The Invisible Border between East and West Germany*

Sebastian Heise†
Federal Reserve Bank of New York

Tommaso Porzio‡
University of California, San Diego and CEPR

February 23, 2018

Abstract

More than 25 years after the fall of the Iron Curtain, Germany is still economically divided into two separate regions. West Germany's average real wage is, controlling for individual characteristics, 20% higher than that of the East. An "invisible border" seems to still separate East and West Germany, thus preventing workers from taking advantage of the large wage difference. In this paper, we unpack this invisible border into several drivers and show that a strong regional identity of workers plays a major role. To reach this conclusion, we leverage rich matched employer-employee data together with a novel theoretical framework that allows us to study workers' mobility patterns across establishments and unemployment, both within and across East and West Germany.

*The views and opinions expressed in this work do not necessarily represent the views of the Federal Reserve Bank of New York. We thank for helpful comments and suggestions Costas Arkolakis, Giuseppe Moscarini, Michael Peters, Todd Schoellman, and seminar participants at Arizona State University, NY Fed, UCLA, and UCSD.

†33 Liberty Street, New York, NY 10045, email: sebastian.heise@ny.frb.org.

‡9500 Gilman Drive #0508, La Jolla, CA 92093-0508, email: tporzio@ucsd.edu.

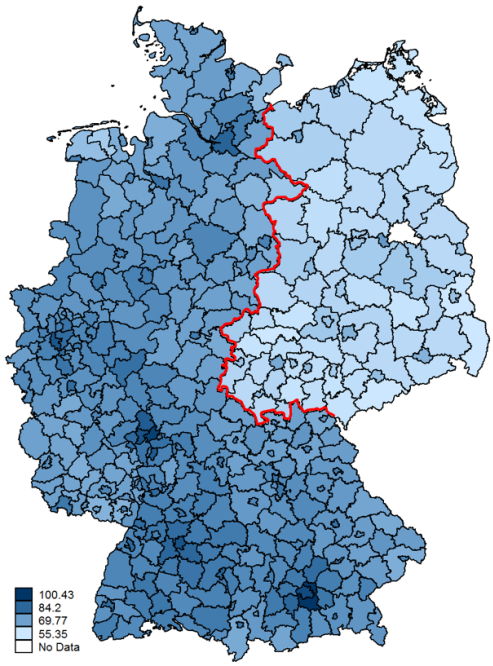
1 Introduction

More than 25 years after the fall of the Iron Curtain, Germany is still economically divided into two separate regions.[1] Although East and West Germany have the same labor market regulations and no restrictions on worker mobility, a large wage gap remains. In *real* terms, the average wage level jumps discretely by 20% at the former border (see Figure 1).[2] This wage gap is not driven by sorting on individual characteristics. As we will show, individuals that work in both East and West Germany earn on average 20% more while working in the West. What prevents workers from taking advantage of this wage difference? The persistent wage gap is surprising because Germany is, in theory, a fully integrated and very dynamic labor market, with about one fifth of workers changing jobs in any given year. Solving this puzzle provides a significant opportunity: if the source of the migration friction could be identified and resolved, large welfare gains are possible.

In this paper, we make progress towards resolving the puzzle by identifying the drivers of the "invisible border" that still separates East and West Germany. We consider two sets of reasons that could allow East German firms to retain workers despite their lower average wage. On the one hand, we examine *spatial frictions*, which prevent workers in East Germany from moving West due to either a comparative advantage at East German firms, relatively fewer job offers from West German firms, or a utility cost of moving away from their home region.[3] On the other hand, we consider *reallocation frictions*, which generically prevent workers from moving freely to more productive firms, for example due to hiring costs which constrain firms' vacancy posting. Considering both types of frictions is important. For example, if West German firms on average have higher productivity, but moving from low to high productivity firms is generally difficult, then the limited East-West mobility is not due to a friction that prevents migration but instead merely reflects a general lack of labor market dynamism affecting worker reallocation also within each region. Our main finding, based on detailed micro data on individuals' employment histories, is that the invisible border results from workers' strong preference to live in their birth-region. Importantly, we do not identify this preference parameter, or home bias, as residual to explain the lack of mobility, as often in the literature, but rather from the observed wage gaps for individuals that do move across borders. We estimate that East born workers require a wage increase by about 27% more than West born workers to move from East to West Germany. Workers' strong regional identity has general equilibrium effects: it generates a segmented labor market in which East German firms paying low real wages are able to become relatively large, and in which East German unemployment can remain relatively high.

[1] See, e.g., Washington Post: "The Berlin Wall fell 25 years ago, but Germany is still divided", 10/31/2014.
[2] We document this result more formally in Section 3.
[3] We will refer to East and West Germany as regions.

Figure 1: Average Real Wage by County, 2009-2014



Notes: Average daily wages are obtained for a 50% random sample of establishments in Germany via the Establishment History Panel (BHP) of the Institute for Employment Research (IAB). Wages are adjusted by county-specific price levels from the Federal Institute for Building, Urban Affairs and Spatial Development (BBSR) in 2009, and deflated forward and backward in time using state-level price deflators from the Statistical Offices of the States. Former East-West border is drawn in red for clarification, there is no border today.

Our analysis encompasses three distinct steps: first, we provide evidence that East and West Germany are in fact two distinct labor markets, separated by an invisible border. Next, we develop a novel labor search and matching framework with multiple types of workers which search for jobs across heterogeneous establishments that are located in multiple geographical regions. This framework allows us to unpack the wage gap into the different spatial and reallocation frictions. Third, we estimate the model with our data and run a structural decomposition to estimate the roles played by the various frictions in explaining the wage gap.

Our work draws on administrative matched employer-employee records provided by the German Institute for Employment Research (IAB) via the longitudinal version of the Linked Employer-Employee Dataset (LIAB). The data cover the entire employment biography of about 1.9 million individuals during employment and unemployment spells, matched to characteristics of the establishments at which these workers are employed, and allow us to track individuals as they move across establishments and space. We complement these data with establishmenty-level information for a 50% random sample of establishments in Germany drawn from the Establishment History Panel (BHP).

In Section 3 we begin our empirical analysis by decomposing workers' wages into an individ-

ual and an establishment component, using the methodology of Abowd, Kramarz, and Margolis (1999). The analysis highlights that the wage gap in Germany is mostly driven by the establishment component, while sorting of high skilled workers to the West only plays a minor role. This result contrasts for example with the case of the urban-rural wage gap, where spatial sorting is important (e.g., Young (2013)). Next, we show that Germany is not an integrated labor market in which the wage gap is only the result of a higher average productivity of West German establishments. Using the BHP, we find that establishments in East Germany are systematically larger, by about 1-2 workers, than West German establishments paying the same real wage.[4] This finding suggests that East German establishments are able to hire and retain more workers for a given wage, and hence face a slacker local labor market. To confirm this result, we use the LIAB to estimate a gravity equation that explains worker flows between counties as a function of distance as well as origin and destination county characteristics. The regression reveals that conditional on distance and county characteristics, workers are significantly less likely to move between East and West Germany than within them. For example, a worker born and employed in East Germany is about twice as likely to move 200km to another firm in the East than to the West. On the other hand, when an East German worker is employed in West Germany, she is about twice as likely to move 200km back to the East than to move the same distance to another firm in the West. Overall, we find that irrespective of their current location, workers are disproportionally attracted to their own region. This finding highlights that establishments in East Germany are indeed not fully exposed to competition for workers from the West, which allows them to hire and retain workers relatively more easily.

Section 4 develops a theoretical framework that allows us to unpack the invisible border into its different components. The nature of our data calls for a model with two types of labor reallocation: (i) spatial movements across East and West Germany; (ii) reallocation within each region across heterogeneous firms. Our main building block is a standard heterogenous firm job-posting model à la Burdett-Mortensen (e.g., Burdett and Mortensen (1998)), which we extend to a setting with an arbitrary number of regions and arbitrary many worker types. Each region is characterized by an exogenous productivity distribution of firms. Each worker type is characterized by a vector of region-specific skills, preferences, and wedges that define the relative chance of receiving a job offer from a given region. Firms choose their optimal wage and decide how many job vacancies to open, subject to a region-specific cost. Workers randomly receive offers and accept the offer that yields the higher utility, moving across firms both within and across regions. Despite the rich heterogeneity, we derive a tractable solution represented by a system of two sets of differential equations with several boundary conditions.

[4]This finding holds for all establishments except in the right tail, where East Germany has only relatively few establishments.

The unique feature of our model vis a vis the previous literature is its ability to distinguish between spatial frictions and reallocation frictions, and hence to compare how the mobility of workers across regions differs from the mobility of workers across firms within a given region. Reallocation frictions are the standard frictions present in frictional labor market models, and are generated in our setup by firms' cost of posting new vacancies and exogenous separation rates. The spatial frictions, which are our main focus, are generated by three type- and region-specific parameters: (i) workers' productivity (e.g., West German workers are more productive); (ii) workers' location preferences, and (iii) workers' probability of receiving an offer from each region. The allocation of labor across regions could be distorted by any of these previous margins. For example, within the spatial frictions, East German workers might be disproportionately attracted to the East if either they have a comparative advantage in East Germany, a stronger preference for the East, or are more likely to receive offers from East German firms.

We quantitatively estimate the model in Section 5. While all parameters are jointly estimated, we begin by performing reduced-form regressions to show that the different types of frictions can be separately identified in our data. First, we show that workers' relative productivity can be identified by comparing the wages of observationally equivalent East and West German individuals employed at the same establishment. We find that West German workers' residual wage, after controlling for individual characteristics, is about 3% higher at both East and West German establishments. This result suggests a slightly higher absolute productivity of West German workers, but no comparative advantage of either type of worker in either region. Second, we estimate workers' relative location preferences by comparing the wage gains of East- and West-born workers when they switch jobs both within and across regions. We find that the average wage gain of an East-born job switcher from East to West Germany is about 45 percentage points higher than the same worker's wage gain when she switches jobs within East Germany, while a West-born worker only receives a 19 percentage point higher wage increase from moving across regions than from moving within the East. This difference suggests that East-born workers' relative preference for East Germany amounts to a 27 percentage point wage premium. Finally, we estimate workers' relative probability of receiving offers from each region using the mobility patterns of workers of each type. We find that flows of East German workers from the East to the West would need to increase by about a factor of ten to be consistent with an integrated market. We are still working on a quantitative estimation to determine the relative importance of the frictions.

While Germany's example is particularly stark, persistent spatial wage gaps also exist within many countries, such as the Italian Mezzogiorno or the South-West of Spain. A large literature has sought to explain the existence of these gaps, arguing that they are only a manifestation of different local amenities (Rosen (1979), Roback (1982)), the result of sorting of workers

with different ability (Young (2013)), or due to mobility frictions making it costly for workers to relocate (Bryan and Morten (2017)). Our work is, to our knowledge, the first to bring to bear detailed matched employer-employee data to disentangle these three types of frictions in a unified framework, and to relate frictions across space to frictions across establishments. Our finding that the "invisible border" is mostly driven by a strong regional identity of workers may have far reaching implications. First of all, it casts doubts on the efficacy of policies aimed at an economic integration of the East and West German labor markets. Through the lens of our model, neither better infrastructure, nor labor market subsidies or even training programs, would have a discernible effect on the spatial wage gap or on the aggregate productivity. Instead, our results suggest that the two regional labor markets could be truly integrated only through the slow process of cultural and social assimilation necessary to reach a common German identity. Furthermore, we have shown that workers' preferences for East Germany shape the distribution of active firms in the East, and allow firms paying low wages to become relatively large. Our results thus provide a theory of spatial differences in firms' productivity distribution that takes as primitive worker preferences. By accepting to work for a lower wage in order to be able to stay in their home region, East born workers effectively keep alive firms that would be uncompetitive in a truly unified German labor market.

Literature. We are not the first to study spatial wage gaps. A large literature, at least since the work of Harris and Todaro (1970) on the rural-urban wage gap, has sought to explain the large observed differences in average wages across space. The literature can be broadly divided into two sets of papers. The first category assumes free labor mobility and homogenous workers and solves for spatial equilibria along the lines of the seminal work by Rosen (1979), Roback (1982), and more recently of Allen and Arkolakis (2014). The assumption of a spatial equilibrium implies that utility is equalized across space, and therefore the observed differences in wage gaps are simply a reflection of differences in local amenities. The second category, instead, has studied spatial wage gaps as a possible symptom of misallocation of labor across space. A core debate in this literature has been to distinguish between sorting of heterogenous workers based on their comparative advantages and frictions to labor mobility that generate wedges along the lines of the work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).[5] Our paper belongs to this second category of papers. As the more recent work in this literature, we allow for both sorting and frictions to explain the wage gap between East and West Germany. Our main contribution is to unpack the "spatial wedges" into several components, and hence to open up the black box of labor mobility frictions. In order to pursue this task we apply the toolset and the datasets of the frictional labor literature. In particular, our model adapts

[5]See for example Bryan and Morten (2017); Young (2013); Hicks, Kleemans, Li, and Miguel (2017).

the work of Burdett and Mortensen (1998) to a setting with a non-trivial spatial dimension. Moreover, we rely on matched employer-employee data , as now common in the labor literature,[6] and we show that they are crucial to distinguish spatial frictions from the general reallocation frictions across firms, which are the focus of the labor literature. From a purely methodological perspective, our paper bridges the gap between the macro-development misallocation literature and the frictional labor literature. We are – to the best of our knowledge – the first to use the tools of the latter within the context and research questions of the former.

Our work is informed by, and consistent with, the rich literature on migration. The idea that worker identity may be an important drivers of migration decisions is at least as old as the work of Sjaastad (1962), and more recently has been revived by the structural approach of Kennan and Walker (2011). This work has documented an important role for home preferences in explaining the dynamics of migration choices. Our contribution is to embed these core ideas into a labor model with reallocation across space and firms. We show that considering both dimensions of reallocation together is important to properly estimate the spatial frictions.

Last, our work is related to the (quite limited) literature that has examined East German convergence (or the lack thereof) after the reunification (e.g., Burda and Hunt (2001), Burda (2006)). This literature has in particular studied possible drivers behind the wage gap between East and West Germany and the nature of migration between the two regions (Krueger and Pischke (1995), Hunt (2001, 2002, 2006), Fuchs-Schündeln, Krueger, and Sommer (2010)). We use matched employer-employee data to examine the role of worker sorting, productivity differences, spatial, and reallocation frictions in a unified framework; a task that has not been attempted before.

2 Data

Our work relies on confidential micro data provided by the German Federal Employment Agency (BA) via the Institute for Employment Research (IAB). First, we use establishment-level data from the Establishment History Panel (BHP). This dataset contains a 50% random sample of all establishments in Germany with at least one employee liable to social security on the 30th June of a given year. The data are based on mandatory annual social security filings. Government employees and the self-employed are not covered. The BHP defines an establishment as a company's unit with at least one worker liable to social security operating in a distinct county and industry. Since several plants of the same company may operate in the same county and industry, the establishments in the BHP do not always correspond to economic units such as a

[6] See for example Card, Heining, and Kline (2013).

plant (Hethey-Maier and Schmieder (2013)). Throughout the rest of this paper, we use the terms establishment and firm interchangeably to refer to these entities. For each such establishment, the dataset contains information on the establishment's location, number of employees, employee structure by education, age, and occupation, and the wage structure. The underlying population of workers comprises about 80% of the German working population. The data are recorded as annual cross-sections since 1975 for West Germany and since 1991 for East Germany, which we combine to form a panel covering about 650,000 to 1.3 million establishments per year. Unless otherwise noted, we examine the period 2009 to 2014, the last available year in the data, to focus on persistent differences between East and West Germany. We focus on full-time employees only.

Our second, and most important, dataset is matched employer-employee data provided via the longitudinal version of the Linked Employer-Employee Dataset (LIAB). The dataset is a combination of establishment information from the BHP, the IAB Establishment Panel (a representative survey of German establishments), and individual-level information from the Integrated Employment Biographies (IEB). The IEB contains employment information and socio-economic characteristics of all individuals that were employed subject to social security or received social security benefits since 1993. The LIAB data are a representative sample of individuals from the IEB, linked to information about the establishments at which these individuals work. The sample is drawn by taking all establishments that are in the IAB Establishment Panel survey in any year between 2000 and 2008, and selecting all the individuals who were employed in one of these establishments at least one day during the time period from 1999 to 2013. The entire employment history of these individuals is then drawn for the period 1993-2014, including spells at other establishments, unemployment spells, etc., with exact beginning and end dates for each spell. The LIAB sample covers about 1.9 million individuals working for between 2,700 and 11,000 establishments per year. In addition to establishment information, the dataset records an individuals' education, location of residence and location of work, year of birth, occupation and daily wage. As with the BHP, we keep only full-time employment spells, and focus on 2009-2014.

We exploit the panel structure of the LIAB to impute each individual's birth location, which is not recorded in the data. We code an individual as born in East Germany (West Germany) if at the first time she appears in our full dataset since 1993 her residence location is in the East (West). Since, for employed workers, the residence location is not available before 1999, for these workers we use the location of the establishment before that. We compute whether an individual is currently located in the region in which she was born and label her a "native" in that case. We label her as "foreign" if she is currently in the other region.

Our third dataset contains information on cost of living differences across German counties

from a study conducted by the Federal Institute for Building, Urban Affairs and Spatial Development for the year 2009 (BBSR (2009)), which we use to translate all wages into real wages. While regional differences in price levels within Germany are not measured by Germany's statistical offices, the BBSR's study assesses regional price variation across 393 German micro regions covering all of Germany that correspond to counties or slightly larger unions of counties. The data cover about two thirds of the consumption basket, including housing rents, food, durables, holidays, and utilities. Figure 9 in Appendix D shows the map of county-level price levels, and confirms that East Germany on average has lower prices, in particular in the cities. We adjust all wages in the BHP and in the LIAB in 2009 based on the BBSR's local price index, and then deflate the wages forward and backward in time using state-specific GDP deflators from the statistics offices of the German states. We complement our data with publicly available county-level unemployment rates from the Federal Employment Agency.

We next document the presence of an invisible border in Germany today.

3 The Invisible East-West Germany Border

Germany was divided into two separate nations until 1990, governed by two distict economic systems. While West Germany was a market economy, the economy in East Germany (then called the German Democratic Republic, GDR) was planned and followed a communist regime. There was virtually no movement of workers between the two regions, and the border was tightly controlled. In 1990, with the fall of the "Iron Curtain", East Germany reunited with the West and adopted all its legal and economic institutions.[7] Despite no visible constraints to worker mobility, in this section we show that East and West Germany still today have distinct labor markets, as if an "invisible border" separates them. To establish this result, we exploit our rich data along several dimensions. In the first step, we show that the large wage gap between East and West Germany is complemented by a significant difference in unemployment. While our finding is consistent with the hypothesis of distinct East and West German labor markets, it could also be generated by: i) spatial sorting of higher quality workers to the West, ii) a fully integrated frictional labor market in which the average productivity of West German firms is higher, or iii) size dependent policies. We present evidence refuting each of these possibilities in turn.

First, we use the matched employer-employee LIAB data to separate the roles that firms and individual characteristics play in determining wages. We document that most of the differences between East and West are driven by the establishment component, suggesting that West Ger-

[7] Section A in the Appendix provides more background on the reunification process

man firms are more productive, while the role of the individual component and spatial sorting are weak. Second, we investigate how the difference in the firm-level component is generated. We show, using the LIAB and the BHP data, that an East German firm paying a certain wage is larger than a West German one paying the same wage. This finding is inconsistent with a single integrated labor market, and instead points to frictions which lead to local labor supply curves. Finally, we document these frictions directly using information on worker flows between counties from the LIAB. We show that workers born in a given region are reluctant to move to the other region.

County-Level Data. We begin by revisiting the sizeable real wage gap between East and West Germany documented by Figure 1 in the introduction.[8] We construct the figure by taking the average daily wage of full-time workers for each establishment in the BHP in each year, converting it into real wages using the BBSR data as described in the previous section, and averaging across establishments and years for each county, for the period 2009-2014. We highlight in the map the former border in red: wages appear to change discontinously around the border.

The large wage gap is surprising given that workers are free to move across regions, and there is no physical border, language, or legal barrier.[9] One possible explanation for it could be that the higher wage in West Germany compensates workers for a higher unemployment risk, along the lines of theories of dual labor markets as in Harris and Todaro (1970). We investigate this possibility in Figure 2, using data on county-level unemployment rates for civilian workers from the Federal Employment Agency. In fact, average unemployment was about 12% in East Germany during this period, with some counties experiencing unemployment as high as 18%, while the average unemployment rate in the West was only 7%.

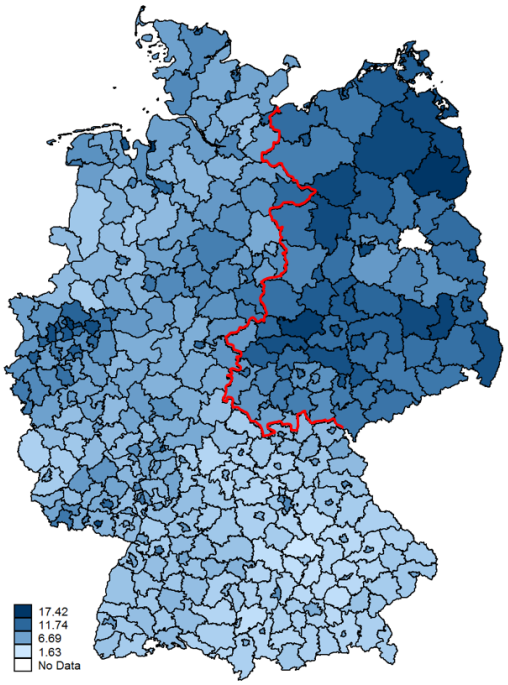
Our results suggest that sizeable frictions preventing East German workers from moving to West Germany might be present. We next show that the results are neither generated by spatial sorting of higher quality workers to the West, nor a fully integrated labor market with on average higher productivity in the West, nor size-dependent policies. We first empirically disprove the sorting hypothesis, using the matched employer-employee data, and decompose the wage gap into worker and establishment components.

Firm-Employee Matched Data. We fit in the individual-level LIAB data a linear model with additive worker and establishment fixed effects, as originally in Abowd, Kramarz, and

[8] All empirical analyses exclude Berlin since it was divided between East and West before the reunification.

[9] Figure 10 in Appendix D shows a time series of real wages in East and West Germany and highlights that after an initial period of rapid convergence in the early 1990s, the gap has remained virtually constant.

Figure 2: Average Unemployment in East and West Germany, 2009-2014



Source: Bundesagentur für Arbeit

Margolis (1999) and, more recently, in Card, Heining, and Kline (2013). The model allows us to quantify the contribution of worker-specific and firm-specific components to the real wage gap. Index full-time workers by k and time by t, and define by $\boldsymbol{Q}(k,t)$ the establishment that employs worker k at time t.[10]

We decompose the log daily real wage $\log w_{kt}$ according to

$$\log w_{kt} = \log \alpha_k + \log \psi_{\boldsymbol{Q}(k,t)} + x'_{kt}\beta + r_{kt}, \tag{1}$$

where α_k is a worker component, $\psi_{\boldsymbol{Q}(k,t)}$ is an establishment component, and x_{kt} is a set of year and age dummies, interacted with education. We specify r_{kt} as in Card, Heining, and Kline (2013) as three separate random effects: a match component $\eta_{k\boldsymbol{Q}(k,t)}$, a unit root component ζ_{kt}, and a transitory error ϵ_{kt},

$$r_{kt} = \eta_{k\boldsymbol{Q}(k,t)} + \zeta_{kt} + \epsilon_{kt}.$$

In this specification, the mean-zero match effect $\eta_{k\boldsymbol{Q}(k,t)}$ represents an idiosyncratic wage premium or discount that is specific to the match, ζ_{kt} reflects the drift in the persistent component of the individual's earnings power, which has mean zero for each individual, and ϵ_{kt} is a mean-

[10]Time is a continous variable, since, if a worker changes multiple firm within the same year, we would have more than one wage observation within the same year.

zero noise term capturing transitory factors. Similar to Card, Heining, and Kline (2013), we estimate the model on the largest connected set of workers in our data. The largest connected set includes approximately 98% of West and East workers.[11]

Denote by $\omega_{kt} = \alpha_k \psi_{\boldsymbol{Q}(k,t)}$ individual k's predicted wage in year t net of the time varying age and education effects. In Appendix B, we show that the average predicted wage in a region or county j, in year t, which we call $\bar{\omega}_{t,j}$, may be decomposed into

$$\bar{\omega}_{t,j} = \bar{\alpha}_{t,j} \bar{\psi}_{t,j} \eta_{t,j} \rho_{t,j}. \tag{2}$$

Here, $\bar{\alpha}_{t,j}$ and $\bar{\psi}_{t,j}$ are the average worker and establishment fixed effects in region j in year t, and $\eta_{t,j}$ is the covariance between establishment size and establishment fixed effect, where both size and fixed effect are normalized relative to their averages in region j. This term captures that the average wage in a region could be high because establishments with a high establishment component employ a relatively large share of workers. The term $\rho_{t,j}$ is the covariance between worker and establishment fixed effect, again both normalized relative to the average. This term reflects the degree of sorting of high fixed effect workers to high fixed effect establishments.

We perform this decomposition separetely for East and West Germany in each year between 2009 and 2014 and then take the expectation of the logs across these years. Thus, for example, $E\left[\log \bar{\omega}_W\right] \equiv \frac{1}{6} \sum_{t=2009}^{2014} \log \bar{\omega}_{W,t}$. We obtain:

$$\begin{aligned} \underbrace{E\left[\log \bar{\omega}_W\right] - E\left[\log \bar{\omega}_E\right]}_{20.2\%} \;=\; & \underbrace{E\left[\log \bar{\alpha}_W\right] - E\left[\log \bar{\alpha}_E\right]}_{4.5\%} + \underbrace{E\left[\log \bar{\psi}_W\right] - E\left[\log \bar{\psi}_E\right]}_{19.7\%} + \\ & \underbrace{E\left[\log \bar{\eta}_W\right] - E\left[\log \bar{\eta}_E\right]}_{\text{-3.0\%}} + \underbrace{E\left[\log \bar{\rho}_W\right] - E\left[\log \bar{\rho}_E\right]}_{-1.0\%}. \end{aligned}$$

The decomposition shows that the majority of the average wage gap is explained by differences in establishment fixed effects. Workers' fixed effects are also larger in West Germany, but this effect contributes only a small fraction to the overall average wage gap. The covariance terms are instead larger in East Germany. However, the size of the effect is relatively small. Moreoever, as discussed in Footnote 11, we should be careful in interpreting the covariance terms due to the fact that the LIAB sample is not a representative sample of establishments. Additionally, the

[11] While the majority of the workers are included in the sample, we are missing a sizable fraction of establishments. Of the establishments included in the LIAB dataset, we miss approximately 40% in East and 48% in the West. We divide the distribution of establishments into deciles based on their average wage paid, and we find that we are more likely to miss establishments that pay lower wages. In fact, of the establishments in the bottom decile of the average wage distribution we miss 62% in the East and 72% in the West, while of the establishments in the top decile we miss 34% of them in the East, and 23% in the West. Last, notice that we miss more establishments than workers since – due to the nature of the exercise – large establishments are more likley to be included in the connected set.

procedure by Abowd, Kramarz, and Margolis (1999) restricts the sample to the set of connected firms, generating a bias towards larger firms.

In Figures 3a and 3b, we repeat the previous decomposition at the county level to visualize the results in a map. The stark discontinuity at the former border is evident for establishment fixed effects, while worker fixed effects are more homogenous across East and West Germany.[12]

The wage decomposition results thus confirm that sorting of high-skilled workers to West Germany does not explain the oberved wage gap. Most of the gap is driven by differences in establishments. In fact, if it were possible to move a randomly selected worker from East Germany to a randomly selected establishment in the West, this worker should on average receive a 19.7% increase in wage: a very large gain. At the same time, the results are still not conclusive on the presence of an invisible border. While we can reject the absence of frictions, the results are also consistent with the hypothesis that East and West Germany are part of one integrated, but frictional, labor market, in which the average West German firm is more productive and thus pays a higher wage. We next turn to establishment-level data to disprove this possibility.

Establishment-Level Data. In a frictional labor market with on the job search, as in Burdett and Mortensen (1998), each firm faces an increasing labor supply curve: by increasing its wage, a firm attracts a larger pool of potential workers that would accept an offer, and manages to retain a larger share of its current employees. Therefore, firms that pay a larger average wage per worker are larger. Moreover, if East and West Germany are one single labor market, then firms should face an identical labor supply curve, which implies that we should observe that conditional on wage, firms are of the same size in each region.

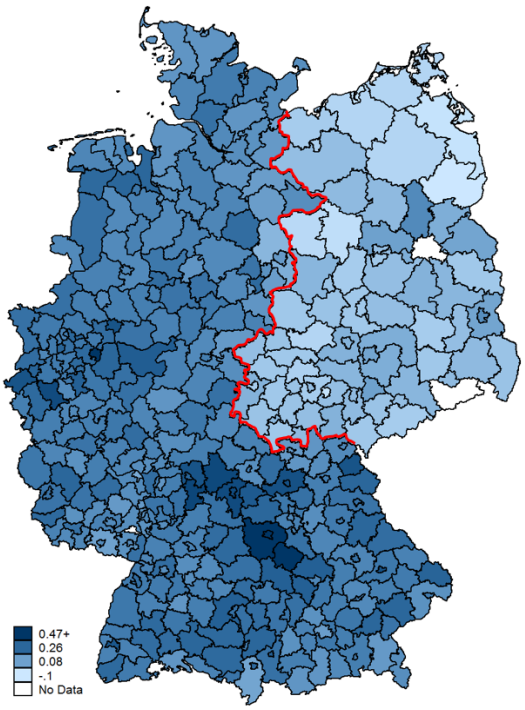
We test whether the wage-size relationship is the same in both East and West Germany and find that this prediction is not supported. In Figure 4, we use the half census data from the BHP to plot establishment size as a function of wage.[13] Using data from 2009 to 2014

[12] A few comments are necessary. First, we use a slightly longer time period from 2005-2014 to increase the sample size for disclosure purposes. Second, in Appendix D we include further results: we plot the distribution of establishment and worker fixed effects in Figures 11a and Figures 11b, and we plot the maps for the other two covariance components of the decomposition in Figures 12 and 13. These results corroborate the main narrative that a shift in the average establishment fixed effects explains the majority of the average wage gap between East and West.

[13] The BHP data has one main limitation: we don't observe workers characteristics and therefore we have to rely on average firm wage rather than calculating wage per efficiency unit. In order to interpret the differences between East and West, we need therefore to assume that the correlation between firm types and worker types is similar in East and West. An alternative path could be to use the matched employer-employee LIAB data. However, this data is not representative at the establishment level, and in fact, we observe that firms that are in the dataset are selected, and differentially so in East and West. We are currently working to match LIAB and BHP datasets together, which ideally could allow us to predict firm fixed effects for the representative distribution.

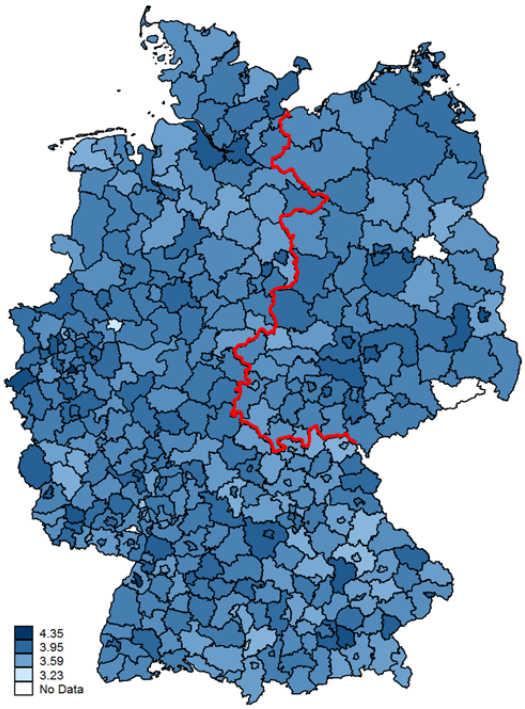
Figure 3: Average Establishment and Worker Component, 2005-2014

(a) Average Establishment Component ($\bar{\psi}$)



Source: LIAB, Authors' Calculations

(b) Average Worker Component ($\bar{\alpha}$)

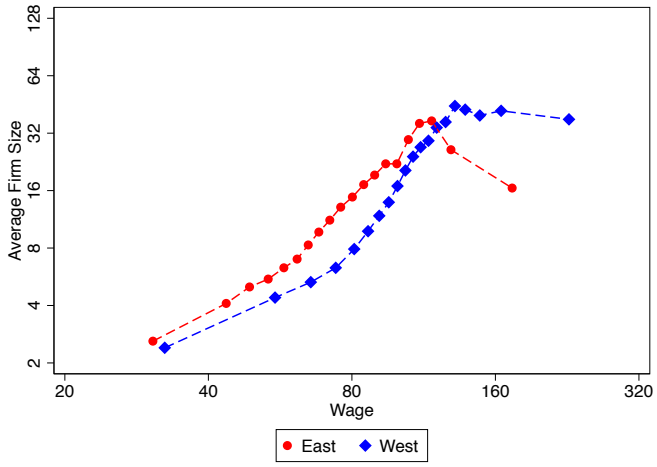


Source: LIAB, Authors' Calculations

we divide, for each region-year pair, the distribution of establishments into 20 equally sized groups based on the average wage that they paid to their employees. We then calculate the average establishment size for each group. Figure 4 shows that, first, there exists substantial heterogeneity in average establishment wages even within each region. Second, along almost the entire distribution, and especially for low paying firms, East German establishments are about $1-2$ workers larger conditional on the real wage paid. This result shows that East firms possibly benefit from supply of local labor that is willing to work for a lower wage than in the West.

Overall, the evidence thus suggests that the two regions are not integrated in one single frictional labor market. At the same time, in interpreting the results, we have so far assumed that all the differences in the wage-size joint distributions are driven by different local labor supply, and not by – for example – different size-dependent policies in the two regions. We finally turn to worker-level data to provide evidence that the labor supply is indeed different in the two regions. Specifically, we show that East born workers are attracted to East counties, and vice versa, which implies that establishments mostly compete for labor with other establishments in the same region.

Figure 4: Firm Size by Average Wage Paid



Worker-Level Data. The LIAB data allow us to track workers as they change jobs over time. We use these data, for the time period 2009-2014, to compute the share of newly hired workers in county d from each origin county o. We show that, conditional on origin and destination county fixed effects and the distance between counties, worker flows are biased towards their home region.

Let $h_{o,d}$ be the total number of workers that were employed in the previous year in county o and are starting this year a job in county d. We then compute the share of workers from county o moving to county d,

$$s_{o,d} = \frac{h_{o,d}}{\sum_{d \in \mathbb{D}} h_{o,d}}$$

where $\mathbb{D}$ is the set of all the 402 counties in both East and West Germany. We use $s_{o,d}$ to study the flows of workers across counties. Out of the approximately 161,000 possible county-origin pairs, we observe at least one worker flow – i.e. a strictly positive $s_{o,d}$ – for $94,546$ of them. We use these pairs to fit the following gravity equation

$$\log s_{o,d} = \delta_o + \gamma_d + g(dist_{o,d}) + \epsilon_{o,d}, \tag{3}$$

where δ_o and γ_d are county of origin and destination fixed effects, which capture the fact that some counties may be more attractive than others, either due to better market conditions or higher amenities, and $\epsilon_{o,d}$ is a mean zero i.i.d. error term. Finally, $g\left(dist_{o,d}\right)$ is a generic function of distance, $dist_{o,d}$, between the origin and destination counties in km. If our hypothesis of an invisible border between East and West Germany is correct, then moving across regions should be "costlier" for workers than moving within-region, and therefore cross-region flows should be smaller than within-region flows conditional on origin and destination fixed effects. We thus

specify the effective distance between any two counties as a weighted average of the distance travelled within a region and distance travelled across regions. Define a set of buckets for the distance travelled, $\mathbb{X}$, which includes 50 km intervals from 50km-100km onward to 350km-400km, and an eighth group for counties that are further than 400 km apart. The omitted category is up to 50km, which includes the origin county. Similarly, we define another set of buckets for moves that are across regions, $\mathbb{Y}$, which includes four groups: 1-100, 100-150, 150-200, and 200+. We then define the function g as

$$g\left(dist_{o,d}\right) = \sum_{x \in \mathbb{X}} \phi_x D^x_{o,d} + \xi_{o,d} \sum_{y \in \mathbb{Y}} \psi_y D^y_{o,d},$$

where D^x_{od} is a dummy that takes value one if the distance between counties o and d is in the distance bucket x, $\xi_{o,d}$ is a dummy variable that takes value one if counties o and d are in different regions, and D^y_{od} is a dummy that takes value one for cross-region moves if the distance between county o and the closest county in the region of county d is in distance group y. If workers treat moves between East and West Germany in the same way as moves within their region, then the coefficients on these dummies, ψ_y, should be zero.

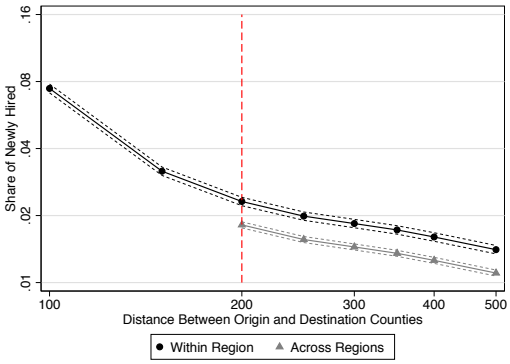
We visualize the results in Figure 5a, which plots for an average county situated at 200 km from the regional border the share of workers that would be hired by a county in the same region at distances $x \in \mathbb{X}$ and by a county in the other region at distances $x \in \mathbb{X}$ for $x \geq 200$.[14] The detailed coefficients from the regression are presented in column (1) of Table 4 in Appendix E. Figure 5a shows, first, that workers are more likely to move to nearby counties, as evidenced by the declining share of workers hired from counties that are further away. This finding corresponds to the standard gravity result in the trade literature. More importantly, we find that even after controlling for county fixed effects, across-region flows are lower than within-region flows at the same distance. In other words, conditional on county-specific characteristics, workers are less likely to move across regions than within region. We have thus identified an "invisible border" effect. This effect is sizeable: at a distance of 200km, workers are about 24% less likely to move to a county in the other region than to an identical county within their own region.

Our data contain additional information which allows us to provide some evidence on the nature of the invisible border. In particular, we are interested in understanding whether it is driven by geographical characteristics, such as worse infrastructure between regions compared to within them, or by characteristics of the labor force, such as workers being more likely to receive job offers or accept them in their birth region. Using workers' imputed location status relative to their birth location as "native" or "foreign", i.e., whether the worker is currently in

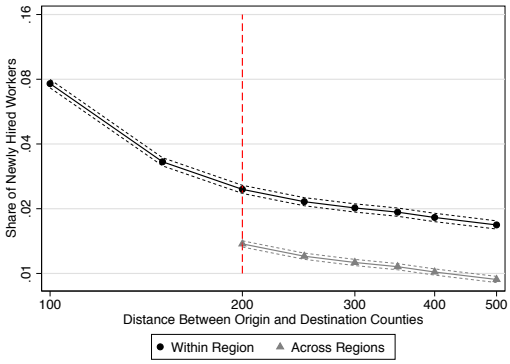
[14] We normalize the coefficients so that they sum to 1 including the excluded category, which is up to 50km.

Figure 5: Flows Between Counties

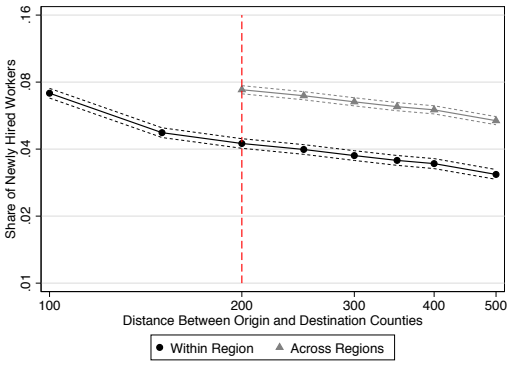
(a) All Workers



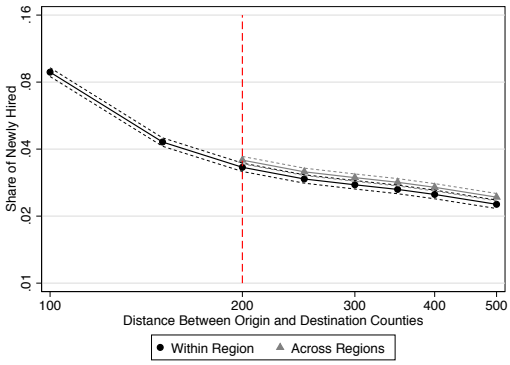
(b) Native Workers



(c) Foreign Workers



(d) All Workers, Birth-Specific FE



the region in which she was born, we run again specification (3) but allow for different behavior of workers that are currently "foreign" (indexed by f). The specification now reads

$$\log s_{o,d,f} = \delta_o + \gamma_d + \zeta_f + g(dist_{o,d,f}) + \zeta_f g^f(dist_{o,d,f}) + \epsilon_{o,d,f}.$$

This regression now includes an additional fixed effect ζ_f, which is equal to one for flows by foreign individuals, and a second distance term appears for foreign workers, where

$$g^f\left(dist_{o,d}\right) = \sum_{x \in \mathbb{X}} \phi_x^f D_{o,d,f}^x + \xi_{o,d,,f}^f \sum_{y \in \mathbb{Y}} \psi_y^f D_{o,d,f}^y,$$

where now the coefficients are different from zero if foreign workers migration patterns are different from the ones of natives. We show the coefficients for natives and foreigners in columns (2) and (3) of Table 4 in Appendix E, respectively, while we repeat the previous thought experiment for a county 200km from the border in Figures 5b and 5c. The results highlight that workers have a strong national identity, which they keep even after they move. Native

workers are very reluctant to move across regions, as visualized by the very low cross-border flows in Figure 5b. On the other hand, foreigners are actually more willing to move across regions (and thus back to their birth region) than within regions, thus suggesting that the "invisible border" does not represent a physical barrier but is rather linked to workers' characteristics. Thus, an East born worker that is currenly in the West is more likely to move 250 km to go back East than to move to an identical county that is more than 100km away in the West.

To further drive home the importance of workers' regional identity, we last approach the data in a more flexible way by allowing the origin and destination fixed effects to vary with the individual's birth-place. Specifically we compute the share of flows for East and West born workers – subscript b – and run in the data

$$\log s_{o,d,b} = \delta_{o,b} + \gamma_{d,b} + g(dist_{o,d,b}) + \epsilon_{o,d,b},$$

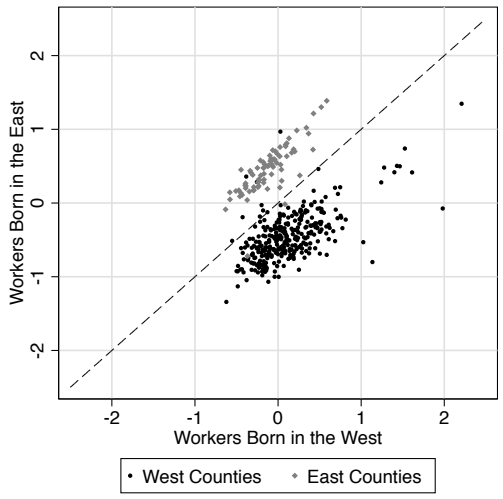
where now the fixed effects $\delta_{o,b}$ and $\gamma_{d,b}$ vary both by county and by birth region of the worker. This last specification gives us two results that corroborate the previous hypothesis. First, in Column (4) of Table 4 in Appendix E and in Figure 5d we show that, once we allow county fixed effect to vary by worker birth region, the distance between any two counties within or across regions has a very similar role, and if anything moving across the border is now actually marginally more likely than within. This result corroborates the idea that the "invible border" is due to workers' birth location: once we control for it, the "invisible border" vanishes. Second, in Figures 6b and 6a, we plot the origin and destination fixed effects for East-born workers as a function of those for West-born workers, color coded to separate East and West counties.[15] To interpret the graph, consider the left panel, which has the destionation fixed effect. If East and West workers had identical preferences across destinations, then all the dots should align on the 45 degrees line. If instead a specific dot is above the 45 degrees line, it means that East workers are more attracted to that county than West workers. The figure perfectly aligns with the invisible border. East and West-born workers mostly agree on the relative ranking of counties within each region, but East workers are more attracted to all East German counties, while West-born workers are more attracted to all West counties. The right panel plots the origin fixed effects and follows the same logic. Here, a high fixed effect means that workers are not attracted to that county and are likely to move out of it. The figure shows that East German workers are more likely to move out of West German counties and West German workers are more likely to move out of East German ones.

The results in this section taken together have established the existence of an "invisible

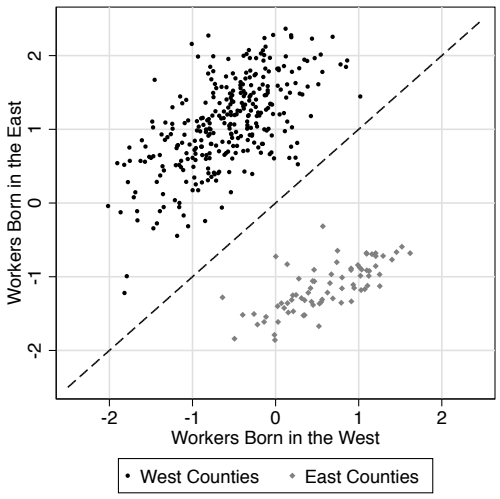
[15]For both origin and destination fixed effect, we normalize them, for both East-born and West-born workers, relatively to the average fixed effect, weighted by the number of within region counties, in such a way to assign equal weight to East and West Germany.

Figure 6: County Fixed Effects by Workers Birthplace

(a) Destination Fixed Effects



(b) Origin Fixed Effects



Notes: in both figures, a dot represents one county. We plot the point estimates for East-born workers as a function of the point estimates for West-born workers.

border": 25 years after the reunification, East and West Germany are still separated into two distinct labor markets. Moreover, the results strongly suggest that such border is driven by persistent characteristics of workers born in East and West, rather than by physical barriers to labor mobility. In the next section, we write down a frictional model of the labor market with multiple regions and workers' identity to investigate the sources and consequences of the "invisible border".

4 A Multi-Region Model of Misallocation and Unemployment

While the East and West German labor market remain distinct, the invisible border is permeable to the extent that some East German born workers move West and vice versa. Moreover, while there exist large average wage gaps across regions, we also observe that establishments paying high wages coexist with low-paying establishments. For these reasons, we next approach the data through the lens of a multi-region model of a frictional labor market with heterogeneous firms and multiple types of workers, who reallocate both across firms within-region and across regions. The model's aims are threefold: (i) it gives theoretical guidance on the types of underlying spatial frictions that can generate the invisible border; (ii) it provides us with a framework that

we can use as a measuring device to identify the relative contribution of the different type of frictions, and distinguish between reallocation frictions across heterogeneous firms and spatial frictions that distort the spatial allocation of workers; (iii) it provides a laboratory to perform counterfactual analysis in general equilibrium, taking into account the endogenous response of firms to changes in the labor supply.

The model we write follows closely the work of Burdett and Mortensen (1998) and of more recent empirical applications such as Moser and Engbom (2017). However, we depart from this previous work along one important dimension: we consider J distinct regional markets, each inhabited by a continuum of heterogeneous firms, and I different types of workers. As in the data, the regions are separeted by permeable borders, and workers are allowed to have region-specific identity and heterogeneous skills. In our framework, workers and firms all interact in one labor market that is subject to different types of reallocation and spatial frictions.

4.1 Model Setup

We first provide a broad overview of the environment, then we study the problem of workers and firms, and last we discuss how the labor market clears.

Environment. Let time be continuous. The economy is inhabited by a continuum of mass 1 of workers of types $i \in \mathbb{I}$, where $\mathbb{I} = \{1, .., I\}$. We denote the mass of workers of type i by $\bar{D}^i$, where $\sum_{i \in \mathbb{I}} \bar{D}^i = 1$. There are J regions in the economy, which could represent East and West Germany, but also smaller units, such as counties. The workers differ in both their ability and in their taste for being in a given region. Specifically, a worker of type i produces θ_j^i units of output per time unit in region j, where we use superscripts for worker types and subscripts for regions. If this worker is employed at wage rate w per efficiency unit, he earns an income of $w\theta_j^i$. Furthermore, worker i has a preference parameter of τ_j^i for being in region j. Assuming linear utility as is standard in models following Burdett and Mortensen (1998), worker i's utility from receiving wage rate w in region j is $u_j^i = w\theta_j^i\tau_j^i$.

Workers operate in a frictional labor market and can either be employed or unemployed. A worker of type i faces an arrival rate of vacancy offers from region j of $\varphi_j^i\lambda_j$, where λ_j is the endogenous arrival rate of offers from region j, determined below, and φ_j^i is an exogenous wedge, which we normalize by $\sum_{i \in \mathbb{I}} \varphi_j^i \bar{D}^i = 1$. This wedge reflects for example the fact that offers from East German firms are more likely to reach workers born in East Germany, due to either physical or social promixity. We assume that the arrival rate of offers does not depend on the region in which the worker is currently located. This is a strong assumption, which is needed to provide analytical tractability. Moreover, it is consistent with one sharp empirical

fact shown in the previous section, namely that once we allow county fixed effects to depend on worker type (i.e. on where they are born), then the current region does not affect the mobility patterns across counties. Workers draw offers from the endogenous distributions of wages F_j in all regions j, and must decide whether to accept an offer as soon as it is received. They separate into unemployment at rate δ^i irrespective of where they are working, and receive a utility flow equal to b^i when unemployed. All agents discount future income at rate r.

On the firm side, there is a continuum of firms of mass one in each region $j \in \mathbb{J}$,[16] distributed over productivity p according to density function $\gamma_j(p)$ with support on the positive real line. In each region j, the support of firms with positive mass is a region-specific closed set $[\underline{p}_j, \bar{p}_j] \subseteq \mathbb{R}^+$. Each firm p in region j decides how many vacancies $v_j(p)$ to post, subject to a vacancy cost $c_j(v)$, and what wage rate $w_j(p)$ to offer. Firms compete for all worker types in one unified labor market. To our knowledge, this is a novel feature of our wage-posting environment. Recent previous work with heterogenous types, see for example Moser and Engbom (2017), assumes that the labor market is segmented by type. In our framework, each firm posts a single wage rate $w_j(p)$, which will determine, endogenously, the composition of worker types it can attract. For example, firms posting a low wage rate in the East may only be able to attract workers born in the East but not West-born ones due to their preference for being in West Germany. Our setup will allow us to analyze how changing the wage impacts the share of East and West German workers hired by a firm. Each vacancy meets workers at a rate that we normalize, without loss of generality, to one.

We next turn to the problems of workers and firms.

Workers Under our assumptions, workers' decisions only depend on the flow utility received by an offer and not on the region they are currently in. Workers move from low utility to high utility jobs when the opportunity arises. However, they may move to a firm that pays a lower wage per efficiency unit if they are moving to a region where they have a stronger comparative advantage, θ_j^i: these type of moves would result in an increase in the worker's overall income but in a decline of the firm's wage rate. Workers may even accept a decrease in their own wage if they are moving to a region that provides them higher utility for a given labor income. The expected discounted lifetime utility of an unemployed worker is the solution to

$$rU^i = b^i + \sum_{x \in \mathbb{J}} \varphi_x^i \lambda_x \left[\int \max\left\{U^i, W^i(\tilde{u})\right\} dF_x\left(\tilde{u}/(\theta_x^i \tau_x^i)\right) - U^i \right], \tag{4}$$

[16] The mass of firms in each region is irrelevant, since the number of vacancies is endogenous. For this reason, we normalize this mass to 1.

where we denote by $W^i(u)$ the value of an employed worker earning utility flow u. Thus, the value of an unemployed worker of type i consists of the worker's flow benefit plus the expected value from finding a job, which is only accepted if this value exceeds the value from continuing search.

The value of an employed worker receiving flow utility u solves

$$rW^i(u) = u + \sum_{x \in \mathbb{J}} \varphi_x^i \lambda_x \left[\int \max \left\{ W^i(u), W^i(\tilde{u}) \right\} dF_x \left(\tilde{u} / (\theta_x^i \tau_x^i) \right) - W^i(u) \right] + \delta^i \left[U^i - W^i(u) \right]. \tag{5}$$

Since $W^i(u)$ is increasing in u, there exists a reservation utility R^i for each type of worker such that $W^i(R^i) = U^i$. From equations (4) and (5), we obtain $R^i = b^i$. The reservation utility corresponds to a reservation firm wage which is region specific: in region j it is given by $\hat{w}_j = \frac{b^i}{\tau_x^i \theta_x^i}$.

Firms. Each firm produces output from each vacancy with the production function $Y_j = Z_j p \sum_{i \in \mathbb{I}} \theta_j^i l_j^i$, where Z_j is a region-specific aggregate productivity shifter and $\sum_{i \in \mathbb{I}} \theta_j^i l_j^i$ is the number of efficiency units of labor used by one vacancy of the firm. Since the production function is linear, the firm-level problem of posting vacancies and choosing wages can be solved separately. Following the literature (e.g., Burdett and Mortensen (1998)), we focus on steady state: employers choose the wage rate that maximizes their steady state profits for each vacancy, which are

$$\pi_j(p) = \max_w (pZ_j - w) \sum_{i \in \mathbb{I}} \theta_j^i l_j^i(w). \tag{6}$$

Just as in the standard Burdett-Mortensen setup, the wage choice is determined by a trade-off between profit margins and firm size. On the one hand, a higher wage rate allows the firm to hire and retain more workers, and hence the steady state firm size increases in w. On the other hand, by offering a higher wage, the firm cuts down its profit margin, $pZ_j - w$. The complementarity between firm size and productivity implies that more productive firms offer a higher wage, just as in the literature. However, unique to our framework, firms need to take into account that their wage posting decision also impacts the types of workers they attract, which introduces a non-convexity, since the labor function $l_j^i(w)$ may be discontinous, as we will show.

Once wages have been determined, firms choose the number of vacancies to post by solving

$$\varrho_j(p) = \max_v \pi_j(p) v - c_j(v),$$

where $\pi_j(p)$ are the maximized profits per vacancy from (6). The size of a firm p in region j is

therefore given by $l_j(w_j(p))\, v_j(p)$. Moreover, the vacancy posting policy from the firm problem gives us the endogenous arrival rate of offers from each region

$$\lambda_j = \int_{\underline{p}_j}^{\bar{p}_j} v_j(p)\, \gamma_j(p)\, dp, \tag{7}$$

and the wage policy gives us the endogenous distributon of offers

$$F_j(w_j(p)) = \frac{1}{\lambda_j} \int_{\underline{p}_j}^{p} v_j(p)\, \gamma_j(p)\, dp. \tag{8}$$

Finally, notice that allowing for the firm size to be affected by both wage and vacancy costs introduces an additional free parameter, which will allow us to match the data by decoupling the relationship between wage and size.

Market Clearing. To close the model, we need to describe how the distribution of workers to firms is determined. Let $\Upsilon^i(u) \equiv D^i(u)/\bar{D}^i$ be the share of workers that need to receive at least a utility of u in order to accept a new offer over their current one. Then $\Upsilon^i(u)$ is a cumulative distribution function (CDF) over workers' reservation utility. Since no worker accepts a wage providing lower utility than unemployment, the support of D^i is bounded below by b^i. The law of motion of $D^i(u)$, for $u \geq b^i$, is

$$\dot{D}^i(u) = \delta^i \left(\bar{D}^i - D^i(u)\right) - \sum_{x \in \mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{u}{\theta_x^i \tau_x^i}\right)\right) D^i(u),$$

where dots represent time derivatives, and F_j comes from the firm problem as just described. The first term captures worker flows from positions offering a higher utility than u into unemployment. The second term represents outflows of workers to positions offering a higher utility than u.

In steady state, $\dot{D}^i(u) = 0$, and we have

$$D^i(u) = \frac{\delta^i \bar{D}^i}{\delta^i + \sum_{x \in \mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{u}{\theta_x^i \tau_x^i}\right)\right)}, \tag{9}$$

if $u \geq b$, and $D^i(u) = 0$ for $u < b$. The unemployment rate of workers of type i is simply $D^i(b)/\bar{D}^i$.

Denote by $l_j^i(w)$ the measure of workers of type i employed at one vacancy posted in region

j offering wage w. For $w\theta_j^i\tau_j^i \geq b^i$ the law of motion of $l_j^i(w)$ is given by

$$\dot{l}_j^i(w) = \varphi_j^i D^i\left(\theta_j^i\tau_j^i w\right) - q^i\left(\theta_j^i\tau_j^i w\right) l_j^i(w).$$

The first term of this expression represents worker inflows. These are given by the probability $\varphi_j^i D^i\left(\theta_j^i\tau_j^i w\right)$ that an offer contacts a worker of type i who is willing to accept, times the arrival rate of workers per vacancy, which is one. Outflows are equal to the mass of workers $l_j^i(w)$ multiplied by the rate at which workers separate either into unemployment or accept other offers,

$$q^i(u) = \delta^i + \sum_{x\in\mathbb{J}} \varphi_x^i \lambda_x \left(1 - F_x\left(\frac{u}{\theta_x^i\tau_x^i}\right)\right). \tag{10}$$

Firms that offer a higher utility flow u are characterized by lower outflows, since workers are less likely to be poached by other firms. In steady state, using expression (9) for $D^i(u)$, we obtain a measure of workers per vacancy of

$$l_j^i(w) = \frac{\varphi_j^i \delta^i \bar{D}^i}{\left[q^i\left(\theta_j^i\tau_j^i w\right)\right]^2} \tag{11}$$

if $\theta_j^i\tau_j^i w \geq b^i$, and zero otherwise.

To conclude the model setup and summarize the discussion, we define the competitive equilibrium.

Definition 1: Stationary Equilibrium. *A stationary equilibrium consists of a set of wage and vacancy posting policies* $\{w_j(p), v_j(p)\}_{j\in J}$, *profits per vacancy* $\{\pi_j(p)\}_{j\in J}$, *firm profits* $\{\varrho(p)\}_{j\in J}$, *arrival rates of offers* $\{\lambda_j\}_{j\in J}$, *wage offer distributions* $\{F_j(w)\}_{j\in J}$, *firm sizes for each worker type* $\left\{l_j^i(w)\right\}_{j\in J, i\in I}$, *separation rates* $\left\{q_j^i(p)\right\}_{j\in J, i\in I}$, *and worker utility distributions* $\{D^i(u)\}_{i\in I}$ *such that*

1. *workers always accept offers that provide higher utility, taking as given the wage offer distributions,* $\{F_j(w)\}_{j\in J}$;

2. *firms set wages to maximize per vacancy profits, and vacancies to maximize overall firm revenues, taking as given the function mapping wage to firm size,* $\left\{l_j^i(w)\right\}_{j\in J, i\in I}$;

3. *arrival rates of offers and wage offer distributions are consistent with vacancy posting and wage policies, according to equations (7) and (8);*

4. *firm sizes and worker distributions satisfy the stationary equations (9) and (11), where* $q_j^i(p) = q^i\left(\theta_j^i\tau_j^i w_j(p)\right)$.

4.2 Characterization

We next proceed to characterize the equilibrium. As mentioned, our model extends the class of job posting models à la Burdett and Mortensen to a setting with J regions and I types of workers that interact in one labor market subject to region-worker specific frictions, preferences, and comparative advantages. In order to understand the structure of our model, it is therefore useful to compare it with the benchmark Burdett-Mortensen model, which would be a special case of our model when all worker heterogeneity is shut down and there is only one region. In this case – as is well known – the equilibrium wage policy is as follows: the lowest productivity firm sets the minimum wage that allows it to hire workers from unemployment – i.e. $w\left(\underline{p}\right) = b$ –, and the wage policy is an increasing and continous function of productivity. We emphasize two key aspects of this solution. First, the equilibrium wage dispersion is given by the fact that firms that pay a higher wage are able to attract and retain more workers, and thus firm size is an increasing function of wage paid. Second, the wage policy must be continous. A discontinuity cannot be optimal, since a discrete jump in wage cannot lead to a discret jump in firm size, the reason being that firm size is purely determined by the ranking of wage offers and not by their level. In our setting these insights generalize, but need to be refined. Due to the presence of several type of workers and regions, discountinuities in the wage policy may arise. In fact, in our setting size of the firm can increase discountinously if the wage increase is sufficient to attract workers of an additional type. More broadly, in our setting, within a given (i, j) pair only the ranking of wage matters for the firm size, as in the benchmark model, but across regions and worker types also the level of the wage is relevant. As a result, we can show that the equilibrium is given by a set of piece-wise differential equations and boundary conditions for optimality.

Proposition 1. *The solution of the stationary equilibrium solves a set of J plus $J \times I$ differential equations*

$$\begin{aligned} \frac{\partial w_j(p)}{\partial p} &= H_j\left(\left\{q_j^i(p)\right\}_{j\in J, i\in I}, \left\{w_j(p)\right\}_{j\in J}\right) \\ \frac{\partial q_j^i(p)}{\partial p} &= K_j^i\left(\left\{q_j^i(p)\right\}_{j\in J, i\in I}, \left\{w_j(p)\right\}_{j\in J}\right) \end{aligned}$$

together with $J \times I$ boundary conditions for $q_j^i(\bar{p}_j)$, $J \times I$ cutoffs for firm types that have the lowest productivity to hire workers of type i in region j, defined by $\hat{p}_j^i = \min p_j$ such that $w_j(p_j) \geq R^i$, and $J \times I$ boundary conditions for $w_j\left(\hat{p}_j^i\right)$.

The proof of the proposition formalizes the previous discussion on the economic forces present in our model, moreover, it will provide closed form solution for the differential equations above, which can then be operationalized to efficiently solve the model for generic J and I.

Proof of Proposition 1. The proof is constructive, and solves the optimization problem to show that it leads to a system of differential equations, for which we provide analytical expressions.

Consider first the wage posting problem (6). Using equation (11), the first-order condition of this problem is

$$\frac{\left(pZ_j - w_j\left(p\right)\right)\left(\sum_{i\in\mathbb{I}}\theta_j^i \frac{\partial l_j^i(w_j(p))}{\partial w}\right)}{\left(\sum_{i\in\mathbb{I}}\theta_j^i l_j^i\left(w_j\left(p\right)\right)\right)} \quad = \quad 1, \tag{12}$$

where for any type i we have $l_j^i(w_j(p)) > 0$ if $\theta_j^i \tau_j^i w_j(p) \geq b^i$ and $l_j^i(w_j(p)) = 0$ otherwise. Define the ordered set for each region j

$$\mathbb{N}\left(j\right) \equiv \left\{ \frac{b^{\iota(j,1)}}{\theta_j^{\iota(j,1)}\tau_j^{\iota(j,1)}}, ..., \frac{b^{\iota(j,I)}}{\theta_j^{\iota(j,I)}\tau_j^{\iota(j,1)}} \right\},$$

where the set $\mathbb{N}\left(j\right)$ ranks worker types according to the minimum wage rate they require to work in region j, starting with the worker type that requires the lowest wage. Specifically, $\iota\left(j,x\right)$ is an operator such that $\iota\left(j,1\right) = \arg\min_{i\in\mathbb{I}} \frac{b^i}{\theta_j^i\tau_j^i}$, $\iota\left(j,2\right) = \arg\min_{i\in\mathbb{I}\setminus\iota(j,1)} \frac{b^i}{\theta_j^i\tau_j^i}$, and so on, up to $\iota\left(j,I\right) = \arg\max_{i\in\mathbb{I}} \frac{b^i}{\theta_j^i\tau_j^i}$.

Next, using equation (11) and re-arranging (12), we obtain a set of differential equations for wages $\left\{ \frac{\partial w_j^{\iota(j,1)}(p)}{\partial p}, ..., \frac{\partial w_j^{\iota(j,I)}(p)}{\partial p} \right\}$ of the form

$$\frac{\partial w_j^{\iota(j,n)}(p)}{\partial p} = -\frac{\left(pZ_j - w_j^{\iota(j,n)}(p)\right)\sum_{i=1}^{n}\theta_j^{\iota(j,i)}\left(\frac{2\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^i\left(\frac{\partial q_j^{\iota(j,i)}(p)}{\partial p}\right)}{\left[q_j^{\iota(j,i)}(p)\right]^3}\right)}{\left(\sum_{i=1}^{n}\theta_j^{\iota(j,i)}\frac{\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^{\iota(j,i)}}{\left[q_j^{\iota(j,i)}(p)\right]^2}\right)} \tag{13}$$

which define firms' optimal wage posting within each non-overlapping interval $w_j^{\iota(j,n)}(p) \in [\frac{b^{\iota(j,n)}}{\theta_j^{\iota(j,n)}\tau_j^{\iota(j,n)}}, \frac{b^{\iota(j,n+1)}}{\theta_j^{\iota(j,n+1)}\tau_j^{\iota(j,n+1)}})$ for $n < I$, and $w_j^{\iota(j,I)} \in [\frac{b^{\iota(j,I)}}{\theta_j^{\iota(j,I)}\tau_j^{\iota(j,I)}}, \infty)$. The overall differential equation for wage, $\frac{\partial w_j(p)}{\partial p} = H_j\left(\left\{q_j^i\left(p\right)\right\}_{j\in J, i\in I}, \left\{w_j\left(p\right)\right\}_{j\in J}\right)$, is going to be given by the union of the piece-wise ones shown in (13). To find this function, we need to know the relevant domains of each region's productivity distribution that map into each wage support.

Towards this aim, we now show how to determine the cutoffs $\hat{p}_j^i$ as defined in Proposition 1, which provide the lowest productivity firm in region j hiring workers of type i, and consequently the boundary conditions for the differential equation. Define $\pi_j^{\iota(j,n)}\left(p_j\right)$ to be the profit function

for a firm p_j that posts a wage high enough to attract workers of type $\iota(j,n)$ and behaves optimally, hence

$$\pi_j^{\iota(j,n)}(p_j) = \max_{w \geq \frac{b^{\iota(j,n)}}{\theta_j^{\iota(j,n)} \tau_j^{\iota(j,n)}}} (p_j Z_j - w) \sum_{i=1}^{n} \theta_j^{\iota(j,i)} l_j^{\iota(j,i)}(w).$$

Next, define $\underline{\tilde{p}}_j = \min p_j \text{ s.t. } p_j \geq \frac{b^{\iota(j,1)}}{\theta_j^{\iota(j,1)} \tau_j^{\iota(j,1)}}$. The firm with productivity $\underline{\tilde{p}}_j$ is the lowest productivity firm active in region j, since firms with $p_j < \underline{\tilde{p}}_j$ would make losses at the wage necessary to attract even the lowest reservation wage workers. Similarly, define for each $n \geq 2$

$$\tilde{p}_j^{\iota(j,n)} = \min p_j \quad s.t. \quad \pi_j^{\iota(j,n)}(p_j) \geq \max_{x<n} \pi_j^{\iota(j,x)}(p_j), \tag{14}$$

where $\tilde{p}_j^n$ is thus the lowest productivity firm that has a weakly higher profit by hiring workers of types $\iota(j,1), ..., \iota(j,n)$ rather than any subset of workers of type lower, in the reservation wage sense, than n. In general, it may be the case that $\tilde{p}_j^{\iota(j,n+1)} < \tilde{p}_j^{\iota(j,n)}$, for example if posting a higher wage and attracting type $n+1$ as well allows a firm to significantly raise its profits relative to the case in which only workers up to type n are hired. Therefore, we define

$$\hat{p}_j^{\iota(j,n)} = \min_{x \geq n} \tilde{p}_j^{\iota(j,x)}. \tag{15}$$

Equation (15) defines the productivity of the marginal firm that hires workers of type i, $\hat{p}_j^i$ for $i \in \mathbb{I}$.

Since it is possible to have $\hat{p}_j^i = \hat{p}_j^{i'}$ for some or even all pairs (i, i') of worker types, we define $\mathbb{D}$ as the set of worker types that have distinct cutoffs. By continuity of the profit function, the $n \in \mathbb{D}$ types are the ones that satisfy

$$\hat{p}_j^{\iota(j,n)} = \tilde{p}_j^{\iota(j,n)}.$$

Due to the usual complementarity argument between productivity and size, it must be that $\hat{p}_j^{\iota(j,n+1)} \geq \hat{p}_j^{\iota(j,n)}$ for all n, that is, higher reservation wage types are hired on average by higher productivity firms within the same region j. Call d_x the x^{th} element of $\mathbb{D}$.[17] We need to find $|\mathbb{D}|$ restrictions to solve for cutoffs values $\hat{p}_j^{\iota(j,d_x)}$ for $d_x \in \mathbb{D}$. Given these $|\mathbb{D}|$ cutoffs, we can find the cutoffs for the remaining types $n \notin \mathbb{D}$ using equation (15).

[17] For example, assume that $I = 4$ and that $\hat{p}_j^{\iota(j,1)} < \hat{p}_j^{\iota(j,2)} = \hat{p}_j^{\iota(j,3)} < \hat{p}_j^{\iota(j,4)}$, therefore the set $\mathbb{D} = \{1, 3, 4\}$, adn $d_1 = 1$, $d_2 = 3$, and $d_3 = 4$.

For the first cutoff, we have

$$\hat{p}_j^{\iota(j,1)} = \underline{\tilde{p}}_j.$$

Then, since the profit function $\pi_j^{\iota(j,n)}(p_j)$ is continous,[18] it must be that for all $d_x \in \mathbb{D}$ with $x \geq 2$, we have

$$\pi_j^{\iota(j,d_{x-1})}\left(\hat{p}_j^{\iota(j,d_{x-1})}\right) = \pi_j^{\iota(j,d_x)}\left(\hat{p}_j^{\iota(j,d_x)}\right),$$

which gives us the remaining $|\mathbb{D}|-1$ restrictions. Thus, we have found the I cutoffs.

The boundaries of the differential equation are then given by, for each $n \in \mathbb{D}$

$$w_j\left(\hat{p}_j^{\iota(j,n)}\right) = \arg \max_{w \geq \frac{b^{\iota(j,n)}}{\theta_j^{\iota(j,n)}\tau_j^{\iota(j,n)}}} (p_j Z_j - w) \sum_{i=1}^{n} \theta_j^{\iota(j,i)} l_j^{\iota(j,i)}(w).$$

Therefore, we have shown that wage function satisfies the following step-wise differential equation

$$\frac{\partial w_j(p)}{\partial p} = -\frac{\left(pZ_j - w_j^{\iota(j,n)}(p)\right)\sum_{i=1}^{n}\theta_j^{\iota(j,i)}\left(\frac{2\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^i\left(\frac{\partial q_j^{\iota(j,i)}(p)}{\partial p}\right)}{\left[q_j^{\iota(j,i)}(p)\right]^3}\right)}{\left(\sum_{i=1}^{n}\theta_j^{\iota(j,i)}\frac{\varphi_j^{\iota(j,i)}\delta^{\iota(j,i)}\bar{D}^{\iota(j,i)}}{\left[q_j^{\iota(j,i)}(p)\right]^2}\right)} \text{for } p \in \left[\hat{p}_j^{\iota(j,n)}, \hat{p}_j^{\iota(j,n+1)}\right),$$

where the cutoffs and the boundaries are as previously described, and we define $\hat{p}_j^{\iota(j,I+1)} = \infty$.

We next turn to the derivation of the derivative of the separation rate with respect to p, $\frac{\partial q_j^i(p)}{\partial p}$, which appears in the differential equation for the wage. From (10), this derivative depends on the distribution of wage offers F from each region x. Since the wage is increasing in firms' productivity, the probability that a worker of type i in region j receiving utility flow $u = \theta_j^i \tau_j^i w_j(p)$ rejects a random offer from region x is given by

$$F_x\left(\frac{\theta_j^i \tau_j^i w_j(p)}{\theta_x^i \tau_x^i}\right) = \frac{\int^{\psi_{jx}^i(w_j(p))} v_x(z)\gamma_x(z)\,dz}{\lambda_x}, \tag{16}$$

[18] This profit function is continous due to the fact that we are defining it keeping constant the set of hired types.

using equation (8). Equation (16) contains a productivity cut-off $\psi^i_{jx}(w)$, which is defined via

$$\begin{cases} \theta^i_x \tau^i_x w_x \left(\psi^i_{jx}(w_j(p))\right) = \theta^i_j \tau^i_j w_j(p) & \text{if } \psi^i_{jx}(w_j(p)) \in \left[\underline{p}_x, \bar{p}_x\right] \\ \psi^i_{jx}(w_j(p)) = \bar{p}_x & \text{if } \theta^i_x \tau^i_x w_x(\bar{p}_x) < \theta^i_j \tau^i_j w_j(p) \\ \psi^i_{jx}(w_j(p)) = \underline{p}_x & \text{if } \theta^i_x \tau^i_x w_x\left(\underline{p}_x\right) > \theta^i_j \tau^i_j w_j(p). \end{cases} \tag{17}$$

Intuitively, $\psi^i_{jx}(w_j(p))$ is the firm with the highest productivity in region x whose offer makes a worker of type i in region j earning $w_j(p)$ just indifferent between accepting and rejecting, provided that such a firm exists. For wage offers made by firms in the same region j as the worker, $\psi^i_{jx}(w_j(p)) = p$. If no firm exists in region x to provide the worker with a sufficiently high utility, the probability that the worker accepts is zero (second line). The acceptance probability is one if any offer would induce the worker to move (third line). For unemployed workers, the expressions are identical with $w_j(p) = \frac{b^i}{\theta^i_j \tau^i_j}$.

Next, rewrite equation (10) as a function of p, by substituting the utility $u = \theta^i_j \tau^i_j w_j(p)$

$$q^i\left(\theta^i_j \tau^i_j w_j(p)\right) = \delta^i + \sum_{x \in \mathbb{J}} \varphi^i_x \lambda_x \left(1 - F_x\left(\frac{\theta^i_j \tau^i_j w_j(p)}{\theta^i_x \tau^i_x}\right)\right),$$

differentiate with respect to p, use (16), and the definition of $q^i_j(p)$, to yield a differential equation for the separation rate $q^i_j(p)$

$$\frac{\partial q^i_j(p)}{\partial p} = -\sum_{x \in \mathbb{J}} \varphi^i_x \left(\frac{\partial \psi^i_{jx}(w_j(p))}{\partial p}\right) v_x\left(\psi^i_{jx}(w_j(p))\right) \gamma_x\left(\psi^i_{jx}(w_j(p))\right). \tag{18}$$

Equation (18) defines the $J \times I$ differential equations for the separation rates: $\frac{\partial q^i_j(p)}{\partial p} = K^i_j\left(\left\{q^i_j(p)\right\}_{j \in J, i \in I}, \{w_j$ The equation shows that the change in the separation rate is negatively related to the change in the marginal firm, which in turn depends on the slope of the wage functions. When the wage schedule in region x is relatively flat compared to the wage schedule in region j, a small change in p can significantly reduce the separation rate of workers from j to x.

We then need to find the $J \times I$ boundary conditions for the separation rate. For each type of worker i, these are given by the rate at which the worker leaves the most productive firm in each region j,

$$q^i_j(\bar{p}_j) = \delta^i + \sum_{x \in \mathbb{J}} \varphi^i_x \int_{\psi^i_{jx}(w_j(p))} v_x(z) \gamma_x(z) \, dz. \tag{19}$$

In the region in which the most productive firm provides the highest utility flow for this type of worker, indexed by $\bar{p}^i_j = \text{argmax}\left\{\theta^i_{j'} \tau^i_{j'} w_{j'}(\bar{p}_{j'})\right\}^{\mathbb{J}}_{j'=1}$, the worker only quits exogenously and

therefore $q_j^i(\bar{p}_j^i) = \delta^i$.

Summing up, we have shown that the solution of the equilibrium satisfies a set of differential equations, with a rich set of boundary conditions. Therefore we have proved Proposition 1.

4.3 Spatial and Reallocation Frictions

The model we describe in the previous section embeds two types of frictions: *spatial* frictions and *reallocation* frictions. We describe each of them and argue that both types of frictions are active in Germany.

Spatial frictions distort the allocation of workers across regions by affecting either workers' location decisions or firms' ability to hire a particular type of worker. Such frictions therefore lead to depressed flows of a particular type of worker across locations, which may generate a different supply curve for labor across regions. Spatial frictions appear in our model through any channel that depends on the region index j. First, they appear through the preference parameter τ_j^i, which directly affects workers' utility from locating in a particular region. Second, they arise from the productivity parameter θ_j^i, which affects workers' effective wage from working in region j. Third, they appear through the wedges φ_j^i, which influence the ability of firms in region j to contact a particular type of worker. Finally, they arise from heterogeneity in the cost of posting vacancies $c_j(v)$ across regions, which gives rise to region-specific contact rates λ_j. Spatial frictions directly generate regional wage differences by affecting workers' ability and willingness to take advantage of them.

Reallocation frictions, on the other hand, distort the allocation of workers across firms in the entire economy in a homogenous way. Since firms are heterogeneous in their productivity, moving workers from low-productivity to high-productivity firms is efficiency-enhancing. In the benchmark economy without reallocation frictions, all workers should be employed by the firm with the highest productivity $\bar{p}_j$ in each region. Reallocation frictions hinder such a reallocation of workers towards more productive firms. They appear in the model through three channels: first, a higher separation rate δ^i makes it more difficult for workers to reach and stay at the most productive firms. Second, higher unemployment benefits b^i raise the wage level in the economy and shrink firms' profits, which lowers the number of vacancies they post. Finally, a higher average cost of posting vacancies $\bar{c(v)} = \frac{1}{J}\sum_j c_j(v)$ lowers the number of vacancies and therefore the average contact rate of workers and firms. Reallocation frictions may interact with spatial frictions to generate wage gaps across regions. For example, a high cost of posting vacancies in region j implies that workers moving to region j find it very difficult to move to the most productive firms in that region, and hence migration to region j may not close the wage gap.

We now discuss two benchmark economies to provide intuition for the properties of the model. We consider the case of infinite spatial frictions and hence no cross-region mobility, and the case of full mobility in which spatial frictions are zero. These cases highlight that regional wage gaps may arise both in the presence of and in the absence of spatial frictions. To assess the specific role of spatial frictions in generating regional wage gaps, we introduce a new concept, which we term the *wage-specific size gap*. The wage-specific size gap measures the difference in size between two firms paying the same wage located in different regions. Formally, let $l_j(w_j(p)) = \sum_i l_j^i(w_j(p))$ be the total number of labor units hired by a firm of productivity p in region j. In the Burdett-Mortensen framework, this function is strictly increasing in the wage. The wage-specific size gap is defined by $\Psi(w) \equiv l_j(w) - l_{j'}(w)$. We show that spatial frictions always manifest themselves in a positivewage-specific size gap.

Consider first the case of infinite spatial frictions between all regions, $\tau_j^i = \varphi_j^i = 0$ for all i and $j \neq i$. Furthermore, let $I = J$ and $\Gamma_j = \Gamma$. We find that if the aggregate productivity parameter Z_j scales unemployment benefits and vacancy costs in all regions, then all regions exhibit the same equilibrium allocation of workers to firms and each region's the wage function is equal to the same baseline function shifted by Z_j. Hence, this economy exhibits a positive wage-specific size gap and as a result a positive aggregate wage gap. We summarize in the following lemma.

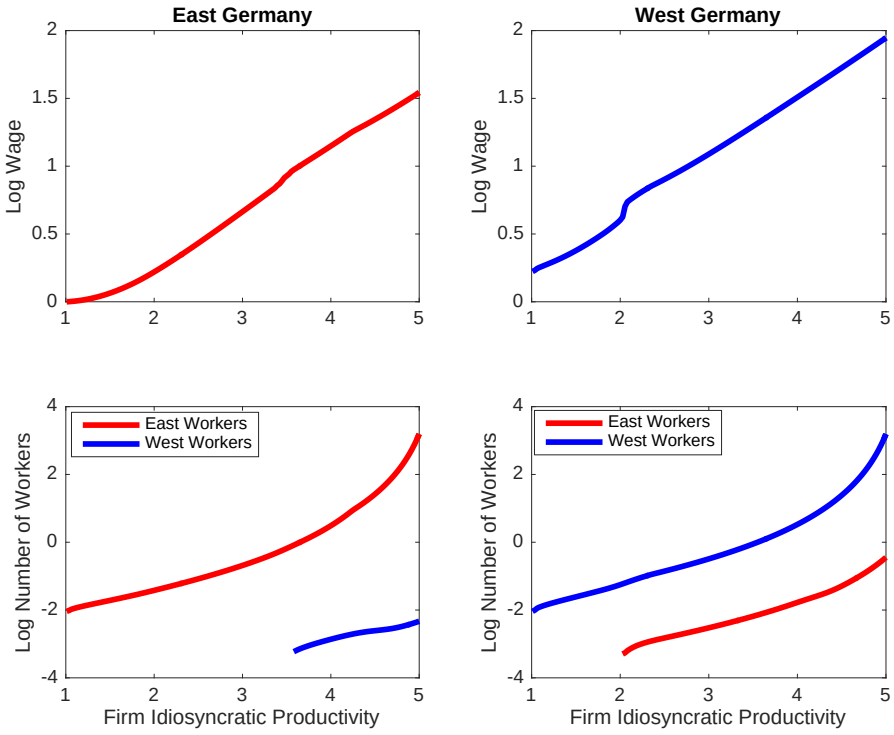
Lemma 1. *Let $I = J$, $\tau_j^i = \varphi_j^i = 0$ for all i and $j \neq i$, and assume that $\Gamma_j = \Gamma$ for all j. Furthermore, let $b^i = Z_i b$ and $c_j(v) = Z_j c(v)$, where b is a constant and $c(v)$ is a non-decreasing function. Then, for all p, $w_j(p) = Z_j w(p)$ and $l_j^i(p) = l^i(p)$, which imply that the wage-specific size gap between any j and j' is constant for each size and equal to $Z_j/Z_{j'}$.*

Proof. See Appendix C.1. □

We next consider the case in which no spatial frictions, and so $\tau_j^i = \tau^i$, $\theta_j^i = \theta^i$, $\varphi_j^i = \varphi^i$, and $c_j(v) = c(v)$ for all j. As documented in Section 3, the average establishment component in West Germany is significantly larger than in the East, and hence differences in the productivity distribution between the two regions seem to be important in driving the aggregate wage gap. We show that when there are no spatial frictions, each firm paying the same wage (which is uniquely determined by the firm's productivity) has the same size, regardless of its location, and hence the wage-specific size gap is equal to zero. However, the aggregate wage gap between any two regions increases in the regions' ratio of aggregate productivities.

Lemma 2. *Let $\tau_j^i = \tau^i$, $\theta_j^i = \theta^i$, $\varphi_j^i = \varphi^i$, and $c_j(v) = c(v)$ for all j. Then, for any two firms located in regions j and j' of the same productivity, $p_j = p_{j'} = p$, the wage-specific size gap is equal to zero, $w_j(p) = w_{j'}(p)$ and $l_j(p) = l_{j'}(p)$. However, the aggregate wage gap between any j and j' increases in $Z_j/Z_{j'}$.*

Figure 7: Wage Functions and Firm Size ($J = 2$, $I = 2$)



Proof. See Appendix C.2. □

In the case of East and West Germany, spatial frictions seem to play a role, as evidenced by the lack of cross-border flows discussed in Section 3, but there are still flows across the border. Furthermore, as shown in Figure 4, the wage-specific size gap is not equal to zero. We therefore next analyze an economy which exhibits finite spatial frictions, and hence lies in between the two cases previously discussed. We consider an economy with $J = 2$ regions, which we name East and West Germany, and $I = 2$ types. These types reflect where a worker was born. We set aggregate productivity to $Z_2 = 1.3Z_1$ and assume that workers have a preference for their home region so that $\tau_j^i = 0.5$ if $i \neq j$ and $\tau_j^i = 1$ if $i = j$. We also assume that there are matching frictions which make it harder for workers to leave their home region, $\varphi_j^i = 0.05$ if $i \neq j$ and $\varphi_j^i = 1$ if $i = j$. We set $b^i = Z_i$, and assume identical productivity distributions and vacancy cost functions in the two regions.

The upper two panels of Figure 7 present the simulated wage in East Germany (left) and West Germany (right) as a function of the idiosyncratic productivity p in the two regions. The panels show that wages are increasing in firm productivity as usual in the Burdett-Mortensen model. The bottom two panels present the log number of workers hired. In both regions, the least productive firms in both regions only hire workers from their own region, since their wages

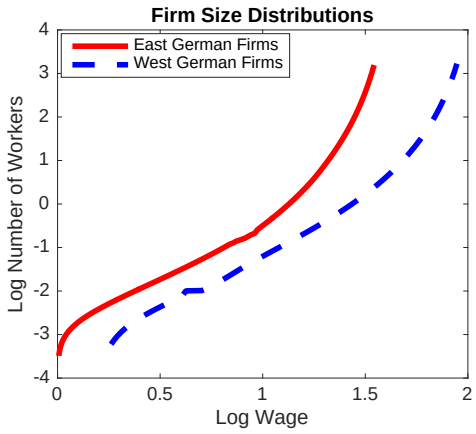
are too low to attract foreign-born workers. In West Germany, these firms pay a higher wage than in the East, since unemployment benefits for West German workers are higher. In contrast to the standard framework, there is a discrete jump in the wage function at the point at which firms start hiring workers from the other region. In particular, the West German firm with idiosyncratic productivity level of around 2 is the marginal firm which is indifferent between hiring only East German workers at a relatively low wage, or paying a discretely higher wage and attracting also West German workers. Any firm with a larger idiosyncratic productivity hires both types of workers and pays a higher wage. Since the aggregate productivity of East Germany is lower and therefore firms pay lower wages, the marginal firm in East Germany has a higher idiosyncratic productivity than in the West. Only comparatively productive East German firms are able to attract West German workers, while East German workers are willing to move even to relatively unproductive firms in the West. We empirically confirm this finding in the next section.

Figure 8a presents the wage-specific size gap in our model economy. Due to the spatial frictions, the size of firms paying the same wage differs across the two regions. In particular, firms paying the same wage are larger in the less productive East Germany than in the West. To gain intuition for this result, consider the lowest productivity firm in the West, which pays a wage of around $w = 0.25$. This firm is only able to hire West German born workers from unemployment. By contrast, an East German firm paying the same wage is significantly further up in the productivity distribution of East German firms. As a result, it is not only able to hire East German born workers from unemployment but also from all other firms below it with lower productivity, which increases its size. The same intuition holds for all other wage levels. Since aggregate productivity is lower in the East, a firm posting a given wage level must be at a higher position in the East German productivity distribution than in the West, which allows it to hire more workers.

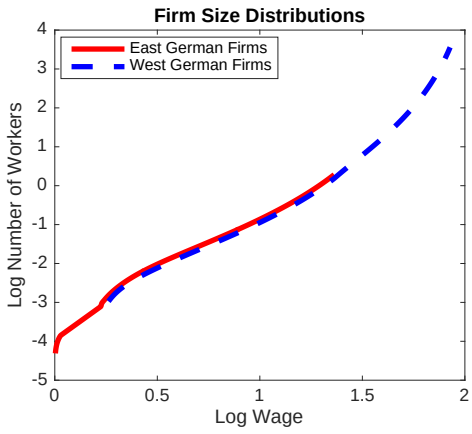
As a contrast, Figure 8b presents the case in which spatial frictions are not present. In this case, firms paying the same wage are exactly of the same size, as shown in Proposition 2. However, since aggregate productivity is higher in West Germany, firms of a given productivity post a higher wage in the West than in the East. This fact shifts the West German firm-size curve to the right, and generates an aggregate wage gap. Empirically, we have shown in Figure 4 in Section 3 that a wage-specific size gap is present in Germany. East German firms posting a given wage are significantly larger than West German firms at the same wage, consistent with local labor supply effects. This result suggests that spatial frictions are present in Germany. We will quantify the size of these frictions in the next section.

Figure 8: Wage-Size Ladders in the Model Economy

(a) Size-Specific Wage Gap with Spatial Frictions



(b) Wage-Specific Size Gap without Spatial Frictions



5 Quantitative Analysis

We now quantify the size of the various frictions in the model for the case of East and West Germany and perform counterfactuals. We proceed in three steps. First, we run reduced-form regressions, which are directly motivated by the model, to build some intuition on how we identify various model frictions in the data and to provide some sense of their general magnitude. Next, we turn to a full structural estimation of the model to match various data moments. Finally, we use the estimated model to run counterfactual simulations.

5.1 Reduced-Form Analysis

We run reduced form regressions to build some intuition for the identification of three spatial friction parameters: the worker productivity component θ_j^i, the preference friction τ_j^i, the contact wedge φ_j^i. We compare firm-level and individual-level wage gap to recover worker productivity. To uncover the preference friction, we use the relative wage gaps of job-to-job movers within and across East and West Germany. Finally, we use workers' mobility patterns to estimate the matching frictions.

Idiosyncratic Productivity Gap

We estimate the relative productivities θ_j^i by comparing the individual-level real wages of workers to the average real wages paid in these workers' establishments. In the model, all workers employed at the same establishment receive the same wage rate w. Deviations in the income of a given type of worker from the average must therefore be due to differences in workers' θ_j^i

Table 1: Idiosyncratic Productivity Differences

	(1) θ_W^W	(2) θ_W^E	(3) West-East born gap	(4) θ_E^W	(5) θ_E^E	(6) West-East born gap
Overall	1	0.97	$\sim 3\%$	1.03	1	$\sim 3\%$

parameters. Intuitively, if the wage of a certain type of worker is systematically lower (higher) than the average wage paid in the establishments that employ these workers, conditional on observable worker characteristics, then this type of worker has a lower (higher) productivity.

To estimate the productivities, we combine the LIAB and the BHP data to compute for each wage record in 2009-2014 the difference between each worker k's log real wage and the log average real wage of the worker's establishment f in the given year. We use this variable, $\Delta \log w_{kf,t} = \log w_{k,t} - \log w_{f,t}$, on the left-hand side in the regression

$$\Delta \log w_{kf,t} = \beta_1 \gamma_{\text{East,k,t}} + \beta_2 \delta_{\text{born East,k}} + \beta_3 \gamma_{\text{East,k,t}} \delta_{\text{born East,k}} + \alpha X_{\text{k,t}} + \xi_t + \nu_{Ind(f)} + \epsilon_{k,t}, \quad (20)$$

where $\gamma_{\text{East,k,t}}$ is a dummy that is equal to one if the establishment at which worker k is employed at time t is in the East, $\delta_{\text{born East,k}}$ is a dummy that is equal to one if worker k was born in the East, $X_{\text{k,t}}$ are characteristics of the worker which include age and age squared, a gender dummy, a dummy for whether a worker has an upper secondary school certificate or not, and dummies for the level of training.[19] Furthermore, ξ_t are time fixed effects, and $\nu_{Ind(f)}$ are fixed effects for establishment f's industry at the 3-digit level of the German National Industry Classification Scheme.[20] We cluster standard errors at the county-level. We can recover the productivity coefficients from the regression parameters: for example, the value for θ_E^E, the productivity of an East German born worker located in East Germany, and is obtained as $\beta_1 + \beta_2 + \beta_3$.

The implied productivity parameters θ_j^i, are presented in the first row of Table 1. We normalize the productivity of West German workers in West Germany to one. The raw regression coefficients are presented in Table 5 in Appendix E. Table 1 shows that a West German worker on average receives a 3% higher real wage than a comparable East German worker employed in the same industry in both regions. Thus, there appear to be meaningful differences in wages based on workers' birth location, which our model attributes to productivity differences.

[19] The categories we consider are 1 =no vocational training, 2 =vocational training or a traineeship, 3 =degree from a university of applied sciences, 4 =university degree.

[20] These codes are based on the WS93 classification from the Federal Employment Agency. The IAB provides a concordance of the WS73, WS03, and WS08 classifications to WS93, which we use (Eberle, Jacobebbinghaus, Ludsteck, and Witter (2011)).

Preference Frictions

We next provide some intuition for the size of the preference frictions, controlling for workers' productivity. From the worker problem in Section 4, holding fixed the productivity level across regions, a worker of type i moves job-to-job from a firm in region j paying wage w to a firm in region j' offering wage w' if and only if $\tau^i_{j'} w' > \tau^i_j w$. Define $G^i_j(w)$ to be the distribution function of workers of type i working in region j. The average wage increase for a randomly drawn worker moving from region j to region j' is then given by

$$\Delta\overline{\log w^i_{j \text{ to } j'}} = \int \left(\int_{(\tau^i_j w)/\tau^i_{j'}} (\log w' - \log w)\, dF_{j'}(w') \right) dG^i_j(w). \tag{21}$$

This expression consists of three parts. First, it depends on the cumulative distribution function of wage offers $F_{j'}$ from firms located in region j'. If the cdf puts a large amount of mass on high wage offers, then the average wage gain from moving between j and j' would be large since workers get very good offers from there. Second, the size of the average wage gain depends on the distribution of workers of type i in region j. If the distribution of these workers puts a lot of mass on low wages, then the average wage gain from moving could be large. Finally, it depends on the preference parameters, since $(\tau^i_j w)/\tau^i_{j'}$ is the lowest wage offer a worker of type i employed in region j at wage w would accept, and which would hence induce a move. For a given $F_{j'}$ and G^i_j, the average wage gap increases in the preference friction $\tau^i_j/\tau^i_{j'}$.

We construct a difference-in-difference regression using workers' wage gains in the LIAB data to isolate the contribution of the preference friction. Consider the task of identifying the preference of East German born workers for East Germany relative to West Germany. We control for the wage offer distribution F_j by subtracting the wage gain of an East German worker moving from East to West from the wage gain of a West German worker making the same move, controlling for individual characteristics. Since F_j does not depend on the type of the worker, a non-zero difference must be either due to differences in the worker distribution or due to preference frictions. We can further control for differences in the worker distribution G^i_j by subtracting the wage gain of an East German worker moving within East Germany from the wage gain of an East German worker moving from East to West. Analogously, we subtract the within-East wage gain for a West German worker from the wage gain of moving East to West. Since workers are drawn from the same wage distribution G^i_j regardless of whether they move within or across region, any remaining wage gain must be due to the preference parameter. Formally, the wage gain for an East German worker from moving from East to West attributable

to preference frictions $\Delta^{\tau}\overline{\log w^{E}_{E\,\text{to}\,W}}$ can be found from

$$\Delta^{\tau}\overline{\log w^{E}_{E\,\text{to}\,W}} = \left(\Delta\overline{\log w^{E}_{E\,\text{to}\,W}} - \Delta\overline{\log w^{E}_{E\,\text{to}\,E}}\right) - \left(\Delta\overline{\log w^{W}_{E\,\text{to}\,W}} - \Delta\overline{\log w^{W}_{E\,\text{to}\,E}}\right). \tag{22}$$

We obtain these variables in the LIAB data using wage gains of job-to-job movers. Let k index an individual and t index time, and consider all wage records from 2009-2014. We compute each worker's log daily real wage change $\Delta \log w_{k,t}$ from one wage record to the next. Since wage records are required to be filed annually, we observe each worker at least once every year, and more often if the worker changes jobs. On these data we run the specification

$$\begin{aligned}\Delta \log w_{k,t} = {} & \beta_1 d_{\text{k,t}} + \beta_2 d_{\text{k,t}}\gamma_{\text{East,k,t}} + \beta_3 d_{\text{k,t}}\delta_{\text{born East,k}} + \beta_4 d_{\text{k,t}}\gamma_{\text{East,k,t}}\delta_{\text{born East,k}} \\ & + \sum_{m\in\{\text{migr,comm}\}}\sum_{x\in\{\text{EW,WE}\}} [\alpha d_{\text{k,t}}\rho_{\text{m,x,k,t}} + \phi d_{\text{i,t}}\rho_{\text{m,x,k,t}}\delta_{\text{born East,k}}] + \xi_t + \nu_k + \epsilon_{k,t}.\end{aligned}$$

Here, $d_{\text{k,t}}$ is a dummy variable that is equal to one if the individual changes jobs at time t, $\gamma_{\text{East},k,t}$ is a dummy that is equal to one if the individual changes jobs within East Germany at time t, and $\delta_{\text{born East,k}}$ is a dummy that is equal to one if the individual is born in East Germany. We furthermore leverage the fact that we observe whether an individual changes her residence or only her work, and define m as an index that records whether an individual migrates or commutes. We record a worker as migrating if she transfers both her place of residence and her location of work from East Germany to West Germany or vice versa. On the other hand, we record a worker as commuting if she only moves her work location to the other region, but maintains her residence. We index by $x = EW$ when a worker moves from East to West and by $x = WE$ if the worker moves from West to East. The variable $\rho_{\text{m,x,k,t}}$ is a dummy that is equal to one if the migration decision is equal to m and the type of move is equal to x for worker k at time t. The regression includes time fixed effects ξ_t and worker fixed effects ν_k. The omitted category is the wage change for workers that remain at the same establishment. Standard errors are clustered at the county-level.

We present the decomposition (22) for East German born and West German born workers in Table 2. The detailed regression results are presented in Table 6 in Appendix E. Table 2 shows that a worker born in East Germany moving from East to West experiences on average a 53.5% real wage gain relative to a worker that stays at the same establishment (column 1). Since we include individual fixed effects, this wage gain is within-worker. It compares to an 8.4% relative real wage gain for workers that move firm within East Germany (column 2). On the other hand, West German workers obtain only a 27% real wage increase when moving East to West, consistent with a relative preference for being in the West (column 3). The within-East wage gain is similar to the one for East German workers (column 4). Based on our

Table 2: Contribution of Taste Friction to Worker-Level Wage Gain

	(1) $\Delta w^i_{i \text{ to } j}$	(2) $\Delta w^i_{i \text{ to } i}$	(3) $\Delta w^j_{i \text{ to } j}$	(4) $\Delta w^j_{i \text{ to } i}$	DiD
i = East	53.5%	8.4%	27%	8.3%	∼26.5%
i = West	37%	12%	11.5%	8.5%	∼22%

decomposition (22), these figures suggest that East German workers demand a 26.5% real wage increase due to preferences for the East to be incentivized to move to West Germany. By a similar decomposition, West German workers require a 22% real wage increase to move to the East. These results show that preferences play an important role in accounting for the wage gap.

Contact Wedge

We estimate the size of the contact wedge using a similar procedure as for the preference parameters. From the model, the rate at which workers of type i move from a job in region j to a job in region j' is

$$\mu^i_{j \text{ to } j'} = \varphi^i_j \lambda_j \times \int \left(\int_{(\tau^i_j w)/\tau^i_{j'}} dF_{j'}(w') \right) dG^i_j(w). \tag{23}$$

The first term, $\varphi^i_j \lambda_j$, is the rate at which workers of type i receive offers from region j, which depends on the relative contact wedge φ^i_j which we seek to estimate. The integral term represents the probability that a worker of type i accepts a random offer from region j, which depends on the offer distribution $F_{j'}$ and on the worker distribution G^i_j.

We estimate the size of the contact wedge using a similar approach as before to control for differences in the two distributions across workers and regions. Since we need the structural model to quantify the role of preferences $\tau^i_j/\tau^i_{j'}$, we only perform a reduced form exercise here, which absorbs the relative size of these frictions into a proportionality constant. Consider for example the wedge for East German workers. We control for differences in the offer distributions $F_{j'}$ by taking the ratio of the flows to West Germany by East German born workers and West German born workers. Since both types of workers face the same offer distribution and the same baseline contact rate λ_j, any deviation of the ratio from one must be due to different wage distributions in East Germany or a different contact wedge. Similarly, we control for differences in the worker distributions G^i_j by dividing the flows of East German workers to West Germany by East German workers' flows within the East, and similarly for West German workers. Thus,

Table 3: Size of the Contact Wedge

	(1) $\mu^i_{i \text{ to } j}$	(2) $\mu^i_{i \text{ to } i}$	(3) $\mu^j_{i \text{ to } j}$	(4) $\mu^j_{i \text{ to } i}$	DiD
i = East	1.3%	12.1%	12.4%	11.1%	~ 0.10
i = West	0.2%	13.9%	5.9%	13.0%	~ 0.03

the overall size of the contact wedge for an East German worker is proportional to

$$\frac{\varphi^E_W/\varphi^E_E}{\varphi^W_W/\varphi^W_E} \sim \frac{\mu^E_{E \text{ to } W}/\mu^E_{E \text{ to } E}}{\mu^W_{E \text{ to } W}/\mu^W_{E \text{ to } E}}. \tag{24}$$

Relatively high flows of East German workers across the border indicate a relatively small contact wedge. The equation for West German workers is analogous.

To implement the estimation, we use the LIAB data and compute the share of job-to-job switches in the total number of observations for each of the four types of transitions for the period 2009-2014. Table 3 shows the reduced-form estimates. The first column in the first row shows that in 1.3% of observations of East German born workers located in East Germany, the worker moves from East Germany to the West, compared to within-East transitions in 12.1% of cases (column 2). For West German born workers located in East Germany, we observe a move from East to the West in 12.4% of cases (column 3) and a within-East job-to-job transition in 11.1% of cases. Overall, the results suggest that the contact wedge is sizeable. Flows of East German workers from the East to the West would need to be larger by about a factor of ten (or flows of West German born workers back to the West correspondingly lower) to eliminate the relative difference in contact rates. Similarly, the flows of West German born workers East would need to be larger by a factor of about 30 in the absence of contact frictions. Through the lens of our model, workers appear to receive relatively more job offers from firms in their birth region.

5.2 Estimation

Work in progress.

6 Conclusion

Our paper has documented that 25 years after reunification, there exists an "invisible border" between East and West Germany. East Germany's real wage level is about 20% below that of the West, and unemployment is significantly higher. We find that this invisible border is

not the result of spatial sorting of more highly skilled East German workers to the West, but arises from a combination of more productive establishments in West Germany and a strong reluctance of workers to leave the region in which they are born. These preferences for workers' birth region affect worker mobility much more strongly than distance frictions. For example, an East German born worker that is currently in the West is as willing to move 100km within the West as she is to move 250km to get back to East Germany. To understand the relative contribution of spatial frictions versus frictions hindering the reallocation of workers to more productive firms, we develop an extension of the model by Burdett and Mortensen (1998) with multiple regions, multiple worker types, and heterogeneous firms. Our quantitative simulations show that preference frictions play an important role in generating the aggregate wage gap.

We believe that these results are of interest beyond the specific case of the German reunification. Large differences remain between regions in many countries, for example in the Italian Mezzogiorno or in Spanish Andalusia. Understanding the forces by which these regions continue to lag economically will enable policymakers to devise better policies to reduce regional gaps.

References

Abowd, J., F. Kramarz, and D. Margolis (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67(2), 251–333.

Allen, T., and C. Arkolakis (2014): "Trade and the Topography of the Spatial Economy," *Quarterly Journal of Economics*, 129(3), 1085–1140.

BBSR (2009): "Regionaler Preisindex," in *Berichte*, vol. 30. BBSR, Bonn.

Bryan, G., and M. Morten (2017): "The Aggregate Productivity Effects of Internal Migration: Evidence from Indonesia," NBER Working Paper No. w23540.

Burda, M. C. (2006): "Factor Reallocation in Eastern Germany after Reunification," *American Economic Review*, 96(2), 368–374.

Burda, M. C., and J. Hunt (2001): "From Reunification to Economic Integration: Productivity and the Labor Market in Eastern Germany," *Brookings Papers on Economic Activity*, 32(2), 1–71.

Burdett, K., and D. T. Mortensen (1998): "Wage Differentials, Employment Size, and Unemployment," *International Economic Review*, 39(2), 257–273.

Card, D., J. Heining, and P. Kline (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *Quarterly Journal of Economics*, 128(3), 967–1015.

Eberle, J., P. Jacobebbinghaus, J. Ludsteck, and J. Witter (2011): "Generation of Time-Consistent Industry Codes in the Face of Classification Changes - Simple Heuristic Based on the Establishment History Panel (BHP)," FDZ Methodenreport, 05/2011, Nuernberg.

Fuchs-Schündeln, N., D. Krueger, and M. Sommer (2010): "Inequality Trends for Germany in the Last Two Decades: A Tale of Two Countries," *Review of Economic Dynamics*, 13(1), 103–132.

Harris, J. R., and M. P. Todaro (1970): "Migration, unemployment and development: a two-sector analysis," *The American economic review*, 60(1), 126–142.

Hethey-Maier, T., and J. F. Schmieder (2013): "Does the Use of Worker Flows Improve the Analysis of Establishment Turnover? Evidence from German Administrative Data," NBER Working Paper No. 19730.

Hicks, J. H., M. Kleemans, N. Y. Li, and E. Miguel (2017): "Reevaluating Agricultural Productivity Gaps with Longitudinal Microdata," Discussion paper, National Bureau of Economic Research.

Hsieh, C.-T., and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124(4), 1403–1448.

Hunt, J. (2001): "Post-Unification Wage Growth in East Germany," *Review of Economics and Statistics*, 83(1), 190–195.

——— (2002): "The Transition in East Germany: When is a Ten-Point Fall in the Gender Wage Gap Bad News?," *Journal of Labor Economics*, 20(1), 148–169.

——— (2006): "Staunching Emigration from East Germany: Age and the Determinants of Migration," *Journal of the European Economic Association*, 4(5), 1014–1037.

Kennan, J., and J. R. Walker (2011): "The effect of expected income on individual migration decisions," *Econometrica*, 79(1), 211–251.

Krueger, A. B., and J.-S. Pischke (1995): "A Comparative Analysis of East and West German Labor Markets: Before and After Unification," in *Differences and Changes in Wage Structures*, ed. by R. B. Freeman, and L. F. Katz, pp. 405–446. University of Chicago Press.

Leiby, R. A. (1999): *The Unification of Germany, 1989-1990*. Greenwood Press, Westport, CT.

Moser, C., and N. Engbom (2017): "Earnings Inequality and the Minimum Wage: Evidence from Brazil," mimeo, Columbia University.

Restuccia, D., and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11(4), 707–720.

Roback, J. (1982): "Wages, Rents, and the Quality of Life," *Journal of Political Economy*, 90(6), 1257–1278.

Rosen, S. (1979): "Wage-Based Indexes of Urban Quality of Life," in *Current Issues in Urban Economics*, ed. by P. Mieszkowski, and M. Straszheim. Johns Hopkins University Press.

Sjaastad, L. A. (1962): "The costs and returns of human migration," *Journal of political Economy*, 70(5, Part 2), 80–93.

Smolny, W. (2009): "Wage Adjustment, Competitiveness and Unemployment - East Germany after Unification," *Jahrbücher für Nationalökonomie und Statistik*, 229(2+3), 130–145.

YOUNG, A. (2013): "Inequality, the Urban-Rural Gap, and Migration," *Quarterly Journal of Economics*, 128(4), 1727–1785.

Appendix

A Historical Overview

East and West Germany were separate countries before 1990. There was virtually no movement of workers between the two regions, and the border was tightly controlled. This separation gave rise to two distinct economic systems. While West Germany was a market economy, the economy in East Germany (then called the German Democratic Republic, GDR) was planned.

The German reunification completely removed the East German institutions of the planned economy and replaced them with West German ones. Starting on July 1, 1990, the two Germanys started a full monetary, economic, and social union, and introduced the regulations and institutions of a market economy to the GDR. These included for example the West German commercial code and federal taxation rules, as well as a reform of the labor market which imposed Western-style institutions (Leiby (1999)). At the same time, the West German Deutschmark (DM) became the legal currency of both halves of Germany. Wages and salaries were converted from Ostmark into DM at a rate of one-to-one, as were savings up to 400DM. While the currency reform implied an East German wage level of about 1/3 the West German level, in line with productivity, the switch meant that East German firms lost export markets in Eastern Europe, since customers there could not pay in Western currency. Additionally, East German customers switched to Western products, which were of much higher quality than East German ones (Smolny (2009)). West German unions negotiated sharp wage increases in many East German industries, which were not in line with productivity gains but driven by a desire to harmonize living conditions across the country (Burda and Hunt (2001), Smolny (2009)). As a consequence, East German unit labor costs rose sharply, and output and employment collapsed (Burda and Hunt (2001)). This trend was further excacerbated by the break-up and transfer of unproductive East German conglomerates to private owners, who usually downsized or closed plants.[21]

B AKM Decomposition

The AKM framework runs the following regression

$$\log w_{t,i} = \log \theta_{t,i} + \log \psi_i + \varepsilon_{t,i}$$

[21] This transfer was done via the *Treuhandanstalt*, a public trust, which was set up by the West German government to manage and ultimately sell the GDR's public companies. West German were initially slow to invest into East German firms. Eventually, most firms were sold at very steep discounts to the highest bidder, usually West German firms, which were often motivated by subsidies and had little interest in keeping their acquisitions alive (Leiby (1999)).

where $\theta_{t,i}$ is the firm fixed effect for the firm where worker i is employed at time t, and ψ_i is the worker fixed effect.

The predicted average wage at time t is therefore given by

$$\bar{w}_t \equiv \frac{1}{N_t} \sum_{i \in \mathbb{I}_r} \theta_{t,i} \psi_i.$$

We can rewrite $\bar{w}_t$ as

$$\bar{w}_t = \eta_t \rho_t \bar{\theta}_t \bar{\psi}_i$$

where

$$\begin{aligned}
\bar{\theta}_t &\equiv \frac{1}{J_t} \sum_{j \in \mathbb{J}_t} \theta_t^j \\
\hat{\theta}_t &\equiv \frac{1}{N_t} \sum_{i \in \mathbb{I}_r} \theta_{t,i} \\
\eta_t &\equiv 1 + \text{Cov}\left(\frac{n_t^j}{\bar{n}_t}, \frac{\theta_t^j}{\bar{\theta}_t} \right) = \frac{\hat{\theta}_t}{\bar{\theta}_t} \\
\rho_t &\equiv 1 + \text{Cov}\left(\frac{\theta_{t,i}}{\hat{\theta}_t}, \frac{\psi_i}{\bar{\psi}} \right) = 1 + \frac{1}{N_t} \sum_i \left(\frac{\theta_{t,i}}{\hat{\theta}_t} - 1 \right) \left(\frac{\psi_i}{\bar{\psi}} - 1 \right)
\end{aligned}$$

and the j superscript indicates the firm, and $\mathbb{J}_t$ is the set of firms.

Each object in the decomposition is easily interpretable. $\bar{\theta}_t$ captures the effect of firm characteristics. $\bar{\psi}$ captures the effect of workers characteristics, η_t captures the correlation between firm size and firm fized effect, and last, ρ_t is the pure effect of sorting, that is, it captures the correlation between firm and workers fixed effects.

We next show the details of the derivation

$$\begin{aligned}
\bar{w}_t &\equiv \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\theta_{t,i}\psi_i \\
&= \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\theta_{t,i}-\hat{\theta}_t\right)\psi_i+\hat{\theta}_t\bar{\psi} \\
&= \frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\theta_{t,i}-\hat{\theta}_t\right)\left(\psi_i-\bar{\psi}_i\right)+\hat{\theta}_t\bar{\psi} \\
&= \bar{\theta}_t\bar{\psi}\left[\frac{1}{\bar{\theta}_t}\frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\theta_{t,i}-\hat{\theta}_t\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)+\frac{\hat{\theta}_t}{\bar{\theta}_t}\right] \\
&= \bar{\theta}_t\bar{\psi}\left[\frac{\hat{\theta}_t}{\bar{\theta}_t}\frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\frac{\theta_{t,i}}{\hat{\theta}_t}-1\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)+\frac{\hat{\theta}_t}{\bar{\theta}_t}\right] \\
&= \bar{\theta}_t\bar{\psi}\frac{\hat{\theta}_t}{\bar{\theta}_t}\left[\frac{1}{N_t}\sum_{i\in\mathbb{I}_r}\left(\frac{\theta_{t,i}}{\hat{\theta}_t}-1\right)\left(\frac{\psi_i}{\bar{\psi}}-1\right)+1\right] \\
&= \bar{\theta}_t\bar{\psi}\frac{\hat{\theta}_t}{\bar{\theta}_t}\left[1+\text{Cov}\left(\frac{\theta_{t,i}}{\hat{\theta}_t},\frac{\psi_i}{\bar{\psi}}\right)\right]
\end{aligned}$$

finaly notice that

$$\frac{\hat{\theta}_t}{\bar{\theta}_t}=\frac{\frac{1}{N_t}\sum_{j\in\mathbb{J}_t}n_t^j\theta_t^j}{\frac{1}{J_t}\sum_{j\in\mathbb{J}_t}\theta_{t,i}}=\frac{\frac{1}{N_t}\sum_{j\in\mathbb{J}_t}\left(n_t^j-\bar{n}_t\right)\theta_t^j+\bar{\theta}_t}{\bar{\theta}_t}=\frac{\frac{1}{J_t}\sum_{j\in\mathbb{J}_t}\left(n_t^j-\bar{n}_t\right)\left(\theta_t^j-\bar{\theta}_t\right)+\bar{\theta}_t}{\bar{\theta}_t}=1+\text{Cov}\left(\frac{n_t^j}{\bar{n}_t},\frac{\theta_t^j}{\bar{\theta}_t}\right)$$

where I've defined $\bar{n}_t$ as average firm size, and I used the fact that $N_t=\bar{n}_tJ_t$.

C Proofs

C.1 Proof of Proposition 1

The proof proceeds in two steps. First, we follow the standard steps in Burdett and Mortensen (1998) to show that the equilibrium in a closed economy can be described by two differential equations. We then show that if we have an equilibrium in the West, then the conjectured equilibrium relationships constitute an equilibrium in the East.

Step 1: Equilibrium in the closed economy

Consider the problem described in the main text for an economy in isolation, i.e., $\varphi = 0$. We normalize the mass of workers in the economy to $N = 1$. The separation rate from equation (**??**) is

$$q(w) = \delta + \theta + \lambda [1 - F(w)]$$

and the hiring rate $h(w)$ from equation (**??**) is

$$h(w) = u + (1 - u)G(w).$$

We have the flow equations

$$\dot{u} = (\delta + \theta)(1 - u) - \lambda u$$

and

$$\dot{D}(w) = \lambda F(w) u - (\delta + \theta) D(w) - \lambda (1 - F(w)) D(w).$$

In steady state, these equations lead to

$$\frac{u}{1 - u} = \frac{\delta + \theta}{\lambda}$$

and

$$G(w) = \frac{\delta F(w)}{\delta + \lambda (1 - F(w))}. \tag{25}$$

As in the main text, equation (**??**), firms maximize profits by choosing vacancies and wages:

$$\max_{(w,v) \geq (b,0)} \left[(p - w) \frac{h(w)}{q(w)} v - c(v) \right].$$

Define profits per vacancy as $\pi(p, w) = (p - w)h(w)/q(w)$. The optimal vacancy posting satisfies

$$\pi^*(p) = c'(v) \quad \Rightarrow \quad v = \xi(\pi^*(p)),$$

where ξ is the inverse marginal cost function and $\pi^*(p)$ are the profits under the profit-maximizing wage policy.

To obtain the wage function, we use the expressions for $h(w)$ and $q(w)$ and the steady state equations for unemployment and $G(w)$ to obtain

$$\pi(p,w) = \frac{(\delta+\theta)(p-w)}{[\delta+\theta+\lambda[1-F(w)]]^2}.$$

The first-order condition of this expression yields

$$\frac{2\lambda F'(w)(p-w)}{\delta+\theta+\lambda[1-F(w)]} = 1. \tag{26}$$

Using the expression for $F(w)$ from equation (**??**) and differentiating with respect to p gives

$$F'(w(p))w'(p) = \frac{v(p)\gamma(p)}{\int_b^{\bar{p}} v(z)\gamma(z)dz} = \frac{v(p)\gamma(p)}{\bar{\lambda}},$$

where $w(p)$ is the wage posting as a function of productivity and the second equality follows from (**??**). Combining this expression with equation (26) yields a differential equation for the wage function

$$w'(p) = \frac{2v(p)\gamma(p)(p-w(p))}{q(p)}. \tag{27}$$

We can re-write this expression by noting that the overall separation rate as a function of productivity is

$$q(p) = \delta+\theta+\lambda[1-F(w(p))] = \delta+\theta+\int_p^{\bar{p}} v(z)\gamma(z)dz,$$

which has the derivative

$$q'(p) = -v(p)\gamma(p).$$

Applying this expression to equation (27) gives

$$w'(p) = \frac{-2q'(p)(p-w(p))}{q(p)},$$

which has the boundary condition

$$w(\underline{p}) = b.$$

On the other hand, from optimal vacancy posting we can re-express the derivative of the separation function as

$$q'(p) = -\xi\left(\frac{(\delta+\theta)(p-w(p))}{[q(p)]^2}\right)\gamma(p), \tag{28}$$

which has boundary condition

$$q(\bar{p}) = \delta+\theta.$$

The two differential equations together with the boundary conditions provide the solution to the problem without cross-regional mobility.

Step 2: Equilibrium in the East

Assume that $\Gamma_E(\kappa p) = \Gamma_W(p)$ with $\kappa < 1$, and $b_E = \kappa b_W$, $\delta_E = \delta_W$, and $\bar{c}_E = \kappa \bar{c}_W$ as given in the proposition. We show that if under these assumptions

$$(\lambda_W, w_W(p), q_W(p), G_W(w(p)), F_W(w(p)))$$

are an equilibrium in the West, then

$$(\lambda_E, w_E(p), q_E(p), G_E(w(p)), F_E(w(p)))$$

with

$$\begin{aligned} \lambda_E &= \lambda_W \\ w_E(\kappa p) &= \kappa w_W(p) \\ q_E(\kappa p) &= q_W(p) \\ G_E(w(\kappa p)) &= G_W(w(p)) \\ F_E(w(\kappa p)) &= F_W(w(p)) \end{aligned}$$

are an equilibrium in the East.

We begin by verifying that our conjectures $q_E(\kappa p) = q_W(p)$ and $w_E(\kappa p) = \kappa w_W(p)$ are correct. Rewriting equation (28) and using the expression for optimal vacancies (**??**), we have for West Germany that

$$q_W(p) = \left(-\frac{\delta(p - w_W(p))}{\bar{c}_W \left[q'_W(p)\right]^{\chi}} \right)^{\frac{1}{2}} \gamma_W(p)^{\chi/2}.$$

If our guess that $q_E(\kappa p) = q_W(p)$ is correct, then $q'_E(\kappa p) = \frac{1}{\kappa} q'_W(p)$. From the assumptions for Γ_E and Γ_w we have that $\gamma_E(\kappa p) = \frac{1}{\kappa}\gamma_W(p)$. Replacing these expressions in the equation for East Germany yields

$$q_E(\kappa p) = \left(-\frac{\delta(\kappa p - w_E(\kappa p))}{\bar{c}_E \left[\frac{1}{\kappa} q'_W(p)\right]^{\chi}} \right)^{\frac{1}{2}} \left(\frac{1}{\kappa}\right)^{\chi/2} \gamma_W(p)^{\chi/2}$$

Given our conjecture for the wage function,

$$q_E(\kappa p) = \left(-\frac{\kappa\delta(p - w_W(p))}{\bar{c}_E \left[q'_W(p)\right]^{\chi}} \right)^{\frac{1}{2}} \gamma_W(p)^{\chi/2}.$$

Using the fact that $\bar{c}_E = \kappa \bar{c}_W$ yields the desired result, and so indeed $q_E(\kappa p) = q_W(p)$.

We also need to verify the guess for the wage. Our conjecture implies that $w'_E(\kappa p) = w'_W(p)$. Using the differential equation for the wage and the relationship between the separation function in the East and in the West, we obtain

$$w'_E(\kappa p) = \frac{-2q'_E(\kappa p)(\kappa p - w_E(\kappa p))}{q_E(\kappa p)} = \frac{-2\frac{1}{\kappa}q'_W(p)(\kappa p - \kappa w_W(p))}{q_W(p)} = w'_W(p).$$

Thus this guess is also verified.

Note that the boundary conditions hold. For the wage function,

$$\kappa w_W(\underline{p}) = \kappa b_W \quad \Leftrightarrow \quad w_E(\kappa \underline{p}) = b_E.$$

For the separation function,

$$q_W(\bar{p}) = q_E(\kappa \bar{p}) = \delta + \theta.$$

We next verify that $\lambda_W = \lambda_E$. From optimal vacancy posting,

$$v_E(\kappa p) = \left[\frac{\delta(\kappa p - w_E(\kappa p))}{\bar{c}_E\left[q_E(\kappa p)\right]^2}\right]^{1/\chi} = \left[\frac{\delta(p - w_W(p))}{\bar{c}_W\left[q_W(p)\right]^2}\right]^{1/\chi} = v_W(p). \tag{29}$$

Given the definition of λ,

$$\lambda_E = \int_{b_E}^{\bar{p}_E} v_E(z)\,\gamma_E(z)\,dz,$$

we obtain

$$\lambda_E = \int_{b_E}^{\bar{p}_E} v_E(x)\,\gamma_E(x)\,dx = \int_{b_E/\kappa}^{\bar{p}_E/\kappa} v_E(\kappa y)\,\gamma_E(\kappa y)\,\kappa dy = \int_{b_W}^{\bar{p}_W} v_W(y)\,\gamma_W(y)\,dy = \lambda_W,$$

as claimed, where the second equality holds by a change in variable.

We finally need to focus on the accounting equations, and show that G and F hold as well. Note that $u_W = u_E$ holds trivially, since $\lambda_W = \lambda_E$, $\theta_E = \theta_W$, and $\delta_W = \delta_E$.To verify that the relationship for F holds, we use that

$$\begin{aligned} F_E(\kappa p) &= \frac{\int_{b_E}^{\kappa p} v_E(x)\gamma_E(x)dx}{\int_{b_E}^{\bar{p}_E} v_E(x)\gamma_E(x)dx} \\ &= \frac{\int_{b_E/\kappa}^{p} v_E(\kappa y)\gamma_E(\kappa y)\kappa dy}{\int_{b_E/\kappa}^{\bar{p}_E/\kappa} v_E(\kappa y)\gamma_E(\kappa y)\kappa dy} \\ &= \frac{\int_{b_W}^{p} v_W(y)\gamma_W(y)dy}{\int_{b_W}^{\bar{p}_W} v_W(y)\gamma_W(y)dy} \\ &= F_W(p). \end{aligned}$$

The condition for G can then be verified using the flow equation (25). This concludes the proof. Since we assumed that the W functions are an equilibrium for the West, than the properly defined E functions are an equilibrium for the East.

C.2 Proof of Proposition 2

If $b_E = b_W$, then from equation (**??**) we have that $U_E = U_W$. Therefore, unemployed workers are indifferent between unemployment in either region. It follows from equation (**??**) that the probability of accepting a job offer when unemployed is equal to $\hat{P}_i(w) = 1$ in both regions. Firms post wages $w \geq b_i$, and the East German firms with $p < b_i$ exit the market and do not post wage offers. From equation (**??**), the steady-state unemployment ration in a region i is then

$$\frac{u_i}{n_i} = \frac{\delta_i + \theta_i}{\bar{\lambda}_i + \bar{\lambda}_j}. \tag{30}$$

Since $\delta_E = \delta_W$ and $\theta_E = \theta_W$ by assumption, the unemployment to employment ratio is the same in both regions.

To see that allocational quality is not necessarily the same in both regions, note that since workers are indifferent between being in either region, they will move to any firm that offers them a higher job. Hence, for a firm posting wage w, the hiring probability $h_i(w)$ and the quit probability $q_i(w)$ will be the same in both regions. From equation (**??**), a firm with the same productivity p will therefore post the same wage w in both regions. However, if the cost of vacancy posting $\bar{c}_i$ differs across regions, then firms of the same productivity level will have a different size in both regions. Furthermore, the lower tail of the East German firms does not post wages since $p < b_E$. Dependent on the shape of the upper tail of the firm productivity distribution, the ratio of the weighted average wage to the unweighted average may either be higher or lower in the East than in the West. Hence, in general, $\rho_E \neq \rho_W$.

D Additional Figures

Figure 9: Price level in 2009

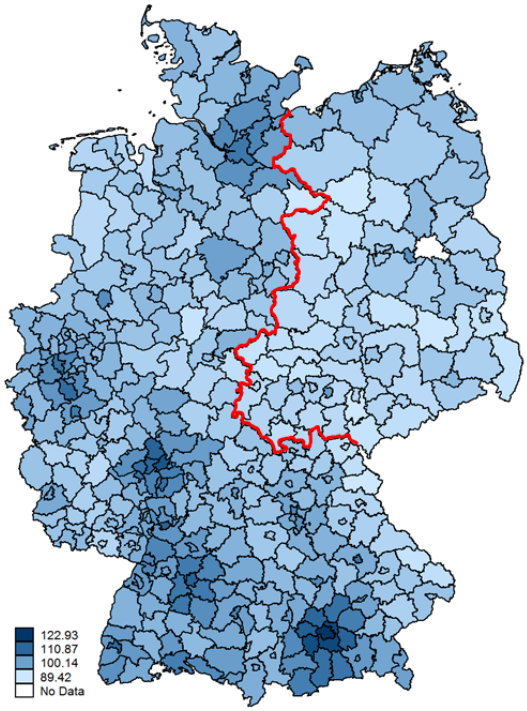
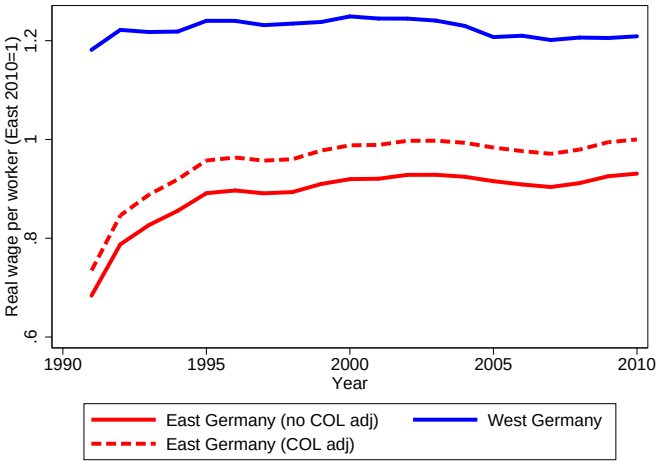


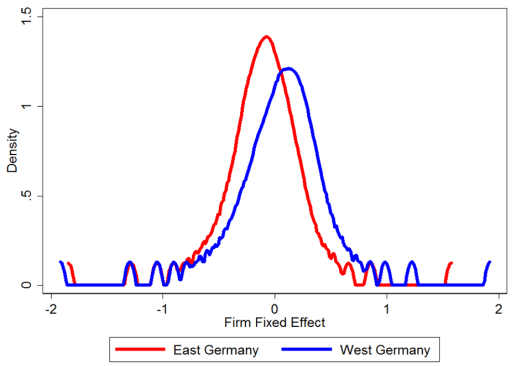
Figure 10: Average real wage with and without cost of living (COL) adjustment



Notes: Wages for East and West Germany (excluding Berlin) are obtained from the statistics offices of the German states and deflated by region-specific price deflators from the same source. We normalize East Germany in 2010 to one. The dashed line is constructed by adjusting the wage level in 2009 based on the population-weighted average price level in East Germany from the BBSR.

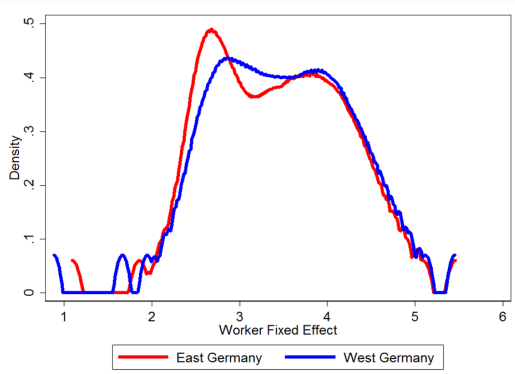
Figure 11: Distribution of Establishme and Workers Component, 2009-2014

(a) Establishment Fixed Effects



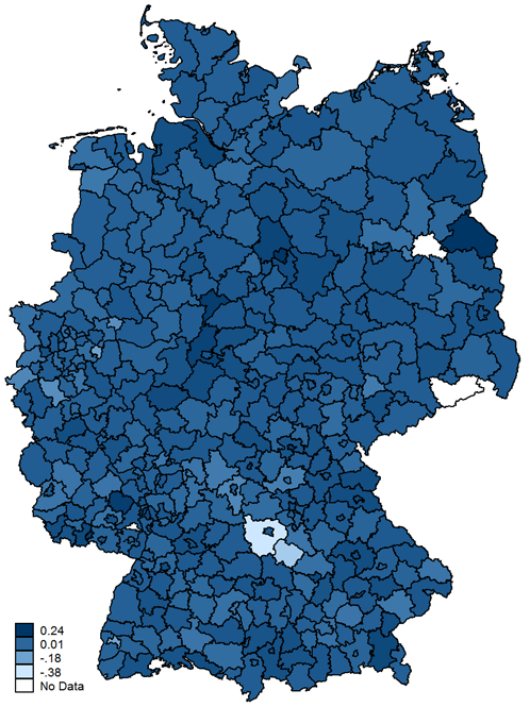
Source: LIAB, Authors' Calculations

(b) Worker Fixed Effects



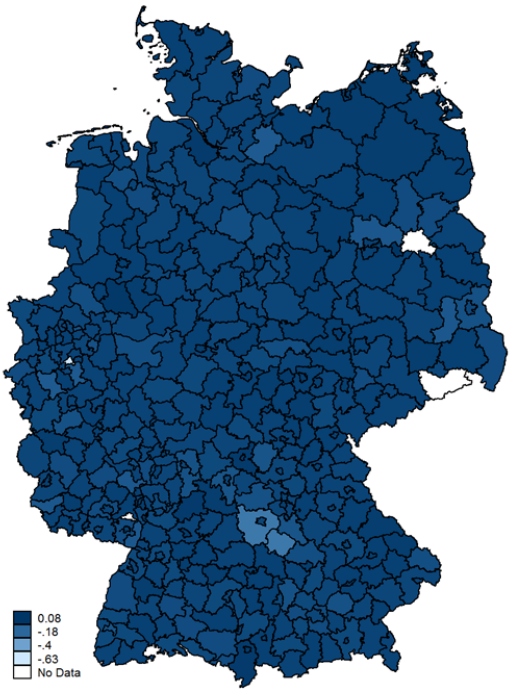
Source: LIAB, Authors' Calculations

Figure 12: Covariance between Establishment Size and Fixed Effect (η)



Source: LIAB, Authors' Calculations

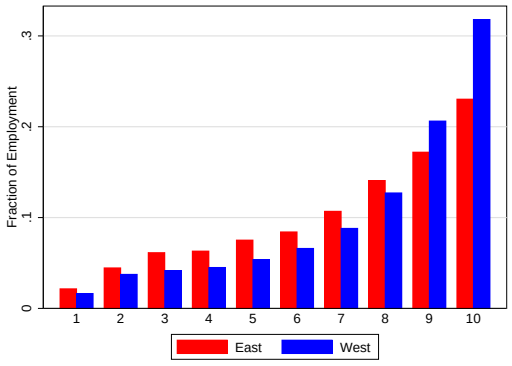
Figure 13: Covariance between Worker and Establishment Component (ρ)



Source: LIAB, Authors' Calculations

Figure 14: Employment by wage decile, 2010

(a) Males



(b) Females

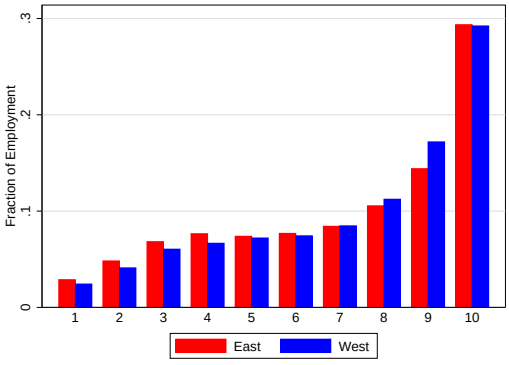
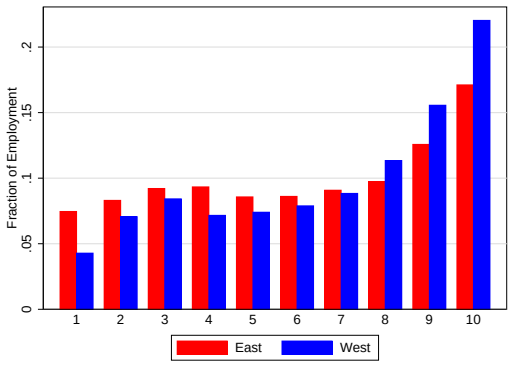
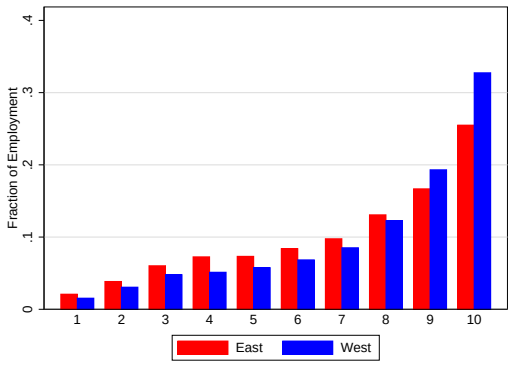


Figure 15: Employment by wage decile, 2010

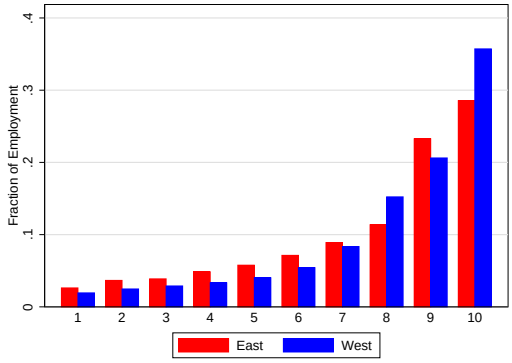
(a) Low-skilled (no vocational training)



(b) Medium-skilled (high-school plus vocational training)



(c) High-skilled (university degree)



E Additional Tables

Table 4: Gravity Equation of Worker Flows

Distance in Km	(1) All Workers	(2) Natives	(3) Foreigners	(4) Birth-Specific FE
	Point Estimates for Effect of Distance Within Region			
50-100	**−2.240*****	**−2.22*****	**−1.355*****	**−1.783*****
	(0.023)	(0.023)	(0.026)	(0.018)
101-150	**−3.096*****	**−3.057*****	**−1.763*****	**−2.506*****
	(0.022)	(0.022)	(0.026)	(0.018)
151-200	**−3.410*****	**−3.352*****	**−1.875*****	**−2.767*****
	(0.022)	(0.022)	(0.025)	(0.018)
201-250	**−3.561*****	**−3.484*****	**−1.937*****	**−2.888*****
	(0.022)	(0.022)	(0.026)	(0.018)
251-300	**−3.638*****	**−3.551*****	**−2.00*****	**−2.948*****
	(0.022)	(0.023)	(0.026)	(0.019)
301-350	**−3.704*****	**−3.596*****	**−2.05*****	**−2.996*****
	(0.022)	(0.023)	(0.026)	(0.019)
351-400	**−3.777*****	**−3.653*****	**−2.083*****	**−3.048*****
	(0.023)	(0.023)	(0.026)	(0.019)
>400	**−3.908*****	**−3.733*****	**−2.195*****	**−3.150*****
	(0.022)	(0.022)	(0.026)	(0.019)
	Point Estimates for Additional Effect of Distance Across Regions			
1-100	**−0.257*****	**−0.405*****	**0.105*****	−0.008
	(0.012)	(0.010)	(0.016)	(0.010)
101-150	**−0.250*****	**−0.425*****	**0.348*****	**0.041*****
	(0.016)	(0.018)	(0.022)	(0.014)
151-200	**−0.242*****	**−0.584*****	**0.556*****	**0.074*****
	(0.016)	(0.019)	(0.022)	(0.015)
>200	**−0.152*****	**−0.471*****	**0.502*****	**0.140*****
	(0.015)	(0.019)	(0.022)	(0.015)

Notes: robust standard errors are in parentheses. $***$ indicates significance at the 1% level. For foreigners, we report the standard error of the interaction terms – i.e. of ϕ^f and ψ^f.

Table 5: Productivity Regression

	All
$\gamma_{\text{East,k,t}}$	**.0284*** **
	(0.0117)
$\delta_{\text{born East,k}}$	**−.0283*** **
	(0.0053)
$\gamma_{\text{East,k,t}}\delta_{\text{born East,k}}$	.0006
	(0.0056)
Obs	5,172,074

$* * * = p < 0.01$. Standard errors clustered at the county-level.

Table 6: Wage Gain Regression

	Coefficient	S.E.
$d_{\text{i,t}}$	**.1199*** **	(0.0057)
$d_{\text{i,t}}\gamma_{\text{East,i,t}}$	**−.0373*** **	(0.0105)
$d_{\text{i,t}}\delta_{\text{born East,i}}$	**−.0358*** **	(0.0060)
$d_{\text{i,t}}\gamma_{\text{East,i,t}}\delta_{\text{born East,i}}$	**.0364*** **	(0.0104)
$d_{\text{i,t}}\rho_{\text{migr,EW,i,t}}$	**.1539*** **	(0.0547)
$d_{\text{i,t}}\rho_{\text{migr,EW,i,t}}\delta_{\text{born East,i}}$	**.2968*** **	(0.0577)
$d_{\text{i,t}}\rho_{\text{migr,WE,i,t}}$	**.2485*** **	(0.0491)
$d_{\text{i,t}}\rho_{\text{migr,WE,i,t}}\delta_{\text{born East,i}}$	**−.2125*** **	(0.0661)
$d_{\text{i,t}}\rho_{\text{comm,EW,i,t}}$	**−.0856** **	(0.0368)
$d_{\text{i,t}}\rho_{\text{comm,EW,i,t}}\delta_{\text{born East,i}}$	**.1241*** **	(0.0325)
$d_{\text{i,t}}\rho_{\text{comm,WE,i,t}}$	−.0094	(0.0169)
$d_{\text{i,t}}\rho_{\text{comm,WE,i,t}}\delta_{\text{born East,i}}$	−.0313	(0.0209)
Obs	6,182,842	

$* * * = p < 0.01$. Standard errors clustered at the county-level.