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Tail-Risk Dumping

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ABSTRACT

This paper analyzes a banking crisis and related policy issues from a viewpoint of equilibrium contracts by providing a micro-foundation of typical macro models with financial frictions. Namely, it characterizes equilibrium contracts and their consequences in a simple one-period general equilibrium model with ex ante identical agents who endogenously choose to be depositors, borrowers, and bankers. Assuming costly state verification, the equilibrium loan and deposit contracts become a standard debt type. Limited liability with a simple asset seizure rule makes the equilibrium bank capital to be positive, creating a sizable banking sector. In equilibrium, income is shared incompletely among depositors, borrowers, and bankers, unlike "big household" assumption in typical macro models. In particular, when a large negative productivity shock hits, borrowers and bankers walk away with predetermined retained assets. Then, depositors have to assume all the tail risk (*tail-risk dumping*). Here, bank bailouts that insure deposits by consumption tax contingent on ex post shocks are welfare improving. This optimality of bailouts relies on the assumption that borrowers can walk away easily from their debts. However, a simple and speedy bankruptcy procedure is itself optimal if otherwise a debt overhang problem emerges.

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I. INTRODUCTION

To analyze a banking crisis, a general equilibrium perspective is important since systemic importance has been stressed as a major reason for bank bailouts in many countries. Also, because regulations and bailout expectations affect the banking sector size, policy implications are better to be studied recognizing endogeneity of the banking sector size. However, in typical general equilibrium macroeconomic models with financial frictions, income risks are assumed to be shared among bankers, borrowers, and depositors, with population of each type exogenously given (e.g., Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Gertler (2010)).[2] Although such a fictitious "big household" assumption is convenient to track macroeconomic dynamics, it is not usual in reality that typical households share their income with bankers. Also, the big household assumption is not theoretically consistent with financial frictions that govern the lending activities in these models.

To understand a banking crisis and related policy issues from a viewpoint of equilibrium contracts, this paper provides a micro-foundation of typical macro models with financial frictions. Namely, it characterizes equilibrium contracts and their consequences in a simple one-period general equilibrium model with depositors, borrowers, and bankers. Assuming costly state verification, the equilibrium loan and deposit contracts become a standard debt type. Limited liability with a simple asset seizure rule, which is often assumed in the literature, makes income sharing incomplete among depositors, borrowers, and bankers, unless the "big household" assumption were made. In particular, when a large negative productivity shock hits, borrowers and bankers walk away with predetermined outputs. Then, depositors have to assume all the tail risk (*tail-risk dumping*).

In this paper, ex ante identical agents choose to work either in the production sector or the banking sector in the beginning of the period. As a result, the expected utility of bankers should equate with those of entrepreneurs in the production sector. Entrepreneurs with a high talent borrows capital from bankers while those with a low talent deposits. Loan and deposit contracts determine how risks are shared among three types of agents. In equilibrium, the bankers own positive capital with which they provide a partial insurance for depositors against aggregate shocks. As the insurance premium, the bankers obtain income from a spread between the deposit and loan rates.

Here, bank bailouts that insure deposits by consumption tax contingent on ex post shocks are welfare improving, even ex ante. This optimality of bailouts relies on the limited liability with a simple asset seizure rule. This is a typically-made assumption in the literature, which allows borrowers to walk away easily from their debts. However, this paper also argues that such a simple and speedy bankruptcy procedure is itself optimal if otherwise a debt overhang problem emerges.

In summary, this paper identifies a reason why many governments ended up to bail out many banks in crises. Moreover, ex ante optimality validates to set up a resolution fund permanently. It is a way to protect depositors against large negative aggregate shocks when the legally allowed retained assets by borrowers and bankers are unconditional on the aggregate output levels. When a tail risk event hits, defaulted borrowers and bankers can still enjoy consumption levels protected by the limited liability but depositors are not (*tail-risk dumping*). By transferring the goods from defaulters to depositors using consumption tax, a government can mitigate the incomplete risk sharing arrangement embedded in the bankruptcy procedure that is efficient in normal times.

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[2] Often banks are not well separated from firms in the literature.

The literature so far identified a few different justifications for bank bailouts. In many theoretical models, as in this paper, distinction between bank depositors and other creditors are not well delineated. Hence, theories to support deposit insurance can also support bailouts. A seminal paper for the need of deposit insurance is Diamond and Divbig (1983). Because a bank borrows short-term funds (deposits) but lends to long-term projects, it cannot repay to all the depositors at once if all of them asked to do so. Knowing this, if depositors expect a bank run, they would try to withdraw their deposits as fast as possible. In this sense, a bank run becomes a self-fulfilling equilibrium. Deposit insurance can eliminate depositors' incentive to withdraw deposits even if they know many other depositors would do so. He and Xiong (2012) extended this analysis to the market based funding. Depending on parameter values, they support public protection of investors in a short-term funding market. However, such protection is never used in equilibrium.

Another reason to bail out banks is its ex post optimality due to a (reduced form) cost of bankruptcy and debt overhang in theory (e.g., Chari and Kehoe, 2013).[3] Indeed, several empirical papers (e.g., Ashcraft, 2005; Peek and Rosengren, 2000) found sizable aggregate costs stemming from banks' bankruptcies. For those mechanisms that thin capital is the source of problem, the recapitalization or other form of bank restructuring would be beneficial to the economy. However, theoretically, the recapitalization by bank itself is often blocked by the shareholders because it benefits mostly debt holders and dilute shareholders' values (Landier and Ueda, 2009). In particular, when the default is imminent, public recapitalization may be worth to pursue (Philippon and Schnabl, 2013). Empirically, some argues that the Japanese lost decade is a result of reluctant government involvement on decisive recapitalization (Hoshi and Kashyap, 2010). The situation may be similar in the recent southern European countries (IMF, 2013). Although numerous papers implicitly or explicitly argue the needs for government intervention to bank restructuring in crisis, to the best of my knowledge, theoretical argument (i.e., better sharing of tail risks) proposed by this paper is a novel one, not articulated before in the literature.[4]

This paper also endogenizes the banking sector size. In most of the macroeconomic models with financial frictions, bank defaults are absent and the capital ratio is not determined endogenously. If the banking sector size were exogenously given in my model, then it would be difficult to identify a full scale of distortions. For example, the capital adequacy ratio requirement would create higher monopoly rents for bankers in an exogenously given banking sector, but such rents would dissipate with endogenous entry of bankers. Only a few papers have investigated the endogenous nature of the financial sector size. The U.S. financial sector has grown over time with increased bankers' wage that compensates increased bankers' income risk (Phillippon, 2008). In an occupational choice model, Bolton, Santos, and Scheinkman (2011) argues that the financial traders attract too many talents due to profitable opportunities in the opaque OTC market. Their papers apparently bring important arguments but have little to say about policies towards deposit-taking banks.

Because it relies on specific financial frictions, this paper obviously misses other important issues relating to bank bailouts, in particular, the moral hazard related to making low efforts or diverting

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[3] A deeper argument of debt overhang problem is as follows. If it is close to bankruptcy, a bank may not lend to profitable projects (Myers, 1977) or it may lend to highly risky projects only (Jensen and Meckling, 1976).

[4] Green (2010) is related. In a model similar to Diamond and Dibvig (1983) but with production by firms (no banks in his model), the only available contract is subject to limited liability. A better contract in terms of incentive to induce higher production is the contract without limited liability. The difference needs to be fill in by the government's tax-subsidy system.

funds by bank mangers. This does not imply that the moral hazard is not important. Rather, when making an actual policy decision, all the issues on bailouts, including the moral hazard, should be carefully considered. Indeed, since Karecken and Walles (1977), the moral hazard of too much risk taking by banks due to deposit insurance and other forms of government protections have been well recognized and called for the regulations. Moreover, Calomiris and Gorton (1991) argue, based on historical evidences, that the bank runs often associated with insolvency and not characterized as random events that models with self-fulfilling equilibria would imply. Demirgüç-Kunt and Detrageache (2002) show more formally that crisis probability is not reduced by the presence of deposit insurance in their regression analysis. Again, the model explored in this paper should be considered as finding one more aspect of the multifaceted nature of the bank bailouts. Still, this paper also shows that, with an explicit bailout expectation, welfare is improved ex ante and more risks are taken but there is no need for a prudential regulation. This prediction departs from previous literature, which often admits ex post optimality of bailouts but recommend ex ante intervention by regulations to control bank risk taking.

Section II sets up the model. Section III studies the optimal contracts. Section IV establishes and characterizes the decentralized equilibrium. Section V shows the socially optimal allocation. Section VI discusses policy implications. Section VII concludes.

II. MODEL SETUP

A. Demography, Utility, and Technology

I analyze a simple one-period model to understand the basic characteristics of a simple default procedure in allocating factors among depositors, borrowers, and bankers. A continuum of ex ante identical agents lives in the interval of $[0, 1]$, as if they are indexed, and endowed with the same initial capital $k_0 > 0$. An agent chooses to become an entrepreneur or a banker endogenously. An entrepreneur then becomes either a depositor or a borrower depending on his talent. I denote bankers' population by μ and entrepreneurs' by $1 - \mu$. I assume bankers locate from 0 to μ on the unit line.

Bankers intermediates the capital market in this paper. In a Walrasian equilibrium, in tradition of Arrow and Debreu (1954), there would be an auctioneer who offers price and matches demand and supply. This paper departs from the Walrasian setup in a few ways.[5] First, instead of an auctioneer, there is a continuum of nonatomic bankers who intermediate capital markets. Second, a banker offers a more general form of "price" in the capital market, that is, deposit and loan repayment schedules. Given the deposit and loan contracts that specify repayment schedules, entrepreneurs decide the amounts of deposits and loans. After the production takes place, goods are allocated among borrowers, depositors, and bankers in accordance with the repayment schedules.

Both entrepreneurs and bankers have the same utility function from consumption. Each agent maximizes the expected utility $E[u(c)]$. For the sake of simplicity, I assume the constant relative risk aversion, that is, $u(c) = c^{1-\sigma}/(1-\sigma)$ with positive relative risk aversion parameter $\sigma > 0$. Note

[5]The equilibrium concept can be regarded as a variant of Prescott and Townsend (1984 a,b) or Ueda (2013).

that, the utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing $u' > 0$ and concave $u'' < 0$ and satisfies the Inada conditions.

Once an agent becomes an entrepreneur, he observes his talent shock e to carry out production.[6] After observing his talent e, he makes an investment decision on endowed capital $k_0 \in \mathbb{R}_{++}$. He has an option to make a financial decision how much to make a deposit $s \in [0, k_0]$ or to take a loan $l \in \mathbb{R}_+$. This capital reallocation is assumed to be intermediated by bankers. An entrepreneur then invest capital and produce outputs. Production is subject to aggregate and idiosyncratic productivity shocks.

In total, there are three types of shocks that each entrepreneur faces: the idiosyncratic talent shock e from the cumulative distribution $F(e) : [\underline{e}, \overline{e}] \to [0, 1]$ with mean one and $\underline{e} > 0$; the idiosyncratic productivity shock ϵ from the distribution $H(\epsilon) : [\underline{\epsilon}, \overline{\epsilon}] \to [0, 1]$ also with mean one and $\underline{\epsilon} > 0$; and the aggregate productivity shock A from the cumulative distribution $G(A) : [\underline{A}, \overline{A}] \to [0, 1]$ with mean greater than one and $\underline{A} > 0$. Without loss of generality, I assume only two levels of talent, e^U and e^D (i.e., up or down) with equal probability $1/2$. Note that $e^U > 1 > e^D > 0$ and $(e^U + e^D)/2 = 1$.

The production function is Cobb-Douglas with capital share $0 < \alpha < 1$ as in a standard macroeconomic model. One unit of labor is assumed to be inelastically supplied by each agent to his own project. The output $y^i \in \mathbb{R}_{++}$, $i \in U, D$, is then expressed as

$$\begin{aligned} y^U &= y(l, e^U, A, \epsilon) = \epsilon A e^U (k_0 + l)^\alpha \quad \text{for those take loans } l; \\ y^D &= y(s, e^D, A, \epsilon) = \epsilon A e^D (k_0 - s)^\alpha \quad \text{for those make deposits } s. \end{aligned} \tag{1}$$

The production function exhibits diminishing marginal returns to capital. Entrepreneurs who received the high talent e^U have higher expected marginal returns on the endowed capital k_0 than the expected loan repayment. Accordingly, they would like to borrow capital l from bankers until the expected marginal returns equate to the expected loan repayment. From a banker's point of view, she lends L to firms. On the other hand, entrepreneurs with low talent e^D have lower expected marginal returns on the endowed capital k_0 than the effective deposit rate. They will deposit some of their endowed capital s to bankers and operate more productive activities in a smaller scale. From a banker's point of view, her deposit intake is S. A depositor's expected marginal return from own business should become equal to the expected deposit return. Note that, with a risk averse utility function, deposit and loan amounts are affected by risk sharing considerations and so do the expected deposit return and loan repayment in equilibrium. With a positive spread between the deposit and loan rates, some entrepreneurs might not engage in transactions with banks. However, in the case with two talents, a sufficient difference between the two are assumed so that the high talent type always become borrowers and the low type always become depositors for a reasonable range of the spread.

After production takes place, a borrower transfers outputs to a banker according to a potentially nonlinear loan repayment schedule, $R^L(l, A, \epsilon) : \mathbb{R}_+ \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \to \mathbb{R}_+$, which is a gross return rate to lenders and potentially depends on the borrower's loan amount. The function space from which R^L is chosen is denoted by Λ. If there is a flat rate portion in the repayment schedule, the return rate is $(1 + \rho^L)$, where $\rho^L \in \mathbb{R}_+$ denote a promised loan rate. For the sake of simplicity and

[6] This e can be also regarded as a business *idea*.

without loss of generality, I assume that capital inputs depreciate 100 percent after they are used in the production process.[7]

A banker can pool idiosyncratic loan repayments from borrowers and thus the deposit repayment depends only on the aggregate shock A. A banker transfers outputs to a depositor according to a potentially nonlinear deposit repayment schedule, $R^D(S, A) : \mathbb{R}_+ \times [\underline{A}, \overline{A}] \to \mathbb{R}_+$, which is a gross return rate to depositors and potentially depends on the bank's deposit intake. The function space from which R^D is chosen is denoted by Δ. If there is a flat rate portion, the return rate is $(1 + \rho^D)$, where $\rho^D \in \mathbb{R}_+$ denotes a promised deposit rate.

For the sake of simplicity, I focus on a symmetric equilibrium, in which all bankers attract the same number of depositors and borrowers.[8] Still, in an off-equilibrium, heterogeneous offers of loan and deposit contracts by bankers are allowed. I denote $\Omega \in \Lambda^\mu$ as the set of loan contract offers from all the bankers and $\Psi \in \Delta^\mu$ as the all deposit contract offers.[9]

At the end of the period, a borrower repays R^L, which includes the principal l, from his own output y^U. A depositor consumes the return from his returned deposits R^D and his own product y^D. A banker, however, may not repay the promised deposit rate ρ^D in full and gross return R^D can be even less than one ex post (i.e., the net return can be negative). In the worst case, a depositor receives nothing from a banker, i.e., $R^D = 0$.

Consumption of a high talent entrepreneur is determined by the budget constraint.It is expressed as a function, if selecting loan amount l and loan contract $R^L(l, A, \epsilon) \in \Omega$ from all available offers, $c^U : \mathbb{R}_+ \times \Lambda \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \to \mathbb{R}_+$.

$$c^U(l, R^L, A, \epsilon) = y(e^U, A, \epsilon) - \tau 1_N - R^L(l, A, \epsilon)l \quad \text{for those take loans;} \tag{2}$$

Consumption of a low talent entrepreneur is expressed similarly.

$$c^D(s, R^D, A, \epsilon) = y(e^D, A, \epsilon) + R^D(S, A)s \quad \text{for those make deposits.} \tag{3}$$

More discussions on costs are given in the next section but, without loss of generality, a borrower ends up paying the verification cost τ when he needs to do so. The cost payment is denoted by an indicator function $1_N \in \{0, 1\}$, i.e., it is $1_N = 1$ if paid, otherwise $1_N = 0$. Note that (**??**) also represents the budget constraint for each type with nonlinear price of loans and deposits. I also assume that cost τ is small relative to endowed capital k_0 so that it can be always paid. This assumption is clarified later.

A banker lends loan l to a borrower funded by her own capital $k_0^B = k_0$ and the deposits that she collected. She then earns, and consumes, the spread income. Banker's consumption is determined by the budget constraint and expressed as a function of her own loan and deposit contract offers given

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[7]More generally, any depreciation rate can be assumed in the model. Theoretical implications would still go through, as long as the retained outputs plus assets are larger than the remained capital net of depreciation.

[8]If the number of bankers is finite and possibly attracts different numbers of depositors and borrowers, strategic actions in both deposit and loan markets could produce a complex equilibrium (Ueda, 2013).

[9]Λ^μ denotes the μ-time Cartesian product $\Lambda \times \Lambda \times \cdots \times \Lambda$.

all offers and aggregate shock, i.e., $c^B : [\underline{A}, \overline{A}] \times \Lambda^\mu \times \Delta^\mu \to \mathbb{R}_+$, if entrepreneurs choose her contracts among other offers.

$$c^B(A, R^L, \Omega, R^D, \Psi) = \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(l, A, \epsilon) L dH(\epsilon) - R^D(S, A)S. \tag{4}$$

If a banker's offer is not selected by any depositors, then the banker's consumption is zero, $c^B(A, R^L, \Omega, R^D, \Psi) = 0$.[10] Note again that the repayment function R^L includes principal l.

B. Equilibrium Definition

Before specifying the institutional setup on financial intermediation, a decentralized equilibrium can be defined generally as follows.

Definition 1. *An equilibrium is the capital allocation, l and s, and the consumption allocations represented by deposit contract $R^D(s, A)$ and loan contract $R^L(l, A, \epsilon)$, that satisfy the following conditions:*

- *Given loan offers Ω, a high talent entrepreneur (i.e., a borrower) chooses the best loan contract $R^{*L} \in \Omega$ and simultaneously decides the loan amount $l \in \mathbb{R}_+$ to maximize his expected utility,*

$$V^U(k_0) = \max_{R^{*L} \in \Omega, l \in \mathbb{R}_+} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^U(l, R^{*L}, A, \epsilon)\right) dH(\epsilon) dG(A). \tag{5}$$

  subject to budget constraint (2).

- *Given deposit offers Ψ, a low talent entrepreneur (i.e., a depositor) chooses the best deposit contract $R^{*D} \in \Psi$ and simultaneously decides the deposit amount $s \in \mathbb{R}_+$ to maximize his expected utility,*

$$V^D(k_0) = \max_{R^{*D} \in \Psi, s \in \mathbb{R}_+} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^D(s, R^{*D}, A, \epsilon)\right) dH(\epsilon) dG(A). \tag{6}$$

  subject to budget constraint (3).

- *A banker offers her deposit contract $\tilde{R}^D \in \Lambda$ and loan contract $\tilde{R}^L \in \Delta$ to maximize her expected utility, given the reaction functions of entrepreneurs (i.e., deposit supply and loan demand functions) and other bankers' loan offers Ω and deposit offers Ψ,*

$$V^B(k_0) = \max_{\tilde{R}^L(l,A,\epsilon) \in \Lambda, \tilde{R}^D(s,A) \in \Delta} \int_{\underline{A}}^{\overline{A}} u\left(c^B(A, \tilde{R}^L, \Omega, \tilde{R}^D, \Psi)\right) dG(A). \tag{7}$$

  subject to budget constraint (4).

[10] More generally, double competition in both deposit and loan markets can be analyzed for possibly a different selection by depositors and borrowers (e.g., Ueda, 2013). Here, however, for the sake of simplicity, I assume a symmetric equilibrium in which a banker can lend the same amount of her collected deposits as long as a banker offers the equilibrium deposit and loan contracts.

- *Before the production takes place, capital market clears. In a symmetric equilibrium, a representative banker takes deposits s and make loans l from a representative depositor and borrower, respectively. She also invests her own capital $k_0^B = k_0$ as a part of loans to high talent entrepreneurs. Adjusting the relative size, the resource constraint in the capital market for the representative bank is expressed as,*

$$\frac{1-\mu}{2\mu}l = L = S + k_0^B = \frac{1-\mu}{2\mu}s + k_0^B. \tag{8}$$

- *Bankers offer the same deposit and loan contracts faced by entrepreneurs (i.e., fixed point conditions)*

$$\tilde{R}^L = R^{*L} = R^L, \quad \tilde{R}^D = R^{*D} = R^D. \tag{9}$$

- *After the production takes place, consumption goods market clears for any realization of aggregate shock $A \in [\underline{A}, \overline{A}]$,*

$$\begin{aligned} &\frac{1-\mu}{2}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(c^U(l, R^L, A, \epsilon) + c^D(s, R^D, A, \epsilon)\right) dH(\epsilon) + \mu c^B(A, R^L, \Omega, R^D, \Psi) \\ =&\frac{1-\mu}{2}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(y(e^U, A, \epsilon) + y(e^D, A, \epsilon)\right) dH(\epsilon). \end{aligned} \tag{10}$$

- *The ex ante arbitrage condition for occupational choice to become a banker or an entrepreneur before observing talent $e = e^U$ or e^L holds,*

$$V^B(k_0) = V^E(k_0) \equiv \frac{1}{2}V^U(k_0) + \frac{1}{2}V^D(k_0). \tag{11}$$

Note that, in accounting, the bank balance sheet is reported differently from the banker's budget constraint (4) as well as the resource constraint (8). In equilibrium, a deposit takes a form of debt contract, which will be proved in Section III. B below. In other words, the deposit contract has a flat payment portion (i.e., a promised payment) and a default region. A typical accounting standard uses the market values for assets but the face values for liabilities. Then, the ex ante accounting balance sheet of the representative bank in a symmetric equilibrium is expressed as

$$\frac{1-\mu}{2\mu}\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(l, A, \epsilon) l dH(\epsilon) dG(A) = \frac{1-\mu}{2\mu}(1+\rho^D)s + w(k_0^B, s), \tag{12}$$

where $w(k_0^B, s)$ is the accounting valuation of the net worth. Ex post, depending on the realization of the aggregate shock, the net worth can become tiny as a bank needs to repay deposits in full if not default.

III. FINANCIAL INTERMEDIATION

A. Institutional Assumptions

Because this paper discusses bankruptcy-related issues, the model's institutional setup on financial intermediation needs to allow debt-type contracts with bankruptcy in equilibrium. This paper

essentially follows the costly state verification setup introduced by Townsend (1979), who shows that costly state verification gives rise to a debt contract, flat payments (i.e., honoring the face value) for good realizations of shocks and state-contingent payments (i.e., default and partial recovery) for bad realizations of shocks. Townsend (1979) introduces one friction in the complete state contingent securities market. That is, information regarding which state is realized can be verified only with a cost. Hence, the state contingent return of a security includes additional endogenous contingency, i.e., its return when state is verified and its return otherwise. Apparently, the return becomes insensitive to state realizations when state is not verified. However, in Townsend (1979), the state contingent return in case verification occurs is prescribed in the security at its issuance as in the Arrow-Debreu market. Townsend (1979) also assumes (implicitly) the enforcement of contracts is free, again as in the Arrow-Debreu market. In summary, both explicit and implicit assumptions of Townsend (1979) on the financial market can be summarized as follows.

Assumption 1. ***[CSV - Contingent Contract Regime]***
(a) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ.
(b-0) [Free Contingent Contracts] It is free to write ex ante contingent loan contracts for all possible states supposing the realized state is verified.
(c-0) [Free Enforcement] Repayment by a borrower is enforced freely according to the contract, including default cases.

Note that, in the literature, the verification cost is sometimes assumed to be paid by borrowers (e.g. hiring an accounting firm) or by a bank (e.g., examining documents), who would however charge the cost to the borrower in equilibrium. In either case, the cost ends up to be deducted from the borrower's income and assets in equilibrium.

While Townsend (1979) and many followers assume such a partially state-contingent contract can be written ex ante and enforced freely ex post, the financial crisis literature has been concerned about costs associated with bankruptcy, (e.g.Chari and Kehoe, 2016).[11] Here, I introduce a cost associated with bankruptcy in a simple way. I assume costly negotiation into the original Townsend Regime as follows.

Assumption 2. ***[CSV - Costly Negotiation Regime]***
(a-1) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ to his lender only.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent loan contracts for any possible states.
(c-I) [Costly Negotiation] Once the shock realization is verified, loan repayment by a borrower is decided by Nash bargaining after the negotiation cost T is subtracted from the output. More specifically, for any realization of triple (l, A, ϵ), a borrower-bank pair maximizes its joint surplus, given a bargaining power parameter ξ^L,[12]

$$\max u(c^U)^{\xi^L} u(c^B)^{(1-\xi^L)}. \tag{13}$$

[11]The bankruptcy cost could be considered as a part of debt overhang cost. The bankruptcy cost could have a spillover effect (e.g., Igan, et. al., 2011). However, this paper assumes away the spill over effect for the sake of simplicity.

[12]The cost of state verification and the cost of negotiation can be defined as a part of c^L, borrower's consumption, as in (**??**).

Assumption (c-1), costly negotiation, is the major assumption I introduce here. Assumption (b-1), incomplete contract, is naturally associated with ex post negotiations because writing contingent contracts ex ante for states that will be verified would make ex post negotiation redundant.[13] Assumption (a-1), costly state verification, is almost the same as in the CSV-Contingent Contract Regime, as it needs to make debt-type contracts exist in equilibrium. A slight clarification is added that the verified information remains private to the borrower and the lender who obtained the information. Note that, under Assumption 1, in the original Townsend (1979) model, it does not matter much if the verified information remains to be private for these two parties or becomes public. This is because contingent contracts can be written ex ante for the disclosed information and enforced freely. However, if the information on the realized state becomes public, a third party could involve ex post in the distressed asset market freely. It is not only inconsistent to Assumptions (b-1) and (c-1) but also to the costs paid (and profits enjoyed) by the real world specialists such as loan servicers and vulture funds.[14]

Note that incomplete contract literature (e.g., Hart, 1995), stresses prohibitive costs of specifying all the states and writing contracts for all the contingencies. Although Hart (1995) points out importance of firm ownership (i.e., equity contracts), this paper focuses on debt contracts. Incompletely prescribed contingent returns on verified states naturally calls for ex post negotiation on splitting outputs and assets of the borrowers. In reality, indeed, disputes and lengthy negotiations for a bankrupt entity between creditors and borrowers often occur (e.g., Djankov, et. al., 2008). Such costs associated with disputes and lengthy negotiations, including any real and opportunity looses, should be considered as a bankruptcy cost, and a part of debt overhang costs.

However, regarding the equilibrium allocations, the CSV-Costly Negotiation Regime is not different from the CSV-Contingent Contract Regime. When he needs to do so, a borrower verifies the realized state of the world. Then, he negotiates with the banker to split the borrower's assets and outputs by Nash bargaining, which is by definition Pareto optimal, as it would be written in the contingent contract in the CSV-Contingent Contract Regime. And, this whole process can be done with cost $\tau + T$. Therefore, the goods allocation in the CSV-Costly Negotiation Regime must be the same as in the CSV-Contingent Contract Regime, except for a cost difference by T in case of default. And, it is obvious that the bank bailouts, which introduces a different goods allocation than the market, would not improve the welfare because Townsend (1979) already shows that the general equilibrium allocation in his model, i.e., what I call the CSV-Contingent Contract Regime, is Pareto optimal.

The CSV-Costly Negotiation Regime, however, still misses an important friction that the real world faces. It is a simple and speedy bankruptcy procedure, which is increasingly adopted in the world since around 2000 and especially after the global financial crisis of 2008. For example, Germany and Japan changed its insolvency related laws and precedents at least a few times to adopt US Chapter 11 like debt restructuring law for private entities (e.g., 1999, 2005, and 2012 in Germany; 2000, 2003, and 2006 in Japan). Before such changes, insolvency in these countries means liquidation and lengthy court process was the norm. And, to avoid it, insolvencies often ignited private negotiations,

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[13] Even if any promise is not honored and requires negotiation ex post, without assumption (b-1), a contingent contract for verified states would be written as a negotiation-proof contract. Hence, if contingent contracts can be written freely, it avoids costly negotiation. But, then, the model misses the important friction that focuses on.

[14] I do not model the distressed asset specialists in this paper. However, as long as they have to pay the costs of verification (and negotiation), they can be considered as one function of bankers in my model. Indeed, Diamond and Rajan (2001) argues that the loan collection skill is a key feature of a bank.

without involving a court, between creditors and a borrower. Moreover, the U.S. adopted an even more simpler and speedier mortgage debt restructuring scheme (the Home Affordable Modification Program) in 2009 to expedite massive mortgage bankruptcy cases to settle. Similar plans were adopted in several other countries in the aftermath of th Global Financial Crisis (IMF, 2013).[15] The International Monetary Fund also recommends crisis-hit countries to adopt such simple and speedy bankruptcy procedures (Claessens, et. al, 2014). Indeed, the law-and-finance literature suggests that a speedy bankruptcy regime is growth and efficiency enhancing (e.g., cross-country panel regression studies by Djankov, et. al., (2008) and by Claessens, Ueda, and Yafeh (2014)).

The simple and speedy allocation of outputs and assets in case of default is often assumed in the literature of macroeconomics with financial frictions (e.g. Gertler and Kiyotaki, 2010). The literature often assume that the defaulter can retain a fixed portion of his assets and the rest is given to the creditors. This assumption is not so much differentfrom the relatively new bankruptcy schemes adopted in many countries in the recent past. Therefore, I assume such a bankruptcy regime in this paper as follows.

Assumption 3. ***[CSV-Speedy Bankruptcy Regime]***
(a-1) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ to his lender only.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent loan contracts for any possible states.
(c-2) [Simple Debt Restructuring Rule] When a borrower declares default, his business is seized by a banker except for the outputs that he is allowed to retain. It is simply assumed worth $\lambda > 0$ portion of his invested capital, λk_0.[16]
(d) [Small Cost] $\lambda k_0 + \tau \leq \underline{\epsilon A} e^U k_0^\alpha$.

Assumption (c-2), simple debt restructuring rule is the one I introduce here. This rule creates a potentially non-optimal allocations, but it allows the creditors and the borrowers to save the costs associated with lengthy negotiations, represented by T.[17] It is a simplest way to represent the *raison d'etre* of a speedy bankruptcy scheme. Note that Assumption (b-1), incomplete contract, is the same as in the CSV-Costly Negotiation Regime, and is naturally associated with the simple debt restructuring rule because writing contingent contracts in case of state verification, which is Pareto optimal, would trash the simple but potentially non-optimal rule. Assumptions (a-1) and (b-1) are the same as in the CSV-Costly Negotiation Regime. Under Assumption (b-1), costly negotiations could occur, and thus it lays the basis for a cost advantage of adopting a speedy bankruptcy rule. For the sake of simplicity, Assumption (d) is made so that the retained outputs and the verification cost can be always secured by the borrower's outputs.

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[15] Chapter 11-type debt restructuring procedure is often characterized not only by a simple rule but also debtor in possession, which allows debtors to keep key assets to run a firm as a going concern or, in household bankruptcy cases (Chapter 13 in the US), to keep key assets such as a house to ensure a minimum consumption level. However, the allowed retained asset values can be large. For example, in the State of Florida, the value of the primary residence that can be retained by a defaulted borrower is almost unlimited as long as it is less than half acre.

[16] If the model were to incorporate a dynamic setup, it would make sense to assume that a defaulted borrower retains his capital to continue production as a going concern or keeps his own house.

[17] Without the simple rule, the optimal contract with CSV implies a potentially non-linear risk sharing between two parities when a default occurs. The literature has so far shown that it becomes a straight line when at least one party is risk neutral (see a review by Fulghieri and Goldman, 2008).

Below, I focus on the CSV-Speedy Bankruptcy Regime. Note that the regime is so far defined for the loan market but, in a later section, essentially the same setup will be also assumed for the deposit market, between depositors as creditors and bankers as borrowers.

B. Loan Contract

Assumption 3 (a-1) makes a loan contract to be possibly contingent on each borrower's combined shock if verified, but never on the aggregate shock alone. The assumption implies that the aggregate shock is not public information if no borrower declares default. Moreover, that the verified information remains private to a borrower and his lender implies that a borrower cannot recognize for sure the other borrowers' combined shocks ϵA and thus the aggregate shock A either.

Lemma 1. ***[Townsend]*** *There is a unique default threshold θ^L defined for combined shocks ϵA. It is determined by borrowers for any given loan rate ρ^L and for any repayment schedules that give a defaulted borrower to retain income that is non-decreasing in shock realization.*

The proof follows Townsend (1979) and is omitted. Essentially, the borrowers would not want to pay the cost to verify (and negotiate) for all realizations. To minimize the cost payment, they would do so only when necessary. But, not verifying states implies the repayment schedule to be non-contingent on state, i.e., there is a flat portion on repayment schedule, which can be viewed as the promised fixed interest rate. The borrowers want to verify states only when they cannot repay the promised interest rate., i.e., when they need to declare default.

Lemma 1 and Assumption 3 restricts the equilibrium loan repayment schedule $R^L(l, A, \epsilon)$ as follows.

- Above threshold θ^L, a borrower repays in full, $(1 + \rho^L)$.
- Below threshold θ^L, a borrower defaults with retaining assets λk_0, and thus a defaulter's repayment schedule has an intercept term and a linearly increased portion with respect to the realized (combined) productivity shock ϵA:

$$\frac{\epsilon A e^U (k_0 + l)^\alpha - \lambda k_0 - \tau}{l}. \tag{14}$$

Accordingly, a borrower consumes, if default,

$$c^U(l, A, \epsilon) = \lambda k_0, \quad \text{if} \quad \epsilon A \in [\underline{\epsilon A}, \theta^L], \tag{15}$$

otherwise

$$c^U(l, A, \epsilon) = \epsilon A e^U (k_0 + l)^\alpha - (1 + \rho^L) l, \quad \text{if} \quad \epsilon A \in [\theta^L, \overline{\epsilon A}]. \tag{16}$$

At the threshold, the rational borrower equates consumption from defaulting and that from not-defaulting. This decision is made after the production takes place given some loan size l.

$$\theta^L e^U (k_0 + l)^\alpha - (1 + \rho^L) l = \lambda k_0. \tag{17}$$

Note that a corner solution $\theta^L = \underline{\epsilon A}$ does not happen in equilibrium because a borrower always defaults in this case and thus he cannot borrow. For another corner solution $\theta^L = \overline{\epsilon A}$, the loan contract is an equity, which is not happen as long as $\overline{\epsilon A}$ is large enough so that the largest output exceeds the retained output λk_0. And, this is the case by Assumption 3 (d).

C. Deposit Contract

Regarding defaults of bankers in the deposit market, institutional assumptions on bankruptcy is made in a manner similar to the loan market. However, for the deposit market, even without costly state verification assumption, the equilibrium deposit contract becomes debt type due to the equilibrium loan repayment schedule.

Lemma 2. *There is a flat portion in an equilibrium deposit repayment schedule.*

Proof. As long as a banker transacts with a measurable set of borrowers, banker's consumption is not affected by idiosyncratic shocks of borrowers. By Lemma 1, the loan default threshold and the loan rate is not contingent on the aggregate shocks. Therefore, when all borrowers repay loans in full, bank revenue would be flat, non-contingent on any shocks. Hence, even if a depositor has an equity-type claim on bank revenue, the deposit return shows a flat portion above a certain threshold level of aggregate shock realization. *Q.E.D.*

Other assumptions regarding the bankruptcy regime remains the same.

Assumption 4. *[Speedy Bankruptcy for Bankers]*
(b-1') [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent deposit contracts for all possible states.
(c-2') [Simple Debt Restructuring Rule] When a banker declares default (i.e., does not pay the flat deposit rate as promised), her business is immediately seized by depositors except for the assets that she is allowed to retain. It is assumed worth $\lambda > 0$ portion of her invested capital, λk_0.

The optimal deposit contract can be characterized as below, following Lemma 2 and Assumption 4:

Lemma 3. *The optimal deposit repayment schedule takes a form of a standard debt-type contract. The deposit repayment schedule $R^D(S, A)$ has a flat portion with full-pay deposit rate ρ^D above the threshold θ^D defined for aggregate shock A. Below θ^D is the bank default region, and repayment depends on the realization of aggregate shocks A.*

The banker's deposit repayment schedule $R^D(s, A)$ is defined as follows. If it can, a banker repays full obligation to a depositor:

$$R^D(S, A) = 1 + \rho^D, \quad \text{if } A \in [\theta^D, \overline{A}]. \tag{18}$$

A defaulted banker retains λ portion of their book capital, if possible. The deposit repayment function reflects revenues from borrowers net of retainment assets, correcting for the relative size of the banking sector:

$$R^D(S, A) = \frac{B(A) - \lambda k_0^B}{S}, \quad \text{if } A \in [\theta^B, \theta^D). \tag{19}$$

where θ^B is the minimum aggregate shock level with which a banker has enough revenue to retain λk_0^B. $B(A)$ denote a banker's gross income from borrowers. It can be expressed as a function of the aggregate shocks only, after correcting for the relative size of the banking sector:

$$\begin{aligned} B(A) &= \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(A,\epsilon) L dH(\epsilon) \\ &= \left(1 - H\left(\frac{\theta^L}{A}\right)\right)(1+\rho^L)L \\ &+ \int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \left(\epsilon A e^U (k_0+l)^\alpha - \lambda k_0 - \tau\right)\frac{L}{l} dH(\epsilon), \end{aligned} \tag{20}$$

In the worst cases, a defaulted banker do not obtain sufficient revenue to cover allowed retained capital, the return to depositors inevitably becomes zero,

$$R^D(s,A) = 0, \quad \text{if } A \in [\underline{A}, \theta^B). \tag{21}$$

In a region where a borrower does not default (i.e., $\epsilon A \geq \theta^L$), repayment from the borrower is constant $(1+\rho^L)$. On the other hand, in a region where a borrower default (i.e., $\epsilon A < \theta^L$), repayment from the borrower is increasing with aggregate shock A with possible zero return. Overall, banker's gross income $B(A)$ is increasing in aggregate shock A, and $B(A) \geq 0$ because of Assumption 3 (d).

D. Bankers' Choice and Consumption

In the beginning of the period, bankers offer deposit and loan rates by equating their expected utility from the spread income to the reservation utility, which is equal to the expected income of being an entrepreneur. When banks offer deposit and loan rates, they rationally expect the possibility of defaults of both borrowers and themselves. A banker's net income is the gross income net of (size-corrected) repayments.

If every borrower repays in full and a banker can do so, a banker enjoys the maximum income from the spread between loan and deposit rates,

$$c^B(A, R^L, \Omega, R^D, \Psi) = (\rho^L - \rho^D)S + (1+\rho^L)k_0^B, \quad \text{if} \quad A \in [\frac{\theta^L}{\underline{\epsilon}}, \overline{A}]. \tag{22}$$

Even if some borrowers cannot repay back the promised loan returns to a banker, a banker can still repay deposits in full to depositors using her own capital buffer,

$$c^B(A, R^L, \Omega, R^D, \Psi) = B(A) - (1+\rho^D)S, \quad \text{if} \quad A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]. \tag{23}$$

In case a banker defaults and can retain allowed amounts at hand,

$$c^B(A, R^L, \Omega, R^D, \Psi) = \lambda k_0^B, \quad \text{if} \quad A \in [\theta^B, \theta^D). \tag{24}$$

In the worst case, a defaulted banker's revenue is less than the allowed retained amounts, $B(A) < \lambda k_0^B$, and his consumption becomes equal to his revenue,

$$c^B(A, R^L, \Omega, R^D, \Psi) = B(A), \quad \text{if} \quad A \in [\underline{A}, \theta^B). \tag{25}$$

IV. Equilibrium Allocation

A. Illustrative Explanation of Equilibrium

I explain illustratively here what are the income and consumption of borrowers, depositors, and bankers. First, think about hypothetical complete contracts in an economy with borrowers and depositors but without bankers. The complete market implies perfect risk sharing, and hence borrowers' and depositors' consumption would converge. Figure 1 shows the consumption schedules for borrowers and depositors with and without capital exchange using hypothetical complete contracts.

Next, still assuming only borrowers and depositors exist, but suppose available contracts are those characterized as a simple debt type that does *not* allow default. Figure 2 shows the consumption schedules for borrowers and depositors in this case. Obviously, without complete contracts, two agent's income risks cannot be perfectly shared. Rather, consumption of borrowers shifts down by the flat loan repayments regardless of states as default is assumed away here, while that of depositors shits up as much as the flat deposit return.

Once the debt contract allows default with retaining some assets, borrower's consumption is bounded by below at their allowed retained assets (see Figure 3). Then, when a large negative aggregate shock (i.e., a tail risk) is realized, the consequences are mostly assumed by depositors (*tail risk dumping*).

The situation is not so much changed if bankers are added. Figure 4 adds banker's consumption as the solid line. Both borrowers and bankers would keep minimum guaranteed consumption from their assets, while depositors cannot do so. However, the bankers provide some insurance by their capital buffers for depositors to mitigate the tail-risk dumping problem.

Below I explain more formally what are the consumption allocations or equivalently the loan repayment and deposit repayment functions in equilibrium. Although the model assumes standard utility and production functions, it allows the deposit and loan repayment schedules to have kinks (Figure 4). Hence, a natural question may arise on existence and uniqueness of equilibrium. One way to analyze is to allow lotteries (i.e., correlated or mixed strategies) to convexify the kinks à la Prescott and Townsend (1984 a, b). This approach can make sure the existence of equilibrium under informational problems if no externality presents (Bisin and Gottardi, 2006). However, this approach gives only a numerical characterizations of equilibrium with possible multiple equilibria, and hence does not necessarily make straightforward policy recommendations. Rather, in this paper, I identify some restrictions on parameter values under which a unique equilibrium with pure strategies is supported and provides analytical characterizations of the equilibrium and associated policy implications. I show the analysis below by a constructive manner on (i) the partial equilibrium in the loan market, (ii) the partial equilibrium in the deposit market, and (iii) the general equilibrium.

Note that rights to consume a portion of the banker's consumption allocation c^B would be called "equity shares" of a bank. In the perfect world, everyone has incentive to sell such equities and hold other people's share (i.e., perfect cross-share holdings) so that "big household" assumption would prevail. However, as stressed at the beginning, the purpose of this paper is to investigate how consumption and investment allocations would be characterized and whether any policy interventions can improve welfare in the segregated household economy without perfect risk sharing. For this focus, the model assumes away the equity issuance or cross-share holdings by bankers or entrepreneurs.[18] However, the non-availability of such equity-type contracts is consistent with the informational assumptions that make debt-type contracts to be chosen optimally. In other words, just assuming equity ownership of banks and firms in an ad hoc manner would not be consistent with the model presented in this paper.

B. Loan Market Partial Equilibrium

Given the deposit repayment schedule, bankers supply loans up to the deposit amounts for a profitable loan repayment schedule. On the other hand, if the loan repayment schedule is unprofitable, bankers would not supply loans. Therefore, given the deposit market partial equilibrium, the loan market is essentially determined by borrowers' decisions.

A borrower at the repayment stage has one decision to make, either repaying in full or verifying the state to pay state-contingent amounts. The default threshold is determined by the borrower as shown by Lemma 1. For any given amount of loans l, the *iso-loan default curve* can be drawn on the θ^L-ρ^L plane (Figure 5) based on the default condition (17):

$$1+\rho^L = \theta^L e^U \frac{(k_0+l)^\alpha}{l} - \lambda\frac{k_0}{l}. \tag{26}$$

Lemma 4. *For any given amount of loans l, the iso-loan default curve on the θ^L-ρ^L plane is monotonically increasing in default threshold θ^L.*

This is easy to see because the direction of the slope is determined by the derivative of the right hand side of the iso-loan default function (26) with respect to θ^L and it is positive:

$$e^U \frac{(k_0+l)^\alpha}{l} > 0. \tag{27}$$

Knowing his own behavior at the repayment stage, a borrower choses his demand for a loan at the borrowing stage. Let $\eta \equiv \epsilon A$ denote the combined shock with the cdf $M \equiv G \circ H$. The first order condition for the borrower's problem to ask for a loan (5) is

$$\int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta e^U (k_0+l)^{\alpha-1} - (1+\rho^L)\right) u'(c^L) dM(\eta) = 0. \tag{28}$$

[18] If bankers' income were shared among people perfectly, no demonstrations would have occurred against the bank failure and bailouts in the aftermath of the global financial crisis.

This is essentially the optimal leverage problem for a limited-liability entrepreneur. An entrepreneur borrows capital until the expected marginal productivity of capital equals to the loan rate but only for the non-default region because, if defaulted, the borrower would consume only the retained initial wealth regardless of the loan amount.

The *iso-loan demand curve* of the loan rate ρ^L with respect to default threshold θ^L given loan amount l is expressed as an implicit function of this first order condition,

$$\chi(\theta^L, l) \equiv \int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta e^U (k_0 + l)^{\alpha-1} - (\rho^L + \delta)\right) u'(c^L) dM(\eta) = 0. \tag{29}$$

Lemma 5. *For any given amount of loans l, the iso-loan demand curve on the θ^L-ρ^L plane is monotonically increasing in default threshold θ^L.*

The proof is provided in Appendix. On the θ^L-ρ^L plane, the *iso-loan demand function* can be drawn like Figure 5. A question may arise if a unique pair of loan rate and default threshold is demanded by a borrower given loan amount. This is the case.

Proposition 1. *In a partial equilibrium of the loan market, there exists a unique loan contract (loan repayment schedule), characterized by a unique set of loan rate and default threshold (ρ^L, θ^L), for each loan amount l. Both the loan rate ρ^L and the default threshold θ^L are lower with a higher loan amount.*

See the proof in Appendix.

C. Credible Deposit Demand by Bankers

When a person decides to be a banker, he offers a deposit contract, which specifies a deposit repayment schedule. As long as the spread income is positive, a banker is happy to take as much deposits as possible. That is, the deposit demand by a bank is inelastic to any given profitable pair of positive spread and default thresholds.[19]

A banker is supposed to repay deposit in full $(1 + \rho^D)$ under a deposit contract $R^D(S, A)$ until the aggregate shock A hits the default threshold θ^D. However, this repayment schedule is credible only if a banker chooses the default threshold in his own behalf. Default means for a banker to give up his revenue to depositors and thus higher default threshold θ^D for a given spread does not necessarily provide a banker a higher profit. Indeed, given loan rate ρ^L and deposit rate ρ^D (or spread

[19] In more general double competition in deposit and loan markets, bank competition brings zero rents at least when banks compete for deposits first (Stahl (1988), Ueda (2013)). In this paper, no rents implies that bankers earn the same expected income as entrepreneurs. When deciding a deposit market strategy, a banker takes into account his reservation utility by switching jobs. And hence there should exist positive expected profits from loan and deposit repayment schedules in equilibrium.

$\pi = \rho^L - \rho^D$), a banker can maximize his utility by choosing threshold θ^D such that consumption under default (24) is equal to consumption under full deposit repayment (23):[20]

$$\lambda k_0^B = B(\theta^D) - (1+\rho^D)S. \tag{30}$$

Given deposits s, loan rate ρ^L, initial capital k_0^B, and banker population μ, this constraint (30) should hold and appears as *credible deposit contract offer curve* $s^d(\theta^D, \pi)$ on the θ^D-π plane as (see Figure 6),

$$-(1+\rho^D) = \pi - \rho^L - 1 = -\frac{B(\theta^D) - \lambda k_0^B}{S}. \tag{31}$$

Lemma 6. *The credible deposit contract offer curve is strictly decreasing in θ^D on the θ^D-π plane.*

D. Deposit Supply by Depositors

Proposition 2 (Optimal Deposit Size)**.** *Given a deposit contract $R^D(S, A)$, a depositor determines deposit amount s uniquely to maximize his utility (6).*

Note that

$$c^D(s, A, \epsilon) = \epsilon A e^D (k_0 - s)^\alpha + R^D(S, A)s, \tag{32}$$

where $R^D(s, A)$ is defined in (18) to (21). The first order condition (FOC) for depositor's problem (6) with respect to deposits s is

$$\begin{aligned} 0 = &- \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u'(c^D(s, A, \epsilon)) \alpha \epsilon A e^D (k_0 - s)^{\alpha-1} dH(\epsilon) dG(A) \\ &+ \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S, A) u'(c^D(s, A, \epsilon)) dH(\epsilon) dG(A) \\ &\equiv \Phi(\theta^D, \pi). \end{aligned} \tag{33}$$

See the rest of the proof in Appendix.

For given k_0 and parameter values of the production and utility as well as the deposit repayment schedule R^D, the utility level is determined in equilibrium by optimally chosen deposit s. For a specific utility level, the first order condition (33) gives us the deposit supply as a function of the deposit repayment schedule characterized by the default threshold θ^D and the full-pay deposit rate ρ^D. For given full-pay loan rate ρ^L, the full-pay deposit rate is just lower as much as the spread, that is, $\rho^D = \rho^L - \pi$. Here, on the θ^D-π plane, the deposit supply function can be expressed as $s^s(\theta^D, \pi|\overline{u})$.

If a depositor allocates more capital to deposit and less capital to own business, he raises the marginal product of capital of his own business. Then, the absolute value of the first term in (33)

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[20] The first order condition for (7) with respect to θ^D is

$$u(\lambda k_0^B) = u(B(\theta^D) - (1+\rho^D)S),$$

which is simplified to (30).

increases. On the other hand, she would allocate more to deposit if the deposit contract becomes safer (i.e., the default threshold θ^D becomes smaller) for the same deposit rate ρ^D. Thus, the deposit supply function $s^s(\theta^D, \pi|\overline{u})$ is decreasing in default threshold θ^D. Apparently, given the default threshold, it is increasing in deposit rate ρ^D $(= \rho^L - \pi)$, i.e., decreasing in spread $\pi = \rho^L - \rho^D$ for a fixed loan rate.

Overall, if the deposit rate is lowered, depositors ask to lower default threshold for compensation to deposit the same amount in a bank compared to returns from own business. This relation can be drawn on θ^D-π plane as *iso-deposit supply curve* for given deposit s (see Figure 6). The deposits (and the utility level) are larger if the iso-deposit supply curve is closer to the origin (lower spread and lower default threshold at the same time).

Given a FOC-satisfying deposit level $\bar{s}$ fixed, the slope of the iso-deposit supply curve on the θ^D-π plane is determined by the first order condition (33). For the same level of deposits $\bar{s}$, the slope of the iso-deposit supply curve is given by the implict function theorem applied to the FOC (33):[21]

$$\frac{d\pi}{d\theta^D} = \frac{\partial\Phi(\theta^D, \pi)/\partial\theta^D}{-\partial\Phi(\theta^D, \pi)/\partial\pi}. \tag{34}$$

Lemma 7. *The iso-deposit supply curve on the θ^D-π plane has zero slope in the neighborhood of the credible deposit contract offer curve.*

Proof. The numerator of (34) is expressed as

$$\frac{\partial\Phi(\theta^D, \pi)}{\partial\theta^D} = \left\{\left(B(\theta^D) - \lambda k_0^B\right) - (1+\rho^D)\right\} g(\theta^D) \int_{\underline{\epsilon}}^{\overline{\epsilon}} u'(c^D(s, \theta^D, \epsilon) dH(\epsilon), \tag{35}$$

where $g(A)$ is pdf for cdf $G(A)$.

The term inside the brace is zero if θ^D is also satisfying the credible deposit contract offer (31). For a larger θ^D given π, the brace term is positive and vice versa. *Q.E.D.*

E. Deposit Market Partial Equilibrium

Proposition 3. *In a deposit market partial equilibrium, deposit rate with associated deposit default threshold (ρ^D, θ^D) is uniquely determined, given deposit amount $\overline{s}$ and loan rate with associated loan default threshold (ρ^L, θ^L).*

Proof. Given the denominator of (34) is not zero, Lemma 7 means that the iso-deposit supply curve has a zero slope only in the neighborhood of the credible deposit contract offer curve.

[21] The derivative changes at default threshold θ^D depending on whether depositors expect default or not by banks at the threshold. The derivative is taken from the right side for the non-default neighborhood and from the left side for the default neighborhood.

The denominator of (34) is indeed not zero. Using $c^D = (1+\rho^D)\overline{s} + y^D$ for the non-default region,

$$\begin{aligned} &-\frac{\partial \Phi(\theta^D, \pi)}{\partial \pi} = \frac{\partial \Phi(\theta^D, \pi)}{\partial \rho^D} \\ &= \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u' dH(\epsilon) dG(A) + \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \bar{s}(1+\rho^D) u'' dH(\epsilon) dG(A) \\ &= \int_{\underline{\epsilon}}^{\overline{\epsilon}} \sigma u' \left\{ \frac{1}{\sigma} - \frac{(1+\rho^D)\bar{s}}{c^D} \right\} dH(\epsilon) dG(A) \end{aligned} \tag{36}$$

where the last line uses the definition of relative risk aversion, $\sigma = -c^D u''/u'$. This is not zero in general. Note that, since $c^D = (1+\rho^D)\overline{s} + y^D$, $1 \geq (1+\rho^D)\overline{s}/c^D$. This means that $\sigma < 1$ is a (strong) sufficient condition for (36) to be positive. However, for more general σ, the sign of (36) is indeterminate.

In the case with positive (36), Lemma 7 and (35) implies that the slope (34) of the iso-deposit supply curve is positive on the right of the credible deposit contract offer curve and negative on the left. With zero slope when the iso-deposit supply curve crosses the credible deposit contract offer curve (Lemma 7), the iso-deposit supply curve has a parabola-like shape in the neighborhood of the crossing point (see Figure Y).[22]

Given a crossing point, there could be a case that the iso-deposit supply curve once again crosses the deposit contract offer curve in the upper left region of the original crossing point, as the credible deposit contract offer curve is strictly decreasing (Lemma 6). However, this cannot happen. Lemma 7 states that the iso-deposit supply curve has a zero slope at the crossing point. This contradicts to the above-discussed implication of equation (35): it must have a negative slope in the upper left region of the original crossing point. Therefore, two curves crosses only once.

In the case with negative (36), a similar argument goes through. *Q.E.D.*

Corollary 1. *In the deposit market partial equilibrium, larger deposit amount s is associated with higher deposit rate ρ^D, which can be supported by higher loan rate ρ^L.*

Proof. The deposit amount is determined by the optimal portfolio allocation by depositors between the bank deposits and own production (Proposition 2). A larger deposit amount is thus supported by a better deposit return function, higher deposit rate ρ^D and/or lower bank default threshold θ^D. Indeed, a larger deposit amount implies that an iso-deposit supply curve is closer to the origin of the θ^D-π plane. And, Lemma **??** means that the higher loan rate is needed to match with the higher deposit supply by shifting down the credible deposit contract curve. *Q.E.D.*

[22]The parabola-like shape for given deposit may be puzzling because the upper right is high risk and low return while the upper left is low risk and low return. This reflects the concave shape of the deposit repayment function, which is higher than the own production around the default threshold but lower for very high and low aggregate shock realizations. Hence, low risk means there is not much gains from ???

F. General Equilibrium

Proposition 4. *Given banker population μ, there exists unique equilibrium loan rate ρ^L, deposit amount s, and loan amount l. Moreover, these pin down unique equilibrium loan default threshold θ^L, deposit rate ρ^D (and spread π), and deposit default threshold θ^D.*

Proof. Given banker population μ, in the deposit market partial equilibrium, the deposit amount s is increasing with loan rate ρ^L (Corollary 1 of Proposition 3). On the other hand, in the loan market partial equilibrium given μ, the loan amount l is decreasing with higher loan rate ρ^L associated with higher default threshold θ^L (Proposition 1). These "deposit" curve and "loan" curve can be drawn in the ρ^L-(s, l) plane and they cross once (see Figure 7).

In the general equilibrium, the resource constraint needs to be met:

$$l^* = s^* + \frac{2\mu}{1-\mu} k_0^B. \tag{37}$$

Shifting up the "deposit" curve by $2\mu/(1-\mu)k_0^B$, the unique general equilibrium loan rate and loan amount (ρ^{L*}, l^*) can be found at the cross point of the shifted-up deposit curve and the loan curve. Given (ρ^{L*}, l^*), the loan market partial equilibrium pins down the general equilibrium loan default threshold θ^{L*}.

Also, the general equilibrium deposit amount s^* is derived by (37). Given the equilibrium loan market variables as well as the equilibrium deposit amount, the deposit market partial equilibrium provides the general equilibrium deposit rate and default threshold (ρ^{D*}, θ^{D*}) and associated spread π^*. *Q.E.D.*

Given banker population μ, the general equilibrium loan and deposit market variables, $(\rho^{L*}, \theta^{L*}, l^*)$ and $(\rho^{D*}, \theta^{D*}, s^*)$, determine the utility of each type, V^L, V^D, and V^B. Now, one question remains if the occupational arbitrage pins down the equilibrium banker population.

First, I characterize how the small changes in banker population μ affects the loan and deposit market partial equilibrium in Lemma 8 and 9 below. Second, I investigate general equilibrium effects in Lemma 11 and 10. Proofs are shown in Appendix.

Lemma 8. *Given a loan amount, the loan-market partial-equilibrium contract is not affected by a change in banker population μ, i.e., $\partial\rho^L/\partial\mu = 0$ and $\partial\theta^L/\partial\mu = 0$.*

Lemma 9. *Given a deposit amount, with larger banker population μ, deposit rate ρ^D is lower and default threshold θ^D is higher in the deposit-market partial equilibrium, i.e., $\partial\rho^D/\partial\mu < 0$ and $\partial\theta^D/\partial\mu > 0$. This implies that the deposits should decrease, $\partial s/\partial\mu < 0$.*

Two lemmas above, however, predicts the spread π increases with banker population μ, while over all spread income πs is uncertain. Here, I assume away an extremely increasing case.

Assumption 5. *Total spread income is at most doubled with a slight change in the banker population:*

$$\frac{\partial(\pi s)}{\partial\mu} \leq \pi s. \tag{38}$$

Note that this assumption also includes in the case which the change in spread is negative due to larger banker population. Also, note that this assumption together with Lemma 8 implies that $\|\partial(\rho^D s)/\partial\mu\| \leq \rho^D s$.

Lemma 10. *Under Assumption 5, with larger banker population μ, the utility of a banker, V^B, strictly decreases, i.e., $\partial V^B/\partial\mu < 0$.*

As for entrepreneurs, the effect is opposite.

Lemma 11. *The ex ante entrepreneur's utility $V^E = V^L/2 + V^D/2$ is strictly increasing with higher banker population μ.*

Proposition 5. *The banker population μ is determined uniquely in the decentralized equilibrium.*

Proof. Both entrepreneur's utility V^E and banker's utility V^B are apparently continuous functions on banker population μ. Lemma 11 says that the V^E is strictly increasing in μ while Lemma 10 says that the V^B is strictly decreasing (see Figure 8).

As $\mu \to 0$, a tiny number of bankers would receive all the transaction fees and utility of a banker, i.e., $V^B \to \infty$, while the entrepreneur's utility V^E is bounded by the finite value achieved in the Walrasian equilibrium allocation.[23] On the other hand, as $\mu \to 1$, only a tiny portion of people produce and thus spread income is close to zero and banker's utility $V^B \to 0$. But, the entrepreneur is bounded below by the autarkic level, and thus $V^E > 0$. Both V^B and V^E are apparently continuous functions with respect to μ. Consider a function $V^E - V^B$. By Lemmas 10 and 11, it is a contraction with respect to μ on the compact domain $[0, 1]$. Therefore, by the Contraction Mapping Theorem, there is a unique fixed point μ at which $V^B = V^E$. *Q.E.D.*

V. Socially Optimal Allocation

A. Welfare Theorem

The institutional assumptions restrict the shape of the loan and deposit repayment schedules R^L and R^D, which agents in this economy as well as the social planner obeys. Because R^L and R^D also dictates the consumption allocation through (**??**), the constrained social planner can determine the consumption allocation indirectly by choosing the loan and deposit repayment schedules.

The social planner's problem can be expressed as maximizing a representative banker's utility, similar to (7), but also with controls μ, l, and s in addition to R^L and R^D:

$$\max_{\mu,l,s,R^L,R^D} \int_{\underline{A}}^{\overline{A}} u\left(c^B(A, \tilde{R}^L, \Omega, \tilde{R}^D, \Psi)\right) dG(A), \tag{39}$$

subject to the representative borrower's utility maximization, i.e., an incentive constraint, (5), the representative depositor's utility maximization, i.e., another incentive constraint, (6), the

[23] See Section V. B. for more explanation.

occupational choice, i.e., yet another incentive constraint, (11), and resource constraints (8) and (10). Note that the social planner can set the all the loan and deposit contracts to be the same $\Omega = R^{L\mu}$ and $\Psi = R^{D\mu}$, i.e., μ-Cartesian products of R^L and R^D, respectively.

Definition 2. *The constrained social optimal allocation is the solution to the social planner's problem in which the social planner faces the same restrictions as the private agents given in Assumptions 2 to* **??**.

Proposition 6. *The decentralized equilibrium achieves the constrained social optimum. That is, it achieves the constrained social optimal allocation given banker's population μ, which then is determined optimally.*

The proof is straightforward and sketched here. Inside the social planning problem, the incentive constraints for borrowers (5), for depositors (6), and for occupational choice (11) are exactly the same problems that are solved in the decentralized equilibrium under the same resource constraints and Assumptions. Only difference is the problem for bankers. In decentralized equilibrium, a banker maximizes his utility by choosing loan and deposit repayment schedules given the reaction functions (i.e., loan demand and deposit supply functions) from borrowers and depositors. The social planner achieves the social optimum by maximizing the representative banker's utility by selecting by choosing loan and deposit repayment schedules and also by selecting loans and deposits. However, those are chosen from the sets restricted by incentive constraints. Those restrictions are the same as reaction functions in the decentralized equilibrium. Moreover, the occupational arbitrage constraint implies that banker's spread income plus any rents from selling deposit contracts are equated with the income of entrepreneurs. The extra rents to sell deposit contracts are uniquely determined at zero. And, there is no distinction between social and private solutions. Therefore, the social planner's problem and the decentralized equilibrium brings the same allocations. This is true for both Regime I and II.

Note that there is no externality to break the link between the decentralized equilibrium and the social optimum in the model. First, there is no technological externality that affect Lemma **??**. Second, because the occupational arbitrage equates the banker's and entrepreneur's utility and everyone is fully employed, there is no externality associated with the number of bankers in Proposition 5.

The allocation of assets and outputs are not the first best in default states as it would differ from the Nash bargaining solution in the regime with bargaining or from the original CSV equilibrium in Townsend (1979).[24] This is not because the linear nature of the rule but the one-fits-all rule of splitting the assets. In this regard, Regime II mimics US Chapter 11 like speedy restructuring procedure, while the first regime mimics private negotiations on splitting defaulters' assets.

[24]If the borrower ask to give back some portion after the banker seize her assets and observe the state, the borrower and the banker have to go through verification of the state and negotiation of splitting assets with cost τ. If they agree to do this ex ante and can commit, this regime becomes essentially Regime I. However, a banker has no incentive to give back a portion of assets to the borrower after seizing them. Therefore, if a banker cannot commit, Regime II prevails.

B. Walrasian Equilibrium as the Benchmark

For a reference, in this section I analyze a complete market case as the limit of the model. Here, the verification and negotiation costs are close to zero, or equivalently all the information is almost public. So, there will be no use of Regime II. Accordingly, I assume here no retained assets. I can still keep assuming prohibitive cost of writing ex ante contingent contracts but with ex post Nash bargaining occurs without costs. the allocations are equivalent between the economies with and those without writing ex ante contracts. In this limit, the loan and deposit contracts take a form of equity contract as the limit of debt contracts.[25]

Proposition 7. *As the verification and negotiation costs becomes close to zero, deposit and loan contracts become complete, equity-type, contingent contracts. The equilibrium mimics the Warlasian competitive equilibrium and is the first best. Specifically, the equilibrium deposit and loan repayment schedules are the same in the limit and so do the deposit and loan amounts. A small number of (i.e., measure zero) banks intermediate the capital. Accordingly, the optimal capital ratio of banks are (almost) zero. As a result, consumption is almost perfectly shared equally among all households.*

Proof. (Sketch). The positive spread between deposit and loan rates is a friction to the production sector. The smaller the spread, the larger is the outputs. Thus, a smallest number of banks should intermediate capital from the social planner's point of view. In the limit, the deposit-loan rate spread becomes zero with the same deposit and loan size. With zero spread and compete contracts, the idiosyncratic shocks are shared perfectly among all agents. As Proposition 6 still applies, this allocation is also supported by a decentralized equilibrium. *Q.E.D.*

C. Optimal Bank Size and Capital Ratio

A question remains on the optimal size of the banking sector in an economy with sizable financial frictions. A depositor faces shock-contingent income up to threshold θ^D and then flat income ρ^D from full deposit repayment. With large enough variations in the aggregate shocks, there are sizable chances that deposits are not repaid in full. Then, the depositors would prefer a more insured contract that provides a same expected return with a lower deposit rate but also with a lower default risk. A banker also prefers to provide this insurance contract, implying that he will have strictly positive capital.

Proposition 8. *In the ex ante social optimum and hence in the decentralized equilibrium, the banking sector is sizable $\mu^* > 0$. This implies that the optimal capital ratio is strictly positive.*

Proof. Suppose that the optimal banking sector size μ is measure zero (as is the case with the optimal complete market in Proposition 7). Then, by construction, the capital ratio becomes (almost) zero, and the expected loan and deposit repayment schedules becomes (almost) the same by arbitrage. Specifically, there is one threshold $\hat{\theta}^D = \hat{\theta}^L/\underline{\epsilon}$ below which borrowers start to default and bankers default. Then, the full-pay deposit and loan rate becomes (almost) equal and the spread is zero, $\hat{\pi} = \hat{\rho}^L - \hat{\rho}^D = 0$. I show below that this deposit contract is not constrained Pareto optimal.

[25] This result follows in spirit that of Townsend (1978) on costly bilateral exchange.

First, a depositor prefers the deposit contract which gives the same expected return with less volatile repayment. Here, less volatile means a lower full repayment (i.e., lower deposit rate $\underline{\rho}^D$) but with lower threshold and higher repayment in case of banker's default. This contract (partially) insures depositors' income for the combined idiosyncratic and aggregate productivity shocks and thus it is preferred by a risk averse depositor.

Second, this partially insured contracts can be indeed designed so that the overall repayment has the same expected values. Note that the depositor's expected utility is larger with the same expected return but more insured income risk. A banker can make such a contract by limiting deposits and loans given his endowed capital. Essentially he uses his capital as a buffer to depositors, so that the default threshold θ^D will be lowered. Because this new contract is assumed to give the same expected repayments to depositors, the banker is offering a lower deposit rate $\underline{\rho}^D$ for the same loan rate $\hat{\rho}^L$. This change of the deposit rate is denoted by the increase in the spread $\triangle\pi$ from zero.

Third, I can show that, even if deposit per bank falls, a banker strictly prefers the new contract—the lower expected value with less volatile deposit repayments—given prevailing loan contracts $R^{L*}(l, A, \epsilon)$ and the amount of deposit $\hat{s}$ per depositor. Consider drawing the original and new deposit repayment schedules with the aggregate shock realization on the x-axis and the repayment on the y-axis (see Figure 9). Let $\underline{\theta}^D$ denote the new threshold of default by the banker.

(i) At the worst case at the origin, $A = 0$, repayment is the seized asset adjusted for relative size $w(1-\lambda)(1-\delta)k_0^B$. Under the measure zero banking sector, this is equal to zero. So, the depositor's gain at the origin is $\hat{w}(1-\lambda)(1-\delta)k_0^B$, where $\hat{w}$ is the relative size of banking sector under the new contract. This is the amount that shifts up linearly the recovery rate of deposit contract in the default region in Figure 9. The new default region is from zero to $\underline{\theta}^D$. Overall gains include the additional area stemming from the difference of default thresholds under the original and new contracts. The rectangle excluding such additional area is smaller than the overall gain. Therefore, depositor's gain is bigger than the increase in recovery in case of bank default:

$$Gain > \underline{Gain} = \int_0^{\underline{\theta}^D} \hat{w}(1-\lambda)(1-\delta)k_0^B dH = \hat{w}(1-\lambda)(1-\delta)k_0^B H(\underline{\theta}^D). \tag{40}$$

(ii) On the other hand, the depositor's loss is strictly lower than the upperbound of the loss, which is measured by the rectangle made by the change in the spread and the cumulative probability above the new threshold in Figure 9. That is,

$$Loss < \overline{Loss} = \triangle\pi(1 - H(\underline{\theta}^D)). \tag{41}$$

(iii) Because the new contract is assumed to have the same expected returns for a depositor, the gain and the loss must be the same. Therefore, the lowerbound of the gain must be strictly lower than the upperbound of the loss: That is,

$$\triangle\pi > \hat{w}(1-\lambda)(1-\delta)k_0^B \frac{H(\underline{\theta}^D)}{1 - H(\underline{\theta}^D)}. \tag{42}$$

(iv) Because the current banking sector size is almost zero, I evaluate this at the limit $\hat{\mu} \to 0$ (i.e., $\hat{w} \to 0$) to see the profit of stemming from a new contract only slightly different from the current one. The limit value is strictly positive:

$$\lim_{\hat{\mu}\to 0} \triangle\pi > 0. \tag{43}$$

This implies that the increase in the spread by introducing a slightly more insuring contract is strictly positive—the spread is literary an insurance premium. With the new contract, a banker will have a higher income per deposit in case not defaulting in addition to a lower default probability.

(v) By limiting loans and deposits, the total income could become less because the total income is affected by the spread times deposits. But, recall that the spread under the original contract is zero. Thus, the total income also increases from zero to a positive sum with a slightly different contract.

In summary, both a depositor and a banker prefer a new contract, given the same loan contract (i.e., the same utility for borrowers). Therefore, the optimal capital ratio must be positive and banker population μ is significantly larger than zero in the (ex ante) social optimum. *Q.E.D.*

VI. Policy Implications

A. Optimal Bank Bailouts

A bailout policy is defined as guaranteeing a banker's income in case that a banker would default without the bailout policy in order to enable a banker to repay deposits in full. The government finances the transfer to a banker by taxing everyone ex post. This description represents actual bailouts (see e.g., Landier and Ueda, 2009).

Assumption 6. *A government can collect tax (e.g., consumption tax) from those who defaulted.*

It is legally and politically difficult to collect funds from those who defaulted. However, in reality, the bailout funds are financed by government bonds, which the government repays over time, for example, by consumption tax.[26] To avoid additional distortion in the model, I focus on lump-sum taxation of consumption goods. The key point is that those who defaulted contribute to the bailout expenditure.

The government is still assumed to follow the information structure of the model. It is, however, assumed to observe the aggregate shock when it bails out banks. It then can determine the tax rate contingent on the aggregate shock realization. Otherwise (i.e., under good shock realizations), it cannot know the aggregate shock. In other words, this paper does *not* naively assume that the government can do any redistribution policy with perfect information.

Definition 3. *A "transparent" bailout transfers funds to depositors via bankers without benefiting bankers directly, while an "untransparent" bailout benefits bankers directly.*

[26] In the real world, other examples include inflation tax on monetary assets or income tax on human capital, though they are not modeled in this paper.

A transparent bailout is a good insurance for depositors at the cost of borrowers. Depositors can have the perfectly constant deposit repayment with the bailout policy for any realizations of the aggregate shocks but they also need to pay lump-sum tax $\kappa(A)$ ex post to finance the bailouts. The net-of-tax deposit repayments are then not perfectly constant:

$$R^D_{BO}(s, A) = (\rho^D + \delta)s - \kappa(A), \quad \text{for } \forall A \in [\underline{A}, \overline{A}]. \tag{44}$$

A borrower needs to pay tax, too. His consumption is changed to

$$\epsilon A e^H (k_0 + l)^\alpha - R^L(l, A, \epsilon) - \kappa(A). \tag{45}$$

Lemma 12. *If the government could access all the information that banks possess when bails them out, and tailor the tax rate conditional also on idiosyncratic shocks of defaulted borrowers, which notationally can be written as $\kappa(A, \epsilon)$, then, the ex ante designed transparent bailout scheme can bring a Pareto superior allocation than in the economy without such scheme.*

Proof. If a government can tax each defaulted borrower at a rate conditional on realized aggregate and idiosyncratic shocks, then the government can relax a constraint, the simple debt restructuring rule, of the social planner's problem that does not allow tax on defaulters (see Figure 10). *Q.E.D.*

However, it seems natural to assume that the government can know only the aggregate shocks when it bails out bankers (i.e., only bankers know idiosyncratic shocks of client firms through costly verification.) Then, the tax-transfer system based on idiosyncratic shocks is impossible to implement. Hence, I consider a conventional tax policy, which is not discriminatory among people and can be conditional only on the aggregate shocks. Transfers are also assumed to be conditional on the aggregate shocks only but can be targeted, for example, to depositors in the case of deposit insurance.

Assumption 7. *The government can know only the aggregate shocks when bailing out bankers.*

Under this assumption, the optimality of bailout is less obvious. I would like to focus now on a simple bank bailout scheme and examine whether everyone voluntarily joins this bailout scheme or not. As the consumption tax is assumed not discriminatory to anyone, everyone pays $\kappa(A)$ to the government under this scheme. Then, the government transfers $2\kappa(A)/(1-\mu)$ to depositors via bankers. Here, bank's participation constraint to the bailout scheme needs to be met:

$$V^B_{Bailout} \geq V^B_{NoBailout}. \tag{46}$$

Under a transparent bailout, when a banker faces default, the government transfers funds to a banker just to repay the deposit in full so that the banker's consumption schedule $c^B(A)$ remains unchanged. The transfer occurs only when the aggregate shock is lower than the banker's default threshold, $A < \theta^D_{BO}$. Per depositor transfer is the difference between the banker's retained asset (24) and what a

banker would consume without retained asset (23), correcting for relative population of bankers to depositors:

$$\begin{aligned} \frac{2}{1-\mu}\kappa(A) &= \frac{2\mu}{1-\mu}\left(\lambda(1-\delta)k_0^B - B(A) + \frac{1-\mu}{2\mu}(1+\rho^D)s\right), \quad \text{if} \quad A \in [\underline{A}, \theta^D_{BO}]; \\ \kappa(A) &= 0, \quad \text{otherwise.} \end{aligned} \tag{47}$$

Lemma 13. *A sudden, unexpected transparent bank bailout is welfare improving.*

Proof. The repayment function R^D and R^L were not reoptimized with this unexpectd transparent bailout. When the ex post bank participation constraint (46) is met with equality, bankers do not gain or lose. The depositors and borrowers would share the cost of bank defaults. Without the bailout scheme, the borrowers would not share the cost. The depositors would be better off but the borrowers would be worse off. However, an entrepreneur ends up reducing the consumption volatility stemming from uncertainty for talent shocks that makes an entrepreneur either a borrower or a depositor. In particular, this risk sharing occurs at the low consumption level with high marginal utility (i.e., $\lim_{c\to 0} u'(c) = \infty$). Therefore, better sharing the risk between a depositor and a borrower for a large negative aggregate shock is welfare improving for an entrepreneur, when assessed ex ante using an equal-weight utilitarian social welfare function, i.e.,

$$V^E_{Bailout} > V^E_{NoBailout}. \tag{48}$$

Q.E.D.

The debt contract with limited liability implies that the borrowers and bankers are well insured for a very low realization of aggregate shock while the depositors face the consequences. If there is a way to redistribute the borrowers' retained assets to the depositors, the ex ante overall welfare improves. Given the limited liability laws, one of a few ways is to use the tax system.

Many argue that expectation of bank bailouts would induce distorted behavior by banks and increase the probability of bank defaults. Indeed, if the bailout is ex ante designed and expected, it is the case in the new equilibrium as shown in Proposition 9 below. Still, the overall welfare is improved with transparent bailouts, which makes the economy closer to the original Townsend CSV economy and allow more optimal risk taking by entrepreneurs and bankers due to a better risk sharing.[27]

Proposition 9. *Introducing an optimal transparent bank bailout scheme ex ante is also welfare improving. In this case, expectation for bank bailouts cause more bank defaults but with less defaults of borrowers. More precisely, the effects are the followings: higher bank default threshold θ^D; lower deposit rate ρ^D; larger deposits s per depositor; larger loan amount l per borrower; lower loan rate ρ^L; lower loan default threshold θ^L; lower banker population μ; and higher bank leverage.*

Proof. In the deposit market, given banker population μ, iso-deposit supply curve becomes a flat, straight line sharing the same intercept (ρ^D_1 in Figure 11) as before, on the θ^D-π plane, because the deposit return is no longer affected by bank default threshold θ^D thanks to bailouts. Hence, in Figure 11, the dot-dash curve becomes the flat dashed line.

[27] Obstfeld (1994) also shows in a different model setup (i.e., perfect information with Epstein-Zin ( ) preference) that a better risk sharing makes people to invest higher-risk-higher-return projects optimally and improves welfare.

Here, because the credible deposit contract offer curve is strictly decreasing (Lemma 6), the deposit market equilibrium is achieved as the corner solution. The corner equilibrium is achieved with a higher credible deposit contract offer (the decreasing dashed line in Figure 11) on the θ^D-π plane at the highest possible default threshold $\overline{\theta}^D$, if the loan rate remains the same at ρ_0^L.[28]

However, on the dashed credible deposit contract offer curve, deposit demand from banks is larger than the original level, meaning that it does not reflect the equilibrium offer. Rather, the deposit and loan rates need to adjust and the offer curve needs to shift down (to a solid line in Figure 11). Then, accordingly, the equilibrium iso-deposit supply curve must shift down (to the solid line in Figure 11), with the deposit rate lower than the intercept of the original curve.

The level of the deposit rate compared to the original rate is determined in the equilibrium. Indeed, the loan rate must increase (to ρ_2^L) to lower the deposit demand by banks (through lower loan demand). As long as the increase in deposit rate ρ^D from the corner solution, i.e., the intercept of the original curve (ρ_1^D), is small, the overall change in deposit rate is negative, that is, the new equilibrium rate ρ_2^D becomes below the intercept of the original curve (ρ_1^D) but above the original deposit rate (ρ_0^D) on the π-θ^D plane. Indeed, this is the case, because the sensitivity of deposit supply to deposit rate is in its full force as the rate guaranteed for depositors. On the other hand, the sensitivity of deposit demand by banks to loan rate is quite low given the high level of bank default threshold. The increase in spread income is only enjoyed fully by a bank in the small region where a bank does not default.

In summary, a new partial equilibrium in the deposit market will be characterized by a higher bank default threshold, a lower deposit rate, larger amounts of deposits and loans, and an associated higher loan rate and a loan default threshold (see Proposition 1).

As for the utility of bankers, a higher loan rate and a lower deposit rate imply a higher spread. If the deposit and loan amounts are the same as before, the deposit rate would dictate the slope of dissipation of the capital buffer from the point where some firms start to default to the point where a banker defaults (see the dot-dashed line in Figure 12). A lower deposit rate $\rho_2^D < \rho_0^D$ or a higher loan rate $\rho_2^L > \rho_0^L$ means that the slope is less steep (the dashed line in Figure 12), suggesting a lower default threshold than the original. In this case, a higher spread with a lower default probability makes a banker better off clearly.

However, in an equilibrium, deposit and loan amounts are larger—increase in deposits from s_0 to s_2 in Figure 12. In this case, as discussed, default threshold becomes higher, because deposit amount that a bank has to honor is larger. But, the total spread income is also larger. The total spread income can be divided into two regions on the c^B-A plane. The full income portion is a rectangle for the region $[\theta^L/\underline{\epsilon}, \overline{A}]$ with the height determined by spread π. The partial income position is a triangle for the region $[\theta^D, \theta^L/\underline{\epsilon}]$ with the positive slope at rate $\pi(1-\mu)/2\mu$. With more deposits (and loans), the total income increases by this spread income per unit of deposits. But, the spread income decreases only marginally due to less buffer against increased deposits, appearing as shrinkage of triangle. Still, the total expected income grows fast with the larger amount of savings and loans. Since the cost appears only as the marginal steepening of the slope, the increase in total income should dominate the risk concern about the shrinkage of capital buffer. Hence, the banker's utility V^B also

[28] If every borrower repays, a banker would not declare default. With own capital as a buffer, the new default threshold θ_{BO}^D is at maximum $\overline{\theta}^D$, which is less than, but close to, $\theta^L/\underline{\epsilon}$ (see banker's consumption (22)).

increases. Essentially, given the banker population, bankers also gain a portion of benefits from government guarantees on deposits once the contracts are endogenously determined.

The increase in the banker's utility V^B stems from indirect, general equilibrium effects. Given the banker population μ, since depositors are more protected and the borrowers face better loan contract terms, V^E is higher than before. This increase in the entrepreneur's V^E is a direct result of the tail-risk insurance provided by the government. Hence, the increase in V^E is larger than that in V^B. As a result, less agents become bankers, that is, endogenously determined μ is lower than before and bank leverage is higher. *Q.E.D.*

[TBD: MATH AT LEAST LAST TWO PARA]

B. Deposit Insurance and Double Liability

A deposit insurance can be defined as a protection for depositors' income in case that a banker would default. It is usually financed by taxing the bankers, ex ante. This "taxing bankers ex ante" is the difference from the bailout, which is "taxing everyone ex post."

Consider a case that the depositors will not lose the face value of the deposit, that is, a full coverage deposit insurance. The full coverage deposit insurance with ex ante fees does not improve welfare. Here is the sketch of the proof. The full coverage deposit insurance with ex ante fee is essentially the same as restricting the bankers' offer of deposit contracts to be very safe, close to zero default $\theta^D \approx 0$, associated with high spread π to pay the insurance fee. This restriction on the bankers' offers of deposit contracts is an obvious distortion to the economy and the associated social planner's problem. Therefore, such a scheme cannot improve the social welfare.

Note that a partial, but substantial, coverage deposit insurance—which covers the full amount down to the government-set threshold θ^D_G—with ex ante fees would create the similar distortion as the full coverage version. Essentially, the bankers are constrained to choose the deposit repayment schedule and thus the welfare decreases.

The unlimited liability or "double liability" of bankers as in the pre-Great Depression in the U.S. would not work well either.[29] Under the double liability regime, in essence, bankers always had to pay deposits in full, otherwise they were jailed (i.e., their consumption level is almost zero). In this regime, bankers were the ones that assumed all the tail risks. This would not be the optimal risk sharing among different types of agents, and thus is not socially optimal. The key friction is not the limited liability itself, but rather the limited liability being non-contingent to the aggregate shocks. A transparent bailout scheme can fine-tune the limited liability, making it contingent to the aggregate shocks.

[29] See a history and theory paper by Kane and Wilson (1998) and an empirical work by Grossman (2001).

C. Prudential Regulations

Following the discussion on the optimal bank capital level in Section C, a natural policy implication can be made as a corollary to Proposition 8. Introduction of the capital adequacy ratio regulation as defined in (49) below is either redundant or welfare decreasing in an economy without possibility of bailouts (i.e., without Assumption 6).

$$q \equiv \frac{k_0^B}{D} \geq \hat{q}. \tag{49}$$

Proof. Proposition 8 says the optimal capital ratio is positive in an economy with debt contracts. By Proposition 5, bankers hold strictly positive capital by themselves in an equilibrium which is the constrained social optimum. Therefore, the capital adequacy ratio requirement is either binding (i.e., welfare decreasing) or not binding (i.e., redundant). *Q.E.D.*

Note that there are two sources of inefficiency. First, with the capital ratio requirement, there will be more bankers with less customer base and a higher spread to compensate less customer base. Second, with a sizable positive spread and resulting wedge between the loan and deposit rates, the marginal product of capital would be less equated between the borrowers and depositors. When $\tau = 0$ (complete market case), with the capital adequacy ratio requirement, the economy cannot reach the limit that mimics the first best, Walrasian equilibrium.

Corollary 2. *When the capital adequacy ratio requirement is introduced to the economy with the bailout policy with tax system $\kappa(A)$ defined in (47), there will be more bankers with a higher capital ratio and a lower spread but also with a higher probability of bank bailouts. The overall welfare becomes worse.*

Proof. The bailout scheme alone takes care of tail risks. With a binding capital adequacy ratio requirement, banker's leverage becomes lower than the optimal level. Then, a banker needs a higher spread to satisfy the same default threshold and to keep the same utility. The banker's deposit demand $s^d(\theta^D, \pi)$ shifts upward and flattens as a result. Even if there were no changes in the loan market, the deposit market equilibrium would change so that deposit rate ρ^D becomes lower with uncertain movement on threshold θ^D. This leads to a lower utility for depositors. *Q.E.D.*

D. Bad Bailouts and Income Shifting

So far, I have focused on the optimal bailout scheme in a realistic institutional setup and find that a bank bailout, if designed well, is welfare improving. However, in the real world, there can be a bad bailout. In particular, the literature (and newspaper articles) often discuss about corruption and other

problems like moral hazard.[30] In other words, ex post "looting" opportunity may also be available for banks if banks can seize a part of bailout funds. In this case, bailouts are not transparent as defined in Definition 3, but include some hidden subsidies to banks. I call this *untransparent bank bailouts*. Some of them may be necessary to persuade bank owners to agree on bailouts (e.g., Landier and Ueda, 2009) but others may well be a result of political influence by bank lobby (e.g., Igan, Mishra, and Tressel, 2011). Here, I do not attempt to theorize the underlying mechanism of such practices in this paper but characterize the implications of this bad bailout scheme.

Under an *untransparent* bailout policy, the banker's consumption increases by extra transfer $\underline{\kappa}$ for low realizations of aggregate shock, $A < \hat{\theta}^D_{BO}$, where the bank default threshold $\hat{\theta}^D_{BO}$ could be higher than θ^D_{BO} under a transparent bailout scheme, because a bank has a little more incentive to declare default to receive extra transfer $\underline{\kappa}$. The overall transfer is simply shifted upwards by this extra transfer:

$$\begin{aligned} &\kappa(A) = \frac{2\mu}{1-\mu}\left(\lambda(1-\delta)k_0^B - B(A) + \frac{1-\mu}{2\mu}(1+\rho^D)s + \underline{\kappa}\right), \quad \text{if} \quad A \in [\underline{A}, \hat{\theta}^D_{BO}]; \\ &\kappa(A) = \underline{\kappa}, \quad \text{otherwise.} \end{aligned} \tag{50}$$

Since bankers are enriched by bailouts, bailout expectations will create distorted incentives for people to become bankers rather than productive entrepreneurs. As a result, there will be too many bankers and too little production. Lower production implies lower entrepreneurs' utility, and so is bankers' utility through occupational arbitrage in a general equilibrium. I call this *income shifting* problem.[31]

Almost tautologically, this problem requires a policy to limit bank profits so as not to attract too many people to become bankers. Introducing the capital adequacy ratio can mitigate a unnecessarily high incentive to become a banker by lowering bankers' utility. Introducing a bank levy to lower the (present value of) transfer works as well. With these regulations to countervail bad transfers $\underline{\kappa}$, an untransparent bank bailout could still become an optimal response to tail-risk events in the presence of a simple debt restructuring rule.

VII. CONCLUSION

I develop a simple macro model with realistic financial frictions in bank lending, namely, costly state verification in Regime I and limited liability with simple and speedy bankruptcy procedure in

[30] Distortions in the presence of the government protection in the financial system has been discussed mostly in a partial equilibrium framework. For example, the risk shifting problem induced by deposit insurance requires prudential regulations such as a capital adequacy ratio requirement in Kareken and Wallace (1978), Keeley (1991), and Allen and Gale (2007). The moral hazard problem from expected bailouts requires prudential regulations in Chari and Kehoe (2009) or tax in Kocherlakota (2010) although Chari and Kehoe (2009) admit the bailout of firms via banks is ex post efficient to avoid assumed fixed costs associated with bankruptcy. In a general equilibrium framework, Van den Heuvel (2008) argues that the capital adequacy ratio requirement is costly as it limits the liquidity available in the general equilibrium. Related issue is the effect of competition policy as regulations such as capital adequacy ratio requirement reduces competition. Some argue that risk taking becomes too excessive under freer competition (Allen and Gale, 2000) because monopolistic rents limit the banks' risk taking behavior. The others argue the opposite (Boyd and De Nicolo, 2005) because bank's higher monopolistic rents implies firms' lower rents that lead to higher risk taking at the firm level.

[31] This income shifting problem in a general equilibrium setup has not been much, if any, referred in the literature so far.

Regime II. These frictions prohibit perfect income risk sharing among bankers, borrowers, and depositors, in particular in Regime II with a simple bankruptcy rule. Moreover, I assume endogenous bank size—a banker is an occupation under a strict assumption of not sharing income (i.e., ownership). Ex ante, banker's income is equated in expectation with entrepreneur's in expectation. Entrepreneurs are further sorted to either borrowers or depositors depending on the idea shocks they draw. Overall outputs are affected by financial frictions and possible policy distortions. Inefficiency can appear in the labor allocation (i.e., the occupational choice) and the capital allocation (i.e., deposits and loans), which are affected by the spreads, set endogenously by bankers.

I show that the optimal loan and deposit contracts take a form of a standard debt contract in both Regime I and II, following costly state verification literature. The optimal bank capital is shown to be positive to provide a buffer to depositors and bankers themselves. And, the banking sector is sizable. However, when a large negative shock hits, both borrowers and bankers would walk away with retained assets because of limited liability protection in Regime II. The depositors would assume all the tail risk. This *tail-risk dumping* problem creates too risky consumption profile ex post and thus make the occupational choice too risky ex ante. This is the new perspective to the existing literature on macro models with financial frictions.

A government can play some role in the economy for the tail-risk dumping in Regime II. Once the government is allowed to tax on consumption, it can de facto relax the limited liability constraint and the simple asset split rule. The government can then make trasfers to be contingent on the aggregate shocks. This transparent bank bailout can mitigate the *tail-risk dumping* problem and improve the social welfare. The deposit insurance, if funded ex post by tax, can mimic such a transparent bailout. In other words, however, a bailout is welfare improving only when banks and borrowers are too protected by limited liability. As such, *bail-in* of defaulters is called for in case of a tail event. Or, if negotiation costs is negligible, Regime I is preferred because the ex post split of assets are done optimally through Nash Bargaining. However, in reality, negotiation is often costly, which many authors suggests one of the debt overhang costs. Therefore, in the presence of sizable verification and negotiation costs, the simple and speedy bankruptcy procedure (Regime I) is preferred to the costly negotiation (Regime II), and in this case the transparent bank bailout is welfare improving.

In summary, this paper provides a more solid microfoundation for the macroeconomic models with financial frictions by looking at the incomplete consumption sharing among different types of households as the logical consequence of financial frictions. With this new framework, the financial sector policies—bank bailouts, deposit insurance, and regulations—can be examined as redistribution policies, which could be welfare improving.

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Figure 1. Capital Exchange with Complete Contracts

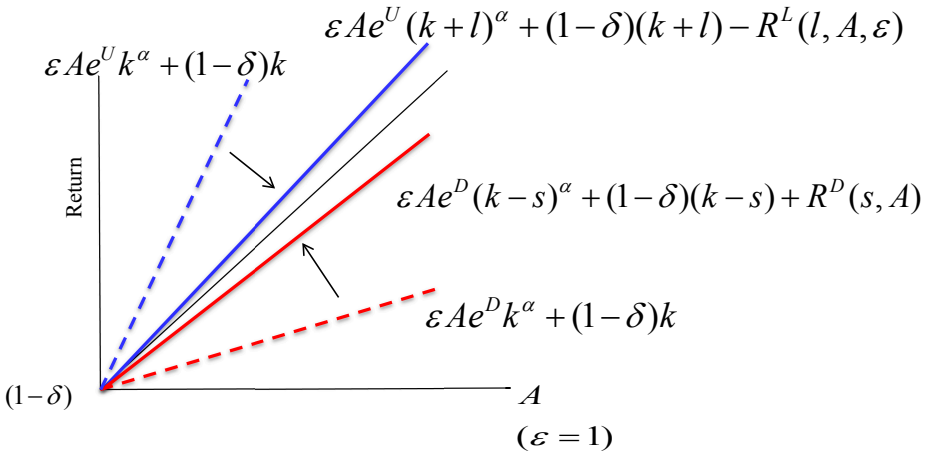


Figure 2. Capital Exchange with Simple Debt Contracts

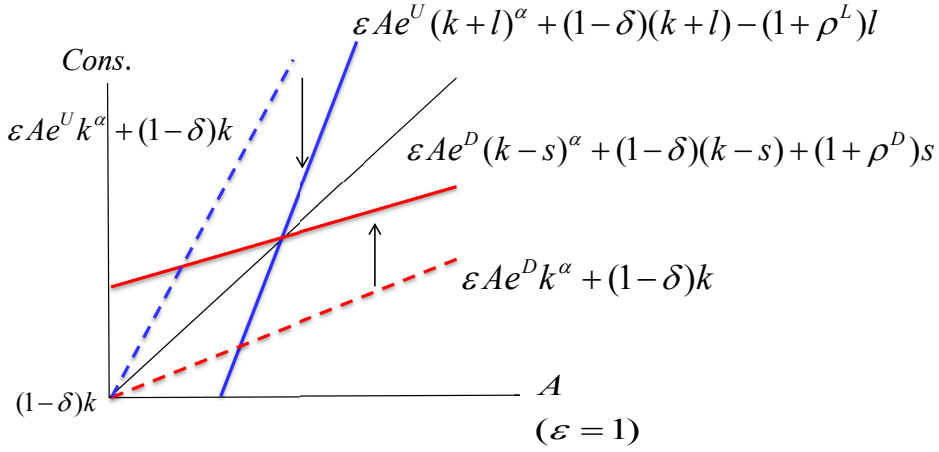


Figure 3. Debt Contracts with Limited Liability

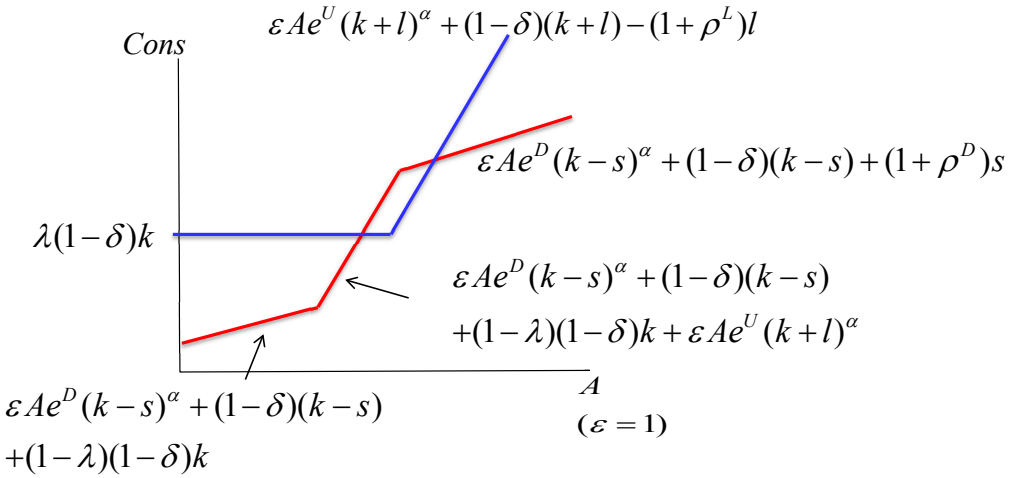


Figure 4. Debt Contracts and Banks

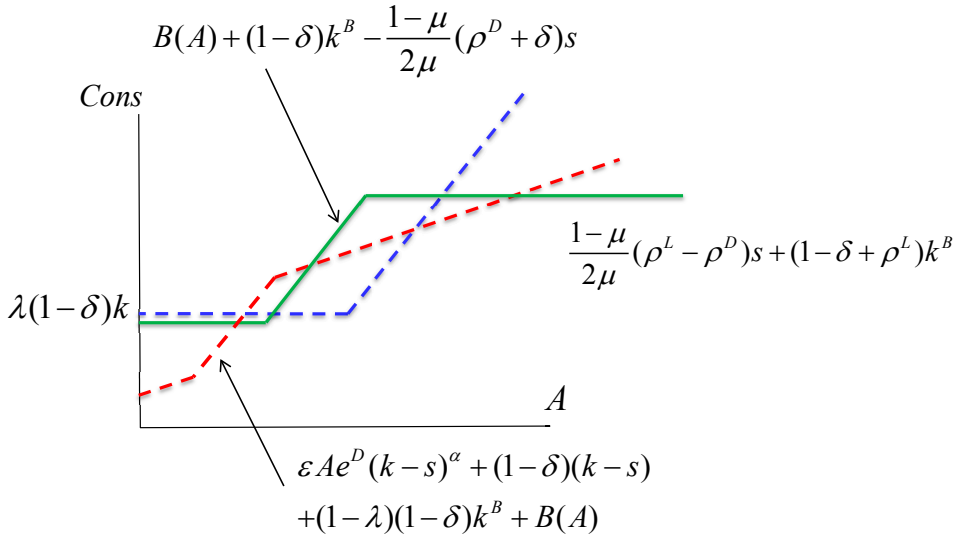


Figure 5. Loan Market Partial Equilibrium

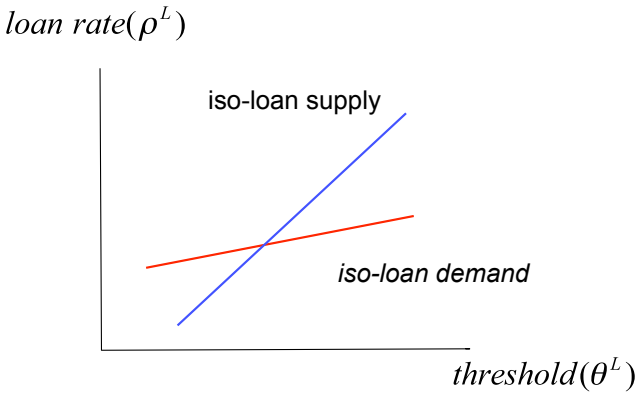


Figure 6. Deposit Market Partial Equilibrium

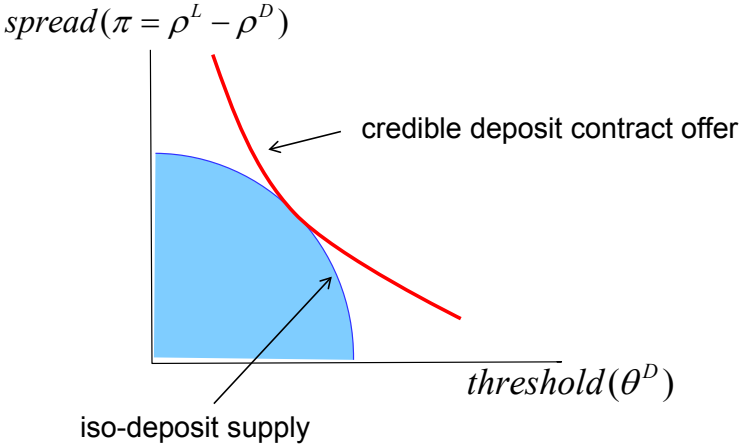


Figure 7. General Equilibrium given Bank Population μ

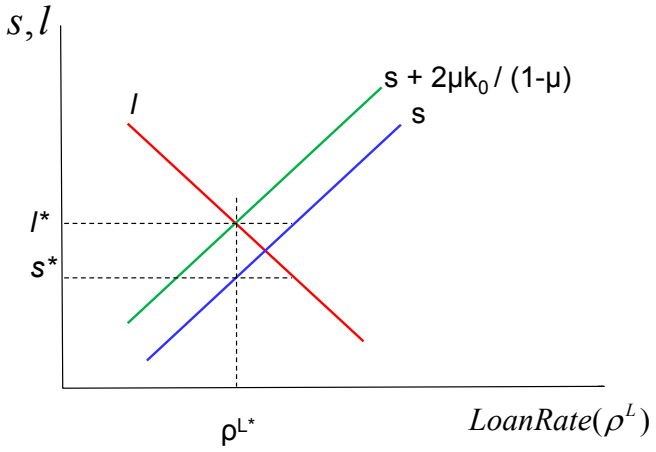


Figure 8. Unique Bank Population in General Equilibrium

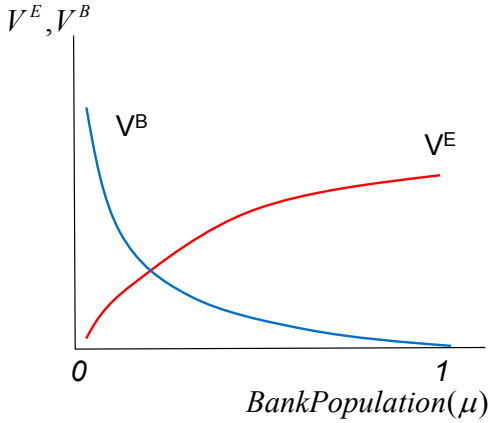


Figure 9. Deposit Return per Depositor

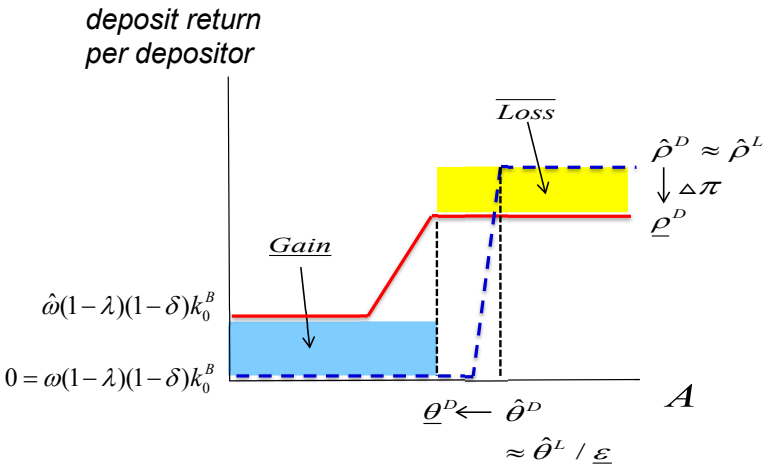


Figure 10. Optimal Tax-Transfer System

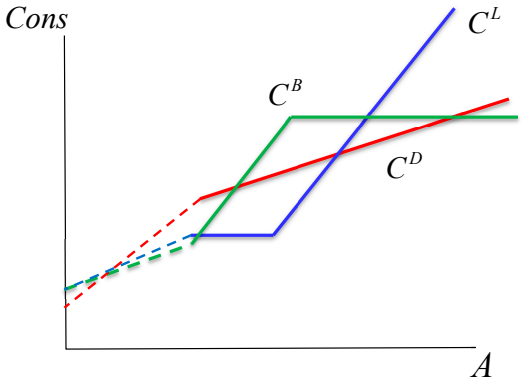


Figure 11. Deposit Risk-Return with Expected Bailout

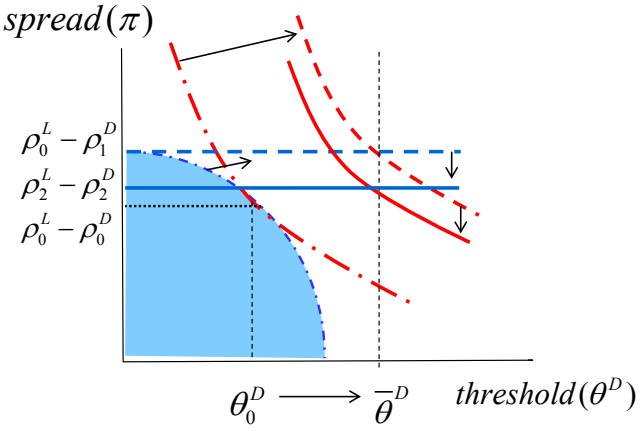
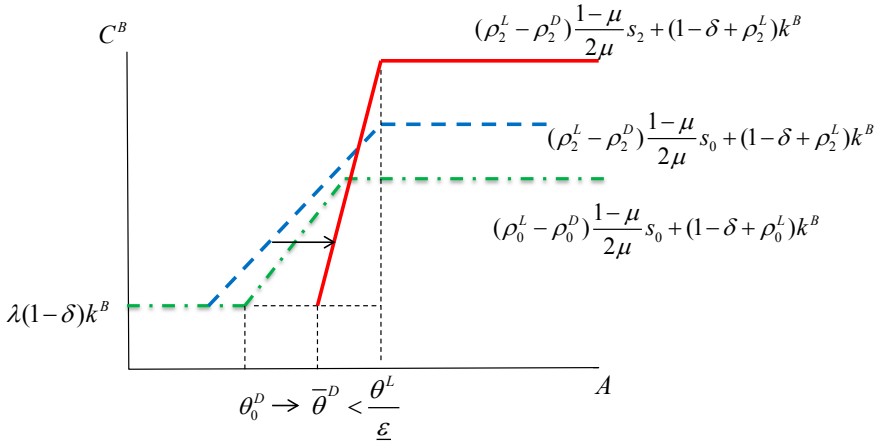


Figure 12. Bank Consumption with Expected Bailout



APPENDIX I. PROOFS

A. Proof for Lemma 5

Proof. The derivative of the iso-loan demand function with respect to θ^L is expressed as

$$\frac{\partial \chi(\theta^L, l)}{\partial \theta^L} = \left((1+\rho^L) - \alpha \theta^L e^U (k_0 + l)^{\alpha-1}\right) u'(\underline{c}^L) m(\theta^L) > 0, \tag{A1}$$

where $\underline{c}^L$ is evaluated at $\eta = \theta^L$ and m is the pdf. This is positive: Looking at the relationship between the marginal product of capital and the loan rate in the first order condition (29), at threshold θ^L, the marginal product of capital must be lower than the loan rate.

The derivative with respect to ρ^L is

$$\begin{aligned}
\frac{\partial \chi(\theta^L, l)}{\partial \rho^L} &= \int_{\theta^L}^{\overline{\epsilon A}} \left(-u'(c^L) - (\alpha \eta e^U (k_0 + l)^{\alpha-1} - (1+\rho^L)) l u''(c^L)\right) dM \\
&= -\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM + \sigma l \int_{\theta^L}^{\overline{\epsilon A}} (\alpha \eta e^U (k_0 + l)^{\alpha-1} - (1+\rho^L)) \frac{u'(c^L)}{c^L} dM \\
&< 0.
\end{aligned} \tag{A2}$$

Note that inside the second integral in the penultimate line has a "weight" of u'/c^L, which has higher weights for the lower realization of shocks and lower weights for the higher realization of shocks compared to the weight u' in the first order condition (29). Because the second integral is different only in this "weight" from the borrower's first order condition (29), the second integral must be negative.

In summary,

$$\frac{d\rho^L}{d\theta^L} = -\frac{\partial \chi / \partial \theta^L}{\partial \chi / \partial \rho^L} > 0. \tag{A3}$$

Q.E.D.

B. Proof for Proposition 1

Proof. On the θ^L-ρ^L plane, the iso-loan default curve is increasing (Lemma 4) and the iso-loan default curve is also increasing (Lemma 5).

As $\theta^L \to 0$, the iso-loan default curve converges to the intercept: $-1 - \lambda k_0 / l$, which is lower than -1. For the iso-loan demand curve, the first order condition (29) implies$(1+\rho) > 0$ no matter what the level of θ^L. Hence, the intercept at $\theta^L = 0$ is bigger than -1 on the θ^L-ρ^L plane.

Now, I show that the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve. This means that the iso-loan default curve crosses the iso-loan demand curve from below only once on the θ^L-ρ^L plane. Therefore, for each loan amount l, the pair (θ^L, ρ^L) is determined uniquely in equilibrium.

First, I simplify the second integral of (A2), which is

$$\begin{aligned} &\alpha\eta e^U(k_0+l)^{\alpha-1} - (1+\rho^L) \\ <&\eta e^U(k_0+l)^{\alpha-1} - (1+\rho^L) \\ <&\eta e^U(k_0+l)^{\alpha-1} - (1+\rho^L)\frac{l}{k_0+l} = \\ &\frac{c^L}{k_0+l}. \end{aligned} \tag{A4}$$

This implies

$$\begin{aligned} -\frac{\partial\chi(\theta^L,l)}{\partial\rho^L} &> \int_{\theta^L}^{\overline{\epsilon A}} u'(c^L)dM - \sigma l\int_{\theta^L}^{\overline{\epsilon A}} \frac{c^L}{k_0+l}\frac{u'(c^L)}{c^L}dM \\ &= \left(1-\frac{\sigma l}{k_0+l}\right)\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L)dM. \end{aligned} \tag{A5}$$

Hence, the slope of iso-loan demand curve is

$$\begin{aligned} \frac{d\rho^L}{d\theta^L} &= -\frac{\partial\chi/\partial\theta^L}{\partial\chi/\partial\rho^L} \\ &< \frac{\left((1+\rho^L)-\alpha\theta^L e^U(k_0+l)^{\alpha-1}\right)u'(\underline{c}^L)m(\theta^L)}{\left(1-\frac{\sigma l}{k_0+l}\right)\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L)dM} \\ &< \frac{\left((1+\rho^L)-\alpha\theta^L e^U(k_0+l)^{\alpha-1}\right)}{1-\frac{\sigma l}{k_0+l}}. \end{aligned} \tag{A6}$$

Note that the last line uses the apparent relation

$$u'(\underline{c}^L)m(\theta^L) < \int_{\theta^L}^{\overline{\epsilon A}} u'(c^L)dM. \tag{A7}$$

Hence, the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve if

$$\frac{\left((1+\rho^L)-\alpha\theta^L e^U(k_0+l)^{\alpha-1}\right)}{1-\frac{\sigma l}{k_0+l}} < \frac{e^U(k_0+l)^{\alpha}}{l}, \tag{A8}$$

or equivalently,

$$\frac{\left((1+\rho^L)-\alpha\theta^L e^U(k_0+l)^{\alpha-1}\right)}{k_0+l-\sigma l} < \frac{e^U(k_0+l)^{\alpha-1}}{l}. \tag{A9}$$

Here, I first show that the numerator of the left-hand side is smaller than that of the right-hand side. And, then later, I show the denominator relation is the opposite.Note that the first order condition (29) can be expressed as

$$1+\rho^L = \frac{\int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta e^U(k_0+l)^{\alpha-1}\right) u'(c^L)dM(\eta)}{\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L)dM(\eta)}. \tag{A10}$$

Because the marginal product of capital is increasing in productivity shock η while the marginal utility is decreasing in the same shock, covariance of these two terms are negative and hence[32]

$$\begin{aligned} 1+\rho^L &< \frac{\int_{\theta^L}^{\overline{\epsilon A}} \alpha\eta e^U(k_0+l)^{\alpha-1}dM(\eta)u'(c^L)\int_{\theta^L}^{\overline{\epsilon A}} dM(\eta)}{\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L)dM(\eta)} \\ &= \int_{\theta^L}^{\overline{\epsilon A}} \alpha\eta e^U(k_0+l)^{\alpha-1}dM(\eta). \end{aligned} \tag{A11}$$

Hence, to prove the numerator of the right hand side of (A9) is smaller than that of the left hand side, it suffices to show

$$\int_{\theta^L}^{\overline{\epsilon A}} \alpha\eta e^U(k_0+l)^{\alpha-1}dM(\eta) - \alpha\theta^L e^U(k_0+l)^{\alpha-1} < e^U(k_0+l)^{\alpha-1}, \tag{A12}$$

or equivalently,

$$\alpha\left(\int_{\theta^L}^{\overline{\epsilon A}} \eta dM(\eta) - \theta^L\right) = \alpha\left(1-M(\theta^L)-\theta^L\right) < 1. \tag{A13}$$

But, because the inside of the parenthesis is less than 1, this is satisfied for any $\alpha \in [0,1]$, which is always the case.

Next, it needs to prove the denominator of the right hand side of (A9) is larger than or equal to that of the left hand side,

$$\begin{aligned} k_0+l-\sigma l &\geq l \\ k_0-\sigma l &\geq > 0 \\ \frac{k_0}{l} &\geq \sigma. \end{aligned} \tag{A14}$$

The loan to capital ratio is at most 2, if $\mu \leq 1/3$ and $l \leq s + k_0^B = 2k_0$. Then, the condition (A14)

[32]Recall that, for any two random variables ξ and ν, $E[\xi,\nu] = E[\xi]E[\nu] + cov(\xi,\nu)$.

becomes simply

$$\frac{k_0}{l} \geq \frac{1}{2} \geq \sigma. \tag{A15}$$

If the risk aversion parameter is small $\sigma \leq 1/2$, then the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve. This is true regardless of other parameter values.

However, it appears too restrictive and I investigate more for the other values of σ. Recall that a borrower's loan comes from savings of depositors and bankers' capital. Again, it is at most $l \leq 2k_0$ and $l = 2s$ in this case. Also, the capital exchange happens at most under the first best allocation, in which the marginal product of capital equates in each state, i.e.,

$$\alpha\eta e^U (k_0 + l)^{\alpha-1} = \alpha\eta e^D (k_0 - s)^{\alpha-1}. \tag{A16}$$

This can be simplified to

$$\frac{k_0 + l}{k_0 - s} = \frac{e^U}{e^D}^{\frac{1}{1-\alpha}} \equiv Z. \tag{A17}$$

Using $l \leq 2s$ and Z,

$$\frac{l}{k_0} = \frac{Z-1}{\frac{Z}{2}+1}. \tag{A18}$$

Then, the condition (A14) becomes simply

$$\frac{\frac{Z}{2}+1}{Z-1} \geq \sigma. \tag{A19}$$

But as e^U/e^D becomes 1, the left hand side increases to ∞, and as e^U/e^D approaches ∞, the left hand side decreases to $1/2$. Therefore, this condition is satisfied if $\sigma > 1/2$. Together with the condition (A15), there is no restriction on σ. This implies that the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve. *Q.E.D.*

C. Proof for Lemma 6

Proof. Using (20), the credible deposit contract offer curve (31) is expressed as

$$\begin{aligned} \pi = 1 + \rho^L + \frac{2\mu}{(1-\mu)s}\lambda k_0^B \\ - \frac{1}{s}\left(1 - H\left(\frac{\theta^L}{\theta^D}\right)\right)(1+\rho^L)l \\ - \frac{1}{s}\int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}} \left(\epsilon\theta^D e^U (k_0+l)^\alpha - \lambda k_0 - \tau\right) dH(\epsilon). \end{aligned} \tag{A20}$$

Multiply both sides of (A20) by s and take a derivative of the right hand side with respect to θ^D:

$$\begin{aligned}
&-(1+\rho^L)l\frac{\theta^L}{\theta^{D2}}h+\left(\frac{\theta^L}{\theta^D}\theta^D e^U(k_0+l)^\alpha-\lambda k_0-\tau\right)\frac{\theta^L}{\theta^{D2}}h-\int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}}\epsilon e^U(k_0+l)^\alpha dH(\epsilon)\\
=&-\left[(1+\rho^L)l-\left(\theta^L e^U(k_0+l)^\alpha-\lambda k_0-\tau\right)\right]\frac{\theta^L}{\theta^{D2}}h-\int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}}\epsilon e^U(k_0+l)^\alpha dH(\epsilon) \qquad \text{(A21)}\\
=&-\tau\frac{\theta^L}{\theta^{D2}}h-\int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}}\epsilon e^U(k_0+l)^\alpha dH(\epsilon)<0,
\end{aligned}$$

where pdf h is evaluated at θ^L/θ^D. This is negative and thus the credible deposit contract offer curve is strictly decreasing. Note that, the last line follows the fact that the full loan repayment $1+\rho^L$ is equal to the all the outputs plus net-of-retained portion of depreciated capital of a borrower at the default threshold of a loan (i.e., ϵA is at θ^L) as shown in (17). *Q.E.D.*

D. Proof for Proposition 2

Proof. The first order condition (33) essentially is the optimal portfolio problem of allocating capital so as to equate the internal marginal product from own business (*MPK*) to the outside opportunity, which is the deposit to banks, weighted by the marginal utility, *u'*, that is,

$$0=-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}}(MPK)u'dH(\epsilon)dG(A)+\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}}R^D(S,A)u'dH(\epsilon)dG(A) \qquad \text{(A22)}$$

To secure the uniqueness, I show below that the second order condition with respect to deposit s is negative.

$$\begin{aligned}
&-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}}u'\frac{\partial MPK}{\partial s}dH(\epsilon)dG(A)-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\frac{\partial u'}{\partial s}MPKdH(\epsilon)dG(A)\\
&+\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}}R^D(S,A)\frac{\partial u'}{\partial s}dH(\epsilon)dG(A).
\end{aligned} \qquad \text{(A23)}$$

Note that

$$\frac{\partial MPK}{\partial s}=-(\alpha-1)\alpha\epsilon Ae^D(k_0-s)^{\alpha-2}=\frac{1-\alpha}{k_0-s}MPK, \qquad \text{(A24)}$$

and

$$\frac{\partial u'}{\partial s}=u''\frac{\partial c^D}{\partial s}=u''\left(-MPK+R^D(S,A)\right). \qquad \text{(A25)}$$

Then, the second order condition (A23) becomes

$$
\begin{aligned}
&-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\frac{1-\alpha}{k_0-s}(MPK)u'dH(\epsilon)dG(A)+\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\bar{\epsilon}}(-MPK)(-MPK+R^D)u''dH(\epsilon)dG(A)\\
&+\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\bar{\epsilon}}R^D(-MPK+R^D)u''dH(\epsilon)dG(A)\\
=&-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\frac{1-\alpha}{k_0-s}(MPK)u'dH(\epsilon)dG(A)+\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\bar{\epsilon}}(R^D-MPK)^2u''dH(\epsilon)dG(A)\\
<&0.
\end{aligned}
$$

Q.E.D.

E. Proof for Lemma 8

Proof. The iso-loan supply function (26) does not have any term involving banker population μ and thus is not affected by a change in μ. This is because the loan market contracts are offered by banks based solely on the optimization taking into account the financial frictions, not on their own capital amount or on the size of the banking sector.

Similarly, the iso-loan demand function (29) is not affected by a change in μ either. This is because the borrowers do not care about the balance sheet condition of the lenders.

Therefore, for any given loan amount l, the equilibrium loan contract (ρ^{L*}, θ^{L*}) is not affected by a change in μ. *Q.E.D.*

F. Proof for Lemma 9

Proof. As for the credible deposit contract offer curve, with larger banker population μ, it shifts up on the θ^D-π plane for a given deposit amount: i.e., deposit rate offer declines for the same default threshold. This is because the credible deposit contract offer (31) has a term with banker population μ. It is the intercept represented by bank capital per depositor:

$$\frac{2\mu}{(1-\mu)s}\lambda(1-\delta)k_0^B. \tag{A27}$$

This is apparently increasing with μ. Essentially, a bank insures depositors more with less leverage. This cost is compensated by higher spread π, implying lower deposit rate (given loan rate) for the same default threshold of a banker.

On the other hand, with larger banker population μ, the iso-deposit supply curve shifts towards northeast on the θ^D-π plane for a given deposit amount. Note that depositors' consumption increases

due to larger capital buffer of banks in case of bank defaults as shown in (32). This means that the return from the bank deposit, in particular for the tail risk events, becomes higher. Accordingly, in their portfolio decision, depositors will allocate more funds to banks. Indeed, in their first order condition (33), banker population μ appears only when the aggregate shock is below threshold, $A < \theta^D$. This matters only for the left-hand-side, inside consumption c^D, which affects portfolio choice via the marginal utility weights $u'(c^D)$. Higher consumption in tail risk events means lower utility weights in the left-hand-side. To compensate, more funds are allocated to banks and less funds are allocated to own production, whose marginal product of capital increases in the left-hand-side of (33) to offset (partially) the lower utility weight.

In essence, depositors are better off with larger bank capital buffer because they can recover their deposits more in case of bank default. To make the same deposits, depositors accept lower deposit rate and higher default threshold.

In summary, a larger bank population μ induces the credible deposit contract offer curve to shit up and the iso-deposit supply curve to shift northeast on the θ^D-π plane (Figure 6). Therefore, deposit rate must become lower and default threshold higher in the deposit market partial equilibrium given deposit amount. *Q.E.D.*

G. Proof for Lemma 11

Proof. The above proof for Lemma 9 shows that additional bank capital buffer improves depositors' consumption in tail events. That is, V^D becomes larger. With larger μ, depositors would deposit more for the same deposit contract or even with slightly lower deposit rate with slightly higher default threshold.

With larger deposit per depositor and larger bank capital per entrepreneur, the loan amount l must becomes larger in the new equilibrium. Then, Proposition 1 implies that both loan rate ρ^L and borrowers' default threshold θ^L are lower in the new equilibrium. With larger loan supply per borrower and better loan contract, borrower's utility V^L becomes larger. *Q.E.D.*

H. Proof for Lemma 10

Proof. Take the derivative of consumption of a banker (24) – (23) with respect to banker population μ. If every bank defaults, the effect of banker population to consumption of a banker is zero: For $A \in [\underline{A}, \theta^D]$, based on (24),

$$\frac{\partial C^B(A, R^L, \Omega, R^D, \Psi)}{\partial \mu} = 0. \tag{A28}$$

If every bank honors the deposit contract and every borrowers repays in full to banks, a larger number of bankers implies less profits per banker: For $A \in [\frac{\theta^L}{\underline{\epsilon}}, \overline{A}]$, based on (22),

$$\begin{aligned}
&\frac{\partial C^B(A; R^L, R^D)}{\partial \mu} \\
=&\frac{-(1-\mu)^2-\mu}{2\mu^2}(\pi s) + \frac{1-\mu}{2\mu}\frac{\partial(\pi s)}{\partial \mu} \\
=&\frac{1}{2\mu^2}\left((1-\mu)\left(\frac{\partial(\pi s)}{\partial \mu} - (1-\mu)(\pi s)\right) - \mu(\pi s)\right).
\end{aligned} \tag{A29}$$

Note that, in the second line, the first term is the direct negative effect due to more competitors for the same spread income per entrepreneur. The second term is the indirect positive effect due to more exchange of capital between entrepreneurs. This somewhat offsets the negative effect but not perfectly. Overall, it is negative if

$$\begin{aligned}
\frac{\partial(\pi s)}{\partial \mu} <& \frac{\mu}{1-\mu}(\pi s) + (1-\mu)(\pi s) \\
<& \left(1 + \frac{\mu^2}{1-\mu}\right)(\pi s).
\end{aligned} \tag{A30}$$

This is true under Assumption 5.

In between, bank honors the deposit contract but some borrowers default: For $A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]$, borrowers with high idiosyncratic shock $\epsilon \geq \theta^L/A$ would not default, and thus the same condition as above apply. In this case, bankers income per entrepreneur, net of deposit repayment, is decreasing with larger number of bankers.

For the same aggregate shock $A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]$, borrowers with low idiosyncratic shocks $\epsilon < \theta^L/A$ would default and the others would not. Based on (23),

$$\begin{aligned}
&\frac{\partial C^B(A, R^L, \Omega, R^D, \Psi)}{\partial \mu} \\
=&\frac{-(1-\mu)^2-\mu}{2\mu^2}\int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} R^L(l, A, \epsilon) dH(\epsilon) - (1+\rho^D)s \\
+&\frac{1-\mu}{2\mu}\int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \left(\alpha \epsilon A e^H (k_0 + l)^{\alpha-1} + (1-\delta)\right)\frac{\partial l}{\partial \mu} dH(\epsilon) \\
+&\frac{1-\mu}{2\mu}\frac{\partial(\rho^D s)}{\partial \mu} \\
<&0.
\end{aligned} \tag{A31}$$

Note that the integration part of the first line is the expected income of a banker and is positive because a banker in this range of shock $A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]$ would not default. This means the first line is

negative. The second and third lines are also negative because $\partial \rho^D / \partial \mu < 0$ and $\partial l / \partial \mu = \partial s / \partial \mu < 0$ by Lemma 9. *Q.E.D.*