A Theory of Small Campaign Contributions*

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February 25, 2018

Abstract

We present a model of electorally-motivated, small campaign contributions, and show it can explain observations that are hard to reconcile with a simple consumption motive. The analysis uncovers interesting effects across small donors and has novel implications for the effect of income inequality on total contributions and election outcomes. We also study the impact of different forms of campaign finance laws, on contribution behavior, probabilities of electoral outcomes, and welfare. We show that our results are consistent with more behaviorally motivated donors when contributions are driven by parties' solicitation of funds. We also show how the model and its results may have important implications for empirical work on campaign contributions.

*Acknowledgements: we hugely benefited from the insights, comments and suggestions by Scott Ashworth, Ethan Bueno de Mesquita, Anthony Fowler, Debraj Ray, Howard Rosenthal, Konstantin Sonin, Richard Van Weelden, by seminar participants at Harris School (U. Chicago), Harvard, LSE, Namur, Pompeu Fabra and Royal Holloway, as well as from comments on a previous step in this research project, in particular the audiences at Georgetown, Warwick, Mannheim, the Priorat Workshop in Theoretical Political Science, and EPSA conference in Edimburgh.This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 637662). Drazen's research is supported by the National Science Foundation, grant SES 1534132.

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1 Introduction

The role of campaign contributions in elections is a central issue in democracies. Both the popular and academic discussion have largely concentrated on large donors, but small donors account for a large fraction of total contributions. In the 2012 U.S. presidential campaign, the Federal Election Commission reports that out of a cost of campaigns of about $1.3 billion to the main candidates, small contributions (less than $200 each), added up to $621 million, and those between $200 and $1000 added up to another $243 million.[1] The numbers tilted slightly more towards small contributions in the 2016 presidential race: Bernie Sanders, for example, raised 202 million dollars from small contributions, for a total budget of 223 million. Hillary Clinton and Donald Trump also each had more than 2 million small donors in the recent election cycle. Interestingly, towards the end of the campaign cycle, contributions come almost exclusively from small donors, as can be seen in Figure 1, which plots histograms of the distribution of the number of contributions for Clinton by dollar value. (In the Appendix, we also show the share of contributions by size.)[2]

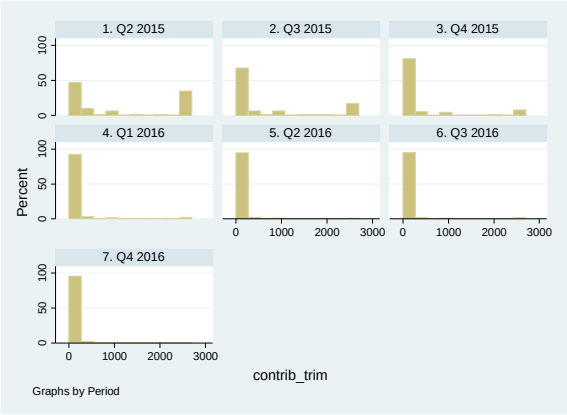


Figure 1: Quarter-by-quarter distribution of the 3'471'316 individual contributions to Hillary Clinton's campaign, from Q2 2015 until Q4 2016 (Source: FEC data). The data displayed here lump together the contributions above $2700, to ease readability.

Small donors are important in other countries as well. In Canada, they represent

[1] http://www.fec.gov/disclosurep/pnational.do;jsessionid=5E34A548A5EEB1D08BBECEA07049DF53.worker1 and http://www.fec.gov/disclosurep/pnational.do

[2] PACs and super-PACs do provide large contributions, but one should note that they are also heavily financed by small contributions.

about a third of total funds raised for recent campaigns. The figure is similar in the United Kingdom, where a significant share of party funding comes from membership dues and small donations (for instance, the Labour party reported £19.2 million in donations and £9.5 million in membership dues in 2015).[3] In Germany, they represent about 53% of campaign resources in the 2012 cycle, with about half of that amount reflecting party membership dues).[4] Small contributions account for such a significant fraction of total funding because the number of small donors is enormous.

The theoretical literature, however, has focused on large donors and a policy influence motive for contributing ("quid pro quo").[5] To the best of our knowledge, there is no formal modeling of small campaign contributions, that is, a model which puts the choices of small donors on whether and how much to contribute into an explicit game-theoretic framework. In large part this appears to reflect the view that small campaign contributions are a pure consumption good for those who contribute, analogous to charitable contributions. The basic reasoning is that because such contributions are so small relative to total campaign funding, donors cannot be motivated either by an attempt to buy influence nor by any effect their contributions may have on election outcomes. A consumption motive wins almost by default because of the atomistic nature of individual small donations.[6]

The aim of this paper is to study small campaign contributions in a more formal game-theoretic model where small donors are motivated by the desire to affect election outcomes. In our model, "small" means that a donor takes both the policy of candidates as given (*i.e.*, there is no motive of trading contributions for policy favors) and the behavior of other donors as given. Hence, "small" can refer to donors who make substantial contributions in dollar terms, but who expect neither to receive policy favors in return nor to influence

[3] http://search.electoralcommission.org.uk/Api/Accounts/Documents/17488

[4] Most of the rest is public funding; medium and large contributions made up only about 9% of total resources.

[5] The leading theoretical model is that of Grossman and Helpman (1994, 1996). The empirical literature finds mixed support (Stratmann, 1992; Ansolabehere, de Figueiredo and Snyder, 2003; Gordon, Hafer, and Landa 2007; Chamon and Kaplan 2013, DellaVigna et al. 2015). Hence, it is not clear to what extent large contributions "buy"policy favors or even access to elected politicians. Given our focus on small contributors in this paper, we take no stand on that empirical debate.

[6] Ansolabehere, de Figueiredo, and Snyder (2003) have stressed this view, arguing that the "tiny size of the average contribution made by private citizens suggests that little private benefit could be bought with such donations"(p117). They support their argument with the finding "income is by far the strongest predictor of giving to political campaigns and organizations, and it is also the main predictor of contributing to nonreligious charities" like other normal consumption goods.

other donors directly.[7] As we argue in Section 2, electoral motives can coexist with, or even be stimulated by, consumption motives for contributing. Also, even though our main model focuses on purely rational-instrumental donors, we show how our approach is also consistent with behavioral approaches to small donor behavior (Sections 2 and 6). Our model should thus be seen as an analysis of small donors' behavior when the electoral motive plays a role, either for purely rational-instrumental reason, or for more behavioral ones.

Because of the strategic interactions that must characterize any model giving a role to electorally-motivated contributions, individual and total contributions may be quite different than those implied by a model of individual choice that ignores such interactions (*e.g.* a basic model of contributions driven solely by the consumption motive). As a consequence, the equilibrium effects of campaign finance laws may be quite different than conventional wisdom or existing literature suggests.

After laying out the base model of a two-candidate race in section 3, we derive the equilibrium level of individual and total contributions in section 4, as well as the equilibrium probabilities of election. We show that equilibrium contributions increase when the support for the two candidates is more even—a "closeness effect"—and that they display an "underdog effect", whereby equilibrium relative contributions for the advantaged party are smaller than their underlying advantage. This contrasts with the predictions under the influence motive, which leads to a "bandwagon" effect in contributions, that is, the advantaged candidate getting disproportionately higher contributions. We also show that donations are increasing in income, which is also predicted when contributions are driven by the consumption motive. Finally, we study the effects of income inequality on the equilibrium. We show that higher income inequality has significantly different effects on contributions if the increase happens within a donor group versus between two groups, as well as on whether it affects donors of the leading or trailing party.

In section 5 we then analyze the effects of various campaign finance laws. We find that a cap on individual contributions generally favors the party with the largest number of donors and works against the party with the richest contributors, but these effects are not necessarily monotonic. Caps on total campaign spending necessarily hurt the

[7] In a subsequent paper we are considering the effect that a very large donor may have on other donors in the electoral context we present here.

party with the largest budget, but may also incentivize donors from the lagging party to contribute more, so that the party initially ahead loses more than the direct effect of the legal constraint would suggest. This indirect effect may be so strong that total contributions increase when the cap is tightened. Finally, we study the effect of public subsidies to the campaign budget and find that equal block subsidies to both parties help the party that is behind, while matching subsidies or taxes on contributions leave election probabilities unchanged when they affect all donations proportionately.

We also study welfare implications of how money affects election outcomes and of policies to limit the effect of contributions, with a focus on how campaign finance laws may limit the influence of income and may help control the "arms race" of ever-higher aggregate contributions. One result concerns the combination of tax and caps on individual contributions. We identify a tax on contributions that, by discriminating across income levels, completely corrects the effects of income inequalities. When that tax is used, middle-of-the-road policies become suboptimal: the optimum is either to essentially ban contributions, or to let money flow freely.

In Section 6 we show how the same basic results would obtain with "naïve" donors being solicited by electorally-motivated candidates, that is, thinking of donors as being more "behavioral" than in our main model of fully rational donors but where politicians are the optimizing actors. Section 7 presents conclusions, and further material and proofs are in Appendices.

Our findings may be relevant for empirical research. First, the different motives for contributions produce qualitatively different donor behavior responses, which could be leveraged to foster our understanding of donors' motivations (see, e.g., Ansolabehere, de Figueiredo, and Snyder 2003 and Barber *et al.* 2017). For instance, election closeness should have no first-order effect on donors if contributions are simply a consumption good, but will affect contributions that are electorally motivated. Another example is that electorally-motivated donors will be induced to contribute more to a candidate who is lagging behind (an "underdog" effect) whereas the incentive is to give to the candidate who is ahead when contributions are made in exchange for policy favors. Second, our results present a cautionary tale for the estimation of behavioral responses, among other to income or regulatory changes. Estimates of the income elasticity of contributions (see,

e.g., Gordon et *et al.* 2007, and Bonica and Rosenthal 2017) may be biased, and the sign of that bias will depend on whether the candidate is ahead or behind, or on the specifics of the income shock. Estimates of the effects of changes in campaign finance laws (such as caps on individual contributions) on electoral outcomes (see, e.g., Lott 2006 and Stratmann and Aparicio-Castillo 2006) are also delicate. Our model predicts that such effects are non-monotonic and may change sign depending on the source of the difference in popularity between candidates.

2 On the Electoral Motive

Logical as it may sound that small donors are too small to be motivated by anything other than a pure consumption motive, there are both theoretical and empirical reasons why electoral motives for small donors, either directly instrumental or behavioral, should not be rejected out of hand.

From a theoretical perspective, "very small" is not zero. That is, a non-zero effect of an individual's contribution on the election outcome means that an optimizing donor should take this effect, however small, into account. This simple observation proves even more important when we embrace the presence of a consumption motive for campaign contributions. A consumption motive implies that a donor should contribute up to the point in which her marginal utility of consumption C is equal to that of her contributions q: $\partial U/\partial C = \partial U/\partial q$. The marginal utility cost of increasing q above that level is therefore essentially nil. The gist of the argument is now straightforward: any non-zero effect of the contribution on the election outcome ($\partial \pi_P/\partial q > 0$ in the model below) will drive additional contributions. How economically significant this effect can be, is of course another question. But, the empirical arguments below suggest that it may be far from trivial.

Even standard utility functions –without a consumption motive, but with a preference for public goods– may imply contributions that represent a significant fraction of a consumer's budget. It is indeed easy to find utility functions (we provide one example with a CARA utility function in Appendix A1.2.) that lead to contributions displaying strong responses to apparently small changes in the effectiveness of the contribution (in our numerical example, individual contributions increase by \$600 if the marginal effect

of contributions on the election probability increases from 10^{-12} to 10^{-9}). Moreover, the decreasing marginal utility of consumption implies that instrumentally-motivated contributions should be increasing in income. That pattern is thus not specific to the consumption motive (in analogy to a similar finding for charitable contributions; see footnote 6) and should not in itself be taken as empirical evidence for the consumption motive against other motivations.

This argument and the example support the idea that a purely instrumental electoral motive has bite, and can actually be reinforced by the consumption motive. But, our objective here is not to defend that purely instrumental version of the electoral motive, *per se*, as our approach is also consistent with a more behavioral perspective. For instance, individual contributors could overestimate the influence of their contribution on the outcome of the election, or they may derive utility from contributing to races in which contributions are more important for the electoral outcome—maybe because the media cover such races more intensely. Thus, through one or another channel, the electoral effects of contributions may end up influencing contributions, and this is what we want to analyse. In section 6, we formally show that a behaviorally-based consumption model, where it is parties rather than individuals that are taking into account the effect of small contributions on electoral outcomes, yields conceptually the same results as those produced by the baseline model. In that alternative model, donors are "naïve" in that they respond to their party's fund-raising efforts according to a simple behavioral rule. The relevant assumption is that parties believe that money helps them win the election. Then, under simple and intuitive assumptions about the behavioral rule of donors, equilibrium contributions are the same as if we assumed purely instrumental donors.

The importance of a (broadly defined) electoral motive for small donors is consistent with empirical regularities. First of all, in surveys (Brown *et al.* 1995; Francia *et al.* 2003; Barber 2016a) donors overwhelmingly list "to affect an election outcome" as an important motive for giving. Furthermore, there are other types of evidence supporting the idea that electoral motivations do influence contributions. First, numerous studies find that ideological proximity is a strong determinant of contributor behavior in different types of contests (see *e.g.* McCarty, Poole, and Rosenthal 2006; Claasen 2007; Bonica 2014; Barber 2016a; Barber, Canes-Wrone, and Thrower 2017). The closeness of the ideological

positions of donors and candidates matters because donors care about election outcomes.[8] Second, donations are significantly and positively affected by the (perceived) closeness of the election (Barber *et al.* 2017). And this effect is economically significant: "a standard deviation increase [in competitiveness] raises the likelihood a donor gives to that campaign by 43%." (Barber *et al.* 2017, p17). While one cannot reject that this is consistent with a consumption motive with sufficiently rich behavioral responses, at the very least it says that a model of small donors should have the probability of outcomes affecting individual decisions. Again, this is exactly our approach.

There is also significant evidence supporting the existence of a two-way relationship between campaign contributions and the electoral outcome. First, there is evidence that money matters for electoral outcomes. The literature can be divided into two sets of studies: the first focus on the effect of specific campaign spending (*e.g.*, TV ads). Recent studies with a well-defined identification strategy find positive and significant effects (see e.g. Da Silveira and De Mello 2011, Kendall et al. 2015, Larreguy et al. 2014, Spenkuch and Toniatti 2016, and Bekkouche and Cage, 2018). The second set of studies analyze the effects of total spending. There, evidence is mixed: spending by challengers appears more effective than spending by incumbents and, for the latter, there is no consensus on whether or not the effect of money is economically significant (see *e.g.* Levitt 1994, Erikson and Palfrey 1998, 2000, Gerber 2004, Stratmann 2009, Bombardini and Trebbi 2011, and Kawai and Watanabe 2015). A simple way to reconcile this seemingly contradictory evidence is provided by Schuster (2016): using detailed transaction-level data on candidate disbursements, he finds systematic differences in the way challengers and incumbents spend money.

3 Model

We model a contribution game in which a pre-determined set of donors simultaneously decide how much to contribute to their preferred candidate's campaign, in order to increase his chances of election (we identify donors with the pronoun "she" and candidates with

[8] A related observation from Barber *et et al.* (2017) is that contributions are made to legislators who "will represent their professional interests, rather than due to expectations of legislative access or an unsophisticated response to networking."This too is consistent with an electoral motive rather than simply a consumption motive for giving, which is exactly our approach

"he"). This captures a situation in which donors are "small" in the sense that they take both platforms and the actions of the other donors as given.[9] Throughout, we focus on the case of perfectly informed donors.

Candidates. We consider an election with two candidates, A and B, who need funding to run their electoral campaign. The total amount of contributions received by a candidate P is Q_P. We summarize through a *contest success function* (Tullock 1980, Hirshleifer 1989, Baron 1994, Skaperdas and Grofman 1995, Esteban and Ray 2001, Epstein and Nitzan 2006, Konrad 2007, Jia *et al.* 2013, *a.o.*) the fact that P's probability of winning the election increases in her funding. This captures the idea that these funds can finance activities such as get out the vote efforts (see Enos and Fowler, 2016) or advertising (as for example in Baron, 1994, Prat, 2002, Coate 2004a, 2004b, and Morton and Myerson, 2012), which increase a candidate's vote totals.

Given total contributions $\mathbf{Q} = \{Q_A, Q_B\} \in \mathbb{R}^2_+$, P's probability of winning the election is given by:

$$\pi_P(\mathbf{Q}) \equiv \frac{(Q_P)^\gamma}{(Q_A)^\gamma + (Q_B)^\gamma} \tag{1}$$

with $\gamma > 0$, such that the winning probability is strictly increasing in Q_P. Note that π_P is everywhere concave in Q_P for $\gamma \leq 1$. Values of $\gamma > 1$ capture the presence of setup costs: π_P is then convex for $Q_P < \bar{Q}_P \equiv \sqrt[\gamma]{\frac{\gamma-1}{\gamma+1}} Q_{-P}$. In words, P's campaign must reach $\bar{Q}_P$ for additional contributions to have maximal effect.[10] Figure 1 illustrates the shape of π_A for $\gamma = 1$ (in blue), $\gamma = 2$ (in red), and $\gamma = 3$ (in black) when $Q_B = 1$.Candidates are passive in our base model: the players of interest are the donors, who contribute to each candidate's campaign. In Section 6, we show that our results also hold in a model where candidates are the players of interest, and donors naïve.

Donors. A large number of donors must, simultaneously and non-cooperatively, decide how much to contribute to their candidate.[11] Each donor i has a two-dimensional type $\left(p^i, y^i\right) \in \{a, b\} \times \mathbb{R}_+$, where $p^i \in \{a, b\}$ identifies who is her favorite candidate/party–

[9] In a separate project, we study the interactions between large and small donors in a multicandidate setup.

[10] Bonneau and Cann (2011) finding that, in US state supreme court elections from 1990 to 2004, campaign spending features diminishing marginal returns. This advocates in favor of values of γ not too much above 1.

[11] A similar setup has been pioneered by Katz *et al.* (1990) for rent-seeking, and by Esteban and Ray (1999, 2001) to analyze conflict situations, in which individuals invest resources to collectively fight over an issue.

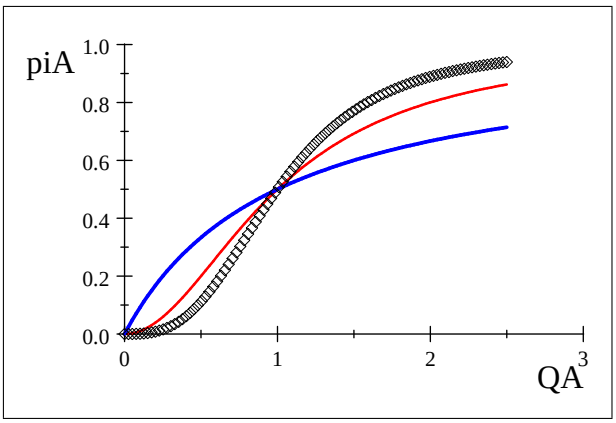


Figure 2: π_A for $Q_B = 1$ and $\gamma = 1$ (blue), $\gamma = 2$ (red), or $\gamma = 3$ (black boxes)

naturally, a-donors support candidate A and a b-donors candidate B: small and capital letters are used to avoid confusion between donors and candidates. y^i represents i's income, which will influence her willingness to contribute.

Income distribution. The n^p donors of type p are distributed in income classes $y^1 < ... < y^G$ according to some (discrete) distribution function $F^p\left(y^i\right)$ with $F^p\left(0\right) = 0$, and $F^p\left(y^G\right) = 1$. The fraction of type-p donors with income y^i is denoted $f^p\left(y^i\right) = F^p\left(y^i\right) - F^p\left(y^{i-1}\right) \geq 0$, and $\bar{y}^p$ is the average income across all p-donors.

Objective function. In line with the motivation discussed in the Introduction, we focus on the electoral motive for contributing to the candidates' electoral campaign. That is, each donor contributes some amount $q_P^i \in [0, \bar{q}]$, where $\bar{q}$ is the legal contribution limit, to influence the outcome of the election. In light of the discussion in Section 2, the marginal cost of contributing must be zero at $q_P^i = 0$ and strictly increasing above that. Assuming iso-elastic cost functions, this amounts to setting $\rho > 1$ in the objective functions (2) and (3):

$$U^a\left(q_A^i; \mathbf{Q}^{-i}\right) = \pi_A\left(q_A^i; \mathbf{Q}^{-i}\right) v^a - \frac{\left(q_A^i\right)^\rho/\rho}{\left(y^i\right)^\theta}, \tag{2}$$

$$U^b\left(q_B^i; \mathbf{Q}^{-i}\right) = \pi_B\left(q_B^i; \mathbf{Q}^{-i}\right) v^b - \frac{\left(q_B^i\right)^\rho/\rho}{\left(y^i\right)^\theta}, \tag{3}$$

where v^p is the intensity of the donors' preference for their candidate and $\mathbf{Q}^{-i}$ is the vector of contributions by all donors other than i.[12] The parameter θ will help parametrize the

[12] It is straightforward that types $p^i = a$ want to contribute 0 to B, and conversely for types b.

elasticity of contributions with respect to income: for $\theta = 0$, the cost of contributing is independent of income. For $\theta > 0$ instead, this marginal cost is strictly decreasing in y^i.

Given individual contributions, the total level of contributions received by party P is:

$$Q_A = \sum_{i=1}^{n^a} q_A^i + \varepsilon_A;\ Q_B = \sum_{i=1}^{n^b} q_B^i + \varepsilon_B,$$

where ε_A and ε_B represent the prior contributions, personal war chest, and/or the voters' initial support of the two candidates.[13,14] In the core of the paper, we set them to $\varepsilon_A = \varepsilon_B \to 0$. In Appendix 2, we show how they influence contributions when we relax that assumption.

4 Equilibrium Analysis

We focus on *pure strategy Nash equilibria* of this contribution game: each donor's contribution must be a best response to the vector of contributions by all other donors. In this section, we study the properties of *unconstrained equilibria*: we assume that the cap $\bar{q}$ on individual contributions is not binding. How constraints imposed by campaign finance laws affect this equilibrium are the focus of Section 5.

4.1 Preliminaries: Donors' Incentives

First, we derive each donor's best response for any given contribution profile by the rest of the population. In the next subsection, we impose consistency, *i.e.* that the contribution profile by the rest of the population is also consistent with individual incentives. Let Q_P^{-i} denote total contributions to candidate P by donors other than i in group a. Deriving first order conditions from (2) and solving for types a and types b's best responses we have:

$$\text{For types } a \quad : \qquad q_A^i = \left(\left(y^i\right)^\theta \ \pi'_A \ v^a \right)^{\frac{1}{\rho - 1}} \tag{4}$$

$$\text{For types } b \quad : \qquad q_B^i = \left(\left(y^i\right)^\theta \ \pi'_B \ v^b \right)^{\frac{1}{\rho - 1}}. \tag{5}$$

[13] With a focus on why money polarizes politics (*i.e.* on how platforms are chosen), Feddersen and Gul (2015) let the probability of winning be a combination of voter support V and monetary contributions Q: $\pi_A = \frac{V_A^{1-\gamma} Q_A^\gamma}{V_A^{1-\gamma} Q_A^\gamma + V_B^{1-\gamma} Q_B^\gamma}$. This formulation amounts to setting $\gamma < 1$ and considering asymmetric marginal effects of contributions (see below).

[14] Technically, winning probabilities are indeterminate for $Q_A = Q_B = \varepsilon_A = \varepsilon_B = 0$. Setting $\varepsilon_A, \varepsilon_B$ positive but small solves that problem.

where π'_P is the –*perceived* in a behavioral model or *actual* in our model– *marginal effect of a donor's contribution* on winning probabilities. Interestingly, these best responses imply that, as long as different donors share a same perception of π'_P, the elasticity of contributions with respect to income will be $\theta/(\rho-1)$. Hence, the fact that contributions rise with income is not in itself evidence of a consumption motive. Here, it only results from the electoral motive.

With rational expectations, the marginal effects result from differentiating (1) with respect to one's own individual contribution q_P^i:

$$\pi'_A \equiv \frac{\partial \pi_A}{\partial q_A^i} = \frac{\gamma}{Q_A}\pi_A\left(1-\pi_A\right) = \frac{\gamma}{Q_A}\pi_A\ \pi_B \text{ and,} \tag{6}$$

$$\pi'_B \equiv \frac{\gamma}{Q_B}\pi_A\ \pi_B. \tag{7}$$

This means that, for any given profile of contributions by the rest of the population $\{Q_A^{-i}, Q_B^{-i}\}$:

Observation 1 *A donor's contribution q_P^i increases in* election closeness $\pi_A\pi_B$ *(maximized in $\pi_A = 0.5$) and decreases in Q_P^{-i} (a* free-riding *effect).*

This observation is in line with the findings of Erikson and Palfrey (2000): the effects of contributions on the election outcome is larger for the trailing candidate only when the race is **not** close. In our model, π_A close to 0.5 requires $Q_A - Q_B$ close to 0. In that case, the money spent by the two candidates have effects of a similar magnitude (*i.e.*, $\pi'_A \simeq \pi'_B$).

4.2 Equilibrium Characterization

The above highlights the two-way relationship between individual best-responses q_P^i and the resulting incentive to contribute π'_P. Indeed, a higher π'_P increases each individual's contribution in (4) and (5), which in turn must increase aggregate contributions Q_P. These influence election closeness and free-riding effect, and hence individual incentives. Importantly, exactly the same strategic interactions would obtain in a model of behaviorally motivated donors who respond to candidate solicitations, as set out in Section 6.

To characterize the equilibrium with rational donors, we first derive the total contributions that are consistent with individual best responses:

$$Q_A = n^a \sum_{i=1}^{G} q_A^i \; f^a\left(y^i\right) = W_A \times \left(\pi'_A\right)^{\frac{1}{\rho-1}} \tag{8}$$

$$Q_B = W_B \times \left(\pi'_B\right)^{\frac{1}{\rho-1}}, \tag{9}$$

$$\text{with: } W_P \equiv \left(v^p\right)^{\frac{1}{\rho-1}} \; n^p \sum_{i=1}^{G} f^p\left(y^i\right) \times \left(y^i\right)^{\frac{\theta}{\rho-1}}. \tag{10}$$

Note crucially that (8) and (9) are composed of two factors of a different nature: the first, W_P, only contains exogenous parameters. We can thus treat W_A and W_B as *parameters of the model*. We call them the *group's willingness to contribute*. The second factor is π'_P, the marginal effect of contributions, which is endogenous to the donors' actions.

Without loss of generality, we label A the candidate who is *Ahead* and B the candidate who is *Behind*, in the sense that $W_A \geq W_B$. Let:

$$\omega \equiv \frac{\left(W_B/W_A\right)^{\gamma\left(1-\frac{1}{\rho}\right)}}{\left(1+\left(W_B/W_A\right)^{\gamma\left(1-\frac{1}{\rho}\right)}\right)^2}, \tag{11}$$

summarize the asymmetry in willingness to contribute between the two parties. Note that ω is strictly increasing in W_B/W_A for $W_B/W_A \leq 1$ (and decreasing for $W_B/W_A > 1$).

Our first proposition identifies sufficient conditions for the existence of a pure strategy equilibrium, characterizes it, and shows that it is unique (most proofs are relegated to Appendix 3):

Proposition 1 *Whenever a pure strategy equilibrium exists, it is unique and characterized by the aggregate contributions:*

$$\left(Q_A^*, Q_B^*\right) = \left(\sqrt[\rho]{\gamma\omega W_A^{\rho-1}}, \sqrt[\rho]{\gamma\omega W_B^{\rho-1}}\right),$$

which result in the winning probabilities:

$$\pi_P^* = \frac{\left(W_P\right)^{\gamma\frac{\rho-1}{\rho}}}{\left(W_A\right)^{\gamma\frac{\rho-1}{\rho}} + \left(W_B\right)^{\gamma\frac{\rho-1}{\rho}}}. \tag{12}$$

Two sufficient conditions for Pure Strategy Equilibrium existence are:
(1) $\gamma \leq \rho$ and, if $\rho < \gamma$, (2) W_A/W_B not too large.

As we already got a hint of (Observation 1), equilibrium contributions are affected by

free-riding. The fact that A is ahead implies that free-riding is stronger among a-donors:

Observation 2 *In any equilibrium, the ratio of contributions for A and B displays an* underdog effect:

$$\frac{Q_A}{Q_B} = \left(\frac{W_A}{W_B}\right)^{\frac{\rho-1}{\rho}}. \tag{13}$$

That is, equilibrium relative contributions for A are always smaller than A's intrinsic advantage, W_A/W_B.

Such an underdog effect has already been identified in turnout models, first by Simon (1954) and Palfrey and Rosenthal (1985), and more recently by Herrera *et al.* (2014) in a model with a contest success function.[15] We are not aware of a similar finding regarding political contributions; to the contrary, the classical policy influence motive would predict that contributions to the advantaged candidate are *larger*. This would lead to a *Bandwagon effect*. Stratmann (1992) finds that PAC contributions display a strong bandwagon effect around a threshold between 30 and 35% of the votes, followed by an underdog effect above that threshold. Bonica (2016, Figure 2) however finds that small donors behave substantially differently from Corporate PACs: their contributions disproportionately flow to underdogs (about 55% of their funds, instead of 15% for Corporate PACs). An underdog effect that characterizes electorally-motivated contributions in this model is difficult to reconcile either with the influence or the consumption motives.

It is crucial to note that free-riding issues cannot reverse A's initial advantage.[16] As a result, A's probability of winning does increase in her intrinsic advantage W_A/W_B, even though this increase is attenuated by free-riding. In the absence of free-riding, her probability of winning would be $W_A^\gamma/(W_A^\gamma + W_B^\gamma) > \pi_A^*$.

4.3 The Effects of Income Inequality

The effects of rising income inequality on elections has become a central issue both in public debate and in academic research. The typical perception is that it skews policies towards

[15] In voting models, the underdog effect results from the fact that pivot probabilities are higher for the underdog (see among others Castanheira (2003), Myatt (2012), Agranov *et al.* (2014)). Here instead, this result is uniquely driven by free riding.

[16] In a different context, Esteban and Ray (2001) show that this is partly due to the shape of the cost function, and partly to the fact that winning the election acts as a public good. We use the qualifier "partly" because they focus on the case in which $\gamma = 1$. For that value of γ, Esteban and Ray (2001, Proposition 3) identify that free-riding effects cannot dominate collective action when payoffs are similar to that of a purely public good, as we have here.

those favored by the rich and unduly favors the party with the richest supporters (see *e.g.* Feddersen and Gul, 2015). Since we are focusing on fixed platforms, our focus is on the latter effect only: how does income inequality influence each party's total contributions?

As seen in Proposition 1, contributions eventually depend on W_A and W_B, and on ω, the asymmetry in willingness to contribute between the two parties as given by (11), which is itself a function of the ratio between W_A and W_B. The following lemma isolates how contributions eventually vary with each W_P, and underpins the effects that we identify in our next propositions on the effects of income inequality:[17]

Lemma 1 *In equilibrium, Q_A^* is increasing in W_A and in W_B. Q_B^* is decreasing in W_A and increasing in W_B.*

Lemma 1 tells us, first, that a higher willingness to contribute for one candidate always increases her contributions: Q_P^* is strictly increasing in W_P. Second, and perhaps more interestingly, such changes in support for one candidate also affect contributions for the *other* candidate, but not always in the same direction. Remember that A is the candidate who is ahead: an increase in A's support thus reinforces her advantage. This *reduces* election closeness, and hence Q_B^*. Conversely, a higher W_B makes the election *closer*, which stimulates contributions both for A and for B.

Having identified these basic forces, we show that income inequality can have a different impact, depending on whether it happens between or within groups and on which group drives the change. First, we show how the effects of an increase in *between-group* inequality on contributions depends on how it comes about:

Proposition 2 *Let $\theta > 0$ and $\bar{y}^a > \bar{y}^b$, so that between-group income inequality initially favors A. A further increase in inequality that results from an increase in the income of a-donors increases Q_A^* and decreases Q_B^*, whereas if it results from a drop in the income of b-donors, it decreases both Q_A^* and Q_B^*.*

Proposition 2 has a clear empirical implication for the estimation of the income elasticity of contributions (as in, *e.g.*, Gordon et al. 2007, and Bonica and Rosenthal 2016). In

[17] While our focus here is on the effects of income inequality, Appendix 2 details additional comparative statics on the importance of money in elections (as parameterized by γ) and on the effect of closeness on total equilibrium contributions ($Q_A^* + Q_B^*$).

terms of our model, the purpose of the estimation is to measure $\frac{\theta}{\rho-1}$. Consider a shock that is exogenous (or properly instrumented for) and impacts donors' incomes. The estimation could still be biased if it failed to control for the effects of this shock on contributions across groups and across income classes. Take for instance a shock that primarily increases the income of the richest a contributors. The observed aggregate reaction by rich-a contributors will be biased below $\frac{\theta}{\rho-1}$: while their contributions increase because of the direct effect, they are reduced by the resulting free-riding and reduced-closeness effects. No less crucial is to control for the contributors' expectations of whether the candidate they support is ahead or behind: the same income shock but on b-donors would result in an upward bias, because of reinforced closeness.

Next, we study the effects of an increase in *within-group* income inequality and how these effects differ depending on which group is affected:

Proposition 3 *If and only if the income elasticity of contributions is larger than 1, a mean-preserving spread:*
(1) of the a-donors' income distribution increases Q_A^* and decreases Q_B^*.
(2) of the b-donors' income distribution increases both Q_A^* and Q_B^*.

The intuition is that, if and only if the elasticity of contributions to income, $\theta/(\rho-1)$, is strictly larger than 1, contributions become a convex function of income. Increasing within-group inequality then increases the aggregate willingness to contribute W_P. However, a given increase in W_p does not have the same effects if it happens in the group supporting the candidate who is ahead or behind (remember Lemma 1). The empirical implications are similar to the one discussed above. These two results indicate that the effects of income inequality on contributions are complex. Because "income inequality" is not a sufficient statistics to capture all these effects, empirical work may benefit from carefully distinguishing between the different shocks to the overall income distribution.

5 Campaign Finance Laws

We study three types of campaign finance laws that are widespread around the world: (1) Caps on individual contributions (used, e.g., in the U.S., Canada, Chile, France, Israel,

and Japan, among others); (2) Caps on total donations/spending (used, *e.g.* , in many countries in Europe, as well as Chile, Israel, New Zealand, and South Korea); (3) Public subsidies to parties (used, *e.g.*, in many countries in Europe, as well as Israel, Japan, and Mexico) either as block subsidies or as subsidies proportional to individual contributions (including tax deductibility of contributions). The logic extends to taxes on contributions, with an interesting difference related to income.

5.1 Rationales for Campaign Finance Laws

Campaign finance laws are, very generally speaking, meant to limit the influence of money in politics. One rationale is that large contributions buy policy influence outside of any direct effect on voting, that is, trading contributions for policy favors in a "quid pro quo", as discussed in footnote 5. Such a rationale, as important as it might be in practice, plays no role here as we abstract from the influence motive.[18]

A second rationale to limit campaign spending is that it is like an "arms race" – what is crucial is the level of total contributions relative to those of one's opponent. Hence, the level of money ratchets up without giving either candidate a relative advantage but drains resources nonetheless.[19] Our model, built around a contest success function, captures well that feature of campaign spending.

A third argument is that a donor's influence on elections is determined by the size of her contribution, so that large contributors have undue electoral influence. In that context, contribution caps are meant to ensure that the "voices of small donors" are also heard (this is sometimes referred to as the "equalization" argument). This is central to our paper, where richer donors contribute more simply because they are richer and, all else equal, have a greater effect on election outcomes.

The debate about campaign finance in the United States, as reflected in U.S. Supreme Court decisions, has been largely framed in terms of issues of 'freedom of speech'. In the famous Buckley v. Valeo, a majority held that limits on campaign spending and individual

[18] Coate (2004) considers such negative welfare effects of contributions because they buy policymaker influence. In his setup, contribution limits may increase social welfare not only because they reduce such influence, but also – and because of this – such limits increase the information value of activities that contributions finance.

[19] Another important factor is the difference between ε_A and ε_B. In particular, incumbency typically provides a substantial exogenous advantage, that a challenger may find easier to overcome with money. See *e.g.* Lott (2006) and Bonneau and Cann (2011).

contributions in the Federal Election Campaign Act of 1971 were unconstitutional because they violated the First Amendment provision on freedom of speech, the argument being that a restriction on spending "necessarily reduces the quantity of expression". Similarly, in the 5-4 majority decision in Citizens United v. FEC, Justice Kennedy argued that limits on corporate and union contributions to PACs should be struck down because such limits interfered with free speech, namely the "right of citizens to inquire, to hear, to speak, and to use information to reach consensus."

Arguments in favor of restrictions also relied on such considerations. In Austin v. Michigan Chamber of Commerce (1990) the court had upheld previous limit on corporate spending, writing "Corporate wealth can unfairly influence elections." Analogously, Justice Stevens, in the minority dissent in Citizens United, reiterated the "unfair influence" argument, writing that "unregulated expenditures will give corporations 'unfair influence' in the electoral process and distort public debate in ways that undermine rather than advance the interests of listeners."

5.2 Campaign Finance Laws: the Positive Effects of Caps and Subsidies

In this section, we study the positive effects of campaign finance laws in the framework of our model and contrast them with the rationales discussed above. The main take away is that, due to the strategic complementarities highlighted in Section 4, the effects of campaign finance laws can have unintended consequences. Among other things, small donors will be affected even if they are not directly capped, an effect almost entirely ignored in the literature. The complementarities central to small donor behavior further suggest that the effects of caps on election outcomes may also be far from simple. Welfare effects are discussed in section 5.3.

5.2.1 Caps on Individual Contributions

The diversity of possible effects is illustrated in the following two propositions: the effects of contribution caps can go in exactly opposite directions, depending on whether the advantage of A results from a larger number of donors (Proposition 4) or from richer donors (Proposition 5). Moreover, the effects need not be monotonic:

Proposition 4 *Consider the case of identical income distributions and preference intensity (v^p) for a- and b-donors, but $n^a > n^b$. In that case:*
(1) π_A will be ***lowest*** *when the cap is not binding;*
(2) π_A will be ***highest*** *when the cap constrains all donors*;
(3) Depending on the shape of the income distribution, the effects of varying the cap can be non-monotonic.

The main driver of the difference between (1) and (2) is the underdog effect (see Observation 2). With $n^a > n^b$, free riding implies that an a-donor with income y^i contributes less than a b-donor with the same income. A binding cap must therefore constrain b-donors more than a-donors. Candidate A is thus better off with a cap than with no cap, and best off when the cap is binding for all donors.

However, this does not imply that the effects of a cap are monotonic, as illustrated in Figure 3.[20] The reason is that capping high-income donors stimulates contributions by low-income donors *and* impacts closeness – remember that closer elections stimulate contributions in both groups. Thus, while the direct effect of the cap favors A (b-donors being more constrained), indirect effects tend to work in the opposite direction, and may dominate.

In the figure, the left pane depicts the equilibrium individual contributions by each donor type (except for high-income b-donors who are capped throughout), given the cap on the horizontal axis. The right pane depicts the probability that A wins as a result of these contributions. As one can tell, indirect equilibrium effects dominate for intermediate caps. In the example, this is due to the fact that small and comparatively large contributions both represent a significant fraction of the total (initially 50%), with no intermediate contributions. This proxies what we typically observe in actual data, where there is a huge number of very small contributions, and another mass at higher levels (typically bunched at legal limits). Technically, when we move from lax to tighter caps, i.e. from right to left on the figure, the cap initially binds for high-income donors only, which corrects for the underdog effect for large contributions, but also increases the weight of

[20] The simulation behind Figure 3 builds on a two-group income distribution with $y_l = 3$ and $y_h = 10$; while we set $\gamma = \rho = 2$, and $v^p = \theta = 1$. The number of low- and high-income donors are: $n_l^a = 60 > n_l^b = 30$ and $n_h^a = 20 > n_h^b = 10$. That is, both income classes are willing to contribute about the same amount (this proxies actual values in the 2015-16 US presidential elections), but there are twice as many a- as b-donors, implying that $W_A = 380$ and $W_B = 190$.

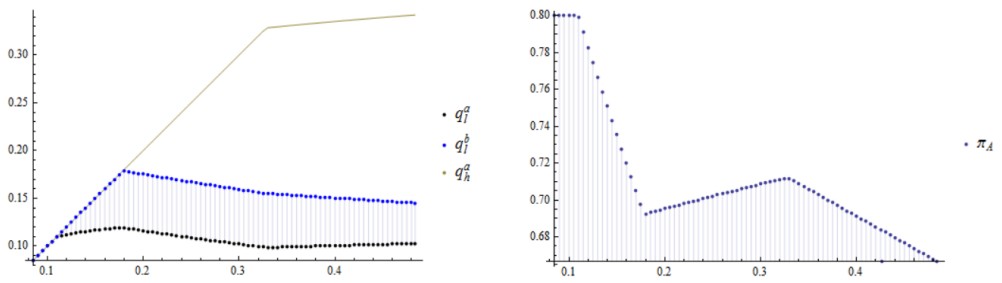


Figure 3: Simulated effect of an individual contribution cap when $n^a > n^b$ but the income distribution is identical across donor groups.

small contributions in the total. When the cap is intermediate (caps between 0.18 and 0.33 in the figure) the underdog effect has been fully addressed among high-income donors, but has been reinforced among low-income donors. Since the latter represent an increasing fraction of the total, tigher caps actually handicap A. In contrast, both lax (above 0.33) and tight (below 0.18) caps primarily reduce the underdog effect, which benefits A.

Now, contrast these results with the case in which the advantage of A is due to *higher donor income*, rather than a numerically larger donor base:

Proposition 5 *Consider the case in which A and B have equal popular support ($n^a = n^b$) and preference intensity, but a-donors benefit from higher income, by a factor $\alpha > 1$ ($f^a\left(\alpha y^i\right) = f^b\left(y^i\right)$, $i = 1, ..., G$). In that case, the effects of a cap are the opposite of the ones in Proposition 4:*

*(1) π_A will be **highest** when the cap is not binding;*

*(2) π_A will be **lowest** when the cap constrains all donors;*

(3) Depending on the income distribution, the effects can be non-monotonic.

The intuition and the mechanism of the proof are similar to those of the previous proposition, with the difference that, if a-donors are richer but no more numerous than b-donors, they must be the first constrained. Hence, there are more a than b constrained donors, and any unconstrained a-donor contributes more than the equivalent b-donor. The initial logic is thus the same as above, with the important difference that closeness and free-riding effects now work in the opposite direction, as illustrated in Figure **??**.[21]

[21] This numerical example also builds on two income classes in each donor group: $y_l^a = 6$ and $y_h^a = 20$, $y_l^b = 3$ and $y_h^b = 10$; $\gamma = \rho = 2$, and $\theta = 1$. Thus a-donors have twice the income of b's, while their numbers are identical: $n_l^p = 30$ and $n_h^p = 10$, $\forall p$. Hence, as in the previous example, $W_A = 380$ and $W_B = 190$.

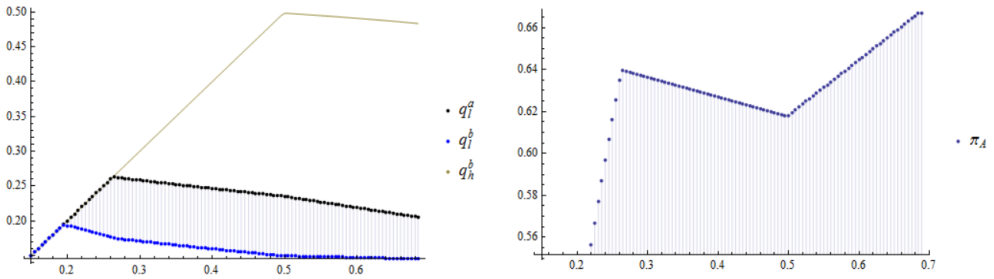


Figure 4: Simulated effect of an individual contribution cap when $y^{i,a} = 2y^{i,b}$ but the number of donors is identical across donor groups.

The empirical literature on the effects of caps on individual contributions finds seemingly contradicting evidence. Stratmann and Aparicio-Castillo (2006) find that, for elections to US state Assemblies (lower house of a bicameral legislature) between 1980 and 2001, caps on individual contributions led to closer elections.[22] Lott (2006) finds the opposite result for elections to US state Senates (upper house) from 1984 to 2002: caps led to less close elections.[23] Propositions 4 and 5 suggest avenues to reconcile these findings. First, empirical studies inevitably focus on the effects of "local" changes in caps on contributions. But, Propositions 4 and 5 show that such local effects need not be monotonic. Estimates as in Stratmann and Aparicio-Castillo (2006) and Lott (2006) may thus have opposite signs simply because the specific cap changes under study affect different parts of the distribution of donors. Second, these propositions also highlight how the effects of caps on individual contributions change sign depending on the main source of differences in support for the candidates. Our model thus suggests to explore in more details these sources for US state legislature elections. For instance, do we observe significant differences in the median number and value of donations for the candidates in those elections?[24]

[22] They also find that both the share and the absolute level of total contributions going to the incumbent decrease significantly. This is also in line with the result in Proposition 5. Stratmann (2006) find that, for the same elections, campaign spending by candidates (both incumbents and challengers) are more effective, and converge one towards the other, in elections with campaign contribution limits. This is also in line with what our model predicts when the cap on contribution has a positive (or nill) effect on the closeness of the race. Indeed, the marginal effect of contributions increase when the total contributions to both parties go down (because of the free-riding effect), and their returns become more equal when $Q_A \to Q_B$.

[23] Similarly, Bonneau and Cann (2011) find that, in US state supreme court elections from 1990 to 2004, campaign finance restrictions (more broadly defined) hurt challengers more than incumbents.

[24] Electoral districts for state house and senate are different. In the vast majority of cases, state senate electoral districts are more populated than house ones. Another difference between state representative and senators is the term length: it is usually longer for senators.

5.2.2 Caps on total spending

Caps on *total* campaign spending, either by parties or by individual candidates, are observed in several countries (Ohman, 2012). In our model, campaign spending by a candidate is equal to total contributions by her supporters, so that we could think of limits on the total size of campaign spending as a cap on total contributions. When the cap on total contributions is binding for both candidates, their total contributions are necessarily identical. We thus focus on the interesting case in which the cap only constrains A (a cursory look at campaign spending by candidates in French presidential elections suggest that not all candidates are constrained by the cap on total spending):

Proposition 6 *Capping total contributions for A increases contributions for B. Therefore, A's probability of winning decreases by more than the direct effect of the cap would imply. Total contributions $Q_A + Q_B$ may increase as a result.*

A cap affecting only A increases elections closeness, which stimulates contributions for B, further favoring the latter. This crowding-in effect on Q_B can be so strong that total contributions $(\bar{Q} + Q_B)$ actually increase when the cap $\bar{Q}$ is tightened. This typically happens when A's lead is initially large (see Appendix 3).[25]

5.2.3 Campaign subsidies

Finally, consider the effects of campaign subsidies. We study two of them: *(i)* a *block subsidy*, where the government gives a lump-sum of s dollars to both candidates' campaigns; and *(ii)* a *matching subsidy*, where for each dollar of contributions, the government adds m cents. In the presence of both types of subsidies, total contributions become:

$$\tilde{Q}_A = \sum_{i=1}^{n^b} (1+m)\, q_A^i + s + \varepsilon_A; \text{ and } \tilde{Q}_B = \sum_{i=1}^{n^b} (1+m)\, q_B^i + s + \varepsilon_B. \tag{14}$$

Consider first a block subsidy s alone, so that $m = 0$ in (14):

[25] Note that this effect is different from the one in Che and Gale (1998), who consider an all pay auction. In that auction, expected total contributions are everywhere (weakly) increasing in the cap, except at a point of discontinuity. When the cap is above that level, the high-valuation bidder can make such aggressive bids that the low-valuation bidder shaves his bids significantly. That reduces total contributions.

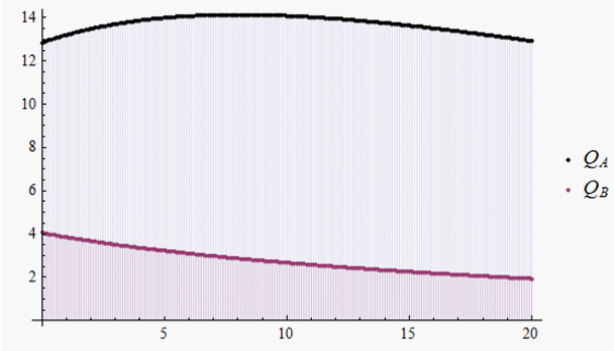


Figure 5: Simulated effect of a block subsidy on total individual contributions when $y^a = y^b$ and $n^a = 100$ and $n^b = 10$.

Proposition 7 *Set $\rho = 2$. Block subsidies then increase the relative voluntary contributions for A, but decrease the probability that A wins: $\frac{d(Q_A/Q_B)}{ds} > 0$, $\frac{d\pi_A}{ds} < 0$.*

A block subsidy has a direct negative effect on the probability that the most popular party, A, wins. This should not be surprising, since an equal subsidy to both candidates "levels the playing field". However, this direct effect is attenuated by the different reactions of a-donors and b-donors. Somewhat surprisingly, a block subsidy can have a crowding-in effect on individual donations by a-donors. This happens when the induced effects of closeness are strong enough, as illustrated by the following example: we consider the case of a single level of income: $y^a = 10 = y^b$ but there are 10 times more a-donors than b-donors: $n^a = 100 > n^b = 10$ (like in the other examples, $\gamma = \rho = 2$ and $\theta = 1$). As one can see on Figure 5, Q_A increases in s when s is low, and decreases in s when s is large.[26]

One direct implication of this proposition is that, neither crowding-in nor crowding-out effects of public subsidies may compensate the direct (negative) effect of the subsidy on the probability that A wins. Moreover, for both parties, the sum of total individual contributions plus the block subsidy always increases with the size of the subsidy.

Consider now a matching subsidy m (which may be negative, that is, a tax on contributions) with no block subsidy ($s = 0$ in (14)):[27]

[26] We did not find any example in which a block subsidy has a crowding-in effect on individual contributions by b-donors.

[27] Instead, a matching subsidy which applies only to contributions below a certain level can affect election probabilities.

Proposition 8 *A matching subsidy m that applies to all contributions has no effect on the behavior of donors, nor on the outcome of the election.*

The first part of the proposition may not be entirely surprising, given the form of our contest success function. Since the matching subsidy increases each (and hence total) contributions by the same fraction m for both candidates, it has no effect on the *relative* position of the two candidates, and hence no effect on election probabilities. Matching subsidies may affect outcomes for other specifications of the contest success function, but the mechanism behind Proposition 8 makes clear why a general matching subsidy will not have a major effect as it has little or no effect on relative candidate positions. Analogously, there is no reason to anticipate that it should either systematically increase or systematically decrease individual contributions.

A matching subsidy that only applies to contributions below a certain level,[28] on the other hand, will generally have an effect. If the aggregate amount of matched contributions (contribution plus matching funds) rises, contributions of those above the matching threhold will decrease. The overall impact on the election could however go either way.

Turning to *taxes* on contributions, making them dependent on the size of the contribution acts like a *negative* size-dependent matching subsidy. Since contributions depend positively on income, this would be like a differential tax on contributions, that is a function of income. Such a tax has the possibility of reducing or even eliminating the effect of income on contributions:

Proposition 9 *A tax on contributions equal to* $\left[\left(y^i\right)^{\theta/\rho} - 1\right] q_P^i$ *removes the effect of income inequalities from equilibrium contributions.*

The tax considered in Proposition 9 increases with income in such a way that all donors, rich and poor, eventually face the same marginal cost of contribution. As a consequence, the size of individual contributions only depends on preference intensity (and the features of the electoral environment, such as the closeness of the race).

Though such a tax seems distant from what is observed in existing campaign finance regulations across countries, a regulation broadly mimicking such a policy is technically

[28] In New York City campaigns, for example, donations up to $175 from New York City residents are matched at a rate of 6:1. In 2013, small donations and matching funds accounted for 71 percent of the individual contributions in the city's elections. See https://nyccfb.info/program/impact-of-public-funds

feasible and in line with existing tax laws, for example in the U.S., in the following sense. Suppose campaign contributions were deductible from income tax liabilities (including perhaps a subsidy or "negative deductibility" as in the previous footnote), but where the allowed deduction was a decreasing function of income. In the United States, for example, allowed itemized deductions as a whole fall with income for high income taxpayers, with deductions in specific categories differentially limited by income. Suppose further that an income-adjusted deductibility specifically for political contributions as described in the sentence above were combined with an increase in tax rates overall. The net effect would be a tax on campaign contributions which increased with the size of the contribution.

Of course the political feasibility of such a change is a separate question. Any proposal framed as a tax on contributions that increases with income would have little prospect of being adopted in the U.S. In contrast, deductibility of contributions that gets phased out as income increases seems far more politically viable, especially since such income-based phase-outs are an accepted part of the U.S. tax code.

5.3 Campaign finance laws: welfare considerations

We now consider the welfare implications of campaign finance laws. As discussed in section 5.1 above, a key rationale for such restrictions is that unlimited contributions give rich donors disproportionate influence on election outcomes. Another argument was to limit the overall explosion of the size of campaign spending. These two arguments can directly be formalized in the framework of our model, as we show here. Below, we extend the discussion to the case in which the number of non-contributing voters does not correlate well with the number of active donors.

Focusing for now on the sum of the donors' individual utilities, we could consider the following objective function for the social planner (SP):

$$U^{SP} = n^a v^a \pi_A - \sum_{i \in a} \frac{\left(q_A^i\right)^\rho}{\left(y^i\right)^\theta \rho} + n^b v^b \pi_B - \sum_{i \in b} \frac{\left(q_B^i\right)^\rho}{\left(y^i\right)^\theta \rho}.$$

In light of the above arguments, however, such a welfare function seems inappropriate: contribution costs being lower for richer donors would produce the result that richer donors deserve disproportionate influence on the election outcome. Correcting this bias requires

setting $\theta = 0$ in the social welfare function:

$$\begin{aligned} U^{SP} &= n^a v^a \pi_A - \sum_{i \in a} \frac{\left(q_A^i\right)^\rho}{\rho} + n^b v^b \pi_B - \sum_{i \in b} \frac{\left(q_B^i\right)^\rho}{\rho} \\ &= \left(n^a v^a - n^b v^b\right) \pi_A - \sum_{i \in a} \frac{\left(q_A^i\right)^\rho}{\rho} - \sum_{i \in b} \frac{\left(q_B^i\right)^\rho}{\rho} + n^b v^b. \end{aligned} \tag{15}$$

The free-speech argument amounts to saying that the group, a or b, with the largest $n^p v^p$ "deserves" winning with the highest possible probability, either because they are more numerous (larger n^p) or because they have more intense preferences (a larger v^p; presumably an influence meant to be protected under the First Amendment). However, this requires allowing them to contribute to the campaign of their candidate, which has a cost $\sum_i \left[\left(q_A^i\right)^\rho + \left(q_B^i\right)^\rho\right]/\rho$ in the social welfare function. Limiting the size of the campaign spending may thus conflict with the former objective.

5.3.1 Contribution caps

Importantly, caps on individual contributions need not produce any such trade-off. To simplify the argument, we focus our attention on the case of no significant exogenous advantage for either candidate: $\varepsilon_A = \varepsilon_B \to 0$.

First, consider the simple case in which preference intensities are symmetric among the two groups, that is, $v^a = v^b$. When all individual donors care equally about election outcomes, differences in group preferences only reflect size, and perhaps income (which we treat as uncorrelated with one another). We find that, in this case, individual contribution caps are an appropriate instrument:

Proposition 10 *When individual preferences are symmetric ($v^a = v^b$), a tight cap on individual contributions necessarily increases social welfare as defined in* (15).

An interesting aspect of this result is that restricting individual contributions actually increases the weight of donor preferences in the election. This stems from the combination of two effects. First, tightening the cap erases the influence of income differences: all donors end up contributing a same amount, the legal maximum. Second, when the advantage of a candidate is driven by a larger group of supporters, individual caps correct the underdog effect, which works against group a. In other words, a tight cap brings us back to the "one

man, one vote benchmark," which is the implicit objective in the first term of (15) when $v^a = v^b$. On top of this, individual caps produce a second dividend: they also decrease "waste," measured by the absolute size of the campaign. This result sheds a new light campaign finance laws that essentially restrict campaign financing to membership dues (Germany is a case in point).

A more delicate, but perhaps more interesting, case is when v^a and v^b differ, so that contributions may reflect both differences in income and in preference intensities. Consider first the case in which there are no income disparities between groups. Then contribution differences only reflect preferences, and the same cap as above would now *reduce* the donors' capacity to convey their preferences. On the other hand, it still reduces waste. Intuitively, for $v^a >> v^b$, the social planner will prefer sufficiently lax caps—the free speech argument. For $v^a \simeq v^b$ instead, the benefit of reducing waste must dominate the cost of a less precise measurement of preferences.

It remains to consider the more difficult case in which the differences in contributions stem both from income and preference differences. In this case, capping contributions is simply too stark a tool because it cannot separate signal (v^p) from noise $\left(y^i\right)$. But this can be addressed by combining a cap with a tax on contributions.

5.3.2 Combining caps with taxes on contributions

Although a tax on contributions has not been considered in practice as part of campaign finance legislation, we show it can help address this conundrum. Under the tax to contributions set out in Proposition 9, equilibrium behavior actually leads to contributions that are independent of income. However, there is still a trade-off between the cost of campaign contributions and the revelation of information about preference intensity. The following proposition show that a cap may be used to address that trade-off:

Proposition 11 *Fix $\gamma = 2 = \rho$, set $n^a \simeq n^b$ and let contributions be taxed like in Proposition 9. Then, (1) equilibrium contributions are the same as if $\theta = 0$; (2) when the population of donors is sufficiently large ($n^a = n^b > v^b/(v^a - v^b)$), the social welfare function* (15) *displays two local optima: one is with $\bar{q} \to 0$ and minimal campaign costs. The other one is with $\bar{q} = \max_i \left[q_P^{i*}\right]$ and, effectively, free speech. But any cap in between these two levels must be welfare inferior to one of these two extreme solutions.*

The intuition for this result is that, thanks to the tax, a cap constrains contributions of all donors in a same group in the same way. With $v^a > v^b$, the cap first constrains all a donors. If it is tightened further, there is a level, call it χ, for which all donors are capped. It follows immediately that, for any cap $\bar{q} < \chi$, winning probabilities are constant. Any cap tightening is then a Pareto improvement.

For $\bar{q} > \chi$ instead, a cap tightening reduces the probability that A wins, which reinforces the initial underdog effect. The question is whether this negative impact of the cap is more than compensated by the decrease in the costs of the campaign. When the number of donors is large, free-riding among a-donors is already severe. This means that the social planner would prefer to increase q_A and reduce q_B. The cap does exactly the opposite, which reduces social welfare. By contrast, we find that, when the number of donors is small, this free-riding effect need not dominate. Also, remember from Proposition 10, that the maximum is unique at $\bar{q} \to 0$ if $n^a >> n^b$.

Proposition 11 has a simple policy implication: when differences in candidate support stem mostly from differences in preference intensities, and that candidates do not benefit from other extraneous advantages ($\varepsilon_A \simeq \varepsilon_B \to 0$), caps on individual contributions should either limit considerably the presence of money in politics, or let it flow freely. Middle-of-the-road policies are suboptimal, even when we only focus on donors' welfare.

5.3.3 Beyond donors' welfare

Typically, donors only represent a relatively small fraction of the total number of supporters for a party. The wider set of supporters also contains passive supporters and active voters. Free-riding, coordination, and information problems may however produce subobtimally low levels of turnout, which the political campaign may help address: this is the implicit logic of the contest success function (1).

Call S^p the total number of supporters behind candidate p. As long as donors represent a fixed proportion of those supporters, $n^a/S^a = n^b/S^b$, and as long as their preference intensity reflects that of the wider group of supporters, the above analysis applies as such. The issue we want do discuss here is when the number of donors may only imperfectly correlated with the size of the wider group, in which case donors' preference may cease to be representative of the population at large (see *e.g.* Barber 2016b).

In this extended setup, the social welfare function should become:

$$U^{SP} = \left(S^a v^a - S^b v^b\right) \pi_A - \sum_{i \in a} \frac{\left(q_A^i\right)^\rho}{\rho} - \sum_{i \in b} \frac{\left(q_B^i\right)^\rho}{\rho} + S^b v^b. \tag{16}$$

If the social planner views the signal conveyed by money as having social value, his first objective should now be to align the number of active donors with the size of the wider population. For this purpose, a limited subsidy to individual contributions starts to make sense. In contrast, it was strictly dominated by a cap on individual contributions in our bechnmark case. Here, the individual cap usefully complements that subsidy: it limits waste and the influence of large donors, as before, and can only crowd in very small donors who may not contribute at all otherwise. We acknowledge that our model is not designed to capture the extensive margin of contributions, and that this argument is thus largely heuristic.It has already been argued that matching small donor funds can provide a counterweight to "big money", even more so with the growing importance internet and social media fundraising.[29] Second, imagine that donors and voters have "sluggish preferences", in the sense that their probability of mutating from preferring A to preferring B between elections is relatively low. In this case, the size of each candidate's campaign should be a combination of past and current signals when the correlation between n^p and S^p is positive but below 1. The lower the correlation between n^p and S^p, and the higher the correlation of S^p with past performance, the higher the weight of the past signal. This can be achieved by a combination of block subsidies proportionate to past electoral performance and "spot" donations.

It is interesting to note that, in all the arguments here, a cap on total contributions seems dominated by the other instruments, possibly used in combination. Such a cap may only benefit B, and does not always address the "arms race" problem with campaign contributions—we saw that it may instead lead to an increase in total contributions.

[29] https://www.brennancenter.org/analysis/money-politics-101-what-you-need-know-about-campaign-finance-after-citizens-united#39

6 A Model of Naïve Donors and Party Fund-Raising

One may argue that modeling donors as fully rational and strategic in their instrumental behavior lacks realism. That is, in their electorally-driven giving, small donors may display more "behavioral" motivations. For example: (1) donors may mechanically react to media attention and/or party fund-raising efforts, and the media or parties focus more on tighter races[30] – we investigate this possibility below; (2) free-riding effects could be rationalized by individual donors enjoying "feeling important" – they would therefore contribute less if other donors contribute more (note that "herding" effects in consumption would produce the opposite result); (3) candidates may intensify their fund-raising effort on small donors when large donors have contributed less: this would also be consistent with a free-riding result.[31]

The purpose of this section is to show that our key results are fully consistent with such behavioral motivations. Comparative statics go in the same direction or can even be identical. We show that a reasonable functional representation of behavioral responses lead to the same first-order conditions, and hence identical results. Hence, whether individual behavior is driven by a purely instrumental electoral motive as above, or by another type of behavioral-instrumental motive, the strategic interactions identified in the previous section are key to understanding how aggregate contributions are determined in equilibrium.

To formalize this point, we assume in this section that small donors are "behavioral" in the sense that they mechanically respond to party nudges. Parties, on their side, need to exert a costly effort in order to induce their supporters actually to contribute to their campaign. This change in perspective transforms our model into a "demand-side" model in which parties are the strategic actors, rather than a "supply-side" model in which donors were the strategic actors.

Such an alternative model could be as follows. As in our base model, consider n^p donors of type p, distributed in income classes $y^1 < ... < y^G$ according to some (discrete) distribution function $F^p\left(y^i\right)$, that satisfies the same assumptions as in section 3. We assume that donor i reacts mechanically to her party's (costly) fund-raising effort, denoted e_P^i. Her contribution q_P^i is increasing and concave in both e_P^i and y^i. We represent this

[30] In other words, one could consider the case in which $\pi'_P(\mathbf{Q})$ enters directly or indirectly the utility function of the consumer of political races.

[31] We thank Debraj Ray for suggesting some of these alternative scenarios consistent with our results.

functionally by:

$$\text{For types } a \quad : \quad q_A^i = \left(\left(y^i \right)^\theta v^a e_A^i \right)^{\frac{1}{2}} \tag{17}$$

$$\text{For types } b \quad : \quad q_B^i = \left(\left(y^i \right)^\theta v^b e_B^i \right)^{\frac{1}{2}}, \tag{18}$$

where θ parameterizes the donors' elasticity of contributions exactly like in the instrumental model. The Cobb-Douglas specification is chosen both for simplicity and to relate with the main model.

Parties choose e_P^i to maximize their probability of winning net of the cost of fundraising (where, for simplicity, we let the cost of soliciting a donor be e_P^i):

$$P \text{ maximizes} \quad : \quad \frac{Q_P^\gamma}{Q_A^\gamma + Q_B^\gamma} - \sum_i e_P^i,$$
$$s.t.\ Q_P = \sum_i q_P^i.$$

It follows that:

$$e_P^{i*} = \left(\frac{\pi'_P}{2} \right)^2 \left(y^i \right)^\theta v^p.$$

Substituting these equilibrium levels of party effort into the donors' contribution functions (17) and (18) yield:

$$q_A^{i*} = \frac{\pi'_A}{2} \left(y^i \right)^\theta v^a,$$
$$q_B^{i*} = \frac{\pi'_B}{2} \left(y^i \right)^\theta v^b,$$

which is identical (but for the factor $\frac{1}{2}$) to (4) and (5) when $\rho = 2$.

In other words, there exists some form of response by behavioral donors and strategic parties such that the equilibrium level of individual and aggregate contributions are the same as with strategic donors and passive parties. Hence, although it is a perfectly valid empirical question to ask, "How rational are small donors?", allowing them to be "behaviorally motivated" rather than fully rationally instrumental does not qualitatively change our findings on how electoral motives (here on the part of parties) determine individual contributions, nor on how economic variables and legal constraints would influence total contributions and the feedback loops between aggregate and individual contributions.

7 Conclusions

Small contribuions to political campaigns have become increasingly important. Conventional wisdom is that such contributions are a consumption good to the donors. In large part this is a conclusion by default, the basic reasoning being that because each donation is so small relative to total campaign donations, small donors cannot be motivated either by an attempt to buy influence nor by any effect they may have on election outcomes. In this paper, we argue that significant aggregate contributions by small donors can be motivated by an electoral motive, either by their wanting to affect election probabilities directly, or, in a more behavioral sense, to their responding to party-fundraising efforts.

We find that in a model of small donors driven by the electoral motive, the equilibrium displays a number of features not predicted by explanations of contributions relying on a simple consumptin motive nor on an influence motive. There is a "closeness" effect in which equilibrium contributions increase when the support for the two candidates is more even, as well as an "underdog effect", whereby equilibrium relative contributions for the advantaged party are smaller than their underlying advantage. (These are in contrast to a "bandwagon" effect in an influence motive, and no predicted effect in the simple consumption motive.) The model also makes novel predictions about the effects of increases in income inequality on campaign contributions and probable election outcomes depending on the source of inequality.

Our model of small donors motivated by the desire to affect election outcomes gives insights into the effects of campaign finance laws, both positive and normative. Our model suggests that such laws will often have complicated effects. We also derive a novel proposal for a feasible income-based tax on contributions to lessen the effect of money in politics.

We view this paper as a first step in better understanding small political contributions by moving away from the common view that they must be a consumption good for the donors. As discussed in the paper, we believe an electoral motive for such contributions can better explain some empirical regularities, as well as providing some guidance to further empirical work. for example on the effect of income inequality on political outcomes. The next step, in our opinion, is to understand the interaction of small and large donors – for example, the latter "jump starting" a campaign by giving small donors greater incentive to give. Only be looking at such interactions can one better choose optimal campaign

finance restructions on large donations. Hence, any analysis based on the desire to limit the influence of large donors must be based on a model that considers small donors. This is the next step in our research agenda.

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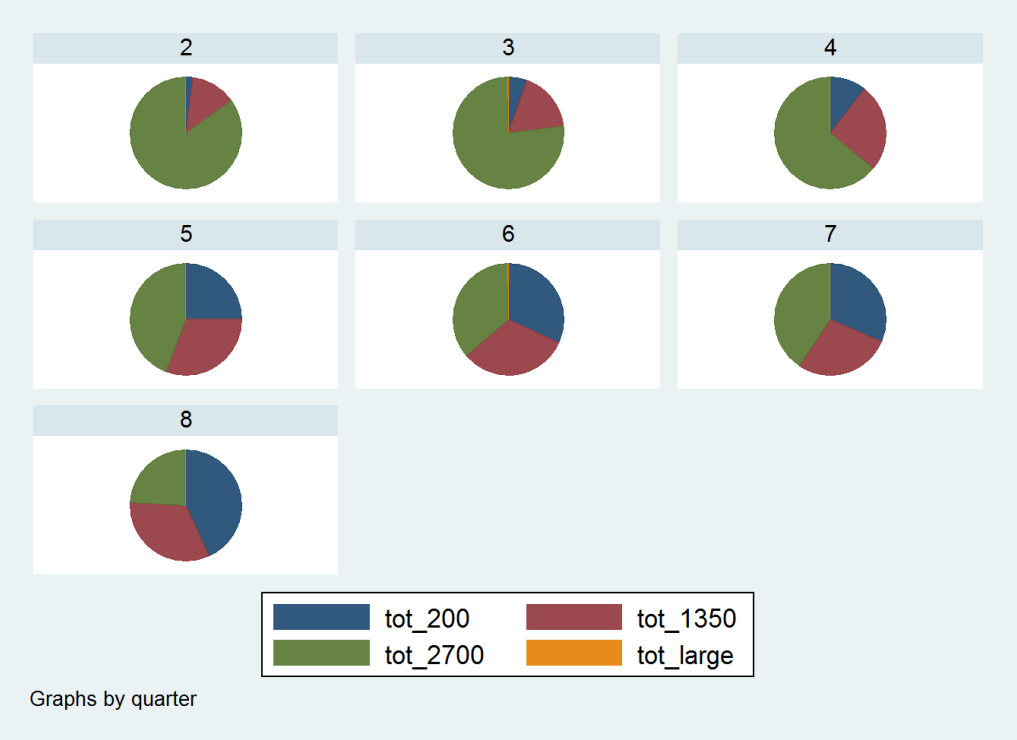
Appendix

Appendix 1.

A1.1. Distribution of contributions to Hillary Clinton

In the following graph, we subdivided contributions in 4 groups: [0,200], [201,1350], [1351,2700], and [2701,∞]. The first group are for contributions that do not require reporting. The legal limit to individual contributions are $2700 per election (once for the primary and once for presidential election), which can thus reach a total of $5400. Out of 3'471'316 contributions, we count three that are above that level, in addition to a number of regular transfers from other financial arms of the Hillary campaign, which we discarded from this analysis.

The quarter-per-quarter results of this decomposition are in the following graph:



One can see that, in 2015, the bulk of campaign finance was coming from contributions between 1350 and 2700. Contributions below $201 represented respectively 2% and 5% of the total in Q2 and Q3. In 2016, instead, the sum of all contributions above 1350 fell to less than 50% of the total, and represented less than 25% of the flow of contributions by the end of the campaign. Instead, the total value of contributions smaller than $201 end up representing 43% of the total in Q4 2016.

But the evolution of contributions over the campaign cycle clearly shows that, even if the contribution game may be one of face-to-face meetings and relatively large individual contributions early in the campaign, there is a shift to a more anonymized game among

an extremely large number of very small individual contributions (the 75th percentile of contributions drops from \$2700 in Q2 2015 to \$75 by Q1 2016) . It is hard to believe that, in this later phase, an individual donor's main motivation could be to influence the candidate's platform.

A1.2. CARA Example

The electoral motive for political contributions can be shown to derive from standard CARA preferences for public and private goods. Consider the following simple example: an individual has preferences over private consumption C and public goods G. That is, we are stacking the deck against our base model by assuming here that she does not derive any direct utility from contributing (no consumption motive for contributions) nor from influencing the electoral outcome: her preferences can be written as $U(C;G) = -\left(e^{-\beta C} + e^{-\rho G}\right)$, where the semi-colon is meant to make clear that she takes the proposed supply of public goods as a given; she has no influence motive since she cannot induce politicians to modify their policy platform. This is thus a standard model of consumption between two types of goods: those that are purchased privately, and those that are publicly provided.

Knowing that the US federal government budget per capita was \$20600 in 2016, whereas the US median income was \$52000 in 2014, and that donors' incomes are typically above that level, we set the parameters of the utility function to $\rho = .04$, and $\beta = .01$. The 4-to-1 ratio between ρ and β ensures that individuals value private goods consumption more than public goods consumption, whereas their absolute values imply that the marginal utility of either consumption is relatively small.

Now, assume that party A proposes a level of public good spending \$1000 above the observed level, and party B a level \$1000 below it: $g_A = \$21600$, $g_B = \$19600$. Then, for $d\pi(q)/dq = 10^{-12}$, the optimal contribution is $q^* = \max[0, y - \$80703]$. That is, even though the probability of affecting π with an extra dollar of contribution is vanishingly small, someone with about 1.5 times the median US income would make a non-negligible contribution. The entire contribution locus increases by about \$600 if $d\pi(q)/dq = 10^{-9}$. As argued in Section 2, these contributions would be even higher if donors also had a direct consumption motive such that $\partial U/\partial q > 0$.

To be clear, this CARA example cannot be interpreted as an actual calibration of actual voters' and donors' preferences. It instead shows that the space of "reasonable" parametrizations for utility functions is so large that it provides very few constraints to produce (too) high predicted levels of contributions.

Appendix 2. Additional comparative statics

HETEROGENEOUS POPULAR SUPPORT

This appendix uses the results of Section 4.1. In most of the paper, we assume that the election probability of each party only depends on its campaign spending. This proxies

a symmetric situation in which the two parties' popular support is symmetric, meaning that their probability of winning would be 1/2 if campaign spending were to drop to zero. Here, we show that the incentives and some properties we identified extend to the case in which popular support is asymmetric.

We capture the influence of each party's popular support on its probability of winning as a shifter $\varepsilon_P\,(>0)$ in the contest success function:

$$\pi_P\left(\mathbf{Q}\right) \equiv \frac{\left(Q_P\right)^{\gamma}+\varepsilon_P}{\left(Q_A\right)^{\gamma}+\left(Q_B\right)^{\gamma}+\varepsilon_A+\varepsilon_B}.$$

In that case, the MEC becomes:

$$\pi'_P = \frac{\gamma}{Q_P}\left(\pi_A\pi_B - \frac{\varepsilon_P}{D}\right),$$

where $D = \left(Q_A\right)^{\gamma}+\left(Q_B\right)^{\gamma}+\varepsilon_A+\varepsilon_B$. It is thus essentially the same as in the base model, except for the fact that party-P donors' incentive to contribution is proportionately reduced by the popularity shock experienced by the party.

The underdog effect therefore applies in a complementary way:

$$\frac{\pi'_A}{\pi'_B} = \frac{Q_B}{Q_A}\cdot\frac{1+\frac{\varepsilon_B}{Q_B^{\gamma}}}{1+\frac{\varepsilon_A}{Q_A^{\gamma}}},$$

where the first factor is the same underdog effect as in the base model, and the second shows that a popularity boost for party A further reinforces the initial underdog effect. Conversely, a popularity boost for party B reduces it – and may reverse it if ε_B increases so much that it compensates B's financial disadvantage (normalizing ε_A to zero, the condition becomes: $Q_A \gtrless Q_B\left(1+\varepsilon_B/Q_B^{\gamma}\right)$).

THE INFLUENCE OF MONEY IN ELECTIONS

Proposition 1 also informs us of how the sensitivity of election outcomes with respect to campaign funding (as captured by the parameter γ) influences the total size of the campaign and the election probabilities:

Observation 3 *A higher γ may translate into costlier or cheaper campaigns ($Q_A^*+Q_B^*$ can be increasing or decreasing in γ). In all cases, a higher γ reinforces the advantage of A (π_A^* converges monotonically to 1 as $\gamma\to\infty$).*

One effect of election probabilities being more sensitive to contributions (a higher γ) is that it traps donors into a larger "arms race". Both donor groups are compelled to contribute more, because the marginal effect of contributions increases. That effect always favors A because collecting funds is comparatively less costly for a. This, in turn, produces a second effect that works against the former: the increasing gap between A and

B reduces the marginal effect of contributions: $\pi_A\pi_B$ in π'_P falls. This *reduces* the incentive to contribute. Which effect eventually dominates determines whether total contributions increase or decrease in γ.[32]

THE INFLUENCE OF CLOSENESS ON CAMPAIGN SIZE

Observation 4 *Ceteris paribus, total campaign spending $Q_A^* + Q_B^*$ strictly decreases in W_A/W_B, and hence strictly increases in the closeness of the election, $\pi_A^*\pi_B^*$.*

Proof. Fix $W_A + W_B = \bar{W}$. For that case, we prove that:

$$\frac{d(Q_A^* + Q_B^*)}{dW_A} < 0, \quad \forall W_A > W_B.$$

Indeed:

$$Q_A^* + Q_B^* = \sqrt[\rho]{\gamma\omega W_A^{\rho-1}} + \sqrt[\rho]{\gamma\omega W_B^{\rho-1}} = \gamma^{1/\rho}\omega^{1/\rho}\left(W_A^{\frac{\rho-1}{\rho}} + W_B^{\frac{\rho-1}{\rho}}\right)$$

Hence:

$$\frac{\partial\left(Q_A^* + Q_B^*\right)}{\partial W_A} = \frac{1}{\rho}\gamma^{1/\rho}\omega^{\frac{1-\rho}{\rho}}\left(W_A^{\frac{\rho-1}{\rho}} + W_B^{\frac{\rho-1}{\rho}}\right)\frac{d\omega}{dW_A} + (\gamma\omega)^{1/\rho}\left(W_A^{\frac{-1}{\rho}} - W_B^{\frac{-1}{\rho}}\right),$$

where we used the fact that $dW_B/dW_A = -1$. From the definition of ω, the first term is zero for $W_A = W_B$ and strictly negative for $W_A > W_B$. The same holds for the second term. ■

Appendix 3. Proofs of the Propositions.

Proof of Proposition 1. We are focusing on pure strategies. Even when the pure strategy equilibrium does not exist, there must be a mixed strategy equilibrium, since payoff functions are continuous and bounded above. We are not interested in such MSE, because they are not realistic in our context.

Plugging (6) and (7) into (8) and (9), then taking the ratio between Q_A and Q_B shows that $\frac{Q_A}{Q_B} = \left(\frac{W_A}{W_B}\right)^{\frac{\rho-1}{\rho}}$ in a pure strategy equilibrium. We can therefore substitute for Q_B in (8), and solve for the equilibrium value of Q_A as a function of the exogenous parameters of the game, W_A,

[32] A similar phenomenon occurs, for example, in "lobby competition" where expected outcomes depend on expenditures relative to competing lobbies. In particular, see Che and Gale (1998a and b). It also relates to the contest effect in Herrera *et al.* (2016).

W_B, and γ:

$$\begin{aligned} Q_A &= W_A \times (\pi'_A)^{1/(\rho-1)} = W_A \times \left(\frac{\gamma}{Q_A} \times \frac{Q_A^\gamma}{Q_A^\gamma + Q_B^\gamma} \times \frac{Q_B^\gamma}{Q_A^\gamma + Q_B^\gamma} \right)^{1/(\rho-1)} \\ &= W_A \times \left(\frac{\gamma}{Q_A} \times \frac{Q_A^\gamma}{Q_A^\gamma + \left(Q_A \left(W_B/W_A \right)^{\frac{\rho-1}{\rho}} \right)^\gamma} \times \frac{\left(Q_A \left(W_B/W_A \right)^{\frac{\rho-1}{\rho}} \right)^\gamma}{Q_A^\gamma + \left(Q_A \left(W_B/W_A \right)^{\frac{\rho-1}{\rho}} \right)^\gamma} \right)^{\frac{1}{\rho-1}} \\ &= W_A \times \left(\frac{\gamma}{Q_A} \times \frac{\left(\left(W_B/W_A \right)^{\frac{\rho-1}{\rho}} \right)^\gamma}{\left(1 + \left(\left(W_B/W_A \right)^{\frac{\rho-1}{\rho}} \right)^\gamma \right)^2} \right)^{\frac{1}{\rho-1}} = W_A \times \left(\frac{\gamma}{Q_A} \times \omega \right)^{\frac{1}{\rho-1}} = \left(\gamma \omega W_A^{\rho-1} \right)^{\frac{1}{\rho}}. \end{aligned}$$

Q_B^* is derived following the same steps, and from the fact that $\frac{x^y}{(1+x^y)^2} = \frac{x^{-y}}{(1+x^{-y})^2}$. The latter implies that ω is identical for A and for B.

Second, equilibrium existence of a pure strategy equilibrium depends on the second order conditions being satisfied for this vector of total contributions. After some simplifications, the SOC for type-a donors can be expressed as:

$$-\gamma \frac{\pi_A^* \pi_B^*}{Q_A^2} \left(1 + \gamma \left(\pi_A^* - \pi_B^* \right) \right) < \left(\rho - 1 \right) \frac{\left(q_A^i \right)^{\rho-2}}{\left(y^i \right)^\theta},$$

which is always satisfied since $\pi_A^* \geq \pi_B^*$. A similar condition must hold for b donors:[33]

$$-\gamma \frac{\pi_A^* \pi_B^*}{Q_B^2} \left(1 + \gamma \left(\pi_B^* - \pi_A^* \right) \right) < \left(\rho - 1 \right) \frac{\left(q_B^i \right)^{\rho-2}}{\left(y^i \right)^\theta}. \tag{19}$$

[33] Second order condition amounts to looking at different points of the contest function for a and for b donors. Since a donors contemplate a higher winning probability than b, their SOC is automatically satisfied: they are in the concave part of the CSF. Instead, b donors may be in a spot in which the CSF is convex. That is, a slight decrease in their contribution base would also decrease their individucal incentives to contribute. For sufficiently high values of γ, this would reinforce the drop in individual incentives so markedly that total contributions may be driven to 0. In that case, there is no pure strategy equilibrium. The proposition shows that this can never happen if γ is no larger than ρ, or –for γ larger– if the contribution bases are not too asymmetric.

Noting that $\pi_A^*\pi_B^* = \omega$, we can rewrite this condition as follows:

$$\gamma\omega\left(\gamma\left(\pi_A^* - \pi_B^*\right) - 1\right) < (\rho - 1)\frac{\left(\left(y^i\right)^\theta \ \pi_B'\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^\theta}Q_B^2 = (\rho-1)\frac{\left(\pi_B'\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}\left(\gamma\omega\right)^{\frac{2}{\rho}}W_B^{\frac{2(\rho-1)}{\rho}}$$

$$\gamma\omega\left(\gamma\left(\pi_A^* - \pi_B^*\right) - 1\right) < (\rho - 1)\frac{\left(\frac{\gamma\omega}{Q_B}\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}\left(\gamma\omega\right)^{\frac{2}{\rho}}W_B^{\frac{2(\rho-1)}{\rho}}$$

$$\gamma\omega\left(\gamma\left(\pi_A^* - \pi_B^*\right) - 1\right) < (\rho - 1)\frac{\left(\left(\gamma\omega/W_B\right)^{\frac{\rho-1}{\rho}}\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}\left(\gamma\omega\right)^{\frac{2}{\rho}}W_B^{\frac{2(\rho-1)}{\rho}} = (\rho-1)\frac{\gamma\omega W_B}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}$$

$$(\gamma - 1 \geq)\ \gamma\left(\pi_A^* - \pi_B^*\right) - 1 < (\rho-1)\frac{\sum n^p f^p\left(y^j\right)\left(y^i\right)^{\frac{\theta}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}\left(> \rho - 1\right).$$

This is automatically satisfied for $\rho \geq \gamma$ (since $\pi_A^* - \pi_B^* \leq 1$), and when $\pi_A^* - \pi_B^* \leq 1/\gamma$ for any other value of ρ and γ. ■

Proof of Lemma 1. From Proposition 1 and the definition of ω, we have:

$$Q_A^* = \frac{\sqrt{\gamma W_A^{\gamma/2+1}\ W_B^{\gamma/2}}}{W_A^{\gamma/2} + W_B^{\gamma/2}} \text{ and } Q_B^* = \frac{\sqrt{\gamma W_A^{\gamma/2}\ W_B^{\gamma/2+1}}}{W_A^{\gamma/2} + W_B^{\gamma/2}}$$

Taking derivatives and simplifying yields:

$$\frac{\partial Q_A^*}{\partial W_A} \propto W_A^{\gamma/2}(2-\gamma) + W_B^{\gamma/2}(2+\gamma) \text{ and } \frac{\partial Q_A^*}{\partial W_B} \propto W_A^{\gamma/2} - W_B^{\gamma/2}.$$

The latter always positive, and the former is necessarily positive for $\gamma \leq 2$. For $\gamma > 2$, we need to invoke the second order condition for equilibrium existence: we saw that for ν_h^b sufficiently large (one can actually check that the same holds for n_l^b and n_h^b large enough), it can be approximated by: $\pi_A^* - \pi_B^* < 1/\gamma$ (see (19), p41).

Now, the sign of $\frac{\partial Q_A^*}{\partial W_A}$ must be the same as that of:

$$\pi_A^*(2-\gamma) + \pi_B^*(2+\gamma) = 2\left(\pi_A^* + \pi_B^*\right) + \gamma\left(\pi_B^* - \pi_A^*\right),$$

since $W_P^{\gamma/2}$ is the numerator of π_P^*. Substituting the SOC in the second term shows that the latter cannot be smaller than -1, whereas the former is equal to 2. Hence, $\frac{\partial Q_A^*}{\partial W_A}$ is always be positive when an equilibrium exists. Next,

$$\frac{\partial Q_B^*}{\partial W_B} \propto W_B^{\gamma/2}(2-\gamma) + W_A^{\gamma/2}(2+\gamma) \text{ and } \frac{\partial Q_B^*}{\partial W_A} \propto W_B^{\gamma/2} - W_A^{\gamma/2},$$

where the former is always positive and the latter always negative.

$$\frac{W_A^{\gamma/2}}{W_B^{\gamma/2}}(2-\gamma)+(2+\gamma)=\frac{\pi_A^*}{\pi_B^*}(2-\gamma)+(2+\gamma)>0$$

■

Proof of Proposition 2. Using the effects of income on W_P in (10), follow the logic of the proof of Lemma 1. ■

Proof of Proposition 3. Remember that $W_P \equiv (v^p)^{\frac{1}{\rho-1}}\ n^p\ \sum_{i=1}^{G} f^p\left(y^i\right) \times \left(y^i\right)^{\frac{\theta}{\rho-1}}$ A mean-preserving spread of the income distribution is such that $\sum_{i<\bar{y}^p} \Delta f^p\left(y^i\right) \times y^i = -\sum_{i>\bar{y}^p} \Delta f^p\left(y^i\right) \times y^i$, where $\bar{y}^p$ is the subgroup with mean income in group p, and $\Delta f^p\left(y^i\right)$ is the change in density of each income class. If and only if $\frac{\theta}{\rho-1} > 1$, this implies that $\left|\sum_{i<\bar{y}^p} \Delta f^p\left(y^i\right) \times \left(y^i\right)^{\frac{\theta}{\rho-1}}\right| < \left|\sum_{i>\bar{y}^p} \Delta f^p\left(y^i\right) \times \left(y^i\right)^{\frac{\theta}{\rho-1}}\right|$ and hence that W_P increases. Applying the proof of Proposition 1 then demonstrates the result. ■

Proof of Proposition 4. Remember that $y^i \in \left[\underline{y}, \bar{y}\right]$ with $\underline{y} > 0$ and $\bar{y}$ positive and finite. In that case, there exist two cutoffs q_0 and q_1 for the cap on individual contributions $\bar{q}$, such that: $\forall \bar{q} > q_1$, no donor is constrained and $\forall \bar{q} < q_0$ all donors are constrained. By Proposition 1, for $\bar{q} > q_1$, the ratio of total contributions must be:

$$\frac{Q_A^*}{Q_B^*} = \left(\frac{W_A}{W_B}\right)^{\frac{\rho-1}{\rho}} = \left(\frac{n^a}{n^b}\right)^{\frac{\rho-1}{\rho}},$$

and winning probabilities are the ones in Proposition 1. For $\bar{q} < q_0$, all donors contribute $\bar{q}$. Therefore, $Q_A = n^a\bar{q}$ and $Q_B = n^b\bar{q}$. The contribution ratio is then $\frac{n^a}{n^b}$, and it is immediate to derive that A's winning probability is then $\pi_A^0 = (n^a)^\gamma / \left((n^a)^\gamma + \left(n^b\right)^\gamma\right)$.

For $\bar{q} \in (q_0, q_1)$, Q_A must always be strictly larger than Q_B, otherwise $q_A\left(y^i\right) \geq q_B\left(y^i\right),\ \forall y^i$, with a set of income levels such that $q_A^i > q_B^i$, a contradiction. If follows that:

(1) there is a (possibly empty) set of income levels y^i such that neither a nor b-donors are capped: $q_A^i < q_B^i$

(2) there is a non-empty set of income levels y^i such that a-donors are uncapped and b-donors are capped: $q_A^i < q_B^i = \bar{q}$

(3) there is a (possibly empty) set of income levels y^i such that both a and b-donors are capped, $q_A^i = q_B^i = \bar{q}$.

Parts (1) and (2) imply that $\pi_A\left(\bar{q}\right)$ must be strictly less than π_A^0. The fact that proportionately more b-donors than a-donors are capped when $\bar{q} > q_0$ implies that their joint contribution capacity is reduced more than a's. This amounts to letting W_B drop because of a reduction in top b incomes. Following Proposition 1, this increases $\pi_A\left(\bar{q}\right)$ above π_A^*. The proof of non-monotonicity is provided by the example in the main text. ■

Proof of Proposition 5. Define $y^{i,a} = \alpha y^{i,b},\ \forall i = 1, ..., G$. Remember that, for any two

donors i and j who support the same candidate and are unconstrained by the cap, we must have: $q^p\left(y^{i,p}\right)/q^p\left(y^{j,p}\right)=\left(y^{i,p}/y^{j,p}\right)^\theta$. The equilibrium is thus fully characterized by two income cutoff levels $\bar{y}^a(\bar{q})$ and $y^b(\bar{q})$ and two "lowest contribution levels" $q^a\left(y^{1,a}\right)$ and $q^b\left(y^{1,b}\right)$ such that:

$$\begin{aligned}\text{for } y^{i,p} &< \bar{y}^p(\bar{q}),\ q^p\left(y^{i,p}\right)=q^p\left(y^{1,p}\right)\left(y^{i,p}/y^{1,p}\right)^\theta,\\ \text{for } y^{i,p} &> \bar{y}^p(\bar{q}),\ q^p\left(y^{i,p}\right)=\bar{q}.\end{aligned}$$

First, we show that $q^a\left(y^{i,a}\right)>q^b\left(y^{i,b}\right)$ for all unconstrained donors of some income group i, and hence that more a- than b-donors will be constrained. To prove this, note that a necessary condition for the fraction of constrained a-donors to be smaller than that of b-donors is to have $\bar{y}^a(\bar{q})>\alpha\bar{y}^b(\bar{q})$. This would require that $q^b\left(\bar{y}^b(\bar{q})\right)>q^a\left(\alpha\bar{y}^b(\bar{q})\right)=\alpha^{\theta/(\rho-1)}\ q^a\left(\bar{y}^b(\bar{q})\right)$, and thence $q^b\left(y^i\right)>\alpha^{\theta/(\rho-1)}q^a\left(y^i\right)$ for any $y^i<\bar{y}^b(\bar{q})$. But this leads to a contradiction: such contributions would aggregate into $Q_A(\bar{q})<Q_B(\bar{q})$, which would produce best-response contributions $q^b\left(\bar{y}^b(\bar{q})\right)<q^a\left(\alpha\bar{y}^b(\bar{q})\right)$, because of free riding.

This establishes that $q^a\left(y^{i,a}\right)\geq q^b\left(y^{i,b}\right)$ for all $i=1,...,G$, and the inequality must be strict for some i. Then, following the same steps as for the proof of Proposition 4 leads to Proposition 5. ■

Proof of Proposition 6 . Applying the same logic as for the proof of Proposition 5, a reduction in Q_A, whether it is the result of a drop in v_h^a or of a legal constraint, must increase contributions q_h^b and q_l^b. The impact on winning probabilities follows immediately.

We use numerical simulations to prove the fact that total contributions may increase or decrease: consider the following example, again with $\gamma=\rho=2$ and $\theta=1$, two income groups and the same number of a- and b-donors at each level of income: $n_l^a=30=n_l^b$, and $n_h^a=10=n_h^b$. The difference with the previous examples is that the high-income a are *much* richer than the high-income b: $y_l^a=10$, $y_h^a=100$, $y_l^b=1$, and $y_h^b=10$. Figure **??** displays total contributions: one can readily see that relaxing a tight cap produces the expected effect of increasing total contributions (Q_A+Q_B). However, the effect is reversed for $\bar{Q}>13.75$: it is then a *tightening* of the cap that increases total contributions. ■

Proof of Proposition 7. The *Marginal Effect of i's Contribution* to P can now be written as (for $\varepsilon_A,\varepsilon_B\to 0$):

$$\pi'_P=\frac{\gamma}{Q_P+s}\ \pi_A(\mathbf{Q},s)\ \pi_B(\mathbf{Q},s). \tag{20}$$

Thus, for any s, the two FOCs give:

$$\frac{Q_A}{Q_B}\frac{Q_A+s}{Q_B+s}=\frac{W_A}{W_B}\ (>1) \tag{21}$$

This requires that $Q_A>Q_B$. Note also that $\left(\frac{Q_A+s}{Q_B+s}\right)^\gamma=\frac{\pi_A}{\pi_B}=\frac{\pi_A}{1-\pi_A}\ (>1)$, and hence that

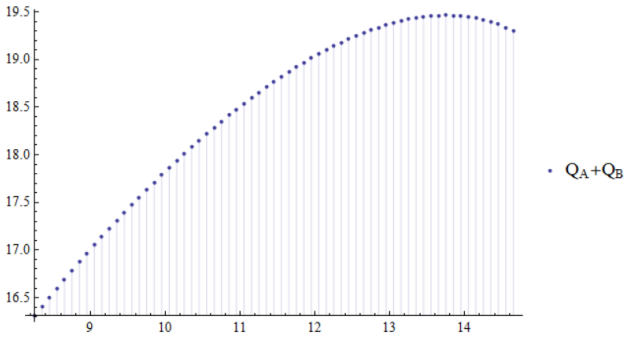


Figure 6: Simulated effect of a cap on total contributions when $y_l^a = 10$, $y_h^a = 100$, $y_l^b = 1$, and $y_h^b = 10$, and $n_l^a = 30 = n_l^b$, and $n_h^a = 10 = n_h^b$.

the former and the latter must move in the same direction as π_A. Note also that $sign\left(\frac{d\pi_A}{ds}\right) \neq sign\left(\frac{d\pi_A \pi_B}{ds}\right)$ since $\pi_A > 1/2$.

Now, we show that $\frac{d\pi_A}{ds} < 0$ by contradiction. From (21), we have: $\frac{d\frac{Q_A}{Q_B}}{ds} < 0 \Leftrightarrow \frac{d\frac{Q_A+s}{Q_B+s}}{ds} > 0$ with:

$$\begin{aligned} \frac{d\frac{Q_A+s}{Q_B+s}}{ds} &= \frac{(Q'_A+1)(Q_B+s)-(Q_A+s)(Q'_B+s)}{(Q_B+s)^2}, \text{ and} \\ \frac{d\frac{Q_A}{Q_B}}{ds} &= \frac{Q'_A Q_B - Q'_B Q_A}{(Q_B)^2} \end{aligned} \tag{22}$$

$\frac{d\frac{Q_A}{Q_B}}{ds} < 0$ would impose:

$$Q'_A Q_B < Q'_B Q_A, \tag{23}$$

and we have two cases: (1) $Q'_B < 0$, which would then require that $Q'_A < 0$ as well (since $Q'_A < Q'_B \frac{Q_A}{Q_B} < 0$), and (2) $Q'_B > 0$, which would then require that $(0 \lesseqgtr) Q'_A < Q'_B \frac{Q_A}{Q_B}$.

Case (1): by (22), $\frac{d\frac{Q_A+s}{Q_B+s}}{ds} > 0$ iff

$$\underbrace{0 >}_{by\ (23)} Q'_A Q_B - Q'_B Q_A > Q_A - Q_B + s(Q'_B - Q'_A)$$

To show the contradiction, we prove that the RHS is positive. Since $Q_A - Q_B > 0$, a SC is: $Q'_B > Q'_A$. By (23):

$$Q'_A < Q'_B \frac{Q_A}{Q_B},$$

which is thus more negative than Q'_B. Hence: $Q'_A < Q'_B \frac{Q_A}{Q_B} < Q'_B$.

Case (2): Remember that, by (20),

$$Q_B = \frac{\gamma W_B}{Q_B + s} \pi_A \pi_B.$$

Hence,

$$\frac{dQ_B}{ds} = -Q_B \frac{\frac{dQ_B}{ds} + 1}{Q_B + s} + \frac{\gamma W_B}{Q_B + s} \frac{d\left(\pi_A \pi_B\right)}{ds},$$

where the first term is necessarily negative when $\frac{dQ_B}{ds} > 0$, and so is the second term if $\frac{d(\pi_A \pi_B)}{ds} < 0$, i.e. if $\frac{d\pi_A}{ds} > 0$.

This contradicts that $\frac{d\pi_A}{ds}$ can be positive (or zero), for any value of $\frac{dQ_B}{ds}$. ■

Proof of Proposition 8. For $\varepsilon \to 0$, we can rewrite these total contributions as functions of the total contributions without the matching subsidies:

$$\tilde{Q}_P = (1+m) \sum_{i=1}^{n^p} q_P^i = (1+m) Q_P.$$

Plugging that into party P's probability of winning the election, we get

$$\pi_P \left(\tilde{\mathbf{Q}}\right) \equiv \frac{\left((1+m) Q_P\right)^\gamma}{\left((1+m) Q_A\right)^\gamma + \left((1+m) Q_B\right)^\gamma} = \frac{Q_P^\gamma}{Q_A^\gamma + Q_B^\gamma} = \pi_P \left(\mathbf{Q}\right).$$

As a consequence, incentives, and therefore the equilibrium, are the same for any $m \lesseqgtr 0$. ■

Proof of Proposition 9. With this tax, the cost of contributing q_P^i for a donor with income y^i becomes:

$$\left(q_P^i + \left[\left(y^i\right)^{\theta/\rho} - 1\right] q_P^i\right)^\rho / \left[\left(y^i\right)^\theta \rho\right] = \left(q_P^i\right)^\rho / \rho.$$

■

Proof of Proposition 11. Given the tax, there are exactly two contribution levels: $q_A^i = q_A$, $\forall p^i = a$ and $q_B^i = q_B$, $\forall p^i = b$. Denote their unconstrained levels q_A^* and q_B^*. Call χ the threshold such that, for all $\bar{q} \in (\chi, q_A^*]$, q_B remains unconstrained ($q_B(\bar{q}) < \bar{q}$) whereas q_A is constrained ($q_A(\bar{q}) = \bar{q}$) and such that $q_B(\bar{q}) = \bar{q}\ \forall \bar{q} < \chi$ (because of the closeness effect, $\chi > q_B^*$).

For any $\bar{q} < \chi$, and for $\varepsilon_A = \varepsilon_B$, $\pi_A = 1/2$. Therefore, social welfare becomes:

$$U^{SP} = \frac{n^a v^a + n^b v^b}{2} - \left(n^a + n^b\right) \frac{\bar{q}^\rho}{\rho},$$

which is unambiguously decreasing in $\bar{q}$.

For $\bar{q} \in (\chi, q_A^*]$, we have that:

$$\begin{aligned} U^{SP} &= n\left(v^a - v^b\right) \frac{\bar{q}^2}{\bar{q}^2 + \left(\sqrt{\bar{q}\left(\sqrt{\frac{2}{n}v^b} - \bar{q}\right)}\right)^2} - n\frac{\bar{q}^2}{2} - n\frac{\left(\sqrt{\bar{q}\left(\sqrt{\frac{2}{n}v^b} - \bar{q}\right)}\right)^2}{2} + nv^b \\ &= -\frac{n}{2}\sqrt{2}\frac{\bar{q}}{v^b}\sqrt{\frac{1}{n}v^b}\left(v^b - nv^a + nv^b\right) + nv^b. \end{aligned}$$

Differentiating with respect to the cap then yields:

$$\frac{dU^{SP}}{d\bar{q}} = \frac{\sqrt{n}}{\sqrt{2v^b}}\left(nv^a - (n+1)v^b\right),$$

which is strictly positive for any $n > v^b/(v^a - v^b)$. ■