Can the Market Multiply and Divide?

Non-Proportional Thinking in Financial Markets

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Abstract

When pricing financial assets, rational agents should think in terms of proportional price changes, i.e., returns. However, stock price movements are often reported in dollar rather than percentage units, which may cause investors to think that news should correspond to a dollar change in price rather than a percentage change in price. Non-proportional thinking in financial markets can lead to return underreaction for high-priced stocks and overreaction for low-priced stocks. Consistent with a simple model of non-proportional thinking, we find that total volatility, idiosyncratic volatility, and absolute market beta are significantly higher for stocks with low share prices, controlling for size. To identify a causal effect of price, we show that volatility increases sharply following stock splits and drops following reverse stock splits. The economic magnitudes are large: non-proportional thinking can explain a significant portion of the "leverage effect" puzzle, in which volatility is negatively related to past returns, as well as the volatility-size and beta-size relations in the data. We also show that non-proportional thinking biases reactions to news that is itself reported in nominal rather than scaled units. Investors react to nominal earnings per share surprises, after controlling for the earnings surprise scaled by share price. The reaction to the nominal earnings surprise reverses in the long run, consistent with correction of mispricing.

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1 Introduction

Rational agents should think in terms of proportional rather than nominal price changes in financial markets. The nominal price level of any financial security has no real meaning; its price can easily be changed through stock splits or reverse splits. What matters for financial securities is returns, i.e., the proportional change in price. However, changes in the value of stocks are frequently reported in dollar units rather than or in addition to percentage returns. For example, the print version of the Wall Street Journal historically only displayed the daily dollar change in share prices and modern apps such as the Apple iPhone stock application display only the dollar change in prices as the default option. Given the emphasis on dollar changes in share prices in the financial media, we hypothesize that investors may mistakenly think that a given piece of news should correspond to a certain dollar change in price rather than a percentage change in price. In other words, investors engage in non-proportional thinking.

For example, consider two otherwise identical stocks, one trading at \$20/share and another trading at \$30/share. Investors may think the same piece of good news should correspond to a dollar increase in price for both stocks. Thinking about this news in dollar rather than return units leads to relative return underreaction for the high-priced stock at \$30/share and relative overreaction for the low-priced \$20/share stock. For a given sequence of news, non-proportional thinking would then lead to higher return volatility for low-priced stocks and lower return volatility for high-priced stocks. Similarly, non-proportional thinking may lead investors to overreact to relevant macro news for low-priced stocks, leading to higher absolute market beta for lower priced stocks. Our hypothesis is also motivated by experimental evidence in Svedsäter, Gamble, and Gärling (2007) showing that laboratory subjects report what amounts to a higher expected percentage change in price in reaction to news for hypothetical firms with lower nominal share prices. In this paper, we test whether these predictions hold in real financial markets and explore how non-proportional thinking can affect

volatility and other pricing patterns.

Consistent with the predictions from a simple model of non-proportional thinking, we find that lower nominal share price is associated higher volatility, measured in three ways: total return volatility, idiosyncratic volatility, and absolute market beta. The economic magnitudes are large: a doubling in share price corresponds to a 20-30 percent reduction in these three measures of volatility. Of course, the negative relation between volatility and nominal share prices could be caused by other factors. In particular, it is widely known in the asset pricing literature that small-cap stocks tend to have higher total volatility, idiosyncratic volatility, and market beta, possibility because small-cap stocks are fundamentally more risky. Small-cap stocks also tend to have lower nominal share prices, so the price-volatility relation in the data could be driven by size. However, we find that the negative price-volatility relation remains equally strong after introducing flexible control variables for size. Moreover, the negative relation between size and volatility flattens by more than 80% after we introduce a single control variable for nominal share price. Thus, the results suggest that non-proportional thinking may explain the size-volatility relation rather than the reverse.

Overall, we find that the negative volatility-price relation is robust and remains stable in magnitude after controlling for other potential determinants of return volatility such as volume turnover, market-to-book, leverage, and sales volatility. The results hold in the cross section and in panel regressions that control for fixed characteristics of each stock. The results hold for stocks in the recent time period and among stocks with high share prices. We also show that the results cannot be driven by historical tick-size limitations. The magnitude of the volatility-price relation declines with institutional ownership and size, suggesting that the volatility-price relation represents a form of mispricing that is weaker among stocks that are easier to arbitrage. Finally, we find that lower priced stocks exhibit greater return reactions to large market movements, and these return reactions revert in the long run. This pattern is consistent with over-reaction to news among low-priced

stocks.

While this collection of facts is consistent with non-proportional thinking, we remain concerned that an omitted factor may drive the negative relation between price and volatility. For example, low nominal share price can be the result of negative past returns, and poor past performance may directly be associated with higher volatility and risk. To better account for potential omitted factors, we conduct a regression discontinuity and event study around stock splits. Following a standard 2-for-1 stock split, the share price falls by half. While the occurrence of a split in a given quarter is unlikely to be random (e.g., firms often choose to split following good performance), the fundamentals that drive the split decision are likely to be slow-moving since most splits are pre-announced one month ahead of the split event. Our tests only require that firm fundamentals don't change dramatically after the split, relative to the day before. We find a sharp discontinuity around stock splits: the stock's return volatility, idiosyncratic volatility, and absolute market beta increase by approximately 30 percent immediately after the split. Further, the volatility remains high with a gradual monotonic decline over the course of the next six months. We further find sharp declines in volatility following reverse stock splits (e.g., when 2 shares become 1 share), in which the share price jumps up.

We also show that our results are unlikely to be explained by a change in investor base or media coverage that could accompany splits. Previous research has argued that low prices, and splits in particular, attract speculative retail investors, who could push up volatility. Along the same lines, media coverage of the firm usually increases around splits, which could contribute to volatility. We argue that these factors are unlikely to explain the change in volatility for four reasons. First, we observe an immediate jump in volatility after the split, even though the investor base is unlikely to change dramatically in a single day (we also find that institutional ownership remains approximately constant after the split). Second, the jump in volatility persists for many months, so it is unlikely to

be caused by a temporary increase in media coverage. Third, simple models of speculative investors (e.g., Brandt et al., 2009) predicts higher idiosyncratic volatility, but not necessarily overreaction to market news. However, we find a sharp increase in absolute beta following splits, which is consistent with non-proportional thinking leading to overreaction to market news for low-priced stocks. Fourth, speculation and increased media coverage should lead to increased volume turnover following the split. Instead, we observe a sharp and persistent decline in volume following splits and the opposite pattern for reverse splits. This change in volume is instead consistent with a model in which some investors naively trade a fixed number of shares for each stock. Following a split, the share float doubles, so the number of shares traded relative to the float declines after splits and rises after reverse splits.

Our empirical results so far are consistent with a simple model of non-proportional thinking in which investors react to news with a reference point for a share price in mind. Investors observe the magnitude of the news and choose a dollar reaction to the news that approximately translates to the correct percentage price change to the news if the share price equaled the reference price. This reference price could the price of a typical stock in the market or the share price just before a stock split. The fact that volatility sharply rises and then gradually declines following stock splits is further consistent with a model in which investors gradually update the reference price toward the current stock price. To explore the rate at which investors update a stock's reference price, we look at the relation between volatility and the stock's past returns over various return windows. By holding the total return over various time horizons fixed, we can vary the rate at which prices have changed. We find that the negative relation between past returns and subsequent realized volatility becomes weaker the farther back the return window is extended. In other words, a stock that has doubled in value in the past two months is significantly more volatile than a stock that doubled in value over the last year. These results suggest that investors gradually and incompletely

update reference prices toward the current price level, implying that misreaction to news should be greater for stocks that have experienced recent large absolute returns. These results also show that non-proportional thinking may contribute to the well-known "leverage effect," in which volatility is negatively related to past returns (e.g., Black, 1976; Glosten, Ravi, and Runkle, 1993). While a number of papers (e.g., Christie, 1982) argue that the negative return-volatility relation may be due to leverage (as asset values decline and debt stays approximately constant, the equity becomes more leveraged and therefore more risky), other research (e.g., Figlewski and Wang, 2001) cast doubt on the leverage explanation for the leverage effect. We show that non-proportional thinking offers a compelling alternative explanation for this empirical pattern: as prices decline, volatility increases because investors react to news in dollar units based upon a higher reference price and thereby overreact in percentage units.

In the final part of the paper, we explore a related prediction relating to non-proportional thinking. We hypothesize that investors may neglect to scale news that is itself reported in nominal rather than the appropriate proportional units. In the case of firm earnings announcements, the best measure of the news is likely to be the nominal value of the earnings surprise, *scaled* by the firm's price just before the news is released. For example, earnings news in which a firm beats analyst expectations by 5 cents per share is a greater positive surprise if the firm's share price is \$20/share than if the firm's share price is \$30/share. However, investors may mistakenly focus on the nominal earnings surprise of 5 cents per share because that is the value that is most commonly reported in the financial press. We find that investors react strongly to nominal earnings per share surprises, after controlling for the earnings surprise scaled by share price. If prices move toward fundamentals in the long run, we expect the initial return reaction to the nominal earnings surprise to reverse over time as the mispricing is corrected. Because investors react to the nominal earnings surprise, they also underreact to the scaled earnings surprise, so we expect the scaled earnings surprise to

predict future drift in prices. Consistent with these predictions, we find that returns drift in the direction of the scaled earnings surprise and against the direction of the nominal earnings surprise in the long run.

Our results contribute to the literature in four ways. First, we document a new way in which thinking about value in the wrong units (i.e., dollars instead of percents) can affect behavior and prices in financial markets. In related work, Shue and Townsend (2017) show that the tendency to think about executive option grants in terms of the *number* of options granted rather than the Black-Scholes value contributed to the dramatic rise in CEO pay starting in the late 1990s. Birru and Wang (2015, 2016) show that nominal price illusion causes investors to mistakenly believe that low-priced stocks have more "room to grow." Finally, our research is related to Baker and Wurgler(2004ba,b), Baker, Nagel, and Wurgler (2006), and Hartzmark and Solomon (2017, 2018), which show that investors fail to incorporate dividend payouts when evaluating total returns.[1]

Second, we contribute to the literature on proportional (or relative) thinking (e.g., Thaler, 1980; Tversky and Kahneman, 1981; Pratt, Wise, and Zeckhauser, 1979; Azar, 2007; and Bushong, Rabin, and Schwartzstein, 2015). This literature has largely focused on instances in which households think in proportional units when they should think in levels. For example, consumers may be willing to travel to a different store to get a $5 discount on a cheap product, but not for the same $5 discount on an expensive product. These consumers incorrectly focus on the $5 discount as a proportion of the good's retail price. In contrast, we explore a financial markets setting in which investors should think in proportional units, and yet they sometimes focus on levels and fail to scale by price.

Third, our findings shed light on the potential origins of volatility in financial markets. Since Shiller (1981), academics have explored the question of what factors determine volatility and risk. Our results suggest that non-proportional thinking may be an important part of the explanation and

[1]Our research is similar in spirit to the money illusion literature, which shows that households confuse the nominal and real value of money (e.g., Fisher, 1928; Benartzi and Thaler, 1995). In this paper, we show that investors focus on nominal units instead of proportional units.

that well-known asset pricing facts such as the leverage effect and the size-volatility and size-beta relations in the data can be reinterpreted through the lens of non-proportional thinking.

Fourth, we offer a new explanation of over- and underreaction to news and subsequent drift patterns in asset prices. The existing literature in behavioral finance has mainly viewed over- and underreaction to news through the lens of limited attention (e.g., Hirshleifer and Teoh, 2003), incorrect weighting of news relative to one's priors (e.g., Barberis, Shleifer, and Vishny, 1998), or mistaken beliefs regarding extrapolation and reversals (e.g., Hong and Stein, 1999). Non-proportional thinking offers a complementary explanation: over and under-reaction to news and consequent drift can also be caused by investors thinking about asset values and news in the wrong units.

2 A simple model

Consider a stock with current share price P. Let P_0 be the reference price for the stock in the minds of investors. P_0 could be the price of a typical stock in the stock market, the price of the stock in a previous time period, or the price of the stock prior to a stock split. Suppose news Z is released that contains information relevant for the valuation of the stock. If markets are fully efficient and rational, the release of news Z should imply a δ fractional change in the price of the stock, i.e., δ is the rational return reaction to the news. However, non-proportional thinking may lead investors to apply a heuristic and think that news Z should move prices by a nominal amount X. The dollar movement of X is such that it roughly equals the rational return reaction if the stock's price equaled the reference price P_o, i.e., $X = \delta P_0$. Thus, non-proportional thinking implies the return reaction to news Z is $\frac{X}{P} = \frac{\delta P_0}{P}$. If we allow investors to partially engage in non-proportional thinking, the

return reaction the news Z can be expressed as:

$$r = \theta \frac{\delta P_0}{P} + (1 - \theta)\delta \tag{1}$$

$\theta \in [0, 1]$ measures the extent to which investors engage in non-proportional thinking. If investors are fully rational, $\theta = 0$, and the return reaction $r = \delta$. If investors fully suffer from non-proportional thinking, $\theta = 1$, and the return reaction to the news behaves as though the stock had an reference price P_0, leading to $r = \delta P_0/P$.

This simple framework delivers a number of testable predictions. First, whether investors under- or overreact to news will depend on the ratio of the reference price to the current price: P_0/P. If the stock's price is high relative to the reference price, then investors will underreact to the news, leading to $|r| < |\delta|$. If the stock's price is low, then investors will overreact to the news, leading to $|r| > |\delta|$. Second, this initial under- or overreaction represents mispricing, which implies drift patterns if we believe that prices correctly incorporate news Z in the long run. Specifically, if the stock's price is high relative to the reference price, we expect continued drift to correct for the initial underreaction. If the stock's price is low, we expect a long run reversal to correct for the initial overreaction. Third, for a given sequence of news over time, we expect the return volatility of the a stock to be higher when the stock's price is lower related to the reference price. The higher volatility arises from the return overreaction to each piece of news. Finally, we expect the absolute value of the market beta of a stock to be higher if its price is lower relative to the reference price, because the return for the stock will overreact to market-level news. Note that non-proportional thinking amplifies the absolute value of beta rather than beta. For example, if a stock's true beta is negative and the market news is positive, the stock's share price should drop and the share price should drop by more if investors overreact to the news.

To test these predictions, we examine cases in which P_0/P is likely to be low or high. First, P_0 may be a simple constant representing a typical share price in the market, e.g. \$25/share. If so, P_0/P is high for stocks with low nominal share price and low for stocks with high nominal share price. Second, P_0 may be the price of a stock just prior to a stock split event. After a 2-for-1 stock split $P_0/P = 0.5$, so we expect return overreaction, leading to higher return volatility and higher absolute beta. Finally, investors may think of each stock's reference price as its price at some period in the past. Therefore, stocks that have decreased in value may be more likely to have $P_0/P > 1$, so we would again expect overreaction to news, leading to higher return volatility and higher absolute beta.

To summarize, when prices are low (high) relative to a reference price, we expect:

1. Initial overreaction to news (initial underreaction to news)
2. Long run reversal (long run drift)
3. Higher total volatility and idiosyncratic (lower total volatility and idiosyncratic volatility)
4. Higher absolute beta (lower absolute beta).

Because we don't always observe the arrival of specific pieces of news, we will focus our baseline analysis on the third and fourth predictions, which can be tested even if the news itself is not observed. In supplemental analysis, we attempt to isolate large news shocks and test for initial under- or over-reaction and subsequent corrections through either long-run drift or reversals. Like many other behavioral models with a reference price, we also face the limitation that we do not directly observe P_0. Therefore, we present baseline tests for the simple case in which P_0 is an unobserved constant representing a typical stock in the market. In later tests, we look at cases in which the reference price may change over time.

We also hypothesize that non-proportional thinking may lead investors to exhibit biased reactions to news that is itself reported in nominal rather than scaled units. We consider the case of earnings surprises, which are usually reported by the financial media as the nominal surprise (the raw difference between actual earnings and analyst consensus forecasts) rather than the scaled surprise (the nominal surprise divided by the share price just before the news is released). If investors are fully rational, they should only react to the the scaled surprise. However, if investors fixate on the nominal surprise, we predict that short run returns will also react to the nominal surprise. If prices correctly incorporate real news in the long run, then we expect that the long run return reaction will only depend on the scaled surprise and not on the nominal surprise.

3 Data

The sample period for our baseline analysis runs from 1926–2016. However, the beginning of the sample period for each empirical test varies depending on when coverage begins for supplementary data sources used in the analysis. We also show that our results are robust across different time periods. Summary statistics of our data can be found in Table 1.

3.1 Stock Market Data

We obtain stock market data from CRSP, which offers information relating to returns, nominal share prices, stock splits, daily high and low, volume, and market capitalization. Data on the market excess return, risk-free rate, SMB, HML, UMD, and size category cutoffs come from the Ken French Data Library. We measure the return for day t as the return from market close on day $t-1$ to market close on day t.

The sample is restricted to stocks that are publicly traded on the NYSE, American Stock Exchange, or NASDAQ. We also restrict the sample to assets that are classified as common equity

(CRSP share codes 10 and 11). To reduce the influence of outlier share prices, we exclude the top and bottom 1% of the sample in each year-month period in terms of 1-month lagged shared price from the analysis.

In our baseline tests, we measure firm i's total return volatility in month t as vol_{it}, equal to the annualized standard deviation of daily returns within each calendar month. We require at least 15 trading days in each month to have non-missing return data in CRSP to compute total volatility. We drop observations with zero monthly total volatility, i.e., stock-months in CRSP where the stock price is exactly the same for all trading days in a month. We also apply these sample restrictions to our measures of $beta_{it}$ and $ivol_{it}$. We measure each firm's monthly market beta as $beta_{it}$, equal to the covariance between daily firm excess returns and market excess returns divided by the variance of daily market excess returns within each calendar month. We measure each firm's idiosyncratic volatility as $ivol_{it}$, equal to the standard deviation of the firm's daily abnormal returns, where abnormal return is defined as the firm return minus $beta_{it}$ multiplied by the market return.

Our baseline tests use observations at the firm-month level. To control for each firm's market capitalization, we match each firm's size at the end the previous month to size categories during the same time period defined using the NYSE size cutoff data from the Ken French Data Library. To classify firms by nominal share price, past returns, etc., we always use past information.

3.2 Firm Accounting Data

We use accounting data to control for firm characteristics. These data come from the COMPUSTAT Quarterly Fundamentals file. Coverage begins in 1961. The primary control variables we construct are sales volatility, market-to-book ratio, and leverage. We define sales volatility as the standard deviation of year-over-year quarterly sales growth over the previous four quarters. That is, for each quarter, we compute the growth of sales (sale) over the year ago quarter. We consider year-over-year

sales growth to be undefined if sales were reported to negative in one of the two quarters. We then compute the standard deviation of year-over-year sales growth over the previous four quarters. In cases where data are missing for some of the quarters, we compute the standard deviation based on the non-missing quarters, assuming there are more than one. We define the market-to-book ratio as market capitalization (csho*prcc_f) plus the book value of assets (at) less shareholder equity (seq), all divided by the book value of assets (at). We define leverage as the ratio of short-term and long-term debt (dlc+dltt) to the book value of assets (at).

3.3 Institutional Ownership

Data on institutional ownership come from the Thomson Institutional Manager Holdings file, which is based on quarterly 13f filings. Coverage begins in 1980. Each quarter, we sum up the number of shares of each stock held by 13f filers and divide by shares outstanding to get institutional ownership percentages.

3.4 Option-Implied Volatility

Data on option-implied volatility come from OptionMetrics, which computes implied volatility over different horizons based on traded options of varying maturities. Coverage begins in 1995.

3.5 Earnings Announcements

We use the I/B/E/S detail history file for data on analyst forecasts as well as the values and dates of earnings announcements. Coverage begins in 1983. The sample is restricted to earnings announced on calendar dates when the market is open. Day t refers to the date of the earnings announcement listed in the I/B/E/S file. We examine the quarterly forecasts of earnings per share.

The two key variables in our analysis are the nominal surprise for a given earnings announcement

and the scaled surprise.[2] Broadly defined, the earnings surprise is the difference between announced earnings and the expectations of investors prior to the announcement. We follow a commonly-used method in the accounting and finance literature and measure expectations using analyst forecasts prior to announcement.[3]

Following the methodology in Hartzmark and Shue (2018) and related studies of investor reactions to earnings announcements, we take each analyst's most recent forecast, thereby limiting the sample to one forecast per analyst, and then take the median of this number within a certain time window for each firm's earnings announcement. In our base specification, we take all analyst forecasts made between two and thirty days prior to the announcement of earnings. We choose thirty days to avoid stale information and still retain a large sample of firms with analyst coverage. Our results remain qualitatively similar if we use alternative windows of 15 or 45 days prior to announcement.

We define the nominal earnings surprise as the dollar difference between actual earnings and the median analyst forecast:

$$nominal\,surprise_{it} = actual\,earnings_{it} - median\,estimate_{i,[t-30,t-2]}. \tag{2}$$

We define the scaled earnings surprise as the nominal earnings surprise divided by the share price of the firm three trading days prior to the announcement:

$$scaled\,surprise_{it} = \frac{\left(actual\,earnings_{it} - median\,estimate_{i,[t-30,t-2]}\right)}{price_{i,t-3}}. \tag{3}$$

[2]We follow the literature on earnings announcements in characterizing earnings news as the surprise relative to expectations. We focus on surprise rather than levels because whether a given level of earnings is good or bad news depends on the level relative to investor expectations. Moreover, the financial press typically reports earnings announcement news in terms of how much earnings beat or missed forecasts. Therefore, the earnings surprise is likely to be the measure of earnings news that is most salient to investors.

[3]Analysts are professionals who are paid to forecast future earnings. While there is some debate about how unbiased analysts are (e.g., Hong and Kubik, 2003 and So, 2013), our tests only require that such a bias is not correlated with the difference between the nominal and scaled earnings surprises.

Most of the existing academic literature exploring return reactions to earnings surprises focus on the scaled surprise measure. Scaling by price accounts for the fact that a given level of earnings surprise implies different magnitudes of news shocks depending on the price per share. However, many media outlets report earnings surprises as the nominal (unscaled) difference between actual earnings and analyst forecasts, and investors may mistakenly pay attention to the nominal surprise. Therefore we compare how markets react to the nominal and scaled surprise measures. To reduce the influence of outliers, which may be relatively more problematic for the scaled surprise measure because it is measured as a ratio, we measure both the scaled and nominal surprise as percentile rank variables within each year-quarter.

We then construct measures of returns over various event windows around the earnings announcement. We measure the direct short-term reaction to the earnings announcement as the firm's abnormal return in the window $[t-1, t+1]$, i.e., the firm's return from market close on day $t-2$ to market close on $t+1$, minus the market return over the same period. We can also test for long-run drift and reversals by examining the firm's abnormal returns over longer event windows.

4 Results

4.1 Baseline Results

4.1.1 Prices, Total Volatility, Idiosyncratic Volatility, and Market Beta

We begin by exploring how return volatility varies with share price. Using data at the stock-month level, we estimate the following regression:

$$log\left(vol_{it}\right) = \beta_0 + \beta_1 log\left(price_{i,t-1}\right) + controls + \tau_t + \epsilon_{it}. \tag{4}$$

We regress each stock i's volatility in month t on the stock's nominal share price at the end of the previous month, calendar-year-month fixed effects, and additional control variables. Volatility can represent total volatility, idiosyncratic volatility, or absolute market beta. We measure volatility and nominal share price in logarithm form because a simple model of non-proportional thinking with a constant reference price implies that volatility should change proportionately with the share price. Control variables can include the log of the firm's size (measured as total market equity) in the previous month or indicator variables for 20 size categories based on the market capitalization of the stock relative to the size breakpoints for each period from the Ken French Data Library. The sample excludes observations with extreme lagged prices (the bottom and top 1% of prices each month). To account for correlated observations, we double-cluster standard errors by stock and year-month.

We present our baseline results in Table 2. Consistent with the predictions from a simple non-proportional thinking model, we find that higher nominal share price is associated with lower total return volatility. The negative coefficient on price remains highly significant and stable in magnitude as we introduce control variables for size (either as the log of lagged market capitalization or with 20 size category indicators based on lagged market capitalization). The results hold in the cross section (with time fixed effects and without stock fixed effects) and in the time-series (with both time and stock fixed effects). The economic magnitudes are also quite large. With the full set of control variables in column (4), a doubling in share price is associated with a 34% decline in volatility in the cross section and a 27% decline in volatility in the time-series (i.e., within stock over time).

In Table 3, we find very similar empirical patterns after replacing the dependent variable with idiosyncratic volatility and absolute market beta. The economic magnitudes are again large. With the full set of control variables in column (4), a doubling in share price is associated with 35% decline in idiosyncratic volatility and a 31% decline in absolute market beta. As discussed previously, we

use the absolute value of market beta instead of the raw level of beta because non-proportional thinking should lead to overreaction to market news for low priced stocks, resulting in larger betas for positive-beta stocks and more negative betas for negative-beta stocks. However, one may be concerned that stocks with measured betas in the negative range may simply be stocks where beta is measured with error. To show that this does not drive our results, Appendix Table A1 restricts the sample to observations with positive estimated market betas. We continue to find similar results in this subsample.

4.1.2 Size and risk

The empirical patterns shown so far are consistent with non-proportional thinking. However, share prices are not randomly assigned, so an omitted factor could determine both price and volatility. Our results can already reject one key alternative explanation involving size: It is well-known in the asset pricing literature that small-cap stocks, i.e., stocks with low market capitalization, tend to have higher return volatility, idiosyncratic volatility, and market beta. The size-volatility relations in the data may even be viewed by some as unsurprising, given that it seems plausible that small stocks may be fundamentally more risky. Since small-cap stocks also tend to have low nominal share prices, size may simultaneously determine share price and volatility.

However, we showed in Tables 2 and 3 that the coefficient on lagged share price remains stable in magnitude and significant after controlling for the logarithm of lagged market capitalization or after controlling non-parametrically for size with 20 size category indicators. We also see in columns (2) and (3) of each table that, while size negatively predicts volatility if we do not control for price, the size-volatility relation flattens toward zero once we control for lagged nominal share price.

As an alternative way to illustrate these results, we note that size is equal to the product of price and the number of shares. Therefore, we can examine whether the negative volatility-size relation is

driven by price or the number of shares, by regressing volatility on lagged price and lagged number of shares. Appendix Table A2 shows that the majority of the negative volatility-size relation is driven by price.[4]

We explore the relation between size and volatility in more detail in Figure 1. Panel A shows the coefficients from a regression of log volatility on 20 size category indicators (the largest size category is the omitted one), after controlling for year-month fixed effects. As expected, we find a strong negative relation between size and volatility. In Panel B, we report the same set of coefficients for the 20 size indicators, after adding a single control variable for the log of the lagged nominal share price to the regression. We see that the relation between size and volatility flattens dramatically. In the range between size categories 4 and 20, size continues to negatively predict volatility. However, the magnitude of the slope shrinks by more than 80 percent. Thus, the results suggest that non-proportional thinking may explain a significant portion of the well-known size-volatility and size-beta empirical relation, rather than the reverse.

4.1.3 Robustness and Heterogeneity

Additional Controls

We have already shown that our results are robust to controlling for size. In Table 4, we repeat our baseline analysis including additional controls that could determine volatility. In column (1), we begin by again including minimal controls, as in column (1) of Table 2 Panel A. In column (2), we control for size even more thoroughly than before by controlling for both the logarithm of lagged market capitalization as well as the 20 size category indicator variables and all interactions between the two. This set of flexible control variables addresses the possibility that the effect of price that we are estimating when we control for size category indicators is driven by within-size-category variation

[4]Idiosyncratic volatility is significantly related to the number of shares, but the magnitude of the correlation is small. Absolute beta is related to the number of shares, controlling for price, but in the opposite direction.

in size that is correlated with price. Using these flexible size controls, we continue to estimate a similar coefficient on price. In column (3), we add an additional control for sales volatility. This is measured as the standard deviation of year-over-year quarterly sales growth in the four most recently completed quarters. In column (4) we include a control for the stock's market-to-book ratio. In column (5) we control for volume turnover, defined as the volume in the previous month divided by shares outstanding. In column (6) we control for leverage, defined as debt (current liabilities + long term debt) divided by the book value of assets. While many of these controls load strongly, suggesting that they are indeed related to volatility, their inclusion has minimal effect on the estimated price coefficient. Therefore our results do not, for example, appear to be driven by low-priced stocks having higher fundamental sales volatility or higher trading volume.

Tick Size

A potential concern with our findings is that the negative price-volatility relation may be driven by tick-size limitations. A tick size is the minimum price movement for a financial security. Tick size as a fraction of share price is larger for stocks with lower nominal share price, which may artificially inflate the measured volatility of low-priced stocks.

In Figure 2, we explore the shape of the price-volatility relation in more detail, as a way of ruling out the possibility that tick-size limitations drive our results. We plot the coefficients of a regression of volatility on 20 equally spaced bins in nominal share price, controlling for 20 size category bins, and time fixed effects. All plotted coefficients measure the difference in volatility within each share price bin relative to the omitted bin of 20 (the largest share price). We observe a strong monotonic negative relation between volatility and share price. The negative relation holds even in the range of very high nominal share price bins, when tick size limits should have minimal impact. The strong monotonic pattern in this figure also shows that our findings of a negative relation between volatility

and nominal share price are unlikely to be driven by a few outlier observations. Rather, the negative relation holds between any two adjacent nominal price bins.

As another way of addressing tick-size issues, we create an alternative measure of volatility that takes tick-size into account. Specifically, on a day where a stock's price increases from the previous closing price, we subtract half a tick from that day's closing price. On days where a stock's price decreases, we add half a tick to that days closing price. These artificial prices round to the actual prices, given tick-size constraints, but compress returns, and therefore volatility, as much as possible.[5] Thus, computing volatility based on the artificial prices gives a *lower bound* of what true volatility would have been absent tick-size constraints. We expect the difference between actual volatility and this lower-bound to be greatest for low-priced stock. If tick-size effects drive our results, the price-volatility relation should disappear when we use this conservative alternative volatility measure. However, Table 5 Panel A shows that, in fact, we continue to find similar results with this alternative volatility measure.

Zero Leverage Subsample

Although we control for leverage in Table 4, one may still be concerned that our findings are driven by a negative relation between priced and leverage, and a positive effect of leverage on volatility. To further rule out this possibility, in Table 5 Panel B, we limit the sample to include only stocks associated with firms with zero debt (current liabilities + long term debt) reported in their most recent quarterly financial statements. We continue to find similar results in this subsample. Note that these results also point away from leverage as a complete explanation for the "leverage effect," which we discuss in later sections.[6]

[5]Tick size on NYSE, AMEX, and NASDAQ was 1/16 prior to 2001 when it became 0.01.

[6]We acknowledge that even firms with zero debt may still have operating leverage, which may increase the risk of equity. It is not the goal of this paper to show that leverage cannot contribute to a leverage effect. Rather, we will argue in later sections that the leverage effect can also be explained by non-proportional thinking.

Institutional Ownership

Institutional investors may be more sophisticated than non-institutional investors and thus less likely to suffer from non-proportional thinking. If so, the price-volatility relation should be weaker for stocks with higher institutional ownership. To explore this, we repeat our baseline analysis, allowing the effect of price to interact with institutional ownership. As is standard in the literature, we define institutional ownership as the percent of outstanding shares reported to be held by institutions in quarterly 13f filings.[7] The results are shown in Table 6. Consistent with the idea that institutional investors are more sophisticated, we estimate a positive coefficient on the interaction term. Thus, volatility declines with price less when a stock has higher institutional ownership. The magnitude of the coefficient implies that as a stock moves from 0% institutional ownership to 100%, the effect of price on volatility is reduced by approximately 44%.

This analysis also addresses another potential alternative explanation for our results, which is that lower-priced stocks may be held by unsophisticated noise traders or speculators who generate high volatility for reasons unrelated to non-proportional thinking. Table 6 shows that, indeed, stocks are more volatile when held be more unsophisticated investors, as we estimate a negative coefficient on the uninteracted institutional ownership variable. However, even controlling for this, the effect of price remains. That is, even among stocks with the same institutional ownership, lower-priced stocks are still more volatile.

Size Subsamples

While we have controlled for size to ensure that the estimated relation between price and volatility is not actually a size-volatility relation, we have not examined how the price-volatility relation

[7] The institutional ownership variable is updated quarterly, while our observations are at the monthly level. As before, we double cluster standard errors by stock as well as year-month. The stock clustering should address the mechanical serial correlation in institutional ownership induced by the quarterly updating (as well as any other source of serial correlation in the error term of a given stock over time).

varies with size. In Table 7, we repeat our baseline analysis in 20 subsamples based on our 20 size categories. These size category bins come from Ken French's ME Breakpoints file. The breakpoints for a given month are based on the size distribution of stocks traded on the New York Stock Exchange. In particular, each group corresponds to every fifth percentile. However, observations in our data are not equally distributed across the size categories, because our sample includes all stocks traded on the NYSE, AMEX, and NASDAQ exchanges.

As can be seen, our main finding is not merely a "micro-cap phenomenon" or even a "small-cap phenomenon." The negative relation between price and volatility continues to hold even among stocks in the top 5th percentile of the NYSE size distribution. Not surprisingly, though, the magnitude of the volatility-price relation does decline with size, consistent with mispricing being less prevalent for large cap stocks which may suffer less from limits to arbitrage.

Time Period Subsamples

Finally, in Table 8, we explore how the price-volatility relation has changed over time by repeating our baseline analysis in different time period subsamples corresponding to each decade since the 1920s, up until the end of our sample period in 2016. We find that the coefficient is relatively stable across these different time periods and there are no secular trends. Thus, it does not seem that the relation has disappeared in recent year or is weakening over time. This also serves as additional evidence that tick-size limitations do not drive our results, because tick sizes have declined over time.

4.1.4 Short Run Under- and Overreaction and Long Run Correction

For a given news shock, our simple model predicts that stock returns for low priced stocks will overreact in the short run, and reverse in the long run as the mispricing is corrected. We also

predict that high priced stocks will underreact to the news in the short run, and then drift in the direction of the news in the long run as the mispricing is corrected.

We test these predictions using market-level news shocks. For the same large market movement in a given month, we expect that higher priced stocks will move less in the direction of the shock in the short run, and drift more in the direction of the market shock in the long run, all else equal. Thus, we estimate the following regression:

$$r_{i,[t+a,t+b]} = \beta_1 log(price_{i,t-1}) \times r_{mkt,t} + controls + \epsilon_{it}$$

We limit the sample to observations in which the absolute market return in month t exceeds 10%. We regress firm stock returns over various time horizons on the interaction between market returns in month t and the stock's share price in month $t-1$. We expect β_1 to be negative in the time interval of (and shortly after) the market news shock, and we expect β_1 to be positive in the time interval further away from the market news shock. To verify that differences in return reactions to market shocks are due to price rather than size (which is correlated with price), we also control for the interaction between the market return in month t and 20 size category indicators. In an ideal test, we would isolate periods in which there was major market news in month t and no news in the months thereafter. In that case, we could attribute firm returns over the long run as continued drift or reversal with respect to the market news released in month t. In reality, market news shocks arrive continuously and may be serially correlated. Therefore, we also control for the interaction between future market movements (over the same horizon as the dependent variable) and share price in $t-1$ and size categories in $t-1$.

The results are shown in Table 9. As predicted by the model, we find that β_1 is negative for short run horizons. In other words, higher priced stocks move significantly less in the direction of

the market return shock in the short run (in the month of the market news shock, as well as in the 2 months after). Starting at around month 3 after the large market new shock, the mispricing begins to correct (β_1 becomes positive), with a significant correction in the period from months 7 to 9 after the shock.

4.1.5 Past returns

In this section, we explore the relation between a stock's volatility and past returns. If a stock has experienced negative past returns, then it's current share price is more likely to be low relative to the reference price in the minds of investors. Therefore, we expect a negative relation between past returns and volatility. Examining past returns over various windows also allows us to see if the evidence is consistent with a model in which investors use a stock's past share price as a reference price. To explore the rate at which investors update a stock's reference price level, we look at the relation between volatility and the stock's past returns over various return windows. By holding fixed the total return over various time horizons, we can vary the rate at which prices have changed.

Table 10 shows the regression results. We estimate the following regression:

$$log\left(vol_{i,t}\right) = \beta_0 + \beta_1 r_{i,[t-x,t-1]} + \tau_t + \epsilon_{it}, \tag{5}$$

where $r^i_{t-x,t-1}$ represents past returns over the past 2, 4, 6, 8, 10, and 12 month windows. Consistent with non-proportional thinking, we find a strong negative relation between past returns and volatility. We also find that the negative relation between past returns and subsequent realized volatility becomes weaker the farther back the return window is extended. In other words, a stock that has doubled in the last two months is significantly more volatile than a stock that doubled in value over the last six months, which is in turn more volatile than a stock that has doubled in value over the the last year. These results suggest that investors gradually and incompletely update

reference prices toward the current share price over time.

These results also show that non-proportional thinking may contribute to the well-known "leverage effect," in which volatility is negatively related to past returns (e.g., Black, 1976). While a number of papers including Christie (1982) argue that the negative relation between returns and volatility may be due to leverage (as asset values decline and debt stays approximately constant, the equity becomes more leveraged and therefore more risky), other research (e.g., Figlewski and Wang, 2001) cast doubt on the leverage explanation for the leverage effect. We show that non-proportional thinking offers an alternative explanation for this empirical pattern: as prices decline, volatility increases because investors react to news in dollar units based upon a higher reference price and thereby overreact in percentage units.

4.2 Stock splits

Despite the fact that we have controlled by many observable factors that could affect volatility, it remains possible that omitted variables may drive the negative relation between price and volatility. To better account for potential omitted factors, we conduct a regression discontinuity and event study around stock splits. While stock splits are not completely randomly assigned across firms, the fundamentals of each firm are unlikely to change exactly on the day of each stock split. Therefore, we can credibly attribute changes in volatility immediately after the split to the change in share price.

4.2.1 Daily Analysis

We begin with granular daily stock data to estimate a regression discontinuity around the date of the stock split. For the regression discontinuity, we change our measure of volatility from the standard deviation of daily returns within each calendar month to the scaled intraday price range, defined

as the difference between the intraday high and low, scaled by the share price at market close on the previous day. We omit the actual day of the split from the analysis, as it is not clear whether the split takes place at the beginning of the trading day or the end. [8]We begin by considering any positive stock split in which one old share is converted into two or more new shares (the results remain similar if we restrict the definition of an event to 2-for-1 stock splits, the most common type of split).

In Figure 3 (regression results in Table 11), we find that the scaled intraday price range increases by 0.015 immediately after the split, an approximate 40 percent increase relative to the pre-split scaled intraday price range. The jump in intraday price range persists with a small decay over the next 40 trading days. These magnitudes are very similar regardless of the exact regression discontinuity method that we adopt. We fit local linear or local quadratic regressions on either side of the regression discontinuity, using either a triangular or Epanechnikov kernel, and the rule-of-thumb optimal bandwidth.

4.2.2 Monthly Analysis

We also conduct event studies examining changes in total volatility, idiosyncratic volatility, and absolute beta around stock splits using monthly data. To explore how these measures of volatility change after splits, we estimate the following regression:

$$log\left(vol_{it}\right) = \beta_0 + \beta_1 Post_{it} + \tau_t + \nu_i + \epsilon_{it}. \tag{6}$$

Observations at the stock-month level, and the sample is limited to the six months before and after a split.[9] We again consider any positive stock split in which one old share is converted into two or

[8]In principle, one could also use intraday trading data from TAQ to address this question, but those data are only available for more recent years and we see no reason that using such data would lead to different conclusions.

[9]We limit the sample to splits that are neither preceded by another split in the previous 12 months, nor followed by another split in the subsequent 12 months, so that our estimation windows do not overlap with other splits. The

more new shares. The coefficient of interest is β_1, which measures the difference in volatility in the six months after the split relative to the six months before. If the drop in share prices following a stock split leads to an increase in volatility, we expect that $B_1 > 0$. To examine how volatility varies with event time in greater detail, we also present results in which the $Post_{it}$ indicator is replaced with event-month indicators. In all specifications, we control for year-month and stock fixed effects and double cluster standard errors by year-month and stock.

We find that volatility rises significantly after stock splits. Table 12 shows that total volatility, idiosyncratic volatility, and absolute beta increase by approximately 20 percent in the 6-month period after the split relative to the 6-month period before the split. If we introduce event month indicators for the post-split period as in the even-numbered columns in Table 12, we find that volatility sharply increases in the first month after the stock split and remains high relative to the pre-period for the next 6 months. There is also evidence of a monotonic decay over time: the initial jump in volatility in the first month after the stock split falls by approximately 21% over the next 6 months.

In Figure 4 Panel A, we explore volatility around stock splits in more detail. We plot the coefficients for each month in event time, relative to the omitted category of 6 months prior to the split. We omit the split month from these figures, as split months contain both pre-split and post-split days. We find that there is a slight pre-trend in that volatility rises in the six months leading up to the split. This pretend is consistent with the view that splits are not entirely random. Firms choose to engage in stock splits following periods of good performance, which may coincide with small increases in volatility. However the direction and magnitude of the pre-trend in volatility cannot explain the sudden large jump in volatility after the split, nor the slow monotonic decay in volatility over the next 6 months.

same sample restrict was applied to the earlier daily analysis.

In Figure 4 Panel B, we similarly explore patterns of volatility following reverse stock splits, defined as events in which two more stocks are converted to one stock. We expect volatility to decline following reverse stock splits because the nominal share price increases by two-fold or more following the reverse split. Consistent with these predictions, we find that volatility drops by more than 20 percent in the month following reverse splits and the drop remains persistent over the next 6 months. As with positive splits (which we refer to as "splits" for short), we observe a positive pre-trend in volatility in the months leading up to the split. However, the direction and small absolute magnitude of the pre-trend in volatility cannot explain the sudden and persistent drop in volatility following the reverse split.

4.2.3 Addressing Alternative Explanations

A potential alternative explanation for our results is that splits, and low share prices in general, may draw a different investor base that is more speculative and retail-dominated, which may directly push up volume. A change to the investor base is unlikely to explain our results for four reasons. First, we observe an immediate jump in volatility after the split, even though the investor base is unlikely to change dramatically in a single day. Second, simple models of speculative investors predict higher idiosyncratic volatility Brandt et al. (2009), but not necessarily overreaction to market news. However, we find a large increase in absolute beta following splits in Table 12, which is consistent with non-proportional thinking leading to overreaction to market news for low-priced stocks. Third, speculation should lead to increased volume turnover (defined as number of shares traded divided by total share float) following the split. Instead, we find in Figure 5 that there is a sharp and persistent decline in volume turnover following splits and the opposite pattern for reverse splits. This change in volume is instead consistent with the view that investors naively tend to trade a fixed number of shares for each stock, e.g., 100 shares. Following a split, the share float doubles, so the number of

shares traded relative to the float will decline after splits and rise after reverse splits. Fourth, we can directly check for changes in investor base after splits. We compare institutional ownership before a split (based on the last observed 13f filing leading up to the split) and after a split (based on the first observed 13f filing following the split). In Table 13, we find that institutional ownership declines very slightly (from 47.3% to 46.3%) and the decline is not statistically significant. Moreover, it seems implausible that a 1% decline in institutional ownership would account for a 20% or more increase in volatility.

Another potential alternative explanation is that splits draw increased media attention which may lead to increased volatility. We find this explanation implausible because the change in volatility after a split persists for many months, so it is unlikely to be caused by a temporary increase in media coverage. Further, investor attention should also increase following reverse splits which also receive significant media coverage, and yet we find in Figure 4 Panel B that volatility declines following these reverse stock splits, consistent with a non-proportional thinking model.

One may also be concerned that splits are timed in way that coincides with fundamental changes in firm volatility. Again, we argue that it is unlikely that firm fundamentals can change quickly over the course of a single day after the split. We also directly check for changes in fundamental volatility. In Table 13, we compare mean sales volatility before a split (based on the the last four quarters leading up to the split) and after a split (based on the first four quarters following the split). As can be seen, mean sales volatility is very similar before and after a split, and the difference is not statistically significant.

Finally, one may be concerned that the results relating to splits are driven by a handful of small cap stocks. In Appendix Figures A1 and A2, we show that similar empirical patterns exist for intraday price range, total volatility, idiosyncratic volatility, absolute beta, and volume turnover for a subsample restricted to large cap stocks in size categories 10 through 20, according to the Fama

French NYSE size cutoffs.

4.2.4 Implied Volatility

Given the above findings, we are also interested in the extent to which option traders anticipate the change in volatility following splits and how quickly they update their beliefs about volatility after the split. If option traders are very sophisticated, we expect that implied volatility (which reflects option traders' expectations of volatility over some future period) should increase prior to a split, as splits are usually announced in advance. While many of the splits in our sample either pre-date the OptionMetrics data or are associated with stocks with few traded options, we are able to obtain option data for 921 split events. Panel A of Figure 6 plots 30-day implied volatility and 30-day realized volatility around splits.[10] Implied volatility is calculated as a linear combination of implied volatilities from call options with approximate 30-day maturities, and realized volatility represents the realized volatility over the same 30-day window. Panel B plots the log difference between these two lines. We find that option traders anticipate some increase in volatility but undershoot by a substantial margin. After the split, the 30-day implied volatility remains below the 30-day realized volatility for over 100 trading days and then converges. This shows that option traders do not fully anticipate the change in volatility around splits, and they do not immediately notice ex-post that volatility has increased.

In Panel C, we find similar results using implied volatility estimated from data on put options. Overall, the results suggest that a profitable trading strategy that exploits non-proportional thinking would involve going long option straddles (equivalent to buying both a call and put option) prior to pre-announced split dates. Option straddles pay off when realized volatility exceeds implied

[10]The figure displays a a cyclical pattern that repeats approximately every three months. A possible explanation for this pattern is that volatility and implied volatility increase around earnings announcements which occur once each quarter. Splits are often pre-announced during the earnings seasons and occur one month later. The figure also shows that on average, implied volatility exceeds realized volatility. This is a general feature of options data and may be explained by investors demanding compensation for risk.

volatility, which is what we observe in the data following stock splits.

Because OptionMetrics data is only available for larger and more liquid stocks, these results also show that our findings related to splits are not a small-cap phenomenon. Even for large cap stocks with options data, realized volatility jumps up significantly around splits, and implied volatility also increases, albeit with a lag consistent with option traders reacting with a delay.

4.3 Reactions to News Reported in Nominal Units

So far, we have shown empirical support for a simple model of non-proportional thinking in which investors overreact to news for lower priced stocks. The news shocks considered could be firm-specific, such as the announcement of a CEO transition or a new product, or economy-wide, such as the announcement of a trade war with China. In this section, we explore a related prediction from a simple model of non-proportional thinking. We hypothesize that non-proportional thinking may distort investors' reactions to news if the news itself is reported in nominal rather than the appropriate proportional units. In the case of firm earnings announcements, the right measure of the magnitude of the news is likely to be the nominal value of the earnings surprise, scaled by the firm's price just before the news is released. For example, earnings news in which a firm beats analyst expectations by 10 cents per share is a greater positive surprise if the firm's share price is \$20/share than if the firm's share price is \$30/share.

However, the financial press commonly reports the nominal earnings surprise of 10 cents per share, without scaling by share price. Therefore, non-proportional thinking may lead investors to react to the nominal earnings surprise instead of, or in additional to, the scaled earnings surprise. To test this prediction, we estimate the following regression:

$$CAR_{i,[t-1,t+1]} = \beta_0 + \beta_1 nominal\, surprise_{it} + \beta_2 scaled\, surprise_{it} + \epsilon_{it}$$

$CAR_{i,[t-1,t+1]}$ is the firm's cumulative abnormal return over the three-day window around the earnings announcement. If markets are fully rational, we expect to find that $\beta_2 > 0$ because bigger real surprises should correspond to larger return reactions, and β_1 to be close to zero, because the nominal surprise should contain no information beyond the scaled surprise.[11] On the other hand, non-proportional thinking predicts that $\beta_1 > 0$.

When estimating this regression, we measure the nominal surprise and the scaled surprise as percentile rankings for two reasons. First, percentile rankings allows for a direct comparison of the relative magnitudes of the return reaction to each type of earnings surprise. Second, percentile rankings reduce the potential influence of outliers, particularly for the measure of the scaled surprise which can take on very large values when the denominator approaches zero. Expressing earnings surprise in ranked form also follows the convention in the earnings literature (e.g., Dellavigna and Pollet, 2009; Hartzmark and Shue, 2018).

The first panel of Table 14 presents the results. We find that investors indeed react strongly to nominal surprises. If fact, the return reaction to the nominal surprise is slightly larger in magnitude and more statistically significant than the reaction to the scaled surprise when both are measured as percentiles. These results hold in the full sample, as well as the subsamples for large-cap and small-cap firms in columns (3) and (4), respectively. This shows that these patterns are not driven only by small firms. However, the relative return reaction to the nominal surprise is much larger for the sample of small-cap firms, consistent with a story in which the investor base for small-cap firms is less sophisticated or a story in which arbitrage frictions or shorting constraints are more likely to apply to small cap firms. In columns (5) and (6), we show that the same patterns hold before and after the year 2001. In the more recent time period, I/B/E/S data records the announcement

[11] If markets are rational and the scaled surprise variable is a sufficient statistics for news, it is still possible that the coefficient on nominal surprise would be non-zero. It could be that the rational relation between CAR and scaled surprise is non-linear. Since the nominal surprise is correlated with the scaled surprise, β_1 may be non-zero to pick up part of this nonlinear relation. To address this possibility, we focus on long run reversals, which provide more direct evidence of mispricing and a subsequent correction.

date and time of earnings announcement with greater accuracy. Finding similar results in the more recent sample period shows that these patterns cannot be explained by date recording errors.

If prices correctly reflect real news in the long run, we expect the initial return reaction to the nominal earnings surprise to reverse over time as the mispricing is corrected. Because investors react to the nominal earnings surprise, they also underreact to the real measure of news (the scaled earnings surprise), so we expect the scaled earnings surprise to predict future drift in returns. Consistent with these predictions, we find that returns continue to drift in the direction of the scaled earnings surprise and against the direction of the nominal earnings surprise in the long run. Table 14 Panel B shows the long run return reactions to the scaled surprise and the nominal surprise. We find over the course of 100 trading days after earnings announcement that the return reaction to the nominal surprise converges toward zero and the return reaction to the scaled surprise increases in magnitude, consistent with a correction of the initial overreaction to the nominal surprise and underreaction to the scaled surprise.

We also note an interesting divergence in predictions regarding how non-proportional thinking affects over- and underreaction to news. For the general class of news that is not reported in distorted nominal units, non-proportional thinking predicts underreaction to the news for high priced stocks and overreaction for low priced stocks. However, the direction of the predictions can potentially flip in situations in which the news is itself reported in nominal units, depending on whether the nominal amount is too big or small relative to the real news. In the case of earnings announcements, the real news is the scaled earnings surprise and the reported news is the nominal earnings surprise. The reported news (e.g. 10 cents per share) is "too big" relative to the real news (e.g. 10 cents per share divided by share price) for firms with high share prices. Thus, we expect overreaction to the real earnings news for high priced firms in the case of earnings announcements. This contrasts with our general prediction, which is that returns for high priced firms underreact to news.

5 Conclusion

We hypothesize that investors in financial markets engage in non-proportional thinking—they think that news should correspond to a dollar change in price rather than a percentage change in price, leading to return underreaction for high-priced stocks and overreaction for low-priced stocks. Consistent with a simple model of non-proportional thinking, we find that total volatility, idiosyncratic volatility, and absolute market beta are significantly higher for stocks with low share prices or negative past returns. To identify a causal effect of price, we show that volatility increases sharply following stock splits and drops following reverse stock splits. The economic magnitudes are large: non-proportional thinking can explain a significant portion of the "leverage effect" puzzle, in which volatility is negatively related to past returns, as well as the volatility-size and beta-size relations in the data. We also show that non-proportional thinking distorts investor reactions to news that is itself reported in nominal rather than the proper scaled units. Investors react to nominal earnings per share surprises, after controlling for the earnings surprise scaled by price. The reaction to the nominal earnings surprise reverses in the long run, consistent with correction of mispricing.

Our analysis sheds light on the determinants of volatility in financial markets. Our results suggest that non-proportional thinking may be an important determinant of cross-sectional variation in volatility and that well-known asset pricing facts such as the leverage effect and the size-volatility and size-beta relations in the data can be reinterpreted through the lens of non-proportional thinking. Our analysis also offers a new explanation of over- and under-reaction to news and subsequent drift patterns in asset prices. The existing behavioral finance literature has mainly focused on limited attention or belief errors regarding the persistence of news shocks or the strength of one's priors to explain these patterns. Non-proportional thinking offers a complementary explanation: over and under-reaction to news and consequent drift can also be caused by investors thinking about asset values and news in the wrong units.

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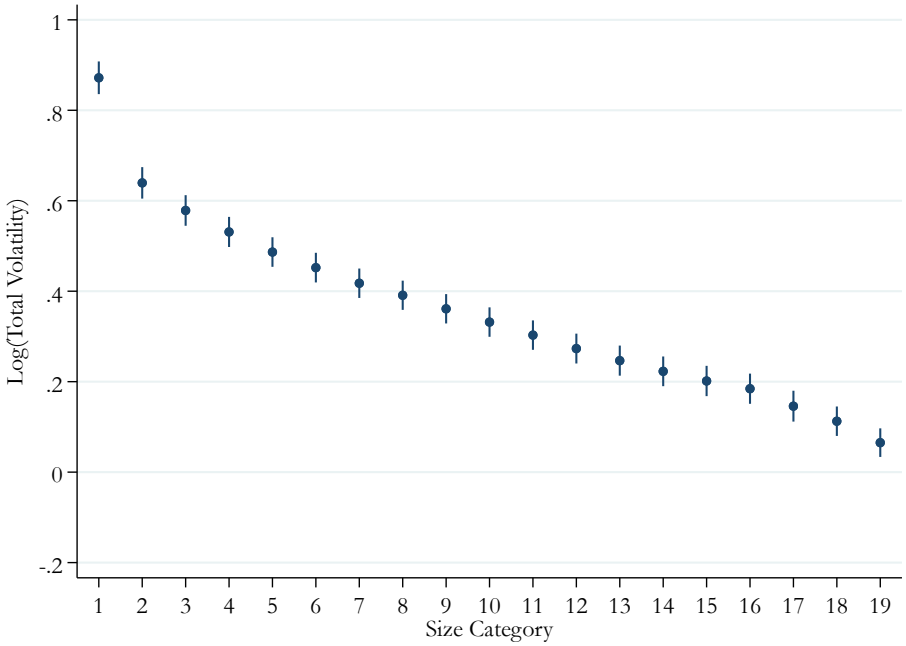
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Figure 1
Size and Volatility

This figure explores the relation between size (market capitalization) and volatility. Panel A shows the coefficients from a regression of log volatility on 20 bins for size (the largest size bin is the omitted category), after controlling for year-month fixed effects. In Panel B, we report the same set of coefficients for the 20 bins representing size, after adding in a single additional control variable for the log of the lagged nominal share price. The dots represent the coefficient estimates and the lines represent 95% confidence intervals. Standard errors are clustered by stock and year-month.

Panel A: Size and Volatility, No Stock Price Control



Panel B: Size and Volatility, Stock Price Control

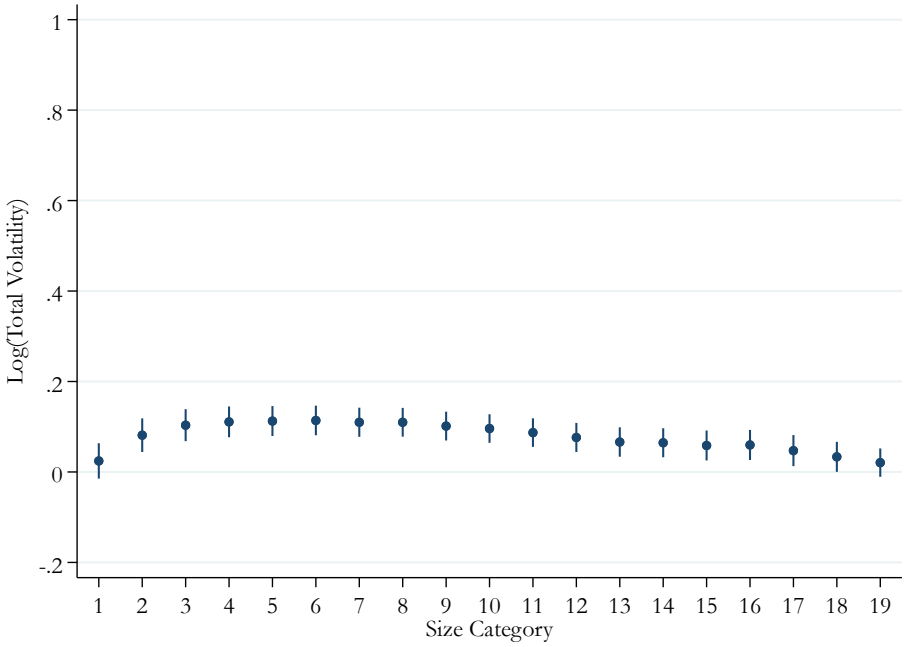


Figure 2
Shape of Volatility-Price Relation

This figure explores the shape of the volatility-price relation by binning lagged prices into 20 equally spaced categories and repeating the regression from Panel A of Table 2, column (4), replacing the continuous *Log(Lagged Price)* variable with these category dummies. The resulting coefficients are plotted with 95% confidence intervals. Category 20 is omitted. Standard errors are clustered by stock and year-month.

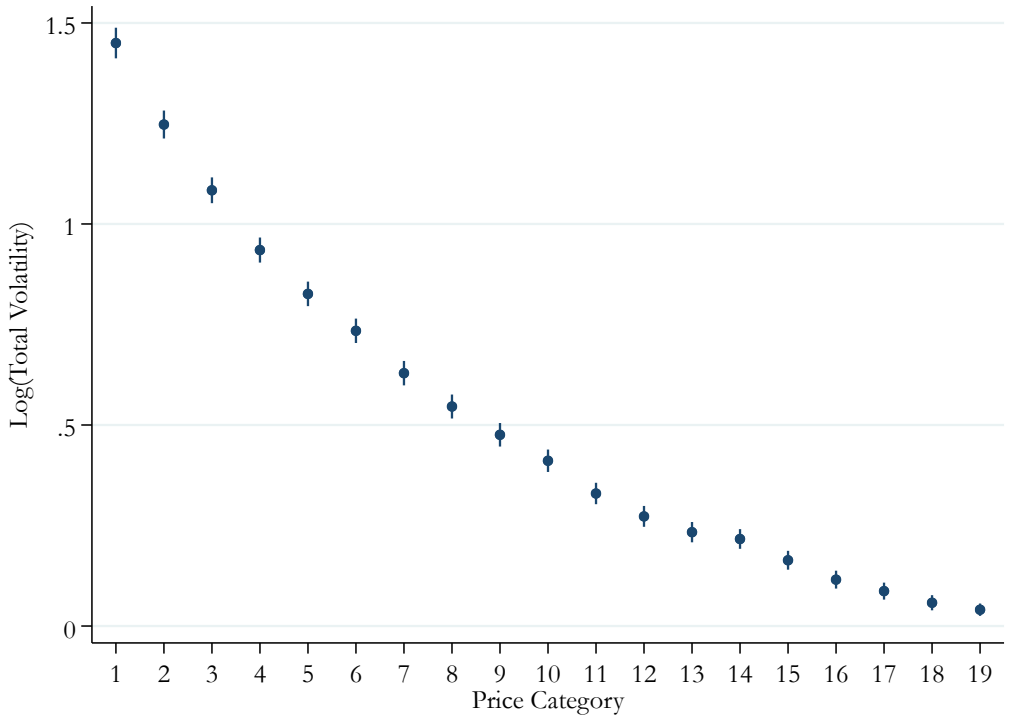


Figure 3

Regression Discontinuity: Scaled Intraday Price Range Around Stock Splits

In this figure, we explore the pattern of volatility around 2-for-1 stock splits or greater (e.g., 3-for-1, 4-for-1, etc.). We examine 45 days before and after the split. The outcome, scaled intraday price range, is defined as difference between the intraday high and intraday low, normalized by the lagged closing price. The thick lines represent non-parametric estimates of the mean on a given day, estimated using a local linear regression with a triangular kernel and MSE-optimal bandwidth. The thin lines represent 95% confidence intervals. The dot shows raw means for each event day.

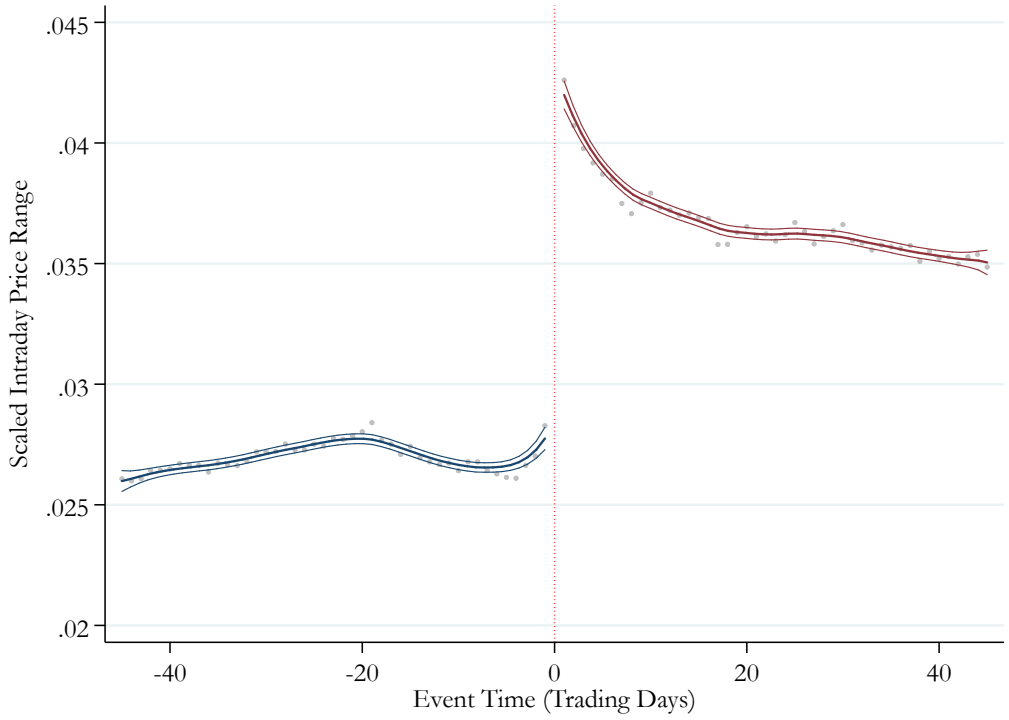
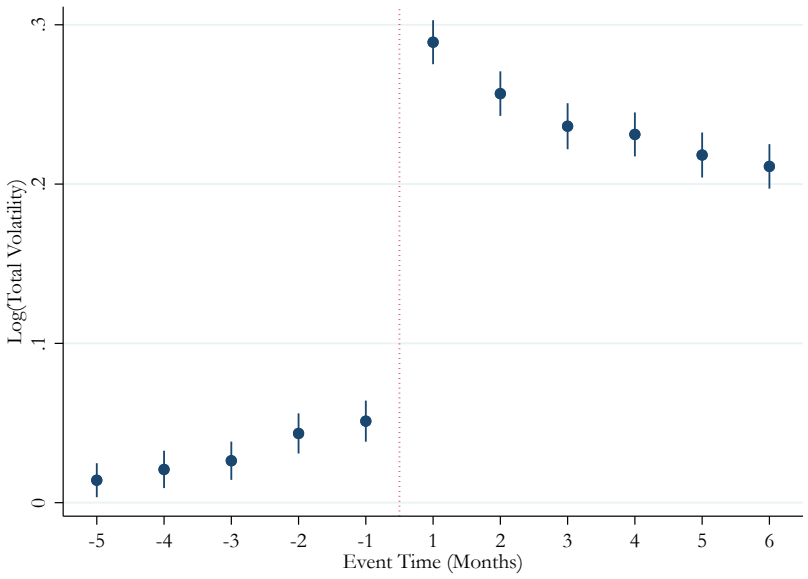


Figure 4

Event Study: Volatility Around Stock Splits and Reverse Stock Splits

Panel A shows total volatility around 2-for-1 stock splits or greater (e.g., 3-for-1, 4-for-1, etc.). Panel B shows total volatility around reverse stock splits: 1-for-2 stock splits or greater (e.g., 1-for-3, 1-for-4, etc.). We plot the coefficients for each month in event time, relative to the omitted category of 6 months prior to the split. The regressions contain stock and year-month fixed effects. The dots represent the point estimates and the lines represent 95% confidence intervals. Standard errors are clustered by stock and month.

(a) **Positive Stock Splits**



(b) **Reverse Stock Splits**

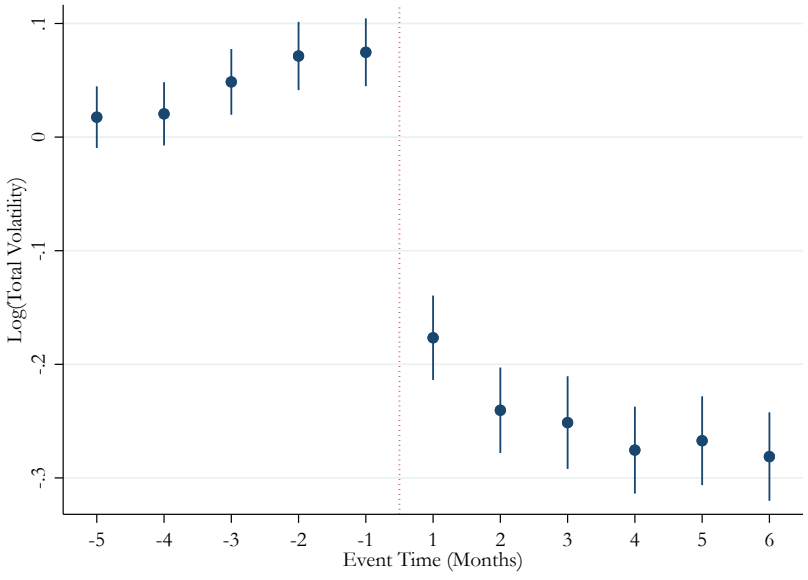
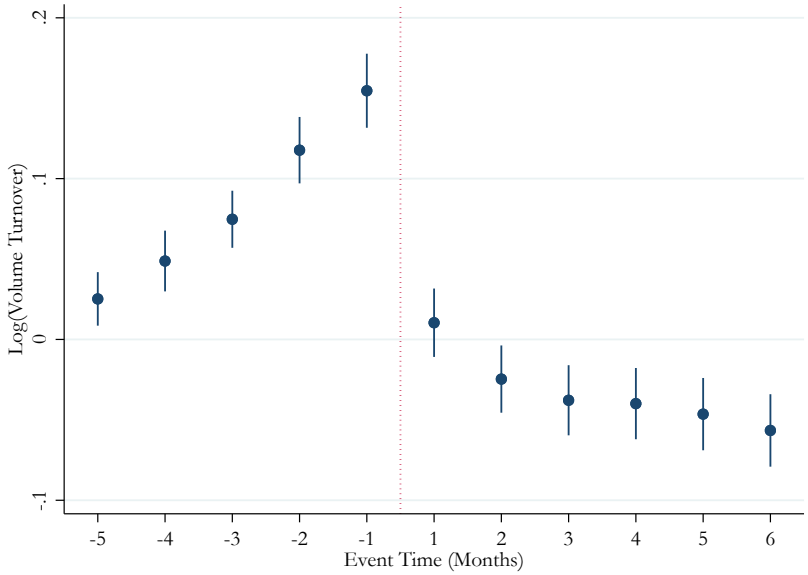


Figure 5

Event Study: Volume Around Stock Splits and Reverse Stock Splits

Panel A shows the pattern of volume around 2-for-1 stock splits or greater (e.g., 3-for-1, 4-for-1, etc.). Panel B shows the pattern of volume around reverse stock splits: 1-for-2 stock splits or greater (e.g., 1-for-3, 1-for-4, etc.). Volume turnover is number of shares traded in each month divided by the total number of shares outstanding. We plot the coefficients for each month in event time, relative to the omitted category of 6 months prior to the split. The regressions contain stock and month fixed effects. The dots represent the point estimates and the lines represent 95% confidence intervals. Standard errors are clustered by stock and month.

(a) **Positive Stock Splits**



(b) **Reverse Stock Splits**

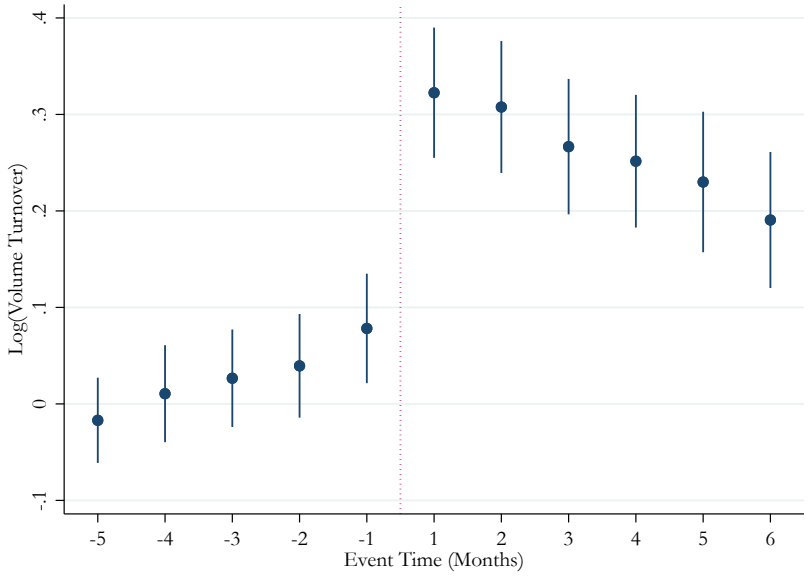
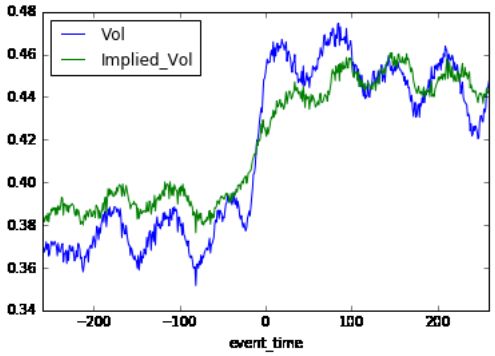


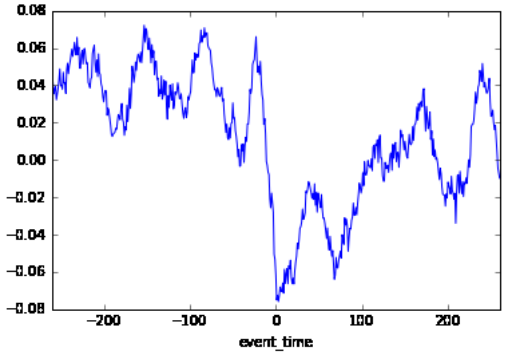
Figure 6
Implied Volatility Around Splits

Panel A plots 30 day implied volatility and 30 day realized volatility from call options. 30 day implied volatility is calculated by OptionMetrics as a linear combination of implied volatilities from call options with approximately 30-day maturities. 30 day realized volatility represents the realized volatility over the subsequent 30-day period. Panel B plots the log difference of these two lines. Panel C plots a similar series as Panel A using data from put options. Event time is trading days relative to the split date. The sample is limited to 2-for-1 stock splits or greater (e.g., 3-for-1, 4-for-1, etc.). It includes 921 firm-split events from 1995-2015 where data are available from OptionMetrics.

(a) **Implied and Realized Volatility (Call Options)**



(b) **Difference Between Implied and Realized Volatility (Call Options)**



(c) **Implied and Realized Volatility (Put Options)**

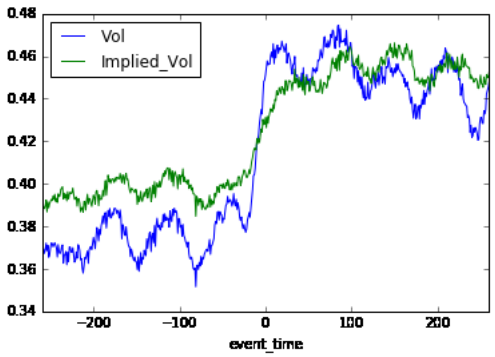


Table 1
Summary Statistics

This table summarizes the variables used in our analyses. Note that sample sizes are reduced for variables that must be linked from CRSP to COMPUSTAT.

	Obs	Mean	Median	Std Dev
Total Volatility (Annualized)	3,254,302	0.510	0.390	0.450
Idiosyncratic Volatility (Annualized)	3,254,302	0.474	0.355	0.436
Market Beta	3,254,302	0.743	0.668	1.434
Price	3,254,302	18.85	13.50	19.13
Market Capitalization (Millions)	3,254,302	1179.9	64.47	8800.6
Insitutional Ownership	2,165,251	0.346	0.272	0.292
Sales Volatility	2,316,178	0.273	0.0966	0.691
Volume Turnover	2,996,292	0.0882	0.0382	0.204
Market-to-Book Ratio	2,240,583	1.977	1.252	18.19
Book Leverage	2,130,604	0.232	0.191	0.278

Table 2
Baseline Results: Total Volatility

This table explores how return volatility varies with the share price. Using data at the stock-month level, we estimate the following regression:

$$log\left(vol_{it}\right) = \beta_0 + \beta_1 log\left(price_{i,t-1}\right) + controls + \tau_t + \epsilon_{it}.$$

We regress each stock i's volatility in month t on the stock's nominal share price at the end of the previous month, indicator variables for 20 size categories using the stock's market capitalization at the end of the previous month relative to other stocks in the same time period, calendar-year month fixed effects, and stock fixed effects. Volatility is estimated using daily returns from month t. The sample excludes observations with extreme lagged price (the bottom and top 1%). To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Cross-Section

	Log(Total Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.326***		-0.332***	-0.339***
	(0.00339)		(0.00446)	(0.00405)
Log(Lagged Size)		-0.146***	0.00431	
		(0.00235)	(0.00311)	
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.442	0.328	0.442	0.445
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Panel B: Time Series

	Log(Total Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.260***		-0.261***	-0.274***
	(0.00395)		(0.00477)	(0.00403)
Log(Lagged Size)		-0.160***	0.000476	
		(0.00334)	(0.00383)	
Stock FE	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.588	0.565	0.588	0.588
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Table 3

Baseline Results: Idiosyncratic Volatility and Absolute Market Beta

This table repeats the analysis of Table 2 Panel A, using idiosyncratic volatility and absolute market beta as the outcome variable. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Idiosyncratic Volatility

	Log(Idiosyncratic Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.357***		-0.330***	-0.345***
	(0.00317)		(0.00432)	(0.00397)
Log(Lagged Size)		-0.171***	-0.0211***	
		(0.00216)	(0.00308)	
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.469	0.363	0.471	0.474
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Panel B: Absolute Market Beta

	Log(|Beta|)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.109***		-0.334***	-0.305***
	(0.00565)		(0.00642)	(0.00538)
Log(Lagged Size)		0.0232***	0.174***	
		(0.00383)	(0.00468)	
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.056	0.047	0.085	0.085
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Table 4
Additional Controls

This table repeats the analysis of Table 2 (Panel A), but includes additional controls. Sales volatility is computed as the standard deviation for year-over-year quarterly sales growth in the four most recently completed quarters. Market-to-book is is the book ratio as of the most recently completed quarter. Volume turnover is volume in the most recent month divided by shares outstanding. Leverage is debt (current liabilities + long term debt) divided by the book value of assets. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Log(Total Volatility)					
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Lagged Price)	-0.326***	-0.344***	-0.308***	-0.305***	-0.282***	-0.288***
	(0.00339)	(0.00441)	(0.00533)	(0.00539)	(0.00533)	(0.00544)
Log(Lagged Sales Volatility)			0.0574***	0.0379***	0.0260***	0.0268***
			(0.00191)	(0.00151)	(0.00133)	(0.00139)
Log(Lagged Market-to-Book)				0.192***	0.151***	0.145***
				(0.00744)	(0.00652)	(0.00671)
Log(Lagged Volume Turnover)					0.110***	0.109***
					(0.00228)	(0.00235)
Log(Lagged Leverage)						-0.00521***
						(0.00146)
Log(Lagged Size)	No	Yes	Yes	Yes	Yes	Yes
Size Category FE	No	Yes	Yes	Yes	Yes	Yes
Log(Lagged Size) × Size Category FE	No	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.442	0.446	0.456	0.473	0.503	0.512
Observations	3,254,302	3,254,302	1,944,230	1,777,933	1,750,796	1,457,187

Table 5

Robustness: Tick-Size Adjusted Volatility and Zero Leverage Subsample

Panel A repeats the analysis of Table 2 Panel A adjusting for tick-size effects. Specifically, on a day where a stock's price increases from the previous closing price, we subtract half a tick from that day's closing price. On days where a stock's price decreases, we add half a tick to that days closing price. We then calculate volatility based on these artificial prices. Panel B repeats the analysis of Table 2 Panel A, on the subsample of stocks with zero. Zero leverage firms are ones with zero current liabilities and zero long term debt (not including missing values as zeros). To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Tick-Size Adjusted Volatility

	Log(Total Tick-Size Adjusted Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.268***		-0.266***	-0.275***
	(0.00348)		(0.00460)	(0.00434)
Log(Lagged Size)		-0.122***	-0.00157	
		(0.00231)	(0.00325)	
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.358	0.285	0.358	0.361
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Panel B: Zero Leverage Subsample

	(1) Log(Total Volatility)	(2) Log(Idiosyncratic Volatility)	(3) Log(|Beta|)
Log(Lagged Price)	-0.286***	-0.290***	-0.256***
	(0.00775)	(0.00772)	(0.0100)
Month FE	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes
R-squared	0.337	0.364	0.093
Observations	224,571	224,571	224,571

Table 6
Institutional Ownership

This table repeats the analysis of Table 2 Panel A, but includes institutional ownership in the previous month as a control, as well as institutional ownership interacted with price. Institutional ownership is computed as the number of shares held by institutions as reported in 13f filings divided by shares outstanding. Because 13f filings are only quarterly, the numerator of this ratio only changes on a quarterly basis. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	(1) Log(Total Volatility)	(2) Log(Idiosyncratic Volatility)	(3) Log(|Beta|)
Log(Lagged Price)	-0.384***	-0.380***	-0.387***
	(0.00509)	(0.00510)	(0.00635)
Log(Lagged Price) × Lagged Inst. Ownership	0.169***	0.136***	0.234***
	(0.0108)	(0.0106)	(0.0152)
Lagged Inst. Ownership	-0.311***	-0.274***	-0.0939*
	(0.0323)	(0.0313)	(0.0494)
Month FE	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes
R-squared	0.432	0.463	0.103
Observations	2,113,118	2,113,118	2,113,118

Table 7
Size Subsamples

This table repeats the analysis of Table 2 Panel A, on 20 subsamples corresponding to 20 different size categories based on market capitalization (prc×shrout). Size1 corresponds to the smallest size category, and Size20 the largest. The size categories are based on the monthly ME Breakpoints from Ken French's data library. The breakpoints for a given month use all NYSE stocks that have a CRSP share code of 10 or 11 and have good shares and price data, excluding closed end funds and REITs. There are 20 groups corresponding the every fifth percentile. Observations in our data are not equally distributed across the categories, because our sample includes all stocks on NYSE, AMEX, and NASDAQ with a share code of 10 or 11, rather than only NYSE. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Log(Total Volatility)									
	(1) Size1	(2) Size2	(3) Size3	(4) Size4	(5) Size5	(6) Size6	(7) Size7	(8) Size8	(9) Size9	(10) Size10
Log(Lagged Price)	-0.363***	-0.386***	-0.370***	-0.364***	-0.346***	-0.325***	-0.310***	-0.298***	-0.281***	-0.270***
	(0.00562)	(0.00659)	(0.00746)	(0.00771)	(0.00800)	(0.00816)	(0.00870)	(0.00870)	(0.00949)	(0.00996)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.393	0.368	0.363	0.362	0.354	0.349	0.347	0.351	0.343	0.350
Observations	1,111,769	333,979	226,006	173,581	146,796	128,577	117,145	107,699	99,812	92,354

	Log(Total Volatility)									
	(1) Size11	(2) Size12	(3) Size13	(4) Size14	(5) Size15	(6) Size16	(7) Size17	(8) Size18	(9) Size19	(10) Size20
Log(Lagged Price)	-0.248***	-0.227***	-0.207***	-0.201***	-0.184***	-0.168***	-0.166***	-0.124***	-0.123***	-0.139***
	(0.00969)	(0.0103)	(0.0118)	(0.0123)	(0.0135)	(0.0139)	(0.0179)	(0.0165)	(0.0206)	(0.0215)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.355	0.359	0.351	0.357	0.359	0.388	0.403	0.412	0.450	0.526
Observations	85,267	81,440	78,053	75,213	72,894	70,567	67,662	65,314	62,743	57,431

Table 8
Time Period Subsamples

This table repeats the analysis of Table 2 Panel A, on subsamples from each decade from the 1920s through the 2010s. The sample ends in 2016, so column (10) corresponds to the years 2010-2016 only. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Log(Total Volatility)									
	(1) 1920s	(2) 1930s	(3) 1940s	(4) 1950s	(5) 1960s	(6) 1970s	(7) 1980s	(8) 1990s	(9) 2000s	(10) 2010s
Log(Lagged Price)	-0.227***	-0.275***	-0.350***	-0.191***	-0.324***	-0.462***	-0.317***	-0.369***	-0.353***	-0.251***
	(0.0140)	(0.0108)	(0.00989)	(0.0147)	(0.0126)	(0.00862)	(0.00646)	(0.00768)	(0.00995)	(0.00554)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.560	0.652	0.609	0.289	0.396	0.353	0.315	0.440	0.449	0.356
Observations	22,843	82,661	97,620	118,906	209,217	452,864	624,639	751,051	586,932	307,569

Table 9
Short Run and Long Run Reactions to Market News

This table shows results from estimating regressions of the form:

$$r_{i,[t+a,t+b]} = \beta_1 log(price_{i,t-1}) \times r_{mkt,t} + controls + \epsilon_{it}$$

We limit the sample to observations in which the absolute market return in month t exceeds 10%. The variable $SizeCat$ refers to indicators for the 20 size categories defined in Table 2. The variable $Ret^{mkt}_{t+a,t+b}$ represents the market return over the same time interval specified in the column titles. *Main effects* refers to the uninteracted versions of all of the control variables. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)
	$\text{Ret}_{\text{i,t}}$	$\text{Ret}_{\text{i,[t+1,t+2]}}$	$\text{Ret}_{\text{i,[t+3,t+5]}}$	$\text{Ret}_{\text{i,[t+6,t+8]}}$	$\text{Ret}_{\text{i,[t+9,t+11]}}$
$\text{Log(Price}_{\text{i,t}-1}) \times \text{Ret}_{\text{mkt,t}}$	-0.258***	-0.0353	0.0481	0.152**	-0.0295
	(0.0519)	(0.0789)	(0.0691)	(0.0630)	(0.0642)
$\text{Log(Price}_{\text{i,t}-1}) \times \text{Ret}_{\text{mkt,[t+a,t+b]}}$	Yes	Yes	Yes	Yes	Yes
$\text{SizeCat}_{\text{i,t}-1} \times \text{Ret}_{\text{mkt,t}}$	Yes	Yes	Yes	Yes	Yes
$\text{SizeCat}_{\text{i,t}-1} \times \text{Ret}_{\text{mkt,[t+a,t+b]}}$	Yes	Yes	Yes	Yes	Yes
Main Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.498	0.253	0.280	0.345	0.381
Observations	123,284	123,284	123,284	123,284	123,284

Table 10
Volatility and Past Returns

This table shows regressions of the form:

$$log\left(vol_{it}\right) = \beta_0 + \beta_1 r_{i,[t-x,t-1]} + \tau_t + \epsilon_{it},$$

where $returns_{i,[t-x,t-1]}$ represents past returns from $t-x$ to $t-1$. Volatility is estimated using daily returns from month t. The sample excludes observations with extreme lagged priced (the bottom and top 1%). To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Cross Section

	Log(Total Volatility)					
	(1)	(2)	(3)	(4)	(5)	(6)
	2-Month	4-Month	6-Month	8-Month	10-Month	12-Month
Lagged Return	-0.114***	-0.0952***	-0.0835***	-0.0745***	-0.0647***	-0.0557***
	(0.0253)	(0.0204)	(0.0158)	(0.0126)	(0.0106)	(0.00899)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.184	0.185	0.185	0.185	0.186	0.186
Observations	2,966,196	2,966,196	2,966,196	2,966,196	2,966,196	2,966,196

Panel B: Time Series

	Log(Total Volatility)					
	(1)	(2)	(3)	(4)	(5)	(6)
	2-Month	4-Month	6-Month	8-Month	10-Month	12-Month
Lagged Return	-0.102***	-0.0811***	-0.0684***	-0.0595***	-0.0512***	-0.0438***
	(0.0117)	(0.00986)	(0.00773)	(0.00601)	(0.00500)	(0.00423)
Stock FE	Yes	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.547	0.547	0.548	0.548	0.548	0.548
Observations	2,966,196	2,966,196	2,966,196	2,966,196	2,966,196	2,966,196

Table 11

Regression Discontinuity: Scaled Intraday Price Range Around Stock Splits

In this table, we explore the pattern of volatility around 2-for-1 stock splits or greater (e.g., 3-for-1, 4-for-1, etc.). We examine 45 days before and after the split. The outcome, scaled intraday price range, is defined as difference between the intraday high and intraday low, normalized by the lagged closing price. Control functions on each side of the cutoff are estimated non-parametrically using local linear regression. Bandwidths are selected using one common MSE-optimal bandwidth selector, two different MSE-optimal bandwidth selectors on each side of the cutoff, or one common MSE-optimal bandwidth selector for the sum of regression estimates (as opposed to the difference thereof). The kernel is either Triangular or Epanechnikov as labeled. The estimated coefficient represents the size of the discontinuity at the split date, as illustrated in Figure 3.

	Scaled Intraday Price Range			
	(1)	(2)	(3)	(4)
Discontinuity at Split	0.0146***	0.0149***	0.0146***	0.0150***
	(0.000529)	(0.000984)	(0.000560)	(0.00107)
Degree Local Poly	1	2	1	2
Bandwidth	7.074	7.074	6.160	6.160
Kernel	Triangular	Triangular	Epanechnikov	Epanechnikov
Observations	646,700	646,700	646,700	646,700

Table 12
Volatility Around Stock Splits

To explore how volatility changes after splits, we estimate the following regression:

$$log\left(vol_{it}\right) = \beta_0 + \beta_1 Post_{it} + \tau_t + \nu_i + \epsilon_{it}.$$

The regression sample uses observations at the stock-month level. We consider any positive stock split in which one old share is converted into two or more new shares. The month of a stock split counts as event date 0, and the sample is restricted to observations in the window $[t-6, t+6]$ around the event date, conditional on the observation not being within 12 months of another stock split for the same stock. To examine how volatility varies with event time in greater detail, we also present results in which the $Post_{it}$ indicator is replaced with event-month indicators. In all specifications, we control for year-month and stock fixed effects and double cluster standard errors by stock and time. To account for correlated observations, we double-cluster standard errors by stock and year-month. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Log(Total Volatility)		Log(Idiosyncratic Volatility)		Log(|Beta|)	
	(1)	(2)	(3)	(4)	(5)	(6)
Post Split	0.216***		0.214***		0.230***	
	(0.00452)		(0.00467)		(0.00836)	
Post Split (0 Month)		0.225***		0.225***		0.230***
		(0.00553)		(0.00566)		(0.0139)
Post Split (1 Month)		0.262***		0.257***		0.295***
		(0.00588)		(0.00608)		(0.0130)
Post Split (2 Month)		0.230***		0.228***		0.251***
		(0.00564)		(0.00579)		(0.0136)
Post Split (3 Month)		0.210***		0.206***		0.234***
		(0.00606)		(0.00620)		(0.0137)
Post Split (4 Month)		0.205***		0.202***		0.214***
		(0.00580)		(0.00598)		(0.0130)
Post Split (5 Month)		0.192***		0.190***		0.195***
		(0.00611)		(0.00624)		(0.0145)
Post Split (6 Month)		0.185***		0.184***		0.184***
		(0.00612)		(0.00621)		(0.0142)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
Stock FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.633	0.634	0.621	0.622	0.283	0.283
Observations	88,584	88,584	88,584	88,584	88,584	88,584

Table 13
Stock Characteristics Around Splits

This table shows mean institutional ownership and sales volatility before and after stock splits, as well as the difference. Institutional ownership is as defined in Table 6. Sales volatility is as defined in Table 4. Before (after) split institutional ownership refers to institutional ownership based on the last (first) observed 13f filing for each stock prior to (following) the split. Before (after) split sales volatility refers to sales volatility based on the most last (first) four completed quarters prior to (following) the split.

	Before Split			After Split			Difference		
	Obs	Mean	Std Dev	Obs	Mean	Std Dev	Obs	Mean	Std Dev
Inst. Ownership	4,531	0.473	0.290	4,610	0.463	0.279	9,141	0.009	0.006
Sales Volatility	4,484	0.201	1.566	4,691	0.209	1.939	9,175	-0.008	0.037

Table 14
Response to Nominal Earnings Surprises

Panel A of this table shows the results from estimating regressions of the form:

$$CAR_{i,[t-1,t+1]} = \beta_0 + \beta_1 nominal\, surprise_{it} + \beta_2 scaled\, surprise_{it} + \epsilon_{it}$$

$CAR_{i,[t-1,t+1]}$ is the firm's cumulative abnormal return over the three-day window around the earnings announcement. *Nominal Surprise* is defined as the difference between announced earnings per share and analyst consensus. *Scaled Surprise* is defined as *Nominal Surprise* divided by the lagged stock price (measured 3 trading days prior to announcement). Analyst consensus is defined as the median analyst earnings forecast among analysts that make a forecast within 30 days of the announcement. We measure both the the nominal surprise and the scaled surprise as percentile rankings in columns (2)-(6). Large and Small Cap are defined as size categories 11-20 and 1-10, respectively. Panel B shows the results from the same regression, using extended return windows for the dependent variable. Standard errors are clustered by date. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Immediate Impact

	Cummulative Abnormal Return [-1,1]					
	(1) All	(2) All	(3) Large Cap	(4) Small Cap	(5) Year $\geq$ 2001	(6) Year $<$ 2001
Surprise Scaled	25.34***					
	(3.497)					
Surprise Nominal	7.917***					
	(0.240)					
Percentile Surprise Scaled		0.0346***	0.0285***	0.0191***	0.0397***	0.0281***
		(0.00206)	(0.00365)	(0.00250)	(0.00309)	(0.00267)
Percentile Surprise Nominal		0.0349***	0.0277***	0.0615***	0.0465***	0.0226***
		(0.00192)	(0.00300)	(0.00266)	(0.00285)	(0.00256)
R-squared	0.034	0.070	0.059	0.078	0.095	0.043
Observations	176,155	176,155	80,606	95,549	95,433	80,722

Table 14
(Continued)
Panel B: Long Term Reversal

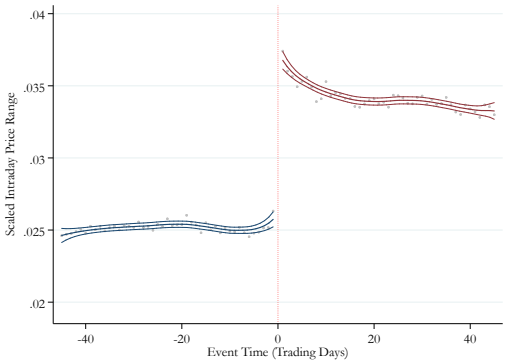
	Cummulative Abnormal Return				
	(1) [-1,1]	(2) [-1,25]	(3) [-1,50]	(4) [-1,75]	(5) [-1,100]
Percentile Surprise Scaled	0.0346***	0.0549***	0.0713***	0.0787***	0.0875***
	(0.00206)	(0.00469)	(0.00645)	(0.00747)	(0.00852)
Percentile Surprise Nominal	0.0349***	0.0292***	0.0239***	0.0212***	0.0126
	(0.00192)	(0.00438)	(0.00605)	(0.00700)	(0.00798)
R-squared	0.070	0.028	0.020	0.015	0.011
Observations	176,155	176,155	176,155	176,155	176,155

APPENDIX

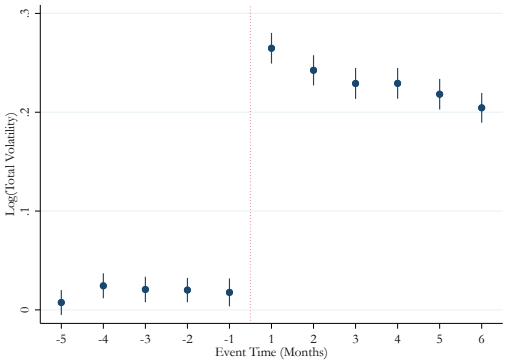
Figure A1
Splits: Large Cap Subsample

This figure shows that the patterns relating to splits discussed in Section 4.2 are not only driven by small-cap stocks. The data used to generate these figures is restricted to firms in Fama French size categories 11 to 20 as of the month prior to the split. We examine positive stock splits only (reverse stock splits are rare for the sample of large cap stocks).

(a) **Scaled Intraday Price Range**



(b) **Total Volatility**



(c) **Idiosyncratic Volatility**

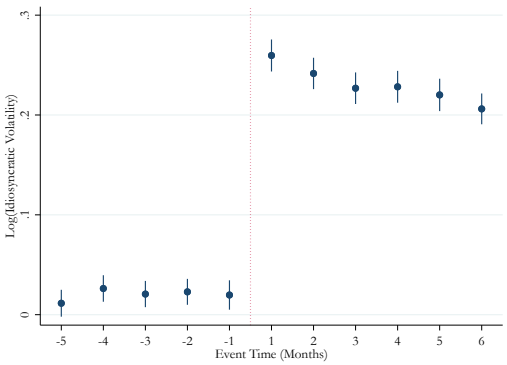
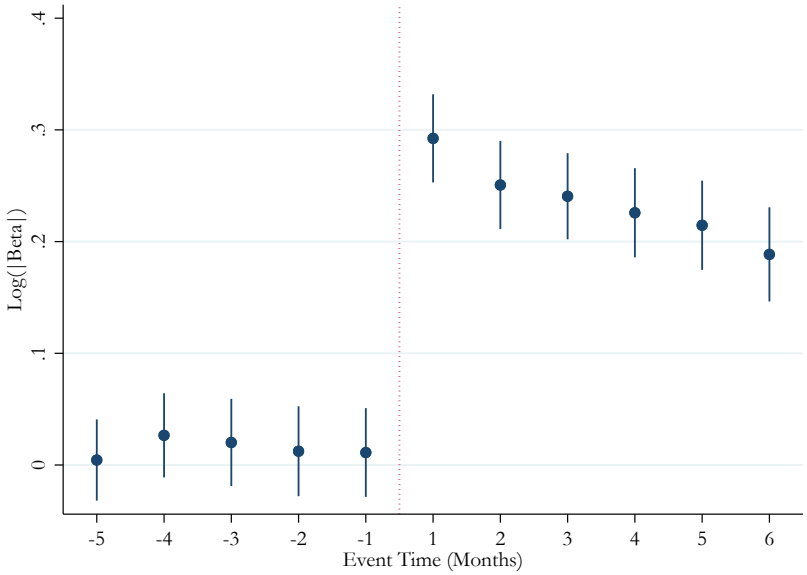


Figure A2
Splits: Large Cap Subsample (Continued)

This figure shows that the patterns relating to splits discussed in Section 4.2 are not only driven by small-cap stocks. The data used to generate these figures is restricted to firms in Fama French size categories 11 to 20 as of the month prior to the split. We examine positive stock splits only (reverse stock splits are rare for the sample of large cap stocks).

(a) **Absolute Market Beta**



(b) **Volume Turnover**

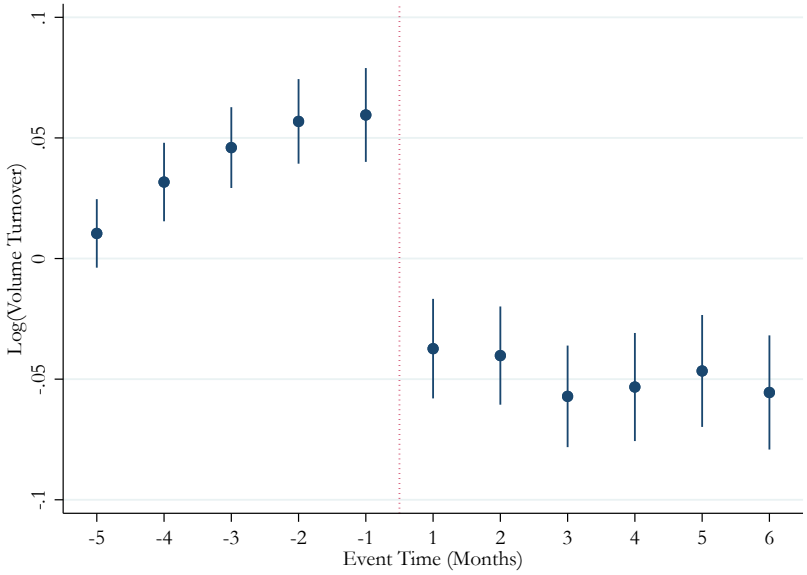


Table A1
Baseline Results: Market Beta - Positive Only

This table repeats the analysis of Table 3 Panel B, limiting the sample to observations with positive estimated market betas.

	Log(Beta)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.0907***		-0.292***	-0.277***
	(0.00475)		(0.00660)	(0.00545)
Log(Lagged Size)		0.0238***	0.151***	
		(0.00312)	(0.00452)	
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.054	0.048	0.081	0.082
Observations	2,541,752	2,541,752	2,541,752	2,541,752

Table A2
Baseline Results: Controlling for Shares Outstanding
This table repeats the cross-sectional analysis of Tables 2 and 3 replacing the control for Log(Lagged Size) with a control for Log(Lagged Shares Outstanding).

	Log(Total Volatility)		Log(Idiosyncratic Volatility)		Log(|Beta|)	
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Lagged Price)	-0.326***	-0.327***	-0.357***	-0.351***	-0.109***	-0.159***
	(0.00339)	(0.00327)	(0.00317)	(0.00299)	(0.00565)	(0.00526)
Log(Lagged Shares Outstanding)		0.00431		-0.0211***		0.174***
		(0.00311)		(0.00308)		(0.00468)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.442	0.442	0.469	0.471	0.056	0.085
Observations	3,254,302	3,254,302	3,254,302	3,254,302	3,254,302	3,254,302