Research Proposal

Gradient-Based Structural Estimation

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April 22, 2018

1. Introduction

Structural estimation links rigorous economic models to data, and as such, is a fundamental tool in many fields of economics, from asset pricing and corporate finance to macroeconomics and industrial organization.[1] It is also thought to be a particularly difficult endeavor since it involves solving complicated nested problems. A typical structural estimation procedure entails stochastic dynamic optimization and Monte Carlo approximation for the model-implied moments for a fixed set of parameters in the *inner* loop, and the minimization of a *distance* function of the parameters in the *outer* loop.

This problem is computationally intensive for at least two reasons. Standard approaches to structural estimation solve a dynamic optimization problem and compute simulated moments for each possible configuration of the model parameters. In addition, the optimization algorithm used in the outer loop is oblivious to the mapping between the model parameters and the model-implied moments. In this paper I propose a solution for these two problems.

I leverage new powerful approximate dynamic programming techniques that allows for the global solution of high-dimensional stochastic dynamic problems that previously could be solved only with local approximation methods.[2] Following Norets (2012), I augment the state space by treating the model parameters as state variables and solve this high-dimensional problem using an extension of the algorithm proposed in Duarte (2018). This makes it unnecessary to solve a complex dynamic programming problem thousands of times.

I then show that one can approximate the model-implied moments without the computation of a single moment. Instead, a deep neural network learns the moments by observing raw outcomes of simulations. This eliminates the need to compute moments for every fixed set of parameters. More importantly, since the network learns the mapping between parameters and moments (the *binding function* of Gourieroux, Monfort, and Renault (1993)), the optimizer in the outer loop can explore this structure to find the model parameters that minimize the corresponding loss function. Specifically, the optimizer has access to analytical gradients and hessians of the loss function.

The second contribution of the paper is to generalize the algorithm proposed in Duarte (2018) to cases where the optimal controls are defined only implicitly. I do so by parametrizing the policy function using a neural network and updating it with policy gradients. The algorithm doesn't rely on first-order conditions and can accommodate problems with constraints.

The method advanced here fits into the class of moment-based estimators. Hansen (1982) proposed the Generalized Method of Moments (GMM) to estimate deep parameters of stochastic dynamic models from the Euler equations when these can be obtained on closed-form. McFadden (1989), Pakes and Pollard

[1] See Durham (2006), Hennessy and Whited (2005), Smith (2005) and Berry, Levinsohn, and Pakes (1995) for examples.
[2] See Scheidegger and Bilionis (2017) and Duarte (2018) for examples.

(1989), Lee and Ingram (1991) and Duffie and Singleton (1993) proposed extensions of the conventional methods of moments that can handle generic moment conditions by using simulation. The main difference between these methods and the approach described in this paper is the method used to approximate the model-implied moments. Simulated Method of Moments (SMM) approximate model-implied moments by Monte Carlo simulations for each set of parameters, whereas I learn a mapping between parameters and moments for a whole range of parameters directly from simulated observations. This subtle distinction has big implications in terms of computing time, as the simulation part can be removed from the optimization loop.

2. Technical Details

The code is written in Python, and makes extensive use of TensorFlow. As any machine learning software, it would greatly benefit from the use of GPU acceleration or parallel computing as all operations are vectorized. GPU acceleration also allows for the use of larger training sets, that produce higher quality gradients, which is important in the two compute intensive parts of the algorithm: iterating a high-dimensional value function and training the moments network.

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