Money Markets, Collateral and Monetary Policy*

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Abstract

We analyze the impact of money market frictions on the macroeconomy and on the conduct of monetary policy. We focus on two key developments in European money markets: i) declining activity in the unsecured market segment, and ii) increased exposure to secured funding and to fluctuations in collateral value. We build a general equilibrium model with secured and unsecured money markets, and a central bank that can conduct open market operations as well as lend to banks against collateral. We find that reduced access to the unsecured market leads to moderate output contractions as long as banks can substitute into secured funding. If secured money market funding is limited, due to high haircuts or scarcity of collateral assets, output contractions can be substantial. A central bank that expands the size of its balance sheet is able to mitigate such adverse impact. A policy of QE that aims at stabilizing inflation is more effective than a policy of unlimited liquidity provision against collateral.

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1 Introduction

Money markets are essential to banks' liquidity management. They also play a key role in the transmission and implementation of monetary policy. These markets have undergone substantial changes over the past fifteen years. Perhaps the most striking change has been a declining importance of the unsecured money market segment compared to the secured money market segment (see Figure 1). While the total turnover was split about equally between unsecured and secured market segments in 2003, turnover in the unsecured market was just one tenth of total by 2015. Increased reliance on the secured funding shifts banks' asset composition away from lending and towards assets that can be used as collateral and, in turn, exposes banks to fluctuations in the value and the supply of collateral assets.

What is the macroeconomic impact of a decline in access to the unsecured money market segment? What is the impact of changes in the availability of secured money market funding due to haircut changes and scarcity of safe collateral assets? How should monetary policy react to the evolving money market landscape?

To answer these questions, we develop a general equilibrium model featuring heterogeneous banks, interbank money markets for both secured and unsecured credit, and a central bank that can conduct open market operations as well as lend to banks against collateral. In the calibrated model, we find that the decline in unsecured money market transactions leads to moderate contractions in lending and output as long as banks can substitute unsecured funding with secured funding. However, when secured funding is also limited, due to high haircuts or safe asset scarcity, contractions can be substantial. We analyze alternative central bank policies and find that policies that expand the size of the central bank balance sheet are able to mitigate such adverse impact. Furthermore, a policy of asset purchases that aims at stabilizing inflation is more effective than a policy of unlimited liquidity supply at a fixed interest rate and against collateral (reminiscent of the fixed rate full allotment liquidity policy implemented by the ECB since 2008).

Our modelling ingredients aim to capture two key developments that have changed the functioning of European money markets over the past fifteen years: declining activity in the unsecured money market segment and increasing recourse to secured market funding which exposes banks to the risk of fluctuations in collateral value.

The first development, a protracted decline in unsecured money market transactions, is documented in Figure 1. The decline started several years before the Global Financial crisis,

and further steepened with the onset of the financial and sovereign debt crisis in the euro area. Importantly, unsecured market activity can be expected to remain at low levels for years to come, for two reasons. First, unsecured funding may be less attractive due to new Basel III liquidity regulations, in particular the Liquidity Coverage Ratio, which applies a 100 percent run-off rate for unsecured short-term money market funding and requires banks to hold large liquidity buffers against such funding (Bucalossi et al. (2016)). Second, secured funding has become more attractive due to the growing importance of central counterparty (CCP) clearing which reduces counterparty risk, enables collateral savings, and defines quality standards for clearing members and for accepted collateral. Indeed, more than 65% of secured money market transactions were cleared by CCPs in 2015 (see Figure 2). By modelling both unsecured and secured money markets, we can assess consequences of the shift away from the unsecured and towards secured money markets.

The second key development is the increased recourse to the secured money market segment which has exposed banks to the risk of fluctuations in collateral value. Collateral value reductions occurred for two reasons in recent years. First, during the euro area sovereign debt crisis, haircuts on the government debt of some countries increased substantially and abruptly, reaching levels of 80 percent or higher for some peripheral countries. At the same time, haircuts applied by the ECB on the same collateral were much lower than private market haircuts and remained largely stable (see Table 1). By modelling both private secured borrowing and borrowing from the central bank, we are able to capture such differences in haircuts and analyze their macroeconomic implications. Second, rating downgrades of several euro area sovereigns reduced the availability of high-quality collateral that could be used in the secured market. The amount of safe (AAA-rated) government debt fell from 60 percent of total debt outstanding in 2003 to 20 percent in 2017 (Figure 3). We investigate the impact of the decrease in the average quality of available collateral by examining the effects of haircut increases in the secured market, as lower-rated collateral commands higher haircuts.

We build a model to assess these developments. At the core of our framework is a bank liquidity management problem. Banks raise deposits which exposes them to idiosyncratic withdrawal shocks. To satisfy withdrawal shocks, they can obtain funding in the interbank money markets. Banks face an exogenous probability of being "connected," defined as the ability to borrow in the unsecured market. Those banks that are unable to borrow in the unsecured market, the "unconnected" banks, can borrow in the secured market. To do so,

banks need to hold government bonds which can be pledged as collateral. All banks can raise funding from the central bank which also requires collateral to back the loan. Collateralized borrowing is subject to a haircut, with haircuts in the private market being potentially different from haircuts set by the central bank.

At the beginning of each period, after knowing whether they are connected or unconnected, banks choose their liabilities (how much deposit and central bank funding to raise) and their assets (choosing between loans, bonds and cash). Bank asset-liability choices are subject to a leverage constraint. After choosing their assets and liabilities, banks face idiosyncratic deposit withdrawal shocks. Those experiencing low withdrawals can lend funds in the secured or unsecured market. Those experiencing high withdrawals can cover them with unsecured borrowing (connected banks), or the combination of collateralized borrowing and cash buffers (unconnected banks).

One novel aspect of our model is that the various constraints faced by banks are not assumed to be binding. As the severity of a particular money market friction varies, due to, e.g., reduced access to unsecured markets or increased haircuts in secured markets, several constraints (e.g., on leverage, withdrawals, central bank funding, cash holdings, bond holdings) can switch from being binding to non-binding or vice versa. Banks react to new money market conditions by changing the composition of their assets and liabilities which, in turn, changes asset prices as well as the tightness of various bank constraints. Policies pursued by the central bank also affect the supply of cash and collateral, and the price of the assets, thus affecting the tightness of the constraints. Complex interactions between several occasionally binding constraints - which we examine in two comparative statics exercises capturing different money market frictions - are a key feature of our framework.

Another contribution of our paper is the analysis of the effectiveness of a rich set of central bank policies. Monetary policy instruments we consider are the interest rate on central bank loans, the haircuts on collateral, and the amount of government bonds held by the central bank. The interest rate on central bank loans, together with the central bank collateral policy, jointly determine whether or not banks tap into central bank funding. Central bank holdings of government bonds and cash injected into the economy affect the amount of liquid assets banks have to satisfy the withdrawal shocks. We map these central bank instruments into three types of monetary policies implemented by the ECB in recent years: i) a pre-crisis policy characterized by a constant balance sheet; ii) a fixed rate full allotment policy (FRFA henceforth), whereby

the size of the balance sheet is determined by the demand for funding of the banking sector at a given policy rate; and iii) a policy of asset purchases (QE henceforth), whereby the central bank changes the stock of bonds on its balance sheet to achieve a certain inflation goal.

We calibrate the model to the euro area data and analyze the macroeconomic impact and the effects of central bank policies under two alternative scenarios: 1) reduced access to the unsecured money market; and 2) reductions in collateral value in the secured market.

In the first scenario - when access to the unsecured money market is reduced - a higher proportion of banks becomes unconnected. This reduces investment and output via two channels. First, since unconnected banks need to satisfy withdrawal shocks by holding bonds and/or by holding cash, they can invest less in the productive asset (capital). Therefore, as the share of unconnected banks in the economy increases, capital and output decrease. Second, as more banks become unconnected, bonds and cash can become more scarce, tightening the withdrawal constraint and forcing unconnected banks to reduce deposit intake (and, as a result, their investment in capital). Central bank policy cannot do anything about the first channel. However, it can mitigate the second channel by providing cash to banks at a low opportunity cost. QE policy is the most effective in achieving this goal as it expands money supply while maintaining constant inflation. In our benchmark calibration, potential withdrawal outflows in the afternoon are moderate and unconnected banks are not very constrained. An increase in the share of unconnected banks from 0.58 to 0.79 (pre- to post-2008 average share of secured turnover in total) generates a decline in output of around 0.5 percent. In this parameter range, the first channel dominates and therefore there is not much difference in economic outcomes under alternative policies. However, if the share of unconnected bank increased to 0.9 (share of secured turnover in total in 2015), then the contraction in output would be 1.5 percent in the constant balance sheet or FRFA cases, and only 1 percent in the QE case.

In the second scenario - when collateral value is reduced due to an increase in haircuts in the secured market - the type of policy pursued by the central bank makes a big difference to economic outcomes. Under constant balance sheet policy, as private haircuts increase, bond collateral value in the private market decreases. Cash becomes increasingly scarce as unconnected banks increase their demand for money but the central bank keeps its balance sheet constant. Unconnected banks become so severely constrained by the withdrawal constraint that they have to dramatically reduce their deposit intake. Their leverage constraint turns slack, and investment in capital decreases. Under our benchmark calibration, an increase in

private haircuts from 3 to 40 percent leads to an output contraction of 3.5 percent. The key to stabilizing output is either for unconnected banks to reduce deposit funding and substitute it with central bank funding (FRFA policy) or to replace bonds that become less valuable as collateral in the private market with cash so that banks can self-insure against withdrawal shocks (QE policy). Both of these policies prevent leverage constraint from turning slack and mitigate the reduction in capital and output. The output contraction is 0.58 percent under the FRFA policy and even lower - just 0.047 percent - under the QE policy.

The paper proceeds as follows. In section 2, we relate our paper to the existing literature. In section 3, we describe the model. In section 4, we define the equilibrium. In section 5, we characterize the system of equilibrium conditions. In section 6, we describe the steady state and present some analytical results. Section 7 illustrates the model predictions through a numerical analysis. Section 8 concludes.

2 Related literature

Our paper is related to the broad literature that investigates the implications of financial frictions for the macroeconomy and for monetary policy as well as to the literature which focuses on frictions in secured and unsecured interbank trade. We now discuss in more detail how various elements in our analysis relate to these literatures.

Bank balance sheet constraints and monetary policy

In the aftermath of the financial crises, many papers have emphasized the role of banks' balance sheet and leverage constraints for the provision of credit to the real economy and for the transmission of standard and non-standard monetary policies (Curdia and Woodford (2011), Gertler and Karadi (2011), and Gertler and Kiyotaki (2011)). As in some of these papers, banks in our model face an enforcement problem and balance sheet constraints. Additionally, they solve a liquidity management problem that further constrains their actions.

From a methodological perspective, we deviate from this literature in that we do not impose the various constraints to be binding at all times. Recent papers have shown the importance of using non-linear solution methods in order to allow for occasionally binding constraints (Brunnermeier and Sannikov (2014); He and Krishnamurthy (2016); Mendoza (2010); Bocola (2016); Justiniano et al. (2017)). Typically, however, only one or few occasionally binding constraints are considered. In our calibrated model, a combination of seven constraints can

switch from binding to slack and vice versa, interacting in complex ways and determining the effectiveness of monetary policy.

Interbank markets and bank liquidity management

There is an extensive literature in banking on the role of interbank markets in banks' liquidity management, starting with Bhattacharya and Gale (1987). A number of recent papers focus on analyzing frictions that prevent interbank markets from distributing liquidity efficiently within the banking system. Frictions include asymmetric information about banks' assets (Flannery (1996); Freixas and Jorge (2008); Heider et al. (2015)), imperfect cross-border information (Freixas and Holthausen (2005)), banks' free-riding on liquidity provision by the central bank (Repullo (2005)), and multiplicity of Pareto-ranked equilibria (Freixas et al. (2011)). Papers in this literature tend to be partial equilibrium and static, with links to the macroeconomy modeled in a reduced-form fashion. More recent papers build general equilibrium models which include interbank trade. Afonso and Lagos (2015) and Atkeson et al. (2015) analyse the trading decisions of banks in an OTC market. Bruche and Suarez (2010) study the macroeconomic impact of money market freezes, focusing on the unsecured money market segment. Gertler et al. (2016) point to the exposure of highly leveraged financial institutions to borrowing from other banks as a main source of the breakdown of the financial system in 2007-2009. Our paper contributes to this literature by considering both unsecured and secured interbank markets, and collateralized lending by the central bank. In our setup, frictions in the unsecured money market segment may in principle be offset by an increased recourse to private secured markets or to central bank funding.

Bank liquidity management and monetary policy

Some recent papers investigate frictions in the unsecured money markets and their interaction with monetary policy. Bianchi and Bigio (2013) build a model where banks are exposed to liquidity risk and manage it by holding a precautionary buffer of reserves. They show that monetary policy affects lending and the real economy by supplying reserves and thus by changing banks' trade-off between profiting from lending and incurring greater liquidity risk. In a general equilibrium model that features the same search frictions in the interbank market as in Bianchi and Bigio (2013), Arce et al. (2017) show that a policy of large central bank balance sheet that uses interest rate policy to react to shocks achieves similar stabilization properties to a policy of lean balance sheet, where QE is occasionally used when the interest rate hits the zero-lower bound. We contribute to this literature by adding to Bianchi and Bigio's liquidity

management problem the possibility to obtain secured funding in a private market by pledging government bonds. Secured funding gives rise to a collateral premium for assets that can be used as collateral. Collateral premium is one of the key determinants of the macroeconomic impact of money market frictions and the effectiveness of central bank policies in our model.

Collateral and monetary policy

A number of recent papers study frictions in the secured markets during the recent crisis, including increases in haircuts for some asset classes and "run"-like phenomena (e.g, Gorton and Metrick (2012); Krishnamurthy et al. (2014); Martin et al. (2014)). Ranaldo et al. (2016) show that fragility in collateralized markets can spill-over into the uncollateralized market and study which central bank and regulatory policies can reduce such fragility. Piazzesi and Schneider (2017) build a model in which the use of inside money by agents for transaction purposes requires banks to be able to handle payments instructions. Banks thus borrow or lend in the interbank market, which requires collateral, or use central bank reserves. The authors show that key to the efficiency of a payment system is the provision and allocation of collateral. Unconventional policy that exchanges reserves for lower quality collateral can be beneficial when high quality collateral is scarce. Our model considers the interaction between unsecured and secured interbank market funding. Moreover, it focuses on the role of collateral value for lending and real activity as well as for asset prices.

Collateral and sovereign risk

In our framework, increases in haircuts on sovereign bonds capture in a reduced-form the impact of sovereign default risk on collateral value. Our paper thus relates to the literature on the impact of sovereign default risk on financial intermediation and the macroeconomy. Recent contributions study the impact of sovereign risk on the funding ability of banks and their lending decisions (Bocola (2016)) as well as the link between government default and financial fragility, including the question of why the banking system may become exposed to government bonds (Gennaioli et al. (2014)). We do not model sovereign default risk explicitly, focusing instead on the implications of changes in collateral value due to increased haircuts on government bonds for banks' ability to borrow.

Scarcity of safe assets and the size of central bank balance sheet The emergence of a shortage of safe assets has been documented and analyzed in a number or recent works (see e.g. Caballero et al. (2017), Andolfatto and Williamson (2015), Gorton and Laarits (2018), and Carlson et al. (2016)). Some papers discuss the implications of scarcity for monetary policy. Caballero and

Farhi analyze a situation of a deflationary safety trap. They point to policies of "helicopter drops" of money, safe public debt issuances, swaps of private risky assets for safe public debt, or increases in the inflation target, as possible ways to mitigate the negative impact of safe asset scarcity. Carlson et al. (2016) argue that the central bank could maintain a large balance sheet and conduct monetary policy using a floor system, as large holdings of long-term assets are financed by large amounts of reserves that are safe and liquid assets for the banking system. In our model, monetary policy can accommodate the increased demand for liquid assets through an expansion of the balance sheet.

3 The model

The economy is inhabited by a continuum of households, firms and banks. There is a government and a central bank.

Time is discrete, $t = 0, 1, 2, \ldots$. We think of a period as composed of two sub-periods, "morning" and "afternoon". Let us describe each in turn.

At the beginning of each period (in the morning), aggregate shocks occur. Households receive payments from financial assets and allocate their nominal wealth among money and deposits at banks. Households also supply labor to firms in their country, receiving wages in return. The government taxes the labor income of the households in its country, makes payments on its debt and may change the stock of outstanding debt. Banks accept deposits from households and the central bank and make dividend payments to households. After accepting deposits, banks learn their afternoon type in the morning. This latter can be either "connected," in which case banks can borrow in the unsecured interbank market, or "not connected," in which case they cannot, and the only possibility is to borrow by pledging assets in the secured interbank market. Banks then lend to firms (more precisely, finance their capital) and they hold government bonds and reserves ("cash"). The central bank provides funding to banks that wish to borrow against collateral. As an additional policy tool, the central bank can choose "haircuts" on the collateral pledged to access those funds.

During the afternoon, firms use labor and capital to produce a homogeneous output good which is consumed by households. Banks experience idiosyncratic deposit withdrawal shocks which average out to zero across all banks. Conceptually, these relate to random idiosyncratic consumption needs, additional economic activity and immediate payment for these services,

which we shall refrain from modelling.[1] Banks can accommodate those shocks by using their existing reserves, by borrowing from other banks in the unsecured market, or by pledging bonds and borrowing in the secured market. They can only access the unsecured market, however, if they are "connected". Banks are assumed to always position themselves so as to meet these liquidity withdrawals, i.e., bank failures are considered too costly and not an option. All banks meet as "one big banker family" at the end of the period. One can think of it as follows. First, the same bank-individual liquidity shock happens "in reverse", so that banks enter the banker-family meeting in the same state they were in at the beginning of the afternoon. However, there would then still be bank heterogeneity left. Thus, banks all equate their positions at that point and restart the next period with the same portfolio. Alternatively, and equivalently, one can think that there are securities markets which open at the end of the period and allow banks to equate their portfolios. Banks during the period therefore are only concerned with the marginal value of an additional unit of net worth they can produce for the next period.

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by bank managers who run a bank on behalf of their owning households. We deviate from those papers in that we assume that banks pay a fixed fraction of their net worth to households as a dividend in the morning of every period.

3.1 The households

There is a representative household, indexed by $i \in (0,1)$. At the beginning of time t, households hold an amount of cash, $\widetilde{M}^h_{t-1}$, brought from period $t-1$. They also receive repayment from banks of deposits opened in the previous period gross of the due interest, $R^D_{t-1}D_{t-1}$. Holding an amount H_t of nominal wealth at hand, each household chooses how to allocate it among existing nominal assets, namely money, M^h_t, and deposits, D_t.

During the day, beginning-of-period money balances are increased by the value of households' revenues and decreased by the value of their expenses. The amount of nominal balances brought by household i into period $t+1$, $\widetilde{M}^h_t$, is thus

$$\widetilde{M}^h_t = M^h_t + (1-\tau_t)\, W_t l_t + E_t - P_t c_t, \tag{1}$$

[1] We follow a long tradition in the banking literature of focusing on the role of interbank money markets in smoothing out idiosyncratic liquidity shocks, as in Bhattacharya and Gale (1987) and Allen and Gale (2000). While analytically convenient, in reality interbank relationships may exhibit more persistent patterns, with some banks being structural borrowers and others structural lenders (Craig and Ma (2018)).

where P_t is the price of the consumption good, l_t is hours worked, τ_t is the labor tax rate, W_t is the nominal wage level, and E_t is the profit payout ("earnings") by banks.

The nominal wealth available at the beginning of period $t+1$ for investment in nominal assets is given by

$$H_{t+1} = R_t^D D_t + \widetilde{M}_t^h. \tag{2}$$

The households then choose $c_t > 0, l_t > 0, D_t \geq 0, M_t^h \geq 0$ to maximize their objective function[2]

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, l_t\right) + v\left(\frac{M_t^h}{P_t}\right) \right] \tag{3}$$

subject to (1), (2) and

$$D_t + M_t^h \leq H_t.$$

3.2 Firms

A representative final-good firm uses capital k_{t-1} and labor l_t to produce a homogeneous final output good y_t according to the production function

$$y_t = \gamma_t k_{t-1}^{\theta} l_t^{1-\theta}$$

where γ_t is a country-specific productivity shock. It receives revenues $P_t y_t$ and pays wages $W_t l_t$. Capital is owned by the firms, which are in turn owned by banks: effectively then, the banks own the capital, renting it out to firms and extracting a real "rental rate" r_t per unit of capital or total nominal rental rate payments $P_t r_t k_{t-1}$ on their capital k_{t-1}.

Capital-producing firms buy old capital k_{t-1} from the banks and combine it with final goods I_t to produce new capital k_t, according to

$$k_t = (1-\delta) k_{t-1} + I_t.$$

New capital is then sold back to banks. Alternatively and equivalently, one may directly assume that the banks undertake the investments.

[2] In the numerical analysis, we assume the following functional form of the utility function

$$u\left(c_t, l_t\right) + v\left(\frac{M_t^h}{P_t}\right) = log(c) + \frac{1}{\chi} log(\frac{M_t^h}{P_t}) - l_t.$$

3.3 The government

The government has some outstanding debt with face value $\overline{B}_{t-1}$. It needs to purchase goods g_t and pays for it by taxing labor income as well as issuing discount bonds with a face value $\Delta\overline{B}_t$ to be added to the outstanding debt next period, obtaining nominal resources $Q_t\Delta\overline{B}_t$ for it in period t. We assume that some suitable no-Ponzi condition holds. The government discount bonds are repaid at a rate κ.

The outstanding debt at the beginning of period $t+1$ will be

$$\overline{B}_t = (1-\kappa)\,\overline{B}_{t-1} + \Delta\overline{B}_t \tag{4}$$

The government budget balance at time t is

$$P_t g_t + \kappa\overline{B}_{t-1} = \tau_t W_t l_t + Q_t\Delta\overline{B}_t + S_t \tag{5}$$

where S_t are seigniorage payments from the central bank and g_t is an exogenously given process for government expenditures.

The government conducts fiscal policy so as to stabilize the stock of debt at a targeted level $\overline{B}^*$, by adopting the following rule for the income tax:

$$\tau_t - \tau^* = \alpha\left(B_t - \overline{B}^*\right), \tag{6}$$

where τ_t increases above its target level τ^*, if the debt level is above $\overline{B}^*$. We assume that α is such that the equilibrium is saddle-path stable and that the fiscal rule ensures a gradual convergence to the desired stock of debt, following aggregate disturbances.[3] The target value τ^* is the level of the income tax necessary to stabilizes the debt at $\overline{B}^*$.[4]

3.4 The central bank

The central bank chooses the total money supply $\overline{M}_t$ and interacts with banks in the "morning", providing them with funds. Banks come into the period with total liabilities (F="funds

[3] In our quantitative section, we provide a comparison of steady state equilibria: in that analysis, the parameter α plays no role.

[4] Notice that τ^* can be obtained by combining equation (4) and (5) in steady state, together with the rule $B = \overline{B}^*$, to get

$$\tau^*(1-\theta)\,y = g + \kappa(1-Q)\,\frac{\overline{b}^*}{\pi} - Q\left(1-\frac{1}{\pi}\right)\overline{b}^* - s.$$

Here $\overline{b}^* = \frac{\overline{B}^*}{P}$, $s = \frac{S}{P}$ and π is the steady state inflation rate.

from the central bank") at face value $\overline{F}_{t-1}$. Banks make payments $\kappa^F \overline{F}_{t-1}$ on these liabilities and obtain new funds, at face value $\Delta\overline{F}_t$. Thus,

$$\overline{F}_t = \left(1-\kappa^F\right)\overline{F}_{t-1} + \Delta\overline{F}_t \tag{7}$$

Banks obtain funds $Q_t^F \Delta\overline{F}_t$ for these new liabilities, at the common price or discount factor Q_t^F. This discount factor is a policy parameter set by the central bank. The central bank furthermore buys and sells government bonds outright. Let B_{t-1}^C be the stock of government bonds held by the central bank ("C") at the beginning of period t. The government makes payments on a fraction of these bonds, i.e., the central bank receives cash payments κB_{t-1}^C. The remaining government bonds in the hands of the central banks are $(1-\kappa)\, B_{t-1}^C$. The central bank then changes its stock to B_t^C, at current market prices Q_t, using cash. Thus,

$$B_t^C = (1-\kappa)\, B_{t-1}^C + \Delta B_t^c$$

The central bank balance sheet looks as follows at time t:

Assets	Liabilities
$Q_t^F \overline{F}_t$ (loans to banks)	M_t^H (currency held by HH)
$Q_t B_t^C$ (bond holdings)	M_t (bank reserves)
	S_t (seigniorage)

Let

$$\overline{M}_t = M_t^h + M_t$$

be the total money stock before seigniorage is paid. Note that the seigniorage is paid to the government at the end of the period and therefore becomes part of the currency in circulation next period. The flow budget constraint of the central bank is given by:

$$\begin{aligned} \overline{M}_t - \overline{M}_{t-1} &= S_{t-1} + Q_t^F \left(\overline{F}_t - \left(1-\kappa^F\right)\overline{F}_{t-1}\right) - \kappa^F \overline{F}_{t-1} \\ &\quad + Q_t \left(B_t^C - (1-\kappa)\, B_{t-1}^C\right) - \kappa B_{t-1}^C. \end{aligned} \tag{8}$$

Seigniorage can then be calculated as the residual balance sheet profit,

$$S_t = Q_t^F \overline{F}_t + Q_t B_t^C - \overline{M}_t. \tag{9}$$

3.5 Banks

There is a continuum of banks ("Lenders"), indexed by $l \in (0,1)$. Consider a bank l.

3.5.1 Assets and liabilities

At the end of the morning, after earning income on its assets, paying interest on its liabilities and retrading, but just before paying dividends to share holders, the bank holds four type of assets. It additionally and briefly holds an asset in the afternoon, for a total of five. As an overview, the end-of-morning balance sheet of that bank is

Assets	Liabilities
$P_t k_{t,l}$(capital held)	$D_{t,l}$ (deposits by HH)
$Q_t B_{t,l}$ (bond holdings)	$Q_t^F F_{t,l}$ (secured loans)
$E_{t,l}$(cash dividends)	$N_{t,l}$ (net worth)
$M_{t,l}$(cash reserves)	

In detail:

1. Capital $k_{t,l}$ of firms, or, equivalently, firms, who in turn own the capital. Capital can only be acquired and traded in the morning. Capital evolves according to

$$k_{t,l} = (1-\delta)\, k_{t-1,l} + \Delta k_{t,l}$$

where $\Delta k_{t,l}$ is the gross investment of bank l in capital.

2. Bonds with a nominal face value $B_{t,l}$. A fraction κ of the government debt will be repaid. The bank changes its government bond position per market purchases or sales ("-") $\Delta B_{t,l}$ in the morning, so that

$$B_{t,l} = (1-\kappa)\, B_{t-1,l} + \Delta B_{t,l}$$

at the end of the morning. If the bank purchases (sells) bonds on the open market, it pays (receives) $Q_t \Delta B_{t,l}$. We allow $\Delta B_{t,l}$ to be negative, indicating a sale. In the afternoon and after the first bank-individual liquidity shock, the bank can get funding by pledging bonds in a secured repo market, vis-a-vis other banks. To that end, it is useful to introduce haircut parameters $0 \leq \widetilde{\eta}_t \leq 1$, imposed by other lending banks. The bank then pledges an amount $\widetilde{B}_{t,l} \leq B_{t,l}$ of bonds and receives in return the cash amount $\widetilde{\eta}_t Q_t \widetilde{B}_{t,l}$ in the first of two transactions, repaying the same amount in the second. The end bond position is therefore the one held in the morning, $B_{t,l}$. Taken literally, there

is no risk here that this haircut could reasonably insure against, but this is just due to keeping the model simple. The interest rate is zero.

3. Cash $E_{t,l}$ earmarked to be distributed to shareholders (E = "earmarked" or "earnings") at the end of the morning. Note that this does not mean that the households end up being forced to hold money, as everything happens "simultaneously" in the morning. If they want to hold those extra earnings as extra deposits, then D_t would simply already be higher before they receive the earnings from the banks, in "anticipation" of these earning payments.

4. Reserves (M="money") $M_{t,l} \geq 0$. They may add to cash (not earmarked for paying shareholders) in the morning,

$$M_{t,l} = M_{t-1,l} + \Delta M_{t,l} \geq 0$$

as well as in the afternoon,

$$\widetilde{M}_{t,l} = M_{t,l} + \widetilde{\Delta M}_{t,l} \geq 0$$

reversing the first-liquidity-shock transaction when the reverse liquidity shock hits,

$$M_{t,l} = \widetilde{M}_{t,l} - \widetilde{\Delta M}_{t,l}$$

5. Unsecured claims on other banks at face value, obtained during the first liquidity shock in the afternoon. They are repaid at zero interest rate during the second reverse-liquidity shock.

Bank l has four types of liabilities:

1. Deposits $D_{t,l}$. This is owed to household and subject to aggregate withdrawals and additions $\Delta D_{t,l}$ in the morning, so that

$$D_{t,l} = R^D_{t-1} D_{t-1,l} + \Delta D_{t,l}$$

where R^D_{t-1} is the return on one unit of deposits, agreed at time $t-1$. Additionally, there are idiosyncratic withdrawals and additions in the afternoon, to be described.

2. Secured loans (F="funding") from the central bank at face value $F_{t,l}$. Secured loans require collateral. A bank l with liabilities $F_{t,l}$ to the central banks needs to pledge an

amount $0 \leq B^F_{t,l} \leq B_{t,l}$ of government bonds $B_{t,l}$ satisfying the collateral constraint

$$F_{t,l} \leq \eta_t \; Q_t B^F_{t,l} \tag{10}$$

where η_t is a haircut parameter and is set by the central bank. The collateral constraints are set in terms of the market value of securities, as is the case in ECB monetary policy operations. Secured loans from the central bank are obtained in the morning. The change in the secured loans $\Delta F_{t,l}$ provide the banks with change in liquidity ("cash") of $Q^F_t \Delta F_{t,l}$, in addition to the liquidity carried over from the previous period. Liquidity is needed in the afternoon. Therefore, the discount rate Q^F_t will not only relate to an intertemporal trade-off, as is common in most models, but importantly also to the intratemporal tradeoff of obtaining potentially costly liquidity in the morning in order to secure sufficient funding in the afternoon.

3. Outstanding unsecured liabilities to other banks issued at the time of the first liquidity shock in the afternoon. Only "connected" banks can issue them. They are repaid at zero interest rate at the time of the reverse liquidity shock.

4. Net worth $N_{t,l}$.

The sum of assets equals the sum of liabilities, at any point in time.

3.5.2 Liquidity needs in the afternoon

At the core of our model there is a bank liquidity management problem. At the beginning of the afternoon, households hold total deposits D_t with banks. We seek to capture the daily churning of deposits at banks, due to cross-household and firm-household payment activities with inside money. We use a modelling device introduced by Bianchi and Bigio (2017). At the start of the afternoon in period t, deposits get reshuffled across banks so that bank l with pre-shuffle end-of-morning deposits $D_{t,l}$ experiences a withdrawal $\omega_{t,l} D_{t,l}$. Here, $\omega = \omega_{t,l} \in (-\infty, \omega^{\max}]$, with $0 \leq \omega^{\max} \leq 1$, is a random variable, which is iid across banks l and is distributed according to $F(\omega)$. The remaining post-shuffle beginning-of-afternoon deposits $\widetilde{D}_{t,l}$ are thus

$$\widetilde{D}_{t,l} = (1 - \omega_{t,l}) \, D_{t,l}$$

In order to meet withdrawals, banks need to have enough reserves at hand to cover them. We assume that banks will always find defaulting on the withdrawals worse than any precautionary measure they can take against it, and thus rule out withdrawal caps and bank runs by assumption. Reserves can be obtained in the morning by various trades, resulting in bank holdings $M_{t,l}$. In the afternoon, additional reserves can be obtained only by new unsecured loans from other banks, maturing at the end of the afternoon, or by pledging government bonds in the secured private market. Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank funding in the morning in precaution to withdrawal demands in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen "fast", compared to central bank liquidity provision.

The withdrawal shock is exactly reversed with a second reverse liquidity shock, so that banks exit the period with the original level of deposits $D_{t,l}$ and can thus repay their unsecured loans or buy back the government securities originally sold. The same holds if the signs are reversed. Thus, the first liquidity shock creates only a very temporary liquidity need that banks must satisfy.

New unsecured loans can only be obtained by "connected" banks. Banks face an exogenous iid probability ξ_t of being connected and being able to borrow on the unsecured loan market. We assume this probability to be iid across banks and time. The draw of the type of the bank (i.e., "connected" or "not connected", with probability ξ_t) happens early in the morning: thus, banks know in the morning, whether they are able to potentially borrow in the afternoon or whether they need to potentially sell government bonds instead. Every bank can lend unsecured, if they so choose.

If banks do not have access to the unsecured loan market, they will need to pledge government bonds in the private secured market, in case of liquidity needs. They can only do so with the portion that has not yet been pledged to the central bank. With $\omega^{\max}$ as the maximal withdrawal shock, non-connected banks therefore have to hold government securities satisfying

$$\omega^{\max} D_{t,l} - M_{t,l} \leq \widetilde{\eta}_t Q_t \left(B_{t,l} - B^F_{t,l} \right) \tag{11}$$

where $0 \leq \widetilde{\eta}_t \leq 1$ is the haircut imposed by other lending banks, and where the constraint is in terms of the unpledged portion of the government bond holdings $B_{t,l} - B^F_{t,l}$.

As all the afternoon transactions are reversed at the end of the afternoon and since all within-afternoon interest rates are zero, banks will be entirely indifferent between using any of the available sources of liquidity: what happens in the afternoon stays in the afternoon. The only impact of these choices and restrictions is that banks need to plan ahead of time in the morning to make sure that they have enough funding in the afternoon, in the worst case scenario. If a bank is unconnected, that worse-case scenario is particularly bad, as it needs to have enough of cash reserves plus unpledged bonds to meet the maximally conceivable afternoon deposit withdrawal.

3.5.3 Objective function and leverage constraints

Banks are owned by households in their country. If net worth is nonnegative, they repay a portion ϕ of their net worth to households each period,

$$E_{t,l} = \phi N_{t,l}.$$

In terms of aggregate bank equity N_t and resulting dividend payments, the profit payments by banks are $E_t = \phi N_t$, if $N_t \geq 0$. If net worth is negative, banks declare bankruptcy. In that case, all assets are sold, and the proceeds are returned pro rata to the holders of bank liabilities. We shall consider only shocks and scenarios, so that net worth remains positive.

The net worth of bank l before payments to shareholders satisfies

$$\begin{aligned} N_{t,l} &= \max\{0, P_t\left(r_t + 1 - \delta\right) k_{t-1,l} + M_{t-1,l} + \left(\left(1-\kappa\right) Q_t + \kappa\right) B_{t-1,l} - R^D_{t-1} D_{t-1,l} - \kappa^F F_{t-1,l}\} \\ &= \max\{0, P_t k_{t,l} + \ Q_t B_{t,l} + M_{t,l} - D_{t,l} - Q^F_t F_{t,l} + E_{t,l}\} \end{aligned}$$

where the first equation is the net worth calculated on the balance of assets and their earnings and payments before the bank makes its portfolio decision, while the second equation exploits the equality of assets to liabilities after the portfolio decision.

From these two equations, one can calculate

$$\Delta M_{t,l} = M_{t,l} - M_{t-1,l}.$$

Given the draw of the type according to $\xi_t = P\left(\text{“connected”}\right)$, bank l can be either “connected” or “unconnected” (denoted with the subscripts “c” or “u”, respectively).

Aggregate net worth at the beginning of the period is

$$\begin{aligned} N_t = \max\{0, \quad & P_t\left(r_t + 1 - \delta\right)\left(\xi_{t-1}k_{t-1,c} + \left(1-\xi_{t-1}\right)k_{t-1,u}\right) \\ & + \left(\xi_{t-1}M_{t-1,c} + \left(1-\xi_{t-1}\right)M_{t-1,u}\right) \\ & + \left(\left(1-\kappa^F\right)Q_t^F + \kappa^F\right)\left(\xi_{t-1}F_{t-1,c} + \left(1-\xi_{t-1}\right)F_{t-1,u}\right) \\ & + \left(\left(1-\kappa\right)Q_t + \kappa\right)\left(\xi_{t-1}B_{t-1,c} + \left(1-\xi_{t-1}\right)B_{t-1,u}\right) \\ & - R_{t-1}^D\left(\xi_{t-1}D_{t-1,c} + \left(1-\xi_{t-1}\right)D_{t-1,u}\right)\} \end{aligned}$$

which implies that $N_t = \xi_t N_{t,c} + \left(1-\xi_t\right)N_{t,u}$.

We shall impose that sub-banks get the same net worth, regardless of type ("connected", "unconnected"), effectively assuming that the net worth is assigned before the type is known[5], $N_{t,c} = N_{t,u} = N_t$, where $N_{t,c}$ is the net worth per connected bank, i.e., the total net worth in all connected banks is $\xi_t N_{t,c}$, the total net worth in all unconnected banks is $\left(1-\xi_t\right)N_{t,u}$. Correspondingly, all assets and liabilities are likewise distributed equally, regardless of type (again, assuming that this redistribution is done before the new type is drawn for each sub-bank).

Summing this and imposing the two previous equations shows that total net worth is N_t, as it should be. Therefore, we shall drop the distinction between $N_{t,c}$, $N_{t,u}$ and N_t. The sub-bank budget constraint is

$$P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t = D_{t,l} + Q_t^F F_{t,l} + N_t \tag{12}$$

As in Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), we assume that there is a moral hazard constraint in that bank managers may run away with a fraction of their assets in the morning, after their asset trades are completed and after dividends are paid to the household. The constraint is

$$\lambda\left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}\right) \leq V_{t,l}$$

[5]If the net worth could be assigned after the type is known, obviously only connected banks would get any net worth, and the model would become rather uninteresting.

where $0 \leq \lambda \leq 1$ is a leverage parameter. Implicitly, we assume that the same leverage parameter holds for all assets, and that bankers can run away with all assets, including government bonds that may have been pledged as collateral vis-a-vis the central bank[6].

3.6 The rest of the world

We assume that a share of the stock of government bonds is held by the rest of the world and that foreigners have an elastic demand for those bonds.[7] Because unconnected banks can buy or sell bonds to foreigners, they can change their bond holdings independently from the government's outstanding stock of debt.

We do not wish to model the foreign sector explicitly. We simply assume that international investors have a demand for domestic bonds that reacts to movements in the real return on these bonds,

$$B_t^w = P_t \left(\varkappa - \frac{1}{\varrho} \log Q_t \pi_t \right), \tag{13}$$

where $\varrho > 0$ and $\varkappa \geq 0$. Notice that this functional form allows foreign bond holdings to become negative, e.g., in case domestic bond demand exceeds government bond supply, while Q_t is always positive. If $\varrho = 0$, bond demand becomes infinitely elastic. In that case, the real return $1/(Q_t \pi_t)$ is fixed and foreign holdings take whatever value is needed to clear the bond market. The flow budget constraint of the foreign sector is

$$Q_t B_t^w + P_t c_t^w = \left[\kappa + (1 - \kappa) Q \right] B_{t-1}^w. \tag{14}$$

4 Equilibrium

An equilibrium is a vector of sequences such that:

[6] Alternatively, one may wish to impose that banks cannot run away with assets pledged to the central bank as collateral. In that case, the collateral constraint would be

$$\lambda \left[P_t \left(k_{t,l} - k_{t,l}^F \right) + Q_t \left(B_{t,l} - B_{t,l}^F \right) + M_{t,l} \right] \leq V_{t,l}$$

or a version in between this and the in-text equation. Since collateral pledged to the central bank typically remains in the control of banks, we feel that the assumption used in the text is more appropriate.

[7] We introduce the elastic foreign sector demand for two reasons. First, a large fraction of euro area sovereign debt is held by non-euro area residents, and these bondholders actively rebalance their bond positions. Koijen et al. (2016) document that during the Public Sector Purchase Programme implemented by the ECB since March 2015, for each unit of sovereign bonds purchased by the ECB, the foreign sector sold 0.64 of it. Second, when solving the model we will focus on the parameter space in which connected banks choose not to hold bonds. In a closed economy, therefore, unconnected banks would have to absorb whatever amount of bonds is issued by the government (after deducting the fixed amount held by the central bank). The price of the bond would have to adjust to clear the market. Such direct link between the bond market and the unconnected banks' decisions would be unrealistic.

1. Given $P_t, \tau_t, W_t, R_{t-1}^D, E_t$, the representative household chooses $c_t > 0, l_t > 0, D_t \geq 0, M_t^h \geq 0$ to maximize their objective function

$$\max E_t \left[\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, l_t\right) + v\left(\frac{M_t^h}{P_t}\right) \right] \right]$$

subject to

$$D_t + M_t^h \leq H_t$$

where

$$H_{t+1} = R_t^D D_t + M_t^h + \left(1 - \tau_t\right) W_t l_t + E_t - P_t c_t.$$

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology

$$y_t = \gamma_t k_{t-1}^{\theta} l_t^{1-\theta}.$$

3. Capital-producing firms choose how much old capital k_{t-1} to buy from banks and to combine with final goods I_t to produce new capital k_t, according to the technology

$$k_t = \left(1 - \delta\right) k_{t-1} + I_t.$$

4. Bank families aggregate the assets and liabilities of the individual family members:

$$V_t = \xi_t V_{t,c} + \left(1 - \xi_t\right) V_{t,u} \tag{15}$$

$$k_t = \xi_t k_{t,c} + \left(1 - \xi_t\right) k_{t,u} \tag{16}$$

$$D_t = \xi_t D_{t,c} + \left(1 - \xi_t\right) D_{t,u} \tag{17}$$

$$B_t = \xi_t B_{t,c} + \left(1 - \xi_t\right) B_{t,u} \tag{18}$$

$$F_t = \xi_t F_{t,c} + \left(1 - \xi_t\right) F_{t,u} \tag{19}$$

$$M_t = \xi_t M_{t,c} + \left(1 - \xi_t\right) M_{t,u} \tag{20}$$

5. Given the stochastic paths for the endogenous variables c_t, l_t, r_t, P_t, Q_t, Q_t^F, η_t, and stochastic exogenous sequence for $\widetilde{\eta}_t$ and the draw of the type according to ξ_t, the

representative date-t connected bank chooses $k_{t,c}$, $B_{t,c}$, $B^F_{t,c}$, $F_{t,c}$, $D_{t,c}$, $M_{t,c}$ and the representative date-t unconnected bank chooses $k_{t,u}$, $B_{t,u}$, $B^F_{t,u}$, $F_{t,u}$, $D_{t,u}$, $M_{t,u}$ to maximize the banks' objective function, i.e. to maximize

$$V_{t,l} = P_t E\left[\phi \sum_{s=0}^{\infty} (\beta(1-\phi))^s \frac{u_c(c_{t+s}, l_{t+s})}{u_c(c_t, l_t)} \frac{N_{t+s}}{P_{t+s}}\right] \tag{21}$$

where

$$\begin{aligned} N_t = \max\{0, \quad & P_t(r+1-\delta)(\xi_{t-1}k_{t-1,c} + (1-\xi_{t-1})k_{t-1,u}) \\ & + (\xi_{t-1}M_{t-1,c} + (1-\xi_{t-1})M_{t-1,u}) \\ & + \left(\left(1-\kappa^F\right)Q^F_t + \kappa^F\right)(\xi_{t-1}F_{t-1,c} + (1-\xi_{t-1})F_{t-1,u}) \\ & + ((1-\kappa)Q_t + \kappa)(\xi_{t-1}B_{t-1,c} + (1-\xi_{t-1})B_{t-1,u}) \\ & - R^D_{t-1}(\xi_{t-1}D_{t-1,c} + (1-\xi_{t-1})D_{t-1,u})\} \end{aligned} \tag{22}$$

s.t. for $l = c, u$,

$$\begin{aligned} V_{t,l} &\geq \lambda(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \\ 0 &\leq B_{t,l} - B^F_{t,l} \\ P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t &= D_{t,l} + Q^F_t F_{t,l} + N_t \\ F_{t,l} &\leq \eta_t Q_t B^F_{t,l} \end{aligned}$$

as well as

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \widetilde{\eta}_t Q_t \left(B_{t,u} - B^F_{t,u}\right)$$

for the unconnected banks.

6. The central bank chooses the total amount of money supply $\overline{M}_t$, the haircut parameter η_t, the discount factor on central bank funds Q^F_t , the bond purchases B^C_t as well as the seigniorage payment S_t. It satisfies the balance sheet constraint

$$S_t = Q^F_t \overline{F}_t + Q_t B^C_t - \overline{M}_t \tag{23}$$

and the budget constraint

$$\begin{aligned}\overline{M}_t &= Q_{t-1}^F \overline{F}_{t-1} + Q_{t-1} B_{t-1}^C + Q_t^F \left(\overline{F}_t - \left(1-\kappa^F\right)\overline{F}_{t-1}\right) \\ &\quad -\kappa^F \overline{F}_{t-1} + Q_t \left(B_t^C - (1-\kappa) B_{t-1}^C\right) - \kappa B_{t-1}^C\end{aligned} \tag{24}$$

7. The government satisfies the debt evolution constraint, the budget constraint and the tax rule

$$\overline{B}_t = (1-\kappa)\overline{B}_{t-1} + \Delta\overline{B}_t \tag{25}$$

$$P_t\, g_t + \kappa\overline{B}_{t-1} = \tau_t W_t l_t + Q_t \Delta\overline{B}_t + S_t \tag{26}$$

$$\tau_t\, W_t l_t = \alpha\overline{B}_{t-1}. \tag{27}$$

8. The foreign sector chooses the amount of domestic bonds to hold

$$B_t^w = \varkappa - \frac{1}{\varrho}\log Q_t \pi_t, \tag{28}$$

and satisfies the budget constraint

$$Q_t B_t^w + P_t c_t^w = \left[\kappa + (1-\kappa) Q\right] B_{t-1}^w. \tag{29}$$

9. Markets clear:

$$c_t + g_t + I_t + c_t^w = y_t \tag{30}$$

$$\overline{B}_t = B_t + B_t^C + B_t^w \tag{31}$$

$$\overline{F}_t = F_t \tag{32}$$

$$\overline{M}_t = M_t + M_t^h \tag{33}$$

5 Analysis

We characterize the decision of households, firms and banks in turn.

5.1 Households

The household budget constraint at time t writes as

$$D_t + M_t^h \leq R_{t-1}^D D_{t-1} + M_{t-1}^h + (1-\tau_{t-1}) W_{t-1} l_{t-1} + E_{t-1} - P_{t-1} c_{t-1} \tag{34}$$

Note also that $c_t > 0, l_t > 0$, $M_t^h > 0$ and $D_t \geq 0$. We do not list these constraints separately for the following reasons. For $c_t > 0$, $l_t > 0$, and $M_t^h > 0$, we can assure nonnegativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to $D_t > 0$, despite the possibility in principle that it could be zero or negative when allowing for more generality[8].

Let μ_t^{HH} denote a Lagrange multiplier on the period-t household budget constraint (34). The optimality conditions are given by:

$$\begin{aligned} -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} &= (1-\tau_t)\frac{W_t}{P_t} \\ v_M\left(m_t^h\right) &= u_c(c_t, l_t)\left(R_t^D - 1\right) \\ \frac{u_c(c_{t-1}, l_{t-1})}{P_{t-1}} &= \beta R_t^D\left[\frac{u_c(c_t, l_t)}{P_t}\right]. \end{aligned}$$

5.2 Firms

First-order conditions arising from the problem of the firms are

$$\begin{aligned} y_t &= \gamma_t k_{t-1}^{\theta} l_t^{1-\theta}, \\ W_t l_t &= (1-\theta) P_t\, y_t, \\ r_t k_{t-1} &= \theta y_t, \\ k_t &= (1-\delta) k_{t-1} + I_t. \end{aligned}$$

5.3 Banks

The run-away constraint (assuming it always binds) is

$$V_{t,l} = \lambda\left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}\right) \tag{35}$$

The value of the mother bank is V_t, which is given by

$$V_t = \xi_t V_{t,c} + (1-\xi_t) V_{t,u} \tag{36}$$

[8]We have not yet fully analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

Proposition 1 (linearity) *The problem of bank l is linear in net worth and*

$$V_{t,l} = \psi_t N_{t,l} \tag{37}$$

for any bank l and some factor ψ_t. In particular, $V_{t,l} = 0$ if $N_{t,l} = 0$.

Proof: Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus the value of the bank is twice as large, giving the linearity above.

We need to calculate $V_{t,l}$. The proposition above implies

$$V_t = \psi_t N_t \tag{38}$$

giving us a valuation of a marginal unit of net worth at the beginning of period t, for a representative bank.

Suppose, at the end of the period, the representative "mother bank" has various assets, k_t, B_t, and M_t, brought to it by the various sub-banks as they get together again at the end of the period. The end-of-period value $\tilde{V}_t$ of the "mother bank" then satisfies

$$\begin{aligned}\tilde{V}_t &= \beta(1-\phi) E_t\left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} N_{t+1}\right] \\ &= \tilde{\psi}_{t,k}\ P_t k_t + \tilde{\psi}_{t,B} B_t\ + \tilde{\psi}_{t,M}\ M_t - \tilde{\psi}_{t,D} D_t - \tilde{\psi}_{t,F} F_t\end{aligned} \tag{39}$$

Per inspecting (22), we obtain

$$\tilde{\psi}_{t,k} = \beta(1-\phi) E_t\left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \psi_{t+1}(r_{t+1} + 1 - \delta)\right] \tag{40}$$

$$\tilde{\psi}_{t,B} = \beta(1-\phi) E_t\left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1}\ ((1-\kappa) Q_{t+1} + \kappa)\right] \tag{41}$$

$$\tilde{\psi}_{t,D} = \beta(1-\phi) E_t\left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1}\ R_t^D\right] \tag{42}$$

$$\tilde{\psi}_{t,F} = \beta(1-\phi) E_t\left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1}\left(\left(1-\kappa^F\right) Q_{t+1}^F + \kappa^F\right)\right] \tag{43}$$

$$\tilde{\psi}_{t,M} = \beta(1-\phi) E_t\left[\frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1}\right] \tag{44}$$

For the sub-banker of type l, write

$$V_{t,l} = \phi N_t + \tilde{V}_{t,l} \tag{45}$$

The sub-bankers contribute to $\tilde{V}_t$ per

$$\tilde{V}_{t,l} = \tilde{\psi}_{t,k} k_{t,l} + \tilde{\psi}_{t,B} B_{t,l} + \tilde{\psi}_{t,M} M_{t,l} - \tilde{\psi}_{t,D} D_{t,l} - \tilde{\psi}_{t,F} F_{t,l} \tag{46}$$

The run-away constraint for bank l can then be rewritten as

$$\phi N_t + \tilde{V}_{t,l} \geq \lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} \right) \tag{47}$$

Banks will pledge just enough collateral to the central bank to make the collateral constraint binding, nothing more (even if indifferent between that and pledging more: then, "binding" is an assumption). For both types of banks,

$$F_{t,l} = \eta_t Q_t B^F_{t,l} \tag{48}$$

with

$$0 \leq B_{t,l} - B^F_{t,l} \tag{49}$$

There are also nonnegativity constraints for investing in cash, in bonds, and for financing from the central bank, for both types of banks:

$$0 \leq M_{t,l} \tag{50}$$

$$0 \leq B_{t,l} \tag{51}$$

$$0 \leq F_{t,l}. \tag{52}$$

Note that we are interested in cases where banks choose to raise deposits and to extend loans. The former requirement ensures that banks have liquidity shocks in the afternoon and thus provides a meaningful role for interbank markets. The latter requirement generates an active link between financial intermediation and real activity in our economy.

We can have cases, however, when banks decide not to raise central bank finance, as in the case of connected banks that can always get afternoon zero-interest rate unsecured loans from other banks, if the need arises (this is assuming that $Q^F_t \leq 1$, otherwise there would be arbitrage possibilities for banks!). Similarly, banks can decide not to hold bonds, if their liquidity value is too low and the cost of satisfying the afternoon constraint with cash is sufficiently low. Alternatively, they can decide not to hold cash, if they have access to afternoon unsecured or

secured finance, and if the expected return on capital is higher than the expected return on money.

To simplify the analysis, we assume (and verify in Appendix A) that the economy is in an interior equilibrium for $D_{t,l}$ and $k_{t,l}$ in all the interesting cases we consider. In light of the considerations above, we explicitly allow for corner solutions for $F_{t,l}$, $B_{t,l}$ and $M_{t,l}$.

As for the afternoon, there is no need to keep track of trades, except to make sure that the afternoon funding constraints for the unconnected banks, equation (11), holds.

Banks $l = u$ and $l = c$ who are given N_t maximize (46) subject to the sub-bank budget constraint (12) and the run-away constraint (47), the collateral constraints (48), (49), as well as (11) only for the unconnected banks. Let $\mu_{t,l}^{BC}$ denote the Lagrange multiplier on the budget constraint (12), $\mu_{t,l}^{RA}$ the Lagrange multiplier on the run-away constraint (47), $\mu_{t,l}^{CC}$ the Lagrange multiplier on the collateral constraint (48), $\mu_{t,u}$ the Lagrange multiplier on the afternoon funding constraint of the unconnected banks, $\mu_{t,l}^{M} \geq 0$, $\mu_{t,l}^{F} \geq 0$, $\mu_{t,l}^{C} \geq 0$ and $\mu_{t,l}^{B} \geq 0$ the Lagrange multipliers on the constraints $M_{t,l} \geq 0$, $F_{t,l} \geq 0$, the collateral constraint (49), and the non-negativity constraint for bonds $B_{t,l}$, respectively.

The first-order conditions characterizing banks' choices for capital, bonds, and money, are

$$\left(1+\mu_{t,l}^{RA}\right)\frac{\tilde{\psi}_{t,k}}{P_t} = \mu_{t,l}^{BC} + \lambda\mu_{t,l}^{RA}$$

$$\begin{aligned}
\left(1+\mu_{t,l}^{RA}\right)\tfrac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA}\lambda - \mu_{t,l}^{C} \quad \text{for } l = c \\
\left(1+\mu_{t,l}^{RA}\right)\tfrac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA}\lambda - \mu_{t,l}^{C} - \mu_{t,u}\widetilde{\eta}_t \quad \text{for } l = u
\end{aligned}$$

$$\begin{aligned}
\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,M} + \mu_{t,c}^{M} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA}\lambda \quad \text{for } l = c \\
\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,M} + \mu_{t,c}^{M} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA}\lambda - \mu_{t,u} \quad \text{for } l = u
\end{aligned}$$

Those characterizing banks' choices for deposits, central bank funding, and bonds to be pledged at the central bank, are

$$\begin{aligned}
\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} \quad \text{for } l = c \\
\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,D} &= \mu_{t,l}^{BC} - \omega^{\max}\mu_{t,u} \quad \text{for } l = u
\end{aligned} \tag{53}$$

$$\left(1+\mu_{t,l}^{RA}\right)\tilde{\psi}_{t,F} = \mu_{t,l}^{BC}Q_t^F - \mu_{t,l}^{CC} + \mu_{t,c}^{F} \tag{54}$$

$$\begin{aligned}
\mu_{t,l}^{CC}\eta_t &= \mu_{t,l}^{C} \quad \text{for } l = c \\
\mu_{t,l}^{CC}\eta_t &= \mu_{t,u}\widetilde{\eta}_t + \mu_{t,l}^{C} \quad \text{for } l = u
\end{aligned}$$

The complementary slackness conditions are

$$\mu_{t,l}^{F} F_{t,l} = 0 \tag{55}$$

$$\mu_{t,l}^{M} M_{t,l} = 0 \tag{56}$$

$$\mu_{t,l}^{C} \left(B_{t,l} - B_{t,l}^{F} \right) = 0 \tag{57}$$

$$\mu_{t,l}^{RA} \left[\phi N_{t,A} + \tilde{V}_{t,l} - \lambda \left(P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} \right) \right] = 0 \tag{58}$$

$$\mu_{t,l}^{B} B_{t,l} = 0 \tag{59}$$

for $l = u, c$, and

$$\mu_{t,u} \left[\omega^{\max} D_{t,u} - M_{t,u} - \widetilde{\eta}_t Q_t \left(B_{t,u} - B_{t,u}^{F} \right) \right] = 0$$

for unconnected banks only.

These are linear programming problem, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions.

6 Steady state analysis

We characterize a stochastic steady state where prices grow at the rate π and all shocks are zero except for the idiosyncratic liquidity shock ω faced by banks. We denote with small letters all real variables, i.e. the corresponding variables in capital letter divided by the price of the consumption good, P_t. The steady state is characterized by the set of conditions reported in Appendix A.

In what follows, we provide some analytical results for the bank problem in the steady state. We focus on the set of parameters such that:

1. Both bank types choose to extend loans and to raise deposits, $k_l > 0$ and $d_l > 0$. The requirement $k_l > 0$ ensures an active link between activity of all banks and the real activity. This requires capital to be sufficiently productive compared to the cost of deposits, $\tilde{\psi}_k > \tilde{\psi}_D$, which after substituting for $\tilde{\psi}_k$ and $\tilde{\psi}_D$ yields:

$$\theta \frac{y}{\left(\xi k_c + (1 - \xi) k_u \right)} + 1 - \delta > \frac{1}{\beta}. \tag{60}$$

The requirement that $d_l > 0$ means that both bank types will be subject to liquidity shocks in the afternoon and thus liquidity management will play an important role for both bank types.

Different bank types may still choose to manage their liquidity differently (through interbank markets and/or by borrowing from the central bank and saving cash for the afternoon). For households to deposit with banks we need that $R^D > 1$ or, equivalently,

$$\frac{\pi}{\beta} > 1. \tag{61}$$

2. Connected banks do not borrow from the central bank, $\mu_c^F > 0$ and $f_c = 0$. In reality, when banks can easily borrow unsecured, they use central bank funding only to manage their expected liquidity needs, like reserve requirements. Those are set to zero in the model. In our model, banks will only access central bank funding when their access to interbank markets is impaired. Indeed, historically, banks have made precautionary use of central bank funding to satisfy (unexpected) liquidity needs only in crisis periods.

A sufficient condition for $\mu_c^F > 0$ and $f_c = 0$ is $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$ or, equivalently,

$$\frac{\left(1-\kappa^F\right)Q^F + \kappa^F}{Q^F} > \frac{\pi}{\beta} = R^D. \tag{62}$$

Note that for $\kappa^F = 1$ (which is the case we will consider in the numerical analysis), this condition is equivalent to $\frac{1}{Q^F} > \frac{\pi}{\beta} = R^D$. The condition is intuitive: if the interest rate on central bank funding is higher than the rate on deposits, central bank funding will not be used. It is both more expensive in terms of the interest rate and it requires collateral. Note that the condition above is a sufficient condition for connected banks not to borrow from the CB. A necessary condition for connected banks not to borrow from the CB is

$$\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right)\frac{1}{\eta} + \left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right)Q^F > 0. \tag{63}$$

Note that this inequality allows for $\frac{\tilde{\psi}_F}{Q^F} \leq \tilde{\psi}_D$ or, when $\kappa^F = 1$, for $\frac{\pi}{\beta} \geq \frac{1}{Q^F}$. Still - when it is satisfied - it implies that $\mu_c^F > 0$ and $f_c = 0$.[9]

We can characterize decisions of connected banks as follows (the proof is in the Appendix B).

Proposition 2 (connected banks) *Suppose conditions (60), (61), and either (62) or (63) hold. Then, a connected bank does not borrow from the central bank. A connected bank does*

[9] In our calibrated steady state $\frac{\pi}{\beta} > \frac{1}{Q^F}$ but condition (63) is satisfied.

not hold any cash. Moreover, if the afternoon constraint of unconnected banks binds, $\mu_u > 0$, then a connected bank does not hold any bonds, i.e., $b_c = 0$.

Connected banks have access to the unsecured market in which they can smooth out liquidity shocks without a need for collateral. If central bank funding is more expensive than deposits, connected banks do not use it for funding purposes (condition (62)). If central bank funding is cheaper than deposits but the opportunity cost of holding bonds is sufficiently large (i.e. the collateral premium, $\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}$, is high), connected banks would still prefer to fund themselves by raising deposits (condition (63)).

Similarly, connected banks will not hold any precautionary cash reserves since holding cash carries an opportunity cost. Whenever the afternoon constraint of unconnected banks binds, physical return on bonds is lower than the return on capital as bonds command a collateral premium. As connected banks do not need any collateral, they prefer to invest solely in capital.

Decisions of unconnected banks are as follows (the proof is in the Appendix B).

Proposition 3 (unconnected banks) *Suppose conditions (60), (61), and either (62) or (63) hold. If the afternoon constraint is slack, $\mu_u = 0$, then an unconnected bank does not borrow from the central bank, $\mu_u^F > 0$ and $f_u = 0$. Instead, if the afternoon constraint binds and condition*

$$\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M} \tag{64}$$

holds, then an unconnected bank borrows from the central bank and satisfies its afternoon liquidity needs solely by holding money.

If the afternoon constraint is slack, unconnected banks are unconstrained in their afternoon borrowing in the secured market. Therefore, they do not borrow from the central bank. By contrast, whenever (64) holds, private haircut $\widetilde{\eta}$ is so unfavorable that unconnected banks do not use secured market and borrow from the central bank instead. Since unconnected banks borrow from the central bank, their afternoon constraint binds, $\mu_u > 0$. It follows that their money holdings are positive

$$m_u = \omega^{\max} d_u > 0.$$

7 Numerical analysis

In this section, we calibrate the model to the euro area data and analyze the macroeconomic impact and the effects of central bank policies under two alternative scenarios: 1) reduced

access to the unsecured money market; and 2) reductions in collateral value in the secured market.[10]

Our results highlight the complex interactions between various occasionally binding constraints, which is the novel feature of our model. The model features eleven such constraints (equations (47) and (49)-(52), for $l = u, c$, and equation (11) for $l = u$). In the numerical analysis that follows, we restrict our attention to regions of the parameter space where conditions (60)-(61) and (63) are satisfied, connected banks hold neither bonds nor money, nor do they borrow at the central bank, i.e. $b_c = b_c^F = m_c = f_c = 0$. This effectively limits the number of interacting occasionally binding constraints to seven (equations ((47) for $l = u, c$, and (49)-(52) and (11) for $l = u$). When a single parameter changes, constraints can turn from binding to slack, and then to binding again, due to the interaction with other constraints. The particular constraint that binds is typically crucial for determining the effectiveness of policy interventions.

7.1 Calibration

In the model, each period is a quarter. We set the depreciation rate at $\delta = 0.02$, the capital income share θ at 0.33 and the discount factor at $\beta = 0.994$.[11] The fraction of government bonds repaid each period, κ, is 0.042, corresponding to an average maturity of the outstanding stock of euro area sovereign bonds of 6 years.[12] The parameters determining the value of collateral in the private market and at the central bank reflect the data shown in Table 1. The haircuts on government bonds in private markets and at the central bank are set equal to each other, at $1 - \widetilde{\eta} = 1 - \eta = 0.03$ (corresponding to a 3% haircut). The private haircut value is taken from LCH Clearnet, a large European-based multi-asset clearing house, and refers to an average haircut on French, German and Dutch bonds across all maturities in 2010. The value for the central bank haircut matches the haircut imposed by the ECB on sovereign bonds with credit quality 1 and 2 (corresponding to a rating AAA to A-) in 2010.

[10] In Appendix C, we present results of an additional scenario, a reduced supply of government bonds. This exercise aims to capture the effects of safe asset scarcity, a concern which became particularly pronounced in the aftermath of the Global Financial Crisis. In the euro area, the share of AAA-rated sovereign bonds in GDP declined from 30% pre-crisis to just 14% in 2017 (here a country is taken as AAA-rated if it is AAA-rated by at least one of the following rating agencies: Moody's, Fitch, S&P).

[11] The inverse of the discount factor $1/\beta$ determines the real rate on household deposits. This rate has been very low in the euro area (in fact, it was negative for overnight deposits both before and after the onset of the financial crisis). To match this stylized fact, we choose a relatively high discount rate β.

[12] Average maturity is computed as a weighted average of all maturities of euro area government bonds, with weights given by outstanding amounts in year 2011. Source: Bloomberg, ECB and authors' calculations. Bond level data used in Andrade et al. (2016) give a similar average maturity in 2015, pointing to a stable maturity structure of euro area debt over time.

Two novel parameters of our model, which capture frictions in the funding markets and are key to determining banks' choices, are the share of "unconnected" banks, $1-\xi$, and the maximum fraction of deposits that households can withdraw in the afternoon, $\omega^{\max}$.

We compute the average pre-crisis value of $1-\xi$ using data from the Euro Money Market Survey, which underlie Figure 1. We set $1-\xi = 0.58$, corresponding to the 2003-2007 average share of cumulative quarterly turnover in the secured market in the total turnover, which sums up the turnover in the secured and in the unsecured segments (where 2003 is the first available observation in the survey while 2007 is the last year before the Global Financial Crisis). To assess the impact of the observed decline in unsecured market access, we compute the same average for post-2008 period, i.e., the average over 2008-2015 (where 2015 is the last available observation in the survey). The average value for that period is 0.79.

We determine $\omega^{\max}$ using the information embedded in the liquidity coverage ratio (LCR) - a prudential instrument that requires banks to hold high-quality liquid assets (HQLA) in an amount that allows banks to meet 30-days liquidity outflows under stress. As we are interested in maximum outflows, the "stressed" scenario as considered in the LCR appears to be an appropriate empirical counterpart for $\omega^{\max}$. We compute $\omega^{\max}$ using the European Banking Authority report from December 2013, which provides LCR data for 2012Q4 and covers 357 EU banks from 21 EU countries. Their total assets sum to EUR 33000 billion, the aggregate HQLA to EUR 3739 billion and their net monthly cash outflows to EUR 3251 billion. We take $\omega^{\max}$ to be the ratio of the net monthly cash outflows over total assets so that $\omega^{\max} = 0.1$.[13]

We choose the parameter of the foreign demand for bonds, $\varkappa$, to ensure that, if foreign bond holdings take a value consistent with their observed share in total debt, then Q and π also take their average value at that steady state (.955 and 1.005, respectively). The steady state calibration cannot inform us about the elasticity of foreign bond demand ϱ, so we pick a value that produces an elasticity which is in line with available empirical evidence. We take data reported by Koijen et al. (2016) on average foreign holdings of euro area government bonds over the periods 2013Q4-2014Q4 and 2015Q2 to 2015Q4. We compute the percentage decrease

[13] In our model, whenever the afternoon constraint binds, banks hold liquid assets in the amount of $M_u + \widetilde{\eta} Q \left(B_u - B_u^F\right)$ to cover afternoon withdrawals $\omega^{\max} D$. Since $F = 0$ in our calibrated steady state, and net worth is a small fraction of total liabilities, D can be approximated with total assets.

Alternatively, we can approximate $\omega^{\max}$ using the run-off rates on deposits, as specified in the LCR regulation (e.g., run-off rate of 10% means that 10% of the deposits are assumed to possibly leave the bank in 30 days). Run-off rates for deposits range between 5% for the most stable, fully insured deposits to 15% for less stable deposit funding. Our calculation of $\omega^{\max} = 0.1$ based on data from the European Banking Authority is consistent with these rates.

in foreign holdings between the two periods to be -3.3%. We then calculate the percentage change between the same periods in the average real return on euro area government bonds to be 38%.[14] We then set ϱ to replicate the observed elasticity of foreign bond holdings with respect to changes in the real return on bonds, i.e. $\varrho = 1.76$. We check robustness to alternative values (not reported) and find little impact on our quantitative analysis.

We are left with six parameters that we calibrate to match the model-based predictions on some key variables from their empirical counterparts: the share of net worth distributed by banks as dividends, ϕ, the share of assets bankers can run away with, λ, the coefficient determining the utility from money holdings for households, χ, the expenditure on public goods, g, the amount of government bonds purchased by the central bank, B^C, and the targeted stock of debt in the economy, $\overline{B}^*$. The targeted variables are: i) average debt to GDP; ii) bank leverage; iii) share of banks' bond holdings in total debt; iv) share of foreign sector's bond holdings in total debt; v) government bond spread; and vi) average inflation. Table 2 summarizes all parameter values. Table 3 reports the value taken by the six variables in the data (computed over the pre-crisis period, 1999-2006, unless otherwise indicated) and the model prediction under the chosen parameterization.[15]

7.2 Macroeconomic impact and central bank policies

We assess the implications of the changes in the money market landscape we outlined in the introduction for the macroeconomy and for central bank policies by means of a comparative statics analysis.[16]

[14] Notice that the period 2015Q2-2015Q4 coincides with the introduction of the Public Sector Purchase Programme, which was implemented by the ECB in March 2015.

[15] The average debt to GDP is computed using data on debt securities issued by euro area (EU12) governments from Eurostat (Annual Financial Accounts for General Government). The value of bank leverage is taken from Andrade et al. (2016). The share of banks' bond holdings in total debt is set at the value reported in Koijen et al. (2016) for 2015, 23%. To compute the share of the foreign sector's bond holdings, we first use data from SDW (the ECB database) to calculate the share of central bank's holdings in total government debt. We impute to this item not only outright purchase of government bonds but also collateralized loans extended in refinancing operations (the main instrument through which the ECB injects liquidity in normal times). The ratio to total sovereign debt is 10%. Koijen et al. (2016) report that households hold 3% of government bonds. We then inpute to the foreign sector the remaining share, which amounts to 64%. The government bond spread is computed using data from SDW. We build average government bond yields by weigthing yields of all euro area government bonds, for all maturities, with the respective amounts in 2011. We then build the spread relative to the overnight rate, the Eonia. Average inflation is computed using quarterly changes of the HICP index taken from SDW.

[16] As the money market developments we investigate are structural and, as we argue, long-lasting, we conduct a comparative statics exercise. A dynamic analysis under several occasionally binding constraints, if feasible, may require log-linearizing the model around a particular steady-state. This may not be a suitable approach, as some of the changes in money markets we document are large.

We consider the following monetary policy instruments: the interest rate on central bank loans, Q_F, the haircut on collateral charged by the central bank, η, and the stock of government bonds on its balance sheet, b^{CB}. We map these central bank instruments into three types of monetary policies implemented by the ECB in recent years: i) a pre-financial crisis policy characterized by a constant balance sheet; ii) a FRFA policy whereby the size of the balance sheet is determined by the demand for funding of the banking sector at a given policy rate; and iii) a QE policy whereby the central bank changes the stock of bonds on its balance sheet to achieve an inflation goal of 2%.

Our benchmark central bank policy is the constant balance sheet policy. We compare outcomes under the benchmark policy to outcomes under a FRFA policy and to a QE policy of maintaining constant inflation.

7.2.1 Reduced access to the unsecured market

The first exercise we conduct aims at analyzing the macroeconomic effects of a shrinking unsecured money market segment. In this comparative statics exercise, the share of unconnected banks, $1-\xi$, increases from 0.58 to 0.9. Figures 4 and 5 show the results for the constant balance sheet policy and for the QE policy, respectively.

In both figures, the solid red line denotes the share of unconnected banks under our benchmark calibration ($1-\xi = 0.58$). In Figure 4, the green dashed lines indicate the level of $1-\xi$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. In addition, in Figure 5, the orange dotted lines indicate the level of $1-\xi$ at which unconnected banks stop holding bonds. As we shall see, these two constraints will play a major role in this exercise.

In the calibrated steady-state (at the red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. The amount of deposits raised by connected and unconnected banks is of a broadly comparable magnitude. Unconnected banks, however, invest less in capital than connected banks, as they need to invest part of the funds in bonds to be pledged in the secured market in the afternoon. At this point, the return on bonds is higher than the return on money (not shown), and unconnected banks choose not to hold money to satisfy their afternoon liquidity needs.

As the share of unconnected banks increases moving rightward in both figures, a larger number of banks faces an afternoon withrawal constraint, which raises the aggregate demand for bonds and the bond price. In the region where $1-\xi < 0.66$, the real return on bonds falls

for foreign investors, inducing them to sell part of their bond holdings to domestic banks. The amount of bonds held by each unconnected bank, b_u, nonetheless falls, as more banks need to hold bonds as collateral, and the supply of bonds is fixed. When the share of unconnected banks increases further, i.e., when $1-\xi$ exceeds 0.66, the high price of bonds lowers the return on bonds to the point when it is equalized with the return on money. From this point onward (indicated by the green dashed lines), unconnected banks also use money to self-insure against afternoon withdrawals. That is, their demand for money increases.

Under the constant balance sheet policy (Figure 4), the supply of money is fixed. Higher demand for money by unconnected banks is accommodated by an increase in the nominal interest rate (the deposit rate), which induces households to reduce their money holdings. Scarce money balances are therefore reallocated from households to unconnected banks. A higher nominal rate requires an increase in inflation,[17] which raises the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts downward pressure on aggregate capital and, correspondingly, upward pressure on the return on capital. The resulting tightening of the run-away constraint induces also connected banks to reduce their investment in capital and their deposit intake. Therefore, aggregate deposits and capital fall and so does output. Quantitatively, an increase in the share of unconnected banks from 0.6 to 0.79 (pre- to post-2008 average share of secured turnover in total) generates a decline in output of around 0.64 percent.

Under the FRFA policy, the outcome is the same as under the constant balance sheet policy. This is because central bank funding is not used in this case (and therefore the central bank balance sheet remains constant) as deposit funding is preferred to central bank funding.

Under the QE policy (Figure 5), the central bank can expand its balance sheet by purchasing bonds and thus increase the supply of money to help relax the afternoon constraint of unconnected banks. When $1-\xi$ exceeds 0.68 (indicated by the orange dashed lines) and the price of bonds is high, unconnected banks sell off their entire bond holdings to the central bank and choose to hold money instead to satisfy the afternoon constraint. As inflation is kept constant, the opportunity cost of holding money is constant (and low) as well. However, aggregate capital and output still fall simply because the share of unconnected banks - who

[17]This is an artefact of our steady-state analysis in which the Fisher equation holds. An alternative way to think about the adjustment in response to a higher demand for real money balances when the nominal money supply is fixed is that the price level must decrease so that the real money supply increases. That is, increased demand for scarce money balances necessitates deflation in the short-run.

invest less in capital - increases in the economy. As this effect is driven by the change in the relative share of banks in the economy, it is not something that the central bank can affect. Quantitatively, an increase in the share of unconnected banks from 0.58 to 0.79 (pre- to post-2008 average share of secured turnover in total) generates a decline in output of around 0.58 percent.

In sum, reduced access to the unsecured market can reduce investment and output via two channels. First, since unconnected banks need to satisfy withdrawal shocks by holding bonds and/or by holding money, they can invest less in capital. Therefore, as the share of unconnected banks in the economy increases, capital and output decrease. Central bank policy cannot do anything about this channel. Second, as more banks become unconnected, bonds and money can become more scarce, tightening the withdrawal constraint, reducing aggregate deposits, investment in capital and, consequently, output. Central bank policy can mitigate the second channel if it provides money to banks at a low opportunity cost by maintaining constant low inflation (QE policy). When the share of unconnected banks changes from 0.58 to 0.79, the first channel dominates and therefore there is not much difference between policies. However, if the share of unconnected bank increased to 0.9 (share of secured turnover in total in 2015), then the contraction in output would be 1.5 percent in the constant balance sheet or FRFA case, and only 1 percent in the QE case.

7.2.2 Reductions in collateral value

In this subsection, we analyze the macroeconomic effects of changing collateral value through an increase in private haircuts in the secured market. In this comparative statics exercise, the private haircut moves from the benchmark pre-crisis value of 3 percent to 70 percent. Figures 6, 7 and 8 show the results for the constant balance sheet, the FRFA, and the QE policy, respectively.

In these figures, the solid red line denotes the secured market haircut under our benchmark calibration ($1-\widetilde{\eta}=0.03$). The green dashed lines indicate the level of $1-\widetilde{\eta}$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. The blue dashed lines indicate the level of $1-\widetilde{\eta}$ at which the leverage constraint of unconnected banks turns slack and the multiplier μ_u^{RA} becomes zero. The cyan dashed lines indicate the level of $1-\widetilde{\eta}$ at which unconnected banks start borrowing from the central bank so that the multiplier μ_u^F becomes zero. The magenta dashed lines indicate the level of $1-\widetilde{\eta}$ at which unconnected banks pledge their entire bond holdings at the central bank and no longer use secured market

($b_u = b_u^F$ and the collateral constraint binds). The orange dotted lines indicate the level of $1-\widetilde{\eta}$ at which unconnected banks no longer hold bonds. As we shall see, these five constraints will play a major role in this exercise.

In the calibrated steady-state (at the red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. As the haircut in the secured market increases moving rightward in all figures, it becomes more difficult for unconnected banks to satisfy their liquidity needs in the secured market.

Under the constant balance sheet policy (Figure 6), as private haircuts increase, bond collateral value in the private market decreases and unconnected banks start demanding money to self-insure against afternoon withdrawal shocks (as of $1-\widetilde{\eta} = 0.07$, indicated by the green dashed lines). As the supply of money is fixed under this policy, higher demand for money by unconnected banks is accommodated by the decrease of money holdings by households. This is facilitated by the increase in the deposit rate, which is proportional to inflation. Higher inflation increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts a downward pressure on aggregate capital and, correspondingly, an upward pressure on the return on capital. For the connected banks, this tightens their run-away constraint and, therefore, they reduce their investment in capital and their deposit intake. When the haircut reaches 0.28, unconnected banks are very constrained in the secured market but they cannot increase their money holdings any further as households' money holdings are at a minimum. At this point, unconnected banks become so constrained in the afternoon that they dramatically reduce their deposit intake. Their leverage constraint turns slack. Bond prices collapse. From here onwards unconnected banks' deposit intake and therefore investment in capital continues to fall. Connected banks are able to pick up some of the deposits from unconnected banks but only up to a limit as they are constrained by the leverage constraint. As a result, aggregate deposits, capital and output decline. Quantitatively, an increase in private haircuts from 3 to 40 percent leads to an output contraction of 3.5 percent.

Both the FRFA policy and the QE policy are able to substantially mitigate output contractions in this case by preventing the leverage constraint of unconnected banks from turning slack.

Under the FRFA policy (Figure 7), this is achieved by unconnected banks accessing central bank funding as haircut in the secured market reaches 0.23 (indicated by the cyan dashed lines). Unconnected banks reduce their deposit funding (as their afternoon constraint is tight due to the high secured market haircut) and substitute it with the central bank funding (which is subject to a much more favorable haircut of 0.03). As the central bank provides funding to banks, its balance sheet expands and so does the money supply. Therefore, unconnected bank can further increase their money holdings, without the need for a reallocation of money holdings from households (indeed, households increase their money holdings again as the nominal interest rate declines). As the private haircut increases above 0.38 (indicated by the magenta dashed lines), unconnected banks pledge all their bond collateral at the central bank and stop using the secured market to manage their afternoon liquidity needs, relying solely on money holdings instead. From this point onwards, the economy is insulated from further increases in the secured market haircut. Deposits, capital, and output stabilize. Quantitatively, an increase in the private haircut from 3% to 40% leads to an output contraction of just 0.58%.

Under the QE policy (Figure 8), the central bank prevents the leverage constraint of unconnected banks from turning slack by purchasing bonds and thus increasing the supply of money which helps relax the afternoon constraint of unconnected banks. When private haircut reaches 0.07, unconnected banks start selling bonds to the central banks and - to a much smaller extent - to foreigners. The bond price decreases. When the private haircut reaches 0.15 (indicated by the orange dotted lines), unconnected banks sell off their entire bond holdings to the central bank and choose to hold money instead to satisfy the afternoon constraint. From this point onwards, the economy is insulated from further increases in the secured market haircut. Deposits, capital, and output stabilize. Quantitatively, an increase in the private haircut from 3% to 40% leads to an output contraction of just 0.05 percent. Note that the output drop is even lower than under the FRFA policy as the output stabilization is achieved much sooner (for a lower level of the haircut).

In sum, the key to stabilizing output when haircuts in the private market increase is to expand the central bank balance sheet either through a provision of collateralized loans to banks (using more favorable haircuts and the FRFA policy) or through bond purchases which replace bonds that become less valuable as collateral in the private market with money so that banks can self-insure against liquidity shocks (QE policy). Both of these policies prevent

the leverage constraint from turning slack and mitigate the reduction in deposits, capital and output.

8 Conclusions

We presented a general equilibrium model in which banks can fund themselves through deposits or through collateralized central bank loans. Deposits funding is subject to withdrawal shocks which can be managed in the unsecured or secured interbank money markets. We calibrated the model to the euro area data and used it to evaluate the macroeconomic impact of recent developments in the European money markets. In particular, we investigated the impact of the reduced access of banks to the unsecured market, higher haircuts in the secured market, and increased scarcity of high-quality collateral assets.

Our findings show that the availability of secured funding mitigates the adverse macroeconomic impact of reduced access to the unsecured market. However, when high haircuts or safe asset scarcity make it difficult for banks to shift to secured funding, output contractions can be substantial.

The central bank can play a key role in shielding the economy from money market frictions. It can mitigate the fall in capital and output by expanding its balance sheet. In our numerical experiments, a policy of QE that aims at stabilizing inflation is more effective than a policy of fixed rate full allotment of liquidity.

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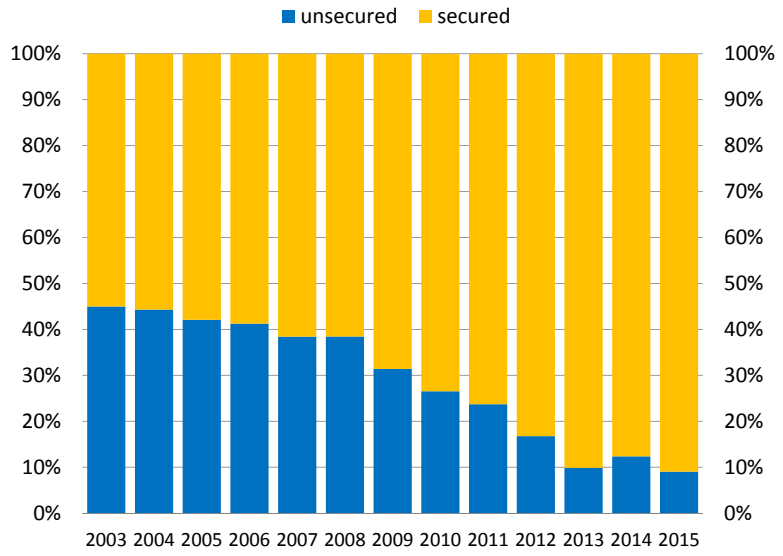


Figure 1: Shares of unsecured and secured money market transactions in total

Breakdown of the cumulative quarterly turnover in the euro area unsecured and secured money market segments (percentages of total). Source: Euro Area Money Market Survey 2015. The survey was conducted once a year, with each data point corresponding to the second quarter of the respective year. The panel comprised 98 euro area credit institutions. The survey was discontinued in 2015.

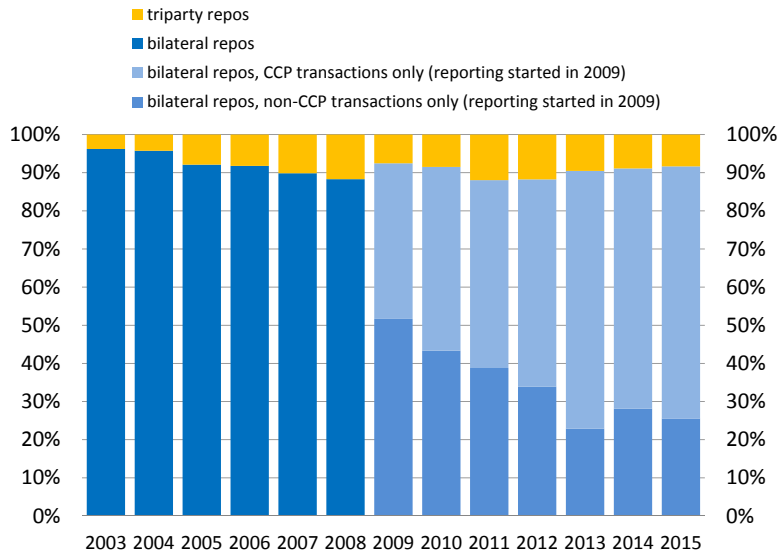


Figure 2: Shares of bilateral, triparty and CCP-cleared secured transactions in total

Breakdown of total secured market (percentages of total). Source: Euro Area Money Market Survey 2015. The survey was conducted once a year, with each data point corresponding to the second quarter of the respective year. The panel comprised 98 euro area credit institutions. The survey was discontinued in 2015.

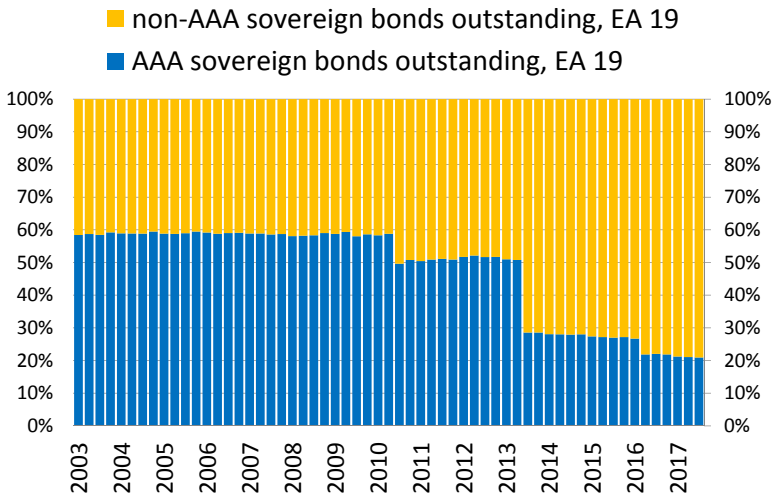


Figure 3: Share of safe (AAA-rated) euro area government debt in total

Breakdown of euro area government debt outstanding according to the credit rating (percentages of total). Country is taken as AAA-rated if the country is AAA-rated by at least one of the following three rating agencies: Moody's, Fitch, S&P. The kinks in the chart correspond to dates when specific countries moved from "at least one AAA" to "no AAA". This happened in 2009 Q3 for Ireland, in 2010 Q3 for Spain, in 2013 Q3 for France, and in 2016 Q2 for Austria. Source: ECB.

Table 1: ECB vs private haircuts on sovereign bonds

	ECB		Private	
	CQS1-2	CQS3	Germany	Portugal
2010	2.8	7.8	2.7	8.1
2011	2.8	7.8	3.0	10.1
2012	2.8	7.8	3.0	80.0
2013	2.8	7.8	3.0	80.0
2014	2.2	9.4	3.0	80.0

ECB haircuts: CQS1-2 refers to sovereign bonds with credit quality 1 and 2, corresponding to a rating AAA to A-; CQS3 refers to bonds with credit quality 3, corresponding to a rating BBB+ to BBB-. Private haircuts: column 'Germany' refers to an average haircut on bonds from Germany, France, and the Netherlands. Source: ECB and LCH Clearnet.

Table 2: Calibration targets

Variable	**Data**	**Model**
Debt/GDP	0.57	0.57
Bank leverage	6.00	6.06
Govt bond spread (annual)	0.008	0.008
Share bonds unconnected banks	0.23	0.24
Share bonds foreign sector	0.64	0.60
Inflation (annual)	0.02	0.02

Table 3: Parameter values

Parameter	**Description**	**Value**
θ	Capital share in income	0.33
δ	Capital depreciation rate	0.02
β	Discount rate households	0.994
χ^{-1}	Coefficient in households' utility	0.005
g	Government spending	0.181
κ^{-1}	Average maturity bonds (years)	6.0
ϕ	Fraction net worth paid as dividends	0.038
ξ	Fraction banks with access to unsecured market	0.42
$\widetilde{\eta}$	Haircut on bonds set by banks	0.97
η	Haircut on bonds set by central bank	0.97
λ	Share of assets bankers can run away with	0.149
$\omega^{\max}$	Max possible withdrawal as share of deposits	0.1
B_C	Bonds held by central bank	0.93
B^*	Stock of debt	6.0
ϱ	Parameter foreign bond demand	0.176
Q^F	Price central bank loans	0.997

Figure 4: Comparative statics: changing the share of banks with no access to unsecured funding, $1-\xi$, constant balance sheet policy

Red solid lines denote the calibrated steady state. Green dashed lines denote the share of unconnected banks at which the non-negativity conditions on their cash holdings become slack. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation and percent deviation of output from steady-state.

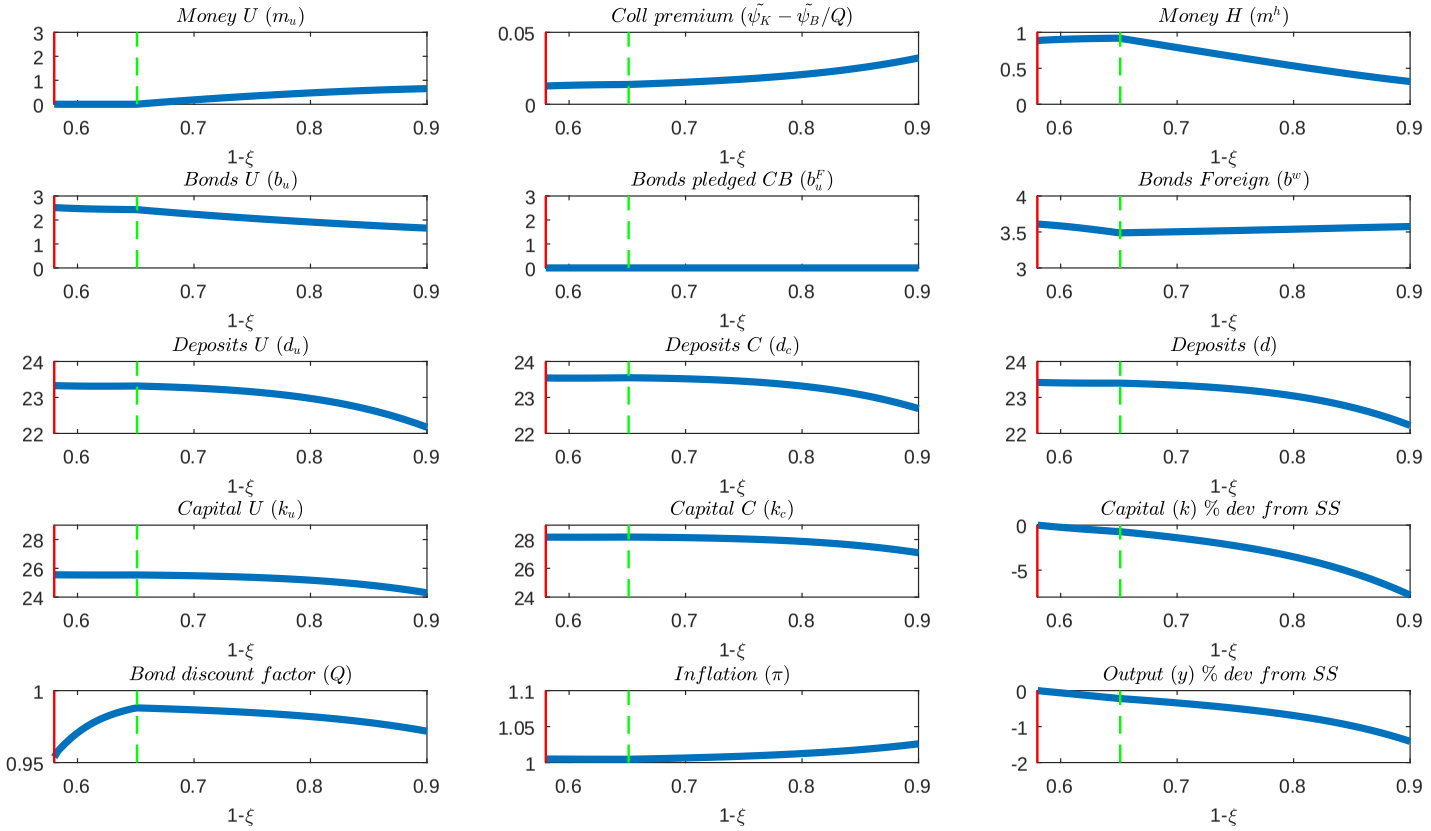


Figure 5: Comparative statics: changing the share of banks with no access to unsecured funding, $1-\xi$, QE policy

Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank's bond holdings, and percent deviation of output from steady-state.

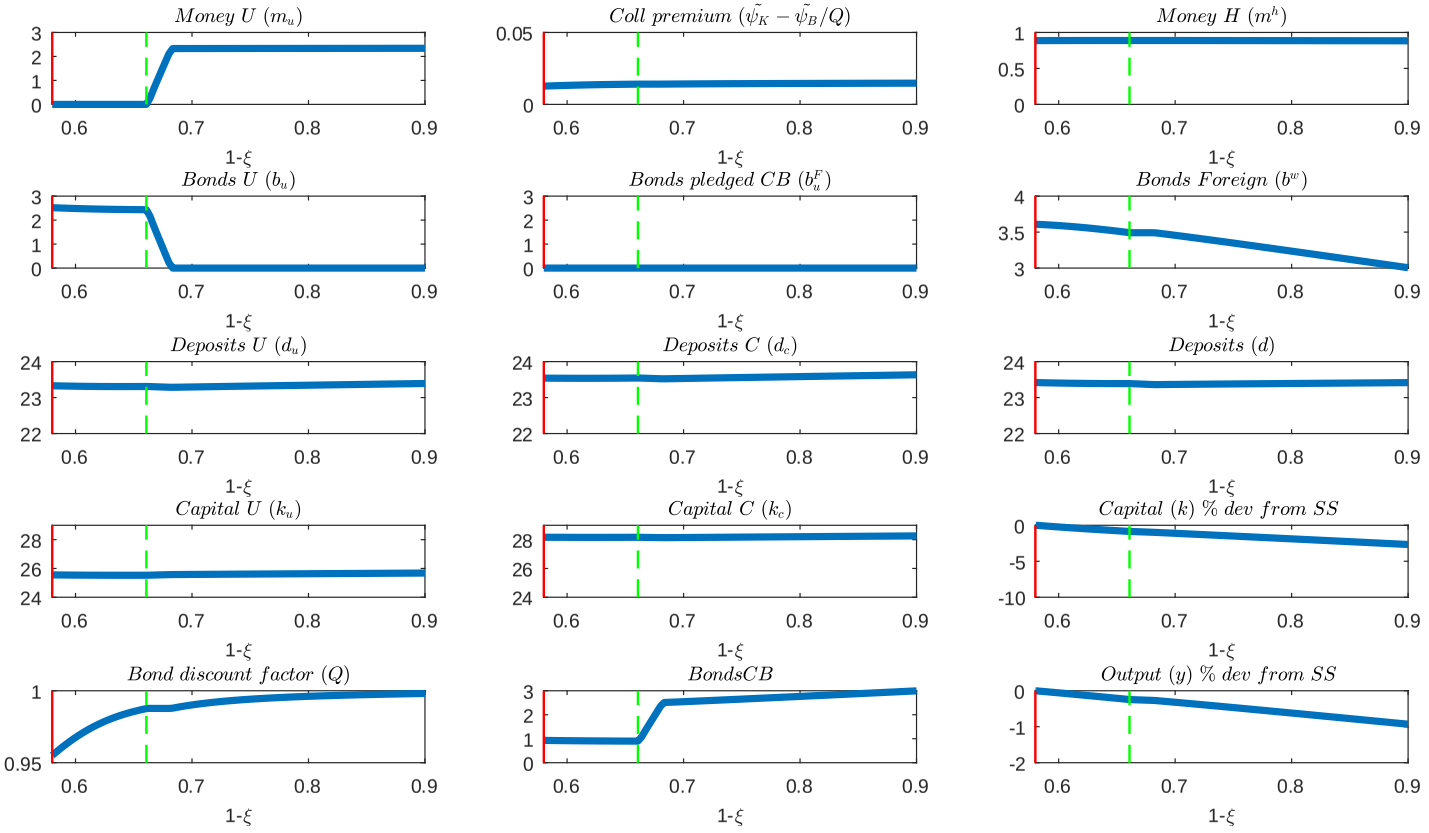


Figure 6: Comparative statics: changing the value of the private haircut, $1 - \widetilde{\eta}$, constant balance sheet policy.

Red solid lines denote the calibrated steady state. Green and blue dashed lines denote the level of private haircuts at which the non-negativity conditions on cash holdings and the run-away constraint of unconnected banks become slack, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation and percent deviation of output from steady-state.

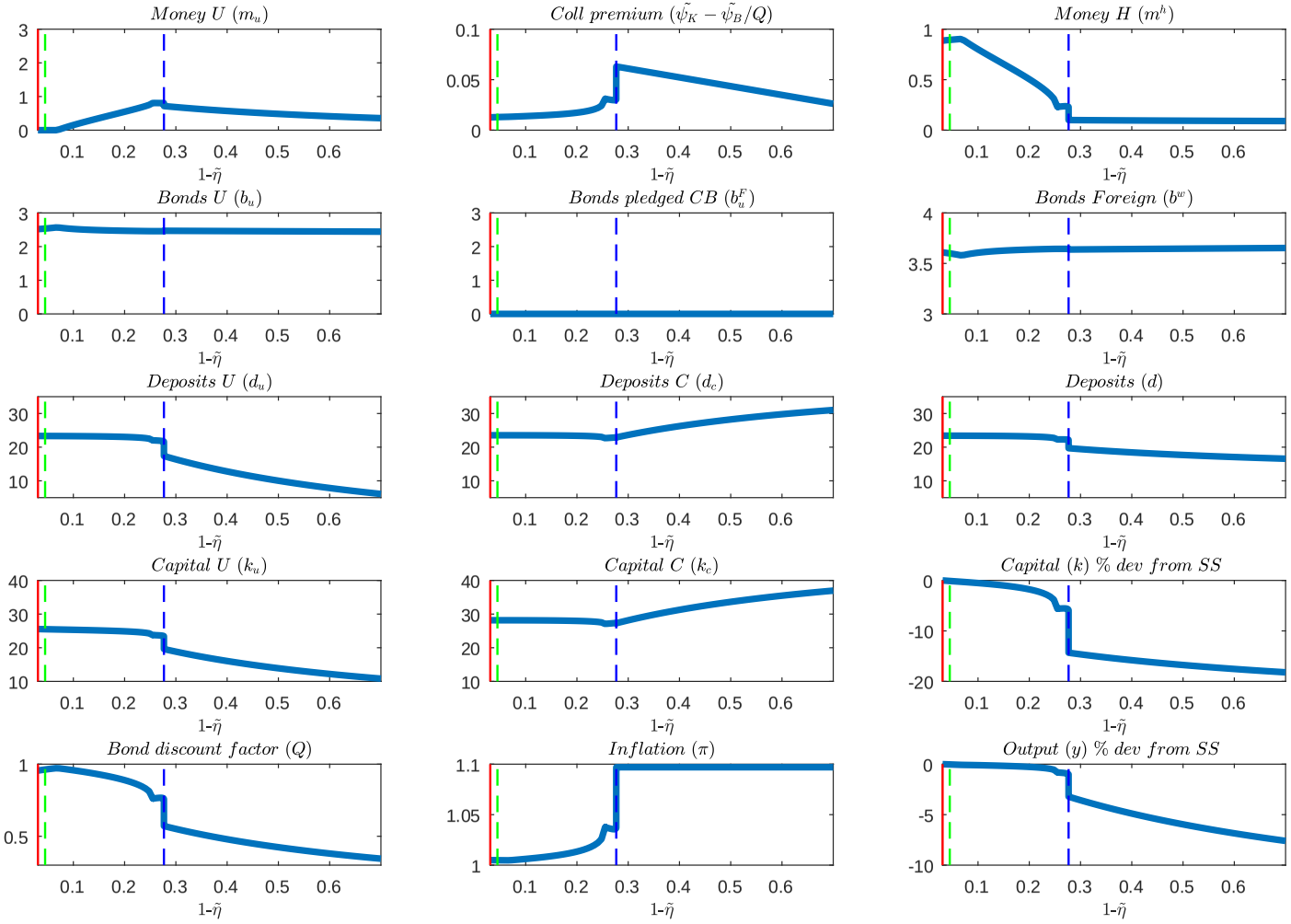


Figure 7: Comparative statics: changing the value of the private haircut, $1-\widetilde{\eta}$, FRFA policy

Red solid lines denote the calibrated steady state. Green, cyan and magenta dashed lines denote the level of private haircuts at which the non-negativity conditions on cash holdings and on bonds pledged at the central bank become slack, and the constraint on bonds pledged at the central bank not exceeding bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation and percent deviation of output from steady-state.

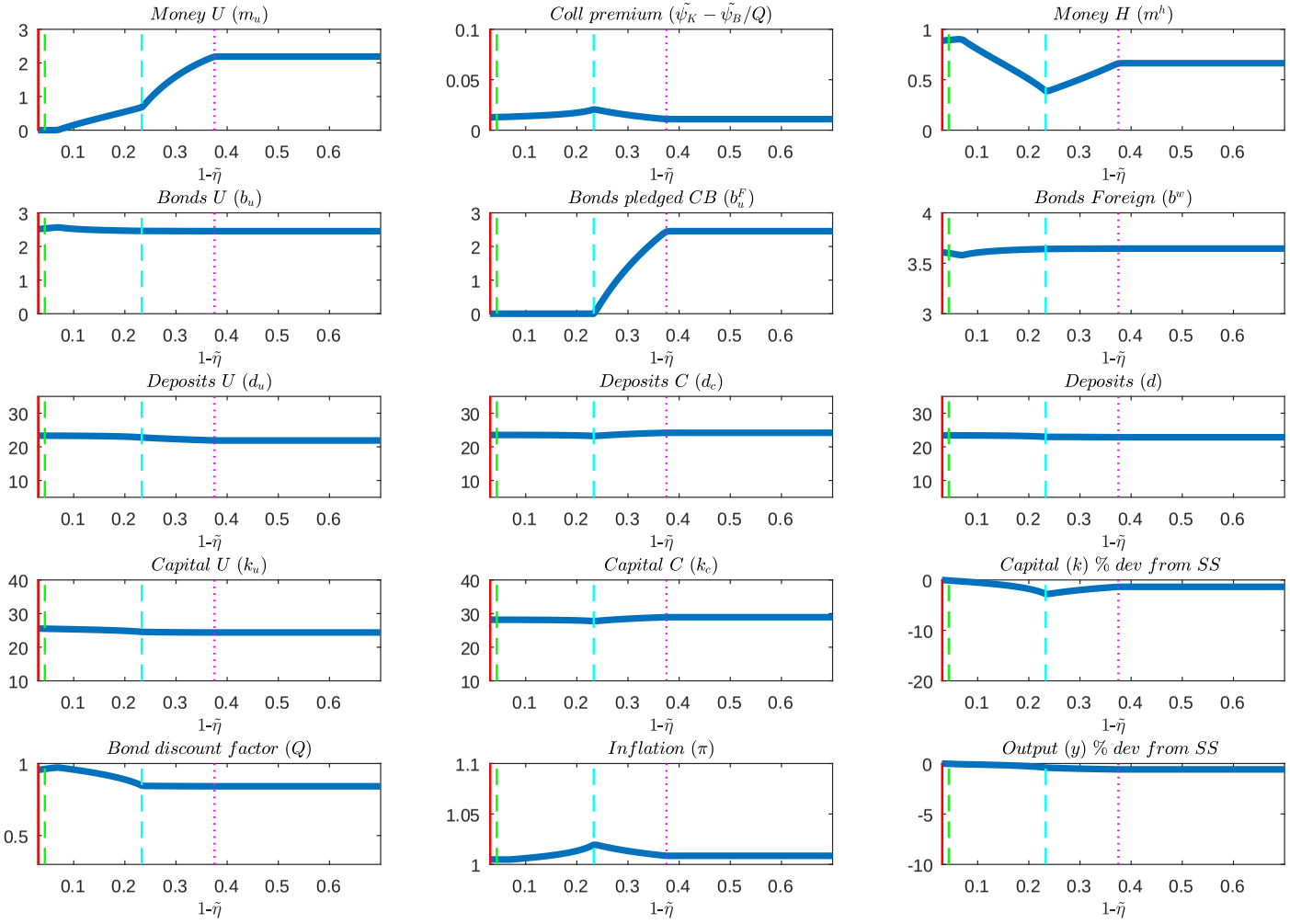
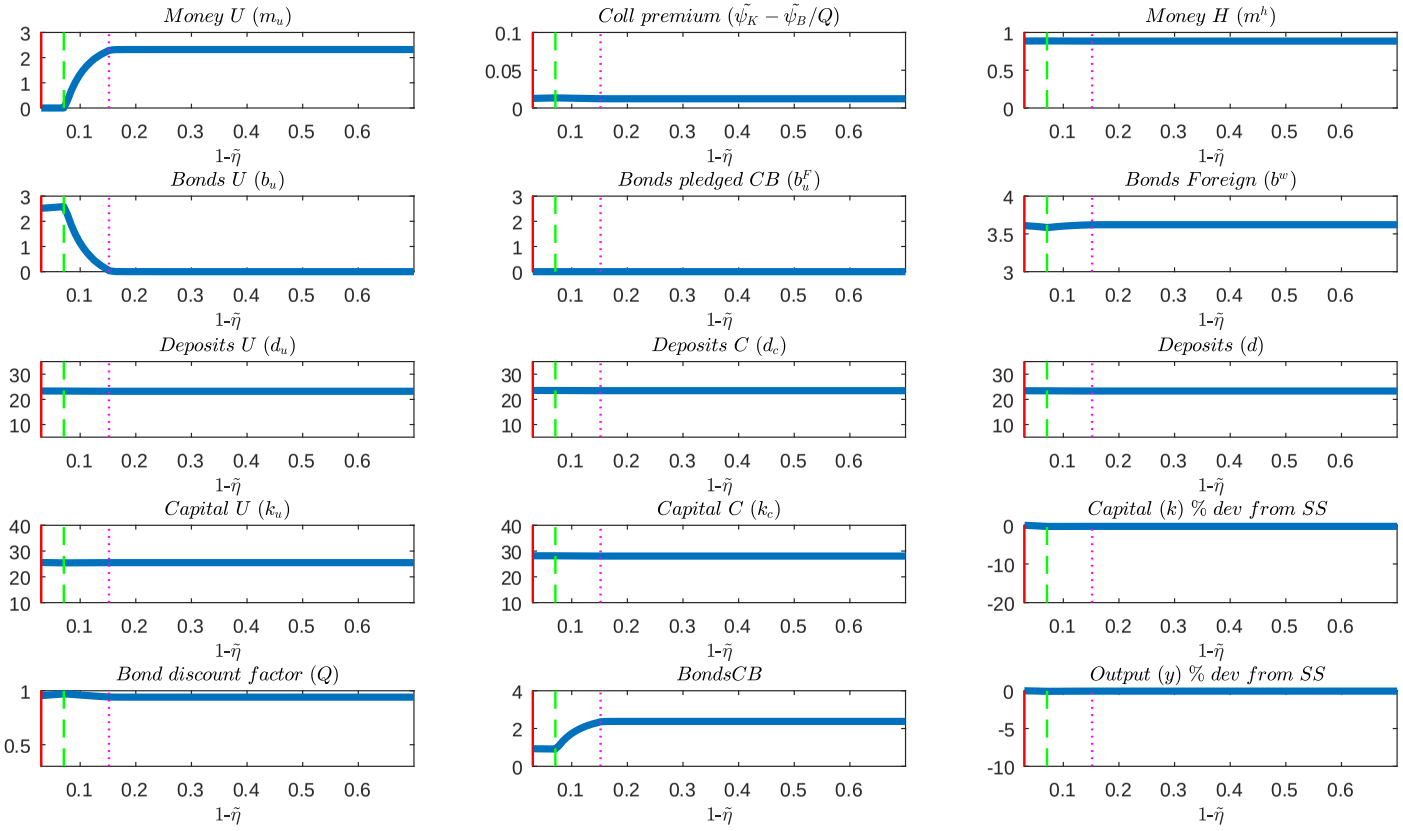


Figure 8: Comparative statics: changing the value of the private haircut, $1-\widetilde{\eta}$, QE policy

Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank's bond holdings, and percent deviation of output from steady-state.



A The equations characterizing the steady state

We characterize the steady state of the model. For simplicity, we focus on the case when capital is not accepted as collateral at the central bank, $\eta^K = 0$. Recall that we already assume that capital is not accepted as collateral in the private secured market, $\widetilde{\eta}^k = 0$.

Define a generic variable as the corresponding capital letter variable, divided by the contemporaneous price level, i.e. $x_t = \frac{X_t}{P_t}$. The steady state is characterized by the following conditions:

1. 4 household equations:

$$R^D = \frac{\pi}{\beta}$$

$$-\frac{u_l(c,l)}{u_c(c,l)} = (1-\tau)\, w$$

$$v_M\left(m^h\right) = u_c(c,n)\left(R^D - 1\right)$$

$$c = (1-\tau)\, wl + \left(\frac{1}{\beta} - 1\right)\pi d + (1-\pi)\, m^h + \phi n$$

2. 3 firms' equations:

$$y = \gamma k^{\theta} l^{1-\theta}$$

$$wl = (1-\theta)\, y$$

$$rk = \theta y$$

and

$$I = \delta k.$$

3. 5 central bank equations: 2 equations

$$s = Q^F \overline{f} + Q b^{CB} - \overline{m}$$

$$\overline{m} = \left[Q^F - \kappa^F \frac{1}{\pi}\left(1 - Q^F\right)\right]\overline{f} + \left[Q - \kappa \frac{1}{\pi}(1-Q)\right] b^{CB}$$

plus the value of 3 variables (policy instruments): η, Q^F, b^{CB}.

Note that the seigniorage revenue of the central bank is given by the interest rate payments on its assets:

$$s = \kappa^F \frac{1}{\pi}\left(1 - Q^F\right)\overline{f} + \kappa \frac{1}{\pi}(1-Q)\, b^{CB}.$$

4. 2 government equations:

$$\overline{b} = \overline{b}^*$$
$$\tau^* (1-\theta) y = g + \kappa (1-Q) \frac{\overline{b}^*}{\pi} - Q \left(1 - \frac{1}{\pi}\right) \overline{b}^* - s.$$

where g is exogenous.

5. 4 market clearing equations:

$$\overline{f} = f$$
$$\overline{m} = m + m^h$$
$$\overline{b} = b + b^{CB} + b^w$$
$$y = c + c^w + g + I$$

where the market clearing condition for the goods market (last equation above) is redundant due to the Walras law.

6. 45 bank equations:

8 equations common to c and u banks,

$$\nu = \psi n$$

$$\begin{aligned} n = \max \{0,\ & (r + 1 - \delta) (\xi k_c + (1-\xi) k_u) \\ & + (\xi m_c + (1-\xi) m_u) \frac{1}{\pi} \\ & + ((1-\kappa) Q + \kappa) (\xi b_c + (1-\xi) b_u) \frac{1}{\pi} \\ & - \left(\left(1-\kappa^F\right) Q^F + \kappa^F\right) (\xi f_c + (1-\xi) f_u) \frac{1}{\pi} \\ & - \frac{1}{\beta} (\xi d_c + (1-\xi) d_u) \Big\}, \end{aligned}$$

$$\widetilde{v} = \tilde{\psi}_k k + \tilde{\psi}_B b + \tilde{\psi}_M m - \tilde{\psi}_D d - \tilde{\psi}_F f$$

$$\tilde{\psi}_k = \beta(1-\phi)\psi(r+1-\delta)$$
$$\tilde{\psi}_B = \beta(1-\phi)\frac{1}{\pi}\psi[(1-\kappa)Q+\kappa]$$
$$\tilde{\psi}_D = \beta(1-\phi)\frac{1}{\beta}\psi$$
$$\tilde{\psi}_F = \beta(1-\phi)\frac{1}{\pi}\psi\left(\left(1-\kappa^F\right)Q^F+\kappa^F\right)$$
$$\tilde{\psi}_M = \beta(1-\phi)\frac{1}{\pi}\psi$$

18 equations for $l = c, u$:

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n$$

$$\phi n + \widetilde{v}_l = \lambda(k_l + Qb_l + m_l)$$

$$v_l = \phi n + \widetilde{v}_l$$

$$\widetilde{v}_l = \tilde{\psi}_k k_l + \tilde{\psi}_B b_l + \tilde{\psi}_M m_l - \tilde{\psi}_D d_l - \tilde{\psi}_F f_l$$

$$f_l = \eta Q b_l^F$$

$$\mu_l^F f_l = 0$$

$$\mu_l^M m_l = 0$$

$$\mu_l^C\left(b_l - b_l^F\right) = 0$$

$$\mu_l^B b_l = 0$$

7 equations for unconnected banks:

$$\left(1+\mu_u^{RA}\right)\tilde{\psi}_k = \mu_u^{BC} + \lambda\mu_u^{RA} \tag{65}$$
$$\left(1+\mu_u^{RA}\right)\frac{\tilde{\psi}_B}{Q} + \mu_u^B = \mu_u^{BC} + \lambda\mu_u^{RA} - \mu_u^C - \mu_u\widetilde{\eta} \tag{66}$$
$$\left(1+\mu_u^{RA}\right)\tilde{\psi}_M + \mu_u^M = \mu_u^{BC} + \lambda\mu_u^{RA} - \mu_u \tag{67}$$
$$\left(1+\mu_u^{RA}\right)\tilde{\psi}_D = \mu_u^{BC} - \omega^{\max}\mu_u \tag{68}$$
$$\left(1+\mu_u^{RA}\right)\frac{\tilde{\psi}_F}{Q^F} = \mu_u^{BC} - \mu_u^C\frac{1}{Q^F} + \mu_u^F\frac{1}{Q^F} \tag{69}$$
$$\mu_u^C = \mu_u^{CC}\eta - \mu_u\widetilde{\eta} \tag{70}$$

$$\mu_u\left[\omega^{\max}d_u - m_u - \widetilde{\eta}Q\left(b_u - b_u^F\right)\right] = 0 \tag{71}$$

6 equations for connected banks:

$$\left(1+\mu_c^{RA}\right)\tilde{\psi}_k = \mu_c^{BC} + \lambda\mu_c^{RA} \tag{72}$$

$$\left(1+\mu_c^{RA}\right)\frac{\tilde{\psi}_B}{Q} + \mu_c^{B} = \mu_c^{BC} + \lambda\mu_c^{RA} - \mu_c^{C} \tag{73}$$

$$\left(1+\mu_c^{RA}\right)\tilde{\psi}_M + \mu_c^{M} = \mu_c^{BC} + \lambda\mu_c^{RA} \tag{74}$$

$$\left(1+\mu_c^{RA}\right)\tilde{\psi}_D = \mu_c^{BC} \tag{75}$$

$$\left(1+\mu_c^{RA}\right)\frac{\tilde{\psi}_F}{Q^F} = \mu_c^{BC} - \mu_c^{C}\frac{1}{Q^F} + \mu_c^{F}\frac{1}{Q^F} \tag{76}$$

$$\mu_c^{C} = \mu_c^{CC}\eta \tag{77}$$

6 bank aggregation equations:

$$v = \xi v_c + (1-\xi)\, v_u$$

$$k = \xi k_c + (1-\xi)\, k_u$$

$$d = \xi d_c + (1-\xi)\, d_u$$

$$b = \xi b_c + (1-\xi)\, b_u$$

$$f = \xi f_c + (1-\xi)\, f_u$$

$$m = \xi m_c + (1-\xi)\, m_u.$$

7. 2 rest of the world equations

$$b^w = \varkappa - \frac{1}{\varrho}\log Q_t\pi_t$$

$$Qb^w + c^w = \left[\kappa + (1-\kappa)\, Q\right]\frac{b^w}{\pi}.$$

These are 66 equations (one redundant by the Walras law) in 65 endogenous variables:

$$\left\{ \begin{array}{c} y, k, c, c^w, l, d, n, m^h, b, b^w, f, m, v, \widetilde{v}, \overline{b}, \tau^*, \\ \psi, \tilde{\psi}_k, \tilde{\psi}_B, \tilde{\psi}_M, \tilde{\psi}_D, \tilde{\psi}_F, \mu_u, w, r, Q, R^D, \pi, I, s, \overline{f}, \overline{m} \end{array} \right\}$$

plus

$$\left\{k_l, m_l, f_l, b_l, b_l^F, d_l, v_l, \widetilde{v}_l, \mu_l^F, \mu_l^M, \mu_l^{RA}, \mu_l^{BC}, \mu_l^{CC}, \mu_l^C, \mu_l^B\right\},$$

plus the value of the three policy instruments

$$\eta^A, Q^F, b^{CB},$$

and of the following exogenous variables: g, ξ.

The bank first-order conditions can be further simplified as follows. For the unconnected banks, conditions (72)-(77) can be simplified to:

$$\mu_u \left[\omega^{\max} d_u - m_u - \widetilde{\eta} Q \left(b_u - b_u^F\right)\right] = 0 \tag{78}$$

$$\omega^{\max} \mu_u = \left(1 + \mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_D\right) - \lambda \mu_u^{RA} \tag{79}$$

$$\mu_u^{CC} \eta = \left(1 + \mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_u^B \tag{80}$$

$$\mu_u^M = \left(1 + \mu_u^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) - \mu_u \tag{81}$$

$$\mu_u^F \frac{1}{Q^F} = \left(1 + \mu_u^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) - \omega^{\max} \mu_u + \mu_u^{CC} \frac{1}{Q^F} \tag{82}$$

For the connected banks, conditions (72)-(77) can be simplified to,

$$\mu_c^{RA} = \frac{\tilde{\psi}_k - \tilde{\psi}_D}{\lambda - \left(\tilde{\psi}_k - \tilde{\psi}_D\right)} \tag{83}$$

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_c^{CC} \eta \tag{84}$$

$$\mu_c^M = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \tilde{\psi}_M\right) \tag{85}$$

$$\mu_c^F \frac{1}{Q^F} = \left(1 + \mu_c^{RA}\right)\left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D\right) + \mu_c^{CC} \frac{1}{Q^F} \tag{86}$$

B Proofs

Proof of Proposition 2

We first show that if conditions (62) or (63) hold, we have that $\mu_c^F > 0$ and $f_c = 0$. First note that (86) states that

$$\mu_c^F \frac{1}{Q^F} = \left(1 + \mu_c^{RA}\right) \left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) + \mu_c^{CC} \frac{1}{Q^F}$$

so that, if $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$, we have $\mu_c^F > 0$ and $f_c = 0$. This is the sufficient condition (62).

Now consider the case when $\frac{\tilde{\psi}_F}{Q^F} \leq \tilde{\psi}_D$. Then, we distinguish between two cases: either connected banks choose not to hold any bonds so that $\mu_c^B \geq 0$ or they choose to hold bonds so $\mu_c^B = 0$.

If connected banks choose not to hold any bonds so that $\mu_c^B \geq 0$, then they cannot borrow from the CB as they do not have any collateral to pledge.

If they choose to hold bonds so $\mu_c^B = 0$, then we have, by combining (72), (73), and (77):

$$\left(1 + \mu_c^{RA}\right) \left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) = \mu_c^{CC} \eta.$$

Using this in (86), we get

$$\mu_c^F = \left(1 + \mu_c^{RA}\right) \left[\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) \frac{1}{\eta} + \left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) Q^F \right]$$

If

$$\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) \frac{1}{\eta} + \left(\frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) Q^F > 0$$

holds, then $\mu_c^F > 0$ and $f_c = 0$ and connected banks do not borrow from the CB.

We next show that connected banks do not hold any cash. First-order condition (85) for connected banks implies that whenever

$$\tilde{\psi}_k > \tilde{\psi}_M$$

holds, we have $\mu_{A,c}^M > 0$ and thus $m_{A,c} = 0$. The above condition is equivalent to

$$\theta \frac{y}{\left(\xi k_c + (1 - \xi) k_u\right)} + 1 - \delta > \frac{1}{\pi}.$$

Given (60) and (61), the condition above is always satisfied.

Finally, we show that if the afternoon constraint of unconnected banks binds, $\mu_u > 0$, then a connected bank does not hold any bonds, i.e., $b_c = 0$. Combining (65) with (66), we get

$$\mu_u^B = \left(1 + \mu_u^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_u^C - \mu_u\widetilde{\eta}.$$

Since $\mu_u > 0$, $\mu_u^C \geq 0$ and $\mu_u^B \geq 0$, it follows that

$$\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q} \tag{87}$$

must hold.

Now turning to the connected banks, combine (72) and (73), to get

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) - \mu_c^C. \tag{88}$$

We now show that $\mu_c^C = 0$ must hold. Intuitively, if a bank does not borrow from the central bank, it cannot be collateral-constrained at its borrowing from the central bank. Consider the following complementary slackness conditions in the bank problem:

$$\mu_c^C\left(b_c - b_c^F\right) = 0.$$

Since $f_c = 0$, we have that $b_c^F = 0$ since $f_c = \eta Q b_c^F$ and $b_c^F \geq 0$. Therefore, the above complementary slackness condition simplifies to

$$\mu_c^C b_c = 0.$$

There are two possibilities: either the bond holdings are positive, $b_c > 0$ or they are zero, $b_c = 0$. In the former case, it follows that $\mu_c^C = 0$, which proves the claim. In the latter case, we have $b_c = 0$, and a bank does not hold any bonds, does not pledge any bonds, and is not constrained by the bond collateral constraint, $\mu_c^C = 0$. Therefore, $\mu_c^C = 0$ holds for a bank that does not borrow from the central bank. Therefore, (88) simplifies to

$$\mu_c^B = \left(1 + \mu_c^{RA}\right)\left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right).$$

Since $\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q}$, we have that $\mu_c^B > 0$ implying that $b_c = 0$.

Proof of Propositon 3

We first show that if the afternoon constraint is slack, $\mu_u = 0$, and either condition (62) or condition (63) hold, then unconnected banks do not borrow from the central bank, $\mu_u^F > 0$ and $f_u = 0$. For $\mu_u = 0$, the proof follows the same lines as the proof of Proposition 2 for connected banks since the relevant first order conditions of the unconnected banks simplify to (72), (73), and (77).

We now show that if $\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}$ holds, then unconnected banks do not borrow in the secured market and instead they borrow only from the central bank and hold money, $m_u > 0$. We prove the claim by contradiction: Suppose that $\widetilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}$ and yet unconnected banks use bonds to borrow from the secured market so that $b_u^F < b_u$ and $\mu_u^C = 0$. Since $b_u > 0$, we have $\mu_u^B = 0$. Since $\mu_u^C = 0$, we have by (70) that $\mu_u^{CC} = \mu_u \frac{\widetilde{\eta}}{\eta}$. Using this to substitute out μ_u^{CC} in (80), we have:

$$\mu_u = \left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) \frac{1}{\widetilde{\eta}}$$

By (81), we have

$$\mu_u^M = \left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \tilde{\psi}_M\right) - \mu_u \geq 0$$

so that

$$\left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \tilde{\psi}_M\right) \geq \mu_u.$$

Then, we have that

$$\left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \tilde{\psi}_M\right) \geq \left(1 + \mu_u^{RA}\right) \left(\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}\right) \frac{1}{\widetilde{\eta}}$$

or, equivalently,

$$\widetilde{\eta} \geq \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}.$$

A contradiction.

Since unconnected banks borrow from the central bank, their afternoon constraint binds, $\mu_u > 0$. Since they do not borrow from the secured market, we have $b_u^F = b_u$. The binding afternoon constraint then implies that

$$m_u = \omega^{\max} d_u > 0.$$

This completes the proof.

C Additional comparative statics

In this Appendix, we present results of a comparative statics exercise which aims to capture the effects of safe asset scarcity, a concern which became particularly pronounced in the aftermath of the Global Financial Crisis. In the euro area, the share of AAA-rated sovereign bonds in GDP declined from 30% pre-crisis to just 14% in 2017 (a country is taken as AAA-rated if the country is AAA-rated by at least one of the following three rating agencies: Moody's, Fitch, S&P). To analyze the macroeconomic effects of this development, the supply of government bonds $\overline{b}$ in our model varies between 6 units (the steady-state level) and 3 units. Figures 9 and 10 show the results for the constant balance sheet and the QE policy, respectively.

In both figures, the solid red line denotes the supply of government bonds under our benchmark calibration ($\overline{b} = 6$). The green dashed lines indicate the level of $\overline{b}$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. The blue dashed lines indicate the level of $\overline{b}$ at which the leverage constraint of unconnected banks turns slack and the multiplier μ_u^{RA} becomes zero. The orange dotted lines indicate the level of $\overline{b}$ at which unconnected banks stop holding bonds. As we shall see, these three constraints will play a major role in this exercise.

In the calibrated steady-state (at the red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. As the supply of government bonds shrinks moving rightward in both figures, it becomes more difficult for unconnected banks to obtain collateralized funding of any kind.

Under the constant balance sheet policy (Figure 9), the figures resemble what happens as private haircuts increase. In particular, as bonds become more scarce (as of $\overline{b} = 5.82$, indicated by the green dashed lines), unconnected banks start demanding money to self-insure against afternoon liquidity shocks. As the supply of money is fixed under this policy, higher demand for money by unconnected banks is accommodated by the decrease of money holdings by households. This is facilitated by the increase in the nominal rate (the deposit rate), which is proportional to inflation. Higher inflation increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts a downward pressure on aggregate capital and, correspondingly, an upward pressure on the return on capital. For the connected banks, this tightens their run-away constraint and, therefore, they reduce their investment in capital and their deposit intake. When the supply of

bonds drops to 4.97 units, unconnected banks are very constrained in the secured market but they cannot increase money holdings any further as households reduced their money holdings to a minimum. At this point, unconnected banks become so constrained in the afternoon that they must reduce their deposit intake. Their leverage constraint turns slack. Connected banks are able to pick up some of the deposits from unconnected banks but only up to a limit as they are constrained by the leverage constraint. As a result, aggregate deposits, capital and output decline. Quantitatively, if the stock of bonds is halved, output contracts by 3.4 percent.

Under the FRFA policy, the outcome is the same as under the constant balance sheet policy. This is because providing collateralized central bank funding through FRFA when bonds are scarce cannot mitigate output contractions.

By contrast, QE policy is very effective in stabilizing output in this case. It achieves this by substituting scarce bonds with another liquid asset - money - while maintaining the opportunity cost of holding money low. Specifically, as the stock of bonds declines from 6 to 5.7, unconnected banks sell their entire bond holdings to the central bank. From this point onwards, further decrease in the stock of government bonds is reflected only in the reduction in foreign bond holdings. Quantitatively, if the stock of bonds is halved, output drop is only 0.1 percent under the QE policy.

Figure 9: Comparative statics: changing the stock of government bonds, $\overline{b}$, constant balance sheet policy

Red solid lines denote the calibrated steady state. Green, blue and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack, their run-away constraint becomes slack, and the non-negativity condition on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation, and percent deviation of output from steady-state.

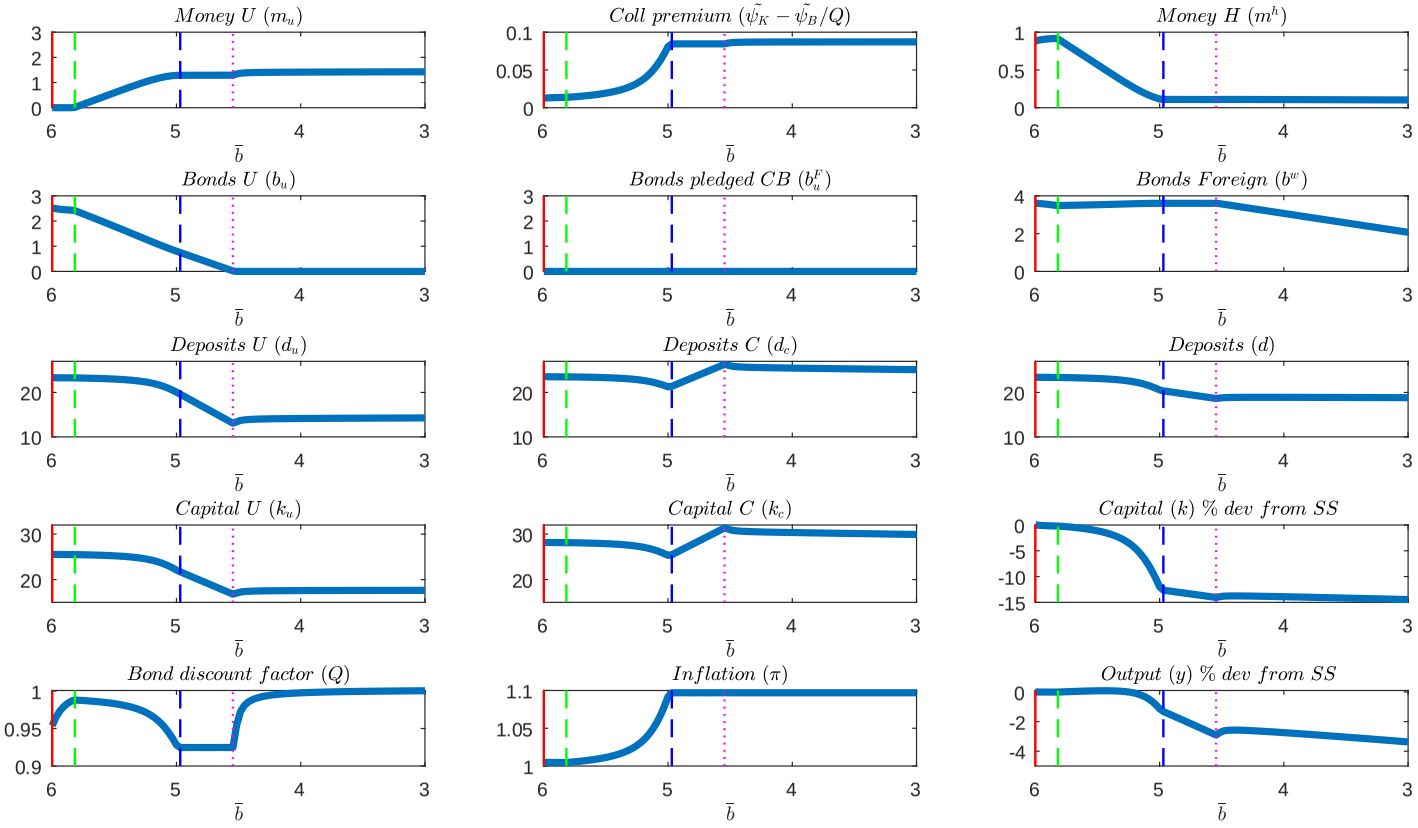


Figure 10: Comparative statics: changing the stock of government bonds, $\overline{b}$, QE policy

Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation, and percent deviation of output from steady-state.

