Income Inequality, Financial Crises, and Monetary Policy*

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Abstract

We construct a general equilibrium model in which income inequality results in insufficient aggregate demand, deflation pressure and excessive credit growth by allocating income to agents featuring low marginal propensity to consume, and if excessive, can lead to an endogenous financial crisis. This economy generates distributions for equilibrium prices and quantities that are highly skewed to the downside due to financial crises and the liquidity trap. Consequently, symmetric monetary policy rules designed to minimize fluctuations around fixed means become inefficient. A simultaneous reduction in inflation volatility and mean unemployment rate is feasible when an asymmetric policy rule is adopted.

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"It's revealing that in public debates, advocates for workers...have tended to favor the continuation of easy money, while the typical op-ed about the adverse effects of easy money on the distribution of income and wealth is written by a hedge fund manager, banker, or right-wing politician...That political alignment—workers' groups in support of easy money, financiers in favor of higher interest rates—is of course the historical pattern in the United States, going back to William Jennings Bryan and beyond." – Bernanke (2017)

1 Introduction

The distribution of income in the United States has become more unequal over the last several decades, reaching unprecedented levels as documented by Atkinson, Piketty, and Saez (2011). The concern regarding inequality has recently become the center of discussions by policymakers (Yellen, 2014; Bernanke, 2017). This paper explores a possibility that income inequality may be at the root of many problems that the U.S. economy could face such as secular stagnation, deflation pressure, excess credit growth and financial crises. Using a theoretical model, we show that an economy suffering from such problems may face disproportionately large downside risks, making traditional monetary policy framework aiming to minimize the volatilities of endogenous quantities and prices around fixed means inefficient. Our goal is to uncover robust monetary policy rules that can correct the biases of the means due to the left-skewed distributions and improve macroeconomic outcomes.

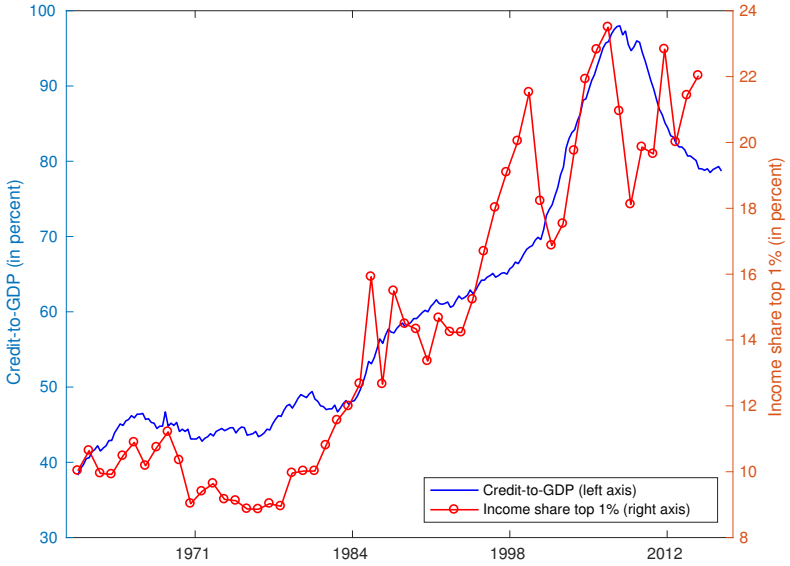
Income inequality may play three essential roles in the macroeconomy, each of which will be an important building block in the construction of our theoretical model. First, income inequality may directly affect aggregate demand if there exists substantial heterogeneity of Marginal Propensity to Consume (MPC) and MPCs are negatively correlated with income levels.[1] According to Jappelli and Pistaferri (2014), the average MPC of the most affluent income group is substantially lower than the MPCs of lower-income groups. In fact, half of the most affluent income group identified by Jappelli and Pistaferri (2014) has the MPC out of transitory income equal to zero. If the economy allocates a growing share of national income to a group with the lowest MPC, it can produce insufficient aggregate demand.[2] In this regard, Summers (2015), in his effort to revive the underconsumption theory of Hansen (1939), points to income inequality as one of the most important factors driving secular stagnation. The concern for the link between income inequality and underconsumption dates back to almost a century ago:

"...society was so framed as to throw a great part of the increased income into the control of the class least likely to consume it. The new rich...preferred the power which investment gave them

[1] John Maynard Keynes considered income distribution as a central element of his theory of effective demand: "*Moreover, each level of effective demand will correspond to a given distribution of income.*" (Keynes, 1936) because "*People's propensity to spend (as I call it) is influenced by many factors such as the distribution of income, their normal attitude to the future and - tho probably in a minor degree - by the rate of interest.*" (Keynes, 1937)

[2] The negative correlation between MPCs and income levels (or liquid wealth holdings) and its implication for aggregate demand and stabilization policies are broadly supported by a growing body of empirical literature: Blundell, Pistaferri, and Preston (2008); Parker, Souleles, Johnson, and McClelland (2013); Broda and Parker (2014); Kaplan and Violante (2014); Carroll, Slacalek, Tokuoka, and White (2017); and Auclert (2017).

Figure 1: Income Inequality and Credit-to-GDP Ratio



Sources: Household credit-to-GDP ratio is from the Financial Accounts of the United States, Federal Reserve System. The income share of top 1 percent is from the 2016 update of Piketty and Saez (2003).

> *to the pleasures of immediate consumption...And so the cake increased; but to what end was not clearly contemplated... the virtue of the cake was that it was never to be consumed, either by you nor by your children after you.*" – Keynes (1919)

Second, another important macroeconomic consequence of income inequality is the deflation pressure. The labor income share, a different gauge of income inequality, has been declining for more than three decades for several reasons studied by Elsby, Hobijn, and Sahin (2013), Karabarbounis and Neiman (2013), Solow (2015), and Koh, Santaeullia-Llopis, and Zheng (2016). Why does that matter for inflation? The labor income share represents a good measure of real marginal costs and accurately captures inflation dynamics, according to the New Keynesian literature such as Galí and Gertler (1999), Galí, Gertler, and López-Salido (2001), Woodford (2001), and Sbordone (2002). Thus, if the decline of the labor share continues as real wage growth fails to catch up with labor productivity growth, it will be much more difficult for a central bank to achieve a certain inflation target.

Third, and perhaps most importantly, income inequality plays a crucial role for the overall stability of the financial system. When a growing share of national income is allocated to a group with the lowest MPC, unused income has to be stored in financial claims. Those claims may represent investment in real assets such as physical capital or borrowing of lower-income groups. In this latter case, a link between income inequality and excessive credit expansion that endangers the soundness of financial system might arise. Indeed, Figure 1 shows a positive correlation between income inequality, measured by the income share of the top 1 percent earners based on data collected by Piketty and Saez (2003), and the household sector credit-to-GDP ratio. During the 1960s and 1970s, both time series moved more or less sideways. Since the early 1980s, however, both series continued to rise over more than three decades. In 2007, on the eve of the Global Financial Crisis, the credit-to-GDP ratio reached an

unprecedented level and the top 1 percent income share reached 24 percent, a level unseen since 1928, which was also on the eve of the Great Depression.

Many researchers view the credit growth in the late 1990s and early 2000s as "excessive" and responsible for the financial vulnerability that ended up in the Global Financial Crisis. For example, Drehmann, Borio, Gambacorta, Jimenez, and Trucharte (2010) and Drehmann, Borio, and Tsatsaronis (2011) point out that the excessive credit growth is a high-quality indicator for the likelihood of financial instability. Jorda, Schularick, and Taylor (2011) and Schularick and Taylor (2012) established a statistical link between credit growth and the probability of financial crises. Thus, if excessive credit growth is a good predictor of financial crises, and if income inequality is the main driver of excessive credit growth, income inequality would then predict financial crises. Indeed, Paul (2017), in an empirical study that employs a macro-financial database of 17 advanced economies over the 1870–2013 period, finds that income inequality outperforms credit in the predictive power of crises.

In this paper, we build a macroeconomic model in which the three aforementioned channels of inequality-macroeconomy nexus play important roles in determining aggregate demand and financial vulnerability. To create a meaningful degree of income inequality, we follow the Kaleckian tradition in which there are two agents, shareholders and workers, with their income shares determined by bargaining power, not by their marginal contribution to production.

We assign so-called *Weberian* preferences to the shareholders such that they earn direct utility from accumulating financial assets. The Weberian preferences, which we adopt from Kumhof, Rancière, and Winant (2015) (KRW henceforth), play two crucial roles in creating links among income inequality, deflation pressure and financial instability. First, the Weberian preferences lower the MPC of the shareholders such that any innovations that make income distribution more favorable to them lead to underconsumption and deflation pressure. Second, the Weberian preferences contribute to financial instability by inducing shareholders to over-accumulate credit to a point where borrowers have strong incentives to partially renege on their debt obligation. As in KRW, a financial crisis erupts endogenously when the leverage of the borrowers reaches unsustainable levels.

While the endogenous financial crisis mechanism is borrowed from KRW, our model economy is fundamentally different from theirs. First, KRW is an endowment economy in which income distribution is determined by exogenous forces. In contrast, our model is a production economy whose income distribution is determined by bargaining powers of the two agents in sharing the production rents. Second, more importantly, KRW is a real business cycle economy, and hence Say's Law holds: supply always creates its demand regardless of the degree of income inequality. In contrast, our economy features nominal rigidities, which generate a link between income inequality and aggregate demand. Finally, the presence of nominal rigidities allows for monetary policy intervention and the size of crises becomes endogenous. This feature enables us to study the dilemma of the monetary authority whose stabilization function is severely distorted during crises by the zero lower bound (ZLB) constraint.

The distributions of equilibrium prices and quantities in such an economy are highly skewed to the downside owing to the two important sources of nonlinearity: occasional eruptions of financial crises and the liquidity trap. These two sources of nonlinearity work only to the downside, making

the unconditional means of inflation and output deviate to the downside from the levels that would prevail in the absence of crises and the ZLB constraint. This generates first order welfare losses, the magnitude of which we show are an increasing function of the strength of the Weberian preferences.

In line with the consensus widely held among monetary economists we find that appointing Rogoff (1985)'s conservative central banker that "places a large, but finite, weight on inflation" for such an economy may maximize the welfare of a society if the central banker's policy choice is not subject to the ZLB constraint. One benefit of a conservative central banker is that she provides a decisive monetary stimulus during crises. However, in the presence of the ZLB constraint, we find that such a conservative central banker may minimize the inflation volatility only at the costs of increasing the probability of financial crises and the deflation bias, and shifting the distributions of inflation and output more to the downside.

The two sides of Rogoff (1985)'s conservative central banker highlight the inefficiency of a symmetric monetary policy rule in the sense that a simultaneous reduction in inflation volatility and mean unemployment rate is feasible when an asymmetric policy rule is adopted. In this paper, we find that an asymmetric policy rule that prescribes a lenient response against inflation during normal times, but provides decisive accommodation during financial crises by temporarily yet persistently raising the inflation target can correct the deflation bias and improve macroeconomic outcomes. We also find that such an asymmetric policy rule can bring large welfare gains when implemented in the form of price-level targeting than in the form of inflation targeting.

We emphasize that monetary policy cannot eliminate income inequality. However, we find that optimal monetary policy breaks the link between income inequality and aggregate demand, which exists only in the presence of nominal rigidities under a suboptimal monetary policy rule. To the extent that optimal monetary policy achieves this goal, the economy should behave as if Say's Law applied. While income inequality still creates a tendency of underconsumption, optimal monetary policy offsets the fall in aggregate demand by sufficiently lowering the real interest rate to recover the aggregate demand at the potential level of output. Whether or not actual monetary policy can achieve this depends on the frequency of the binding ZLB constraint.

Finally, we find that the optimized monetary policy rules for a loss function that assigns a large weight to the reduction in unemployment improves the welfare of workers only at the cost of bringing substantial welfare losses to shareholders. Overall, these findings suggest that monetary policy can have distributional consequences, in line with the empirical results of Coibion, Gorodnichenko, Kueng, and Silvia (2017) and the "political alignment" quoted at the beginning may have a real root.

The rest of the paper is organized as follows. Section 2 provides a brief summary of related literature. Section 3 develops the model, which is then calibrated in Section 4. Section 5 shows the model dynamics using impulse response functions, highlighting the role of financial crises and the ZLB constraint. Section 6 discusses our main findings regarding the linkage between income inequality, aggregate demand and financial stability. Section 7 analyzes the relationship between monetary policy and the macroeconomy using alternative monetary policy rules. Section 8 concludes. A complete description of the non-stochastic steady state, the solution method, the complete system

of equations and additional results are described in Appendix A, B, C, and D, respectively.

2 Related Literature

Our paper contributes to three strands of literature. First, it contributes to the theoretical literature that formalizes the link between income inequality and aggregate demand. Auclert and Rognlie (2016), Kaplan, Moll, and Violante (2018) and McKay and Reis (2016) construct models that feature rich heterogeneity a la Bewley-Huggett-Aiyagari and nominal rigidities. Auclert and Rognlie (2016) show that a permanent increase in income inequality can lead to a permanent Keynesian recession. Kaplan, Moll, and Violante (2018) show that monetary policy transmission channel can be fundamentally different for the rich and the poor. In turn, McKay and Reis (2016) and Mitman, Manovskii, and Hagedorn (2017) analyze the large stabilization role of fiscal and social insurance policies in an economy suffering from insufficient aggregate demand due to income inequality. While our modeling of income inequality is much more stylized compared with this literature, our stylized model allow us to study the link among income inequality and financial vulnerability in the sense of endogenous financial crises and the policy dilemma facing a monetary authority trying to secure price stability and financial stability under the ZLB constraint, which are absent in this literature. Finally, none of this literature exploits the bargaining power between firms and workers in a search and matching framework as a driver of income inequality.[3]

Second, our paper contributes to the theoretical literature that analyzes the causes and consequences of asymmetric distributions of equilibrium prices and quantities. In particular, our two sources of nonlinearity are able to generate what Adrian, Boyarchenko, and Giannone (2016) and Adrian and Duarte (2016) call "vulnerable growth", i.e., highly left-skewed distribution of GDP growth correlated with financial leverage. Our contribution is to generate such vulnerable growth in a completely structural way. The nature of asymmetric distribution in our framework is comparable to Dupraz, Nakamura, and Steinsson (2017), who generate an asymmetric distribution of the unemployment rate by combining downward nominal wage rigidity and search frictions but abstract from income inequality and financial crises. Relatedly, Kocherlakota (2000) and Jensen, Petrella, Ravn, and Santoro (2017) show that occasionally binding financial constraints in a highly leveraged economy can be an important source of business cycle asymmetries. While our paper also highlights the role of credit in generating asymmetric distributions, Kocherlakota (2000) and Jensen, Petrella, Ravn, and Santoro (2017) abstract from nominal rigidities, which prevent the study of monetary policy. Importantly, to the best of our knowledge, the interaction between financial crises and the binding ZLB constraint in our paper is only found in Guerrieri and Lorenzoni (2017). In particular, they model an heterogeneous-agent incomplete-market economy in which deleveraging after a shock to borrowing capacity forces the real interest rate to fall to zero even with flexible prices. However, credit crunch is exogenous in Guerrieri

[3] Krueger, Mitman, and Perri (2016) is unique in this literature. They analyze the role of endogenous changes in income/wealth inequality as an amplification/propagation channel in an equilibrium business cycle framework without nominal rigidities. What is remarkable in their work is that the model and the calibration strategy allow Krueger, Mitman, and Perri (2016) to match the stylized fact in the data that the bottom 40% of households hold no net worth.

and Lorenzoni (2017), whereas financial crises arise endogenously in our paper.

Finally, our paper contributes to the theoretical literature that analyzes the distributional consequences of monetary policy. In particular, our results are in line with what Gornemann, Kuester, and Nakajima (2016) find in assessing the welfare implications for "Main Street" and "Wall Street"of increasing the monetary policy reaction to the unemployment gap in a New Keynesian model with heterogeneous agents in incomplete market. Even though our modeling of income distribution is much more stylized than Gornemann, Kuester, and Nakajima (2016), our framework allows us to study optimal monetary policy when the economy features occasional financial crises and is subject to the ZLB constraint. Our paper is also related to Auclert (2017) that analyzes the different redistribution channels through which monetary policy affects macroeconomic aggregates due to heterogeneity in MPCs. In particular, our model economy features both the earnings heterogeneity channel and the Fisher channel of monetary policy described in Auclert (2017), but in a model economy that features financial crises and ZLB constraint.

3 The Model

The model features two groups of agents: shareholders, denoted by superscript K, and workers, denoted by superscript W. Each group contains a continuum of agents and forms a large family that shares the budget and consumption. The population shares of shareholders and workers are denoted by χ and $1-\chi$, respectively.

We assume a segmented asset market structure such that only shareholders own production firms and accumulate physical capital. Shareholders also accumulate private bonds and government bonds. Workers do not participate in capital markets and the only instrument available to them to smooth consumption is borrowing from the private bond market. In equilibrium, shareholders lend money to workers. Monetary policy determines the interest rate on government bonds. While only shareholders accumulate government bonds, monetary policy also affects workers' consumption profiles due to general equilibrium effects.

3.1 Workers

Preferences of workers are specified as:

$$U_t^W = \mathbb{E}_t \sum_{t=0}^{\infty} (\beta^W)^t \left\{ \frac{(c_t^W - s c_{t-1}^W)^{1-1/\sigma_c}}{1-1/\sigma_c} \right\}, \tag{1}$$

where β^W is the time discount factor, c_t^W denotes per capita consumption level of workers, s is the degree of external habit formation, and σ_c is the elasticity of intertemporal substitution. Per capita composite consumption is given by $c_t^W = \left[\int_0^1 c_t^W(i)^{1-1/\gamma} di\right]^{1/(1-1/\gamma)}$, where γ is the elasticity of substitution between different good varieties denoted by i.

Workers earn wage incomes by providing labor when employed and search for new jobs and earn

unemployment benefits when unemployed. We assume that the only financial instrument that workers can use to smooth consumption is issuance of defaultable discount bonds. We denote per capita private bond issuance of workers by b_t and the price of the discount bond by q_t^B. If borrowers do not default, the bond delivers a real return of $\mathbb{E}_t[1/\pi_{t+1}]$ to lenders in the next period, where π_t is the gross inflation rate. If borrowers default, lenders get back only $(1-h)\mathbb{E}_t[1/\pi_{t+1}]$, where h is the haircut associated with the default. Thus, the actual payment can be denoted as:

$$l_t = (1 - h\delta_t^B)\frac{b_{t-1}}{\pi_t}, \tag{2}$$

where $\delta_t^B \in \{0, 1\}$ is the default indicator that takes 1 upon default and 0 otherwise.

Default involves pecuniary and non-pecuniary costs, the latter in terms of direct loss of borrowers' utility. The size of the pecuniary default cost is given by a fraction ν_t of aggregate output y_t that follows the following process:

$$\nu_t = \rho_\nu \nu_{t-1} + \gamma_\nu \delta_t^B, \tag{3}$$

where the impact effect of a default is given by γ_ν and ρ_ν governs the decay rate in the absence of further defaults. Default occurs when the borrowers' utility gain from defaulting is greater than the utility cost of default χ_t, which follows an iid process with its cumulative distribution function denoted by $\Xi(\cdot)$. Formally, default occurs when $\chi_t < U_t^D - U_t^N$, where U_t^D and U_t^N denote the workers' values of default and non-default, which will be given formal definitions below. The probability of default is then simply given by:

$$p_t^\delta \equiv \text{prob}(\delta_t^B = 1) = \Xi(U_t^D - U_t^N). \tag{4}$$

The distribution of the utility cost of default χ_t is assumed to follow a modified logistic distribution as in KRW:

$$\Xi(\chi_t) = \left\{ \begin{array}{cc} \dfrac{\varrho}{1+\exp(-\varsigma\chi_t)} & \text{if } \chi_t < \infty \\ 1 & \text{if } \chi_t = \infty \end{array} \right\}, \tag{5}$$

where $\varrho < 1$. The parameters ϱ, ς, γ_ν and ρ_ν are calibrated to match the empirical evidence on financial crises.

Per capita budget constraint for workers can then be expressed as:

$$c_t^W = q_t^B b_t - l_t + \frac{1}{1-\chi}\left[\int_0^1 w_t(i)n_t(i)di + (1-\chi-n_t)b^U - \nu_t y_t\right], \tag{6}$$

where $w_t(i)n_t(i)$ is wage income of the workers employed by firm i, $n_t = \int_0^1 n_t(i)di$ is total employment, and b^U are unemployment benefits. Denoting the shadow value of budget constraint by Λ_t^W and using the probability of default, the first-order conditions (FOCs) for workers can be expressed as:

$$\Lambda_t^W = (c_t^W - sc_{t-1}^W)^{-1/\sigma_c} \tag{7}$$

and

$$q_t^B = \beta^W \mathbb{E}_t \left[\frac{\Lambda_{t+1}^W}{\Lambda_t^W} (1 - hp_{t+1}^{\delta}) \frac{1}{\pi_{t+1}} \right]. \tag{8}$$

For later use, we define the stochastic discounting factor of workers as $m_{t,s}^W \equiv (\beta^W)^{s-t} \Lambda_s^W / \Lambda_t^W$.

Regardless of the default decision, the consumption level of the workers is determined by equation (6) at any point in time. However, we need to construct a few auxiliary devices to analyze the probability of default, which relies on hypothetical values of default and non-default. To construct the borrowers' hypothetical values of default and non-default, we define the consumption levels under default and non-default, c_t^D and c_t^N as:

$$c_t^D = q_t^B b_t - (1-h) \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1 - \chi - n_t) b^U - (\rho_\nu \nu_{t-1} + \gamma_\nu) y_t \right] \tag{9}$$

and

$$c_t^N = q_t^B b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1 - \chi - n_t) b^U - \rho_\nu \nu_{t-1} y_t \right]. \tag{10}$$

Denoting the continuation values conditioned upon default and non-default by V_t^D and V_t^N, respectively, we define the value of default U_t^D and the value of non-default U_t^N as the sum of one period utility and the continuation value in each case:

$$U_t^D = \frac{(c_t^D - s c_{t-1}^W)^{1-1/\sigma_c}}{1 - 1/\sigma_c} + V_t^D \tag{11}$$

and

$$U_t^N = \frac{(c_t^N - s c_{t-1}^W)^{1-1/\sigma_c}}{1 - 1/\sigma_c} + V_t^N. \tag{12}$$

To construct the conditional continuation values, we define c_{t+1}^{NN}, c_{t+1}^{ND}, c_{t+1}^{DN} and c_{t+1}^{DD} as the consumption value tomorrow in the following four cases. c_{t+1}^{NN}, consumption level in the case of non-default tomorrow after non-default today is given by:

$$c_{t+1}^{NN} = q_{t+1}^B b_{t+1} - \frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi} [w_{t+1} n_{t+1} + (1 - \chi - n_{t+1}) b^U - \rho_\nu \rho_\nu \nu_{t-1} y_{t+1}]. \tag{13}$$

c_{t+1}^{ND}, consumption level in the case of default tomorrow after non-default today is:

$$c_{t+1}^{ND} = q_{t+1}^B b_{t+1} - (1-h) \frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi} [w_{t+1} n_{t+1} + (1 - \chi - n_{t+1}) b^U - (\rho_\nu \rho_\nu \nu_{t-1} + \gamma_\nu) y_{t+1}]. \tag{14}$$

c_{t+1}^{DN}, consumption level in the case of non-default tomorrow after today's default is:

$$c_{t+1}^{DN} = q_{t+1}^B b_{t+1} - \frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi} [w_{t+1} n_{t+1} + (1 - \chi - n_{t+1}) b^U - \rho_\nu (\rho_\nu \nu_{t-1} + \gamma_\nu) y_{t+1}]. \tag{15}$$

Finally, c_{t+1}^{DD}, consumption level in the case of default tomorrow after today's default is given by:

$$c_{t+1}^{DD} = q_{t+1}^B b_{t+1} - (1-h) \frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi} \{ w_{t+1} n_{t+1} + (1 - \chi - n_{t+1}) b^U - [\rho_\nu (\rho_\nu \nu_{t-1} + \gamma_\nu) + \gamma_\nu] y_{t+1} \}. \tag{16}$$

The continuation value under default today is then given by a weighted average of the value under default and the value under non-default tomorrow such that:

$$V_t^D = \beta^W \mathbb{E}_t \left[p_{t+1}^{\delta} \left(\frac{(c_{t+1}^{DD} - sc_t^D)^{1-1/\sigma_c}}{1-1/\sigma_c} + V_{t+1}^D \right) + (1-p_{t+1}^{\delta}) \left(\frac{(c_{t+1}^{DN} - c_t^D)^{1-1/\sigma_c}}{1-1/\sigma_c} + V_{t+1}^N \right) \right]. \quad (17)$$

The continuation value under non-default today can be constructed in a symmetric way such that:[4]

$$V_t^N = \beta^W \mathbb{E}_t \left[p_{t+1}^{\delta} \left(\frac{(c_{t+1}^{ND} - sc_t^N)^{1-1/\sigma_c}}{1-1/\sigma_c} + V_{t+1}^D \right) + (1-p_{t+1}^{\delta}) \left(\frac{(c_{t+1}^{NN} - sc_t^N)^{1-1/\sigma_c}}{1-1/\sigma_c} + V_{t+1}^N \right) \right]. \quad (18)$$

3.2 Shareholders

Per capita consumption level of shareholders is denoted by c_t^K. We denote per capita government bond holdings by b_t^G. In this notation, per capita holdings of private bonds by shareholders is denoted by $b_t(1-\chi)/\chi$. Shareholders maximize the following intertemporal utility function:

$$U_t^K = \mathbb{E}_t \sum_{t=0}^{\infty} (\beta^K)^t \left\{ \frac{(c_t^K - sc_{t-1}^K)^{1-1/\sigma_c}}{1-1/\sigma_c} + \psi^B \frac{[1+b_t(1-\chi)/\chi]^{1-1/\sigma_b}}{1-1/\sigma_b} + \psi^G \frac{(1+b_t^G)^{1-1/\sigma_g}}{1-1/\sigma_g} \right\}, \quad (19)$$

where β^K is the discount factor of shareholders and the composite consumption good is given by $c_t^K = \left[\int_0^1 c_t^K(i)^{1-1/\gamma} di \right]^{1/(1-1/\gamma)}$. The parameters ψ^B and ψ^G are strictly positive weights given to the utility from asset holdings, while σ_b and σ_g determine how fast the marginal utility of financial asset holdings decline. These preferences exhibit "the spirit of capitalism" of Weber (2013) in that financial assets are not simply means to utility maximization but become direct goal of utility maximization. These preferences generalize KRW's preferences by introducing external consumption habits and utility from holding government bonds in addition to private bonds. Note that government bonds are needed to generate a transmission channel for monetary policy.

Per capita budget constraint for shareholders can then be expressed as:

$$c_t^K = \frac{b_{t-1}^G}{\pi_t} - \frac{b_t^G}{1+i_t} + (l_t - q_t^B b_t) \frac{1-\chi}{\chi} - \frac{1}{\chi} \left\{ q_t^K \left[k_t - (1-\delta) k_{t-1} \right] + r_t k_{t-1} + \Pi_t^Y + \Pi_t^K - T_t + \nu_t y_t \right\}, \quad (20)$$

where i_t is the nominal interest rate controlled by the monetary authority, r_t is the rental rate of capital, k_t/χ is per capita capital stock, δ is the depreciation rate of capital, q_t^K is the relative price of capital, Π_t^Y/χ is the per capita profit of intermediate-goods firms, Π_t^K/χ is the per capita profits of investment-goods firms, and T_t/χ is the per capita lump sum tax.

As indicated by the last term in the shareholders' budget constraint, we assume that the pecuniary default cost of workers is transferred to shareholders in lump-sum fashion. In this sense, the haircut is not completely lost. The default provides a large one-time release in the workers' budget, but generates a persistent cost. Hence the transfer of default costs can be viewed as debt restructuring.[5]

[4] The forward iteration of the recursive formulation given by equations (11)∼(18) can cover the entire event tree in the future.

[5] The expected present value of the lump-sum transfer of the pecuniary default costs amounts to 2/3 of the haircut, implying 66 percent recovery rate in our calibration.

We denote the shadow value of the shareholders' budget constraint by Λ_t^K. The FOCs of shareholders can be expressed as follows:

$$\Lambda_t^K = (c_t^K - sc_{t-1}^K)^{-1/\sigma_c}, \tag{21}$$

$$q_t^B = \beta^K \mathbb{E}_t \left[\frac{\Lambda_{t+1}^K}{\Lambda_t^K} (1 - hp_{t+1}^\delta) \frac{1}{\pi_{t+1}} \right] + \frac{\psi^B}{\Lambda_t^K} \left[1 + b_t \left(\frac{1-\chi}{\chi} \right) \right]^{-1/\sigma_b}, \tag{22}$$

$$1 = \beta^K \mathbb{E}_t \left[\frac{\Lambda_{t+1}^K}{\Lambda_t^K} \left(\frac{r_{t+1} + (1-\delta) q_{t+1}^K}{q_t^K \phi_t} \right) \right] \tag{23}$$

and

$$\frac{1}{1+i_t} = \beta^K \mathbb{E}_t \left[\frac{\Lambda_{t+1}^K}{\Lambda_t^K} \frac{\phi_t}{\pi_{t+1}} \right] + \frac{\psi^G}{\Lambda_t^K} (1 + b_t^G)^{-1/\sigma_g}, \tag{24}$$

where we assume that the economy is subject to Smets and Wouters (2007)'s risk premium shock to create aggregate demand disturbances, which is denoted by ϕ_t and follows an AR(1) process:[6]

$$\log \phi_t = \rho_\phi \log \phi_{t-1} + \epsilon_{\phi,t}, \quad \epsilon_{\phi,t} \sim N(-0.5\sigma_\phi^2, \sigma_\phi^2). \tag{25}$$

Note that $\psi^B > 0$ and $\psi^G > 0$ create liquidity premium for financial assets as opposed to real asset. Shareholders earn non-pecuniary benefits from holding financial claims in addition to pecuniary returns, which makes them to accept lower interest rate and provide more credit to financial markets. For later use, we define the stochastic discounting factor of shareholders as $m_{t,s}^K \equiv (\beta^K)^{s-t} \Lambda_s^K / \Lambda_t^K$.

3.3 Symmetric Equilibrium

Note that c_t^D and c_t^N defined by equations (9) and (10) differ from each other only in aspects related to default, i.e., $l_t = (1-h)b_{t-1}/\pi_t$ and $\nu_t = \rho_\nu \nu_{t-1} + \gamma_\nu$ in one case, and $l_t = b_{t-1}/\pi_t$ and $\nu_t = \rho_\nu \nu_{t-1}$ in the other case. Otherwise all elements of the budget constraint are symmetric across the two cases. This does not imply that macroeconomic variables, such as the market price of bond q_t^B, total wage income $w_t n_t$, and aggregate output y_t, are not affected by the crisis. As will be shown, the dynamics of these macroeconomic variables are influenced by the crisis in an important way.

However, individuals take the macroeconomic variables as given while making their individual default decision. The bond market is characterized as a competitive equilibrium with a continuum of agents and "the actions of a single individual are negligible" (Aumann, 1975). In our symmetric default or non-default equilibrium, every individual makes an identical choice, believing that their actions will not affect macroeconomic outcomes. However, with everyone making the same choice, default decisions impact the economy in equilibrium. It is for the same reason that neither the lender's efficiency condition nor the borrower's efficiency condition (equations (8) and (22), respectively) incorporate the

[6] The risk premium shock appears in the denominator of Lucas tree equation (23) such that the expected real return falls. However, the same risk premium shock appears in the numerator of the consumption Euler equation (24) such that the shock is equivalent with monetary policy shock implemented by "nature"and not by the monetary authority. Fisher (2015) provides a structural interpretation of such a shock as "money demand shock."

effect of increasing debt on the probability of default or the price of bond. In other words, both agents behave as if $\partial p^{\delta}_{t+1}/\partial b_t = \partial q^B_t/\partial b_t = 0$ because they view their individual actions inconsequential for the competitive equilibrium in debt market: they are price takers.

In our model economy, $\delta^B_t \in \{0, 1\}$, has both endogenous and exogenous natures. δ^B_t is endogenous in the sense that the probability of $\delta^B_t = 1$ is endogenously determined. However, given the probability of default, whether or not the default occurs is stochastic. In this latter sense, δ^B_t can be viewed as an exogenous variable. In simulating the model economy, we employ a 'guess and verify' strategy. We first simulate the economy under the guess that default does not occur today, i.e., we set $\delta^B_t = 0$ and simulate the model economy. The probability of default is computed as a result of this simulation. Equations (9)∼(12) are the devices that allow us to compute the probability of default under the assumption $\delta^B_t = 0$. We then verify if our guess is valid. We take a random draw of the utility cost of default χ_t. If the utility gain $U^D_t - U^N_t$ is less than the random draw, we conclude that the guess was correct. If not, i.e., if $U^D_t - U^N_t > \chi_t$, we conclude that the default indeed happens today. We then set the exogenous variable $\delta^B_t = 1$ and simulate the economy again (see Appendix B for more details).

3.4 Labor Market

There exists a continuum of firms indexed by $i \in [0, 1]$ and owned by shareholders. Firms post vacancies to hire new workers at the beginning of each period. The matching process between vacancies and unemployed workers is assumed to be governed by a constant returns to scale matching function:

$$m(v_t, u_t) = \zeta v^{\epsilon}_t u^{1-\epsilon}_t, \tag{26}$$

where $v_t \equiv \int v_t(i)di$ denotes the total measure of vacancies and u_t denotes the measure of unemployed workers searching for a job at the beginning of period t defined as:

$$u_t = 1 - \chi - (1 - \rho)n_{t-1}, \tag{27}$$

with $\rho \in (0, 1)$ being the exogenous separation rate. The parameter ζ stands for matching efficiency and $1 - \epsilon$ is the matching function elasticity with respect to unemployment. The probability for an unemployed worker to meet a vacancy and the probability for a vacancy to meet with an unemployed worker are given by, respectively:

$$p(\theta_t) = \frac{m(v_t, u_t)}{u_t} = \zeta\theta^{\epsilon}_t, \tag{28}$$

$$q(\theta_t) = \frac{m(v_t, u_t)}{v_t} = \zeta\theta^{\epsilon-1}_t, \tag{29}$$

where $\theta_t \equiv v_t/u_t$ is labor market tightness. Note that firms consider these flow probabilities as given when deciding optimal employment.

3.5 Firm problem

Each firm i produces a differentiated good, using an identical Cobb-Douglas production function with capital $k_t(i)$ and labor $n_t(i)$. The production technology is represented by:

$$y_t(i) = z_t k_{t-1}(i)^{\alpha} n_t(i)^{1-\alpha}, \tag{30}$$

where z_t is an aggregate productivity shock following an AR(1) process:

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(-0.5\sigma_z^2, \sigma_z^2). \tag{31}$$

Firms make three decisions: a pricing decision subject to infrequent price adjustments, a hiring decision in a frictional labor market, and a capital rental decision. The timing of events for the firm's problem is summarized as follows. At the end of time $t-1$, a fraction ρ of workforce is exogenously separated from the firm. Then, aggregate shocks realize, firms make pricing decisions, which determines production scale. Firms then post vacancies $v_t(i)$ at a flow vacancy posting cost ξ per period, which will be filled with probability $q(\theta_t)$. Then, firms make the capital rental decision, which is assumed to be frictionless.

It is assumed that vacancies posted at the beginning of the period can be filled in the same period before production takes place, and that recently separated workers can search and find a job in the same period. Thus, the law of motion for the firm's workforce is given by:

$$n_t(i) = (1-\rho) n_{t-1}(i) + q(\theta_t) v_t(i). \tag{32}$$

After the matching process is complete, the wage is determined through Nash wage bargaining. Finally, production takes place, and wages, capital rents and dividends are paid.

3.5.1 Cost minimization

The firm's cost minimization problem can be separated from the optimal pricing problem. A firm i minimizes its production costs subject to equations (30) and (32), formalized by the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^K \left[w_s(i) n_s(i) + \frac{\vartheta}{2} \left(\frac{w_s(i)}{w_{s-1}(i)} - 1 \right)^2 w_{s-1}(i) n_s(i) + r_s k_{s-1}(i) + \xi v_s(i) \right] \\ + \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^K J_s(i) \{ n_s(i) - [(1-\rho) n_{s-1}(i) + q(\theta_s) v_s(i)] \} \\ + \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^K \mu_s(i) \{ y_s(i) - z_s k_{s-1}(i)^{\alpha} n_s(i)^{1-\alpha} \},\end{aligned} \tag{33}$$

where we assume that firms face quadratic adjustment cost of changing the real wage. $\mu_t(i)$ and $J_t(i)$ are the shadow values of constraints (30) and (32), respectively. Thus, $\mu_t(i)$ represents the real marginal cost of production and $J_t(i)$ the real marginal value of employment. The associated

first-order conditions with respect to $v_t(i)$, $n_t(i)$, and $k_t(i)$ are, respectively:

$$\xi = q(\theta_t) J_t(i), \tag{34}$$

$$J_t(i) = \mu_t(i)(1-\alpha)\frac{y_t(i)}{n_t(i)} - w_t(i) - \frac{\vartheta}{2}\left(\frac{w_t(i)}{w_{t-1}(i)} - 1\right)^2 w_{t-1}(i) + (1-\rho)\mathbb{E}_t[m^K_{t,t+1} J_{t+1}(i)] \tag{35}$$

and

$$r_t = \mu_t(i)\alpha\frac{y_t(i)}{k_{t-1}(i)}. \tag{36}$$

Equation (34) equalizes the costs of posting a vacancy with the expected benefit of employing a new worker, which is given by equation (35) and equals the gap between the cost reduction of an additional worker and the wage, plus the continuation value of the employment relationship. Given that the capital allocation decision is frictionless, equation (36) equals the real rental rate to the cost reduction due to renting an additional unit of capital.

3.5.2 Wage bargaining

The worker's surplus at a firm i, denoted by $W_t(i)$, is given by the sum of the surplus of the contract wage $w_t(i)$ over the worker's outside option value $\underline{w}_t$ and the continuation value at the firm i:

$$W_t(i) = w_t(i) - \underline{w}_t + (1-\rho)\mathbb{E}_t[m^W_{t,t+1} W_{t+1}(i)]. \tag{37}$$

The worker's outside option $\underline{w}_t$ is given by the sum of unemployment benefits and the expected value of finding a new job next period:

$$\underline{w}_t = b^U + (1-\rho)\mathbb{E}_t\left[m^W_{t,t+1} p(\theta_{t+1}) \int_0^1 \frac{v_{t+1}(j)}{v_{t+1}} W_{t+1}(j) dj\right]. \tag{38}$$

The matched firm and worker bargain over the wage to maximize the Nash product:

$$w_t(i) = \arg\max_{w_t(i)} W_t(i)^{\eta_t} J_t(i)^{1-\eta_t}, \tag{39}$$

where η_t is the bargaining power of the worker, which is assumed to follow an AR(1) process:

$$\log \eta_t = (1-\rho_\eta)\log\eta + \rho_\eta \log \eta_{t-1} + \epsilon_{\eta,t}, \quad \epsilon_{\eta,t} \sim N(-0.5\sigma^2_\eta, \sigma^2_\eta). \tag{40}$$

The solution to the Nash bargaining problem is given by the surplus sharing rule:

$$0 = \eta_t \frac{\partial W_t(i)}{\partial w_t(i)} \frac{J_t(i)}{W_t(i)} + (1-\eta_t)\frac{\partial J_t(i)}{\partial w_t(i)}, \tag{41}$$

where $\partial W_t(i)/\partial w_t(i) = 1 \equiv -\Gamma^W_t$ and

$$\frac{\partial J_t(i)}{\partial w_t(i)} = -1 - \vartheta\left(\frac{w_t(i)}{w_{t-1}(i)} - 1\right) + (1-\rho)\mathbb{E}_t\left\{m^K_{t,t+1}\frac{\vartheta}{2}\left[\left(\frac{w_{t+1}(i)}{w_t(i)}\right)^2 - 1\right]\right\} \equiv \Gamma^J_t.$$

We now define the generalized workers' bargaining power Ω_t as:

$$\Omega_t \equiv \frac{\eta_t}{\eta_t + (1-\eta_t)\Gamma^J_t/\Gamma^W_t}.$$

Note that if there is no real wage rigidity, $\Gamma^J_t/\Gamma^W_t = 1$ and $\Omega_t = \eta_t$. Using the generalized bargaining power, the surplus sharing rule (41) can be rewritten as a generalization of the standard Nash-wage bargaining condition:

$$\Omega_t J_t(i) = (1-\Omega_t)W_t(i). \tag{42}$$

Without loss of generality, we focus on a symmetric equilibrium in which the real wage is equalized across all firms, i.e., $w_t(i) = w_t$. The surplus sharing condition (42), the cost minimization conditions (34) and (35), worker's surplus value function (37) and outside option value (38) jointly imply the following equilibrium wage:[7]

$$\begin{aligned} w_t = {} & \Omega_t\mu_t(1-\alpha)\frac{y_t}{n_t} + (1-\Omega_t)b^U - \Omega_t\frac{\vartheta}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} \\ & + (1-\rho)\mathbb{E}_t\left[\left(\Omega_t m^K_{t,t+1} - (1-\Omega_t)m^W_{t,t+1}[1-p(\theta_{t+1})]\frac{\Omega_{t+1}}{1-\Omega_{t+1}}\right)\frac{\xi}{q(\theta_{t+1})}\right]. \end{aligned} \tag{43}$$

3.5.3 Optimal pricing

Each firm i produces a differentiated good and faces an identical isoelastic demand curve given by:

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} y_t, \tag{44}$$

where $P_t(i)$ is the firm's price, P_t is the aggregate price level, and y_t is the aggregate demand.

We assume that firms set prices according to a variant of the formalism proposed by Calvo (1983). In particular, each firm has a constant probability $1-\varphi$ to reset its price in any given period, which is independent across firms and time. Firms that cannot reset their price in a given period partially index their price to lagged inflation. Thus, the firm's price in period t is

$$P_t(i) = \begin{cases} P^*_t & \text{with probability } 1-\varphi \\ P_{t-1}(i)\pi^{\varepsilon}_{t-1} & \text{with probability } \varphi \end{cases}$$

where P^*_t is the reset price, $\pi_t = P_t/P_{t-1}$ is the inflation rate, and ε is the degree of indexation.

A firm that reoptimizes its price in period t chooses the reset price P^*_t that maximizes the expected

[7] If we assume a single representative agent such that $m^K_{t,t+1} = m^W_{t,t+1} = m_{t,t+1}$ and further assume a constant bargaining power and no real wage rigidity, the equilibrium wage is given by the more conventional wage equation: $w_t = \eta\mu_t(1-\alpha)y_t/n_t + (1-\eta)b^U + \eta(1-\rho)\mathbb{E}_t[m_{t,t+1}\xi\theta_{t+1}]$.

present value of the profits generated while the price remains effective. Formally, P_t^* satisfies:

$$P_t^* = \arg\max_{P_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} \varphi^{s-t} m_{t,s}^K \left(\frac{P_t^*}{P_s} \prod_{j=t}^{s-1} \pi_j^{\varepsilon} - \mu_s \right) \left(\frac{P_t^*}{P_s} \prod_{j=t}^{s-1} \pi_j^{\varepsilon} \right)^{-\gamma} y_s, \tag{45}$$

with $0 < \varepsilon < 1$ and $\prod_{j=t}^{s-1} \pi_j^{\varepsilon} = 1$ for $s \leq t$.

The first order condition to (45) is expressed as the optimal reset price inflation rate $p_{0,t} \equiv P_t^*/P_{t-1}$:

$$p_{0,t} = \mathcal{P}_t^N / \mathcal{P}_t^D, \tag{46}$$

where $\mathcal{P}_t^N$ and $\mathcal{P}_t^D$ satisfy the following recursions:

$$\mathcal{P}_t^N = \pi_t^{(1-\varepsilon)\gamma} \left\{ \pi_t^{\varepsilon\gamma} \gamma \mu_t y_t + \varphi \mathbb{E}_t \left[m_{t,t+1}^K \mathcal{P}_{t+1}^N \right] \right\}, \tag{47}$$

and

$$\mathcal{P}_t^D = \pi_t^{(1-\varepsilon)(\gamma-1)} \left\{ \pi_t^{\varepsilon(\gamma-1)} (\gamma - 1) y_t + \varphi \mathbb{E}_t \left[m_{t,t+1}^K \mathcal{P}_{t+1}^D \right] \right\}. \tag{48}$$

Combining the aggregate price index implied by the Dixit-Stiglitz aggregator and the partial indexation assumption, one can summarize the aggregate inflation dynamics as:

$$\pi_t = \left[(1-\varphi) p_{0,t}^{1-\gamma} + \varphi \pi_{t-1}^{\varepsilon(1-\gamma)} \right]^{1/(1-\gamma)}. \tag{49}$$

3.6 Closing the model

3.6.1 Investment-good industry

We assume that there exists a representative firm in a competitive investment-good producing industry, which maximizes the following profit given a CRS technology:

$$\max_{x_s} \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^K \left\{ q_s^K x_s - \left[x_s + \frac{\kappa}{2} \left(\frac{x_s}{x_{s-1}} - 1 \right)^2 x_{s-1} \right] \right\}.$$

x_t is the investment level at time t and κ is the investment adjustment cost parameter. The first order condition to the above problem is given by an investment Euler equation:

$$q_t^K = 1 + \kappa \left(\frac{x_t}{x_{t-1}} - 1 \right) - \mathbb{E}_t \left\{ m_{t,t+1}^K \frac{\kappa}{2} \left[\left(\frac{x_{t+1}}{x_t} \right)^2 - 1 \right] \right\}. \tag{50}$$

3.6.2 Government Budget Constraint

We assume a balanced budget each period. Government spending is financed by lump sum taxes on shareholders and is composed of two elements: unemployment benefits and interest expenses on

government debt. The balanced budget constraint then implies:

$$T_t = (1 - \chi - n_t)b^U + \chi\left(\frac{b^G_{t-1}}{\pi_t} - \frac{b^G_t}{1+i_t}\right). \tag{51}$$

In all computations considered in this paper, it is assumed that the amount of government bond issuance remains constant. This implies that when the monetary authority changes the interest rate, it is the demand for government bonds that should adjust to clear the bond market.

3.6.3 Market clearing

We define aggregate consumption as $c_t = (1-\chi)c^W_t + \chi c^K_t$. Imposing a balanced budget condition yields the following aggregate resource constraint:

$$y_t = c_t + x_t + \xi v_t + \frac{\vartheta}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1}n_t + \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2 x_{t-1}. \tag{52}$$

Equating equations (30) and (44) and integrating the equality yields:

$$y_t = \Delta_t^{-1} z_t k^{\alpha}_{t-1} n_t^{1-\alpha}, \tag{53}$$

where

$$\Delta_t = (1-\varphi)\left(\frac{P^*_t}{P_t}\right)^{-\gamma} + \varphi\left(\frac{P_{t-1}\pi^{\varepsilon}_{t-1}}{P_t}\right)^{-\gamma} = \pi^{\gamma}_t[(1-\varphi)p^{-\gamma}_{0,t} + \varphi\pi^{-\varepsilon\gamma}_{t-1}] \tag{54}$$

is due to the price dispersion under staggered pricing.[8]

3.6.4 Monetary Policy

In the absence of shocks that affect the output gap and inflation gap in opposite directions, such as price markup shocks, the model economy exhibits the divine coincidence.[9] For this reason, the monetary authority is assumed to follow an inflation targeting regime in our baseline economy:

$$i_t = \max\left\{0, i^* + \rho_{\pi}\left(\frac{\pi^Y_t - \pi^*}{4}\right)\right\}, \tag{55}$$

where i^* is the steady state interest rate, $\pi^Y_t \equiv \pi_t\pi_{t-1}\pi_{t-2}\pi_{t-3}$ is the annual inflation rate, and π^* is the annual inflation target.

As shown by equation (55), the ZLB constraint is imposed on the policy interest rate. Thus we can analyze, first, the effects of ZLB on the ability of the central bank to control the probability of

[8] Since we solve our model around zero trend inflation rate, the price dispersion term does not play any active role.

[9] We define the divine coincidence in terms of inflation gap and natural output gap – the distance of actual output from the output of flexible economy. This is different from Blanchard and Galí (2007), where they define the welfare relevant output gap as the distance of actual output from efficient level of output that will prevail without the distortions due to nominal/real rigidities and market power. For simplicity, we assume that the central bank is concerned about minimizing the output from their natural levels. However, as Blanchard and Galí (2007) showed, the presence of real wage rigidity in our model may break down the divine coincidence when the central bank is concerned about the gap from the efficient level of output.

financial crises and to stabilize inflation during financial crises; and second, to analyze the effects of the strength of inflation targeting given by ρ_π on the frequency of financial crises and binding ZLB constraint. To satisfy the ZLB constraint, we use a combination of current monetary policy shocks and anticipated news shocks. For this purpose, we express the monetary policy rule (55) as:

$$i_t = i^* + \rho_\pi \left(\frac{\pi_t^Y - \pi^*}{4} \right) + \sum_{j=0}^{n} \epsilon_{j,t-j}, \tag{56}$$

where $\epsilon_{0,t}$ is the current monetary policy shock and $\epsilon_{j,t-j}$ is $j-$periods ahead news shock to the policy rate. While both $\epsilon_{0,t}$ and $\epsilon_{j,t-j}$ are called "shocks", we use them only to satisfy the ZLB constraint, which can be expressed as a combination of $i_t \geq 0$, $\mathbb{E}_t[i_{t+1}] \geq 0, \ldots,$ and $\mathbb{E}_t[i_{t+n}] \geq 0$, which are $n+1$ constraints. We ensure that the ZLB constraint is satisfied not only in the current period, but also in agent's expectations such that economic agents understand the nature of the constraint. For a sufficiently large value of n, $\mathbb{E}_t[i_{t+n+1}] \geq 0$ can be safely assumed to be non-binding. In each period, $n+1$ shocks should be *endogenously* determined to satisfy the $n+1$ constraints.[10]

As an alternative monetary policy rule, we consider the following form of price-level targeting:[11]

$$i_t = i^* + \rho_\Pi \frac{\Pi_t}{4} + \sum_{j=0}^{n} \epsilon_{j,t-j}, \tag{57}$$

where Π_t is defined as the cumulative miss in inflation target, i.e.,[12]:

$$\Pi_t \equiv \sum_{s=0}^{\infty} (\pi_{t-s}^Y - \pi^*) = \Pi_{t-1} + \pi_t^Y - \pi^*. \tag{58}$$

4 Calibration

The model is calibrated at a quarterly frequency. Table 1 summarizes our choices for the structural parameters determining preferences and default, which closely follow the calibration strategy in KRW. Table 2 summarizes the parameters pertaining to the structure of production, labor markets, nominal rigidities, monetary policy and exogenous shock processes.

Preferences and Default: We use the same haircut due to default ($h = 0.10$) and the persistence of the default cost ($\rho_\nu = 0.65^{0.25}$) as in KRW. The output loss upon default is set to $\gamma_\nu = 0.045$, which is slightly above the KRW value of 0.04. For this choice, we target a cumulative output loss of 26

[10] See Appendix B for the solution method. Lindé, Smets, and Wouters (2016) recently used the same methodology.

[11] We do not consider nominal GDP targeting as an alternative monetary policy rule because there is no shock in the model that moves the output gap and the inflation gap in opposite directions.

[12] Note that we use the annualized inflation rate to define the cumulative miss in hitting the inflation target. If we use the quarterly inflation rate, and respecify the monetary policy rate as $i_t = i^* + \rho_\Pi \Pi_t + \sum_{j=0}^{n} \epsilon_{j,t-j}$ this rule becomes literally a price-level targeting rule. To see this, one could rewrite

$$\Pi_t \equiv \sum_{s=0}^{\infty} (\pi_{t-s} - \pi^*) = \log \frac{P_t}{P_{t-1}} - \pi^* + \log \frac{P_{t-1}}{P_{t-2}} - \pi^* + \ldots + \log \frac{P_1}{P_0} - \pi^* = \log \left[\frac{P_t}{P_0 (\pi^*)^t} \right].$$

Since we use the 4-quarter annual rate, the formula becomes slightly different, which creates inconsequential difference.

Table 1: Structural Parameters for Preferences and Default

Parameter	Target/Source	Value
Haircut	KRW	$h = 0.1$
Persistence of default cost	KRW	$\rho_\nu = 0.65^{0.25}$
Impact default cost	Output loss during the Great Recession	$\gamma_\nu = 0.045$
Default cost parameter	KRW	$\varsigma = 18$
Default cost parameter	Empirical default probability	$\varrho = 0.055$
Population share of shareholders	5 percent	$\chi = 0.05$
Steady state interest rate	Literature	$i^* = 0.005$
Utility weight on private bond	Debt to income ratio of bottom 95 percent income earners = 1.5	$\psi^B = 0.45$
Utility weight on government bond	Same as private bond	$\psi^G = 0.45$
Discount factor of workers	Literature	$\beta^W = 0.99$
Discount factor of shareholders	Income share of top 5 percent income earners = 0.36	$\beta^K = 0.89$
Wealth elasticity of shareholders	KRW	$\sigma_b = 1.09$
Wealth elasticity of shareholders	Annual nominal interest rate of 2 percent	$\sigma_g = 0.328$
Elasticity of intertemporal subs.	Literature	$\sigma_c = 1$
Elasticity of subs. between goods	Literature	$\gamma = 5$

percent during the nine years following a financial crisis. This corresponds to the observed output loss suffered by the U.S. economy from 2008 to 2016 relative to its potential as estimated by the CBO in 2017.[13] Regarding the parameters of the modified logistic distribution of the utility cost of default, we calibrate $\varsigma = 18$ as in KRW and set $\varrho = 0.055$ to match a quarterly default probability of 1.3 percent, which is consistent with the annual default probability in the data computed by Schularick and Taylor (2012) on the eve of the Great Recession.

The steady state risk-free interest rate is set to 0.5 percent quarterly. In contrast to KRW, in which the risky private bond is the only financial investment, in our model economy the risk-free bond is an important financial investment vehicle for monetary policy transmission. Once we calibrate σ_g, equation (24) and the target interest rate pin down the level of per capita government bond holdings.

We calibrate the steady state debt-to-income ratio of workers as 150 percent, which is close to the debt-to-income ratio of the bottom 95 percent income earners on the eve of the Great Recession. This choice leads us to set a higher utility weight on private bond holdings of shareholders ($\psi^B = 0.45$). For the government bond holdings, we use the same utility weight ψ^G as with private bonds. We choose a slightly lower value of the discount factor for shareholders than KRW such that there is a greater incentive to invest in financial assets. This allows us to match 150 percent debt-to-income ratio. However, we make the same choice as in their analysis for the wealth elasticity and set $\sigma_b = 1.09$. With these choices, the income share of shareholders is 0.36 in the steady state, a similar value observed on the eve of the Great Recession for the top 5 percent income earners. Also, the marginal propensity to save (MPS) of shareholders in our baseline economy equals 0.332, close to the value of 0.397 used in

[13] Note that this output loss can be considered a conservative estimate, given the significant downward revisions to potential output undertaken by the CBO during the years after the Great Recession.

Table 2: Structural Parameters for Technology, Labor Markets, Nominal Rigidities and Shock Processes

Parameter	Target/Source	Value
Capital share of production	Labor income share = 0.60	$\alpha = 0.20$
Investment adjustment cost	Relative volatility of investment	$\kappa = 12$
Habit in consumption	Literature	$s = 0.45$
Depreciation rate	Literature	$\delta = 0.025$
Separation rate	CPS	$\rho = 0.37$
Matching efficiency	CPS, job finding rate	$\zeta = 0.91$
Matching function elasticity	Literature	$\epsilon = 0.50$
Worker's bargaining power	Literature	$\eta = 0.50$
Unemployment benefit	$b^U/w = 0.83$	$b^U = 0.5$
Vacancy posting cost	Literature	$\xi = 0.11$
Real wage stickiness	S.D. of real comp. 4-quarter growth of nonfarm business sector = 1.5 percent	$\vartheta = 50$
Inflation indexation	Literature	$\varepsilon = 0.5$
Price stickiness	S.D. of inflation = 0.64 percent	$\varphi = 0.935$
Taylor rule: Inflation gap	Literature	$\rho_\pi = 1.5$
Persistence of technology shock	Literature	$\rho_z = 0.85$
Persistence of bargaining power shock	Near Random Walk	$\rho_\eta = 0.90$
Persistence of risk premium shock	Literature	$\rho_\phi = 0.85$
Std. Dev. of technology shock	1/3 of output variance share and ZLB frequency	$\sigma_z = 0.005$
Std. Dev. of bargaining power shock	1/3 of output variance share and ZLB frequency	$\sigma_\eta = 0.0135$
Std. Dev. of risk premium shock	1/3 of output variance share and ZLB frequency	$\sigma_\phi = 0.00075$

KRW for the top 5 percent income earners that matches empirical MPS.[14]

We specify a log utility, $\sigma_c = 1$, and an elasticity of substitution between differentiated goods of $\gamma = 5$ to generate a 20 percent markup in steady state. We set the degree of habit formation equal to $s = 0.45$, which is close to the mean value of 0.43 from 597 estimates of habit formation in the literature reported by Havranek, Rusnak, and Sokolova (2017).

Production: The capital share of production equals $\alpha = 0.2$ to match a labor income share of 0.60 in steady state. This value is somewhat lower than the convention of $0.3 \sim 0.4$. However, a conventional calibration results in a too-low labor share in our environment given the existence of rents due to market power, which are divided into different agents according to bargaining power. We set the coefficient of investment adjustment cost equal to $\kappa = 12$ to make investment three times more volatile than output as in the data. The depreciation rate of capital stock is set to $\delta = 0.025$.

Labor markets: The efficiency of the matching function is set to $\zeta = 0.91$ to hit a quarterly job finding rate of 85 percent as in the Current Population Survey (CPS). The exogenous gross separation rate is calibrated to $\rho = 0.37$, so that the quarterly net separation rate equals 5.6 percent as in the CPS. For the elasticity of the Cobb-Douglas matching function with respect to vacancies, we follow the evidence reported in Pissarides and Petrongolo (2001) and set $\epsilon = 0.5$. We follow much of the literature and set the steady state workers' bargaining power to $\eta = 0.5$. Unemployment benefits

[14] To compute the MPS of top earners in our model economy, we compare the increase in savings of shareholders versus the increase in their income that occur in steady state when aggregate productivity rises 1 percent permanently.

equal $b^U = 0.5$, which represent 83 percent of the equilibrium wage in steady state. Finally, we set the vacancy posting cost equal to $\xi = 0.11$, about 11 percent of labor productivity, essentially the same as in Hagedorn and Manovskii (2008) and very similar to other values used in the literature.

Price and wage rigidities: We set the real wage adjustment cost $\vartheta = 50$ to match the standard deviation of the 4-quarter growth rate of real compensation in the non-farm business sector. Regarding the degree of nominal rigidities, we choose a low value for the probability of resetting the price $(1 - \varphi = 0.065)$ to be consistent with so-called flat Phillips curve. For the same reason, we choose a substantial degree of price indexation $(\varepsilon = 0.5)$. With these choices, together with the calibration of real wage rigidity, inflation in the baseline is as volatile as PCE price inflation in the post-war period.

Monetary policy rule: As mentioned earlier, we let the monetary policy rule react only to the inflation gap given that the nature of the shocks considered are consistent with the so-called divine coincidence. In the baseline calibration, the inflation coefficient is set to $\rho_\pi = 1.5$ following Taylor (1999). Note that in an economy with the divine coincidence, this parametrization without the reaction term for the output gap implies a more lenient monetary policy rule than in Taylor (1999). However, we study a range of values for ρ_π in our analysis.

Exogenous shock processes: The persistence parameters of the technology shock and the risk premium shock are chosen to be the same and equal to $\rho_z = \rho_\phi = 0.85$. We set the persistence of the bargaining power shock to a slightly higher value of $\rho_\eta = 0.9$. The reason is that we view changes in bargaining power as a slow-moving process representing changes in social norms and labor market institutions. We choose the volatility of the three exogenous shocks such that they have equal variance decomposition shares for output in the long run, without considering financial crises or ZLB constraint. This gives us two restrictions. The third ones comes from matching the frequency of being at the ZLB of around 5 percent, as in Coibion, Gorodnichenko, and Wieland (2012). These three restrictions determine the values for σ_z, σ_ϕ, and σ_η. Note that we use the monetary policy shock only to satisfy the ZLB constraint. Overall the volatility of the cyclical component of output in the baseline economy with financial crises and the ZLB constraint is 2.2 percent, slightly above the 1.6 percent observed in the United States during the post-war period.

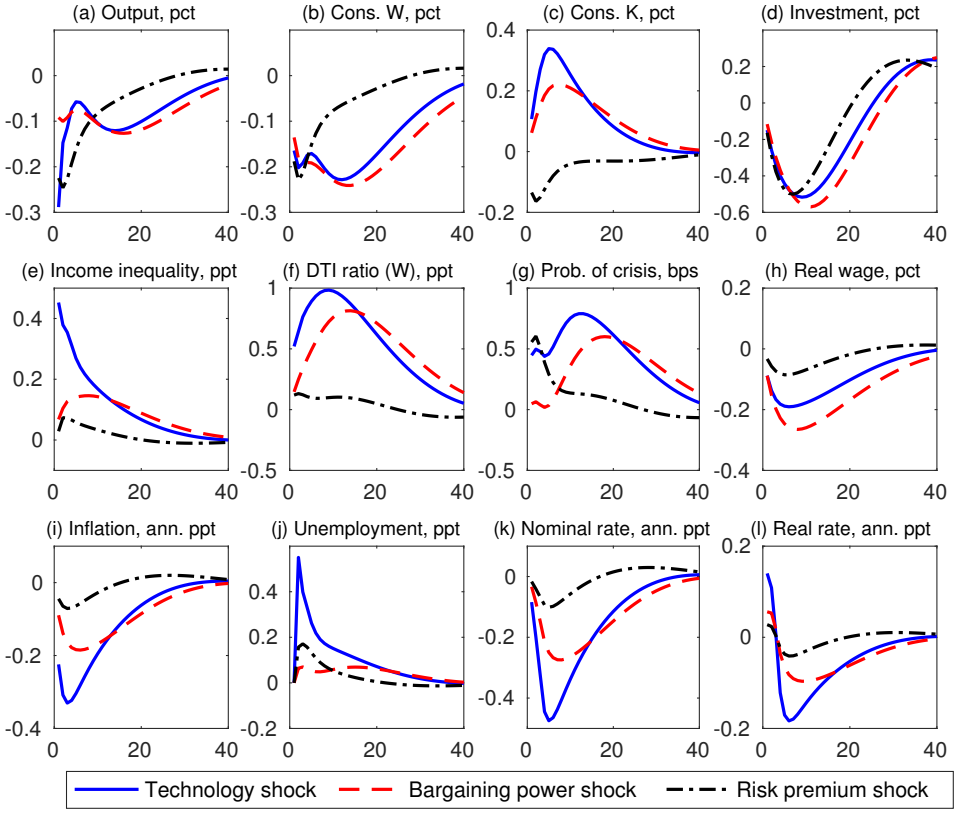
5 Model Dynamics

This section characterizes the dynamics of the model using impulse response functions. In doing so, we first assume that neither default nor a binding ZLB constraint occur. We then show how these two sources of nonlinearity modify model dynamics.

5.1 Without Crises and ZLB

Figure 2 depicts the effects of a positive technology shock (blue solid line), a negative shock to workers' bargaining power (red dashed line), and a positive shock to the risk premium (black dash-dotted line) when neither default nor a binding ZLB constraint occur.

Figure 2: Impulse Response Functions: Without Crises and ZLB



Notes: This figure plots impulse response functions to a positive technology shock, a negative shock to worker's bargaining power, and a positive risk-premium shock. It is assumed that neither default nor a binding ZLB constraint occur.

Technology shock: Panel (a) shows that, despite the technology shock being positive, the response of aggregate output is negative. This is because our baseline monetary policy fails to provide enough monetary accommodation (see Galí (1999)). The positive technology shock lowers the marginal cost and generates deflationary pressure as shown in panel (i). In response, the policy rate is lowered in panel (k). However, monetary accommodation is not strong enough and, as a result, the real interest rate initially rises as shown in panel (l), which then leads to a decline of real investment (panel (d)) and aggregate output.

The decline of aggregate demand and higher technology then lead to a fall in job creation, rising unemployment rate in panel (j), and a fall in the real wage in panel (h). Accordingly, workers' consumption declines persistently in panel (b). To smooth out the decline in consumption, workers increase borrowing and their debt-to-income ratio increases in panel (f), which then raises the probability of crisis in panel (g). In contrast, the consumption level of shareholders increases persistently owing to the increased profits. The decline of real investment also contributes to the increase in the consumption of shareholders because the reduced investment level creates a slack in the budget constraint of this group. Finally, the combination of lower wage income and higher profits results in a substantial increase in income inequality defined as the income share of shareholders in panel (e), which is correlated with increases in the debt-to-income ratio of workers.

Bargaining power shock: In the literature, a shock to workers' bargaining power affects aggregate output through the labor market channel. For instance, a lower bargaining power leads to an improvement in the job creation condition, expanding both employment and investment (see Gertler, Sala, and Trigari (2008) and Drautzburg, Fernández-Villaverde, and Guerrón-Quintana (2017)). Such a labor market channel is active in our model economy. However, the negative bargaining power shock also redistributes income from workers to shareholders. This redistribution may lead to a decline in aggregate demand if the marginal propensity to save of the shareholders is strong enough such that the reduced consumption of workers is not completely offset by the increased consumption of shareholders. Panels (a) to (c) of Figure 2 show that this is indeed the case in our baseline economy.

Despite the fundamental improvement in job creation condition, employment actually decreases because of the fall in aggregate demand. For the same reason, investment decreases. Since lower bargaining power reduces the real wage persistently, marginal cost declines and the economy faces a mild deflation pressure.[15] Monetary policy reacts by lowering the nominal interest rate, but the real interest rate is slightly increased. Workers try to smooth out the decline in consumption by issuing more bonds. Since shareholders have to increase lending, they do not increase consumption sufficiently.

The rise in income inequality generated by the negative bargaining power shock correlates with the increase in the debt-to-income ratio of workers. As the debt-to-income ratio goes up persistently, the probability of default rises.

Risk premium shock: Panel (a) of Figure 2 shows that the risk premium shock leads to a persistent decline in aggregate output. This is driven by the decline in the consumption of shareholders through the consumption Euler equation and the decline in investment through the investment Euler equation. While this shock does not directly affect workers' consumption through their consumption Euler equation, the shock also leads to a decline in their consumption because the decline of aggregate demand leads to higher unemployment and lower wages. The debt issuance of workers declines as the borrowing cost is elevated by this shock. The decline of income and consumption in the context of higher borrowing cost increases the incentive to default and the default probability goes up, but not substantially since the borrowers undergo deleveraging in response to the risk-premium shock.

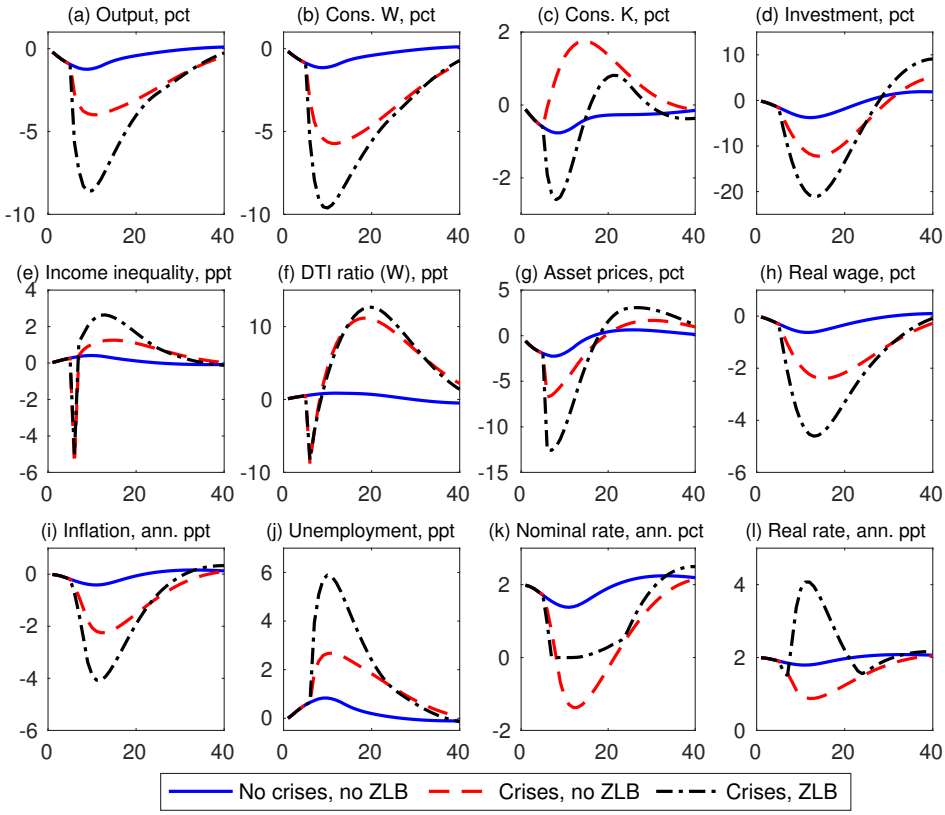
5.2 With Crises and ZLB

Figure 3 illustrates how occasional eruptions of financial crises make a difference in the response of endogenous variables to a given series of random shocks. It also makes clear how the ZLB constraint works in conjunction with the endogenous crisis mechanism. To create an environment in which a financial crisis and a binding ZLB constraint may occur, we assume a sequence of adverse risk premium

[15] The change in aggregate demand due to redistribution of aggregate income among different income groups with heterogeneous MPCs is very much in the *Kaleckian* tradition:

> "If an increase in bargaining capacity is demonstrated by spectacular achievements, there is a downward shift in function $f(\bar{p}/p)$ and the mark-ups decline. A redistribution of national income from profits to wages will take place then...But, as there is a redistribution of income from profits to wages...there is a rise in employment and output there"–(Kalecki (1971), p. 162).

Figure 3: Nonlinear Effects of ZLB and Financial Crises



Notes: We use a sequence of adverse risk-premium shocks such that the economy undergoes a recession. The blue solid line shows the case where a financial crisis does not occur and the ZLB is not binding. The red dashed line and the black dash-dotted line are the cases in which a financial crisis occurs in period 7 without and with the ZLB constraint, respectively.

shocks that bring the economy into a recession.[16]

First, we focus on the blue solid line which shows the response of the economy under the assumption that a financial crisis does not occur during this episode. Whether or not a financial crisis occurs depends on the specific random draw that the economy is given for the utility cost of default and the difference between the values of default and non-default for workers. Panels (a)∼(c) of Figure 3 show that aggregate output and consumption for both types of agents decline about 1∼2 percent from their steady state values. In panel (d), investment in physical assets declines 5 percent. Panel (j) shows that unemployment rate rises up to 1 percentage point above its steady state level, bringing downward pressure on the real wage. Note that without a financial crisis, the ZLB constraint does not bind in this example, as shown by panel (k).

Next, the red dashed line shows the case where the economy is given a particularly low realization of the random draw for the utility cost of default in period 7 and a financial crisis occurs. This case assumes that the ZLB constraint does not exist as indicated by the fact that the level of the nominal interest rate falls nearly to -2 percent. Overall, the responses of all endogenous variables are much stronger except the consumption level of shareholders, which increases in response to the crisis. The

[16] In particular, we assume $\epsilon_{\phi,1:12} = \sigma_\phi \times [\ 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.75 \quad 0.6 \quad 0.45 \quad 0.3 \quad 0.15\ ]'$.

reason is that shareholders reduce investment in physical assets and financial assets up to a point where there exists a large surplus cash flow for increased consumption expenditures.

Finally, the black dash-dotted line shows the case when the ZLB constraint exists in addition to the crisis happening in period 7, and thus the ZLB constraint binds endogenously in response to the crisis. It is somewhat difficult to see in the figure, but the ZLB constraint does not bind immediately after the crisis erupts. This is because the response of inflation rate is hump-shaped. However, the ZLB constraint binds in future periods in expectations immediately and the economy faces anticipated news shocks to monetary policy. In this environment, the economy undergoes a much deeper recession.[17] Aggregate output, for instance, can deviate as much as 6 percentage points from the case without the crisis and the binding ZLB constraint. In the presence of binding ZLB constraint, the size of the financial crisis roughly doubles in terms of output loss, unemployment increase and deflation.[18]

6 Simulation Results: Income Inequality, Aggregate Demand and Financial Crises

In this section, we first perform a thought experiment to highlight the link between income inequality and aggregate demand in the model. We then use stochastic simulations to analyze the key properties of the model economy.

6.1 Understanding the Mechanism

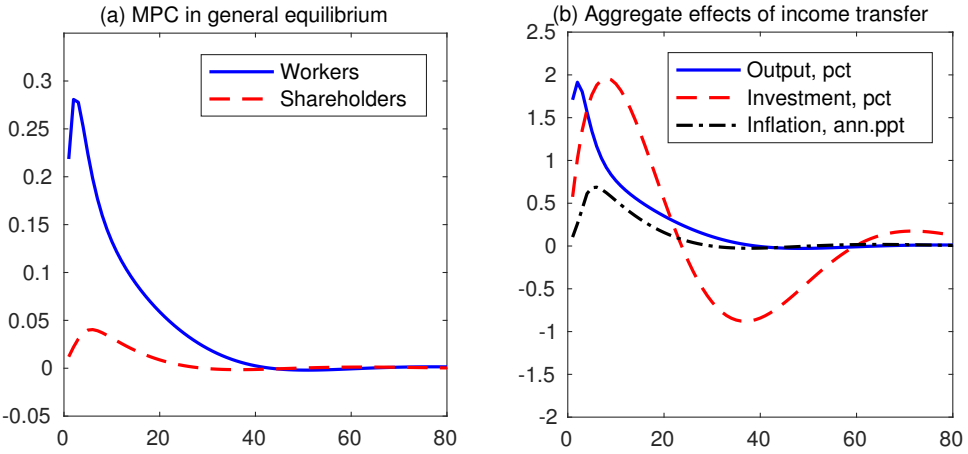
Before describing the main results of the paper, we illustrate the key mechanism that provides links between income inequality, insufficient aggregate demand and deflation pressure. The fundamental reason why increases in income inequality lead to insufficient aggregate demand in the model is the composition of aggregate MPC: Shareholders have a strictly lower MPC than workers due to the *Weberian* preferences of "the spirit of capitalism" of shareholders. In this situation, the redistribution of national income towards workers leads to increases in aggregate demand.

In order to illustrate that the model features this property, Figure 4 plots the effects of a fictitious government policy that redistributes part of shareholders' income to workers. This redistribution is assumed to be a one-off event with no persistence. The per capita income transfer for workers is denoted by ω, and is calibrated as 10 percent of wage income of workers in the steady state. Per capita transfer for shareholders is then given by $-\omega(1-\chi)/\chi$. Note that the aggregate resource constraint is not

[17] The sizes of these anticipated news shocks are large on the order of 3∼5 percentage points.

[18] Note that the binding ZLB constraint can be thought of as contractionary monetary policy shocks executed by nature rather than by the monetary authority. The difference between the black dash-dotted lines and the red dashed lines in panels (b) and (c) of Figure 3 show that the monetary policy transmission channel is starkly different for workers and shareholders for the same reason mentioned as for the effects of the risk premium shock. In the case of shareholders, the difference is mainly driven by the intertemporal substitution effect owing to the higher real interest rate during the binding ZLB episode. In contrast, the reduction in workers' consumption is primarily driven by the general equilibrium effects of reduced job creation. While our model is a very stylized, two-agent general equilibrium model, this difference in monetary transmission channels between shareholders and workers is essentially identical to that obtained in the much richer model of Kaplan, Moll, and Violante (2018), with heterogeneous agents and incomplete markets with multiple assets of different liquidities.

Figure 4: Effects of a Transitory Income Transfer Shock



Notes: This figure plots the effects of a fictitious government policy that redistributes part of shareholders' income to workers. This redistribution is assumed to be a one-time event with no persistence and is calibrated to 10 percent of wage income of workers in the steady state.

affected because the population weighted sum of the transfer is zero, i.e. $(1-\chi)\omega+\chi[-\omega(1-\chi)/\chi]=0$.[19]

Panel (a) of Figure 4 shows the marginal effects of income redistribution on consumptions of the two agents, i.e., dc^W/ω (blue solid line) and $dc^K/[-\omega(1-\chi)/\chi]$ (red dashed line).[20] Note that $dc^K/[-\omega(1-\chi)/\chi]$ is positive because both the numerator and the denominator are negative. In other words, the transitory income shock for shareholders leads to a decline in their consumption. Panel (a) shows that the marginal effects of the transitory income shock are about six times larger for workers than for shareholders. Panel (b) displays the effects on aggregate output (blue solid line), investment (red dashed line) and inflation rate (black dash-dotted line). The change in the income distribution in favor of those agents with greater MPC brings a persistent boom by changing the composition of aggregate MPC, even when the shock is completely transitory.

Note that the investment decision for capital accumulation is made by shareholders. Thus, the fact that the negative income shock for shareholders leads to an increase in investment is a remarkable result: while shareholders observe that the government confiscates part of their income in this experiment, they also expect a persistent boost in aggregate demand and increase investment spending.[21]

6.2 Main Results

Table 3 summarizes the unconditional moments of the economy under the baseline monetary policy rule, which takes the form of inflation targeting with $\rho_\pi = 1.5$. The moments are computed using

[19] This experiment is for illustration purposes and such government redistribution does not occur in our simulations.

[20] Note that these marginal effects are not the MPC discussed in the microeconomics literature. This is because the income redistribution in this experiment brings large general equilibrium effects as shown by the aggregate output response in panel (b) of Figure 4.

[21] While our focus is on monetary policy issues, the transfer experiment shown in Figure 4 suggests much larger stabilization roles for tax-and-transfer and social insurance (see McKay and Reis (2016) and Mitman, Manovskii, and Hagedorn (2017)).

Table 3: Moments of Key Variables under the Baseline Monetary Policy Rule

	No crises No ZLB	No crises ZLB	Crises No ZLB	Crises ZLB
1. Corr(income inequality, y)	-0.78	-0.88	-0.57	-0.82
2. Corr(income inequality, c)	-0.61	-0.82	-0.48	-0.80
3. Corr(income inequality, x)	-0.68	-0.75	-0.55	-0.77
4. Corr(income inequality, π)	-0.97	-0.97	-0.80	-0.88
5. Corr(income inequality, b/y)	0.82	0.86	0.72	0.84
6. Corr(income inequality, p^{δ})	0.75	0.85	0.62	0.81
7. Corr(consumption inequality, y)	-0.79	-0.82	-0.92	-0.90
8. Corr(consumption inequality, c)	-0.72	-0.80	-0.89	-0.90
9. Corr(consumption inequality, x)	-0.73	-0.72	-0.77	-0.74
10. Corr(consumption inequality, π)	-0.94	-0.92	-0.92	-0.88
11. Corr(consumption inequality, b/y)	0.94	0.95	0.91	0.94
12. Corr(consumption inequality, p^{δ})	0.87	0.90	0.81	0.92
13. Corr(b/y, p^{δ})	0.98	0.99	0.96	0.98
14. $E(p^{\delta})$	1.29	1.29	1.32	1.33
15. $E(i)$, quarterly, percent	0.50	0.55	0.33	0.49
16. $E(\pi)$, quarterly, percent	0.00	-0.03	-0.11	-0.23
17. $E(y)$	0.91	0.90	0.90	0.89
18. $E(u)$, percent	5.88	6.32	6.73	8.15
19. E(debt-to-income ratio of workers)	1.52	1.53	1.55	1.56
20. S.D.(y), percent	0.91	1.82	1.61	4.36
21. S.D.(π), quarterly, ppts	0.30	0.38	0.35	0.64
22. corr(y, π)	0.74	0.87	0.79	0.93
23. Frequency of ZLB, percent	-	3.24	-	5.41
24. Mean MP shock, annual bps	-	1.01	-	3.22
25. Skewness(i)	0.01	0.52	-0.33	0.70
26. Skewness(π)	0.01	-1.13	-0.33	-2.04
27. Skewness(y)	0.02	-2.39	-0.96	-2.59
28. Skewness(u)	0.00	1.80	0.47	2.52

Notes: The moments are computed using 250,000 simulations with identical set of random draws for the shocks in each column. The baseline monetary policy is assumed in all cases. Mean MP shock is the annualized mean of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint.

250,000 periods and 40 periods of news shocks to monetary policy such that the agents of the model understand the nature of the ZLB constraint.[22] This exercise is meant to uncover important links between income inequality, low aggregate demand, deflation pressure and the likelihood of financial crises. The links have been conjectured but not yet quantified in the existing literature.

In order to better understand the separate effects of financial crisis and the ZLB constraint, Table 3 reports four set of results, using an identical set of random draws for shocks in each column. The four columns show the cases (i) with no crises and no ZLB, (ii) with no crises and the ZLB, (iii) with crises and no ZLB, and (iv) with crises and the ZLB, which is our baseline case. In our explanation of the results, we focus on the baseline case for the sake of space. However, we will draw readers' attention to alternative cases to emphasize the role of a particular mechanism.

Lines 1∼3 of Table 3 show that income inequality, defined as the share of shareholders' income

[22]Increasing the number of simulation periods to 500,000 does not change the moments in any meaningful way. Also, using 60 periods of news shocks to monetary policy leads to roughly identical moments.

relative to total income, is negatively correlated with aggregate output, consumption, and investment, and the degrees of correlations are strong in the baseline case.[23] Since correlation is not causation, one cannot state that the income inequality causes insufficiency of aggregate demand. Rather, causation runs in both directions. Owing to the lower MPC of shareholders, aggregate demand falls when a growing share of national income is distributed to those agents. Conversely when aggregate demand rises, labor income rises as more jobs are created and wage is boosted. Since labor income is the major income source for workers, a boost in aggregate demand leads to a decline in income inequality. As shown in lines 7∼9, the link between inequality and aggregate demand is also confirmed when inequality is measured in terms of consumption. These results indicate that secular stagnation due to income inequality and underconsumption, originally raised by Hansen (1939) and recently re-taken by Summers (2015), can be a real possibility.

Line 4 of Table 3 shows a strong, negative correlation between income inequality and the inflation rate. Again, causation runs in both direction. Since income inequality is negatively correlated with aggregate demand, it is also negatively correlated with inflation. Conversely, deflation elevates the real debt burden of workers and redistributes income towards shareholders, increasing the degree of income inequality. We want to emphasize the role played by nominal rigidities. Without them, a negative correlation between income inequality and aggregate demand cannot exist in KRW's original framework as Say's Law holds regardless of the degree of income inequality.

Lines 5 and 11 of Table 3 show that the strong correlation between income/consumption inequality and excess credit, defined as the deviations of credit-to-GDP ratio from its steady state, such as seen in the data, can be replicated by our model. More importantly, lines 6 and 12 indicate a strong correlation between inequality and the probability of financial crises. This arises due to the near perfect linear relationship between excess credit and the probability of financial crises regardless of the presence of the ZLB constraint, as shown in line 13. Such a linear relationship replicates the empirical findings by Jorda, Schularick, and Taylor (2011) and Schularick and Taylor (2012).[24]

Interestingly, the tie between income inequality and aggregate demand is much stronger in the presence of the ZLB constraint.[25] This indicates that a failure to provide a sufficient degree of monetary accommodation during economic downturns strengthens the nexus between income inequality and aggregate demand. The presence of the ZLB constraint also slightly increases the probability of financial crises, due to slightly higher debt-to-income ratio of workeres.

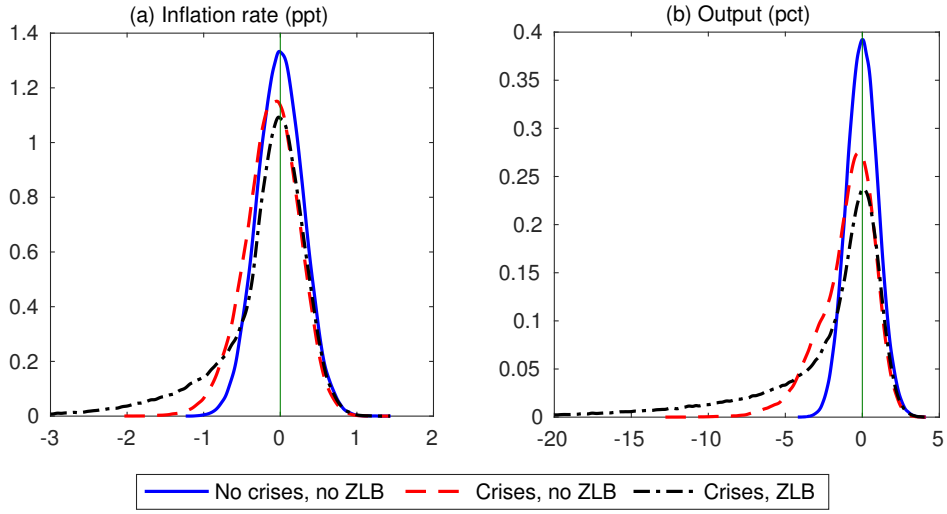
Importantly, both financial crises and the presence of the ZLB constraint modify the distributions

[23] Note that it is the case with no ZLB but with financial crises, shown in the third column of Table 3, where these correlations are the weakest. This is because without the ZLB constraint, financial crises tend to lower the overall comovement property of the economy. As illustrated by the red line in panel (c) of Figure 3, financial crisis works like a financial shock that tends to lower the comovement between consumption and investment when the ZLB is not present. That is, shareholders increase consumption in response to a financial crisis as the crisis reduces overall aggregate demand and they respond by reducing real investment substantially. The increase in consumption of shareholders in this case lowers the negative correlation between income inequality and aggregate demand.

[24] The linear relationship also justifies the reduced form approach that assumes a (log) linear relationship between the excess credit and the probability of disaster without structural modeling, an approach recently taken by Gourio, Kashyap, and Sim (2018).

[25] Figure 11 in Appendix D shows the unconditional and conditional means of monetary policy shocks (both contemporaneous and anticipated news shocks) needed to satisfy the ZLB constraint in the baseline economy.

Figure 5: Skewed Distributions of Inflation and Output



Notes: The figure plots the distributions of inflation and output in deviations from their non-stochastic steady state depending on the presence of financial crises and the ZLB constraint using kernel density estimates of simulated data.

of equilibrium quantities and prices. In particular, with no financial crises and no ZLB constraint (first column of Table 3), the skewness of endogenous variables (see lines 26∼28) is basically zero, which should not be surprising given the assumed Gaussian shocks. This also implies that the mean of their distributions coincides with the non-stochastic steady state of each variable (see lines 16∼18). When financial crises are included but not the ZLB constraint (third column), the distributions of inflation, aggregate output and unemployment rate become skewed to the downside and their means deviate from the non-stochastic steady state values. Thus, financial crises create skewness to the distributions. The combination of crises and the ZLB constraint (fourth column) creates extraordinary degrees of skewness for the endogenous variables. Since negative skewness assigns more probability mass to the downside of the economy, the mean levels of the inflation rate and aggregate output are substantially lower. Indeed, the presence of the ZLB constraint doubles the deflation bias created by financial crises and lowers the mean level of output by 1 percent.[26] Figure 5 shows how the exact shapes of the distributions change depending on the presence of financial crises and the ZLB constraint using kernel density estimates of simulated data for inflation rate in panel (a) and aggregate output in panel (b). Additionally, the ZLB constraint makes the distribution of the policy rate more skewed to the upside (see lines 15 and 25), creating additional deflation bias. Both sources of nonlinearity also increase the volatility of inflation and output (see lines 20 and 21).

Interestingly, the presence of the ZLB constraint strengthens the comovement property of the economy as measured by the correlation between aggregate output and inflation (see line 22). Note that, in our model economy, rational expectations imply that current inflation is the present value of future expected real marginal costs and current consumption is the negative present value of future

[26] Note that the skewness of the unemployment rate without the ZLB constraint is 0.47 in our economy, which is the same in the U.S. data from 1948 to 1999. The skewness goes up to 0.62 if the sample is expanded to 2017 to include the binding ZLB episode.

expected real interest rates. When the ZLB constraint is expected to bind in the future, agents expect real marginal costs to be lower and the real interest rate to be higher. This situation leads to an immediate decline of both inflation and consumption in line with the results in McKay, Nakamura, and Steinsson (2016), strengthening the comovement property of the economy. The presence of the ZLB constraint therefore introduces nominal shocks, which are considered to be the ones with stronger comovement property in the DSGE literature.[27]

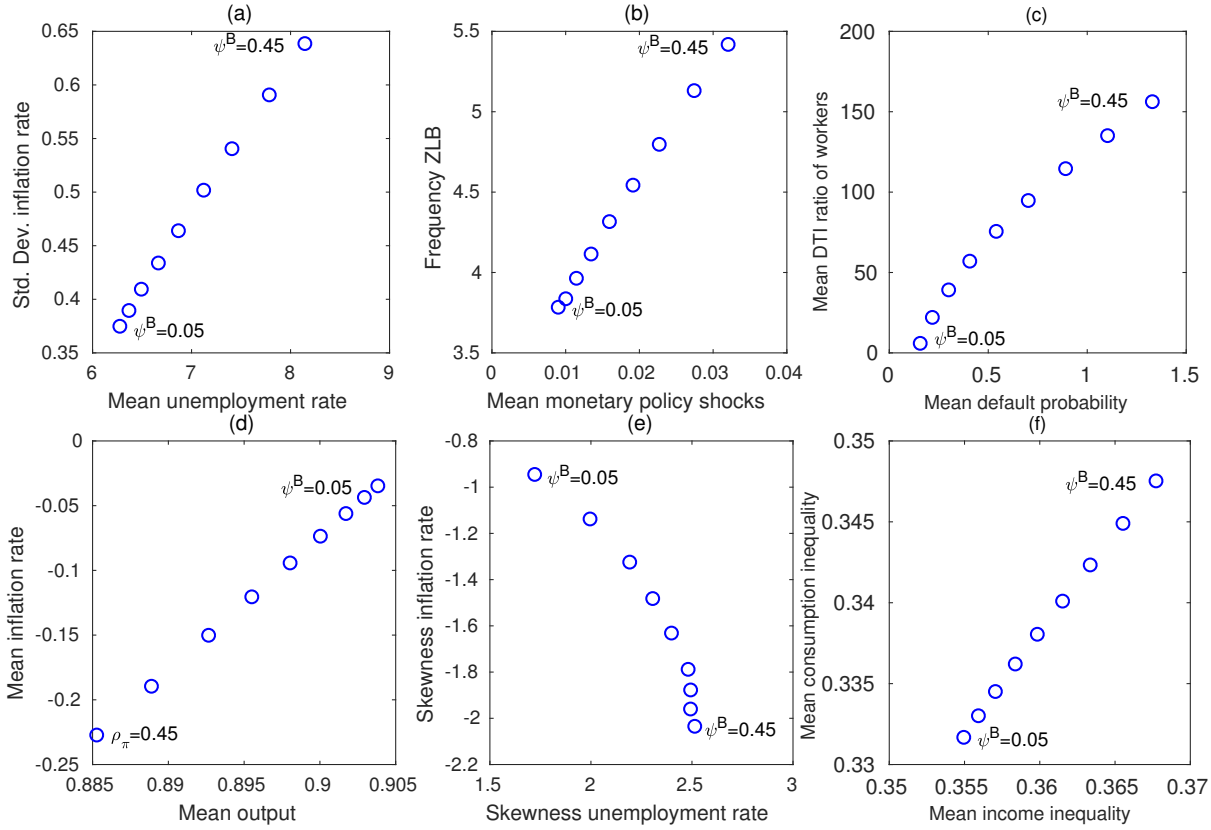
The results in this section show that income inequality generates insufficient aggregate demand and financial fragility, creating deflation bias and elevating the mean unemployment rate. In our model economy, the negative link between income inequality and aggregate demand is created by the desire of shareholders to overaccumulate financial wealth, motivated by their *Weberian* preferences. In order to illustrate this mechanism, Figure 6 reports the effects of changing the weight given to the direct utility from holding financial assets, ψ^B. As the weight is lowered from the baseline value $\psi^B = 0.45$, which was chosen to match the observed debt-to-income ratio of workers, toward zero, both the standard deviation of inflation and mean unemployment rate fall linearly in panel (a), and the mean inflation rate is raised from -0.23 percent in the baseline towards zero in panel (d). Panel (c) show the effects on the debt-to-income ratio of workers and the probability of financial crises: they both converge to essentially zero as the "love of wealth" preferences disappear. In panel (b), as the deflation risk falls, the frequency of the ZLB constraint declines linearly and the size of the monetary policy shocks needed to satisfy the ZLB constraint also falls. One can also see that the mean level of aggregate output would be more than 2 percent higher with much weaker "love of wealth" preferences (panel d), and the distributions of inflation and the unemployment rate would be much less skewed to the downside (panel e). Finally, the mean levels of both income and consumption inequality would be much lower if the "love of wealth" preferences disappear (panel f). Overall, the results in Figure 6 indicate that the Weberian preferences are the key mechanism that creates a link between income inequality, deflation pressure, excess credit and financial fragility in our model economy.

7 Monetary Policy Strategies

This section discusses monetary policy strategies for an economy that suffers from insufficient aggregate demand due to income inequality and faces occasional financial crises due to excess credit. We first describe the monetary policy dilemma facing a central bank that follows either an inflation targeting rule or a price-level targeting rule. We then examine the role of the ZLB constraint in generating the

[27] McKay, Nakamura, and Steinsson (2016) show that the precautionary saving motive in incomplete markets can make the response of aggregate consumption less sensitive to the forward guidance. Our model economy can be viewed as a two-agent incomplete market economy. The ZLB constraint is implemented by anticipated news shocks to monetary policy, which can be viewed as reverse forward guidance. Despite the stylized incomplete market setting, the forward guidance in our model economy is as powerful as in a representative agent economy for two reasons. First, shareholders in our economy are well-insured, making the forward guidance powerful. Second, once the forward guidance becomes powerful through reduction in consumption and investment of shareholders, this brings a large general equilibrium effects on labor market, which then puts direct downward pressure on workers' consumption. Furthermore, correlation between credit and workers' consumption is very close to zero in the baseline economy. This means that on average, the credit instrument is not very useful for workers' consumption smoothing, making them more like the agents under extreme asset illiquidity in Werning (2016) rather than the agents with precautionary savings in McKay, Nakamura, and Steinsson (2016).

Figure 6: The Role of the Weberian Preferences



Notes: The figure shows the loci of the moments of selected endogenous variables for different values of the utility weight on private bond ψ^B. The baseline economy uses $\psi^B = 0.45$. The horizontal axis in panel (b) shows the annualized unconditional mean in percent of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint.

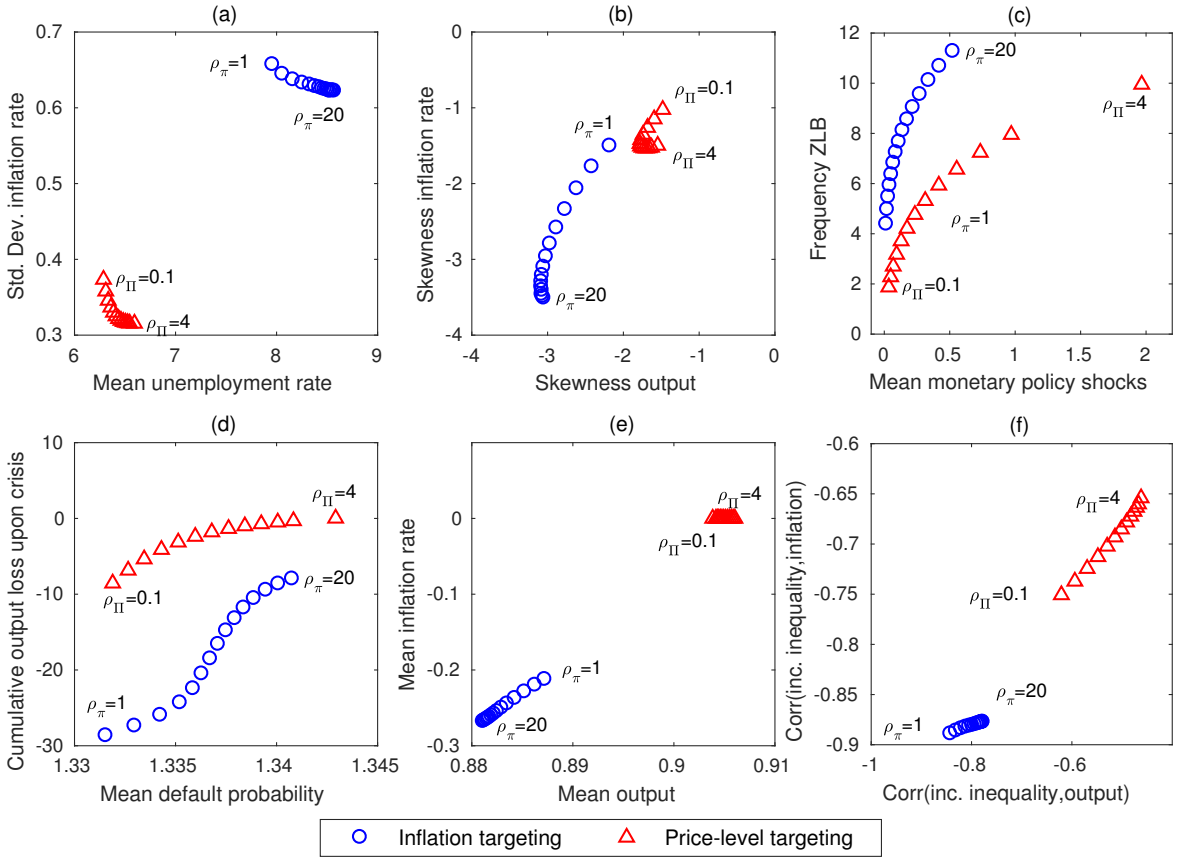
monetary policy dilemma. Finally, we discuss asymmetric monetary policy rules that can mitigate the degree of skewed distributions of endogenous quantities and prices due to occasional financial crises and binding ZLB constraint.

7.1 The Costs and Benefits of Appointing a Conservative Central Banker

Clarida, Galí, and Gertler (1999) showed that Rogoff (1985)'s conservative central banker is optimal in the context of an optimal monetary policy rule without commitment. As the weight on the output gap in the central bank's loss function converges to zero, which is equivalent with assuming the divine coincidence due to the lack of markup shocks, the inflation reaction coefficient should approach infinity. As pointed out earlier, our model economy features the divine coincidence. However, when the economy faces disproportionately large downside risks due to financial crises and the stabilizing capacity of monetary policy is severely distorted by the presence of the ZLB constraint, can one reach the same conclusion? We re-investigate the benefits and costs of a conservative central banker.

Figure 7 shows how key moments of the model economy change as the central bank increases either the inflation reaction coefficient (blue circles) or the reaction coefficient for price-level deviation (red

Figure 7: The Consequences of Conservatism in Monetary Policy



Notes: The figure shows the loci of the moments of selected endogenous variables as the central bank increases either the inflation reaction coefficient ρ_π (inflation targeting) or the reaction coefficient for price-level deviation ρ_Π (price-level targeting). The horizontal axis in panel (c) shows the annualized unconditional mean in percent of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint.

triangles). Panel (a) uncovers an important tradeoff: A conservative central banker achieves lower inflation volatility only at a cost of elevating the mean unemployment rate. This is due to the tendency of a conservative central banker to aggravate the skewed distributions of output (employment) and inflation as shown by panel (b). The welfare loss due to the increase in mean unemployment rate and the decrease in mean output may dominate any welfare gain from reducing the volatility of inflation rate.

Panel (c) of Figure 7 presents another challenge facing a conservative central banker. The panel shows how different degrees of policy conservatism affects the frequency of the ZLB constraint (vertical axis) and the unconditional mean of monetary policy shocks (both contemporaneous and anticipated) required to satisfy the ZLB constraint (horizontal axis). The former measures the time spent at the ZLB constraint while the latter measures how much damage is inflicted to the economy in terms of monetary policy shocks to satisfy the ZLB constraint. As it can be seen, the frequency of binding ZLB constraint increases monotonically for both inflation targeting and price-level targeting as policy conservatism is strengthened. Furthermore, the mean level of monetary policy shocks used to satisfy the ZLB constraint, which can be interpreted as the shadow value of the ZLB constraint, also increases

monotonically with the degree of policy conservatism.

Panel (d) of Figure 7 shows two aspects of policy conservatism regarding the probability of financial crises (horizontal axis) and the cumulative output loss upon a financial crisis (vertical axis). For both rules, a conservative central banker tends to increase the probability of financial crises. This is because strong reactions in monetary policy rates to inflation tends to elevate debt-servicing costs of borrowers. Furthermore, the increase in unemployment rate due to hawkish monetary policy as shown in panel (a) also elevates the debt-to-income ratio and thus strengthens the incentive of borrowers to default. However, the same conservative central banker has a virtue of providing a massive amount of monetary accommodation during a financial crisis, limiting the size of the cumulative output drop.[28]

It is notable that while the price-level targeting regime faces the same type of tradeoff between stabilization of inflation volatility and reduction of the mean unemployment rate, it substantially improves the nature of the tradeoff. This can be seen in the fact that the frontier associated with the price-level targeting is located much lower and more to the left in panel (a) of Figure 7. Furthermore, as shown in panel (b), the absolute degrees of skewness of the distributions are also much smaller with the price-level targeting as compared with the inflation targeting.

Price-level targeting can also be much more effective in controlling the deflation bias. Panel (e) of Figure 7 shows the mean inflation rate on the vertical axis and the mean aggregate output on the horizontal axis. The results suggest that, in contrast to the inflation targeting, price-level targeting eliminates the deflation bias almost completely regardless of the degree of the conservatism of monetary policymakers. It also suggests that simply switching from the inflation targeting with $\rho_\pi = 20$ to the price-level targeting with $\rho_\Pi = 0.1$ can result in an almost 3 percent increase in the mean level of aggregate output, which has significant implications for the welfare of both types of agents.[29] Panel (e) also suggests that if the price-level targeting cannot be implemented, adopting the most lenient inflation targeting regime can minimize the deflation bias at the cost of increased volatility of inflation rate.

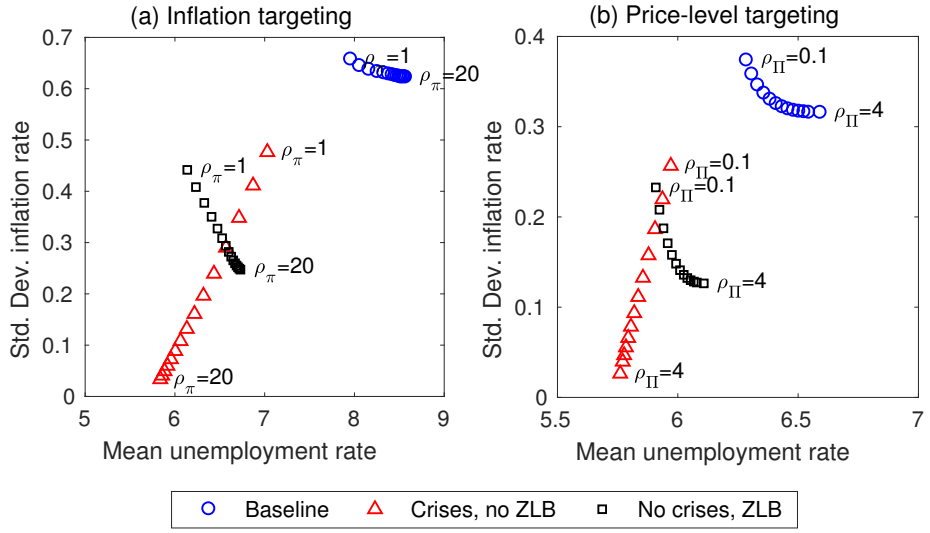
Finally, panel (f) of Figure 7 shows how monetary policy conservatism affects the correlation between income inequality and the inflation rate (vertical axis) and the correlation between income inequality and output (horizontal axis). The results point out that price-level targeting can be much more effective in breaking the link between income inequality and aggregate demand in the model. In fact, no level of inflation targeting is nearly as successful as the price-level targeting in breaking the tie between income inequality and aggregate demand.

Summing up, the investigation of various aspects of the economy under alternative monetary policy rules leads to two conclusions regarding the desirability of monetary policy conservatism. First, regardless of optimality of a conservative central banker, price-level targeting outperforms inflation targeting in many dimensions such as containing the deflation bias created by occasional financial

[28] Figure 12 in Appendix D replicates the experiment shown in Figure 3 when both financial crises and the ZLB constraint are present for a range of calibrated inflation targeting and price-level targeting rules.

[29] Output loss associated with appointing a conservative central bank has two sources: skewed distributions of production inputs and price dispersion due to the deflation bias. The latter does not exist for price-level targeting regardless of policy conservatism as the rule eliminate the bias almost completely.

Figure 8: The Benefits of Conservative Central Bankers
without Financial Crises and ZLB



Note: The figure shows the loci of the moments of selected endogenous variables as the central bank increases either the inflation reaction coefficient ρ_π in panel (a) (inflation targeting) or the reaction coefficient for price-level deviation ρ_Π in panel (b) (price-level targeting).

crises and binding ZLB constraint. Second, conservatism in monetary policy may lead to greater deflation bias and more frequent financial crises in an economy suffering from insufficient aggregate demand due to income inequality, though the same conservatism may have a benefit of providing a protection against downside risk conditioned upon an actual financial crisis. The second conclusion seems to suggest that in an economy in which an active link exists among income inequality, deflation pressure and financial crises, a mix of a dovish central banker and extraordinary stabilization measures that are implemented only during financial crises may improve upon symmetric monetary policy rules.

7.2 The Role of the ZLB Constraint

Before we move onto the analysis of optimal monetary policy, we want to emphasize that the tradeoff facing a conservative central banker is due to the presence of the ZLB constraint. In other words, the presence of occasional financial crises alone does not create the tradeoff in monetary policy. To illustrate that, Figure 8 shows what happens to the tradeoff between inflation stabilization and bias reduction in the unemployment rate if there is no ZLB constraint (red triangles) and if there is no financial crises (black squares) with panel (a) showing the case of inflation targeting and panel (b) the case of price-level targeting. For comparison, the two panels also show the baseline cases with both ZLB and financial crises (blue circles). Figure 8 clearly indicates that if there is no ZLB constraint (red triangles), the trade off does not exist. In contrast, as indicated by black squares in the two panels, even without financial crises, a conservative central banker still faces the same type of monetary policy dilema in the presence of the ZLB constraint.[30] In reality, financial crises contribute to the monetary

[30] This shows that the presence of ZLB constraint creates tradeoff in implementing the dual mandate, which is consistent with what Nakata and Schmidt (2015) finds in a canonical New Keynesian model.

Table 4: Moments of Key Variables under Optimal Monetary Policy with Crises but without ZLB

	Inflation targeting $\rho_\pi = 1e+10$	Price-level targeting $\rho_\Pi = 1e+10$
1. Corr(income inequality, y)	-0.02	-0.02
2. Corr(income inequality, π)	0.00	0.00
3. $E(\pi)$, quarterly, percent	0.00	0.00
4. S.D.(π)	0.00	0.00
5. E(income share of shareholders)	0.36	0.36
6. E(debt-to-income ratio of workers)	1.57	1.57
7. $E(p^\delta)$	1.32	1.32

Notes: The moments are computed using 250,000 simulations with identical set of random draws for the shocks in each column, with financial crises but without the ZLB constraint.

policy dilemma only to the extent that financial crises make the ZLB constraint more likely to bind for a given monetary policy rule.

From a monetary policymaker's perspective, financial crises are no different from large 'negative' demand disturbances. As such, they can be completely accomodated by sufficiently lowering the interest rate. Hence, the desirability of conservative central bankers is maintained in this environment. In fact, complete elimination of inflation volatility and bias in the unemployment rate can be achieved by a very conservative central banker both in the inflation targeting regime and the price-level targeting regime. This experiment also makes it clear that the rationale for price-level targeting hinges on the presence of the ZLB constraint, not on the presence of endogenous financial crises. The inefficiency of inflation targeting in the presence of the ZLB constraint is clearly due to its principle of *bygones are bygones*.

Note that we have emphasized the link between income inequality and aggregate demand. However, such link cannot exist in the model without nominal rigidities. In other words, such link exists only when nominal rigidities exist and monetary policy is suboptimal in eliminating their effects. To verify this claim, in Table 4, we ignore the ZLB constraint and set the monetary policy coefficients at an extremely large level, which is optimal given the lack of cost-push shock.

Lines 3 and 4 of Table 4 show that in the absence of the ZLB constraint, such monetary policy rules eliminate completely the effects of nominal rigidities, making both mean and standard deviation of inflation equal to zero. Lines 1 and 2 confirm that in an economy with no effects coming from nominal rigidities, there exists no comovement between income inequality and output on the one hand and between income inequality and inflation on the other hand. This is not because the optimal monetary policy eliminates income inequality. As shown by lines 5∼7, the main financial characteristics of the economy are no different from the baseline economy. This shows that optimal monetary policy cannot eliminate financial crises due to income inequality, but can well insulate the business cycle from the effects of inequality.

To build intuition, as Keynes (1936) did, let us assume that consumption is a linear function of income for each agent, where each agent's income is given by fractions α and $1-\alpha$ of aggregate income y : $c^W = k^W \alpha y$ and $c^K = k^K(1-\alpha)y$ with $0 < k^K < k^W \leq 1$. Consider a simplified version

of aggregate income indentity with no adjustment cost: $y = c^W + c^K + x$. For now we treat x "autonomous" component of aggregate demand. Combining the consumption functions and aggregate income identity yields

$$y = \frac{x}{1 - [\alpha k^W + (1-\alpha)k^K]} \equiv \frac{x}{1 - k(\alpha)}$$

where $1/(1-k(\alpha))$ is our version of what Keynes called "investment multiplier". Now consider a new income redistribution $\alpha' < \alpha$ that lowers the share of the income that accrues to the agent with the higher MPC. It is then clear that

$$y' = \frac{x}{1 - k(\alpha')} < \frac{x}{1 - k(\alpha)} = y,$$

since $k(\alpha') = \alpha' k^W + (1-\alpha')k^K < \alpha k^W + (1-\alpha)k^K = k(\alpha)$. For a given level of autonomous investment spending x, the ratio $y/y' > 1$ because $[1 - k(\alpha')]/[1 - k(\alpha)] > 1$, which is the ratio of the marginal propensities to save in the two economies. This is what we call inequality multiplier in which the famous *Paradox of Thrift* manifests itself through the increased income inequality ($\alpha' < \alpha$).

The inequality multiplier would not be operative if the real investment can no longer be considered "autonomous". The inequality multiplier is operative only to the extent that prices fail to clear the market immediately due to the mix of nominal rigidities and suboptimal monetary policy. In an economy with no nominal rigidities, the real "rate of interest falls at a sufficiently rapid rate" (Keynes (1936), p. 31) to induce greater investment immediately (as well as consumptions of both agents, not just because of lower real interest rate but because the increase in investment implies a greater income), completely offsetting the decline of aggregate demand due to the increased income inequality. Say's Law holds: supply creates its own demand and the channel between income inequality and aggregate demand disappears.[31] Line 1 and 2 of Table 4 shows that in the absence of the ZLB constraint, optimal monetary policy can actually recover Say's Law. Finally, panel (f) of Figure 7 shows two things. First, the ZLB constraint prevents the monetary authority from reaching such efficient allocation. Second, facing the ZLB constraint, price-level targeting is much more effective in reducing the magnitude of inequality multiplier than inflation targeting.

7.3 Dual Mandate and Optimal Monetary Policy

This section studies optimal monetary policy when the central bank has a dual mandate to stabilize the inflation rate and minimize the bias in the unemployment rate. This dual mandate is justified in our model economy because occasional financial crises and binding ZLB constraint generate a tradeoff between the two objectives (see panel (a) of Figure 7), even when the economy is not subject to shocks that move inflation and output gap in opposite directions. These considerations support the following form of loss function:

$$\min\{\lambda \mathbb{E}[\pi - \mathbb{E}(\pi)]^2 + (1-\lambda)[\mathbb{E}(u) - u^*]^2\}, \tag{59}$$

[31] How much fall in aggregate demand is offset by increases in consumption and investment due to the decline of real interest rate in this scenario depends on the degree of investment adjustment friction, which is a real adjustment friction.

where $\lambda \in [0,1]$ is a preference parameter for the central bank and u^* is the level of the unemployment rate in the absence of nominal rigidities, aggregate shocks and financial crises.

We restrict our study within the boundary of optimized simple rules that minimize the loss function (59). By varying λ from 0 to 1 and optimizing the monetary policy rules to minimize the loss function for a given λ, we can derive an "efficient policy frontier" (EPF) in the spirit of Taylor (1979), but between inflation volatility and unemployment bias, instead of between inflation volatility and unemployment volatility.

We start by optimizing ρ_π for the symmetric inflation targeting rule (56) and ρ_Π for the symmetric price-level targeting rule (57). In addition, we also study the benefits of adopting asymmetric monetary policy rules with the potential of mitigating the degree of skewness in the distributions of endogenous quantities and prices that occasional financial crises and binding ZLB constraint generate. Specifically, we optimize the inflation targeting rule (56) and price-level targeting rule (57) under the assumption that the inflation target of these policy rules follows a time-varying law of motion:

$$\pi_t^* = (1-\rho_{\pi^*})\pi^* + \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\delta_t^B, \tag{60}$$

where $0 \leq \rho_{\pi^*} < 1$ and $\sigma_{\pi^*} > 0$.[32] We refer to the alternative monetary policy rules as asymmetric rules because the inflation target deviates from its long run value π^* only to the upside when the crisis indicator δ_t^B is activated. Under the two asymmetric policy rules, the central bank announces an increase in the inflation target by σ_{π^*} in response to an eruption of a financial crisis and brings it back to the long-run target π^* only gradually. As shown previously, a conservative central banker has the benefit of mitigating the effects of financial crises only at the cost of increasing the degrees of skewed distributions and thereby increasing the mean level of the unemployment rate. The asymmetric policy rules allow the economy to have such policy conservatism only during financial crises. In our optimization, we fix $\sigma_{\pi^*} = 0.01$ so that the annual inflation target is raised by 1 percentage point immediately after a financial crisis hits the economy. We then optimize (ρ_π, ρ_{π^*}) for asymmetric inflation targeting and (ρ_Π, ρ_{π^*}) for asymmetric price-level targeting. If such an asymmetric adjustment of inflation targeting brings more harm than good, our optimization procedure will choose a value for ρ_{π^*} close to zero.

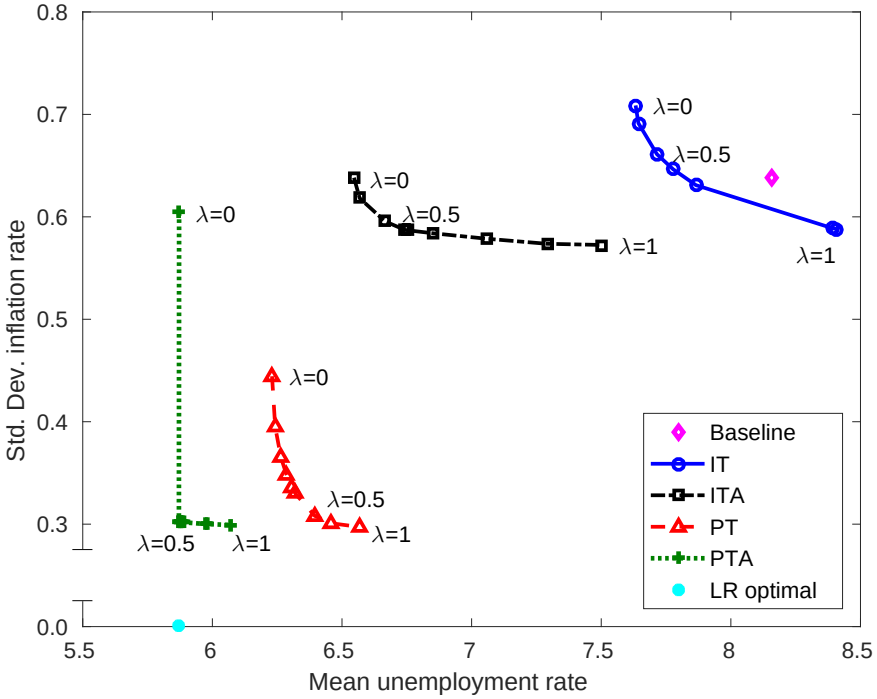
Figure 9 compares four EPFs: inflation targeting (blue circles); asymmetric inflation targeting (black squares); price-level targeting (red triangles); and asymmetric price-level targeting (green pluses).[33] The pink diamond bullet locates the moments of the baseline economy. The cyan circled bullet marks the location of socially optimal allocation where inflation volatility is zero and the level of mean unemployment rate is given by u^*.

Since these are all efficient frontiers, all frontiers are located inward from the baseline economy. It is not difficult to see how the asymmetric inflation targeting rule improves upon the symmetric

[32] While we call equation (60) time-varying inflation target, the same term can be interpreted as time-varying intercept adjustment with the opposite sign. That is, a monetary policy rule that adjusts the constant term i^* downward in response to financial crises.

[33] In this optimization, we impose $0.5 \leq \rho_\pi \leq 20$ and $0.01 \leq \rho_\Pi \leq 4$. The upper bounds never bind, while the lower bounds bind in some instances (see Tables 5 and 6).

Figure 9: Efficient Policy Frontiers:
Standard Deviation of Inflation Rate and Mean Unemployment Rate



Note: IT, ITA, PT and PTA stand for inflation targeting, asymmetric inflation targeting, price-level targeting and asymmetric price-level targeting, respectively. The pink diamond bullet locates the moments of the baseline economy. The cyan circled bullet marks the location of socially optimal allocation where inflation volatility is zero and the level of the unemployment rate is given by its long-run natural rate.

counterpart in terms of the reduced mean unemployment rate. Even when the central bank assigns zero weight on the unemployment bias term, which can be found at the right-most point of the EPF (black squares), it achieves lower unemployment than the level achieved when the symmetric inflation targeting rule is optimized under the full weight on the bias term.

What is remarkable in Figure 9 is that the strict price-level targeting rule (red triangles) can improve upon both inflation targeting rules in terms of reducing the variance of inflation and the mean unemployment rate regardless of the weights given to the loss function. In other words, when the weight given to the variance of inflation rate is zero, it still achieves much lower inflation volatility than any points in the two EPFs of the inflation targeting rules. Furthermore, when the weight given to the bias term is zero, it still achieves lower unemployment rate than most of the points on the two EPFs of inflation targeting rules.

Finally, green pluses show that a large reduction in the mean unemployment rate is possible if the central bank adopts the asymmetric price-level targeting rule and assigns a nontrivial weight on the bias correction term. In particular, when a strictly positive weight is given to the bias correction term, the asymmetric price-level targeting can achieve a large reduction in the mean level of the unemployment rate on the order of 2.2 percentage points versus the baseline economy. In fact, when the bias reduction is given a weight close to $\lambda = 0.5$, it attains the long-run natural rate of unemployment. Assigning a weight greater than this level to the bias correction results in a large increase in inflation volatility

Table 5: Optimized Rules:
Inflation Targeting vs. Price-Level Targeting

	Inflation Targeting			Price-Level Targeting		
1. λ	0.00	0.50	1.00	0.00	0.50	1.00
2. ρ_π	0.50	0.77	12.51	-	-	-
3. ρ_Π	-	-	-	0.04	0.40	3.87
4. ρ_{π^*}	-	-	-	-	-	-
5. $E(u)$	7.63	7.72	8.41	6.23	6.40	6.57
6. $E(\pi)$	-0.18	-0.19	-0.25	0.00	0.00	0.00
7. S.D.(u)	5.91	5.44	5.09	4.28	2.95	2.81
8. S.D.(π)	0.71	0.66	0.59	0.44	0.31	0.30
9. Skewness(u)	1.02	1.54	3.44	0.76	1.95	2.12
10. Skewness(π)	-0.76	-1.14	-3.40	-0.61	-1.50	-1.54
11. $E(p^\delta)$, pct.	1.32	1.33	1.34	1.33	1.33	1.34
12. Frequency of ZLB., pct.	2.08	3.42	9.74	0.77	4.02	9.35
13. Mean MP. shock, bps.	0.37	2.58	30.74	0.72	26.44	178.78
14. Loss function value	3.10	3.88	3.46	0.13	0.61	0.88
15. Welfare of workers, CE	0.01	0.01	-0.03	0.38	0.37	0.38
16. Welfare of shareholders, CE	-0.29	-0.24	0.47	-2.23	-2.24	-2.23
17. Aggregate welfare, CE	-0.01	-0.01	-0.01	0.25	0.24	0.25

Note: We impose $0.5 \leq \rho_\pi \leq 20$ and $0.01 \leq \rho_\Pi \leq 4$ in the optimization, while ρ_{π^*} and σ_{π^*} are set equal to zero. Mean MP shock is the annualized mean of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint. CE stands for consumption equivalent that needs to be given to the baseline economy to achieve the same level of welfare.

without further reduction in the unemployment rate beyond its long-run natural rate.[34]

Table 5 shows the optimized coefficients of symmetric inflation and price-level targeting rules for selected values of λ together with key moments of the model economy under each policy rule. Not surprisingly, as the loss function assigns a greater weight on the stabilization of the inflation gap, the reaction coefficient to the inflation gap and price-level gap are increased (lines 2 and 3).[35] Consequently, the frequency of a binding ZLB constraint and the sizes of anticipated monetary policy shocks are substantially higher (lines 12 and 13), and the flip side is a larger bias in the mean unemployment rate (line 5). The price-level targeting rule dominates inflation targeting in terms of bias reduction in the unemployment rate regardless of the value of λ. Furthermore, the optimized price-level targeting rules eliminate deflation bias in contrast to inflation targeting rules (line 6). Finally, higher order moments shown in lines 7$\sim$10 also confirm the improved stabilization properties of the price-level targeting rule in terms of variance reduction of the unemployment rate and the inflation rate and in terms of correcting skewed distributions of these key macroeconomic variables. Overall, smaller loss function values (line 14) associated with price-level targeting summarize the better performance of this rule against the inflation targeting rule in our model economy [36]

[34] In Appendix D, we consider an alternative loss function that penalizes the level of unemployment rather than the bias term. In this case, the EPF shows that a larger reduction in the mean level of unemployment rate on the order of 4 percentage points from the baseline economy is possible at some costs of inflation bias and increased inflation volatility.

[35] It is notable that so called Taylor principle is not applied in the case of inflation targeting. In fact, inflation targeting rule with $\lambda = 0$ hits the lower bound $\rho_\pi = 0.50$.

[36] Loss function values cannot be compared across different values of λ as they represent different preferences. However for a given λ, loss function values can be compared for different policy rules.

Table 6: Optimized Rules:
Asymmetric Inflation Targeting vs. Asymmetric Price-Level Targeting

	Asymmetric Inflation Targeting			Asymmetric Price-Level Targeting		
1. λ	0.00	0.50	1.00	0.00	0.50	1.00
2. ρ_{π}	0.68	1.22	2.11	-	-	-
3. ρ_{Π}	-	-	-	0.01	0.96	0.98
4. ρ_{π^*}	0.99	0.99	0.98	0.94	0.96	0.95
5. $E(u)$	6.55	6.67	7.50	5.87	5.97	6.08
6. $E(\pi)$	-0.03	-0.01	-0.13	0.06	0.08	0.06
7. S.D.(u)	5.29	4.91	4.77	5.64	2.73	2.74
8. S.D.(π)	0.64	0.60	0.57	0.60	0.30	0.30
9. Skewness(u)	1.21	1.82	2.56	0.42	1.67	1.76
10. Skewness(π)	-0.83	-1.34	-2.10	-0.33	-0.81	-0.95
11. $E(p^{\delta})$, pct.	1.33	1.34	1.34	1.33	1.34	1.34
12. Frequency of ZLB., pct.	3.99	5.70	6.23	0.16	6.05	5.92
13. Mean MP. shock, bps.	1.16	3.16	5.74	0.11	61.70	61.45
14. Loss function value	0.45	2.09	3.28	0.00	0.46	0.90
15. Welfare of workers, CE	0.48	0.61	0.22	0.65	0.58	0.54
16. Welfare of shareholders, CE	-1.98	-2.11	-0.89	-2.83	-3.13	-2.95
17. Aggregate welfare, CE	0.36	0.48	0.17	0.47	0.39	0.36

Note: We impose $0.5 \leq \rho_{\pi} \leq 20$, $0.01 \leq \rho_{\Pi} \leq 4$ and $0.0 \leq \rho_{\pi^*} \leq 0.99$ in the optimization. σ_{π^*} is calibrated at 0.01. Mean MP shock is the annualized mean of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint. CE stands for consumption equivalent that needs to be given to the baseline economy to achieve the same level of welfare.

Lines 15 and 16 in Table 5 report the consumption equivalents that need to be given to the agents in the baseline economy to be indifferent between the economy under the optimized policy rules and the baseline economy. As such, positive numbers represent welfare improvement and negative numbers welfare deterioration with respect to the baseline economy.[37] What is notable about the welfare results is that a particular monetary policy rule that improves the welfare of one type of agent may not improve the welfare of the other type of agent. In particular, a rule that improves the welfare of workers in turn deteriorates the welfare of shareholders. Overall, these findings suggest that monetary policy can have distributional consequences, in line with the empirical results of Coibion, Gorodnichenko, Kueng, and Silvia (2017). As for the population weighted welfare (line 17), the price-level targeting rule brings welfare gains as the rule is more effective in controlling deflation bias and unemployment bias.

Finally, Table 6 summarizes the optimization results for the asymmetric inflation and price-level targeting rules. Qualitative features remain the same as in Table 5 except for three features. First, larger welfare gains can be achieved by using a time-varying inflation target during financial crises, and these gains are particularly larger for inflation targeting compared to price-level targeting. This result was expected given that the EPF in Figure 9 moves inward more substantially when the inflation targeting rule is combined with the time-varying inflation target. The symmetric price-level targeting rule already does a good job in correcting for asymmetric business cycles, thus the additional gains

[37] All welfare values are based on a first order approximation. The presence of multiple anticipated news shocks to satisfy the ZLB constraint in expectations creates a large number of auxiliary variables to limit the lag structure of the model to one period. This large number of auxiliary variables are not suitable for second order approximation.

from adopting the asymmetric component are relatively modest.

Second, the optimized coefficients ρ_π and ρ_Π tend to be smaller with the time-varying inflation target. This tendency confirms our conjecture that a mix of a dovish central banker and a time-varying inflation target is likely to be optimal in an economy that suffers from insufficient aggregate demand due to income inequality. The fact that the frequency of a binding ZLB constraint and the mean size of monetary policy shocks required to satisfy the ZLB constraint tend to be lower with the asymmetric policy rules also suggests that such opportunistic adjustment of the inflation target can be an effective tool in fighting the deflation bias imposed by the ZLB constraint.

Third, the population weighted welfare gains indicate that a conservative central banker tends to reduce the size of overall welfare. In particular, the gains for workers generally decline as the weight given to inflation stabilization is increased. Given that wage income is the major income source for low income earners, a crucial factor for their welfare gains is how effective a particular monetary policy rule is in reducing the bias in the unemployment rate.

8 Conclusion

The increase in income inequality in the United States during the recent decades has become a reason for concern. This paper shows the important role that income inequality plays in the macroeconomy. By using a two-agent general equilibrium model, we show that income inequality can generate low aggregate demand, deflation pressure, excessive credit growth and financial instability. The main mechanism to deliver those results is the desire to accumulate financial wealth by shareholders motivated by their "love of wealth" preferences. The paper shows that both occasional eruptions of endogenous financial crises and the zero lower bound constraint on nominal interests generate distributions of equilibrium prices and quantities that are highly skewed to the downside. In that environment, a conservative central banker faces a tradeoff between stabilizing inflation volatility and increasing the mean level of the unemployment rate. This tradeoff highlights the inefficiency of symmetric monetary policy rules in the sense that a simultaneous reduction in inflation volatility and the mean unemployment rate is feasible when an asymmetric policy rule is adopted.

Our results highlight the close link between income inequality and financial instability. While the focus in this paper is on monetary policy issues, our findings suggest an important role for fiscal policy as a macroprudential tool. In particular, redistribution policies that ameliorate income inequality can potentially bring additional benefits in terms of preventing excessive credit growth, reducing the probability of financial crises and further improving macroeconomic outcomes. We leave the study of fiscal and macroprudential policies in our model economy for future research.

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Online Appendix - Not Intended for Publication

A Non-Stochastic Steady State

A.1 Pinning Down the Endogenous Variables

To determine the steady state of the model, we guess the values of the borrowing level b and the default probability p^{δ}. We assume a zero inflation steady state, i.e., $\pi = 1$. Equation (49) then satisfies:

$$1 = \left[(1-\varphi)p_0^{1-\gamma} + \varphi\right]^{1/(1-\gamma)}, \tag{A.1}$$

resulting in $p_0 = 1$. Equations (46)$\sim$(48) imply:

$$p_0 = \frac{\gamma\mu y\pi^{\gamma}/(1-\beta\varphi)}{(\gamma-1)y\pi^{(\gamma-1)}/(1-\beta\varphi)} = 1, \tag{A.2}$$

which results in:

$$\mu = \frac{\gamma-1}{\gamma}. \tag{A.3}$$

In the steady state $q^K = 1$ from equation (50). Then, equation (23) determines:

$$r = 1/\beta^K - (1-\delta). \tag{A.4}$$

Using this in equation (36), we define the steady state output-capital ratio as:

$$\rho_{y/k} \equiv \frac{y}{k} = \frac{1/\beta^K - (1-\delta)}{\mu\alpha}. \tag{A.5}$$

From the production function, we have:

$$\frac{y}{k} = z\left(\frac{n}{k}\right)^{1-\alpha},$$

or equivalently

$$\rho_{n/k} \equiv \frac{n}{k} = \left(\frac{\rho_{y/k}}{z}\right)^{1/(1-\alpha)}. \tag{A.6}$$

Again from the production function, we derive output-labor ratio as:

$$\rho_{y/n} \equiv \frac{y}{n} = z\left(\frac{n}{k}\right)^{-\alpha} = z\rho_{n/k}^{-\alpha}. \tag{A.7}$$

These ratios can be used to determine the levels of output and capital once the level of employment n is determined.

We determine the steady state job market tightness by calibrating the job finding rate from the data, i.e., $p(\theta) = p$. Using equation (A.7), we simplify the expression for the wage as:

$$w = \eta\mu(1-\alpha)\rho_{y/n} + (1-\eta)b^U + (1-\rho)\eta[\beta^K - \beta^W(1-p)]\frac{\xi}{q}. \tag{A.8}$$

Equations (34) and (35) imply in steady state:

$$\frac{\xi}{q} = \mu(1-\alpha)\rho_{y/n} - w + (1-\rho)\beta^K\frac{\xi}{q}. \tag{A.9}$$

Equations (A.8) and (A.9) provide us with two equations for two unknowns, w and ξ/q. Substituting equation (A.8) in equation (A.9) and solving it for ξ/q yields:

$$J = \frac{\xi}{q} = \frac{(1-\eta)\left[\mu(1-\alpha)\rho_{y/n} - b^U\right]}{1-(1-\rho)\beta^K\{1-\eta[1-(1-p)\beta^W/\beta^K]\}}. \tag{A.10}$$

Using this in equation (43) yields the steady state wage.

Since $\xi/q = \xi\theta/p(\theta) = \xi\theta/p$, we have:

$$\theta = \frac{p}{\xi}\left[\frac{(1-\eta)\left[\mu(1-\alpha)\rho_{y/n} - b^{U}\right]}{1-(1-\rho)\beta^{K}\{1-\eta[1-(1-p)\beta^{W}/\beta^{K}]\}}\right]. \tag{A.11}$$

This determines the matching function efficiency as:

$$\zeta = \frac{p}{\theta^{\epsilon}}. \tag{A.12}$$

Note that we pin down ζ endogenously by matching the job finding rate of the model with the data. This requires us to treat ζ as an endogenous variable, which always takes the same value as in the steady state.

From equation (32), we have the steady state employment stock as:

$$n = \frac{q}{\rho}v = \frac{q}{\rho}\theta u. \tag{A.13}$$

Substituting this expression in equation (27) yields:

$$u = 1 - \chi - (1-\rho)\frac{q}{\rho}\theta u.$$

Solving this for u yields

$$u = \frac{1-\chi}{1+(1-\rho)q\theta/\rho}.$$

The vacancy posting is determined as:

$$v = \theta u,$$

and using this in equation (A.13) gives us the steady state employment stock. The levels of capital and output are then given by:

$$y = \rho_{y/n} n, \tag{A.14}$$

and

$$k = \rho_{n/k}^{-1} n. \tag{A.15}$$

In a steady state where a default event does not occur:

$$c^{W} = c^{N} = (q^{B} - 1)\, b + \frac{1}{1-\chi}\left[wn + (1-\chi-n)b^{U}\right], \tag{A.16}$$

and

$$c^{K} = b\,(1-q^{B})\,\frac{1-\chi}{\chi} - q^{K}\frac{\delta k}{\chi} + \frac{rk}{\chi} + \frac{\Pi^{Y}}{\chi} + \frac{\Pi^{K}}{\chi} + \frac{T}{\chi}, \tag{A.17}$$

where

$$\Pi^{Y} = y - wn - rk - \xi v, \tag{A.18}$$

and

$$\Pi^{K} = q^{K}x - x = 0. \tag{A.19}$$

Substituting equations (A.18) and (A.19), the steady state version of the FOC for borrowing given by:

$$q^{B} = \beta^{B}(1 - p^{\delta}h), \tag{A.20}$$

and the balanced budget constraint for government given by:

$$T = -(1-\chi-n)b^{U} + \chi b^{G}\left(1 - \frac{1}{1+i}\right)$$

into equation (A.17), we express the steady state consumption level of shareholders as:

$$c^K(b) = \frac{1}{\chi}\{b[1-\beta_W(1-p^\delta h)](1-\chi) - q^K\delta k + y - wn - \xi v - (1-\chi-n)b^U\}. \tag{A.21}$$

Note that the right-hand side of the above has only one unknown, b. In order to pin down b, we equate the FOCs of the two agents regarding borrowing and lending, which results in

$$(\beta^W - \beta^K)(1-p^\delta h) = \frac{\psi}{[(1-s)c^K(b)]^{-1/\sigma_c}}\left[1 + b\left(\frac{1-\chi}{\chi}\right)\right]^{-1/\sigma_b}. \tag{A.22}$$

Our initial guess b should satisfy equation (A.22) given our guess p^δ. We now discuss how the guess solution p^δ can be validated.

A.2 Probability of Default

We envision a steady state in which there has been no default for a long time such that the lagged value of pecuniary default cost has converged to zero, i.e., $\nu_{-1} = 0$. In such an environment, if a default occurs today, the value of default cost becomes $\nu = \gamma_\nu$. If a default does not occur today, the value of default cost becomes $\nu = 0$. If a default occurs again tomorrow after an episode of default today, $\nu' = (1+\rho_\nu)\gamma_\nu$. If a default does not occur tomorrow after an incidence of default, $\nu' = \rho_\nu\gamma_\nu$. If a default occurs tomorrow after no incidence of default today, $\nu' = \gamma_\nu$. If a default does not occur either today or tomorrow, $\nu' = 0$. Note that all of these four cases are hypothetical in that a default does not occur in our steady state.

We denote four cases of consumption level tomorrow under the four scenarios by c^{DD}, c^{DN}, c^{ND} and c^{NN}. These are given by:

$$c^{DD} = [q^B - (1-h)]b + \frac{1}{1-\chi}\left[wn + (1-\chi-n)b^U - (1+\rho_\nu)\gamma_\nu y\right], \tag{A.23}$$

$$c^{DN} = (q^B - 1)b + \frac{1}{1-\chi}\left[wn + (1-\chi-n)b^U - \rho_\nu\gamma_\nu y\right], \tag{A.24}$$

$$c^{ND} = [q^B - (1-h)]b + \frac{1}{1-\chi}\left[wn + (1-\chi-n)b^U - \gamma_\nu y\right], \tag{A.25}$$

and

$$c^{NN} = (q^B - 1)b + \frac{1}{1-\chi}\left[wn + (1-\chi-n)b^U\right]. \tag{A.26}$$

Note that all right-hand side variables are already determined above under the initial guess solution of p^δ. However, this guess solution needs to be verified.

In the steady state, the continuation values of default and non-default are given by:

$$V^D = \beta^W p^\delta\left(\frac{(c^{DD} - sc^D)^{1-1/\sigma_c}}{1-1/\sigma_c} + V^D\right) + \beta^W(1-p^\delta)\left(\frac{(c^{DN} - sc^D)^{1-1/\sigma_c}}{1-1/\sigma_c} + V^N\right) \tag{A.27}$$

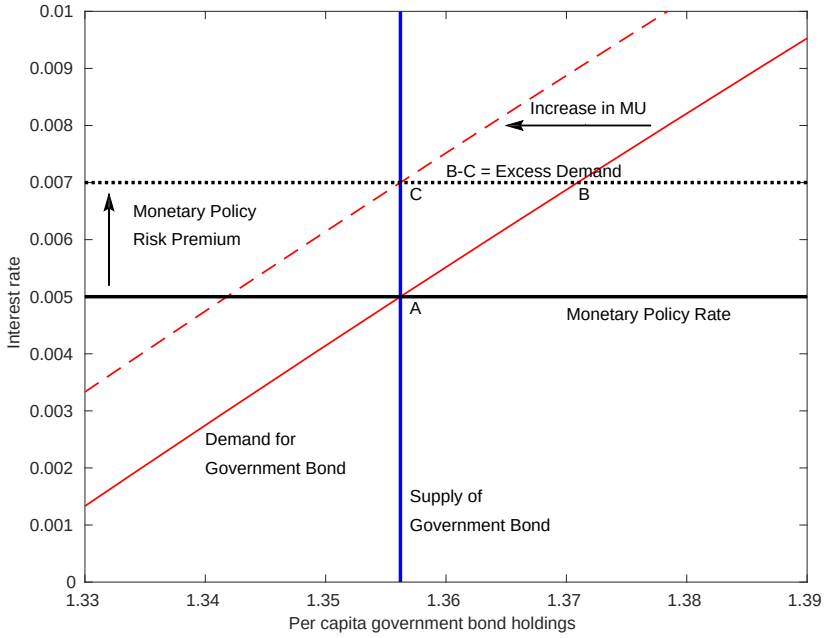
and

$$V^N = \beta^W p^\delta\left(\frac{(c^{ND} - sc^N)^{1-1/\sigma_c}}{1-1/\sigma_c} + V^D\right) + \beta^W(1-p^\delta)\left(\frac{(c^{NN} - sc^N)^{1-1/\sigma_c}}{1-1/\sigma_c} + V^N\right). \tag{A.28}$$

By subtracting equation (A.27) from equation (A.28), one can derive $\Delta V(p^\delta)$ as:

$$\begin{aligned}\Delta V(p^\delta) &= V^D - V^N \\ &= \beta^W p^\delta\left(\frac{(c^{DD} - sc^D)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c^{ND} - sc^N)^{1-1/\sigma_c}}{1-1/\sigma_c}\right) \\ &\quad + \beta^W(1-p^\delta)\left(\frac{(c^{DN} - sc^D)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c^{NN} - sc^N)^{1-1/\sigma_c}}{1-1/\sigma_c}\right).\end{aligned}$$

Figure 10: Monetary Policy and Government Bond Market Clearing



Hence $\Delta U(p^\delta)$ is given by:

$$\Delta U(p^\delta) = \frac{(c^D - sc^W)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c^N - sc^W)^{1-1/\sigma_c}}{1-1/\sigma_c} + \Delta V(p^\delta).$$

Note that we express ΔU and ΔV functions of our initial guess p^δ. The default probability is then given by:

$$p^\delta = \frac{\varrho}{1+\exp(-\varsigma \Delta U(p^\delta))} \tag{A.29}$$

Equation (A.29) can then be viewed as a fixed point problem. Hence, the steady state needs to be pinned down by the simultaneous equations (A.22) and (A.29). Once this is done, the consumption level of shareholders can be determined by equation (A.21). Using this consumption level, we can then evaluate the marginal utility of shareholders and then finally determine the level of government debt consistent with the calibrated risk-free bond rate i:

$$b^G = \left\{ \frac{\Lambda^K}{\psi^G} \left[\frac{1}{(1+i^*)\phi} - \beta^K \right] \right\}^{-\sigma_g} - 1. \tag{A.30}$$

A.3 Monetary Policy Transmission

Equation (A.30) describes the shape of demand for government bonds. Figure 10 shows that the demand for government bonds is an increasing function of the risk-free rate i^* and the risk premium ϕ. Since the supply of government bonds is fixed, the equilibrium interest rate is determined as the intersection of the upward sloping demand curve and the vertical supply curve as marked by point A in Figure 10. When either the policy rate is raised or the risk premium is elevated, the horizontal line shifts from the black solid line to the black dotted line. Given that the supply of government bonds is fixed, this generates excess demand for government bonds, marked by B-C. To clear the market, the marginal utility of shareholders Λ^K has to increase, i.e., the consumption level of shareholders has to decline such that the demand curve moves to the left as indicated by the red dashed line. Thus the new equilibrium is found at point C.

B Solution Method

The model is inherently nonlinear for two reasons: the ZLB constraint and the binary nature of default. However, given the large number of state variables, we simulate the model piecewise linearly. This section illustrates how such simulations can be executed. Suppose that we solve the model linearly, ignoring the ZLB constraint and default. In our model economy, $\delta_t^B \in \{0,1\}$, has both endogenous and exogenous natures. δ_t^B is endogenous in the sense that the probability of $\delta_t^B = 1$ is endogenously determined. However, given the probability of default, whether or not the default occurs is stochastic. In this latter sense, δ_t^B can be viewed as an exogenous variable.

Step 1. We linearize the model around the non-stochastic steady state in which we assume that the economy has not experienced a crisis for a long period of time and the default cost ν has converged to zero. We can express the state space representation for the system of equations as:

$$\begin{bmatrix} \mathbf{s}_t \\ \mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{s}_{t-1} + \mathbf{B}\boldsymbol{\epsilon}_t \\ \mathbf{C}\mathbf{s}_t \end{bmatrix}, \boldsymbol{\epsilon}_t = \begin{bmatrix} \boldsymbol{\epsilon}_t^{EX} \\ \boldsymbol{\epsilon}_t^{EN} \end{bmatrix} \tag{B.1}$$

and

$$\begin{aligned} \boldsymbol{\epsilon}_t^{EX} &= \begin{bmatrix} \epsilon_{z,t} & \epsilon_{\eta,t} & \epsilon_{\phi,t} \end{bmatrix}' \\ \boldsymbol{\epsilon}_t^{EN} &= \begin{bmatrix} \delta_t^B & \epsilon_{0,t} & \epsilon_{1,t} & \cdots & \epsilon_{n,t} \end{bmatrix}' \end{aligned} \tag{B.2}$$

where $\mathbf{s}_t$ and $\mathbf{x}_t$ are state and policy vectors with dimension Note that $\boldsymbol{\epsilon}_t^{EX}$ follows Gaussian distributions. Although $\boldsymbol{\epsilon}_t^{EN}$can be viewed as 'shocks' from a mechanical point of view, we treat $\boldsymbol{\epsilon}_t^{EN}$ as endogenous variables since the sizes of the shocks should be determined endogenously. In particular, the contemporaneous and anticipated news shocks to monetary policy should satisfy

$$\epsilon_{k,t} = \max\left\{0, -\left[i^* + \rho_\pi\left(\frac{\pi_t^Y - \pi^*}{4}\right)\right] - \sum\nolimits_{j=k+1}^{n} \epsilon_{j,t-j}\right\}, \tag{B.3}$$

for $k = 0, ..., n$. In words, we use the current monetary policy shock ($k = 0$) to satisfy the ZLB today and the news shocks ($k > 0$) to satisfy the ZLB in expectations. Note that $\mathbb{E}_t[i_{t+j-1}]$ and $\mathbb{E}_t[\pi_{t+j}]$ for $j = 1, ..., n$ should be consistent with $\epsilon_{0,t}$ and $\epsilon_{j,t}$ in the sense that when $\epsilon_{0,t}$ and $\epsilon_{j,t}$ are given to the economy, the agents' expectations about $\mathbb{E}_t[i_{t+j-1}]$ and $\mathbb{E}_t[\pi_{t+j}]$ for $j = 1, ..., n$ should be identical with those showing up on the right hand sides of equation (B.3).

Step 2. In each period, we simulate the economy with random draws for $\boldsymbol{\epsilon}_t^{EX}$ under the assumption that $\boldsymbol{\epsilon}_t^{EN} = 0$. The system of equations will return the default probability

$$p_t^{\delta(0)} = \Xi(\Delta U_t^W) = \frac{\varrho}{1 + \exp(-\varsigma \Delta U_t)},$$

and the endogenous component of monetary policy

$$i_t^{(0)} = i^* + \rho_\pi\left(\frac{\pi_t^Y - \pi^*}{4}\right).$$

Step 3. We then need to check if a default occurs and/or the ZLB is violated. To see if the ZLB is violated or not, we simply need to check if i_t is negative. We also need to check whether or not $\mathbb{E}_t[i_{t+j}] \geq 0$ is satisfied. A default can be detected in the following way. Take a random draw from a uniform distribution over a support $[0, 1]$. If this draw is strictly greater than $p_t^{\delta(0)}$, we conclude that a default did not occur. If the draw is strictly less than $p_t^{\delta(0)}$, we conclude that a default did occur. There can be four cases: (i) no default, no ZLB violation; (ii) default, no ZLB violation; (iii) no default, ZLB violation; (iv) default, ZLB violation.

Case (i). Since there is no default and the ZLB is not violated, we move to the next period.

Case (ii). Since a default occurs, we simulate the economy again with $\delta_t^B = 1$ and check if no ZLB violation is still valid both in current period and in expectations. If the ZLB constraint binds or is expected to bind in future, then move to Case (iv). Otherwise we move to the next period.

Case (iii). Since the ZLB constraint is violated ($i_t^{(0)} < 0$), where the superscript (0) indicates the number of iterations in the ZLB loop, we set $\epsilon_{0,t}^{(1)} = -i_t^{(0)}$ $\epsilon_{j,t}^{(1)} = \max\{0, -\mathbb{E}_t[i_{t+j}^{(0)}]\}$ to satisfy $\mathbb{E}_t[i_{t+j}^{(1)}] \geq 0$. We then simulate the economy again and check if the ZLB in current period or in expectations is satisfied. This process cannot be done in one step because once the monetary policy shock or news shock is given to the system, the economy reacts to the shocks, and the initial amounts of monetary policy shocks to satisfy the ZLB may not be enough. If the constraint is still not satisfied, we set $\epsilon_{0,t}^{(2)} = -i_t^{(0)} - i_t^{(1)}$ and $\epsilon_{j,t}^{(2)} = -\mathbb{E}_t[i_{t+j}^{(0)}] - \mathbb{E}_t[i_{t+j}^{(1)}]$, and simulate the economy again and check if the ZLB is satisfied in current or in future expectations. If the ZLB is finally satisfied after S simulations, the final level of the monetary policy shock is given by:

$$\epsilon_{0,t}^{(S)} = -\sum_{s=0}^{S} i_t^{(s)} \text{ and } \epsilon_{j,t}^{(S)} = -\sum_{s=0}^{S} \mathbb{E}_t[i_{t+j}^{(s)}] \text{ for } j = 1, \dots N. \tag{B.4}$$

An important thing in this iteration is to check whether or not a default occurs owing to the endogenous reaction of the economy to the monetary policy shock to satisfy the ZLB for a given simulation number j. If a default occurs for this reason, then move to Case (iv). If not, move to the next period.

Case (iv). In this case a crisis occurs and the ZLB binds. Hence we set $\delta_t^B = 1$ and determine the size of the monetary policy shock according to equation (B.4). For a given simulation number j, we need to check if a crisis occurs or not. In principle, it is logically possible, although not likely, that the economy gets itself out of the default mode due to the monetary policy shock. If this happens, move to Case (iii).

C System of Equations

There are variables 50 endogenous variables within the system:

$$X_t \equiv \begin{bmatrix} c_t^K & b_t & l_t & c_t & \pi_t & q_t^B & i_t & q_t^K & k_t & r_t \\ T_t & \Lambda_t^K & \Lambda_t^W & w_t & n_t & \nu_t & y_t & c_t^W & m_{t,t+1}^K & \Delta U_t^W \\ p_t^\delta & v_t & u_t & \theta_t & p(\theta_t) & q(\theta_t) & z_t & m_{t,t+1}^W & J_t & \mu_t \\ \underline{w}_t & W_t & \eta_t & p_{0,t} & \mathcal{P}_t^N & \mathcal{P}_t^D & x_t & \phi_t & \Gamma_t^W & \Delta_t \\ c_t^D & c_t^N & \Pi_t^Y & \Pi_t^K & c_t^{DD} & c_t^{DN} & c_t^{ND} & c_t^{NN} & \Omega_t & \Gamma_t^J \end{bmatrix}$$

The following provides the complete list of 50 equations for 50 endogenous variables:

$$l_t = (1 - h\delta_t^B)\frac{b_{t-1}}{\pi_t}, \tag{C.1}$$

$$c_t = \chi c_t^K + (1 - \chi)c_t^W, \tag{C.2}$$

$$T_t = (1 - \chi - n_t)b^U + \chi\left(\frac{b_{t-1}^G}{\pi_t} - \frac{b_t^G}{1 + i_t}\right), \tag{C.3}$$

$$\Lambda_t^K = (c_t^K - sc_{t-1}^K)^{-1/\sigma_c}, \tag{C.4}$$

$$m_{t,t+1}^K = \beta^K \frac{\Lambda_{t+1}^K}{\Lambda_t^K}, \tag{C.5}$$

$$1 = \mathbb{E}_t\left[m_{t,t+1}^K\left(\frac{r_{t+1} + (1 - \delta)q_{t+1}^K}{q_t^K \phi_t}\right)\right], \tag{C.6}$$

$$\frac{1}{1 + i_t} = \mathbb{E}_t\left[m_{t,t+1}^K \frac{\phi_t}{\pi_{t+1}}\right] + \frac{\psi^G}{\Lambda_t^K}(1 + b_t^G)^{-1/\sigma_g}, \tag{C.7}$$

$$q_t^B = \mathbb{E}_t\left[m_{t,t+1}^K(1-hp_{t+1}^\delta)\frac{1}{\pi_{t+1}}\right] + \frac{\psi^B}{\Lambda_t^K}\left[1+b_t\left(\frac{1-\chi}{\chi}\right)\right]^{-1/\sigma_b}, \tag{C.8}$$

$$q_t^B = \mathbb{E}_t\left[m_{t,t+1}^W(1-hp_{t+1}^\delta)\frac{1}{\pi_{t+1}}\right], \tag{C.9}$$

$$c_t^W = q_t^B b_t - l_t + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - \nu_t y_t\right], \tag{C.10}$$

$$\nu_t = \rho_\nu \nu_{t-1} + \gamma_\nu \delta_t^B, \tag{C.11}$$

$$\Lambda_t^W = (c_t^W - sc_{t-1}^W)^{-1/\sigma_c}, \tag{C.12}$$

$$m_{t,t+1}^W = \beta^W \frac{\Lambda_{t+1}^W}{\Lambda_t^W}, \tag{C.13}$$

$$\begin{aligned}\Delta U_t = {} & \frac{(c_t^D - sc_{t-1}^W)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c_t^N - sc_{t-1}^W)^{1-1/\sigma_c}}{1-1/\sigma_c} \\ & +\beta^W \mathbb{E}_t\left[p_{t+1}^\delta\left(\frac{(c_{t+1}^{DD} - sc_t^D)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c_{t+1}^{ND} - sc_t^N)^{1-1/\sigma_c}}{1-1/\sigma_c}\right)\right] \\ & +\beta^W \mathbb{E}_t\left[(1-p_{t+1}^\delta)\left(\frac{(c_{t+1}^{DN} - sc_t^D)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c_{t+1}^{NN} - sc_t^N)^{1-1/\sigma_c}}{1-1/\sigma_c}\right)\right],\end{aligned} \tag{C.14}$$

$$c_t^D = q_t^B b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - (\rho_\nu \nu_{t-1} + \gamma_\nu)y_t\right], \tag{C.15}$$

$$c_t^N = q_t^B b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu \nu_{t-1} y_t\right], \tag{C.16}$$

$$c_t^{DD} = q_t^B b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - [\rho_\nu(\rho_\nu \nu_{t-2} + \gamma_\nu) + \gamma_\nu]y_t\right], \tag{C.17}$$

$$c_t^{DN} = q_t^B b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu(\rho_\nu \nu_{t-2} + \gamma_\nu)y_t\right], \tag{C.18}$$

$$c_t^{ND} = q_t^B b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - (\rho_\nu \rho_\nu \nu_{t-2} + \gamma_\nu)y_t\right], \tag{C.19}$$

$$c_t^{NN} = q_t^B b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu \rho_\nu \nu_{t-2} y_t\right], \tag{C.20}$$

$$p_t^\delta = \frac{\varrho}{1+\exp(-\varsigma \Delta U_t)}, \tag{C.21}$$

$$u_t = 1 - \chi - (1-\rho)n_{t-1}, \tag{C.22}$$

$$p(\theta_t) = \zeta \theta_t^\epsilon, \tag{C.23}$$

$$q(\theta_t) = \zeta \theta_t^{\epsilon-1}, \tag{C.24}$$

$$\frac{\xi}{q(\theta_t)} = J_t, \tag{C.25}$$

$$J_t = \mu_t(1-\alpha)\frac{y_t}{n_t} - w_t - \frac{\vartheta}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} + (1-\rho)\mathbb{E}_t[m^K_{t,t+1}J_{t+1}], \tag{C.26}$$

$$\Gamma^W_t = -1, \tag{C.27}$$

$$\Gamma^J_t = -1 - \vartheta\left(\frac{w_t}{w_{t-1}} - 1\right) + (1-\rho)\mathbb{E}_t\left\{m^K_{t,t+1}\frac{\vartheta}{2}\left[\left(\frac{w_{t+1}}{w_t}\right)^2 - 1\right]\right\}, \tag{C.28}$$

$$\Omega_t = \frac{\eta_t}{\eta_t + (1-\eta_t)\Gamma^J_t/\Gamma^W_t}, \tag{C.29}$$

$$W_t = \frac{\Omega_t}{1-\Omega_t}J_t, \tag{C.30}$$

$$\underline{w}_t = b^U + (1-\rho)\mathbb{E}_t\left[m^W_{t,t+1}p(\theta_{t+1})\frac{\Omega_{t+1}}{1-\Omega_{t+1}}J_{t+1}\right], \tag{C.31}$$

$$\begin{aligned} w_t = {} & \Omega_t\mu_t(1-\alpha)\frac{y_t}{n_t} + (1-\Omega_t)b^U - \Omega_t\frac{\vartheta}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} \\ & + (1-\rho)\mathbb{E}_t\left[\left(\Omega_t m^K_{t,t+1} - (1-\Omega_t)m^W_{t,t+1}(1-p(\theta_{t+1})\frac{\Omega_{t+1}}{1-\Omega_{t+1}}\right)\frac{\xi}{q(\theta_{t+1})}\right], \end{aligned} \tag{C.32}$$

$$\mathcal{P}^N_t = \pi_t^{(1-\varepsilon)\gamma}\left\{\pi_t^{\varepsilon\gamma}\gamma\mu_t y_t + \varphi\mathbb{E}_t\left[m^K_{t,t+1}\mathcal{P}^N_{t+1}\right]\right\}, \tag{C.33}$$

$$\mathcal{P}^D_t = \pi_t^{(1-\varepsilon)(\gamma-1)}\left\{\pi_t^{\varepsilon(\gamma-1)}(\gamma-1)y_t + \varphi\mathbb{E}_t\left[m^K_{t,t+1}\mathcal{P}^D_{t+1}\right]\right\}, \tag{C.34}$$

$$p_{0,t} = \frac{\mathcal{P}^N_t}{\mathcal{P}^D_t}, \tag{C.35}$$

$$\pi_t = \left[(1-\varphi)p_{0,t}^{1-\gamma} + \varphi\pi_{t-1}^{\varepsilon(1-\gamma)}\right]^{1/(1-\gamma)}, \tag{C.36}$$

$$q^K_t = 1 + \kappa\left(\frac{x_t}{x_{t-1}} - 1\right) - \mathbb{E}_t\left\{m^K_{t,t+1}\frac{\kappa}{2}\left[\left(\frac{x_{t+1}}{x_t}\right)^2 - 1\right]\right\}, \tag{C.37}$$

$$i_t = i^* + \rho_\pi\left(\frac{\pi_t\pi_{t-1}\pi_{t-2}\pi_{t-3} - \pi^*}{4}\right) + \sum_{j=0}^{n}\epsilon_{j,t-j}, \tag{C.38}$$

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_{z,t}, \tag{C.39}$$

$$\log \eta_t = (1-\rho_\eta)\log\eta + \rho_\eta \log\eta_{t-1} + \epsilon_{\eta,t}, \tag{C.40}$$

$$\log \phi_t = \rho_\phi \log\phi_{t-1} + \epsilon_{\phi,t}, \tag{C.41}$$

$$y_t = \Delta_t^{-1} z_t k_{t-1}^{\alpha} n_t^{1-\alpha}, \tag{C.42}$$

$$\Delta_t = \pi_t^{\gamma}[(1-\varphi)p_{0,t}^{-\gamma} + \varphi\pi_{t-1}^{-v\gamma}], \tag{C.43}$$

$$n_t = (1-\rho)n_{t-1} + q(\theta_t)v_t, \tag{C.44}$$

$$k_t = (1-\delta)k_{t-1} + x_t, \tag{C.45}$$

$$r_t = \mu_t \alpha \frac{y_t}{k_t}, \tag{C.46}$$

$$\theta_t = \frac{v_t}{u_t}, \tag{C.47}$$

$$\Pi_t^Y = y_t - w_t n_t - \frac{\vartheta}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} n_t - r_t k_{t-1} - \xi v_t, \tag{C.48}$$

$$\Pi_t^K = q_t^K x_t - \left[x_t + \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2 x_{t-1}\right], \tag{C.49}$$

$$c_t^K = \frac{b_{t-1}^G}{\pi_t} - \frac{b_t^G}{1+i_t} + (l_t - q_t^B b_t)\frac{1-\chi}{\chi} - \frac{1}{\chi}\left\{q_t^K\left[k_t - (1-\delta)k_{t-1}\right] + r_t k_{t-1} + \Pi_t^Y + \Pi_t^K - T_t + \nu_t y_t\right\}. \tag{C.50}$$

D Additional Results

D.1 Replication of Paul (2017)

In what sense can the moments shown in Table 3 can be considered representative of reality? To answer this question, we replicate the empirical exercise of Paul (2017): we test the predictive power of credit in forecasting the probability of financial crises, with and without controlling for income inequality using our simulated data. Specifically, we replicate Paul (2017)'s early warning signal model using the binary data of crisis indicator during our stochastic simulation. The logit model is specified as:

$$\log\left(\frac{P[\delta_t^B = 1|\cdot]}{P[\delta_t^B = 0|\cdot]}\right) = \alpha_k + \beta_1 Z_{t-4} + \beta_2 X_{t-4} + u_t$$

where Z_{t-4} is the income inequality and X_{t-4} is other control variables. The following table shows the estimation results using our simulation data.

Table 7: Replicating the Logit Estimation Using Simulated Data

	(1)	(2)	(3)
Credit_{t-4}	1.342**	-	1.007*
	(0.5927)	-	(0.597)
	[0.017]		[0.013]
$\text{Shareholders Income Share}_{t-4}$	-	2.349***	2.111**
	-	(0.774)	(0.780)
		[0.030]	[0.027]

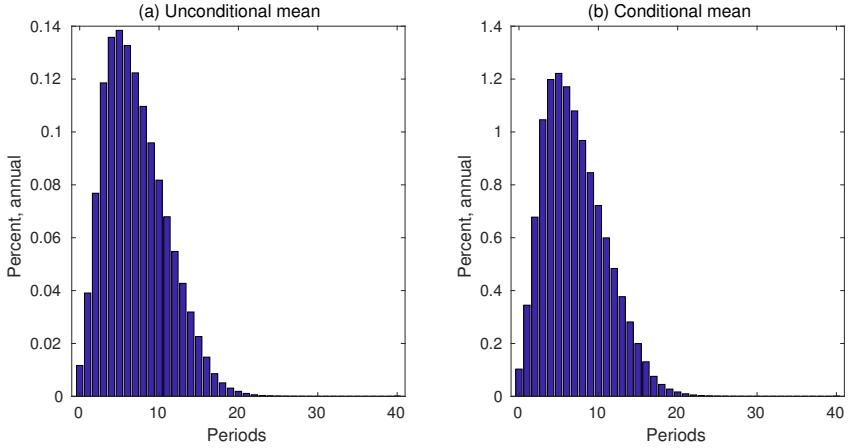
Note: Parentheses are for standard errors and square brackets are for marginal effects.

As in Paul (2017), the credit variable predicts financial crises in our simulated economy as reported in column (1). However, column (2) shows that the share of shareholders predicts financial crises as well. Finally, column (3) reports that once we control for income inequality, the credit variable loses its predictive power. These are the stylized facts emphasized by Paul (2017) and our model can replicates them quite well. In particular, the marginal effects of credit and income inequality shown in squared brackets are quite comparable to those reported in his study.

D.2 Monetary Policy Shocks to Satisfy the ZLB Constraint

Figure 11 shows the annualized unconditional and conditional means of monetary policy shocks (both contemporaneous and anticipated news shocks) that are used to satisfy the ZLB constraint in our baseline economy. The former measures unconditional tightening effects due to the ZLB constraint. As such, the sum of unconditional means represents a constant deflationary force facing the economy due to the ZLB constraint for any given period. The latter measures tightening effects conditioned upon any of 41 shocks being positive due to either a presently binding ZLB constraint or the past expectations that the constraint would be binding in the near term. As such, the sum of the conditional means represents a typical magnitude of tightening during a binding ZLB episode.

Figure 11: Means of Monetary Policy Shocks to Satisfy the ZLB Constraint

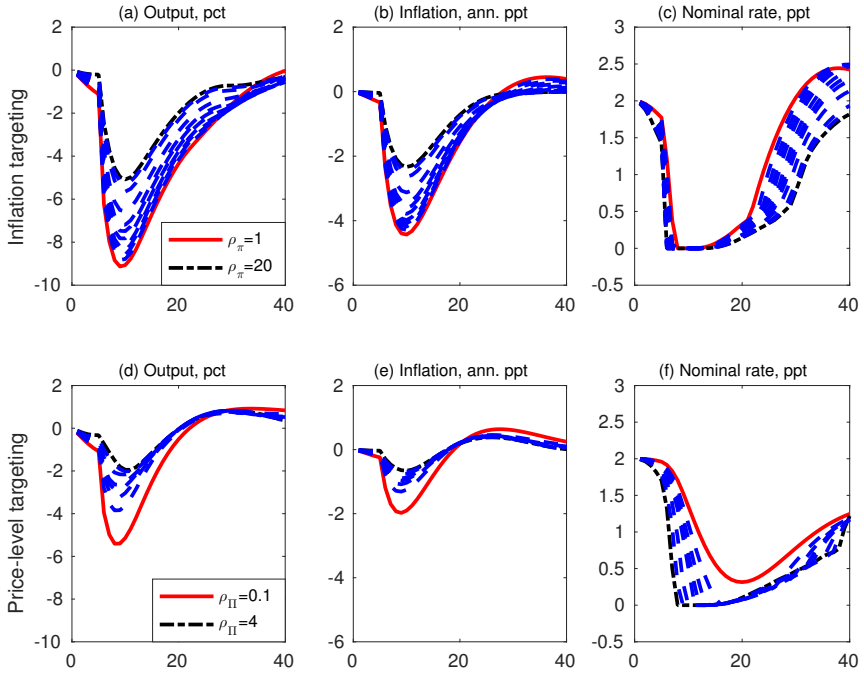


Notes: The figure shows the annualized unconditional and conditional means of monetary shocks used to satisfy the ZLB constraint in our baseline economy with financial crises and the ZLB constraint. The period marked as 0 on the horizontal axis corresponds to contemporaneous shocks. The rest of the periods show the sizes of 40 anticipated news shocks at each horizon.

D.3 Monetary Accomodation During Financial Crises

Figure 12 replicates the experiment shown in Figure 3 when both financial crises and the ZLB constraint are present for ranges of inflation targeting rules between 1 and 20 (upper three panels) and price-level targeting rules between 0.1 and 4 (lower three panels) and shows the reactions of aggregate output, the inflation rate and the path of the policy rate. The results shown in Figure 12 appear to suggest that for both inflation targeting and price-level targeting rules, the more hawkish the central banker is, the more monetary accommodation she provides to the economy during a financial crisis.

Figure 12: Monetary Policy Accommodation During a Financial Crisis



Notes: We use the same sequence of adverse risk-premium shocks as in Figure 3 when both financial crises and the ZLB constraint are present for different values of the inflation reaction coefficient ρ_π in the top panels and for different reaction coefficient for price-level deviation ρ_Π in the bottom panels.

D.4 Distributions of Key Endogenous Variables under Optimized Rules

Figures 13 and 14 show the exact shapes of the distributions for inflation and output in deviations from their non-stochastic steady state under the optimized symmetric and asymmetric rules, respectively.

Figure 13: Distributions of Key Endogenous Variables under Optimized Inflation and Price-Level Targeting Rules with $\lambda = 0.5$

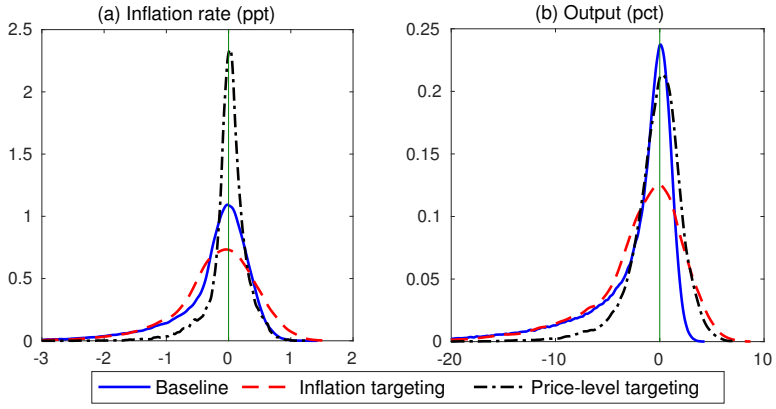
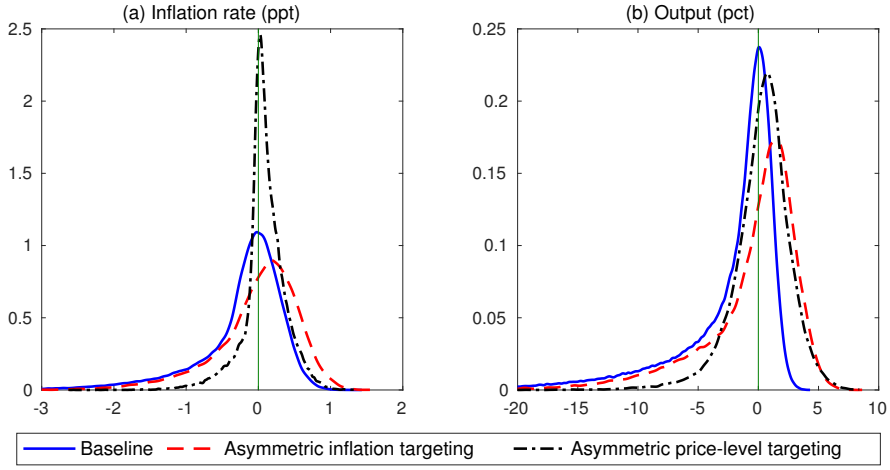


Figure 14: Distributions of Key Endogenous Variables under Optimized Asymmetric Inflation and Price-Level Targeting Rules with $\lambda = 0.5$



D.5 Alternative EPF

The EPFs shown in Figure 15 assume the following alternative loss function:

$$\min \lambda\{\mathbb{E}[\pi - \mathbb{E}(\pi)]^2\}^{1/2} + (1-\lambda)\mathbb{E}(u), \qquad \lambda \in [0,1], \tag{D.1}$$

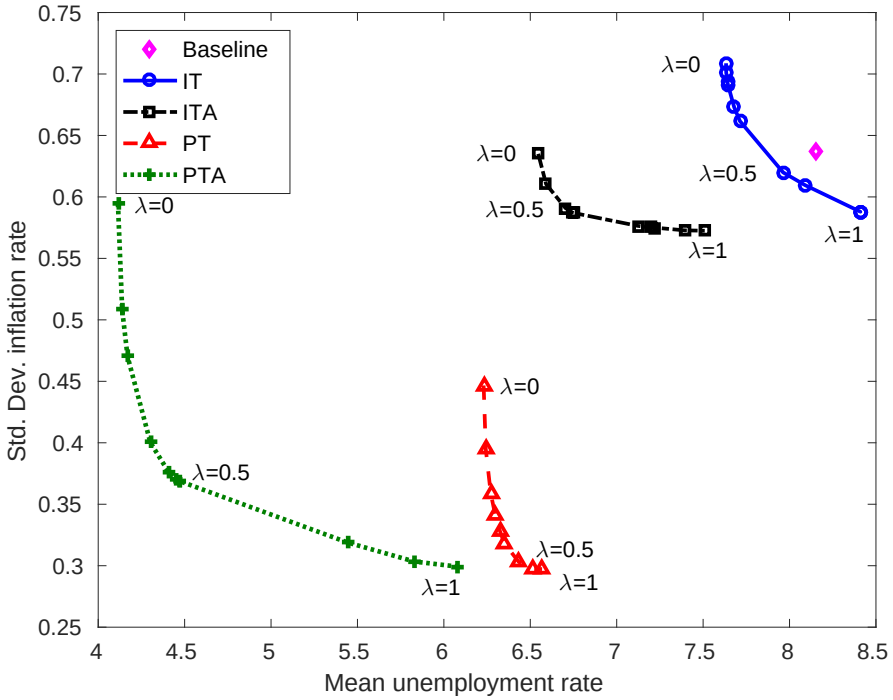
in which the central bank penalizes the *level* of the unemployment rate in contrast to the bias minimization shown in the main text. This can be considered James Tobin's loss function.[38] Roughly speaking, the loss function (59) in the main text works like a constrained optimization of equation (D.1), i.e.,

$$\min \lambda\{\mathbb{E}[\pi - \mathbb{E}(\pi)]^2\}^{1/2} + (1-\lambda)\mathbb{E}(u) \text{ subject to } \mathbb{E}(u) \geq u^*.$$

[38] Joseph E. Stiglitz is known to have said, "I did a paper where I analysed the optimal unemployment rate...Tobin went livid over the idea. To him the optimal unemployment rate was zero" (http://www.bbc.com/news/magazine-15276765).

This is the reason why the asymmetric price-level targeting in Figure 9 appears as a vertical line at $\mathbb{E}(u) = u^*$ as the constraint starts binding at this level of the mean unemployment rate. Finally, Tables 8 and 9 report the optimized coefficients and key moments under both symmetric and asymmetric inflation and price-level targeting for selected values of λ.

Figure 15: Efficient Policy Frontiers under Alternative Loss Function



Note: IT, ITA, PT and PTA stand for inflation targeting, asymmetric inflation targeting, price-level targeting and asymmetric price-level targeting, respectively. The diamond bullet locates the moments of the baseline.

Table 8: Optimized Rules under Alternative Loss Function:
Inflation Targeting vs. Price-Level Targeting

	Inflation Targeting			Price-Level Targeting		
1. λ	0.00	0.50	1.00	0.00	0.50	1.00
2. ρ_π	0.50	0.77	12.51	-	-	-
3. ρ_Π	-	-	-	0.04	0.40	3.87
4. ρ_{π^*}	-	-	-	-	-	-
5. $E(u)$	7.63	7.72	8.41	6.23	6.40	6.57
6. $E(\pi)$	-0.18	-0.19	-0.25	0.00	0.00	0.00
7. S.D.(u)	5.91	5.44	5.09	4.28	2.95	2.81
8. S.D.(π)	0.71	0.66	0.59	0.44	0.31	0.30
9. Sewness(u)	1.02	1.54	3.44	0.76	1.95	2.12
10. Skewness(π)	-0.76	-1.14	-3.40	-0.61	-1.50	-1.54
11. $E(p^\delta)$, pct.	1.32	1.33	1.34	1.33	1.33	1.34
12. Frequency of ZLB., pct.	2.08	3.42	9.74	0.77	4.02	9.35
13. Mean MP. shock, bps.	0.37	2.58	30.74	0.72	26.44	178.78
14. Loss function value	3.10	3.88	3.46	0.13	0.61	0.88
15. Welfare of workers, CE	0.01	0.01	-0.03	0.38	0.37	0.38
16. Welfare of shareholders, CE	-0.29	-0.24	0.47	-2.23	-2.24	-2.23
17. Aggregate welfare, CE	-0.01	-0.01	-0.01	0.25	0.24	0.25

Note: We impose $0.5 \leq \rho_\pi \leq 20$ and $0.01 \leq \rho_\Pi \leq 4$ in the optimization, while ρ_{π^*} and σ_{π^*} are set equal to zero. Mean MP shock is the annualized mean of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint. CE stands for consumption equivalent that needs to be given to the baseline economy to achieve the same level of welfare.

Table 9: Optimized Rules under Alternative Loss Function:
Asymmetric Inflation Targeting vs. Asymmetric Price-Level Targeting

	Asymmetric Inflation Targeting			Asymmetric Price-Level Targeting		
1. λ	0.00	0.50	1.00	0.00	0.50	1.00
2. ρ_π	0.70	1.36	2.18	-	-	-
3. ρ_Π				0.01	0.43	1.05
4. ρ_{π^*}	0.99	0.99	0.98	0.99	0.99	0.95
5. $E(u)$	6.55	6.70	7.51	4.11	4.45	6.08
6. $E(\pi)$	-0.02	-0.01	-0.13	0.32	0.32	0.06
7. S.D.(u)	5.26	4.87	4.77	5.45	3.07	2.74
8. S.D.(π)	0.64	0.59	0.57	0.59	0.37	0.30
9. Skewness(u)	1.24	1.93	2.59	0.34	0.92	1.76
10. Skewness(π)	-0.86	-1.44	-2.14	-0.18	-0.03	-0.95
11. $E(p^\delta)$, pct.	1.33	1.34	1.34	1.34	1.35	1.34
12. Frequency of ZLB., pct.	4.07	5.96	6.32	0.69	5.98	6.10
13. Mean MP. shock, bps.	1.24	3.67	6.00	0.77	39.92	66.24
14. Loss function value	6.55	6.31	5.73	4.11	4.08	2.99
15. Welfare of workers, CE	0.49	0.63	0.23	1.70	1.58	0.54
16. Welfare of shareholders, CE	-1.99	-2.11	-0.89	-5.61	-5.71	-2.95
17. Aggregate welfare, CE	0.37	0.50	0.17	1.33	1.22	0.36

Note: We impose $0.5 \leq \rho_\pi \leq 20$, $0.01 \leq \rho_\Pi \leq 4$ and $0.0 \leq \rho_{\pi^*} \leq 0.99$ in the optimization. σ_{π^*} is calibrated at 0.01. Mean MP shock is the annualized mean of monetary policy shocks (both contemporaneous and anticipated news shocks) required to satisfy the ZLB constraint. CE stands for consumption equivalent that needs to be given to the baseline economy to achieve the same level of welfare.