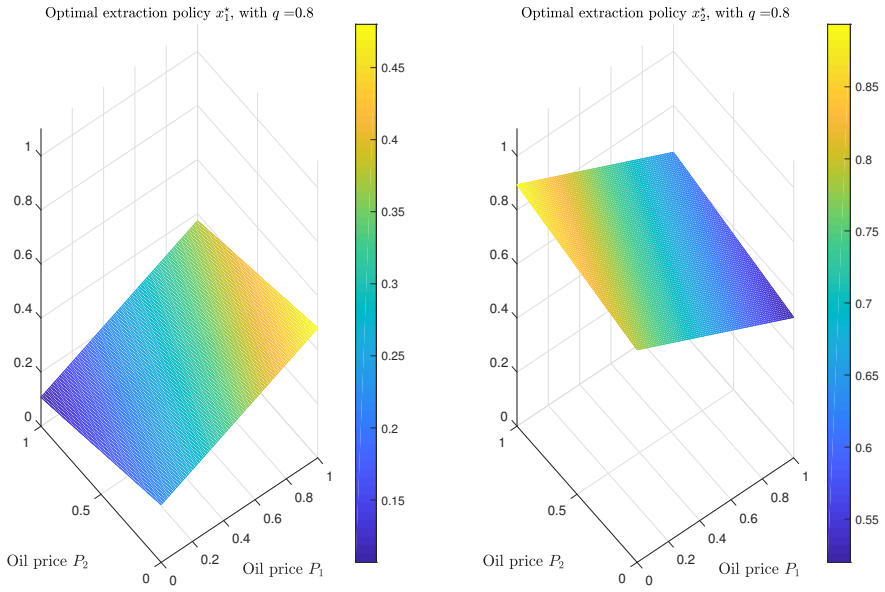
$$\hat{p} = q\left(\frac{\bar{p}' - \varphi}{2}\right) + \frac{\varphi\left(\gamma + 1\right)}{2}.$$

Likewise, for $x \in (0, \bar{s})$, tomorrow's optimal extraction, x'^*, is increasing in future prices, p', and $\bar{s}$, but decreasing in p. Similarly, there is a level of the future expected price $E\hat{p'}$ such that at prices higher than this level it is optimal to extract all the available oil reserves in the second period

$$E\hat{p'} = 2\frac{p}{q} + \varphi.$$

Figure 8 illustrates the optimal extraction policies for a given q.

Figure 8: Optimal extraction policies



Substituting for the optimal extraction policy in the second period in Equation 12, and letting H be the cdf of p' we can compute the probability of default $H\left(\tilde{p'}\left(b', \bar{s}, p\right)\right)$

$$\delta = \tilde{H} = \frac{\tilde{p'}}{\bar{p'}} = \frac{\phi}{\left(1 - \left[\frac{2p - q(b')(\bar{p}' - \varphi)}{(1+\gamma)\varphi}\right]^{\frac{1}{\gamma}}\right)\bar{s}\bar{p}'},$$

where $\phi = \frac{b'}{(\theta - 1)}$.

Since in equilibrium the price of sovereign bonds must satisfy the arbitrage equation, $q = \bar{q}\left(1 - \tilde{H}\right)$ then,

$$q = \bar{q}\left(1 - \frac{\phi}{\left(1 - \left[\frac{2p - q(b')(\bar{p}' - \varphi)}{(1+\gamma)\varphi}\right]^{\frac{1}{\gamma}}\right)\bar{s}\bar{p}'}\right).$$

Figure 9 shows the solution to q:

Figure 9: Bond price implied by the no-arbitrage condition

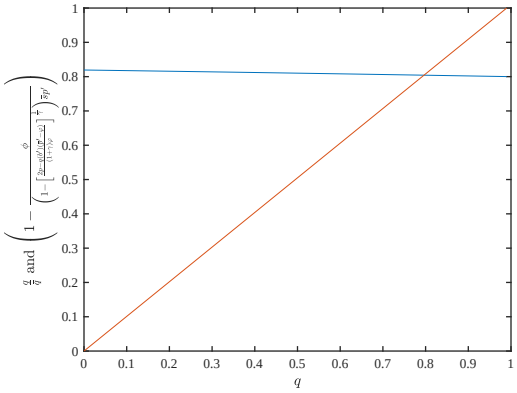
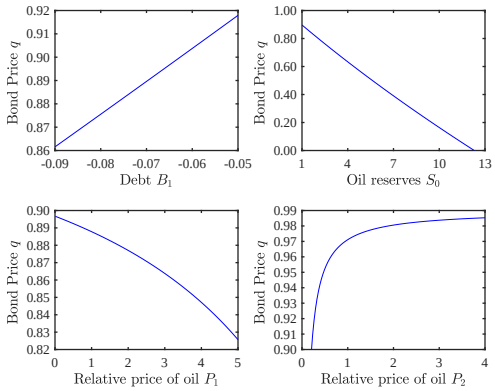


Figure 10 shows the comparative statics for the two period case. As is expected, higher levels of debt imply lower $q's$, or higher risk premia (first subplot in Figure 10). As was previously explained a higher price today would imply that the threshold price tomorrow that is needed to repay is higher, so a higher price today implies a lower q(left bottom subplot in Figure 10), and a higher price tomorrow implies a higher ability to repay and hence lower risk premia, higher q(right subplot in the bottom of Figure 10).

More surprising is the relationship between oil reserves and sovereign risk. As can be seen from the left upper subplot in Figure 10, the more oil reserves, the higher sovereign risk. So much so, that there is a level of oil reserves such that at that point the sovereign will default for sure. This is pretty intuitive, and is in line with what we found in the empirical section, if the sovereign has enough oil then its not necessary to resort to debt to smooth out consumption, as such higher oil reserves increase the value of autarky and sovereign risk goes up.

Figure 10: Comparative Statics



4 Calibration and Solution Method

In this section we describe the calibration and method that we use to solve the model. We start by describing the parametrization.

We calibrate the model to the Russian economy at an annual frequency.

On the sovereign's side, the model has six parameters: the sovereign's discount factor, β, the degree of risk aversion, μ,[7], the price of the risk-free bond, $\bar{q}$, the trade penalty that it faces when in default, θ, the probability of reentering financial markets when in the default state, λ, and a scaling parameter A that accounts for investment and government spending in the budget constraint of the sovereign.

Parameters μ and $\bar{q}$ are set to standard values in the literature. The risk aversion is set to 2, and the price of the risk-free bond implies a risk-free interest rate $\bar{r} = 0.00775$, which corresponds to the average real (ex-post US-CPI deflated 1955-2014) yield on a 3-month U.S. Treasury bill—around 0.8%—the value reported in Mehra and Prescott (1985).

We set the probability of reentry at $\lambda = 0.332$ as in Mendoza and Yue (2012). In addition, we set the fraction of oil exports that can be exported while the economy is in default θ to 0.2, which implies a trade penalty of 8% of the steady state GDP, matching the average trade penalty estimate in terms of GDP found by Rose (2005). We estimate β using a simulated method of moments which we discuss later on in this section.

The rescaling parameter, A, was set to $A = 1 + rb - c$, where b and c are the external debt to GDP ratio and the total consumption share in GDP. For the debt to GDP ratio we take the values from our panel database for Russia, while for consumption we use the World Bank and OECD Statistics which period covers 1989-2016 for Russia.[8]

On the side of the oil producing company there are three parameters: two parameters associated to extraction adjustment costs, φ, and γ, and the oil discovery rate, d. The discovery parameter d was set to match the share of oil production in Russia's GDP which is 10%.

The parameter that determines the curvature of the extraction cost function, γ, is estimated using a simulated method of moments and it will be used to determine the scale parameter of the extraction cost, φ (see Equation 3).

In the simulated method of moments we estimate β and γ to match as closely as possible two moments in the data: a default rate of 3% and a 20% ratio of external debt stocks (public and publicly guaranteed) to GDP for the period 1992-2015. We find $\beta = 0.78$ and $\gamma = 2.25$.

The remaining parameter, φ, is set such that in the deterministic steady-state the extraction cost per unit is 32%. Table 3 summarizes all the parameter values mentioned above.[9]

[7]The utility function is assumed to be $u(c) = \frac{c^{1-\mu}}{1-\mu}$

[8]The value of r is set to $\bar{r} + spread$, were $spread$ corresponds to the average value of JP Morgan's EMBI+ GSS spread for the period 1979-2016 (777 bp). Since the JP Morgan data starts in 1998 but we have the Institutional Investors database, we estimate a linear regression model between the Institutional Investor Index and the EMBI for the period 1998-2016 to calculated the (within-sample) fitted EMBI for the period 1979-1997

[9]In the deterministic steady-state $\varphi = n^{\gamma}/(1 + \gamma - (1/(1 + r))\gamma/n)$, where n is the ratio of Russian oil reserves to production (in years) for the period 1997-2016. In accordance with the US Energy Information Administration (EIA) we set $n = 18.5$ years.

Table 4: VAR Process for Non-Oil Output and Oil Prices

Parameter	Description	Value
ρ_p	oil price auto-correlation	0.89
ρ_y	non-oil output auto-correlation	0.67
ρ_{py}	oil price non-oil output correlation	0.15
σ_p^2	variance oil price	0.062
σ_y^2	variance non-oil output	0.002

Table 3: Summary of Parameters

Parameter	Description	Value
β	discount factor	0.78
μ	risk aversion	2
$\bar{q}$	risk-free debt price	0.99
θ	trade penalty	0.2
d	discovery rate	0.10
λ	redemption probability	0.33
γ	curvature extraction cost	2.25
φ	scale extraction cost	224
A	rescaling parameter	0.331

Recall that there are also two exogenous processes. One for the non-oil output (the non-storable consumption good received by the sovereign) and the price of oil. We estimate a VAR for these two variables using annual data for the real Brent price of crude oil (average of the daily price) and the real GDP (excluding oil rents) as a proxy of our non-oil endowment for the period 1979-2016. The estimated VAR(1) model for p_t and y_t, imposing the restriction that Russian non-oil GDP does not impact international oil prices, is:

$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.8901 & 0 \\ 0.1499 & 0.6723 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_p e_{p,t} \\ \sigma_y e_{y,t} \end{bmatrix}$$

where $e_{p,t}$ and $e_{y,t}$ are mean-zero, i.i.d. random variables. The diagonal of the estimated covariance matrix of the innovations is a matrix with variances σ_p^2 and σ_y^2. Table 4 shows the results of the estimation.

We use the following algorithm to solve the model.

1. Assume an initial value for the price of the bond $q_0(B', S, y, p)$ as well as initial values for the functions v_0^d and V_0.

2. Use q_0 and solve equation (4) to find the optimal extraction policy, $x_0^*(p, s; B')$. We use a time-iteration algorithm to find the function x_0^*.

3. Use q_0 and x_0^* to solve the sovereign problem, that is solve (5) subject to (6) and (7) subject to (8), and update the value functions v_1^d and V_1.

4. Find the default set using equation (9) and compute the probability of default using (10).

5. Update the price of the bond using the following equation:

$$q_1 = \bar{q}\left(1 - d\left(b'; S, y, p\right)\right).$$

6. Use the updated price of the bond q_1 and the updated value functions v_1^d and V_1 to repeat steps 1-5 until $\|q_1 - q_o\| < \epsilon$, $\left\|v_1^d - v_o^d\right\| < \epsilon$ and $\|V_1 - V_o\| < \epsilon$, where ϵ is a small number.

5 Results and Quantitative Analysis

In this section we show the comparison between the model generated data and the observed data for Russia, as well as the default sets generated by the model. We follow by analyzing the model dynamics around default episodes and the default costs associated with it in our model.

5.1 Quantitative Analysis

Table 5 shows the comparison between certain data moments and the those same moments generated by the model. As was mentioned in Section 4, we target the average external debt to GDP ratio and the default rate in our estimation, and as such we can see that the model generates 20% external debt over gdp and a default rate of 3%. The standard deviation of oil GDP ranges between 40% and 70% depending on the detrending method. In the table we write down the lower bound which corresponds to the one calculated using the Hodrik and Prescott filter (41%) and the one generated by the model is 56%.

The model generates average reserves in years of 21 while in the data its 18.5. Note that this number is indirectly targeted as was mentioned in the calibration section, and the discrepancy between model and data comes from the fact that we targeted it in the deterministic steady state and the number generated by the model in Table 5 comes from the stochastic simulation of the model.

Finally, we calculate the model spread as $\frac{q}{\bar{q}}$ and compare it to the volatility of the EMBI spread in the data. The model underestimates both the mean and volatility of the EMBI, the mean is 3.5% in the model versus 8% in the data and the standard deviation is 8%in the data versus 3% in the model.

Even though we have used the Institutional Investor Index as our measure of country risk, it is not a price but an index, so we considered appropriate to compare our results against an actual spread such as the EMBI.

Table 5: Data vs Model Moments

Description	Data	Model
Average External Debt to GDP	0.20	0.21
Standard Deviation of Oil GDP	0.41	0.56
Average Reserves (in years)	18.5	21
Default Rate	0.03	0.03
Average Spread	0.08	0.035
Standard Deviation of Spread	0.08	0.03

Estimates of the proven reserves for Russia correspond to those of the US Energy Information Administration.

Table 6 shows the comparison between other volatilities and correlations. The correlation between oil GDP and total GDP is 55% in the data and 54% in the model. The volatility of the overall GDP is 7% in the data and 9% in the model.

Total consumption (including government) has a correlation to total GDP of 71% in the data and in the model it is 87%, while its volatility in the data is 5% but 15% in the model. As pointed out by Chatterjee and Eyigungor (2012) one-period sovereign debt default models exhibit a counterfactual consumption volatility. Debt refinancing of all outstanding debt during a given period implies that changes in bond prices q tend to imply large changes in consumption, especially if b is a relatively high share of output. In our benchmark calibration debt as a share of GDP is relatively large (20% of GDP) compared to other models of short-term sovereign debt (usually at most 10% of GDP). Introducing long-term debt smooths consumption because it reduces refinancing needs, for a given volatility of bond prices. We do not introduce long-term debt as it is beyond the scope of this paper.

Table 6: Summary of Comovements - Russia vs. Model

1979-2016. HP filter Log-Data*

	Std. Dev.		Correlation vs GDP	
	Data	Model	Data	Model
Oil GDP	0.41	0.56	0.55	0.54
Total GDP	0.07	0.09	1.00	1.00
Total Consumption	0.05	0.15	0.71	0.87
Trade Balance	0.05	0.06	−0.48	−0.43
Spread	0.08	0.035	−0.58	−0.16

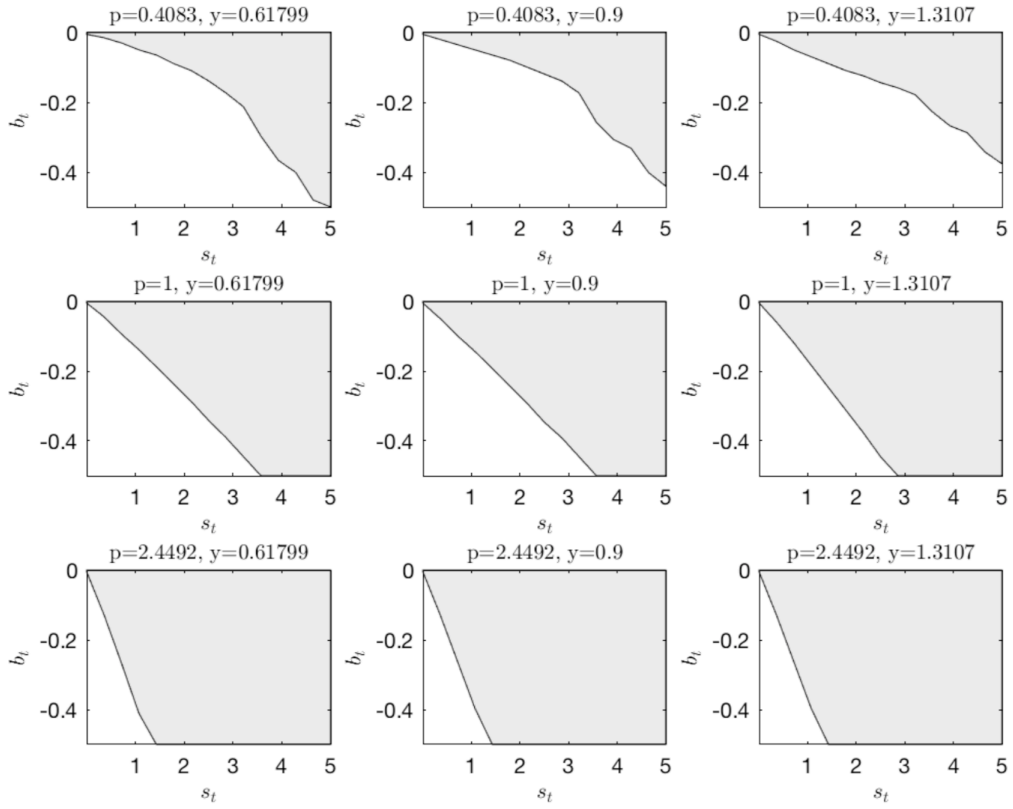
*Except TB which is relative to GDP and EMBI which is in levels (basis points).

The model does very well in terms of the cyclicality and volatility of the trade balance. It has a standard deviation of 5% in the data compared to 6% in the model, and in the data the correlation of the trade balance to GDP is -0.48 versus -0.43 in the model. Finally, even though the model misses in terms of the volatility of the spread it still captures the counter cyclicality.

5.2 Dynamic of Default Episodes

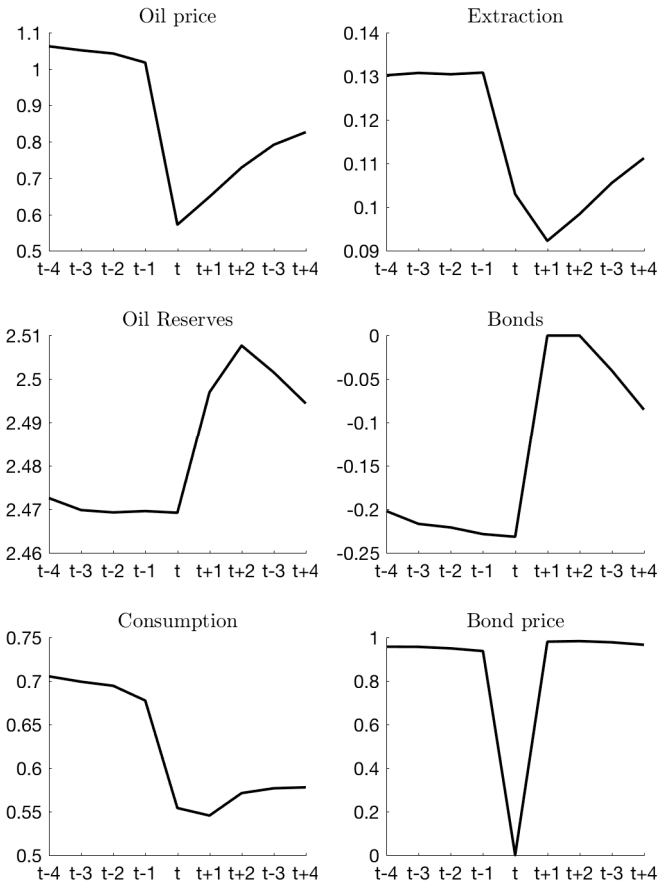
Figure 11 shows the resulting default sets associated with high, medium and low oil prices as well as high, medium and low non-oil output. The default set is increasing in debt and oil reserves for all states.

Figure 11: Default Sets



We can also use the model to study the dynamics around default episodes. Using simulated data, we average out over the default episodes and look at the path of the different variables four periods (years) before and four periods after. Figure 12 shows the results. Not surprisingly the main driver of a default in the model is the price of oil and interestingly, the dynamics of the model resemble the typical boom-boost pattern of a financial crises in an emerging economy: the price of a commodity is booming—oil in this case—, oil extraction is high and reserves are falling, consumption grows faster that GDP implying trade and current account deficits. When oil prices collapse, oil extraction falls and there is a sharp adjustment in both consumption and GDP. The sovereign defaults, bonds go to zero and debt prices collapse.

Figure 12: Model Dynamics Four Years Before and After Default Episodes



In this model for a given price of oil, x^d (extraction in default) is higher than that under continuation (x^*) which in turn is weakly higher than that under the risk free rate (x^{rf}):[10] $x^d(p) > x^*(p) \geq x^{rf}(p)$. So in this model there is no output default cost, in fact there is an incentive to default because oil revenues for a given state p are higher under default. However there is a difference between oil output, x, and oil revenues, px. Suppose that we come from a long sequence of drawing the mean oil price $\bar{p}$ and zero default risk so that oil output is x^{rf}. Hence at $t-1$ we have $\bar{p}$, x^{rf} and $\bar{p}x^{rf}$ for the price of oil, oil output and oil revenues. Then a very large negative shock on p hits the economy (say, p_l, and triggers a default. Now at t we have p_l, $x^d(p_l)$, and $p_l x^d(p_l)$. So what can we say about the values of t relative to $t-1$?

[10]Extraction under the risk free rate, x^{rf}, is that extraction that results from solving the firm's problem using the risk free bond price $\bar{q}$ as the discount factor.

We know for sure that $p_l < \bar{p}$, that is p at t is lower than at $t-1$, but it is less obvious what happens to the output and oil revenues, because that hinges on whether $x^d(p_l)$ is higher or lower than $x^{rf}(\bar{p})$. We only know that $x^{rf}(p_l)$ is lower than both $x^{rf}(\bar{p})$ and $x^d(p_l)$, but this leaves open the possibility that $x^d(p_l)$ could be higher than $x^{rf}(\bar{p})$. In other words, while it can be that oil output falls at t relative to $t-1$, it can also be higher.

Another thing that seems to follow is that in the economy without default risk oil output and oil revenue would be more volatile than when there is default risk. This is because without default we know that at t $x^{rf}(p_l) < x^{rf}(\bar{p})$, at least for shocks with positive autocorrelation. This implies that default risk endogenously reduces the volatility of oil revenues and oil production. In the simulated data of this economy one would associate the stability of the debt and country risk to more stable oil revenues, but in fact it is the other way around, or more precisely the two things support each other. The sovereign is generating a smoother income path by responding to the incentive of being more likely to default when prices are low because it can yield more revenue by defaulting in those states.

To make this point, note that in equilibrium extraction can be written as $x(p, q(b', y, s, p))$: p affects directly through the marginal benefit of extracting an extra unit as well as indirectly via the effect of p on country risk. As such, we can write the derivative dx/dp in the following way:

$$dx/dp = dx/dp + (dx/dq)(dq/dp).$$

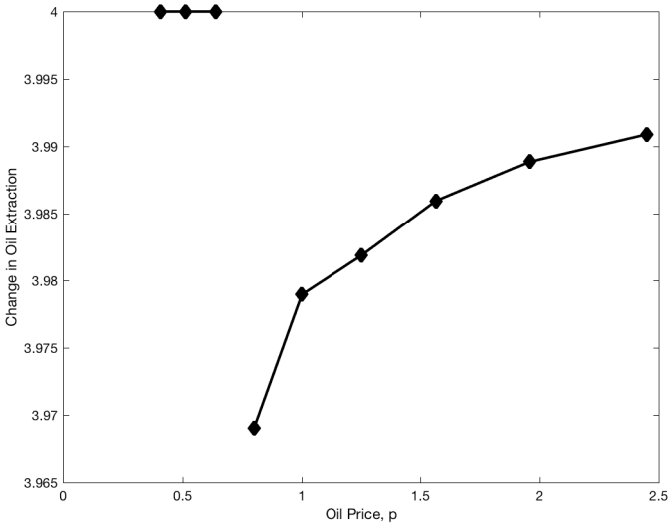
In this expression we know that $dx/dp > 0$, $dx/dq < 0$, and $dq/dp \geq 0$. This last one is greater or equal to zero because in the interval where the probability of default is zero $q = \frac{1}{1+\bar{r}}$ where $\bar{r}$ is the risk free rate, and q is independent of p in this interval. But this implies that the sign of dx/dp is ambiguous. When the probability of default is zero or close to zero, dq/dp is zero or very small, and $dx/dp > 0$. But as q goes down, and given that we know from the literature that dq/dp can be very high because q has a very steep profile, it can be that dq/dp is large, and so it can be that $dx/dp < 0$ when country risk is high. This means that in the equilibrium dynamics, extraction can increase or decrease in response to a lower price, and it is more likely that it increases when the price drop occurs in a state where country risk is higher.

On the side of the oil revenue, something similar happens:

$$d(px)/dp = x + p\left[dx/dp + (dx/dq)(dq/dp)\right],$$

the difference is now that it is harder for $d(px)/dp$ to be negative when dq/dp is large, because x is adding up to the expression. In other words, in terms of oil revenues one would expect them to go down with the oil price, even though extraction were to go up.

Figure 13: Default Cost



6 Counterfactuals

Table 7: Target and model moments

Description	Data	Benchmark	Profits	Stoch. Disc.
Average external Debt to GDP	0.20	0.21	0.14	0.25
Std dev of oil GDP	0.41	0.56	0.54	0.44
Average reserves (in years)	18.5	21	21	19
Default rate	0.03	0.03	0.02	0.01
Average Institutional Investor Index	51	93	95	97
Std dev Intitutional Investor Index	0.20	0.17	0.15	0.10
Average EMBI spread	0.08	0.035	0.025	0.01
Std dev of EMBI spread	0.08	0.03	0.025	0.01

Estimates of the proven reserves for Russia correspond to those of the US Energy Information Administration.

Table 8: Summary of Comovements - Russia vs. Model

1979-2016. HP filter Log-Data*

	Std. Dev.				Correlation vs GDP			
	Data	Benchmark	Profits	Stoch. Disc.	Data	Benchmark	Profits	Stoch. Disc.
Real Oil Price	0.21	0.23	0.23	0.23	0.53	0.65	0.76	0.66
Non-Oil GDP	0.06	0.07	0.07	0.07	0.92	0.80	0.73	0.81
Oil GDP	0.41	0.56	0.54	0.44	0.55	0.54	0.64	0.59
GDP	0.07	0.09	0.09	0.09	1.00	1.00	1.00	1.00
Consumption	0.05	0.15	0.13	0.14	0.71	0.87	0.93	0.91
Trade Balance	0.05	0.06	0.06	0.09	−0.48	−0.43	−0.35	−0.44
Inst. Inv. Index	0.20	0.17	0.15	0.10	0.51	0.08	0.07	0
EMBI spread	0.08	0.035	0.025	0.01	−0.58	−0.16	0	0.09

*Except TB which is relative to GDP and EMBI which is in levels (basis points).

7 Conclusion

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Appendix A

Figure 14 plots the relationship between the III and oil production value to GDP ratio, for each country, over the period 1979-2010. One feature stands out from Figure 14: when oil production value to GDP ratio is high, the country risk index tends to improve. Note that there are countries where the correlation is not significant, such as Iran, United Kingdom, Egypt or Gabon.

Figure 14: Institutional Investor Index (X-Axis) and oil production value to GDP (%, Y-Axis).

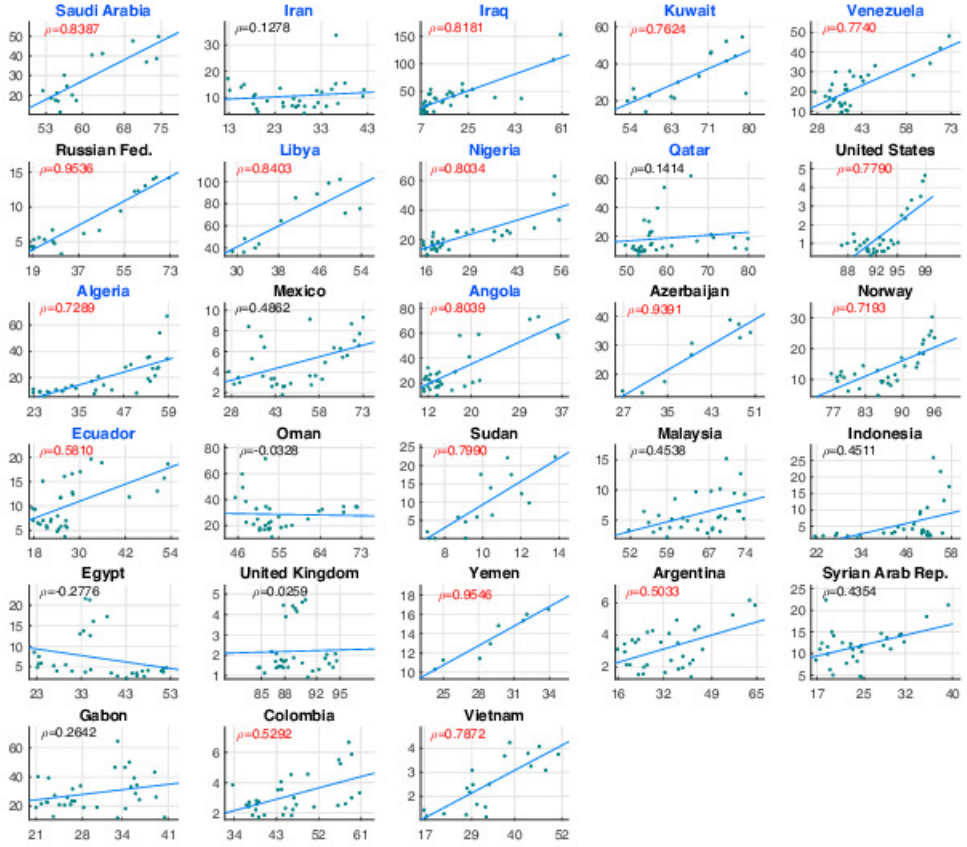
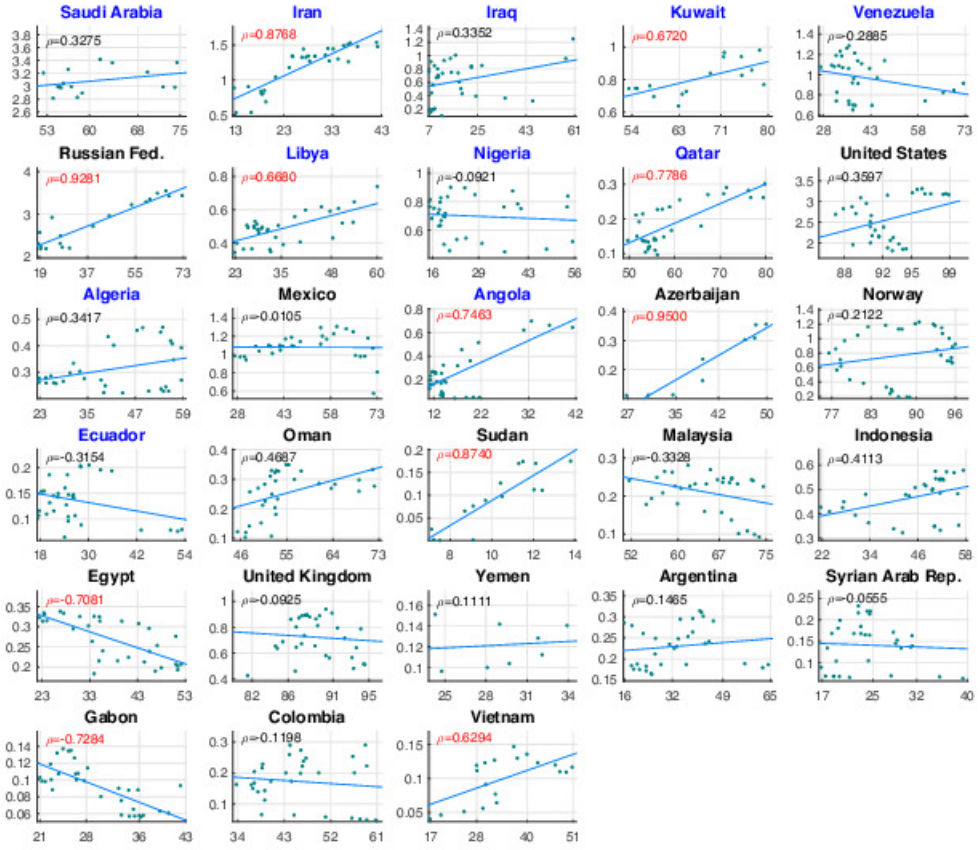


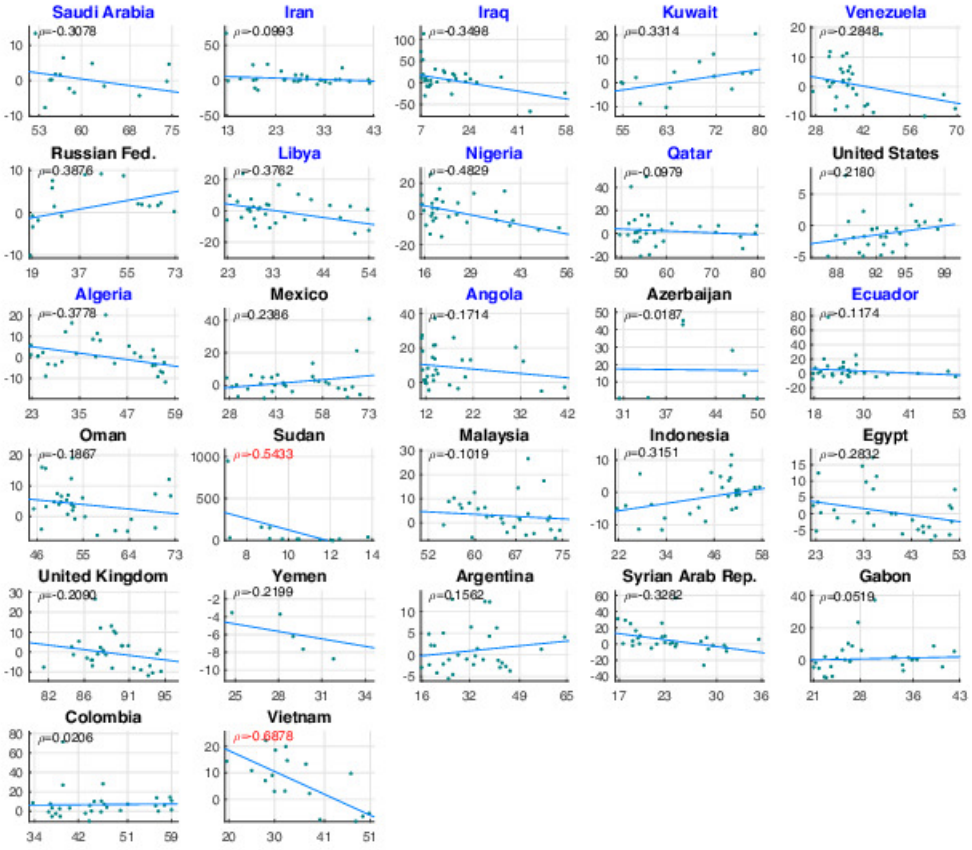
Figure 15 presents the III versus oil production (in billion barrels per year).

Figure 15: Institutional Investor Index (X-Axis) and oil production (billion barrels per year, Y-Axis).



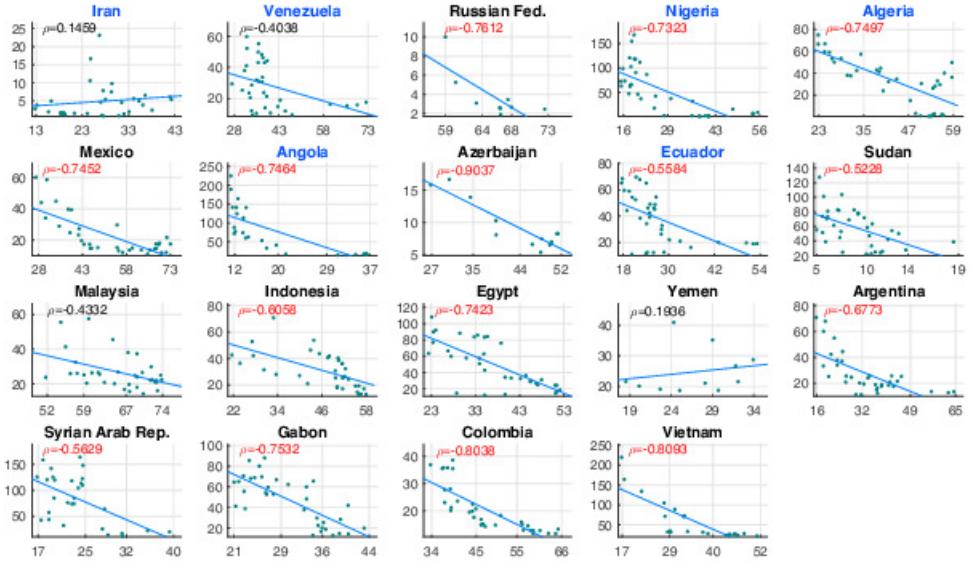
In this figure, absolute value of correlation coefficients greater than 0.5 are displayed in red. As we can see, there is not a clear pattern, since there are some countries for which the relationship is clearly positive, while for others it is negative or zero. This suggests that oil price is the "main driving force" behind changes in the country risk index (and not oil production). In Figure 16 we document the association between III and the oil production growth rate.

Figure 16: Institutional Investor Index (X-Axis) and oil production growth rate (%, Y-Axis).



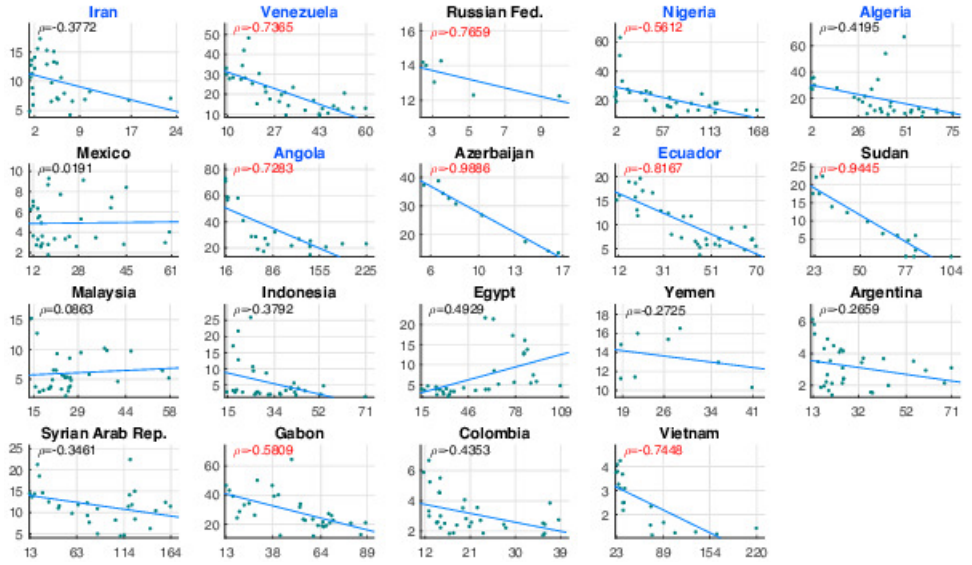
In this case, correlation coefficients lower than -0.5 are displayed in red. The results point in the direction that there is not any association between these two variables, although a negative relationship is observed for Sudan and Vietnam. Additionally, Figure 17 shows the relationship between the III and total public external debt to GDP ratio.

Figure 17: Institutional Investor Index (X-Axis) and total external public debt to GDP (%, Y-Axis).



Note that for most countries, correlation coefficients are displayed in red, which means that these are lower than -0.5. As we can see, III goes down when total public external debt increases. Additionally, Figure 18 shows the association between total external public debt to GDP ratio and oil production value to GDP ratio.

Figure 18: Total external public debt to GDP (%, X-Axis) and oil production value to GDP (%, Y-Axis).

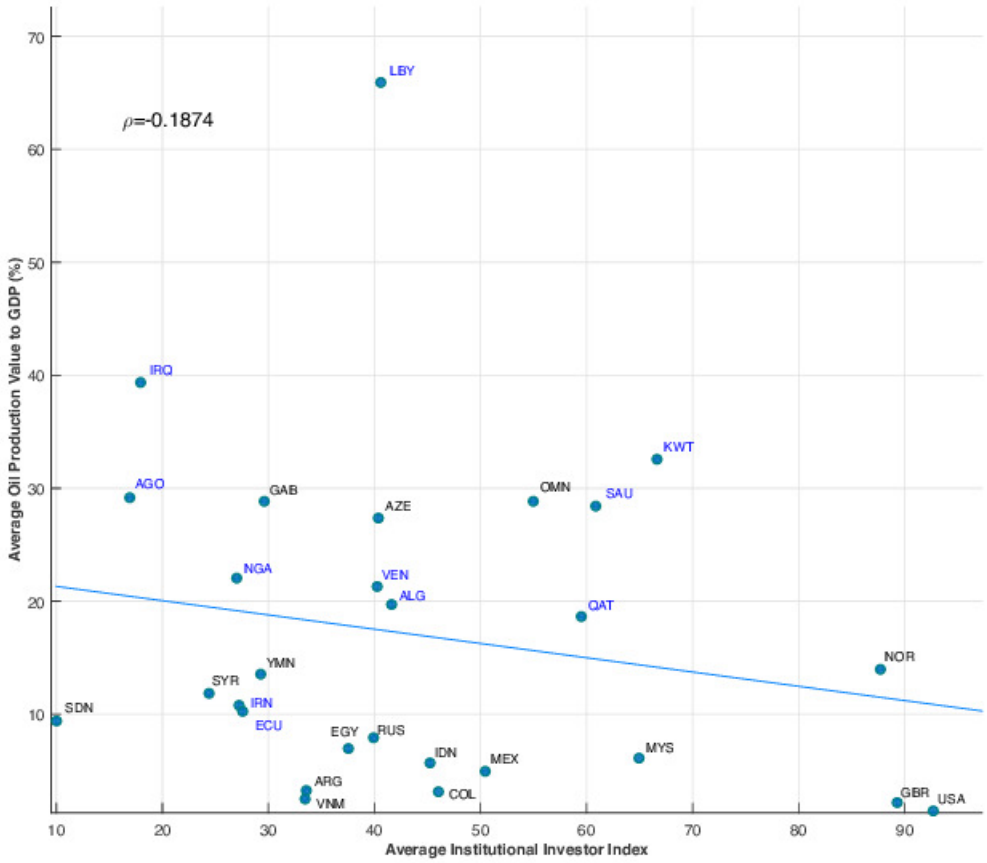


As we can see, for 9 countries there is a negative correlation, which implies than when oil production

value to GDP is high, total public external debt tends to be low. Nevertheless, such a contention is not reinforced by the rest of countries in the sample, since no significance is observed. Moreover, in the case of Egypt, the estimated coefficient shows almost a positive and statistically strong effect.

Moreover, Figure 19 plots the average III against average oil production value to GDP: In this case, we compute a low correlation coefficient (-0.187). The negative trend indicates that countries with high oil production value to GDP over time show a high country risk (or a low average III). It is important to mention that average oil production value to GDP may be low because historical GDP is very high when compared with the historical oil production value, such as in USA or Norway. Furthermore, this negative relationship may also be driven by exceptional cases such as Libya or Iraq, which have average oil production value to GDP of about 67 and 39 percent, and average III of about 41 and 17, respectively.

Figure 19: Average Institutional Investor Index (X-Axis) and average oil production value to GDP (Y-Axis): 1979-2010.



Appendix B

Estimation Approach

Before proceeding to dynamic panel data models, we need to verify that all variables are integrated of the same order. In doing so, we have used the test of the panel unit root of Im. *et al.* (2003, IPS henceforth), which is based on averaging individual unit root test statistics for panels. Specifically, they proposed a test based on the average of augmented Dickey-Fuller statistics (ADF henceforth) computed for each group in the panel. In accordance with some survey on panel unit root tests (such as those discussed in Banerjee (1999)), this test is less restrictive and more powerful than others that do not allow for heterogeneity in the autoregressive coefficient. IPS test permit solving serial correlation problem by assuming heterogeneity between units (in this case, countries) in a dynamic panel framework, as considered here. The basic equation of IPS test is as follows:

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \sum_{j=1}^{p} \phi_{ij} \Delta y_{it-j} + \epsilon_{it} \tag{15}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where N refers to the number of countries in the panel and T refers to the number of observations over time. In this case, y_i stands for each variable under consideration in our model (for example, III, total public debt, oil GDP or non-oil GDP), α_i is the individual fixed effect and p is the maximum number of lags included in the test. The null hypothesis then becomes $\beta_i = 0$ for all i, against the alternative hypothesis, which is that $\beta_i < 0$ for some $i = 1, ..., N_1$ and $\beta_i = 0$ for $i = N_1 + 1, ..., N$, where N_1 denote the number of stationary panels. Therefore, IPS statistic can be written as follows:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i^{ADF} \tag{16}$$

where t_i^{ADF} is the ADF t-statistic for country i, taking into account the country specific ADF regression, given by (15). The $\bar{t}$ statistic has been shown to be normally distributed under H_0. Table 1 reports the outcome for the global sample of this test.

As we can see, each variable is integrated of order one. Once the order of stationary has been defined, we estimated a country risk equation on the basis of cross-country panel data. In particular, we focus on three estimation methods which are consistent when both T and N are large. At one extreme, the usual practice is either to estimate N separate regressions and compute the mean of the estimated coefficients across countries, which is called the Mean Group (MG) estimator. Pesaran and Smith (1995) show that the MG estimator will produce consistent estimates of the average of the parameters, but ignores the fact that certain parameters are the same across countries.

At the other extreme are the traditional pooled estimators (such as dynamic fixed effects estimators), where the intercepts are allow to differ across countries while all other coefficients and error variances are constrained to be the same. In this case, the model controls for all time-invariant differ-

Table 9: Im *et al.* (2003) panel unit root test outcome: 1979-2010

	Levels		Logs	
	t-statistic	P-value	t-statistic	P-value
Inst. Inv.	-1.396	0.635	-1.356	0.741
Δ Inst. Inv.	-3.600	0.000	-3.613	0.000
Oil GDP	-0.603	1.000	-1.208	0.949
Δ Oil GDP	-3.486	0.000	-3.745	0.000
Non-oil GDP	0.630	1.000	-1.150	0.981
Δ Non-oil GDP	-2.839	0.000	-3.249	0.000
Oil reserves	-0.974	0.999	-1.234	0.940
Δ Oil reserves	-3.812	0.000	-3.871	0.000
Tot. pub. debt	-1.397	0.572	-1.277	0.855
Δ Tot. pub. debt	-2.926	0.000	-2.940	0.000
NFA	-1.227	0.943	-	-
Δ NFA	-3.044	0.000	-	-

Note: When computing NFA outcome, we excluded Iraq because of data limitations.

ences between countries, so the estimated coefficient cannot be biased because of omitted time-invariant characteristics. An intermediate technique is the Pooled Mean Group (PMG) estimator, proposed by Pesaran *et al.* (1999), which relies on a combination of pooling and averaging of coefficients, allowing the intercepts, short-run coefficients and error variances to differ freely across countries, but the long-run coefficients are constrained to be the same.

Therefore, for the implementation of these methods we consider the following model:

$$III_{it} = \theta_{0i} + \theta_{1i} OilGDP_{it} + \theta_{2i} NonOilGDP_{it} + \theta_{3i} OilR_{it} + \theta_{4i} X_{it} + \mu_i + \epsilon_{it} \tag{17}$$

Again, each observation is subscripted for the country i and the year t. In this case, $X \in \{TotPubD, NFA\}$. The variable III is the log of Institutional Investor's country credit ratings, $OilGDP$ is the log of oil GDP, $NonOilGDP$ is the log of non-oil GDP, $OilR$ is the log of oil reserves stock, $TotPubD$ is the log of total public debt, and NFA corresponds to net foreign assets to GDP ratio. Additionally, μ_i is a set of country fixed effects (such as geographical or institutional factors) and ϵ_{it} is the idiosyncratic error term.

Now, with a maximum lag of one for all variables, we construct the autorregresive distributive lag (ARDL) (1,1,1,1,1) dynamic panel specification of (17):

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \mu_i + \epsilon_{it} \quad (18)$$

Then, the error correction equation of (18) is:

$$\Delta III_{it} = \phi_i \left(III_{i,t-1} - \hat{\theta_{0i}} - \hat{\theta_{1i}} OilGDP_{it} - \hat{\theta_{2i}} NonOilGDP_{it} - \hat{\theta_{3i}} OilR_{it} - \hat{\theta_{4i}} X_{it} \right) - \delta_{11i} \Delta OilGDP_{it} - \delta_{21i} \Delta NonOilGDP_{it} - \delta_{31i} \Delta OilR_{it} - \delta_{41i} \Delta X_{it} + \epsilon_{it} \quad (19)$$

where

$$\hat{\theta_{0i}} = \frac{\mu_i}{1-\lambda_i}; \hat{\theta_{1i}} = \frac{\delta_{10i} + \delta_{11i}}{1-\lambda_i}; \hat{\theta_{2i}} = \frac{\delta_{20i} + \delta_{21i}}{1-\lambda_i}$$

$$\hat{\theta_{3i}} = \frac{\delta_{30i} + \delta_{31i}}{1-\lambda_i}; \hat{\theta_{4i}} = \frac{\delta_{40i} + \delta_{41i}}{1-\lambda_i}; \phi_i = -(1-\lambda_i)$$

In this case, ϕ_i is the error correction speed of adjustment parameter, and we would expect ϕ_i to be negative if the variables exhibit a return to long-run equilibrium[11].

[11] Replacing $\hat{\theta}_i$-parameters and ϕ_i in equation (17) we get:

$$\Delta III_{it} = -(1-\lambda_i)\Big(III_{i,t-1} - \frac{\mu_i}{1-\lambda_i} - \frac{\delta_{10i} + \delta_{11i}}{1-\lambda_i} OilGDP_{it} - \frac{\delta_{20i} + \delta_{21i}}{1-\lambda_i} NonOilGDP_{it} - \frac{\delta_{30i} + \delta_{31i}}{1-\lambda_i} OilR_{it} - \frac{\delta_{40i} + \delta_{41i}}{1-\lambda_i} X_{it} \Big) - \delta_{11i} \Delta OilGDP_{it} - \delta_{21i} \Delta NonOilGDP_{it} - \delta_{31i} \Delta OilR_{it} - \delta_{41i} \Delta X_{it} + \epsilon_{it}$$

Removing similar terms, the above expression is as follows:

$$\Delta III_{it} = -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i} + \delta_{11i}) OilGDP_{it} + (\delta_{20i} + \delta_{21i}) NonOilGDP_{it} + (\delta_{30i} + \delta_{31i}) OilR_{it} + (\delta_{40i} + \delta_{41i}) X_{it} - \delta_{11i} \Delta OilGDP_{it} - \delta_{21i} \Delta NonOilGDP_{it} - \delta_{31i} \Delta OilR_{it} - \delta_{41i} \Delta X_{it} + \epsilon_{it}$$

Rewriting:

$$\begin{aligned} III_{it} - III_{i,t-1} = & -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i} + \delta_{11i}) OilGDP_{it} + (\delta_{20i} + \delta_{21i}) NonOilGDP_{it} + (\delta_{30i} + \delta_{31i}) OilR_{it} \\ & + (\delta_{40i} + \delta_{41i}) X_{it} - \delta_{11i} (OilGDP_{it} - OilGDP_{i,t-1}) - \delta_{21i} (NonOilGDP_{it} - NonOilGDP_{i,t-1}) \\ & - \delta_{31i} (OilR_{it} - OilR_{i,t-1}) - \delta_{41i} (X_{it} - X_{i,t-1}) + \epsilon_{it} \end{aligned}$$

Again, simplifying this equality we obtain:

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \mu_i + \epsilon_{it}$$

Note that this expression is equivalent to (18). For a long-run relationship to exist, we require that $\phi \neq 0$.

Estimation results

In this subsection we estimate the PMG, MG and DFE estimators for model (19). In order to obtain reliable estimators and seeking to maintain a large data sample, we include information for United States, China, India, Brazil and Australia, since these countries have large proven oil reserves, although these have not been oil net exporters in the time interval considered here. When deciding about model selection, we apply the Hausman test to see whether there are significant differences among these three estimators. The null of this test is that the difference between DFE and MG, DFE and PMG or PMG and MG is not significant. Consider, for example, the test between DFE and PMG. If the null is not rejected, the DFE estimator is recommended since it is efficient. The alternative is that there is a significant difference between PMG and DFE, and the null is rejected. Specifically, the Hausman statistic is:

$$H = (\beta_{DFE} - \beta_{PMG})' \left[\text{var}(\beta_{DFE}) - \text{var}(\beta_{PMG})\right]^{-1} (\beta_{DFE} - \beta_{PMG}) \sim \chi^2$$

where β_j is the vector of coefficients and $\text{var}(\beta_j)$ is the covariance matrix of β_j, estimated using the $j-$technique, for $j =$DFE, PMG. Under the null hypothesis, H has asymptotically the χ^2 distribution. Table 2 reports the results of Hausman test, in which Model (1) corresponds to equation (19), excluding X_i from regressors, while Model (2) and Model (3) represent the same equation, considering $X_i = TotPubD$ and $X_i = NFA$, respectively.

Table 10: Hausman test outcome: 1979-2010

	Model (1)		Model (2)		Model (3)	
	χ^2-stat.	P-value	χ^2-stat.	P-value	χ^2-stat.	P-value
DFE vs. MG	0.01	0.999	0.00	1.000	0.01	1.000
DFE vs. PMG	0.02	0.999	0.10	0.998	0.03	0.999
PMG vs. MG	1.68	0.640	2.87	0.579	3.50	0.477

Under the current specification, the hypothesis that the country risk equation (equation (19)) is adequately modeled by a PMG or MG model is resoundingly rejected. In general, when considering Model (1) the results in the table above suggest that it is not possible to reject the null hypothesis of the homogeneity restriction on regressors (in the short and long run), since P-values are both 0.999, which indicates that DFE is more efficient estimator than MG and PMG, respectively. Notice that this conclusion holds for Model (2) and Model (3), because P-values associated to these tests are close to 1. Because of this, we choose to employ the DFE estimator.

Appendix C

Table 11: Oil Price Upswings and Downswings

Downswings		Upswings	
Period	Number of Months	Period	Number of Months
NOV 75 - OCT 78	36	NOV 78 - JAN 81	27
FEB 81 - JUL 86	66	AUG 86 - JUL 87	12
AUG 87 - NOV 88	16	DEC 88 - OCT 90	23
NOV 90 - DEC 93	38	JAN 94 - OCT 96	34
NOV 96 - DEC 98	26	JAN 99 - SEP 00	21
OCT 00 - DEC 01	15	JAN 02 - JUL 08	79
AUG 08 - MAY 10	22		
TOTAL	219	TOTAL	196

Appendix D

Correlation between credit ratings of a country a oil companies.