Learning about the Neighborhood*

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Abstract

We develop a model of neighborhood choice to analyze information aggregation and learning in housing and commercial real estate markets. In the presence of pervasive informational frictions, housing prices serve as important signals to households and commercial developers about the economic strength of a neighborhood. Through this learning channel, noise from housing market supply and demand shocks can propagate from housing prices to the local economy, distorting not only migration into the neighborhood, but also supply of commercial facility. Our analysis also provides testable, nuanced implications on how the magnitudes of these noise effects vary across neighborhoods with different elasticity of housing supply and degree of complementarity of their industries.

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Widely spread optimism is recognized by the literature as an important driver of the U.S. housing cycle in the 2000s that led to the subsequent financial crisis and the Great Recession, e.g., Cheng, Raina and Xiong (2014) and Kaplan, Mitman, and Violante (2017). How was the optimism developed? The literature often associates housing market optimism with some form of extrapolation, e.g., Case and Shiller (2003), Glaeser, Gyourko, and Saiz (2008), Piazzesi and Schneider (2009), and Glaeser and Nathanson (2017). To the extent that coastal cities like San Francisco and New York with the most inelastic housing supply and the largest price appreciations did not experience the most pronounced boom-and-bust cycle, it requires a more nuanced mechanism to fully explain the rich cross-section of this housing cycle across U.S. cities.

In this paper, we develop a theoretical model to analyze how information frictions affect the learning of households and developers about a neighborhood, which in turn drives both housing market dynamics and other investment decisions in the neighborhood. The model features a continuum of households in an open neighborhood, which can be viewed as a county. Each household has a choice of whether to move into the neighborhood by purchasing a house, and has a Cobb-Douglas utility function over its consumption of its own good and its aggregate consumption of the goods produced by other households in the neighborhood. This complementarity in households' consumption motivates each household to learn about the unobservable economic strength of the neighborhood, which determines the common productivity of all households and, consequently, their desire to live in the neighborhood. Each household requires both labor, which it supplies, and commercial facility, to produce its good according to a Cobb-Douglas production function of these two inputs. Since the price of commercial facility depends on its marginal product across households in the neighborhood, competitive commercial developers must also form expectations about the unobservable economic strength of the neighborhood when determining how much commercial facility to develop.

The local housing market provides a useful platform for aggregating private information about the economic strength of the neighborhood. It is intuitive that the traded housing price reflects the net effect of demand and supply factors, in a similar spirit of the classic models of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets. Different from the linear equilibrium in these models, our model features an important neighborhood selection, through which only households with private signals above

a certain equilibrium cutoff choose to live in the neighborhood. This selection makes our model inherently non-linear. Nevertheless, we are able to derive the equilibrium in analytical forms, building on the cutoff equilibrium framework developed by Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2014, 2015) for asset markets. Specifically, despite each household's housing demand being non-linear, the Law of Large Numbers allows us to aggregate their housing demand, and to derive a cutoff equilibrium for the housing market. Each household possesses a private signal regarding the neighborhood common productivity. By aggregating the households' housing demand, the housing price aggregates their private signals. The presence of unobservable supply shocks, however, prevents the housing price from perfectly revealing the neighborhood strength and acts as a source of informational noise in the housing price.

Our model allows us to analyze how informational frictions affect not only the housing price but also the households' neighborhood, labor and production choices, which, in turn, determine their demands for housing and commercial facility. The housing price plays a key role in affecting agents' expectations. Through this learning channel, noise in the housing price, originated from either demand or supply side of the housing market, may affect agents' choices of neighborhood, housing and commercial facility. That is, by pushing up the housing price, a noise factor may lead to more households moving into the neighborhood, a more pronounced housing cycle, and, interestingly, greater over-supply of commercial facility and a more pronounced commercial real estate cycle in the neighborhood.

Our analysis highlights how the magnitudes of these noise effects may vary across different neighborhoods along two dimensions: 1) elasticity of housing supply in the neighborhood and 2) the degree of households' consumption complementarity. In particular, the noise effects induced by agents' learning from the housing price are hump-shaped with respect to housing supply elasticity, due to the following reason. At one end with housing supply being perfectly inelastic, housing price is fully determined by housing demand and thus fully reveals the strength of the neighborhood; at the other end with housing supply being perfectly elastic, housing price is fully determined by housing supply and is thus un-informative of the strength of the neighborhood. At both ends, noise in housing price does not affect agents' expectations. As a result, the noise effects are strongest at intermediate supply elasticties. This insight reflects that supply-side characteristics of the local housing market may interfere with the informativeness of the housing price about housing demand and

neighborhood strength, leading to nuanced empirical predictions. For example, our analysis shows that noise induced population inflow and housing and commercial real estate cycles being most pronounced in areas with intermediate supply elasticties, rather than areas with the most inelastic housing supply. This result helps explain why Las Vegas had a more dramatic housing cycle than New York and San Francisco. Our analysis also shows that the noise effects are increasing with the households' consumption complementarity because greater complementarity makes learning about the neighborhood strength a more important part of the households' decisions. These comparative statistics results give rise to testable hypotheses in the cross-section when sorting areas by supply elasticity or the degree of complementarity of their industries.

Our analysis also highlights a learning externality in that when making housing choices, households do not internalize the subsequent effects on the expectations of commercial developers. In addition to misallocation of resources ex ante, this learning channel also gives rise to persistent distortions to local economies ex post. To the extent that any overbuilding of offices and commercial infrastructure is difficult to reverse in the short or medium-term, the excess supply can have prolonged, overhang effects on the local economies. Gao, Sockin, and Xiong (2017), for instance, find consistent evidence that supply overhang in housing markets helped transmit the adverse impact of housing speculation to the real economy during the recent bust.

1 Related Literature

The existing literature has emphasized the importance of accounting for home buyers' expectations in understanding dramatic housing boom and bust cycles, e.g., Case and Shiller (2003); Glaeser, Gyourko, and Saiz (2008); and Piazzesi and Schneider (2009). Much of the analyses and discussions, however, are made in the absence of a systematic framework that anchors home buyers' expectations to their information aggregation and learning process. In this paper, we help fill this gap by developing a model for analyzing information aggregation and learning in housing markets, and its spillover to commercial real estate. By doing so, we are able to uncover a novel interaction in which distortions to housing prices can impact the supply of commercial real estate through expectations about future rents. We also demonstrate that supply elasticity, beyond its role in driving housing supply, determines the informational content of the housing price, and how households weight the price signal

compared to other public signals of demand when learning. This learning effect implies non-monotonic patterns in housing price volatility and real activity.

The literature has offered several other explanations for the comovement of residential and commercial real estate cycles. Rosen (1979) and Roback (1982) link housing and commercial real estate in spatial equilibrium settings in which land can either be developed for residential or commercial use. As a consequence, both markets are driven by similar local fundamentals, as prices, on the margin, reflect both the value of amenities to households and productivity to firms. Our analysis links these two markets instead through the production technology with which households produce output and, as a result, both housing and commercial real estate are driven by expectations about future neighborhood productivity. Gyourko (2009a) emphasizes the role of irrational investor and lender optimism, citing that much of the recent run-up in commercial real estate prices was driven by investors competing over the same income rents, rather than rising cash flows, who had easier access to capital because of declining loan underwriting standards. Levitin and Wachter (2013) explain this recent parallel boom in construction real estate as a change in investor demographics and a deterioration in securitization underwriting standards. To the extent that more accommodative lending and securitization standards enabled optimistic commercial developers to act on their expectations of future rents, we view these two channels as complementary to our learning channel.

Our model also differs from Burnside, Eichenbaum, and Rebelo (2013), which offers a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions. Our learning-based mechanism is also different from Nathanson and Zwick (2017), which studies the hoarding of land by home builders in certain elastic areas as a mechanism to amplify price volatility in the recent U.S. housing cycle. Glaeser and Nathanson (2017) presents a model of biased learning in housing markets, building on current buyers not adjusting for the expectations of past buyers, and instead assuming that past prices reflect only contemporaneous demand. This incorrect inference gives rise to correlated errors in housing demand forecasts over time, which in turn generate excess volatility, momentum, and mean-reversion in housing prices. In contrast to this model, informational frictions in our model anchor on the interaction between the demand and supply sides, and feedback to both housing price and real outcomes. This key feature is also different from the amplification to price volatility induced by dispersed

information and short-sale constraints featured in Favara and Song (2014).

By focusing on information aggregation and learning of symmetrically informed households with dispersed private information, our study differs in emphasis from those that analyze the presence of information asymmetry between buyers and sellers of homes, such as Garmaise and Moskowitz (2004) and Kurlat and Stroebel (2014). Neither does our model emphasize the potential asymmetry between in-town and out-of-town home buyers, which is shown to be important by Chinco and Mayer (2015).

Our work features a tractable cutoff equilibrium framework, similar to that in Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2014, 2015), which employ risk-neutral agents, normally distributed asset fundamentals, and position limits to deliver tractable nonlinear equilibria. Goldstein, Ozdenoren, and Yuan (2013) investigate the feedback to the investment decisions of a single firm when managers, but not investors, learn from prices. Albagli, Hellwig, and Tsyvinski (2014, 2015) focus on the role of asymmetry in security payoffs in distorting asset prices and firm investment incentives when future shareholders learn from prices to determine their valuations. In contrast, we focus on the feedback induced by learning from housing prices to household neighborhood choice and labor decisions in an equilibrium production setting, and its spillover to the investment decisions of commercial developers. The interaction between learning and supply elasticity can potentially help explain why relatively unconstrained areas recently experienced more pronounced house price boom-bust cycles, as documented in, for instance, Davidoff (2013), Glaeser (2013), and Nathanson and Zwick (2017).

In addition, there are extensive studies in the housing literature highlighting the roles played by both demand-side and supply-side factors in driving housing cycles. On the demand side, Himmelberg, Mayer, and Sinai (2005) focus on interest rates, Poterba, Weil, and Shiller (1991) on tax changes, Mian and Sufi (2009) on credit expansion, and DeFusco, Nathanson, and Zwick (2017) and Gao, Sockin and Xiong (2017) on investment home purchases. On the supply side, Glaeser, Gyourko, Saiz (2008) emphasize supply as a key force in mitigating housing bubbles, Haughwout, Peach, Sporn and Tracy (2012) provide a detailed account of the housing supply side during the U.S. housing cycle in the 2000s, and Gyourko (2009b) systematically reviews the literature on housing supply. By introducing informational frictions, our analysis shows that supply-side and demand-side factors are not mutually independent. Supply shocks can affect housing and commercial real estate demand by act-

ing as informational noise in learning, and influence households' and commercial developers' expectations of the strength of the neighborhood.

2 The Model

The model has two periods $t \in \{1, 2\}$. There are three types of agents in the economy: households looking to buy homes in a neighborhood or elsewhere, home builders, and commercial real estate developers. Suppose that the neighborhood is new and all households purchase houses from home builders in a centralized market at $t = 1$ after choosing whether to live in the neighborhood. Households choose their labor supply and demand for commercial facilities, such as offices and warehouses, to complete production, and consume consumption goods at $t = 2$. Our intention is to capture the decision of a generation of home owners to move into a neighborhood, and we view the two periods as representing a long period in which they live together and share amenities, as well as exchange their goods and services.

2.1 Households

We consider a pool of households, indexed by $i \in [0, 1]$, each of which can choose to live in a neighborhood or elsewhere. We can divide the unit interval into the partition $\{\mathcal{N}, \mathcal{O}\}$, with $\mathcal{N} \cap \mathcal{O} = \varnothing$ and $\mathcal{N} \cup \mathcal{O} = [0, 1]$. Let $H_i = 1$ if household i chooses to live in the neighborhood, i.e., $i \in \mathcal{N}$, and $H_i = 0$ if it chooses to live elsewhere.[1] If household i at $t = 1$ chooses to live in the neighborhood, it must purchase one house at price P. This reflects, in part, that housing is an indivisible asset and a discrete purchase, consistent with the insights of Piazzesi and Schneider (2009).

Household i in the neighborhood has a Cobb-Douglas utility function over consumption of its own good C_i and its consumption of the goods produced by all other households in the neighborhood $\{C_j\}_{j \in \mathcal{N}}$:

$$U\left(\{C_j\}_{j \in \mathcal{N}}; \mathcal{N}\right) = \left(\frac{C_i}{1 - \eta_c}\right)^{1-\eta_c} \left(\frac{\int_{\mathcal{N}/i} C_j dj}{\eta_c}\right)^{\eta_c}. \tag{1}$$

The parameter $\eta_c \in (0, 1)$ measures the weights of different consumption components in the utility function. A higher η_c means a stronger complementarity between the consumption of household i and its consumption of the composite good produced by the other households

[1] See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.

in the neighborhood. As we will discuss later, this utility specification implies that each household cares about the strength of the neighborhood, i.e., the productivities of other households in the neighborhood. This assumption is motivated by the empirical findings of Ioannides and Zabel (2003), and leads to strategic complementarity in each household's housing demand.[2]

The production function of household i is also Cobb-Douglas $e^{A_i}K_i^{\alpha}l_i^{1-\alpha}$, where l_i is the household's labor choice and A_i is its productivity. Different from the usual production function of having capital as an input, we introduce another factor K_i as commercial facility with a share of $\alpha \in (0,1)$ in the production function. We broadly interpret the commercial facility as office space, infrastructure, or other investment households can use for their productive activities in the neighborhood. As we describe later, the households rent commercial facility from commercial developers. When households are more productive in the neighborhood, the marginal productivity of the commercial facility is higher, and consequently commercial develops would be able to develop more facility at higher prices. Introducing commercial facility allows us to discuss how information frictions affect price and supply of not only residential housing but also commercial real estate and other related investment in the neighborhood.[3]

Household i's productivity, A_i, is comprised of a component, A, common to all households in the neighborhood and an idiosyncratic component, ε_i:

$$A_i = A + \varepsilon_i,$$

where $A \sim \mathcal{N}\left(\bar{A}, \tau_A^{-1}\right)$ and $\varepsilon_i \sim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$ are both normally distributed and independent of each other. Furthermore, we assume that $\int \varepsilon_i d\Phi\left(\varepsilon_i\right) = 0$ by the Strong Law of Large Numbers. The common productivity, A, represents the strength of the neighborhood, as a higher A implies a more productive neighborhood. As A determines the households' aggregate demand for housing, it represents the demand-side fundamental. One can view τ_θ as a measure of household diversity.

As a result of realistic informational frictions, A is not observable to households at $t = 1$ when they need to make the decision of whether to live in the neighborhood. Instead, each

[2]There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2003).

[3]One can extend our analysis to consider K to be a public good, in which case its price is the tax a local government that faces a balanced budget can raise to offset the cost of construction. Our model then has implications for how housing markets impact local government fiscal policy.

household observes its own productivity A_i, after examining what it can do if it chooses to live in the neighborhood. Intuitively, A_i combines the strength of the neighborhood A and the household's own attribute ε_i. Thus, A_i also serves as a noisy private signal about A at $t = 1$, as the household cannot fully separate its own attribute from the opportunity provided by the neighborhood. The parameter τ_θ governs both the diversity in the neighborhood, or dispersion in productivity, and the precision of this private signal. As $\tau_\theta \to \infty$, the households' signals become infinitely precise and the informational frictions about A vanish. Households care about the strength of the neighborhood because of complementarity in their demand for consumption. Consequently, while a household may have a fairly good understanding of its own productivity when moving into a neighborhood, complementarity in consumption demand motivates it to pay attention to housing prices to learn about the average level A for the neighborhood.

We start with each household's problem at $t = 2$ and then go backwardly to describe its problem at $t = 1$. At $t = 2$, we assume that A is revealed to all agents. Furthermore, we assume that each household experiences a disutility for labor $\frac{l_i^{1+\psi}}{1+\psi}$, and that a household in the neighborhood $\mathcal{N}$ maximizes its utility at $t = 2$ by choosing labor l_i, commercial facility K_i, and its consumption demand $\{C_j\}_{j\in\mathcal{N}}$:

$$\begin{aligned} U_i \;=\; & \max_{\left\{\{C_j\}_{j\in\mathcal{N}}, l_i, K_i\right\}} U\left(\{C_j\}_{j\in\mathcal{N}}; \mathcal{N}\right) - \frac{l_i^{1+\psi}}{1+\psi} \\ & \text{such that } \; p_i C_i + \int_{\mathcal{N}/i} p_j C_j(i)\, dj + P + RK_i = p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} + \Pi_i, \end{aligned} \tag{2}$$

where p_i is the price of the good it produces, P is the housing price in the neighborhood, and R is the unit price of commercial facility. Households behave competitively and take the prices of their goods as given. For simplicity, we assume that each home builder and commercial developer is part of a household in the neighborhood, similar to the Lucas household paradigm, and that the builder and commercial developer bring home their profit $\Pi_i = P + RK_i$ to the household after construction has taken place. This allows us to focus on distortions to household behavior from informational frictions at $t = 1$ in the absence of the mechanical impact on wealth from the purchase of a house. As a result, at $t = 2$, household i's budget constraint simplifies to $p_i C_i + \int_{\mathcal{N}/i} p_j C_j(i)\, dj = p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$, although this simplification does not affect the household's choice of K_i. We further normalize the interest rate from $t = 1$ to $t = 2$ to be zero.

At $t = 1$, before choosing its consumption, commercial facility usage, and labor supply, a household need to decide whether to live in the neighborhood. In addition to their private signals, all households and commercial developers observe a noisy public signal, Q, about the strength of the neighborhood A :

$$Q = A + \tau_Q^{-1/2} \varepsilon_Q,$$

where $\varepsilon_Q \sim \mathcal{N}(0, 1)$ is independent of all other shocks. As τ_Q becomes arbitrarily large, A becomes common knowledge to all agents.

In addition to the utility flow U_i at $t = 2$ from final consumption, we assume that households have quasi-linear expected utility at $t = 1$, and incur a linear utility penalty equal to the housing price P if they choose to live in the neighborhood and thus have to buy a house. Given that households have Cobb-Douglas preferences over their consumption, they are effectively risk-neutral at $t = 1$, and their utility flow is then the value of their final consumption bundle less the cost of housing.[4] Households make their neighborhood choice subject to a participation constraint that their expected utility from moving into the neighborhood $E[U_i|\mathcal{I}_i] - P$ must (weakly) exceed a reservation utility, which we normalize to 0. One can interpret the reservation utility as the expected value of getting a draw of productivity from another potential neighborhood less the cost of search. Household i makes its neighborhood choice:

$$\max_{H_i} \{E[U_i|\mathcal{I}_i] - PH_i, 0\} \tag{3}$$

The choice of neighborhood is made at $t = 1$ subject to each household's information set $\mathcal{I}_i = \{A_i, P, Q\}$, which includes its private productivity signal A_i, the public signal Q, and the housing price P.[5]

2.2 Commercial Developers

In addition to households, there is a continuum of risk-neutral commercial developers that develop commercial facilities at $t = 1$, and sells them to households for their production at $t = 2$. The representative developer cares about R, the price of commercial facility at $t = 2$, which depend on the marginal productivity of the facility. This, in turn, depends on the

[4] As each household takes the transfer from the home builder in its family as given, the utility cost of buying the house can be thought of as a mental cost.

[5] We do not include the volume of housing transactions in the information set as a result of a realistic consideration that, in practice, people observe only delayed reports of total housing transactions at highly aggregated levels, such as national or metropolitan levels.

strength of the neighborhood, and which households choose to live in the neighborhood. The housing price in the neighborhood serves as a useful signal to the developer when deciding how much commercial facilities to develop at $t = 1$.

To simplify our analysis, and distinguish our mechanism from that of Rosen (1979) and Roback (1982), we decouple the supply of residential housing from the supply of commercial real estate. We assume that commercial developers can develop K units of commercial facility by incurring a convex effort cost $\frac{1}{\lambda}K^{\lambda}$, where $\lambda > 1$.

We assume that households buy commercial facility from commercial developers when production occurs at $t = 2$, and that commercial developers must forecast this demand when choosing how much commercial facility K to develop at $t = 1$. The representative commercial developer takes the commercial facility price R as given, and chooses K to the maximize its expected profit:

$$\Pi_c = \sup_K E\left[RK - \frac{1}{\lambda}K^{\lambda}\middle|\,\mathcal{I}^c\right] \tag{4}$$

where $\mathcal{I}^c = \{P, Q\}$ is the public information set, which include the housing price P and the public signal Q. It then follows that the optimal choice of commercial facility sets the marginal cost, $K^{\lambda-1}$, equal to the marginal benefit, $E\left[R|\,\mathcal{I}^c\right]$:

$$K = E\left[R|\,\mathcal{I}^c\right]^{\frac{1}{\lambda-1}}.$$

The choice of commercial facility is influenced by the expectation of the commercial developer about future neighborhood productivity, which is affected by the realization of the housing price P. Market-clearing in the market for commercial facility at $t = 2$ requires that:

$$\int_{\mathcal{N}} K_i di = K \int_{\mathcal{N}} di, \tag{5}$$

where $\int_{\mathcal{N}} di$ represents the population of households that live in the neighborhood.

The commercial developers' decision to develop commercial facility at $t = 1$ gives another source of amplification for informational frictions. In addition to distorting neighborhood choice of potential household entrants, informational frictions in housing markets also distort costly investment in resources that foster economic growth in the neighborhood. In the absence of informational frictions, housing prices may distort neighborhood choice but not expectations about neighborhood productivity.

2.3 Home Builders

There is a population of home builders, indexed on a continuum $[0,1]$, in the neighborhood. Home builders also face uncertainty about the aggregate strength of the neighborhood and the ability of the supply side to respond to the demand. Specifically, builder i builds a single house subject to a disutility from labor:

$$e^{-\frac{1}{1+k}\omega_i} S_i,$$

where $S_i \in \{0,1\}$ is the builder's decision to build and

$$\omega_i = \xi + e_i$$

is the builder's productivity, which is correlated across builders in the neighborhood through ξ. We assume that $\xi = k\zeta$, where $k \in (0,\infty)$ is a constant parameter, and ζ represents an unobserved, common shock to building cost in the neighborhood. From the perspective of households and builders, $\zeta \sim \mathcal{N}\left(\bar{\zeta}, \tau_\zeta^{-1}\right)$. Then $\xi = k\zeta$ can be interpreted as a supply shock with normal distribution $\xi \sim \mathcal{N}\left(\bar{\xi}, k^2\tau_\zeta^{-1}\right)$ with $\bar{\xi} = k\bar{\zeta}$. Furthermore, $e_i \sim \mathcal{N}\left(0, \tau_e^{-1}\right)$ such that $\int e_i d\Phi\left(e_i\right) = 0$ by the Strong Law of Large Numbers.

Builders in the neighborhood at $t = 1$ maximize their revenue:

$$\Pi_s\left(S_i\right) = \max_{S_i} \left(P - e^{-\frac{1}{1+k}\omega_i}\right) S_i. \tag{6}$$

Since builders are risk-neutral, it is easy to determine the builders' optimal supply curve:

$$S_i = \begin{cases} 1 & \text{if } P \geq e^{-\frac{k\zeta+e_i}{1+k}} \\ 0 & \text{if } P < e^{-\frac{k\zeta+e_i}{1+k}} \end{cases}. \tag{7}$$

The parameter k measures the supply elasticity of the neighborhood. A more elastic neighborhood has a larger supply shock, i.e., the supply shock has greater mean and variance. In the housing market equilibrium, the supply shock ξ not only affects the supply side but also the demand side, as it acts as informational noise in the price signal when the households use the price to learn about the common productivity A.

2.4 Noisy Rational Expectations Cutoff Equilibrium

Our model features a noisy rational expectations cutoff equilibrium, which requires clearing of the two real estate markets that is consistent with the optimal behavior of households, homer builders and commercial developers:

- Household optimization: each household chooses H_i at $t = 1$ to solve its maximization problem in (3), and then chooses $\left\{\{C_i\}_{i\in\mathcal{N}}, l_i, K_i\right\}$ at $t = 2$ to solve its maximization problem in (2).

- Commercial developer optimization: the representative developer chooses K at $t = 1$ to solve its maximization problem in (4).

- Builder optimization: each builder chooses S_i at $t = 1$ to solve his maximization problem in (6).

- At $t = 1$, the residential housing market clears:

$$\int_{-\infty}^{\infty} H_i\left(A_i, P, Q\right) d\Phi\left(\varepsilon_i\right) = \int_{-\infty}^{\infty} S_i\left(\omega_i, P, Q\right) d\Phi\left(e_i\right),$$

  where each household's housing demand $H_i\left(A_i, P, Q\right)$ depends on its productivity A_i, the housing price P, and the public signal Q, and each builder's housing supply $S_i\left(\omega_i, P, Q\right)$ depends on its productivity ω_i, the housing price P, and the public signal Q. The demand from households and supply from builders are integrated over the idiosyncratic components of their productivities $\{\varepsilon_i\}_{i\in[0,1]}$ and $\{e_i\}_{i\in[0,1]}$, respectively.

- At $t = 2$, the market for each household's good clears:

$$\int_{\mathcal{N}} C_i\left(j\right) dj = e^{A_i} K_i^{\alpha} l_i^{1-\alpha}, \quad \forall\ i \in \mathcal{N},$$

  and the market for commercial facility clears:

$$\int_{\mathcal{N}} K_i di = K \int_{\mathcal{N}} di.$$

3 Equilibrium

In this section, we analyze a symmetric cutoff equilibrium, in which the choice of each household to live in the neighborhood is monotonic with respect to its own productivity A_i.

3.1 Choices of Households and Commercial Developers

We first analyze household choices. At $t = 2$, households need to make their production and consumption decisions, after the strength of the neighborhood A is revealed to the public, and home builders and commercial developers have also made their choices at $t =$

1. Household i has $e^{A_i}K_i^{\alpha}l_i^{1-\alpha}$ units of good i for consumption and trading with other households. It maximizes its utility function given in (2). The following proposition describes the household's consumption, labor, and commercial facility choices. Its marginal utility of goods consumption also gives the equilibrium goods price.

Proposition 1 *Households i's optimal goods consumption and labor supply at $t=2$ are:*

$$\begin{aligned}
C_i(i) &= (1-\eta_c)\, e^{A_i}K_i^{\alpha}l_i^{1-\alpha},\ C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\eta_c e^{A_j}K_j^{\alpha}l_j^{1-\alpha},\\
\log l_i &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\,\psi+(1+\alpha\psi)\,\eta_c}\frac{\eta_c}{\psi}A+\frac{1-\eta_c}{(1-\alpha)\,\psi+(1+\alpha\psi)\,\eta_c}A_i-\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log R\\
&\quad+\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}+l_0.
\end{aligned}$$

The price of the good produced by household i is:

$$p_i = e^{\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c(A-A_i)+\frac{1}{2}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}}\left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\right)^{\eta_c},$$

and its optimal choice of commercial facilities is:

$$\begin{aligned}
\log K_i &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\,\psi+(1+\alpha\psi)\,\eta_c}\frac{1+\psi}{\psi}\eta_c A+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\,\psi+(1+\alpha\psi)\,\eta_c}A_i-\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log R\\
&\quad+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}+h_0.
\end{aligned}$$

Furthermore, the expected utility of household i at $t=1$ is given by:

$$E\left[U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right)-\frac{l_i^{1+\psi}}{1+\psi}\middle|\,\mathcal{I}_i\right]=\frac{\psi+\alpha}{1+\psi}E\left[p_i e^{A_i}K_i^{\alpha}l_i^{1-\alpha}\middle|\,\mathcal{I}_i\right],$$

with constants l_0 and h_0 given in the Appendix.

Proposition 1 shows that each household spends a fraction $1-\eta_c$ of its wealth (excluding housing wealth) on consuming its own good $C_i(i)$ and a fraction η_c on goods produced by its neighbors $\int_{\mathcal{N}/i}C_j(i)\,dj$. When $\eta_c=1/2$, the household consumes its own good and the goods of its neighbors equally. The price of each good is determined by its output relative to that of the rest of the neighborhood. One household's good is more valuable when the rest

of the neighborhood produces more, and thus each household needs to take into account the labor decisions of the other households in its neighborhood when making its own decision. The proposition demonstrates that the labor chosen by a household is determined by not only by its own productivity e^{A_i} but also the aggregate production of other households in the neighborhood. This latter component arises from the complementarity in the utility function of the household. Note that in the expressions above A^* is the equilibrium threshold for each household to enter the neighborhood.

Proposition 1 also reveals that the optimal choice of labor for each household is log-linear with the strength of the neighborhood, A, its own productivity, A_i, and the logarithm of the commercial real estate price, $\log R$. The final (nonconstant) term reflects selection, in that only households with productivity above A^* enter the neighborhood. Since A is the mean of the distribution of household productivity, it shows up in this truncation. This proposition also demonstrates that household i's optimal choice of commercial facilities has a similar functional form, with the last (nonconstant) term reflecting selection into the neighborhood. Household i's optimal labor choice and demand for commercial facilities are increasing in the strength of the neighborhood A because of consumption complementarity, since a higher A represents improved trading opportunities, while they are decreasing in the price of commercial facilities $\log R$.

We now discuss each household's decision on whether to live in the neighborhood at $t = 1$ when it still faces uncertainty about A. As a result of Cobb-Douglas utility, the household is effectively risk-neutral over its aggregate consumption, and its optimal choice reflects the difference between its expected output in the neighborhood and the cost of living there, which is the price P to buy a house. This is because the household views the transfer from its home builder Π_i and its commercial developers RK_i as exogenous to whether it lives in the neighborhood. It then follows that household i's neighborhood decision is given by:

$$H_i = \begin{cases} 1 & \text{if } \frac{\psi+\alpha}{1+\psi} E\left[p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} \middle| \mathcal{I}_i\right] \geq P \\ 0 & \text{if } \frac{\psi+\alpha}{1+\psi} E\left[p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} \middle| \mathcal{I}_i\right] < P \end{cases}.$$

This decision rule for neighborhood choice supports our conjecture to search for a cutoff strategy for each household, in which only households with productivity above a critical level, A^*, enter the neighborhood. This cutoff is eventually solved as a fixed point in the equilibrium.

Given each household's equilibrium cutoff A^* at $t = 1$ and optimal choices at $t = 2$, we can impose market-clearing in the market for commercial facilities to arrive at its price R at $t = 2$. Commercial developers forecast this price when choosing their optimal stock of commercial facilities to develop at $t = 1$. These observations are summarized by the following proposition.

Proposition 2 *Given K units of commercial facilities developed by commercial developers, the price of commercial facilities R at $t = 2$ takes the log-linear form:*

$$\begin{aligned}\log R &= \frac{1+\psi}{\psi+\alpha}A - (1-\alpha)\frac{\psi}{\psi+\alpha}\log K + \frac{1+\psi}{\psi+\alpha}\eta_c \log \frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)} \\ &\quad + (1-\alpha)\frac{\psi}{\psi+\alpha}\log \frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)} + r_0,\end{aligned}$$

with constant r_0 given in the Appendix. The optimal supply of commercial facilities by commercial developers at $t = 1$ is given by:

$$\begin{aligned}\log K &= \frac{1}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}}\log E\left[e^{\frac{1+\psi}{\psi+\alpha}A}\left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\right)^{\frac{1+\psi}{\psi+\alpha}\eta_c}\right. \\ &\quad \left.\cdot\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\right)^{\frac{\psi(1-\alpha)}{\psi+\alpha}}\middle|\,\mathcal{I}^c\right] + k_0,\end{aligned} \tag{8}$$

with constant k_0 given in the Appendix.

Proposition 2 reveals that the commercial real estate price at $t = 2$ is increasing in the strength of the neighborhood, A, with the last two (nonconstant) terms reflecting selection by households into the neighborhood, and is decreasing in the supply of commercial facilities K. It also demonstrates that the optimal supply of commercial facilities reflects expectations over not only the strength of the neighborhood, A, but also the impact of truncation from the neighborhood choice of households on the expected price of commercial facilities at $t = 2$. The expectation term captures not only the expected productivity from the terms-of-trade (relative prices of household goods) in the first ratio, but also the dispersion in labor productivity in the second ratio.

3.2 Perfect-Information Benchmark

With perfect information, all households, home builders, and commercial developers observe the strength of the neighborhood A when making their neighborhood choice decision. It is straightforward to show that the optimal choice of commercial facility K simplifies to:

$$\begin{aligned}\log K &= \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A+\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\eta_c\left\{\log\left[\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right]\right.\\ &\left.+(1-\alpha)\frac{\psi}{1+\psi}\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\log\left[\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right]\right\}+k_0,\end{aligned}$$

where k_0 is given in the Appendix and $\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}>0$ since $\lambda-\alpha\frac{1+\psi}{\psi+\alpha}>\lambda-1>0$.

Similar to the labor choice of households from Proposition 1, the supply of commercial facility is log-linear with respect to the strength of the neighborhood, A, with a correction term for the truncation in the household population that occurs because of household selection into the neighborhood. This truncation term reflects two forces. The first is that a smaller population implies less demand for a given choice of commercial facility per household, while the second reflects that the price at which households charge each other for their goods, p_i, is also affected by this truncation.

We now characterize the neighborhood choice of households and the housing price. Households will sort into the neighborhood according to a cutoff equilibrium determined by the net benefit of living in the neighborhood, which trades off household income opportunities with other households in the neighborhood with the price of housing. Despite the inherent nonlinearity of our framework, we derive a tractable, unique cutoff equilibrium that is characterized by the solution to a fixed-point problem over the endogenous cutoff of entry in the neighborhood, A^*. This is summarized in the following proposition.

Proposition 3 *In the absence of informational frictions, there exists a unique cutoff equilibrium in which the following hold: 1) household i follows a cutoff strategy in its neighborhood choice such that:*

$$H_i=\begin{cases}1 & \text{if } A_i\geq A^*\\ 0 & \text{if } A_i<A^*\end{cases},$$

where $A^*(A,\xi)$ solves equation (17) in the Appendix; 2) the housing price takes the log-linear form:

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}(A - A^*) - \xi\right);$$

3) the cutoff productivity $A^*(A,\xi)$ is monotonically decreasing in ξ and increasing in A; 4) the population that enters the neighborhood is monotonically increasing in both A and ξ; and 5) the housing price P, and consequently the utility of the household with the cutoff productivity, A^*, is increasing and convex in A.

Proposition 3 characterizes the cutoff equilibrium in the economy in the absence of informational frictions, and confirms the optimality of a cutoff strategy for households in their neighborhood choice. Households sort based on their individual productivity into the neighborhood, with the most productive, who expect the most gains from living in the neighborhood, entering and participating in production at $t = 2$. This determines the supply of labor at $t = 2$, and, through this channel, the price of commercial facilities at $t = 2$.

Given a cutoff productivity $A^*(A,\xi)$, the housing price P positively loads on the strength of the neighborhood, A, since a higher A implies stronger demand for housing, and loads negative on the supply shock ξ, reflecting that a discount is needed to ensure that a positive shift in housing supply is absorbed by a larger household population. As one would expect, the cutoff A^* enters negatively into the price since households above the cutoff sort into the neighborhood. The higher the cutoff, the fewer the households that enter the neighborhood, and the lower the housing price that is needed to clear the market with the lower housing demand. Despite its log-linear representation, the housing price is actually a generalized linear function of $\sqrt{\frac{\tau_\theta}{\tau_e}}A - \xi$, since A^* is an implicit function of A and $\log P$.

Finally, the last part of the proposition provides comparative statics of the cutoff household that enters the neighborhood with productivity $A^*(A,\xi)$. This cutoff is decreasing in ξ, since a lower house price causes more households to enter the neighborhood for a given neighborhood strength A, and consequently a higher population enters the neighborhood as ξ increases. The cutoff, in contrast, is increasing in neighborhood strength A, since a higher A implies a higher housing price, and can also raise the price of commercial facilities, depending on the supply response of commercial developers. This dominates the countervailing force that a higher A also signals more gains from trade from complementarity in household consumption decisions, and more commercial facilities developed by commercial developers. Though the cutoff productivity increases, more households ultimately enter the

neighborhood because a higher A shifts right (in the sense of FOSD) the distribution of households more than it moves the cutoff.

As a result of endogenous selection into the neighborhood, the productivity of the neighborhood is determined by which households choose to live there. The aggregate productivity of the neighborhood $A_{\mathcal{N}}$ is given by:

$$A_{\mathcal{N}} = \log \int_{A^*}^{\infty} e^{A_j} d\Phi\left(\varepsilon_j\right) = A + \frac{1}{2}\tau_\theta^{-1} + \log \Phi\left(\tau_\theta^{-1/2} + \frac{A - A^*}{\tau_\theta^{-1/2}}\right).$$

The first two terms would be what one would expect without neighborhood choice, while the third term reflects that productivity is truncated by selection. Importantly, since $A^* = A^*\left(A, \xi\right)$, it follows that A^* depends on the housing price in the neighborhood, introducing feedback from housing markets to real decisions. Similar aggregation results exist for total income $\int_{\mathcal{N}} e^{A_j} p_i K^{\alpha} l_j^{1-\alpha} d\Phi\left(\varepsilon_j\right)$ and labor supply $\int_{\mathcal{N}} l_j d\Phi\left(\varepsilon_j\right)$ as well.

3.3 Cutoff Equilibrium with Informational Frictions

Having characterized the perfect-information benchmark equilibrium, we now turn to the equilibrium at $t = 1$ in the presence of informational frictions. With informational frictions, households and developers must now forecast the strength of the neighborhood, A, and the realized price of commercial facilities R, when choosing whether to live in the neighborhood, and when deciding the stock of commercial facilities to develop. We again conjecture a cutoff equilibrium and, in addition, now conjecture that the housing price contains a sufficient statistic, $z\left(P\right)$ that is linear in A and in the supply shock ξ :

$$z\left(P\right) = A + \frac{1}{z_\xi}\left(\xi - \bar{\xi}\right).$$

In addition to the housing price, each household's type, A_i, also acts as a private signals about the strength of the neighborhood, A. Since types are positively correlated with this common productivity, higher types also have more optimistic expectations about A. As such, we anticipate that households will again follow a cutoff strategy when deciding whether to live in the neighborhood.

By solving for the learning of households and commercial developers, and clearing the aggregate housing demand of the households with the supply from home builders, we derive the housing market equilibrium. The following proposition summarizes the housing price from each household's housing demand, and the supply of commercial facilities in this equilibrium.

Proposition 4 *There exists a cutoff equilibrium in the presence of informational frictions in which the following hold: 1) the posterior of household i after observing housing price P, the household public signal Q, and its own productivity A_i is conditionally Gaussian with conditional mean $\hat{A}_i$ and variance $\hat{\tau}_A$ given by:*

$$\begin{aligned}\hat{A}_i &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A}+\tau_Q Q+\frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(\log P+\bar{\xi}\right)+A^*\right)+\tau_\theta A_i\right),\\ \hat{\tau}_A &= \tau_A+\tau_Q+\frac{\tau_\theta}{\tau_e}\tau_\xi+\tau_\theta,\end{aligned}$$

and the posterior of commercial developers is conditionally Gaussian with conditional mean $\hat{A}^c$ and variance $\hat{\tau}_A^c$ given by:

$$\begin{aligned}\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A}+\tau_Q Q+\frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(\log P+\bar{\xi}\right)+A^*\right)\right),\\ \hat{\tau}_A^c &= \tau_A+\tau_Q+\frac{\tau_\theta}{\tau_e}\tau_\xi;\end{aligned}$$

2) household i follows a cutoff strategy in its neighborhood choice such that:

$$H_i=\begin{cases}1 & \text{if } A_i\geq A^*\\ 0 & \text{if } A_i<A^*\end{cases},$$

where $A^*(A,Q,\xi)$ solves equation (19) in the Appendix; 3) the equilibrium commercial facility supply takes the form:

$$\log K=\frac{1}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A^*+k_0.$$

where $F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)$ is given in the Appendix, and $\log K$ is increasing in the conditional belief of the commercial developers, $\hat{A}^c$; 4) the housing price takes the log-linear form:

$$\log P=\frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A-A^*\right)-\xi\right);$$

and 5) the equilibrium converges to the perfect-information benchmark in Proposition 3 as $\tau_Q\nearrow\infty$.

Proposition 4 establishes the analogue of the cutoff equilibrium in the presence of informational frictions. Households now must form beliefs about the strength of the economy when deciding whether to live in the neighborhood. From equation (18) in the Appendix, we can express $A^*(A,Q,\xi)$ as $A^*(\log P,Q)$, which verifies that the cutoff is indeed measurable to households when making their decisions at $t=1$. Consequently, after observing the

housing price P and the public signal Q, they know the exact value of A^* and form their conditional estimate of A.

Importantly, informational frictions provide a channel for housing prices to feed into the supply decision of commercial developers through learning. With perfect information, housing prices only impact the supply of commercial facility by altering the cost that households pay to enter the neighborhood. Since the price of commercial facility depends on the imperfectly observed strength of the neighborhood, commercial developers use the housing price as a signal when forming their beliefs about this strength. Consequently, any noise in this signal will distort commercial developers' optimal choice of how much commercial facilities to develop.

Though optimal policies are qualitatively similar to those in the perfect-information benchmark, these policies are distorted by informational frictions. The cutoff household productivity A^* now reflects that households and commercial developers imperfectly observe the neighborhood's strength, and use the housing price to form their beliefs. The distortion to both beliefs and the cutoff productivity then feed back into the optimal choice of commercial facilities supplied at $t = 1$, amplifying the impact of informational frictions.

The complementarity between households is even more important for a cutoff strategy in the presence of informational frictions. Without complementarity, a stronger neighborhood, i.e., higher A, is bad news for households, since a higher A raises housing prices, and can also raise the price of commercial facilities depending on the supply response of commercial developers. Furthermore, a higher household type A_i signals a stronger neighborhood, which gives a household with a high type A_i less incentive to enter the neighborhood. With complementarity, however, a stronger neighborhood is also good news for households, since it signals that other households in the neighborhood will be more productive, and household i will get a better terms-of-trade when it trades with other households at $t = 2$. A higher A_i, consequently, can be good news since it signals a stronger neighborhood. This latter effect reinforces the cutoff equilibrium.

Informational frictions distort housing prices by shifting the productivity cutoff $A^* = A^*(\log P, Q)$. With perfect information, $\frac{dA^*(\log P,Q)}{d\log P} > 0$ captures only the direct effect that a higher price deters households from entering the neighborhood because of the cost effect, shifting up the productivity of the marginal household. In the presence of informational frictions, however, $\frac{dA^*(\log P,Q)}{d\log P}$ also reflects an information effect that prices act as a positive

signal about the strength of the neighborhood, which lowers $\frac{dA^*(\log P,Q)}{d\log P}$ compared to its perfect-information benchmark value because it also encourages more households to enter the neighborhood.

The sensitivity of the price to changes in $z(P) = A - \sqrt{\frac{\tau_e}{\tau_\theta}}\xi$ (i.e., the sufficient statistic of P), by the Implicit Function Theorem, is:

$$\frac{d\log P}{dz} = \left((1+k)\sqrt{\frac{\tau_e}{\tau_\theta}} + \frac{dA^*(\log P, Q)}{d\log P}\right)^{-1}.$$

As $\frac{d\log P}{dz}$ is greater in the presence of informational frictions, this effect can make housing prices more volatile, as highlighted by Albagli, Hellwig, and Tsyvinski (2015) in their analysis of the cutoff equilibrium in an asset market. This interesting feature also differentiates our cutoff equilibrium from other type of non-linear equilibrium with asymmetric information, such as the log-linear equilibrium developed by Sockin and Xiong (2015) to analyze commodity markets. In their equilibrium, prices become less sensitive to their analogue of z in the presence of informational frictions. This occurs because households, on aggregate, underreact to the fundamental news in the their private signals because of noise, and this is reflected in a lower weight on z in prices compared to perfect-information.

To understand how housing prices impact learning, it is instructive to consider two polar cases for supply elasticity. When $k = 0$, then housing prices are only a function of the strength of the neighborhood, A, and prices are fully revealing to households and commercial developers. Informational frictions, as a result, unravel when supply is infinitely inelastic. When, instead, $k \to \infty$, then prices converge to $\log P = -\zeta$, and the housing price is driven only by the supply shock.[6] When supply is perfectly elastic, then prices contain no information about demand, and therefore no information about the strength of the neighborhood. Consequently, the information content of prices, and the weight that households and commercial developers assign to prices in forming their conditonal estimates of the neighborhood strength, dissipates as supply elasticity increases.

4 Model Implications

We now investigate several implications of our model regarding how informational frictions affect the dynamics of housing and commercial real estate markets. We provide comparative

[6] It is straightforward to see from equation (19) that A^* remains finite a.s. as $k \to \infty$, allowing us to take the limit.

statistics to illustrate how several key aspects of the neighborhood and its real estate markets vary across two dimensions: 1) supply elasticity k, and 2) the degree of consumption complementarity in household utility η_c. Supply elasticity is a natural candidate for classifying the cross-section of housing markets. It has been emphasized in the literature, in work including Malpezzi and Wachter (2005) and Glaeser, Gyourko, and Saiz (2008), to help explain certain features of housing cycles, such as housing price volatility. Similarly, the degree of complementarity captures the agglomeration and spillover effects that lead to coordination among firms and industries that locate in one area, such as the financial industry in New York City, the technology sector in San Francisco, the Research Triangle in North Carolina, and the oil industry in Houston. As emphasized, for instance, by Dougal, Parsons, and Titman (2015), employers and/or workers can benefit from locating in close proximity to competitors, either from knowledge spillovers or from the implicit insurance in labor markets.

While we have analytical expressions for most equilibrium outcomes, the key equilibrium cutoff A^* needs to be numerically solved from the fixed-point condition in equation (19). We therefore analyze the equilibrium properties of A^* and other variables through a series of numerical illustrations. The benchmark parameters we choose for the numerical examples are provided in Table 1. For the share of commercial facility in households' production, we treat it as being similar to capital, and select the typical estimate of $\alpha = 0.33$. For the Frisch elasticity of labor supply, we choose $\psi = 2.5$, which is within the typical range found in the literature. We set τ_ζ to be four-fold larger than τ_A to ensure that with perfect information, the log housing price variance is monotonically declining in supply elasticity, as is observed empirically. We set $\lambda = 1.1$ to have commercial facility be in elastic supply, and avoid having convexity in its production function. We choose for the neighborhood fundamentals $A = \zeta = -0.5$, though the qualitative patterns we highlight hold more generically for a wide range of shock values. In addition, we set the public signal Q to 0.

τ_A	0.50	τ_ζ	2.00	τ_θ	0.20
τ_θ	0.20	τ_Q	1.00	η_c	0.50
α	0.33	ψ	2.50	k	0.50
λ	1.10	$\overline{A}$	0	$\overline{\zeta}$	0

Table 1: Benchmark Parameters for Numerical Illustrations

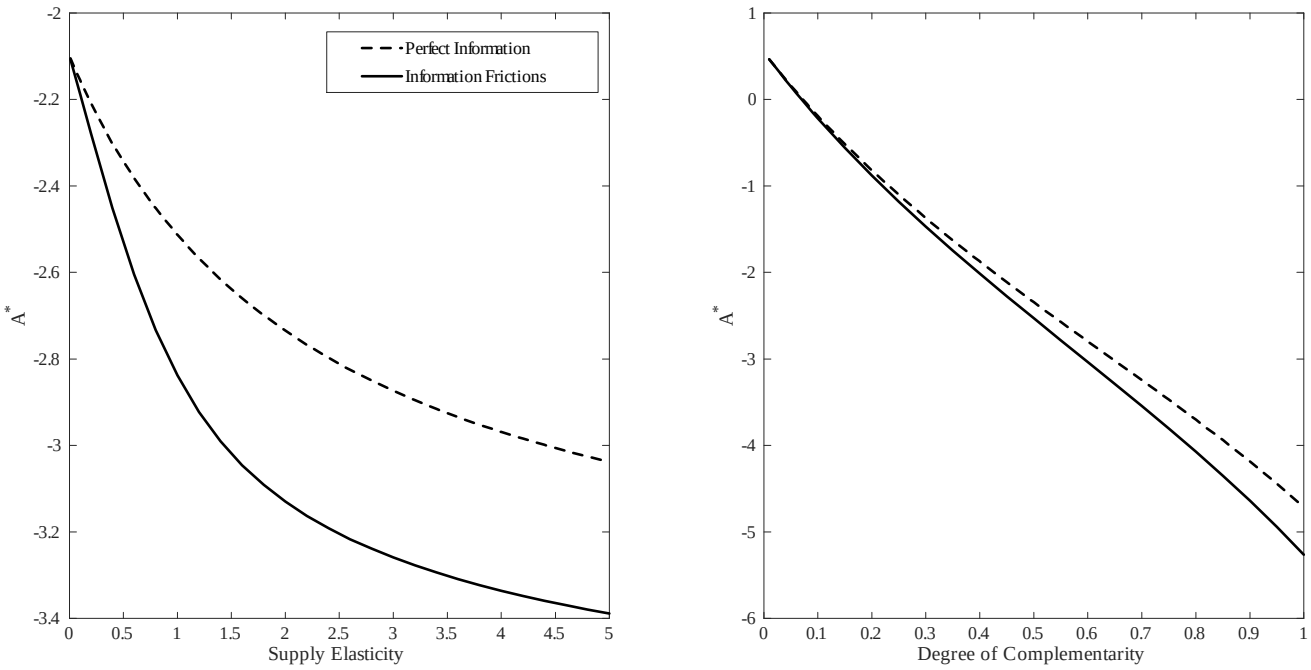


Figure 1: Equilibrium cutoff productivity across housing supply elasticity (left) and degree of complementarity (right).

4.1 Equilibrium Cutoff

Our model features an equilibrium cutoff productivity for the marginal household to enter the neighborhood, which hinges on the households' learning process about the neighborhood. This, in turn, determines the population flow into the neighborhood, and the dynamics of both housing and commercial real estate markets. As a consequence, the equilibrium productivity cutoff serves as a channel for informational frictions to impact the local economy.

In Figure 1, we illustrate how the equilibrium cutoff varies across housing supply elasticity (left panel) and the degree of households' consumption complementarity (right panel). To highlight the effects of informational frictions, we provide a benchmark with perfect information in both panels. With perfect information, as housing supply becomes more elastic, more households enter the neighborhood because housing is cheaper. This shifts down the cutoff productivity A^*, above which households enter the neighborhood. As the degree of consumption complementarity increases, the cutoff productivity also falls. This reflects that households derive more of their consumption from trading with other households, and thus benefit more from participating in the neighborhood for the same neighborhood strength A.

Interestingly, informational frictions increase the population flow into the neighborhood

by lowering the cutoff productivity, and this holds across both supply elasticity and the degree of consumption complementarity. Holding constant each household's productivity, uncertainty about the productivity of other households in the neighborhood (i.e., about A) benefits the household as follows. If ex post it turns out that other households are more productive, a higher A, then the household can sell its good at a higher price for other goods. On the other hand, if other households are less productive, the household is not hurt as much because it can purchase other goods at lower prices, even though its own good also now commands a lower price. In addition, the price at which it purchases commercial land is also cheaper when A is lower. Consequently, each household's utility has a convex shape with respect to A, and this intuition is confirmed for the perfect-information case in Proposition 3. This convexity implies that uncertainty induced by informational frictions lowers the cutoff productivity, by raising each household's utility, and this causes more households to enter the neighborhood. As consumption complementarity strengthens the households' incentives to trade goods, a greater degree of complementarity further intensifies this convexity and lowers the cutoff productivity, as shown in the right panel of Figure 1. Furthermore, as households use the observed housing price to learn about A, and the informativeness of the housing price deteriorates with supply elasticity, the uncertainty faced by households regarding A increases as supply elasticity increases. As a result, the left panel shows that the cutoff productivity decreases with supply elasticity further than in the perfect information case.

Figure 2 further illustrates how the cutoff productivity responds to random shocks. We focus on two types of shocks, a noisy demand shock and a housing supply shock. The first row considers a random shock to the public signal Q, which we consider as our noisy demand shock, and computes the response of the equilibrium cutoff to the shock by taking its partial derivative with respect to Q. In the absence of any informational frictions, there is no need for anyone to use the public signal to infer the neighborhood productivity A, and consequently the shock has no impact on households' neighborhood choice. In the presence of informational frictions, however, the shock affects households' expectations about A, as they use the public signal to infer the value of A. By making households more optimistic about A, a positive shock to Q raises each household's utility, and this lowers the cutoff productivity of the marginal household that enters the neighborhood. This induces a greater population flow to the neighborhood.

Interestingly, this learning effect is stronger when supply elasticity is greater (the upper-

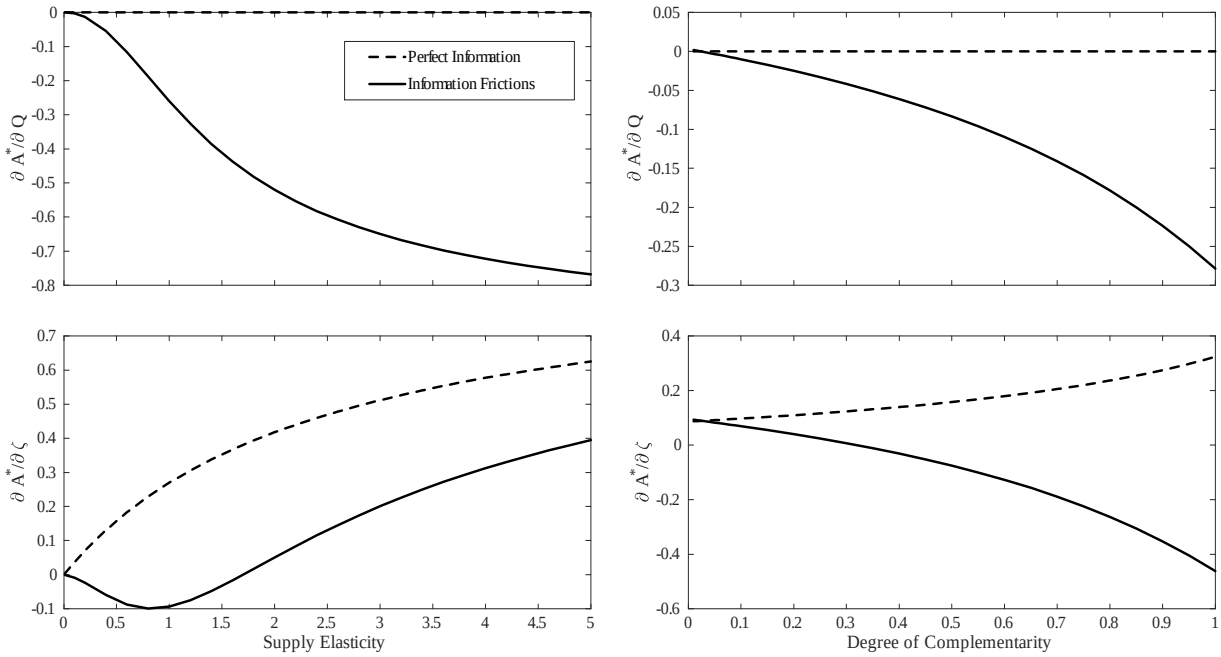


Figure 2: The Response of the equilibrium cutoff productivity to a noise demand shock (the first row) and a negative supply shock (the second row) across housing supply elasticity (left) and degree of complementarity (right).

left panel of Figure 2), or when the households' consumption elasticity is greater (the upper-right panel of Figure 2). The former result results from the fact that greater supply elasticity makes the housing price more dependent on supply-side factors, and therefore less informative of the neighborhood productivity. Consequently, households place a greater weight on the public signal Q in their learning about A, and this amplifies the effect of the noise shock to Q. The latter result is driven by the greater role that household learning plays as consumption complementarity increases, as a higher complementarity makes each household more concerned about the neighborhood's productivity.

The second row of Figure 2 considers another shock: a negative shock to the housing building cost ζ. In the perfect-information benchmark, the housing price increases with this negative supply shock, and the higher housing price, in turn, raises the cutoff productivity for the marginal household to move into the neighborhood. Furthermore, this price increase is greater when supply elasticity is larger, or if consumption complementarity is larger, further increasing the cutoff productivity. Interestingly, in the presence of informational frictions, there is an offsetting effect to the higher cost of housing. As the higher housing price is

partially interpreted as a signal for stronger neighborhood productivity, this learning effect makes households more willing to move into the neighborhood, putting downward pressure on the cutoff productivity. The lower-left panel shows that this learning effect is particularly strong for intermediate values of supply elasticity, and makes the cutoff productivity U-shaped with respect to supply elasticity, in contrast to the monotonically increasing shape in the perfect-information benchmark.

To understand this non-monotonic learning effect, it is useful to consider two limiting cases. As supply elasticity $k \to 0$, the equilibrium housing price is fully determined by the demand side and thus fully reveals the neighborhood productivity A, leaving no room for the supply shock to affect the households' expectations about A. At the other end, as $k \to \infty$, the equilibrium housing price is fully determined by the supply side and is uninformative of A. Consequently, the supply shock cannot affect the households' expectation of A through the learning channel either. Given the muted effect at these two ends, the supply shock has its greatest effect through the learning channel when supply elasticity is in an intermediate range.

The lower-right panel of Figure 2 shows that this learning effect strengthens monotonically with consumption complementarity, as complementarity makes each household more concerned about the neighborhood's productivity when choosing whether to enter the neighborhood. Interestingly, complementarity has such a strong impact on the role of learning in household entry that it makes the cutoff productivity decrease with the degree of complementarity, in contrast to the perfect-information benchmark, as the learning effect dominates the increased housing cost.

Relating the model to empirical predictions, the noise shock to Q represents a non-fundamental shift in housing demand. One may broadly interpret this non-fundamental demand shock, in practice, as originating from different sources. For instance, it can be noise in public information, as featured in Morris and Shin (2002) and Hellwig (2005), housing market optimism, as in Ferreira and Gyourko (2011), Gao, Sockin and Xiong (2017), and Kaplan, Mitman, and Violante (2017), or credit expansion from the banking sector, as in Mian and Sufi (2009). The supply cost shock represents another shock to housing markets that is independent of the neighborhood's economic strength. Our analysis thus illustrates a mechanism for these non-fundamental shocks to induce greater population flow into the neighborhood through the households' learning channel. One may be able to test

this effect across different regions with properly designed measures of these non-fundamental shocks. Housing supply elasticity can be measured, for instance, as in Saiz (2010), while complementarity in an area can be measured by its industry complementarity, as suggested by Dougal, Parsons, and Titman (2015). One can consequently directly test the cross-sectional implication of our model that non-fundamental shocks, such as the noise demand shock and the housing supply shock, have a greater impact in inducing stronger population inflow to areas with greater industry complementarity and intermediate supply elasticities.

4.2 Housing Cycle

We now examine the reaction of the housing market to the noise demand shock and the housing supply shock. Figure 3 illustrates the impacts of the noise shock to Q on the housing price and housing stock in the neighborhood, by computing their partial derivatives with respect to Q across different values of supply elasticity k in the two left panels, and across different values of the degree of consumption complementarity η_c in the two right panels. As we discussed earlier, in the absence of informational frictions, this shock has no effect on the housing market. In the presence of informational frictions, the noise shock raises both housing price and housing stock relative to the perfect-information benchmark, because the shock boosts the agents' expectations about the neighborhood's productivity. Interestingly, the upper-left panel shows that this effect on housing price is hump-shaped with respect to supply elasticity, and peaks at an intermediate value. This results from the non-monotonicity of the learning effect that we discussed earlier. When housing supply is either perfectly elastic or inelastic, the noise shock has a muted effect on households' expectations. This panel again illustrates that the learning distortion effect is strongest when supply elasticity is in an intermediate range.

The upper-right panel of Figure 3 shows that the effect of the noise shock on the housing price is increasing with the complementarity. As complementarity rises, each household cares more about trading goods with other households. This makes households' expectations about the neighborhood's productivity a greater determinant of the equilibrium housing price, since the housing price is equal to the utility of the marginal household with the cutoff productivity. This, in turn, causes the noise shock to have a greater effect on the housing price as complementarity increases. That the growth in housing stock is hump-shaped reflects that, near perfect complementarity, almost all households are already entering the neighborhood.

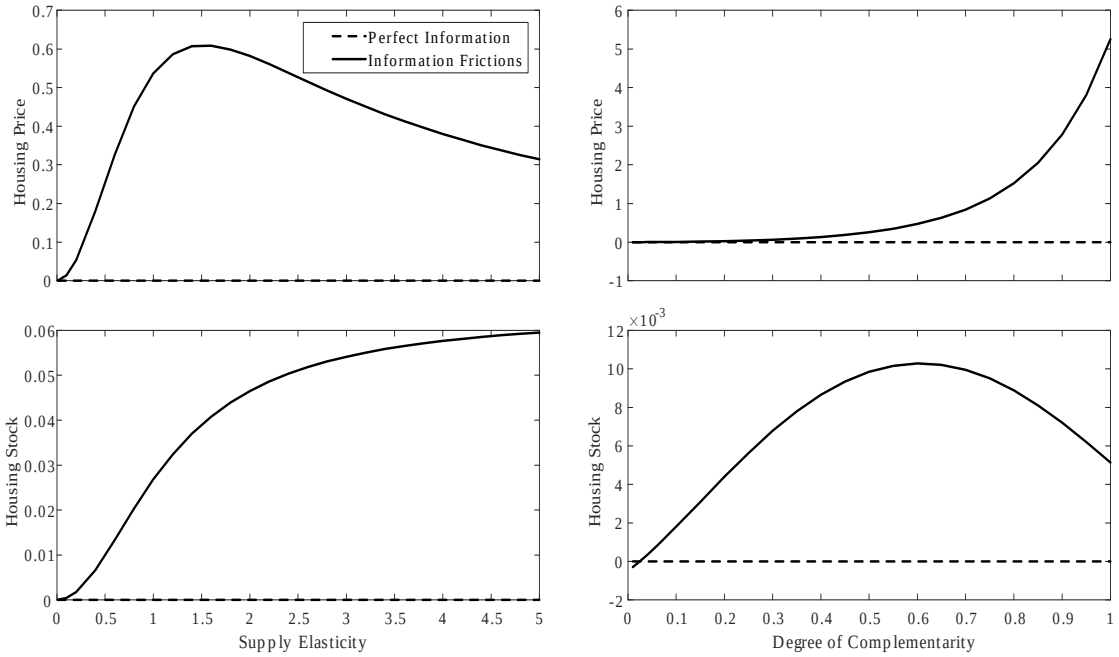


Figure 3: Housing responses to a noise shock to the public signal Q across supply elasticity (left) and degree of complementarity (right).

We now consider a negative shock to the building cost shock ζ. Figure 4 displays the responses of the housing price and housing stock to this shock across different values of supply elasticity k in the two left panels, and across different degrees of consumption complementarity η_c in the two right panels. In the perfect-information benchmark, the housing price increases with a negative supply shock, and the price increase rises with supply elasticity. In contrast, the housing stock falls with the negative supply shock since the higher housing price discourages more households from entering, and the supply drop is greater when supply elasticity is larger.

In the presence of informational frictions, the negative supply shock is, in part, interpreted by households as a positive demand shock when they observe a higher housing price. This expectational effect, in turn, pushes up the housing price and housing stock, relative to the perfect-information benchmark, as shown in the left panels of Figure 4. Across supply elasticity, these distortions are hump-shaped because the impact of learning from the housing price is most pronounced at intermediate supply elasticities, and, consequently, the response of the housing price and housing stock also peak at an intermediate range. As consumption

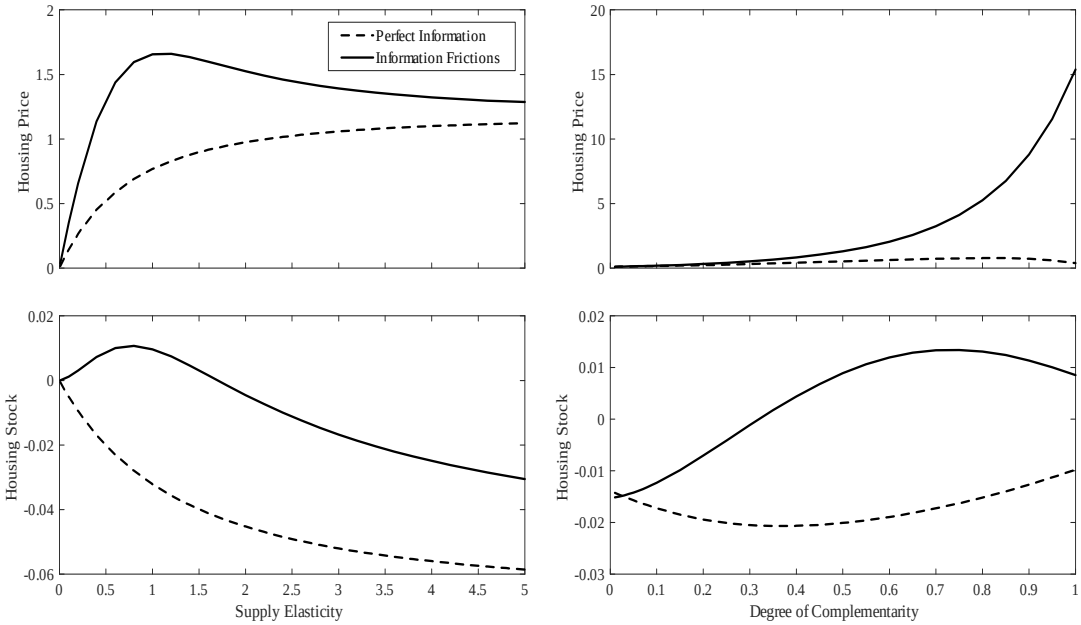


Figure 4: Housing responses to a negative supply shock across supply elasticity (left) and degree of complementarity (right).

complementarity increases, the learning effect from the negative supply shock is amplified, since households put more weight on the neighborhood's strength when determining whether to enter the neighborhood. This is shown in the upper-right panel of Figure 4. Similar to the noise demand shock, the impact on the housing stock is hump-shaped, since most households are already entering the neighborhood as η_c nears perfect complementarity.

While our model is static and cannot deliver a boom-and-bust housing cycle across periods, one may intuitively interpret the deviation of housing price induced by the noise demand shock and the supply shock from its value in the perfect-information benchmark in Figures 3 and 4 as a non-fundamental driven price boom, which would eventually reverse. Then, we have a testable implication for housing cycle—non-fundamental shocks, such as the noise demand shock and the supply shock, can lead to a more pronounced housing cycle in areas with greater industry complementarity and intermediate housing supply elasticities. This implication helps to explain why during the recent U.S. housing cycle, Las Vegas and Pheonix experienced more pronounced housing price boom and bust cycles than San Francisco and New York, which have more inelastic housing supply.

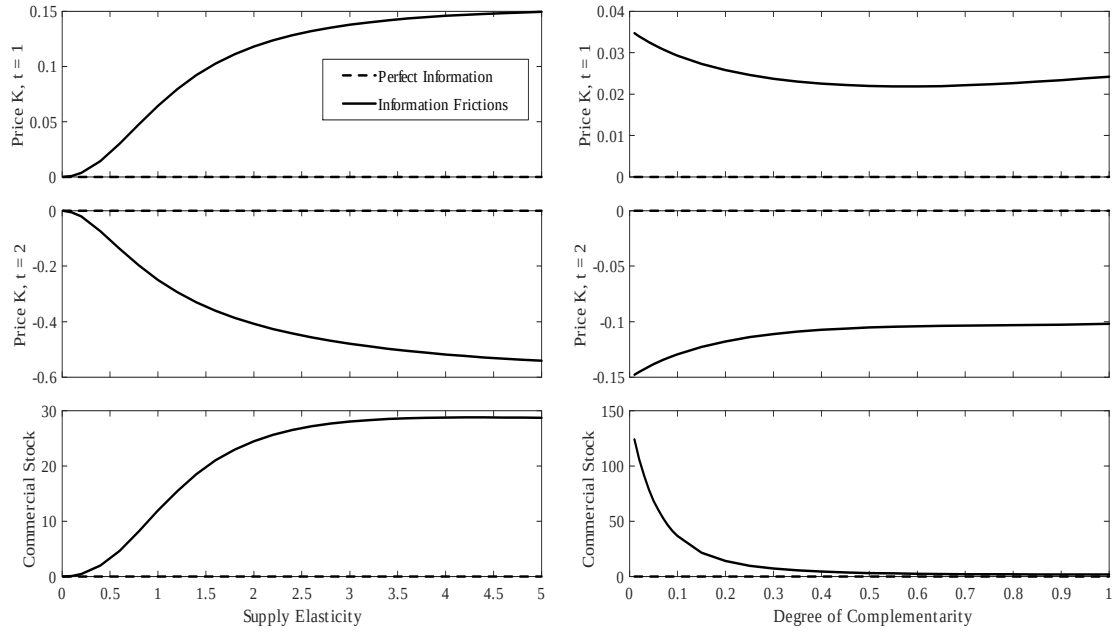


Figure 5: Neighborhood responses to a noise shock to the public signal Q across supply elasticity (left) and degree of complementarity (right).

4.3 Commercial Real Estate Cycle

By affecting agents' expectations, informational frictions not only distort the housing price and housing stock but also other investment decisions related to the neighborhood. The commercial real estate market featured in our model allows us to analyze such effects. We first analyze in Figure 5 how the price and stock of commercial facilities react to the noise demand shock across different values of supply elasticity k in the three left panels, and across the degree of consumption complementarity η_c in the three right panels. While households acquire commercial facility only at $t = 2$ at the price R, we can also compute the shadow price of commercial facility at $t = 1$ as the commercial developers' marginal development cost $K^{\lambda-1}$ when they develop the facility. This shadow price reflects the developers' expectations about the price that will prevail at $t = 2$. We depict this shadow price of the commercial facility at $t = 1$ in the first row of Figure 5, its market price R at $t = 2$ in the second row, and the stock of commercial facility K built by the developers at $t = 1$ in the third row.

As we discussed before, in the perfect-information benchmark, the noise demand shock has no impact on agents' expectations, and consequently no impact on the price and stock of commercial facilities. In the presence of informational frictions, however, the noise shock

boosts agents' expectations about A. As a result, it pushes up both the shadow price and supply of commercial facilities at $t = 1$, relative to the perfect-information benchmark. When the households come at to buy the commercial facility at $t = 2$, the market price is determined by their realized productivity, and thus falls to reflect that A had been overestimated at $t = 1$. As a result, the noise shock causes a boom in the market for the commercial facility at $t = 1$, in terms of both price and supply, and a bust at $t = 2$ when the price reverses.

Interestingly, the magnitude of this boom-and-bust cycle, measured by the deviation of the price response at either $t = 1$ or $t = 2$ from the perfect-information benchmark, is monotonically increasing with supply elasticity. As we discussed before, as supply elasticity rises, the housing price is driven more by supply side factors, and is thus less informative about the neighborhood productivity A. Consequently, the public signal Q gets a greater weight in the agents' learning process about A, giving the noise shock to Q a larger impact on the market for commercial facilities. With respect to consumption complementarity, the demand shock has the largest impact on commercial real estate at lower levels of complementarity. This occurs because the price of commercial facilities at $t = 2$ is less sensitive to the neighborhood strength A when there is more coordination among households in their production decisions, and because the average marginal product of commercial facilities is lower the more households that enter the neighborhood. In contrast to the housing market, the largest boom and bust in the shadow price and the subsequent market price R occurs at low levels of complementarity.

We now analyze how the commercial real estate market reacts to a negative supply shock ζ to the housing market in Figure 6, which shows the responses of the commercial facility's shadow price at $t = 1$ in the first row, its market price at $t = 2$ in the second row, and its supply at $t = 1$ in the third row, across housing supply elasticity in the three left panels and across households' consumption complementarity in the three right panels.

In the perfect-information benchmark, the negative supply shock only impacts the housing price, and, through this channel, the cutoff productivity of the households that enter the neighborhood. As is apparent, this direct effect has only a modest impact on the commercial real estate market. In the presence of informational frictions, its impact on the commercial real estate market is substantially larger. This occurs because the negative supply shock is partially interpreted by the agents as a positive shock to the neighborhood productivity when they learn from the housing price about the neighborhood productivity A. Consequently,

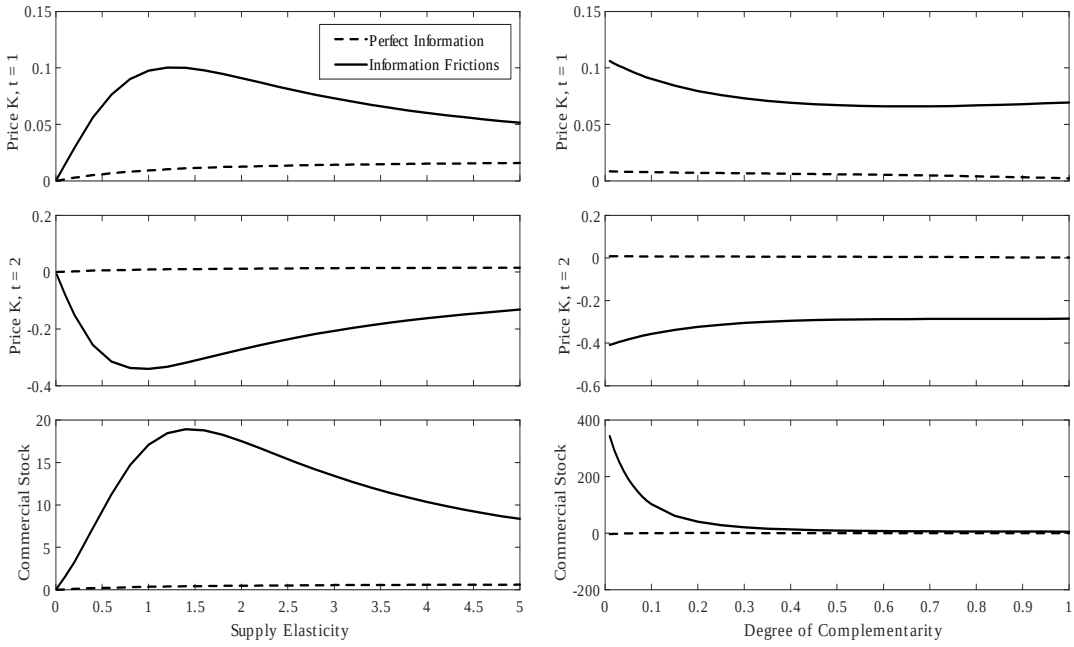


Figure 6: Neighborhood responses to a negative supply shock across supply elasticity (left) and degree of complementarity (right).

it distorts the agents' expectations about A upwardly, leading to overoptimism about the local economy. This results in both a higher shadow price and more supply of commercial facilities at $t = 1$, and a greater price reversal at $t = 2$. Interestingly, the magnitudes of these effects are all hump-shaped with respect to housing supply elasticity, as a result of the hump-shaped distortion to agents' expectations caused by their learning from the housing price. Similar to the noise demand shock, the negative supply shock distorts the commercial real estate market by leading to overoptimism about A, and it is most pronounced at low levels of consumption complementarity.

Our analysis shows that non-fundamental shocks to the housing market lead to not only a housing cycle, but also to a boom and bust in the market for commercial facilities. This is consistent with Gyourko (2009a), who highlights that the recent U.S. housing cycle was accompanied by a similar boom and bust in commercial real estate. Though also characterized by a dramatic run-up and collapse in prices, this second boom and bust, and its relation to local economic outcomes, have received less attention. It is difficult to simply attribute this commercial real estate boom to the subprime credit expansion that had played an important role for the housing boom, as the credit expansion was mainly targeting households. One

may attribute it to widespread optimism, and our model provides a coherent explanation for the shared optimism in both housing and commercial real estate markets. Specifically, our model shows that non-fundamental shocks may lead to joint cycles in housing and commercial real estate markets, especially in areas with intermediate values of housing supply elasticity.

5 Conclusion

In this paper, we introduce a model of information aggregation in housing and commercial real estate markets, and examine its implications for not only housing prices, but also economic outcomes such as neighborhood choice and the supply of commercial real estate. We provide empirical predictions for the expected response of neighborhoods to noise demand and supply shocks across supply elasticity and the degree of consumption complementarity, and offer a rationale for the synchronized boom and bust cycles observed in the U.S. housing and commercial real estate markets during the 2000s.

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Appendix Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions of household i's optimization problem in (2) respect to $C_i(i)$ and $C_j(i)$ at an interior point are:

$$C_i(i) \quad : \quad \frac{1-\eta_c}{C_i(i)} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_i, \tag{9}$$

$$C_j(i) \quad : \quad \frac{\eta_c}{\int_{\mathcal{N}/i} C_j dj} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_j, \tag{10}$$

where θ_i is the Lagrange multiplier for the budget constraint. Rewriting (10) as

$$\frac{\eta_c C_j}{\int_{\mathcal{N}/i} C_j dj} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) \;=\; \theta_i p_j C_j$$

and integrating over $\mathcal{N}$, we arrive at

$$\eta_c U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i \int_{\mathcal{N}/i} p_j C_j dj.$$

Dividing equations (9) by this expression leads to $\frac{\eta_c}{1-\eta_c} = \frac{\int_{\mathcal{N}/i} p_j C_j(i) dj}{p_i C_i(i)}$, which in a symmetric equilibrium implies $p_j C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \frac{\eta_c}{1-\eta_c} p_i C_i(i)$. By substituting this equation back to the household's budget constraint in (2), we obtain:

$$C_i(i) = (1-\eta_c) e^{A_i} K_i^\alpha l_i^{1-\alpha}.$$

The market-clearing for the household's good requires that $C_i(i) + \int_{\mathcal{N}/i} C_i(j)\, dj = e^{A_i} K_i^\alpha l_i^{1-\alpha}$, which implies that $C_i(j) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \eta_c e^{A_i} K_i^\alpha l_i^{1-\alpha}$.

The first order condition in equation (9) also gives the price of the good produced by household i. Since the household's budget constraint in (2) is entirely in nominal terms, the price system is only identified up to θ_i, the Lagrange multiplier. We therefore normalize θ_i to 1. It follows that:

$$p_i = \frac{1-\eta_c}{C_i(i)} U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right) = \left(e^{A_i} l_i^{1-\alpha} K_i^\alpha\right)^{-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} l_j^{1-\alpha} K_j^\alpha dj\right)^{\eta_c}. \tag{11}$$

Furthermore, given equation (1), it follows since $C_i(i) = (1-\eta_c)\, e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$ and $C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \eta_c e^{A_j} K_j^{\alpha} l_j^{1-\alpha}$ that:

$$\begin{aligned} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) &= \left(e^{A_i} l_i^{1-\alpha} K_i^{\alpha}\right)^{1-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} K_j^{\alpha} l_j^{1-\alpha} dj\right)^{\eta_c} \\ &= p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha}, \end{aligned}$$

from substituting with the household's budget constraint at $t = 2$.

The first-order conditions for household i's choice of l_i at an interior point is:

$$l_i^{\psi} = (1-\alpha)\,\theta_i p_i e^{A_i} \left(\frac{K_i}{l_i}\right)^{\alpha} = (1-\alpha)\frac{1-\eta_c}{C_i(i)} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) e^{A_i} \left(\frac{K_i}{l_i}\right)^{\alpha}. \tag{12}$$

from equation (9). Imposing $C_i = (1-\eta_c)\, e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$ in equation (12), and $U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$, it follows that:

$$\log l_i = \frac{1}{\psi+\alpha+(1-\alpha)\,\eta_c} \log(1-\alpha) + \frac{1}{\psi+\alpha+(1-\alpha)\,\eta_c} \log\left(\left(e^{A_i} K_i^{\alpha}\right)^{(1-\eta_c)} \left(\frac{\int_{\mathcal{N}/i} e^{A_j} K_j^{\alpha} l_j^{1-\alpha} dj}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\right)^{\eta_c}\right). \tag{13}$$

The optimal labor choice of household i, consequently, represents a fixed point problem over the optimal labor strategies of other households in the neighborhood.

Noting that $K_i = \left(\frac{\alpha p_i e^{A_i} l_i^{1-\alpha}}{R}\right)^{\frac{1}{1-\alpha}}$, we can substitute in the price function p_i to arrive at:

$$\begin{aligned} \log K_i &= \frac{1}{1-(1-\eta_c)\,\alpha} \log\left(\left(e^{A_i} l_i^{1-\alpha}\right)^{1-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} l_j^{1-\alpha} K_j^{\alpha} dj\right)^{\eta_c}\right) \\ &\quad - \frac{1}{1-(1-\eta_c)\,\alpha} \log R + \frac{1}{1-(1-\eta_c)\,\alpha} \log \alpha, \end{aligned} \tag{14}$$

which is a fixed-point problem for the optimal choice of commercial facility.

Given the optimal labor supply of household i l_i and optimal demand for commercial land K_i jointly satisfy the functional fixed-point equations (13) and (14), let us conjecture for i for which $A_i \geq A^*$, so that $i \in \mathcal{N}$ is in the neighborhood, that:

$$\log l_i = l_0 + l_A A + l_s A_i + l_R \log R + l_\Phi \log \frac{\Phi\left(\left(1+(\alpha h_s + (1-\alpha)\,l_s)\right)\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)},$$

where R is the rental rate of commercial land, and that capital satisfies:

$$\log K_i = h_0 + h_A A + h_s A_i + h_R \log R + h_\Phi \log \frac{\Phi\left(\left(1+(\alpha h_s + (1-\alpha)\,l_s)\right)\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)},$$

Substituting these conjectures into the fixed-point recursion for labor, equation (13), we arrive, by the method of undetermined coefficients, at the coefficient restrictions:

$$\begin{aligned}
cons &: (\psi+\alpha)\, l_0 = \log(1-\alpha) + \alpha h_0 + \frac{1}{2}\eta_c \left(1+\alpha h_s + (1-\alpha)\, l_s\right)^2 \tau_\theta^{-1}, \\
A &: (\psi+\alpha)\, l_A = \alpha h_A + \left(1+\alpha h_s + (1-\alpha)\, l_s\right)\eta_c, \\
A_i &: (\psi+\alpha+(1-\alpha)\,\eta_c)\, l_s = (1-\eta_c)\left(1+\alpha h_s\right), \\
\log R &: (\psi+\alpha)\, l_R = \alpha h_R, \\
\Phi &: (\psi+\alpha)\, l_\Phi = \eta_c + \alpha h_\Phi.
\end{aligned}$$

Similarly, substituting these conjectures into the fixed-point recursion for commercial land, equation (14), we arrive at the coefficient restrictions:

$$\begin{aligned}
cons &: (1-\alpha)\, h_0 = (1-\alpha)\, l_0 + \frac{1}{2}\eta_c \left(1+\alpha h_s + (1-\alpha)\, l_s\right)^2 \tau_\theta^{-1} + \log\alpha, \\
A &: (1-\alpha)\, h_A = (1-\alpha)\, l_A + \eta_c \left(1+\alpha h_s + (1-\alpha)\, l_s\right), \\
A_i &: (1-(1-\eta)\,\alpha)\, h_s = (1-\eta_c)\left(1+(1-\alpha)\, l_s\right), \\
\log R &: (1-\alpha)\, h_R = (1-\alpha)\, l_R - 1, \\
\Phi &: (1-\alpha)\, h_\Phi = (1-\alpha)\, l_\Phi + \eta_c.
\end{aligned}$$

We consequently have ten linear equations and ten coefficients, from which follows that:

$$\begin{aligned}
l_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\left(\frac{1+\psi}{(1-\alpha)\,\psi + (1+\alpha\psi)\,\eta_c}\right)^2 \tau_\theta^{-1} + \frac{\alpha}{1-\alpha}\frac{1}{\psi}\log\alpha + \frac{1}{\psi}\log(1-\alpha), \\
l_A &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\,\psi + (1+\alpha\psi)\,\eta_c}\frac{\eta_c}{\psi}, \\
l_s &= \frac{1-\eta_c}{(1-\alpha)\,\psi + (1+\alpha\psi)\,\eta_c}, \\
l_R &= -\frac{\alpha}{1-\alpha}\frac{1}{\psi}, \\
l_\Phi &= \frac{1}{1-\alpha}\frac{\eta_c}{\psi},
\end{aligned}$$

and

$$\begin{aligned}
h_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha),\\
h_A &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c,\\
h_s &= \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c},\\
h_R &= -\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi},\\
h_\Phi &= \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c,
\end{aligned}$$

which confirms the conjectures.

Consequently, we find that, for $A_i \geq A^*$:

$$\begin{aligned}
\log l_i &= \frac{1}{2}\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha)\\
&+\frac{1-\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{\eta_c}{\psi}A\\
&-\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log R+\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)},
\end{aligned}$$

and:

$$\begin{aligned}
\log K_i &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha)\\
&+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c A\\
&-\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log R+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}.
\end{aligned}$$

Substituting this functional form for the labor supply and commercial labor demand of household i into equation (11), the price of household $i's$ good then reduces to:

$$p_i = e^{\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c(A-A_i)+\frac{1}{2}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}}\left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\right)^{\eta_c}.$$

Finally, that $U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right)=p_i e^{A_i}K_i^\alpha l_i^{1-\alpha}$, implies:

$$E\left[U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right)-\frac{l_i^{1+\psi}}{1+\psi}\middle|\mathcal{I}_i\right]=\frac{\psi+\alpha}{1+\psi}E\left[p_i e^{A_i}K_i^\alpha l_i^{1-\alpha}\middle|\mathcal{I}_i\right].$$

A.2 Proof of Proposition 2

Substituting the optimal demand for commercial land K_i into the market-clearing condition for the rental market (5) reveals that the rental rate R is given by:

$$\begin{aligned}\log R &= \frac{1+\psi}{\psi+\alpha}A-(1-\alpha)\frac{\psi}{\psi+\alpha}\log K+\frac{1+\psi}{\psi+\alpha}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\\ &\quad+(1-\alpha)\frac{\psi}{\psi+\alpha}\log\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}+r_0,\end{aligned}$$

where K is the total amount of commercial land supplied by commercial developers at $t=2$, and:

$$r_0=\log\alpha+\frac{1-\alpha}{\psi+\alpha}\log\left(1-\alpha\right)+\frac{1}{2}\left(\frac{1+\psi}{\psi+\alpha}\eta_c+\left(1-\alpha\right)\frac{\psi}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\right)^2\tau_\theta^{-1}.$$

Since market-clearing in the market for commercial land imposes that $K\int_{i\in\mathcal{N}}di=\int_{i\in\mathcal{N}}K_idi$, it follows from equation (4) that the optimal choice of how much commercial land commercial developers create is given by equation (8) and constant k_0 is given by:

$$k_0=\frac{\log\alpha+\frac{1-\alpha}{\psi+\alpha}\log\left(1-\alpha\right)+\frac{1}{2}\left(\frac{1+\psi}{\psi+\alpha}\eta_c+\left(1-\alpha\right)\frac{\psi}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}$$

A.3 Proof of Proposition 3

When all households and builders observe A directly, there are no longer information frictions in the economy. Substituting for prices, the optimal labor and commercial land choices of household i, the realized rental rate R, and commercial land demand K_i from Proposition 2, the utility of household i at $t=1$ from choosing to live in the neighborhood is then:

$$\begin{aligned}E\left[U_i|\mathcal{I}_i\right] &= \frac{\psi+\alpha}{1+\psi}e^{u_0+u_AA+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i}\cdot\left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{u_\Phi}\\ &\quad\times\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{(1-\lambda)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}},\end{aligned}$$

where:

$$
\begin{aligned}
u_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\lambda\eta_c\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}-\frac{\alpha\left(\lambda-1\right)}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{\psi\left(1-\alpha\right)}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\right)^2\tau_\theta^{-1} \\
&\quad +\frac{1}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\alpha}{1-\alpha}\left(1+\psi\right)\log\alpha+\lambda\log\left(1-\alpha\right)\right) \\
u_A &= \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\eta_c-\left(\lambda-1\right)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\right), \\
u_\Phi &= \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\eta_c>0.
\end{aligned}
$$

Since the household with the critical productivity A^* must be indifferent to its neighborhood choice at the cutoff, it follows that $U_i - P = 0$, which implies:

$$
\begin{aligned}
& e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i}\left(\frac{\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{u_\Phi}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}} \\
= \;& \frac{1+\psi}{\psi+\alpha}e^{-u_0-u_AA}P,\ A_i=A^* \qquad (15)
\end{aligned}
$$

which implies the difference in benefit of living with more productive households must be offset by the differences in the cost of living in the neighborhood.

Fixing the critical value A^* and price P, we see that the LHS of equation (15) is increasing in monotonically in A_i, since $\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\left(1-\eta_c\right)>0$. This confirms the optimality of the cutoff strategy that households for which $A_i \geq A^*$ enter the neighborhood, and households for which $A_i < A^*$ to live somewhere else. Since $A_i = A + \varepsilon_i$, it then follows that a fraction $\Phi\left(-\sqrt{\tau_\theta}\left(A^*-A\right)\right)$ enter the neighborhood, and a fraction $\Phi\left(\sqrt{\tau_\theta}\left(A^*-A\right)\right)$ choose to live somewhere else. As one can see, it is the integral over the idiosyncratic productivity shocks of households ε_i that determines the fraction of households in the neighborhood.

From the optimal supply of housing by builder i in the neighborhood (7), there exists a critical value ω^* :

$$
\omega^* = -\left(1+k\right)\log P, \qquad (16)
$$

such that builders with productivity $\omega_i \geq \omega^*$ build houses. Consequently, a fraction $\Phi\left(-\sqrt{\tau_e}\left(\omega^*-\xi\right)\right)$ build households in the neighborhood.

Imposing market-clearing, it must be the case that:

$$
\Phi\left(-\sqrt{\tau_\theta}\left(A^*-A\right)\right)=\Phi\left(-\sqrt{\tau_e}\left(\omega^*-\xi\right)\right).
$$

Since the CDF of the normal distribution is montonically increasing, we can invert the above market-clearing conditions, and impose equation (16) to arrive at:

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A - A^*\right) - \xi\right).$$

Substituting for prices, we can express equation (15) as:

$$\begin{aligned} & e^{\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{\sqrt{\tau_\theta/\tau_e}}{1+k}\right)A^*}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}\frac{\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{u_\Phi}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{u_\Phi}} \\ = \quad & \frac{1+\psi}{\psi+\alpha}e^{\left(\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}-u_A\right)A-\frac{1}{1+k}\xi-u_0}. \end{aligned} \tag{17}$$

Taking the derivative of the log of the LHS of equation (17) with respect to A^*, which is a monotonic transformation of the LHS:

$$\begin{aligned} \frac{d\log LHS}{dA^*} \quad = \quad & u_\Phi\frac{1}{\tau_\theta^{-1/2}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}-\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right)+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \\ & +\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}-\frac{1}{\tau_\theta^{-1/2}}\frac{(\lambda-1)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}-\frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right). \end{aligned}$$

The term in parentheses are nonnegative by the properties of the normal CDF. The last term is nonpositive, since $\lambda > 1$, and attains its minimum at $A^* \to \infty$, from which follows, substituting for u_Φ, that:

$$\frac{d\log LHS}{dA^*} \to_{A^*\to\infty} \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}+\frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} > 0.$$

Consequently, since $\frac{d\log LHS}{dA^*} > 0$ when the last term attains its (nonpositive) minimum, it follows that $\frac{d\log LHS}{dA^*} > 0$. Therefore, $\log LHS$, and consequently LHS, since log is a monotonic transformation of the LHS, is monotonically increasing in A^*. Since the RHS of equation (17) is fixed for all A^*, it follows that the LHS and RHS of equation (17) intersect at most once. Therefore, the can be, at most, one cutoff equilibrium. Furthermore, since the LHS of equation (17) tends to 0 as $A^* \to -\infty$, and the RHS is nonnegative, it follows that a cutoff equilibrium always exist. Therefore, there exists a unique cutoff equilibrium in this economy.

Finally, it is straightforward to apply the Implicit Function Theorem to (17) to conclude that

$$\begin{aligned}\frac{dA^*}{dA} &= \frac{\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} - \frac{d\log LHS}{dA} - u_A}{\frac{d\log LHS}{dA^*}} \\ \frac{dA^*}{d\xi} &= -\frac{1}{1+k}\frac{1}{\frac{d\log LHS}{dA^*}} < 0,\end{aligned}$$

where:

$$\begin{aligned}\frac{d\log LHS}{dA} &= -u_\Phi \frac{1}{\tau_\theta^{-1/2}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)} - \frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right) \\ &\quad + \frac{1}{\tau_\theta^{-1/2}}(\lambda-1)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)} - \frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right).\end{aligned}$$

Notice that the nonpositive term in $\frac{d\log LHS}{dA}$ achieves its minimum at $A \to -\infty$, at which:

$$\frac{d\log LHS}{dA} \to_{A\to-\infty} \left((\lambda-1)\alpha(1-\eta_c) - \lambda\eta_c\right)\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}.$$

Then, as $A \to -\infty$, the numerator of $\frac{dA^*}{dA}$ converges to:

$$\begin{aligned}\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} - \frac{d\log LHS}{dA} - u_A &\to_{A\to-\infty} -\frac{(1+\psi)\left(\frac{((\lambda-1)\alpha(1-\eta_c)-\lambda\eta_c)\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\right)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \\ &\quad + \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{(\lambda-1)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}},\end{aligned}$$

which is positive. Consequently:

$$\frac{dA^*}{dA} \geq 0.$$

Finally, we can rewrite equation (17) as:

$$\begin{aligned}&e^{-\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}\right)s}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} + \frac{s}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{s}{\tau_\theta^{-1/2}}\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}\frac{\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} + \frac{s}{\tau_\theta^{-1/2}}\right)^{u_\Phi}}{\Phi\left(\frac{s}{\tau_\theta^{-1/2}}\right)^{u_\Phi}} \\ &= \frac{1+\psi}{\psi+\alpha}e^{-\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A-\frac{1}{1+k}\xi-u_0},\end{aligned}$$

where $s = A - A^*$ determines the population that enter the neighborhood. It is straightforward to see, with some manipulation, that:

$$\frac{d\log LHS}{ds} = -\frac{d\log LHS}{dA^*} < 0,$$

and therefore:

$$\begin{aligned}
\frac{ds}{d\xi} &= -\frac{\frac{1}{1+k}}{\frac{d\log LHS}{ds}} > 0, \\
\frac{ds}{dA} &= -\frac{\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}{\frac{d\log LHS}{ds}} > 0.
\end{aligned}$$

Conequently, the population that enters, $\Phi\left(\sqrt{\tau_\theta}s\right)$, is increasing in A and ξ. Furthermore, it follows that:

$$\frac{d\log P}{dA} = \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}\frac{ds}{dA} > 0,$$

and therefore the log housing price is increasing in A. Since the housing price is a monotonic transformation of the log housing price, it follows that $\frac{dP}{dA} \geq 0$.

Finally, we recognize that:

$$\frac{d^2P}{dA^2} = \left(\frac{ds}{dA}\right)^2 P + \frac{d^2s}{dA^2}P = \left(\frac{ds}{dA}\right)^2 P + \frac{\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}{\left(\frac{d\log LHS}{ds}\right)^2}\frac{ds}{dA}\frac{d^2\log LHS}{ds^2}P,$$

where $\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{ds}{dA} > 0$ by the above arguments. As $s \to \infty$, it follows that:

$$\frac{d^2\log LHS}{ds^2} \to_{s\to\infty} \left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi + (1+\alpha\psi)\eta_c} + \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}\right)\frac{1}{\tau_\theta^{-1/2}}\lim_{s\to\infty} s = \infty,$$

and $P \to \infty$, and consequently that $\frac{d^2P}{dA^2} \to_{s\to\infty} \infty$. Furthermore, as $s \to -\infty$:

$$\frac{d\log LHS}{ds} \to_{s\to-\infty} -\left(\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} + \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\right),$$

and:

$$\begin{aligned}
\frac{d^2\log LHS}{ds^2} \to & \lim_{s\to-\infty}\left(\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} + \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\right)\frac{1}{\tau_\theta^{-1/2}}s \\
&+2(1+\psi)\frac{1}{\tau_\theta^{-1}}\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{(\lambda-1)\alpha(1-\eta_c)-\lambda\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}\frac{s}{\tau_\theta^{-1/2}} \\
&+\frac{1}{\tau_\theta^{-1}}\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left((\lambda-1)\alpha(1-\eta_c)^2-\lambda\eta_c\right)\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}\right)^2,
\end{aligned}$$

which is linear in s, while $P \to 0$ at an exponential rate. Consequently, $\frac{d^2P}{dA^2} \to_{s\to-\infty} 0$. Since $\frac{d^2P}{dA^2}$ is continuous, it follows that $\frac{d^2P}{dA^2} \geq 0$. Consequently, P is convex in A. Since, in equilibrium, the housing price is equal to the utility of the household with the cutoff productivity, it follows that this utility is also convex and increasing in A.

A.4 Proof of Proposition 4

We first conjecture that each household's housing purchasing, each builder's housing supply, and the housing price follow a cutoff strategy, and that learning by households and capital owners is linear as derived in the main text. Given our assumption about the sufficient statistic in prices, each household's posterior about A is Gaussian $A\,|\mathcal{I}_i \sim \mathcal{N}\left(\hat{A}_i, \hat{\tau}_A^{-1}\right)$ with conditional mean and variance

$$\begin{aligned}
\hat{A}_i &= \bar{A} + \tau_A^{-1}\left[\begin{array}{ccc} 1 & 1 & 1 \end{array}\right]\left[\begin{array}{ccc} \tau_A^{-1}+\tau_Q^{-1} & \tau_A^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1}+z_\xi^{-2}\tau_\xi^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1} & \tau_A^{-1}+\tau_\theta^{-1} \end{array}\right]^{-1}\left[\begin{array}{c} Q-\bar{A} \\ z\left(P\right)-\bar{A} \\ A_i-\bar{A} \end{array}\right] \\
&= \hat{\tau}_A^{-1}\left(\tau_A\bar{A}+\tau_Q Q+z_\xi^2\tau_\xi z\left(P\right)+\tau_\theta A_i\right), \\
\hat{\tau}_A &= \tau_A+\tau_Q+z_\xi^2\tau_\xi+\tau_\theta.
\end{aligned}$$

We recognize that the conditional estimate of $\hat{A}_i$ of household i is increasing in its own productivity A_i. Similarly, the posterior for capital owners about A is Gaussian $A\,|\mathcal{I}^c \sim \mathcal{N}\left(\hat{A}^c, \hat{\tau}_A^{c-1}\right)$, where

$$\begin{aligned}
\hat{A}^c &= \bar{A} + \tau_A^{-1}\left[\begin{array}{cc} 1 & 1 \end{array}\right]\left[\begin{array}{cc} \tau_A^{-1}+\tau_Q^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1}+z_\xi^{-2}\tau_\xi^{-1} \end{array}\right]^{-1}\left[\begin{array}{c} Q-\bar{A} \\ z\left(P\right)-\bar{A} \end{array}\right] \\
&= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A}+\tau_Q Q+z_\xi^2\tau_\xi z\left(P\right)\right), \\
\hat{\tau}_A^c &= \tau_A+\tau_Q+z_\xi^2\tau_\xi.
\end{aligned}$$

This completes our characterization of learning by households and capital owners.

Let us now turn to the optimal commercial land decision of commercial developers. Since the posterior for $A-A^*$ of households is conditionally Gaussian, it follows that the expectations in the expression for K in Proposition 2 is a function of the first two conditional moments, $\hat{A}_i - A^*$ and $\hat{\tau}_A^{-1}$. Let:

$$F\left(\hat{A}^c-A^*, \hat{\tau}_A^c\right) = E\left[\left.\left(\frac{e^{(A-A^*)}\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c+\frac{\psi(1-\alpha)}{1+\psi}}\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{-\frac{\psi(1-\alpha)}{1+\psi}}}\right)^{\frac{1+\psi}{\psi+\alpha}}\right|\mathcal{I}^c\right],$$

where $F\left(\hat{A}^c - A^*, \hat{\tau}_A\right) \geq 0$. Define $z = \frac{A-A^*}{\tau_\theta^{-1/2}}$, and the function $f(z)$:

$$f(z) = e^{\tau_\theta^{-1/2} z} \frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)^{\eta_c}}{\Phi(z)^{\eta_c}} \left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)}{\Phi(z)}\right)^{\frac{\psi}{1+\psi}(1-\alpha)}.$$

Then it follows that:

$$\begin{aligned}\frac{1}{f(z)}\frac{df(z)}{dz} &= \tau_\theta^{-1/2} + \eta_c\left(\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)} - \frac{\phi(z)}{\Phi(z)}\right) \\ &\quad + \frac{\psi}{1+\psi}(1-\alpha)\left(\frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)} - \frac{\phi(z)}{\Phi(z)}\right).\end{aligned}$$

Notice that $\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)} - \frac{\phi(z)}{\Phi(z)}$ achieves its minimum as $z \to -\infty$. Applying L'Hospital's Rule, it follows that the minimum of $\frac{1}{f(z)}\frac{df(z)}{dz}$ is given by:

$$\begin{aligned}\lim_{z\to-\infty}\frac{1}{f(z)}\frac{df(z)}{dz} &= \tau_\theta^{-1/2} + \lim_{z\to-\infty}\eta_c\left(\frac{\frac{d}{dz}\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2} + z\right)}{\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2} + z\right)} - \frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right) \\ &\quad + \frac{\psi}{1+\psi}(1-\alpha)\left(\frac{\frac{d}{dz}\phi\left(\frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2} + z\right)}{\phi\left(\frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2} + z\right)} - \frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right) \\ &= \alpha\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}(1-\eta_c)\tau_\theta^{-1/2} \\ &> 0\end{aligned}$$

from which follows that $\frac{1}{f(z)}\frac{df(z)}{dz} \geq 0$ for all z, and therefore $\frac{df(z)}{dz} \geq 0$, since $f(z) \geq 0$. Consequently, since $f(z)^{\frac{1+\psi}{\psi+\alpha}}$ is a monotonic transformation of $f(z)$, it follows that $\frac{dF}{dx}(x, \hat{\tau}_A) \geq 0$ since this holds for all realizations of $A - A^*$. This establishes that the optimal choice of commercial land is increasing in the conditional estimate of commercial developers, $\hat{A}^c$, since $f(z)$ is increasing for each realization of z.

The optimal choice of commercial land then takes the form

$$\log K = \frac{1}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}}\log F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right) + \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}}A^* + k_0.$$

Substituting this expression into the functional form for commercial land, the utility of

household i is then given by:

$$E\left[U_i|\mathcal{I}_i\right] = \frac{\psi+\alpha}{1+\psi}e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{1+\psi}{\psi+\alpha}A^*\right)+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)A^*+u_0}$$
$$\times E\left[\left(\frac{e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)(A-A^*)}\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c-\alpha}\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\alpha}}\right)^{\frac{1+\psi}{\psi+\alpha}} \mid \mathcal{I}_i\right].$$

where u_0 is given in the proof of Proposition 3.

Since the posterior for $A-A^*$ of households is conditionally Gaussian, it follows that the expectations in the expressions above are functions of the first two conditional moments $\hat{A}_i-A^*$ and $\hat{\tau}_A^{-1}$. Let:

$$G\left(\hat{A}_i-A^*,\hat{\tau}_A\right)=E\left[\left(\frac{e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)(A-A^*)}\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c-\alpha}\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\alpha}}\right)^{\frac{1+\psi}{\psi+\alpha}} \mid \mathcal{I}_i\right],$$

where $G\left(\hat{A}_i-A^*,\hat{\tau}_A\right)\geq 0$. Define $z=\frac{A-A^*}{\tau_\theta^{-1/2}}$, and the function $g(z)$:

$$g(z)=e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2}z}\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)^{\eta_c}}{\Phi(z)^{\eta_c}}\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)^{-\alpha}}{\Phi(z)^{-\alpha}}.$$

Then it follows that:

$$\begin{aligned}\frac{1}{g(z)}\frac{dg(z)}{dz} &= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2}\\ &-\eta_c\left(\frac{\phi(z)}{\Phi(z)}-\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}\right)\\ &+\alpha\left(\frac{\phi(z)}{\Phi(z)}-\frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}\right).\end{aligned}$$

Notice that $\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}-\frac{\phi(z)}{\Phi(z)}$ achieves its minimum as $z\to-\infty$. Applying

L'Hospital's Rule, it follows that the minimum of $\frac{1}{g(z)}\frac{dg(z)}{dz}$ is given by:

$$\begin{aligned}
\lim_{z\to-\infty}\frac{1}{g(z)}\frac{dg(z)}{dz} &= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2}\\
&\quad+\eta_c\lim_{z\to-\infty}\left(\frac{\frac{d}{dz}\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}{\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}-\frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right)\\
&\quad+\alpha\lim_{z\to-\infty}\left(\frac{\frac{d}{dz}\phi(z)}{\phi(z)}-\frac{\frac{d}{dz}\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}\right)\\
&= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2}\\
&\quad+\left((1-\eta_c)\alpha-\eta_c\right)\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}.
\end{aligned}$$

With some manipulation, the above expression collapsed to:

$$\lim_{z\to-\infty}\frac{1}{g(z)}\frac{dg(z)}{dz}=0,$$

and it follows that $\frac{1}{g(z)}\frac{dg(z)}{dz}\geq 0$, and therefore $\frac{dg(z)}{dz}\geq 0$, since $g(z)\geq 0$. Consequently, since $g(z)^{\frac{1+\psi}{\psi+\alpha}}$ is a monotonic transformation of $g(z)$, it follows that $\frac{dG}{dx}(x,\hat{\tau}_A)\geq 0$ since this holds for all realizations of $A-A^*$.

Since the household with the critical productivity A^* must be indifferent to its neighborhood choice at the cutoff, it follows that $U_i-P=0$, which implies

$$\begin{aligned}
&e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{1+\psi}{\psi+\alpha}A^*\right)+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)A^*+u_0}G\left(\hat{A}_i-A^*,\hat{\tau}_A\right)\\
=\;&\frac{1+\psi}{\psi+\alpha}P,\ A_i=A^* \qquad (18)
\end{aligned}$$

which does not depend on the unobserved A or the supply shock ξ. As such, $A^*=A^*(\log P,Q)$. Furthermore, since $\hat{A}_i^*$ is increasing in A_i and $G\left(\hat{A}_i^*-A^*,\tau_A\right)$ is (weakly) increasing in $\hat{A}_i$, it follows that the LHS of equation (18) is (weakly) monotonically increasing in A_i, confirming the cutoff strategy assumed for households is optimal. Those for which the RHS is nonnegative enter the neighborhood, and those for which it is negative go choose to live elsewhere.

It then follows from market-clearing that

$$\Phi\left(-\sqrt{\tau_\theta}\left(A^*-A\right)\right)=\Phi\left(-\sqrt{\tau_e}\left(\omega^*-\xi\right)\right).$$

Since the CDF of the normal distribution is montonically increasing, we can invert the above market-clearing conditions, and impose equation (16) to arrive at

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A - A^*\right) - \xi\right)$$

from which follows that

$$z\left(P\right) = \sqrt{\frac{\tau_e}{\tau_\theta}}\left(1+k\right)\log P + A^* = A - \sqrt{\frac{\tau_e}{\tau_\theta}}\xi,$$

and therefore $z_\xi = \sqrt{\frac{\tau_\theta}{\tau_e}}$. This confirms our conjecture for the sufficient statistics in housing prices and that learning by households is indeed a linear updating rule. As a consequence, it follows we can express the conditional estimates of household i as:

$$\begin{aligned}
\hat{A}_i &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(1+k\right)\log P + A^*\right) + \tau_\theta A_i\right), \\
\hat{\tau}_A &= \tau_A + \tau_Q + \frac{\tau_\theta}{\tau_e}\tau_\xi + \tau_\theta,
\end{aligned}$$

and for capital owners as:

$$\begin{aligned}
\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(1+k\right)\log P + A^*\right)\right), \\
\hat{\tau}_A^c &= \tau_A + \tau_Q + \frac{\tau_\theta}{\tau_e}\tau_\xi.
\end{aligned}$$

Substituting for prices, and simplifying A^* terms, we can express equation (18) as:

$$e^{\left(\frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{\sqrt{\tau_\theta/\tau_e}}{1+k}\right)A^*} G\left(\hat{A}^* - A^*, \hat{\tau}_A\right) F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right)^{\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}} = \frac{1+\psi}{\psi+\alpha} e^{\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}A - \frac{1}{1+k}\xi - u_0}, \tag{19}$$

where:

$$\begin{aligned}
\hat{A}_i^* &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(1+k\right)\log P + A^*\right) + \tau_\theta A^*\right), \\
\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(1+k\right)\log P + A^*\right)\right).
\end{aligned}$$

Notice that the LHS of equation (19) is continuous in A^*. It follows, as $A^* \to -\infty$, that the LHS of equation (19) tends to:

$$\begin{aligned}
\lim_{A^*\to-\infty} LHS &= \left[e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A^* + u_0} E\left[e^{\frac{1+\psi}{\psi+\alpha}A}\middle|\mathcal{I}^c\right]^{\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}} E\left[e^{\frac{1+\psi}{\psi(1-\alpha)}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} - \alpha\frac{1+\psi}{\psi+\alpha}\right)A} \mid \mathcal{I}_i\right]\right]_{A^*\to-\infty} \\
&= 0.
\end{aligned}$$

Furthermore, by L'Hospital's Rule and the Sandwich Theorem, one also has that:

$$\lim_{A^* \to \infty} LHS = \infty.$$

Since the LHS of equation (19) is continuous in A^* and the RHS is fixed for all A^*, it follows that the LHS and RHS intersect at least once. Therefore, a cutoff equilibrium in the economy with informational frictions exists.

Finally, notice that, as $\tau_Q \nearrow \infty$, that $\hat{A}^c$ and $\hat{A}^i$ converge to A a.s., since $\hat{\tau}_A^c,\ \hat{\tau}_A^i \nearrow \infty$. Taking the limit along a sequence of τ_Q, it is straightforward to verify that equation (18) converges to equation (17), and therefore A^* converges to its perfect-information benchmark value. Taking similar limits for the expressions for capital and labor supply verify that they also converge to their perfect-information benchmark values, and therefore the noisy rational expectations cutoff equilibrium converges to the perfect-information benchmark economy as $\tau_Q \nearrow \infty$.